Conservation Laws and The Ballistic Pendulum

Introduction: Many times it is impossible to measure a quantity directly. Perhaps the equipment has insufficient resolution, or perhaps the measuring device hasn't been invented. Sometimes the equipment isn't at hand, or it is too expensive. In any case, the scientist has to improvise. In this lab we shall measure the speed of a projectile indirectly using conservation laws.

The Theory: Conservation laws apply here to energy and momentum. Conservation means a quantity (energy, momentum) is the same before and after an event.

Mechanical energy can be divided into two categories: potential energy and kinetic energy. Potential energy is the energy of position. It may be a position in a gravitational or electromagnetic field or compressed position against a spring. It is important to realize that an object does not have an absolute quantity of this energy, it only has an amount measured against some reference point. Hence the potential energy of an object varies according to what you chose as a reference or zero point. The formula for potential energy (U) in a gravitational field is:

$$U = mgh$$

eq. 1

where *m* is mass, *g* is the acceleration by gravity and *h* is the change in height measured from the reference point. Consider an object suspended above a table. It has a certain *U* referenced to the table but a different *U* referenced to the floor. What is inviolate is a change in potential energy. If the object falls through a distance $y < h$ (so it doesn't hit the table or floor), the change is *U* is *mgy* regardless of its reference point.

Kinetic energy is an energy of motion. Again, an object does not have an absolute amount of kinetic energy (*K*), only an amount relative to some point. The formula for *K* is:

$$K = \tfrac{1}{2} mv^2$$

eq. 2

where *m* is still mass and *v* is velocity. Since motion is relative (ask Einstein) an object in motion can only have some relative amount of *K*. However, if the object changes its velocity, its corresponding change in K is the same regardless of reference.

In a mechanical system where one kind of energy is present, the other kind is 'waiting', for lack of a better term. For instance, if one drops the object suspended above the table it moves, obviously. As it falls, it loses *U* (*h* gets smaller) but gains *K* as it moves faster. The conservation law here states that the increase in *K* is exactly equal to the loss in *U*, neglecting friction and other dissipating factors. (It can be shown that the energy lost to these dissipating factors will equal any discrepancy in the *U/K* values). In other words, the energy changing from *U* to *K* is conserved. If one reverses the conditions and throws the object up, the *U* increases in the exact amount that the *K* decreases. We call this simple system without dissipative forces a conservative system.

Momentum (*P*) is another quantity that is conserved. Momentum is simply the product of mass and velocity:

$$P = mv$$

eq. 3

It is not an energy, although it is related to *K*. Momentum is more closely related to inertia, the resistance of an object to a change in motion. The more massive an object is the more momentum it has, in direct proportion. Of course, the faster the object travels the more momentum it has, in direct proportion. It also has more *K*, but the increase in *K* is not linear. Momentum is also a relative quantity since it is tied to velocity. Conservation of momentum means the amount of momentum before an event equals the amount of momentum after an event. This event is called a collision.

A collision is not necessarily the catastrophic event which occurs between two cars. It could be one billiard ball striking another, or a dog catching a frisbee. Each of these situations describes a type of collision.

<u>Elastic collision:</u> a collision where neither object is deformed permanently by the collision, such as in billiards. In this case, kinetic energy as well as momentum is conserved.

<u>Inelastic collision:</u> a collision where one or both objects is permanently deformed. Here momentum is conserved but *K* isn't, since energy is required to deform the object.

<u>Perfectly inelastic:</u> a collision where one or both objects is permanently deformed AND they stick together. Once again, momentum is conserved but *K* is not.

In the case of a perfectly inelastic collision between two objects the conservation equation is:

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v'$$

eq. 4

where v' is the speed of the combined masses after the collision and v_1 is the speed of the bullet as it leaves the gun. Note that since the objects stick together, their masses are summed on the right side of the equation. If m_2 is stationary before the collision, the product $m_2 v_2 = 0$. In our experiment, the catcher will be at rest before the collision.

The study of collisions and conservation laws can and should be further pursued. For now, considering the equations and concepts presented, you are ready for the experiment.

Task:

To find the velocity of a projectile. N.B. You are looking for a **single** value **two** different ways using **two** different techniques and **two** different sets of equations. Don't mix them up!

Equipment:

- ballistic pendulum
- meterstick
- triple beam balance
- carbon paper
- apple boxes
- plumb bob
- C-clamp

Procedure:

Description of the ballistic pendulum: The device consists of a spring gun, a small steel projectile (a ball), a free-swinging pendulum catcher, and a ratchet ramp. The spring gun fires the projectile into the catcher which then swings up the ramp where it is caught by the notches. The height it rises is determined by the K the combined ball and catcher system has after the collision. Since we assume a conservative system after the collision, the final U of the system will equal the initial K, after the collision. Since the collision is perfectly inelastic, we cannot equate the initial K of the ball with the final U of the system. However, since momentum is conserved, we can equate the momentum of the ball alone with the momentum of the ball and catcher. Then you can find the speed v_1 of the ball as it leave the gun.

Part 1

1) Mass (weight) the ball on the triple beam balance. **DO NOT mass the catcher!** <u>It's mass is written on the base of the unit.</u> Leave the catcher in the pendulum and leave the pivot alone; it is delicately adjusted.

2) Compress the ball on the gun and fire a few test shots. Be sure all students are out of the way, and make especially sure the teacher is safe! The catcher should catch the ball, swing up and catch on a notch. If it doesn't, ask the teacher for assistance.

3) There is a small sharp projection on the out-facing side of the catcher. This is the center of gravity for the ball/catcher system. It is to this point that you will measure height. Choose a convenient zero point (the table, floor or pendulum base will do) and measure to the center of gravity. Then fire the gun and note in which notch the system comes to rest. Repeat this 4 more times.

4) Place the catcher in the 'average' notch and measure the height from the zero point to the sharp projection on the catcher.

<u>**You are now finished with the first part of the lab.**</u>

Part 2

<u>**(Remember--this is a different technique that won't use your data or equations from Part 1!)**</u>

Another way of determining the initial speed of the ball is to use some basic kinematics principles and simple measurements. If one fires the ball, not at the catcher but out into space, it will eventually fall to the ground. By knowing the initial height of the ball one can calculate the time of fall, since vertical and horizontal components of velocity are completely independent. If the horizontal range has been measured, one can use the time calculation to compute the horizontal velocity.

1) Swing the catcher up on the ramp, out of the way.

2) Aim the gun out toward a clear and safe area of the room. Be sure *the teacher* is out of the way!

3) Fire the projectile and note approximately where it lands.

4) Place a piece of paper down on the landing spot, then tape the carbon paper carbon side down on top of the paper. This way, when the ball lands on the spot it will make a mark signifying the horizontal range. Use the apple box as a backstop.

5) Fire 5 shots, making sure that they all land on the paper. Carefully peel away the carbon paper, leaving the marked paper in place.

6) Using the plumb bob and meterstick, find the spot under the ballistic pendulum (and the table) which marks the starting horizontal position of the ball. Measure from this spot to each mark on the paper (*X*) and average the results.

13) Measure the vertical drop (*Y*) of the ball. This will be from the middle of the ball on the gun to the floor.

14) Now continue to the calculations section.

Calculations:

1) Using Conservation of Energy to calculate the speed of the bob and the ball system immediately after the collision

2) Using conservation of momentum to calculate the speed of the ball just before the impact

3) Compute the time of flight of the ball for part 2 using kinematics equations. **Do NOT use equations or measurements from Part 1 here.**

4) Calculate the initial velocity of the bullet for part 2 using the time from the previous calculation and kinematics equations. **Do NOT use equations or measurements from Part 1 here.**