3-6 Basic Skills and Concepts

Calculating Factorials, Combinations, Permutations. *In Exercises 1–8, evaluate the given expressions and express all results using the usual format for writing numbers (instead of scientific notation).*

1. $6!$

2. $15!$

3. ${}_{25}P_2$

4. ${}_{100}P_3$

5. ${}_{25}C_2$

6. ${}_{100}C_3$

7. ${}_{52}C_5$

8. ${}_{52}P_5$

Probability of Winning the Lottery. *This section included an example showing that the probability of winning the Maine lottery is 1/5,245,786. In Exercises 9–12, find the probability of winning the indicated lottery.*

9. Massachusetts Mass Millions: Select the winning six numbers from 1, 2, . . . , 49.

10. Pennsylvania Super 6 Lotto: Select the winning six numbers from 1, 2, . . . , 69.

11. New York Lotto: Select the winning six numbers from 1, 2, . . . , 59.

12. New York Take Five: Select the winning five numbers from 1, 2, . . . , 39.

13. Age Discrimination The Pitt Software Company reduced its sales staff from 32 employees to 28. The company claimed that four employees were randomly selected for job termination. However, the four employees chosen are the four oldest employees among the original sales force of 32. Find the probability that when four employees are randomly selected from a group of 32, the four oldest are selected. Is that probability low enough to charge that instead of using random selection, the Pitt Software Company actually fired the oldest employees?

14. Computer Design In designing a computer, if a *byte* is defined to be a sequence of 8 bits and each bit must be a 0 or 1, how many different bytes are possible? (A byte is often used to represent an individual character, such as a letter, digit, or punctuation symbol. For example, one coding system represents the letter *A* as 01000001.) Are there enough different bytes for the characters that we typically use, including lowercase letters, capital letters, digits, punctuation symbols, dollar sign, and so on?

15. Maine Lottery The probability of winning the Maine lottery is 1/5,245,786. What is the probability of winning if the rules are changed so that in addition to selecting the correct six numbers from 1 to 42, you must now select them in the same order as they are drawn?

16. Testing a Claim Mike claims that he has developed the ability to roll a 6 almost every time that he rolls a die. You test his claim by having Mike roll a die five times, and he gets a 6 each time. If Mike has no ability to affect the outcomes, find the probability that he will roll five consecutive 6s when a die is rolled five times. Is that probability low enough to support Mike's claim?

17. Selection of Treatment Group Walton Pharmaceuticals wants to test the effectiveness of a new drug designed to relieve allergy symptoms. The initial test will be conducted by treating six people chosen from a pool of 15 volunteers. If the treatment group is randomly selected, what is the probability that it consists of the six youngest people in the pool? If the six youngest are selected, is there sufficient evidence to conclude that instead of being random, the selection was based on age?

18. He Did It His Way Singing legend Frank Sinatra recorded 381 songs. From a list of his top-10 songs, you must select three that will be sung in a medley as a tribute at the next MTV Music Awards ceremony. The order of the songs is important so that they fit together well. If you select three of Sinatra's top-10 songs, how many different sequences are possible?

19. Air Routes You have just started your own airline company called Air America (motto: "Where your probability of a safe flight is greater than zero"). You have one plane for a route connecting Austin, Boise, and Chicago. One route is Austin-Boise-Chicago and a second route is Chicago-Boise-Austin. How many other routes are possible? How many different routes are possible if service is expanded to include a total of eight cities?

20. Social Security Numbers Each social security number is a sequence of nine digits. What is the probability of randomly generating nine digits and getting *your* social security number?

21. Electrifying When testing for electrical current in a cable with five color-coded wires, the author used a meter to test two wires at a time. How many tests are required for every possible pairing of two wires?

22. Elected Board of Directors There are 12 members on the board of directors for the Newport General Hospital.

a. If they must elect a chairperson, first vice chairperson, second vice chairperson, and secretary, how many different slates of candidates are possible?

b. If they must form an ethics subcommittee of four members, how many different subcommittees are possible?

23. Jumble Puzzle Many newspapers carry "Jumble," a puzzle in which the reader must unscramble letters to form words. For example, the letters TAISER were included in newspapers on the day this exercise was written. How many ways can the letters of TAISER be arranged? Identify the correct unscrambling, then determine the probability of getting that result by randomly selecting an arrangement of the given letters.

24. Finding the Number of Possible Melodies In Denys Parsons' *Directory of Tunes and Musical Themes,* melodies for more than 14,000 songs are listed according to the following scheme: The first note of every song is represented by an asterisk *, and successive notes are represented by *R* (for repeat the previous note), *U* (for a note that goes up), or *D* (for a note that goes down). Beethoven's Fifth Symphony begins as *RRD. Classical melodies are represented through the first 16 notes. With this scheme, how many different classical melodies are possible?

25. Combination Locks A typical "combination" lock is opened with the correct sequence of three numbers between 0 and 49 inclusive. (A number can be used more than once.) What is the probability of guessing those three numbers and opening the lock with the first try?

26. Five Card Flush A standard deck of cards contains 13 clubs, 13 diamonds, 13 hearts, and 13 spades. If five cards are randomly selected, find the probability of getting a flush. (A flush is obtained when all five cards are of the same suit. That is, they are all clubs, or all diamonds, or all hearts, or all spades.)

27. Probabilities of Gender Sequences

a. If a couple plans to have eight children, how many different gender sequences are possible?

b. If a couple has four boys and four girls, how many different gender sequences are possible?

c. Based on the results from parts (a) and (b), what is the probability that when a couple has eight children, the result will consist of four boys and four girls?

28. Is the Researcher Cheating? You become suspicious when a genetics researcher randomly selects groups of 20 newborn babies and seems to consistently get 10 girls and 10 boys. The researcher explains that it is common to get 10 boys and 10 girls in such cases.

a. If 20 newborn babies are randomly selected, how many different gender sequences are possible?

b. How many different ways can 10 boys and 10 girls be arranged in sequence?

continued

c. What is the probability of getting 10 boys and 10 girls when 20 babies are born?
d. Based on the preceding results, do you agree with the researcher's explanation that it is common to get 10 boys and 10 girls when 20 babies are randomly selected?

29. Finding the Number of Area Codes *USA Today* reporter Paul Wiseman described the old rules for telephone area codes by writing about "possible area codes with 1 or 0 in the second digit. (Excluded: codes ending in 00 or 11, for toll-free calls, emergency services, and other special uses.)" Codes beginning with 0 or 1 should also be excluded. How many different area codes were possible under these old rules?

30. Cracked Eggs A carton contains 12 eggs, 3 of which are cracked. If we randomly select 5 of the eggs for hard boiling, what is the probability of the following events?
a. All of the cracked eggs are selected.
b. None of the cracked eggs are selected.
c. Two of the cracked eggs are selected.

31. California Lottery In California's Super Lotto Plus lottery game, winning the jackpot requires that you select the correct 5 numbers between 1 and 47 and, in a separate drawing, you must also select the correct single number between 1 and 27. Find the probability of winning the jackpot.

32. N.C.A.A. Basketball Tournament Each year, 64 college basketball teams compete in the N.C.A.A. tournament. Sandbox.com recently offered a prize of $10 million to anyone who could correctly pick the winner in each of the tournament games. The president of that company also promised that, in addition to the cash prize, he would eat a bucket of worms. Yuck.
a. How many games are required to get one championship team from the field of 64 teams?
b. If someone makes random guesses for each game of the tournament, find the probability of picking the winner in each game.
c. In an article about the $10 million prize, the *New York Times* wrote that "Even a college basketball expert who can pick games at a 70 percent clip has a 1 in _____ chance of getting all the games right." Fill in the blank.

3-6 Beyond the Basics

33. Finding the Number of Computer Variable Names A common computer programming rule is that names of variables must be between 1 and 8 characters long. The first character can be any of the 26 letters, while successive characters can be any of the 26 letters or any of the 10 digits. For example, allowable variable names are A, BBB, and M3477K. How many different variable names are possible?

34. Handshakes and Round Tables
a. Five managers gather for a meeting. If each manager shakes hands with each other manager exactly once, what is the total number of handshakes?
b. If n managers shake hands with each other exactly once, what is the total number of handshakes?
c. How many different ways can five managers be seated at a round table? (Assume that if everyone moves to the right, the seating arrangement is the same.)
d. How many different ways can n managers be seated at a round table?