0-3 Simple Probability

A die is rolled. Find the probability of each outcome.

1. P(less than 3)

SOLUTION:

The probability of an event A is the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

There are 6 possible outcomes: 1, 2, 3, 4, 5, and 6.
There are 2 numbers less than 3: 1 and 2.
Therefore,

$$\begin{aligned} P(\text{less than } 3) &= \frac{2}{6} \\ &= \frac{1}{3} \\ &\approx 33\%. \end{aligned}$$

2. P(even)

SOLUTION:

The probability of an event A is the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

There are 6 possible outcomes: 1, 2, 3, 4, 5, and 6.
There are 3 even numbers: 2, 4, and 6.
Therefore,

$$\begin{aligned} P(\text{even}) &= \frac{3}{6} \\ &= \frac{1}{2} \\ &= 50\%. \end{aligned}$$

3. P(greater than 2)

SOLUTION:

The probability of an event A is the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

There are 6 possible outcomes: 1, 2, 3, 4, 5, and 6.
There are 4 numbers greater than 2: 3, 4, 5, and 6.
Therefore,

$$\begin{aligned} P(\text{greater than } 2) &= \frac{4}{6} \\ &= \frac{2}{3} \\ &\approx 67\%. \end{aligned}$$

4. P(prime)

SOLUTION:

The probability of an event A is the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

There are 6 possible outcomes: 1, 2, 3, 4, 5, and 6.
There are 3 prime numbers: 2, 3 and 5. (Remember that 1 is not prime!)
Therefore,

$$\begin{aligned} P(\text{prime}) &= \frac{3}{6} \\ &= \frac{1}{2} \\ &= 50\%. \end{aligned}$$

5. P(4 or 2)

SOLUTION:

The probability of an event A is the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

There are 6 possible outcomes: 1, 2, 3, 4, 5, and 6.
There are two favorable outcomes: 2 and 4.
Therefore,

$$\begin{aligned} P(4 \text{ or } 2) &= \frac{2}{6} \\ &= \frac{1}{3} \\ &\approx 33\%. \end{aligned}$$

6. P(integer)

SOLUTION:

The probability of an event A is the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

There are 6 possible outcomes: 1, 2, 3, 4, 5, and 6. All the possible outcomes are integers, that is, all are favorable outcomes.
Therefore,

$$\begin{aligned} P(\text{integer}) &= \frac{6}{6} \\ &= 1 \\ &= 100\%. \end{aligned}$$

A jar contains 65 pennies, 27 nickels, 30 dimes, and 18 quarters. A coin is randomly selected from the jar. Find each probability.

7. P(penny)

SOLUTION:

The probability of an event A is the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

There are a total of 65 + 27 + 30 + 18 = 140 coins in the jar.
Out of 140 coins 65 are pennies. So, there are 140 possible outcomes and there are 65 favorable outcomes.
Therefore,

$$\begin{aligned} P(\text{penny}) &= \frac{65}{140} \\ &= \frac{13}{28} \\ &\approx 46\%. \end{aligned}$$

8. P(quarter)

SOLUTION:

The probability of an event A is the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

There are a total of 65 + 27 + 30 + 18 = 140 coins in the jar.
Out of 140 coins 18 are quarters. So, there are 140 possible outcomes and 18 favorable outcomes.
Therefore,

$$\begin{aligned} P(\text{quarter}) &= \frac{18}{140} \\ &= \frac{9}{70} \\ &\approx 13\%. \end{aligned}$$

9. P(not dime)

SOLUTION:

The probability of an event A is the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$
$$P(\text{ not } A) = 1 - P(A)$$

There are a total of 65 + 27 + 30 + 18 = 140 coins in the jar.
Out of 140 coins 30 are dimes. Therefore,

$$\begin{aligned} P(\text{dime}) &= \frac{30}{140} \\ &= \frac{3}{14}. \end{aligned}$$
$$\begin{aligned} P(\text{not dime}) &= 1 - P(\text{dime}) \\ &= 1 - \frac{3}{14} \\ &= \frac{11}{14} \\ &\approx 79\% \end{aligned}$$

10. P(penny or dime)

SOLUTION:

The probability of an event A is the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

There are a total of 65 + 27 + 30 + 18 = 140 coins in the jar.
Out of 140 coins 65 are pennies and 30 are dimes. So, there are 140 possible outcomes and 65 + 30 = 95 favorable outcomes.
Therefore,

$$\begin{aligned} P(\text{penny or dime}) &= \frac{95}{140} \\ &= \frac{19}{28} \\ &\approx 68\%. \end{aligned}$$

11. P(value greater than \$0.15)

SOLUTION:

The probability of an event A is the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

There are a total of 65 + 27 + 30 + 18 = 140 coins in the jar.
A penny is worth \$0.01, a nickel, \$0.05, a dime \$0.1 and a quarter is worth \$0.25. So, selecting a coin with value greater than \$0.15 is same as selecting a quarter.
Out of 140 coins 18 are quarters. So, the number of possible outcomes is 140 and that of favorable outcomes is 18.
Therefore,

$$\begin{aligned} P(\text{quarter}) &= \frac{18}{140} \\ &= \frac{9}{70} \\ &\approx 13\%. \end{aligned}$$

12. P(not nickel)

SOLUTION:

The probability of an event A is the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$
$$P(\text{ not } A) = 1 - P(A)$$

There are a total of 65 + 27 + 30 + 18 = 140 coins in the jar.
Out of 140 coins 27 are nickels. Therefore,

$$P(\text{nickel}) = \frac{27}{140}$$
$$\begin{aligned} P(\text{not nickel}) &= 1 - P(\text{nickel}) \\ &= 1 - \frac{27}{140} \\ &= \frac{113}{140} \\ &\approx 81\%. \end{aligned}$$

13. P(nickel or quarter)

SOLUTION:

The probability of an event A is the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

There are a total of 65 + 27 + 30 + 18 = 140 coins in the jar.
Out of 140 coins 27 are nickels and 18 are quarters.
So, there are 140 possible outcomes and 27 + 18 = 45 favorable outcomes.
Therefore,

$$\begin{aligned} P(\text{nickel or quarter}) &= \frac{45}{140} \\ &= \frac{9}{28} \\ &\approx 32\%. \end{aligned}$$

14. P(value less than \$0.20)

SOLUTION:

The probability of an event A is the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

There are a total of 65 + 27 + 30 + 18 = 140 coins in the jar.
A penny is worth \$0.01, a nickel, \$0.05, a dime \$0.1 and a quarter is worth \$0.25. So, selecting a coin with value less than \$0.20 is same as selecting a coin other than a quarter.
Out of 140 coins 18 are quarters. Therefore,

$$\begin{aligned} P(\text{quarter}) &= \frac{18}{140} \\ &= \frac{9}{70}. \end{aligned}$$

$$\begin{aligned} P(\text{not quarter}) &= 1 - P(\text{quarter}) \\ &= 1 - \frac{9}{70} \\ &= \frac{61}{70} \\ &\approx 87\% \end{aligned}$$

PRESENTATIONS The students in a class are randomly drawing cards numbered 1 through 28 from a hat to determine the order in which they will give their presentations. Find each probability.

15. *P*(13)

SOLUTION:

The probability of an event A is the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

The cards are numbered 1 through 28. So, the total number of possible outcomes is 28 and there is only one favorable outcome, 13.
Therefore,

$$\begin{aligned} P(13) &= \frac{1}{28} \\ &\approx 3.6\%. \end{aligned}$$

16. *P*(1 or 28)

SOLUTION:

The probability of an event A is the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

The cards are numbered 1 through 28. So, the total number of possible outcomes is 28 and there are only two favorable outcomes, 1 and 28.
Therefore,

$$\begin{aligned} P(1 \text{ or } 28) &= \frac{2}{28} \\ &= \frac{1}{14} \\ &\approx 7\%. \end{aligned}$$

17. *P*(less than 14)

SOLUTION:

The probability of an event A is the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

The cards are numbered 1 through 28 and there are 13 cards numbered less than 14. So, there are 28 total possible outcomes and 13 favorable outcomes.
Therefore,

$$\begin{aligned} P(\text{less than } 14) &= \frac{13}{28} \\ &\approx 46\%. \end{aligned}$$

18. *P*(not 1)

SOLUTION:

The probability of an event A is the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

The cards are numbered 1 through 28 and there are 27 cards numbered other than 1. So, there are 28 total possible outcomes and 27 favorable outcomes.
Therefore,

$$\begin{aligned} P(\text{not } 1) &= \frac{27}{28} \\ &\approx 96\%. \end{aligned}$$

19. *P*(not 2 or 17)

SOLUTION:

The probability of an event A is the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

The cards are numbered 1 through 28 and there are 26 cards numbered other than 2 and 17. So, the total number of possible outcomes is 28 and the number of favorable outcomes is 26.
Therefore,

$$\begin{aligned} P(\text{not 2 or 17}) &= \frac{26}{28} \\ &= \frac{13}{14} \\ &\approx 93\%. \end{aligned}$$

20. *P*(greater than 16)

SOLUTION:

The probability of an event A is the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

The cards are numbered 1 through 28 and there are 12 cards numbered greater than 16. So, the total number of possible outcomes is 28 and the number of favorable outcomes is 12.
Therefore,

$$\begin{aligned} P(\text{greater than 16}) &= \frac{12}{28} \\ &= \frac{3}{7} \\ &\approx 43\%. \end{aligned}$$

The table shows the results of an experiment in which three coins were tossed.

Outcome	HHH	HHT	HTH	THH	TTH	THT	HTT	TTT
Tally	卌	卌	卌 I	卌 I	卌 II	卌	卌 III	卌 III
Frequency	5	5	6	6	7	5	8	8

21. What is the experimental probability that all three of the coins will be heads? The theoretical probability?

SOLUTION:

The experimental probability is the ratio of the number of times the favorable event occurs to the total number of trials.

The total number of trials is 50 and “three heads” occurred 5 times.

$$\begin{aligned} \text{experimental probability} &= \frac{5}{50} \\ &= \frac{1}{10} \\ &= 10\%. \end{aligned}$$

The probability of an event A is the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

There are 8 possible outcomes and the only favorable outcome is HHH.

$$\begin{aligned} P(3\text{ heads}) &= \frac{1}{8} \\ &= 12.5\%. \end{aligned}$$

22. What is the experimental probability that at least two of the coins will be heads? The theoretical probability?

SOLUTION:

The experimental probability is the ratio of the number of times the favorable event occurs to the total number of trials.
The total number of trials is 50 and at least two heads occurred 5 + 5 + 6 + 6 = 22 times.

$$\text{experimental probability} = \frac{22}{50} = \frac{11}{25} = 44\%.$$

The probability of an event A is the ratio of the number of favorable outcomes to the total number of outcomes.

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

There are 8 possible outcomes and there are 4 favorable outcomes HHH, HHT, HTH, and THH.

$$P(\text{at least 2 heads}) = \frac{4}{8} = \frac{1}{2} = 50\%.$$

23. **DECISION MAKING** You and two of your friends have pooled your money to buy a new video game. Describe a method that could be used to make a fair decision as to who gets to play the game first.

SOLUTION:

Sample answer: Assign each friend a different colored marble: red, blue, or green. Place all the marbles in a bag and without looking, select a marble from the bag. Whoever’s marble is chosen gets to go first.

24. **DECISION MAKING** A new study finds that the incidence of heart attack while taking a certain diabetes drug is less than 5%. Should a person with diabetes take this drug? Should they take the drug if the risk is less than 1%? Explain your reasoning.

SOLUTION:

Sample answer: With either a less than 5% or 1% chance of having a heart attack, a person would still need to weigh the benefits of the drug versus the small chance of having a heart attack. A chance of less than 1% versus a chance of less than 5% should make a person who is considering taking the drug more likely to risk taking the drug to control their diabetes.