## Grade 12 Pre-Calculus Mathematics (40S)

Final Practice Exam

## Grade 12 Pre-Calculus Mathematics

## Final Practice Exam

Name:

Student Number:

Attending $\square$ Non-Attending

Phone Number: $\qquad$

Address: $\qquad$
$\qquad$
$\qquad$

## Instructions

The final examination will be weighted as follows:
Modules 1-8
100\%
Time allowed: 3 hours
Note: You are allowed to bring the following to the exam: pens/pencils (2 or 3 of each), blank paper, a ruler, a scientific calculator, and your Final Exam Resource Sheet. Your Final Exam Resource Sheet must be handed in with the exam.

Show all calculations and formulas used. Use all decimal places in your calculations and round the final answers to the correct number of decimal places. Include units where appropriate. Clearly state your final answer.

## General Marking Principles

- Concepts learned in Grade 12 are worth 1 mark each. Concepts learned earlier (unless they are retaught under the curriculum, e.g., absolute value, reciprocals) are worth 0.5 mark each.
■ Some errors are deducted only once (e.g., not putting arrow heads on graphs).
- Errors are followed through (e.g., if an arithmetic error is made in the first line, it is still possible for the student to receive nearly full marks).
■ Many types of communication errors receive a 0.5 mark deduction, but 0.5 mark is the maximum communication error deduction for the entire exam.

Name: $\qquad$

Answer all questions to the best of your ability. Show all your work.
Long-Answer Questions (100 marks)

1. Given that $f(x)=\left|\frac{1}{x}\right|$ and $g(x)=x^{3}+6 x-3$, find the following.
$(2 \times 1$ mark each $=2$ marks $)$
a) $f(f(x))$
b) $g(f(-2))$
2. Divide, using long division or synthetic division, and write in the form given by the division algorithm. (3 marks)

$$
\left(-x^{3}-4 x^{2}+7 x+4\right) \div(x+3)
$$

3. Rewrite $2 \log x+\frac{1}{2} \log 5-\frac{1}{3} \log (x+2)$ as a single logarithm statement. (3 marks)

Name: $\qquad$
4. Given the sketch of $f(x)$ drawn below, sketch the following functions.

a) $\begin{aligned} & y=f(x-2) \quad(1 \text { mark })\end{aligned}$

|  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |

b) $y=|f(x)|+2 \quad$ (1 mark)

|  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |

Name: $\qquad$
5. Using the sketch of $f(x)$, sketch the following. Express the transformation algebraically or in words.

a) $y=-\frac{1}{2} f(x)$
(2 marks)

|  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |

b) $y=f(-2 x) \quad$ (2 marks)

|  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |

c) $y=f^{-1}(x) \quad$ (2 marks)

|  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |

Name: $\qquad$
6. Write and simplify the fifth term of the expansion of $(x+1)^{8}$. $(2$ marks $)$
7. There are 9 boys and 11 girls in a Grade 12 English class. In how many ways can 5 students be chosen for a group project if the group must have 3 female members and 2 male members? ( 2 marks)
8. Convert $1265^{\circ}$ to radians. Write the exact answer. (1 mark)
9. You know that $\sin \alpha=-\frac{2}{7}$ and $\pi<\alpha<\frac{3 \pi}{2}$. You also know that $\mathrm{P}(\beta)$ is in Quadrant IV and $\cos \beta=\frac{4}{5}$. Find $\sin (\alpha+\beta) \cdot(3$ marks $)$

Name: $\qquad$
10. Consider the function $f(x)=-(x-1)(x+3)(x-7)$.
a) Determine the end behaviour of the function. (1 mark)
b) Find all $x$ - and $y$-intercept(s). (2 marks)
c) Sketch the function. (2 marks)

11. Sketch the function $g(x)=9-3^{x}$ and state its range, $x$-intercept, and equation of asymptote. (4 marks)

|  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

12. Sketch the function $f(x)=\log _{3} x-2$. (1 mark)


Name: $\qquad$
13. Graph the following function using transformations. State the domain and range of the function. (3 marks)

$$
g(x)=2 \sqrt{x}+4
$$


14. Given the following graph of $f(x)$, graph $\sqrt{f(x)}$. (2 marks)



Name: $\qquad$
15. Solve the following radical equation for $x$ algebraically. Check your solution for extraneous roots. (2 marks)

$$
0=\frac{1}{2} \sqrt{(x+2)}-1
$$

16. Graph the following function. Pay attention to whether the graph should have a point of discontinuity or a vertical asymptote. (3 marks)

$$
y=\frac{4 x}{x-1}
$$

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Name: $\qquad$
17. Graph the following function. Pay attention to whether the graph should have a point of discontinuity or a vertical asymptote. (5 marks)

$$
y=\frac{x+1}{x^{2}-4 x-5}
$$

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

18. Solve for the variable in the equation $\log _{\sqrt{2}} 64=x$. (2 marks)
19. Solve the exponential equation. Round the final answer to the nearest thousandth. (3 marks)
$e^{3 x+2}=5^{x+1}$

Name: $\qquad$
20. Solve the following equations. Your answer should be exact, whenever possible. Otherwise, round to two decimal places.
a) $5\left(3^{x}\right)=e^{x-1}$
(3 marks)
b) $\log _{2}(x-4)+\log _{2}(x-3)=1$
(3 marks)
21. A bacteria culture is growing according to the formula $y=1000 e^{0.6 t}$, where $t$ is the time in days.
a) Determine the number of bacteria after 7 days. (1 mark)
b) How long will it take the bacteria culture to triple in size? (3 marks)
22. Determine all of the angles that are coterminal with $\frac{2 \pi}{3}$ over the domain $[-2 \pi, 4 \pi]$. (2 marks)

Name: $\qquad$
23. Determine all of the angles that are coterminal with $-261^{\circ}$ in general form. ( 1 mark)
24. Determine the exact value of each of the following expressions. State the coterminal values, where necessary. Show all work. ( $2 \times 2$ marks each $=4$ marks $)$
a) $\sec \left(-\frac{2 \pi}{3}\right)$
b) $\cot \left(630^{\circ}\right)$
25. State the amplitude, phase shift, period, domain, range, and $y$-intercept of the function. Sketch the following curve. (6 marks)

$$
y=-\frac{1}{2} \sin \left(\frac{\pi}{2} x\right)-1
$$



Name: $\qquad$

26 Find the equation of the following graph as a cosine function. (2 marks)

27. Solve the following equations over the indicated intervals. Provide exact answers wherever possible. Write answers as exact values.
a) $2 \sin ^{2} \theta+3 \cos \theta=3$, where $-2 \pi \leq \theta \leq 0 \quad$ ( 5 marks)
b) $6 \sin ^{2} x+5 \sin x+1=0$, where $0 \leq x<2 \pi$. ( 4 marks)

Name: $\qquad$

28 Explain the difference between a trigonometric identity and a trigonometric equation. (1 mark)

29 Consider the equation $\sin (2 x)=-\cos \left(2\left(x+\frac{\pi}{4}\right)\right)$.
a) Graph $y=\sin (2 x)$ and $y=-\cos \left(2\left(x+\frac{\pi}{4}\right)\right)$ on the coordinate grids below. (4 marks)

b) Using the graph you created in (b), do you believe this demonstrates an identity? (1 mark)
30. Prove the identity $\frac{2 \sin x}{\sin 2 x}=\sec x$. (2 marks)

31 Prove the identity $\cos 2 \theta+2 \sin ^{2} \theta=1$. (2 marks)
32. Find the exact value of $\frac{\tan 80^{\circ}+\tan 55^{\circ}}{1-\tan 80^{\circ} \cdot \tan 55^{\circ}}$.

