NPV calculation

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NPV calculation

- PV calculation

a. Constant Annuity
b. Growth Annuity
c. Constant Perpetuity
d. Growth Perpetuity

- NPV calculation

a. Cash flow happens at year 0
b. Cash flow happens at year n

NPV Calculation – basic concept

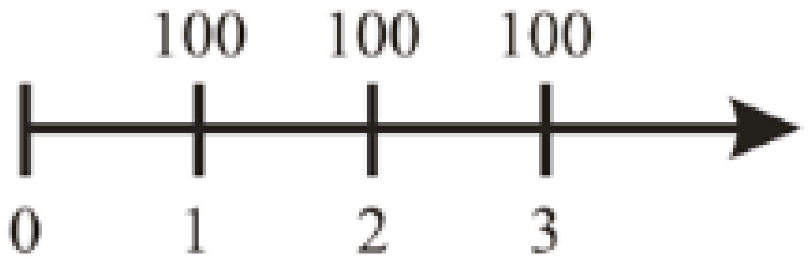
Annuity:

An annuity is a series of equal payments or receipts that occur at evenly spaced intervals.

Eg. loan, rental payment, regular deposit to saving account, monthly home mortgage payment, monthly insurance payment

http://en.wikipedia.org/wiki/Annuity_(finance_theory)

Constant Annuity Timeline



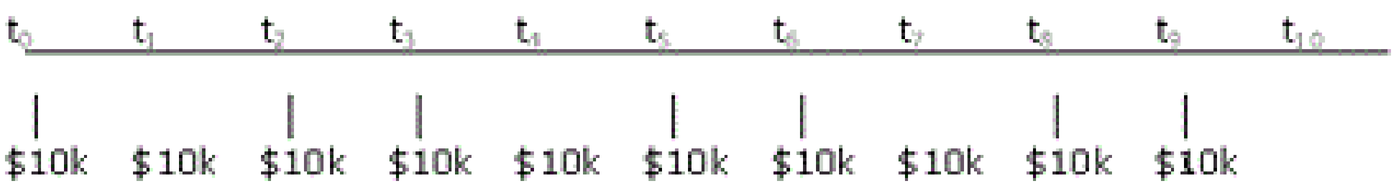
NPV Calculation – basic concept

- Perpetuity:

A constant stream of identical cash flows with no end. The concept of a perpetuity is used often in financial theory, such as the dividend discount model (DDM), by Gordon Growth, used for stock valuation.

http://www.investopedia.com/terms/p/perpetuity.asp

Constant Perpetuity Timeline



NPV Calculation – basic concept

PV(Present Value):

PV is the current worth of a future sum of money or stream of cash flows given a specified rate of return.

Future cash flows are discounted at the discount rate, and the higher the discount rate, the lower the present value of the future cash flows.

Determining the appropriate discount rate is the key to properly valuing future cash flows, whether they be earnings or obligations.

http://www.investopedia.com/terms/p/presentvalue.asp

NPV Calculation – basic concept

NPV(Net Present Value):

The difference between the present value of cash inflows and the present value of cash outflows.

http://www.investopedia.com/terms/n/npv.asp

PV of Constant annuity

- Eg. 1

N=3 yrs, r=6% annually,

PMT = $100 annually,

start at the end of the 1st year

What is the PV?

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PV of Constant annuity

- Answer

$$PV = \frac{a}{r}\left(1 - \frac{1}{(1+r)^n}\right)$$

= 100/6% * (1 – 1/(1+6%)^3)

PV of Constant annuity

- Eg. 2

N=3 yrs, r=6% annually,

PMT = $100 semiannually,
starts at the end of the first 6 months

What is the PV?

PV of Constant annuity

- Answer

- r(half a year) = 6% / 2 = 3%
- n= N*t = 3*2=6

$$PV = \frac{a}{r}\left(1 - \frac{1}{(1+r)^n}\right)$$

= 100/3% * (1 – 1/(1+3%)^6)

PV of Constantly growing annuity

- Eg. 3

N=3 yrs, r=6% annually, g = 4% annually,

PMT = $100 per year, starts at the end of the 1st year

What is the PV?

PV of Constantly growing annuity

- Answer

$$PV = \frac{a}{r-g}\left(1 - \frac{(1+g)^n}{(1+r)^n}\right)$$

= 100/(6%-4%) * (1 – (1+4%)^3 / (1+6%)^3)

PV of Constantly growing annuity

- Eg. 4

N=3 yrs, r=6% annually, g=4% annually

PMT = $100 every 6 months, start at the end of the first 6 months

What is the PV?

PV of Constantly growing annuity

- Answer

- r(half year) = 6% / 2 = 3%
- n= N*t = 3*2=6
- g = 4% /2 = 2%

$$PV = \frac{a}{r-g}\left(1 - \frac{(1+g)^n}{(1+r)^n}\right)$$

= 100/(3%-2%) * (1 – (1+2%)^6 / (1+3%)^6)

PV of constant Perpetuity

- Eg. 5

N:endless years, r=6% annually PMT = $100 annually start at the end of the 1st year

What is the PV?

PV of constant Perpetuity

- Answer

$$PV = \frac{a}{r}$$

$= 100 / 6\%$

PV of constant Perpetuity

- Eg. 6

N: endless years, r=6% annually

PMT = $100 per quarter, starts at the end of the first 3 months

What is the PV?

PV of constant Perpetuity

- Answer

- r (quarterly) = 6% / 4 = 1.5%

$$PV = \frac{a}{r}$$

= 100 / 1.5%

PV of Constantly growing perpetuity

- Eg. 7

r=6% annually, compounded semiannually,

g=4% annually

PMT = $100 annually, start at the end of the first year

What is the PV?

PV of Constantly growing perpetuity

- Answer

1. Calculate the semiannual interest rate

Compounded semiannual interest rate

(1+6%/2) ^2 = 1+R annually.

So R annually = 6.09%

PV of Constantly growing perpetuity

- Answer (continued)

- R = 6.09%
- g = 4%
- PMT = 100

$$PV = \frac{a}{r-g}$$

= 100/(6.09% - 4%)

Formulas Summary

- Constant annuity:

$$PV = \frac{a}{r}\left(1 - \frac{1}{(1+r)^n}\right)$$

- Constant perpetuity:

$$PV = \frac{a}{r}$$

- Constant growing annuity:

$$PV = \frac{a}{r-g}\left(1 - \frac{(1+g)^n}{(1+r)^n}\right)$$

- Constant growing perpetuity:

$$PV = \frac{a}{r-g}$$

PV Calculation

Scenario 1:

If the first payment starts at the end of the year (eg. Year n) , how to calculate the PV in different conditions?

PV Calculation

1. Calculate the PV at beginning of the year (eg, year n -1)

2. Discounted it back to now (year 0)

PV Calculation

- Eg. 8 Constant annuity

N = 10 years

PMT = $100, starts at the end of year 5

r (annually) = 6%

What is the PV of today?

PV Calculation

- Answer:
- PV (beginning of year 5)

$$PV = \frac{a}{r}\left(1 - \frac{1}{(1+r)^n}\right)$$

= 100/6% * (1 – 1/(1+6%)^10)

- PV (year 0)

= PV (beginning of year 5) / (1+r)^4

PV Calculation

- Summary:
- If PMT does not happen at year 1, we first calculate the PV at the year that payment happens, then we should discounted the PV back to year 0, today.

PV Calculation

Scenario 2

- We know the PV at year 0, the number of periods to pay and the interest rate. How to calculate the payment?

- What if they have different interest rate in the whole payment period, what are the different payments?

PV Calculation

- Eg. 9

Constant annuity

N = 10 years

PV = $5000 at year 0 (now)

$r_{1(\text{annually})}$ = 6% for first 3 years

Then, suddenly change interest policy:

$r_{2(\text{annually})}$ = 8% for last 7 years

What is the PMT of today?

PV Calculation

- Answer
- According to the formula:

$$PV = \frac{a}{r}\left(1 - \frac{1}{(1+r)^n}\right)$$

1. $5000 = PMT_1 / 6\% * (1-1/(1+6\%)^{10})$
   $PMT_1 = \$679.34$

2. $PV_1 = PMT_1 / r_1 * (1-1/(1+r)^{n})$
   $= 679.34 / 6\% * (1-1/(1+6\%)^{3})$
   $= 1815.88$

PV Calculation

- Answer (continued)

3. $PV = PV_{1(\text{first 3 yrs})} + PV_{2\ (\text{last 7 yrs})}$

$PV_{2\ (\text{last 7 yrs})} = PV - PV_{1(\text{first 3 yrs})}$

$= 5000 - 1815.88 = 3184.12$

$PV_2 = PV_2$

$PV_2 = (PMT_2 / r_2 * (1-1/(1+r_2)^{\wedge}n_2))/(1+ r_1)$

$= PMT_2 / 8\% *(1-1/(1+ 8\%)^{\wedge}7) / (1+ 6\%)$

So we will get PMT_2

PV Calculation

- Answer (continued)

Alternative solution for calculating PMT2

3'. $PV2 = PMT1 / r1 * (1-1/(1+r)^{n'})$

$= 679.34 / 6\% * (1-1/(1+6\%)^{7})$

$PV2 = PMT2 / r2 * (1-1/(1+r2)^{n_2})$

$= PMT2 / 8\% * (1-1/(1+8\%)^{7})$

PV2 = PV 2

So we will get PMT2

NPV Calculation

If we know all the cash flow and PVs at time 0, we calculate NPV in this way:

NPV = cash inflows – cash out flows + PV

PV could be negative or positive. If it is negative, it is cash outflow, and vise versa

NPV Calculation

- Eg 10
- Investing in machine A to produce shoes.
- Annually profit is $100,000, starts from the end of the first year
- N = 10 yrs
- r = 5% annually
- Maintenance expense is $5000 every time. It happens two times at year 0 and the end of year 5.
- What is the NPV of the project?

NPV Calculation

1. Calculate PV of the profit in the next 10 years

$$PV = \frac{a}{r}\left(1 - \frac{1}{(1+r)^n}\right)$$

= 100,000 / 5% * (1 – 1/(1+5%) ^ 10)

= 772173.4929

NPV Calculation

2. Calculate cash out flow

COF = 5000 + 5000/ (1+r) ^ n'

= 5000 + 5000/(1+5%) ^5

= 8917.63

NPV Calculation

3. NPV = PV - cash out flows

   = 772173.4929 - 8917.63

   = 763255.8629