Mathematics Lesson Plan for 3rd, 4th, and 5th grade

For the lessons on March 3, 4, 5, and 6 - 2009
At the Mills College Children's School, Oakland, CA
Instructor: Akihiko Takahashi

a. Title of the Lesson: Fractions

b. Goals of the Unit:

Students will understand the meaning and the representations of fractions in simple cases and appropriately use them.

- To understand that fractions are used to express an amount obtained as a result of equal partitioning and are used to express quantities less than 1
- To understand that a fraction can be considered as a collection of unit fractions
- To understand fraction notation
- To become aware that a fraction can also be put on a number line like whole numbers
- To become aware that addition and subtraction can also be applied to fractions

c. Relationship of the Lesson to the Standards

Prior to this unit:

- Students understand the concepts of whole numbers that includes how to represent them and how to put them on a number line, and developed the ability to use numbers.
- Students become aware that fractions represent one portion of an equally divided object or a fractional part of some quantity from their everyday life.
- Students understand the concepts of length and capacity, and to measure them in simple cases.
    - Students know about the units to be used in measuring length (millimeter (*mm*), centimeter (*cm*), meter (*m*), and customary units such as mile).
    - Students know about the units to be used in measuring capacity (milliliter (*ml*), and liter (*l*).
- To understand the meaning of division and to use it.

This Unit

After this unit

- Students will deepen their understanding of the meaning of fractions and be able compute fractions in simple cases.
- Students will deepen their understanding of the representation of fractions and their meanings. Furthermore, in simple cases to pay attention to the fact that there are equivalent fractions.
- Students will be able to add and subtract fractions with a common denominator.

d. Unit Plan

Day 1	Introduction to the unit	*Displaying the dates* Students will deepen their understanding of decimal notation through solving a problem related to children's everyday life.
Day 2	How can we express fractional parts (1) *Mathematics for elementary school 3B* (Hironaka H. et al., 2006) pp.57-58.	Students will become aware that fractions can be seen in students' everyday life. Students will understand that fractions are used to express an amount obtained as a result of equal partitioning and are used to express quantities less than 1 (only unit fractions).
Day 3	How can we express fractional parts (2) *Mathematics for elementary school 3B* (Hironaka H. et al., 2006) pp.58-59.	Students will understand that a fraction can be considered as a collection of unit fractions. Students will know fraction notation.
Day 4	The size of fraction *Mathematics for elementary school 3B* (Hironaka H. et al., 2006) pp.60	Student will become aware that a fraction can be put on a number line.

e. Instruction of the Lesson

Fraction is an important topic in the elementary grades. At the same time, it is one of the most challenging topics for students to understand (U.S. Department of Education, 2008). Although many students have seen fractions in everyday life, e.g. a half mile on the highway road signs and a quarter pound in a fast-food-chain menu, these students may not be able to see fractions as numbers and use them comfortably like whole numbers. Researchers argue that the concept of fractions may be well introduced in second grade with manipulatives but they need to go through a gradual process moving from the concrete, the semi-concrete, and the abstract in order for them to see fractions as numbers (Gunderson & Gunderson, 1957).

According to the Japanese Course of Study Teaching Guide (Takahashi, Watanabe, & Yoshida, 2004), fractions should be introduced to represent one portion of an equally divided object, or to represent a fractional part of some quantity. After this kind of introduction, the idea that $\frac{2}{3}$ represents a collection of two $\frac{1}{3}$ units should be taught, as if $\frac{1}{3}$ is thought of as a unit (Thompson & Saldanha, 2003). Researchers also argue that students who work with fractions well use words in the beginning rather than the symbol, i.e., writing "2 thirds" rather than $\frac{2}{3}$ so that students can see "one third" as a unit just like measurement unit like miles (Gunderson & Gunderson, 1957). This shows that a fraction is the number that represents some portion of equally divided 1; that is, a fraction has the meaning of $\frac{a}{b} = 1 \div b \times a$.

It is also important for students to understand that fractions, just like whole numbers and decimal numbers, are used to represent not only size of numbers and quantities but also proportion of numbers and quantities. The Singapore National Curriculum (Ministry of Education, 2006) emphasizes that fractions are introduced as part of a whole interpretation in the primary 2 and the fraction of a set of objects, which is to represent proportion of numbers and quantities, should not be introduced until primary 4. Japanese Course of Study also mentions that part of a whole interpretation should be introduced at the beginning while using fractions to represent proportion of numbers and quantities should not be introduced until grade 5 (Takahashi, Watanabe, & Yoshida, 2008).
Based on the above discussion, the present research lesson unit, which consists of four lessons in four days, is designed for students to deepen their understanding of fractions in order for them to see that fractions are numbers. This unit is designed based on the English translation of the Japanese mathematics textbook series for the elementary grades, the most widely used public school mathematics textbook in Japan (Hironaka & Sugiyama, 2006). This series of research lessons support the following key ideas from the textbook:

- Fractions are introduced as part of whole interpretation, which is to express an amount obtained as a result of equal partitioning.
- Fractions are used to express quantities less than 1 in measurement contexts.
- Diagrams such as tape diagrams and area diagrams are used for students to understand that a fraction can be considered as a collection of unit fractions.
- Tape diagrams and number lines are used for students to see fractions are numbers just like whole numbers.

Since the Japanese textbooks are originally written in Japanese and designed for Japanese children who live in Japan, the followings are added to the contents of the Japanese textbook in order to maximize the benefits of learning for the English speaking children who live in the US.

- Some examples of fractions from students' everyday life will be shown in order to encourage students to see that fractions are often used in American society.
- Students will be given opportunities to write fractions using not only the symbol but also the word, i.e., writing "2 thirds" in addition to $\frac{2}{3}$ so that students can see "one third" as a unit.

f. Plan of the Lessons
Day 1 – March 3, 2009
Goal of the lesson:

- Students will deepen their understanding of decimal notation through solving a problem related to children's everyday life

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher's Support	Points of Evaluation
1. Introduction Encourage students to see the benefit of displaying dates on the wall in each classroom. Rather than writing the date everyday, it might be a good idea to prepare some cards to display dates so that you can use all the time.	Help student awareness that you can save your effort if you prepare cards to display dates. Help student awareness that you do not need too many cards to display dates. Assemble groups of students from different grades; maybe one 3rd grader, one 4th grader, and one 5th grader. Ask older students to help younger students understand.	If all the students understand that we do not need only 31 cards to display dates because the largest dates in a month is 31.
2. Posing the Problem Think about how many cards we need to display dates on the wall in your classroom. **3. Anticipated Student Responses** 20 cards 13 cards 12 cards 11 cards	Help student awareness that we might not need 31 cards if we put a digit on each card and use them for ones and tens. Provide students actual cards if students request.	Do students recall what they learned previously?
4. Comparing and Discussing **a. 20 cards** Cards for ones: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 Cards for tens: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 **b. 13 cards** Cards for ones: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 Cards for tens: 1, 2, 3 **c. 13 cards** Cards for ones: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 Cards for tens: 0, 1, 2 **d. 12 cards** Cards for ones: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 Cards for tens: 1, 2 **e. 12 cards** Cards for ones: 1, 2, 3, 4, 5, 6, 7, 8, 0 Cards for tens: 1, 2, 3	Begin discussion with the group with the larger number of cards. Encourage students to share with other groups not only how many cards but also what kind of cards they want to prepare. Help other groups to understand how the group came up the idea and see if the method works for all the dates. For the fewest number of the cards, let each student make the cards and try one by one to make sure all the dates of a month can be displayed with 11 cards. (If a student does not want to use 6 to display 9, he/she can use 12 cards).	Does each student understand other students' ideas? Does each student understand that only 11 cards with a digit in each card are necessary to display all the dates of each month?

f. 11 cards Cards for ones: 1, 2, 3, 4, 5, 6, 7, 8, 0 Cards for tens: 1, 2		
4. Extending the problem How many cards do we need to display all the dates if you can use both sides of each card? **5. Anticipated Student Responses** **6 cards** • Front 1, 2, 3, 4, 5, 6 • Back 0, 1, 2, 7, 8, 9 **6 cards** • Any combination would work if the following pair of digits are not put on the same cards 1 & 1, 2 & 2, 3 & 0	Provide students enough cards so that students can actually try to find how their ideas work. Help students become aware there will be multiple correct solutions to this problem.	Does each student find a way to display all the dates of a month by using both sides of six cards?
6. Summing up • Let each student writes what he/she learned today.	Encourage students to use the board writing as an example in order to summarize what they learned.	

Evaluation:

- Do students see that a digit can be used for not only ones but also tens place, i.e., 3 can be used not only for displaying 23rd but also 31st?
- Do students recall their understanding of decimal notation?
- Do students understand the nature of the problem solving approach?

Day 2 – March 4, 2009
Goal of the lesson:

- Students become aware that fractions can be seen in students' everyday life.
- Students will understand that fractions are used to express an amount obtained as a result of equal partitioning and are used to express quantities less than 1 (only unit fractions).

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher's Support	Points of Evaluation
1. Introduction Showing road signs with fractions to help students become aware that fractions can be found in everyday life. Ask students what they know about fractions • What does each number on the road sign represent? Students will discuss about what the fractions on the road sign represent by using their prior knowledge regarding whole numbers, measurements of distance, and fractions.	If some of the students seem not familiar with the term fraction, avoid using the term until the class informally defines the term. Encourage students to help each other to share their prior knowledge.	Is each student comfortable using term "fraction"? Does each student see what the fractions on the road sign represent?
2. Posing the Problem The length of the tape strip represents the length around trunk of a tree on the campus. The length of the tape strip is a bit longer than 1 *m*. How can we express the length of the fractional part of this tape strip using 1 *m* tape strip as a reference. **3. Anticipated Student Responses** • About a half meter • About a quarter meter • The length the fractional part is the same as the length of a portion that obtained by dividing 1 *m* into three equal parts. • One of the third of 1 *m* • $\frac{1}{3}$ • Some students might want to use their personal references such as the length of their hands or belongings. Some others may want to use yard or feet.	Each student will work with a partner. Each pair of students will use the actual length of the tape strips, one is 1 *m* and another is $1\frac{1}{3}$ *m*. The actual tape strips are similar to the diagrams on the textbook page. Encourage students to use 1 *m as* a reference to create their own unit in order to express the length of the fractional part.	Does each understand that the fractional part can be expressed by using 1 *m* as a reference?
4. Comparing and Discussing • To understand each approach to express the fractional part of the tape strip • To understand that a fraction can be used to express the length of a fractional part of the tape strip	Through the discussion encourage students to see that using a formal unit, such as meter, as a reference is a good idea to express quantities. Encourage students to write the length of the fractional part in the words, "1 third of 1 meter" or "1 third meter".	Does each student understand that the fractional part can be expressed using third meter as a unit

5. Apply the learning to the similar situation How to express the following parts of 1 *m* using the similar approach that you learned from the previous problem? • Find the length of a tape strip (1 half meter) • Find the length of a tape strip (1 fifth meter) Let's make 1 quarter-meter tape strip from 1 meter tape strip. • Ask a couple of younger grade students to explain how he/she made the tape strip • Ask other students to verify if the tape strips are 1 quarter-meter length.	Provide students actual length of tape strips. Encourage students to work with their partners. Once they find the length of each tape strip, let students write down the length in the words. Provide 1 meter tape strip for each student so that each of them can make own 1 quarter meter.	Does each student understand how to express the length and write it in the words? Does each student make a 1 quarter-meter length tape strip?
6. Summing up • Let each student write what he/she learned today.	Encourage students to use the board writing as an example in order to summarize what they learned.	

Evaluation:

- Do students understand that an amount obtained as a result of equal partitioning can be used to express quantities less than 1?
- Do students understand how to express the length of fractional parts by using words such as 1 third?

Picture of the road sign

Day 3 – March 5, 2009

Goal of the lesson:

- Students will understand that a fraction can be considered as a collection of unit fractions.
- Students will know fraction notation.

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher's Support	Points of Evaluation
1. Introduction Ask some students to share what they wrote in their notebook the day before.		
2. Posing the Problem The length of the tape strip is a bit shorter than 1 *m*. How can we express the length of this tape strip using 1 *m* tape strip as a reference. **3. Anticipated Student Responses** • A bit longer than a half meter • Twice as long as 1 third meter • Two of the third of 1 *m* • $\frac{2}{3}$ • 2 thirds meter.	Each student will work with a partner. Each pair of students will use the actual length of the tape strips, one is 1 *m* and another is $\frac{2}{3}$ *m*. The actual tape strips are similar to the diagrams in the textbook page but do not have dots line to show 1 third. Encourage students to use the tape strips from the day before lesson as reference.	Does each understand that the fractional part can be expressed by using 1 *m* as a reference?
4. Comparing and Discussing • To understand that the length of the fractional part is twice as long as the 1 third meter tape strip. • To understand that the fractional part can be expressed as a collection of third. • To understand the fractional part can be express by using the words, **2 thirds meter.**	Encourage students to write the length of the fractional part in the words, "2 thirds of 1 meter" or "2 thirds meter".	Does each student understand that the fractional part can be expressed using third meter as a unit?
5. Apply the learning to another situation How to express the following parts of 1 *liter* using the similar approach that you learned from the previous problem? • Find the amount of the water in the picture (2 fifths liter) • Find the amount of the water in the picture (1 fourth liter or 1 quarter liter) • Find the amount of the water in the picture (4 sixth liter)	Provide students a picture of the liter cup for each problem. Encourage students to work with their partners. Once they find the amount of the water, let students write down the amount in the words.	Does each student understand how to express the amount and write it in the words?
7. Summing up • Introduce fraction notation by replacing the words to express fractional parts. *4 sixth liter* can be written as $\frac{4}{6}$ liter.		

Numbers like $\frac{4}{6}$ are called **fractions.** **6 is called denominator to express the unit, sixth.** **4 is called numerator to express how many of the unit.** • Let students to use fraction notation to express the fractional part that they expressed in the words during the prior activities. • Do the exercises 2 & 3 on the textbook p.59. • Let each student write what he/she learned today.	Encourage students to use the board writing as an example in order to summarize what they learned.	Is each student able to express the quantities of the fractional parts by using fraction notation?

Evaluation:

- Do students understand that a fraction can be considered as a collection of unit fractions?
- Are students able to write fractions by using fraction notation?

Day 4 – March 6, 2009

Goal of the lesson:

- Student will aware that a fraction can be put on a number line.

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher's Support	Points of Evaluation
1. Introduction Ask some students to share what they wrote in their notebook the day before.		
2. Posing the Problem By using the tape strips we have had so far, lets create various length of the tape strips. Students can use the following unit fractions to express variety of fractions. · Half (meter) · Third (meter) · Quarter/fourth (meter) · Fifth (meter) · Sixth (liter/meter) **3. Anticipated Student Responses** • 2 halves meter • 2 thirds meter, 3 thirds meter • 2 fourths meter, 3 fourths meter, 4 fourths meter, 2 quarters meter, 3 quarters meter, 4 quarters meters • 2 fifths meters, 3 fifths meters, 4 fifths meters, 5 fifths meters • 2 sixths meters, 3 sixths meters, 4 sixths meters, 5 sixths meters, 6 sixths meters • $\frac{2}{2},\frac{2}{3},\frac{3}{3},\frac{2}{4},\frac{3}{4},\frac{4}{4},\frac{2}{5},\frac{3}{5},\frac{4}{5},\frac{5}{5},\frac{2}{6},\frac{3}{6},\frac{4}{6},\frac{5}{6},\frac{6}{6}$	Each student will work with a partner. Each pair of students will use the tape strips from the previous days as a unit fraction in order to make several different lengths of tape strip. Each group will have a cash register paper role to make tape strips.	Does each understand that the fractional part can be expressed by using 1 *m* as a reference?
4. Comparing and Discussing • Through sharing the fractions, students will have opportunities to express, interpret, validate the fractions in the word and the fraction notation. • Organize fractions according to the size of the unit fraction, put the fractions with the same denominator on the same number line. • Compare the size of fractions with the same denominators.	Encourage students to write the length of the fractional part in the words, "2 thirds of 1 meter" or "2 thirds meter" and by the fraction notation.	Does each student understand that the fractional part can be expressed both in the words and fraction notation?
8. Summing up • Do the exercise in the textbook p.60. • Let each student write what he/she learned today.	Encourage students to use the board writing as an example in order to summarize what they learned.	

Evaluation:

- Are students able to put fractions on number lines?

References:

Gunderson, A., & Gunderson, E. (1957). Fraction Concepts Held by Young Children. Arithmetic Teacher, 4 (October 1957), 168 - 173.

Hironaka, H., & Sugiyama, Y. (Eds.). (2006). Mathematics for Elementary School. Tokyo, Japan: Tokyo Shoseki Co., Ltd.

Ministry of Education, S. (2006). Mathematics Syllabus Primary.

Takahashi, A., Watanabe, T., & Yoshida, M. (2004). Elementary School Teaching Guide for the Japanese Course of Study: Arithmetic (Grade 1-6). Madison, NJ: , Global Education Resources.

Takahashi, A., Watanabe, T., & Yoshida, M. (2008). English Translation of the Japanese Mathematics Curricula in the Course of Study, March, 2008, Grades 1-9. Madison, NJ: Global Education Resources.

Thompson, P. W., & Saldanha, L. i. A. (2003). Fractions and Multiplicative Reasoning. In J. Kilpatrick, W. G. Martin & D. Schifter (Eds.), A Research Companion to Principles and Standards for School Mathematics (pp. 95-111): National Council of Teachers of Mathematics.

U.S. Department of Education (2008). Foundations for Success: The Final Report of the National Mathematics Advisory Panel. Washington, DC.