Homework 1 Problems
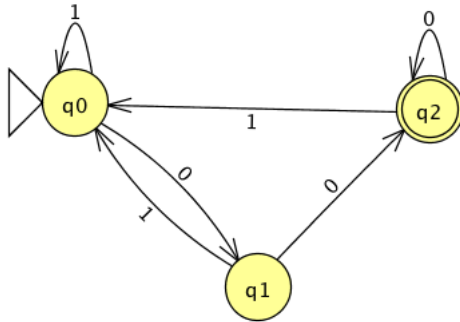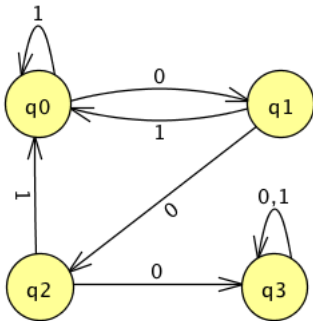
1. Exercise 2.2.4 on page 53 of Hopcroft et al.
   Give DFA's accepting the following languages over the alphabet $\{0, 1\}$.
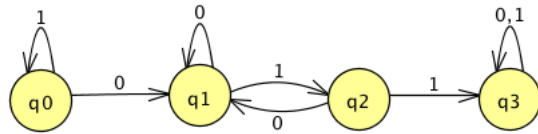
   (a) The set of all strings ending in 00.

   (b) The set of all strings with three consecutive 0's (not necessarily at the end).

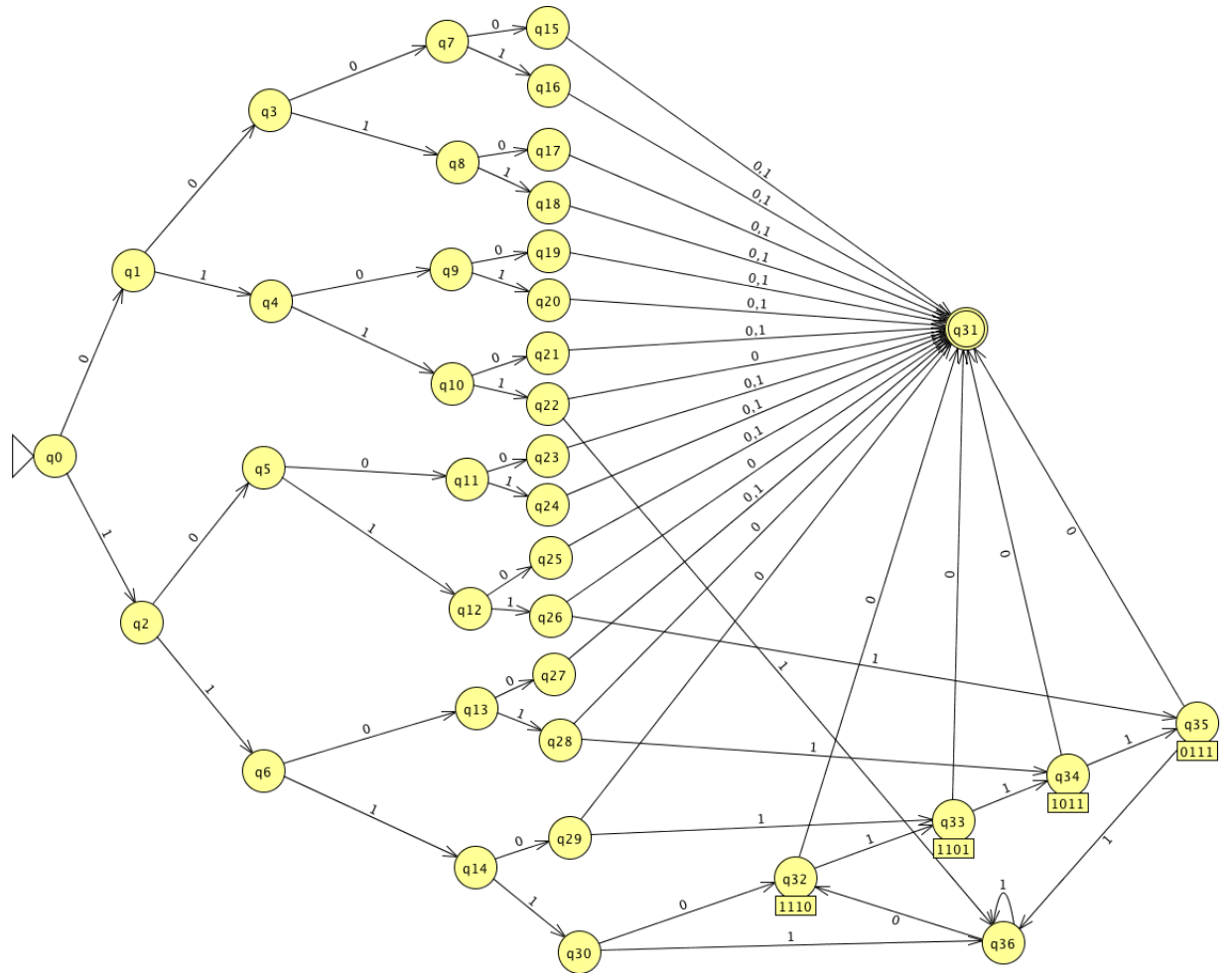   (c) The set of strings with 011 as a substring.

2. Exercise 2.2.5 on pages 53-54 of Hopcroft et al.

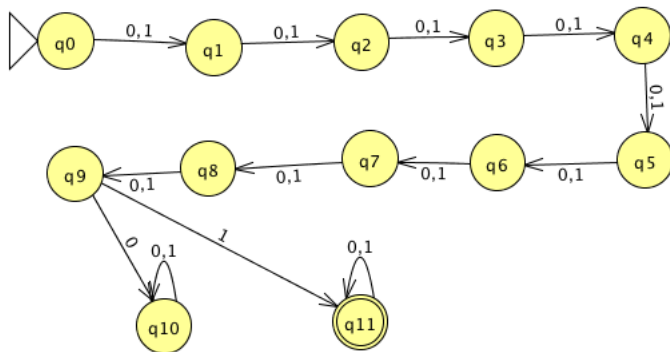   Give DFA's accepting the following languages over the alphabet $\{0, 1\}$.

   (a) The set of all strings such that any block of five consecutive symbols contains at least two 0's.
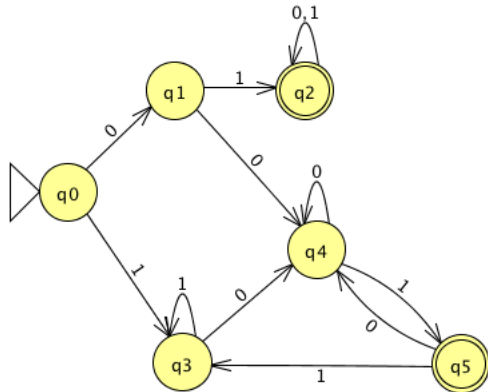
   (I'm going to interpret this to mean that there exists a block of five consecutive symbols containing at least two 0's.)

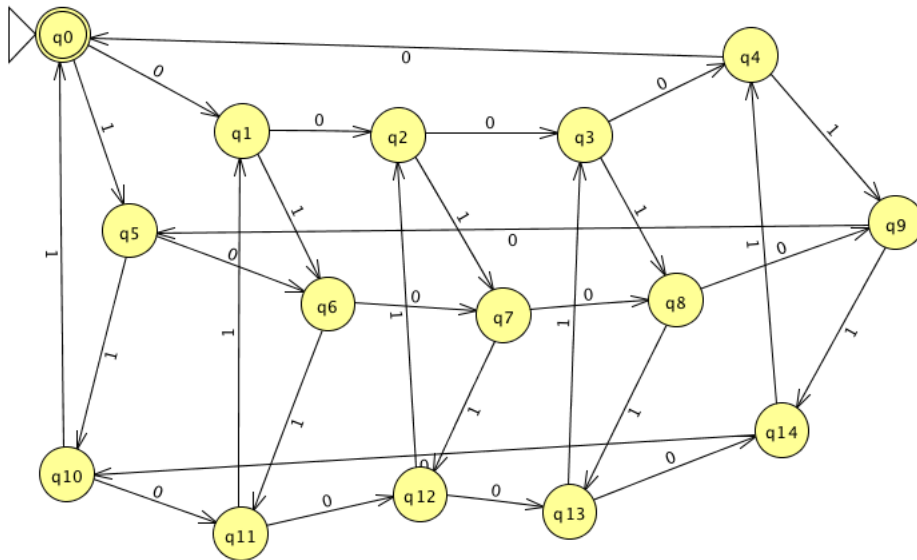(b) The set of all strings whose tenth symbol from the left end is a 1.

(c) The set of strings that either begin or end (or both) with 01.

(d) The set of strings such that the number of 0's is divisible by five, and the number of 1's is divisible by 3.

3. Exercise 2.2.8 on page 54 of Hopcroft et al.

Let A be a DFA and a a particular input symbol of A, such that for all states q of A we have $\delta(q, a) = q$.

(a) Show by induction on n that for all $n \geq 0$, $\hat{\delta}(q, a^n) = q$, where a^n is the string consisting of n a's.

Base case: $n = 0$
$\hat{\delta}(q, \epsilon) = q$, by definition.

Induction step: Assume that $\hat{\delta}(q, a^k) = q$.

$$\hat{\delta}(q, a^{k+1}) = \hat{\delta}(\hat{\delta}(q, a^k), a)$$
$$= \hat{\delta}(q, a)$$
$$= q$$

Therefore, by induction, $\hat{\delta}(q, a^n) = q$ for all $n \geq 0$.

(b) Show that either $\{a^*\} \subseteq L(A)$ or $\{a^*\} \cap L(A) = \emptyset$.

By the previous part, for all states q, for all $n \geq 0$, $\hat{\delta}(q, a^n) = q$. In particular, this is true for the initial state q_0. So for any input in $\{a^*\}$, we will end in state q_0.

First, suppose that our initial state q_0 is a final state. Then we accept every string in $\{a^*\}$, so $\{a^*\} \subseteq L(A)$.
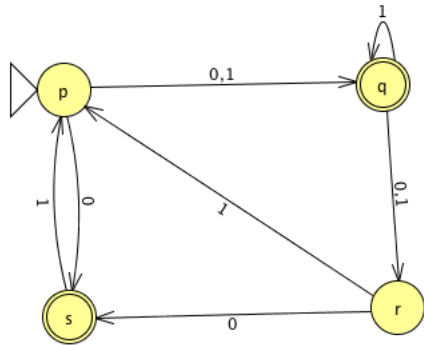
Otherwise, suppose that q_0 is not a final state. Then we reject every string in $\{a^*\}$, so $\{a^*\} \cap L(A) = \emptyset$.

4. Exercise 2.3.2 on page 66 of Hopcroft et al.
Convert to a DFA the following NFA:

	0	1
$\rightarrow p$	$\{q, s\}$	$\{q\}$
$*q$	$\{r\}$	$\{q, r\}$
r	$\{s\}$	$\{p\}$
$*s$	$\emptyset$	$\{p\}$

Our NFA is drawn below.

The table for the equivalent DFA using the subset construction is:

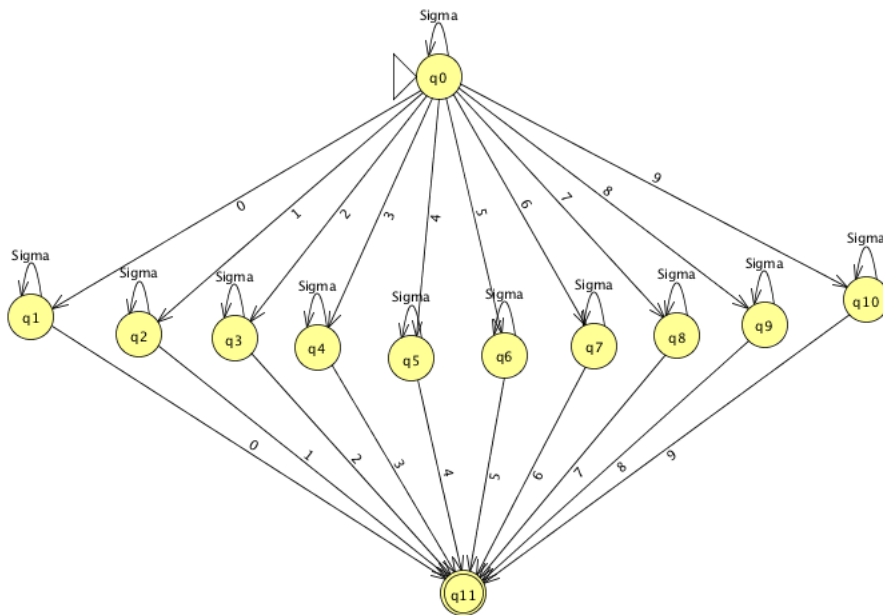|	0	1
$\to \{p\}$	$\{q,s\}$	$\{q\}$
$*\{q,s\}$	$\{r\}$	$\{p,q,r\}$
$*\{q\}$	$\{r\}$	$\{q,r\}$
$\{r\}$	$\{s\}$	$\{p\}$
$*\{p,q,r\}$	$\{q,r,s\}$	$\{p,q,r\}$
$*\{q,r\}$	$\{r,s\}$	$\{p,q,r\}$
$*\{s\}$	$\emptyset$	$\{p\}$
$*\{q,r,s\}$	$\{r,s\}$	$\{p,q,r\}$
$*\{r,s\}$	$\{s\}$	$\{p\}$
$\emptyset$	$\emptyset$	$\emptyset$

This DFA is drawn below:
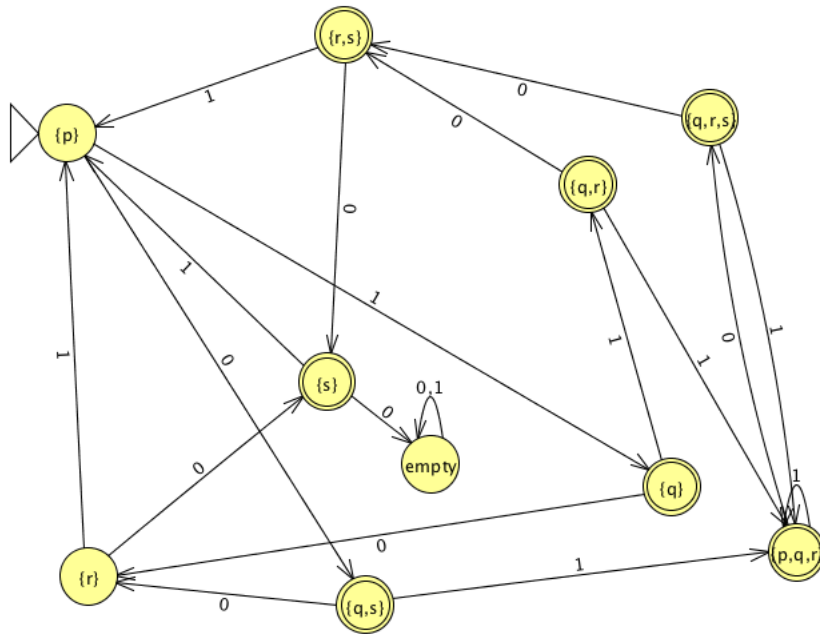
5. Exercise 2.3.4 on pages 66-67 of Hopcroft et al.
   Give nondeterministic finite automata to accept the following languages. Try to take advantage of nondeterminism as much as possible.

   (a) The set of strings over alphabet $\{0, 1, \ldots, 9\}$ such that the final digit has appeared before.

(b) The set of strings over alphabet $\{0, 1, \ldots, 9\}$ such that the final digit has *not* appeared before.
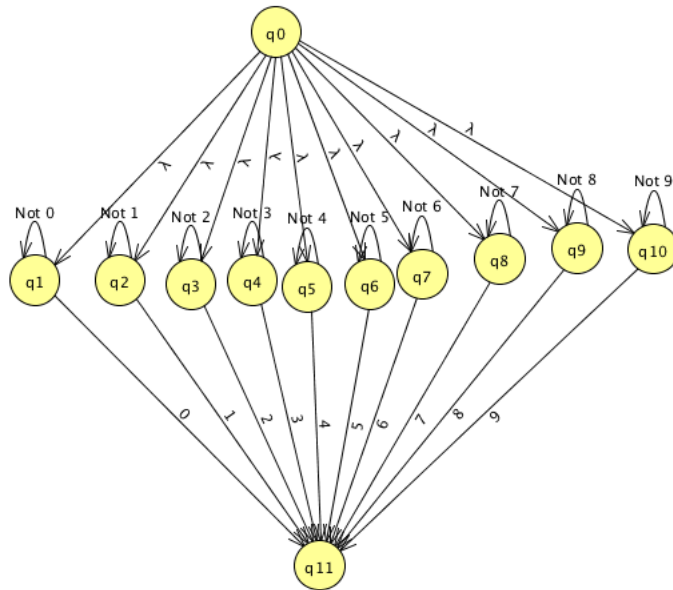
(c) The set of strings of 0's and 1's such that there are two 0's separated by a number of positions that is a multiple of 4. Note that 0 is an allowable multiple of 4.