

Fourth Set of Homework for Math 05

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Please note: You should fully justify your answers.

1 Translating to algebra

1. Write an algebraic expression for each of the following English phrases. If you introduce variables, state clearly what the stand for.

(a) The product of negative eight and an unknown number.

Answer. $-8x$ where, x stand for the unknown number.

(b) The quotient of x and 3.

Answer. $\frac{x}{3}$, where x stands for the unknown number.

(c) Three fifths of an unknown number.

Answer. $\frac{3}{5}x$, where x stands for the unknown number.

(d) The difference of negative nine and the product of -4 and a . $-9 - (-4a)$

(e) The sum of a number and its square.

Answer. $x + x^2$, where x stands for the unknown number.

(f) Three times a number is subtracted from five.

Answer. $5 - 3x$, where x stands for the unknown number.

(g) Seven less than twice the sum of an unknown number and six.

Answer. $2(x + 6) - 7$, where x stands for the unknown number.

(h) The third power of a number is subtracted from the product of eight and the number.

Answer. $8x - x^3$, where x stands for the unknown number.

(i) Eleven more than the square of the sum of twice a number and three.

Answer. $(2x + 3)^2 + 11$, where x stands for the unknown number.

(j) The difference of the quotient of the sum of twice a number and three and seven and seven times the sum of the number and six.

Answer. $\frac{2x + 3}{7} - 7(x + 6)$, where x stands for the unknown number.

2. Translate the following sentences into Mathematics. If you introduce variables, state clearly what the stand for.

(a) The sum of two consecutive integers is nine.

Answer. $n(n + 1) = 9$, where n stands for the smaller number.

An other solution: $n + (n - 1) = 9$, where n stands for the larger number.

- (b) The product of two consecutive integers is equal to twenty two more than ten times the smaller of the two numbers.

Answer. $n(n + 1) = 10n + 22$, where n stands for the smaller integer.

Another solution: $n(n + 1) = 10n + 22$, where n stands for the larger integer. □

- (c) The width of a rectangle is three more than twice its length.

Answer. $w = 2l + 3$, where w stands for the width and l stands for the length. □

- (d) Three times the sum of a number and six equals the difference of the number and eight.

Answer. $3x + 18 = x - 8$, where x stands for the unknown number. □

- (e) The sum of twice a number and twenty is smaller than three times the number plus seven.

Answer. $2x + 20 < 3x + 7$, where x stands for the unknown number. □

- (f) The difference of two thirds of an unknown number and eleven is greater than the sum of five halves of the number and nine.

Answer. $\frac{2}{3}x - 11 > \frac{5}{2}x + 9$, where x stands for the unknown number. □

- (g) The absolute value of five less than six times a number is equal to twenty three.

Answer. $|6x - 5| = 23$, where x stands for the unknown number. □

2 Solving Linear equation in one unknown

1. Is the given value a solution to the given equation?

(a) $2x - 7 = 3x - 11$; $x = -4$ No

(b) $3x - 4 = 8x - 9$; $x = 1$ Yes

(c) $3x - 4 = 8x - 9$; $x = -1$ No

(d) $3x - 7 = 2 - 7x$; $x = \frac{9}{10}$ Yes

(e) $5x + 10 = \frac{1}{2}$; $x = -\frac{1}{2}$ Yes

(f) $3x - 4 = 7x + -94$; $x = -9$ Yes

(g) $5(2x - 6) = 3x - 11$; $x = 3$ No

(h) $x^2 - x = 6$; $x = -3$ No

(i) $x^3 - 2x^2 = x - 2$; $x = 1$ Yes

(j) $x^3 - 2x^2 = x - 2$; $x = -1$ Yes

(k) $x^3 - 2x^2 = x - 2$; $x = 2$ Yes

2. Solve each of the following linear equations. After solving you should verify your solution by substituting into the equation.

(a) $2x = -8$ $x = -4$

(b) $3x = 5$ $x = \frac{5}{3}$

(c) $-7x = 0$ $x = 0$

(d) $-11x = 66$ $x = -6$

(e) $-x = 3$ $x = -3$

- (f) $-5x = -75$ $x = 15$
- (g) $\frac{3}{2}x = 5$ $x = \frac{10}{3}$
- (h) $\frac{5}{7}x = -\frac{3}{4}$ $x = -\frac{21}{20}$
- (i) $x + 3 = -5$ $x = -8$
- (j) $x - 7 = 11$ $x = 18$
- (k) $x - \frac{2}{3} = -\frac{5}{6}$ $x = -\frac{1}{6}$
- (l) $-5x + 7 = 0$ $x = \frac{7}{5}$
- (m) $11x - 8 = -8$ $x = 0$
- (n) $7x - 8 = 13$ $x = 3$
- (o) $\frac{1}{2}x - 2 = -3$ $x = -2$
- (p) $-8x - 3 = 7$ $x = -\frac{5}{4}$
- (q) $-x + 3 = -\frac{5}{2}$ $x = \frac{11}{2}$
- (r) $\frac{3}{4}x - \frac{2}{3} = -\frac{7}{12}$ $x = \frac{1}{9}$

3. What is the value of the real number a if $x = -2$ is a solution to the following equation:

$$ax = 3x - 4$$

$$a = 5$$

4. Find the real number b if $x = 3$ is a solution to the following equation:

$$x^2 - 7 = bx$$

$$b = \frac{2}{3}$$