# Children Making Sense During Word Problem Solving

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**Abstract**
This is a workshop on word problems in primary mathematics. The workshop is based on a paper derived from an investigation into children' responses to standard and non-standard mathematics word problems before and after an intervention programme. Standard word problems can be solved by identifying the correct operation and performing the necessary computation. The story context does not affect the solution. In solving non-standard word problems the story context is important in obtaining a correct solution. Primary Three children in five Singapore schools participated in a year-long intervention where their teachers used several lessons that included non-standard problems. The children were asked to solve standard and non-standard word problems at the start and at the end of the school year. Among these word problems, there were those that were similar to, those that were similar in the mathematical structure to but different in the superficial features from, and those that were different in mathematical structure from the problems in the intervention programme. The responses from four intact classes were selected for analysis. It was found that the children were able to make sense of their computation results. However, in situations that went beyond computation, many children were not able to make sense. Intervention and use of concrete materials were found to encourage sense-making.

**Introduction**

*There are 26 sheep and 10 goats on a ship. How old is the captain?*

cited in Verschaffel, Greer & De Corte, 2000

Word problems are used extensively in mathematics classroom in Singapore and elsewhere. Local studies (Yeap & Kaur, 2001a, 2001b; Yeap, Ferrucci & Carter, 2002) have found that textbooks used in Singapore schools contain a narrow range of typical word problems. Research studies elsewhere (cited in Verschaffel, Greer & De Corte, 2000) have indicated that children suspended their ability to make sense when they solved mathematics word problems. Kilpatrick and his colleagues (Kilpatrick & Swafford, 2002) included sense-making as one of the five components of mathematical proficiency. According to them, "engaging oneself with mathematics requires frequent opportunities to make sense of it [and] to experience the rewards of making sense of it" (Kilpatrick & Swafford, 2002, p.16). Verscaffel, Greer and De Corte (2000) argued that there are many advantages in using word problems to help children engage in mathematical modelling. Word problems which require children to consider problem context can be used to engage them in mathematical modelling. In this paper, sense-making is defined to be the ability to consider problem context during the problem-solving process.

Verscaffel, Greer and De Corte (2000) conducted an extensive review of the literature and located studies that experimented with different ways to engage children in sense-making. Specifically, the collection of studies from different cultural settings studied the effects of providing prompts, the effects of referring a test by names such as puzzle test rather than mathematics test, effects of showing children that realistic responses were legitimate responses (including "I can't do this problem") to a non-standard problems, and effects of effects of intervention. They concluded that the effects were not that significant. The researchers proceeded to suggest increasing the authenticity of the word problems.

**Research Problem & Methodology**
The present study aimed to investigate children' responses to standard and non-standard mathematics word problems before and after an intervention programme. Standard word problems can be solved by identifying the correct operation and performing the necessary computation. The story context does not affect the solution. In solving non-standard word problems the story context is important in obtaining a correct solution.

Primary Three children in five Singapore schools participated in a year-long intervention where their teachers used several lessons that included non-standard problems. Twelve lessons were provided to the teachers. Other than brief teaching notes and a briefing, teachers did not undergo any form of training. Teachers were invited to use the lessons in any way they deemed fit. Some teachers used all the lessons while others did not. Some teachers used the lessons as they were while others wrote other lessons using the lessons as a model.

The children were asked to solve standard and non-standard word problems at the start and at the end of the school year. Among these word problems, there were those that were similar to, those that were similar in the mathematical structure to but different in the superficial features from, and those that were different in mathematical structure from the problems in the intervention programme. Children in a control school solved the same word problems at the end of the school year. The word problems were presented in the paper-and-pencil format and the children responses were in written form. There were ten problems in the test taken at the start of the school year (referred to as Test One) and eleven problems in the test taken at the end of the school year (referred to as Test Two).

In this paper, children’s responses to four related problems were analysed to give insights into the extent of the problem of suspension of sense-making amongst children in Singapore and the effects of an intervention programme. The responses from four intact classes were selected for initial analysis. The initial analysis aimed to help the research team to develop and refine a coding scheme to be used on responses of the entire data set. Two classes went through the intervention programme while the two classes from the control school did not.

The children in these classes did not have much difficulty with computations. Table 1 provides evidence for this. The children had not been taught division-with-remainder computation when Test One was taken. Even then the success rate for division computation items was 2 in 3. In this paper, we investigate children’s ability to engage in sense-making given that they were competent in their computation.

Table 1
Success rate in division computation items

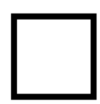
| Group | Test One | | Test Two | |
|---|---|---|---|---|
| | Correct Computation | Incorrect Computation | Correct Computation | Incorrect Computation |
| Control | - | - | 140 (96%) | 6 |
| Intervention | 77 (65%) | 41 | 110 (93%) | 8 |
| Total | 77 (65%) | 41 | 250 (95%) | 14 |

Question 1

36 ? 4
17 ? 3

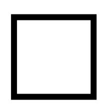
Question 2

Azlan uses 4 sticks to make a square.
How many such squares, at most, can he make using 13 sticks?
Show your method.



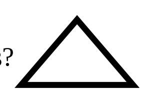
Question 3

Betty uses 4 sticks to make a square.
How many such squares, at most, can she make using 12 sticks?
Show your method.



Question 4

Chelvi uses 3 sticks to make a triangle.
How many such triangles, at most, can she make using 11 sticks?
Show your method.



Chelvi says she can make 5 triangles. Is this possible? Explain.

Figure 1 Tasks done at the start of the school year (Test One)

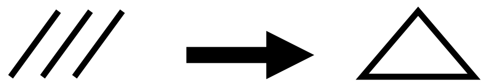
Question 1

27 ? 3

29 ? 4

Question 2

Ailing uses 3 sticks to make a triangle.

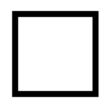


At most, how many such triangles can she make using 16 sticks?
Show your method.

Question 3

Like Ailing, Bala uses 3 sticks to make a triangle.
At most, how many such triangles can he make using 16 sticks?
Show your method.

Question 4



Chris uses 4 sticks to make a square.
At most, how many such squares can she make using 16 sticks?
Show your method.

Chris says she can make 5 such squares using 16 sticks. Is this possible?
Explain.

Figure 2 Tasks done at the end of the school year (Test Two)

**Making Sense During Word Problem Solving**

In solving the word problems shown in Figures 1 and 2, children had opportunities to make sense, or consider problem context, in three different ways. Firstly, they were required to deal with the remaining sticks. Secondly, they were required to consider the possibility of making triangles and squares that share sides. Thirdly, they were given situations where breaking the sticks was possible.

The children were very competent in making sense of the remainder, even prior to the intervention. Only 5 children out of 132 demonstrated some degree of inability to handle the remainder. Only one child demonstrated consistent inability to handle the remainder. This child used the computation results as the solution. Thus, this child gave solutions such as 3 remainder 1 squares. The other four demonstrated lapses in this ability. Two of them rounded up their computation answers. Another child gave the solution in mixed numbers. One of them used the remainder as the solution. All others were able to state that there were *a* squares or triangles when the computation result is *a* remainder *b*. Each of these five cases was observed in the Test One. By the Test Two, no child was suspending their ability to make sense in this manner anymore.

Some children were not able to make sense of a situation where the shapes might have shared sides. These children used the number of sticks and divided that by 3 (for triangles) or 4 (for squares). Even when prompted to create a situation where some shapes might have shared sides, these children continued to respond that it was not possible for 16 sticks to form 5 squares because there were insufficient sticks.

Table 2 shows the results for children in two classes in the control schools. These children did not experience any intervention programme. Nearly 40% of the children in these two classes did not think of the situation even when prompted. They treated the problem as a division problem.

Table 2
Responses from two classes in the control school

| Class | Number of children who did not consider the situation where shapes share sticks | Number of children who considered the situation where shapes shared sticks at least once |
|---|---|---|
| 6301 | 12 | 27 |
| 6302 | 16 | 18 |
| Total | 28 (38%) | 45 (62%) |

Table 3 shows the results for children in two classes in the intervention schools. These children had done a lesson that required them to form figures using sticks and to find the perimeter. The activity sheet for the lesson is included in Figure 3. In one of these classes (3305), children were given sticks to use during the Test One. Nine out of ten of the children were already able to make sense of the situation where shapes share sticks during the Test One. In the Test Two, the sticks were not available for use during the test. The proportion of children who were able to make sense of the situation where shapes share sticks was maintained (8 out of 10).

Table 3
Responses from two classes in two intervention schools

| Class | Number of children who did not consider the situation where shapes share sticks | | Number of children who considered the situation where shapes shared sticks at least once | |
|---|---|---|---|---|
| | Test One | Test Two | Test One | Test Two |
| 1309 | 18 (81%) | 3 (14%) | 4 (19%) | 19 (86%) |
| 3305 | 4 (11%) | 8 (22%) | 33 (89%) | 29 (78%) |

The situation was different when the children did not have sticks to use during the Test One. In one class (1309) where sticks were not available during the Test One, only about 20% of the children considered the situation where shapes shared sticks at least once. After the intervention, the proportion increased to about 90%.

Triangles

Use 3 sticks to make a triangle.
A triangle this big is called a unit triangle.

Draw the unit triangle.

What is the perimeter? ______ units

Asrina says that a shape that is made up of 2 triangles has a perimeter of 6 units. Is she correct?

Benny says that a shape that is made up of 4 unit triangles has a perimeter of 12 units

1 triangle ✍ 3 units
4 triangles ✍ 4 x 3 = 12 units

Why is he not correct?

Use 9 sticks to make a shape that has 4 unit triangles.

Do it in three different ways.

What is the perimeter of each shape?
Which sticks do you count?
Which sticks do you not count?

Can you think of a way to use 6 sticks to make 4 unit triangles?

Figure 3 One intervention activity

The lack of sense-making when solving division-with-remainder word problems that were observed in many previous studies (e.g., Cai & Silver, 1995) was not seen in the present study. Children in the present study were already able to make sense of the remainder even before, or without, the intervention.

More children were not able to consider situations where the shapes might have shared sides. Children's engagement with the situation increased when (a) they had concrete materials to use during problem solving, (b) they were prompted, and (c) they had the previous experience with the situation during the intervention programme.

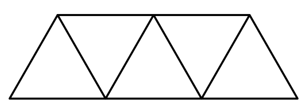
**Strategies Used and Sense-Making**

Two main strategies were used by the children in the two intervention classes. They were drawing a diagram and performing a computation. It was observed that children who used the computation strategy exclusively tended not to engage in sense-making. Among those who used diagrams, there were two categories. In one category, the children were able to consider situations where shapes shared sides. In another, the children continued not to consider that situation as possible.

It was also observed that when children were given sticks to act the situation out during the test, most of them used drawing rather than computation to obtain an answer.

A few children, especially in Test One, felt compelled to use computation to justify their correct solution which was based on drawings. Figure 4 shows one such example. After obtaining a possible solution by drawing a diagram, the child felt compelled to justify his solution by doing a computation. The child showed that there were two types of triangles – the first one which requires three sticks and the others which require two additional sticks each. The division sentence was to find the number of second type of triangle. The addition sentence was to find the total number of triangles of both types.

How many such triangles, at most, can she make using 11 sticks?
Show your method.



11 – 3 = 8

8 ? 2 = 4

1 + 4 = 5

Figure 4 A child justify his drawing using computation

It would be interesting to explore the relationship between the strategies used and extent of sense-making. It is also interesting to investigate what children perceived as legitimate methods in the mathematics classroom.

**Conclusion**

In the present investigation, children's ability to make sense during word problem solving increased after the intervention. This effect was observed for word problems that had situations similar to those in the intervention programme. Would the effects be the same if the word problems include situations that are different from those in the intervention programme? Would the effects be the same if the word problems and intervention tasks had situations that are similar in mathematical structure but different in surface features? In other words, would the children be able to do near transfer? Would the effects be the same if the word problems had totally unfamiliar situations? In other words, would the children be able to do far transfer?

It was also found that the use of concrete materials increased the ability of children to engage in sense-making. The use of authentic materials increased the authenticity of the problem as well as allowed children to use strategies other than a computational one.

In the present study, only children who had little difficulty with computations were included. The extent to which children who struggle with computation engage with sense-making may be different.

## References

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