MATH 373
TEST 1
Fall 2017

September 26, 2017

1. Amber has a loan which will be repaid with a lump sum at the end of five years. The amount of the loan is 40,000 and has an interest rate of 12% compounded quarterly.

   Calculate the amount of interest that Amber will pay on this loan.

   Solution:

   After five years, Amber will have:

$$(40,000)\left(1+\frac{0.12}{4}\right)^{(4)(5)} = 72,244.45$$

   The amount of interest will be $72,244.45 - 40,000.00 = 32,244.45$

2. Megan buys a US Treasury Bill with a maturity period of 120 days. The quoted rate on the US Treasury Bill is 0.09000 and the price is 9506.

   Megan also buys a Canadian Treasury Bill with a maturity period of 120 days

   The US Treasury Bill and the Canadian Treasury Bill have the same maturity value and the same price.

   Determine the quoted rate on the Canadian Treasury Bill.

   Solution:

   For the US Treasury Bill,

$$QR = 0.09 = \left(\frac{360}{120}\right)\left(\frac{MV - 9506}{MV}\right) ==> 0.03MV = MV - 9506$$

$$==> 9506 = MV - 0.03MV ==> MV = \frac{9506}{0.97} = 9800$$

   For the Canadian Treasury Bill,

$$QR = \left(\frac{365}{120}\right)\left(\frac{9800 - 9506}{9506}\right) = 0.09407$$

3. Covadonga loans 10,000 to Summer to be repaid with three level annual payments of Q at the end of years one, two and three. Covadonga reinvests each payment at an annual effective interest rate of 7.2%.

   Taking into account reinvestment, Covadonga realizes a return on the loan of an annual effective rate of 6.5%.

   Determine Q .

   Solution:

$$10,000(1.065)^3 = Q(1.072)^2 + Q(1.072) + Q$$

$$Q = \frac{10,000(1.065)^3}{(1.072)^2 + (1.072) + 1} = 3750.02$$

4. You are given that $v(t) = \dfrac{1}{1+0.002t^2}$.

Calculate $1000(\delta_5 - d_5)$.

Solution:

$$v(t) = \frac{1}{1+0.002t^2} \text{ ==> } a(t) = 1+0.002t^2$$

$$\delta_t = \frac{a'(t)}{a(t)} = \frac{0.004t}{1+0.002t^2} \text{ ==> } \delta_5 = \frac{(0.004)(5)}{1+0.002(5)^2} = \frac{0.02}{1.05} = 0.01905$$

$$d_5 = \frac{a(5)-a(4)}{a(5)} = \frac{1+0.002(5)^2 - \left[1+0.002(4)^2\right]}{1+0.002(5)^2} = \frac{1.05-1.032}{1.05} = 0.01714$$

$$Answer = (1000)(0.01905 - 0.01714) = 1.91$$

5. Wang National Bank makes five year loans for college students. Wang wants to receive an annual interest rate 3.5% compounded continuously without taking into account defaults and inflation.

Wang expects that inflation for the next five years will be at an annual rate of 2.5% compounded continuously. However, since this inflation assumption is only an expectation, inflation could be higher or lower. Therefore, Wang also charges an annual rate of 0.4% compounded continuously on all loans to compensate for the uncertainty of the inflation expectation.

College students have a high default rate. Wang believes that 5% of the students will default on the loan at the end of five years. Wang also believes that the bank will be able to recover 45% of the amount owed on defaults.

Calculate the credit spread that Wang needs to charge as an annual rate compounded continuously.

Solution:

$$R^{WithoutDefaults} = 0.035 + 0.025 + 0.004 = 0.064$$

$$R^{WithDefautls} = 0.064 + s$$

$$e^{0.064(5)} = (0.95)e^{(0.064+s)(5)} + (0.05)(0.45)e^{(0.064+s)(5)}$$

$$e^{0.064(5)} = (0.9725)e^{(0.064+s)(5)} ==> \frac{e^{(0.064+s)(5)}}{e^{0.064(5)}} = \frac{1}{0.9725} ==> e^{5s} = \frac{1}{0.9725}$$

$$5s = \ln\left[\frac{1}{0.9725}\right] ==> s = \frac{\ln\left[\frac{1}{0.9725}\right]}{5} = 0.00558$$

6. Giacomo, Yuchen, and Cai enter into a financial agreement. Under the agreement, Giacomo pays Yuchen 10,000 now. Additionally, he pays Cai 12,000 at the end of N years.

Yuchen pays Cai 4000 at the end of two years and pays 9000 to Giacomo at the end of 2N years.

Cai also pays Giacomo 20,000 at the end of 2N years.

Giocomo realizes an annual effective return of 12% on this financial arrangement.

Determine N. (Note: N is not an integer.)

Solution:

First you must find Giacomo's payments:

Giacomo will pay 10,000 at time 0 and 12,000 at time N. He will receive 20,000 plus 9000 at time 2N.

Now set up our equation of value:

$$-10,000(1.12)^{2N} - 12,000(1.12)^{N} + 29,000 = 0$$

Let $x = (1.12)^N$

$$-10,000(x)^2 - 12,000x + 29,000 = 0$$

$$==> x = \frac{-(-12,000) \pm \sqrt{(-12,000)^2 - (4)(-10,000)(29,000)}}{(2)(-10,000)} = 1.205547$$

$$1.205547 = (1.12)^N \;==>\; N = \frac{\ln(1.205547)}{\ln(1.12)} = 1.64948$$

7. Sue lends 100,000 to Nathan. The loan is for four years and includes inflation protection. Nathan will repay an annual interest rate of 5.2% compounded continuously plus the rate of inflation. The 5.2% compounded continuously already reflects the cost of inflation protection and the cost of defaults.

   The rate of inflation in the first year was 2.3% compounded continuously. The rate of inflation in the second and third years was $x\%$ compounded continuously. The rate of inflation in the last year of the loan was 3.5% compounded continuously.

   At the end of four years, Nathan repays the loan with a payment of 143,000.

   Calculate x.

   Solution:

   $$(100,000)e^{0.052+0.023+0.052+x+0.052+x+0.052+0.035} = 143,000$$

   $$e^{0.266+2x} = 1.43$$

   $$0.266 + 2x = \ln(1.43) ==> x = \frac{\ln(1.43) - 0.266}{2} = 0.04584$$

8. Brandon borrows 4000 at a simple interest rate. At the end of 8 years, Brandon repays the loan with a payment of 5800.

Calculate the effective interest rate for the last two years of the loan which is $i_{[6,8]}$.

Solution:

$$a(t) = 1 + st$$

$$4000(1+8s) = 5800 ==> 4000 + 32,000s = 5800$$

$$==> s = \frac{5800 - 4000}{32,000} = 0.05625$$

$$i_{[6,8]} = \frac{a(8) - a(6)}{a(6)} = \frac{1 + (0.05625)(8) - \left[1 + (0.05625)(6)\right]}{1 + (0.05625)(6)} = \frac{1.45 - 1.3375}{1.3375} = 0.08411$$

9. Winnie invests in the Chow Fund. On January 1, 2016, her account has a balance of 75,000 in the Fund. On April 30, 2016 Winnie has a balance of 80,000 and decides to withdraw 20,000 to go to Europe for the summer. On August 30, 2016, Winnie has a balance of 55,000 in the Fund. She decides to deposit 12,000 in the Fund at that time.

   Finally, on August 1, 2017, Winnie withdraws 15,000 to pay her tuition. Prior to the withdrawal, Winnie had a balance of 72,000.

   On December 31, 2017, Winnie has a balance of 60,000.

   Estimate Winnie's annual dollar weighted return using the simple interest estimate that we learned.

   Solution:

$$A + C + I = B \implies 75,000 + (-20,000 + 12,000 - 15,000) + I = 60,000$$

$$I = 8000$$

$$j = \frac{8000}{75,000 - 20,000\left(1 - \frac{4}{24}\right) + 12,000\left(1 - \frac{8}{24}\right) - 15,000\left(1 - \frac{19}{24}\right)} = 0.1265656$$

$$1 + i = (1 + j)^{1/T} = (1.1265656)^{1/2} = 1.061398 \implies Answer = 0.061398$$

10. Zhang LTD is going to build a new factory. Zhang invests 10 million (10,000,000) at time zero. In return for this investment, Zhang expects the following cash flows:

Time t	Cash Flow
1	6 million
2	X million
3	2 million

After three years, the factory will be obsolete and no longer generate cash flows.

The internal rate of return is 10%.

Calculate the net present value at 12%.

Solution:

In Millions:

$$-10 + 6(1.1)^{-1} + x(1.1)^{-2} + 2(1.1)^{-3} = 0$$

$$==> x(1.1)^{-2} = 3.042824944 ==> x = 3.68181818$$

$$NPV = -10 + 6(1.12)^{-1} + 3.68181818(1.12)^{-2} + 2(1.12)^{-3} = -0.28417374 \text{ millions}$$

$$Answer = -284{,}173.74$$

11. Adam has the choice of the following car loans:

    a. Bradley Bank will loan Adam 13,000 for five years to be repaid at an annual interest rate of 8% compounded continuously.

    b. Chen Bank will loan Adam 13,000 for five years to be repaid at an annual interest rate equivalent to a discount rate of 8% compounded monthly.

    State which loan Adam should choose. Demonstrate that the loan that you chose is better for Adam than the other loan.

    Solution:

    Under Option A:

    $$\text{Amount to be Repaid} = (13,000)(e^{(0.08)(5)}) = 19,393.72$$

    Under Option B:

    $$\text{Amount to be Repaid} = (13,000)\left(1 - \frac{0.08}{12}\right)^{-12(5)} = 19,419.71$$

    Adam should choose the loan that will result in the smallest repayment so he should choose Option A which is the loan from Bradley Bank.

12. Christine has the choice of two investments:

    a. With Investment A, Christine will invest 10,000 today. The investment will earn an annual effective interest rate of i. Using the Rule of 72, Christine believes that she will have 40,000 at the end of 17 years.

    b. Investment B is a US Treasury Bill which has a price of 10,000 today and a maturity value of 10,240.16. The quoted rate on the Treasury Bill is i.

Calculate the annual effective interest rate earned by Investment B.

Solution:

The amount quadruples in 17 years so it doubles in 8.5 years.

Using the Rule of 72==> $\frac{72}{i} = 8.5$ ==> $i = \frac{72}{8.5} = 8.47059\%$

For the US Treasury Bill, we need to find its duration.

$$QR = 0.0847059 = \left(\frac{360}{x}\right)\left(\frac{240.16}{10,240.16}\right) \text{==> } x = 99.6742$$

Then we solve for the annual effective interest rate.

$$(10,000)(1+i)^{\frac{99.6742}{365}} = 10,240.16$$

$$1+i = (1.02416)^{\frac{365}{99.6742}} = 1.09079 \text{ ==> } Answer = 0.09079$$

Some students rounded the number of days to 100 and completed the calculation which is reasonable.

13. Billy borrows 10,000 to be repaid with three payments. The first payment is P at the end of two years. The second payment is $2P$ at the end of 4 years. The final payment is $3P$ at the end of five years.

The loan has an annual effective interest rate of 6%.

Determine P .

Solution:

$$(10,000)(1.06)^5 = P(1.06)^3 + 2P(1.06) + 3P$$

$$P = \frac{(10,000)(1.06)^5}{(1.06)^3 + 2(1.06) + 3} = 2120.46$$