10.1 Tangents to Circles

What you should learn

GOAL 1 Identify segments and lines related to circles.

GOAL 2 Use properties of a tangent to a circle.

Why you should learn it

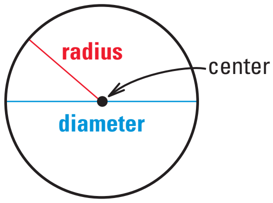
▼ You can use properties of tangents of circles to find **real-life** distances, such as the radius of the silo in **Example 5**.

REAL LIFE

GOAL 1 Communicating About Circles

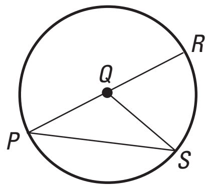
A **circle** is the set of all points in a plane that are equidistant from a given point, called the **center** of the circle. A circle with center P is called "circle P", or $\odot P$.

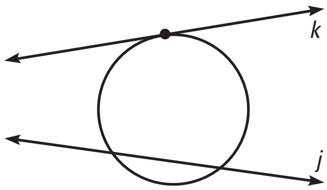
The distance from the center to a point on the circle is the **radius** of the circle. Two circles are **congruent** if they have the same radius.



The distance across the circle, through its center, is the **diameter** of the circle. The diameter is twice the radius.

The terms *radius* and *diameter* describe segments as well as measures. A **radius** is a segment whose endpoints are the center of the circle and a point on the circle. $\overline{QP}$, $\overline{QR}$, and $\overline{QS}$ are radii of $\odot Q$ below. All radii of a circle are congruent.





A **chord** is a segment whose endpoints are points on the circle. $\overline{PS}$ and $\overline{PR}$ are chords.

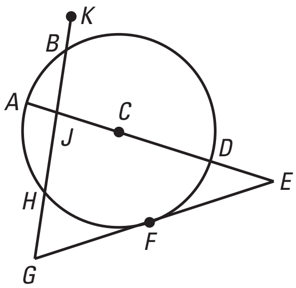
A **diameter** is a chord that passes through the center of the circle. $\overline{PR}$ is a diameter.

A **secant** is a line that intersects a circle in two points. Line j is a secant.

A **tangent** is a line in the plane of a circle that intersects the circle in exactly one point. Line k is a tangent.

EXAMPLE 1 *Identifying Special Segments and Lines*

Tell whether the line or segment is best described as a *chord*, a *secant*, a *tangent*, a *diameter*, or a *radius* of $\odot C$.



a. $\overline{AD}$ **b.** $\overline{CD}$

c. $\overleftrightarrow{EG}$ **d.** $\overline{HB}$

Solution

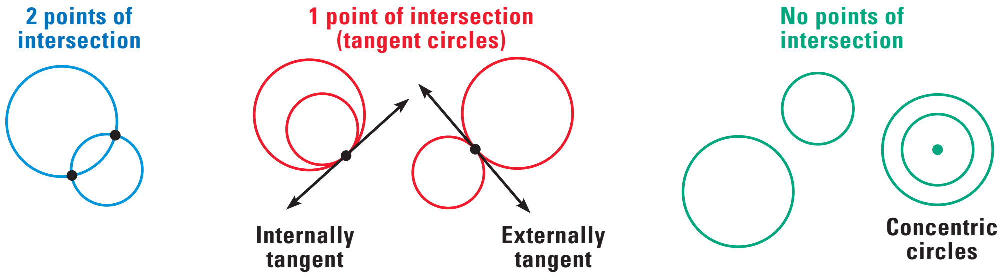
a. $\overline{AD}$ is a diameter because it contains the center C.

b. $\overline{CD}$ is a radius because C is the center and D is a point on the circle.

c. $\overleftrightarrow{EG}$ is a tangent because it intersects the circle in one point.

d. $\overline{HB}$ is a chord because its endpoints are on the circle.

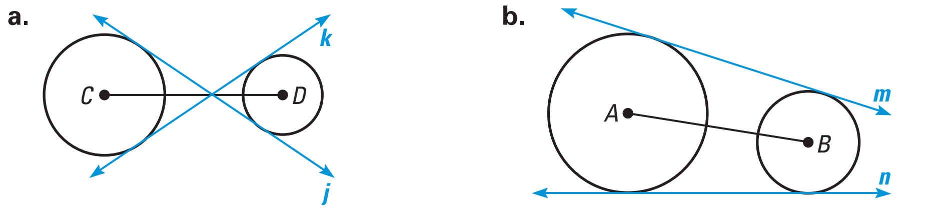
In a plane, two circles can intersect in two points, one point, or no points. Coplanar circles that intersect in one point are called **tangent circles**. Coplanar circles that have a common center are called **concentric**.



A line or segment that is tangent to two coplanar circles is called a **common tangent**. A *common internal tangent* intersects the segment that joins the centers of the two circles. A *common external tangent* does not intersect the segment that joins the centers of the two circles.

EXAMPLE 2 *Identifying Common Tangents*

Tell whether the **common tangents** are *internal* or *external*.



SOLUTION

a. The lines j and k intersect $\overline{CD}$, so they are common internal tangents.

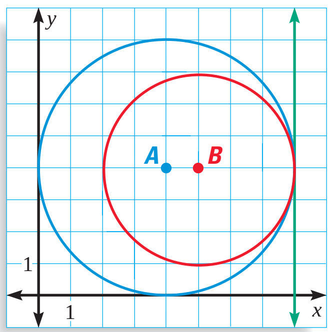
b. The lines m and n do not intersect $\overline{AB}$, so they are common external tangents.

. . . .

In a plane, the **interior of a circle** consists of the points that are inside the circle. The **exterior of a circle** consists of the points that are outside the circle.

EXAMPLE 3 *Circles in Coordinate Geometry*

Give the center and the radius of each circle. Describe the intersection of the two circles and describe all common tangents.



SOLUTION

The center of $\odot A$ is $A(4, 4)$ and its radius is 4. The center of $\odot B$ is $B(5, 4)$ and its radius is 3. The two circles have only one point of intersection. It is the point $(8, 4)$. The vertical line $x = 8$ is the only common tangent of the two circles.

STUDENT HELP

INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

GOAL 2 USING PROPERTIES OF TANGENTS

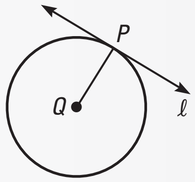
The point at which a tangent line intersects the circle to which it is tangent is the **point of tangency**. You will justify the following theorems in the exercises.

THEOREMS

THEOREM 10.1

If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

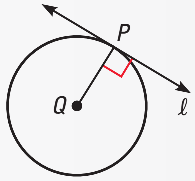
If ℓ is tangent to $\odot Q$ at P, then $\ell \perp \overline{QP}$.



THEOREM 10.2

In a plane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

If $\ell \perp \overline{QP}$ at P, then ℓ is tangent to $\odot Q$.

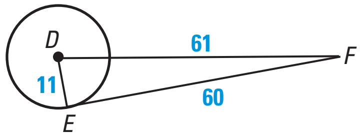


STUDENT HELP

Study Tip
A secant can look like a tangent if it intersects the circle in two points that are close together.

EXAMPLE 4 *Verifying a Tangent to a Circle*

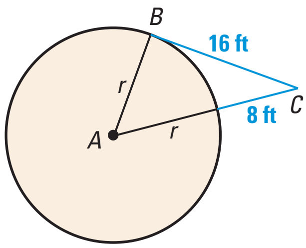
You can use the Converse of the Pythagorean Theorem to tell whether $\overleftrightarrow{EF}$ is tangent to $\odot D$.



Because $11^2 + 60^2 = 61^2$, $\triangle DEF$ is a right triangle and $\overline{DE}$ is perpendicular to $\overline{EF}$. So, by Theorem 10.2, $\overleftrightarrow{EF}$ is tangent to $\odot D$.

EXAMPLE 5 *Finding the Radius of a Circle*

You are standing at C, 8 feet from a grain silo. The distance from you to a point of tangency on the tank is 16 feet. What is the radius of the silo?



SOLUTION

Tangent $\overleftrightarrow{BC}$ is perpendicular to radius $\overline{AB}$ at B, so $\triangle ABC$ is a right triangle. So, you can use the Pythagorean Theorem.

$(r + 8)^2 = r^2 + 16^2$ — Pythagorean Theorem

$r^2 + 16r + 64 = r^2 + 256$ — Square of binomial

$16r + 64 = 256$ — Subtract r^2 from each side.

$16r = 192$ — Subtract 64 from each side.

$r = 12$ — Divide.

▶ The radius of the silo is 12 feet.

STUDENT HELP

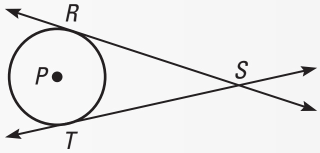
Skills Review
For help squaring a binomial, see p. 798.

From a point in a circle's exterior, you can draw exactly two different tangents to the circle. The following theorem tells you that the segments joining the external point to the two points of tangency are congruent.

THEOREM

THEOREM 10.3

If two segments from the same exterior point are tangent to a circle, then they are congruent.



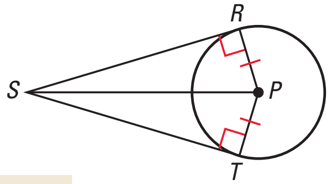
If $\overleftrightarrow{SR}$ and $\overleftrightarrow{ST}$ are tangent to $\odot P$, then $\overline{SR} \cong \overline{ST}$.

EXAMPLE 6 ***Proof of Theorem 10.3***

Proof

GIVEN ▶ $\overleftrightarrow{SR}$ is tangent to $\odot P$ at R.
$\overleftrightarrow{ST}$ is tangent to $\odot P$ at T.

PROVE ▶ $\overline{SR} \cong \overline{ST}$



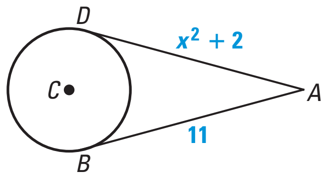
- ***$\overleftrightarrow{SR}$ and $\overleftrightarrow{ST}$ are both tangent to $\odot P$.*** — Given
- ***$\overline{SR} \perp \overline{RP}$, $\overline{ST} \perp \overline{TP}$*** — Tangent and radius are $\perp$.
- ***$RP = TP$*** — Def. of circle
- ***$\overline{RP} \cong \overline{TP}$*** — Def. of congruence
- ***$\overline{PS} \cong \overline{PS}$*** — Reflexive Property
- ***$\triangle PRS \cong \triangle PTS$*** — HL Congruence Theorem
- ***$\overline{SR} \cong \overline{ST}$*** — Corresp. parts of $\cong$ △ are $\cong$.

EXAMPLE 7 ***Using Properties of Tangents***

Using Algebra

$\overleftrightarrow{AB}$ is tangent to $\odot C$ at B.
$\overleftrightarrow{AD}$ is tangent to $\odot C$ at D.

Find the value of x.



SOLUTION

$AB = AD$	**Two tangent segments from the same point are ≅.**
$11 = x^2 + 2$	**Substitute.**
$9 = x^2$	**Subtract 2 from each side.**
$\pm 3 = x$	**Find the square roots of 9.**

▶ The value of x is 3 or -3.

GUIDED PRACTICE

Vocabulary Check ✓

1. Sketch a circle. Then sketch and label a *radius*, a *diameter*, and a *chord*.

2. How are chords and secants of circles alike? How are they different?

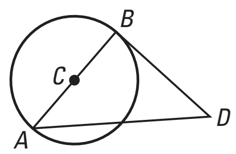
Concept Check ✓

3. $\overleftrightarrow{XY}$ is tangent to $\odot C$ at point P. What is $m\angle CPX$? Explain.

4. The diameter of a circle is 13 cm. What is the radius of the circle?

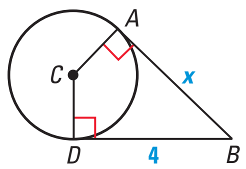
Skill Check ✓

5. In the diagram at the right, $AB = BD = 5$ and $AD = 7$. Is $\overleftrightarrow{BD}$ tangent to $\odot C$? Explain.



$\overleftrightarrow{AB}$ is tangent to $\odot C$ at A and $\overleftrightarrow{DB}$ is tangent to $\odot C$ at D. Find the value of x.

6.



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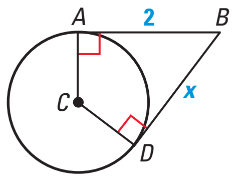

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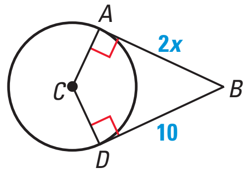


PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 821.

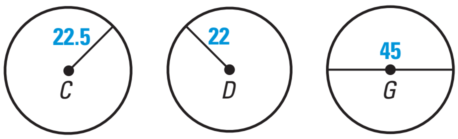
FINDING RADII **The diameter of a circle is given. Find the radius.**

9. $d = 15$ cm **10.** $d = 6.7$ in. **11.** $d = 3$ ft **12.** $d = 8$ cm

FINDING DIAMETERS **The radius of $\odot C$ is given. Find the diameter of $\odot C$.**

13. $r = 26$ in. **14.** $r = 62$ ft **15.** $r = 8.7$ in. **16.** $r = 4.4$ cm

17. CONGRUENT CIRCLES Which two circles below are congruent? Explain your reasoning.



MATCHING TERMS **Match the notation with the term that best describes it.**

18. $\overline{AB}$ **A.** Center

19. H **B.** Chord

20. $\overleftrightarrow{HF}$ **C.** Diameter

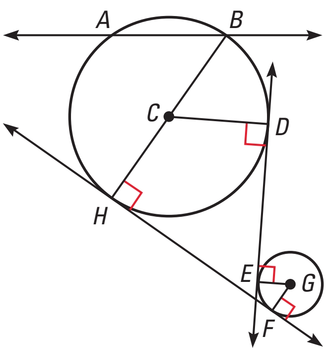
21. $\overline{CH}$ **D.** Radius

22. C **E.** Point of tangency

23. $\overline{HB}$ **F.** Common external tangent

24. $\overleftrightarrow{AB}$ **G.** Common internal tangent

25. $\overleftrightarrow{DE}$ **H.** Secant



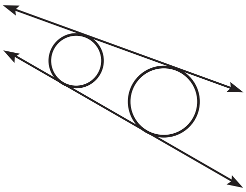
STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 18–25, 42–45
Example 2: Exs. 26–31
Example 3: Exs. 32–35
Example 4: Exs. 36–39
Example 5: Exs. 40, 41
Example 6: Exs. 49–53
Example 7: Exs. 46–48

IDENTIFYING TANGENTS **Tell whether the common tangent(s) are *internal* or *external*.**

26.



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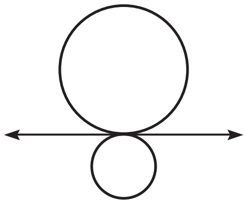

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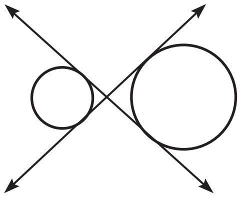
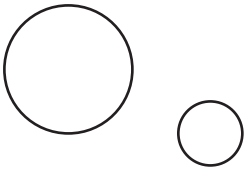


DRAWING TANGENTS **Copy the diagram. Tell how many common tangents the circles have. Then sketch the tangents.**

29.



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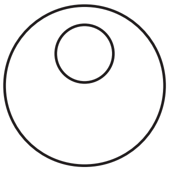

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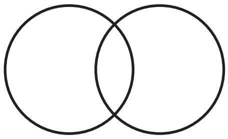
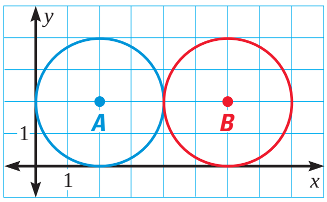


COORDINATE GEOMETRY **Use the diagram at the right.**



32. What are the center and radius of $\odot A$?

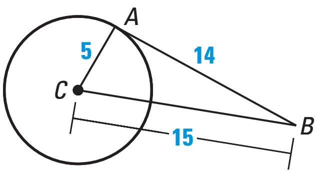
33. What are the center and radius of $\odot B$?

34. Describe the intersection of the two circles.

35. Describe all the common tangents of the two circles.

DETERMINING TANGENCY **Tell whether $\overleftrightarrow{AB}$ is tangent to $\odot C$. Explain your reasoning.**

36.



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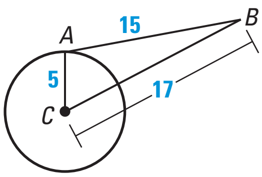

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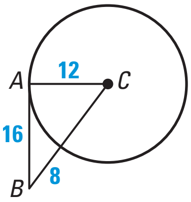

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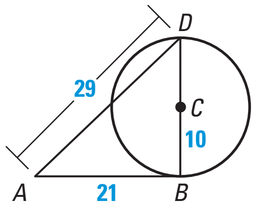
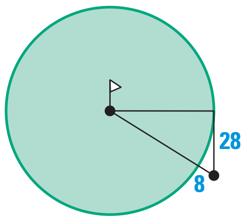


FOCUS ON PEOPLE

REAL LIFE **TIGER WOODS** At age 15 Tiger Woods became the youngest golfer ever to win the U.S. Junior Amateur Championship, and at age 21 he became the youngest Masters champion ever.

GOLF **In Exercises 40 and 41, use the following information.**
A green on a golf course is in the shape of a circle. A golf ball is 8 feet from the edge of the green and 28 feet from a point of tangency on the green, as shown at the right. Assume that the green is flat.

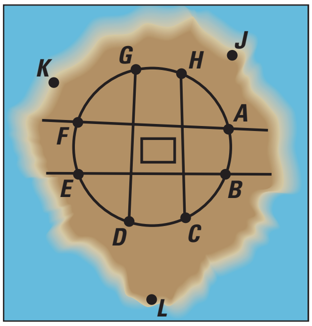


40. What is the radius of the green?

41. How far is the golf ball from the cup at the center?

Mexcaltitlán Island, Mexico

MEXCALTITLÁN **The diagram shows the layout of the streets on Mexcaltitlán Island.**



42. Name two secants.

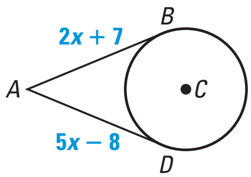
43. Name two chords.

44. Is the diameter of the circle greater than HC? Explain.

45. If $\triangle LJK$ were drawn, one of its sides would be tangent to the circle. Which side is it?

USING ALGEBRA **$\overleftrightarrow{AB}$ and $\overleftrightarrow{AD}$ are tangent to $\odot C$. Find the value of x.**

46.



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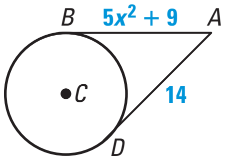

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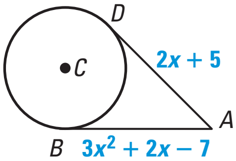
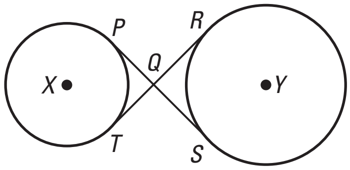


49. **PROOF** Write a proof.

GIVEN ▶ $\overleftrightarrow{PS}$ is tangent to $\odot X$ at P.
$\overleftrightarrow{PS}$ is tangent to $\odot Y$ at S.
$\overleftrightarrow{RT}$ is tangent to $\odot X$ at T.
$\overleftrightarrow{RT}$ is tangent to $\odot Y$ at R.

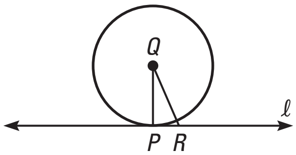
PROVE ▶ $\overline{PS} \cong \overline{RT}$



PROVING THEOREM 10.1 **In Exercises 50–52, you will use an indirect argument to prove Theorem 10.1.**

GIVEN ▶ ℓ is tangent to $\odot Q$ at P.

PROVE ▶ $\ell \perp \overline{QP}$



50. Assume ℓ and $\overline{QP}$ are not perpendicular. Then the perpendicular segment from Q to ℓ intersects ℓ at some other point R. Because ℓ is a tangent, R cannot be in the interior of $\odot Q$. So, how does QR compare to QP? Write an inequality.

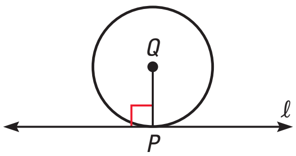
51. $\overline{QR}$ is the perpendicular segment from Q to ℓ, so $\overline{QR}$ is the shortest segment from Q to ℓ. Write another inequality comparing QR to QP.

52. Use your results from Exercises 50 and 51 to complete the indirect proof of Theorem 10.1.

53. **PROVING THEOREM 10.2** Write an indirect proof of Theorem 10.2. (*Hint:* The proof is like the one in Exercises 50–52.)

GIVEN ▶ ℓ is in the plane of $\odot Q$.
$\ell \perp$ radius $\overline{QP}$ at P.

PROVE ▶ ℓ is tangent to $\odot Q$.



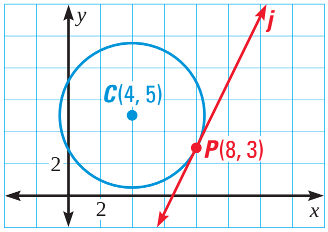
LOGICAL REASONING **In $\odot C$, radii $\overline{CA}$ and $\overline{CB}$ are perpendicular. $\overleftrightarrow{BD}$ and $\overleftrightarrow{AD}$ are tangent to $\odot C$.**

54. Sketch $\odot C$, $\overline{CA}$, $\overline{CB}$, $\overline{BD}$, and $\overline{AD}$.

55. What type of quadrilateral is *CADB*? Explain.

Test Preparation

56. MULTI-STEP PROBLEM In the diagram, line j is tangent to $\odot C$ at P.



a. What is the slope of radius $\overline{CP}$?

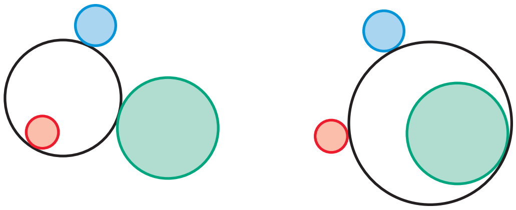
b. What is the slope of j? Explain.

c. Write an equation for j.

d. *Writing* Explain how to find an equation for a line tangent to $\odot C$ at a point other than P.

★ Challenge

57. CIRCLES OF APOLLONIUS The Greek mathematician Apollonius (c. 200 B.C.) proved that for any three circles with no common points or common interiors, there are eight ways to draw a circle that is tangent to the given three circles. The red, blue, and green circles are given. Two ways to draw a circle that is tangent to the given three circles are shown below. Sketch the other six ways.



EXTRA CHALLENGE
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MIXED REVIEW

58. TRIANGLE INEQUALITIES The lengths of two sides of a triangle are 4 and 10. Use an inequality to describe the length of the third side. **(Review 5.5)**

PARALLELOGRAMS **Show that the vertices represent the vertices of a parallelogram. Use a different method for each proof.** **(Review 6.3)**

59. $P(5, 0)$, $Q(2, 9)$, $R(-6, 6)$, $S(-3, -3)$

60. $P(4, 3)$, $Q(6, -8)$, $R(10, -3)$, $S(8, 8)$

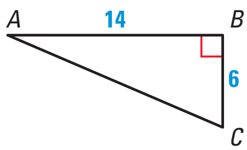
SOLVING PROPORTIONS **Solve the proportion.** **(Review 8.1)**

61. $\frac{x}{11} = \frac{3}{5}$ **62.** $\frac{x}{6} = \frac{9}{2}$ **63.** $\frac{x}{7} = \frac{12}{3}$ **64.** $\frac{33}{x} = \frac{18}{42}$

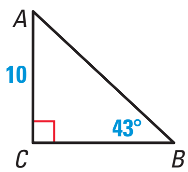
65. $\frac{10}{3} = \frac{8}{x}$ **66.** $\frac{3}{x+2} = \frac{4}{x}$ **67.** $\frac{2}{x-3} = \frac{3}{x}$ **68.** $\frac{5}{x-1} = \frac{9}{2x}$

SOLVING TRIANGLES **Solve the right triangle. Round decimals to the nearest tenth.** **(Review 9.6)**

69.



70.



71.