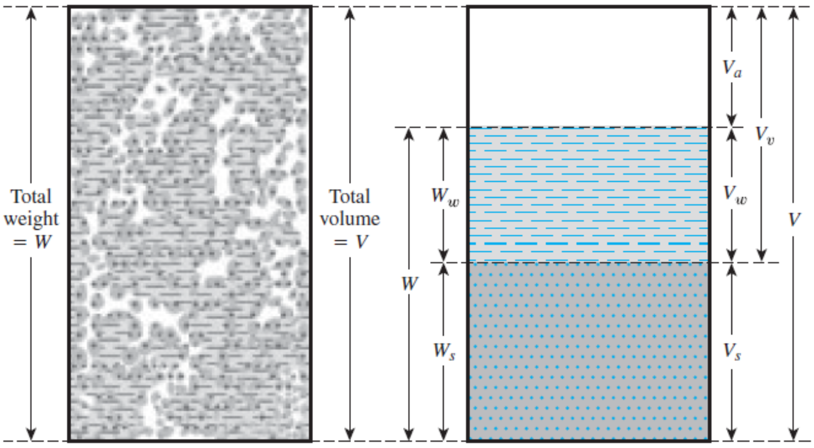
Chapter 3

Weight–Volume Relationships



Useful Formulas

- ✓ $V = V_s + V_v$

  $V_v = V_w + V_a$

- ✓ $W = W_s + W_w$
- ✓ Void Ratio

  (is defined as the ratio of the volume of voids to the volume of solids.)

  $e = \frac{Vv}{Vs}$

- ✓ Porosity

  (is defined as the ratio of the volume of voids to the total volume)

  $n=\frac{Vv}{V}$

- ✓ Degree Of Saturation

  (is defined as the ratio of the volume of water to the volume of voids,)

  $S=\frac{Vw}{Vv}$

  - $e=\frac{Vv}{Vs}=\frac{Vv}{V-Vv}=\frac{\frac{Vv}{V}}{\frac{V}{V}-\frac{Vv}{V}}=\frac{n}{1-n}$
  - $n=\frac{Vv}{V}=\frac{Vv}{Vs+Vv}=\frac{\frac{Vv}{Vs}}{\frac{Vs}{Vs}+\frac{Vv}{Vs}}=\frac{e}{1+e}$

- ✓ Moisture Content

  (is also referred to as water content and is defined as the ratio of the weight of water to the weight of solids in a given volume of soil)

  $w\%=\frac{Ww}{Ws}$

- ✓ Unit Weight

  (is the weight of soil per unit volume)

  $\gamma=\frac{W}{V}=\frac{Ww+Ws}{V}=\frac{Ws(1+\frac{Ww}{Ws})}{V}=\frac{Ws(1+w)}{V}$

- ✓ Dry Unit Weight

  $\gamma_d = \frac{Ws}{V}$

  $\gamma = \gamma_d(1+w)$

- ✓ Density

  $\rho = \frac{M}{V}$ kg/m3

- ✓ Dry Density

  $\rho_d = \frac{Ms}{V}$

- ✓ Specific gravity

  $\gamma = \frac{GS * \gamma w(1+w)}{1+e}$

  $\gamma_d = \frac{GS * \gamma w}{1+e}$

- ✓ S.e=GS.w

- ✓ For Saturation
  - $\gamma sat = \frac{\gamma w(Gs+e)}{1+e}$
  - S=100% , so e=Gs.w

✓ Relative density, $D_r = \frac{e_{max} - e}{e_{max} - e_{min}}$

Or $D_r = \frac{\rho_d - \rho_{d(min)}}{\rho_{d(max)} - \rho_{d(min)}} * \frac{\rho_{d(max)}}{\rho_d}$

where

$\gamma_{d(min)}$: dry unit weight in the loosest condition (at a void ratio of emax).

γ_d : in situ dry unit weight (at a void ratio of e).

$\gamma_{d(max)}$: dry unit weight in the densest condition (at a void ratio of emin).

3.4 A 0.4-m3 moist soil sample has the following:

- **Moist mass = 711.2 kg**
- **Dry mass = 623.9 kg**
- **Specific gravity of soil solids = 2.68**

Estimate:

a. Moisture content
b. Moist density
c. Dry density
d. Void ratio
e. Porosity

Solution

a) $w\% = \frac{Ww}{Ws}$

Ww=W-Ws=711.2-623.9=87.3 g

$w\% = \frac{87.3}{623.9} = 13.99\%$

b) $\rho = \frac{M}{V} = \frac{711.2}{0.4} = 1778 kg/m3$

$\rho_d = \frac{Ms}{V} = \frac{623.9}{.4} = 1559.75 kg/m3$

c) e=??

$\rho d = \frac{GS * \rho w}{1+e}$

$1559.75 = \frac{2.68*1000}{1+e}$

Solve for e=0.7182

d) $n = \frac{e}{1+e} = \frac{0.7182}{1+.7182} = 0.418$

3.7 The saturated unit weight of a soil is 19.8 kN/m3. The moisture content of the soil is 17.1%. Determine the following:

a. Dry unit weight

b. Specific gravity of soil solids

c. Void ratio

Solution

$\gamma_{sat.}$=19.8KN/m3, w%=17.1%

a) γ_d=??

$$\gamma sat. = \gamma d(1 + w)$$

$$\gamma d = \frac{19.8}{1+0.171} = 16.9 \text{ KN/m3}$$

b) & c) Gs=??, e=??

S.e=GS.w

1*e=Gs*0.171 ……..(1)

$$\gamma d = \frac{GS * \gamma w}{1+e}$$

$$16.9 = \frac{GS * 9.81}{1+e}$$

16.9e+16.9=9.81Gs……(2)

Solve eq. 1 and 2 for e and Gs

e=0.4176

Gs=2.442

3.22 For a given sandy soil, the maximum and minimum dry unit weights are 108 lb/ft3 and 92 lb/ft3, respectively. Given Gs = 2.65, determine the moist unit weight of this soil when the relative density is 60% and the moisture content is 8%.

Solution

$\gamma_{d(max)}$ =108 Ib/ft3, ɣd(min) =92 Ib/ft3,

Gs=2.65, ɣ=??, Dr=60% , w%=8%

$$Dr=\frac{\rho d-\rho d(min)}{\rho d(max)-\rho d(min)}*\frac{\rho d(max)}{\rho d}$$

$$0.6=\frac{\gamma d-92}{108-92}*\frac{108}{\gamma d}$$

ɣd=100.975 Ib/ft3

$$\gamma d=\frac{GS*\gamma w}{1+e}$$

$$100.975=\frac{2.65*62.4}{1+e}$$

e =0.637

$$\gamma=\frac{GS*\gamma w(1+w)}{1+e}=\frac{2.65*62.4(1+.08)}{1+.637}$$

$$=109.095\ Ib/ft3$$

3.24 A loose, uncompacted sand fill 6 ft in depth has a relative density of 40%. Laboratory tests indicated that the minimum and maximum void ratios of the sand are 0.46 and 0.90, respectively. The specific gravity of solids of the sand is 2.65.

a. What is the dry unit weight of the sand?

b. If the sand is compacted to a relative density of 75%, what is the decrease in thickness of the 6-ft fill?

Solution

Depth=6 ft, Dr=40%, emax=0.9, emin=0.46, Gs=2.65

Ɣd=? ?

$Ɣd= \frac{GS* Ɣw}{1+e}$ ...e=?

$Dr = \frac{emax-e}{emax-emin} = \frac{0.9-e}{0.9-.46} = 0.4$

e=0.724

$Ɣd= \frac{2.65* 62.4}{1+0.724} = 95.916 \text{ Ib/ft3}$

decrease in thickness=??

$Ɣd=95.916= \frac{Ws}{V} = \frac{Ws}{area*6}$

Ws=area*6*95.916…….(1)

After compaction:

$Dr=0.75, Dr = \frac{emax-e}{emax-emin} = \frac{0.9-e}{0.9-0.46} = 0.75$

e=0.57

$Ɣd= \frac{GS* Ɣw}{1+e} = \frac{2.65* 62.4}{1+0.57} = 105.32 \text{ Ib/ft3}$

$Ɣd=105.32= \frac{Ws}{V} = \frac{Ws}{area*thick.}$

Ws=105.32*area*thick…..(2)

Solve 1 and 2
Thick.=5.4643
decrease in thickness=6-5.4643=0.5357 ft

Q2(Exam 2011)

An undisturbed sample of clayey soil is found to have a wet weight of 285 N, a dry weight of 250 N, and a total Volume of $14*10^3$ cm3.If the specific gravity of soil solid is 2.7, determine the water content, void ratio, and degree of saturation.

Solution

W=0.285KN, Ws=0.25KN,

$V=14*10^3*10^{-6}=14*10^{-3}m^3$

$$W\%=\frac{Ww}{Ws}=\frac{0.285-0.25}{0.25}=0.14$$

e=?? $Ɣd=\frac{GS*Ɣw}{1+e}=\frac{Ws}{V}=$

$$\frac{0.25}{14*(10)^{\wedge}-3}=17.857Kn/m3$$

$$17.857=\frac{2.7*9.81}{1+e}$$

e=0.483

S=??

S.e=GS.w

$$S=\frac{2.7*0.14}{0.483}=0.782$$

Q1 (Exam 2012)
A soil sample has a void ratio of 0.72 , moisture content =12% and Gs=2.72 determine the following:
a)Dry Unit Weight, moist unit weight (KN/m3)
b)Weight of water in KN/m3 to be added for 80% degree of saturation.
c)Is it possible to reach a water content of 30% without change the present void ratio.

Solution

a)

$$\text{Ɣd}= \frac{GS*\text{Ɣw}}{1+e} = \frac{2.72*9.81}{1+0.72} = 15.513 \text{ KN/m3}$$

$$\text{Ɣ}= \text{Ɣd}(1+w) = 15.513*(1+0.12) = 17.375 \text{ KN/m3}$$

b)

S.e=GS.w

S*0.72=2.72*0.12

S=45.33%

$$\text{Ɣ}=\frac{Wt}{V} = 17.375 \text{ KN/m3}$$

Wt1=17.375 KN if 1 m3 volume

$$\text{Ɣd}= \frac{Ws}{V} = 15.513 \text{ KN/m3}$$

Ws=15.513 KN

Ww1=Wt-Ws= 1.862 KN

If S=80%
S.e=GS.w
0.8*0.72=2.72w
W%=21.17%
$\gamma = \gamma d(1 + w) = 15.513 * (1 + 0.2117) =$
18.797 KN/m3
$\gamma = \frac{Wt}{V} = 18.797 KN/m3$

Wt2=18.797 KN if 1 m3 volume

Ww2=Wt2-Ws= 3.284 KN
Weight of water in KN/m3 to be added
=3.284-1.862=1.422 KN in 1m3

c)w=30%
S.e=GS.w
S*0.72=2.72*0.3
S=1.133% >1
No.