## Reteach Book <br> Grade 5

Houghton Mifflin Harcourt


PROVIDES Tier 1 Intervention for Every Lesson

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## Place Value and Patterns

You can use a place-value chart and patterns to write numbers that are 10 times as much as or $\frac{1}{10}$ of any given number.

Each place to the right is $\frac{1}{10}$ of the value of the place to its left.

|  | $\frac{1}{10}$ of the <br> hundred <br> thousands <br> place | $\frac{1}{10}$ of the <br> ten thousands <br> place | $\frac{1}{10}$ of the <br> thousands <br> place | $\frac{1}{10}$ of the <br> hundreds <br> place | $\frac{1}{10}$ of the <br> tens place |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Hundred <br> Thousands | Ten <br> Thousands | Thousands | Hundreds | Tens | Ones |
| 10 times <br> the ten <br> thousands <br> place | 10 times the <br> thousands <br> place | 10 times the <br> hundreds <br> place | 10 times the <br> tens place | 10 times the <br> ones place |  |

Each place to the left is 10 times the value of the place to its right.
Find $\frac{1}{10}$ of 600.
$\frac{1}{10}$ of 6 hundreds is 6 tens.
So, $\frac{1}{10}$ of 600 is $\qquad$ 60 .

Find 10 times as much as 600.
10 times as much as 6 hundreds is 6 thousands.
So, 10 times as much as 600 is 6,000 .

## Use place-value patterns to complete the table.

| Number | 10 times as <br> much as | $\frac{\mathbf{1}}{\mathbf{1 0}}$ of |
| :---: | :---: | :---: |
| 1. 200 |  |  |
| 2. 10 |  |  |
| 3. 700 |  |  |
| 4. 5,000 |  |  |


| Number | $\mathbf{1 0}$ times as <br> much as | $\frac{\mathbf{1}}{\mathbf{1 0}}$ of |
| :---: | :---: | :---: |
| 5. 900 |  |  |
| 6. 80,000 |  |  |
| 7. 3,000 |  |  |
| 8. 40 |  |  |

## Place Value of Whole Numbers

You can use a place-value chart to help you understand whole numbers and the value of each digit. A period is a group of three digits within a number separated by a comma.

| Millions Period |  |  | Thousands Period |  |  | Ones Period |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Hundreds | Tens | Ones | Hundreds | Tens | Ones | Hundreds | Tens | Ones |
|  |  | 2, | 3 | 6 | 7, | 0 | 8 | 9 |

Standard form: 2,367,089
Expanded Form: Multiply each digit by its place value, and then write an addition expression.
$(2 \times 1,000,000)+(3 \times 100,000)+(6 \times 10,000)+(7 \times 1,000)+(8 \times 10)+(9 \times 1)$
Word Form: Write the number in words. Notice that the millions and the thousands periods are followed by the period name and a comma.
two million, three hundred sixty-seven thousand, eighty-nine
To find the value of an underlined digit, multiply the digit by its place value.
In $\underline{2}, 367,089$, the value of 2 is $2 \times 1,000,000$, or $2,000,000$.

Write the value of the underlined digit.

1. $153,732,991$
$\qquad$
2. 264,807
$\qquad$
Write the number in two other forms.
3. $2 \underline{3} 6,143,802$
$\qquad$
4. $78,209,146$
$\qquad$
5. $40,023,032$
$\qquad$
$\qquad$

## Algebra • Properties

Properties of operations are characteristics of the operations that are always true.

| Property | Examples |
| :--- | :--- |
| Commutative Property of <br> Addition or Multiplication | Addition: $3+4=4+3$ <br> Multiplication: $8 \times 2=2 \times 8$ <br> Associative Property of <br> Addition or Multiplication |
| Addition: $(1+2)+3=1+(2+3)$ |  |
| Multiplication: $6 \times(7 \times 2)=(6 \times 7) \times 2$ |  |
| Identity Property of Addition | $8 \times(2+3)=(8 \times 2)+(8 \times 3)$ |
| Identity Property of Multiplication | $9+0=9 \quad 0+3=3$ |

Use properties to find $37+24+43$.

$$
\begin{aligned}
37+24+43 & =24+\underline{37}+43 & & \begin{array}{l}
\text { Use the Commutative Property of Addition } \\
\text { to reorder the addends. } \\
\text { Use the Associative Property of Addition }
\end{array} \\
& =24+(37+43) & & \begin{array}{l}
\text { Us group the addends. } \\
\text { to } \\
\text { Use mental math to add. }
\end{array} \\
& =24+\underline{80} & &
\end{aligned}
$$

Grouping 37 and 43 makes the problem easier to solve because their sum, 80 , is a multiple of 10 .

## Use properties to find the sum or product.

1. $31+27+29$
2. $41 \times 0 \times 3$
3. $4+(6+21)$

Complete the equation, and tell which property you used.
4. $(2 \times$ $\qquad$ ) $+(2 \times$ $2)=2 \times(5+2)$
5. $\qquad$ $\times 1=15$

## Algebra • Powers of 10 and Exponents

You can represent repeated factors with a base and an exponent.
Write $10 \times 10 \times 10 \times 10 \times 10 \times 10$ in exponent form.
10 is the repeated factor, so 10 is the base.
The base is repeated 6 times, so 6 is the exponent.
$10 \times 10 \times 10 \times 10 \times 10 \times 10=10^{6}$
A base with an exponent can be written in words.

Write $10^{6}$ in words.
The exponent 6 means "the sixth power."
$10^{6}$ in words is "the sixth power of ten."

You can read $10^{2}$ in two ways: "ten squared" or "the second power of ten."
You can also read $10^{3}$ in two ways: "ten cubed" or "the third power of ten."

Write in exponent form and in word form.

1. $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$
exponent form: $\qquad$ word form: $\qquad$
2. $10 \times 10 \times 10$
exponent form: $\qquad$ word form: $\qquad$
3. $10 \times 10 \times 10 \times 10 \times 10$
exponent form: $\qquad$ word form: $\qquad$

Find the value.
4. $10^{4}$
5. $2 \times 10^{3}$
6. $6 \times 10^{2}$

## Algebra • Multiplication Patterns

You can use basic facts, patterns, and powers of 10 to help you multiply whole numbers by multiples of 10,100 , and 1,000 .

Use mental math and a pattern to find $90 \times 6,000$.

- $9 \times 6$ is a basic fact. $\quad 9 \times 6=54$
- Use basic facts, patterns, and powers of 10 to find $90 \times 6,000$.

$$
\begin{aligned}
9 \times 60 & =(9 \times 6) \times 10^{1} \\
& =54 \times 10^{1} \\
& =54 \times 10 \\
& =540
\end{aligned}
$$

$9 \times 600=(9 \times 6) \times 10^{2}$

$$
=54 \times 10^{2}
$$

$$
=54 \times 100
$$

$$
=5,400
$$

$9 \times 6,000=(9 \times 6) \times 10^{3}$

$$
=54 \times 10^{3}
$$

$$
=54 \times 1,000
$$

$$
=54,000
$$

$$
\begin{aligned}
90 \times 6,000 & =(9 \times 6) \times(10 \times 1,000) \\
& =54 \times 10^{4} \\
& =54 \times 10,000 \\
& =540,000
\end{aligned}
$$

So, $90 \times 6,000=540,000$.

## Use mental math to complete the pattern.

1. $3 \times 1=3$
$3 \times 10^{1}=$ $\qquad$
$3 \times 10^{2}=$ $\qquad$
$3 \times 10^{3}=$ $\qquad$
2. $8 \times 2=16$
$(8 \times 2) \times 10^{1}=$ $\qquad$
$(8 \times 2) \times 10^{2}=$ $\qquad$
$\qquad$ $(8 \times 2) \times 10^{3}=$ $\qquad$
3. $4 \times 5=20$
$(4 \times 5) \times \ldots=200$
$(4 \times 5) \times \ldots=2,000$
$(4 \times 5) \times \ldots=20,000$
4. $7 \times 6=$ $\qquad$
$(7 \times 6) \times \ldots=420$
$(7 \times 6) \times \ldots=4,200$
$(7 \times 6) \times$ $\qquad$

## Multiply by 1-Digit Numbers

You can use place value to help you multiply by 1-digit numbers.
Estimate. Then find the product. $378 \times 6$
Estimate: $400 \times 6=2,400$


So, $378 \times 6=2,268$.

## Complete to find the product.

1. $7 \times 472$

Multiply the ones.
Estimate: $7 \times$ $\qquad$ $=$ $\qquad$
Multiply the tens.
Multiply the hundreds.

$$
472
$$

$\times 7$

$$
\begin{array}{r}
1 \\
472 \\
\times \quad 7 \\
\hline
\end{array}
$$

51
472
$\times 7$

Estimate. Then find the product.
2. Estimate:

3. Estimate:
809
$\times \quad 8$
4. Estimate:

| 932 |
| ---: |
| $\times \quad 7$ |

5. Estimate:
$\qquad$

## Multiply by 2-Digit Numbers

You can use place value and regrouping to multiply.
Find $29 \times 63$.
Step 1 Write the problem vertically.
Multiply by the ones.

$$
\left.\begin{array}{r}
\stackrel{2}{63} \\
\times \quad 29 \\
\hline 567 \longleftarrow 63 \times 9
\end{array}\right)(\underline{60} \times 9)+(\underline{3} \times 9)
$$

Step 2 Multiply by the tens.

$$
\begin{array}{r}
\begin{array}{r}
2 \\
63 \\
\times \quad 29
\end{array} \\
\hline 567 \\
1,260 \longleftarrow 63 \times 20
\end{array}=(\underline{60} \times 20)+(\underline{3} \times 20)
$$

Step 3 Add the partial products.

$$
\begin{array}{r}
63 \\
\times \quad 29 \\
\hline 567 \\
+1,260 \\
\hline 1,827
\end{array}
$$

So, $63 \times 29=1,827$.

## Complete to find the product.


4. Find $26 \times 69$. Estimate first.

69

| $\times 26$ |
| :--- |

Estimate: $\qquad$

## Relate Multiplication to Division

Use the Distributive Property to find the quotient of $56 \div 4$.

## Step 1

Write a related multiplication sentence for the division problem.

$$
\begin{aligned}
& 56 \div 4=\square \\
& 4 \times \square=56
\end{aligned}
$$

## Step 2

Use the Distributive Property to break apart
$(40+16)=56$ the product into lesser numbers that are multiples of the divisor in the division problem.
$(4 \times 10)+(4 \times 4)=56$
Use a multiple of 10 for one of the multiples.
$4 \times(10+4)=56$
Step 3
To find the unknown factor, find the sum of the numbers $10+4=14$ inside the parentheses.

## Step 4

$4 \times 14=56$
Write the multiplication sentence with the unknown factor you found. Then, use the multiplication sentence $56 \div 4=14$ to complete the division sentence.

Use multiplication and the Distributive Property to find the quotient.

1. $68 \div 4=$ $\qquad$
2. $75 \div 3=$ $\qquad$
3. $96 \div 6=$ $\qquad$
$\qquad$
$\qquad$
4. $80 \div 5=$ $\qquad$
5. $54 \div 3=$ $\qquad$
6. $105 \div 7=$ $\qquad$

## Problem Solving • Multiplication and Division

In Brett's town, there are 128 baseball players on 8 different teams. Each team has an equal number of players. How many players are on each team?

| Read the Problem | Solve the Problem |
| :---: | :---: |
| What do I need to find? <br> I need to find $\qquad$ how many players are on each team in Brett's town <br> What information do I need to use? <br> There are $\qquad$ 8 teams with a total of 128 players. <br> How will I use the information? <br> I can divide $\qquad$ the total number of players by the number of teams. I can use a simpler problem to $\qquad$ divide. | - First, I use the total number of players. $128 \text { players }$ <br> - To find the number of players on each team, I will need to solve this problem. $128 \div 8=?$ <br> - To find the quotient, I break 128 into two simpler numbers that are easier to divide. $\begin{aligned} 128 \div 8 & =(80+\underline{48}) \div 8 \\ & =\left(\frac{80}{10} \div 8\right)+\left(\frac{48}{10} \div 8\right) \\ & =\frac{16}{6} \\ & =\frac{16}{} \text { players on each team. } \end{aligned}$ |

1. Susan makes clay pots. She sells 125 pots per month to 5 stores. Each store buys the same number of pots. How many pots does each store buy?

$$
\begin{aligned}
125 \div 5 & =(100+\ldots) \div 5 \\
& =(100 \div 5)+(\ldots \div 5) \\
& =\square 5 \\
& =
\end{aligned}
$$

2. Lou grows 112 rosemary plants. He ships an equal number of plants to customers in 8 states. How many rosemary plants does he ship to each customer?

$$
\begin{aligned}
112 \div 8 & =(80+\ldots) \div 8 \\
& =(\square)+(\ldots-\ldots) \\
& =\square \\
& =
\end{aligned}
$$

## Algebra • Numerical Expressions

Write words to match the expression.

$$
6 \times(12-4)
$$

Think: Many word problems involve finding the cost of a store purchase.
Step 1 Examine the expression.
-What operations are in the expression? multiplication and subtraction
Step 2 Describe what each part of the expression can represent when finding the cost of a store purchase.
-What can multiplying by 6 represent? buying 6 of the same item
Step 3 Write the words.

- Joe buys 6 DVDs. Each DVD costs $\$ 12$. If Joe receives a $\$ 4$ discount on each DVD, what is the total amount of money Joe spends?

1. What is multiplied and what is subtracted?
2. What part of the expression is the price of the item?
3. What can subtracting 4 from 12 represent?

## Write words to match the expression.

4. $4 \times(10-2)$
5. $3 \times(6-1)$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Algebra • Evaluate Numerical Expressions

A numerical expression is a mathematical phrase that includes only numbers and operation symbols.

You evaluate the expression when you perform all the computations to find its value.

## Order of Operations

1. Parentheses
2. Multiply and Divide
3. Add and Subtract

To evaluate an expression, use the order of operations.
Evaluate the expression $(10+6 \times 6)-4 \times 10$.
Step 1 Start with computations inside the $10+6 \times 6$ parentheses.

Step 2 Perform the order of operations inside the parentheses.

Step 3 Rewrite the expression with the $46-4 \times 10$ parentheses evaluated.

Step 4 Multiply and divide from left to right.
Step 5 Add and subtract from left to right.
Multiply and divide from left to right.

$$
10+6 \times 6=10+36
$$

Add and subtract from left to right.

$$
10+36=
$$

So, $(10+6 \times 6)-4 \times 10=6$.

## Evaluate the numerical expression.

1. $8-(7 \times 1)$
2. $5-2+12 \div 4$
3. $8 \times(16 \div 2)$
4. $4 \times(28-20 \div 2)$
5. $(30-9 \div 3) \div 9$
6. $(6 \times 6-9)-9 \div 3$
7. $11 \div(8+9 \div 3)$
8. $13 \times 4-65 \div 13$
9. $9+4 \times 6-65 \div 13$

## Algebra•Grouping Symbols

Parentheses ( ), brackets [ ], and braces \{ \}, are different grouping symbols used in expressions. To evaluate an expression with different grouping symbols, perform the operation in the innermost set of grouping symbols first. Then evaluate the expression from the inside out.

Evaluate the expression $2 \times[(9 \times 4)-(17-6)]$.
Step 1 Perform the operations in the parentheses first.


Step 2 Next perform the operations in the brackets.


Step 3 Then multiply.

$$
2 \times 25=50
$$

So, $2 \times[(9 \times 4)-(17-6)]=\underline{50}$

## Evaluate the numerical expression.

1. $4 \times[(15-6) \times(7-3)]$
$4 \times[9 \times$ $\qquad$ ]
2. $40-[(8 \times 7)-(5 \times 6)]$
3. $60 \div[(20-6)+(14-8)]$
$4 \times[$ $\qquad$ ]
4. $5+[(10-2)+(4-1)]$
5. $3 \times[(9+4)-(2 \times 6)]$
6. $32 \div[(7 \times 2)-(2 \times 5)]$

## Place the First Digit

When you divide, you can use estimation or place value to place the first digit of the quotient.

## Divide.

$6 \longdiv { 1 , 2 6 6 }$

- Estimate. $1,200 \div 6=200$, so the first digit of the quotient is in the hundreds place.
- Divide the hundreds.
- Divide the tens.
- Divide the ones.

So, 1,266 $\div 6=211$.
06 $-\frac{6}{0}$
Since 211 is close to the estimate, 200, the answer is reasonable.

## Divide.

$8,895 \div 8$
1,111r7
8 8,895

- Use place value to place the first digit.
- Look at the first digit.

If the first digit is less than the divisor, then the first digit of the quotient will be in the hundreds place.
If the first digit is greater than or equal to the divisor, then
the first digit of the quotient will be in the thousands place.
Since 8 thousands can be shared among 8 groups, the first digit of the quotient will be in the thousands place. Now divide.
So, $8,895 \div 8$ is $1,111 \mathrm{r} 7$.

Divide.

1. $3 \longdiv { 6 2 7 }$
2. $5 \longdiv { 7 , 4 3 3 }$
3. $4 \longdiv { 5 , 3 6 7 }$
4. $9 \longdiv { 6 , 4 7 0 }$
5. $8 \longdiv { 2 , 8 6 9 }$
6. $6 \longdiv { 1 , 2 9 9 }$
7. $4 \longdiv { 8 9 3 }$
8. $7 \longdiv { 4 , 4 1 8 }$

## Divide by 1-Digit Divisors

You can use compatible numbers to help you place the first digit in the quotient. Then you can divide and check your answer.

Divide. 4 $\longdiv { 7 5 7 }$

Step 1 Estimate with compatible numbers to decide where to place the first digit.

$$
\begin{aligned}
& 757 \div 4 \\
& \downarrow \\
& 800 \div 4=200
\end{aligned}
$$

The first digit of the quotient is in the hundreds place.

Step 2 Divide.
189 r 1
$4 \longdiv { 7 5 7 }$
$\begin{array}{r}-4 \downarrow \\ \hline 35\end{array}$
35
-32
-37
$-36$

Step 3 Check your answer.
$189 \leftarrow$ quotient
$\times 4 \longleftarrow$ divisor
756

| $+\quad 1$ | remainder |
| ---: | :--- |
| 757 | dividend |

Since 189 is close to the estimate of 200 , the answer is reasonable. So, $757 \div 4$ is 189 r 1 .

## Divide. Check your answer.

1. $8 \longdiv { 1 3 6 }$
2. $7 \longdiv { 2 9 7 }$
3. $5 \longdiv { 8 , 1 2 6 }$
4. $7 \longdiv { 4 , 9 7 3 }$
5. $3 \longdiv { 7 4 1 }$
6. $7 \longdiv { 4 5 6 }$

## Division with 2-Digit Divisors

You can use base-ten blocks to model division with 2-digit divisors.

Divide. $154 \div 11$

Step 1 Model 154 with base-ten blocks.

|  |  |  |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |


$\bullet \square \square \square$

Step 2 Make equal groups of 11. Each group should contain $\quad 1$ ten and $\quad 1$ one.

You can make 4 groups of 11 without regrouping.


Step 3 Regroup 1 hundred as 10 tens Regroup 1 ten as $\qquad$


Step 4 Use the regrouped blocks to make as many groups of 11 as possible. Then count the total number of groups.

There are 14 groups. So, $154 \div 11=14$.


Divide. Use base-ten blocks.

1. $192 \div 12$ $\qquad$ 2. $182 \div 14$ $\qquad$

## Partial Quotients

## Divide. Use partial quotients.

$858 \div 57$

Step 1 Estimate the number of groups of 57 that are in 858 . You know $57 \times 10=570$. 858 Since $570<858$, at least 10 groups of 57

## Quotient

10
288 are in 858 . Write 10 in the quotient column, because 10 groups of the divisor, 57 , are in the dividend, 858.

Step 2 Now estimate the number of groups of 57 that are in 288 . You know $60 \times 4=240$. So at least 4 groups of 57 are in 288 . Subtract

288
-228
60
228 from 288, because $57 \times 4=228$. Write 4 in the quotient column, because 4 groups of the divisor, 57, are in 288.

Step 3 Identify the number of groups of 57 that are in $60.57 \times 1=57$, so there is 1 group of 57 in 60 . Write 1 in the quotient 60 column.

Step 4 Find the total number of groups of the divisor, 57 , that are in the dividend, 858 , by adding the numbers in the quotient column. Include the remainder in your answer.

Answer: 15 r3

## Divide. Use partial quotients.

1. $1 7 \longdiv { 4 7 6 }$
2. $1 4 \longdiv { 3 6 5 }$
3. $2 5 \longdiv { 7 5 3 }$
4. $462 \div 11$
5. $1,913 \div 47$
6. $1,085 \div 32$

## Estimate with 2-Digit Divisors

You can use compatible numbers to estimate quotients. Compatible numbers are numbers that are easy to compute mentally.
To find two estimates with compatible numbers, first round the divisor. Then list multiples of the rounded divisor until you find the two multiples that are closest to the dividend. Use the one less than and the one greater than the dividend.

Use compatible numbers to find two estimates. $4,125 \div 49$
Step 1 Round the divisor to the nearest ten.
49 rounds to $\qquad$ 50

Step 2 List multiples of 50 until you get the two closest to the dividend, 4,125. Some multiples of 50 are:
$\begin{array}{llllllll} & 1,000 & 1,500 & 2,000 & 2,500 & 3,000 & 3,500 & 4,000\end{array} 4,500$ 4,000 and 4,500 are closest to the dividend.

Step 3 Divide the compatible numbers to estimate the quotient.

$$
4,000 \div 50=\underline{80} \quad 4,500 \div 50=90
$$

The more reasonable estimate is $4,000 \div 50=80$, because 4,000 is closer to 4,125 than 4,500 is.

## Use compatible numbers to find two estimates.

1. $4 2 \longdiv { 1 , 5 7 8 }$
2. $7 3 \longdiv { 4 , 8 5 8 }$
3. $5 4 \longdiv { 3 4 3 }$
4. $4,093 \div 63$
5. $4,785 \div 79$
6. $7,459 \div 94$

## Use compatible numbers to estimate the quotient.

7. $847 \div 37$
8. $6,577 \div 89$
9. $218 \div 29$

## Divide by 2-Digit Divisors

When you divide by a 2-digit divisor, you can use estimation to help you place the first digit in the quotient. Then you can divide.

Divide. 53 $\longdiv { 2 , 3 6 9 }$
Step 1 Use compatible numbers to estimate the quotient. Then use the estimate to place the first digit in the quotient.
$5 0 \longdiv { 2 , 0 0 0 }$
The first digit will be in the tens place.
Step 2 Divide the tens.

$$
\begin{gathered}
4 \\
5 3 \longdiv { 2 , 3 6 9 } \\
-\frac{212}{24}
\end{gathered}
$$

Step 3 Bring down the 9 ones.
Then divide the ones.

$$
\begin{array}{r}
5 3 \longdiv { 2 , 3 6 9 } \text { r37 } \\
-212 \downarrow \\
\hline 249 \\
-212 \\
\hline 37
\end{array}
$$

So, $2,369 \div 53$ is 44 r 37 .

## Think:

Divide: 236 tens $\div 53$
Multiply: $53 \times 4$ tens $=212$ tens
Subtract: 236 tens -212 tens
Compare: $24<53$, so the first digit of the quotient is reasonable.

## Think:

Divide: 249 ones $\div 53$
Multiply: $53 \times 4$ ones $=212$ ones
Subtract: 249 ones - 212 ones
Compare: $37<53$, so the second digit of the quotient is reasonable.

Write the remainder to the right of the whole number part of the quotient.

## Divide. Check your answer.

1. $5 2 \longdiv { 6 1 2 }$
2. $6 3 \longdiv { 9 1 7 }$
3. $8 9 \longdiv { 1 , 5 9 7 }$
4. $4 3 \longdiv { 6 4 1 }$
5. $2 7 \longdiv { 4 , 6 8 4 }$
6. $6 4 \longdiv { 8 , 4 5 5 }$

## Interpret the Remainder

## Erin has 87 ounces of trail mix. She puts an equal number of ounces in each of 12 bags. How many ounces does she put in each bag? <br> $1 2 \longdiv { 8 7 }$

First, divide to find the quotient and remainder. Then, decide how to use the quotient and the remainder to answer the question.

- The dividend, 87 , represents the total number of ounces of trail mix.
- The divisor, $\frac{12}{2}$, represents the total number of bags.
- The quotient, $\frac{7}{7}$, represents the whole-number part of the number of ounces in each bag.
- The remainder, $3^{3}$, represents the number of ounces left over.

Divide the 3 ounces in the remainder by the divisor, 12 , to write the remainder as a fraction: $\frac{3}{12}$

Write the fraction part in simplest form in your answer.
So, Erin puts $7 \frac{1}{4}$ ounces of trail mix in each bag.

## Interpret the remainder to solve.

1. Harry goes on a canoe trip with his scout troop. They will canoe a total of 75 miles and want to travel 8 miles each day. How many days will they need to travel the entire distance?
2. There are 103 students eating lunch in the cafeteria. Each table seats 4 students. All the tables are full, except for one table. How many students are sitting at the table that is not full?
3. Hannah and her family want to hike 8 miles per day along a 125-mile-long trail. How many days will Hannah and her family hike exactly 8 miles?
4. Emily buys 240 square feet of carpet. She can convert square feet to square yards by dividing the number of square feet by 9 . How many square yards of carpet did Emily buy? (Hint: Write the remainder as a fraction.)

## Adjust Quotients

When you divide, you can use the first digit of your estimate as the first digit of your quotient. Sometimes the first digit will be too high or too low. Then you have to adjust the quotient by increasing or decreasing the first digit.

| Estimate Too High |  | Estimate Too Low |  |
| :---: | :---: | :---: | :---: |
| Divide. $271 \div 48$ |  | Divide. $2,462 \div 27$ |  |
| Estimate. $300 \div 50=6$ |  | Estimate. 2,400 $\div 30=80$ |  |
| Try 6 ones. | Try 5 ones. | Try 8 tens. | Try 9 tens. |
| 6 | 5 r31 | 8 | 91 r5 |
| $4 8 \longdiv { 2 7 1 }$ | 48) 271 | 27) 2 ,462 | 27) 2 2,462 |
| - 288 | - 240 | -216 | -243 |
|  | 31 | 30 | 32 -27 |
| You cannot subtract |  |  | 5 |
| 288 from 271. So, |  | 30 is greater than |  |
| the estimate is too high. | $\begin{aligned} & \text { So, } 271 \div 48 \text { is } \\ & 5 \mathrm{r} 31 \text {. } \end{aligned}$ | the divisor. So, the estimate is too low. | So, $2,462 \div 27$ is 91 r5. |

Adjust the estimated digit in the quotient, if needed. Then divide.

1. $5 8 \longdiv { 1 , 3 2 5 }$
2. $3 7 \longdiv { 2 4 1 }$
3. $2 9 \longdiv { 2 , 2 7 6 }$

## Divide.

4. $1 6 \longdiv { 8 4 5 }$
5. $2 4 \longdiv { 2 1 7 }$
6. $3 7 \longdiv { 4 , 8 1 9 }$

## Problem Solving • Division

Sara and Sam picked apples over the weekend. Sam picked nine times as many apples as Sara. Together, they picked 310 apples. How many apples did each person pick?

| Read the Problem |  |  |
| :---: | :---: | :---: |
| What do I need to find? $\qquad$ <br> I need to find number of apples each person picked. | What information do I need to use? <br> I need to know that Sam and Sara picked a total of $\qquad$ apples. I need to know that Sam picked $\qquad$ times as many apples as Sara. | How will I use the information? I can use the strategy draw a diagram to organize the information. I can draw and use a bar model to write the division problem that will help me find the number of apples Sam and Sara each picked. |
| Solve the Problem |  |  |
| My bar model needs to have o picked and nine boxes for the divide the total number of appl <br> Sara 31 $\begin{array}{l\|l\|l\|l\|} \hline 31 & 31 & 31 & 3 \\ \hline \end{array}$ <br> So, Sara picked $\qquad$ | ne box for the number of ap umber of apples Sam picked. s picked by the total numbe $\begin{array}{\|l\|l\|l\|l\|} \hline 31 & 31 & 31 & 31 \\ \hline \end{array}$ <br> apples and Sam picked | ples Sara d. I can $r$ of boxes. $\begin{array}{r} 31 \\ 1 0 \longdiv { 3 1 0 } \\ -30 \\ \hline 10 \\ -10 \\ \hline 0 \end{array}$ apples. |

Solve each problem. To help, draw a bar model on a separate sheet of paper.

1. Kai picked 11 times as many blueberries as Nico. Together, they picked 936 blueberries. How many blueberries did each boy pick?
2. Jen wrote 10 times as many pages of a school report as Tom. They wrote 396 pages altogether. How many pages did each student write?

## Thousandths

Thousandths are smaller parts than hundredths. If one hundredth is divided into 10 equal parts, each part is one thousandth.

Write the decimal shown by the shaded parts of the model.
One column of the decimal model is shaded. It represents one tenth, or $\qquad$ 0.1 .

Two small squares of the decimal model are shaded. They represent two hundredths, or 0.02 .

A one-hundredth square is divided into 10 equal parts, or thousandths. Three columns of the thousandth square
 are shaded. They represent 0.003 .

So, 0.123 of the decimal model is shaded.
The relationship of a digit in different place-value positions is the same for decimals as for whole numbers.

Write the decimals in a place-value chart.

| Ones | - | Tenths | Hundredths |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 8 |  |
| Thousandths |  |  |  |
| 0 | 0 | 0 | 8 |
| 0 | 0 | 0 | 8 |
| 0 | 0.08 is $\frac{1}{10}$ of 0.8 |  |  |

1. Write the decimal shown by the shaded parts of the model.


## Use place-value patterns to complete the table.

| Decimal | 10 times as much as | $\frac{1}{10}$ of |
| :--- | :--- | :--- |
| 2. 0.1 |  |  |
| 3. 0.03 |  |  |
| 4. 0.5 |  |  |


| Decimal | 10 times as much as | $\frac{1}{10}$ of |
| :--- | :--- | :--- |
| 5. 0.02 |  |  |
| 6. 0.4 |  |  |
| 7. 0.06 |  |  |

## Place Value of Decimals

You can use a place-value chart to find the value of each digit in a decimal.
Write whole numbers to the left of the decimal point.
Write decimals to the right of the decimal point.

| Ones | Tenths | Hundredths | Thousandths |
| :---: | :---: | :---: | :---: |
| 3 | 8 | 4 | 7 |
| $3 \times 1$ | $8 \times \frac{1}{10}$ | $4 \times \frac{1}{100}$ | $7 \times \frac{1}{1,000}$ |


| 3.0 | o | 0.8 | 0.04 |
| :--- | :--- | :--- | :--- |

The place value of the digit 8 in 3.847 is tenths.
The value of 8 in 3.847 is $8 \times \frac{1}{10}$, or 0.8 .
You can write a decimal in different forms.
Standard Form: $\underline{3.847}$
Expanded Form: $3 \times 1+\frac{8}{3} \times \frac{1}{10}+\frac{4}{} \times\left(\frac{1}{100}\right)+\underline{7} \times\left(\frac{1}{1,000}\right)$
When you write the decimal in word form, write "and" for the decimal point.
Word Form: three and eight hundred forty-seven thousandths

1. Complete the place-value chart to find the value of each digit.

| Ones | Tenths | Hundredths | Thousandths |
| :---: | :---: | :---: | :---: |
| 2 | 6 | 9 | 5 |
| $2 \times 1$ |  | $9 \times \frac{1}{100}$ |  |
|  | 0.6 |  |  |

Value
Write the value of the underlined digit.
2. $0.7 \underline{9} 2$
3. 4.691
4. $3.80 \underline{5}$

## Compare and Order Decimals

You can use a place-value chart to compare decimals.
Compare. Write $<,>$, or $=$.
4.375
 4.382

Write both numbers in a place-value chart. Then compare the digits, starting with the highest place value. Stop when the digits are different and compare.

| Ones | Tenths | Hundredths | Thousandths |
| :---: | :---: | :---: | :---: |
| 4 | 0 | 3 | 7 |
| 4 | $\bullet$ | 3 | 8 |



The ones digits The tenths digits The hundredths are the same. are the same. digits are different.

The digits are different in the hundredths place.
Since 7 hundredths $<8$ hundredths, 4.375 4.382.

1. Use the place-value chart to compare the two numbers. What is the greatest placevalue position where the digits differ?

| Ones | Tenths | Hundredths | Thousandths |
| :---: | :---: | :---: | :---: |
| 2 | $\bullet$ | 8 | 6 |
| 2 | $\bullet$ | 8 | 6 |

Compare. Write $<,>$, or $=$.
2. 5.37

5.370
3. 9.425
 9.417
4. 7.684

7.689

Name the greatest place-value position where the digits differ.
Name the greater number.
5. $8.675 ; 8.654$
6. $3.086 ; 3.194$
$\qquad$
$\qquad$
7. $6.243 ; 6.247$
$\qquad$
$\qquad$

Order from least to greatest.
8. $5.04 ; 5.4 ; 5.406 ; 5.064$

## Round Decimals

Rounding decimals is similar to rounding whole numbers.
Round 4.682 to the nearest tenth.

Step 1 Write 4.682 in a place-value chart.

| Ones | o Tenths | Hundredths | Thousandths |
| :---: | :---: | :---: | :---: |
| 4 | 6 | 6 | 8 |

Step 2 Find the digit in the place to which you want to round.
Circle that digit.
The digit $\quad 6$ is in the tenths place, so circle it.

Step 3 Underline the digit to the right of the circled digit.
The digit $\quad 8$ is to the right of the circled digit, so underline it.
Step 4 If the underlined digit is less than 5, the circled digit stays the same. If the underlined digit is 5 or greater, round up the circled digit.
$\underline{8}>5$, so round 6 up to 7 .
Step 5 After you round the circled digit, drop the digits to the right of the circled digit.
So, 4.682 rounded to the nearest tenth is $\underline{4.7}$.

Write the place value of the underlined digit. Round each number to the place of the underlined digit.

1. $0.3 \underline{9} 2$
2. 5.714
3. 16.908
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Name the place value to which each number was rounded.
4. 0.825 to 0.83
5. 3.815 to 4
6. 1.546 to 1.5

## Decimal Addition

You can use decimal models to help you add decimals.
Add. $1.25+0.85$

Step 1 Shade squares to represent 1.25 .


Step 2 Shade additional squares to represent adding 0.85 .


Step 3 Count the total number of shaded squares.
There are 2 whole squares and 10 one-hundredths squares shaded. So, 2.10 wholes in all are shaded.
So, $1.25+0.85=\underline{2.10}$.

Add. Use decimal models. Draw a picture to show your work.

1. $2.1+0.59$
2. $1.4+0.22$
3. $1.27+1.15$
4. $0.81+0.43$

## Decimal Subtraction

You can use decimal models to help you subtract decimals.
Subtract. $1.85-0.65$

Step 1 Shade squares to represent 1.85.


Step 2 Circle and cross out 65 of the shaded squares to represent subtracting 0.65 .


Step 3 Count the shaded squares that are not crossed out.
Altogether, 1 whole square and 20 one-hundredths squares, or 1.20 wholes, are NOT crossed out.
So, $1.85-0.65=\underline{1.20}$.

Subtract. Use decimal models. Draw a picture to show your work.

1. $1.4-0.61$
$\qquad$
2. $0.84-0.17$
3. $1.6-1.08$
$\qquad$
4. $1.39-1.14$
$\qquad$

## Estimate Decimal Sums and Differences

You can use rounding to help you estimate sums and differences.

Use rounding to estimate $1.24+0.82+3.4$.

Round to the nearest whole number. Then add.


So, the sum is about $\quad 5$.

## Remember:

If the digit to the right of the place you are rounding to is:

- less than 5 , the digit in the rounding place stays the same.
- greater than or equal to 5 , the digit in the rounding place increases by 1 .

Use benchmarks to estimate $8.78 \mathbf{- 0 . 3 0}$.

| 8.78 | $\longrightarrow \quad 8.75$ | Think: 0.78 is between 0.75 and 1 . |
| :---: | :---: | :---: |
| - 0.30 | $\longrightarrow-0.25$ | It is closer to 0.75 . |
|  | 8.5 | Think: 0.30 is between 0.25 and 0.50 |
|  |  | It is closer to 0.25 . |

So, the difference is about $\xrightarrow[8.5]{ }$.

## Use rounding to estimate.

1. $\begin{aligned} & 51.23 \\ & -28.4\end{aligned}$
2. $\$ 29.38$
$-28.4$ $+\$ 42.75$
3. 7.6
$-2.15$
4. 0.74
$+0.20$
5. 2.08
0.56
$+0.41$

Use benchmarks to estimate.
6. $\quad 6.17$
$-3.5$
7. 1.73
1.4
$+3.17$
8. 3.28
$-0.86$
9. $\quad 15.27$
$+41.8$
10. $\$ 23.07$
$-\$ 7.83$
11. $0.427+0.711$
12. $61.05-18.63$
13. $40.51+30.39$

## Add Decimals

Add. $4.37+9.8$
Step 1 Estimate the sum.


Step 2 Line up the place values for each number in a place-value chart.
Then add.
$\left.+\begin{array}{|r|c|c|}\hline \text { Ones } & \text { Tenths } & \text { Hundredths } \\ \hline 4 & \bullet & 3 \\ \hline 9 & \bullet & 8 \\ \hline 14 & \emptyset & 1\end{array}\right]$
$\leftarrow$ sum

Step 3 Use your estimate to determine if your answer is reasonable.
Think: 14.17 is close to the estimate, 14 . The answer is reasonable.
So, $4.37+9.8=\underline{14.17}$.

## Estimate. Then find the sum.

1. Estimate:
1.20
$+0.34$
2. Estimate: $\qquad$
1.52
+1.21
$\begin{array}{r}+1.21 \\ \hline\end{array}$
3. Estimate: $\qquad$
12.25
$+11.25$
4. Estimate:
10.75
$+1.11$
5. Estimate:
22.65
$+18.01$
6. Estimate: $\qquad$
34.41
$+15.37$

## Subtract Decimals

Subtract. $12.56-4.33$
Step 1 Estimate the difference.
$\begin{array}{cc}12.56-4.33 \\ \downarrow \\ \text { Estimate: } & 13-\quad \downarrow=9\end{array}$

Step 2 Line up the place values for each number in a place-value chart.
Then subtract.

-\begin{tabular}{|c|c|c|}
\hline Ones \& Tenths \& Hundredths <br>
\hline 12 \& $\bullet$ \& 5 <br>
\hline 4 \& $\bullet$ \& 3 <br>
\hline 8 \& $\bullet$ \& 2 <br>
\hline

$\leftarrow$

\hline
\end{tabular}$\leftarrow$ difference

Step 3 Use your estimate to determine if your answer is reasonable.
Think: 8.23 is close to the estimate, 9 . The answer is reasonable.
So, $12.56-4.33=\underline{8.23}$.

Estimate. Then find the difference.

1. Estimate: $\qquad$
2. Estimate: $\qquad$

| 1.97 |
| ---: |
| -0.79 |

4.42
$-1.26$
3. Estimate:
10.25
$-8.25$
$\qquad$

Find the difference. Check your answer.
4. 5.75
$-1.11$
5. 25.21
$-19.05$
6. $\quad 42.14$
$-25.07$

## Algebra • Patterns with Decimals

Marla wants to download some songs from the Internet. The first song costs $\$ 1.50$, and each additional song costs $\$ 1.20$. How much will 2,3 , and 4 songs cost?


Step 1 Identify the first term in the sequence.
Think: The cost of 1 song is $\$ 1.50$. The first term is $\$ 1.50$.
Step 2 Identify whether the sequence is increasing or decreasing from one term to the next.
Think: Marla will pay $\$ 1.20$ for each additional song.
The sequence is increasing.
Step 3 Write a rule that describes the sequence. Start with $\$ 1.50$ and add \$1.20.
Step 4 Use your rule to find the unknown terms in the sequence.

| Number of <br> Songs | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| Cost | $\$ 1.50$ | $1.50+1.20=\$ 2.70$ | $2.70+1.20=\$ 3.90$ | $3.90+1.20=\$ 5.10$ |

So, 2 songs cost $\$ 2.70$, 3 songs cost $\$ 3.90$, and 4 songs cost $\$ 5.10$.

Write a rule for the sequence.

1. $0.4,0.7,1.0,1.3, \ldots$
2. $5.25,5.00,4.75,4.50, \ldots$

Rule: $\qquad$ Rule: $\qquad$

Write a rule for the sequence, then find the unknown term.
3. $26.1,23.8,21.5, \ldots, 16.9$
4. 4.62, 5.03, $\qquad$ , 5.85, 6.26

## Problem Solving • Add and Subtract Money

At the end of April, Mrs. Lei had a balance of $\$ 476.05$. Since then she has written checks for $\$ 263.18$ and $\$ 37.56$, and made a deposit of $\$ 368.00$. Her checkbook balance currently shows $\$ 498.09$.
Find Mrs. Lei's correct balance.


1. At the end of June, Mr. Kent had a balance of $\$ 375.98$. Since then he has written a check for $\$ 38.56$ and made a deposit of $\$ 408.00$. His checkbook shows a balance of $\$ 645.42$. Find Mr. Kent's correct balance.
2. Jordan buys a notebook for himself and each of 4 friends. Each notebook costs $\$ 1.85$. Make a table to find the cost of 5 notebooks.

## Choose a Method

There is more than one way to find the sums and differences of whole numbers and decimals. You can use properties, mental math, place value, a calculator, or paper and pencil.

Choose a method. Find the sum or difference.

- Use mental math for problems with fewer digits or rounded numbers.

$$
\begin{array}{r}
2.86 \\
-1.2 \\
\hline 1.66
\end{array}
$$

- Use place value for larger numbers.
$\$ 15.79$
$\begin{array}{r}\text { + } \$ 32.81 \\ \hline \$ 48.60\end{array}$
- Use a calculator for difficult numbers or very large numbers.


Find the sum or difference.

1. 73.9
$\begin{array}{r}+4.37 \\ \hline\end{array}$
2. 127.35
$+928.52$
3. 10
$+2.25$
4. 0.36

| +1.55 |
| :--- |

5. 71.4

| +11.5 |
| :--- |

6. $\quad 90.4$
$+88.76$
7. 3.3
$+5.6$
8. 14.21
1.79
$+15.88$
9. $68.20-42.10$
10. $2.25-1.15$
11. $875.33-467.79$
12. $97.26-54.90$

## Algebra • Multiplication Patterns with Decimals

You can use patterns and place value to help you place the decimal point.

To multiply a number by a power of 10, you can use the exponent to determine how the position of the decimal point changes in the product.

$$
\begin{aligned}
& 10^{\circ} \times 5.18=\frac{5.18}{51.8} \\
& 10^{1} \times 5.18=\frac{5}{518} \\
& 10^{2} \times 5.18=\underline{5,180} \\
& 10^{3} \times 5.18=\underline{2}
\end{aligned}
$$

| Exponent | Move decimal point: |
| :---: | :--- |
| 0 | 0 places to the right |
| 1 | 1 place to the right |
| 2 | 2 places to the right |
| 3 | 3 places to the right |

You can use place-value patterns to find the product of a number and the decimals 0.1 and 0.01 .
$1 \times 2,457=\underline{2,457}$
$0.1 \times 2,457=\frac{245.7}{24.57}$
$0.01 \times 2,457=\underline{24.57}$

Multiply by: Move decimal point:
1
$0.1 \quad 1$ place to the left
$0.01 \quad 2$ places to the left

Complete the pattern.

1. $10^{\circ} \times 25.89=$
$\qquad$
$10^{2} \times 25.89=$ $\qquad$
$10^{3} \times 25.89=$ $\qquad$

## Multiply Decimals and Whole Numbers

You can draw a quick picture to help multiply a decimal and a whole number.

Find the product. $4 \times 0.23$
Draw a quick picture. Each bar represents one tenth, or 0.1.
Each circle represents one hundredth, or 0.01.

Step 1
Draw 4 groups of 2 tenths and 3 hundredths.


So, $4 \times 0.23=\underline{0.92}$

## Step 2

Combine the tenths. Then combine the hundredths.


Step 3
There are $\frac{12}{10}$ hundredths. Rename 10 hundredths as 1 tenth. Then you will have 9 tenths and 2 hundredths.


Find the product. Use a quick picture.

1. $2 \times 0.19=$ $\qquad$ 2. $3 \times 0.54=$ $\qquad$
2. $4 \times 0.07=$ $\qquad$
3. $3 \times 1.22=$ $\qquad$

## Multiplication with Decimals and Whole Numbers

To find the product of a one-digit whole number and a decimal, multiply as you would multiply whole numbers. To find the number of decimal places in the product, add the number of decimal places in the factors.

To multiply $6 \times 4.25$, multiply as you would multiply $6 \times 425$.

Step 1
Multiply the ones.

$$
\begin{array}{r}
3 \\
425 \\
\times \quad 6 \\
\hline 0
\end{array}
$$

Step 2
Multiply the tens.

$$
\begin{array}{r}
13 \\
425 \\
\times \quad 6 \\
\hline 50
\end{array}
$$

## Step 3

Multiply the hundreds. Then place the decimal point in the product.


So, $6 \times 4.25=\underline{25.50}$.

Place the decimal point in the product.

1. 8.23 Think: The place
$\begin{array}{r}\times \quad 6 \\ \hline 49.38\end{array}$
value of the decimal factor is hundredths.
2. 6.3
$\begin{array}{r}\times \quad 4 \\ \hline 252\end{array}$
3. $\begin{array}{r}16.82 \\ \times \quad 5 \\ \hline 8410\end{array}$

Find the product.
4. 5.19
$\times 3$
5. 7.2
$\times 8$
6. $\quad 37.46$
$\times \quad 7$

## Multiply Using Expanded Form

You can use a model and partial products to help you find the product of a two-digit whole number and a decimal.

Find the product. $13 \times 6.8$
Step 1 Draw a large rectangle. Label its longer side 13 and its shorter side 6.8. The area of the large rectangle represents the product, $13 \times \underline{6.8}$.

Step 2 Rewrite the factors in expanded form. Divide the large rectangle into four smaller rectangles. Use the expanded forms to label the smaller rectangles.

Step 3 Multiply to find the area of each small rectangle.

$10 \times 6=\underline{60} \quad 10 \times 0.8=\underline{8} \quad 3 \times 6=\underline{18} \quad 3 \times 0.8=\underline{2} 4$

Step 4 Add to find the total area.
$\underline{60}+\underline{8}+\underline{18}+\underline{2.4}=\underline{88.4}$
So, $13 \times 6.8=\underline{88.4}$.

Draw a model to find the product.

1. $18 \times 0.25=$ $\qquad$ 2. $26 \times 7.2=$

Find the product.
3. $17 \times 9.3=$ $\qquad$
4. $21 \times 43.5=$ $\qquad$
5. $48 \times 4.74=$

## Problem Solving • Multiply Money

Three students in the garden club enter a pumpkin-growing contest. Jessie's pumpkin is worth $\$ 12.75$. Mara's pumpkin is worth 4 times as much as Jessie's. Hayden's pumpkin is worth $\$ 22.25$ more than Mara's. How much is Hayden's pumpkin worth?

| Read the Problem | Solve the Problem |
| :---: | :---: |
| What do I need to find? <br> I need to find how much Hayden's pumpkin is worth | The amount that Hayden's and Mara's pumpkins are worth depends on how much Jessie's pumpkin is worth. Draw a diagram to compare the amounts without calculating. Then use the diagram to find how much each person's pumpkin is worth. |
| What information do I need to use? <br> I need to use the worth of Jessie's pumpkin to find how much Mara's and $\qquad$ Hayden's pumpkins are worth. |  |
| How will I use the information: <br> I can draw a diagram to show how much Jessie's and Mara's pumpkins are worth to find how much Hayden's pumpkin is worth. | Jessie: \$12.75 <br> Mara: $\quad 4 \times \underline{\$ 12.75}=\underline{\$ 51.00}$ <br> Hayden: $\underline{\$ 51.00}+\$ 22.25=\underline{\$ 73.25}$ |
| So Hayden's pumpkin is worth \$73.25 |  |

1. Three friends go to the local farmers' market. Latasha spends $\$ 3.35$. Helen spends 4 times as much as Latasha. Dee spends $\$ 7.50$ more than Helen. How much does Dee spend?
2. Alexia raises $\$ 75.23$ for a charity. Sue raises 3 times as much as Alexia. Manuel raises $\$ 85.89$. How much money do the three friends raise for the charity in all?

## Decimal Multiplication

You can use decimal squares to multiply decimals.
Multiply. $0.2 \times 0.9$
Step 1 Draw a square with 10 equal rows and 10 equal columns.

Step 2 Shade 9 columns to represent $\underline{0.9}$.
Step 3 Shade 2 rows to represent $\underline{0.2}$

Step 4 Count the number of small squares where the shadings overlap: 18 squares, or 0.18 .

The shadings
overlap in 18
squares, or 0.18 .


9 columns represent 0.9.
So, $0.2 \times 0.9=\underline{0.18}$.

Multiply. Use the decimal model.

1. $0.3 \times 0.2=$ $\qquad$ 2. $0.9 \times 0.5=$ $\qquad$ 3. $0.1 \times 1.8=$ $\qquad$


2. $0.4 \times 0.4=$ $\qquad$
3. $0.6 \times 0.5=$ $\qquad$
4. $0.4 \times 1.2=$ $\qquad$


## Multiply Decimals

Multiply. $9.3 \times 5.27$
Step 1 Multiply as with whole numbers.

$$
\begin{array}{r}
26 \\
2 \\
527 \\
\times \quad 93 \\
\hline 1,581 \\
+47,430 \\
\hline 49,011
\end{array}
$$

Step 2 Add the number of decimal places in the factors to place the decimal point in the product.

$$
\begin{aligned}
& \quad 5.27 \longleftarrow+\frac{2}{1} \text { decimal places } \\
& \frac{\times 9.3}{1,581} \longleftarrow+\frac{1}{2} \text { decimal place } \\
& +\frac{47,430}{49.011} \longleftarrow \quad-3 \text { decimal places }
\end{aligned}
$$

So, $9.3 \times 5.27=\underline{49.011}$.

Place the decimal point in the product.

1. 1.6
$\begin{array}{r}\times \quad 0.7 \\ \hline 112\end{array}$
2. $\quad 14.2$
$\times 7.6$
10792
3. 3.59
$\times 4.8$
17232

Find the product.
4.
$\begin{array}{r}5.7 \\ \times \quad 0.8 \\ \hline\end{array}$
5. 35.1

| $\times 8.4$ |
| :--- |

6. $\quad 2.19$
$\begin{array}{r}\times 6.3 \\ \hline\end{array}$

## Zeros in the Product

Sometimes when you multiply two decimals, there are not enough digits in the product to place the decimal point.

Multiply. $0.9 \times 0.03$
Step 1 Multiply as with whole numbers.

$$
\begin{array}{r}
3 \\
\times \quad 9 \\
\hline 27
\end{array}
$$

Step 2 Find the number of decimal places in the product by adding the number of decimal places in the factors.


Step 3 Place the decimal point.
0,027 There are not enough digits in the product to place the decimal point. Write zeros as needed to the left of the product to place the decimal point.
So, $0.9 \times 0.03=\underline{0.027}$.

Write zeros in the product.

1. 0.8
$\begin{array}{r}0.1 \\ \times \quad 0.1 \\ \hline \quad 8\end{array}$
2. 0.04
0.7
$\times \quad$
28
3. 0.03
$\times \quad 0.3$
$\times \quad 9$

Find the product.
4. $\$ 0.06$
$\begin{array}{r} \\ \times \quad 0.5 \\ \hline\end{array}$
5. 0.09
$\times 0.8$
$\times$
6. 0.05
$\begin{array}{r}\times \quad 0.7 \\ \hline\end{array}$

## Algebra • Division Patterns with Decimals

To divide a number by 10,100 , or 1,000 , use the number of zeros in the divisor to determine how the position of the decimal point changes in the quotient.

$$
\begin{aligned}
& 147 \div 1=\underline{147} \\
& 147 \div 10=\underline{14.7} \\
& 147 \div 100=\underline{1.47} \\
& 147 \div 1,000=\underline{0.147}
\end{aligned}
$$

Number of zeros: Move decimal point:
$0 \quad 0$ places to the left
1
1 place to the left
2 places to the left
3 places to the left

To divide a number by a power of 10, you can use the exponent to determine how the position of the decimal point changes in the quotient.

$$
\begin{aligned}
& 97.2 \div 10^{0}=\frac{97.2}{9.72} \\
& 97.2 \div 10^{1}=\underline{9.72} \\
& 97.2 \div 10^{2}=\underline{0.972}
\end{aligned}
$$

Exponent Move decimal point:
0 places to the left
1 place to the left
2 places to the left

Complete the pattern.

1. $358 \div 10^{0}=$ $\qquad$
2. $102 \div 10^{0}=$ $\qquad$ 3. $99.5 \div 1=$ $\qquad$

$$
358 \div 10^{1}=
$$

$102 \div 10^{1}=$ $\qquad$
$\qquad$
$358 \div 10^{2}=$

$$
99.5 \div 100=
$$

$358 \div 10^{3}=$ $\qquad$
$102 \div 10^{2}=$ $\qquad$
$102 \div 10^{3}=$ $\qquad$
$\qquad$

## Divide Decimals by Whole Numbers

You can draw a quick picture to help you divide a decimal by a whole number.
In a decimal model, each large square represents one, or 1. Each bar represents one-tenth, or 0.1.

Divide. $1.2 \div 3$

Step 1 Draw a quick picture to represent the dividend, 1.2 .


Step 2 Draw 3 circles to represent the divisor, 3 .


Step 4 Share the tenths equally among 3 groups.


Each group contains 0 ones and 4 tenths.
So, $1.2 \div 3=\underline{0.4}$.

## Divide. Draw a quick picture.

1. $2.7 \div 9=$ $\qquad$
2. $4.8 \div 8=$ $\qquad$
3. $2.8 \div 7=$
4. $7.25 \div 5=$ $\qquad$
5. $3.78 \div 3=$ $\qquad$
6. $8.52 \div 4=$ $\qquad$

## Estimate Quotients

You can use multiples and compatible numbers to estimate decimal quotients.

Estimate. $249.7 \div 31$
Step 1 Round the divisor, 31, to the nearest 10.
31 rounded to the nearest 10 is 30

Step 2 Find the multiples of 30 that the dividend, 249.7, is between.
249.7 is between $\underline{240}$ and $\underline{270}$.

Step 3 Divide each multiple by the rounded divisor, 30.

$$
240 \div 30=8 \quad 270 \div 30=9
$$

So, two possible estimates are $\quad 8$ and $\underline{\square}$.

Use compatible numbers to estimate the quotient.

1. $23.6 \div 7$
2. $469.4 \div 62$
$\qquad$ $\div$ $\qquad$ $=$ $\qquad$
$\qquad$ $\div$ $\qquad$
$\qquad$

## Estimate the quotient.

3. $338.7 \div 49$
4. $75.1 \div 9$
5. $674.8 \div 23$
6. $61.9 \div 7$
7. $96.5 \div 19$
8. $57.2 \div 8$

## Division of Decimals by Whole Numbers

Divide. $19.61 \div 37$

Step 1 Estimate the quotient.
2,000 hundredths $\div 40=$ $\qquad$ hundredths, or 0.50 .
$3 7 \longdiv { 1 9 . 6 1 }$
So, the quotient will have a zero in the ones place.

$$
\begin{gathered}
05 \\
3 7 \longdiv { 1 9 . 6 1 } \\
-185 \\
\hline 11
\end{gathered}
$$

Use the estimate. Try 5 in the tenths place.
Multiply. $\quad 5 \times 37=\underline{185}$
Subtract. $196-\underline{185}=\underline{11}$
Check. $11<37$
Step 3 Divide the hundredths.
Estimate: 120 hundredths $\div 40=3$ hundredths.
$\begin{array}{r}0.53 \\ \hline 19.61\end{array}$
Multiply. $\underline{3} \times 37=\underline{111}$
$\begin{array}{r}-185 \\ \hline 111\end{array}$
Subtract. $\underline{111}-\underline{111}=\underline{0}$
Check. 0
Place the decimal point in the quotient.
So, $19.61 \div 37=\underline{0.53}$.

Write the quotient with the decimal point placed correctly.

1. $5.94 \div 3=198$
2. $48.3 \div 23=21$

Divide.
3. $9 \longdiv { 6 1 . 2 }$
4. $1 7 \longdiv { 8 3 . 3 }$
5. $9 \longdiv { 7 . 3 8 }$

## Decimal Division

You can use decimal models to divide tenths.
Divide. $1.8 \div 0.3$.
Step 1 Shade 18 tenths to represent the dividend, 1.8 .

Step 2 Divide the 18 tenths into groups of $\underline{3}$ tenths to represent the divisor, 0.3


Step 3 Count the groups.
There are $\underline{6}$ groups of 0.3 in 1.8 . So, $1.8 \div 0.3=\underline{6}$

You can use decimal models to divide hundredths.
Divide. $0.42 \div 0.06$
Step 1 Shade 42 squares to represent the dividend, $\underline{0.42}$.

Step 2 Divide the 42 small squares into groups of 6 hundredths to represent the divisor, 0.06

Step 3 Count the groups.
There are $\quad 7$ groups of 0.06 in 0.42 . So, $0.42 \div 0.06=\underline{7}$.

Use the model to complete the number sentence.

1. $1.4 \div 0.7=$


2. $0.15 \div 0.03=$ $\qquad$


Divide. Use decimal models.
3. $2.7 \div 0.3=$ $\qquad$ 4. $0.52 \div 0.26=$
5. $0.96 \div 0.16=$ $\qquad$

## Divide Decimals

You can multiply the dividend and the divisor by the same power of 10 to make the divisor a whole number. As long as you multiply both the dividend and the divisor by the same power of 10 , the quotient stays the same.

## Example 1: Divide. $0.84 \div 0.07$

Multiply the dividend, $\underline{0.84}$, and the divisor, $\underline{0.07}$, by the power of 10 that makes the divisor a whole number.


Since $84 \div 7=12$, you know that $0.84 \div 0.07=\underline{12}$.

Example 2: Divide. $4.42 \div 3.4$
Multiply both the dividend and the divisor by 10 to make the divisor a whole number.

Divide as you would whole numbers. Place the decimal point in the quotient, above the decimal point in the dividend.
$3 . 4 \longdiv { 4 . 4 2 } \longrightarrow \begin{array} { c } { \text { Multiply } 3 . 4 } \\ { \text { and } 4 . 4 2 } \\ { \text { both by } 1 0 } \end{array} \longrightarrow 3 4 \longdiv { 4 4 . 2 }$

So, $4.42 \div 3.4=\underline{1.3}$.
$3 4 \longdiv { 4 4 . 2 }$

| -34 |
| :---: |
| 102 |

$\begin{array}{r}-102 \\ \hline 0\end{array}$

## Copy and complete the pattern.

1. $54 \div 6=$ $\qquad$

$$
5.4 \div \square=9
$$

$$
-\quad \div 0.06=9
$$

2. $184 \div 23=$ $\qquad$
$18.4 \div$ $\qquad$ $=8$
3. $138 \div 2=$ $\qquad$
$13.8 \div$ $\qquad$ $=69$

- 

$$
\div 0.02=69
$$

## Divide.

4. $1 . 4 \longdiv { 9 . 8 }$
5. $0 . 3 \longdiv { 0 . 6 }$
6. $3.64 \div 1.3$

## Write Zeros in the Dividend

When there are not enough digits in the dividend to complete the division, you can write zeros to the right of the last digit in a decimal number in the dividend. Writing zeros to the right of the last digit will not change the value of the dividend or the quotient.

Divide. $5.2 \div 8$
Step 1 Divide as you would whole numbers. Place the decimal point in the quotient above the decimal point in the dividend.

The decimal point in the


$$
\begin{array}{r}
-48 \\
4
\end{array}
$$

Step 2 The difference is less than the divisor. Write a 0 in the dividend to the right of the last digit and continue to divide.


So, $5.2 \div 8=\underline{0.65}$. Then continue to divide.

Write the quotient with the decimal point placed correctly.

1. $3 \div 0.4=75$
2. $25.2 \div 8=315$
3. $60 \div 25=24$
4. $8.28 \div 0.72=115$

## Divide.

5. $6 \longdiv { 4 3 . 5 }$
6. $1 . 4 \longdiv { 7 . 7 }$
7. $3 0 \longdiv { 7 2 }$
8. $0 . 1 8 \longdiv { 0 . 6 3 }$

## Problem Solving • Decimal Operations

Rebecca spent $\$ 32.55$ for a photo album and three identical candles. The photo album cost $\$ 17.50$ and the sales tax was $\$ 1.55$. How much did each candle cost?


## Use a flowchart to help you solve the problem.

1. Maria spent $\$ 28.69$ on one pair of jeans and two T-shirts. The jeans cost $\$ 16.49$. Each T-shirt cost the same amount. The sales tax was \$1.62. How much did each T-shirt cost?
2. At the skating rink, Sean and Patrick spent $\$ 17.45$ on admission and snacks. They used one coupon for $\$ 2$ off the admission. The snacks cost $\$ 5.95$. What is the regular admission cost for one?

## Addition with Unlike <br> Denominators

Karen is stringing a necklace with beads. She puts green beads on $\frac{1}{2}$ of the string and purple beads on $\frac{3}{10}$ of the string. How much of the string does Karen cover with beads?

You can use fraction strips to help you add fractions with unlike denominators. Trade fraction strips of fractions with unlike denominators for equivalent strips of fractions with like denominators.

Use fraction strips to find the sum. Write your answer in simplest form.
$\frac{1}{2}+\frac{3}{10}$
Step 1 Use a $\frac{1}{2}$ strip and three $\frac{1}{10}$ strips to model fractions with unlike denominators.
Step 2 Trade the $\frac{1}{2}$ strip for five $\frac{1}{10}$ strips.
$\frac{1}{2}+\frac{3}{10}=\frac{5}{10}+\frac{3}{10}$


Step 3 Add the fractions with like denominators.

$$
\frac{5}{10}+\frac{3}{10}=\frac{8}{10}
$$

Step 4 Write the answer in simplest form.

$$
\frac{8}{10}=\frac{4}{5}
$$

So, Karen covers $\frac{4}{5}$ of the string with beads.

## Use fraction strips to find the sum. Write your answer in

 simplest form.1. $\frac{3}{8}+\frac{3}{4}$
2. $\frac{2}{3}+\frac{1}{4}$
3. $\frac{5}{6}+\frac{7}{12}$

## Subtraction with Unlike Denominators

You can use fraction strips to help you subtract fractions with unlike denominators. Trade fraction strips of fractions with unlike denominators for equivalent strips of fractions with like denominators.

Use fraction strips to find the difference. Write your answer in simplest form.
$\frac{1}{2}-\frac{1}{10}$
Step 1 Use a $\frac{1}{2}$ fraction strip to model the first fraction.

Step 2 Trade the $\frac{1}{2}$ strip for
 five $\frac{1}{10}$ strips.
$\frac{1}{2}-\frac{1}{10}=\frac{5}{10}-\frac{1}{10}$
Step 3 Subtract by taking away $\frac{1}{10}$.
$\frac{5}{10}-\frac{1}{10}=\frac{4}{10}$


So, $\frac{1}{2}-\frac{1}{10}=\frac{4}{10}$. Written in simplest form, $\frac{4}{10}=\frac{2}{5}$.
Use fraction strips to find the difference. Write your answer in simplest form.

1. $\frac{7}{8}-\frac{1}{2}$
2. $\frac{2}{3}-\frac{1}{4}$
3. $\frac{5}{6}-\frac{1}{3}$
4. $\frac{1}{2}-\frac{1}{3}$
5. $\frac{9}{10}-\frac{4}{5}$
6. $\frac{2}{3}-\frac{5}{12}$

## Estimate Fraction Sums and Differences

You can round fractions to 0 , to $\frac{1}{2}$, or to 1 to estimate sums and differences.

Estimate the sum. $\frac{4}{6}+\frac{1}{9}$
Step 1 Find $\frac{4}{6}$ on the number line.
Is it closest to $0, \frac{1}{2}$, or 1 ?
The fraction $\frac{4}{6}$ is closest to $\frac{1}{2}$.


Step 2 Find $\frac{1}{9}$ on the number line. Is it closest to $0, \frac{1}{2}$, or 1 ?
The fraction $\frac{1}{9}$ is closest to 0 .


Step 3 To estimate the sum $\frac{4}{6}+\frac{1}{9}$, add the two rounded numbers.

$$
\frac{1}{2}+0=\frac{1}{2}
$$

So, $\frac{4}{6}+\frac{1}{9}$ is about $\frac{1}{2}$.

Estimate the sum or difference.


1. $\frac{4}{6}+\frac{1}{8}$
2. $\frac{2}{6}+\frac{7}{8}$
3. $\frac{5}{6}-\frac{3}{8}$
4. $\frac{4}{6}+\frac{3}{8}$
5. $\frac{7}{8}-\frac{5}{6}$
6. $\frac{1}{6}+\frac{7}{8}$

## Factors

The factors of a number are the numbers that divide evenly into it.
Prime factors are the factors of a given number that are prime.
A prime number has exactly two factors, 1 and itself. A composite number has more than two factors.

You can use division to find the factors of a number.
Find the factors of 45 .
Step 1 You know that 45 is an odd number so
it cannot be divided by 2 . Try dividing by 3 .
$45 \div 3=15$
So, $3 \times 15=45$.
3 and 15 are factors of 45 .
Step 2 Identify the factors as prime or composite numbers.
3 is a prime number. Its factors are 1 and itself.
15 is a composite number. It's factors are: $1,3,5$, and itself.
Step 3 You can divide 15 further because it is not a prime number.
$15 \div 3=5$
3 and 5 are prime factors.
So, the prime factors of 45 are: 3,3 , and 5
Step 4 You can write 45 as a product of its prime factors.
Write them in order from least to greatest.
$3 \times 3 \times 5=45$

Write the number as the product of prime factors.

1. 8
2. 15
3. 30
4. 50

## Common Denominators and Equivalent Fractions

You can find a common denominator of two fractions.
A common denominator of two fractions is a common multiple of their denominators.

Find a common denominator of $\frac{1}{6}$ and $\frac{7}{10}$. Rewrite the pair of fractions using a common denominator.

Step 1 Identify the denominators.
The denominators are 6 and 10.
Step 2 List the multiples of the greater denominator, 10.
Multiples of 10: 10, 20, 30, 40, 50, 60, ...
Step 3 Check if any of the multiples of the greater denominator are evenly divisible by the other denominator.

Both 30 and 60 are evenly divisible by 6.
Common denominators of $\frac{1}{6}$ and $\frac{7}{10}$ are 30 and 60.
Step 4 Rewrite the fractions with a denominator of 30.
Multiply the numerator and the denominator of each fraction by the same number so that the denominator results in 30.
$\frac{1}{6}=\frac{1 \times 5}{6 \times 5}=\frac{5}{30} \quad \frac{7}{10}=\frac{7 \times 3}{10 \times 3}=\frac{21}{30}$

Use a common denominator to write an equivalent fraction for each fraction.

1. $\frac{5}{12}, \frac{2}{9}$
common denominator:
2. $\frac{3}{8}, \frac{5}{6}$
common denominator: $\qquad$
3. $\frac{2}{9}, \frac{1}{6}$
common denominator: $\qquad$
4. $\frac{3}{4}, \frac{9}{10}$
common denominator: $\qquad$

## Add and Subtract Fractions

To add or subtract fractions with unlike denominators, you need to rename them as fractions with like denominators. You can do this by making a list of equivalent fractions.

Add. $\frac{5}{12}+\frac{1}{8}$
Step 1 Write equivalent fractions for $\frac{5}{12}$. $\frac{5}{12}, \frac{10}{24}, \frac{15}{36}, \frac{20}{48}$
Step 2 Write equivalent fractions for $\frac{1}{8}$. $\frac{1}{8}, \frac{2}{16}, \frac{3}{24}$ )
Step 3 Rewrite the problem using the equivalent fractions.
Then add.
$\frac{5}{12}+\frac{1}{8}$ becomes $\frac{10}{24}+\frac{3}{24}=\frac{13}{24}$.

Stop when you find two fractions with the same denominator.

Subtract. $\frac{9}{10}-\frac{1}{2}$
Step 1 Write equivalent fractions for $\frac{9}{10}$. $\quad 10, \frac{1}{20}, \frac{27}{30}, \frac{3}{40}$
Step 2 Write equivalent fractions for $\frac{1}{2}$. $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}$
Step 3 Rewrite the problem using the equivalent fractions.
Then subtract.
$\frac{9}{10}-\frac{1}{2}$ becomes $\frac{9}{10}-\frac{5}{10}=\frac{4}{10}$. Written in simplest form, $\frac{4}{10}=\frac{2}{5}$.

Find the sum or difference. Write your answer in simplest form.

1. $\frac{2}{9}+\frac{1}{3}$
2. $\frac{1}{2}+\frac{2}{5}$
3. $\frac{1}{4}+\frac{1}{6}$
4. $\frac{1}{5}+\frac{3}{4}$
5. $\frac{7}{8}-\frac{1}{4}$
6. $\frac{3}{4}-\frac{2}{3}$
7. $\frac{9}{10}-\frac{4}{5}$
8. $\frac{8}{9}-\frac{5}{6}$

## Add and Subtract Mixed Numbers

When you add or subtract mixed numbers, you may need to rename the fractions as fractions with a common denominator.

Find the sum. Write the answer in simplest form. $5 \frac{3}{4}+2 \frac{1}{3}$
Step 1 Model $5 \frac{3}{4}$ and $2 \frac{1}{3}$.

| 1 | 1 | 1 | 1 | 1 | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | $\frac{1}{3}$ |  |  |  |  |  |  |

Step 2 A common denominator for $\frac{3}{4}$ and $\frac{1}{3}$ is 12, so rename $5 \frac{3}{4}$ as $5 \frac{9}{12}$ and $2 \frac{1}{3}$ as $2 \frac{4}{12}$.


Step 3 Add the fractions.

$$
\frac{9}{12}+\frac{4}{12}=\frac{13}{12}
$$

Step 4 Add the whole numbers
$5+2=7$
Add the sums. Write the answer in simplest form.

$$
\frac{13}{12}+7=7 \frac{13}{12}, \text { or } 8 \frac{1}{12}
$$

So, $5 \frac{3}{4}+2 \frac{1}{3}=8 \frac{1}{12}$.

Find the sum or difference. Write your answer in simplest form.

1. $2 \frac{2}{9}+4 \frac{1}{6}$
2. $10 \frac{5}{6}+5 \frac{3}{4}$
3. $11 \frac{7}{8}-9 \frac{5}{6}$
4. $18 \frac{3}{5}-14 \frac{1}{2}$

## Subtraction with Renaming

You can use a common denominator to find the difference of two mixed numbers.
Estimate. $9 \frac{1}{6}-2 \frac{3}{4}$
Step 1 Estimate by using $0, \frac{1}{2}$, and 1 as benchmarks.
$9 \frac{1}{6}-2 \frac{3}{4} \rightarrow 9-3=6$
So, the difference should be close to 6 .
Step 2 Identify a common denominator.
$9 \frac{1}{6}-2 \frac{3}{4} \quad$ A common denominator of 6 and 4 is 12 .
Step 3 Write equivalent fractions using the common denominator.
$9 \frac{1}{6}=9+\frac{1 \times 2}{6 \times 2}=9 \frac{2}{12}$
$2 \frac{3}{4}=2+\frac{3 \times 3}{4 \times 3}=2 \frac{9}{12}$
Step 4 Rename if needed. Then subtract.
Since $\frac{2}{12}<\frac{9}{12}$, rename $9 \frac{2}{12}$ as $8 \frac{14}{12}$.
Subtract. $8 \frac{14}{12}-2 \frac{9}{12}=6 \frac{5}{12}$
So, $9 \frac{1}{6}-2 \frac{3}{4}=6 \frac{5}{12}$.
Since the difference of $6 \frac{5}{12}$ is close to 6 , the answer is reasonable.

## Estimate. Then find the difference and write it in simplest form.

1. Estimate:

$\qquad$
2. Estimate:

$$
8 \frac{2}{3}-2 \frac{7}{9}
$$

$\qquad$
5. Estimate: $\qquad$

$$
7 \frac{3}{16}-1 \frac{5}{8}
$$

$\qquad$
2. Estimate:
$7 \frac{1}{4}-2 \frac{5}{12}$ $\qquad$
4. Estimate: $\qquad$
$9 \frac{2}{5}-3 \frac{3}{4}$ $\qquad$
6. Estimate:
$2 \frac{4}{9}-1 \frac{11}{18}$ $\qquad$

## Algebra • Patterns with <br> Fractions

You can find an unknown term in a sequence by finding a rule for the sequence.

Find the unknown term in the sequence.
$1 \frac{2}{5}, 1 \frac{7}{10}, 2$, $\qquad$ , $2 \frac{3}{5}$

Step 1 Find equivalent fractions with a common denominator for all of the terms.

The denominators are 5 and 10 . A common denominator is 10 .
$1 \frac{2}{5}=1 \frac{4}{10}$ and $2 \frac{3}{5}=2 \frac{6}{10}$

Step 2 Write the terms in the sequence using the common denominator.
$1 \frac{4}{10}, 1 \frac{7}{10}, 2, \ldots, 2 \frac{6}{10}$

Step 3 Write a rule that describes the pattern.
The sequence increases. To find the difference between terms, subtract at least two pairs of consecutive terms.
$1 \frac{7}{10}-1 \frac{4}{10}=\frac{3}{10} \quad 2-1 \frac{7}{10}=\frac{3}{10}$
So, a rule is to add $\frac{3}{10}$.

Step 4 Use the rule to find the unknown term.
Add $\frac{3}{10}$ to the third term to find the unknown term.
$2+\frac{3}{10}=2 \frac{3}{10}$

Write a rule for the sequence. Then, find the unknown term.

1. $2 \frac{2}{3}, 3 \frac{1}{2}$,
 $5 \frac{1}{6}, 6$
2. $4 \frac{1}{2}, 3 \frac{7}{8}, 3 \frac{1}{4}, \longrightarrow, 2$

Rule: $\qquad$

Rule: $\qquad$

## Problem Solving • Practice Addition and Subtraction

Makayla walks for exercise. She wants to walk a total of 6 miles.
On Monday, she walked $2 \frac{5}{6}$ miles. On Tuesday, she walked $1 \frac{1}{3}$ miles.
How many more miles does Makayla need to walk to reach her goal?

| Read the Problem | Solve the Problem |
| :---: | :---: |
| What do I need to find? <br> I need to find the distance that | - Start with the equation. $6=2 \frac{5}{6}+1 \frac{1}{3}+x$ |
| Makayla needs to wallk. | Subtraction is the inverse operation of addition. |
| What information do I need to use? <br> I need to use the distance she | - Use subtraction to work backward and rewrite the equation. $6-2 \frac{5}{6}-1 \frac{1}{3}=x$ |
| wants to walk and the distance she has already walked. | - Subtract to find the value of $x$. $6=5 \frac{6}{6} \quad 3 \frac{1}{6}=2 \frac{7}{6}$ |
| How will I use the information? <br> First I can write an equation | $-2 \frac{5}{6}=\frac{-2 \frac{5}{6}}{3 \frac{1}{6}} \quad-1 \frac{1}{3}=\frac{-1 \frac{2}{6}}{1 \frac{5}{6}}$ |
| $6=2 \frac{5}{6}+1 \frac{1}{3}+x$ | Estimate to show that your answer is reasonable. |
| Then II can worlk backward to | $3+1+2=6$ |
| solve the problem. | ayla has to walk $1 \frac{5}{6}$ more miles |

1. Ben has $5 \frac{3}{4}$ cups of sugar. He uses $\frac{2}{3}$ cup of sugar to make cookies. Then he uses $2 \frac{1}{2}$ cups of sugar to make fresh lemonade. How many cups of sugar does Ben have left?
2. Cheryl has 5 ft of ribbon. She cuts a $3 \frac{3}{4}$-ft strip to make a hair bow. Then she cuts a $\frac{5}{6}$-ft strip for a border on a scrapbook page. Is there enough ribbon for Cheryl to cut two $\frac{1}{3}$-ft pieces to put on a picture frame? Explain.

## Algebra • Use Properties of Addition

You can use the properties of addition to help you add fractions with unlike denominators.

## Use the Commutative Property and the Associative Property.

Add. $\quad\left(3 \frac{2}{5}+1 \frac{7}{15}\right)+2 \frac{1}{5}$

$$
\begin{array}{rlrl}
\left(3 \frac{2}{5}+1 \frac{7}{15}\right)+2 \frac{1}{5} & =\left(1 \frac{7}{15}+3 \frac{2}{5}\right)+2 \frac{1}{5} & \leftarrow \begin{array}{l}
\text { Use the Commutative Property to } \\
\text { order fractions with like denominators. }
\end{array} \\
& =1 \frac{7}{15}+\left(3 \frac{2}{5}+2 \frac{1}{5}\right) & \leftarrow \leftarrow \begin{array}{l}
\text { Use the Associative Property to group } \\
\text { fractions with like denominators. }
\end{array} \\
& =1 \frac{7}{15}+5 \frac{3}{5} & & \leftarrow \begin{array}{l}
\text { Use mental math to add the fractions } \\
\text { with like denominators. }
\end{array} \\
& =1 \frac{7}{15}+5 \frac{9}{15} & & \leftarrow \begin{array}{l}
\text { Write equivalent fractions with like } \\
\text { denominators. Then add. }
\end{array} \\
& =6 \frac{16}{15}=7 \frac{1}{15} & & \leftarrow \text { Rename and simplify. }
\end{array}
$$

Use the properties and mental math to solve. Write your answer in simplest form.

1. $\left(\frac{5}{7}+\frac{3}{14}\right)+\frac{4}{7}$
2. $\left(\frac{2}{5}+\frac{5}{9}\right)+\frac{7}{9}$
3. $\left(3 \frac{7}{10}+5 \frac{3}{4}\right)+\frac{3}{4}$ $\qquad$ 4. $2 \frac{5}{12}+\left(4 \frac{2}{3}+3 \frac{7}{12}\right)$ $\qquad$
4. $3 \frac{3}{8}+\left(2 \frac{1}{5}+5 \frac{1}{8}\right)$ $\qquad$ 6. $\left(4 \frac{3}{7}+2 \frac{1}{6}\right)+3 \frac{5}{7}$ $\qquad$

## Find Part of a Group

Lauren bought 12 stamps for postcards. She gave Brianna $\frac{1}{6}$ of them. How many stamps did Lauren give to Brianna?


Find $\frac{1}{6}$ of 12.
Step 1 What is the denominator in the fraction
of the stamps Lauren gave to Brianna? 6
So, divide the 12 stamps into 6 equal groups. Circle the groups.


Step 2 Each group represents $\frac{1}{6}$ of the stamps.
How many stamps are in 1 group? 2
So, $\frac{1}{6}$ of 12 is $\underline{2}$, or $\frac{1}{6} \times 12$ is $\underline{2}$.
So, Lauren gave Brianna $\underline{2}$ stamps.

## Use a model to solve.

1. $\frac{3}{4} \times 12=$ $\qquad$ 2. $\frac{1}{3} \times 9=$ $\qquad$
2. $\frac{3}{5} \times 20=$ $\qquad$
3. $\frac{4}{6} \times 18=$

## Multiply Fractions and Whole Numbers

Find the product. $\frac{3}{8} \times 4$
Step 1 Draw 4 rectangles to represent the factor 4.


Step 2 The denominator of the factor $\frac{3}{8}$ is 8 . So, divide the 4 rectangles into 8 equal parts.


Step 3 The numerator of the factor $\frac{3}{8}$ is 3 . So, shade 3 of the parts.


Step 4 The 4 rectangles have 3 shaded parts. Each rectangle is divided into 2 equal parts. So, $\frac{3}{2}$ of the rectangles are shaded.

So, $\frac{3}{8} \times 4$ is $\frac{3}{2}$, or $1 \frac{1}{2}$.

## Find the product.

1. $\frac{5}{12} \times 4=$ $\qquad$ 2. $8 \times \frac{3}{4}=$
2. $\frac{7}{9} \times 3=$ $\qquad$
3. $5 \times \frac{4}{7}=$ $\qquad$
4. $\frac{9}{10} \times 5=$ $\qquad$ 6. $3 \times \frac{3}{4}=$ $\qquad$
5. $\frac{7}{12} \times 6=$ $\qquad$
6. $12 \times \frac{2}{9}=$ $\qquad$ 9. $\frac{2}{9} \times 3=$ $\qquad$

## Fraction and Whole Number Multiplication

Find the product. $3 \times \frac{5}{6}$

$$
\begin{array}{rlrl}
3 \times \frac{5}{6} & =\frac{3}{10} \times \frac{5}{6} & & \text { Write the whole-number factor, 3, as } \frac{3}{1} . \\
& =\frac{3 \times \frac{5}{1 \times 6}}{} & \begin{array}{l}
\text { Multiply the numerators. Then multiply the } \\
\text { denominators. }
\end{array} \\
& =\frac{15}{6} & \\
& =2 \frac{3}{6}, \text { or } 2 \frac{1}{2} & \begin{array}{l}
\text { Write the product as a mixed number in } \\
\text { simplest form. }
\end{array}
\end{array}
$$

So, $3 \times \frac{5}{6}$ is $\underline{2 \frac{1}{2}}$.

Find the product. Write the product in simplest form.

1. $\frac{2}{3} \times 8=\frac{2}{3} \times \frac{8}{\square}$
2. $4 \times \frac{2}{9}=$ $\qquad$
$=\frac{\square \times \square}{\square \times \square}$
$=\frac{\square}{\square}$, or
3. $6 \times \frac{3}{4}=$ $\qquad$
4. $\frac{4}{9} \times 3=$ $\qquad$
5. $5 \times \frac{3}{8}=$ $\qquad$
6. $9 \times \frac{2}{3}=$ $\qquad$ 7. $2 \times \frac{5}{6}=$
7. $7 \times \frac{4}{10}=$ $\qquad$

## Multiply Fractions

You can use a model to help you multiply two fractions.
Multiply. $\frac{1}{3} \times \frac{4}{5}$
Step 1 Draw a rectangle. Divide it into 5 equal columns.
To represent the factor $\frac{4}{5}$, shade
4 of the 5 columns.
Step 2 Now divide the rectangle into 3 equal rows.


Shade $\frac{1}{3}$ of the $\frac{4}{5}$ you already shaded.
The rectangle is divided into 15 smaller rectangles. This is the denominator of the product.

There are 4 smaller rectangles that contain
 both types of shading. So, 4 is the numerator of the product.

So $\frac{4}{15}$ of the rectangles contain both types of shading.
Think: What is $\frac{1}{3}$ of $\frac{4}{5}$ ? $\frac{1}{3} \times \frac{4}{5}=\underline{\frac{4}{15}}$.

Find the product. Draw a model.
1.


$$
\frac{1}{4} \times \frac{2}{3}=
$$

$\qquad$
3.

$$
\frac{2}{5} \times \frac{3}{4}=
$$

$\qquad$
2.

$\frac{3}{5} \times \frac{5}{8}=$ $\qquad$
4.

$$
\frac{2}{3} \times \frac{3}{8}=
$$

$\qquad$

## Compare Fraction Factors and Products

You can use a model to determine how the size of the product compares to the size of one factor when multiplying fractions.

The factor is $1: \frac{2}{3} \times 1$

- Draw a model to represent the factor 1.

Divide it into 3 equal sections.

- Shade 2 of the 3 sections to represent the factor $\frac{2}{3}$.
$\frac{2}{3}$ of the rectangle is shaded. So, $\frac{2}{3} \times 1$ is equal to $\frac{2}{3}$.


The factor is greater than $1: \frac{2}{3} \times 2$

- Draw two rectangles to represent the factor 2.

Divide each rectangle into 3 equal sections.

- Shade 2 of 3 sections in each to represent the factor $\frac{2}{3}$.

In all, 4 sections are shaded, which is greater than the number
 of sections in one rectangle. So, $\frac{2}{3} \times 2$ is greater than $\frac{2}{3}$.

The factor is less than $1: \frac{2}{3} \times \frac{1}{6}$

- Draw a rectangle. Divide it into 6 equal columns.

Shade 1 of the 6 columns to represent the factor $\frac{1}{6}$.

- Now divide the rectangle into 3 equal rows. Shade 2 of the


3 rows of the section already shaded to represent the factor $\frac{2}{3}$.
The rectangle is divided into 18 sections. 2 of the sections are shaded twice. 2 sections is less than the 3 sections that represent $\frac{1}{6}$.
So, $\frac{2}{3} \times \frac{1}{6}$ is less than $\frac{1}{6}$.

Complete the statement with equal to, greater than, or less than.

1. $\frac{3}{7} \times \frac{2}{5}$ will be $\qquad$
$\frac{3}{7}$
2. $\frac{7}{8} \times 3$ will be $\frac{7}{8}$.
3. $\frac{1}{6} \times \frac{5}{5}$ will be $\longrightarrow \frac{1}{6}$.
4. $5 \times \frac{6}{7}$ will be $\qquad$ 5.

## Fraction Multiplication

To multiply fractions, you can multiply the numerators, then multiply the denominators. Write the product in simplest form.
Multiply. $\frac{3}{10} \times \frac{4}{5}$
Step 1 Multiply the numerators. Multiply the denominators.

$$
\begin{aligned}
\frac{3}{10} \times \frac{4}{5} & =\frac{3 \times 4}{10 \times 5} \\
& =\frac{12}{50}
\end{aligned}
$$

Step 2 Write the product in simplest form.

$$
\begin{aligned}
\frac{12}{50} & =\frac{12 \div 2}{50 \div 2} \\
& =\frac{6}{25}
\end{aligned}
$$

So, $\frac{3}{10} \times \frac{4}{5}$ is $\underline{\frac{6}{25}}$

Find the product. Write the product in simplest form.

1. $\frac{3}{4} \times \frac{1}{5}$
2. $\frac{4}{7} \times \frac{5}{12}$
3. $\frac{3}{8} \times \frac{2}{9}$
4. $\frac{4}{5} \times \frac{5}{8}$
5. $\frac{1}{3} \times 4$
6. $\frac{3}{4} \times 8$
7. $\frac{5}{8} \times \frac{2}{3}$
8. $\frac{5}{6} \times \frac{3}{8}$

## Area and Mixed Numbers

You can use an area model to help you multiply mixed numbers.
Find the area. $1 \frac{4}{5} \times 2 \frac{1}{3}$
Step 1 Rewrite each mixed-number factor as the sum of a whole number and a fraction.
$1 \frac{4}{5}=1+\frac{4}{5}$ and $2 \frac{1}{3}=2+\frac{1}{3}$
Step 2 Draw an area model to show the original multiplication problem.


Step 3 Draw dashed lines, and label each section to show how you broke apart the mixed numbers in Step 1.

Step 4 Find the area of each section.

$$
\begin{aligned}
& 1 \times 2=\frac{2}{3} \\
& 1 \times \frac{1}{3}=\frac{1}{3} \\
& \frac{4}{5} \times 2=\frac{8}{5} \\
& \frac{4}{5} \times \frac{1}{3}=\frac{4}{15}
\end{aligned}
$$



Step 5 Add the areas of each of the sections to find the total area of the rectangle.

$$
\begin{aligned}
2+\frac{1}{3}+\frac{8}{5}+\frac{4}{15} & =\frac{30}{15}+\frac{5}{15}+\frac{24}{15}+\frac{4}{15} \\
& =\frac{63}{15}, \text { or } 4 \frac{1}{5}
\end{aligned}
$$

So, $1 \frac{4}{5} \times 2 \frac{1}{3}$ is $4 \frac{1}{5}$.

## Use an area model to solve.

1. $1 \frac{2}{3} \times 2 \frac{1}{4}$
2. $1 \frac{3}{4} \times 2 \frac{3}{5}$
3. $2 \frac{1}{2} \times 1 \frac{1}{3}$

## Compare Mixed Number Factors and Products

Complete each statement with equal to, greater than, or less than.
$1 \times 1 \frac{3}{4}$ is ? $1 \frac{3}{4}$.
The Identity Property of Multiplication states that the product of
1 and any number is that number. So, $1 \times 1 \frac{3}{4}$ is equall to $1 \frac{3}{4}$.
$\frac{1}{2} \times 2 \frac{1}{4}$ is


Draw three rectangles. Divide each rectangle into 4 equal columns.

Shade completely the first two rectangles
 and one column of the last rectangle to represent $2 \frac{1}{4}$.

Divide the rectangles into 2 rows. Shade one row to represent the factor $\frac{1}{2}$.
18 small rectangles are shaded. 9 rectangles have both types of shading.
9 rectangles is less than the 18 rectangles that represent $2 \frac{1}{4}$.
So, $\frac{1}{2} \times 2 \frac{1}{4}$ is less than $2 \frac{1}{4}$.
When you multiply a mixed number by a fraction less than 1 ,
the product will be $\qquad$ the mixed number.
$1 \frac{1}{4} \times 1 \frac{3}{4}$ is ? $1 \frac{1}{4}$.
Use what you know about the product of two whole numbers greater than 1 to determine the size of the product of two mixed numbers.

So, $1 \frac{1}{4} \times 1 \frac{3}{4}$ is greater than $1 \frac{1}{4}$ and greater than $1 \frac{3}{4}$.
When you multiply two mixed numbers, their product is $\qquad$ either factor.

## Complete the statement with equal to, greater than, or less than.

1. $\frac{3}{5} \times 1 \frac{2}{7}$ is $\qquad$ $1 \frac{2}{7}$
2. $\frac{6}{6} \times 3 \frac{1}{3}$ is $\qquad$ $3 \frac{1}{3}$
3. $2 \frac{1}{5} \times 1 \frac{1}{4}$ is $\qquad$ $1 \frac{1}{4}$.
4. $\frac{8}{9} \times 4 \frac{3}{4}$ is $\longrightarrow 4 \frac{3}{4}$.

## Multiply Mixed Numbers

You can use a multiplication square to multiply mixed numbers.
Multiply. $1 \frac{2}{7} \times 1 \frac{3}{4}$ Write the product in simplest form.

Step 1 Write the mixed numbers outside the square.

Step 2 Multiply the number in each column by the number in each row.

| $\times$ | 1 | $\frac{2}{7}$ |
| :---: | :---: | :---: |
| 1 | $1 \times 1$ | $\frac{2}{7} \times 1$ |
| $\frac{3}{4}$ | $1 \times \frac{3}{4}$ | $\frac{2}{7} \times \frac{3}{4}$ |
|  |  |  |

Step 3 Write each product inside the square.

| $\times$ | 1 |  |
| :---: | :---: | :---: |
| $\frac{2}{7}$ |  |  |
| 1 | 1 | $\frac{2}{7}$ |
| $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{14}$ |
|  |  |  |

Step 4 Add the products inside the multiplication square.

$$
1+\frac{2}{7}+\frac{3}{4}+\frac{3}{14}
$$

Find the least common denominator.

$$
\frac{28}{28}+\frac{8}{28}+\frac{21}{28}+\frac{6}{28}=\frac{63}{28}
$$

Simplify.

$$
\frac{63}{28}=2 \frac{7}{28}, \text { or } 2 \frac{1}{4}
$$

So, $1 \frac{2}{7} \times 1 \frac{3}{4}$ is $2 \frac{1}{4}$.

Find the product. Write the product in simplest form.

1. $2 \frac{5}{8} \times 1 \frac{1}{7}$
2. $3 \frac{1}{2} \times 12$
3. $10 \frac{5}{6} \times \frac{3}{5}$
4. $7 \frac{7}{10} \times \frac{10}{11}$

Use the Distributive Property to find the product.
5. $12 \times 2 \frac{1}{2}$
6. $15 \times 5 \frac{1}{3}$

## Problem Solving • Find Unknown

## Lengths

Zach built a rectangular deck in his backyard. The area of the deck is 300 square feet. The length of the deck is $1 \frac{1}{3}$ times as long as the width. What are the dimensions of the deck?

| Read the Problem |  |  |
| :---: | :---: | :---: |
| What do I need to find? <br> I need to find the dimensions of the deck | What information do I need to use? <br> The deck has an area of $\qquad$ square feet, and the length is $1 \frac{1}{3}$ $\qquad$ as long as the width. | How will I use the information? <br> I will gluess the length and width of the deck. Then I will check my guess and revise it if it is not correct. |
| Solve the Problem |  |  |
| I can try different values for the length of the deck, each that is $1 \frac{1}{3}$ times as long as the width. Then I can multiply the length and width and compare to the correct area. |  |  |


| Guess |  | Check | Revise |
| :---: | :---: | :---: | :---: |
| Width <br> (in feet) | Length (in feet) <br> (1 $\frac{1}{3}$ times the width) | Area of Deck <br> (in square feet) |  |
| 12 | $1 \frac{1}{3} \times 12=\underline{16}$ | $12 \times 16=\underline{\underline{192} \text { too low }}$ | Try a longer width. |
| 18 | $1 \frac{1}{3} \times 18=\underline{24}$ | $18 \times 24=\underline{432}$ too high | Try a shorter width. |
| 15 | $1 \frac{1}{3} \times 15=\underline{20}$ | $15 \times 20=\underline{300}$ correct |  |

So, the dimensions of the deck are 20 feet by $\qquad$ feet.

1. Abigail made a quilt that has an area of 4,800 square inches. The length of the quilt is $1 \frac{1}{3}$ times the width of the quilt. What are the dimensions of the quilt?
2. The width of the mirror in Shannon's bathroom is $\frac{4}{9}$ its length. The area of the mirror is 576 square inches. What are the dimensions of the mirror?

## Divide Fractions and Whole Numbers

You can use a number line to help you divide a whole number by a fraction.
Divide. $6 \div \frac{1}{2}$
Step 1 Draw a number line from 0 to 6. Divide the number line into halves. Label each half on your number line, starting with $\frac{1}{2}$.


Step 2 Skip count by halves from 0 to 6 to find $6 \div \frac{1}{2}$.
Step 3 Count the number of skips. It takes 12 skips to go from 0 to 6 . So the quotient is 12 .

$$
6 \div \frac{1}{2}=\underline{12} \text { because } \underline{12} \times \frac{1}{2}=6
$$

You can use fraction strips to divide a fraction by a whole number.
Divide. $\frac{1}{2} \div 5$
Step 1 Place a $\frac{1}{2}$ strip under a 1-whole strip.
Step 2 Find 5 fraction strips, all with the same denominator, that fit exactly under the $\frac{1}{2}$ strip.

$$
\text { Each part is } \frac{1}{10} \text { of the whole. }
$$

Step 3 Record and check the quotient.


$$
\frac{1}{2} \div 5=\underline{\frac{1}{10}} \text { because } \frac{\frac{1}{10}}{1} \times 5=\frac{1}{2} .
$$

So, $\frac{1}{2} \div 5=\underline{\overline{10}}$.

## Divide. Draw a number line or use fraction strips.

1. $1 \div \frac{1}{2}=$ $\qquad$
2. $2 \div \frac{1}{3}=$ $\qquad$
3. $4 \div \frac{1}{4}=$ $\qquad$
4. $\frac{1}{5} \div 3=$ $\qquad$
5. $\frac{1}{3} \div 2=$ $\qquad$
6. $4 \div \frac{1}{5}=$
$\qquad$

## Problem Solving• Use Multiplication

Nathan makes 4 batches of soup and divides each batch into halves. How many $\frac{1}{2}$-batches of soup does he have?

| Read the Problem | Solve the Problem |
| :--- | :--- |
| What do I need to find? <br> I need to find the number of | Since Nathan makes 4 batches of soup, <br> my diagram needs to show 4 circles to <br> represent the 4 batches. I can divide <br> each of the 4 circles in half. |
| $\frac{1}{2}$-batches of soulp Nathan |  |,

## Draw a diagram to help you solve the problem.

1. A nearby park has 8 acres of land to use for gardens. The park divides each acre into fourths. How many $\frac{1}{4}$-acre gardens does the park have?
2. Clarissa has 3 pints of ice tea that she divides into $\frac{1}{2}$-pint servings. How many $\frac{1}{2}$-pint servings does she have?

## Connect Fractions to Division

You can write a fraction as a division expression.

$$
\frac{4}{5}=4 \div 5 \quad \frac{15}{3}=15 \div 3
$$

There are 8 students in a wood-working class and 5 sheets of plywood for them to share equally. What fraction of a sheet of plywood will each student get?

Divide. $5 \div 8 \quad$ Use a drawing.
Step 1 Draw 5 rectangles to represent 5 sheets of plywood. Since there are 8 students, draw lines to divide each piece of plywood into eighths.


Each student's share of 1 sheet of plywood is $\frac{1}{8}$.
Step 2 Count the total number of eighths each student gets.
Since there are 5 sheets of plywood, each student will get 5 of the $\qquad$ eighths or $\qquad$ $\frac{5}{8}$.

Step 3 Complete the number sentence. $5 \div 8=\underline{\frac{5}{8}}$

Step 4 Check your answer.
Since $\underline{\frac{5}{8}} \times \underline{8}=\underline{5}$, the quotient is correct.
So, each student will get $\qquad$ of a sheet of plywood.

## Complete the number sentence to solve.

1. Ten friends share 6 pizzas equally. What fraction of a pizza does each friend get?

$$
6 \div 10=
$$

$\qquad$
2. Four students share 7 sandwiches equally. How much of a sandwich does each student get?
$7 \div 4=$ $\qquad$

## Fraction and Whole-Number Division

You can divide fractions by solving a related multiplication sentence.
Divide. $4 \div \frac{1}{3}$
Step 1 Draw 4 circles to represent the dividend, 4.


Step 2 Since the divisor is $\frac{1}{3}$, divide each circle into thirds.


Step 3 Count the total number of thirds.
When you divide the $\qquad$ circles into thirds, you are finding the number of thirds in 4 circles, or finding 4 groups of 3 .
There are $\qquad$ thirds.

Step 4 Complete the number sentence.
$4 \div \frac{1}{3}=4 \times \underline{3}=\underline{12}$

Use the model to complete the number sentence.

$3 \div \frac{1}{5}=3 \times$ $\qquad$ $=$ $\qquad$
2.

$\frac{1}{4} \div 2=\frac{1}{4} \times$ $\qquad$

Write a related multiplication sentence to solve.
3. $2 \div \frac{1}{5}$
4. $\frac{1}{3} \div 3$
5. $\frac{1}{6} \div 2$
6. $5 \div \frac{1}{4}$

## Interpret Division with Fractions

You can draw a diagram or write an equation to represent division with fractions.

Beatriz has 3 cups of applesauce. She divides the applesauce into $\frac{1}{4}$-cup servings. How many servings of applesauce does she have?

One Way Draw a diagram to solve the problem.
Draw 3 circles to represent the 3 cups of applesauce. Since Beatriz divides the applesauce into $\frac{1}{4}$-cup servings, draw lines to divide each "cup" into fourths.


To find $3 \div \frac{1}{4}$, count the total number of fourths in the 3 circles.
So, Beatriz has $\underline{12}$ one-fourth-cup servings of applesauce.

Another Way Write an equation to solve.
Write an equation.
Write a related multiplication equation.
Then solve.

$$
\begin{aligned}
3 \div \frac{\frac{1}{4}}{4} & =n \\
3 \times \frac{4}{12} & =n \\
\frac{12}{} & =n
\end{aligned}
$$

So, Beatriz has 12 one-fourth-cup servings of applesauce.

1. Draw a diagram to represent the problem. Then solve.

Drew has 5 granola bars. He cuts the bars into halves. How many $\frac{1}{2}$-bar pieces does he have?
2. Write an equation to represent the problem. Then solve.
Three friends share $\frac{1}{4}$ of a melon. What fraction of the whole melon does each friend get?

## Line Plots

A line plot is a graph that shows the shape of a data set by placing $X$ s above each data value on a number line. You can make a line plot to represent a data set and then use the line plot to answer questions about the data set.

Students measure the lengths of several seeds.
The length of each seed is listed below.
$\frac{1}{2}$ inch, $\frac{3}{4}$ inch, $\frac{1}{2}$ inch, $\frac{1}{4}$ inch, $\frac{3}{4}$ inch, $\frac{3}{4}$ inch, $\frac{3}{4}$ inch, $\frac{1}{4}$ inch, $\frac{1}{2}$ inch
What is the combined length of the seeds that are $\frac{1}{4}$ inch long?

Step 1 To represent the different lengths of the seeds, draw and label a line plot with the data values $\frac{1}{4}, \frac{1}{2}$, and $\frac{3}{4}$. Then use an $X$ to represent each seed. The line plot


## Length of Seeds (in inches)

 has been started for you.Step 2 There are_ 2 Xs above $\frac{1}{4}$ on the line plot.
Multiply to find the combined length of the seeds:
$2 \times \frac{\frac{1}{4}}{2}=\frac{2}{4}$, or $\frac{1}{2}$ inch

The combined length of the seeds that are $\frac{1}{4}$ inch long is $\frac{1}{2}$ inch.
You can use the same process to find the combined lengths of the seeds that are $\frac{1}{2}$ inch long and $\frac{3}{4}$ inch long.

## Use the data and the line plot above to answer the questions.

1. What is the total length of all the seeds that the students measured?
2. What is the average length of one of the seeds that the students measured?

## Ordered Pairs

A coordinate grid is like a sheet of graph paper bordered at the left and at the bottom by two perpendicular number lines. The x-axis is the horizontal number line at the bottom of the grid. The $y$-axis is the vertical number line on the left side of the grid.

An ordered pair is a pair of numbers that describes the location of a point on the grid. An ordered pair contains two coordinates, $x$ and $y$. The $\mathbf{x}$-coordinate is the first number in the ordered pair, and the $\boldsymbol{y}$-coordinate is the second number.
$(x, y) \longrightarrow(10,4)$
Plot and label $(10,4)$ on the coordinate grid.
To graph an ordered pair:

- Start at the origin, $(0,0)$.
- Think: The letter $x$ comes before $y$ in the alphabet. Move across the $x$-axis first.
- The $x$-coordinate is 10 , so move 10 units right.
- The $y$-coordinate is 4 , so move 4 units up.
- Plot and label the ordered pair (10, 4).


Use the coordinate grid to write an ordered pair for the given point.

1. $G \longrightarrow$
2. $H$ $\qquad$
3. J
4. $K$

Plot and label the points on the coordinate grid.
5. $A(1,6)$
6. $B(1,9)$
7. $C(3,7)$
8. $D(5,5)$
9. $E(9,3)$
10. $F(6,2)$

## Graph Data

## Graph the data on the coordinate grid.

| Plant Growth |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| End of Week | 1 | 2 | 3 | 4 |
| Height (in inches) | 4 | 7 | 10 | 11 |

- Choose a title for your graph and label it. You can use the data categories to name the $x$ - and $y$-axis.
- Write the related pairs of data as ordered pairs.



End of Week

- Plot the point for each ordered pair.

Graph the data on the coordinate grid. Label the points.
1.

| Distance of Bike Ride |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Time (in minutes) | 30 | 60 | 90 | 120 |
| Distance (in miles) | 9 | 16 | 21 | 27 |

Write the ordered pair for each point.


Distance of Bike Ride


Time (in minutes)
2.

Bianca's Writing Progress

| Time (in minutes) | 15 | 30 | 45 | 60 |
| :---: | :---: | :---: | :---: | :---: |
| Total Pages | 1 | 3 | 9 | 11 |

Write the ordered pair for each point.


Bianca's Writing Progress


Time (in minutes)

## Line Graphs

A line graph uses a series of line segments to show how a set of data changes over time. The scale of a line graph measures and labels the data along the axes. An interval is the distance between the numbers on an axis.

## Use the table to make a line graph.

- Write a title for your graph. In this example, use Average Monthly High Temperature in Sacramento.
- Draw and label the axes of the line graph.

| Average Monthly High Temperature <br> in Sacramento, California |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Month | Jan. | Feb. | Mar. | April | May |
| Temperature ( ${ }^{\circ}$ F) | 53 | 60 | 65 | 71 | 80 | Label the horizontal axis Month. Write the months.

Label the vertical axis Temperature ( ${ }^{\circ} \mathrm{F}$ ).

- Choose a scale and an interval. The range is 53-80, so a possible scale is $0-80$, with intervals of 20.
- Write the related pairs of data as ordered pairs: (Jan, 53); (Feb, 60); (Mar, 65); (April, 71); (May, 80).

1. Make a line graph of the data above.


Use the graph to determine between which two months the least change in average high temperature occurs.
2. Make a line graph of the data in the table.

| Average Low Temperature in San Diego, California |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Month | Mar. | April | May | June | July |
| Tempe | perature ( ${ }^{\circ} \mathrm{F}$ ) | 51 | 51 | 60 | 62 | 66 |
| Average Low Temperature in San Diego |  |  |  |  |  |  |
|  |  | ril | May <br> Month | June | July |  |

Use the graph to determine between which two months the greatest change in average low temperature occurs.

## Numerical Patterns

A soccer league has 7 teams. How many players are needed for 7 teams? How many soccer balls are needed by the 7 teams?

|  | Number of Teams | 1 | 2 | 3 | 4 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Add 8 | Number of Players | 8 | 16 | 24 | 32 | 56 |
| Add 4 | Number of Soccer Balls | 4 | 8 | 12 | 16 | 28 |

Step 1 Find a rule that could be used to find the number of players for the number of teams.

Think: In the sequence $8,16,24,32$, you add 8 to get the next term.
As the number of teams increases by 1 , the number of players increases by 8 . So the rule is to add 8.

Step 2 Find a rule that could be used to find the number of soccer balls for the number of teams.

Think: In the sequence $4,8,12,16$, you add 4 to get the next term.
As the number of teams increases by 1 , the number of soccer balls needed increases by 4 . So the rule is to add 4.
Step 3 For 7 teams, multiply the number of players by $\frac{1}{2}$ to find the number of soccer balls.

So, for 7 teams, 56 players will need 28 soccer balls.

Complete the rule that describes how one sequence is related to the other. Use the rule to find the unknown term.

| Number of <br> Teams | 1 | 2 | 3 | 4 | 8 | 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> Players | 15 | 30 | 45 | 60 | 120 |  |
| Number of <br> Bats | 5 | 10 | 15 | 20 |  | 50 |

1. Divide the number of players by $\qquad$ to find the number of bats.
2. Multiply the number of bats by $\qquad$ to find the number of players.

## Problem Solving • Find a Rule

Samantha is making a scarf with fringe around it. Each section of fringe is made of 4 pieces of yarn with 2 beads holding them together. There are 42 sections of fringe on Samantha's scarf. How many wooden beads and how many pieces of yarn are on Samantha's scarf?

| Read the Problem | Solve the Problem |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| What do I need to find? <br> Possible answer: I need to find | Sections of Fringe | 1 | 2 | 3 | 4 | 6 | 42 |
| the number of beads and the number of pieces of yarn on | Number of <br> Beads | 2 | 4 | 6 | 8 | 12 | 84 |
| Samantha's scarf. | Pieces of Yarn | 4 | 8 | 12 | 16 | 24 | 168 |
| What information do I need to use? Possible answer: I need to use the number of sections on the scarf, and that each section has 4 pieces of yarn and 2 beads. | Possible answer: I can multiply the number of sections by 2 to find the number of beads. Then, I can multiply the number of sections by 4 , or the |  |  |  |  |  |  |
| How will I use the information? <br> I will use the information to search for patterns to solve a simpler problem. | numb the nu So, Sa 42, or 168 pi | nb | be | 'sids | by ces car <br> nd . | $\frac{\frac{2}{2, t o}}{\frac{\text { ha }}{4 \times}}$ | find yarn. s $2 \times$ 42, or |

1. A rectangular tile has a decorative pattern of 3 equal-sized squares, each of which is divided into 2 same-sized triangles. If Marnie uses 36 of these tiles on the wall behind her kitchen stove, how many triangles are displayed?
2. Leta is making strawberry-almond salad for a party. For every head of lettuce that she uses, she adds 5 ounces of almonds and 10 strawberries. If she uses 75 ounces of almonds, how many heads of lettuce and how many strawberries does Leta use?

## Graph and Analyze Relationships

The scale on a map is $1 \mathrm{in} .=4 \mathrm{mi}$. Two cities are 5 inches apart on the map. What is the actual distance between the two cities?

Step 1 Make a table that relates the map distances to the actual distances.

| Map Distance (in.) | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Actual Distance (mi) | 4 | 8 | 12 | 16 | $?$ |

Step 2 Write the number pairs in the table as ordered pairs.

$$
(1,4),(2,8),(3,12),(4,16),(5, ?)
$$

Step 3 Graph the ordered pairs.
Connect the points with a line from the origin.

Possible rule: Multiply the map distance by $\underline{4}$ to get the actual distance.

Step 4 Use the rule to find the actual distance between the two cities.

So, two cities that are 5 inches apart on the map are actually $5 \times 4$, or $\underline{20}$ miles apart.

Plot the point $(5,20)$ on the graph.


Graph and label the related number pairs as ordered pairs. Then complete and use the rule to find the unknown term.

1. Multiply the number of yards by $\qquad$ to find the number of feet.

| Number of Yards | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Number of Feet | 3 | 6 | 9 | 12 |  |



## Customary Length

You can convert one customary unit of length to another customary unit of length by multiplying or dividing.

Multiply to change from larger to smaller units of length.
Divide to change from smaller to larger units of length.
Convert 3 feet to inches.

## Customary Units of Length

1 foot (ft) $=12$ inches (in.)
1 yard (yd) $=3$ feet
1 mile (mi) $=5,280$ feet
1 mile $=1,760$ yards

## Step 1

Decide:
Multiplyor Divide
$\begin{array}{ll}\text { feet } \\ \text { larger } & \longrightarrow \text { inches } \\ \text { smaller }\end{array}$

## Step 2

Think:
$1 \mathrm{ft}=12 \mathrm{in}$., so
$3 \mathrm{ft}=(3 \times \underline{12}) \mathrm{in}$.

## Step 3

Multiply.
$3 \times 12=36$

So, 3 feet $=\underline{36}$ inches.

Convert 363 feet to yards.

Step 1
Decide:
Multiply or Divide
feet $\longrightarrow$ yards
smaller $\longrightarrow$ larger

Step 2
Think:
$3 \mathrm{ft}=1 \mathrm{yd}$,
so $363 \mathrm{ft}=(363 \div$ $\qquad$ ) yd.

## Step 3

Divide.

$$
363 \div 3=121
$$

So, 363 feet $=\underline{121}$ yards.

## Convert.

1. $33 \mathrm{yd}=$ $\qquad$ ft
2. $300 \mathrm{mi}=$ $\qquad$ yd
3. 46 in. $=$ $\qquad$ ft $\qquad$ in.
4. $96 \mathrm{yd}=$ $\qquad$ ft
5. $48 \mathrm{ft}=$ $\qquad$ yd
6. $2 \mathrm{mi} 20 \mathrm{yd}=$ $\qquad$ yd

Compare. Write $<,>$, or $=$.
7. 2 yd

8. 67 mi
 $117,920 \mathrm{yd}$
9. 250 yd
 800 ft
10. 14 yd 2 ft
 16 ft 11. 34 ft 10 in .
 518 in. 12. 5 mi 8 ft

## Customary Capacity

You can convert one unit of customary capacity to another by multiplying or dividing.

Multiply to change from larger to smaller units.
Divide to change from smaller to larger units.

## Convert 8 cups to quarts.

Customary Units of Capacity
1 cup $(\mathrm{c})=8$ fluid ounces (fl oz)
1 pint $(\mathrm{pt})=2$ cups
1 quart $(\mathrm{qt})=2$ pints
1 quart $=4$ cups
1 gallon $=4$ quarts

Step 1
Decide:
Multiply or Divide

|  |
| :--- | :--- | :--- |
| cups |
| smaller $\longrightarrow$ larger |\(\quad \begin{aligned} \& 4 \mathrm{c}=1 \mathrm{qt}, <br>

\& so 8 \mathrm{c}=(8 \div 4) \mathrm{qt.}\end{aligned}\)
So, 8 cups $=\underline{2}$ quarts.

Convert 19 gallons to quarts.

## Step 1

Decide:
Multiply)or Divide
$\begin{array}{lll}\text { gallons } & \longrightarrow & \text { quarts } \\ \text { larger } & & \text { smaller }\end{array}$

Step 2
Think:
$1 \mathrm{gal}=4 \mathrm{qt}$,
so $19 \mathrm{gal}=(19 \times \xrightarrow{4}) \mathrm{qt}$.

Step 3
Divide.
$8 \div \underline{4}=\underline{2}$

So, 19 gallons = 76 quarts.

## Convert.

1. $14 \mathrm{pt}=$ $\qquad$ qt
2. $32 \mathrm{qt}=$ $\qquad$ C
3. $7 \mathrm{c}=$ $\qquad$ fl oz
4. $28 \mathrm{c}=$ $\qquad$ pt
5. $9 \mathrm{gal}=$ $\qquad$ qt
6. $16 \mathrm{c}=$ $\qquad$ qt

Compare. Write $<,>$, or $=$.
7. 16 qt
 60 c
8. 88 fl oz
 11 c
9. 3 gal
 10 qt
 54 c
11. 66 fl oz
 9 c
12. 16 gal
 64 qt

## Weight

You can convert one customary unit of weight to another by multiplying or dividing.

Multiply to change from larger to smaller units.
Divide to change from smaller to larger units.

## Customary Units of Weight

1 pound $(\mathrm{lb})=16$ ounces $(\mathrm{oz})$
1 ton $(T)=2,000$ pounds

Convert 96 ounces to pounds.
Step 1
Decide:
Multiply or Divide
ounces $\longrightarrow$ pounds
smaller $\longrightarrow$ larger

Step 1
Multiply or Divide
ounces $\longrightarrow$ pounds
smaller $\longrightarrow$ larger

Step 2
Think:
$16 \mathrm{oz}=1 \mathrm{lb}$ so 96 oz $=(96 \div \underline{16})$ lb.

So, 96 ounces $=\underline{6}$ pounds.

Convert 4 pounds to ounces.

Step 1
Decide:
Multiplyor Divide
pounds $\longrightarrow$ ounces larger $\longrightarrow$ smaller

Step 2
Think:
$\left.\begin{array}{l}1 \mathrm{lb}=16 \mathrm{oz}, \\ \text { so } 4 \mathrm{lb}=(4 \times \ldots\end{array}\right) \mathrm{Cz}$.

## Step 3

Multiply.
$4 \times \underline{16}=\underline{64}$

So, 4 pounds $=\underline{64}$ ounces.

## Convert.

1. $14 \mathrm{lb}=$ $\qquad$ oz
2. $12,000 \mathrm{lb}=\square \mathrm{T}$
3. $2 \mathrm{~T}=$ $\qquad$ lb
4. $7 \mathrm{lb}=$ $\qquad$ oz
5. $22 \mathrm{lb}=$ $\qquad$ oz
6. $16 \mathrm{oz}=$ $\qquad$ lb

Compare. Write $<,>$, or $=$.
7.
7. 1 T

8. 3 lb

9. 5 T

10. 3 T

11. 6 lb
 96 oz
12. 16 T


## Multistep Measurement Problems

## An ice cream parlor donated 6 containers of ice cream to a local elementary school. Each container holds 3 gallons of ice cream. If each student is served 1 cup of ice cream, how many students can be served?

Step 1 Record the information you are given.
There are 6 containers of ice cream.
Each container holds 3 gallons of ice cream.
Step 2 Find the total amount of ice cream in the 6 containers.
$6 \times 3$ gallons $=$ $\qquad$ gallons of ice cream

Step 3 Convert from gallons to cups.
There are 4 quarts in 1 gallon, so 18 gallons $=\underline{72}$ quarts.
There are 2 pints in 1 quart, so 72 quarts $=\underline{144}$ pints.
There are 2 cups in 1 pint, so 144 pints $=\underline{288}$ cups.
So, 288 students can be served 1 cup of ice cream.

## Solve.

1. A cargo truck weighs 8,750 pounds. The weight limit for a certain bridge is 5 tons. How many pounds of cargo can be added to the truck before it exceeds the weight limit for the bridge?
2. Larry has 9 gallons of paint. He uses 10 quarts to paint his kitchen and 3 gallons to paint his living room. How many pints of paint will be left?
3. A plumber uses 16 inches of tubing to connect each washing machine in a laundry to the water source. He wants to install 18 washing machines. How many yards of tubing will he need?
4. Ketisha is practicing for a marathon by running around a track that is 440 yards long. Yesterday she ran around the track 20 times. How many miles did she run?

## Metric Measures

The metric system is based on place value. To convert between units, you multiply or divide by a power of 10 . You multiply to change larger units to smaller units, such as liters to centiliters. You divide to change smaller units to larger units, such as meters to kilometers.

## Convert 566 millimeters to decimeters.

- Think about how the two units are related.

1 decimeter $=100$ millimeters

| Metric Units of Length |
| :---: |
| 1 centimeter $(\mathrm{cm})=10$ millimeters $(\mathrm{mm})$ |
| 1 decimeter $(\mathrm{dm})=10$ centimeters $(\mathrm{cm})$ |
| 1 meter $(\mathrm{m})=1,000$ millimeters $(\mathrm{mm})$ |
| 1 kilometer $(\mathrm{km})=1,000$ meters $(\mathrm{m})$ |

- Think: Should I multiply or divide?

Millimeters are smaller than decimeters.
So divide, or move the decimal point left for each power of 10 .
$\begin{gathered}566 \quad \div \\ \text { millimeters }\end{gathered} \quad 100 \quad=\underline{5 m \text { in } 1 \mathrm{dm}} \quad \begin{aligned} & \text { total decimeters }\end{aligned}$

|  |  |  |  | 5 | 6 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| kilo- <br> (k) | hecto- <br> (h) | deka- <br> (da) | meter <br> liter <br> gram | deci- <br> (d) | centi- <br> (c) | milli- <br> (m) |

So, $566 \mathrm{~mm}=\underline{5.66} \mathrm{dm}$.

Complete the equation to show the conversion.

1. 115 km

$10=$ $\qquad$ hm
$115 \mathrm{~km} \int 100=\square$ dam
2. 418 cL
 $10=$ $\qquad$ dL
418 cL
 $100=$
$\qquad$
418 cL
 $1,000=$ $\qquad$ daL

## Convert.

3. $40 \mathrm{~cm}=$ $\qquad$ mm
4. $500 \mathrm{~mL}=$ $\qquad$ dL
5. $6 \mathrm{~kg}=$ $\qquad$
6. $5,000 \mathrm{cL}=$ $\qquad$ L
7. $4 \mathrm{~kg}=$ $\qquad$ hg
8. $200 \mathrm{~mm}=$ $\qquad$ cm

## Problem Solving • Customary and Metric Conversions

You can use the strategy make a table to help you solve problems about customary and metric conversions.

Jon's faucet is dripping at the rate of 24 centiliters in a day. How many milliliters of water will have dripped from Jon's faucet in 24 hours?

| Read the Problem |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| What do I need to find? <br> I need to find how many millilliters of water will have dripped from Jon's faucet in 24 hours. | Conversion Table |  |  |  |  |
|  |  | L | dL | cL | mL |
|  | 1 L | 1 | 10 | 100 | 1,000 |
| What information do I need to use? <br> I need to use the number of cL that have dripped in 24 hr and the number of mL in a cL. | 1 dL | $\frac{1}{10}$ | 1 | 10 | 100 |
|  | 1 cL | $\frac{1}{100}$ | $\frac{1}{10}$ | 1 | 10 |
|  | 1 mL | $\frac{1}{1,000}$ | $\frac{1}{100}$ | $\frac{1}{10}$ | 1 |
| How will I use the information? <br> I will make a table to show the relationship between the number of $\qquad$ centiliters and the number of milliliters $\qquad$ | I can us number There | the C f milli 10 |  | Table entilit <br> in 1 | find the <br> tiliter. |
|  | cL | 1 | 2 | 4 | 24 |
|  | mL | 10 | 20 | 40 | 240 |

So, 240 milliliters of water will have dripped from Jon's faucet in 24 hours.
Make a table to help you solve the problems.

1. Fernando has a bucket that holds 3 gallons of water. He is filling the bucket using a 1-pint container. How many times will he have to fill the pint container in order to fill the bucket?
2. Lexi has a roll of shelf paper that is 800 cm long. She wants to cut the paper into 1-m strips to line the shelves in her pantry. How many 1-meter strips can she cut?

## Elapsed Time

You can solve elapsed time problems by converting units of time.

Starting at 4:20 р.м., Connie practiced piano for 90 minutes. At what time did Connie stop practicing piano?

Convert 90 minutes to hours and minutes. Then find the end time.

Step 1 To convert minutes to hours, divide.
$90 \div 60$ is 1 r 30

## Units of Time

| 60 seconds $(\mathrm{s})=1$ minute $(\mathrm{min})$ |
| :---: |
| 60 minutes $=1$ hour $(\mathrm{hr})$ |
| 24 hours $=1$ day $(\mathrm{d})$ |
| 7 days $=1$ week $(\mathrm{wk})$ |
| 52 weeks $=1$ year (yr) |
| 12 months $(\mathrm{mo})=1$ year |
| 365 days $=1$ year |

$90 \min =$ $\qquad$ hr $\qquad$ min

Step 2 Count forward by hours until you reach 1 hour.

Step 3 Count forward by minutes until you reach 30 minutes.
$4: 20 \rightarrow 5: 20=1$ hour
$5: 20 \rightarrow 5: 30=1$ hour 10 minutes
$5: 30 \rightarrow 5: 40=1$ hour 20 minutes $5: 40 \rightarrow 5: 50=1$ hour 30 minutes Connie stops practicing piano at $5.50 \mathrm{P} . \mathrm{M}_{\|}$

## Convert.

1. $480 \mathrm{~min}=$ $\qquad$ hr
2. $4 \mathrm{~d}=$ $\qquad$ hr
3. $125 \mathrm{hr}=$ $\qquad$ d $\qquad$ hr

Find the start, elapsed, or end time.
4. Start time: 7:15 A.M.

Elapsed time: 2 hr 20 min
End time: $\qquad$
6. Start time: $\qquad$
Elapsed time: 5 hr 50 min
End time: 7:55 P.M.
5. Start time: 6:28 A.m.

Elapsed time:
End time: 10:08 A.m.
7. Start time: 5:24 р.м.

Elapsed time: 6 hr

End time:

## Polygons

A polygon is a closed plane figure formed by three or more line segments that meet at points called vertices. You can classify a polygon by the number of sides and the number of angles that it has.
Congruent figures have the same size and shape. In a regular polygon, all sides are congruent and all angles are congruent.

## Classify the polygon below.



| Polygon | Sides | Angles | Vertices |
| :--- | :---: | :---: | :---: |
| Triangle | 3 | 3 | 3 |
| Quadrilateral | 4 | 4 | 4 |
| Pentagon | 5 | 5 | 5 |
| Hexagon | 6 | 6 | 6 |
| Heptagon | 7 | 7 | 7 |
| Octagon | 8 | 8 | 8 |
| Nonagon | 9 | 9 | 9 |
| Decagon | 10 | 10 | 10 |

How many sides does this polygon have?

## 5 sides

How many angles does this polygon have? 5 angles
Name the polygon.
pentagon
Are all the sides congruent? $\qquad$ no

Are all the angles congruent? no

So, the polygon above is a pentagon. It is not a regular polygon.
Name each polygon. Then tell whether it is a regular polygon or not a regular polygon.

1. $G$

2. 


3.

$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
4.

$\qquad$

## Triangles

You can classify triangles by the length of their sides and by the measure of their angles. Classify each triangle.

Use a ruler to measure the side lengths.

## - equilateral triangle

All sides are the same length.

- isosceles triangle

Two sides are the same length.

- scalene triangle

All sides are different lengths.


Use the corner of a sheet of paper to classify the angles.

- acute triangle

All three angles are acute.

## - obtuse triangle

One angle is obtuse. The other two angles are acute.

- right triangle

One angle is right. The other two angles are acute.

Classify the triangle according to its side lengths. It has two congruent sides.

## The triangle is an isosceles triangle.

Classify the triangle according to its angle measures. It has one right angle.
The triangle is a right triangle.


Classify each triangle. Write isosceles, scalene, or equilateral.
Then write acute, obtuse, or right.
1.

2.

3.

4.

5.

6.


## Quadrilaterals

You can use this chart to help you classify quadrilaterals.


Classify the figure.


The figure has 4 sides, so it is a quadrilateral. The figure has exactly one pair of parallel sides, so it is a trapezoid.
quadrilateral, trapezoid

Classify the quadrilateral in as many ways as possible. Write quadrilateral, parallelogram, rectangle, rhombus, square, or trapezoid.
1.

2.

3.

4.


## Three-Dimensional Figures

A polyhedron is a solid figure with faces that are polygons.
You can identify a polyhedron by the shape of its faces.

A pyramid is a polyhedron with one polygon base. The lateral faces of a pyramid are triangles that meet at a common vertex.

| triangular <br> pyramid | The base <br> and faces are <br> triangles. |
| :--- | :--- |
| rectangular <br> pyramid | The base is a <br> rectangle. <br> square <br> pyramid |
| The base is a <br> pentagonal <br> pyramid | The base is a <br> pentagon. <br> hexagonal <br> pyramid |
| The base is a <br> hexagon. |  |

A prism is a polyhedron with two congruent polygons as bases. The lateral faces of a prism are rectangles.

| triangular | The two |
| :--- | :--- |
| prism | bases are |
|  | triangles. |


rectangular prism

All faces are rectangles.

square prism or cube
pentagonal prism
hexagonal prism

All faces are squares.

The two bases are pentagons.


The two bases are hexagons.


A solid figure with curved surfaces is not a polyhedron.

| cone | The one base |
| :--- | :--- |
| is a circle. |  |


sphere
There is no base.
cylinder
The two bases are circles.


Classify the solid figure. Write prism, pyramid, cone, cylinder, or sphere.
The solid figure has one base.
The rest of its faces are triangles.
So, the solid figure is a $\qquad$
pyramid


Classify each solid figure. Write prism, pyramid, cone, cylinder, or sphere.
1.

2.

3.

4.


## Unit Cubes and Solid Figures

A unit cube is a cube that has a length, width, and height of 1 unit. You can use unit cubes to build a rectangular prism.


Count the number of cubes used to build the rectangular prism.


The length of the prism is made up of $\qquad$ unit cubes.

The width of the prism is made up of $\qquad$ unit cubes.

The height of the prism is made up of $\qquad$ unit cube.

The number of unit cubes used to build the rectangular prism is 16 .

Count the number of unit cubes used to build each solid figure.

$\qquad$ unit cubes
3.

$\qquad$ unit cubes
2.

4.

.
$\qquad$ unit cubes
$\qquad$ unit cubes

## Understand Volume

The volume of a rectangular prism is equal to the number of unit cubes that make up the prism. Each unit cube has a volume of 1 cubic unit.

Find the volume of the prism. 1 unit cube $=1$ cubic inch


Step 1 Count the number of unit cubes in the bottom layer of the prism.
There are $\qquad$ 4 unit cubes that make up the length of the first layer.

There are $\qquad$ 2 unit cubes that make up the width of the first layer.

There is 1 unit cube that makes up the height of the first layer.
So, altogether, there are 8 unit cubes that make up the bottom layer of the prism.

Step 2 Count the number of layers of cubes that make up the prism.
The prism is made up of $\qquad$ layers of unit cubes.

Step 3 Find the total number of cubes that fill the prism.
Multiply the number of layers by the number of cubes in each layer.

$$
3 \times 8=\underline{24} \text { unit cubes }
$$

Each unit cube has a volume of 1 cubic inch. So, the volume of the prism is $24 \times 1$, or 24 cubic inches.

## Use the unit given. Find the volume.

1. 



Each cube $=1 \mathrm{cuft}$
Volume $=$ $\qquad$ Cu $\qquad$
2.


Each cube $=1 \mathrm{cu} \mathrm{cm}$
Volume $=$ $\qquad$ cu $\qquad$

## Estimate Volume

You can estimate the volume of a larger box by filling it with smaller boxes.

Mario packs boxes of markers into a large box. The volume of each box of markers is 15 cubic inches. Estimate the volume of the large box.


The volume of one box of markers is 15 cubic inches.
Use the box of markers to estimate the volume of the large box.

- The large box holds 2 layers of boxes of markers, a top layer and a bottom layer. Each layer contains 10 boxes of markers. So, the large box holds about $2 \times 10$, or $\underline{20}$ boxes of markers.
- Multiply the volume of 1 box of markers by the estimated number of boxes of markers that fit in the large box.

$$
\underline{20} \times \underline{15}=\underline{300}
$$

So, the volume of the large box is about $\underline{300}$ cubic inches.

## Estimate the volume.

1. Each box of toothpaste has a volume of 25 cubic inches.


There are $\qquad$ boxes of toothpaste in the large box.
The estimated volume of the large box is $\qquad$ $\times 25=$ $\qquad$ cubic inches.
2. Volume of CD case: 80 cu cm


Volume of large box: $\qquad$

## Volume of Rectangular Prisms

Jorge wants to find the volume of this rectangular prism. He can use cubes that measure 1 centimeter on each side to find the volume.

Step 1 The base has a length of 2 centimeters and a width of 3 centimeters. Multiply to find the area of the base.


Base $=\underline{2} \times \underline{3}$
Base $=6 \mathrm{~cm}^{2}$
Step 2 The height of the prism is 4 centimeters. Add the number of cubes in each layer to find the volume.


Remember: Each layer has 6 cubes.

Step 3 Count the cubes. $\qquad$ cubes
Multiply the base and the height to check your answer.
Volume $=$ $\qquad$ $\times$ $\qquad$
Volume $=\underline{24}$ cubic centimeters
So, the volume of Jorge's rectangular prism is $\qquad$ cubic centimeters.


## Find the volume.

1. 



Volume: $\qquad$
3.


Volume: $\qquad$
2.


Volume: $\qquad$
4.


Volume: $\qquad$

## Algebra • Apply Volume Formulas

You can use a formula to find the volume of a rectangular prism.

$$
\begin{aligned}
\text { Volume } & =\text { length } \times \text { width } \times \text { height } \\
V & =(I \times w) \times h
\end{aligned}
$$

Find the volume of the rectangular prism.
Step 1 Identify the length, width, and height of the rectangular prism.
 length $=$ $\qquad$ in. width $=$ $\qquad$ in. $\quad$ height $=$ $\qquad$ 4 in.

9 in.
Step 2 Substitute the values of the length, width, and height into the formula.

$$
\begin{aligned}
& V=(I \times w) \times h \\
& V=(\underline{9} \times \underline{3}) \times \underline{4}
\end{aligned}
$$

Step 3 Multiply the length by the width.

$$
\begin{aligned}
& V=(9 \times 3) \times 4 \\
& V=\underline{27} \times 4
\end{aligned}
$$

Step 4 Multiply the product of the length and width by the height.

$$
\begin{aligned}
V & =27 \times \underline{4} \\
& =\underline{108}
\end{aligned}
$$

So, the volume of the rectangular prism is 108 cubic inches.

## Find the volume.

1. 


$V=$ $\qquad$
2.

$V=$ $\qquad$

## Problem Solving • Compare Volumes

A company makes aquariums that come in three sizes of rectangular prisms. The length of each aquarium is three times its width and depth. The depths of the aquariums are 1 foot, 2 feet, and 3 feet.
What is the volume of each aquarium?


1. Jamie needs a bin for her school supplies. A blue bin has a length of 12 inches, a width of 5 inches, and a height of 4 inches. A green bin has a length of 10 inches, a width of 6 inches, and a height of 5 inches. What is the volume of the bin with the greatest volume?
2. Suppose the blue bin that Jamie found had a length of 5 inches, a width of 5 inches, and a height of 12 inches. Would one bin have a greater volume than the other? Explain.

## Find Volume of Composed Figures

A composite figure is a solid made up of two or more solids. To find the volume of a composite figure, first find the volume of each solid that makes up the figure. Then find the sum of the volumes of the figures.
Find the volume of the composite figure at right.
Step 1 Break apart the composite figure into two rectangular prisms. Label the dimensions of each prism.

Prism 1
Prism 2


Step 2 Find the volume of each prism.

Prism 1

$$
\begin{aligned}
& V=(I \times w) \times h \\
& V=4 \times 8 \times \underline{4}
\end{aligned}
$$

$$
V=128 \text { in. }^{3}
$$

Prism 2

$$
\begin{aligned}
& V=(I \times w) \times h \\
& V=\underline{20} \times \underline{8} \times \underline{4}
\end{aligned}
$$

$$
V=640 \mathrm{in}^{3}
$$

Step 3 Find the sum of the volumes of the two prisms.
Volume of Prism $1+$ Volume of Prism 2 = Volume of Composite Figure $\begin{aligned} 128 \mathrm{in}_{土}^{3} & \frac{640 \mathrm{in}_{土}^{3}}{768 \mathrm{in}_{-}^{3}}\end{aligned}=$ Volume of Composite Figure

So, the volume of the composite figure is $768 \mathrm{in}^{3}{ }^{3}$

## Find the volume of the composite figure.


$V=$ $\qquad$

$V=$ $\qquad$

## Compare Fractions and Decimals

Three friends compare the thicknesses of their textbooks. Julio's science book is 1.35 inches thick. Hannah's math book is $1 \frac{3}{5}$ inches thick. Gabriela's history book is 1.9 inches thick.
Who has the textbook with the least thickness?
You can use a number line to compare fractions and decimals.

Remember: On a number line, the number farthest to the left from 0 has the least value.

Step 1 Draw a number line. Locate some benchmarks on the number line.

Benchmark decimals: $1,1.25,1.5,1.75,2, \ldots$
Benchmark mixed numbers: $1,1 \frac{1}{4}, 1 \frac{1}{2}, 1 \frac{3}{4}, 2, \ldots$
Step 2 Mark the thickness of each textbook on the number line.
Find the locations of $1.35,1 \frac{3}{5}$, and 1.9.


Since $1.35<1 \frac{3}{5}<1.9$, Julio's textbook has the least thickness.

For 1-2, identify the points on the number line. Then write the greater number.

1. point $A$ as a fraction $\qquad$
2. point $B$ as a decimal $\qquad$

$\qquad$ is greater than $\qquad$ .

Locate each number on a number line. Then complete the sentence.
3. $1 \frac{3}{5}, 1.85,1.1$

The number with the greatest value is $\qquad$ .

## Order Fractions and Decimals

You can use a number line to help you order decimals, fractions, and mixed numbers.

In one day, a bakery sells 5.2 apple pies, $4 \frac{3}{5}$ cherry pies, $5 \frac{1}{3}$ blueberry pies, and 5.45 pumpkin pies. Order the number of pies the bakery sells from least to greatest.

Step 1 Draw a number line. Locate some benchmarks on the number line.

Benchmark decimals: $4,4.25,4.5,4.75,5,5.25,5.5, \ldots$
Benchmark mixed numbers: $4,4 \frac{1}{4}, 4 \frac{1}{2}, 4 \frac{3}{4}, 5,5 \frac{1}{4}, 5 \frac{1}{2}, \ldots$
Step 2 Locate $5.2,4 \frac{3}{5}, 5 \frac{1}{3}$, and 5.45 on the number line.


Step 3 Order the fractions and decimals.
Remember: The point farthest to the left is the least value. The point farthest to the right is the greatest value.

So, the number of pies the bakery sells from least to greatest is $4 \frac{3}{5}, 5.2,5 \frac{1}{3}$, and 5.45 .

For 1-2, locate each set of numbers on a number line.
Then write the numbers in order from least to greatest.

1. $2.32,2 \frac{3}{4}, 2.16,2 \frac{3}{10}$
2. $\frac{4}{7}, 0.4, \frac{1}{4}, 0.28$

## Factor Trees

You can use a factor tree to show the factors of a number that are all prime numbers. Remember a prime number must be greater than 1, and have only 1 and itself as factors.

Use a factor tree to find the prime number factors that have a product of 18.

Step 1 Draw two branches below 18.
Step 2 Choose any two factors of 18 . Try $6 \times 3$.
Write the factors under the branches. Include the multiplication sign.

Step 3 Check if 6 and 3 are prime numbers.
Think: $6=2 \times 3$ and $3=3 \times 1$.
Draw branches below 6 and write the factors.


Since 3 has only 1 and itself as factors, do not draw any branches below 3.

Step 4 Check if 2 and 3 are prime numbers.
Think: $2=2 \times 1$ and $3=3 \times 1$.
Each factor has only 1 and itself as a factor.
Do not draw any more branches.
Write the factors from least to greatest. Use each factor that has only 1 and itself as a factor.

So, $18=2 \times 3 \times 3$

## Use a factor tree to find the prime number factors.

1. 

12
2.
30

3. 50


## Model Percent

Percent means "per hundred" or "out of 100." For example, 40 percent means 40 out of 100 . You can write 40 percent as $40 \%$.

You can use a decimal model like the one below to represent percents.
The model has 100 squares. Each small square represents $1 \%$.
All 100 squares represent $100 \%$.


Use the model to write the percent.
How many whole rows and single squares are shaded?
$\qquad$
rows: 4
What percent is shaded?
4 rows: $4 \times 10=$ $\qquad$ single squares: $3 \times 1=$ $\qquad$ single squares: 3


Total: $40+3=43$ out of 100 squares, or $43 \%$ is shaded.

## Shade the grid to show the percent.

1. 16 percent

2. $83 \%$

3. 97 percent


## Relate Decimals and Percents

Decimals and percents are two ways of expressing a number. You can express a decimal as a percent and a percent as a decimal.

Model 0.26 . Write 0.26 as a percent.
Step 1 Write the decimal as a ratio.
$0.26=26$ hundredths $=26$ out of 100.
Step 2 Make a model that shows 26 out of 100.


Remember: 1 square represents 1 hundredth, or $1 \%$.
Step 3 Use the model to write a percent.
26 shaded squares $=$ $\qquad$ 26 percent, or $\qquad$
Model 13 percent. Write $13 \%$ as a decimal.
Step 1 Write the percent as a fraction.
$13 \%=\frac{13}{100}$
Step 2 Make a model that shows 13 out of 100 .

$13 \%=0.13$
Step 3 Use the model to write a decimal.
13 shaded squares out of 100 squares $=\underline{0.13}$

Use the model. Complete each statement.
1a. $0.89=$ $\qquad$ out of 100

1b. How many squares are shaded? $\qquad$

1c. What percent is shaded? $\qquad$


Write the percents as decimals.
2. $67 \%$
3. $14 \%$

## Fractions, Decimals, and Percents

## You can write a percent and a decimal as a fraction.

You can also write a fraction as a decimal and as a percent.
Write the percent that is equivalent to $\frac{17}{20}$.
Step 1 Set up the equivalent fraction with a denominator of 100.

$$
\frac{17 \times ?}{20 \times ?}=\frac{}{100}
$$

Step 2 Ask: By what factor can you multiply the denominator, 20, to get 100?
$\frac{17 \times ?}{20 \times 5}=\frac{}{100}$ Multiply the denominator by 5.
Step 3 Multiply the numerator by the same factor, 5 .
$\frac{17 \times 5}{20 \times 5}=\frac{85}{100}$
Step 4 Write the fraction as a percent.
$\frac{85}{100}=85$ percent.
So, $\frac{17}{20}$ equals $85 \%$.

Write $\frac{7}{20}$ as a decimal.
Step 1 Write an equivalent fraction with a denominator of 100.

$$
\frac{7 \times 5}{20 \times 5}=\frac{35}{100} \longleftarrow \begin{aligned}
& \text { Multiply the } \\
& \text { numerator and } \\
& \text { denominator by } 5
\end{aligned}
$$

Step 2 Write the fraction as a decimal.

$$
\frac{35}{100}=0.35
$$

Write $15 \%$ as a fraction in simplest form.

Step 1 Write $15 \%$ as a fraction.

$$
15 \%=\frac{15}{100}
$$

Step 2 Simplify.

$$
15 \%=\frac{15 \div 5}{100 \div 5}=\frac{3}{20}
$$

Write a decimal, a percent, or a simplified fraction.

1. $\frac{1}{5}$ as a decimal
2. $\frac{7}{10}$ as a percent
3. $60 \%$ as a fraction

## Divide Fractions by a Whole Number

You can use a model to help you divide a fraction by a whole number.
Divide. $\frac{2}{5} \div 3$

Step 1 The denominator of the dividend is $\underline{5}$. So divide a rectangle into five equal-size parts, or fifths. The numerator of the dividend is 2 . So shade 2 of the fifths.


Step 2 The divisor is 3 . So divide the rectangle into thirds by drawing horizontal lines. Shade $\frac{1}{3}$ of $\frac{2}{5}$.


Step 3 The rectangle is now divided into 15 equal parts. Each part is $\frac{1}{15}$ of the rectangle.
Step 4 Of the 15 equal parts, 2 parts are shaded twice. So $\frac{\frac{2}{15}}{}$ of the rectangle is shaded twice.
So, $\frac{2}{5} \div 3=\underline{\underline{2}}$.

## Use the model to find the quotient. Write the quotient in simplest form.

1. $\frac{3}{4} \div 4=$

2. $\frac{1}{2} \div 3=$
$\qquad$

3. $\frac{5}{6} \div 7=$

4. $\frac{4}{5} \div 3=$ $\qquad$

## Ratios

A ratio compares two numbers.
Shawna is decorating a picture frame by repeating the tile pattern shown below.


What is the ratio of triangles to circles?
Step 1 Count the number of triangles and circles. triangles: $\qquad$ circles: $\qquad$ 3

Step 2 Use the numbers to write a ratio of triangles to circles. $\qquad$ 4 to 3

So, the ratio of triangles to circles is $\qquad$ 4 to 3 .

You can also write this ratio as $4: 3$ and $\frac{4}{3}$.

Find the ratio of rectangles to circles.
1a. How many rectangles are there?
$\qquad$


1b. How many circles are there?
1c. What is the ratio of rectangles to circles?

## Write the ratio.

2. dark circles to white circles

3. total rectangles to light rectangles

$\qquad$

## Equivalent Ratios

Equivalent ratios are equal forms of the same ratio. You can use multiplication or division to write equivalent ratios.

## Write the equivalent ratio.

4 to $7=$ ? to 21
Step 1 Write the ratios as fractions.

$$
\frac{4}{7}=\frac{?}{21}
$$

Step 2 Compare the denominators.

$$
\frac{4}{7}=\frac{?}{21} \quad \text { Think: } 21>7, \text { so multiply. }
$$

Step 3 Multiply the numerator and denominator by the same number.

$$
\frac{4 \times ?}{7 \times ?}=\frac{?}{21} \quad \text { Think: } 7 \times 3=21
$$

$$
\frac{4 \times \mathbf{3}}{7 \times \mathbf{3}}=\frac{12}{\mathbf{2 1}}
$$

So, 4 to 7 is equivalent to 12 to 21 .

8 to $10=4$ to ?
Step 1 Write the ratios as fractions.

$$
\frac{8}{10}=\frac{4}{?}
$$

Step 2 Compare the numerators.

$$
\frac{8}{10}=\frac{4}{?} \quad \text { Think: } 4<8, \text { so divide. }
$$

Step 3 Divide the numerator and denominator by the same number.
$\frac{8 \div ?}{10 \div ?}=\frac{4}{?} \quad \begin{aligned} & \text { Think: } 8 \div 2=4 \\ & \text { so divide by } 2\end{aligned}$
$\frac{8 \div 2}{10 \div 2}=\frac{4}{5}$
So, 8 to 10 is equivalent to 4 to 5 .

## Write equivalent or not equivalent.

1. 2 to 3 and 8 to 12
2. 15 to 20 and 3 to 5
3. 5 to 6 and 25 to 36
4. 18 to 10 and 9 to 5

Write the equivalent ratio.
5. 28 to $32=$ $\qquad$ to 8
6. 9 to $8=63$ to $\qquad$ 7. $13: 5=$ $\qquad$ :15
$\qquad$
$\qquad$
$\qquad$

## Rates

A rate is a special kind of ratio. It compares two numbers with different units. A unit rate has a 1 as its second term.

Find the unit rate of 12 apples in 3 pounds.
Step 1 Write a rate in fraction form. $\frac{12}{3}$


Step 2 Divide the apples into 3 equal groups. Each group of apples weighs 1 pound.

Step 3 Show your work by writing an equivalent rate with 1 in the denominator.

$\frac{12 \div 3}{3 \div 3}=\frac{4}{1} \longleftarrow$ unit rate
So, the unit rate is 4 apples for 1 pound.
You can read this as 4 apples per pound.

Find the unit rate.

1. 20 oranges in 5 pounds
2. 180 miles in 3 hours
3. 140 pages in 7 days
4. $\$ 100$ for 10 hours
5. 400 miles on 20 gallons
6. $\$ 16$ for 2 books
$\qquad$
7. $\$ 15$ for 5 boxes
8. 225 pages in 5 hours
9. 210 miles in 7 hours
10. $\$ 7.50$ for 3 pounds
11. 84 miles on 7 gallons of gas

## Distance, Rate, and Time

You can use the formula $d=r \times t$ to solve a problem about distance, rate, or time. In the formula, $d$ stands for distance, $r$ stands for rate (or speed), and $t$ stands for time.

A car travels 300 miles in 5 hours. What is the car's speed?
Step 1 Write the formula.

$$
d=r \times t
$$

Step 2 Replace the values you know in the formula.

$$
300=r \times 5
$$

distance: $d=300$
time: $t=5$

Step 3 Use patterns and the inverse operation, division, to solve.

So, the car's speed is $\qquad$ 60 miles per hour.
$300 \div 5=r$
Think: $30 \div 5=$ $\qquad$
$300 \div 5=\underline{60}$

## Use the formula $d=r \times t$ to solve. Include the units in your answer.

1. A rower travels 750 feet in 5 minutes. What is the rower's speed?
2. A snake travels 60 feet in 10 minutes. What is the snake's speed?
3. A cyclist travels at a speed of 7 miles per hour. How long does it take the cyclist to travel 35 miles?
4. A walker travels 3 miles per hour for 4 hours. What distance does the walker travel?
5. A bus travels 15 hours at 60 miles per hour. How far does the bus travel?
6. A plane travels at an average speed of 300 miles per hour. How long does it take the plane to travel 1,200 miles?

## Understand Integers


#### Abstract

You can use positive and negative integers to represent real world quantities. You have used a number line to show 0 and the whole numbers greater than 0 . You can also use a number line to represent the opposites of whole numbers.


Opposites are two numbers that are the same distance from 0 on the number line but in opposite directions. For example, 3 and -3 are opposites. The whole numbers,
 negative integers positive integers their opposites, and 0 are called integers.

You use a negative sign, - , to represent negative integers. You can use a positive sign, + , or no sign, to represent positive integers.

The elevation of Mt. Washington is 6,288 feet above sea level. Write an integer to represent the situation. Then, tell what 0 represents.

Step 1 Decide whether the integer is positive or negative.
In this example, positive integers represent elevation above sea level.
Negative integers represent elevation below sea level. So, the word
above tells me that the integer is $\qquad$
Step 2 Write the integer: $\qquad$ 6,288 or $\qquad$ 6,288 .

So, the elevation of Mt. Washington is $\qquad$ 6,288 .

Step 3 Decide what 0 represents.
0 represents $\qquad$ at sea level .

Write an integer to represent the situation. Then, tell what 0 represents.
Situation

1. The helicopter hovered 150 feet above the ground.
2. Miriam earned 25 bonus points.
3. Pete dove 15 feet into the water.

## Algebra • Write and Evaluate Expressions

An expression is a mathematical phrase made up of numbers, variables, and operation symbols. A variable is a symbol that represents one or more numbers. You evaluate an expression by replacing each variable with a number and simplifying.

Maura sells handmade soap at the farmers' market for $\$ 4.00$ per bar.

- Write an expression for how much Maura earns selling bars of soap.
- Evaluate the expression to determine how much money she will earn if she sells 26 bars of soap.

Step 1 Choose a variable and explain Let $s=$ the number of bars of soap what it stands for.

Step 2 Write a word expression.
\$4 earned for each bar of soap sold
Step 3 Replace the word expression
$4 \times s$ with a multiplication expression using $s$.
Step 4 Replace $s$ with 26.
$4 \times 26$
Step 5 Multiply to evaluate.
$4 \times 26=104$
So, Maura will earn $\$ 104$ if she sells 26 bars of soap.

## Write an expression.

1. Jack's dog weighs $p$ pounds and his puppy weighs 15 pounds less. How much does the puppy weigh?
$\qquad$
Evaluate each expression for the value given.
2. $n-17$ for $n=50$
3. $27+t$ for $t=30$
4. $q \times 15$ for $q=7$
5. $88 \div p$ for $p=4$
6. Paul saved $d$ dollars. Sally saved $\$ 25$ more than Paul saved. How much did Sally save?
$\qquad$

## Algebra•Understand Inequalities

An inequality is a mathematical sentence that compares two quantities.
An inequality contains an inequality symbol: $<,>, \leq, \geq$, or $\neq$.

| Inequality Symbols |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :---: |
| $<$ less than | $>$ greater than | $\leq$ less than <br> or equal to | $\geq$ greater than <br> or equal to | $\neq$ not equal to |  |

The speed limit on a certain road is 45 miles per hour. A driver does not want to exceed the speed limit. Write an inequality using a variable to represent the driver's speed.

Step 1 Write the inequality in words. speed is less than or equal to 45
Step 2 Replace speed with the variable s. $s$ is less than or equal to 45
Step 3 Replace less than or equal to with $\leq . \quad s \leq 45$
So, the inequality $s \leq 45$ represents a driver's speed if he doesn't want to exceed the speed limit of 45 miles per hour.

Of $4,8,12$, and 16 , which numbers are solutions for $f \geq \mathbf{8}$ ? Graph the solutions on a number line.

Step 1 In $f \geq 8$, replace $f$ with 4 . Repeat the process for $f=8,12,16$.
Step 2 Identify the values that make $f \geq 8$ true. $f \geq 8$

$$
4 \geq 8 \text { false }
$$

True values are solutions: $f=8,12,16 \quad 8 \geq 8$ true
$12 \geq 8$ true
$16 \geq 8$ true
Step 3 Graph the solutions on a number line. Use filled circles.


Of 3,5 , and 8 , which numbers are solutions for the inequality $k>5$ ? Graph the solutions on the number line.

1. Replace $k$ with 3 . True or false? $\qquad$
2. Replace $k$ with 5 . True or false? $\qquad$

3. Replace $k$ with 8 . True or false? $\qquad$

## Polygons on a Coordinate Grid

Isabella is designing a quilt on a coordinate grid. The quilt is made up of polygons sewn together. The vertices of one of the polygons can be graphed using the coordinates shown in the table. Plot and describe the polygon.

Plot the points on a coordinate grid.

| $\mathbf{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| 1 | 6 |
| 3 | 3 |
| 7 | 3 |
| 9 | 6 |
| 7 | 9 |
| 3 | 9 |

Step 1 Write ordered pairs.
Use each row of the table to write an ordered pair.
$(\underline{1}, \underline{6}),(\underline{3}, \underline{3}),(\underline{7}, \underline{3}),(\underline{9}, \underline{6}),(\underline{7}, \underline{9}),(\underline{3}, \underline{9})$
Step 2 Graph a point for each pair on the coordinate grid.

Step 3 Connect the points.

So, the polygon has the shape of a $\qquad$ hexagon


Plot the polygon with the given vertices on a coordinate grid. Identify the polygon.

1. $(1,4),(8,1),(6,9)$

2. $(1,1),(1,5),(9,5),(9,1)$


## Area of a Parallelogram

The area of a parallelogram is the product of its base and its height.

$$
A=b \times h
$$

You can use any side as the base of the parallelogram. The height of the parallelogram is the length of a line segment that is perpendicular to the base and has endpoints on the base and the side or vertex opposite the base.

Find the area of the parallelogram.


Step 1 Use the formula for the area of a parallelogram.

$$
A=b \times h
$$

Step 2 Substitute 3 for $b$ and $\underline{7}$ for $h$.

$$
A=\underline{3} \times \underline{7}
$$

Step 3 Multiply.

$$
A=\underline{21}
$$



So, the area of the parallelogram is $\underline{21}$ square feet, or 21 sq ft .

Find the area of the parallelogram.
1.

2.

Area $=$ $\qquad$ Area $=$ $\qquad$
3.

Area $=$ $\qquad$
4.

Area $=$ $\qquad$

## Median and Mode

The median of a set of data is the middle value when the data are written in order.


If a set of data contains an even number of items, the median is the sum of the two middle terms divided by 2.

The mode of a set of data is the data value or values that occur most often. A set of data may have no mode, one mode, or more than one mode.

$$
0, \quad 1, \quad 4, \quad 2, \quad 3, \quad 1
$$

In the data set above, 1 is the mode because it occurs the most often.
The list shows the numbers of books 12 students read during summer vacation.

$$
2,3,4,1,4,5,3,6,2,4,3,4
$$

What are the median and mode of the data?
Step 1 Order the numbers from least to greatest.

$$
1,2,2,3,3,3,4,4,4,4,5,6
$$

Step 2 To find the median, circle the middle value. Since there are 12 values, circle the two middle values. Find the sum of the two middle values and divide by 2 .

$$
\underline{3}+\underline{4}=7 \quad 7 \div \underline{2}=\underline{3.5}
$$

So, the median is 3.5 books.
Step 3 To find the mode, identify the data value that occurs most often.
4 occurs $\underline{4}$ times. So, the mode is $\underline{4}$ books.

Find the median and mode of the data.

1. number of minutes to run $1 \mathrm{mi}: 7,9,8$, 9, 7, 9, 8
median: $\qquad$
mode: $\qquad$
2. Callie's quiz scores: $95,87,93,100$, 87, 95
median: $\qquad$
mode: $\qquad$

## Finding the Average

An average of a set of data is the sum of the data values divided by the total number of data values.

For example, suppose you have the data set $4,0,24,28$, and 14.
The sum of the data values is $4+0+24+28+14$, or 70 .
There are a total of 5 data values. So the average is $70 \div 5$, or 14 .
Several friends are participating in a walk-a-thon for charity. The table at the right shows the amount of money each friend raised. What is the average amount of money raised by each friend?

Step 1 Find the total amount of money the friends raised.
$85+90+\underline{100}+\underline{75}+\underline{115}=\underline{465}$

| Name | Amount of Money <br> Raised (\$) |
| :---: | :---: |
| Aki | 85 |
| Stephen | 90 |
| Lainie | 100 |
| Janelle | 75 |
| Azumi | 115 |

Step 2 Determine how many friends raised money for the walk-a-thon.

| Aki | Stephen | Lainie | Janelle | Azumi |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 |

A total of $\underline{5}$ friends raised money.
Step 3 Divide the total amount of money, 465, by the total number of friends, $\underline{5}$, who raised the money.
$\underline{465} \div \underline{5}=\underline{93}$
So, the average amount of money raised by each friend is $\$ 93$.

Ana Lisa's runs batted in (RBI) record is shown for this month. What was the average number of runs that Ana Lisa batted in per game?

1. Find the total number of runs Ana Lisa batted in.

| Game | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Number of <br> RBIs | 3 | 4 | 1 | 0 | 2 | 2 | 2 | 3 | 1 | 2 |

2. In how many games did Ana Lisa play?
3. Divide the sum by the number of games.

What is the average number of runs batted in per game?

Find the average of the set of numbers.
4. $16,22,19,14,24$ $\qquad$
5. $40,36,51,36,29,18$ $\qquad$

## Histograms

A histogram is a graph that uses bars to show the number of data values that occur within equal intervals. The table below shows the test scores of the students in Omar's science class.

| Science Test Scores |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 82 | 76 | 92 | 65 | 84 | 80 | 98 | 81 | 89 | 90 | 94 | 78 | 91 |
| 100 | 74 | 90 | 76 | 95 | 68 | 75 | 83 | 92 | 85 | 85 | 83 | 94 |

## Use the data to make a histogram.

Step 1 Make a frequency table, using intervals of 10, and then start a bar graph. Write the intervals on the horizontal axis of the graph and label the axis.

| Scores | Tally | Frequency |
| :---: | :---: | :---: |
| 61-70 | \| | 2 |
| 71-80 | H I | 6 |
| 81-90 | H HH | 10 |
| 91-100 | HI III | 8 |

## Science Test Scores

Step 2 Choose a scale for the vertical axis that works with the frequencies. Use a scale from 0 to12 with intervals of two. Label the axis.

Step 3 Draw a bar for each interval. The bar's height is determined by the frequency.


For 1-2, use the data below.
The ages of the children in a swim club are given below.
$6,8,11,10,7,9,8,8,7,7,12,8,8,10,10,11$, $12,10,9,13,14,10,11$

1. Complete the frequency table. Use 3 years for each interval.

| Ages | Tally | Frequency |
| :--- | :--- | :--- |
| $6-8$ |  |  |
|  |  |  |
|  |  |  |

2. Complete the histogram.


## Analyze Histograms

A histogram shows how often data occur within intervals. You can use a histogram to compare the frequency of the data within each interval.

The histogram shows the number of students in Mr. Lee's class who walked 4 miles within the range of each interval.

How many students walked between 60 and 62 minutes?

Step 1 Find the interval labeled 60-62.
Step 2 Find the frequency by reading the height of the bar. The bar ends halfway between 10 and 12 . It ends at 11.

So, 11 students walked between 60 and 62 minutes.

How many students walked between 54 and 59 minutes?

Step 1 Find the intervals for the range of times: 54-56 and 57-59.

Step 2 Find the frequency for each interval
by reading the height of each bar.
Step 2 Find the frequency for each interva
by reading the height of each bar.


54-56: 2 students
57-59: 8 students

Step 3 Add the frequencies to find the total. $2+8=10$
So, 10 students walked between 54 and 59 minutes.

## For 1-2, use the histogram at the right.

The histogram shows the number of hours of TV that students watched last week.

1. How many students watched between 10 and 14 hours of TV last week?
2. How many students watched less than 10 hours of TV last week?

Number of Hours of TV Last Week


