

Exploring Probability:
Permutations and Combinations

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Introduction

This mini unit was designed and implemented for a 6th grade advanced math class at Walker Charter Academy in Walker, Michigan. Since this is an advanced math class, the material covered is considered 7th grade material and aligns with National Heritage Academy Crosswalk Standards for 7th grade math. This unit consists of three lessons covering a subset of probability: permutations and combinations. Technology, student interaction, and higher level thinking are all included in the unit and will be described further below.

Standards

The standards in this lesson come from the National Heritage Academy Crosswalk Standards for 7th grade math. They include:

Measurement Topic: Probability

The student will understand and apply basic concepts of probability.
(Use permutations and combinations to find possible arrangements.)

Objectives

Each lesson contains one objective to align with the standards mentioned above. The objectives are as follows:

- TLW find possible arrangements using permutations.
- TLW find possible arrangements of objects using combinations.
- TLW determine if a permutation or combination is needed to solve a probability problem.

Instructional Delivery

This unit uses a variety of instructional methods. First of all, the lessons rely heavily on real world examples. Students will be asked to come in front of the class to act out problems as a way to solve them as well as come up with their own real world examples that relate to the problems. Students will spend a large chunk of each lesson engaged in guided practice with the teacher and other students. This will help the teacher become aware of misconceptions that students may have. Independent practice will engage students' higher level thinking and encourage them to come to their own understanding of the concepts. Technology will be integrated through the use of projectors and videos, as well as interactive worksheets that students may complete online for additional practice.

Technology

In this unit, videos from the Kahn Academy will be used to highlight important concepts from the lessons. In having students watch these videos, they will gain another perspective on the subject and perhaps view the information in a way that the teacher cannot present. Students may also use online worksheets for additional practice.

Assessment Tools

At the end of each lesson, students will participate in independent practice, which will serve as a formative assessment for the teacher. The information gained in the assessment will guide instruction for the following lessons in the unit. At the end of the unit, students will complete a summative assessment consisting of six questions related to material covered in the unit.

Assessment Data & Instructional Changes

The information gathered from the assessment is essential in guiding instructions and making changes for future lessons. After students turned in their independent practice from the second lesson, I realized that some students were struggling to make the change from finding permutations to finding combinations. We spent extra time in class coming up with ways we could explore word problems to decide whether we needed to use a permutation or a combination formula. Students decided that it would be helpful to come up with certain steps to follow when solving each type of problem, which I have now added to each guided practice worksheet for future instruction.

Overall Reflection

This mini unit is intended to be implanted a part of a larger unit on probability. These three lessons were completed in four days, including the summative assessment. Students enjoyed the hands-on aspect of the first lesson and were engaged when students were chosen to come to the front of the class and act out our word problems. Students seemed to easily grasp the concept of permutation and were successful in completing the guided and independent practice. In the second lesson, students had some difficulty making the transition from permutations to combinations. As a class, we came up with a list of steps in order to make identifying what makes a problem a combination problem easier. Creating this list of steps made the third lesson simple for the students. They enjoyed using the whiteboards to show what they had learned regarding the differences between permutations and combinations. The summative assessment was appropriate for the objectives of the lessons, but before doing this unit again, I would adjust the assessment to include higher level thinking questions.

Lesson 1: Permutations

1. Standards

Measurement Topic: Probability

The student will understand and apply basic concepts of probability.

- Use permutations and combinations to find possible arrangements.

2. Objective

TLW find possible arrangements using permutations.

*I can find arrangements with permutations.

3. Anticipatory Set

TTW bring 3 students to the front of the classroom. The students will arrange themselves in as many different orders as possible while the class writes these orders down.

4. Input

- Anticipatory Set
- TTW ask students how many different arrangements were possible. Then, TTW ask students if it would be easy or difficult to do this with 10 students.
- TTW introduce students to permutations. A permutation is an arrangement of objects in which the order matters.
- To find an arrangement of a large number, say 10, students will multiply $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$. This is also known as 10 factorial, or $10!$
- TTW also show this video from the Kahn Academy (<http://www.youtube.com/watch?v=oQpKtm5TtxU>) to illustrate the concept in a different way.
- As a class, students will complete the guided practice questions with the teacher.
- TLW complete the independent practice at the end of the lesson.

Thinking Levels

Knowledge: TLW define permutation.

Comprehension: TLW explain how to find a factorial.

Application: TLW solve probability problems using permutations.

Learning Styles

Visual-Spatial: TLW sketch diagrams to solve problems.

Logical-Mathematical: TLW explore patterns and relationships.

Accommodations

Remediation: Students who struggle finding factorials and permutations will be given hands-on materials in order to manipulate arrangements.

Extensions: Students will be given challenge word problems to solve for possible arrangements.

Differentiated Instruction

In this lesson, students will use real world examples to solve problems. Students will work with their peers and as a class to solve problems as well. Classroom discussion will be key in student understanding during this lesson. Students will also watch a short video lesson from the Kahn Academy in order to learn from a different perspective.

Materials

- Guided and Independent practice for each student
- Student calculators
- Access to youtube video and projector
<http://www.youtube.com/watch?v=oQpKtm5TtxU>

5. Modeling

During guided practice, TTW show students how to use factorials to find permutations.

6. Checking for Understanding

During guided practice, TTW do a visual check of all students' answers to guide instruction before independent practice.

7. Guided Practice

Students will complete the two guided practice questions as a class. Students will first try problems on their own and then they will discuss the answers and how to find them as a class.

8. Independent Practice

TLW complete the independent practice problems. These problems will be turned in once completed and checked for correctness.

9. Closure

After independent practice, students will check their answers and the class will discuss possible misconceptions.

10. Assessment

The independent practice problems in this lesson will serve as a formative assessment for the teacher. Summative assessment will follow the third lesson in the unit.

11. Reflection

In this lesson, students enjoyed coming up to the front of their class with their classmates in order to illustrate the problems we were trying to solve. While showing the video from the Kahn Academy, some students were disengaged while others were very interested. I would still show this video in future lessons even though some

students did not enjoy it. I think it was important for those students who struggled with my instruction and needed to hear it from a different point of view.

PERMUTATIONS

GUIDED PRACTICE

Example: In how many different ways can Daisy, Eddie, Ian, Casey, and MaKenna stand in a row?

- (1) Identify the number of objects (5)
- (2) Use $5!$ in your calculator OR
- (3) Solve $5 \times 4 \times 3 \times 2 \times 1$
- (4) Answer: 120

Practice 1: In how many different ways can a red, blue, and green marker be arranged on the board?

Example: Myles has 6 cross county medals. He wants to display 3 of them on his wall. How many different ways could he display them?

- (1) Identify the number of objects (6)
- (2) Identify how many you want to arrange (3)
- (3) How many do you have to choose from for your first display? (6) How many for the second? (5) How many for the third? (4)
- (4) Solve $5 \times 4 \times 3$
- (5) Answer: 120

Practice 2: There are 12 contestants in a horse race. 3 ribbons will be awarded. How many different ways could the ribbons be awarded?

PERMUTATIONS

1. There are 7 finalists in the WCA academic competition. In how many ways can the judges select a winner?
2. How many different arrangements can be made using the letters C D F R E G H S R ?
3. Miss Hunt has 20 pieces of candy. She is going to pass them out to 8 students. In how many different ways can she pass out the candy?
4. An artist paints 10 portraits. How many ways can he give these portraits to 4 people?
5. A software company makes serial numbers for computers. How many different serial numbers could they make out of the numbers 1n 4 6 3 2 5 7 8 9 0 ?

Lesson 2: Combinations

1. Standards

Measurement Topic: Probability

The student will understand and apply basic concepts of probability.

- Use permutations and combinations to find possible arrangements.

2. Objective

TLW find possible arrangements of objects using combinations.

*I can find arrangements using combinations.

3. Anticipatory Set

The class will define permutation as discussed in lesson 1.

4. Input

- Anticipatory set
- TTW pose a question to students. "If you are going on a trip and are packing your suitcase with a scarf, a shirt, and boots, does it matter what order you pack the 3 items in?" The answer is no. You can pack the scarf, the shirt, then the boots (or any other possible combination).
- The class will define combination as: an arrangement of objects in which the order does not matter.
- TTW introduce this example: There are 5 flavors of ice cream: chocolate, vanilla, strawberry, banana, and superman. You can select 3 of these flavors for your sundae. First, you find the permutation of the larger group ($5 \times 4 \times 3 = 60$). Then, you find the permutation of the smaller group ($3 \times 2 \times 1 = 6$). Lastly, divide the larger group by the smaller group ($60/6=10$).
- As a class, TLW complete the guided practice questions.
- TLW complete the independent practice at the end of the lesson.

Thinking Levels

Knowledge: TLW define combination.

Comprehension: TLW explain how to solve a combination problem.

Application: TLW solve probability problems using combinations.

Learning Styles

Visual-Spatial: TLW sketch diagrams to solve problems.

Logical-Mathematical: TLW explore patterns and relationships.

Accommodations

Remediation: Students who struggle to use the combination formula will first be assessed on if they can use the permutation formula. Without knowing how to use the permutation formula, it will not be possible to solve combination problems. Once this

issue is addressed, students will be asked to come up with real world problems in which a combination would be used in order to solve it.

Extension: Students will be given challenge word problems and asked to identify what information will be needed to solve the problem and then solve the problem using combinations.

Differentiated Instruction

In this lesson, students will be encouraged to find real world examples to help them solve problems. Students will also be able to use manipulatives, diagrams, and sketches to help them understand and solve the problem. Students will be able to work with classmates during guided practice as a way to scaffold instruction.

Materials

- Guided and independent practice for each student
- Student calculators

5. Modeling

During guided practice, TTW model for students how to solve a problem using combinations.

6. Checking for Understanding

During modeling and guided practice, TTW do a visual check of students' work on the guided practice worksheet to see if there are areas of struggle.

7. Guided Practice

Students will try the guided practice questions on their own. Once completed, the class will discuss the answers and how students found them. This will be a time for students to ask questions about the problems and to clear up misconceptions and confusions they may have.

8. Independent Practice

Students will complete the independent practice worksheet on their own. Once completed, this worksheet will be turned into the teacher and checked for correctness.

9. Closure

At the end of the lesson, the class will define combination and how to find it as a way to recap the lesson.

10. Assessment

The independent practice will serve as a formative assessment for this lesson. The summative assessment for the unit will follow lesson 3.

11. Reflection

Some students struggled to make the transition from permutations to combinations. During the lesson, we created steps that students should follow to solve the various

types of problems. These steps are now included on the guided practice worksheets for each lesson.

COMBINATIONS

GUIDED PRACTICE

Example: There are 11 names in a jar. 6 names will be drawn to win a prize. How many different combinations of names can be drawn?

1. Identify how many names will be drawn (6)
2. Find permutation of the larger group
 $11 \times 10 \times 9 \times 8 \times 7 \times 6 = 332,640$
3. Find permutation of the smaller group
 $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$
4. Divide the larger group by the smaller group
 $332,640 / 720 = 462$ combinations

Practice 1: The volleyball team has 12 players but only 6 can be on the court at a time. How many combinations of player could be on the court?

Practice 2: There are 13 possible pizza toppings and the local pizza place. You can have 3 toppings on a large pizza. How many combinations of pizza toppings are possible?

COMBINATIONS

1. You and your family are packing your suitcase for your spring break trip. Your mom asks you to pack your shoes, shirt, pants, scarf, and gloves. How many combinations of clothes are possible?
2. Eddie selected 6 books from the library. However, he can only check out 4 books at a time. How many possible selections can he make?
3. You are making a sandwich. There are 8 items for you to put on your sandwich. You decide to only put 4 items on the sandwich. How many combinations of items could you have?
4. There are four balls: red, green, purple, and blue. If 2 balls are picked, what is the possible number of combinations?

Lesson 3: Permutations vs. Combinations

1. Standards

Measurement Topic: Probability

The student will understand and apply basic concepts of probability.

- Use permutations and combinations to find possible arrangements.

2. Objective

TLW determine if a permutation or combination is needed to solve a probability problem.

*I can decide whether to use a permutation or a combination.

3. Anticipatory Set

TLW will do a think-pair-share with someone sitting near them to define and tell the difference between a permutation and a combination.

4. Input

- Anticipatory set
- As a class, students will review the differences between permutations and combinations using specific examples discussed in previous lesson.
- Students will be asked to determine which way a probability problem should be solved by participating in a white board game.
- TTW give each student a white board, marker, and eraser. TTW read probability problems from the previous two lessons to students and they will write down P for permutation and C for combination on their white board. When they have their answer, they will raise their white board so the teacher can see if they have the correct answer or not.
- After each question, students will be asked to come to the board and show the class how to solve the problem using the correct formula.
- After this activity, students will complete the independent practice by writing at least 2 permutation problems and 2 combination problems.

Thinking Levels

Knowledge: TLW define combination. TLW define permutation.

Comprehension: TLW explain how to solve a combination problem as well as a permutation problem.

Application: TLW solve probability problems using combinations and permutations.

Analysis: TLW determine whether a permutation or combination should be used to solve a problem.

Learning Styles

Visual-Spatial: TLW sketch diagrams to solve problems.

Logical-Mathematical: TLW explore patterns and relationships.

Interpersonal: TLW work with a partner during the think-pair-share

Intrapersonal: TLW work independently to participate in the white board activity.

Accommodations

Remediation: Students who struggle to find permutations and combinations will first be asked to look at problems and determine which formula should be used to solve it. They will focus on finding key words that will help them identify which way to solve the problem.

Extensions: Students who easily distinguish between permutations and combinations will be asked to come up with their own questions and share these questions with a partner.

Materials

- Student calculators
- Student white boards and markers

5. Modeling

During white board practice, TTW assist students in solving problems if they are having difficulty with the formulas.

6. Checking for Understanding

As students hold up their white board with their answers, TTW be able to assess their progress on an informal basis. This will give the teacher an understanding of how to class as a whole is doing with the lesson.

7. Guided Practice

As students participate in the white board activity, TTW assist students as they solve the problems on the board.

8. Independent Practice

For independent practice, students will write 2 permutation problems and 2 combination problems. They will be encouraged to trade with another student to solve the problems.

9. Closure

At the end of the lesson, TTW ask students to describe how to tell the difference between a permutation and a combination.

10. Assessment

During white board practice, TTW be able to assess students' individual understanding of the lesson on an informal basis. Summative assessment for this unit will be given on the next day.

11. Reflection

This unit was easily completed in four days. This final lesson took two days to complete. One day for the instruction and review and a second day for the summative assessment. Students were very engaged during the whiteboard review. This worked out great

because I was able to use this information they showed on their whiteboard as an assessment tool.

4. There are 10 possible pizza toppings and the local pizza place. You can have 4 toppings on a large pizza. How many combinations of pizza toppings are possible?

5. There are 26 names in a jar. 6 names will be drawn to win a prize. How many different combinations of names can be drawn?

6. What is the difference between a permutation and a combination?