Section F.4: Present Value of Annuities and Amortization

Example 1: Suppose your rich uncle has agreed to give you a lump sum of money that you will use to pay for your child's daycare until they are school age in five years. You will invest this amount in an account now that earns 3.2% annual interest compounded biweekly, so that you can make biweekly withdrawals of $400 for 5 years to pay for daycare. How much money do you need from your rich uncle now?

$P = $ the present value of an ordinary annuity.

$R = $ the payment made at the end of each compounding period.

$i = r/m$ is the interest rate per period.

$n = mt$ is the number of payments.

Present Value of an Ordinary Annuity

$$P = R \times \frac{[(1 + i)^n - 1]}{i(1 + i)^n}$$

Example 1 cont.

Amortization: A lender loans a borrower a lump sum of money called the principal and the borrower pays back the loan by making equal payments at some regular intervals. We assume the payments are made at the end of the compounding period and the payment is large enough to cover periodic interest and reduce some of the balance. The future value FV of the payments must equal the future value of investing the principal PV in an account with the same interest rate and compounding. Thus the present value of an annuity formula holds and can be rewritten in terms of the payments.

$$R = P \times \frac{i(1 + i)^n}{[(1 + i)^n - 1]}$$

Example 2: Finding Payments on a Loan The Federal Perkins loan is a low-interest student loan available to students with exceptional financial need. The loan has an interest rate of 5% compounded monthly with monthly payments and the borrower has up to 10 years to repay. Suppose a student has a $19,000 Perkins Loan and will take 10 years to repay it.

(a) What are the monthly payments?

(b) What is the balance after 4 years?

(c) What is the total amount of payments over the life of the loan?

(d) How much interest is paid over the life of the loan?

Example 2: Amortization Schedule You get a mortgage of $200,000 amortized over 30 years at an annual interest rate of 6.4% compounded monthly with monthly payments.

(a) Complete the following amortization table for the first 4 months.

End of Period	Payment	Interest charged	Payment toward principal	Outstanding principal

(b) If you wish to pay off the mortgage after 20 years, how much outstanding principle is left?

(c) How much of the first payment in the 15th year goes to principal?