## CONSERVATION OF ANGULAR MOMENTUM, STATIC EQUILIBRIUM

- 1. **Rotating Rod**: A "one-dimensional" rod (no width to consider) of mass *M*, length *L* and moment-of-inertia *I* is nailed to a wall at one end. The rod is free to pivot about the nail which acts as a frictionless hinge. The rod is held at rest horizontally and then released, at which point gravity accelerates it downward as indicated in the figure.
  - (a) What is the magnitude of the net torque acting on the rod as soon as it is released?
  - (b) Using conservation of energy, calculate the angular velocity  $\omega$  of the rod when it reaches the lowest (vertical) position.
  - (c) Calculate the angular momentum  $\mathcal{L}$  of the rod when it reaches the vertical position.



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- (d) Is angular momentum conserved in the motion? If so, why? If not, why not? (In one short sentence, concisely explain the relevant physical concept).
- 2. **Rotating Block**: A small block with mass 0.130 kg is attached to a string passing through a hole in a frictionless, horizontal surface. The block is originally revolving in a circle with a radius of 0.800 m about the hole with a tangential speed of 4.00 m/s. The string is then pulled slowly from below, shortening the radius of the circle in which the block revolves. The breaking strength of the string is 30.0 N. What is the radius of the circle when the string breaks?
- 3. **Bullet**: A uniform rod of length L rests on a frictionless horizontal surface The rod pivots about a fixed frictionless axis at one end. The rod is initially at rest. A bullet traveling parallel to the horizontal surface and perpendicular to the rod with speed v strikes the rod at its center and becomes embedded in it. The mass of the bullet is one-fourth the mass of the rod.
  - (a) What is the final angular speed of the rod?
  - (b) What is the ratio of the kinetic energy of the system after the collision to the kinetic energy of the bullet before the collision?
- 4. **Marbles**: Two uniform, 75.0 g marbles 2.00 cm in diameter are stacked as shown in the figure in a container that is 3.00 cm wide.
  - (a) Find the force that the container exerts on the marbles at the points of contact A, B, and C.
  - (b) What force does each marble exert on the other?
- 5. **Dropped Ball**: A ball with mass *m* is dropped from a height of *h* above one end of a uniform bar that pivots at its center. The bar has mass *M* and is *L* in length. At the other end of the bar sits another ball with mass *m*, unattached to the bar. The dropped ball sticks to the bar after the collision. How high will the other ball go after the collision?

- 6. **Rings on a Rod:** A uniform, rod with mass M of length L rotates in a horizontal plane about a fixed axis through its center and perpendicular to the rod. Two small rings, each with mass m, are mounted so that they can slide along the rod. They are initially held by catches at positions x on each side of the center of the rod, and the system is rotating at  $\omega$ . With no other changes in the system, the catches are released, and the rings slide outward along the rod and fly off at the ends. What is the angular speed
  - (a) of the system at the instant when the rings reach the ends of the rod;
  - (b) of the rod after the rings leave it?
- 7. Ladder: A uniform ladder of mass M and length L is placed against a frictionless vertical wall and makes an angle  $\theta$  with respect to the horizontal as shown in the figure. The base of the ladder rests on a rough horizontal floor where friction keeps the ladder from sliding. A bucket of mass m is suspended from a rung that is 3/4 of the way up the ladder.
  - (a) Use the figure to make a free-body diagram of the ladder, clearly labelling all forces acting on it at the correct position.
  - (b) What is the magnitude of the normal force that the wall exerts on the top of the ladder? Be sure to specify your reference point for calculating the torques.
  - (c) What is the minimum value that the coefficient of static friction between the ladder and the floor can be such that the ladder is still in equilibrium?
- 8. **Mountain Climbing**: Mountaineers often use a rope to lower themselves down the face of a cliff (this is called rappelling). They do this with their body nearly horizontal and their feet pushing against the cliff. Suppose that an 82.0 kg climber, who is 1.90 m tall and has a center of gravity 1.1 m from his feet, rappels down a vertical cliff with his body raised 35.0° above the horizontal. He holds the rope 1.40 m from his feet, and it makes a 25.0° angle with the cliff face.
  - (a) What tension does his rope need to support?
  - (b) Find the horizontal and vertical components of the force that the cliff face exerts on the climber's feet.
  - (c) What minimum coefficient of static friction is needed to prevent the climber's feet from slipping on the cliff face if he has one foot at a time against the cliff?





