LESSON 13

Overview | Find Equivalent Ratios

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

2 Reason abstractly and quantitatively.

5 Use appropriate tools strategically.

* See page 1q to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Identify and generate equivalent ratios using models, double number lines, and tables with addition and multiplication.
- Find missing values in tables of equivalent ratios.
- Solve problems with equivalent ratios.
- Generate ordered pairs from tables of equivalent ratios and plot them in the coordinate plane.

Language Objectives

- Demonstrate understanding of equivalent ratios by completing models and responding to written questions.
- Interpret word problems involving equivalent ratios by identifying the relationship among the quantities.
- Describe how to represent equivalent ratios on a coordinate plane using the lesson vocabulary.
- Ask clarifying questions to deepen understanding during partner and class discussions.

Prior Knowledge

- Understand what a ratio is, how to describe a ratio using ratio language, and that order in a ratio matters (*a* to *b* is not the same as *b* to *a*).
- Represent a ratio using a diagram.
- Plot ordered pairs and interpret coordinates of points in the context of a situation.

Vocabulary

Math Vocabulary

equivalent ratios two ratios that express the same comparison. Multiplying both numbers in the ratio $a : b$ by a nonzero number n results in the equivalent ratio $na : nb$.

Review the following key terms.

coordinate plane a two-dimensional space formed by two perpendicular number lines called axes.

ordered pair a pair of numbers, (x, y), that describes the location of a point in the coordinate plane. The x-coordinate gives the point's horizontal distance from the y-axis, and the y-coordinate gives the point's vertical distance from the x-axis.

***x*-axis** the horizontal number line in the coordinate plane.

***x*-coordinate** the first number in an ordered pair. It tells the point's horizontal distance from the y-axis.

***y*-axis** the vertical number line in the coordinate plane.

***y*-coordinate** the second number in an ordered pair. It tells the point's vertical distance from the x-axis.

Academic Vocabulary

graph (noun) a diagram that shows data or relationships between values or quantities.

graph (verb) to show something with a graph.

Learning Progression

In Grade 5, students extended their use of multiplication to scale a quantity, which means to increase or decrease by multiplying by a factor.

In the previous lesson, students were introduced to the concept of ratio, ratio notation, and ratio language. They compared quantities and examined relationships using ratios.

In this lesson, students merge their understanding of multiplication as scaling and ratio concepts to identify and generate equivalent ratios. Students find missing values in tables of equivalent ratios and solve problems using double number lines or tables. They represent ratios as ordered pairs and then graph them as points in the coordinate plane.

Later in Grade 6, students will use tables to compare ratios, and they will apply their understanding of ratio to the ideas of rate and unit rate. They will analyze relationships between dependent and independent variables using graphs.

In Grade 7, students will apply ratio reasoning to explore proportional relationships and calculate probabilities.

Pacing Guide

Items marked with are available on the **Teacher Toolbox**.

	MATERIALS	DIFFERENTIATION
SESSION 1 **Explore** Equivalent Ratios (35–50 min)		
• **Start** (5 min) • **Try It** (5–10 min) • **Discuss It** (10–15 min) • **Connect It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 283–284)	**Math Toolkit** connecting cubes, counters, grid paper Presentation Slides	**PREPARE** Interactive Tutorial **RETEACH or REINFORCE** Hands-On Activity **Materials** For each group: 25 two-color counters
SESSION 2 **Develop** Finding Equivalent Ratios (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 289–290)	**Math Toolkit** connecting cubes, counters, double number lines, grid paper Presentation Slides	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each group: 24 two-color counters **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 3 **Develop** Graphing a Table of Equivalent Ratios (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 295–296)	**Math Toolkit** connecting cubes, counters, double number lines, graph paper Presentation Slides	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: 24 two-color counters, Activity Sheet *Coordinate Plane: First Quadrant* **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 4 **Develop** Using Equivalent Ratios (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 301–302)	**Math Toolkit** connecting cubes, counters, double number lines, graph paper Presentation Slides	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: 30 two-color counters **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 5 **Refine** Finding Equivalent Ratios (45–60 min)		
• **Start** (5 min) • **Monitor & Guide** (15–20 min) • **Group & Differentiate** (20–30 min) • **Close: Exit Ticket** (5 min)	**Math Toolkit** Have items from previous sessions available for students. Presentation Slides	**RETEACH** Hands-On Activity **Materials** For each student: 30 two-color counters, Activity Sheet *Double Number Lines* **REINFORCE** Problems 4–8 **EXTEND** Challenge **PERSONALIZE** i-Ready
Lesson 13 Quiz or **Digital Comprehension Check**		**RETEACH** Tools for Instruction **REINFORCE** Math Center Activity **EXTEND** Enrichment Activity

Overview | Find Equivalent Ratios

Connect to Culture

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.

SESSION 1 ■ □ □ □ □

Try It Henna is a plant that grows in hot climates. Women of many Indian, African, and Middle Eastern cultures use henna paste to paint designs on their hands and feet in preparation for a variety of celebrations. One such celebration is *Diwali,* a Hindu festival of lights. Diwali takes place at the end of the lunar month Ashvina into the start of the lunar month Karttika (late October/early November). Ask students about other festivals of light they may know of and to describe traditions of dress and decoration associated with celebrations in their cultures.

SESSION 2 ■ ■ □ □ □

Try It In 1872, Yellowstone National Park was established by Congress. Yellowstone was not only the first national park in the United States but the first national park in the world. Many people enjoy seeing nature's beauty as they visit a national park. Maintaining and preserving a national park are not simple tasks. It takes a large number of people to clean the parks and enforce the rules so that everyone can enjoy the natural beauty of the park. Ask students about their experience with national, state, or local parks or for other ways they enjoy nature.

SESSION 3 ■ ■ ■ □ □

Try It The way we listen to music is constantly changing with technology. Many years ago, the only way music could be heard was live and in person. As technology evolved, records, cassette tapes, and CDs were invented, which allowed music to be heard at any time. Now, in the twenty-first century, the means of listening to music have evolved even further, as one of the most popular methods today is listening to music that is streamed through the Internet. Ask students to share their experiences with listening to music in different ways other than on the Internet. Ask for a show of hands if anyone has family members that own listening devices such as a record, tape, CD, or MP3 player.

SESSION 4 ■ ■ ■ ■ □

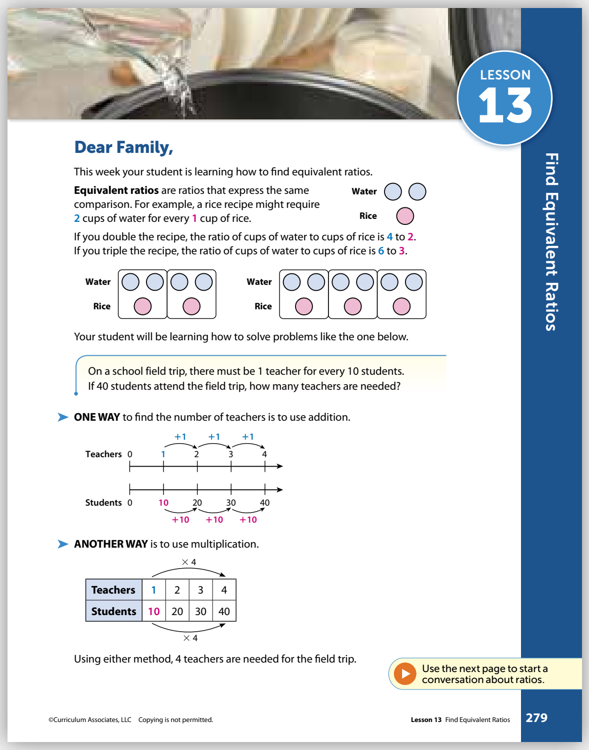
Try It A unicycle is like a bicycle, but it only has one wheel. Instead of using handles to steer and balance, a rider must balance by pedaling and shifting their weight. Unicycles were invented in the 1800s after the creation of the penny-farthing bicycle, which has a huge front wheel and a tiny back wheel. Ask students about their experiences riding bicycles and tricycles, and if they have ever seen anyone riding a unicycle.

SESSION 5 ■ ■ ■ ■ ■

Apply It **Problem 6** Dairy cows eat an average of 100 pounds of feed and drink 40 to 50 gallons of water per day. These cows need to eat and drink a lot in order to produce an average of 8 gallons of milk on a daily basis. Whether you actually drink milk or not, chances are that you consume products made with milk, because more than 1,000 new dairy products are put on the market each year. They include new varieties of yogurt, cheese, butter, and ice cream. Ask students to share their favorite dairy products or dairy alternatives they like to eat.

Connect to Family and Community

After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.


LESSON 13

Find Equivalent Ratios

Dear Family,

This week your student is learning how to find equivalent ratios.

Equivalent ratios are ratios that express the same comparison. For example, a rice recipe might require 2 cups of water for every 1 cup of rice.

If you double the recipe, the ratio of cups of water to cups of rice is 4 to 2.
If you triple the recipe, the ratio of cups of water to cups of rice is 6 to 3.

Your student will be learning how to solve problems like the one below.

On a school field trip, there must be 1 teacher for every 10 students. If 40 students attend the field trip, how many teachers are needed?

ONE WAY to find the number of teachers is to use addition.

ANOTHER WAY is to use multiplication.

$\times 4$

Teachers	1	2	3	4
Students	10	20	30	40

$\times 4$

Using either method, 4 teachers are needed for the field trip.

Use the next page to start a conversation about ratios.

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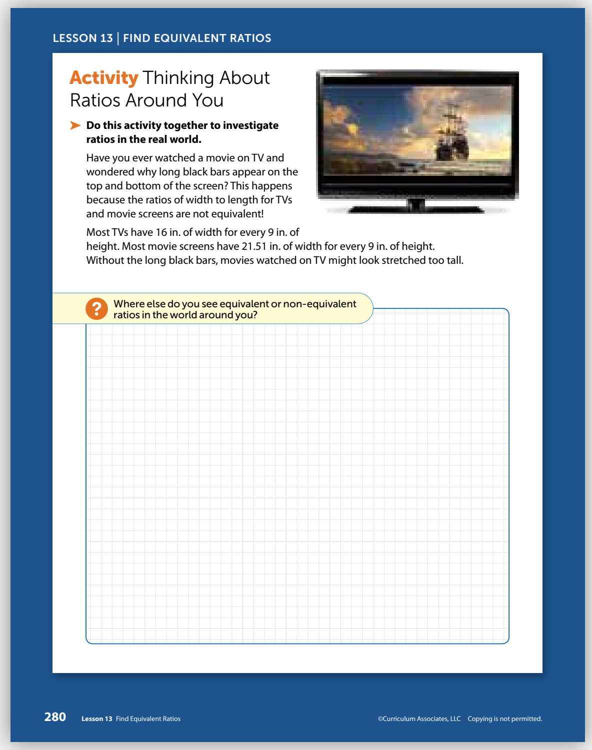
LESSON 13 | FIND EQUIVALENT RATIOS

Activity Thinking About Ratios Around You

Do this activity together to investigate ratios in the real world.

Have you ever watched a movie on TV and wondered why long black bars appear on the top and bottom of the screen? This happens because the ratios of width to length for TVs and movie screens are not equivalent!

Most TVs have 16 in. of width for every 9 in. of height. Most movie screens have 21.51 in. of width for every 9 in. of height. Without the long black bars, movies watched on TV might look stretched too tall.

Where else do you see equivalent or non-equivalent ratios in the world around you?

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Connect to Language

For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 1** **Connect It**

ACADEMIC VOCABULARY

Combine means to form by adding two or more things or amounts together.

Levels 1–3: Speaking/Writing

Prepare students to identify and explain equivalent ratios. Read Connect It problem 2a aloud. Use **Act It Out** to model the meanings of *equal groups* and *combine*. Show how to make several equal groups of counters and combine them. Have students work with a partner to model the equivalent ratios presented in problem 2a. Ask them to identify and describe the ratios using the terms *equal groups*, *combine*, and *equivalent ratios*. Call on a volunteer to share ideas. Provide a sentence frame to support writing:

- *For each ratio, there are always* _____ *for every* _____.

Levels 2–4: Speaking/Writing

Prepare students to identify and explain equivalent ratios. Read Connect It problem 2a chorally with students. Have students work with a partner to **Act It Out**, using counters to show the meaning of *combine equal groups*. Ask partners to talk about the models using the phrase *for every*.

Provide sentence frames to support writing:

- *The model shows that the ratio* _____ *is a group of* _____.
- *It also shows that the ratio* _____ *combines* _____.
- *I know the ratios are equivalent because they both compare* _____.

Levels 3–5: Speaking/Writing

Prepare students to identify and explain equivalent ratios. Have partners read Connect It problem 2a. Ask them to discuss the meanings of *same comparison, combine equal groups,* and *equivalent ratios.* Encourage them refer to the model to support their discussion.

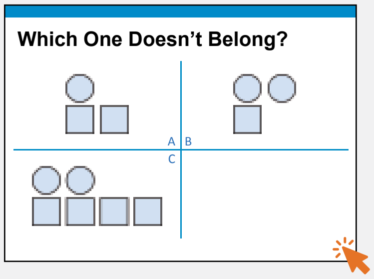
Have students write their responses to problem 2a using precise language and complete sentences. Remind students that they can use words from the question to frame their response. Call on volunteers to suggest words that might be used in their written answers. Record and display the terms for students to refer to as they write.

Explore Equivalent Ratios

Purpose

- **Explore** the idea that two different ratios can express the same comparison.
- **Understand** that you can determine whether ratios are equivalent by using diagrams and tables to prove that they represent the same comparison.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

A because it shows only 1 circle instead of 2 circles.

B because it shows twice as many circles instead of twice as many squares.

C because it is the only one that shows quantities that can both be separated into two equal groups.

WHY? Support students' understanding of using ratios to compare quantities.

TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Three Reads** to help them make sense of the problem. Students should recognize that the quantities of each ingredient is given in the picture. After the third read, listen for students to identify that the amounts of oil and henna powder are important quantities for this problem, but the amounts of sugar and water are not.

DISCUSS IT SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding of:

- 2 : 1 as the ratio of tablespoons of henna powder to teaspoons of oil for 1 batch.
- 6 : 3 as the ratio of tablespoons of powder to teaspoons of oil for 3 batches.
- the two ratios as naming the same comparison.

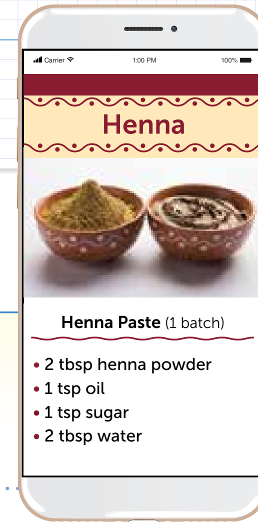
LESSON 13 | SESSION 1

Explore Equivalent Ratios

Previously, you learned how to compare quantities by using ratios. In this lesson, you will learn about equivalent ratios.

➤ **Use what you know to try to solve the problem below.**

Veda uses henna paste to paint designs on her friends' hands and feet as they prepare to celebrate Diwali, a festival of lights. What is the ratio of tablespoons of henna powder to teaspoons of oil if Veda makes 3 batches of paste?

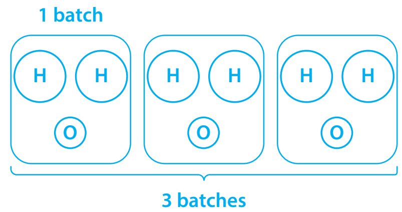


TRY IT

Math Toolkit connecting cubes, counters, grid paper

Possible work:

SAMPLE A



The ratio of tablespoons of henna powder to teaspoons of oil for 3 batches is 6 : 3.

SAMPLE B

Henna Powder (tbsp)	Oil (tsp)
2	1
4	2
6	3

+ 2, + 2 (henna powder); + 1, + 1 (oil)

The ratio of tablespoons of henna powder to teaspoons of oil is 6 to 3.

DISCUSS IT

Ask: How does your model show 3 batches of paste?

Share: My model shows that . . .

Learning Targets SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6

Use ratio and rate reasoning to solve real-world and mathematical problems.

- Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

281

Common Misconception Listen for students who increased the number of one quantity in the ratio but not the other. As students share their strategies for how they know how many tablespoons of oil to add for each batch, ask how their strategy shows that the numbers for both quantities are increased by 3 equal groups of 1 batch each to form 3 complete batches.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- physical models or drawings that use equal groups to show the quantity of each ingredient in 3 batches
- (misconception) strategies that only increase one of the quantities in the ratio for each additional batch
- tables that show the total amounts of henna powder and oil used in different numbers of batches
- equations that show the quantity of each ingredient in 1 batch multiplied by the number of batches

Facilitate Whole Class Discussion

Call on students to share selected strategies. Prompt students to participate actively and listen for understanding by looking at the speaker and asking clarifying questions.

Guide students to **Compare and Connect** the representations. Use turn and talk to help students think through their responses before sharing with the group.

ASK *How does [student name]'s model represent the amounts of henna powder and oil in each batch?*

LISTEN FOR The model shows that each batch contains 2 tbsp of henna powder and 1 tsp of oil.

CONNECT IT

SMP 2, 4, 5

1 Look Back Look for understanding that the ratio of tablespoons of henna powder to teaspoons of oil is 6 to 3 or an equivalent form of that ratio—specifically, that for every 2 tablespoons of henna powder, there is 1 teaspoon of oil.

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Model ratios that express the same comparison.

If students are unsure about the concept of equivalent ratios, then use this activity to help them visualize how to calculate ratios using equal groups.

Materials For each group: 25 two-color counters

- Distribute two-color counters and have students make a row of red counters and a row of yellow counters to show a 4 : 3 ratio. [4 red counters, 3 yellow counters]
- Tell students to make another equal group of red and yellow counters. Ask: *What is the ratio of red counters to yellow counters in both of the equal groups?* [8 to 6]
- Guide students to see how the ratios 8 : 6 and 4 : 3 name the same relationship. Ask volunteers to describe each relationship using ratio language. [8 red for every 6 yellow is the same as 4 red for every 3 yellow.]
- Have students describe how to add a third equal group to find another ratio that names the same relationship as 8 to 6 and 4 to 3. [Add another group of 4 red counters and 3 yellow counters to show the ratio 12 to 9.]
- Support students in describing how all their ratios represent the same comparison. There are always 4 red counters for every 3 yellow counters.

LESSON 13 | SESSION 1

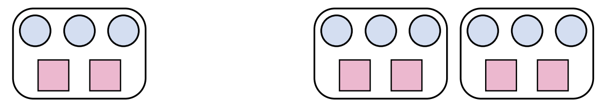
CONNECT IT

1 Look Back What is the ratio of tablespoons of henna powder to teaspoons of oil for 3 batches of henna paste? Explain how you know.

6 to 3 (or equivalent); Possible explanation: Each time Veda makes a batch, she adds 2 tbsp of powder and 1 tsp of oil. With 3 batches, she uses 6 tbsp of powder and 3 tsp of oil.

2 Look Ahead **Equivalent ratios** are ratios that express the same comparison.

a. To find a ratio that is equivalent to the ratio *3 to 2*, you can combine equal groups of 3 circles and 2 squares. How does the model show that the ratios 3 : 2 and 6 : 4 are equivalent ratios?

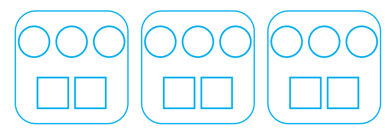


3 circles to 2 squares   6 circles to 4 squares

It shows that for each ratio, there are always 3 circles for every 2 squares, so the ratios represent the same comparison.

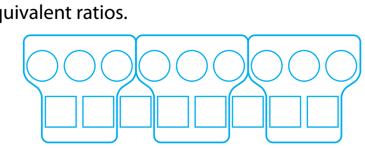
b. Find another ratio that is equivalent to 3 : 2. Use a model to support your answer.

Possible answer: 9 : 6



c. Explain why 3 : 2 and 9 : 8 are not equivalent ratios.

Possible answer: In a model of 9 : 8, there are not 3 circles for every 2 squares. There are some squares left over.



3 Reflect How can you tell whether two ratios are equivalent?

Possible answer: Draw a model of the ratio with greater numbers in it. Then check whether you can separate the model into equal groups that show the other ratio, with no shapes left over.

282

2 Look Ahead Point out that equivalent ratios are ratios that show the same comparison. Students should recognize that combining equal groups (or batches) creates equivalent ratios.

Ask a volunteer to rephrase the definition of *equivalent ratios*. Support student understanding by showing that the number of circles to squares is the same in each group. You can describe the quantities in equivalent ratios as being in the *same ratio*.

CLOSE EXIT TICKET

3 Reflect Look for understanding that two ratios are equivalent if a model of one ratio can be organized in equal groups that show the other ratio, with no shapes left over, or an additional identical group that shows the same relationship is added.

Common Misconception If students think that two ratios are equivalent if the order of the quantities is switched (thinking $a : b$ is equivalent to $b : a$), then ask them to draw a model and explain how the order of the numbers in a ratio is important based on the model they drew.

Prepare for Finding Equivalent Ratios

Support Vocabulary Development

Assign **Prepare for Finding Equivalent Ratios** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *ordered pair* by discussing what *ordered* means and what *pair* means. Provide support as needed, helping students use their previous knowledge of coordinate planes to guide their thinking.

Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers and prompt a whole-class comparative discussion of the descriptions and examples that students provide.

Have students look at the ordered pairs in problem 2 and discuss with a partner what each value in the ordered pair represents in the coordinate plane. Students may use the terms *x-coordinate* and *y-coordinate* in their explanation.

Problem Notes

1. Students should understand that an ordered pair is a pair of numbers that describes the location of a point in the coordinate plane. Student responses may include that the first number is the *x*-coordinate and the second number is the *y*-coordinate. Students may recognize that ordered pairs can be represented in tables, on a coordinate plane, or as numbers in parentheses separated by a comma.
2. Students should recognize the importance of order in an ordered pair. Student responses may include that the first number, or *x*-coordinate, tells how far to move horizontally from the origin, and the second number, or *y*-coordinate, tells how far to move vertically from the origin.

 Name:

Prepare for Finding Equivalent Ratios

1. Think about what you know about ordered pairs. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

Possible answers:

What Is It?

a pair of numbers that describes the location of a point in the coordinate plane

The first number is the *x*-coordinate, and the second number is the *y*-coordinate.

What I Know About It

The *x*-coordinate gives the horizontal distance from the *y*-axis.

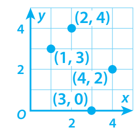
The *y*-coordinate gives the vertical distance from the *x*-axis.

ordered pair

Examples

x	*y*	Ordered Pair
2	4	(2, 4)
1	3	(1, 3)
3	0	(3, 0)
4	2	(4, 2)

Examples



Examples

The ordered pair for the origin is (0, 0).

2. Do the ordered pairs (1, 4) and (4, 1) represent the same point in the coordinate plane? Explain.

No; Possible explanation: The order of the coordinates matters. To graph (1, 4), move 1 unit right and 4 units up from the origin. To graph (4, 1), move 4 units right and 1 unit up from the origin.

283

REAL-WORLD CONNECTION

Architects use ratios to draw structures that are much smaller than the actual structure. Some of their drawings are done on a coordinate plane that shows the top, front, and side views of their structure. Most blueprints like this are not created on paper anymore because Computer-Aided Design (CAD) has become a more efficient way to make and share designs. Architects use this CAD software to make not only the two-dimensional drawings but an entire three-dimensional model of the structure that can be rotated and made larger and smaller. If a client requests that part of a building or house be made larger or smaller, architects can use ratios to make changes to the structures. Ask students to think of other real-world examples when ratios or coordinate planes might be useful.

3 Problem 3 provides another look at equivalent ratios. This problem is similar to the problem about the henna paste recipe. In both problems, a recipe is given for 1 batch. This problem asks for the ratio of cups of flour to tablespoons of peanut butter in 3 batches of dog treats.

Students may use a table or a diagram to solve the problem.

Suggest that students use **Three Reads**, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What are we trying to find out?*
- *What information is important in this problem?*

LESSON 13 | SESSION 1

3 Felipe has a recipe for peanut butter dog treats.

Dog Treats (1 batch)	
Ingredient	**Amount**
Peanut Butter	4 tbsp
Flour	1 cup
Egg	1
Water	2 tbsp

a. What is the ratio of cups of flour to tablespoons of peanut butter if Felipe makes 3 batches of dog treats? Show your work.

Possible work:

	Flour (cups)	Peanut Butter (tbsp)	
	1	4	
+ 1	2	8	+ 4
+ 1	3	12	+ 4

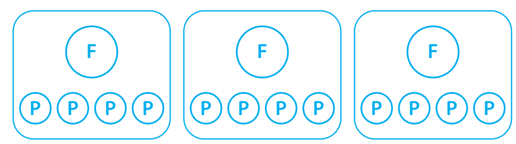
SOLUTION The ratio of cups of flour to tablespoons of peanut butter is 3 : 12 (or equivalent).

b. Check your answer to problem 3a. Show your work.

Possible work:

Each batch should have 1 cup of flour and 4 tbsp peanut butter.

3 batches



The ratio of cups of flour to tablespoons of peanut butter for 3 batches is 3 : 12.

284

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** Apply It

Levels 1–3: Reading/Listening

Adapt **Three Reads** to support students as they make sense of Apply It problem 7. Organize students into pairs to discuss the questions after each reading. Read the problem aloud, and then ask: *What does Hailey make first? What does she want to make?* Call on volunteers to explain the meanings of *necklace, bracelet,* and *beads.*

After the second read, ask: *What quantities do you know? What quantity do you want to find out? What ratio can we write for the necklace?*

After the final read, ask: *What is the ratio between the blue beads and purple beads in the necklace? How can you find the equivalent ratio for the bracelet?*

Levels 2–4: Reading/Listening

Use **Three Reads** to support comprehension of Apply It problem 7. Display the questions before each read. Have partners craft a sentence starter using the words in the question. After each read, provide students with think time to read the question and consider their answer. Have them discuss their answers with partners. Remind students that they can ask their partners clarifying questions:

- *Can you explain more about ____?*
- *What does ____ mean?*

For the final read, ensure that students understand that the problem has two relationships: a *ratio* and an *equivalent ratio.*

Levels 3–5: Reading/Listening

Support understanding of Apply It problem 7 by having students read the problem individually and underline important quantities. Then have students take turns paraphrasing the problem with **Say It Another Way**. Encourage students to ask at least one clarifying question to gather more information about their partner's paraphrase.

Ask students to write down the clarifying questions that they ask their partner. Then compile all of the questions into a class list. Note similarities and differences between the questions. Display the list for future reference during partner and class discussions.

Develop Finding Equivalent Ratios

Purpose

- **Develop** strategies for generating equivalent ratios.
- **Recognize** that you can produce an equivalent ratio by multiplying both quantities in the ratio by the same nonzero number.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

A	B
3, 6, 9, 12, . . .	8, 16, 24, 32, . . .
C	
24, 48, 72, 96, . . .	

Possible Solutions

Each shows multiples of the first number in the list.

A shows multiples of 3.

B shows multiples of 8.

C shows multiples of 24, and 24 is also a common multiple of 3 and 8. So any number in C would also be in both A and B when you continue the pattern of multiples.

WHY? Support students' ability to recognize multiples of a number.

DEVELOP ACADEMIC LANGUAGE

WHY? Support students as they listen to understand a speaker's message.

HOW? Model for students ways to ask clarifying questions or ask for more information during a discussion. Use sentence frames such as: *Can you explain more about ____? What does ____ mean?* During class discussion, highlight and recognize when students ask classmates clarifying questions or ask for more information.

TRY IT

SMP 1, 2, 4, 5, 6

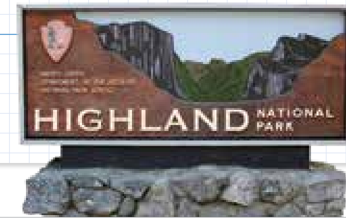
Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, suggest that they use **Three Reads**, asking themselves one of the following questions each time.

- *What is this problem about?*
- *What are you asked to find?*
- *What information is important in this problem?*

LESSON 13 | SESSION 2

Develop Finding Equivalent Ratios



➤ **Read and try to solve the problem below.**

The ratio of picnic tables to garbage cans in a new national park should be 8 : 3. The park design shows plans for picnic tables in a small campground and a large campground. How many garbage cans should there be in each campground?

Number of Picnic Tables

40 in a small campground

120 in a large campground

TRY IT

Math Toolkit connecting cubes, counters, double number lines, grid paper

Possible work:

SAMPLE A

Small Campground

Picnic Tables: 8 8 8 8 8

Garbage Cans: 3 3 3 3 3

Large Campground

Picnic Tables: 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8

Garbage Cans: 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3

The park should have 15 garbage cans in a small campground and 45 garbage cans in a large campground.

SAMPLE B

Small Campground	
Picnic Tables	Garbage Cans
8	3
16	6
24	9
32	12
40	15

Large Campground	
Picnic Tables	Garbage Cans
40	15
80	30
120	45

A small campground should have 15 garbage cans. A large campground should have 45 garbage cans.

DISCUSS IT

Ask: How did you use the ratio 8 : 3 to find the number of garbage cans for 40 picnic tables?

Share: I used the ratio 8 : 3 when I . . .

285

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *How are you keeping track of the information for small and large playgrounds?*
- *How does your model show the ratio 8 : 3?*

Common Misconception Listen for students who think there should be 35 garbage cans in the small campground or 115 in the large campground. They may be think there should be a difference of 5 between the number of picnic tables and garbage cans. As students share their strategies, elicit discussion of what equivalent ratios mean. Encourage students to draw a picture to prove their ratios of tables to garbage cans are equivalent to the given ratio 8 : 3. Ask students how they know their ratios are equivalent. Listen for students who explain how their models show 8 tables for every 3 garbage cans.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- drawing equal groups that represent 8 : 3
- (misconception) assuming there should always be a difference of 5 between the number of picnic tables and garbage cans because 8 − 3 = 5
- using tables or double number lines and ratio reasoning to determine equivalent ratios

Facilitate Whole Class Discussion

Call on students to share selected strategies. After agreeing with a student's statement, add details that add to the idea or increase other students' understanding of the statement.

Allow students time to think by themselves, and then guide students to **Compare and Connect** the representations. Prompt students to connect each representation to the number of picnic tables and garbage cans in the small and large campground.

ASK *How does [student name]'s model show equivalent ratios?*

LISTEN FOR Representations may show equivalent ratios as models of 8 picnic tables and 3 garbage cans, as rows of a table, or as corresponding values on a double number line.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK *How is it possible for these two models to show equivalent ratios if one uses addition and the other uses multiplication?*

LISTEN FOR The double number line uses repeated addition. The table uses multiplication, which is the same as repeated addition.

For the double number line, prompt students to think about how addition is used to generate equivalent ratios.

- *Why is 8 added to get the quantities for the top number line but not for the bottom number line?*
- *What numbers are added to find the quantity of garbage cans when there are 40 picnic tables?*

For the table, prompt students to describe how multiplication can be used to complete the table.

- *What can you multiply 8 by to get 40? To get 120? How does this help you solve the problem?*

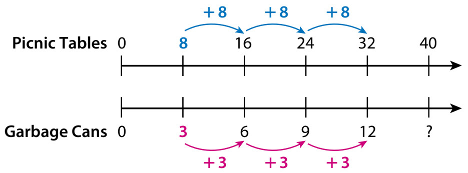
Explore different ways to find equivalent ratios.

The ratio of picnic tables to garbage cans in a new national park should be 8 : 3. The park design shows 40 picnic tables in a small campground and 120 picnic tables in a large campground. How many garbage cans should there be in each campground?

Model It

You can use addition to find equivalent ratios.

One way to show adding groups of 8 picnic tables for every 3 garbage cans is with a double number line.



You can write ratios for number pairs that line up vertically. The double number line shows the equivalent ratios 8 : 3, 16 : 6, 24 : 9, and 32 : 12.

Model It

You can use multiplication to find equivalent ratios.

You can record equivalent ratios in a table.

Picnic Tables	8	16	24	32	40	120
Garbage Cans	3	6	9	12	?	?

DIFFERENTIATION | EXTEND

SMP 2

Deepen Understanding

Making Sense of Quantities and the Relationships Between Them

Prompt students to focus on the relationship between the quantities by changing the numbers in the problem. Then have them use those numbers within the context of the problem to visualize how the models would reflect these changes.

ASK *How would the number of garbage cans change if the number of picnic tables at the small campground is doubled? How many picnic tables and garbage cans would there be?*

LISTEN FOR There would be twice as many picnic tables, so there would be twice as many garbage cans. Both quantities would be multiplied by 2. There would be 80 picnic tables and 30 garbage cans.

ASK *How would the number of picnic tables change if one-fifth of the number of garbage cans is needed at the large campground? How many picnic tables and garbage cans would there be?*

LISTEN FOR Only one-fifth of the number of picnic tables is needed. There would be 24 picnic tables and 9 garbage cans.

Develop Finding Equivalent Ratios

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the ratios of picnic tables to garbage cans are the same in each representation. Explain that they will now use those models to find equivalent ratios.

Before students begin to record and expand on their work in Model It, tell them that problem 3 will prepare them to provide the explanation asked for in problem 4.

Monitor and Confirm Understanding 1 – 2

- The ratios in the double number line are equivalent to 8 : 3 because groups of 8 picnic tables and 3 garbage cans are added to find the next number pair.
- Using double number lines and tables can help you use ratio reasoning to find equivalent ratios.

Facilitate Whole Class Discussion

3 Look for the idea that you can use repeated addition of the quantities in a ratio to find equivalent ratios, and that multiplication is another way to show repeated addition.

ASK *What number can you multiply by to find your answer? What numbers can you repeatedly add to find your answer?*

LISTEN FOR You can multiply by each quantity by 15 or you can add 8 fifteen times and add 3 fifteen times.

4 Look for understanding that you can use either addition or multiplication when finding equivalent ratios.

ASK *How can you use either addition or multiplication to find the number of garbage cans in each campground?*

LISTEN FOR To create another equal group, you can use repeated addition for each quantity, which is the same as multiplying each quantity by the same number in the ratio.

5 Students may recognize that dividing by a number is the same as multiplying by its reciprocal, so equivalent ratios can also be found by dividing both quantities by the same nonzero number.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to find equivalent ratios.**

1 Look at the first **Model It**. How do you know that the ratios from the double number line are equivalent ratios?

You can keep adding equal groups of 8 picnic tables and 3 garbage cans to find the next number pair. In each ratio, there are always 8 picnic tables for every 3 garbage cans.

2 Look at the second **Model It**. What number can you multiply 8 by to get 120? How can you use this number to solve part of the problem?

15; This number tells you that you need 15 groups of 8 picnic tables and 3 garbage cans. Multiply 3 by 15 to find the number of garbage cans.

3 How many garbage cans should be placed in each campground? Explain how you can use addition or multiplication to find the answer.

Small: 15; large: 45; Possible explanation: Multiply both quantities in 8 : 3 by 5 to get 40 : 15, and by 15 to get 120 : 45.

4 Why can you multiply both quantities in a ratio by the same number to find an equivalent ratio?

When you multiply both quantities of a ratio by the same number, you are adding equal groups of the same ratio. The comparison stays the same.

5 Cai says you can divide both quantities in a ratio by the same nonzero number to find an equivalent ratio. Explain why Cai is correct.

Possible answer: You can multiply both quantities by the same number, and dividing by a number is the same as multiplying by its reciprocal.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to find equivalent ratios.

Responses will vary. Check student responses.

287

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use repeated addition and multiplication to form equivalent ratios.

If students are unsure about finding equivalent ratios using multiplication, then use this activity to connect multiplication to repeated addition.

Materials For each group: 24 two-color counters

- Give counters to each group and have them make a row of red counters and a row of yellow counters in a 6 to 2 ratio. Ask: *How do you know your model is correct?* [There are 6 red counters and 2 yellow counters.]
- Have students add 6 red counters and 2 yellow counters to the existing group. Ask: *What is the ratio of red counters to yellow counters?* [12 : 4] *Describe how to find this ratio using multiplication.* [Multiply each quantity by 2.]
- Repeat a third time. Describe how to find this ratio using multiplication based on the original ratio. [Multiply each quantity by 3.]
- Ask: *Suppose you wanted to add another group of 6 red counters and 2 yellow counters. How could you determine the ratio of counters in the entire collection based on the original ratio by using multiplication?* [Multiply both 6 and 2 by 4.]

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; drawing number lines with equal spacing between tick marks can be difficult, and precise measures are not necessary to determine a solution to the problem.

7 **B** **is correct.** Students may divide 24 by 4 to get 6 and then divide 32 by 4 to find the number of purple beads Hailey should use.

A is not correct. This answer is the result of dividing the number of blue beads in the necklace by 6 instead of the number of purple beads by 4.

C is not correct. This answer is the result of adding 8 to the number of blue beads for the bracelet.

D is not correct. This answer is the result of subtracting 6 from the number of blue beads in the necklace, 24.

8 Students may recognize that Kareem added the same value, 8, to each number in the ratio.

Apply It

➤ **Use what you learned to solve these problems.**

7 Hailey makes a necklace with 24 blue beads and 32 purple beads. She wants to make a bracelet that uses the same ratio of blue beads to purple beads. She plans to use 6 blue beads for the bracelet. How many purple beads should Hailey use?

A 4 purple beads

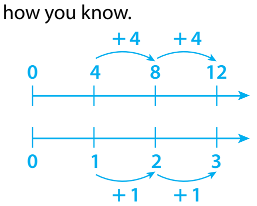
B 8 purple beads

C 14 purple beads

D 18 purple beads

8 Kareem says that the ratio 4 : 1 is equivalent to the ratio 12 : 9 because 4 + 8 = 12 and 1 + 8 = 9. Is Kareem correct? Explain how you know.

No; Possible explanation: The double number line shows that 4 : 1 is equivalent to 12 : 3, not 12 : 9.



9 The table shows that Marta's heart beats 18 times every 15 s. Use equivalent ratios to complete the table. Explain how you found the time in seconds for 180 heartbeats.

Marta's Heartbeats

Time (s)	Number of Beats
15	18
30	**36**
45	**54**
150	180

See table; Possible explanation: You can go from 18 to 180 by multiplying by 10. To find an equivalent ratio, you need multiply 15 by 10.

CLOSE **EXIT TICKET**

9 Students' solutions should show an understanding of:

- equivalent ratios as ratios that name the same multiplicative comparison.
- finding equivalent ratios by multiplying each quantity in the ratio by the same number.

Error Alert If students write 60 as the missing time, then explain that the missing time must form a ratio with second quantity 180 that is equivalent to 15 : 18. Have students compare the number of beats in the original ratio to the quantity 180 heartbeats.

Practice Finding Equivalent Ratios

Problem Notes

Assign **Practice Finding Equivalent Ratios** as extra practice in class or as homework.

1. Students should recognize dividing as the same as multiplying by the reciprocal and understand that either method can be used to find an equivalent ratio. *Basic*

2. **A, D, and E are correct.** Students may solve the problem by multiplying or dividing each number in the ratio 8 : 12 by the same number.

   B is not correct. This answer is the given ratio listed in a different order.

   C is not correct. This answer is found by adding 8 to each value in the ratio 8 : 12. *Basic*

 Name:

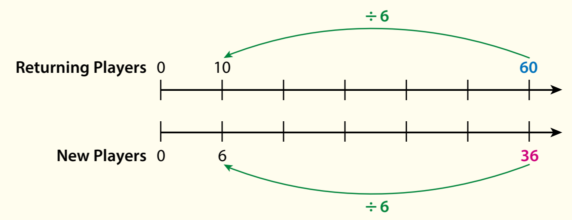
Practice Finding Equivalent Ratios

➤ **Study the Example showing how to find equivalent ratios. Then solve problems 1–5.**

Example

A soccer league has 60 returning players and 36 new players. Each team will have the same ratio of returning players to new players as the league has. How many new players will a team with 10 returning players have?

You can use a double number line to find ratios equivalent to 60 : 36. Number pairs that line up vertically represent equivalent ratios.



You can divide each quantity in 60 : 36 by 6 to find the equivalent ratio 10 : 6.

A team with 10 returning players will have 6 new players.

1. Sophia says that you can solve the problem in the Example by multiplying both quantities in the ratio 60 : 36 by $\frac{1}{6}$. Is Sophia correct? Explain.

   Yes; Possible explanation: Multiplying both quantities by $\frac{1}{6}$ is the same as dividing both quantities by 6.

2. Which ratios are equivalent to 8 : 12? Select all that apply.

   A 4 : 6

   B 12 : 8

   C 16 : 20

   D 24 : 36

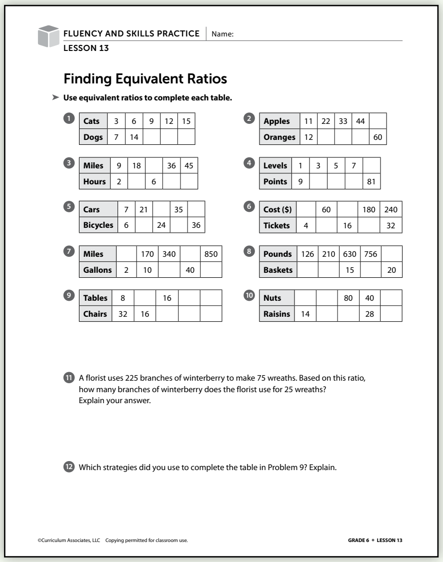
   E 56 : 84

Vocabulary
equivalent ratios two ratios that express the same comparison. Multiplying both numbers in the ratio $a : b$ by a nonzero number n results in the equivalent ratio $na : nb$.

Fluency & Skills Practice

Finding Equivalent Ratios

In this activity, students use equivalent ratios to complete tables and to solve a real-world problem.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 13

Finding Equivalent Ratios

➤ **Use equivalent ratios to complete each table.**

1.

Cats	3	6	9	12	15
Dogs	7	14			

2.

Apples	11	22	33	44	
Oranges	12				60

3.

Miles	9	18		36	45
Hours	2		6		

4.

Levels	1	3	5	7	
Points	9				81

5.

Cars	7	21		35	
Bicycles	6		24		36

6.

Cost ($)		60		180	240
Tickets	4		16		32

7.

Miles		170	340		850
Gallons	2	10		40	

8.

Pounds	126	210	630	756	
Baskets			15		20

9.

Tables	8		16		
Chairs	32	16			

10.

Nuts			80	40	
Raisins	14			28	

11. A florist uses 225 branches of winterberry to make 75 wreaths. Based on this ratio, how many branches of winterberry does the florist use for 25 wreaths? Explain your answer.

12. Which strategies did you use to complete the table in Problem 9? Explain.

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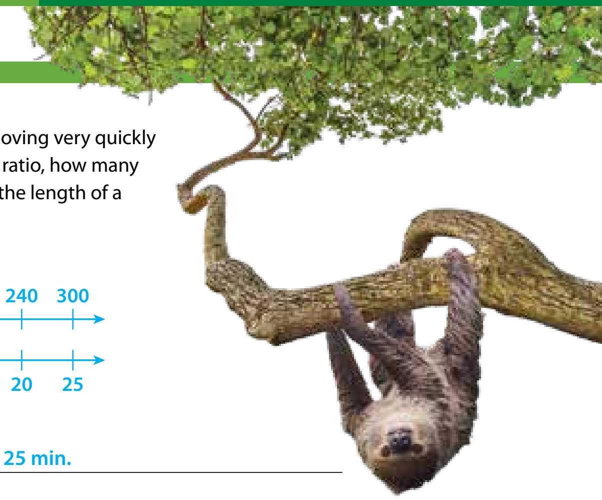
Additional Practice

3 Students may also use a table or repeated addition to write equivalent ratios. ***Medium***

4 **a.** Students may use ratio reasoning or multiplication to find equivalent ratios. ***Medium***

b. Students may divide 63 by 7 to get 9, and then multiply 9 by 1 to find the number of adults. ***Medium***

5 Students may also multiply the number of small T-shirts and large T-shirts by 6 to find the quantities the manager should order. ***Challenge***



3 A football field is 300 ft long. A sloth moving very quickly travels 60 ft every 5 min. Based on this ratio, how many minutes would it take a sloth to travel the length of a football field? Show your work.

Possible work:

Distance (ft) 0 60 120 180 240 300

Time (min) 0 5 10 15 20 25

SOLUTION It would take the sloth 25 min.

4 At a summer camp, the ratio of campers to adults is kept equivalent to 7 : 1.

a. Use equivalent ratios to complete the table.

Campers	7	14	28	210
Adults	1	2	4	30

b. Next week, there will be 63 campers. How many adults should the camp have next week? Show your work. Possible work:

Campers 0 7 14 21 28 35 42 49 56 63

Adults 0 1 2 3 4 5 6 7 8 9

SOLUTION The camp should have 9 adults.

5 A manager of a clothing store always orders 2 small T-shirts and 3 large T-shirts for every 4 medium T-shirts. The manager plans to order 24 medium T-shirts. How many small T-shirts and large T-shirts should the manager order? Show your work.

Possible work:

Small T-Shirts	2	4	6	8	10	12
Medium T-Shirts	4	8	12	16	20	24
Large T-Shirts	3	6	9	12	15	18

SOLUTION The manager should order 12 small shirts and 18 large shirts.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** **Connect It**

Levels 1–3: Speaking/Writing

Prepare students to respond in writing to Connect It problem 4. Read the problem aloud. Point out that the question is about finding equivalent ratios using a point in a coordinate plane. Begin a **Co-Constructed Word Bank** with the terms *coordinate plane* and *equivalent ratios.* Ask students to tell which Model It connects to the problem. Provide think time for students to consider how the Model It graph can help them answer the question. Have partners identify words they can use in their responses. Prompt students to suggest additional words and phrases that can help them discuss the problem. Guide them to include *ordered pairs, x-coordinate,* and *y-coordinate*. Then help students write explanations in short sentences.

Levels 2–4: Speaking/Writing

Prepare students to respond in writing to Connect It problem 4. Read the problem with students. Call on volunteers to rephrase the question.

Have students work with a partner to identify which Model It connects to the problem. Ask partners to list at least three important terms that might be used in their responses. Compile the responses into a **Co-Constructed Word Bank**.

Have students discuss their answers before writing. Ask partners to take turns explaining the steps to finding equivalent ratios on graphs. Remind them to use terms from the word bank. Then have students draft their responses individually.

Levels 3–5: Speaking/Writing

Prepare students to respond in writing to Connect It problem 4. Have students read the problem and discuss what the question is asking. Have partners create a **Co-Constructed Word Bank** of terms they can use to discuss and write about the problem. Allow time for partners to discuss, and then compile the terms into a class word bank.

Have students draft a written response using words from the word bank. Have them use **Stronger and Clearer Each Time** to get feedback from a partner and revise their explanations. Encourage partners to discuss how the explanation was strengthened by precise language.

Develop Graphing a Table of Equivalent Ratios

Purpose

- **Develop** strategies for graphing points that represent equivalent ratios.
- **Recognize** that a graph is another way to represent and generate equivalent ratios.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

A: 1 to 3	B: 3 : 1
C: 3 : 9	D: 9 to 3

Possible Solutions

A and C are equivalent ratios.

B and D are equivalent ratios.

B and C are written using a colon and A and D are written using the word *to*.

All ratios can be written as 1 : 3 or 3 : 1.

WHY? Support students' recognition of equivalent ratios represented in different formats.

DEVELOP ACADEMIC LANGUAGE

WHY? Recognize words, punctuation, and symbols used to express ratios.

HOW? Discuss the relationship between the ordered pair, the words in the ratio, and the mathematical representation in Model It. Ask: *What does the comma separate in the ordered pair? What does each number represent? What word expresses a ratio?* [to] *What replaces the preposition when you write a ratio in mathematical notation?* [:]

TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

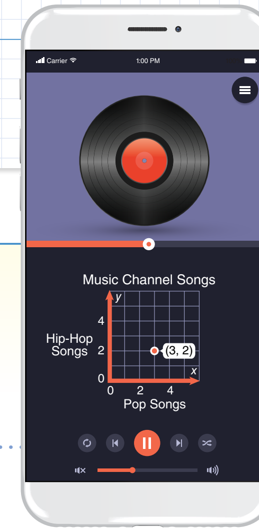
See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Co-Craft Questions** to help them make sense of the problem. If discussion lags, have students turn and talk with a partner about what the point on the graph represents.

LESSON 13 | SESSION 3

Develop Graphing a Table of Equivalent Ratios

Read and try to solve the problem below.

A streaming music channel always plays the same ratio of pop songs to hip-hop songs. The point on the graph shows the number of hip-hop songs played for every 3 pop songs. Based on the relationship in the graph, how many hip-hop songs does the channel play for every 12 pop songs?



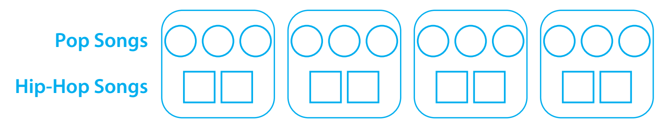
TRY IT

Math Toolkit connecting cubes, counters, double number lines, graph paper

Possible work:

SAMPLE A

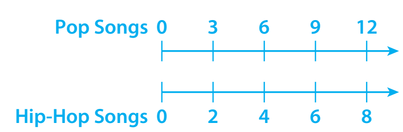
The ordered pair (3, 2) means that for every 3 pop songs, there are 2 hip-hop songs.



The music channel plays 8 hip-hop songs for every 12 pop songs.

SAMPLE B

The point at (3, 2) shows that the station plays 3 pop songs for every 2 hip-hop songs.



For every 12 pop songs, the music channel plays 8 hip-hop songs.

DISCUSS IT

Ask: How does your model use the ordered pair from the graph?

Share: In my model, I used the ordered pair to . . .

291

DISCUSS IT SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *What does the ordered pair (3, 2) mean in this context?*
- *How can you use your model to prove that your answer is correct?*

Common Misconception Listen for students who attempt to generate equivalent ratios by adding the same amount to both quantities. For example, they may add 3 to both quantities in the ratio 3 : 2 and name 6 : 5 and 9 : 8 as equivalent ratios. Point out how repeatedly adding 3 to the quantity of pop songs and 2 to the quantity of hip-hop songs will result in equivalent ratios. Guide students to distinguish the incorrect and correct strategies that use repeated addition. As students share their strategies, ask them to explain what the given ordered pair means in terms of the problem and how to use the ordered pair to find equivalent ratios.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- concrete or pictorial representations of ratios equivalent to 3 : 2
- (misconception) addition of the same number to both quantities in the ratio
- double number lines or tables of equivalent ratios
- coordinate planes that show equivalent ratios as points

Facilitate Whole Class Discussion

Call on students to share selected strategies. Allow think time for students to process the ideas presented.

Guide students to **Compare and Connect** the representations. Call on volunteers to reword any vague or unclear statements.

ASK *How can you express the ordered pair (3, 2) as a ratio?*

LISTEN FOR The music channel plays 3 pop songs for every 2 hip-hop songs. The ratio is 3 to 2.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK *How do the values in the table relate to the points in the coordinate plane?*

LISTEN FOR The number of pop songs corresponds to the x-coordinate of each ordered pair. The number of hip-hop songs corresponds to the y-coordinate of each ordered pair. Each row in the table represents a point on the graph.

For the table, prompt students to think about how the rows of the table show equivalent ratios.

- *How are the values in each row of the table related to the values in the row above? How are they related to the values in the first row?*

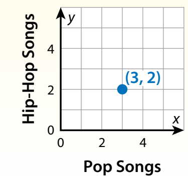
For the coordinate plane, prompt students to think about how the points on the coordinate plane represent the number of songs.

- *How do the ordered pairs show each ratio?*
- *How can you move from one ratio on the graph to an equivalent ratio?*

➤ **Explore different ways to use a graph to show equivalent ratios.**

A streaming music channel always plays the same ratio of pop songs to hip-hop songs. The point on the graph shows the number of hip-hop songs played for every 3 pop songs. Based on the relationship in the graph, how many hip-hop songs does the channel play for every 12 pop songs?

Music Channel Songs



Model It

You can make a table of equivalent ratios from the given ordered pair.

The ordered pair (3, 2) shows that the ratio of pop songs to hip-hop songs is 3 : 2.

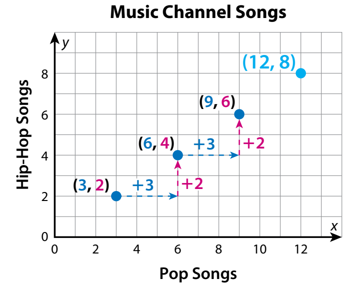
Pop Songs, x	Hip-Hop Songs, y
3	2
6	4
9	6
12	?

+ 3, + 3, + 3 (pop songs); + 2, + 2, + 2 (hip-hop songs)

Model It

You can use the coordinates of the given ordered pair to find other ordered pairs that represent equivalent ratios.

Music Channel Songs



DIFFERENTIATION | EXTEND

SMP 5

Deepen Understanding
Using Tables and Coordinate Planes Strategically

Prompt students to understand the use of each model and compare them.

ASK *How does each x-coordinate in the table compare to the x-coordinate below it? How is this shown in each of the models?*

LISTEN FOR The x-coordinates increase by 3. This is shown by adding 3 from one row to the row below it using +3. On the graph, there is an arrow pointing 3 units to the right along the x-axis between each point.

ASK *How does each y-coordinate in the table compare the y-coordinate below it? How is this shown in each of the models?*

LISTEN FOR The y-coordinates increase by 2. This is shown by adding 2 from one row to the row below it using +2. On the graph, there is an arrow pointing 2 units up along the y-axis between each point.

ASK *Can you move 3 units to the right and 3 units up on the graph to find an equivalent ratio? Explain why or why not.*

LISTEN FOR No; If you moved from (3, 2) to (6, 5), the ratios would not be equivalent.

Develop Graphing a Table of Equivalent Ratios

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about graphing a table of equivalent ratios.

Before students begin to record and expand on their work in Model It, tell them that problem 4 will prepare them to provide the explanation asked for in problem 5.

Monitor and Confirm Understanding 1 – 3

- The *x*-coordinate represents the number of pop songs and the *y*-coordinate represents the number of hip-hop songs.
- Both the table and the graph show the relationship between pop songs and hip-hop songs.
- When 12 pop songs are played, 8 hip-hop songs are played.

Facilitate Whole Class Discussion

4 Look for understanding of moving between points that represent equivalent ratios on the coordinate plane.

> **ASK** *How can you relate moving to the right and moving up on a graph to mathematical operations?*
>
> **LISTEN FOR** Moving to the right is adding to the *x*-coordinate. Moving up is adding to the *y*-coordinate.

5 Look for the idea that you can multiply both values in an ordered pair by the same number to find an equivalent ratio.

> **ASK** *How does multiplying both values in an ordered pair by the same number create an equivalent ratio?*
>
> **LISTEN FOR** Multiplying both quantities by the same number is the same as repeated addition for each quantity.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to use a graph to show equivalent ratios.**

1 How does the graph in the problem show that the ratio of pop songs to hip-hop songs is 3 : 2?

The graph shows the number of pop songs on the *x*-axis and the number of hip-hop songs on the *y*-axis. The point (3, 2) represents 3 pop songs and 2 hip-hop songs.

2 Look at the table in the first **Model It** and the graph in the second **Model It**. How is the addition pattern in the graph related to the addition pattern in the table?

Both patterns show adding equal groups of 3 pop songs and 2 hip-hop songs to find equivalent ratios.

3 How many hip-hop songs does the streaming music channel play for every 12 pop songs? Plot a point on the graph to model this relationship.

8 hip-hop songs. See graph in Model It.

4 How can you use a point on a graph to find another point that represents an equivalent ratio? Explain why your method works.

Possible answer: Start at the point. Move right the same number of units as the *x*-coordinate and up the same number of units as the *y*-coordinate. This is the same as combining two equal groups of the quantities in the ratio.

5 How could you use ordered pairs and multiplication to find equivalent ratios?

If an ordered pair represents a ratio, multiply both coordinates by the same number to find an equivalent ratio.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand using a graph to show equivalent ratios.

Responses will vary. Check student responses.

293

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity
Model generating equivalent ratios with repeated addition.

If students are unsure about the connections between tables and graphs of equivalent ratios, then use this activity to model finding ordered pairs that represent equivalent ratios.

Materials For each pair: 24 two-color counters, Activity Sheet *Coordinate Plane: First Quadrant*

- Distribute counters and an Activity Sheet to each pair. Have students label the *x*-axis *Number of Red Counters* and the *y*-axis *Number of Yellow Counters.*
- Have pairs display 2 red counters and 4 yellow counters. Have students represent this ratio as an ordered pair and as a point in the coordinate plane. [(2, 4)]
- Ask: *How can you use counters to find a ratio equivalent to 2 : 4?* [Add 2 red counters and 4 yellow counters; the equivalent ratio is 4 : 8.]
- Ask: *How can you use the graph to find a ratio equivalent to 2 : 4?* [Move over 2 and up 4 starting from (2, 4).] Have pairs show moving over 2 and up 4 from (2, 4) and plot a point. Ask: *What are the coordinates of the point? What ratio does this point represent?* [(4, 8); 4 : 8]
- Have pairs find additional equivalent ratios with counters and as points on the graph.

Apply It

For all problems, encourage students to use a model to support their thinking.

7. Students may use multiplication or division to find the missing hose lengths and gallons of water in the table. Then they should express each value as an ordered pair.

8. Students may also use the coordinate plane to solve, first moving 8 units to the right and 1 unit up from the given point until reaching 32 on the *x*-axis.

Apply It

➤ **Use what you learned to solve these problems.**

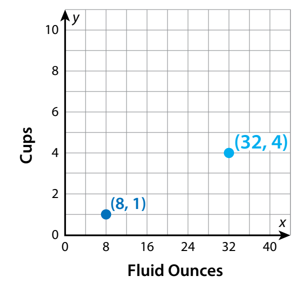
7. The ratio of the length of a fire hose in feet to the number of gallons of water the hose can hold is 100 : 4. Complete the table of equivalent ratios. Then write each ratio as an ordered pair. See table.

Hose Length (ft)	Volume of Water (gal)	Ordered Pair
25	1	(25, 1)
50	2	(50, 2)
100	4	(100, 4)
300	12	(300, 12)

8. Enrique has a container of 32 fl oz of orange juice. He is filling glasses with 1 cup of juice. The point on the graph shows the ratio of fluid ounces to cups. Based on this ratio, how many glasses can Enrique fill from the container? Plot a point on the graph to show the number of cups in 32 fl oz. Show your work.

Possible work: (8, 1) shows that there are 8 fl oz for every 1 cup. Multiply 8 and 1 each by 4 to get (32, 4).

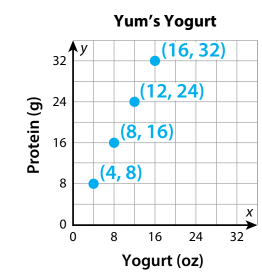
See graph.



SOLUTION Enrique can fill 4 glasses.

9. Every 4-oz serving of Yum's Yogurt contains 8 g of protein. Complete the table of equivalent ratios. Then plot points on the graph to represent the ratios. See table and graph.

Yogurt (oz)	Protein (g)
4	8
8	16
12	24
16	32



294

CLOSE EXIT TICKET

9. Students' solutions should show an understanding of:
   - completing a table of equivalent ratios.
   - plotting ordered pairs that represent equivalent ratios in a coordinate plane.

Error Alert If students switch the order of the numbers in an ordered pair, then remind them to examine the table headings and axis labels to identify the number of ounces of yogurt as the *x*-coordinate and the number of grams of protein as the *y*-coordinate. Have them demonstrate and describe the difference between the locations of the points (12, 24) and (24, 12) in the coordinate plane.

Practice Graphing a Table of Equivalent Ratios

Problem Notes

Assign **Practice Graphing a Table of Equivalent Ratios** as extra practice in class or as homework.

1. Students should recognize that only the *y*-coordinates of the points would change because only the number of pages is different. The number of minutes stays the same. ***Basic***

2. Students may understand that they can find an equivalent ratio by multiplying both coordinates by the same number, not by adding the same number. Students may realize that repeated addition can be used separately for each quantity in a ratio. ***Challenge***

LESSON 13 | SESSION 3 Name:

Practice Graphing a Table of Equivalent Ratios

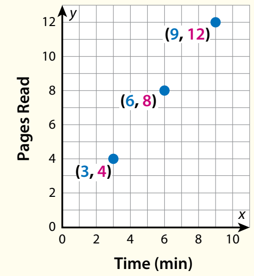
➤ **Study the Example showing how to graph a table of equivalent ratios. Then solve problems 1–5.**

Example

Jade reads 4 pages every 3 min. Make a table of equivalent ratios to show how many pages Jade can read in 3 min, 6 min, and 9 min. Then graph the equivalent ratios.

Record the ratio 3 to 4 in one row of a table. Find equivalent ratios for 6 min and 9 min by multiplying each number in the ratio 3 : 4 by 2 and by 3.

Time (min)	Pages Read
3	4
6	8
9	12



Think of each ratio in the table as an ordered pair (*x*, *y*). The *x*-coordinate is the time in minutes and the *y*-coordinate is the number of pages read.

1. How would the graph in the Example change if Jade reads 5 pages every 3 minutes instead of 4 pages every 3 minutes?

   Possible answer: The *x*-coordinates of the points would stay the same, but the *y*-coordinates would increase. The new ordered pairs would be (3, 5), (6, 10), and (9, 15).

2. The point (7, 8) in the coordinate plane represents a ratio. Adela claims that you can find an equivalent ratio by adding the same number to both coordinates of the point. Is Adela correct? Explain.

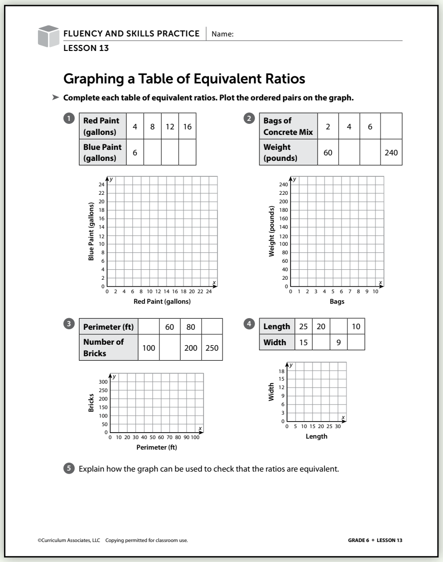
   No; Possible explanation: If you add 7 to the first coordinate, you need to add 8, not 7, to the second coordinate to get an equivalent ratio.

295

Fluency & Skills Practice

Graphing a Table of Equivalent Ratios

In this activity, students use equivalent ratios to complete tables. Then they plot ordered pairs on coordinate grids to represent the ratios.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 13

Graphing a Table of Equivalent Ratios

➤ Complete each table of equivalent ratios. Plot the ordered pairs on the graph.

1.

Red Paint (gallons)	4	8	12	16
Blue Paint (gallons)	6			

2.

Bags of Concrete Mix	2	4	6	
Weight (pounds)	60			240

3.

Perimeter (ft)		60	80	
Number of Bricks	100		200	250

4.

Length	25	20		10
Width	15		9	

5. Explain how the graph can be used to check that the ratios are equivalent.

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Additional Practice

3 **a.** Any plotted point will have an x-coordinate that is a multiple of 10 and a y-coordinate that is a multiple of 35. ***Medium***

b. The x-coordinate represents the number of people that plan to attend the picnic. The y-coordinate represents the number of napkins to bring. ***Medium***

4 **a.** Each value in the first row is a multiple of 2. Each value in the second row is a corresponding multiple of 3. ***Medium***

b. The first row of the table represents the x-coordinates of the points on the graph, and the second row of the table represents the y-coordinates of the points on the graph. ***Medium***

5 **B and E are correct.** Students may solve the problem by multiplying each coordinate of the ordered pair by 2 or by dividing each coordinate by 3.

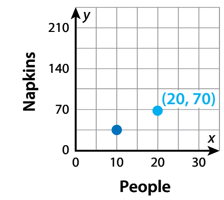
A is not correct. This answer represents dividing each coordinate by 3 and then switching the order of the coordinates.

C is not correct. This answer represents adding 3 to each coordinate instead of multiplying by 3.

D is not correct. This answer represents adding 6 to each coordinate instead of multiplying.

Basic

3 Jordan and Mia are bringing napkins to a back-to-school picnic. They decide to bring 35 napkins for every 10 people who plan to attend. The point on the graph represents this ratio.



a. Plot another point that represents an equivalent ratio. Explain how you found the coordinates of this point.

Possible answer: (20, 70); From the point (10, 35), I moved 10 units to the right and 35 units up to show adding another group of 10 people and 35 napkins.

b. What do the coordinates of the point you plotted represent in this situation?

Possible answer: The coordinates show that Jordan and Mia should bring 70 napkins if 20 people plan to attend the picnic.

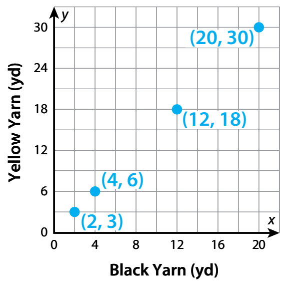
4 Allen is making a scarf for charity. He uses 4 yd of black yarn for every 6 yd of yellow yarn.

a. Complete the table of equivalent ratios.

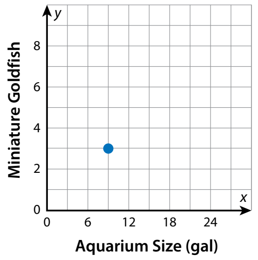
Black Yarn (yd)	2	4	12	20
Yellow Yarn (yd)	3	6	18	30

b. Plot ordered pairs on the graph to represent the ratios.

See graph.



5 An aquarium that holds 9 gal is the correct size for 3 miniature goldfish. The point on the graph represents this ratio relationship. Which ordered pairs represent equivalent ratios that would also be on the graph? Select all that apply.



A (1, 3)

B (3, 1)

C (12, 6)

D (15, 9)

E (18, 6)

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** **Model It**

Levels 1–3: Listening/Reading

Support students in understanding how to use double number lines to find equivalent ratios. Read aloud the first Model It. Have students tell what they notice about the double number lines. Reword any unclear statements and record for reference. Help students identify terms to describe the double number lines, such as *scale* and *marks*.

Read the word problem aloud. Have students circle the marks that help them solve the problem. Provide a sentence frame to help students explain how the double number lines help them solve the problem:

- *The double number line shows the ratio ____ and other ratios that are ____.*

Levels 2–4: Listening/Reading

Support students in understanding how to use double number lines to find equivalent ratios. Read aloud the first Model It. Display:

- *I notice ____.*
- *One way the double number lines are the same is ____.*
- *One way they are different is ____.*

Have students use the frames to tell what they notice about the double number lines. Guide them to identify the scale between the marks on the number lines.

Have partners discuss how the double number lines help them solve the word problem. Support discussions with:

- *I know the ratios are ____ because ____.*

Levels 3–5: Listening/Reading

Support students in understanding how to use double number lines to find equivalent ratios. Have students read the first Model It. Have partners discuss the scale and tell what they notice about the double number lines. Record statements for reference.

Have partners reread the word problem and discuss how the double number lines help them solve the problem. Encourage students to discuss using precise mathematical language and complete sentences. Display the following questions to support discussion:

- *How do the double number lines represent equivalent ratios?*
- *How do you know the ratios are equivalent?*

Develop Using Equivalent Ratios

Purpose

- **Develop** strategies for using equivalent ratios to solve problems.
- **Recognize** that you can solve ratio problems by multiplying and dividing to find equivalent ratios.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

A: 2 to 3	B: 12 : 6
C: 6 : 9	D: 18 to 9

Possible Solutions

A and C are equivalent ratios.

B and D are equivalent ratios.

B and C are written using a colon and A and D are written using the word *to*.

WHY? Support students' facility with recognizing equivalent ratios written in different forms.

DEVELOP ACADEMIC LANGUAGE

WHY? Support understanding of the multiple-meaning word *scale*.

HOW? Display the term *scale*. Have students turn and talk about multiple meanings of the word as they have heard used in everyday language. Possible ideas may include a tool for measuring weight, the covering on fish and insect bodies, and the way notes are organized in music. Read the first Model It. If necessary, point out that the *scale* is a ratio comparing the measurements used on the number lines to the actual measurements of distance and time.

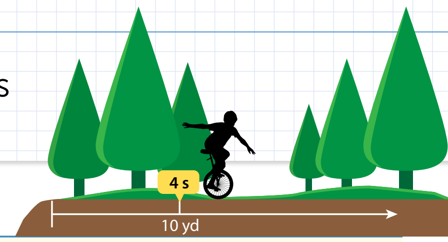
TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Say It Another Way** to help them make sense of the problem. Listen for understanding that the ratio in the problem is 10 to 4.

LESSON 13 | SESSION 4 ■ ■ ■ ■ □

Develop Using Equivalent Ratios



➤ **Read and try to solve the problem below.**

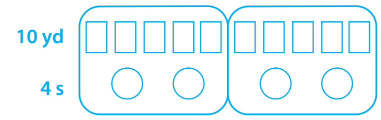
Ian travels 10 yd on his unicycle every 4 s. Based on this ratio, how many seconds does it take Ian to travel 25 yd on his unicycle?

TRY IT

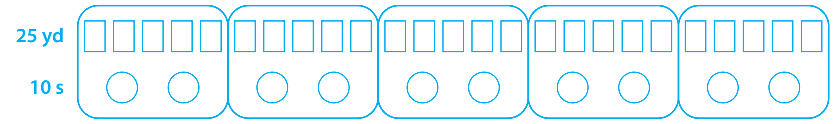
Math Toolkit connecting cubes, counters, double number lines, graph paper

Possible work:

SAMPLE A



It takes Ian 2 s to travel 5 yd.



It takes Ian 10 s to travel 25 yd.

SAMPLE B

+ 10 + 10 + 10 + 10 ÷ 2

Distance (yd)	10	20	30	40	50	25
Time (s)	4	8	12	16	20	10

+ 4 + 4 + 4 + 4 ÷ 2

Ian takes 10 s to travel 25 yd on his unicycle.

DISCUSS IT

Ask: How does your model show that Ian travels 10 yd every 4 s?

Share: I showed this ratio by . . .

297

DISCUSS IT SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *Do you prefer to use a drawing, table, or other model to show your work? Why?*
- *How is 25 yd shown in your model?*

Error Alert If students claim they cannot solve the problem because there is no whole number that they can multiply 10 by to get 25, ask them to think about finding other ratios of whole numbers that are equivalent to 10 : 4. If further prompting is needed, guide them to draw a model of 10 : 4 and divide it into two equal groups.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- drawing a model to represent the ratio 10 : 4
- using a table or coordinate plane to represent the ratio
- using a double number line to represent the ratio

Facilitate Whole Class Discussion

Call on students to share selected strategies. Remind students that when listening for understanding, they can ask the speaker to explain the ideas in a different way.

Guide students to **Compare and Connect** the representations. If ideas are unclear, ask the speaker or another student to repeat or rephrase.

> **ASK** *How does [student name]'s model show how many seconds it takes for Ian to travel 25 yards?*
>
> **LISTEN FOR** The part of the model that represents 25 yards corresponds to 10 seconds.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

> **ASK** *Why does it help to find an equivalent ratio where the distance is less than 10 yards and the time is less than 4 seconds?*
>
> **LISTEN FOR** 25 is not a multiple of 10. It is helpful to find an equivalent ratio that has a first quantity that is a factor of 25.

For the double number line model, prompt students to identify how the number lines are labeled to represent the problem.

- *Why is it helpful to add marks for 5 yd, 15 yd, and 25 yd to the double number line?*
- *How can you use the double number line to find the time it takes Ian to travel 25 yards?*

For the table, prompt students to identify the meaning of the ratios in the table using *for every* language.

- *What does the first ratio represent?*
- *What does the second ratio represent?*

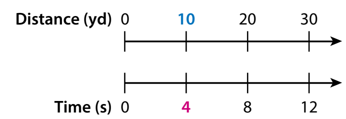
➤ **Explore different ways to use equivalent ratios to solve problems.**

Ian travels 10 yd on his unicycle every 4 s. Based on this ratio, how many seconds does it take Ian to travel 25 yd on his unicycle?

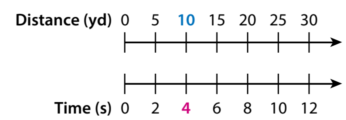
Model It

You can use a double number line to solve the problem.

Choose scales to show that Ian travels 10 yd every 4 s.



Add marks halfway between the existing marks to find additional equivalent ratios.



Model It

You can use a combination of multiplication and division to solve the problem.

Show the ratios in a table. Think of a way to get from 10 yd to 25 yd by using a combination of multiplication and division.

	÷ 2 →	× 5 →
Distance (yd)	10	5	25
Time (s)	4	2	?

÷ 2 → × 5 →

DIFFERENTIATION | EXTEND

SMP 2

Deepen Understanding

Using Tables to Reason Abstractly About Equivalent Ratios

Prompt students to reason about how the table was constructed to form equivalent ratios.

> **ASK** *How do you know that all three ratios in the table are equivalent ratios?*
>
> **LISTEN FOR** Both quantities in the first ratio are divided by 2 to get the second ratio. Both quantities in the second ratio are multiplied by 5 to get the third ratio. Each quantity in the ratio is multiplied or divided by the same number, so the resulting ratios are equivalent.

> **ASK** *What would the table look like if you multiplied by 5 before dividing by 2? Explain why this method still makes equivalent ratios.*
>
> **LISTEN FOR** The first row would show 10, 50, 25 and the second row would show 4, 20, 10. You can multiply and divide in any order. The final equivalent ratio is 25 : 10 whether you divide or multiply first.

Develop Using Equivalent Ratios

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationship between them are the same in each representation. Explain that they will now use this relationship to reason about equivalent ratios.

Before students begin to record and expand on their work in Model It, tell them that problem 4 will prepare them to provide the explanation asked for in problem 5.

Monitor and Confirm Understanding 1 – 2

- Double number lines where quantities line up vertically show equivalent ratios.
- Additional tick marks can be added to a double number line to find other equivalent ratios.
- You can rewrite a ratio as an equivalent ratio where quantities have the same multiple in order to find more equivalent ratios.

Facilitate Whole Class Discussion

3 Look for understanding that finding one equivalent ratio can help you find other equivalent ratios.

> **ASK** *Why does the table show dividing 10 by 2 instead of dividing 10 by 5?*
>
> **LISTEN FOR** When you divide 10 by 2, you get 5, which is a is a factor of 25 and makes it is easier to solve the problem. If you divided 10 by 5, you would get 2, which is not a factor of 25.

4 Look for understanding of the importance of checking an answer for reasonableness.

> **ASK** *Do you think your answer should be greater than or less than 4? Why?*
>
> **LISTEN FOR** Greater than 4 because 25 is more than 2 times 10, so the number of seconds should be more than 2 times 4.

5 Look for the idea that you can solve ratio problems by multiplying and dividing by whole numbers to find equivalent ratios.

> **ASK** *When would you use multiplication and division to find an equivalent ratio?*
>
> **LISTEN FOR** When it is not possible to write an equivalent ratio with whole-number quantities using one operation.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to use equivalent ratios to solve problems.**

1 Look at the first **Model It**. How do you know that 5 : 2, 15 : 6, and 25 : 10 are equivalent to the ratio 10 : 4?

Possible answer: All pairs of numbers that line up vertically on the number lines represent equivalent ratios.

2 Can you solve the **Try It** problem by multiplying both quantities in the ratio 10 to 4 by the same whole number? Why or why not?

No; Because 25 yd is not a multiple of 10 yd.

3 Look at the second **Model It**. It shows that the ratio 10 to 4 can be written as the equivalent ratio 5 to 2. Why is this step helpful?

You can multiply both quantities in the ratio 5 to 2 by the same whole number to solve the problem, but you cannot do this for 10 to 4.

4 How many seconds does it take Ian to ride 25 yd on his unicycle? How do you know that your answer is reasonable?

10 s; Possible explanation: If Ian rides 10 yd in 4 s, then he rides 20 yd in 8 s and 30 yd in 12 s. The time it takes him to ride 25 yd must be between 8 s and 12 s, so my answer of 10 s is reasonable.

5 Why is it sometimes helpful to use a combination of multiplication and division when finding equivalent ratios to solve problems?

Sometimes you cannot get to the ratio you want by multiplying or dividing both quantities by a whole number. Instead, you can combine multiplication and division steps.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Responses will vary. Check student responses.

299

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Model using division and multiplication to find equivalent ratios.

If students are unsure about using equivalent ratios to solve problems, then use this activity to show how both multiplication and division are used to generate equivalent ratios.

Materials For each pair: 30 two-color counters

- Display this problem: *A store owner sells different packs of pencils and pens. The ratio of pencils to pens is always 12 : 8. How many pens would be in a pack of 18 pencils?*
- Have students use counters to represent 12 : 8. Ask: *How does your model represent the situation?* [12 red counters represent 12 pencils. 8 yellow counters represent 8 pens.]
- Ask: *How can you use division to show ratios equivalent to 12 : 8?* [Divide each quantity by 2 to get 6 : 4, or divide each quantity by 4 to get 3 : 2.] Have students show the ratio that they describe by separating their model for 12 : 8 into equal groups.
- Ask: *How can you use your equivalent ratio to find the number of pens for 18 pencils?* [Multiply by 3 or 6 to show how 6 : 4 or 3 : 2 is equivalent to 18 : 12.]
- Ask: *Why is it helpful to first use division to find a ratio equivalent to 12 : 8?* [Dividing finds an equivalent ratio in which the number of pencils is a factor of 18. Then, it is easy to use multiplication to find an equivalent ratio where the number of pencils is 18.]

Apply It

For all problems, encourage students to use a model to support their thinking.

7 Students may also make a table of equivalent ratios.

8 **B is correct.** Students may solve the problem by using a double number line or by using a table of equivalent ratios. Students may also determine that 80 is about 3 times 25, so 3 times 40, or 120, is a good estimate.

A is not correct. This answer is the result of 72 banana muffins subtracted from 96 blueberry muffins to find a difference of 24 muffins, which is then subtracted from the 36 blueberry muffins ($36 - 24 = 12$).

C is not correct. This answer represents placing the numbers in the ratio out of order.

D is not correct. This answer is the result of finding the sum of the Tuesday muffins ($96 + 72 = 168$) and finding the same total for the number of muffins on Wednesday ($36 + 132 = 168$).

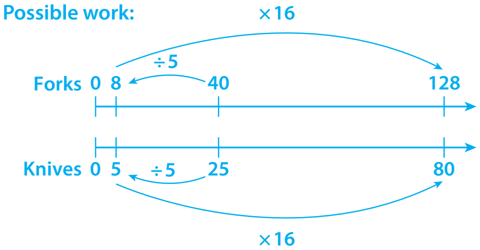
LESSON 13 | SESSION 4

Apply It

➤ **Use what you learned to solve these problems.**

7 A caterer typically uses 40 forks for every 25 knives. The caterer estimates that he will use 80 knives today. Use equivalent ratios to estimate the number of forks the caterer will use today. Show your work.

Possible work:



SOLUTION Possible answer: The caterer will use about 128 forks.

8 Each day, a baker makes the same ratio of blueberry muffins to banana muffins. On Tuesday, she makes 96 blueberry muffins and 72 banana muffins. On Wednesday, she makes 36 blueberry muffins. How many banana muffins does the baker make on Wednesday?

A 12 banana muffins

B 27 banana muffins

C 48 banana muffins

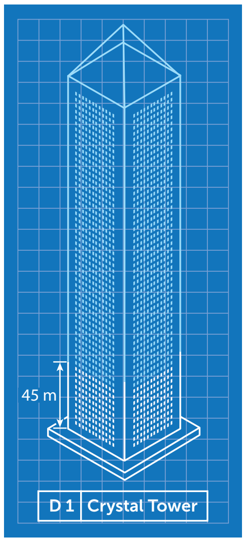
D 132 banana muffins

9 An architect designs a skyscraper with 56 floors. The height of the skyscraper must increase by 45 m for every 10 floors. Based on this ratio, what is the planned height of the skyscraper? Show your work.

Possible work:

	÷ 5		× 28
Floors	10	2	56
Height (m)	45	9	252
	÷ 5		× 28

SOLUTION The planned height is 252 m.



300

CLOSE EXIT TICKET

9 Students' solutions should show an understanding of:

- using double number lines or tables to solve ratio problems.
- using ratio reasoning and multiplication and/or division to find equivalent ratios.

Error Alert If students multiply 56 by 45 to get a solution of 2,520 m, prompt them to list each quantity in the problem and label it with its corresponding units. Point out that they found the height of the skyscraper if there was an increase of 45 m for each floor.

Practice Using Equivalent Ratios

Problem Notes

Assign **Practice Using Equivalent Ratios** as extra practice in class or as homework.

1. Students may also use a table to determine how much the company should charge. ***Medium***
2. Students may use a double number line or table to show that they can divide each quantity in the ratio by 3 and then multiply each quantity in the ratio by 7 to show that the ratio is equivalent. ***Basic***

LESSON 13 | SESSION 4 Name:

Practice Using Equivalent Ratios

➤ **Study the Example showing how to use ratios to solve problems. Then solve problems 1–5.**

Example

A company sells shampoo in two sizes of bottles. The ratio of the capacity of a bottle to its cost is the same for both sizes. A large bottle of shampoo contains 32 fl oz and costs $8. A small bottle contains 12 fl oz. What is the cost of a small bottle of shampoo?

You can use a table of equivalent ratios.

Think of a way to get from 32 to 12 by using a combination of multiplication and division. Then use this combination to find equivalent ratios.

÷ 8 × 3

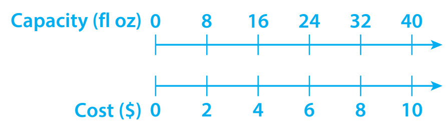
Capacity (fl oz)	32	4	12
Cost ($)	8	1	3

÷ 8 × 3

A small bottle of shampoo costs $3.

1. The company in the Example decides to increase the capacity of its large bottles from 32 fl oz to 40 fl oz. It plans to keep the ratio of capacity to cost the same. How much should the company charge for a bottle that holds 40 fl oz? Show your work.

Possible work:



SOLUTION The company should charge $10 for 40 fl oz.

2. Which ratio is equivalent to 3 : 18?

A 6 : 21

B 5 : 20

C 7 : 42

D 12 : 2

Vocabulary

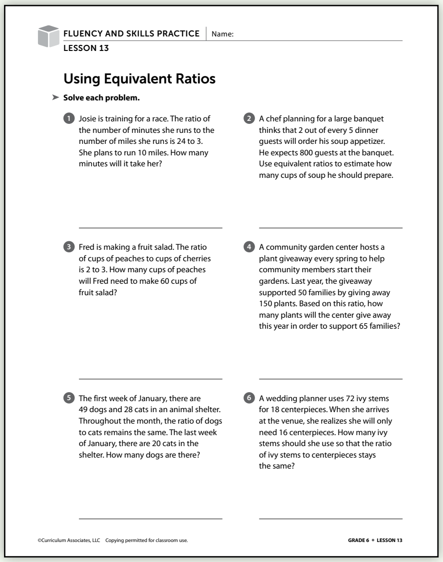
equivalent ratios
two ratios that express the same comparison. Multiplying both numbers in the ratio $a : b$ by a nonzero number n results in the equivalent ratio $na : nb$.

301

Fluency & Skills Practice

Using Equivalent Ratios

In this activity, students use equivalent ratios to solve real-world problems.


FLUENCY AND SKILLS PRACTICE | Name:
LESSON 13

Using Equivalent Ratios

➤ **Solve each problem.**

1. Josie is training for a race. The ratio of the number of minutes she runs to the number of miles she runs is 24 to 3. She plans to run 10 miles. How many minutes will it take her?
2. A chef planning for a large banquet thinks that 2 out of every 5 dinner guests will order his soup appetizer. He expects 800 guests at the banquet. Use equivalent ratios to estimate how many cups of soup he should prepare.
3. Fred is making a fruit salad. The ratio of cups of peaches to cups of cherries is 2 to 3. How many cups of peaches will Fred need to make 60 cups of fruit salad?
4. A community garden center hosts a plant giveaway every spring to help community members start their gardens. Last year, the giveaway supported 50 families by giving away 150 plants. Based on this ratio, how many plants will the center give away this year in order to support 65 families?
5. The first week of January, there are 49 dogs and 28 cats in an animal shelter. Throughout the month, the ratio of dogs to cats remains the same. The last week of January, there are 20 cats in the shelter. How many dogs are there?
6. A wedding planner uses 72 ivy stems for 18 centerpieces. When she arrives at the venue, she realizes she will only need 16 centerpieces. How many ivy stems should she use so that the ratio of ivy stems to centerpieces stays the same?

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Additional Practice

3. Students may recognize that they can divide both 48 and 4 to find the ratio 12 feet for every 1 post, and then multiply both quantities by 250 to find the total number of posts. They may also use a double number line. ***Medium***

4. Students may also use a double number line to find the number of plastic bandages. ***Medium***

5. Students may solve this problem by first recognizing that they need to subtract 2 red tiles from 6 to determine that there are 4 red tiles remaining. Then, find that 4 forms a ratio with 10 and is equivalent to 6 : 15. Finally, subtract 10 yellow tiles from 15 to determine that Lila needs to remove 5 yellow tiles. ***Challenge***

LESSON 13 | SESSION 4

3. A community garden is surrounded by a fence. The total length of the fence is 3,000 ft. For every 48 ft of fence, there are 4 posts. What is the total number of posts in the fence? Show your work.

Possible work:

÷ 4 × 250

Fence Length (ft)	48	12	3,000
Posts	4	1	250

÷ 4 × 250

SOLUTION The fence has 250 posts.

4. A company makes first-aid kits in different sizes. The ratio of fabric bandages to plastic bandages in each kit is 3 to 9. A small kit has 16 fabric bandages. How many plastic bandages should a small kit have? Show your work.

Possible work:

÷ 3 × 16

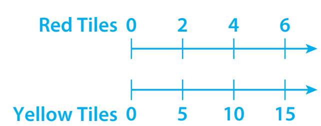
Fabric Bandages	3	1	16
Plastic Bandages	9	3	48

÷ 3 × 16

SOLUTION The kit should have 48 plastic bandages.

5. A bag contains 6 red tiles and 15 yellow tiles. Lilia removes 2 red tiles. How many yellow tiles should she remove so that the ratio of red tiles to yellow tiles in the bag stays equivalent to 6 : 15? Show your work.

Possible work: After Lilia removes 2 red tiles, there are 4 red tiles left. When there are 4 red tiles, there should be 10 yellow tiles.



SOLUTION Lilia should remove 5 yellow tiles.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 5** **Apply It**

Levels 1–3: Reading/Speaking

Use **Three Reads** to help students make sense of Apply It problem 4. Read the problem aloud as students follow along. Help students identify the parts of the problem that will help them answer the question for each read.

After the last read, have students work with a partner to identify and write the ratios in the problem, taking care to include labels for each quantity (for example: *1 pound of chicken for every 5 people*). Then have partners read each statement and identify the ratios that will help them determine if it is true or false. Provide sentence frames.

- *I can use the ratio* _____.
- *I think the statement is* _____ *because* _____.

Levels 2–4: Reading/Speaking

Use **Three Reads** to help students make sense of Apply It problem 4. After the second read, remind students to include the labels when thinking about important quantities. After the last read, ask: *What ratios can you write based on the information provided in the problem?*

Once students have written ratios based on the information provided in the problem, have partners take turns reading each true or false statement. Allow think time for students to consider which of the ratios they wrote corresponds to the statement. Provide a sentence frame to help students justify their thinking:

- *I know this statement is* _____ *because* _____.

Levels 3–5: Reading/Speaking

Have students work with a partner to use **Three Reads** to make sense of Apply It problem 4. After the last read, ask partners to discuss ratios they can write for the problem.

Allow time for students to read and answer each true/false statement individually. Then ask partners to compare answers, taking turns to justify their answers with reasoning. Encourage students to express agreement or disagreement and build on using a sentence frame such as this:

- *I agree because* _____.
- *I disagree because* _____.

Refine Finding Equivalent Ratios

Purpose

- **Refine** strategies for using ratio reasoning to solve multi-step problems.
- **Refine** understanding of using tables, double number lines, or graphs to generate equivalent ratios.

START CHECK FOR UNDERSTANDING

For every 8 raffle tickets a class sells they raise $6 for a class trip.

How much money will the class raise by selling 20 raffle tickets?

Solution

$15

WHY? Confirm students' understanding of finding equivalent ratios in a problem-solving context, identifying common errors to address as needed.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to identify those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.

LESSON 13 | SESSION 5

Refine Finding Equivalent Ratios

➤ **Complete the Example below. Then solve problems 1–9.**

Example

A picture-hanging kit contains 2 short nails for every 8 long nails. There are 28 short nails. How many long nails does the kit contain?

Look at how you could use a table of equivalent ratios.

Short Nails	2	4	6	28
Long Nails	8	16	24	?

In each ratio, the number of long nails is 4 times the number of short nails.

? = 4 × 28

SOLUTION The kit contains 112 long nails.

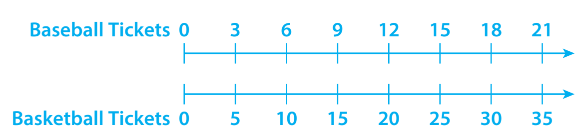
CONSIDER THIS . . . How do you know that the ratios 4 : 16 and 6 : 24 are equivalent to 2 : 8?

PAIR/SHARE How could you solve this problem a different way to check your answer?

Apply It

1 Nicanor keeps the tickets from all the sporting events he attends. The ratio of baseball tickets to basketball tickets in his collection is 3 : 5. Nicanor has 21 baseball tickets. How many more basketball tickets than baseball tickets does he have? Show your work.

Possible work:



Nicanor has 35 basketball tickets.

35 − 21 = 14

SOLUTION He has 14 more basketball tickets than baseball tickets.

CONSIDER THIS . . . How does the ratio given in the problem show that Nicanor has more basketball tickets than baseball tickets?

PAIR/SHARE How did you use equivalent ratios to help you solve this problem?

303

START ERROR ANALYSIS

If the error is . . .	Students may . . .	To support understanding . . .
$18	have added the difference of 20 and 8 to 6.	Ask students to draw a diagram representing 8 tickets and $6. Have them draw lines or circles to divide their diagram into two equal groups, showing 4 tickets and $3. Then, ask students to draw duplicate groups representing 4 tickets and $3 until there is a total of 20 tickets. Prompt students to find the amount of money raised using the diagram and name each equivalent ratio they formed.
$26.67	have used the quantity 20 in the equivalent ratio to represent $20 instead of 20 raffle tickets.	Elicit from students the understanding that the order of the quantities in a ratio is important, particularly when finding equivalent ratios. Ask students to state the quantity in the second ratio that corresponds to the quantity in the first ratio. Ensure that both ratios compare the quantities in the same order, either *dollars to tickets* or *tickets to dollars*.
$30	have multiplied both quantities in the ratio 8 : 6 by 5 to find a number of tickets that is a multiple of 20, but then neglected to divide each quantity by 2.	Ask students to make a table or double number line to compare the amount of money raised to the number of tickets sold. Have students explain the steps in extending their model to show the amount of money raised by selling 20 tickets.

Refine

Example

Guide students in understanding the Example. Ask:

- *How can you show that 2 : 8 is equivalent to 4 : 16?*
- *The number of long nails is how many times the number of short nails?*
- *How can you use the relationship between long and short nails to solve the problem?*

Help all students focus on the Example and responses to the questions by asking them to explain the reasons why their answers make sense.

Look for understanding that the multiplicative relationship between the two quantities can be used to find the number of long nails in the kit.

Apply It

1. Students may also use a table to write equivalent ratios and then subtract to find the difference. *DOK 2*

2. Students may also use the points on the coordinate plane to write a ratio, then divide and multiply to find the number of steps. *DOK 2*

3. **D is correct.** Carson rides 4 miles in 1 day. Divide 80 by 4 to find the number of days to ride 80 miles.

   A is not correct. This answer expresses the number of miles Carson rides in 1 day.

   B is not correct. This answer represents 4 miles for every 1 day added to 3 days.

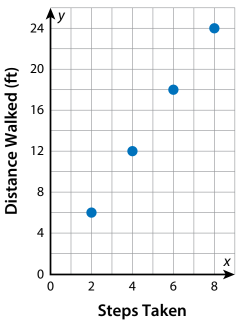
   C is not correct. This answer is the result of dividing 30 by 3 instead of dividing 12 by 3 to find the number of miles Carson rides in 1 day.

   DOK 3

LESSON 13 | SESSION 5

2. The graph shows the relationship between the number of steps Jamila takes and the distance she walks. Based on the equivalent ratios shown in the graph, how many steps does Jamila need to take to walk 120 ft? Show your work.

Jamila's Steps



Possible work:

Steps Taken	Distance Walked (ft)
2	6
4	12
6	18
8	24
?	120

The distance in feet is always 3 times the number of steps.

$120 \div 3 = 40$

SOLUTION Jamila needs to take 40 steps.

CONSIDER THIS . . . How can you show that the ordered pairs in the graph represent equivalent ratios?

PAIR/SHARE How does the graph indicate that the number of steps Jamila must take to walk 120 ft is less than 120?

3. Carson rides his bike for 30 min each day. He rides a total of 12 mi every 3 days. Based on this information, how many days will it take Carson to ride a total of 80 mi on his bike?

   A 4 days

   B 7 days

   C 10 days

   D 20 days

   Galeno chose A as the correct answer. How might he have gotten that answer?

   Possible answer: Galeno found the number of miles Carson rides in 1 day.

CONSIDER THIS . . . How can you find the distance Carson rides in 1 day?

PAIR/SHARE How can you check that your answer is reasonable?

304

GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3.
A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency
- **RETEACH** Hands-On Activity
- **REINFORCE** Problems 5, 8

Meeting Proficiency
- **REINFORCE** Problems 4–8

Extending Beyond Proficiency
- **REINFORCE** Problems 4–8
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket**.

Resources for Differentiation are found on the next page.

Refine Finding Equivalent Ratios

Apply It

4 **a.** The ratio of pounds of beef to people is 3 : 10. For 80 people, Mr. Romano orders 3 × 8, or 24, pounds of beef, not 48 pounds.

b. The ratio of pounds of chicken to people is 1 : 5. The ratio 16 : 80 is equivalent to 1 : 5, because 1 × 16 = 16 and 5 × 16 = 80. So, Mr. Romano orders 16 pounds of chicken.

c. Mr. Romano orders 24 pounds of beef and 16 pounds of chicken. The ratio 24 : 16 is equivalent to 3 : 2.

d. The ratio of pounds of beef to pounds of chicken is 3 : 2. So, the ratio of pounds of chicken to pounds of beef is 2 : 3, not 1 : 3.

DOK 2

5 Students may also find that when Evelyn uses 114 in. of white ribbon, she will have used 76 in. of blue ribbon, meaning she still has blue ribbon left when she uses all of the white ribbon. ***DOK 3***

6 See ***Connect to Culture*** to support student engagement. Students may also make a table of equivalent ratios and use the values in the table to plot the ordered pairs. ***DOK 2***

---

4 Mr. Romano is ordering meat for a family reunion. He knows that 80 people plan to attend. He orders 1 lb of chicken for every 5 people and 3 lb of beef for every 10 people. Tell whether each statement is *True* or *False*.

	True	False
a. Mr. Romano orders 48 lb of beef.	○	●
b. Mr. Romano orders 16 lb of chicken.	●	○
c. The ratio of pounds of beef to pounds of chicken that Mr. Romano orders is 3 : 2.	●	○
d. The ratio of pounds of chicken to pounds of beef that Mr. Romano orders is 1 : 3.	○	●

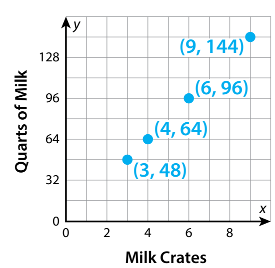
5 Evelyn is making bows from blue and white ribbon. She uses 6 in. of blue ribbon for every 9 in. of white ribbon. Evelyn has 82 in. of blue ribbon and 114 in. of white ribbon. Which color of ribbon will she run out of first? Explain.

White ribbon; Possible explanation:

	$\xrightarrow{\div 3}$	$\xrightarrow{\times 41}$	
Blue Ribbon (in.)	6	2	82
White Ribbon (in.)	9	3	123
	$\xrightarrow{\div 3}$	$\xrightarrow{\times 41}$	

If Evelyn uses 82 in. of blue ribbon, she will need 123 in. of white ribbon. Evelyn only has 114 in. of white ribbon, so she will run out of white ribbon first.

6 A dairy farm ships crates of milk to food stores. There are 48 quarts of milk for every 3 crates shipped. Plot points on the graph to show how many quarts of milk there are for shipments of 3, 4, 6, and 9 milk crates. Label each point with its ordered pair.



305

DIFFERENTIATION

RETEACH

Hands-On Activity

Make a concrete model to visualize equivalent ratios.

Students approaching proficiency with finding equivalent ratios will benefit from building concrete models to represent equivalent ratios.

Materials For each student: 30 two-color counters, Activity Sheet *Double Number Lines*

- Give students 24 two-color counters. Have them display 6 red counters and 10 yellow counters. Ask: *What is the ratio of red counters to yellow counters?* [6 : 10]
- Prompt students to describe how to make a ratio that is equivalent to this ratio but uses fewer counters. [Divide each quantity by 2.]
- Ask: *What is the equivalent ratio?* [3 : 5]
- Prompt students to describe how they can use this equivalent ratio to find the number of red counters when there are 15 yellow counters. [5 yellow counters times 3 equals 15 yellow counters, so multiply 3 red counters by 3 to find 9 red counters.]
- Ask: *How many yellow counters will you need when there are 15 red counters?* Explain. [25 yellow counters; 3 red counters times 5 equals 15 yellow counters, so multiply 5 yellow counters by 5.]
- Have students draw a double number line and then record the equivalent ratios on the double number line.
- Repeat with other examples. For instance, have students display 12 red counters and 9 yellow counters, find an equivalent ratio with fewer counters [4 : 3], and then the number of red counters when there are 12 yellow counters [16].

7 Students need to find an equivalent ratio for 8 : 16 and then calculate the cost of pepper plants. *DOK 2*

8 **C** **is correct.** (5, 24) represents a ratio that is not equivalent to 1 : 6.

A is not correct. This answer represents the ratio 2 : 12, which is equivalent to 1 : 6.

B is not correct. This answer represents the ratio 3 : 18, which is equivalent to 1 : 6.

D is not correct. This answer represents the ratio 6 : 36, which is equivalent to 1 : 6.

DOK 1

CLOSE EXIT TICKET

9 **Math Journal** Look for understanding of how to use an equivalent ratio to solve a problem.

Error Alert If students switch the order of the coordinates when graphing ordered pairs, then remind them that the first number in an ordered pair is the *x*-coordinate and the second number is the *y*-coordinate.

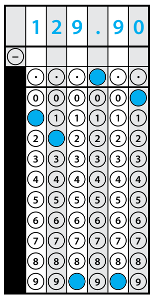
End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting that they use a table to find equivalent ratios for 9 : 4.

SELF CHECK Have students review and check off any new skills on the Unit 3 Opener.

LESSON 13 | SESSION 5

7 Last year, students planted 8 tomato plants and 16 pepper plants in a school garden. This year, the students planted 15 tomato plants. They want to have the same ratio of tomato plants to pepper plants as last year. Pepper plants cost $4.33 each. What is the cost, in dollars, of the pepper plants for this year's garden?



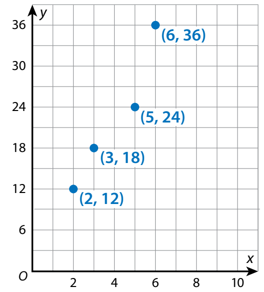
8 The graph shows four ordered pairs that represent ratios. Which ordered pair represents a ratio that is not equivalent to the others?

A (2, 12)

B (3, 18)

C (5, 24)

D (6, 36)



9 **Math Journal** Write a word problem that can be solved by finding an equivalent ratio. Show how to find the answer.

Possible answer: The ratio of green paper clips to silver paper clips in a box is 3 to 8. The box has 60 green paper clips. How many silver paper clips are in the box? Solution:

Green Paper Clips	3	60
Silver Paper Clips	8	160

There are 160 silver paper clips in the box.

End of Lesson Checklist

- [ ] **INTERACTIVE GLOSSARY** Find the entry for *equivalent ratios*. Add two important things you learned about equivalent ratios in this lesson.
- [ ] **SELF CHECK** Go back to the Unit 3 Opener and see what you can check off.

306

REINFORCE

Problems 4–8
Find equivalent ratios.

Students meeting proficiency will benefit from additional work with finding equivalent ratios by solving multi-step problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND

Challenge
Find equivalent ratios with three numbers.

Students extending beyond proficiency will benefit from finding and using equivalent ratios with 3 quantities and 2 ratios.

- Have students work with a partner to solve this problem. *The ratio of triangles to squares is 3 to 5. The ratio of squares to circles is 15 to 12. How many total shapes are there if there are 9 triangles?* [36 total shapes; 9 triangles, 15 squares, and 12 circles]
- Some students will multiply each quantity in the ratio 3 : 5 by 3 to find 9 triangles and 15 squares and use the ratio 15 to 12 to add the number of each shape.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.

Overview | Use Unit Rates to Solve Problems

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

3 Construct viable arguments and critique the reasoning of others.

8 Look for and express regularity in repeated reasoning.

* See page 1q to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Use division to find unit rates.
- Solve unit rate problems, including problems involving constant speed and unit pricing.
- Use unit rates to find unknown values in equivalent ratios when three of four values are given.
- Use unit rates to compare two ratios.
- Use unit rates to convert measurement units.

Language Objectives

- Interpret word problems involving constant speed and unit pricing by analyzing rates and unit rates.
- Explain in writing how a table of equivalent ratios can be used to find a unit rate, solve for unknown quantities, or convert measurement units.
- Discuss ideas about comparing ratios and provide specific reasons when agreeing or disagreeing with a partner's explanation.
- Understand and use *unit rate, convert, rate,* and *constant* when speaking and writing.

Prior Knowledge

- Understand the concept of ratios and rates.
- Find the two rates associated with a ratio relationship.
- Generate and identify equivalent ratios.
- Solve ratio problems to find unknown quantities.
- Find missing values in a table of equivalent ratios and use tables to compare ratios.

Vocabulary

Math Vocabulary

unit rate the numerical part of a rate. For example, the rate 3 miles per hour has a unit rate of 3. For the ratio $a : b$, the unit rate is the quotient $\frac{a}{b}$.

Review the following key terms.

convert to write an equivalent measurement using a different unit.

rate a ratio that tells the number of units of one quantity for 1 unit of another quantity. Rates are often expressed using the word *per*, such as 5 miles per hour or 2 cups per serving.

Academic Vocabulary

constant staying the same.

Learning Progression

Earlier in Grade 6, students generated and identified equivalent ratios and used ratio reasoning to solve problems. They used tables of equivalent ratios to find missing values and solve problems.

In the previous lesson, students built on their work with equivalent ratios and double number lines and tables to consider a rate—a special type of ratio that compares a quantity to 1 of another quantity.

In this lesson, students learn to call the numerical part of a rate a *unit rate* and find unit rates by dividing the two numbers in a ratio. They recognize that equivalent ratios have the same unit rate and that a unit rate is a factor that relates the two quantities in a ratio relationship. Students start to move away from relying on tables and double number lines to find equivalent ratios and compare ratios. They use efficient strategies to determine a better deal, faster speed, or make other comparisons.

Later in Grade 6, students will extend their reasoning with ratios and rates to solve percent problems. They will use equations and graphs to represent two quantities that change in relation to one another.

In Grade 7, students will compute unit rates associated with ratios that include fractions. They will also reason about proportional relationships to solve real-world problems.

Pacing Guide

Items marked with are available on the **Teacher Toolbox**.

	MATERIALS	DIFFERENTIATION
SESSION 1 **Explore** Unit Rates (35–50 min)		
• **Start** (5 min) • **Try It** (5–10 min) • **Discuss It** (10–15 min) • **Connect It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 361–362)	**Math Toolkit** double number lines, graph paper Presentation Slides	**PREPARE** Interactive Tutorial **RETEACH or REINFORCE** Visual Model **Materials** For display: Activity Sheet *1-Centimeter Grid Paper*
SESSION 2 **Develop** Using Unit Rates to Find Equivalent Ratios (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 367–368)	**Math Toolkit** double number lines, graph paper Presentation Slides	**RETEACH or REINFORCE** Visual Model **Materials** For each pair: Activity Sheet *Double Number Lines* **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 3 **Develop** Using Unit Rates to Compare Ratios (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 373–374)	**Math Toolkit** double number lines, graph paper Presentation Slides	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: 35 counters, 8 small paper cups, markers **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 4 **Develop** Using Unit Rates to Convert Measurements (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 379–380)	**Math Toolkit** double number lines, graph paper, ruler Presentation Slides	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: 5 sticky notes or index cards **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 5 **Refine** Using Unit Rates to Solve Problems (45–60 min)		
• **Start** (5 min) • **Monitor & Guide** (15–20 min) • **Group & Differentiate** (20–30 min) • **Close: Exit Ticket** (5 min)	**Math Toolkit** Have items from previous sessions available for students. Presentation Slides	**RETEACH** Hands-On Activity **Materials** For each group: 15 sticky notes **REINFORCE** Problems 4-8 **EXTEND** Challenge **PERSONALIZE** i-Ready
Lesson 16 Quiz or **Digital Comprehension Check**		**RETEACH** Tools for Instruction **REINFORCE** Math Center Activity **EXTEND** Enrichment Activity

Overview | Use Unit Rates to Solve Problems

Connect to Culture

➤ **Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.**

SESSION 1 ■ □ □ □ □

Try It The speed limit is the maximum legal speed that a vehicle can drive on the road. In 1901, Connecticut was the first state to adopt a speed limit law. The maximum speed was 12 miles per hour (mph) on city roads and 15 mph on country roads. At that time, driving was not nearly as safe as it is now. Cars had no seatbelts or lights. Roads did not have stop signs or lines distinguishing separate lanes. As cars and roads have become safer, speed limits have risen. Ask students to generate a list of pros and cons for raising the speed limit on roads in their community.

SESSION 2 ■ ■ □ □ □

Try It Encourage students who run to share their experiences. The great thing about running is that you can start out as slow or as fast as you want. You can run alone or with friends. Many communities have running programs and races for people of all ages. You can also run when playing sports or games. Ask students to share what sports or games they like to play that involve running.

SESSION 3 ■ ■ ■ □ □

Try It Blowing bubbles is a fun activity for people of all ages. You can make bubble solution using dish soap and water. Bubbles pop when the water molecules evaporate. When it is cold outside, it takes longer for the molecules to evaporate. A bubble can even freeze if it is cold enough. Glycerin and corn syrup slow the evaporation down, causing the bubbles to last longer. The world record for the largest free-floating bubble was set by John Erck in 2005. The bubble was 105.4 cubic feet in volume. It was big enough to hold 788 gallons of water, or 13,627 baseballs. If time allows, students can research more bubble world records.

SESSION 4 ■ ■ ■ ■ □

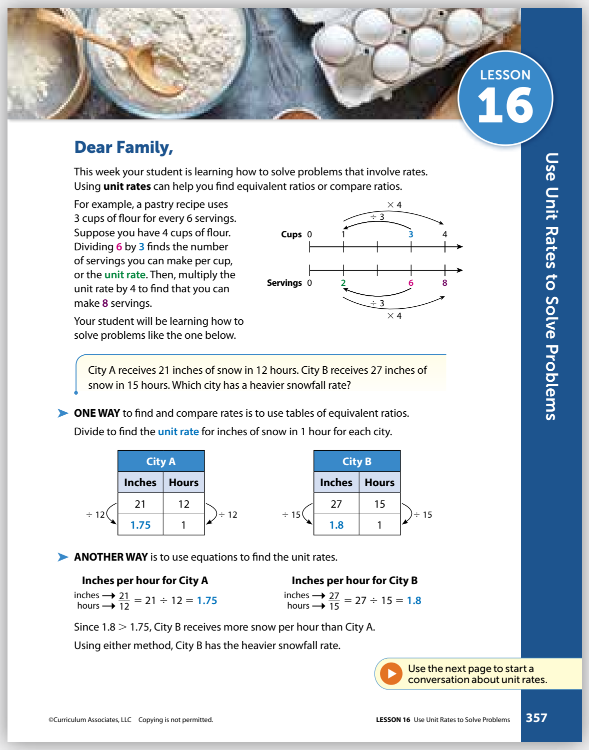
Try It The African American Day Parade is held the third Sunday in September in New York City. Since 1968, the parade has promoted "unity, integrity, and excellence among African Americans." People throughout the United States celebrate African Americans and their accomplishments. Community and political leaders and celebrities march alongside organizations, bands, and dance groups to celebrate African American culture. Have students research parades or other cultural celebrations in their community.

SESSION 5 ■ ■ ■ ■ ■

Apply It **Problem 1** Riding bikes is a great way to get exercise and is a clean form of transportation. You develop strength and balance when you ride a bike. Two common types of bikes are road bikes and mountain bikes. Road bikes are designed with smooth, skinny tires and light frames, which allow for smooth and fast rides on pavement. Mountain bikes are designed with wide, knobby tires and shock absorbers for riding trails with rough terrain. Survey students to see if they enjoy or think they would enjoy riding bikes. If not, ask students to share what other healthy activities they enjoy.

Connect to Family and Community

After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.



LESSON 16

Use Unit Rates to Solve Problems

Dear Family,

This week your student is learning how to solve problems that involve rates. Using **unit rates** can help you find equivalent ratios or compare ratios.

For example, a pastry recipe uses 3 cups of flour for every 6 servings. Suppose you have 4 cups of flour. Dividing 6 by 3 finds the number of servings you can make per cup, or the **unit rate**. Then, multiply the unit rate by 4 to find that you can make 8 servings.

Your student will be learning how to solve problems like the one below.

City A receives 21 inches of snow in 12 hours. City B receives 27 inches of snow in 15 hours. Which city has a heavier snowfall rate?

ONE WAY to find and compare rates is to use tables of equivalent ratios. Divide to find the unit rate for inches of snow in 1 hour for each city.

City A	
Inches	Hours
21	12
1.75	1

City B	
Inches	Hours
27	15
1.8	1

ANOTHER WAY is to use equations to find the unit rates.

Inches per hour for City A

$\frac{21}{12} = 21 \div 12 = 1.75$

Inches per hour for City B

$\frac{27}{15} = 27 \div 15 = 1.8$

Since $1.8 > 1.75$, City B receives more snow per hour than City A.

Using either method, City B has the heavier snowfall rate.

Use the next page to start a conversation about unit rates.

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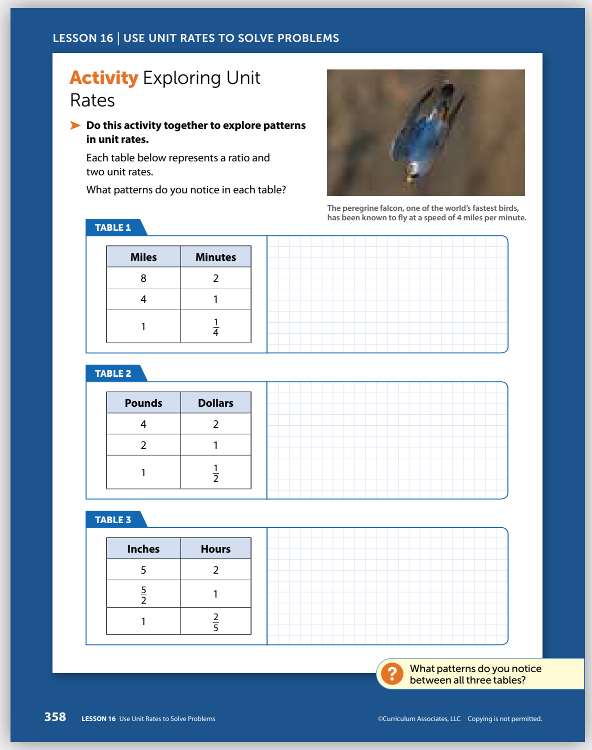


LESSON 16 | USE UNIT RATES TO SOLVE PROBLEMS

Activity Exploring Unit Rates

Do this activity together to explore patterns in unit rates.

Each table below represents a ratio and two unit rates.

What patterns do you notice in each table?

The peregrine falcon, one of the world's fastest birds, has been known to fly at a speed of 4 miles per minute.

TABLE 1

Miles	Minutes
8	2
4	1
1	$\frac{1}{4}$

TABLE 2

Pounds	Dollars
4	2
2	1
1	$\frac{1}{2}$

TABLE 3

Inches	Hours
5	2
$\frac{5}{2}$	1
1	$\frac{2}{5}$

What patterns do you notice between all three tables?

358 LESSON 16 Use Unit Rates to Solve Problems ©Curriculum Associates, LLC Copying is not permitted.

Connect to Language

For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS — Use with **Session 1** **Connect It**

Levels 1–3: Reading/Speaking

Support students as they interpret and discuss the table in Connect It problems 2c–d. Direct students' attention to the table. Review the difference between columns and rows. Point to the fourth row of the table as you read the first sentence of problem 2c aloud. Then read the rest of the problem. Ask: *If Chloe's rate is 60 miles for every 1 hour, where should the number 60 go?* [first column, first row] Have students talk with a partner about any patterns they notice in the first two columns.

For problem 2d, review the term *quotient*. Have students complete the table and then discuss the pattern in the third column with a partner.

Levels 2–4: Reading/Speaking

Support students as they interpret and discuss the table in Connect It problems 2c–d. Before reading problems 2c–d, use **Notice and Wonder** to help students make sense of the table. Record their thoughts, and then read problem 2c. Review the meaning of *unit rate* by referring to Look Ahead and the Interactive Glossary. Discuss things the students noticed and wondered that are relevant to the problem, and have students complete the table individually.

For problem 2d, read the problem aloud. Then have partners **Say It Another Way**. After the students complete all sections of the chart, revisit the ideas from **Notice and Wonder** and place a check mark by ideas that were helpful or relevant to solving the problems.

Levels 3–5: Reading/Speaking

Support students as they interpret and discuss Connect It problems 2c–d. Read each problem separately, and then have students work with a partner to **Say It Another Way**.

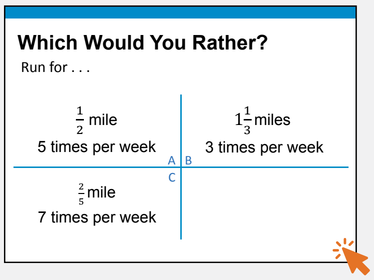
Allow time for students to complete the table independently, and then adapt **Notice and Wonder** by asking partners to look at the completed charts and make a list of things they notice and wonder. Call on volunteers to share the most important thing their partner noticed or wondered.

Explore Unit Rates

Purpose

- **Explore** the idea that a rate contains information about equivalent ratios.
- **Understand** that equivalent ratios have the same unit rate.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

A because it is the least mileage per week.

B because it is the fewest number of times to run.

C because it is healthy to exercise every day.

WHY? Support students' understanding of comparing ratios and rates.

TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Say It Another Way** to help them make sense of the problem. Listen for understanding that Chloe is driving at a constant speed and wants to know if she can reach her destination in less than $3\frac{1}{2}$ hours.

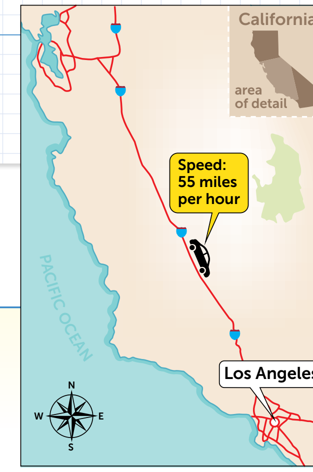
DISCUSS IT SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding of:

- 200 miles is the distance Chloe needs to travel, and 55 miles per hour is her constant speed.
- the question asks if Chloe can drive the distance in less than $3\frac{1}{2}$ hours.
- the need to find the distance Chloe can drive in the given time and compare it with the distance from where she is to Los Angeles.

Explore Unit Rates



Previously, you learned about rates. In this lesson, you will learn how to use rates and unit rates to solve problems.

Use what you know to try to solve the problem below.

Chloe is driving on the freeway. She is 200 miles from Los Angeles. She drives at a constant speed of 55 miles per hour. Can Chloe get to Los Angeles in less than $3\frac{1}{2}$ hours?

TRY IT **Math Toolkit** double number lines, graph paper

Possible work:

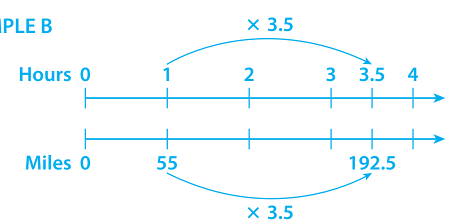
SAMPLE A

Hours	1	2	3	$\frac{1}{2}$	$3\frac{1}{2}$
Miles	55	110	165	$27\frac{1}{2}$	?

$165 + 27\frac{1}{2} = 192\frac{1}{2}$

In $3\frac{1}{2}$ h, Chloe goes $192\frac{1}{2}$ mi. She can't get to Los Angeles in less than $3\frac{1}{2}$ h.

SAMPLE B



It will take Chloe longer than $3\frac{1}{2}$ hours to get to Los Angeles.

DISCUSS IT

Ask: How is your strategy similar to mine? How is it different?

Share: My strategy is similar to yours because . . . It is different because . . .

Learning Targets SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6, SMP 8

- Solve unit rate problems including those involving unit pricing and constant speed.
- Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

359

Common Misconception Listen for students who identify $192\frac{1}{2}$ miles as the distance Chloe can travel in $3\frac{1}{2}$ hours but conclude that since the distance is less than 200, the time to reach the destination would also be less. As students share their strategies, ask them to apply their reasoning to explain the steps they used to solve the problem.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- tables of equivalent ratios that show the number of miles traveled each hour and half hour when moving at a constant speed of 55 miles per hour
- (misconception) strategies that identify the distance of $192\frac{1}{2}$ miles in $3\frac{1}{2}$ hours but conclude that since the distance is less is less than 200 miles, the time to reach the destination would be less
- double number lines that show the number of miles traveled in $3\frac{1}{2}$ hours when traveling at a constant speed of 55 miles per hour
- equations that find the number of miles Chloe can travel in $3\frac{1}{2}$ hours when traveling at a constant speed of 55 miles per hour

Facilitate Whole Class Discussion

Call on students to share selected strategies. Prompt students to describe what they noticed or assumed about the problem, what they decided to do as a result, and why.

Guide students to **Compare and Connect** the representations. Allow time for students to think by themselves before starting the discussion.

ASK *How do the strategies use the rate in the problem?*

LISTEN FOR The ratio table in the first strategy uses the fact that Chloe drives 55 miles in 1 hour to determine how far she can drive in 3 hours and in $\frac{1}{2}$ hour. The double number line in the second strategy shows that both 1 and 55 are multiplied by 3.5 to determine how many miles Chloe can travel in 3.5 hours.

CONNECT IT

SMP 2, 4, 5

1 Look Back Look for understanding that the rate is used to find the distance in miles Chloe can travel in $3\frac{1}{2}$ hours. This distance is then compared with 200 miles to answer the question.

DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Use bar models with rates.

If students are unsure about how to use rates to solve problems, then use this activity to show how bar models can represent rate situations.

Materials For display: Activity Sheet: *1-Centimeter Grid Paper*

- Draw a bar that is 2 squares long on grid paper. Tell students the bar represents the distance Chloe travels in one hour.
- Ask: *How far will Chloe travel in 1 hour? How do you know?* [55 miles; She is traveling at a constant speed of 55 miles per hour.]
- Have a volunteer add a bar to show the number of miles Chloe travels in 2 hours. Ask: *How many squares show 2 hours? What distance does it represent?* [4 squares; 110 miles] Repeat for the miles traveled in 3 hours. [6 squares; 165 miles]
- Ask: *What will you draw next to represent $3\frac{1}{2}$ hours? Why?* [Draw a bar 1 square long because it represents half an hour.] *How can you find the distance this bar represents?* $\left[55 \times \frac{1}{2} = 27\frac{1}{2}\right]$ *How do you find the total distance traveled?* $\left[165 + 27\frac{1}{2} = 192\frac{1}{2}\right]$

LESSON 16 | SESSION 1

CONNECT IT

1 Look Back Can Chloe get to Los Angeles in less than $3\frac{1}{2}$ hours? Explain.

No; Possible explanation: At a rate of 55 miles per hour, Chloe travels $192\frac{1}{2}$ mi in $3\frac{1}{2}$ h. Since Los Angeles is 200 mi away, Chloe cannot get there in less than $3\frac{1}{2}$ h.

2 Look Ahead Chloe's constant speed of 55 miles per hour is a rate. The numerical part of the rate, 55, is called the **unit rate**.

a. What does the unit rate 55 tell you in this situation?

Chloe drives 55 miles in 1 hour.

b. On another trip, Chloe drives at a constant speed of 60 miles per hour. What is Chloe's unit rate? What does the unit rate tell you?

60; Chloe drives 60 miles in 1 hour.

c. The table shows that Choe travels 240 miles in 4 hours. Complete the equivalent ratios in the first two columns. Where do you see Chloe's unit rate?

See first two columns of table; The unit rate is the number 60 in the first row, which shows the rate 60 miles in 1 hour.

d. The third column of the table shows the quotient of the numbers in each equivalent ratio. Complete the third column. What do you notice?

See table; Possible answer: Dividing the two numbers in each equivalent ratio results in the same unit rate, 60.

Miles, a	Hours, b	$\frac{a}{b} = a \div b$
60	1	$\frac{60}{1} = 60 \div 1 = 60$
120	2	$\frac{120}{2} = 120 \div 2 = 60$
180	3	$\frac{180}{3} = 180 \div 3 = 60$
240	4	$\frac{240}{4} = 240 \div 4 = 60$
300	5	$\frac{300}{5} = 300 \div 5 = 60$

3 Reflect How could you use unit rates to help you identify equivalent ratios?

Possible explanation: Find the unit rate for each ratio by dividing the numbers in the ratio. The ratios are equivalent ratios if they have the same unit rate.

360

2 Look Ahead Point out that a unit rate is constant. Students should understand that the given ratio of miles to hours, 240 : 4, is equivalent to 60 : 1, so 60 is the unit rate. As students complete the table, they should recognize that when you divide the pairs of numbers in equivalent ratios, the result is the same unit rate.

Ask a volunteer to rephrase the definition of *unit rate*. Support student understanding by discussing another common rate, such as an hourly wage. A wage of $15 per hour has a unit rate of 15.

CLOSE | EXIT TICKET

3 Reflect Look for understanding of how to use a given rate to generate a series of equivalent ratios, as well as understanding that a unit rate is the same for equivalent ratios.

Common Misconception If students divide the units in the wrong order to find and compare unit rates, then remind them to look for the word *per* in the problem. The quantity that comes before *per* is typically the dividend and the quantity that comes after is typically the divisor. With the constant speed 55 miles per hour, 55 is the dividend and 1 is the divisor.

Prepare for Using Unit Rates to Solve Problems

Support Vocabulary Development

Assign **Prepare for Using Unit Rates to Solve Problems** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *rate*. Connect the words *rate* and *ratio*, eliciting from students that a rate is a specific type of ratio that compares a quantity to 1 unit of another quantity. Provide support as needed, helping students use previous knowledge of ratios, including writing and interpreting ratios, identifying and generating equivalent ratios, and using ratios to solve problems to guide their thinking.

Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers, and prompt a whole-class comparative discussion of the definition, prior knowledge, and examples given.

Have students look at the double number line in problem 2 and discuss with a partner how the labels on each number line can be used to help write ratios. Encourage students to use the placement of the number lines to write one rate, gallons per mile, and then discuss how they can write a second rate using the quantities from the double number line.

Problem Notes

1. Students should understand that a rate is a ratio that tells the number of units of one quantity for 1 unit of the other quantity. Student responses may include rates involving miles per gallon or miles per hour because those are familiar rates to many students. Students should recognize that two rates can be written for any ratio relationship, since either number in a ratio can be 1.

2. Students should recognize that the two rates can be determined by finding the number of units of the one quantity for 1 unit of the other quantity. Remind students to keep the units with the quantity, such as 30 miles per gallon and $\frac{1}{30}$ gallon per mile.

LESSON 16 | SESSION 1   Name:

Prepare for Using Unit Rates to Solve Problems

1. Think about what you know about rates. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

Possible answers:

What Is It?

A rate is a ratio that tells you the number of units of one quantity for 1 unit of the other quantity.

What I Know About It

You can write a rate using the word *per*.

Since either number in a ratio can be 1, you can write two rates for any ratio.

rate

Examples

Ratio:

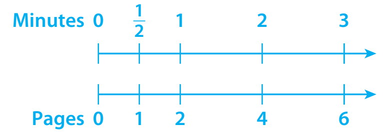
120 ft every 2 min

Rates:

60 feet per minute

$\frac{1}{60}$ minute per foot

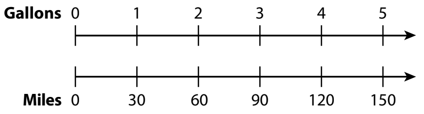
Examples



I read 2 pages per minute.

I take $\frac{1}{2}$ minute to read 1 page.

2. What two rates can you write for the ratios shown by the double number line? What do they tell you?



30 miles per gallon, $\frac{1}{30}$ gallon per mile;

You can drive 30 mi with 1 gal of gas; it takes $\frac{1}{30}$ gal of gas to drive 1 mi.

361

REAL-WORLD CONNECTION

Builders need to budget the costs for new projects before beginning the physical work. The National Association of Home Builders keeps records on the average cost per square foot for homes across the United States. The cost per square foot is calculated by dividing the cost of the land, materials, and labor used to build the home by the number of square feet of the home. The largest factor that influences a home's cost per square foot is the value of the land. For example, a home in a major metropolitan area may have a higher land value than a home in a rural area. Ask students to think of other real-world examples when examining rates might be useful.

3 Problem 3 provides another look at using a given unit rate to generate a series of equivalent ratios. This problem is similar to the problem about Chloe driving on the freeway to Los Angeles. In both problems, a rate is given and then used to analyze a situation. This problem asks for students to apply the rate to a real-world scenario that compares the amount of dog food in the new bag with the amount of food needed to feed the dog for a set period of time.

Students may want to use tables of equivalent ratios or double number lines to solve.

Suggest that students use **Say It Another Way**, asking themselves the following questions to help promote understanding:

- *How could you paraphrase the text?*
- *Is the paraphrase complete and accurate? If it is not, what is inaccurate or missing?*

3 Deon feeds his Great Dane 62 cups of dog food per week. He has a new bag with 160 cups of dog food.

a. Deon will pick up more dog food at the pet store in $2\frac{1}{2}$ weeks. Will the new bag of food last until then? Show your work.

Possible work:

Weeks	1	2	$2\frac{1}{2}$	3
Cups	62	124	?	186

$2\frac{1}{2}$ is halfway between 2 and 3, so ? is halfway between 124 and 186.

$186 - 124 = 62$

$\frac{1}{2} \times 62 = 31$

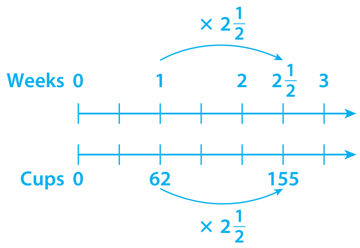
$124 + 31 = 155$

Deon will use 155 cups in $2\frac{1}{2}$ weeks.

SOLUTION Yes; the bag will last for $2\frac{1}{2}$ weeks.

b. Check your answer to problem 3a. Show your work.

Possible work:



Deon needs 155 cups for $2\frac{1}{2}$ weeks, so the new bag of food will be enough.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** Connect It

Levels 1–3: Writing/Speaking

Facilitate discussion about Connect It problem 2 and prepare students to construct a written response.

Ask students to turn and talk with a partner to review the steps as described in Connect It problem 1. Invite a volunteer to share the steps. Listen to confirm understanding that the unit rate was found by dividing the number of minutes by the number of laps. Next, support students as they discuss and write what a fraction represents using the following sentence frames:

- *To find unit rate, you ____.*
- *In this problem, we divided ____ by ____.*
- *A fraction represents ____.*

Levels 2–4: Writing/Speaking

Facilitate discussion about Connect It problem 2 and prepare students to construct a written response.

Have students refer to the table in the first Model It and review the steps used to find the unit rate that they described in Connect It problem 1. Provide a sentence starter:

- *You can find a unit rate by ____.*

Have students work in pairs to identify the unit rate from the table and show it as a fraction. Then ask students to write about the relationship:

- *A fraction can represent a unit rate because ____.*

Levels 3–5: Writing/Speaking

Have students talk with a partner about Connect It problem 2 to prepare to construct a written response.

Have students turn to a partner to discuss how fractions relate to unit rate. As partners discuss, encourage them to make a list of math terms that they use or hear. Lists might include terms such as: *fraction, division, unit rate, ratio, numerator,* and *denominator*.

After adequate discussion time, compile the lists into a class word bank. Then allow students to draft responses independently. Next, ask partners to compare their writing and discuss by providing reasons they agree or disagree.

Develop Using Unit Rates to Find Equivalent Ratios

Purpose

- **Develop** strategies for using unit rates to find an unknown quantity in an equivalent ratio.
- **Recognize** that you can solve ratio problems by dividing numbers in a ratio to find the unit rate and using the unit rate as a multiplier.

START CONNECT TO PRIOR KNOWLEDGE

Which One Doesn't Belong?

A: $171 \div 9$	B: $3 \div 15$
C: $39 \div 2.5$	D: $\frac{1}{8} \times 42$

Possible Solutions

A is the only expression that results in a whole number.

B is the only expression that results in a number less than 1.

C is the only expression that includes a decimal.

D is the only expression that is a product.

WHY? Support students' ability to evaluate expressions with decimals and fractions.

DEVELOP ACADEMIC LANGUAGE

WHY? Clarify the relationships of *ratio, rate,* and *unit rate.*

HOW? Explain that in math, these terms have precise meanings that may not be used in everyday language. Record as volunteers read aloud definitions from the Interactive Glossary, in this order: ratio, rate, unit rate. Using $6 for 2 tickets as an example, discuss the differences between a ratio and a rate and then a rate and a unit rate. Repeat with 5 hours for 3 movies. Post the chart for reference.

TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Three Reads** to help them make sense of the problem. After the third read, have them turn to a partner and answer: *What are the important quantities and relationships in the problem?*

LESSON 16 | SESSION 2

Develop Using Unit Rates to Find Equivalent Ratios

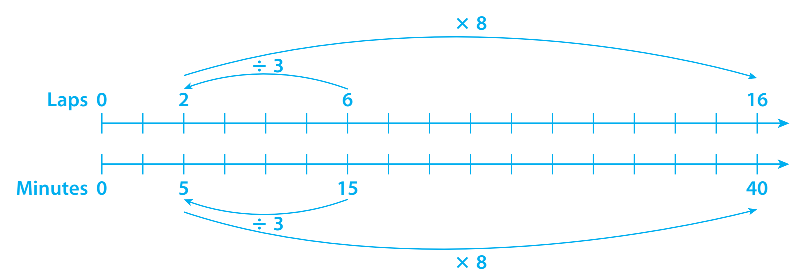
➤ **Read and try to solve the problem below.**

Ashwini jogs on the track at her school. She uses a watch to track her progress. At this rate, how long will it take her to jog 16 laps?

TRY IT Math Toolkit double number lines, graph paper

Possible work:

SAMPLE A



It will take Ashwini 40 min to jog 16 laps.

SAMPLE B

Divide the number of minutes by the number of laps to find the number of minutes per lap.

$15 \div 6 = 2.5$

Multiply the number of laps by the number of minutes per lap.

$16 \times 2.5 = 40$

It will take Ashwini 40 min to jog 16 laps.

DISCUSS IT

Ask: How does your model show Ashwini's rate?

Share: My model shows Ashwini's rate . . .

363

DISCUSS IT SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding that:

- the given ratio of minutes to laps is 15 to 6.
- the ratio can be used to calculate the unit rate at which Ashwini jogs.
- once the unit rate is known, it can be used to determine the time it will take for Ashwini to jog any number of laps.

Error Alert Listen for students who use models with incorrect units for a rate, such as writing minutes instead of minutes per lap or writing the units in the wrong order compared to the quantities. As students share their strategies, encourage them to always include the units with the quantities to make sure the quantities are precisely represented.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- double number lines to represent the problem
- a table of equivalent ratios
- equations that show dividing the number of minutes, 15, by the number of laps, 6, to find the number of minutes per lap, and then multiplying the rate by 16 laps

Facilitate Whole Class Discussion

Call on students to share selected strategies. Review that one way to justify a solution is to try to convince others that the answer makes sense.

Guide students to **Compare and Connect** the representations. Allow time for students to think by themselves before starting the discussion.

ASK *How do [student name]'s and [student name]'s models show Ashwini's rate?*

LISTEN FOR Representations may show the rate expressed as $2\frac{1}{2}$ (or 2.5) minutes per lap. Or, the model may show the rate expressed as $\frac{2}{5}$ (or 0.4) lap per minute.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK *How are the models the same? How are they different?*

LISTEN FOR Both use tables of equivalent ratios. The first Model It uses a table of equivalent ratios of laps to minutes to solve. The second Model It uses an equation to first find the rate for minutes per lap and then displays the information in a table of equivalent ratios, multiplying the number of laps by the unit rate.

For the equivalent ratios table model, prompt students to explain how to find equivalent ratios.

- *Why are the values in the first row divided by 6? Why are the values in the second row multiplied by 16?*

For the equations model, prompt students to think about how a fraction is used to find the unit rate.

- *How do you know which is the numerator and which is the denominator?*

➤ Explore different ways to understand how to use a unit rate to find equivalent ratios.

Ashwini jogs on the track at her school. She uses a watch to track her progress. It takes her 15 minutes to jog 6 laps. At this rate, how long will it take her to jog 16 laps?

Model It

You can use a table of equivalent ratios to solve the problem.

Laps	Minutes
6	15
1	2.5
16	?

÷ 6 ÷ 6
× 16 × 16

Model It

You can find the unit rate and then use it to find equivalent ratios.

Divide the numbers in the ratio 15 : 6 to find the unit rate for minutes per lap.

$$\frac{\text{minutes} \rightarrow 15}{\text{laps} \rightarrow 6} = \frac{5}{2} = 2.5$$

Multiply the number of laps by the unit rate for minutes per lap.

Laps	Minutes
6	15
16	?

× 2.5

DIFFERENTIATION | EXTEND

SMP 8

Deepen Understanding
Using Reasoning to Find and Use Unit Rates

Prompt students to look for the relationships between quantities in a ratio and use fractions and division to find unit rates.

ASK *What fractions can you write with 6 and 15 to show unit rates in this situation? Write each unit rate as a decimal and explain what the unit rate means in this situation.*

LISTEN FOR You can write the fractions $\frac{6}{15}$ and $\frac{15}{6}$. The unit rate 0.4 means 0.4 lap per minute. The unit rate 2.5 means 2.5 minutes per lap.

ASK *Why can you multiply 16 by a unit rate to find the minutes it takes to jog 16 laps?*

LISTEN FOR The unit rate of 2.5 represents 2.5 minutes per lap. Since the number of minutes is unknown, you can find an equivalent ratio for 16 laps by multiplying 16 by the unit rate.

Develop Using Unit Rates to Find Equivalent Ratios

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about how to use rates and unit rates to find an unknown quantity.

Before students begin to record and expand on their work in Model It, tell them that problem 4 will prepare them to provide the description asked for in problem 5. To help students collect their ideas, ask them to turn and talk about all of the models and strategies used in today's lesson.

Monitor and Confirm Understanding 1 – 3

- The unit rate is calculated by dividing each quantity in the ratio by the number of laps.
- A fraction represents division, with the number of minutes in the numerator and the number of laps in the denominator.
- The unit rate is multiplied by the number of laps to find the number of minutes.

Facilitate Whole Class Discussion

4 Look for understanding that unit rate can be used to find the number of minutes it takes to jog any number of laps, or the number of laps that can be jogged in any number of minutes.

ASK *What is one way to find a unit rate from a given ratio?*

LISTEN FOR The unit rate of 2.5 means that it takes 2.5 minutes to run 1 lap, so multiplying any number of laps by 2.5 tells the time for that number of laps.

5 Look for understanding that the unit rate is a factor that relates two quantities in a ratio.

ASK *How are the two quantities in a ratio related?*

LISTEN FOR One quantity in a ratio can be divided by the other to find the unit rate.

ASK *How does a unit rate of a given ratio relate to the unit rate of equivalent ratios?*

LISTEN FOR Equivalent ratios have the same unit rate. So you can multiply or divide by the unit rate to find the missing quantity in an equivalent ratio.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to use a unit rate to find equivalent ratios.**

1 Look at the table in the first **Model It**. Where do you see the unit rate for the ratio of minutes to laps? Describe how the unit rate is found.

The unit rate is the number 2.5 in the second row. Dividing 6 and 15 by 6 finds the rate 2.5 minutes for 1 lap, so the unit rate is 2.5.

2 Look at the second **Model It**. Why can you use a fraction to show the unit rate for the ratio of minutes to laps?

You divide the numbers in a ratio to find the unit rate. A fraction shows the division of the numerator by the denominator.

3 Write a multiplication expression that uses the unit rate to find the missing value of the equivalent ratio. How long will it take Ashwini to jog 16 laps?

16×2.5; It will take Ashwini 40 minutes to jog 16 laps.

4 How long will it take Ashwini to jog 22 laps? Explain how you can use the unit rate to find the total number of minutes it takes Ashwini to run any number of laps.

55 minutes; Possible explanation: The unit rate is 2.5, so the number of minutes is always 2.5 times the number of laps.

5 How does a unit rate relate the two quantities in a ratio? How can a unit rate help you solve problems involving equivalent ratios?

A unit rate is a factor that relates the two quantities in a ratio; You can multiply or divide by a unit rate to find a missing value of an equivalent ratio.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to use a unit rate to find equivalent ratios.

Responses will vary. Check student responses.

365

DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Use a double number line to show using the unit rate as a constant.

If students are unsure about using the unit rate to find an unknown quantity in an equivalent ratio, then use this activity to show the pattern generated by a unit rate.

Materials For each pair: Activity Sheet *Double Number Lines*

- Distribute the Activity Sheet. Have pairs label the top line *Laps* and draw tick marks from 1 to 22 by ones. Label the bottom line *Minutes* and draw tick marks in the same locations.
- Ask: *How many minutes does it take Ashwini to jog one lap?* [2.5 minutes] Have pairs label the first tick mark on the *Minutes* number line with 2.5. *How many minutes will it take Ashwini to jog two laps? How do you know?* [5 minutes; Multiply the unit rate by 2: $2.5 \times 2 = 5$.] *Three laps?* [7.5 minutes; Multiply the unit rate by 3: $2.5 \times 3 = 7.5$.] Have pairs label the *Minutes* number line with 5 and 7.5.
- Ask: *How can you use the unit rate to find the number of minutes it takes to jog 22 laps?* [Multiply the unit rate by 22.]
- Extend by finding the number of minutes it takes to jog 32 laps and 40 laps.

Apply It

For all problems, encourage students to use a model to support their thinking.

7 Students may also use the unit rate 0.8 or $\frac{4}{5}$ to find the pounds per dollar. Then they can divide 7 pounds by the unit rate to find Alejandro's cost.

8 Students may use the rate for dollars per pound, $\frac{5}{4}$ or 1.25, to solve: $8 \div 1.25 = 6.4$.

Apply It

➤ **Use what you learned to solve these problems.**

7 Alejandro is buying chicken for a barbecue. At the rate shown in the Weekly Special, what does 7 lb of chicken cost? Show your work.

Possible work:

Pounds	Dollars
4	5
7	?

$\times 1.25$

$\frac{5}{4} = 1.25$

$7 \times 1.25 = 8.75$

SOLUTION 7 lb of chicken costs \$8.75.

8 Look at problem 7. How much chicken can Alejandro buy for \$8? Show your work.

Possible work:

The unit rate for pounds per dollar is $\frac{4}{5}$, or 0.8.

dollars　　pounds per dollar

$8 \times 0.8 = 6.4$

SOLUTION Alejandro can buy 6.4 lb of chicken for \$8.

9 Anica volunteers to fold T-shirts for the runners at a marathon. She folds 8 T-shirts every 6 minutes. At this rate, how many T-shirts does Anica fold in 45 minutes? Show your work.

Possible work:

Since $8 \div 6 = \frac{8}{6}$, Anica folds $\frac{8}{6}$ T-shirts per minute.

$45 \cdot \frac{8}{6} = 60$

SOLUTION Anica folds 60 T-shirts in 45 minutes.

CLOSE EXIT TICKET

9 Students' solutions should show an understanding of:
- the relationship between the quantities in a ratio and the unit rate.
- using the unit rate to find an unknown value in an equivalent ratio.

Error Alert If students' solution is 33.75 or $33\frac{3}{4}$, then ask them to check their answer for reasonableness. Have them think about whether Anica folds more or less than 1 T-shirt per minute and whether the number of T-shirts folded in any number of minutes will be greater than or less than the number of minutes.

Practice Using Unit Rates to Find Equivalent Ratios

Problem Notes

Assign **Practice Using Unit Rates to Find Equivalent Ratios** as extra practice in class or as homework.

1. Students may also divide the number of rides by the rate $\frac{4}{13}$ to determine the cost of 20 rides. ***Basic***
2. Students may also divide the number of dollars by the rate $3.25 per ride to determine the number of rides Vinh buys. ***Medium***

LESSON 16 | SESSION 2 Name:

Practice Using Unit Rates to Find Equivalent Ratios

➤ **Study the Example showing how to use a unit rate to find an equivalent ratio. Then solve problems 1–5.**

Example

Winona and Reth are adding money to their subway fare cards. Winona pays $26 for 8 rides. Each ride costs the same amount. How much does Reth pay for 7 rides?

The ratio of dollars to rides is 26 : 8. Divide to find the **unit rate**.

$$\begin{matrix}\text{dollars} \rightarrow \\ \text{rides} \rightarrow\end{matrix} \frac{26}{8} = \frac{13}{4} = 3.25$$

The rate is $**3.25** per ride.

Multiply the **number of rides** by the **unit rate** to find the missing value of the equivalent ratio.

$7 \times 3.25 = 22.75$

Reth pays $22.75 for 7 rides.

Dollars	Rides
26	8
?	7

$\times$ 3.25

1. Look at the problem in the Example. Rolando also adds money to his subway fare card. How much does Rolando pay for 20 rides? Show your work.

   Possible work:

   $20 \times 3.25 = \$65$

   SOLUTION Rolando pays $65 for 20 rides.

2. Look at the problem in the Example. Vinh adds $39 to his subway fare card. How many rides does Vinh buy? Explain how you can use the unit rate for rides per dollar to find the answer.

   12 rides; Possible explanation: You can divide the numbers in the ratio 8 : 26 to get a rate of $\frac{8}{26}$ rides per dollar. Then you can multiply the unit rate $\frac{8}{26}$ by $39 to get the number of rides.

Vocabulary

rate
a ratio that tells the number of units of one quantity for 1 unit of another quantity.

unit rate
the numerical part of a rate. For the ratio $a : b$, the unit rate is the quotient $\frac{a}{b}$.

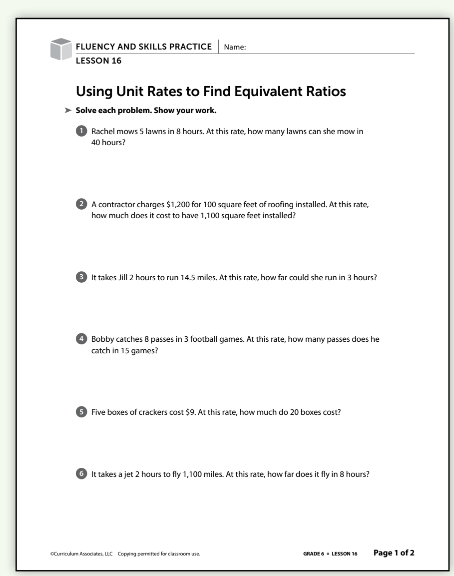
per
for each or *for every*. The word *per* can be used to express a rate, such as $2 per pound.

367

Fluency & Skills Practice

Using Unit Rates to Find Equivalent Ratios

In this activity, students solve problems by calculating unit rates and finding equivalent ratios.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 16

Using Unit Rates to Find Equivalent Ratios

➤ **Solve each problem. Show your work.**

1. Rachel mows 5 lawns in 8 hours. At this rate, how many lawns can she mow in 40 hours?
2. A contractor charges $1,200 for 100 square feet of roofing installed. At this rate, how much does it cost to have 1,100 square feet installed?
3. It takes Jill 2 hours to run 14.5 miles. At this rate, how far could she run in 3 hours?
4. Bobby catches 8 passes in 3 football games. At this rate, how many passes does he catch in 15 games?
5. Five boxes of crackers cost $9. At this rate, how much do 20 boxes cost?
6. It takes a jet 2 hours to fly 1,100 miles. At this rate, how far does it fly in 8 hours?

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Additional Practice

3 Students may write an equation to determine the rate of subscribers per day, $22 \div 4 = 5.5$. Then they would multiply the unit rate by the number of days. ***Medium***

4 Students should recognize that the information given in the problem can be used to calculate the rate in words per minute. Since the question asks if Ximena can type the essay in 1 hour, students will need to use the unit rate to find the number of words Ximena can type in 1 hour and then compare this to the number of words in the essay. ***Challenge***

5 Students should understand that they can use any of the rows from the table to find the unit rate because the problem states that Andrew saves the same amount of money each week. All of the rows will have the same unit rate: $\frac{288}{9} = 32$ and $\frac{352}{11} = 32$. ***Medium***

LESSON 16 | SESSION 2

3 Angela starts a blog about wheelchair basketball. In the first 4 days, the blog gets 22 new subscribers. At this rate, how many new subscribers can Angela expect in 30 days? Show your work.

Possible work:

Days	Subscribers
4	22
30	?

$\times$ 5.5

$\frac{22}{4} = 5.5$

$30 \times 5.5 = 165$

SOLUTION Angela can expect 165 new subscribers.

4 Ximena is typing a 2,500-word essay. In 9 minutes she types 396 words. At this rate, can Ximena type the essay in an hour? Explain.

Yes; Possible explanation: Ximena types 396 words in 9 min, so her rate is $\frac{396}{9}$ words per minute, or 44 words per minute. Her unit rate is 44 and 1 hour is 60 min. So, in 1 hour, the number of words Ximena can type is $60 \times 44 = 2{,}640$. This means she can type the 2,500-word essay in an hour.

5 Andrew saves the same amount of money each week. The table shows the amount he saves in different numbers of weeks. How much money does Andrew save in 40 weeks? Show your work.

Weeks	Dollars
7	224
9	288
11	352

Possible work:

$\frac{\text{dollars} \rightarrow 224}{\text{weeks} \rightarrow 7} = 32$

weeks ↓ dollars per week ↓

$40 \cdot 32 = 1{,}280$

SOLUTION Andrew saves $1,280 in 40 weeks.

368

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** **Model It**

Levels 1–3: Listening/Speaking

Display both sets of tables from the Model Its to support students as they discuss how to compare unit rates.

Write the word *per* on the board. Review that this means *for each*. Point out that the tables in the first Model It compare the two prices *per* ounce. Ask students to work with a partner to rephrase this concept. A sample response might be: *These tables compare how much it costs for 1 ounce of dish soap.* Ask: *Is it a better deal to have a higher or a lower unit rate? Why?* Call on volunteers to share. Extend discourse with details such as: *Yes, it is better to cost less for the same amount.*

Repeat the process for the second set of tables.

Levels 2–4: Listening/Speaking

Display both sets of tables from the Model Its to support students as they discuss how to compare unit rates. Have students work with a partner to look at the tables in the first Model It. Ask them to discuss the tables using the following sentence frames:

- *The unit rates are* ____ *and* ____.
- *That means we are comparing* ____ *for one* ____.
- *The better deal is the brand that is* ____ *because it means* ____.

Repeat this discussion for the second set of tables. Ask students to share ideas and have them extend discourse with details from the tables or life experience.

Levels 3–5: Listening/Speaking

Direct students' attention to the tables in both Model Its and guide them to talk about comparing unit rates.

Have students work with a partner to compare and contrast both sets of tables.

Guide discourse with the following prompts:

- *What unit rate is being compared in each set of tables?*
- *For each set, discuss whether it is a better deal to have a higher or a lower unit rate.*
- *Explain with examples what it means in each set to have a higher or a lower unit rate.*

Call on volunteers to share their ideas and explanations. Encourage other groups to agree and add on with specific details.

Develop Using Unit Rates to Compare Ratios

Purpose

- **Develop** strategies for comparing two or more ratios.
- **Recognize** that rates or unit rates are useful tools for comparing ratios.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

A	B
$1\frac{1}{5}$ lb for \$2.00	1.5 lb for \$2.50
C	**D**
7 lb for \$10.00	12 lb for \$20.00

Possible Solutions

The cost per pound can be found for each set of quantities.

A has the only weight written using a fraction.

B has the only weight written as a decimal.

C has a different unit rate that the other three.

D has two quantities that are even numbers.

WHY? Support students' facility with calculating unit rates.

DEVELOP ACADEMIC LANGUAGE

WHY? Guide students to be specific when they disagree with an idea.

HOW? In Discuss It, prompt students to be specific about the parts of an idea or strategy that they disagree with and tell why they think it is incorrect. Model disagreeing and explaining with a volunteer. It may be helpful to make a two-column chart that suggests what to do and what not to do when disagreeing with a classmate's idea.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. Discuss the phrase *better buy*. In this problem, the better buy is the product with the lower price per fluid ounce. In the real world, other factors can impact what is considered the better buy, such as quantity, quality, or packaging.

LESSON 16 | SESSION 3

Develop Using Unit Rates to Compare Ratios

Read and try to solve the problem below.

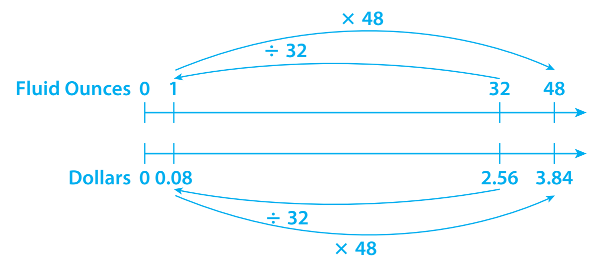
Antonio uses dish soap in his recipe for giant bubbles. He compares the prices of two brands of dish soap. Which brand is the better buy?

TRY IT

Math Toolkit double number lines, graph paper

Possible work:

SAMPLE A



It costs \$3.64 for 48 fl oz of Brand A. This is less than \$4.80 for 48 fl oz of Brand B.

Brand A is the better buy.

SAMPLE B

Brand A

Fluid Ounces	32	16
Dollars	2.56	1.28

÷ 2

Brand B

Fluid Ounces	48	16
Dollars	4.80	1.60

÷ 3

For 16 fl oz, Brand A costs less than Brand B.

Brand A is the better buy.

DISCUSS IT

Ask: What was the first thing you did to compare the prices of the brands?

Share: First I . . .

369

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them explain their work and then respond to Discuss It with a partner. To support students in extending the conversation, prompt them to discuss these questions:

- *Is your answer based on finding the unit rate? How do you know?*
- *Are there other ways to compare the brands besides finding the unit rate?*

Common Misconception Listen for students who compare only the prices and conclude Brand A is the better buy because \$2.56 < \$4.80 or students who compare only the fluid ounces and conclude Brand B is the better buy because 48 fl oz. > 32 fl oz. As students share their strategies, have students explain the reasoning behind their answer and encourage other students to voice any disagreements.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- comparing the price for the same number of fluid ounces by making a double number line or equivalent ratio tables
- (misconception) comparing only the price of the brands without considering the price per unit
- finding the unit rate for each brand by using equations

Facilitate Whole Class Discussion

Call on students to share selected strategies. Remind listeners to be specific when explaining why they disagree with a speaker's idea.

Guide students to **Compare and Connect** the representations. After each strategy, allow individual think time for students to process the ideas.

ASK *How do [student name]'s and [student name]'s models compare the price for both brands?*

LISTEN FOR Models should show using multiplication or division to compare the price per fluid ounce or the number of fluid ounces per dollar.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK *How do the equivalent ratio tables show the same information?*

LISTEN FOR Both pairs of tables show finding a rate for the given ratios. The first pair of tables shows the cost per fluid ounce, and the second pair shows the fluid ounces per dollar.

For the tables with price listed first, prompt students to describe how the tables are organized.

- *Is the organization of the table important? Why or why not?*
- *What is a unit price?*

For the tables with fluid ounces listed first, prompt students to think about how the quantities compare with those in the first set of tables.

- *What numbers are the same in both models?*
- *What numbers are different? Why are they different?*

LESSON 16 | SESSION 3

➤ **Explore different ways to use unit rates to compare ratios.**

Antonio uses dish soap in his recipe for giant bubbles. He compares the prices of two brands of dish soap. Which brand is the better buy?

Dish Soap	Fluid Ounces	Price
Brand A	32	\$2.56
Brand B	48	\$4.80

Model It

You can find the better buy by comparing the unit rates for dollars per fluid ounce.

Use a table to find the price per fluid ounce for each brand.

Brand A (÷ 32)

Price (\$)	2.56	0.08
Fluid Ounces	32	1

Brand B (÷ 48)

Price (\$)	4.80	0.10
Fluid Ounces	48	1

A price for 1 unit, such as 1 fl oz, is called a unit price.

The unit price for Brand A is \$0.08 per fluid ounce.

The unit price for Brand B is \$0.10 per fluid ounce.

Model It

You can find the better buy by comparing the unit rates for fluid ounces per dollar.

Brand A (÷ 2.56)

Fluid Ounces	32	12.5
Price (\$)	2.56	1

Brand B (÷ 4.80)

Fluid Ounces	48	10
Price (\$)	4.80	1

For Brand A, you get 12.5 fluid ounces per dollar.

For Brand B, you get 10 fluid ounces per dollar.

370

DIFFERENTIATION | EXTEND

SMP 3

Deepen Understanding

Constructing Viable Arguments for Deciding on a Solution Method

Prompt students to explain how they might decide to organize a table of equivalent ratios to solve a problem.

ASK *What are the advantages of finding a unit price, as shown in the first Model It?*

LISTEN FOR The calculations involve dividing a decimal by a whole number, which can be easier than dividing by a decimal. Once the unit price is found, you could compare the price of any number of fluid ounces for any size bottle.

ASK *What are the advantages of finding the number of fluid ounces per dollar, as shown in the second Model It?*

LISTEN FOR The calculations for finding the unit rate of fluid ounces per dollar result in quantities greater than 1, which can be easier to compare than decimals less than 1.

Encourage students to explain which solution method they prefer and why.

Develop Using Unit Rates to Compare Ratios

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about how rates or unit rates are used to compare ratios.

Before students begin to record and expand on their work in Model It, tell them that problem 4 will prepare them to provide the description asked for in problem 5. To help students collect their ideas, ask them to turn and talk about the models and strategies presented in today's lesson.

Monitor and Confirm Understanding 1 – 3

- Since the bottles contain different numbers of fluid ounces and different prices, an equivalent ratio that has the same quantity for one of the units for each brand is needed to compare the brands to determine the better buy.
- The unit prices compare the cost of each brand per fluid ounce.
- Using either model, Brand A is the better buy.

Facilitate Whole Class Discussion

4 Students should recognize that both strategies involve finding a unit rate. The interpretation of the unit rate varies between the two Model Its.

> **ASK** *Why is the better buy the lesser unit rate in the first Model It and the greater unit rate in the second Model It?*
>
> **LISTEN FOR** In the first Model It, the lesser unit rate shows that Brand A costs less per fluid ounce. In the second Model It, the greater unit rate shows that Brand A has more fluid ounces per dollar.

5 Look for the idea that unit rates are used to compare ratios because the quantity of one of the units in each ratio is 1.

> **ASK** *What are you finding when you calculate a unit rate? How does that help you compare two ratios?*
>
> **LISTEN FOR** A unit rate is the multiplier used to compare two quantities in a ratio. To find out if a ratio is equivalent, divide to find the unit rate.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to use rates and unit rates to compare ratios.**

1 Look at the first **Model It**. Why do you divide by 32 to find the unit price for Brand A and divide by 48 to find the unit price for Brand B?
For each brand, you want to know the price of 1 fl oz of the dish soap. So, you divide to get 1 fl oz in each table.

2 How can you use the unit prices to find which brand is the better buy?
The unit price for Brand A is less than the unit price for Brand B. That means you pay less for each fluid ounce with Brand A than with Brand B.

3 Look at the second **Model It**. How can you uses the unit rates for fluid ounces per dollar to find which brand is the better buy?
The unit rates show that you get more dish soap for $1 with Brand A than with Brand B.

4 How are the strategies in the two **Model Its** similar? How are they different?
Possible answer: In both Model Its, you find unit rates. In the first Model It, the lesser unit rate tells you the better buy. In the second Model It, the greater unit rate tells you the better buy.

5 Why can you compare two ratios by comparing their unit rates?
Unit rates show the amount for each unit of a given quantity. You can compare ratios where the quantity that is the same in each ratio is 1.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.
Responses will vary. Check student responses.

371

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Make a model using counters to compare two ratios using unit prices.

If students are unsure about how to compare two ratios using unit rates, then use this activity to show how unit prices and unit rates can be compared.

Materials For each pair: 35 counters, 8 small paper cups, markers

- Pose this problem to students: *A bar of Soap A costs $2.00 for 5 ounces. A bar of Soap B costs $1.50 for 3 ounces. Which is the better buy?* Tell students that each counter represents 10 cents.
- Ask: *How can you find the cost of 1 ounce?* [Divide the price by the number of ounces.] Have pairs label 5 cups with A and label 3 cups with B. Have pairs place counters in each cup to represent the unit price per ounce.
- Ask: *What is the unit price per ounce of each brand?* [Soap A: $0.40; Soap B: $0.50] *Which is the better buy?* [Soap A]

Apply It

For all problems, encourage students to use a model to support their thinking. Remind students of the importance of accurate labels when working with ratios to help organize their thinking.

7 Students may also find the rate in seconds per meter. Giraffe: $\frac{20}{280} = \frac{1}{14}$ second per meter; Zebra: $\frac{12}{204} = \frac{1}{17}$ second per meter. Since $\frac{1}{17} < \frac{1}{14}$, this method also shows that the zebra runs faster than the giraffe because it takes a zebra less time to run 1 meter.

8 **A** **is correct.** Students may solve the problem by finding the rate of teaspoons of hot sauce per pint of chili. $\frac{15}{6} = 2.5$ teaspoons per pint, which is greater than the other rates.

B is not correct. This answer is the result of choosing the least rate instead of the greatest: $\frac{18}{15} = 1.2$ teaspoons per pint.

C is not correct. This answer is the result of choosing the least amount of hot sauce.

D is not correct. This answer is the result of choosing the greatest amount of hot sauce or the greatest difference between the quantities in the answer choices.

Apply It

➤ **Use what you learned to solve these problems.**

7 The table shows the top running speeds for a giraffe and a zebra. Which animal can run faster? Show your work.

Animal	Meters	Seconds
Giraffe	280	20
Zebra	204	12

Possible work:

Giraffe:

$\frac{280}{20} = 14$, so the giraffe can run 14 meters per second.

Zebra:

$\frac{204}{17} = 17$, so the zebra can run 17 meters per second.

SOLUTION The zebra can run faster.

8 Four friends make chili for a chili cook-off. Each of them uses a different amount of hot sauce to make the chili spicy. Which ratio of hot sauce to chili makes the spiciest chili?

A 15 tsp hot sauce for 6 pt of chili

B 18 tsp hot sauce for 15 pt of chili

C 12 tsp hot sauce for 8 pt chili

D 24 tsp hot sauce for 10 pt chili

9 DeAndre's laptop downloads a 9 GB (gigabyte) file in 15 seconds. It takes Cheryl's laptop 80 seconds to download a 32 GB file. Whose laptop downloads files at a faster rate? Show your work.

Possible work:

DeAndre (÷ 15)

Gigabytes	9	0.6
Seconds	15	1

÷ 15

Cheryl (÷ 80)

Gigabytes	32	0.4
Seconds	80	1

÷ 80

The laptop with the greater unit rate is the faster laptop.

$0.6 > 0.4$

SOLUTION DeAndre's laptop downloads at a faster rate.

372

CLOSE **EXIT TICKET**

9 Students' solutions should show an understanding of:

- rates or unit rates can be used to compare two or more given ratios to solve problems that determine the best deal, compare speeds, etc.
- ratios can be compared with either unit rate, as long as the same unit has a quantity of 1 in each ratio.

Error Alert If students think that Cheryl's laptop downloads at a faster rate, then have them try to solve the problem using the other rate. For example, if they found the number of gigabytes per second, have them find the number of seconds per gigabyte. [DeAndre's laptop: $1\frac{2}{3}$ seconds per GB; Cheryl's laptop: 2.5 seconds per GB] Students should understand that taking less time to download a gigabyte means that the rate is faster. Discuss how any solution method should show that DeAndre's laptop downloads at a faster rate.

Practice Using Unit Rates to Compare Ratios

Problem Notes

Assign **Practice Using Unit Rates to Compare Ratios** as extra practice in class or as homework.

1. Students should recognize that the problem asks for a specific unit rate. The Example shows square meters per hour, and this problem asks for the hours per square meter. Both solution methods will show that the Blue Team paints faster. ***Basic***

2. Students should recognize that the problem asks for the unit price, which means they need to find the number of dollars per month. In this case, finding the months per dollar will not answer the question asked. Remind students that since the problem involves money, it is easier to work with decimals than fractional amounts. ***Basic***

 Name:

Practice Using Unit Rates to Compare Ratios

➤ **Study the Example showing how to use unit rates to compare ratios. Then solve problems 1–5.**

Example

Two teams of students are painting fences at Lakeside Middle School. The Blue Team paints 15 square meters in 6 hours. The Red Team paints 8 square meters in 4 hours. Which team paints faster?

You can compare the unit rates for square meters painted per hour.

Blue Team

$$\begin{matrix}\text{square meters} \rightarrow \\ \text{hours} \rightarrow\end{matrix} \frac{15}{6} = 2.5$$

Red Team

$$\begin{matrix}\text{square meters} \rightarrow \\ \text{hours} \rightarrow\end{matrix} \frac{8}{4} = 2$$

The team with the greater unit rate paints more square meters per hour.

$2.5 > 2$

The Blue Team paints faster.

1. Show how to solve the problem in the Example by comparing the unit rates for hours per square meter.

   Possible explanation:

   Blue Team

Hours	6	0.4
Square Meters	15	1

   Red Team

Hours	4	0.5
Square Meters	8	1

   The rate for the Blue Team is 0.4 hour per square meter. The rate for the Red Team is 0.5 hour per square meter. The Blue team paints faster because they take less time to paint every 1 square meter.

2. A news site offers a subscription that costs $28.50 for 6 months. What is the unit price per month? Show your work.

   Possible work:

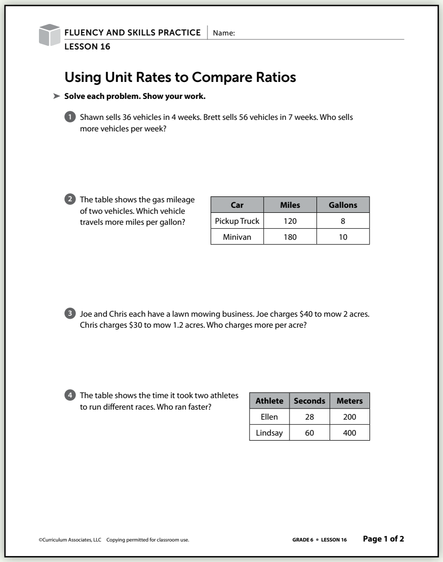
   $$\begin{matrix}\text{dollars} \rightarrow \\ \text{months} \rightarrow\end{matrix} \frac{28.50}{6} = 4.75$$

   SOLUTION The unit price per month is $4.75.

Fluency & Skills Practice

Using Unit Rates to Compare Ratios

In this activity, students compute and compare two unit rates to solve real-world problems.


FLUENCY AND SKILLS PRACTICE | Name:
LESSON 16

Using Unit Rates to Compare Ratios

➤ Solve each problem. Show your work.

1. Shawn sells 36 vehicles in 4 weeks. Brett sells 56 vehicles in 7 weeks. Who sells more vehicles per week?

2. The table shows the gas mileage of two vehicles. Which vehicle travels more miles per gallon?

Car	Miles	Gallons
Pickup Truck	120	8
Minivan	180	10

3. Joe and Chris each have a lawn mowing business. Joe charges $40 to mow 2 acres. Chris charges $30 to mow 1.2 acres. Who charges more per acre?

4. The table shows the time it took two athletes to run different races. Who ran faster?

Athlete	Seconds	Meters
Ellen	28	200
Lindsay	60	400

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3 Students may also choose to compare the feet per dollar, rather than the unit prices. However, working with that rate (feet per dollar) will require students to round the decimals, possibly making it more difficult for them to compare or calculate. ***Medium***

4 Students should recognize that this problem contains an extra step compared to the other problems in this lesson. The problem asks how much less per fluid ounce Brand X costs than Brand Y, so students should first find the unit prices and then find the difference between them. ***Challenge***

5 Students should recognize that they have to find the unit rate for each recipe and then compare all three unit rates. Students may also find the rate of cups of lemonade to cups of lemon juice: Erin: 6 cups lemonade per cup lemon juice; Damita: 4 cups lemonade per cup lemon juice; Jayden: 5 cups lemonade per cup lemon juice. When using this rate, students should understand that the least number of cups of lemonade per cup of lemon juice results in the strongest flavor. ***Medium***

3 Khalid wants to buy a long sandwich for a party. Store A sells a 5-foot sandwich for $42.50. Store B sells a 6-foot sandwich for $49.50. Which store has the better buy? Show your work.

Possible work:

Store A ($\div 5$)

Price ($)	42.50	8.50
Feet	5	1

Store B ($\div 6$)

Price ($)	49.50	8.25
Feet	6	1

The lower unit price is the better deal.

$8.25 < $8.50

SOLUTION Store B has the better buy.

4 A store sells two brands of hand lotion. Brand X costs $3.25 for 5 fluid ounces. Brand Y costs $6 for 8 fluid ounces. How much less per fluid ounce does Brand X cost than Brand Y? Show your work.

Possible work:

Brand X:

dollars fluid ounces

$3.25 \div 5 = 0.65$

$0.65 per fluid ounce

Brand Y:

dollars fluid ounces

$6.00 \div 8 = 0.75$

$0.75 per fluid ounce

$0.75 - 0.65 = 0.1$

SOLUTION Brand X costs $0.10 less per fluid ounce than Brand Y.

5 Three friends make lemonade with different recipes. The table shows the ratio of lemon juice to the total amount of lemonade. Which friend makes lemonade with the strongest lemon flavor? Explain how to use unit rates to decide.

Name	Lemon Juice (cups)	Lemonade (cups)
Erin	2	12
Damita	4	16
Jayden	3	15

Possible explanation: The unit rates for lemon juice to lemonade are $\frac{1}{6}$ for Erin, $\frac{1}{4}$ for Damita, and $\frac{1}{5}$ for Jayden. The greatest unit rate is $\frac{1}{4}$. Damita's lemonade has the strongest lemon flavor.

374

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** Connect It

MATH TERM

Equivalent ratios are ratios that express the same comparison. You can find *equivalent ratios* by multiplying both numbers in a ratio by the same nonzero number. For example, $a : b$ is equivalent to $2a : 2b$.

Levels 1–3: Speaking/Writing

Help students respond in writing to Connect It problem 5. First, read the problem with students and review the Math Term. Then ask students to identify what they need to compare. Help students list words and phrases to write the steps in each process using **Co-Constructed Word Bank**. Add verbs from Model It, such as *convert, find, multiply by,* and *write.*

Next, have students refer to the word bank and write the steps for each process in a numbered list. Reinforce that each step needs a verb. With students, draft sentences that compare the processes.

Levels 2–4: Speaking/Writing

Support students as they prepare to write for Connect It problem 5. Have students read the problem, identify the two processes to compare, and discuss the meaning of *equivalent ratios.* Remind students that they can describe steps in a process using sequence words like *first, next,* and *then.* Ask partners to discuss the steps. Next, help students list and discuss comparison words, such as *the same as, both,* and *similarly.*

Ask students to write in complete sentences and to use sequence and comparison words to make their ideas clear.

Levels 3–5: Speaking/Writing

Support students as they write a response for Connect It problem 5 using **Stronger and Clearer Each Time.** Have students read the problem and think about the steps of each process and describe any similarities. Ask students to draft a response and explain their ideas to partners.

During partner discussions, suggest that students agree and build onto their partner's ideas or disagree and explain using specific examples and details. Ask students to think about the discussions and feedback as they revise their writing.

Develop Using Unit Rates to Convert Measurements

Purpose

- **Develop** strategies for converting measurements by using ratio reasoning within the customary system or metric system.
- **Recognize** that you can use rates and unit rates for conversions just as you would any other ratio problem.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

A: $\frac{5}{6}$ hour	B: 0.75 hour
C: 45 minutes	D: 2,700 seconds

Possible Solutions

All are times less than 1 hour.

A is the only time expressed as a fraction. It is also a different time than the others.

B is the only time expressed as a decimal.

C is the only time expressed in minutes.

D is the only time expressed in seconds.

WHY? Support students' facility with converting between units of time.

DEVELOP ACADEMIC LANGUAGE

WHY? Clarify the terms *unit rate* and *unit of measurement*.

HOW? Display the term *units of measurement* and have students name units of measurement they know. Display the term *unit rate* and discuss that rates and unit rates show a relationship between two units of measurement. Have students give examples of rates or unit rates and identify the units of measurement in each. For example, miles per hour shows the relationship between distance and time.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Co-Craft Questions** to help them make sense of the problem. Once the full problem is revealed, have students compare their questions with the one they are asked to solve.

LESSON 16 | SESSION 4 ■■■■□

Develop Using Unit Rates to Convert Measurements

➤ **Read and try to solve the problem below.**

A band marches in the African American Day Parade in New York City. The band marches 800 meters every 15 minutes. At this rate, how many kilometers does the band march in 1 hour?

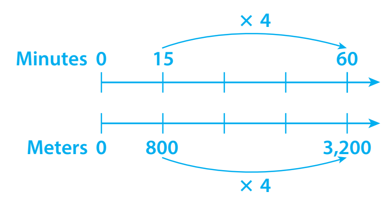
TRY IT

Math Toolkit double number lines, graph paper, ruler

Possible work:

SAMPLE A

1 hour is 60 minutes.



The band marches 3,200 meters in 1 hour.

1 km = 1,000 m

3,200 ÷ 1,000 = 3.2

The band marches 3.2 kilometers in 1 hour.

SAMPLE B

1 km = 1,000 m, so 1 m is $\frac{1}{1{,}000}$ km.

So, 800 m is 800 × $\frac{1}{1{,}000}$ km.

$800 \times \frac{1}{1{,}000} = \frac{800}{1000} = \frac{8}{10}$

The band marches $\frac{8}{10}$ km every 15 minutes.

1 h = 60 min and 4 × 15 = 60.

$4 \times \frac{8}{10} = \frac{32}{10}$, or $3\frac{2}{10}$

The band marches $3\frac{2}{10}$ km in 1 hour.

DISCUSS IT

Ask: How do you know your answer is reasonable?

Share: My answer makes sense because . . .

375

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them explain their work and then respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *What makes an answer reasonable? How does your answer meet that criteria?*
- *What steps can you take if your answer proves to be unreasonable?*

Error Alert If students' solution is $53\frac{1}{3}$ km in 1 hour, then remind students to check the units used in their calculations. This value is the rate of meters per minute, which students need to change to kilometers per hour. Have students reread the problem and make note of the units for each quantity given. Review that there are 60 minutes in 1 hour and 1,000 meters in 1 kilometer. Discuss how students can use this information to solve the problem.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- double number lines that show minutes per meter, which are then used to calculate kilometers per hour
- equations that show conversions from kilometers to meters before finding the rate of kilometers per minute, and equations that show conversion of hours to minutes

Facilitate Whole Class Discussion

Call on students to share selected strategies. Remind students that one way to agree and build on ideas is to give another example that shows that the strategy makes sense.

Guide students to **Compare and Connect** the representations. After each strategy, allow individual think time for students to process the ideas.

> **ASK** *How do [student name]'s and [student name]'s models show the information presented in the problem?*
>
> **LISTEN FOR** Each model uses the rate of meters per minute given in the problem. The units are converted from meters to kilometers and from minutes to hours to give the rate of kilometers per hour.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

> **ASK** *How are the problems similar to other conversion problems you have solved?*
>
> **LISTEN FOR** All conversion problems involve changing a measurement unit from a smaller to a larger unit or vice versa.

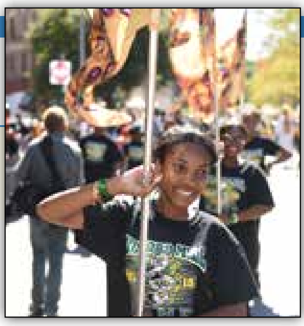
For the tables, prompt students to think about how known conversion facts are used.

- *How do the conversion facts shown above the tables help you convert from one unit of measurement to another?*

For the equations, prompt students to think about how unit rates are related to converting units of measure.

- *How is a conversion fact similar to a unit rate? How is it different?*

LESSON 16 | SESSION 4



➤ **Explore different ways to convert between units of measure.**

A band marches in the African American Day Parade in New York City. The band marches 800 meters every 15 minutes. At this rate, how many kilometers does the band march in 1 hour?

Model It

You can use a table of equivalent ratios to convert between units of measure.

1,000 meters = 1 kilometer

Meters	Kilometers
1,000	1
1	0.001
800	0.8

÷ 1,000 ÷ 1,000
× 800 × 800

60 minutes = 1 hour

Minutes	Hours
60	1
15	$\frac{1}{4}$

÷ 4 ÷ 4

The band marches 0.8 kilometers in $\frac{1}{4}$ hour. Multiply by 4 to find the number of kilometers the band marches in 1 hour.

4×0.8

Model It

You can multiply by a unit rate to convert between units of measure.

Write the rate for kilometers per meter.

1 kilometer = 1,000 meters

$\frac{1}{1,000}$ kilometer per meter

Find the number of meters the band marches in 1 hour.

$4 \times 800 = 3,200$

To convert 3,200 meters to kilometers, multiply by the unit rate.

$$\underset{\text{meters}}{3,200} \times \underset{\text{kilometers per meter}}{\frac{1}{1,000}}$$

376

DIFFERENTIATION | EXTEND

SMP 8

Deepen Understanding
Using Repeated Reasoning with Conversion Facts

Prompt students to use known relationships between measurements to assess the reasonableness of or to condense calculations.

> **ASK** *How does the number of kilometers relate to the number of meters when converting between units?*
>
> **LISTEN FOR** The number of kilometers is always 0.001 or $\frac{1}{1,000}$ the number of meters.

> **ASK** *What operation or operations can you use to convert from meters to kilometers? Is there only one way to solve?*
>
> **LISTEN FOR** You can multiply or divide by a unit rate. For example, to convert from meters to kilometers, you can divide by 1,000 or multiply by 0.001.

Generalize Encourage students to think of other conversions such as hours to minutes and describe how multiplication or division can be used when converting from larger to smaller units. Students should recognize that to convert hours to minutes, you can divide by 60 or multiply by $\frac{1}{60}$. Ask students to provide other examples.

Develop Using Unit Rates to Convert Measurements

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about how a known conversion fact can be used to write a rate and then the unit rate can be used to find a solution.

Before students begin to record and expand on their work in Model It, tell them that problem 3 will prepare them to provide the description asked for in problem 5. Use turn and talk to help students think through their responses before sharing.

Monitor and Confirm Understanding 1 – 2

- Rates can be written for measurement conversions, such as $\frac{1}{1,000}$ kilometer per meter or 1,000 meters per kilometer.
- Rates are used to convert measurements with the same reasoning used in other ratio problems. Multiply or divide by the unit rate to find the missing value.

Facilitate Whole Class Discussion

3 Look for understanding that multiplying the number of meters by the number of kilometers per meter results in the number of kilometers.

ASK How many kilometers are in a meter? How many meters are in a kilometer?

LISTEN FOR There are 1,000 meters in a kilometer and $\frac{1}{1,000}$ kilometer in a meter.

4 Look for understanding that the band marches 3.2 kilometers per hour.

ASK *What units of measurement are provided in the problem? What units of measurement are asked for in the answer?*

LISTEN FOR The problem gives quantities in meters and minutes. The solution should be in kilometers and hours.

5 Look for the idea that measurement conversion problems are similar to other rate problems.

ASK *In what ways are rate problems similar to and different from conversion problems?*

LISTEN FOR Both problem types involve taking a ratio relationship and using a unit rate to write an equivalent ratio. Not all rate problems involve measurements.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to convert between units of measure.**

1 Look at the first **Model It**. How are the relationships 1,000 meters = 1 kilometer and 60 minutes = 1 hour similar to rates?

1,000 meters = 1 kilometer means that there are 1,000 meters per kilometer and 60 minutes = 1 hour means that there are 60 minutes per hour.

2 Look at the second **Model It**. The relationship 1 kilometer = 1,000 meters is used to write the rate $\frac{1}{1,000}$ kilometer per meter. How is this rate shown by a row of the table of meters and kilometers in the first **Model It**?

It is the rate shown by the second row, where you see 0.001 kilometer is the same as 1 meter.

3 There are two rates that relate meters and kilometers. In the second **Model It**, why is $\frac{1}{1,000}$ the unit rate that is used to convert 3,200 meters to kilometers?

You know the number of meters, 3,200. Multiplying the number of meters by the number of kilometers in each meter, $\frac{1}{1,000}$, tells you how many kilometers are in 3,200 meters.

4 How many kilometers does the band march in 1 hour? **3.2 kilometers**

5 How is converting between measurements similar to finding equivalent ratios?

Possible answer: The relationship between two different units of measurement is a rate. You can use the unit rate as the factor that relates a measurement in one unit to its equivalent measurement in the other unit.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to convert between units of measure.

Responses will vary. Check student responses.

377

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Solve rate problems involving converting measurements.

If students are unsure about converting units to solve rate problems, then use this activity to show how to convert multiple units in one problem.

Materials For each pair: 5 sticky notes or index cards

- Pose this problem: *A large earthworm can travel 25 centimeters in 12 seconds. How many meters does the earthworm travel in 1 minute?*
- Have pairs label a sticky note 25 cm. Ask: *How many seconds does 1 sticky note represent? Why?* [12 seconds; The worm moves 25 centimeters in 12 seconds.] *How many sticky notes will you need to model 1 minute? Why?* [5; There are five 12-second intervals in 1 minute because there are 60 seconds in 1 minute.] Have pairs align 5 sticky notes to finish their model.
- Ask: *How can you find the number of centimeters the worm travels in 1 minute?* [Multiply 25 by 5; 125 centimeters] *How do you convert centimeters to meters?* [Divide the number of centimeters by 100.] *What is the earthworm's speed in meters per minute?* [1.25 meters per minute] *How does your model support your answer?* [4 sticky notes are equal to 100 centimeters, or 1 meter, and 1 more sticky note is equal to 25 centimeters, or 0.25 meter.]

Apply It

For all problems, encourage students to use a model to support their thinking.

7 Students should recognize that they have to calculate the conversion fact based on the information in the problem. At the time of Anne's visit, \$13 = £10, so divide \$13 by 10 to find the rate of dollars per pound. Students should also recognize that since Anne wants to spend less than \$20, she needs to convert pounds to dollars rather than dollars to pounds.

8 **D is correct.** Students may solve the problem by using a known conversion fact to find the number of fluid ounces in 4 cups. There are 8 fluid ounces in 1 cup, so multiply to find that there are 32 fluid ounces in 4 cups. Divide the cost of the can of juice by the number of ounces to find the unit price per fluid ounce: $2.56 \div 32 = 0.08$.

A is not correct. This answer is the result of finding $32 \div 2.56 = 12.50$, which is fluid ounces per dollar.

B is not correct. This answer is the result of finding $4 \div 2.56 = 1.5625$ and rounding to the nearest cent, which is cups per dollar.

C is not correct. This answer is the result of finding $2.56 \div 4 = 0.64$, which is the price per cup.

Apply It

➤ **Use what you learned to solve these problems.**

7 The unit of money in England is the pound (£). When Anne visits England, £10 equals \$13. She sees a bike rental that costs £3 per hour. Anne wants to spend less than \$20. Can Anne rent the bike for 5 hours? Explain.

Yes; Possible explanation: A 5-hour bike rental costs 5 × £3 = £15.

\$13 = £10

The rate is \$1.30 per pound.

pounds ↓ dollars per pound ↓

$15 \times 1.3 = 19.5$

The rental costs \$19.50, so Anne will spend less than \$20.

8 A can contains 4 cups of pineapple juice. The can of juice costs \$2.56. What is the unit price in dollars per fluid ounce?

A \$12.50 per fluid ounce

B \$1.56 per fluid ounce

C \$0.64 per fluid ounce

D \$0.08 per fluid ounce

9 A model train takes 10 seconds to travel along a section of track that is 5 yards long. At this rate, how many feet does the model train travel every minute? Show your work.

Possible work: 1 min = 60 s

The distance the train travels in 1 min is 6 • 5 yd = 30 yd.

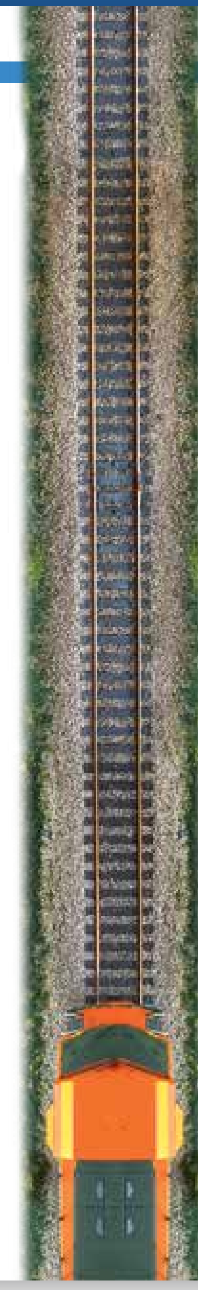
1 yd = 3 ft

The rate is 3 feet per yard.

yards ↓ feet per yard ↓

$30 \cdot 3 = 90$

SOLUTION The train travels 90 ft every minute.



CLOSE EXIT TICKET

9 Students' solutions should show an understanding of:

- how to write a rate for a measurement conversion.
- how to use the unit rate to convert measurements.
- when to make one or more measurement conversions to solve a particular problem.

Error Alert If students' solution includes the rate 0.5 foot per minute, then remind students to pay close attention to the units used for the measurements in the problem and those asked for in the question. Students could draw a double number line. One number line should show 6 sections of 10 to show the 60 seconds in 1 minute. The other would show 5 sections of 3 to show the 3 feet in 1 yard. Discuss how to use the model to solve the problem.

Practice Using Unit Rates to Convert Measurements

Problem Notes

Assign **Practice Using Unit Rates to Convert Measurements** as extra practice in class or as homework.

1. Students should use the information in the Example to compare unit prices. Students may choose to multiply each unit price in the Example by the number of ounces in 1 pound (16) to compare the unit prices.
Brand A: \$0.03 × 16 = \$0.48;
Brand B: \$0.04 × 16 = \$0.64. ***Medium***

Practice Using Unit Rates to Convert Measurements

➤ **Study the Example showing how to solve a measurement conversion problem. Then solve problems 1–4.**

Example

The table shows the prices of two brands of flour. Which brand is the better buy?

Flour	Weight	Price
Brand A	5 pounds	\$2.40
Brand B	48 ounces	\$1.92

Convert the weight of Brand A to ounces.

1 pound = 16 ounces

The rate is 16 ounces per pound.

pounds ounces per pound

5 × 16 = 80

Brand A weighs 80 ounces.

Find the unit prices in dollars per ounce, as shown in the tables.

Brand A ($\div$ 80)

Dollars	2.40	0.03
Ounces	80	1

Brand B ($\div$ 48)

Dollars	1.92	0.04
Ounces	48	1

Brand A costs \$0.03 per ounce.

Brand B costs \$0.04 per ounce.

Brand A is the better buy.

1. Show how you can solve the problem in the Example by comparing the unit prices in dollars per pound.

Convert the weight of Brand B to pounds.

The rate is $\frac{1}{16}$ pound per ounce. $48 \times \frac{1}{16} = 3$, so Brand B weighs 3 lb.

Brand A ($\div$ 5)

Dollars	2.40	0.48
Pounds	5	1

Brand B ($\div$ 3)

Dollars	1.92	0.64
Pounds	3	1

Brand A costs \$0.48 per pound. Brand B costs \$0.64 per pound.

Brand A is the better buy.

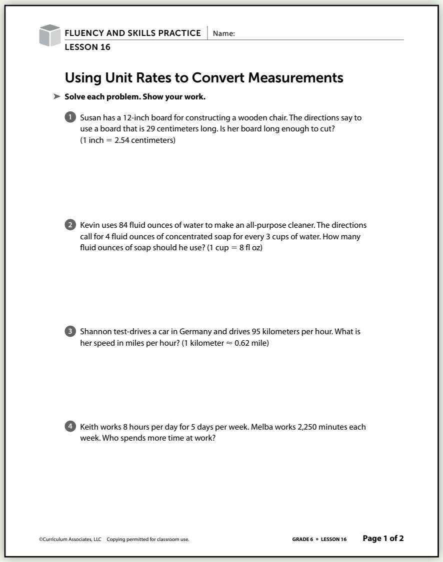
Vocabulary
convert
to write an equivalent measurement using a different unit.

379

Fluency & Skills Practice

Using Unit Rates to Convert Measurements

In this activity, students use their knowledge of ratios and unit rates to solve problems by converting measurements.


FLUENCY AND SKILLS PRACTICE | Name:
LESSON 16

Using Unit Rates to Convert Measurements

➤ **Solve each problem. Show your work.**

1. Susan has a 12-inch board for constructing a wooden chair. The directions say to use a board that is 29 centimeters long. Is her board long enough to cut? (1 inch = 2.54 centimeters)

2. Kevin uses 84 fluid ounces of water to make an all-purpose cleaner. The directions call for 4 fluid ounces of concentrated soap for every 3 cups of water. How many fluid ounces of soap should he use? (1 cup = 8 fl oz)

3. Shannon test-drives a car in Germany and drives 95 kilometers per hour. What is her speed in miles per hour? (1 kilometer ≈ 0.62 mile)

4. Keith works 8 hours per day for 5 days per week. Melba works 2,250 minutes each week. Who spends more time at work?

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Additional Practice

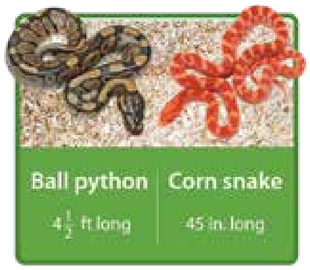
2 Students may also choose to find the number of feet in 45 inches. The conversion rate is $\frac{1}{12}$ foot per inch. Multiply to find the length of the corn snake in feet: $45 \times \frac{1}{12} = 3\frac{3}{4}$. Since $3\frac{3}{4} < 4\frac{1}{2}$, Vivian should get the corn snake. ***Basic***

3 Students should recognize that more than one conversion is needed to solve the problem. Students may to convert from seconds to minutes and then from minutes to hours to write the time correctly. They may choose to do this in one step: $60 \times 60 = 3{,}600$. Or they may choose to do it in two steps at different parts of the solution process. ***Challenge***

4 Students may find the number of milliliters of water that leak out in one hour, $20 \times 80 = 1{,}600$, and then convert this to liters: $\frac{1{,}600}{1{,}000} = 1.6$ liters. They can compare this with the 2 liters of water in the bottle to see that some water would still be in the bottle. ***Medium***

LESSON 16 | SESSION 4

2 Vivian is getting a pet snake. She is choosing between the ball python and the corn snake. Vivian wants the shorter snake. Which snake should she get? Show your work. (12 in. = 1 ft)



Possible work: The rate is 12 inches per foot.

feet ↓ inches per foot ↓

$4\frac{1}{2} \cdot 12 = 54$

The ball python is 54 in. This is longer than the corn snake.

SOLUTION Vivian should get the corn snake.

3 Kenji walks 44 feet in 10 seconds. At this rate, how many miles does Kenji walk in an hour? Show your work. (1 mile = 5,280 feet)

Possible work: 1 min is 60 s, or 6×10 s.

So, Kenji walks 6×44 ft $= 264$ ft in 1 min.

He walks 60×264 ft $= 15{,}840$ ft in 1 h.

The rate is $\frac{1}{5{,}280}$ mile per foot.

$15{,}840 \times \frac{1}{5{,}280} = 3$

SOLUTION Kenji walks 3 miles in an hour.

4 A 2-liter bottle is full of water. The bottle leaks 80 milliliters of water every 3 minutes. Will the bottle be empty in 1 hour? Explain why or why not. (1 liter = 1,000 milliliters)

No; Possible explanation: There are 1,000 milliliters per liter.

$2 \times 1{,}000 = 2{,}000$. So the bottle holds 2,000 mL of water.

1 h = 60 min

× 20

Milliliters	80	1,600
Minutes	3	60

× 20

1,600 mL leaks from the bottle in 1 h, which leaves some water in the bottle.

380

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 5** **Apply It**

Levels 1–3: Reading/Speaking

Help students make sense of Apply It problem 1. Read the problem, and then review the terms *distance, farther, time, faster,* and *rate*. Explain that the ending *-er* in *farther* and *faster* shows a comparison. Guide students to tell what each word compares. Then ask them to identify the units for time and distance in the problem.

Read Pair/Share aloud and ask students to tell the number of miles, or *distance*, for each person. Guide students to compare distances using *farther*. Ask: *How can you find out who goes faster?* Provide a sentence frame to support discussion:

- *I notice the units for ____ are ____.*
- *I can compare the number of ____ per ____.*

Levels 2–4: Reading/Speaking

Have students read Apply It problem 1 and use **Say It Another Way** to confirm understanding.

Ask students to read Pair/Share and tell the word that compares distance [*farther*] and the word that compares time [*faster*]. Provide time for students to think about their answer to Pair/Share before discussing with the class.

Next, have students read Consider This and tell which units measure $\frac{distance}{time}$ or $\frac{time}{distance}$.

Ask partners to discuss the units they can use to compare the rates in the problem, using:

- *I think we should use ____ to compare the rates because ____.*
- *I agree/disagree because ____.*

Levels 3–5: Reading/Speaking

Ask partners to read Apply It problem 1 and confirm understanding using **Say It Another Way**. Then have partners use Consider This and Pair/Share to make sense of the rates in the problem. Ask partners to identify the two words in Pair/Share that compare the quantities from the problem. Then have partners discuss the units they can use to compare the rates. Provide these sentence starters to help students building on to their partner's ideas and/or explain their own ideas:

- *I think you said ____.*
- *I think you are right because ____.*
- *I have a different idea because ____.*

Refine Using Unit Rates to Solve Problems

Purpose

- **Refine** strategies for performing unit conversions.
- **Refine** understanding of how to use rates and unit rates to convert measurements.

START CHECK FOR UNDERSTANDING

A smartwatch records 6 miles in 1 hour. At this rate, how many yards does the smartwatch record in 1 minute?
(1 mile = 1,760 yards)

Solution

176 yards

WHY? Confirm students' understanding of converting measurement units to solve rate problems, identifying common errors to address as needed.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.

Refine Using Unit Rates to Solve Problems

Complete the Example below. Then solve problems 1–9.

Example

An elm tree is 20 feet tall. A poplar tree is 6.4 meters tall. Which tree is taller?

Look at how you could use a rate to convert measurement units.

Convert 6.4 meters to feet.

For every 100 meters there are about 328 feet.

The rate is 3.28 feet per meter. The unit rate is 3.28.

$$\underbrace{6.4}_{\text{meters}} \times \underbrace{3.28}_{\text{feet per meter}} = 20.992$$

The height of the poplar tree is about 21 feet.

SOLUTION The poplar tree is taller.

CONSIDER THIS . . .
The relationship you use to convert between the customary unit *feet* and the metric unit *meters* is an approximation: 100 meters is about 328 feet.

PAIR/SHARE
How would the steps of the solution be different if you compared the heights in meters?

Apply It

1. Lucía and Quinn train for a bike race. Lucía bikes 46 miles in 240 minutes. Quinn bikes 51 miles in 5 hours. Who bikes at a faster rate? Show your work.

Possible work:

There are 60 min in 1 h. 240 ÷ 60 = 4. So, Lucía bikes for 4 h.

Lucía (÷ 4)

Miles	46	11.5
Hours	4	1

Quinn (÷ 5)

Miles	51	10.2
Hours	5	1

11.5 miles per hour is a faster rate than 10.2 miles per hour.

SOLUTION Lucía bikes at a faster rate.

CONSIDER THIS . . .
You can compare the rates in miles per hour, hours per mile, miles per minute, or minutes per mile.

PAIR/SHARE
Does knowing which person bikes farther give you enough information to decide who bikes faster? Why or why not?

START ERROR ANALYSIS

If the error is . . .	Students may . . .	To support understanding . . .
0.1 yard	have divided the number of miles by the number of minutes in one hour without converting the number of miles to yards.	Have students write the key information from the problem using different colored pencils: one for miles, one for hours, one for yards, and one for minutes. Students can check their work by making sure to use the color that matches each unit when writing the quantity.
10,560 yards	have found the number of yards in 6 miles without dividing the number of yards by the number of minutes in 1 hour.	Have students think about the reasonableness of this answer. If a yardstick is available, have students look at the yardstick and decide if 10,560 yards in one minute sounds reasonable. Remind students to also check their answer to make sure it makes sense.
$\frac{9}{44}$ yard	have multiplied the number of miles recorded by the number of minutes in one hour, and then divided 360 by 1,760.	Have students make a two-column chart and write the headings *Time* and *Distance* at the top of each column. Have students list all times in the *Time* column and all distances in the *Distance* column to help them keep track of related measurements.

Example

Guide students in understanding the Example. Ask:

- *What units are used to measure the elm tree? What units are used to measure the poplar tree?*
- *Why will comparing 20 and 6.4 not help you solve the problem?*
- *How do unit rates help you solve this problem?*

Help all students focus on the Example and responses to the questions by reinforcing that good listeners ask questions to clarify ideas or ask for more information during math discussions.

Look for understanding that conversion strategies can also be used to convert from either metric to customary or from customary to metric units.

Apply It

1. See ***Connect to Culture*** to support student engagement. Students may solve the problem by calculating the number of miles each person rides per minute as fractions and then compare them. Lucía: $\frac{46}{240} = \frac{23}{120}$; Quinn: $\frac{51}{300} = \frac{17}{100}$
DOK 3

2. Students may find the weight of the suitcase in kilograms and then compare it to 23 kilograms. The unit rate of kilograms per pound is $\frac{1}{2.2}$, so divide the pounds: $\frac{49}{2.2} \approx 22.3$. This is less than 23 kilograms, so Elisa can take the suitcase on the plane. *DOK 2*

3. **C** **is correct.** Students may solve by finding the unit rate: $\frac{16}{40} = \frac{2}{5} = 0.4$. Divide the number of minutes, 30, by the unit rate to find the number of napkins: $\frac{30}{0.4} = 75$.

A is not correct. This answer is the result of solving the problem *How long will it take to fold 30 napkins?*

B is not correct. This answer is the result of adding 14 to the number of napkins; however, 14 is the number of additional minutes Issay will fold compared to today.

D is not correct. This answer is the result of multiplying the rate 2.5 napkins per minute by 40 (the number of napkins), instead of by 30 (number of minutes).

DOK 3

2. Elisa packs her suitcase before a trip. The suitcase weighs 49 pounds. The airline only allows suitcases that weigh up to 23 kilograms. Can Elisa take the suitcase on the plane? Show your work. (For every 10 kilograms there are about 22 pounds.)

Possible work:

The weight limit is 23 kilograms. Convert the weight limit to pounds.

The rate is 2.2 pounds per kilogram. The unit rate is 2.2.

kilograms ↓ pounds per kilogram ↓

$23 \times 2.2 = 50.6$

The weight limit is 50.6 lb; 49 lb is less than 50.6 lb.

SOLUTION Yes, Elisa can take the suitcase on the plane.

CONSIDER THIS . . . What comparison do you need to make to solve this problem?

PAIR/SHARE How could you solve the problem a different way?

3. Issay works at a restaurant. Today, it takes him 16 minutes to fold 40 napkins. He plans to fold napkins for 30 minutes tomorrow. If he works at the same rate, how many napkins will he fold tomorrow?

A 12

B 54

C 75

D 100

Destiny chose C as the correct answer. How might she have gotten that answer?

Possible answer: Destiny divided the number of napkins by the number of minutes to find the number of napkins per minute.

$40 \div 16 = 2.5$

So, the rate is 2.5 napkins per minute. The unit rate is 2.5. Then she multiplied.

$30 \times 2.5 = 75$

CONSIDER THIS . . . Do you expect the answer to be less than 40 or greater than 40?

PAIR/SHARE How did you decide which strategy or model to use to solve this problem?

GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency
- **RETEACH** Hands-On Activity
- **REINFORCE** Problems 4, 6, 7

Meeting Proficiency
- **REINFORCE** Problems 4–8

Extending Beyond Proficiency
- **REINFORCE** Problems 4–8
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket**.

Resources for Differentiation are found on the next page.

Refine Using Unit Rates to Solve Problems

4 Students may understand that the calculations will be easier if they find the number of fluid ounces in 5 gallons, rather than finding the number of gallons in 40 fluid ounces. Students may also divide the number of fluid ounces by the number of fluid ounces per minute to find the number of minutes it will take to fill the aquarium. *DOK 3*

5 Students should recognize that yards are longer than feet. This leads to recognition that they may divide the number of feet by the number of feet in one yard, or multiply by $\frac{1}{3}$, in order to correct Glen's error. *DOK 3*

6 **a.** The mouse runs at a rate of 3 meters per second, and 30 meters in 14 seconds is a unit rate of $\frac{15}{7}$, not 3.

b. The mouse runs 24 meters in 8 seconds, and $\frac{24}{8} = 3$ meters per second, which is faster than a mouse who runs 26 meters in 13 seconds, or $\frac{26}{13} = 2$ meters per second.

c. The mouse runs 3 meters per second and $\frac{1}{3} \times 5 = \frac{5}{15}$, so it takes 5 seconds for the mouse to run 15 meters.

d. The mouse runs 8 seconds in 24 meters, or $\frac{8}{24}$, which equals $\frac{1}{3}$ second per meter.

DOK 2

4 A desktop aquarium can hold 5 gallons of water. Desiderio fills it at a rate of 40 fluid ounces per minute. How long does it take him to fill the aquarium? Show your work. (1 gallon = 128 fluid ounces)

Possible work:

The rate is 128 fluid ounces per gallon.

$$\underset{\text{gallons}}{5} \times \underset{\text{fluid ounces per gallon}}{128} = 640$$

Fluid Ounces	40	$\xrightarrow{\times 16}$ 640
Minutes	1	16

SOLUTION It takes 16 minutes to fill the aquarium.

5 Glen is asked to convert 21 feet to yards. He knows that 3 feet = 1 yard. His work is shown. Explain Glen's error and show how to use a rate to find the correct solution.

The rate is 3 feet per yard.

$21 \cdot 3 = 63$

So, the length in yards is 63 yards.

Possible explanation: Glen used the incorrect rate and unit rate. He should have used the rate $\frac{1}{3}$ yard per foot. Then $21 \cdot \frac{1}{3} = 7$, so the length is 7 yd.

6 A mouse runs 24 meters in 8 seconds. Tell whether each statement is *True* or *False*.

	True	False
a. At this rate, the mouse runs 30 meters in 14 seconds.	○	●
b. The mouse runs faster than a mouse who runs 26 meters in 13 seconds.	●	○
c. At this rate, it takes the mouse 5 seconds to run 15 meters.	●	○
d. The mouse runs at a rate of $\frac{1}{3}$ second per meter.	●	○

383

DIFFERENTIATION

RETEACH

Hands-On Activity

Make a model to show the relationships between unit conversions and unit rates.

Students approaching proficiency with using unit rates to solve problems will benefit from modeling the process for finding unit rates and using unit conversions.

Materials For each group: 15 sticky notes

- Write and display: *Ayana buys 9 feet of wood for \$2.88. Dara buys 4 yards of wood for \$4.20. Who got the better deal?* Discuss with students that they will need to compare unit prices.
- Have one group find the rate of dollars per foot and the other group find the rate of dollars per yard.
- Give the group that is finding the rate of dollars per foot 4 sticky notes. Have them find the unit conversion. Students should label each sticky note using the unit conversion to find the number of feet equal to 4 yards.
- Repeat with the group that is finding the rate of dollars per yard. Ask: *How many yards will each sticky note represent? How do you know?* [$\frac{1}{3}$ yard; $\frac{1}{3}$ yard is the same as 1 foot.] Have the group model 9 feet in sticky notes. Ask: *What is this length in yards?* [3 yards]
- Next, have each group calculate the unit price. [Ayana: unit price for 1 foot is \$0.32; unit price for 1 yard is \$0.96; Dara: unit price for 1 foot is \$0.35; unit price for 1 yard is \$1.05.]
- Ask: *Who got the better deal and why?* [Ayana; She paid less per foot (or yard).]
- Discuss how students could build similar models to compare prices: *Which is the better deal: 6 pounds of red grapes for \$11.52 or 64 ounces of green grapes for \$8.32?* [Red: \$1.92 per pound or \$0.12 per ounce; Green: \$2.08 per pound or \$0.13 per ounce; Red grapes are the better deal.]

7 Students may find the unit price, which is dollars per ounce, and divide the price by the number of ounces: $2.25 \div 5 = 0.45$. *DOK 1*

8 Students may realize that this problem does not require calculations. Students should reason about the given rates to make a comparison. It may help some students to act out each rate and move 2 feet per second or 2 seconds per foot. *DOK 3*

CLOSE EXIT TICKET

9 **Math Journal** Look for understanding that the rate given can be used to find the unit rate, which can be used to determine a fair price.

Error Alert If students find the unit rate for ride tickets per dollar, then have them think about why using the unit rate for dollars per ticket might be more helpful in this problem. While rates can work in either direction, sometimes the calculations or results are easier to interpret in one direction than the other.

End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting that they use one of the unit rates from the lesson to explain how it is related to a ratio.

SELF CHECK Have students review and check off any new skills on the Unit 4 Opener.

7 Soledad buys 5 ounces of frozen yogurt for \$2.25. What is the unit price of the frozen yogurt in dollars per ounce?

0 . 4 5

8 Which rate is faster, 2 feet per second or 2 seconds per foot, or are both rates the same? Explain.

2 feet per second; Possible explanation: The rate 2 feet per second means you move 2 ft in 1 s. The rate 2 seconds per foot means you take 2 s to move 1 ft, or 4 s to move 2 ft. So, you move a distance of 2 ft more quickly with the first rate. That means the first rate is faster.

9 **Math Journal** At a county fair, a strip of 20 ride tickets costs \$13. The manager thinks they should also sell a strip of 8 ride tickets. Describe how the manager can use a unit rate to decide on a fair price for the strip of 8 tickets.

Possible answer: The rate for the strip of 20 ride tickets is $(13 \div 20)$ dollars per ticket. That is a rate of \$0.65 per ticket. To decide on a fair price for the strip of 8 tickets, the manager can multiply 8 by the unit rate, 0.65; $8 \times 0.65 = 5.20$. So, \$5.20 is a fair price for the strip of 8 tickets.

End of Lesson Checklist

- ☐ **INTERACTIVE GLOSSARY** Find the entry for *unit rate*. Give an example of a unit rate and tell how the unit rate is related to a ratio.
- ☐ **SELF CHECK** Go back to the Unit 4 Opener and see what you can check off.

REINFORCE

Problems 4–8

Solve problems with unit rates to convert measurements.

Students meeting proficiency will benefit from additional work with using unit rates to solve problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND

Challenge

Solve rate problems involving conversions between systems.

Students extending beyond proficiency will benefit from solving rate problems with multiple conversions.

- Have partners research conversion rates to solve this problem: *A car travels 55 miles per hour. What is this speed in kilometers per second, rounded to the nearest thousandth?*
- Some students may first convert miles to kilometers, and then convert hours to seconds. Others may make all conversions at once. [0.025 kilometer per second]
- Repeat with solving the following problem: *A small pool holds 3,785 liters of water. Water flows through a hose at a rate of 1 gallon per minute. About how many hours will it take to fill the pool?* [about $16\frac{2}{3}$ hours]

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.

LESSON 18

Overview | Use Percents to Solve Problems

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

2 Reason abstractly and quantitatively.

5 Use appropriate tools strategically.

* See page 1q to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Use percents to compare ratios.
- Find a percent of a number.
- Find what percent one number is of another number.
- Find the whole, given a part and the percent.

Language Objectives

- Compare ratios using the term *percent*.
- Justify reasoning about percents by explaining how a model represents a percent of a number during partner and class discussion.
- Interpret problems involving percents by identifying the whole, a given part, and the percent.
- Listen to an explanation and confirm understanding by paraphrasing the speaker's ideas and checking that the paraphrase is correct.

Prior Knowledge

- Use ratio and rate reasoning to solve problems.
- Understand the meaning of a percent as a rate per 100.
- Draw and interpret models for percents.
- Represent a percent as a fraction or decimal.

Vocabulary

Math Vocabulary

There is no new vocabulary. Review the following key terms.

percent per 100. A percent is a rate per 100. A percent can be written using the percent symbol (%) and represented as a fraction or decimal. For example, 15% can be represented as $\frac{15}{100}$ or as 0.15.

Academic Vocabulary

survey an activity in which people are asked a question or questions in order to gather information.

Learning Progression

Earlier in Grade 6, students came to understand unit rate, and they learned how to use ratio and rate reasoning to solve problems.

In the previous lesson, students learned that percents are a rate per 100. They visualized percents with models and understood relationships between percents, fractions, and decimals.

In this lesson, students learn that percents are one way to compare two part-to-whole ratios with different first or second quantities. They find a percent of a number, as well as the whole when a part and a percent are known. They apply these concepts and skills as they solve problems.

In Grade 7, students will apply their knowledge of percent as they solve a variety of multi-step problems including problems that involve percent increase or percent decrease. A strong understanding of percent will prepare them to reason about percents greater than 100%, understanding, for example, that 5% more than a number is the same as 105% of that number.

Pacing Guide

Items marked with are available on the **Teacher Toolbox**.

	MATERIALS	DIFFERENTIATION
SESSION 1 **Explore** Percent Problems (35–50 min)		
• **Start** (5 min) • **Try It** (5–10 min) • **Discuss It** (10–15 min) • **Connect It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 401–402)	**Math Toolkit** double number lines, grid paper, hundredths grids Presentation Slides	**PREPARE** Interactive Tutorial **RETEACH or REINFORCE** Visual Model **Materials** For display: Activity Sheet *Aniyah's Oware Record*
SESSION 2 **Develop** Finding a Percent of a Quantity (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 407–408)	**Math Toolkit** base-ten grid paper, double number lines, fraction bars, hundredths grids Presentation Slides	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: scissors, Activity Sheets *Percent Match*; *Hundredths Grids* **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 3 **Develop** Finding the Whole (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 413–414)	**Math Toolkit** base-ten grid paper, double number lines, fraction bars, hundredths grids Presentation Slides	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each student: 10 unit cubes, Activity Sheet *Tenths Grid: Large* **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 4 **Refine** Using Percents to Solve Problems (45–60 min)		
• **Start** (5 min) • **Monitor & Guide** (15–20 min) • **Group & Differentiate** (20–30 min) • **Close: Exit Ticket** (5 min)	**Math Toolkit** Have items from previous sessions available for students. Presentation Slides	**RETEACH** Hands-On Activity **Materials** For each student: 10 sticky notes or index cards **REINFORCE** Problems 4–8 **EXTEND** Challenge **PERSONALIZE** i-Ready
Lesson 18 Quiz or **Digital Comprehension Check**		**RETEACH** Tools for Instruction **REINFORCE** Math Center Activity **EXTEND** Enrichment Activity

Connect to Culture

➤ **Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.**

SESSION 1 ■ □ □ □

Try It Oware (oh-WAH-ruh or oh-WAH-ree) is a traditional game for the Ashanti people of Ghana. Similar to mancala, two players move playing pieces from cup to cup around the game board. The goal is to capture more pieces than your opponent. Because Oware is simple to learn and also involves strategy, it is a game that both children and adults enjoy. Oware can be highly competitive and social, with onlookers cheering for the players. Every year there are national and international Oware tournaments. Have the class create a list of popular games.

SESSION 2 ■ ■ □ □

Try It Ask students to raise their hands if they play an instrument or sing in a choir or chorus. Explain that some schools or community centers offer individual or group lessons. After Try It, students might calculate the percent of the class that plays an instrument or sings.

Apply It **Problem 8** Many water safety courses focus on the skills lifeguards and caregivers need. These safety courses typically involve multiple sessions over several days, and people enrolled in the course must pass a written test at the end. A water safety course may cover how to be safe in the water, whether a person is at a pool or even in a bathtub. Have students share what they know about lifeguarding or other safety courses.

SESSION 3 ■ ■ ■ □

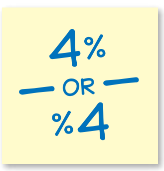
Try It Cross-country bike races are popular in many parts of the country. Bike riders may start at different times, but they travel the same course. During the race they may skid, fall, or need to change tires. The rider who completes the course in the least amount of time wins. Ask students what they can tell about cross-country bike races from the picture and/or about their own experiences with bike races.

SESSION 4 ■ ■ ■ ■

Apply It **Problem 8** Baby seahorses are also known as fry. It is the responsibility of male seahorses to carry fry in their pouches, up to 1,500 at a time! The tiny babies are released into the water and are immediately able to live on their own. Ask students to share what else they know about seahorses or other aquatic animals.

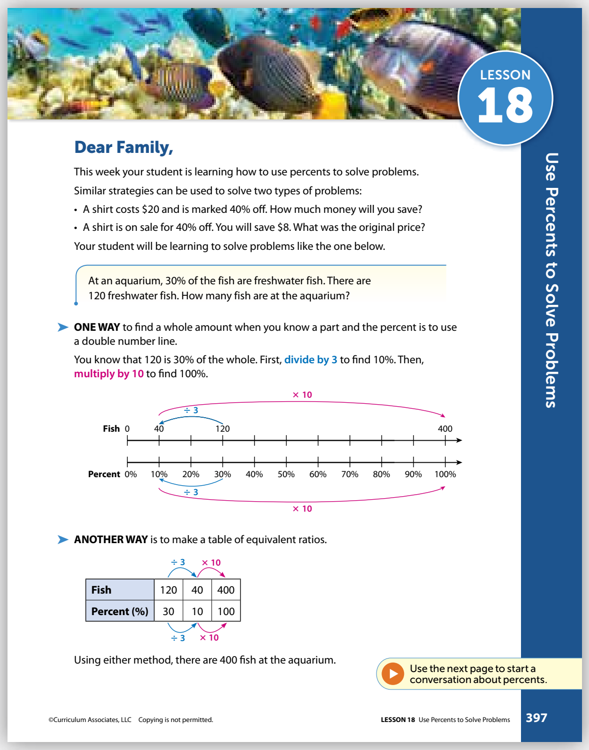
CULTURAL CONNECTION

Alternate Notation In some Middle Eastern cultures, the percent symbol is written before the number, as in %4. There are also cultural differences in the amount of space between the number and the percent symbol. Encourage students who have experience with this notation to share what they know with the class.



Connect to Family and Community

➤ **After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.**



LESSON 18

Use Percents to Solve Problems

Dear Family,

This week your student is learning how to use percents to solve problems.

Similar strategies can be used to solve two types of problems:

- A shirt costs $20 and is marked 40% off. How much money will you save?
- A shirt is on sale for 40% off. You will save $8. What was the original price?

Your student will be learning to solve problems like the one below.

At an aquarium, 30% of the fish are freshwater fish. There are 120 freshwater fish. How many fish are at the aquarium?

➤ **ONE WAY** to find a whole amount when you know a part and the percent is to use a double number line.

You know that 120 is 30% of the whole. First, **divide by 3** to find 10%. Then, **multiply by 10** to find 100%.

➤ **ANOTHER WAY** is to make a table of equivalent ratios.

Fish	120	40	400
Percent (%)	30	10	100

Using either method, there are 400 fish at the aquarium.

Use the next page to start a conversation about percents.

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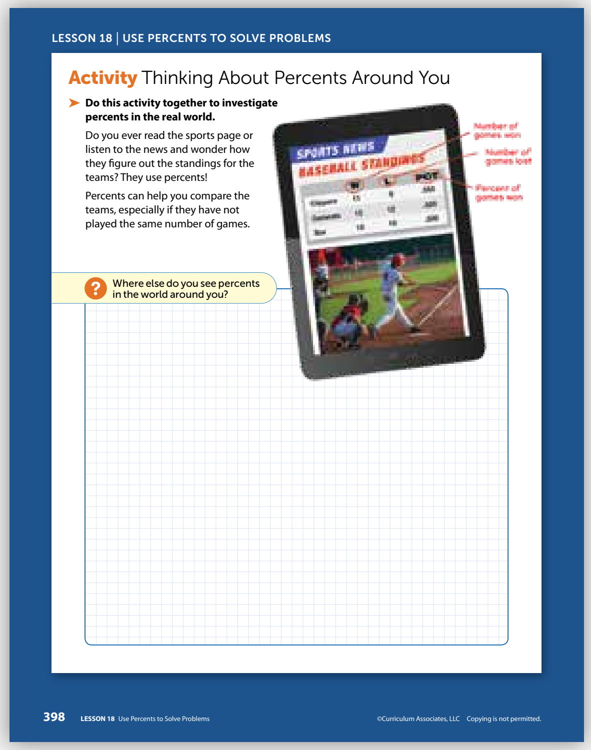

LESSON 18 | USE PERCENTS TO SOLVE PROBLEMS

Activity Thinking About Percents Around You

➤ **Do this activity together to investigate percents in the real world.**

Do you ever read the sports page or listen to the news and wonder how they figure out the standings for the teams? They use percents!

Percents can help you compare the teams, especially if they have not played the same number of games.

Where else do you see percents in the world around you?

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Connect to Language

➤ **For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.**

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 1** **Connect It**

Levels 1–3: Speaking/Writing

Prepare students to talk and write about Connect It problem 2. Read the problem aloud. Review the phrases *winning record* and _____ *out of* _____ *games.* Guide students to indicate which number represents a *part* and which number represent the *whole*. Use **Co-Constructed Word Bank** and include these phrases if students do not mention them.

Facilitate a small-group discussion about the table of equivalent ratios. Encourage students to use terms from the word bank in their responses. Provide frames to help students write their answers:

- *[Name] wins* _____ *out of* _____*. This is* _____.
- *The friend with the best* _____ *is* _____.

Levels 2–4: Speaking/Writing

Prepare students to talk and write about Connect It problem 2. Read the problem and help students discuss the meaning of *winning record*. Ask: *What does a winning record tell?* Create a **Co-Constructed Word Bank** with these and other words from the problem. Then have them turn to partners to talk about the problem. Ask: *What do the friends keep track of? How do they compare their winning records? How do you find each percent? What do you do to compare?* Allow think time for students to process ideas and complete the problem. Guide students to use *won* to explain their answers:

- *[Student name] has the best winning record because [student name] won* _____.

Levels 3–5: Speaking/Writing

Prepare students to talk and write about Connect It problem 2. Have students work with a partner to make sense of the problem and talk about the meaning of *winning record*. Then have partners begin a **Co-Constructed Word Bank** of important words that might be used to talk and write about the problem. Compile the terms into a class bank. Allow think time for students to discuss the problem. Then have them work on problems 2a–c individually. Use **Stronger and Clearer Each Time** to help students respond to problem 2d. When students finish drafting, explaining, and revising their answers, have them share the verbs they used to explain.

Explore Percent Problems

Purpose

- **Explore** the idea that you can use percents to compare ratios.
- **Understand** that writing ratios as percents is one way to compare the ratios.

START CONNECT TO PRIOR KNOWLEDGE

Which One Doesn't Belong?

A: 0.8	B: $\frac{16}{20}$
C: 80%	D: 80

Possible Solutions

A is the only number written as a decimal.

B is the only fraction.

C is the only number written as a percent.

D is the only number not equivalent to 0.8.

WHY? Support students' facility with finding equivalent amounts.

Explore Percent Problems

Previously, you learned about representing percents. In this lesson, you will learn about solving problems with percents.

Use what you know to try to solve the problem below.

Carolina and Aniyah are playing in an Oware tournament. Who has the better winning record so far?

Wins

Carolina	Aniyah
19 out of 25 games	17 out of 20 games

TRY IT **Math Toolkit** double number lines, grid paper, hundredths grids

Possible work:

SAMPLE A

Carolina		
Number of Wins	Total Games	
19	25	× 4
76	100	

Aniyah		
Number of Wins	Total Games	
17	20	× 5
85	100	

If each girl plays 100 games, then based on their winning rates so far, Aniyah will win more games (85) than Carolina (76).

Aniyah has the better winning record so far.

SAMPLE B

Look at the fraction of games played that each girl has won.

Carolina: $\frac{19}{25} = \frac{76}{100}$ Aniyah: $\frac{17}{20} = \frac{85}{100}$

Since $\frac{85}{100} > \frac{76}{100}$, Aniyah has won a greater fraction of her games.

Aniyah's winning record is better than Carolina's winning record.

DISCUSS IT

Ask: How did you decide which player has the better winning record?

Share: At first, I thought . . .

Learning Targets SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6

Use ratio and rate reasoning to solve real-world and mathematical problems.

- Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percent.

TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Three Reads** to help them make sense of the problem and the information in the picture. After the first read, ask students what they know about Oware or mancala. After the second read, clarify the term *winning record* as needed. For the third read, ask: *Do you think the total number of games that each person played will be important? Why or why not?*

DISCUSS IT SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding of:

- 19 and 17 as the number of wins for each player.
- 25 and 20 as the total number of games played for each player.
- the need to use equivalent ratios to find who has the better winning record.

Common Misconception Listen for students who compare the number of wins without considering the total number of games. They may say Carolina has the better record because 19 > 17, or that Aniyah has the better record because she lost only 3 games. As students share their strategies, ask them to apply their reasoning to an example with more or fewer total games, such as 19 wins out of 90 games or 3 wins out of 4 games.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- drawings that compare parts to wholes
- (misconception) strategies that compare the number of wins without considering the total number of games
- tables or double number lines that show who has more wins when the number of games is the same
- equations that convert winning records to ratios that can be compared

Facilitate Whole Class Discussion

Call on students to share selected strategies. Prompt students to participate actively, such as by looking at the speaker and asking clarifying questions.

Guide students to **Compare and Connect** the representations. If the discussion lags, have students turn and talk about what is similar in the strategies before resuming the class discussion.

ASK *How do [student name]'s and [student name]'s models show equivalent ratios?*

LISTEN FOR The models show how the number of wins compare when the total number of games played is the same.

CONNECT IT

SMP 2, 4, 5

1 Look Back Look for understanding that the two winning records are ratios that need to be compared and that, in this problem, finding equivalent part-whole ratios with the same whole allows for comparison.

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Model equivalent ratios.

If students are unsure about equivalent ratios, then use this activity to help them visualize equivalent ratios.

Materials For display: Activity Sheet *Aniyah's Oware Record*

- On the board, draw a table with columns labeled *Games Won* and *Games Played*. Refer to Try It and have a volunteer record the number of games Aniyah won and played. [17; 20] Ask: *What is the ratio of games won to games played?* [17 to 20]
- Display the Activity Sheet, showing only the rectangle that is divided into 20 equal parts. Ask a volunteer to shade the rectangle to show the number of games Aniyah won. [Shade 17 parts.]
- Now show the rectangle that has 40 equal parts. Ask: *Suppose Aniyah played 40 games. How many parts should you shade to show an equivalent ratio? How do you know?* [34; There are twice as many equal parts as in the first rectangle, so twice as many parts should be shaded.]
- Have a volunteer shade the rectangle to show the ratio 34 : 40 and then record the ratio in the table.
- Repeat the activity using the rectangles that are divided into 60, 80, and 100 equal parts to have volunteers show equivalent ratios for 60, 80, and 100 games played. [51 : 60, 68 : 80, 85 : 100]
- Extend by having students identify which ratio shows the percent of games Aniyah won. [85 : 100 shows that she won 85% of her games.]

LESSON 18 | SESSION 1

CONNECT IT

1 Look Back Does Carolina or Aniyah have the better winning record? Explain how you know.

Aniyah; Possible explanation: Write ratios of games won to games played, 19 : 25 and 17 : 20. Write equivalent ratios with the same second quantity, 76 : 100 and 85 : 100. Since 85 > 76, Aniyah has the better winning record.

2 Look Ahead Carolina, Aniyah, and their friend Keith keep track of how many Oware games they play and how many games they win during one month. They can use percents to compare their winning records.

a. Carolina wins 77 out of 100 games. What percent of her games does she win? Explain how you know.

77%; Possible explanation: Carolina's winning rate is $\frac{77}{100}$, or 77%.

b. Kyle wins 14 out of 20 games.

$\frac{14}{20} = \frac{\boxed{70}}{100}$, so Kyle wins 70 % of his games.

c. Aniyah wins 32 out of 40 games. Complete the table of equivalent ratios.

$\div 4$ $\times 10$

Part	32	8	80
Whole	40	10	100

$\div 4$ $\times 10$

What is 32 out of 40 games expressed as a percent? 80%

d. Who has the best winning record? Explain how you know.

Aniyah; Aniyah won the greatest percent of her games.

3 Reflect How can writing ratios as percents help you compare the ratios?

Possible answer: When you write a ratio as a percent, the second quantity, or whole, will be 100. Ratios with the same second quantity are easier to compare.

400

2 Look Ahead Point out that rates can be compared using equivalent ratios and percents. Students should recognize that finding what percent one number is of another can be thought of as finding an equivalent fraction with a denominator of 100, or as finding an equivalent part-to-whole ratio where the whole is 100.

CLOSE EXIT TICKET

3 Reflect Look for understanding that ratios with the same whole are easier to compare and that *percent* means *per 100* or *for every 100*.

Common Misconception If students' explanations do not show clear understanding that writing ratios as percents allows them to find ratios with the same second quantity, then provide hundredths grids and have students compare ratios, such as 6 out of 20 and 4 out of 10, by representing each as a percent.

Prepare for Using Percents to Solve Problems

Support Vocabulary Development

Assign **Prepare for Using Percents to Solve Problems** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *percent* by considering its relationship to rates and ratios. Provide support as needed, having students use previous knowledge of ratios represented as fractions or decimals.

Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers and prompt a whole-class comparative discussion of the words, illustrations, examples, and non-examples given.

Have students look at the model in problem 2 and discuss with a partner how a bar model can be used to represent a percent. Encourage students to think about how bar models and hundredths grids can be used to illustrate ratios and fractions.

Problem Notes

1. Students should understand that percents represent a part-to-whole ratio where the whole is 100. Student responses may include a percent illustrated using a hundredths grid, or examples of how decimals or fractions can be equivalent to a percent, such as 0.1 or $\frac{1}{10}$ being equal to 10%. Students should recognize that a percent is a rate per 100.

2. Students should recognize that 8 out of 10, 80 out of 100, and 80% are all equivalent representations of the same ratio relationship.

LESSON 18 | SESSION 1 Name:

Prepare for Using Percents to Solve Problems

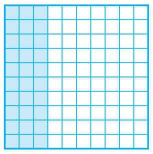
1. Think about what you know about percents. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

Possible answers:

In My Own Words

A percent compares a number of parts to a whole of 100 equal parts. A percent is a rate per 100.

Illustration



30%

percent

Examples

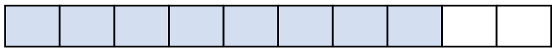
25 parts out of 100 parts is 25%.

The decimal 0.1 represents 10%, because $0.1 = \frac{1}{10}$, or $\frac{10}{100}$.

Non-Examples

35 parts out of 50 parts is not 35% because the second quantity in the ratio must be 100.

2. Explain how the model shows 80%.



Possible explanation: The model shows that 8 of the 10 equal parts are shaded. A ratio of 8 out of 10 is equivalent to 80 out of 100, which is 80%.

401

REAL-WORLD CONNECTION

Scientists use percents to allow them to compare data from samples of different sizes. For example, public health researchers study Ebola recovery rates in West Africa. Researchers collect recovery data on people with different species of the virus. Since there are different numbers of people infected with different species, converting their data to percents allows scientists to compare the recovery rates for different species and use their findings to inform public health protocols. Ask students to think of other real-world examples of when converting to percents might be useful.

Additional Practice

3 Problem 3 provides another look at comparing ratios. This problem is similar to the problem about the Oware tournament. In both problems, two part-whole ratios with different wholes are given. The question asks for a comparison in order to find which group is more in favor of the after-school club.

Students may want to use tables, double number lines, grid paper, or hundredths grids to solve.

Suggest that students use **Three Reads**, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

3 In a survey, 13 out of 20 teachers respond *yes* to a proposal for a new after-school club. In the same survey, 37 out of 50 students respond *yes*.

a. Which group is more in favor of the new after-school club, *teachers* or *students*? Show your work.

Possible work:

	Teachers		
	Said "Yes"	Number Surveyed	
× 5	13	20	× 5
	65	100	

	Students		
	Said "Yes"	Number Surveyed	
× 2	37	50	× 2
	74	100	

If you surveyed 100 teachers and 100 students, then based on these rates, 65 teachers would say "yes" and 74 students would say "yes."

$74 > 65$

SOLUTION Students are more in favor of the new club than teachers.

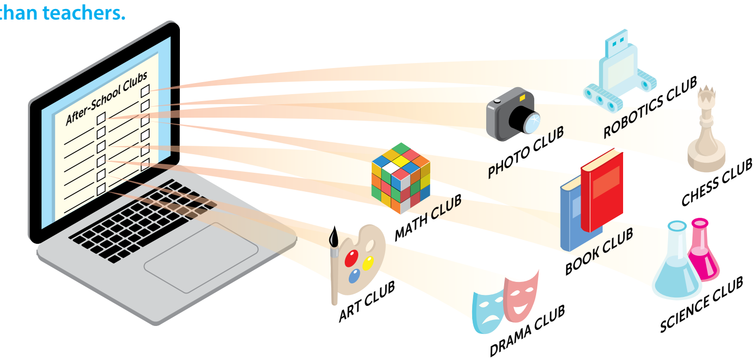
b. Check your answer to problem 3a. Show your work.

Possible work:

Teachers: $\frac{13}{20} \times \frac{5}{5} = \frac{65}{100}$

Students: $\frac{37}{50} \times \frac{2}{2} = \frac{74}{100}$

$\frac{74}{100} > \frac{65}{100}$, so students are more in favor of the after-school club than teachers.



DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** **Discuss It**

Levels 1–3: Listening/Speaking

Support students as they talk about their models with a partner in Discuss It. Provide sentence frames to help students explain their model to a partner:

- *My model shows ____.*
- *The total is____.*
- *This part means ____.*
- *So, ____ students participate.*

Have students write down important words and ideas in the speaker's message. Allow think time for the listeners to craft a paraphrase, and then have them share the paraphrase with the speaker. Have the speaker rate the paraphrase with thumbs up or thumbs down.

Levels 2–4: Listening/Speaking

Support students as they talk about their models with a partner in Discuss It. Before students engage in partner discussion, display these suggestions for effective listening:

- *Use engaged body language.*
- *Ask questions to clarify areas of confusion.*
- *Restate the message and check understanding.*

Encourage students to take notes as they listen to their partner and to use their notes to paraphrase the speaker's message. Provide a frame to help students confirm understanding:

- *I think you said ____. Is that correct?*

Levels 3–5: Listening/Speaking

Support students as they talk about their models with a partner in Discuss It. Before students engage in partner discussion, ask them to generate a list of characteristics of good listeners. Facilitate a small-group discussion, and record student ideas. As students share ideas, paraphrase their responses and confirm understanding with the speaker.

Organize students into partners to explain their models. Encourage students to employ the characteristics of good listeners. Call on volunteers to share ways that their partners demonstrated good listening skills.

Develop Finding a Percent of a Quantity

Purpose

- **Develop** strategies for finding a percent of a quantity.
- **Recognize** that you can use models to find a percent of a quantity, or you can write the percent as a fraction or decimal and multiply.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

A: $\frac{100}{200}$	B: 0.5
C: 50%	D: $\frac{10}{20}$

Possible Solutions

All have the same value.

A and D are both fractions.

B is the only decimal.

C is the only percent.

WHY? Support students' facility with recognizing relationships between fractions, decimals, and percents.

DEVELOP ACADEMIC LANGUAGE

WHY? Model effective listening skills by paraphrasing and confirming understanding.

HOW? Have students share ways they try to understand another person's ideas during a discussion. Then introduce the idea that the listener can take notes as the speaker talks and use the notes to paraphrase the explanation. It may be helpful to model this process in a think-aloud as a volunteer explains his or her ideas. Then, encourage students to confirm understanding by asking, *Did I understand what you said correctly?* Have students practice good listening skills as they share explanations to Connect It problem 2 with a partner.

TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Say It Another Way** to help them make sense of the problem. Listen for understanding that in this problem, 800 is the whole.

LESSON 18 | SESSION 2

Develop Finding a Percent of a Quantity

Read and try to solve the problem below.

At Gordon Middle School, 75% of the 800 students participate in the music program. How many students participate in the music program?

TRY IT

Math Toolkit base-ten grid paper, double number lines, fraction bars, hundredths grids

Possible work:

SAMPLE A

0 ... 560 ... 800

80	80	80	80	80	80	80	80	80	80

0% 10% 20% 30% 40% 50% 60% 70% 80% 90% 100%

7 groups of 80 is 560. Add half a group, which is 40.

$560 + 40 = 600$

So, 600 students participate in the music program.

SAMPLE B

Students 0, 200, 400, 600, 800

Percent 0%, 25%, 50%, 75%, 100%

75% lines up with 600.

There are 600 students who participate in the music program.

DISCUSS IT

Ask: How does your model show 75%?

Share: My model shows 75% by . . .

403

DISCUSS IT SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *Why did you choose the model or strategy you used?*
- *How did your model help you make sense of the amounts in the problem?*

Error Alert If students identify 75% as 75 out of 100 and do not take into consideration that they are finding 75% of 800 students, then remind them to think about the part and the whole in the problem.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- hundredths grids that represent the problem
- bar model representations that show parts, percents, and the whole
- equations that show multiplying 800 by 75% (represented as a fraction or a decimal)

Facilitate Whole Class Discussion

Call on students to share selected strategies. Remind listeners that when something is unclear during a discussion, they can try to paraphrase to check if they understood the speaker's ideas.

Guide students to **Compare and Connect** the representations. To emphasize a key mathematical idea, call on two or three others to rephrase so that students hear the idea in more than one way.

ASK *How does [student name]'s model show the number of students at the school? The number of students in the music program?*

LISTEN FOR 75% is equivalent to 75 out of 100, or 75 hundredths. Representations may include shading 75 out of 100 squares 8 times or finding the product of 0.75 and 800.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK *What is the whole in the hundredths grids? In the expression? Is it the same or different? What does it represent?*

LISTEN FOR 800 is the whole, or total number of students in both representations. It represents the total number of students in the school.

For the hundredths grids, prompt students to identify how hundredths grids are used to represent the problem.

- *What does the shading on the hundredths grids represent?*
- *How can you use the hundredths grids to find the number of students in the music program?*

For the expression, prompt students to connect the words to the numerical expression.

- *Why is 75% rewritten as a fraction?*
- *What operation is used to find the number of students who participate in music? Why?*

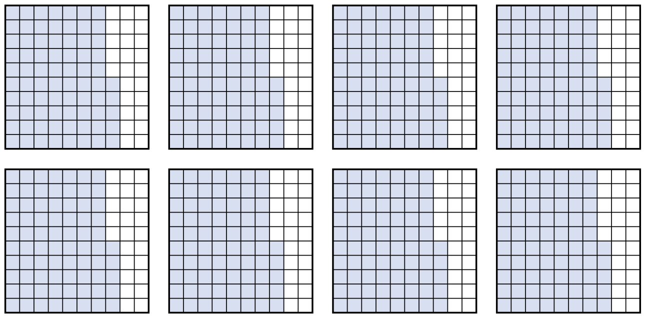
➤ **Explore different ways to understand finding a percent of a quantity.**

At Gordon Middle School, 75% of the 800 students participate in the music program. How many students participate in the music program?

Picture It

You can use hundredths grids to show a percent of a number.

75% means 75 out of every 100, and there are 8 hundreds in 800.



Model It

You can write a multiplication expression to find a percent of a number.

Write the percent as a fraction.

75%	of	800 students
↓	↓	↓
$\frac{75}{100}$	×	800

DIFFERENTIATION | EXTEND

SMP 2

Deepen Understanding

Making Sense of Quantities and the Relationships Between Them

Prompt students to think about the relationship between quantities by having them visualize how the models would reflect changes in the relationship between the quantities.

ASK *What if 100 students move away, but the same percent of the remaining students continue to participate in the music program? How would the hundredths grids change?*

LISTEN FOR There would be fewer total students, but there would also be fewer students who participate in the music program. The number of squares shaded in each hundredths grid would remain the same, but the number of hundredths grids would decrease by one.

ASK *What happen if no one moved away, but enough students left the music program so that only 40% of the students participate in the music program? How would the hundredths grids change?*

LISTEN FOR The number of squares shaded in each hundredths grid would change from 75 to 40. The number of grids would not change.

Develop Finding a Percent of a Quantity

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about how to find a percent of a number.

Before students begin to record and expand on their work in Model It, tell them that problem 4 will prepare them to provide the explanation asked for in problem 5.

Monitor and Confirm Understanding 1 – 3

- The model shows 800 divided into 8 equal groups with 100 in each group.
- For 100 squares to represent 800 students, each square must represent 800 ÷ 100, or 8 students.
- The hundredths grid provides a visual representation for why 75×8 is equivalent to 0.75×800.

Facilitate Whole Class Discussion

4 Look for understanding that a fraction can be written as a whole number times a unit fraction.

> **ASK** *How does writing $\frac{75}{100}$ as $75 \times \frac{1}{100}$ help you show that $\frac{75}{100} \times 800$ is equivalent to 75×8?*
>
> **LISTEN FOR** After rewriting $\frac{75}{100}$ as $75 \times \frac{1}{100}$, you can then multiply $\frac{1}{100}$ by 800 first to get 8, because the order in which factors are multiplied does not change the product.

5 Look for understanding that multiplying by $\frac{p}{100}$ is equivalent to multiplying by 1% and then by p.

> **ASK** *How do you write 1% as a fraction? What is the product of that fraction and p?*
>
> **LISTEN FOR** 1% as a fraction is $\frac{1}{100}$. $\frac{1}{100} \times p = \frac{p}{100}$

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

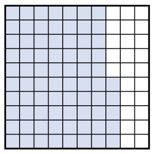
CONNECT IT

➤ **Use the problem from the previous page to help you understand how to find a percent of a quantity.**

1 Look at **Picture It**. Why can you use 8 hundredths grids to show the school's 800 students? How many students does each grid square represent?

The 800 squares show 800 students. Each square represents 1 student.

2 You can also use a single hundredths grid to represent the school's 800 students. How many students does each grid square represent now? What percent of 800 students does each grid square represent? Explain.



8 students; 1%; There are 100 squares, so each square represents 800 ÷ 100, or 8. Each square is 1 part out of 100 equal parts, or 1%.

3 How many students participate in the music program? Explain why you can use the expression 75×8 to find 75% of 800.

600 students; Possible explanation: In the single hundredths grid, each square is 1% of 800, or 8. The 75 shaded squares show 75%, so 75% of 800 is 75×8.

4 Look at **Model It**. How does the hundredths grid in problem 2 represent the expression $\frac{75}{100} \times 800$? Show that the expression is equivalent to 75×8.

$\frac{75}{100}$ of the grid is shaded and the whole grid represents 800.

Possible work: $\frac{75}{100} \times 800 = 75 \times \left(\frac{1}{100} \times 800\right) = 75 \times \frac{800}{100} = 75 \times 8$

5 One way to find p% of a number is to multiply the number by the percent written as the fraction $\frac{p}{100}$. Why does this give the same result as first finding 1% of the number and then multiplying by p?

Possible explanation: To multiply a number by $\frac{p}{100}$, you can divide by 100 to find 1% of the number. Then multiply by p to find p% of the number.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Responses will vary. Check student responses.

405

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Match and illustrate equivalent percents, fractions, and decimals.

If students are unsure how to use fractions and decimals to find the percent of a number, then use this activity to connect percents to fractions, decimals, and visual representations.

Materials For each pair: scissors, Activity Sheets *Percent Match*; *Hundredths Grids*

- Have students cut out the cards from *Percent Match* and sort them into sets with three equivalent ways to represent the same value (fraction, decimal, percent).
- Direct students' attention to the set for 30%. Distribute *Hundredths Grids* and have students shade 30 out of 100 on five of the grids. Ask: *How does this model show $\frac{30}{100} \times 500$?* [There are 5 hundreds in 500. So, shade 30 out of 100 squares 5 times.] *What is 30% of 500?* [150]
- Ask: *If you want to find 30% of 500, what fraction can you multiply 500 by?* $\left[\frac{30}{100}\right]$ *What decimal can you multiply 500 by?* [0.3]
- Extend by having students select another set of an equivalent fraction, a decimal, and a percent. Have them calculate and shade hundredths grids to show that percent of 500.

Apply It

For all problems, encourage students to use a model to support their thinking.

7 Students may also write the equations $0.05 \times 400 = 20$ and $0.95 \times 400 = 380$.

8 **B is correct.** Students may solve the problem by using a hundredths grid or by writing an expression such as 0.8×90 to find 80% of 90.

A is not correct. This answer is 80% of 80 questions.

C is not correct. This answer is the given percent.

D is not correct. This answer is 90% of 80 questions.

Apply It

➤ **Use what you learned to solve these problems.**

7 Bruno is setting up a school garden. His budget is \$400. He spends 5% of the budget on gardening tools. He spends 95% of the budget on plants. How much money does Bruno spend on each? Show your work.

Possible work:

SOLUTION Bruno spends \$20 on tools and \$380 on plants.

8 To pass a test in a water safety course, a student must get 80% of the questions correct. There are 90 questions on the test. How many questions must a student answer correctly to pass the test?

A 64 questions

B 72 questions

C 80 questions

D 81 questions

9 A school will have a fall festival if at least 40% of the 450 students plan to attend. How many students must plan to attend in order for the school to have the festival? Show your work.

Possible work:

40% of 450 students

$\frac{40}{100}$ of 450 students

$0.4 \times 450 = 180$

SOLUTION 180 students must plan to attend the festival.

CLOSE EXIT TICKET

9 Students' solutions should show an understanding of:

- the relationship between percents and fractions or decimals.
- using the part per 100 of the whole quantity to find the percent of a number.

Error Alert If students' solution is 1,800, then review the meaning of percent as a rate where the second quantity is 100. Remind students that 40% expressed as a fraction is $\frac{40}{100}$, and a fraction less than 1 of a total will be less than the total.

Practice Finding a Percent of a Quantity

Problem Notes

Assign **Practice Finding a Percent of a Quantity** as extra practice in class or as homework.

1. **a.** Students may rewrite the fraction $\frac{25}{100}$ as $\frac{1}{4}$. *Basic*

   b. *Basic*

   c. Students should set up and evaluate an appropriate expression (expressing 25% as a fraction or a decimal) to find the solution. *Medium*

   d. Students should compare the result of their multiplication to the bar model in the Example and conclude that they both can be used to find 25% of 500. *Basic*

2. Students should demonstrate understanding that each group represents 25%, so shading 3 groups represents 75%. Each group represents 125 students, so 3 groups represent $3 \times 125 = 375$. *Medium*

3. Students may also use decimals to find $0.30 \times 500 = 150$. *Medium*

LESSON 18 | SESSION 2 Name:

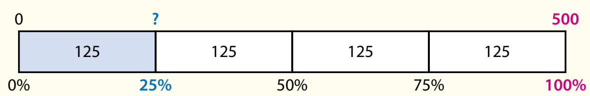
Practice Finding a Percent of a Quantity

➤ **Study the Example showing how to find a percent of a quantity. Then solve problems 1–5.**

Example

There are 500 students who participate in an after-school sports program. Of these students, 25% play field hockey. How many students play field hockey?

You can use a model to find 25% of 500.



The model shows 500 divided into 4 groups of 125. Each group of 125 represents 25% of 500. This means that 25% of 500 is 125.

There are 125 students who play field hockey.

1. **a.** What is 25% written as a fraction? $\frac{25}{100}$, or $\frac{1}{4}$

   b. What is 25% written as a decimal? 0.25

   c. Write and evaluate a multiplication expression that represents 25% of 500.

   Possible work: $0.25 \times 500 = 125$

   d. Compare your answer to problem 1c to the answer in the Example.

   The answers are the same.

2. How could you use the bar model in the Example to find 75% of 500?

   Each group shows 25%, so shade 3 groups to get $3 \times 125 = 375$.

3. Suppose 30% of 500 students play an instrument. Describe one way to find 30% of 500.

   Possible explanation: Divide a bar model for 500 into 10 groups. Each group represents 10%, or 50 people. To show 30%, shade 3 groups which represents 150 students.

Vocabulary

percent

per 100. A percent is a rate per 100. A percent can be written using the percent symbol (%) and represented as a fraction or decimal.

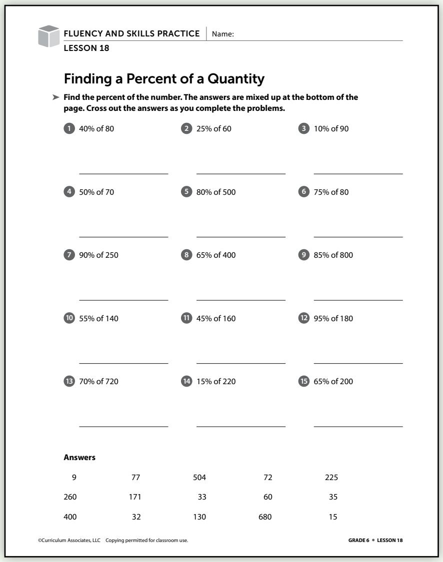
For example, 15% can be represented as $\frac{15}{100}$ or as 0.15.

407

Fluency & Skills Practice

Finding a Percent of a Quantity

In this activity, students practice finding the percent of a number.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 18

Finding a Percent of a Quantity

➤ Find the percent of the number. The answers are mixed up at the bottom of the page. Cross out the answers as you complete the problems.

1. 40% of 80
2. 25% of 60
3. 10% of 90
4. 50% of 70
5. 80% of 500
6. 75% of 80
7. 90% of 250
8. 65% of 400
9. 85% of 800
10. 55% of 140
11. 45% of 160
12. 95% of 180
13. 70% of 720
14. 15% of 220
15. 65% of 200

Answers

9	77	504	72	225
260	171	33	60	35
400	32	130	680	15

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Additional Practice

4 **a.** Students may also write the equation $0.4 \times 300 = 120$ or multiply 300 by $\frac{40}{100}$. ***Medium***

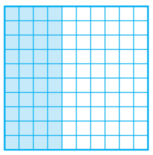
b. Students may also write the equation $0.20 \times 300 = 60$ or multiply 300 by $\frac{20}{100}$. ***Medium***

5 Students may write an expression with a fraction or decimal to find that 55% of 20 is 11, and then subtract 11 from 20 to find how many puzzles are left. Students may also reason that if Magdalena completes 55% of the puzzles, she still has 45% to complete. They could draw a bar model divided into tenths, with each section labeled 2 and 10%, and find that 45% of 20 is 2×4.5, or 9. ***Challenge***

4 The results of a survey show that 40% of 300 students choose recycling as the top priority for their generation.

a. How many students choose recycling? Show your work.

Possible work:



$300 \div 100 = 3$, so each square of the hundredths grid represents 3 students.

$40 \cdot 3 = 120$, so the shaded squares represent 120 students.

SOLUTION 120 students choose recycling.

b. Suppose 20% of 300 students choose recycling. How many students choose recycling? Explain how you found your answer.

60 students; Possible explanation: On a hundredths grid, 20% is shown by 20 squares instead of 40 squares. 20% of 300 is half of 40% of 300, so 20% of 300 is $\frac{1}{2} \cdot 120 = 60$.

5 There are 20 puzzles in Magdalena's puzzle book. Magdalena completes 55% of the puzzles. How many puzzles does Magdalena have left to complete? Show your work.

Possible work:

$0.55 \times 20 = 11$

She completes 6 puzzles.

$20 - 11 = 9$

SOLUTION Magdalena has 9 puzzles left to complete.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** **Apply It**

Levels 1–3: Reading/Speaking

To help students connect Apply It problems 7 and 8, read each problem separately and have students take notes about the information and the question. For problem 7, ask: *What is 150? What is 30%? What do you need to find?*

Repeat with similar questions for problem 8. Guide students to identify the parts and percents in their notes. Then help them tell how they found the whole in each case. Have students turn to partners to connect:

- *Problem _____ is about _____.*
- *Both problems give _____.*
- *Both problems ask for _____.*
- *To find the total, we _____.*

Levels 2–4: Reading/Speaking

To help students connect Apply It problems 7 and 8, read each problem with them. Allow time for students to write down the parts and percents in each case. Ask: *What do you need to find?* Have students turn to partners to discuss their notes. Then have them tell how they found the answers.

Have partners take turns comparing and connecting the problems. Then have them share ideas with the group. Provide sentence starters:

- *Each problem is about _____.*
- *In both cases, _____.*
- *The problems are related because _____.*

Levels 3–5: Reading/Speaking

To help students connect Apply It problems 7 and 8, have them read and take notes about each problem. Then have them turn to partners to **Compare and Connect**. Ask:

- *What is the same about the problems?*
- *What is different?*

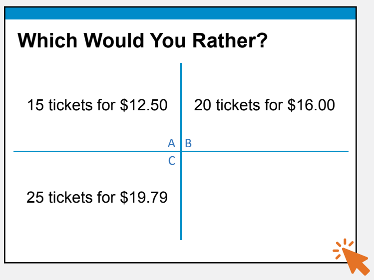
Encourage students to talk about the information, the question, and the steps to solve the problem. Then have them meet with other partners to share and ask each other questions about their connections.

Develop Finding the Whole

Purpose

- **Develop** strategies for finding the whole when a percent and a part are known.
- **Recognize** that you can use equivalent ratios to find the whole.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

A because it is the least total cost.

B because it is a price with dollars only and no cents.

C because it has the lowest unit price per ticket.

WHY? Support students' understanding of comparing ratios.

DEVELOP ACADEMIC LANGUAGE

WHY? Deepen understanding of academic vocabulary by generating synonyms.

HOW? Read Discuss It and have students discuss the question. Call on volunteers to share words that mean the same as *strategy*. [plan, method, procedure, approach] If students struggle to list synonyms, encourage them to consult a thesaurus. Direct students' attention to Discuss It. Ask them to rephrase the question using a synonym for *strategy*.

TRY IT

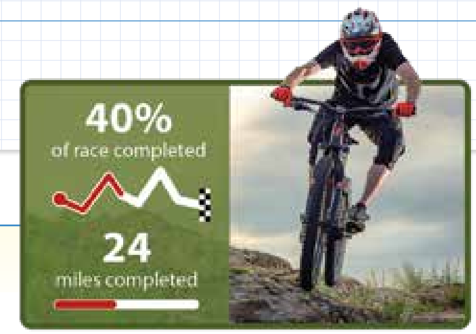
SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Three Reads** to help them make sense of the problem. Draw their attention to the picture to ensure students are noting and interpreting the information correctly.

LESSON 18 | SESSION 3

Develop Finding the Whole



Read and try to solve the problem below.

Akira is checking his progress in a bike race. How many miles is the race?

TRY IT

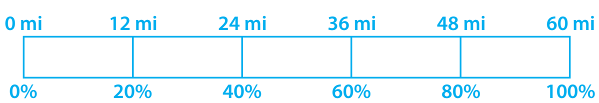
Math Toolkit base-ten grid paper, double number lines, fraction bars, hundredths grids

Possible work:

SAMPLE A

40% is 40 out of 100, and $\frac{40}{100} = \frac{2}{5}$.

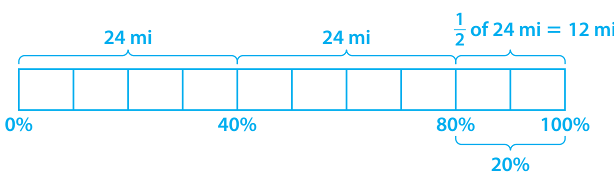
So, $\frac{1}{5}$ is 20%. Divide a bar model into 5 equal parts.



The race is 60 mi long.

SAMPLE B

24 miles is 40% of the race.



$24 + 24 + 12 = 60$

Akira's bike race is 60 mi.

DISCUSS IT

Ask: How is your strategy similar to mine? How is it different?

Share: My model shows . . .

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *How did you know what the problem was asking?*
- *How would you describe your model?*
- *How did your model represent the problem situation?*

Error Alert If students find 40% of 24, then have them identify each known quantity in the problem and the unknown quantity. Have students estimate a reasonable answer for the unknown quantity and explain why it makes sense.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- drawings such as bar models to represent the problem
- double number line representations
- equations with fractions or decimals

Facilitate Whole Class Discussion

Call on students to share selected strategies. Allow think time for students to process the ideas.

Guide students to **Compare and Connect** the representations. If ideas are unclear, ask the speaker or another student to repeat and rephrase.

ASK *How do [student name]'s and [student name]'s models show the number of miles Akira has completed? The total number of miles in the race?*

LISTEN FOR Models should show that 24 miles is 40% of the whole and 60 miles is 100% of the whole.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models and then connect them to the models presented in class.

ASK *What quantity corresponds to 40% in each model? How is 100% shown in each model?*

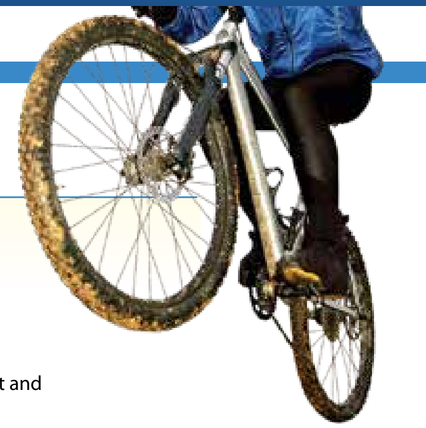
LISTEN FOR Both models show that 40% is 24 and 100% is the whole.

For the double number line model, prompt students to identify how the double number line is labeled to represent the problem.

- *Into how many equal parts is the double number line divided? Why?*
- *What is the greatest percent on the double number line?*
- *Why is the number of miles for 10% shown?*

For the table of equivalent ratios, prompt students to identify the meaning of the ratios in the table.

- *What does the first ratio tell you?*
- *What does the second ratio tell you? Why is this ratio important?*
- *Could another ratio be used to find the total number of miles in the race?*

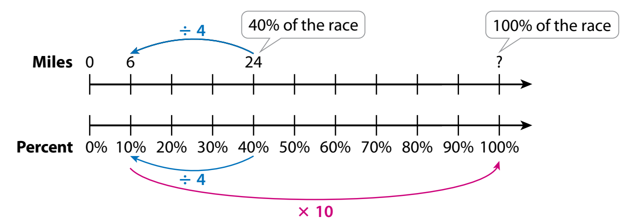


➤ **Explore different ways to understand finding the whole when a part and the percent are given.**

Akira is checking his progress in a bike race. He has biked 24 mi so far, and that is 40% of the race. How many miles is the race?

Model It

You can use a double number line to find the whole when a part and the percent are given.



Model It

You can make a table of equivalent ratios to find the whole when a part and the percent are given.

Start with the ratio 24 : 40.

Miles	Percent (%)
24	40
6	10
?	100

÷ 4   × 10

DIFFERENTIATION | EXTEND

SMP 5

Deepen Understanding
Using Double Number Lines and Tables Strategically

Prompt students to compare the advantages and disadvantages of each model.

ASK *What are the advantages and disadvantages of using a double number line to find the total length of the race? What are the advantages and disadvantages of using a table of equivalent ratios?*

LISTEN FOR The double number line lets you see the relationships between the distances and the percents all the way up to 100%, but it could be difficult to draw precisely. The table lets you find the equivalent ratio with 100 as the second quantity quickly, but it does not show all the percents.

ASK *Which one do you think works better for finding the total length of the race? Why?*

LISTEN FOR The double number line helps me visualize the race, but the table is quicker.

Generalize Encourage students to describe how they might choose an appropriate model when solving a problem. If they understand the problem, they might think about what is most efficient. If they need help understanding the problem, they might want to use a visual representation of the relationship between quantities.

Develop Finding the Whole

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about equivalent ratios and how to find the whole when given a part and the percent.

Before students begin to record and expand on their work in Model It, tell them that problem 4 will prepare them to provide the explanation asked for in problem 5. You may also want to read problem 6 aloud and call on two or three students to rephrase the problem to confirm understanding.

Monitor and Confirm Understanding 1 – 3

- The relationship between 10% and 40% is 1 to 4, or 10 is one fourth of 40.
- To move from 40 to 10, divide by 4. To move from 24 to the unknown distance, divide by 4.
- The relationship between 6 miles and 60 miles is 1 to 10, or 60 is 10 times 6.

Facilitate Whole Class Discussion

4 Students should recognize that this problem asks them to reason about a different problem-solving approach that also involves equivalent ratios.

ASK *Why would Mavis divide 24 by 8? How is this different than the solution shown in the Model Its?*

LISTEN FOR Mavis chose to use 5% instead of 10%. The method of finding the whole is the same.

5 Look for the idea that a percent is a rate per 100.

ASK *How do you write a percent as a ratio? How can you use this to find an equivalent ratio using the part?*

LISTEN FOR You write a percent as a ratio with 100 as the second quantity. Then, you find the unknown whole with the known part.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to find the whole when a part and the percent are given.**

1 Look at the double number line and the table in the **Model Its**. How do both models use the relationship between 40% and 10%?
Possible answer: 10% is $\frac{1}{4}$ of 40%, so the models use dividing 40 by 4 to get to 10%.

2 How many miles make up 10% of the race? Explain how you know.
6 mi; Possible explanation: 10% lines up with 6 miles on the double number line. 6 is also $\frac{1}{4}$ of 24.

3 Suppose Akira completes 100% of the race. How many miles does he bike? Explain how you know.
60 mi; Possible explanation: 100 is 10 times 10. So, multiply the number of miles in 10% of the race, 6, by 10 to get 60.

4 Mavis used another method. First, she divided 24 mi by 8 to get 3 mi. Then, she multiplied 3 mi by 20 to get 60 mi. What percent of the whole race is 3 mi? Why did Mavis multiply 3 mi by 20?
5%; Since $100 = 20 \times 5$, Mavis multiplied 3 mi (5% of the race) by 20 to find the number of miles in 100% of the race.

5 How can you find the whole when you know a part and the percent?
Possible answer: You can use equivalent ratios. The given part and percent form one ratio. Use multiplication and/or division to find the equivalent ratio for which the percent is 100.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you understand finding the whole when you know the part and the percent.
Responses will vary. Check student responses.

411

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Make a model to show the relationship between a part and the whole in a percent problem.

If students are unsure how to find a whole when given a part and the percent, then use this activity to illustrate that the whole is the quantity that corresponds to 100%.

Materials For each student: 10 unit cubes, Activity Sheet *Tenths Grid: Large*

- Distribute the Activity Sheet. Have students label the portions of the grid in order to show percents from 10% to 100%. Then ask students to shade 30%.
- Write and display: *6 is 30% of ____*. Tell students that 6 unit cubes are 30% of a whole grid of cubes. Have students distribute 6 cubes evenly between the 3 shaded portions. Ask: *How many cubes are in each 10% portion? What number is 10% of the total?* [2; 2]
- Have students draw squares in each of the remaining 10% portions to show the whole grid of cubes. Ask: *What is the total number of unit cubes? When 6 is 30% of the whole, what is the whole?* [20; 20] Write 20 in the blank: *6 is 30% of 20*.
- Extend by naming other parts and percents and ask students to find the whole. Prompt students to recognize that when asked to find a whole, the whole is the quantity that corresponds to 100%.

Apply It

For all problems, encourage students to use a model to support their thinking.

7 Students may also draw a percent bar model or make a table of equivalent ratios.

8 **C is correct.** Students may solve the problem by using a double number line or by using a table of equivalent ratios.

A is not correct. This answer is 60% of the 45 points that the home team scores.

B is not correct. This answer is 45 divided by 10% of 60, or 6, incorrectly resulting in 7 before multiplying by 10 to get 70.

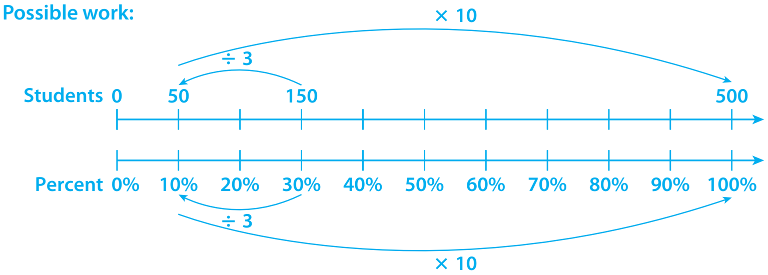
D is not correct. This answer is $75 + (4 \times 45)$ because there are four more jumps of 10% to get from 60% to 100%.

Apply It

➤ **Use what you learned to solve these problems.**

7 At Shaw Middle School, 150 students take part in cleaning up the school. This is 30% of the students that attend the school. How many students attend Shaw Middle School? Show your work.

Possible work:



SOLUTION 500 students attend Shaw Middle School.

8 At a basketball game, the home team scores 60% of the points. The home team scores 45 points. How many points are scored in all?

A 27 points

B 70 points

C 75 points

D 225 points

9 Lamont spends \$120 on groceries. This is 25% of the money he earns this week. How much money does Lamont earn? Show your work.

Possible work: 25% is 25 out of 100 or $\frac{1}{4}$. So, \$120 is $\frac{1}{4}$ of what Lamont earns.

Multiply \$120 by 4 to find the total amount he earns: $4 \times 120 = 480$.

SOLUTION Lamont earns \$480 this week.

CLOSE EXIT TICKET

9 Students' solutions should show an understanding of:

- \$120 as 25%, or one fourth, of the whole.
- the solution as the quantity that corresponds to one whole, or 100%.

Error Alert If students' solution is \$30 (25% of \$120), then prompt them to describe the known quantities. Have them draw a visual model to represent the relationship between the known quantities. Students should recognize that 25% corresponds to \$120. Have them identify where in the model represents the unknown quantity. Students should be able to state that the unknown quantity is the whole, or the amount of money Lamont earned this week.

Practice Finding the Whole

Problem Notes

Assign **Practice Finding the Whole** as extra practice in class or as homework.

1 Students should recognize that they can calculate 100% of Camela's earnings by multiplying 10% of her earnings by 10. ***Basic***

2 Students should understand that they can use equivalent ratios to find the whole. ***Medium***

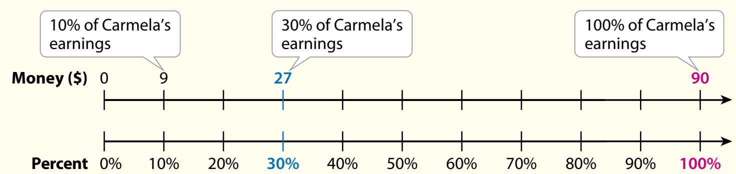
Practice Finding the Whole

➤ **Study the Example showing how to find the whole when a part and the percent are given. Then solve problems 1–5.**

Example

Carmela saves $27. This is 30% of the money she earns. How much does Carmela earn?

You can use a double number line to find the whole when you know a part and the percent. On the number line, 27 lines up with 30%. To find the whole, find the number that lines up with 100%.



Divide $27 by 3 to find 10% of Carmela's earnings: $\$27 \div 3 = \$9$.

Multiply $9 by 10 to find 100% of Carmela's earnings: $\$9 \times 10 = \$90$.

Carmela earns $90.

1 In the Example, why is it helpful to find 10% of Carmela's earnings before finding 100% of her earnings?

You cannot multiply 30 by any whole number to get 100. You can use division to find 10% of her earnings, then multiply by 10 to find 100% of her earnings.

2 Aiden spends $18 on souvenirs during a school trip to New York City. This is 45% of the money he brings on the trip. How much money does Aiden bring on the trip? Show your work.

Possible work:

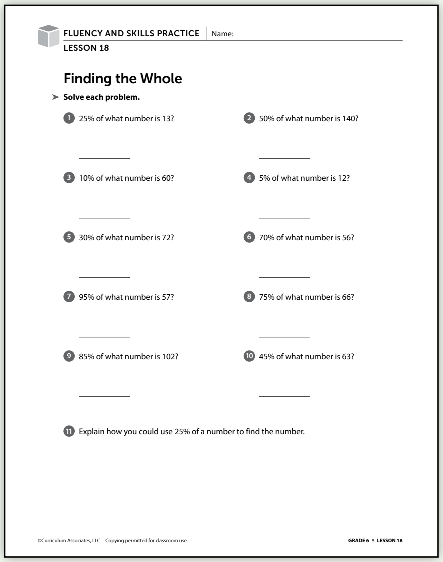
Money ($)	18	2	40
Percent (%)	45	5	100

SOLUTION Aiden brings $40 on the trip.

Fluency & Skills Practice

Finding the Whole

In this activity, students practice finding the whole, given the percent and the part.


FLUENCY AND SKILLS PRACTICE | Name:
LESSON 18

Finding the Whole

➤ Solve each problem.

1 25% of what number is 13?

2 50% of what number is 140?

3 10% of what number is 60?

4 5% of what number is 12?

5 30% of what number is 72?

6 70% of what number is 56?

7 95% of what number is 57?

8 75% of what number is 66?

9 85% of what number is 102?

10 45% of what number is 63?

11 Explain how you could use 25% of a number to find the number.

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Additional Practice

3 Students may also make a table or draw a percent bar model. ***Medium***

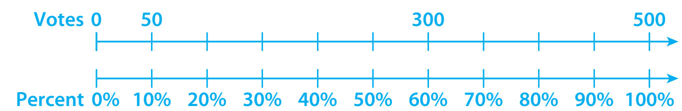
4 Students may also write an expression or draw a percent bar model divided into tenths, with each section labeled 6 and 10%, and find that when 48 is 80%, 60 is 100%. ***Medium***

5 Students may also write an expression or draw a model to represent and solve the problem. ***Challenge***

LESSON 18 | SESSION 3

3 Angel is running for school council president. He receives 300 votes, which is 60% of all the votes. How many students vote in the election? Explain how you found your answer.

500 students; Possible explanation: Use a double number line.



300 ÷ 6 = 50 and 60% ÷ 6 = 10%. 10% of the votes is 50. Multiply 50 by 10 to get 500.

4 Students sell 80% of the books at a book sale. They sell 48 books in all. How many books are at the book sale? Show your work.

Possible work:

	÷ 8	× 10	
Books	48	6	60
Percent (%)	80	10	100
	÷ 8	× 10	

SOLUTION 60 books are at the book sale.

5 Aiyana reads 147 pages of a book. She completes 70% of the book. How many pages does Aiyana still have left to read? Show your work.

Possible work:

Pages	Percent (%)
147	70
21	10
210	100

$210 - 147 = 63$

SOLUTION Aiyana still has 63 pages left to read.

414

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** **Apply It**

Levels 1–3: Reading/Writing

Help students understand and respond to Apply It problem 8. Adapt **Three Reads** by clarifying unfamiliar words and phrases such as *tank, aquarium, male seahorse, eggs,* and *pouches* after the first read. Then ask: *What is the problem about?* Have students turn and talk with a partner after each reading. Call on volunteers to share responses to each question in the routine.

Provide sentence frames to support writing:

- *The expression is incorrect because _____.*
- *The _____ expression is _____.*
- *There are _____ male seahorses with _____.*

Levels 2–4: Reading/Writing

Have students work with a partner to understand Apply It problem 8 and prepare to respond in writing. Adapt **Three Reads** by displaying the questions for each read and having students complete the routine in pairs.

Then display sentence frames for partners to complete by thinking aloud:

- *Kennedy's expression _____ because _____.*
- *Kennedy should have used the expression _____ because _____.*
- *There are _____ with _____.*

Have students draft responses individually and then compare with their partner.

Levels 3–5: Reading/Writing

Support students in understanding Apply It problem 8 and prepare them to respond in writing. Use **Notice and Wonder** to help students make sense of the problem.

Display Kennedy's expression and have them tell what they notice about the expression and what they wonder that can be answered with mathematics.

Use **Stronger and Clearer Each Time** to help students clarify and revise their answers. Encourage students to write a brief paragraph and to use precise language in their explanations. Point out that they may use the conditional verb *should have* in their responses. Allow time for students to think before drafting.

Refine Using Percents to Solve Problems

Purpose

- **Refine** strategies for finding the percent of a number and finding the whole when given a part and a percent.
- **Refine** understanding of finding an equivalent ratio with the second quantity of 100.

START CONNECT TO PRIOR KNOWLEDGE

What is 40% of 25?

Solution

10

WHY? Confirm students' understanding of finding a percent of a number, identifying common errors to address as needed.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.

Refine Using Percents to Solve Problems

➤ **Complete the Example below. Then solve problems 1–9.**

Example

Alison is driving from Houston to Dallas. She drives 180 mi. This is 75% of the trip. What is the distance from Houston to Dallas?

Look at how you could show your work using a table of equivalent part-to-whole ratios.

75% is 75 parts out of a whole made up of 100 equal parts.

÷ 25 × 60

Part (mi)	75	3	180
Whole (mi)	100	4	?

÷ 25 × 60

SOLUTION The distance from Houston to Dallas is 240 mi.

CONSIDER THIS . . . You can find a ratio where the part is 180 and the whole is the distance in miles from Houston to Dallas.

PAIR/SHARE Explain why you cannot solve the problem by multiplying 180 by $\frac{75}{100}$.

Apply It

1 Tarik turns in his third research report. His teacher says that 20% of his reports for the year are done. How many research reports will Tarik complete during the school year? Show your work.

Possible work:

20% is 20 out of 100, or $\frac{1}{5}$.

So, 3 reports is $\frac{1}{5}$ of the total.

$3 \times 5 = 15$

SOLUTION Tarik will complete 15 research reports.

CONSIDER THIS . . . How can representing 20% as a unit fraction help you solve the problem?

PAIR/SHARE Suppose three research reports represent 25% of all the reports for the year. How would your answer change?

START ERROR ANALYSIS

If the error is . . .	Students may . . .	To support understanding . . .
$\frac{25}{40}$ or $\frac{5}{8}$	have used the ratio 25 : 40 rather than using ratios to solve the problem.	Ask students to draw a double number line. Have them divide it into fifths, labeling the bottom line 20%, 40%, 60%, 80%, and 100% and the top line 5, 10, 15, 20, and 25. Prompt students to articulate how the quantities are related.
1	have multiplied 25 by 0.04 or $\frac{4}{100}$.	Ask students to review their answer to see whether it is reasonable. Point out that 40% is a little less than 50%, or $\frac{1}{2}$, and that 40% means $\frac{40}{100}$, or 0.4.
1,000	have multiplied by 40.	Prompt students to recall that 40% means $\frac{40}{100}$, or 0.4, so it is a quantity less than 1. Elicit from students the understanding that when a number is multiplied by a number less than 1, the product will be less than the number. Invite students to test this by solving problems such as 0.5×10 or $\frac{3}{5} \times 50$.

Example

Guide students in understanding the Example. Ask:

- *In the table, what does 75 represent? What does 100 represent?*
- *Why is $\frac{75}{100}$ equivalent to $\frac{3}{4}$?*
- *How do equivalent ratios and percents help you solve this problem? How else might you solve it?*

Help all students focus on the Example and responses to the questions by asking them to agree, disagree, or add on to classmates' responses.

Look for understanding that all ratios of the part to the whole must be equivalent.

Apply It

1. Students may solve the problem by writing 20% as a fraction, such as $\frac{1}{5}$, and using what they know about equivalent ratios to find that Tarik will complete a total of 15 research reports. *DOK 2*

2. Students may solve the problem by writing 5% as a fraction and multiplying by 40. *DOK 2*

3. **C is correct.** Students may solve the problem by multiplying $120 by 0.5 to get $60 and then finding the sum of $120 and $60.

   A is not correct. This answer is the amount raised by Mr. McClary's class.

   B is not correct. This answer adds $120 and 50, which is the percent and not a dollar amount.

   D is not correct. This answer is double the amount that Mrs. Shen's class raised.

   DOK 3

2 Rani takes a 40-question test. She answers 5% of the questions incorrectly. How many questions does she answer incorrectly? Show your work.

Possible work:

$\frac{5}{100} \times 40 = \frac{200}{100}$ or 2

SOLUTION Rani answers 2 questions incorrectly.

CONSIDER THIS . . .
What operation do you use to find a fraction of a number?

PAIR/SHARE
What strategy did you use to solve this problem? Why?

3 Two sixth grade classes are raising money. Mrs. Shen's class raises $120. Mr. McClary's class raises 50% of the amount Mrs. Shen's class raises. How much money do the two classes raise in all?

A $60

B $170

C $180

D $240

Jake chose A as the correct answer. How might he have gotten that answer?

Possible answer: Jake might have found the amount raised by Mr. McClary's class instead of the amount raised by both groups.

CONSIDER THIS . . .
You could use a fraction to help you solve this problem.

PAIR/SHARE
How would you solve the problem if Mr. McClary's class raises 60% of the amount Mrs. Shen's class raises?

GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency
- **RETEACH** Hands-On Activity
- **REINFORCE** Problems 4, 6, 8

Meeting Proficiency
- **REINFORCE** Problems 4–8

Extending Beyond Proficiency
- **REINFORCE** Problems 4–8
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket**.

Resources for Differentiation are found on the next page.

Refine Using Percents to Solve Problems

Apply It

4 **C is correct.** Students may solve the problem by using a table to find an equivalent ratio to $\frac{5}{20}$.

A is not correct. This answer is 5% of 20.

B is not correct. This answer is 20 divided by 5 written as a percent.

D is not correct. This is 5 divided by 20 with 60 added to the percent.

DOK 2

5 Students may write a ratio for each player and then find equivalent ratios where the second part is 100 to compare them. *DOK 2*

6 Students may use a bar diagram or other model to first find the number of minutes that corresponds to 100% if 9 minutes corresponds to 30%. Then subtract 9 from the total time. *DOK 2*

4 What percent of 20 is 5?

A 1%

B 4%

C 25%

D 85%

5 Three basketball players take different numbers of shots during practice. They each record the number of baskets made out of the number of shots taken. The players in order from greatest percent of baskets made to least percent of baskets made are Amare, Diego, and Paula.

Number of Baskets Made

Diego	**21** out of **60** shots
Amare	**16** out of **40** shots
Paula	**17** out of **50** shots

6 Rosa has a limit to the time she may play video games each day. She plays for 9 min on Monday, which is 30% of the time she can play. How many more minutes can Rosa play on Monday? Show your work.

Possible work:

÷ 10 × 3

Part (min)	30	3	9
Whole (min)	100	10	30

÷ 10 × 3

The whole is 30 min.

Subtract the time played from the whole time: $30 - 9 = 21$

SOLUTION Rosa can play for 21 more minutes.

417

DIFFERENTIATION

RETEACH

Hands-On Activity

Make a concrete model to visualize a relationship involving percents.

Students approaching proficiency with finding the whole will benefit from building a concrete model to represent relationships between percents, parts, and the whole.

Materials For each student: 10 sticky notes or index cards

- Write and display: *$12 is 40% of ____*. Discuss with students that $12 is part of a total amount of money and they need to find the total amount.
- Have students place 4 sticky notes in a row. Explain that this represents 40%. Ask: *What percent does each sticky note represent? Why?* [10%; Each sticky note is $\frac{1}{4}$ of 40%.] Have students write 10% on each sticky note.
- Ask: *How many dollars does each sticky note represent? Why?* [$3; Each sticky note is $\frac{1}{4}$ of $12.] Have students write $3 on each sticky note.
- Elicit from students that they can extend the model to show 100% by making a row of 10 sticky notes. Have students place sticky notes to extend the model. Ask: *What is the whole amount of money? Why?* [$30; There are 10 sticky notes, each labeled $3, and $10 \times \$3 = \$30$.]
- Discuss how students can build similar models to find the whole for these two situations: *$24 is 60% of ____* and *$40 is 80% of ____*.
- Have students solve similar problems by using 5 sticky notes and 4 sticky notes to represent 100%.

7 **a.** 80% is 80 out of 100, not 80 out of 90.

b. 45% is 45 per 100; $\frac{45}{100} \times 60 = 27$.

c. 20% and $\frac{1}{5}$ can be written as equivalent fractions, so both values are the same.

d. Multiply 80 by 0.35 or $\frac{35}{100}$; $0.35 \times 80 = 28$, not 25.

DOK 2

8 See ***Connect to Culture*** to support student engagement. Multiply by 0.7 to find that 70% of 20 is 14. *DOK 3*

CLOSE EXIT TICKET

9 **Math Journal** Look for understanding that when given a part and a percent, the whole is the quantity that corresponds to 100%.

Error Alert If students write and solve a problem involving finding the percent of a number (instead of finding the whole), then have them rewrite their problem using a problem in the Student Worktext, such as Session 4 problem 1 or 6, as a guide.

End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting they list what they have learned about percent and then turn and talk to a partner to compare lists.

SELF CHECK Have students review and check off any new skills on the Unit 4 Opener.

7 Tell whether each statement is *True* or *False*.

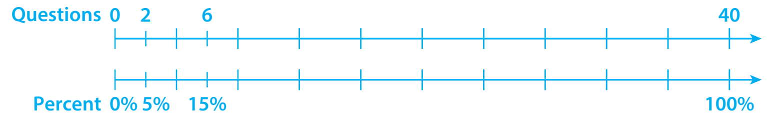
	True	False
a. 80% of 90 is the same as $\frac{8}{9}$ of 90.	○	●
b. 45% of 60 is 27.	●	○
c. 20% of 90 is the same as $\frac{1}{5}$ of 90.	●	○
d. 25 is 35% of 80.	○	●

8 In the seahorse tank at an aquarium, 70% of the male seahorses have eggs in their pouches. There are 20 male seahorses in the tank. Kennedy uses the expression 0.07×20 to find the number of male seahorses with eggs. Explain why Kennedy's expression is incorrect. How many male seahorses have eggs in their pouches?

The expression is incorrect because 70% is 0.70 (or 0.7) as a decimal, not 0.07. The expression should be 0.7×20. There are 14 male seahorses with eggs in their pouches.

9 **Math Journal** Choose one of the following percents: 15%, 35%, or 85%. Write and solve a word problem that uses your percent and involves finding the whole.

Possible answer: On her math test, Ellema gets 6 questions wrong, or 15% of the total questions. How many questions are on the test?



There are 40 questions on the test.

End of Lesson Checklist

☐ **INTERACTIVE GLOSSARY** Find the entry for *percent*. Add two important things you learned about percents in this lesson.

☐ **SELF CHECK** Go back to the Unit 4 Opener and see what you can check off.

REINFORCE

Problems 4–8

Solve percent problems.

Students meeting proficiency will benefit from additional work with percents by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND

Challenge

Solve multi-step percent problems.

Students extending beyond proficiency will benefit from solving multi-step percent problems with an unknown total.

- Have students work with a partner to solve this problem: *25% of 80% of a number is 130. What is the number?*
- Some students may first find that 25% of 80% is 20% and then reason that 130 is 20% of 650. Other students may first reason that 130 is 25% of 520, and then further reason that 520 is 80% of 650. Regardless of their solution method, encourage all students to check their answer.
- Repeat, this time having partners solve the following problem: *5% of 60% of another number is 6.* [200]

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.