

9.3

Adding and Subtracting Radicals

9.3 OBJECTIVES

1. Add and subtract expressions involving numeric radicals
2. Add and subtract expressions involving algebraic radicals

Two radicals that have the same index and the same radicand (the expression inside the radical) are called **like radicals**. For example,

$2\sqrt{3}$ and $5\sqrt{3}$ are like radicals.

$\sqrt{2}$ and $\sqrt{5}$ are not like radicals—they have different radicands.

$\sqrt{2}$ and $\sqrt[3]{2}$ are not like radicals—they have different indices (2 and 3, representing a square root and a cube root).

NOTE “Indices” is the plural of “index.”

Like radicals can be added (or subtracted) in the same way as like terms. We apply the distributive property and then combine the coefficients:

$$2\sqrt{5} + 3\sqrt{5} = (2 + 3)\sqrt{5} = 5\sqrt{5}$$

Example 1

Adding and Subtracting Like Radicals

Simplify each expression.

NOTE Apply the distributive property, then combine the coefficients.

$$(a) \quad 5\sqrt{2} + 3\sqrt{2} = (5 + 3)\sqrt{2} = 8\sqrt{2}$$

$$(b) \quad 7\sqrt{5} - 2\sqrt{5} = (7 - 2)\sqrt{5} = 5\sqrt{5}$$

$$(c) \quad 8\sqrt{7} - \sqrt{7} + 2\sqrt{7} = (8 - 1 + 2)\sqrt{7} = 9\sqrt{7}$$



CHECK YOURSELF 1

Simplify.

$$(a) \quad 2\sqrt{5} + 7\sqrt{5}$$

$$(b) \quad 9\sqrt{7} - \sqrt{7}$$

$$(c) \quad 5\sqrt{3} - 2\sqrt{3} + \sqrt{3}$$

If a sum or difference involves terms that are *not* like radicals, we may be able to combine terms after simplifying the radicals according to our earlier methods.

Adding and Subtracting Radicals

Simplify each expression.

$$(a) \quad 3\sqrt{2} + \sqrt{8}$$

We do not have like radicals, but we can simplify $\sqrt{8}$. Remember that

$$\sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$$

SO

$$\begin{aligned} 3\sqrt{2} + \sqrt{8} &= 3\sqrt{2} + \sqrt{\overbrace{2 \cdot 2}^{\sqrt{8}} \cdot 2} \\ &= (3 + 2)\sqrt{2} = 5\sqrt{2} \end{aligned}$$

NOTE Simplify $\sqrt{12}$.**NOTE** The radicals can now be combined. Do you see why?

$$\begin{aligned} \text{(b)} \quad 5\sqrt{3} - \sqrt{12} &= 5\sqrt{3} - \sqrt{4 \cdot 3} \\ &= 5\sqrt{3} - \sqrt{4} \cdot \sqrt{3} \\ &= 5\sqrt{3} - 2\sqrt{3} \\ &= (5 - 2)\sqrt{3} = 3\sqrt{3} \end{aligned}$$

**CHECK YOURSELF 2***Simplify.*

(a) $\sqrt{2} + \sqrt{18}$

(b) $5\sqrt{3} - \sqrt{27}$

If variables are involved in radical expressions, the process of combining terms proceeds in a fashion similar to that shown in previous examples. Consider Example 3. We again assume that all variables represent positive real numbers.

Example 3**Simplifying Expressions Involving Variables**

Simplify each expression.

NOTE Because like radicals are involved, we apply the distributive property and combine terms as before.**NOTE** Simplify the first term.**NOTE** The radicals can now be combined.

(a) $5\sqrt{3x} - 2\sqrt{3x} = (5 - 2)\sqrt{3x} = 3\sqrt{3x}$

$$\begin{aligned} \text{(b)} \quad 2\sqrt{3a^3} + 5a\sqrt{3a} &= 2\sqrt{a^2 \cdot 3a} + 5a\sqrt{3a} \\ &= 2\sqrt{a^2} \cdot \sqrt{3a} + 5a\sqrt{3a} \\ &= 2a\sqrt{3a} + 5a\sqrt{3a} \\ &= (2a + 5a)\sqrt{3a} = 7a\sqrt{3a} \end{aligned}$$

**CHECK YOURSELF 3***Simplify each expression.*

(a) $2\sqrt{7y} + 3\sqrt{7y}$

(b) $\sqrt{20a^2} - a\sqrt{45}$

CHECK YOURSELF ANSWERS

1. **(a)** $9\sqrt{5}$; **(b)** $8\sqrt{7}$; **(c)** $4\sqrt{3}$ 2. **(a)** $4\sqrt{2}$; **(b)** $2\sqrt{3}$
 3. **(a)** $5\sqrt{7y}$; **(b)** $-a\sqrt{5}$

9.3

Exercises

Simplify by combining like terms.

1. $2\sqrt{2} + 4\sqrt{2}$

2. $\sqrt{3} + 5\sqrt{3}$

3. $11\sqrt{7} - 4\sqrt{7}$

4. $5\sqrt{3} - 3\sqrt{2}$

5. $5\sqrt{7} + 3\sqrt{6}$

6. $3\sqrt{5} - 5\sqrt{5}$

7. $2\sqrt{3} - 5\sqrt{3}$

8. $2\sqrt{11} + 5\sqrt{11}$

9. $2\sqrt{3x} + 5\sqrt{3x}$

10. $7\sqrt{2a} - 3\sqrt{2a}$

11. $2\sqrt{3} + \sqrt{3} + 3\sqrt{3}$

12. $3\sqrt{5} + 2\sqrt{5} + \sqrt{5}$

13. $5\sqrt{7} - 2\sqrt{7} + \sqrt{7}$

14. $3\sqrt{10} - 2\sqrt{10} + \sqrt{10}$

15. $2\sqrt{5x} + 5\sqrt{5x} - 2\sqrt{5x}$

16. $5\sqrt{3b} - 2\sqrt{3b} + 4\sqrt{3b}$

17. $2\sqrt{3} + \sqrt{12}$

18. $5\sqrt{2} + \sqrt{18}$

19. $\sqrt{20} - \sqrt{5}$

20. $\sqrt{98} - 3\sqrt{2}$

21. $2\sqrt{6} - \sqrt{54}$

22. $2\sqrt{3} - \sqrt{27}$

23. $\sqrt{72} + \sqrt{50}$

24. $\sqrt{27} - \sqrt{12}$

Name _____

Section _____ Date _____

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25. $3\sqrt{12} - \sqrt{48}$

26. $5\sqrt{8} + 2\sqrt{18}$

27. $2\sqrt{45} - 2\sqrt{20}$

28. $2\sqrt{98} - 4\sqrt{18}$

29. $\sqrt{12} + \sqrt{27} - \sqrt{3}$

30. $\sqrt{50} + \sqrt{32} - \sqrt{8}$

31. $3\sqrt{24} - \sqrt{54} + \sqrt{6}$

32. $\sqrt{63} - 2\sqrt{28} + 5\sqrt{7}$

33. $2\sqrt{50} + 3\sqrt{18} - \sqrt{32}$

34. $3\sqrt{27} + 4\sqrt{12} - \sqrt{300}$

Simplify by combining like terms.

35. $a\sqrt{27} - 2\sqrt{3a^2}$

36. $5\sqrt{2y^2} - 3y\sqrt{8}$

37. $5\sqrt{3x^3} + 2\sqrt{27x}$

38. $7\sqrt{2a^3} - \sqrt{8a}$

Use a calculator to find a decimal approximation for each of the following. Round your answer to the nearest hundredth.



39. $\sqrt{3} - \sqrt{2}$

40. $\sqrt{7} + \sqrt{11}$

41. $\sqrt{5} + \sqrt{3}$

42. $\sqrt{17} - \sqrt{13}$

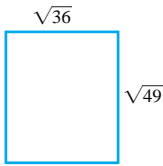
43. $4\sqrt{3} - 7\sqrt{5}$

44. $8\sqrt{2} + 3\sqrt{7}$

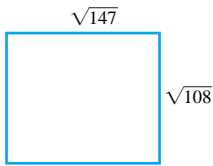
45. $5\sqrt{7} + 8\sqrt{13}$

46. $7\sqrt{2} - 4\sqrt{11}$

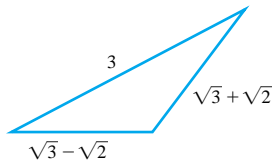
47. **Perimeter of a rectangle.** Find the perimeter of the rectangle shown in the figure.



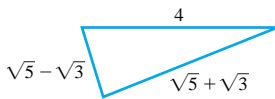
48. **Perimeter of a rectangle.** Find the perimeter of the rectangle shown in the figure. Write your answer in radical form.



49. **Perimeter of a triangle.** Find the perimeter of the triangle shown in the figure.



50. **Perimeter of a triangle.** Find the perimeter of the triangle shown in the figure.



- 47. _____
- 48. _____
- 49. _____
- 50. _____
- a. _____
- b. _____
- c. _____
- d. _____
- e. _____
- f. _____
- g. _____
- h. _____



Getting Ready for Section 9.4 [Section 3.4]

Perform the indicated multiplication.

- (a) $2(x + 5)$
- (b) $3(a - 3)$
- (c) $m(m - 8)$
- (d) $y(y + 7)$
- (e) $(w + 2)(w - 2)$
- (f) $(x - 3)(x + 3)$
- (g) $(x + y)(x + y)$
- (h) $(b - 7)(b - 7)$

Answers

1. $6\sqrt{2}$ 3. $7\sqrt{7}$ 5. Cannot be simplified 7. $-3\sqrt{3}$ 9. $7\sqrt{3x}$
11. $6\sqrt{3}$ 13. $4\sqrt{7}$ 15. $5\sqrt{5x}$ 17. $4\sqrt{3}$ 19. $\sqrt{5}$ 21. $-\sqrt{6}$
23. $11\sqrt{2}$ 25. $2\sqrt{3}$ 27. $2\sqrt{5}$ 29. $4\sqrt{3}$ 31. $4\sqrt{6}$ 33. $15\sqrt{2}$
35. $a\sqrt{3}$ 37. $(5x + 6)\sqrt{3x}$ 39. 0.32 41. 3.97 43. -8.72
45. 42.07 47. 26 49. $2\sqrt{3} + 3$ a. $2x + 10$ b. $3a - 9$
c. $m^2 - 8m$ d. $y^2 + 7y$ e. $w^2 - 4$ f. $x^2 - 9$ g. $x^2 + 2xy + y^2$
h. $b^2 - 14b + 49$