Formulas from Trigonometry:

$\sin^2 A + \cos^2 A = 1$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

$\sin \frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$

$\cos \frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$

$\tan \frac{A}{2} = \frac{\sin A}{1+\cos A}$

$\sin^2 A = \frac{1}{2} - \frac{1}{2}\cos 2A$

$\cos^2 A = \frac{1}{2} + \frac{1}{2}\cos 2A$

$\sin A + \sin B = 2 \sin \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)$

$\sin A - \sin B = 2 \cos \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B)$

$\cos A + \cos B = 2 \cos \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)$

$\cos A - \cos B = 2 \sin \frac{1}{2}(A+B) \sin \frac{1}{2}(B-A)$

$\sin A \sin B = \frac{1}{2}\{\cos(A-B) - \cos(A+B)\}$

$\cos A \cos B = \frac{1}{2}\{\cos(A-B) + \cos(A+B)\}$

$\sin A \cos B = \frac{1}{2}\{\sin(A-B) + \sin(A+B)\}$

$\cos(\theta) = \sin(\theta + \pi/2)$

Differentiation Formulas:

$\frac{d}{dx}(uv) = u\frac{dv}{dx} + \frac{du}{dx}v$

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v(du/dx) - u(dv/dx)}{v^2}$

Chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

$\frac{d}{dx}\sin u = \cos u \frac{du}{dx}$

$\frac{d}{dx}\cos u = -\sin u \frac{du}{dx}$

$\frac{d}{dx}\tan u = \sec^2 u \frac{du}{dx}$

$\frac{d}{dx}\sin^{-1} u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx},\ \left(-\frac{\pi}{2} < \sin^{-1} u < \frac{\pi}{2}\right)$

$\frac{d}{dx}\cos^{-1} u = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx},\ (0 < \cos^{-1} u < \pi)$

$\frac{d}{dx}\tan^{-1} u = \frac{1}{1+u^2}\frac{du}{dx},\ \left(-\frac{\pi}{2} < \tan^{-1} u < \frac{\pi}{2}\right)$

$\frac{d}{dx}e^u = e^u \frac{du}{dx}$

$\frac{d}{dx}\ln u = \frac{1}{u}\frac{du}{dx}$

$\frac{d}{dx}\log_a u = \frac{\log_a e}{u}\frac{du}{dx},\ a \neq 0, 1$

Integration Formulas:

Integration by parts: $\int u\,dv = uv - \int v\,du$

$\int \frac{du}{u} = \ln|u|$

$\int e^u\,du = e^u$

$\int a^u\,du = \frac{a^u}{\ln a}, \quad a > 0,\ a \neq 1$

$\int \sin u\,du = -\cos u$

$\int \cos u\,du = \sin u$

$\int \tan u\,du = -\ln \cos u$

$\int \sin^2 u\,du = \frac{u}{2} - \frac{\sin 2u}{4} = \frac{1}{2}(u - \sin u \cos u)$

$\int \tan^2 u\,du = \tan u - u$

$\int \cos^2 u\,du = \frac{u}{2} + \frac{\sin 2u}{4} = \frac{1}{2}(u + \sin u \cos u)$

$\int \frac{du}{u^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{u}{a}$

$\int \frac{du}{u^2 - a^2} = \frac{1}{2a}\ln\left(\frac{u-a}{u+a}\right)$

$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\frac{u}{a}$

$\int \frac{du}{\sqrt{u^2 + a^2}} = \ln(u + \sqrt{u^2 + a^2})$

$\int \frac{du}{\sqrt{u^2 - a^2}} = \ln(u + \sqrt{u^2 - a^2})$

$\int e^{ax}\sin bx\,dx = \frac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2}$

$\int e^{ax}\cos bx\,dx = \frac{e^{ax}(a \cos bx + b \sin bx)}{a^2 + b^2}$

$\int x \sin ax\,dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a}$

$\int x^2 \sin ax\,dx = \frac{2x}{a^2}\sin ax + \left(\frac{2}{a^3} - \frac{x^2}{a}\right)\cos ax$

$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$

$\int x \cos ax\,dx = \frac{\cos ax}{a^2} + \frac{x \sin ax}{a}$

$\int x^2 \cos ax\,dx = \frac{2x}{a^2}\cos ax + \left(\frac{x^2}{a} - \frac{2}{a^3}\right)\sin ax$

$\int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$

$\int \tan^2 ax\,dx = \frac{\tan ax}{a} - x$

$\int x e^{ax}\,dx = \frac{e^{ax}}{a}\left(x - \frac{1}{a}\right)$

$\int \ln x\,dx = x \ln x - x$

$\int x \ln x\,dx = \frac{x^2}{2}\left(\ln x - \frac{1}{2}\right)$

Rules for Exponents:

$$a^{b+c} = a^b a^c \qquad (ab)^c = a^c b^c$$

$$(a^b)^c = a^{bc} \qquad a^{-b} = \left(\frac{1}{a}\right)^b = \frac{1}{a^b}$$

Taylor Series:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\sin x = 1 - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

Euler's Formula:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \qquad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Rectangular and Polar Form of a Complex Number:

$$z = a + jb = re^{j\theta}$$

$$r = |z| = \sqrt{a^2 + b^2} \qquad a = \text{Re}\{z\} = r\cos\theta$$

$$r^2 = zz^* \qquad a = \text{Re}\{z\} = \frac{z + z^*}{2}$$

$$\theta = \arctan\frac{b}{a} \qquad b = \text{Im}\{z\} = r\sin\theta$$

$$b = \text{Im}\{z\} = \frac{z - z^*}{2j}$$

Phasors:

Complex Signal: $z(t) = Ae^{j(\omega_0 t + \phi)} = Ae^{j\phi}e^{j\omega_0 t}$
Real Signal: $x(t) = \text{Re}\{z(t)\} = A\cos(\omega_0 t + \phi)$
Phasor Representation: $X = Ae^{j\phi}$

Phasor Addition:
Let $x_1(t) = A_1\cos(w_0 t + \phi_1)$, $x_2(t) = A_2\cos(w_0 t + \phi_2)$, and $x(t) = x_1(t) + x_2(t)$.
Then $x(t) = A\cos(w_0 t + \phi)$ and:
the phasor representation for $x(t)$ is $X = Ae^{j\phi} = A_1 e^{j\phi_1} + A_2 e^{j\phi_2}$.

Continuous-Time Unit Impulse and Unit Step:

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1 \qquad \int_{-\infty}^{\infty} x(t)\delta(t)\,dt = x(0) \qquad \int_{-\infty}^{\infty} x(t)\delta(t - t_0)\,dt = x(t_0)$$

$$u(t) = \int_{-\infty}^{t} \delta(t)\,dt$$

Discrete-Time Unit Impulse and Unit Step:

$$\delta[n] = \begin{cases} 1, & n = 0, \\ 0, & \text{otherwise.} \end{cases} \qquad u[n] = \begin{cases} 1, & n \geq 0, \\ 0, & n < 0. \end{cases}$$

Complex Exponential Signals:

$e^{j\omega_0 t}$	$e^{j\omega_0 n}$
Distinct signals for distinct w_0	Identical signals for values of w_0 separated by multiples of 2π
Periodic for any choice of w_0	Periodic only if $w_0/(2\pi) = m/N \in \mathbb{Q}$
Fundamental frequency w_0	Fundamental frequency w_0/m
Fundamental period: $w_0 = 0$: undefined $w_0 \neq 0$: $2\pi/w_0$	Fundamental period: $w_0 = 0$: one $w_0 \neq 0$: $N = 2\pi m/w_0$

Periodicity of Discrete-Time Sinusoids:

$\cos(\omega_0 n)$, $\sin(\omega_0 n)$, and $e^{j\omega_0 n}$ are periodic if and only if $\dfrac{w_0}{2\pi}$ is a ratio of two integers.

If periodic, then write in reduced form: $\dfrac{w_0}{2\pi} = \dfrac{m}{N}$ (no common factors between m and N)

N: Fundamental Period

m: In each period of the discrete-time signal, the graph "goes around" m times.

Summation Formulas:

$$\sum_{k=N_1}^{N_2} \alpha^k = \frac{\alpha^{N_1} - \alpha^{N_2+1}}{1-\alpha}, \quad \alpha \neq 1 \qquad \sum_{k=0}^{\infty} a^k = \frac{1}{1-a}, \quad |a| < 1$$

$$\sum_{k=0}^{\infty} k a^k = \frac{a}{(1-a)^2}, \quad |a| < 1 \qquad \sum_{k=0}^{n} a^k = \frac{1-a^{n+1}}{1-a}, \quad a \neq 1$$

$$\sum_{k=0}^{n} k a^k = \frac{a\{1-(n+1)a^n + n a^{n+1}\}}{(1-a)^2}$$

Time Domain Representation of Discrete-Time Signals:

$$\begin{aligned} x[n] &= \cdots + x[-2]\delta[n+2] + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + x[2]\delta[n-2] + \cdots \\ &= \sum_{k=\infty}^{\infty} x[k]\delta[n-k]. \end{aligned}$$

Systems:

System H is linear if $H\{ax_1[n] + bx_2[n]\} = aH\{x_1[n]\} + bH\{x_2[n]\}$.

System H is time invariant if $H\{x[n-n_0]\} = y[n-n_0]$.

Impulse response: for LTI system H, $h[n] = H\{\delta[n]\}$.

Convolution:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]h[k]$$

Convolution with $\delta[n]$:

$$x[n] * \delta[n] = x[n] \qquad\qquad x[n] * \delta[n-n_0] = x[n-n_0].$$