Mathematics Education in the United States 2012

A Capsule Summary Fact Book

Written for

The Twelfth International Congress on Mathematical Education (ICME-12)

SEOUL, KOREA • JULY 2012

by
John A. Dossey
Katherine Taylor Halvorsen
Sharon Soucy McCrone

under the Auspices of the
National Council of Teachers of Mathematics
and the
United States National Commission on Mathematics Instruction

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The preparation, production, and dissemination of this document were funded by National Science Foundation under EHR Grant: DRL 112759 to the National Council of Teachers of Mathematics, Gail Burrill, Principal Investigator. The opinions expressed herein are those of the authors and do not necessarily reflect the views of the National Science Foundation, the National Council of Teachers of Mathematics, or the U.S. National Commission on Mathematics Instruction.

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Library of Congress Cataloging-in-Publication Data

Dossey, John A.
Mathematics education in the United States 2012 : a capsule summary fact book written for the Twelfth International Congress on Mathematical Education (ICME-12) Seoul, Korea, July 2012 / by John A. Dossey, Katherine Taylor Halvorsen, Sharon Soucy McCrone ; under the auspices of the National Council of Teachers of Mathematics and the United States National Commission on Mathematics Instruction.
p. cm.
Includes bibliographical references.
ISBN 978-0-87353-721-6
1. Mathematics—Study and teaching—United States. I. Halvorsen, Katherine. II. McCrone, Sharon. III. International Congress on Mathematical Education (12th : 2012 : Seoul, Korea) IV. National Council of Teachers of Mathematics. V. United States National Commission on Mathematics Instruction. VI. Title.
QA13.D724 2012
510.71'073--dc23

2012015868

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Printed in the United States of America

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Preface

On January 1, 2012, the population of the United States was approximately 314,232,000. Approximately 18% of these individuals were formally enrolled in an elementary or secondary school, while nearly another 7% were enrolled as students in a degree-granting postsecondary institution. In the entire population, about 64% were aged twenty-five years or older, and of those adults, 87% had completed high school or its equivalent, and about 30% had at least a bachelor's degree (Snyder and Dillow 2011).

No single government agency controls public education in K–grade 12 in the United States. Rather, authority for most educational decisions lies with education agencies in the fifty individual states, which in turn share decision making with the slightly less than 14,000 individual school districts within them. In 2009–10, U.S. public schools accounted for 49,373,307 students, private elementary and secondary schools contributed another 4,700,119 students, and another 1,508,000 students were homeschooled (Broughman, Swaim, and Hryczaniuk, 2011; Snyder and Dillow 2011). Similarly, both public and private institutions exist at the college and university level, with ultimate authority residing at the state level for public institutions and at the institutional level for most private institutions. In the 2009–10 academic year, 4,495 accredited institutions offered degrees at the associate's level or above. These included 1,672 public institutions, 1,624 private not-for-profit institutions, and 1,199 private for-profit institutions. Of the 4,495 institutions, 2,774 awarded degrees at the bachelor's level or higher, and 1,721 offered associate's degrees as their highest degree awarded (Snyder and Dillow 2011).

Determining what is happening in such a large and complex arena is quite difficult, even for those in the United States and others familiar with education in the United States. Furthermore, many at conferences of the International Congress on Mathematical Education (ICME) lack familiarity with education in the United States. Consequently, in 1999, the U.S. National Commission on Mathematics Instruction recommended that the National Council of Teachers of Mathematics (NCTM) request funds from the National Science Foundation (NSF) to bring together available data about mathematics education in the United States for a document to be distributed at the Ninth International Congress on Mathematical Education (ICME-9) in 2000 to provide mathematics educators throughout the world with information about this complex system. This process was repeated in 2004 for ICME-10 and in 2008 for ICME-11, and this publication for ICME-12 now extends the series with information available as of early 2012.

This document begins with some general information about education in the United States. The three kinds of curricula identified in the Second International Mathematics Study—intended, implemented, and attained—are then described (McKnight et al. 1987). A special focus is given to the emergence of a common K–grade 12 curriculum that has been adopted by forty-five states and the District of Columbia. This curriculum, the Common Core State Standards for Mathematics (CCSSM), was developed by a consortium consisting of state governors and chief state education officers (National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA Center and CCSSO] 2010). The adoption of such a set of common outcomes, matching assessments, and similar instructional materials is expected to bring to U. S. mathematics education a level of uniformity that it has never before seen.

As in earlier editions, this publication has sections dealing with programs for high-achieving students, programs for mathematics teacher education, and resources for additional information about U.S. mathematics education. One message that comes through repeatedly in these descriptions is the variety of available programs and thus the inability to characterize them adequately in a brief document like this one. Another message is that all levels of the educational system exhibit great flux, and even though we have attempted to provide the latest available information, we realize that the information presented here will quickly become dated. By listing our sources, we hope to enable the interested reader to obtain updated information.

We would like to acknowledge the efforts of Gail Burrill, who wrote the proposal for the grant under which the funding for this publication was obtained; the insightful and constructive advice of Richelle (Rikki) Blair, Mark Ellis, Francis (Skip) Fennell, Natalie Jakucyn, Patrick (Rick) Scott, and the fine work of Anita Draper and Randy White at NCTM in editing and producing this document. We have tried to be as accurate as we could and apologize for any errors.

PART I: General Information

We begin by offering general information about education in the United States to provide background for our subsequent, more detailed examination of mathematics education in the United States.

Overall Organization of Education in the United States

Figure 1 presents a graphical overview of the structure of education in the United States. The system can be thought of as consisting of four broadly defined levels: elementary school (K–grade 5 or K–grade 6, corresponding to ages 5–10 or 11); middle school or junior high school (grades 6–8 or 7–8, ages 11–13 or 12–13); senior high school (grades 9–12, ages 14–17); and postsecondary, or tertiary, education (grades 13 and above, ages 18 and older). There are variants of the ending and beginning points of each of the levels, owing to state and local school system regulations and preferences.

Students are legally required to start and maintain enrollment in formal education by state-mandated ages. The minimum compulsory ages range from 5 to 8 years (age 5, 8 states; age 6, 24 states; age 7, 16 states; and age 8, 2 states). Standards for the length of compulsory education also vary by state, with minimum allowed school-leaving ages of 16 to 18 (age 16, 19 states; age 17, 11 states; and age 18, 20 states). However, state standards in nearly half of the states allow for variances in their regulations for students who are employed, have a physical or mental condition that makes attendance infeasible, have passed eighth grade successfully, or have the permission of their parents, district court, or school board (Bush 2010). Not all students complete secondary education prior to leaving formal education.

Education is compulsory by law in all states from age 5 to 8 through at least age 16 (Snyder and Dillow 2011). Although the law requires compulsory education, it also allows for homeschooling of students by their parents. The percentage of students completing a public school education can be quantified in many ways. The *average freshman graduation rate* provides an estimate of the proportion of public high school students who graduate from high school four years after having entered the ninth grade. As such, it provides a picture of the percentage of students completing the secondary school program on schedule. Of those who entered high school as ninth graders in the fall of 2004, 74.9% graduated (finished twelfth grade) in the spring of 2008 (Chapman, Laird, and Kewal-Ramani 2010), an improvement of 2.3% since the spring of 2002. Many who do not complete high school with their class earn equivalent diplomas later. The *status completion rate*, another completion ratio, provides data on the percentage of people by age ranges who are not attending a secondary school but who have a high school diploma or have completed a high school equivalency program, irrespective of when either path to completion was accomplished. In the eighteen- through twenty-four-year-old age group in 2008, 89.9% had achieved the status completion rate, compared to 86.6% in 2002. Gender comparisons showed that 90.5% of females and 89.3% of males had achieved a high school diploma or its equivalent, but major differences exist among racial or ethnic subgroups: 95.5% for Asians/Pacific Islanders, 94.2% for both Whites and people of two or more races, 86.9% for Blacks, 82.5% for American Indians/Alaskan Natives, and 75.5% for Hispanics (Chapman, Laird, and Kewal-Ramani 2010).

Students who graduate from high school may enter the workforce, attend a non-university tertiary institution focusing on technical or vocational education, attend a two-year

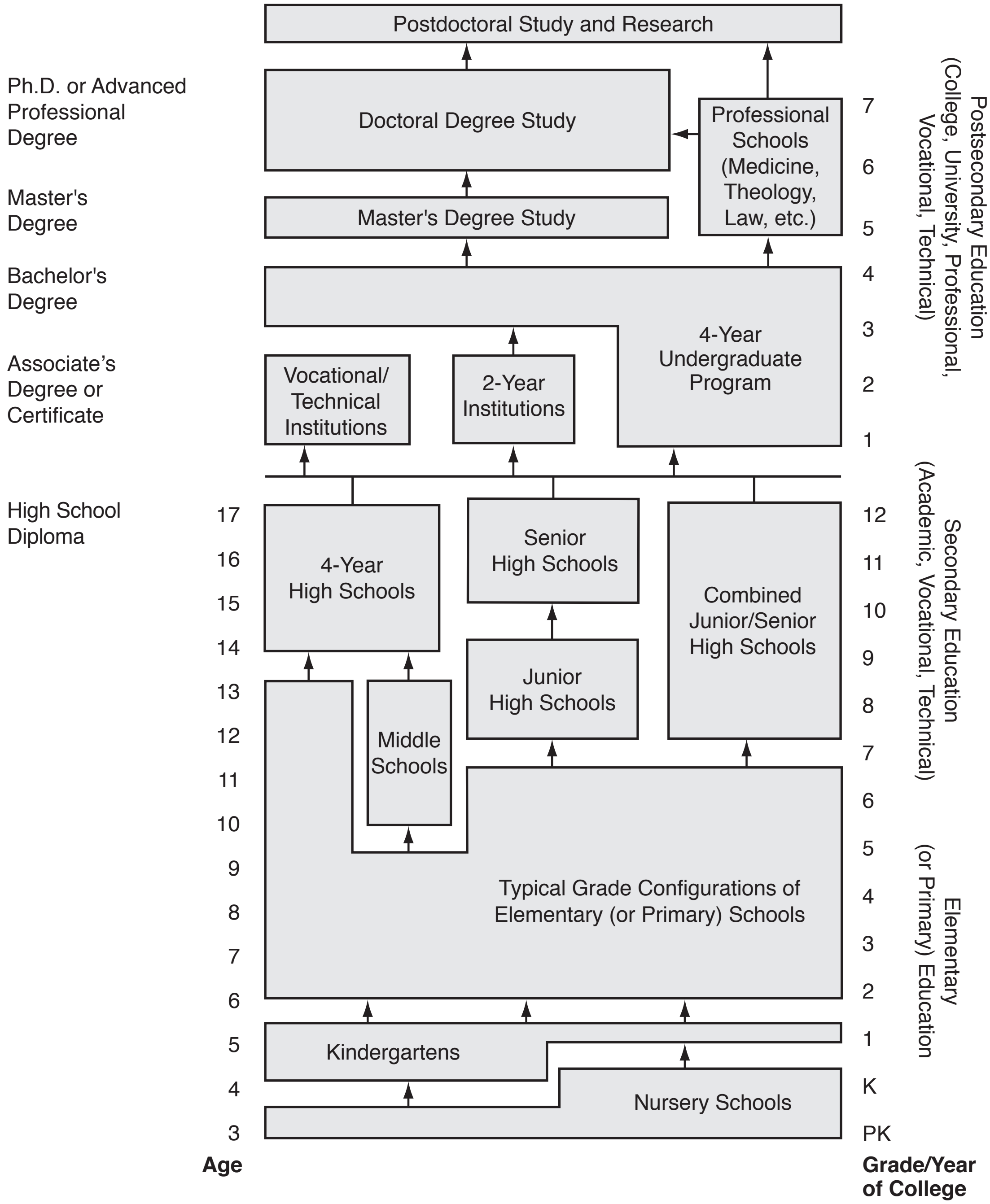


Fig. 1. The structure of education in the United States (Snyder and Dillow 2011)

community college, or attend a four-year college or university. Two-year and community colleges usually offer diverse selections of courses and programs, including the first two years of a four-year college's curriculum along with a number of courses also found in the technical colleges and high schools. In these two-year or community colleges, an associate of arts (AA), associate of sciences (AS), or associate of applied sciences (AAS) degree can usually be earned through the equivalent of two years of full-time study. One-year certificate programs are also offered in various technical fields. In addition, there are a number of vocational or trade schools where students focus on the knowledge and skills needed to perform a particular job. Vocational schools may be integrated with public schools as part of programs that facilitate the transition from school to work. In other instances, these schools are private schools, nonprofit or proprietary, operated outside the public school system. The foci of these schools range from apprenticeship programs for trades to culinary institutes.

The four-year colleges and universities offer bachelor of science (BS) and bachelor of arts (BA) degrees, which can typically be completed in four years of full-time study. In addition, many universities offer graduate programs leading to master's (MS, MA, or MEd) and doctoral (EdD and PhD) degrees. Programs leading to professional degrees (law, medicine, business, etc.) exist both in universities and in institutions that offer no other degree programs. Time to complete post-bachelor degrees varies with the field and institution.

Educational Enterprise

The delivery of elementary and secondary education through the structures shown in figure 1 is a complex enterprise. In the 2009–10 school year, 98,817 public elementary or secondary schools were in operation. Most (89,072) were focused on the standard curriculum, whereas 1,417 provided targeted vocational or technical education, 2,089 provided special education services, and 6,239 offered some alternative form of education. Included in this total number of schools supported by public funding were 4,952 charter schools and 2,213 magnet schools (Chen 2011a, 2011b). Charter schools are public schools that are allowed to operate with freedom from many of the regulations that apply to traditional public schools. Magnet schools are public schools whose curricula address the standard requirements but provide targeted and advanced instruction in such areas as mathematics, science, or the arts. Besides the public schools at the elementary and secondary levels, approximately 33,366 private schools were in operation. The organizing structure of these schools were 21% Catholic, 47% other religious bodies, and 32% nonsectarian schools (Broughman, Swaim, and Hryczaniuk 2011).

In 2009–10, U.S. K–12 public schools accounted for 49,373,307 students (Snyder and Dillow 2011), private elementary and secondary schools contributed another 4,700,119 students (Broughman, Swaim, and Hryczaniuk 2011), and homeschooling accounted for approximately 1,508,000 students (Snyder and Dillow 2011). Thus, slightly more than 55,580,000 students were involved in K–12 education. At the collegiate level, enrollments in 2009–10 started out at 14,810,642 students enrolled in public degree-granting institutions and 5,617,096 in private institutions. Overall, 36% of these were enrolled in two-year colleges, while the remaining 64% were enrolled in four-year colleges or universities. As table 1 suggests, the number of students in U.S. schools has risen steadily since 1985 (Snyder and Dillow 2011).

Projections through 2019 show the total number of students in grades K–12 public schools continuing to increase. Total public and private elementary and secondary school enrollment reached 55 million in 2005, representing a 22% increase since fall 1985. Between fall 2005 and fall 2019, a further increase of 5.5% is expected, with increases projected in both public schools and in private schools. Increases in public school enrollment are expected for the proportions of Hispanics, Asians/Pacific Islanders, and American Indians/Alaska Natives, and decreases are expected in the proportions of Whites and Blacks. Increases in public school enrollment are expected in the South and West, a decrease is expected in the Northeast, and the Midwest is expected to remain steady (Aud et al. 2011, Hussar and Bailey 2011).

Table 1

School and college enrollments and projections over time (in millions)

Type	Year							
	1985**	1990**	1995**	2000**	2005**	2010***	2015***	2019***
K–12 public	39.4	41.2	44.8	47.2	49.1	49.4	50.9	52.3
K–12 private*	5.6	5.6	5.9	6.2	6.1	6.0	6.0	6.2
Postsecondary	12.2	13.8	14.3	15.3	17.5	20.6	22.1	23.4

*Nongovernmental, including parochial schools (governed by religious bodies)

**Snyder and Dillow 2011; Broughman, Swaim, and Hryczaniuk 2011; Hussar and Bailey 2011

***Projections in last three columns since official statistics trail calendar by four years.

Admission to Postsecondary Institutions

Graduates of public or private senior high schools may matriculate to the nation's colleges, but they must apply to individual schools to be considered for admission. Most two-year colleges will accept any secondary school graduate from the geographic area that they serve. Some two-year colleges and most four-year colleges require students to have taken certain numbers of courses in English, mathematics, science, social studies, and foreign language for admission. Many state-supported institutions have formulas for admission that may take into consideration the intended field of study, secondary school course grades, percentile rank in class, scores on college entrance examinations, letters of recommendation, sports and other extracurricular activities, and other information supplied by the high school or the students. Private colleges use some of the same criteria as public institutions but may take other factors, such as family variables, into consideration. More selective schools also consider the difficulty of courses taken in high school and scores that applicants have earned on recognized nationwide standardized examinations, some of which can provide advanced placement on entry to college or even college credit.

The mean costs of college attendance, including tuition, fees, room, and food at four-year public and private colleges in 2009–10 were $15,212 and $35,464, respectively (College Board 2010e). Many students receive scholarships and other types of financial aid from various sources, including the college that they attend, government sources, or private foundations. The College Board scholarship database has more than 2,300 programs that award nearly $3 billion in support of 1.7 million awards annually (College Board 2010d, 2010f). In 2003−4, 63% of all undergraduate postsecondary students received financial aid (grants, loans, or student work programs). The federal government offers tax credits for tuition and certain other education expenses (Bersudskaya and Wei 2011). The costs of attending two-year colleges varies widely, depending on the program selected by a student. In some cases, almost all expenses are borne by the local taxing district; in other cases, the costs are equivalent to those of a public four-year college or university.

From 1990 to 2009 proportionally more college students became enrolled full-time on two-year and four-year campuses (58.3% in 1990, 60.2% in 2000, and 63.4% in 2009). Women overtook men as the majority of college attendees in 1979 and accounted for 56.1%, 57.2%, and 56.8% of postsecondary enrollments in 2000, 2005, and 2009, respectively. Recent data suggest that the percentages of students from underrepresented groups enrolling in two- or four-year programs are increasing. The percentage of Whites declined from 81.4% in 1980 to 62.3% in 2009. Over the same period, the percentages increased for Blacks from 3.9% to 14.3%, for Hispanics from 3.9% to 11.9%, for Asian/Pacific Islanders from 2.4% to 6.8%, and for American Indians/Alaska Natives from 0.7% to 1.0% (Snyder and Dillow 2011). Matriculation to college does not have to occur immediately after high school graduation, nor does attendance in college need to be full-time. For financial and other reasons, many students delay college study, either altogether or by attending only part-time (Horn, Cataldi, and Sikora 2005).

The Governance of Mathematics Education: "No Child Left Behind"

Because the Constitution of the United States does not claim education as a responsibility of the federal government, individual states have considerable leeway in structuring the education of their students. State laws define the boundaries for the compulsory education of students; outline the general framework for required studies in reading, writing, mathematics, science, social science, physical education, and other subjects; define the minimum number of days of school attendance per year; and define the standards for teacher certification and professional development. These laws, however, present little or no regulation or monitoring for homeschooling. State laws also provide the mechanisms by which local schools are recognized by the state government and provide statutes for the founding and accreditation of private schools. In like manner, states have considerable leeway in waiving regulations for charter schools in lieu of their achieving other goals as defined in their charter. These schools thus receive public funds but are not responsible for meeting all the regulations binding other public schools in the state or district.

The United States Department of Education sets standards and provides federal funding for special programs, such as school lunch programs for students in poverty and compensatory programs for students needing special educational assistance. The role of the federal government in education has increased markedly since the establishment of the No Child Left Behind Act (NCLB), passed by Congress in 2001. NCLB authorizes the U.S. Department of Education to manage a program that provides financial incentives for schools with good performance profiles and penalties for schools with poor performance records. The program is unprecedented in the nation's history (U.S. Department of Education 2008).

Three days after taking office in January 2001, President George W. Bush announced No Child Left Behind, his framework for education reform that he described as "the cornerstone of my administration" (Bush 2001). Less than a year later, Congress passed the No Child Left Behind Act of 2001. NCLB has four main thrusts: increased accountability for states, school districts, and schools; greater choice for parents and students, particularly those attending low-performing schools; more flexibility for states and local educational agencies (LEAs) in the use of federal education dollars; and a stronger emphasis on reading, especially for the youngest children. The disaggregation of state and local data required by NCLB mandates that all students, and in particular, special education students of various types, receive a high-quality mathematics education. In short, the success of *all students* does truly mean a focus on *all*. The summary that this report gives is taken from the Executive Summary written by the Department of Education (U.S. Department of Education 2008).

Increased Accountability

NCLB requires all publicly funded schools to implement programs designed to meet accountability systems defined by their state's department of education. These systems must be based on challenging state standards in reading and mathematics, include annual testing for all students in grades 3–8, and set out annual statewide progress objectives ensuring that all groups of students reach proficiency by 2014. Assessment results and state progress objectives must be achieved by all student groups defined by poverty levels, race/ethnicity, disability, and limited English proficiency to ensure that no group is left behind. School districts and schools that fail to make adequate yearly progress (AYP) toward achieving these statewide proficiency goals will, over time, be subject to improvement measures, corrective action, or restructuring measures aimed at getting them back on course to meet state standards. Schools that meet or exceed AYP objectives or close achievement gaps will be eligible for state academic achievement awards. At present, states are able to apply for waivers from the 2014 deadlines for meeting some of the NCLB requirements, but it is not clear whether or when Congress will act to change the overall structure of the law itself.

Federal corrective or restructuring actions include the following consequences for schools failing to meet state student achievement standards:

- **Year 1 of Missing AYP:** A school that misses AYP for one year faces no consequences.
- **Year 2 of Missing AYP:** If a school misses AYP for a second consecutive year, it is identified as "in need of improvement." The school must develop a two-year improvement plan in consultation with parents, school staff, and the school district. The plan should address core academic subjects and any specific subjects in which the school is struggling to make adequate progress. Students enrolled in the school now have the option to transfer to another school within the school district that has not been identified as "in need of improvement." Priority is given to the lowest-achieving students from low-income families enrolled in the school.
- **Year 3 of Missing AYP:** If a school misses AYP for a third consecutive year, the school must continue to offer students the option to transfer to another school and must offer tutoring and other "supplemental education services" to students.
- **Year 4 of Missing AYP:** If a school misses AYP for a fourth consecutive year, the school is identified for "corrective action." Corrective action involves more serious steps to improve the school's academic performance. Steps can include replacing staff, introducing new curricula, bringing in outside consultants to help with school performance, extending the school day or year, or changing the management structure of the school.
- **Year 5 of Missing AYP:** If, after a full year of corrective action, a school misses AYP for a fifth consecutive year, the school will be placed under "restructuring." The school must prepare a plan for an alternative governance arrangement, which can include reopening the school as a charter school, contracting management to a private, outside management group, turning the school over to the state for reorganization, or any other changes to school governance that "make fundamental reforms."
- **Year 6 of Missing AYP:** If the school misses AYP for a sixth consecutive year, it must implement the restructuring plan developed in the prior year (Dillon and Rotherham 2007, p. 3).

Because state standards for AYP vary widely and different states allow specific exceptions as student population parameters change over years, it is difficult to monitor the proportion of the nation's school systems that fall in each of the AYP categories (Usher 2011). The Center on Education Policy (CEP) data on the nation's schools, released in December 2011, provide perhaps the best picture of the situation at the end of 2010–11 (Usher 2011). This report provides estimates of the number of schools within states that had not met the AYP targets for the 2010–11 school testing period. Usher reports that in 2010–11, in twenty-four states at least 50% of the schools did not make AYP. In a majority of the states, at least 25% of the schools did not make AYP. There was great variability in the percentage of public schools failing to make AYP in 2010–11, ranging from 11% in Wisconsin to 89% in Florida. Combining these state-level school reports into a national report, the CEP estimates that 48% of the nation's public schools failed to make AYP in 2010–11. These data provide a signal of the severity of the achievement problems in the nation's schools. However, comparisons among states are difficult because of varied standards for AYP. Further, states differ in the ways in which school districts are formed and grouped into districts. This further complicates numerical analyses of the status of the nation's schools relative to the NCLB strictures (Usher 2011).

In December 2011, Secretary of Education Arne Duncan was striving to provide relief to states and thus to their schools through the issuance of waivers from the NCLB strictures. However, application for the waivers requires states to submit plans for teacher and principal evaluations based on student test scores, embrace the Common Core State Standards, and develop new standards for college and career readiness (Cody 2011). As of the end of 2011, two of the larger states—California and Texas—had not yet submitted waivers, and others submitting waivers were opposed to the conditional requirements outlined above (CEP 2011).

NCLB and Teacher Quality

The No Child Left Behind Act of 2001 combines the Eisenhower Professional Development and Class Size Reduction programs into a new Improving Teacher Quality State Grants program that focuses on using practices grounded in scientifically based research to prepare, train, and recruit high-quality teachers. The new program gives states and local education authorities (LEAs) flexibility to select the strategies that best meet their particular needs for improved teaching to help them raise students' achievement in the core academic subjects. In return for this flexibility, LEAs are required to demonstrate annual progress in ensuring that all teachers of core academic subjects within the state are highly qualified.

One highly focused research study of teacher quality, as defined by NCLB, indicated that the NCLB definition of teacher quality is simply a mandate for change that emphasizes teachers' content knowledge over their knowledge and skills in pedagogical situations. Further, the study indicated significant discrepancies in certification, training, and administration of the teaching profession across different states (Smith and Gorard 2007). A study conducted by CEP (2007), involving a nationally representative sample of 349 school districts taken from the fifty states, combined with interviews of local administrators and focus groups of representative teachers, found little support for the NCLB teacher quality requirements. Data from the states and districts suggested minimal to no correlation between "teacher quality" and student achievement. However, the study did indicate that the distribution of "highly qualified teachers" was becoming more equitable in many districts with significant numbers of minority or poverty-affected students (CEP 2007).

Reactions to NCLB

NCLB remains the most discussed piece of federal legislation a decade after its implementation. Although the need to develop some form of accountability system that monitors K–12 public education is generally accepted, neither the public nor those involved with the operations of K–grade 12 systems believe that the present formulation of NCLB is the best vehicle for reaching that goal. Opponents of NCLB point to the facts that the measurement of students' capabilities in an academic area is based on a single assessment and that the sanctions implemented by the program do not directly address the improvement of and support for the educational programs that provide instruction for the affected children. Although politicians and educators responsible for enacting changes to the law admit that it has significant problems and needs a significant overhaul, they remain far apart in how a reshaping of the legislation should take place.

The results of the *43rd Annual Phi Delta Kappa/Gallup Poll of the Public's Attitudes toward the Public Schools* reflect a similar dissatisfaction with the outcomes associated with NCLB. Globally, the June 2011 respondents reported a high belief in teacher quality, along with responses indicating that they—

- would select higher quality instruction delivered on the Internet over learning in a classroom with a less qualified teacher (50% to 49%);
- think that teaching as a career is a laudable and needed profession—even for their own children (67% to 31%);
- approve of charter schools (70% to 27%) and want choice over which school their children could attend (74% to 25%) but disapprove of allowing private school attendance at public expense (65% to 34%);
- would allow teachers flexibility over requiring them to follow a prescribed curriculum (73% to 26%).

The survey findings suggest that the American public believes that investment in improving teacher quality, involving content knowledge and pedagogical knowledge as well as the capability to use both effectively, has the potential for strengthening student achievement (Bushaw and Lopez 2011).

Mathematics Education in U.S. Schools

The description of mathematics education as an enterprise in U.S. schools is more difficult than the description of education as a whole, because education in mathematics is, in most instances, left to the control of a locally elected board of education in each school district. Each district, operating under its own authority and various state laws, sets standards for, designs delivery programs for, and provides financial support for its own mathematics education program. Given that the United States had 13,867 regular public school districts in 2009–10 along with another 2,356 charter schools, the views of mathematics and its goals and the amount of resources expended on mathematics instruction are diverse and varied. The growing number of students who are homeschooled for all or a portion of their K–12 education only compounds this diversity (Keaton 2011; Snyder and Dillow 2011).

The guidance that states provide to schools in their jurisdictions also varies. All fifty states have standards or curricular frameworks for mathematics as part of their required programs of study. These standards or frameworks for mathematics take a variety of forms in outlining what students should know and be able to do as a result of their study of school mathematics. In most states, the curricular programs of study are not binding, but they may define the boundaries for state assessment programs in mathematics, outlining what mathematical knowledge is expected of students by a certain grade level. At least forty-six states also employ resource individuals (state mathematics supervisors or consultants) who consult with schools on questions concerning the classroom teaching and learning of mathematics and on issues concerning statewide assessment programs. In the end, however, the decisions made in a local school district determine the actual content of the school mathematics program within that district. Research studies have shown that school curricular decisions are influenced by a number of factors, which include the adopted textbook series (Porter et al. 1988) and state and national standards (Reys 2006). The Common Core State Standards Initiative is a current nationwide movement to establish a common curriculum for mathematics within the nation's schools. At present, forty-five states and the District of Columbia have indicated that they have adopted the Common Core Standards for language arts and mathematics (NGA Center and CCSSO 2010a). The realization of this goal will establish a de facto national curriculum for school mathematics (Porter et al. 2011). More details of the Common Core State Standards initiative are provided in parts III, IV, and V.

The picture is much the same at the college and university level. The programs of study that U.S. students complete under the name of mathematics vary greatly. Yet, a great deal of similarity exists in the mathematics curricula offered by schools and universities. Part of this similarity is the result of core recommendations for study in mathematics issued explicitly by state governments and professional societies and implicitly by commercial textbooks and examinations.

PART II: Intended Curriculum in an Age of Standards

The release of NCTM's *An Agenda for Action* in 1980 set the stage for a proactive era of professional input to the reform of mathematics education in the United States. The development of standards in that era had a significant impact on the school mathematics curriculum.

Major Reform Recommendations and Standards Documents since 1980

A decade after the publication of *An Agenda for Action*, NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989) provided focus to content standards across three grade bands (K–4, 5–8, and 9–12) for problem solving, communication, reasoning, and connections. The 1989 Standards were restated in 2000 to reflect the growth of knowledge about learning and practice over the intervening period of time.

This revision and updating, published as *Principles and Standards for School Mathematics* (NCTM 2000), introduced new grade bands of suggested content (pre-K–2, 3–5, 6–8, and 9–12); added representation to the group of mathematical processes addressed in the Process Standards; made specific suggestions for content considerations within grade bands; and merged the previous NCTM Standards on teacher education, professional development, assessment, and evaluation with curricular recommendations.

In 2006, NCTM made the link between curriculum and anticipated outcomes even more specific with its spelling out of grade-level focal points for content consideration in *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence.* In 2009, NCTM completed the work through the secondary level by offering guidance for grades 9–12 mathematics with *Focus in High School Mathematics: Reasoning and Sense Making.*

In 2010, the Council of Chief State School Officers and the National Governors Association Center for Best Practices released *Common Core State Standards for Mathematics* (CCSSM). A grade-by-grade listing of standards and related expectations for grades K–8 and high school standards organized by conceptual categories define the mathematics that students should know to be college or career ready. The Standards for Mathematical Practice complement the content domains and standards for K–8 and the conceptual categories at the high school level by describing ways in which students engage in mathematics. The Standards for Mathematical Practice are based on the NCTM Process Standards (NCTM 2000) and the levels of mathematical proficiency described in *Adding It Up* (Kilpatrick, Swafford, and Findell 2001). The intent of the internationally benchmarked CCSSM was to engage the mathematical community at large and move mathematics education programs across the nation toward a single set of standards. The endeavor has also attempted to add rigor and craft clearer statements of expected outcomes for school mathematics. The nature of these standards and their implementation are discussed later in this book. A reflective look at the period from 1980 to 2010 shows that the shaping of the intended curriculum and the monitoring of its progress during this period of time were influenced by several major reports, along with a growing body of research studies and curricular development and evaluation programs financed by the National Science Foundation (NSF). Among the most influential reports were the following:

1980: NCTM, *An Agenda for Action*

1982: Conference Board of the Mathematical Sciences (CBMS), *The Mathematical Sciences Curriculum K–12: What Is Still Fundamental and What Is Not*

1983: National Commission on Excellence in Education, *A Nation at Risk*

1987: Second International Mathematics Study (SIMS), *The Underachieving Curriculum: Assessing U.S. School Mathematics from an International Perspective* (McKnight et al. 1987)

1989: National Research Council (NRC), *Everybody Counts*

1989: NCTM, *Curriculum and Evaluation Standards for School Mathematics*

1990: National Research Council (NRC), *Reshaping School Mathematics*

1991: NCTM, *Professional Standards for Teaching Mathematics*

1995: NCTM, *Assessment Standards for School Mathematics*

1995: American Mathematical Association of Two-Year Colleges (AMATYC), *Crossroads in Mathematics: Standards for Introductory College Mathematics*

1995–96: *TIMSS: Third International Mathematics and Science Study* (follow-up reports in 2000, 2004, 2008, …)

2000: NCTM, *Principles and Standards for School Mathematics*;
National Commission on Mathematics and Science Teaching for the 21st Century (Glenn Commission), *Before It's Too Late*

2001: CBMS, *The Mathematical Education of Teachers*

2001: NRC, *Adding It Up: Helping Children Learn Mathematics* (Kilpatrick, Swafford, and Findell 2001)

2004: Organization for Economic Cooperation and Development (OECD), *Learning for Tomorrow's World: First Results from PISA 2003* (follow-up reports in 2007, 2010, 2013, …)

2004: OECD, *Problem Solving for Tomorrow's World: First Measures of Cross-Curricular Competencies from PISA 2003*

2004: Mathematical Association of America (MAA) Committee on the Undergraduate Program in Mathematics (CUPM), *Undergraduate Programs and Courses in the Mathematical Sciences*

2005: American Statistical Association (ASA), *Guidelines for Assessment and Instruction in Statistics Education: A Pre-K–12 Curriculum Framework* (GAISE Report) (Franklin et al. 2005)

2006: AMATYC, *Beyond Crossroads: Implementing Mathematics Standards in the First Two Years of College*

2006: College Board, *College Board Standards for College Success: Mathematics and Statistics*

2006: NCTM, *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*

2007: NCTM, *Mathematics Teaching Today: Improving Practice, Improving Student Learning*

2008: National Mathematics Advisory Panel, *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*

2009: NCTM, *Focus in High School Mathematics: Reasoning and Sense Making*

2010: ASA, *Guidelines for Assessment and Instruction in Statistics Education: College Report* (Aliaga et al. 2010)

2010: National Governors Association Center for Best Practices (NGA Center) and Council of Chief State School Officers (CCSSO), *Common Core State Standards for Mathematics*

National Council of Teachers of Mathematics (NCTM) Standards and Focal Points

As the major professional organization dedicated to K–12 mathematics education, NCTM has contributed documents to the list above that provide an evolving picture of the intended school mathematics curriculum. Other professional organizations in the mathematical sciences added to this picture with supporting documents, such as ASA with the GAISE report, CBMS with *The Mathematical Education of Teachers*, and the MAA with *Undergraduate Programs and Courses in the Mathematical Sciences.* Results from the domain of comparative assessments of educational outcomes, like NAEP (National Assessment of Educational Progress, conducted by the U.S. Department of Education), TIMSS (Trends in International Mathematics and Science Studies, administered by the International Association for the Evaluation of Educational Achievement [IEA]), and PISA (Program for International Student Assessment, administered by the Organization for Economic Cooperation and Development [OECD]), provided information on the degree to which professional recommendations were reaching the classroom and the nature of and rate of change in outcomes that might be traced to such recommendations.

NCTM, with its 1989 release of *Curriculum and Evaluation Standards*, provided a path to reform in school mathematics. These initial standards provided broad recommendations for content to be implemented in K–grade 4, grades 5–8, and grades 9–12, as well as suggestions on how such mathematics teaching and learning might be evaluated. This effort opened a national conversation and guided initial steps toward developing and implementing curricula with more focus and a greater balance between conceptual and process-oriented topics, while maintaining the core focus on procedural topics. Recommendations for teachers' preservice and professional development followed, along with an in-depth look at assessment and the roles that it should play in learning and evaluation.

However, it was not until NCTM's 2006 publication of *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* that the Council made specific recommendations reflecting grade-level placement of topics and the major focal points within the teaching of the topics at those grade levels. This was followed in 2009 by the release of *Focus in High School Mathematics: Reasoning and Sense Making*, which details the role that understanding and sense making and the related processes of reasoning, justification, and proof play in secondary school mathematics. This set of recommendations, rather than providing grade-level content listings, indicates how the foci of sense making and reasoning can strengthen the teaching of the major curricular areas of number and measurement, algebra, functions, geometry, and statistics and probability. Although the foci identified for high school are less prescriptive than the pre-K to grade 8 focal points, they provide guidance to policy makers, curricular development staff, and teachers in continuing the movement toward a common curriculum for U.S. schools.

Supporting Standards: The College Board and the American Statistical Association

Moving parallel to NCTM's development of the pre-K to grade 8 focal points, the American Statistical Association (ASA) developed and released *Guidelines for Assessment and Instruction in Statistics Education: A Pre-K–12 Curriculum Framework* (GAISE report) in 2005, and the College Board released *Standards for College Success: Mathematics and Statistics* in 2006. Both of these documents grew out of the perceived need to move recommendations of content considerations closer to the grade levels, along with giving more specific progressions for the development of central concepts.

In the case of the ASA recommendations, school-level statistical learning outcomes had always received short shrift until the NCTM Standards gave them more focus. The GAISE report further sharpened the major concepts and provided structure and pathways to learning for both students and teachers. Most important, the document provides grounding for the statistical content appropriate to K–12 educational programs and introduces levels of exposure to that content that reflect the developmental understanding of the role of variation in the learning and teaching of statistics. Further, content coverage shifts from a sole focus on univariate settings to bivariate contexts, while at the same time creating ties among statistics, algebra, and geometry. Further, the GAISE recommendations provide examples of projects, investigations, and applications that will help ease the way for moving the content into the school program.

The ASA document also does a nice job of delineating the differences between mathematics and statistics and in doing so defines the role of probability as it interacts with both. The GAISE document notes that "probability plays an important role in statistical analysis, but formal mathematical probability should have its own place in the curriculum. Pre-college statistics education should emphasize the ways in which probability is used in statistical thinking; an intuitive grasp of probability will suffice at these levels" (Franklin et al. 2005, p. 9).

At the same time that the ASA document was evolving, the College Board was moving forward with a document detailing the curriculum for grades 6 through 12 as a basis for the development of instructional and assessment materials. The board's equity outreach in urban education was requiring greater specificity in curricular outcomes for teacher professional development, especially in the area of diagnostic and remedial teaching efforts. In addition, such recommendations could provide a better grounding for the board's assessment and evaluation instruments in the school curriculum. *Standards for College Success: Mathematics and Statistics* (College Board 2006) appeared in three versions: a "traditional" approach starting with grade 6 mathematics and continuing to Advanced Placement Calculus in grade 12 by a route including algebra 1 in grade 8 and geometry, algebra 2 and trigonometry, and precalculus at the high school level prior to grade 12; a similar sequence using an integrated curriculum approach, in which the algebra 1 to precalculus courses are replaced by integrated mathematics 1–4; and a third path structured around a three-year sequence prior to algebra 1 and culminating in the study of precalculus in grade 12.

The committees developing the College Board documents and the NCTM Focal Points documents had overlapping membership. In addition, both had close connections with and advice from members of the ASA recommendations writing team. Of these documents, the one that provided the most comprehensive detailed curricular structure was the College Board document, which broke down the grades 6–12 curriculum into eight different strands:

- Operations and equivalent representations
- Algebraic manipulation skills
- Quantity and measurement
- Proportionality
- Relations, patterns, and functions
- Shape and transformation
- Data and variation
- Chance, fairness, and risk

The content of each of these strands was then analyzed and allocated to grade levels, with specified outcomes that would serve to define the development of curricular materials or assessments. The document further provided a role for the teaching of statistical concepts and skills, as well as probability, in the mainline program. These eight strands were specified by means of overarching standards, with three to five related objectives per standard. Each of the objectives then had two to eight performance expectations. For example, in the algebra 2 course, the standard related to systems of equations is stated as follows:

Standard AII.3
Systems of Equations and Inequalities and Matrices

Students construct, solve, and interpret solutions to systems of linear equations in two variables. Students represent cross-categorized data in matrices and perform operations on matrices to model and interpret problem situations. Students model and solve systems of equations with technology. (College Board 2006, p. 37)

Under this standard there are three objectives with performance expectations. The third one of these is the following:

Objective AII.3.3

Student multiplies matrices, verifies the properties of matrix multiplication, and uses the matrix form for a system of linear equations to structure and solve systems consisting of two or three linear equations in two or three unknowns, respectively, with technology.

Performance Expectations

AII.3.3.1 Verifies the properties of matrix multiplication, and multiplies matrices to solve problems.

AII.3.3.2 Constructs a system of linear equations modeling a real-world situation, and represents the system as a matrix equation (A$\boldsymbol{x}$ = **b**), that is,

$$\begin{matrix} ax + by = c \\ dc + ey = f \end{matrix} \quad \Leftrightarrow \quad \begin{bmatrix} a & b \\ d & e \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}.$$

AII.3.3.3 Solves a system consisting of two or three linear equations in two or three unknowns, respectively, by solving the related matrix equation A$\boldsymbol{x}$ = **b**, using technology to find $\boldsymbol{x} = A^{-1}\mathbf{b}$. (College Board 2006, p. 37).

NCTM's *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*, ASA's *Guidelines for Assessment and Instruction in Statistics Education: A Pre-K–12 Curriculum Framework,* and the College Board's *Standards for College Success: Mathematics and Statistics* moved the discussion of standards and their guidance of school mathematics forward with a call for a set of common standards for K–12 mathematics for the nation's public schools.

All the states had some form of state standards, but these standards exhibited significant inconsistencies in the grade levels at which specific topics were introduced and in the span of time allotted for students to develop mastery of the basic number facts and fluency with the algorithmic procedures for numerical calculations. In some cases, these levels were never specifically addressed, leaving the decisions to local districts or even schools (Reys 2006). It was this set of circumstances, together with concerns over school progress toward meeting NAEP-like accountability standards associated with the NCLB legislation, as well as the acceptance, by many states, of the NCTM Curriculum Focal Point recommendations and the related recommendations of the National Mathematics Advisory Panel (2008), which moved the Council of Chief State School Officers to launch the Common Core State Standards Initiative.

The Common Core State Standards

The Common Core State Standards for Mathematics (CCSSM) resulted from a state-led effort, initiated by state leaders, including governors and state commissioners of education. Working with a writing team consisting of mathematics content experts; mathematics educators and supervisors; assessment staff from ACT, the College Board, and Achieve; and experienced teachers, the Council of Chief State School Officers and the Center for Best Practices of the National Governors Association led the development of a K–12 curricular framework drawn from the best of state standards, international curricular frameworks, and research results concerning mathematics teaching and learning, as well as teachers' experiences.

The Common Core State Standards for Mathematics were released in June 2010, following a vetting by an advisory board and other selected members of the mathematics, mathematics education, and policy communities, with feedback from a large number of members of the public. To date, CCSSM has been adopted by forty-five states, the District of Columbia, and some of the U.S. territories as their official curriculum guide for mathematics. The Common Core State Standards include career-readiness standards and other standards that extend the study to provide readiness for university-level study of science, technology, engineering, mathematics, and other quantitatively rich disciplines, such as economics, finance, and emerging areas of the social sciences.

Built around a core of mathematical practices, CCSSM is focused on developing deep student understanding of a smaller set of outcomes than are currently contained in most state standards. This reduced number results from the fact that many objectives in these state standards are repeated in subsequent grades as the curricula on which they are built circle back to the same topics year after year. CCSSM tends to focus on core, or focal, topics for longer periods of study and then integrates further work with those topics with science and other disciplinary applications, both to show their importance and to anchor students' learning more firmly in modeling and the mathematical practices. CCSSM will be further explained and illustrated in part VI.

State Standards and Textbook Adoption Guidelines

As a result of the release of the Common Core State Standards for Mathematics, official state standards are now in flux, as forty-five states are adopting or adapting CCSSM as their state standards for mathematics. At this point, Alaska, Minnesota, Nebraska, Texas, and Virginia have not yet adopted CCSSM. In the adopting states, CCSSM replaces standards that had been crafted over the period from the 1983 report *A Nation at Risk* (National Commission on Excellence in Education) and evolved through revisions in response to NCTM's sequence of recommendations in *Principles and Standards for School Mathematics* (NCTM 2000). The format and language in these existing state standards were similar, in many cases, to that found in the NCTM Standards (2000) and in the majority of the extant instructional materials. As such, a shift to CCSSM in many states is akin to a discontinuity in the curriculum process, even though CCSSM was in circulation and under discussion for a period of more than a year.

The Common Core State Standards for Mathematics arrived without supporting curricular materials, shifting a significant body of number and operation and algebra material downward through the grades, eliminating particular topics from grade levels and shifting some material related particularly to data analysis and probability upward, as well as identifying a need for new teacher skills for coping with all of these changes in real time. To date, this has created a number of issues that need to be resolved rather rapidly by school districts facing the 2014–15 NCLB dictums for schools to have all of their students meeting NCLB proficiency targets.

In essence, the change from the NCTM Standards–based curriculum to CCSSM is more than an evolution in the U.S. school mathematics curriculum. The impact of this abrupt change has yet to be determined, since most teachers and schools have not had a chance to work through the full implications of the changes, which have come through the policy community at the top of state educational agencies, rather than through professional mathematics and mathematics education channels, which historically have provided more professional development and information ahead of the announcement of new standards.

Historically, in most states, local school districts make the end decisions regarding which instructional materials their classrooms will use. For textbooks, this decision process is called *textbook adoption* and may be subject to formal regulations. In twenty-one states, a portion of the state education funds is earmarked for textbooks that are selected or recommended by statewide committees for use in that state's classrooms in accordance with the state's content standards. In these states, all adoptions for a given course or level may take place in the same school year. A 2011 communiqué from the state textbook adoption group within the California Department of Education to the Board of the National Association of State Textbook Administrators (NASTA) indicates the flux brought on by the release of CCSSM:

> California … is committed to implementing the Common Core State Standards (CCSS) adopted by the California State Board of Education (SBE) in August 2010. While it will take a number of years to develop new curriculum frameworks and instructional materials aligned to the CCSS, State Superintendent of Public Instruction Tom Torlakson has invited publishers of state-adopted programs in mathematics and language arts to submit supplemental instructional materials that bridge the gap between their existing programs and the CCSS. Select teachers and content experts will review the supplemental materials, and the California Department of Education (CDE) will list on its Web site those materials found to meet the evaluation criteria. The process is completely voluntary for publishers, and the materials will not be adopted by the SBE. Yet, this information should help school

> districts significantly in their transition to the CCSS in light of the fact that the California State Legislature and Governor recently extended the suspension of SBE instructional materials adoptions until July 2015. (California Department of Education 2011)

Reports from other large states in NASTA on their current textbook adoption processes indicated similar interrupted states of affairs.

In the other twenty-nine states, school districts typically adopt textbooks under less regulated procedures and with more local choice; they may have to adopt every five years, but they may also select from any available materials. Many of these states also receive funding for textbooks from their state department of education, but it does not come with restrictions on the actual textbooks chosen. However, the necessity to sequence such adoptions to accommodate the availability of funds and coming changes in state standards and, possibly, NCLB restrictions, has created problems in these non-textbook adoption states.

Historically, the direction taken by the large textbook adoption states—California, Florida, and Texas—has had a large influence on the market and choices for other states. Hence, schools nationwide are in a holding pattern about what to do in making significant implantation of CCSSM in their schools. They do not want to make changes in their programs until decisions are made about their state standards by their state departments of education, which, in turn, together with other states' departments of education, instigated the CCSS Initiative (Association of American Publishers 2011; National Association of School Textbook Administrators 2011) that led to the development of CCSSM.

This might be likened to a "Standards Spring," as the state departments now have their standards, but it is not clear that they all have the same idea of what they are going to do with them. Some have adopted them in their released form, while others are speaking of adapting them. Will the states move to positions of more uniformity in outcomes across the states? Will the CCSSM program become the potential national curriculum that it was envisioned to be? Will curricular materials be directed toward a more focused curriculum serving both career and higher education needs? The answers to these questions will reside in the decisions made in state departments of public instruction in the coming months and the careful plans for implementation developed to bring the new curricula and instructional materials to the nation's teachers, schoolrooms, and students.

Akin to the adoption of curricular frameworks and instructional materials at the state levels, the setting of mathematics requirements as part of state secondary school graduation requirements has a great influence on the shape of mathematics education programs in grades 9–12. As of 2007, forty-three states had statewide credit requirements for high school graduation, whereas seven transfer this power to local school districts. Four of the forty-three with statewide credit requirements compelled students to earn four credits (years) in mathematics. Seventeen states plus the District of Columbia will do so by 2018. Only six states essentially required algebra 1, geometry, and algebra 2 or its equivalent in 2011; at least sixteen states will have that requirement in place by 2012. Michigan and Kentucky will require all students to take a mathematics course during their final year of secondary school education, starting in 2011 and 2012, respectively (Zinth 2006; Zinth and Dounay 2006, 2007; Dounay 2008). These added graduation requirements may lessen the number of sections of remedial work taught at the two- and four-year college levels. This load could also be lessened by instituting a mathematics course during each year of the high school students' secondary curriculum.

College-Level Curricular Documents

The past decade has seen the focus in collegiate curricular documents shift from a listing of course contents to documents looking introspectively at content and appropriate instructional strategies for specific content in collegiate classrooms, as well as at ways in which the collegiate mathematics curriculum might be linked to the applications of mathematics in partner disciplines.

Building on its publication in 1995 of *Crossroads in Mathematics: Standards for Introductory College Mathematics before Calculus*, the American Mathematical Association of Two-Year Colleges (AMATYC) developed and released *Beyond Crossroads: Implementing Standards in the First Two Years of College* (AMATYC 2006). This document offers a clear discussion of systemic relationships that exist among content; the formation of content into a curriculum; and knowledge of students' learning, classroom pedagogy, and assessment strategies. It also emphasizes the role of continued professional development of the faculty itself, and shows how all of these factors play into developing programs that reach their potential in serving students. It focuses on the fact that what students learn is a function of "how they learn it" (Lutzer et al. 2007).

Across a slightly longer period of time, the Mathematical Association of America's Committee on Curriculum Renewal Across the First Two Years (CRAFTY) has been gathering information through a series of workshops sponsored by its Foundations Project. This effort is directed toward hearing what partner disciplines want from the mathematical sciences and the relationships existing among the courses that the partner disciplines provide for their students. Beyond the content, the focus has been on the culture within the partner disciplines regarding how the mathematical sciences are viewed and the contexts in which and technology through which mathematics is used in applications, problem solving, and research within these domains. Several of the papers from these workshops have been published in the MAA's newsletter *Focus* and are available at the Curriculum Foundations Project: Voices of the Partner Disciplines site: http://www.maa.org/cupm/crafty/cf_project.html.

Most recently, CRAFTY has focused on the partner discipline needs associated with the college algebra course. In the spadework for its 2004 *CUPM Curriculum Guide*, MAA's Committee on the Undergraduate Program in Mathematics (CUPM) had determined that mathematics departments' visions for college algebra were not well matched with the needs of the partner disciplines, even though the partner disciplines were requiring that their students enroll in the course for added mathematics background. Following a series of Curriculum Process II workshops focusing on added undergraduate topics and the college algebra course in particular, CRAFTY in 2011 released *Partner Discipline Recommendations for Introductory College Mathematics and the Implications for College Algebra* (Ganter and Haver 2011). This report provides general recommendations along with case studies of experimental programs under way in several institutions and another program developed by a group of historically Black colleges and universities working together with faculty from the United States Military Academy at West Point, New York. With a program to complement the MAA's efforts in re-envisioning college algebra in postsecondary institutions, the American Mathematical Association of Two-Year Colleges developed course materials focusing on the mathematics curriculum in two-year colleges, *Mathematics Across the Community College Curriculum* (MAC^3). The project creates and disseminates exemplary projects and courses that integrate mathematics into all disciplines and offers a support system for community colleges in integrating math across the disciplines.

A third effort bridging the disciplines and offering goals and methods for an introductory statistics course was developed by the Education Committee of the American Statistical Association (Aliaga et al. 2010). Like the AMATYC, CRAFTY, CUPM, and other collegiate groups, the ASA was aware that introductory statistics offerings were not presenting an accurate picture of the discipline and, at the same time, were not serving their students well. A draft set of recommendations for such a course, listed below, were developed earlier by George Cobb (1992):

- Emphasize statistical literacy and develop statistical thinking
- Use real data
- Stress conceptual understanding, rather than mere knowledge of procedures
- Foster active learning in the classroom
- Use technology for developing conceptual understanding and analyzing data
- Use assessments to improve and evaluate student learning

With these as a start, the *Guidelines for Assessment and Instruction in Statistics Education: College Report* (Aliaga et al. 2010) provides sample questions, activities, investigations, and assessments, which, along with the discussion of the previous recommendations, offer a framework for the introductory statistics course for general education.

From 1995 to the present, the work done at the two- and four-year college levels, as well as at the university level, supports and extends the work started by the MAA's Committee on the Undergraduate Program in Mathematics in its development of its 2004 report, *Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide 2004* and the accompanying *Discussion Papers about Mathematics and the Mathematical Sciences in 2010: What Should Students Know?* (MAA 2004a, 2004b). Undergraduate mathematics departments have become aware that high-quality content is not sufficient to produce high-quality learning. A major portion of what students learn and are able to use resides in the interaction of that content and the ways in which they learned it. Departments from the community college level through graduate schools have realized that the goals must be to have both high-quality curriculum and grounded programs. Further, national scientific organizations, state boards of higher education, and professional groups are stepping forward to assure that resources are available to provide both ongoing professional development for collegiate faculty and support for continued curricular development work (Snook 2004).

PART III: Implemented Curriculum

At present, the implemented mathematics curriculum in U.S. schools remains dictated to a large degree by the textbook that has been adopted by the district where students attend school. That said, there is a great deal of consistency, especially within textbook adoption states, as schools within states are working to help their students attain the same state-level learning outcomes, which, at present, are still based largely on the framework presented by NCTM's *Principles and Standards for School Mathematics* (2000). Although national or state data are not routinely collected on topics covered or textbooks used, some data are available through publishers, assessment organizations, and isolated research studies. Data on textbook usage are not routinely collected by the National Assessment of Educational Progress or other national educational surveys (Reys 2006).

Mathematics Study in Elementary School (K–Grade 5)

An elementary school teacher who teaches reading, science, and social studies and is with the same students almost the entire day almost always also teaches these students their mathematics. These teachers do not have time to create lessons for all these subjects, and as a result, they tend to rely heavily on the mathematics instructional materials purchased by their school district. Central to these is the teacher's edition of the mathematics textbook. According to an industry survey, 92.7% of schools in the country reported in 2010–11 using a basal mathematics series that they either follow very closely or from which they pick and choose as needed. In the past, most publishers would market a coordinated elementary school curricular series to cover all grades, K–8. Beginning in the mid-1980s, these curricula were split into two parts, a K–5 or K–6 elementary school series and a 6–8 middle school series. More recently, the K–5 portions of these programs have been segmented into blocks covering K–grade 2 and grades 3–5 for marketing purposes, to parallel NCTM's *Principles and Standards*.

Any discussion of most commonly used textbooks must be written and read with great care, because textbooks remain in use for several years after their initial purchases. Much of the everyday conversation about what is most popular focuses on what textbooks are currently being adopted. As a result, two distinct perceptions emerge regarding which textbooks U.S. students use. We will base our discussion of the status of textbooks on a national survey of textbooks in service in the school year 2010–11 (Resnick, Sanislo, and Oda 2011).

Data on textbook use collected in 2010–11 indicated that mathematics teachers tend to stay with their textual materials longer than teachers in almost all other domains. On average, across K–grade 12, 28.5% of the programs in use have been in use for more than 5 years, 42% for 3 to 5 years, 14.6% for 1 to 2 years, and the remaining 14% for less than 1 year. Teachers in K–grade 2 tend, on average to be using textbooks that have been adopted for about 3.9 years. The three most frequently used series reported in K–grade 2 in 2010–11 were McGraw-Hill's *Everyday Mathematics*, Harcourt's *Mathematics,* and Scott Foresman-Addison Wesley-Pearson's *Mathematics.* Collectively, these three series accounted for slightly more than 53% of the books in use in K–grade 2 in 2010–11.

Examining the data on textbooks in grades 3–5, the responses suggest that teachers at this level have been using their present textbook for an average of 4.0 years. The overall frequency-of-usage results for 2010–11 showed that the most used series again accounted for a little more than 53% of the books in use. These series were McGraw-Hill's *Everyday Mathematics*, Harcourt's *Mathematics*, and Houghton Mifflin's *Houghton Mifflin Mathematics*.

Mathematics Study in Middle or Junior High School (Grades 6–8)

When the data for grades 6–8 are analyzed, the patterns of usage are much more difficult to summarize because many districts use a mixture of basal series that cover the entire span of content topics for some of the grades but select other textbooks covering algebraic content at either an introductory level or the level that would typically be found in the first year of high school (grade 9). In addition, approximately 5% of the grade-level enrollment is taking a course in geometry. Mathematics teachers at this grade level have had their current textbook, on average, for about 3.6 years.

With respect to the textual materials currently in use in 2010–11, we examine the basal 6–8 textbooks, pre-algebra materials, and algebra 1/geometry textbooks. For the basal series, the three leading series were *Connected Mathematics* from Dale Seymour/Pearson, *Math Applications and Concepts* from Glencoe/McGraw-Hill, and *Prentice Hall Mathematics* from Prentice Hall/Pearson. Volumes in these series were found in 28.2% of schools as textbooks for these grade levels. The pre-algebra textbooks most frequently mentioned were from Glencoe/McGraw-Hill and Prentice Hall/Pearson. These textbooks accounted for another 20.1% of textbooks used in schools at grades 6–8. The algebra 1 textbooks most frequently mentioned were *Algebra I: Concepts and Skills* from McDougal Littell/Houghton Mifflin, *Algebra I* from Glencoe/McGraw-Hill, and *Algebra I* from Holt-Harcourt. These algebra 1 textbooks were found in 25.7% of schools with classes in grades 6–8. The final textbook category sampled at these grade levels was geometry textbooks. The two leading geometry textbooks were *Geometry: Explorations and Applications* from McDougall Littell/Houghton Mifflin and *Geometry* from Prentice Hall/Pearson. These were texts in 4% of the middle schools or junior high schools.

Mathematics Study in High School (Grades 9–12)

High school programs, like programs at the other levels, seem to be on a seven-year rotation program for consideration of textual material replacement. On average, secondary programs reported that they had been using their present materials for 3.4 years. At the high school level, the mainstream core curriculum currently found in U.S. secondary school classrooms is built around a sequence of three full-year courses, algebra 1-geometry-algebra 2 or algebra 1-algebra 2-geometry, beginning in eighth, ninth, or tenth grade, followed by a fourth year of precalculus, usually giving strong attention to functions and trigonometry. Since the mid-1950s, an increasing percentage of students has completed a year of calculus at the high school level. This latter course, especially when it is an Advanced Placement Calculus course, usually covers the content normally found in the first semester of university-level calculus. In about 20% of these cases, this course covers the equivalent of the full first year of university-level calculus. In most school districts where students participate in AP Calculus courses, algebra 1 is taught in the eighth grade.

More than 90% of the secondary schools in the country follow the more traditional course-based curriculum for the majority of their students. Many of them present it at a slower pace for lower-performing students by subdividing some of the courses or altering topics covered. Since 1990, several integrated secondary school mathematics curricula have been developed. These programs blend the content of algebra, geometry, functions, and data analysis in a highly connected and integrated fashion, usually with an emphasis on modeling and applications (Hirsch 2007). Data on the percentage of students studying the integrated curricula are hard to obtain and verify, but common estimates place it at 8% of secondary students.

During the past twenty-five years, high school graduation requirements and college admission requirements have increased along with the percentage of four-year colleges and universities now requiring two years of algebra and a year of geometry for admission. Also, as a result of the impact of technology on everyday lives and worries about U.S. students' lackluster mathematical performances in international studies, the public appears to have become more aware of the role that mathematics can play in the future lives and careers of secondary school students. Furthermore, as the data in tables 2 and 3 indicate, high school mathematics enrollment data, when combined with course-taking data from the middle grades, show that more students are taking courses in algebra before high school. This change has then contributed to a steady and significant increase over time in the percentage of students taking higher-level mathematics courses in high school.

The data in table 2 reflect the transition of students from the regular eighth-grade curriculum to a pre-algebra class and then to an algebra 1 class over time. The bulk of the data for "Other Course" at present is enrollment in the equivalent of a high school geometry course. So when we see the decline in enrollment in high school courses at the level of algebra 1 or below, we are seeing a greater proportion of secondary students enrolling in at least geometry or algebra 2 as their most advanced mathematics course taken in high school.

The data in table 2 are from the NAEP Long-Term Trend Studies, a series of survey assessments using basically the same instruments and questionnaires over time. As a result, the data on course taking here are perhaps most reliable as trend data, in that the questions asked about courses and the framework of courses have remained very constant over time. Unfortunately, the data for the most advanced course in this source merge the percentage of students reporting precalculus with those reporting calculus as their most advanced course taken.

Table 3 contains data from the main NAEP mathematics assessment, an assessment that is given on a different schedule and for which the survey questions, as well as the test items, are revised more frequently. There are differences in the total numbers ascribed to the percentage taking precalculus or calculus in the two versions of NAEP. However, the ratio between precalculus enrollments and calculus enrollments in main NAEP decreases from 2.25 : 1 to 1.33 : 1 over the period of time covered by the main NAEP data. This pattern follows the pattern seen in table 2, with the gradual increase in the selection of more advanced mathematics courses as part of a student's high school courses.

Confirming these data, other sources (Blank, Langesen, and Peterman 2007a) report that the percentage of high school students completing algebra 2 increased 13% from 2000 to 2009, and the percentage completing precalculus increased 24% over the same time period. Data from the 2010 CBMS study of mathematics programs in two-year colleges also indicate a decrease in the number of students enrolling in remedial courses in mathematics covering high school content prior to precalculus. Concurrent with these changes has been an increase in the numbers of students taking advanced placement courses in mathematics (Roey et al. 2007; College Board 2011a; Nord et al. 2011; Kirkman, Blair, and Maxwell 2012).

Table 2

Percentage of students by most advanced mathematics course taken in middle school and taken in high school

M.S. course	**1986**	**1990**	**1994**	**1999**	**2004**	**2008**
Regular 8th-grade math	61	57	43	37	31	31
Pre-algebra	19	23	32	34	32	32
Algebra 1	16	15	20	22	29	30
Other course	5	5	4	6	6	7
H.S. course	**1986**	**1990**	**1994**	**1999**	**2004**	**2008**
Pre-algebra or general mathematics	18	15	9	7	4	3
Algebra 1	18	15	15	11	9	7
Geometry	16	15	15	16	16	17
Algebra 2	40	44	47	51	53	52
Precalculus or calculus	7	8	13	13	17	19

Percentage of students by the most advanced course they were enrolled in during their final year of secondary school, NAEP Long-Term Trend data from the studies in the years shown (Mullis et al. 1991a; Campbell et al. 1996; Campbell et al. 1997; Campbell, Hombo, and Mazzeo 2000; Perie, Moran, and Lutkus 2005; Shettle et al. 2007; and Nord et al. 2011).

Table 3

Percentage of students taking precalculus or calculus as their most advanced class

Course	**1990**	**1996**	**2000**	**2005**	**2009**
Precalculus	9	14	17	21	24
Calculus	4	7	16	18	18

Breakdown of percent data for students enrolled in precalculus or above in data from NAEP national studies in years shown: 1990, 2000, 2005 (Mullis et al. 1991b; Mitchell et al. 1999; Braswell et al. 2001; Grigg, Donahue, and Dion 2007; NCES 2011b).

Much as for the grades 6–8 level, describing the actual textbooks in use in high school classrooms is very difficult beyond offering general comments about the leading publishers. In 2011, the three major publishing houses responsible for the majority of secondary school mathematics textbooks were Glencoe-McGraw Hill, Houghton Mifflin-McDougal Littell-Heath, and Pearson-Prentice Hall-Addison Wesley-Scott Foresman. The hyphenated sequences of publishers reflect the number of acquisitions and mergers that have taken place in the educational products market during the past decade. These changes in the industry have resulted in the merging of product lines and the discontinuation of long-running series. Like those interested in middle school materials, those interested in particular products for high school would be most successful by referring to the websites of the individual publishers as contained in part IX of this fact book. In addition to overlapping use of the same name for books, each of these may have several editions among which it is difficult to distinguish.

None of the high school series developed with initial support from the NSF has the breadth of usage that is enjoyed by the series' elementary and middle school counterparts. Together with the NCTM Standards documents, these NSF project-related series have influenced mainstream texts to include more applications and more work with technology. At the same time, pressure from colleges has influenced these texts to maintain, if not increase, skill work with algebra and functions. At the middle school level, the Dale Seymour/Pearson text series *Connected Mathematics* is a curriculum development project that had its start in one of the NSF-funded projects to build curricula based on the NCTM Standards. Other series have garnered shares of the market, but none have been as successful as the *Connected Mathematics* series.

Pedagogical Patterns in K–Grade 12

Prior to the 1990 shift of the use of the National Assessment of Educational Progress (NAEP) to collect information for state comparisons and for demographics-based monitoring of students' achievement in the nation's schools, a larger focus was given to the collection of information on curriculum and instructional variables in the nation's classrooms. Starting with the 2005 assessment, these data were severely reduced, and many of the long-term lines of data were truncated.

NAEP has tracked the use of calculators at the classroom level since the 1980s. Initial data showed that schools owned sets of calculators for instructional purposes. But by 2005, the data reflected that 76% of the nation's grade 4 students reported owning a regular calculator and 6% reported owning a graphing calculator (NCES 2011d). More recent NAEP assessments have attempted to gather data on student use from both teachers and students.

In 2005 through 2011, fourth-grade teachers were asked about the levels of calculator use that they allowed their students in taking a mathematics test or quiz. Table 4 contains the responses in terms of the percentage of grade 4 students falling into each usage class and the mean scale score for students in that usage class. The results show a slight, but statistically significant, decline in the percentage never using a calculator and a corresponding slight, but statistically significant growth in the percentage using a calculator sometimes from 2005 to 2011. The data also reflect that students using a calculator sometimes score slightly higher than those never using one.

Table 4

Percentages and related mean scale scores of fourth-grade students by their teachers' answers to the question, When you give students a test or quiz, how often do they use a calculator?

Usage class	2005		2007		2009		2011	
	Percent	Scale score	Percent	Scale score	Percent	Scale score	Percent	Scale score
Never	75	238	67	240	71	240	72	240
Sometimes	25	240	32	241	28	241	27	243
Always	0	dna	1	231	1	231	1	232

(Perie, Grigg, and Dion 2005; Lee, Grigg, and Dion 2007; NCES 2009a, 2011d)

At the grade 8 level, teachers were asked about the type, if any, of calculators that their students used during mathematics lessons. The results from the NAEP assessments of 2009 and 2011 showed the data reported in table 5. Here the data are very stable, with

72 to 73 percent of the students reporting that they were using either a scientific calculator without graphing capabilities or a scientific calculator with graphing capabilities. An analysis of these data for both years also shows significant differences in students' scale scores as one moves from the basic four-function level to a scientific (not graphing) calculator and then again to the graphing calculator level. Also interesting is the fact that the use of a basic four-function calculator at the grade 8 level does not advantage a student over a student without the use of a calculator in terms of their scale scores.

Table 5

Percentages and related mean scale scores of eighth-grade students by their teachers' answers to the question, What kind of calculators do your students use in mathematics lessons?

Type of calculator	2009		2011	
	Percent	Scale score	Percent	Scale score
None	10	273	11	275
Basic four-function (+, -, ×, ÷)	17	273	17	274
Scientific (not graphing)	48	286	46	286
Graphing	25	290	26	291

(NCES 2009a, 2011d)

In 2009, students at grades 4, 8, and 12 were asked, "When you take a math test or quiz, how often so you use a calculator?" The data reflecting their answers, as well as their NAEP scale scores, are shown in table 6. An examination of the data shows that as students move from grade 4 to grade 12 they are more inclined to make use of a calculator. Reflecting on the prior tables, we note that this growth is conditioned by the degree to which teachers allow the use of the calculator. As table 6 shows, at grades 4 and 12 students responding that they make use of a calculator either sometimes or always score significantly higher than those students responding that they never use a calculator. At grade 8, a different pattern emerges, with the mean scale scores decreasing as the use increases. Each of the steps down is a significant decrease in performance from the previous usage level.

Table 6

Percentages and related mean scale scores for fourth-, eighth-, and twelfth-grade students according to their responses to the question, When you take a math test or quiz, how often do you use a calculator?

2009	Grade 4		Grade 8		Grade 12	
	Percent	Scale score	Percent	Scale score	Percent	Scale score
Never	66	278	28	242	7	140
Sometimes	33	285	51	237	52	160
Always	2	287	21	208	41	156

(NCES 2009a)

Mathematics Study at the Postsecondary Level

At the postsecondary level, students have a wide variety of options for studying mathematics. Coursework is available through community colleges, universities, and a variety of vocational schools, work-based educational programs, and commercial outlets. The data collected every five years by the Conference Board of the Mathematical Sciences (CBMS) provide the best trend data for curricular programs and enrollments in two- and four-year colleges.

Mathematics courses at these institutions range from arithmetic and pre-algebra to linear algebra and differential equations at vocational and two-year colleges, and from intermediate algebra and precalculus through advanced graduate courses at four-year institutions and universities. Tables 7 and 8 demonstrate this wide range and the change in enrollments over time at two- and four-year colleges, respectively. In these tables, remedial courses include arithmetic, pre-algebra, and elementary and intermediate algebra. Precalculus courses include college algebra and trigonometry as well as finite mathematics, noncalculus-based business mathematics, mathematics for prospective elementary school teachers, and other courses for nonscience majors. Calculus includes both mainstream and nonmainstream courses (e.g., calculus courses tailored to students in other majors, such as life sciences or business). These tables do not include mathematics courses taught outside mathematics and statistics departments. Enrollments are for the fall quarter or semester of the year (Kirkman, Blair, and Maxwell, 2012).

Two-year college enrollments increased over this same period of time and were projected to rise from the nearly 6.50 million enrolled in 2005 to slightly more 7.01 million in the fall of 2009, an increase of about 8% (Snyder and Dillow 2011; Hussar and Bailey 2009). An examination of the data in table 7 shows that this same period saw an increase of more than 18.9% in the number of students enrolled in mathematics. Not only did enrollments increase overall but the increase also occurred across the full range of the two-year college offerings. Remedial enrollments were up by 19.3%, precalculus enrollments were up by 13.7%, calculus enrollments were up by 29.0%, statistics enrollments were up by 16.1%, and enrollments in other courses (liberal arts, math for elementary teachers, and so on) were up by 21.5%. This pattern contrasts with four-year college data over the same time period, as the data in table 8 will show.

Table 7
Estimated enrollment (in thousands) in mathematics courses in two-year colleges

Course	Year						
	1980	**1985**	**1990**	**1995**	**2000**	**2005***	**2010***
Remedial	441	482	724	800	763	964	1150
Precalculus	180	188	245	295	274	321	365
Calculus	86	97	128	129	106	107	138
Statistics	28	36	54	72	74	118	137
Other	218	133	144	160	130	186	226
Total	953	936	1295	1456	1347	1696	2016

*Data in 2005 and forward report by sections by average size rather than by percentage of total students calculation (Kirkman, Blair, and Maxwell, 2012).

Table 7 shows that since 1985, more than half of the mathematics enrollments in two-year colleges have been at the remedial level. The overall increase in number of mathematics courses in two-year colleges is partially a function of the overall increase in enrollments at these institutions. The increase is also, however, partially a function of the increased realization that mathematics enables knowledge. Enrollments increased in the precalculus level by 22.2%, in the calculus level by 30.3%, in statistics by an amazing 44.5%, and in the advanced coursework level by 33.9%. Comparing mathematics enrollment gains to statistics enrollment gains over the same five-year period shows a 19.1% increase for mathematics, whereas statistics grew by 44.5% over the same period (Lutzer et al. 2007; Kirkman, Blair, and Maxwell 2012).

Table 8
Estimated enrollment (in thousands) in undergraduate mathematics and statistics courses in four-year colleges

Course	**Year**						
	1980	**1985**	**1990**	**1995**	**2000**	**2005***	**2010***
Remedial	242	251	261	222	219	201	209
Precalculus	602	593	592	613	723	706	863
Calculus	590	637	647	538	570	587	765
Statistics	n/a	n/a	125	143	171	182	263
Advanced	91	138	119	96	102	112	150
Total	1525	1619	1744	1612	1785	1788	2250

*Data in 2005 and forward report by sections by average size rather than by percentage of total students calculation (Kirkman, Blair, and Maxwell 2012).

The graph in figure 2 shows the consistent growth in enrollment for both two-year and four-year colleges and their contributions to the total number of undergraduate students enrolled in mathematics. Although differences occur in the rates of growth of individual subareas within each subdivision, one can see the increasing percentage of the total contributed by the two-year college enrollments over time.

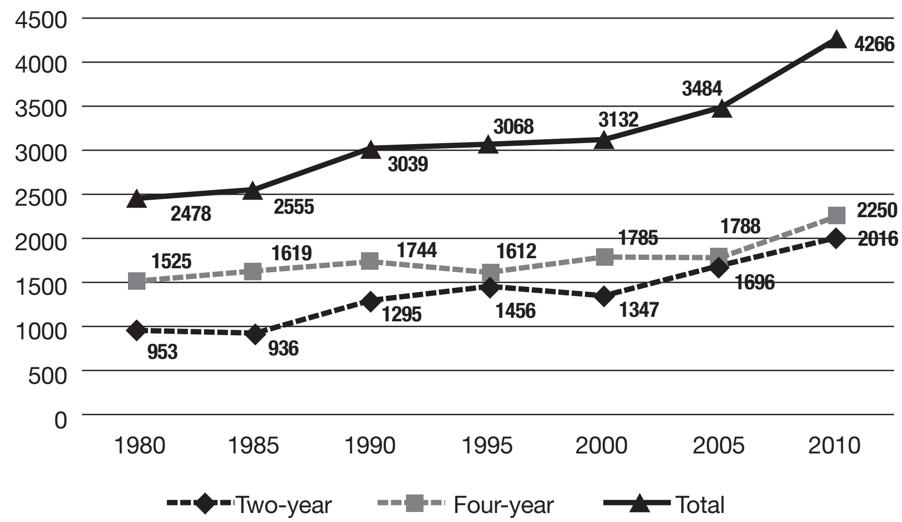


Fig. 2. Undergraduate enrollments (in thousands) in mathematics 1980–2010

Additional data from the CBMS study, still under analysis at the time of this writing, will provide more in-depth information about the enrollments in mainstream university calculus versus nonmainstream business and social science–oriented calculus courses, enrollments in specific advanced courses and statistics courses, and the percentage of sections taught by tenure track and temporary faculty. These additional analyses will be available at the CBMS website (http://www.cbmsweb.org/) later in 2012.

Data from *The American Freshman* study indicate the changes in the percentages of freshmen entering college intending to major in mathematics or statistics over the years: 4.5% (1966), 1.0% (1976), 0.7% (1986), 0.5% (1996), 0.7% (2001), 0.7% (2004), 0.8% (2006), 0.8% (2008), and 0.9% (2010) (Higher Education Research Institute 2011; National Science Board 2012). The percentage in 2010 is significantly lower than in the 1960s, even when computer science majors are included. The mathematical requirements of majors outside the physical sciences, however, have increased significantly in the same time period. Although some of the mathematics needed to fulfill these requirements is taught outside departments of mathematics and statistics, the increases in these requirements are a major factor in the overall increase in the number of courses taken in departments of mathematics and statistics.

PART IV: Attained Curriculum

Central to the measure of the success of a curriculum is the academic attainment of the students who have experienced the instruction associated with it. Unfortunately, we have little information at a national level on student progress in the diverse curricula offered throughout the United States, owing to the lack of centralized curricular oversight. Further, studies have shown that state-level assessments vary greatly in their standards and student expectations. As a result, we examine the attained curriculum through National Assessment of Educational Progress results and mean national results on two college entrance examinations.

The National Assessment of Educational Progress (NAEP)

The U.S. government, with guidance from the National Assessment Governing Board (NAGB) and the expertise of the Department of Education's National Center for Education Statistics (NCES), administers a large-scale assessment known as the National Assessment of Educational Progress (NAEP). This program, which periodically assesses knowledge of, and opportunity to learn, mathematics and other subjects with random samples of American youth, is one of the best measures of mathematics achievement in American schools. This survey of the mathematical abilities of American youth has taken on even more importance with the use of NAEP as a barometer for measuring states' performances relative to the strictures of the NCLB legislation.

In 2009, more than 168,000 students in grade 4 and 161,000 students in grade 8 participated in the mathematics portion of the NAEP assessment of mathematics. These students were randomly selected according to a complex sampling design to form the basis from which results at both national and state levels could be statistically developed and compared. In addition to students from public and private schools in the fifty states, students were also selected to develop scores for the District of Columbia and the Department of Defense Education Activity (DoDEA) schools. The 2009 grade 12 assessment tested representative samples of about 49,000 students from grade 12 in 1,670 schools across the nation. These samples also allowed the reporting of performances of grade 12 students in eleven states that volunteered to participate. Beginning with the 2013 assessment, the NAEP mathematics assessment for grades 4, 8, and 12 will be conducted on an every-other-year basis. NAEP longitudinal data will be collected in years divisible by 4.

The most recent NAEP assessment in mathematics was conducted in 2011 and collected data from a national sample of fourth and eighth graders. With the 2003 assessment, the NAEP program shifted its administration policies to provide expanded accommodations for students as required by law. Figures 3 and 4 contain NAEP data for grades 4 and 8 from studies conducted between 1990 and 2011 (NCES 2011b).

The 2011 assessment result for fourth-grade performance in mathematics was one point higher than that observed in 2009. Although the difference from the performance noted in 2009 was statistically significant, the educational significance of the difference is debatable. Given the large sample sizes involved at a grade level, a small change in the mean performance may be judged as statistically significant even when the actual difference in performance is less than one item on the NAEP assessment. However, when one looks at the mean performances for fourth graders over the set of assessments since 1990, one notes the consistent improvement in student performance. The 28-point increase in performance since 1990 is both statistically and *educationally* significant.

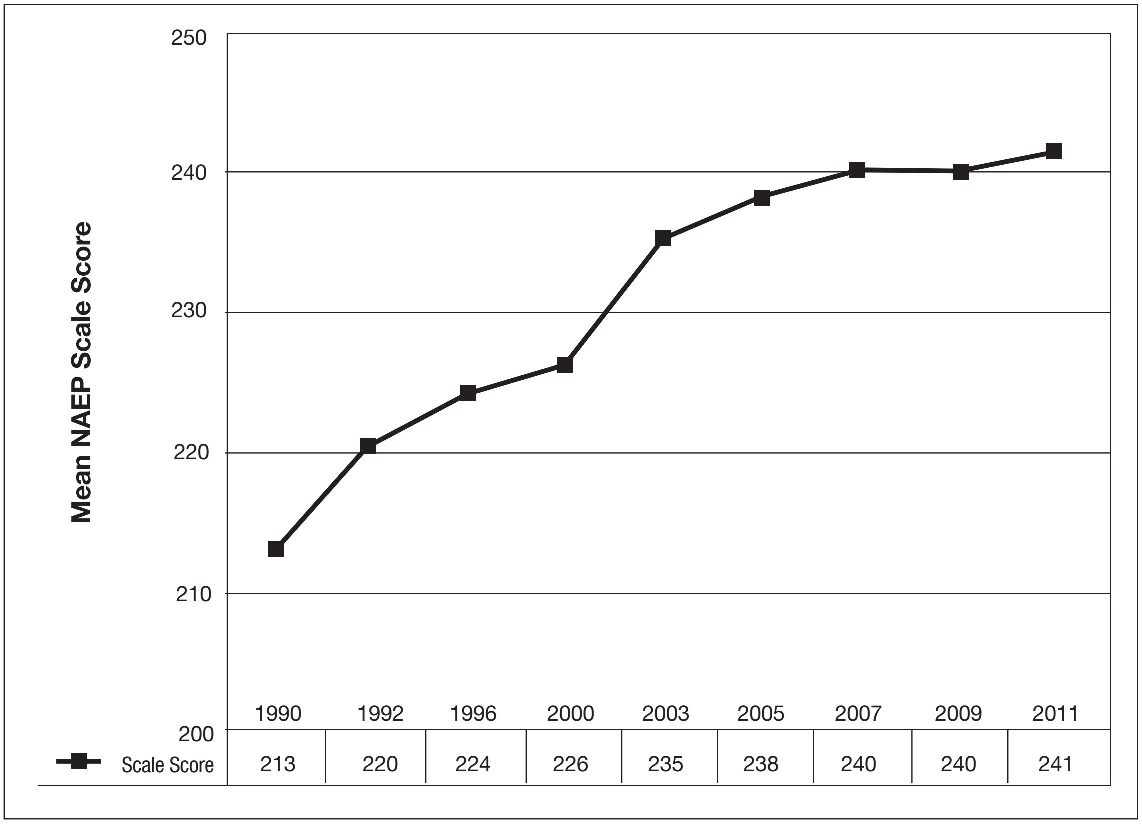


Fig. 3. Trend in fourth-grade NAEP mathematics mean scale scores.
(Mean scaled scores for all years prior to 2011 are significantly different from 2011 score, with $p < 0.05$.)

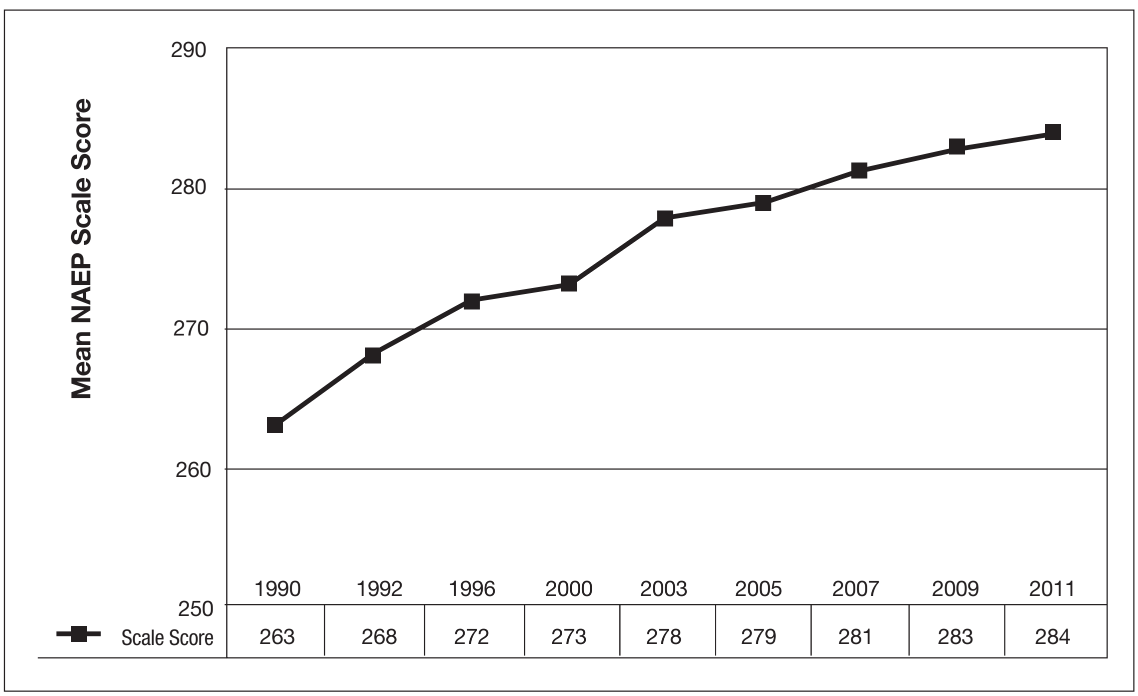


Fig. 4. Trend in eighth-grade NAEP mathematics mean scale scores.
(Mean scaled scores for all years prior to 2011 are significantly different from 2011 score, with $p < 0.05$.)

Other national results observed for fourth graders in 2011 included statistical gains over 2009 performances for White, Black, and Hispanic students; for both male and female students; and for students from lower- and higher-income families. State-by-state results for grades 4 and 8 will be discussed later in part IV (NCES 2011b).

Students' performance on the NAEP mathematics assessment is reported on a 0–500-point scale. Prior to 2005, comparing across grades was possible because the tests were reported on the same scale. However, with the 2005 examination, the cross-grade blocks of items were dropped from the assessment in order to examine more of each grade's individual curriculum in depth. However, a bridge study was conducted that allowed the trend lines at both fourth- and eighth-grade levels to continue to be reported over time in spite of the transition. Statistically, the 2011 performances at both grade levels are significantly higher than any other observed from 1990 to 2009. This pattern of continued growth has also been observed in other assessments carried out by states and local districts.

Figure 4 shows the trend in mean mathematics scores for eighth-grade students on the 2011 NAEP mathematics assessment. The average mathematics score for the nation's eighth graders in 2011 was higher than the scores in the eight previous assessment years. Similar to the pattern observed in the fourth-grade scores, eighth-grade performance showed a 1-point improvement from that noted in 2009. Although this gain was not as numerically large as that observed for the fourth-grade difference, the mean 2011 achievement again is clearly a significant educational gain over the intervening 21 years.

Other national results for the eighth-grade performances in mathematics in 2011 include higher scores in 2011 than in 2009 for Hispanic students, female students, and students from both lower- and higher-income families (NCES 2011b).

The content frameworks for the NAEP mathematics assessments changed in 2005 and again in 2007. Most notable among these changes was the emphasis given to the role of algebra and functions at all grade levels. In addition, more attention was focused on students' proficiency in context-based problem solving, in constructing their own responses, and in knowing when and how to apply technology in solving problems on assessments. In addition, the previous application of cognitive categories of "conceptual," "procedural," and "problem solving" to discuss the level of cognitive demands of an item was discontinued and replaced by a factor that focuses on the task complexity associated with items on the assessment (NAGB 2005). The 2009 NAEP mathematics framework reflected changes taking place in expected outcomes for students in grade 12. Most of these were related to added questions regarding topics in high school geometry and second-year algebra courses (NAGB 2010). However, the content assessed at grades 4 and 8 was based on the same objectives as the 2005 assessment.

Additional changes made to the grade 12 assessment dealt with a variety of concerns associated with differential response rates of twelfth graders to constructed-response items and to the test itself. Some of these differences could be traced to students' overall capabilities, but other concerns were more broadly based in attitudinal issues surrounding the tasks presented in what, for the students, was a low-stakes test administered in the spring of their final year of secondary schooling. Other issues behind the modifications in the grade 12 framework dealt with adding more grade-appropriate content in the areas of algebra, geometry, and problem solving. Hence, the 2009 assessment of grade 12 students continued the trials of the new NAEP framework for grade 12. Thus, from a trend standpoint, the data from the 2009 assessment of these students has to be treated as preliminary or initial. The results from the 2009 NAEP mathematics assessment at grade 12 showed a mean scale score of 153. This was 3 points higher than in 2005, and the difference was judged as statistically significant (NCES 2011a).

A study of the results of the national data for grade 12 performances in mathematics on the 2009 NAEP mathematics assessment showed a significant gain in overall student performance on the examination from the level observed in 2005. Further, significant gains in performance were recorded for all racial and ethnic groups and for both gender groups since 2005. In 2005, 23% of the students in grade 12 performed at or above the "proficient" level and 61% performed at or above the "basic" level. In 2009 these rates improved to 26% and 64%, respectively (NCES 2011a).

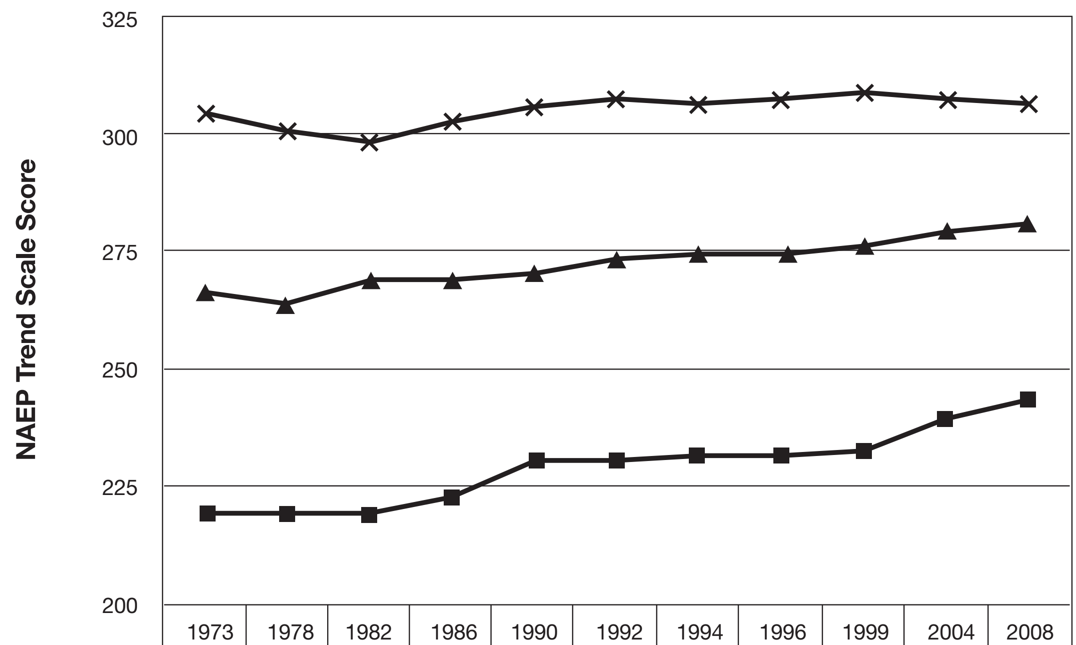
Further, it was observed that students who took more advanced mathematics courses scored higher than those who took lower-level mathematics courses or left the mathematics curriculum earlier in their high school studies. Further, one must take into account the differential rates of students who discontinue their formal education after reaching the age of compulsory education but before graduating from secondary school (NCES 2011a).

The NAEP Longitudinal Study

Although the national NAEP assessments and their frameworks are designed to change as the curriculum and school programs change, the National Center for Education Statistics also administers an additional NAEP assessment, the NAEP Long-Term Trend Assessment, to a nationally representative sample of students. The Long-Term Trend Assessment, which was initiated in 1973, used exactly the same test over time under the same conditions through 1999. Because the early NAEP assessments drew samples of 9-, 13-, and 17-year-olds, instead of fourth-, eighth-, and twelfth-grade students, the Long-Term Trend Assessment has continued to collect data at these age levels. As such, the NAEP trend assessment provides valuable information on whether students' performance on items considered important in 1973 (such as paper-and-pencil computation skills, direct application of measurement formulas in geometric settings, and the use of mathematics in daily living skills involving time and money) has changed over time (Perie, Moran, and Lutkas 2005). Data from the NAEP longitudinal study are shown in figure 5.

Analyses of the significance of the differences observed over time reflect significant differences between the 2008 mean and all scores from 1999 and before for both the grade 4 and grade 8 trend lines. The analysis for grade 12 indicates that only the 1978, 1982, and 1986 scale scores differ significantly from the 2008 scale score for the grade 12 trend line.

Both the 2004 and 2008 studies were conducted by using the new long-term trend assessment framework and assessment. The new assessment can change gradually over time, like main NAEP, contrary to the invariant assessment that was used from 1973 through 1999. A bridge assessment has indicated that the continuation of the trend line between the old and new assessments is appropriate (Perie, Moran, and Lutkas 2005). Figure 5 contains the data for the performances of students at each of the three age ranges on the NAEP Long-Term Trend Assessment. The 2008 level of performance for both 9- and 13-year-old groups is statistically higher than that of the same age groups at every testing period from 1999 or earlier. The trend line for 17-year-olds shows a pattern of insignificant variation from 1990 to the present (Rampey, Dion, and Donahue 2009). These findings indicate that, on average, elementary and middle school students in 2008 had a better command of the fundamental concepts and skills deemed important in 1973 than their age-related peers across the history of the assessment. The 17-year-old group, with the slight exception observed in the 1978–1986 period, showed no appreciable growth or decline in their command of these basic concepts and skills over the period constituting the history of the assessment.



	1973	1978	1982	1986	1990	1992	1994	1996	1999	2004	2008
Age 9	219	219	219	222	230	230	231	231	232	239	243
Age 13	266	264	269	269	270	273	274	274	276	279	281
Age 17	304	300	298	302	305	307	306	307	308	307	306

Fig. 5. Trend in NAEP mathematics average scores for 9-, 13-, and 17-year-old students (Rampey, Dion, and Donahue 2009)

Longitudinal Trends in Performance at NAEP Benchmark Levels

To help in understanding trends in students' knowledge and skills as measured by NAEP, levels of performance were established by anchoring five points on the mathematics scale: 150, 200, 250, 300, and 350. These five levels are accompanied by descriptions that outline the concepts, procedures, and processes associated with performance at each level. These levels are briefly described in the left column of table 9, which gives the results from the 1978 to 2008 assessments with respect to the levels (Perie, Moran, and Lutkus 2005).

Analyses of the data in table 9 show a significant increase in the percentages of students reaching benchmark levels of 200 and 250 for 9-year-olds across the period from 1978 to 2004 and from 250 to 300 for 13-year-olds across the period from 1978 through 2004. Deeper analyses indicate that much of the increase in students' long-term trend results comes from growth in mathematical topics, such as basic number facts and operations, and in reading and interpreting graphs, tables, and charts. The data for 17-year-olds shows no significant growth at any level since 1986. Few students at any age achieved the 350 benchmark, a level that indicates substantial ability in elementary algebra and geometry and in multistep problem solving (Rampey, Dion, and Donahue 2009).

Table 9

Trends in percentages of students at or above five mathematics performance levels, 1978–2008

Performance levels	Age	1978	1986	1990	1996	1999	2004	2008
Level 350 Multistep problem solving and algebra	9	0	0	0	0	0	0	0
	13	1	0	0	1	1	0	0
	17	7	7	7	7	7	7	6
Level 300 Moderately complex procedures and reasoning	9	1	1	1	2	2	0	0
	13	18	16	17	21	23	29	30
	17	52	52	56	60	61	59	59
Level 250 Numerical operations and beginning problem solving	9	20	21	28	30	31	42	44
	13	65	73	75	79	79	83	83
	17	92	96	96	97	97	97	96
Level 200 Beginning skills and understandings	9	70	74	82	82	83	89	89
	13	95	99	99	99	99	99	98
	17	100	100	100	100	100	100	100
Level 150 Simple arithmetic facts	9	97	98	99	99	99	99	99
	13	100	100	100	100	100	100	100
	17	100	100	100	100	100	100	100

(Perie, Moran, and Lutkus 2005)

An analysis was made of the performance of students in the bottom quartile, middle half, and upper quartile of each of the age groups over the same periods of time. The resulting growth patterns for each of the three groups paralleled the increases shown in table 9. This finding indicates that the increases were not an artifact of the performance of the most able students, but rather an increase indicative of change in the students in each of the three groups (Perie, Moran, and Lutkus 2005).

NAEP Results for Various Subgroups

Considerable research has been conducted in recent years on the differences in the performance of students of different racial/ethnic and gender groups, a matter of great concern to policy makers (Willingham and Cole 1997; Oakes 1990; Oakes and Wells 1998; Lubienski and Lubienski 2006; Robinson and Lubienski 2011; Venezia and Maxwell-Jolly 2007).

An analysis of data reporting differences in White and Black students' performances at all three ages between 1973 and 2009 showed a narrowing of the gap between their respective performances. While White students gained 25, 16, and 4 points at ages 9, 13, and 16, respectively, Black students improved 34, 34, and 17 points, respectively. By comparison, Hispanic students improved 32, 29, and 16 points, respectively. However, the majority of these gains were made in the period before the changes in the long-term trend assessment. Examination of the data for changes at ages 9, 13, and 17 for the period from 2004 to 2009 reveals that the gains are 5, 3, and 3 points for Whites, 3, 5, 3 points for Blacks, and 5, 4, 1 points for Hispanic students, respectively. The gaps separating Whites and Blacks and Hispanics significantly narrowed for 9-year-olds, but there was not a significant narrowing of similar gaps in achievement for 13- or 17-year olds.

Some studies have suggested that these differences are the result of opportunities afforded to the students in school and in their homes and communities; other studies point to the atmosphere of encouragement toward education and its role in students' lives (Mullis, Jenkins, and Johnson 1994; Oakes 1990; Eakin and Backler 1993; Venezia and Maxwell-Jolly 2007).

On the 2009 main, or national, NAEP mathematics assessment, although scores for both genders were at their all-time high for grades 4 and 8, male students outscored female students by one point at both levels (NCES 2011b). However, for both grade levels, this was the first time that the difference between the genders has not also indicated a significant difference in favor of males. At grade 12, males outscored females by 3 points, but again, the gap was not significant. Both male and female averages at grade 12 were significantly higher than those observed in the 2005 assessment (NCES 2011b). Robinson and Lubienski (2011) posit that some of the difference may be due to the increasing role that homework plays and the attention given to homework by females over males. Narrowing these gaps, whatever the cause, must remain a central challenge for mathematics education in the United States in the coming years.

State NAEP Results

The data in table 10 illustrate the vast differences that exist among state-level mean achievement scores and the percentages of students reaching the level of "proficient" or above on the 2011 state NAEP assessments at grades 4 and 8. When data were compared with 2009 state-level NAEP scores for grade 4, it was evident that students in Alabama, Arizona, the District of Columbia, Georgia, Hawaii, Maryland, New Mexico, Rhode Island, and Wyoming had done better than they had in 2009. Fourth graders in the state of New York had scored significantly lower than they had in 2009 (NCES 1011b). Comparison of the grade 8 data with same state-data from 2009 showed that students in Arkansas, Colorado, the District of Columbia, Hawaii, Maine, Mississippi, Nevada, New Mexico, Ohio, Oklahoma, Rhode Island, Texas, and West Virginia had scored significantly higher than students in the same states had in 2009. Comparison of average scores for students in Missouri for the 2009 and 2011 assessments showed a significant decline (NCES 2011b). There were no comparable state-level comparisons for grade 12.

Table 10
Selected statistics for U.S. public schools on the 2011 NAEP by nation and states

Jurisdiction	Grade 4		Grade 8	
	Mean scale score	Percent Proficient or above	Mean scale score	Percent Proficient or above
NATION	**240**	**40**	**283**	**34**
Alabama	231	27	269	20
Alaska	236	37	283	35
Arizona	235	34	279	31
Arkansas	238	37	279	29
California	234	34	273	25

Table 10—*Continued*

Jurisdiction	Grade 4		Grade 8	
	Mean scale score	Percent Proficient or above	Mean scale score	Percent Proficient or above
Colorado	244	47	292	43
Connecticut	242	45	287	38
Delaware	240	39	283	32
Florida	240	37	278	28
Georgia	238	37	278	28
Hawaii	239	40	278	30
Idaho	240	39	287	37
Illinois	239	38	283	33
Indiana	244	44	285	34
Iowa	243	43	285	34
Kansas	246	48	290	41
Kentucky	241	39	282	31
Louisiana	231	26	273	22
Maine	244	45	289	39
Maryland	247	48	288	40
Massachusetts	253	58	299	51
Michigan	236	35	280	31
Minnesota	249	53	295	48
Mississippi	230	25	269	19
Missouri	240	41	282	32
Montana	244	45	293	46
Nebraska	240	39	283	33
Nevada	237	36	278	29
New Hampshire	252	57	292	44
New Jersey	248	51	294	47
New Mexico	233	30	274	24
New York	238	36	280	30
North Carolina	245	44	286	37
North Dakota	245	46	292	43
Ohio	244	45	289	39
Oklahoma	237	33	279	27
Oregon	237	37	283	33
Pennsylvania	246	48	286	39
Rhode Island	242	43	283	34
South Carolina	237	36	281	32
South Dakota	241	40	291	42
Tennessee	233	30	274	24
Texas	241	39	290	40
Utah	243	43	283	35
Vermont	247	49	294	46
Virginia	245	46	289	40

Table 10—*Continued*

Jurisdiction	Grade 4		Grade 8	
	Mean scale score	Percent Proficient or above	Mean scale score	Percent Proficient or above
Washington	243	45	288	40
West Virginia	235	31	273	21
Wisconsin	245	47	289	41
Wyoming	244	44	288	37
Other jurisdictions				
District of Columbia	222	22	260	17
DoDEA	241	39	288	37

(NCES 2011b)

NAEP Trial Urban District Assessment (TUDA)

Starting in 2003 and continuing in 2005, 2007, 2009, and 2011, the National Assessment has conducted a Trial Urban District Assessment (TUDA) involving some of the largest school districts in the nation. In light of the fact that one-quarter of the nation's youth live in urban areas, their success in preparing for postsecondary study in science, technology, engineering, and mathematics (STEM) is critical to the nation's future workforce in the coming decade. Hence, special monitoring of these youth to ensure their progress is crucial to our continued economic progress.

The results of TUDA 2011 were quite reassuring in that they showed significant progress in the largest districts that had been involved with prior TUDA assessments and had implemented innovative programs directed toward the goals of STEM program improvement. Table 11 contains data from the 2011 TUDA assessment for the nation, large cities, and ten participating urban districts.

Table 11

*TUDA results for the nation, large cities, and ten urban districts in 2003, 2009, and 2011, with the percentage of students whose performance was "proficient" or higher in 2011**

Students	Grade 4				Grade 8			
	2003	2009	2011	% Prof	2003	2009	2011	% Prof
Nation	234	239	240	39	276	282	283	26
Large Cities	224	231	233	30	262	271	274	20
Atlanta	224	225	228	24	256	259	266	14
Boston	233	236	237	33	276	279	282	24
Charlotte	244	245	247	48	283	283	285	25
Chicago	220	222	224	20	260	264	270	17
Cleveland	215	213	216	10	257	256	256	9
D.C. Public Schools	214	220	222	23	248	251	255	11

Table 11—*Continued*

Students	Grade 4				Grade 8			
	2003	2009	2011	% Prof	2003	2009	2011	% Prof
Houston	234	236	237	32	273	277	279	22
Los Angeles	221	222	223	20	257	258	261	13
New York City	236	237	234	33	270	273	272	18
San Diego	234	236	239	39	272	280	278	24

*Column shading indicates a significant difference ($p < 0.05$) from 2011 mean for row group (NCES 2009b, 2011c).

In 2011 TUDA results for grade 4 students were higher than those from the nation, large cities, and nine of the ten urban districts that also participated in 2003 and higher for four of the same ten districts participating in 2009. The performances of grade 8 urban youth in 2011 significantly exceeded those of their counterparts in nine of the ten urban districts from 2003 and from four of these same ten urban districts that participated in 2009, as well as scores for the nation and large cities in both 2003 and 2009. Also listed for the urban districts are the percentages of students achieving at "proficient" or higher (NCES 2011c). These results are encouraging, but indicate that there is still additional ground to be gained to achieve the necessary STEM levels.

College Entrance Examinations

Typically, a student in the United States applies for college in the twelfth grade, the last year of high school. The selectivity of colleges in the United States varies from community colleges and postsecondary institutions that require no more than a high school diploma or its equivalent to selective colleges at which 10% or fewer of the applicants are accepted. Occasionally, the selectivity of an institution varies with the academic major for which a student applies. Because college entrance examination scores provide the only easily quantifiable and comparable measure for students coming from different high schools and different areas of the country, they are often given great importance by colleges. As a result, most college-intending students in the United States take a college entrance examination during their junior or senior year of secondary school.

Two such major and independent college admission examinations exist. The SAT test, administered by the College Board, is more common in the east, south, and west. The ACT test, administered by the ACT, Inc., is more commonly preferred by institutions in the middle portion of the country. The percentage of graduating seniors having taken the SAT increased to a high of 49% in 2005. Since that time, there has been a slight decrease, to 47% percent of the graduating seniors having taken the examination in the class of 2010. The percentage of graduating seniors having taken the ACT trailed the percentage of those having taken the SAT until 2010. The percentage of high school graduates taking the ACT increased from 38% in 1999 to 47.5% in 2010 (Snyder, Dillow, and Hoffman 2009; Snyder and Dillow 2011). With the class of 2010, slightly more of the graduating seniors had taken the ACT, with 47.5% of the nation's graduating class having an ACT score. The data on the percentage taking the test have been influenced strongly in recent years by some states requiring all of their students to take the ACT at least once during the eleventh or twelfth grade as part of their state assessment program (ACT 2011a; College Board 2011a).

The SAT test assesses high school students' general capabilities in critical reading, mathematics, and writing. A student's results for each of the three sections of the SAT are reported on a 200–800 scale. A student's overall reasoning ability is reported by the sum of the individual scale scores on a 0–2400 scale. The mathematics test employs multiple-choice items and items in response to which students grid in their answers on an optically scanned answer sheet. The mathematics portion of the test covers number and operations; algebra and functions; geometry; and statistics, probability, and data analysis. Beginning with the March 2005 administrations of the SAT, the coverage of the mathematics test was increased to include more items testing the content of second-year algebra and more advanced topics from geometry. However, the focus in these items, as in those in the previous versions, remains on students' critical reasoning skills (College Board 2011b; Korbin and Schmidt 2007).

The ACT test assesses high school students' general subject matter knowledge and college or workforce readiness in four skill areas: English, mathematics, reading, and science. The test is composed entirely of multiple-choice items, and each of the four skill areas is reported on a 1–36 scale. A general summary score, also on the same 1–36 scale, is used to report a student's overall skill level (ACT 2011b). Graduating seniors' mean mathematics performance on both the SAT and ACT has shown substantial improvement since 1995 (see table 12). The asterisks indicate scores on the new form of the SAT beginning with 2006 data. These scores result from a bridge study that provides the basis for them to be reported on the same scale in a valid and reliable fashion for interpretation and comparison.

The general public has come to view these mean scores as a barometer of how well the education system is performing as a whole, despite the fact that the examinations were not designed for that purpose and have obvious shortcomings when used as a single measure of students' mathematics competence and overall quantitative literacy.

Table 12
Mean grade 12 scores for mathematics and English on the SAT and ACT

Year	Test			
	SAT—Math	SAT—Reading	ACT—Math	ACT English
1995	506	504	20.2	20.2
1996	508	505	20.2	20.3
1997	511	505	20.6	20.3
1998	512	505	20.8	20.4
1999	511	505	20.7	20.5
2000	514	505	20.7	20.5
2001	514	506	20.7	20.5
2002	516	504	20.6	20.2
2003	519	507	20.6	20.3
2004	518	508	20.7	20.4
2005	520	508	20.7	20.4
2006	518*	503*	20.8	20.6

Table 12—*Continued*

Year	Test			
	SAT—Math	**SAT—Reading**	**ACT—Math**	**ACT English**
2007	515*	502*	21.0	20.7
2008	515*	502*	21.0	20.6
2009	515*	501*	21.0	20.6
2010	516*	501*	21.0	20.5
2011	514*	497*	21.1	20.6

*SAT content upgraded, but all table data report on comparable score scale values.

PART V: Issues and Developments in U.S. Mathematics Education, K–12: Mathematical Reasoning and Common Core State Standards

In the past two years, mathematics educators in the United States have shifted their focus from the K–8 level to the secondary school curriculum. At the same time, there has been a concentrated movement toward a national K–12 curriculum. NCTM's efforts have mirrored this trend. With its 2006 publication of *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*, NCTM sorted and refined earlier curricular recommendations for three grade bands (pre-K–2, 3–5, and 6–8) to make specific, targeted recommendations, called "Focal Points," for individual grades, pre-K–8, thus clarifying what content was the major focus of instruction in each grade level. Other less central content at each grade was related to the main Focal Points with supporting connections to build a platform for consideration in later grades. With this pre-K through grade 8 framework in place, NCTM launched a new effort emphasizing the importance of developing reasoning and sense making in grades 9–12.

At the same time, the Council of Chief State School Officers (CCSSO) and the National Governors Association Center for Best Practices (NGA Center) launched the Common Core State Standards Initiative. The standards that this project developed for English language arts and mathematics present expectations for student knowledge and skills that high school graduates need to master to succeed in college and careers.

NCTM's *Focus in High School Mathematics: Reasoning and Sense Making*

In 2006, NCTM's 9–12 Curriculum Task Force recommended development of a framework to guide the future of high school mathematics. Rather than simply restating lists of curricular content expectations for grades 9–12, the writers identified common processes for doing and learning mathematics, and they used these to develop a framework that focuses on the core processes of reasoning and sense making in the classroom. The resulting core document, *Focus in High School Mathematics*, highlights reasoning and sense making as foundational to learning mathematics and proposes instructional approaches for making the development of reasoning and sense making happen in the classroom (NCTM 2009).

As stated by the authors of this framework for high school mathematics, "A focus on reasoning and sense making, when developed in the context of important content, will ensure that students can accurately carry out mathematical procedures, understand why those procedures work, and know how they might be used and their results interpreted" (NCTM 2009, p. 3). In a broader sense, a focus on reasoning and sense making is needed to prepare students for using mathematics in the classroom; in the workplace, including scientific and technical communities; and in life.

Central Purpose

NCTM's framework sees reasoning and sense making as the foundation for all mathematics processes, such as those identified by the NCTM Standards documents (NCTM 1989, 2000): problem solving, reasoning and proof, communication, connections, and representation. Even as these processes are seen as valuable to the learning of all mathematics at all levels, the teaching of mathematics at the high school level has historically reserved reasoning (and formal proof) for select areas of the curriculum, such as proof of geometry theorems. And sense making is often pushed aside as teachers struggle to prepare students for standardized testing situations by emphasizing procedural competence and quickness.

Focus in High School Mathematics is intended to refocus the teaching of high school mathematics and to provide teachers with examples for guiding reasoning and sense making in ways that help students build meaning and support further study of mathematics. NCTM also recognizes that these opportunities for reasoning and sense making must be provided to *all* students, not just those who intend to continue the study of mathematics and sciences beyond high school. All students must be provided with opportunities to learn, and a focus on reasoning and sense making, with expectations that all students can make sense of mathematical ideas, is a first step.

Overview of the Document

Focus in High School Mathematics defines reasoning as "the process of drawing conclusions on the basis of evidence or stated assumptions" (NCTM 2009, p. 4). Sense making is defined as "developing understanding of a situation, context, or concept by connecting it with existing knowledge" (NCTM 2009, p. 4). The interplay between reasoning and sense making can be complex, but it is important to understand that increased sophistication in reasoning ability can lead to increased capacity to make sense of new or old ideas. The reciprocal relationship is also true.

Although habits of mathematical reasoning have long been discussed in the literature on teaching and learning in mathematics (e.g., Cuoco, Goldenberg, and Mark 1996; Pólya 1954, 1957, 1967), *Focus in High School Mathematics* identifies components of essential reasoning habits, provides classroom vignettes that exemplify these habits, and discusses connections between an increasing sophistication of reasoning habits and mathematical sense making. Reasoning habits outlined in the document are the following:

- *Analyzing a problem* by identifying relevant mathematical concepts, defining relevant variables, and seeking patterns and relationships
- *Implementing a strategy* through purposeful use of known procedures, organizing, making logical deductions, and monitoring progress
- *Seeking and using connections* across mathematical domains
- *Reflecting on a solution* by interpreting results, considering the reasonableness of the result, justifying, refining, and generalizing (NCTM 2009, pp. 9–10)

Focus in High School Mathematics reemphasizes the importance of making sense of mathematics while developing valuable reasoning habits that cut across all areas of mathematics.

Content chapters in the core document include the familiar areas of number and measurement, geometry, algebra, functions, and statistics and probability. Additional chapters address reasoning and sense making across the overall high school mathematics program. These chapters discuss how a focus on reasoning and sense making can help address issues of equity and coherence (e.g., vertical alignment).

The document's five content chapters are organized by key elements within the focal content area. These key elements are not intended to be an exhaustive list of concepts and procedures to be taught, but instead they represent a broad view of the content area as taught with a focus on reasoning and sense making. For example, chapter 5, "Reasoning with Algebraic Symbols," identifies the following five key elements:

- Meaningful use of symbols
- Mindful manipulation
- Reasoned solving
- Connecting algebra with geometry
- Linking expressions and functions (NCTM 2009, p. 31)

For each of these key elements, the document provides an example of one or more mathematical tasks and illustrates how they might be used in the classroom to promote the key element as well as reasoning habits.

Additional Resources

Supporting publications quickly followed the release of the core document, *Focus in High School Mathematics: Reasoning and Sense Making.* Three companion publications provide more specific examples of what reasoning and sense making can look like in the mathematics classroom in the areas of statistics and probability (Shaughnessy, Chance, and Kranendonk 2009), algebra and functions (Graham, Cuoco, and Zimmermann 2010), and geometry (McCrone, King, Orihuela, and Robinson 2010).

Focus in High School Mathematics: Reasoning and Sense Making in Algebra (Graham, Cuoco, and Zimmermann 2010), for instance, offers ideas for developing formal algebra and work on functions with connections across content areas and to hands-on experiences as ways to help students make sense of algebraic concepts. One example shows how students might develop and make sense of the area formula for the trapezoid. A case study shows how exploratory work in cutting and rearranging trapezoids to create parallelograms, combined with classroom dialogue such as that modeled in a sample in the text, can encourage sense-making related to the area formula for a triangle and other basic geometric figures. Such explorations may offer needed practice as well with simplifying and finding equivalent algebraic expressions. The connections between the algebra formulas and the geometric figures also allow students to actively exercise reasoning habits. All examples and samples of classroom dialogue in the book highlight the reasoning habits and key content elements displayed in the examples.

A more recent supporting publication, *Fostering Reasoning and Sense Making for All Students* (Strutchens and Quander 2011), addresses issues of equity in the teaching and learning of mathematics, supporting and furthering the Equity Principle set forth in NCTM's *Principles and Standards for School Mathematics* (2000). Diversity in schools and mathematics classrooms covers a range of differences, including disparities in academic ability, language differences, economic differences, and ethnic and home-life differences. It is often the case that these differences translate to differences in educational opportunities, whether intentional or not. The message of *Fostering Reasoning and Sense Making for All Students* is that all students should be given access to rich mathematics and intellectual challenges through mathematical reasoning. All students can develop reasoning skills when such skills and processes are fostered in the classroom. This message is also highlighted in the fifth (and final) companion volume to *Focus in High School Mathematics: Reasoning and Sense Making—Focus in High School Mathematics: Technology to Support Reasoning and Sense Making* (Dick and Hollebrands 2011).

Connections and Support for *Focus in High School Mathematics*

Although the overall message is not new, *Focus in High School Mathematics: Reasoning and Sense Making* and its supporting publications offer a fresh look at the teaching of mathematics in the secondary school. At the same time, these documents provide ideas for teachers of mathematics at all levels. Mathematics should be a sense-making activity for all students. These ideas appear as the Process Standards in NCTM's Standards documents (1989, 2000). But with so much attention to student attainment of skills and procedures as assessed in state-level yearly standardized tests, the teaching of mathematics often does not involve valuable sense-making and reasoning processes.

NCTM has promoted this focus on reasoning and sense making through workshops and discussions at national and regional meetings, as well as through intensive professional development workshops. Support from mathematics educators across the United States is evident in the number and range of teachers and university faculty contributing to the development of the documents and the promotion of these ideas through professional development opportunities.

Common Core State Standards for Mathematics

A parallel development that has occurred in mathematics education over the past four years is the Common Core State Standards Initiative, a state-led effort to offer common expectations at each grade level for all adopting states in the United States. This initiative, coordinated by the Council of Chief State School Officers (CCSSO) and the National Governors Association Center for Best Practices (NGA Center), resulted in the creation of English language arts standards for all grade levels and mathematics standards for all grade levels, as well as literacy standards in the areas of history/social studies, science, and technical subjects for grades 6–12.

The U.S. education system is typically characterized as a non-cohesive system that relies on state and sometimes local funding and control. Thus, for example, two school districts within one state could have different graduation expectations for their students. Grade-level expectations from state to state vary to an even greater degree. Thus, the Common Core movement is intended to provide a framework for more closely aligned expectations within individual states and across states. The standards were developed in collaboration with teachers, school administrators, and experts to provide a clear and consistent framework "to prepare all children for college and the workforce" (NGA Center and CCSSO 2010a). Since the release of the Common Core State Standards (CCSS) in 2010, forty-five out of fifty states have adopted all aspects of the document as their state educational standards or goals. These standards were created with the hope of developing common educational experiences across school districts and across states in the United States. With their broad adoption, they are on their way to becoming a de facto national curriculum for English and mathematics (Porter, McMaken, Hwang, and Yang 2011).

Central Purpose

Why are common standards so important in mathematics? As noted above, there is no standard U.S. curriculum for school mathematics, and state-mandated curricula often lack focus and coherence. Results from international assessments of mathematics have shown that nations scoring highest are those whose curricula are more focused and coherent. A focused curriculum is one that addresses a limited number of topics in a given calendar year. A coherent curriculum is one that demonstrates the development of mathematical concepts in ways that make sense. In other words, coherence means that one idea leads to another, connected idea throughout the course of focused study. With ongoing changes in technology and alterations in the requirements of our national workforce, the need for change and movement toward more advanced mathematics in schools is constant. Thus, if our intention as educators is to prepare all students to be college and career ready by the time that they leave K–12 education, mathematics education needs to provide students with a solid background and tools to be lifelong learners.

A second main reason for the development of such standards is to increase collaboration across schools, districts, and states adopting the Common Core State Standards. With common expectations for content and mathematical practices at all grade levels, K–12, states can develop common curricula and assessments. Commercial curriculum developers can also use the common standards for developing textbooks that are a more reasonable size and focus on deep understanding of fewer key mathematical concepts. And, perhaps most important, students whose parents move from place to place during their formative education years will be less likely to miss opportunities to learn content as a result of non-aligned grade-level mathematics curricular programs.

Overview of the Document

The Common Core State Standards for Mathematics (CCSSM) recognize that it is important for students to understand both the content of the discipline and how knowledge is organized and created within the discipline. Thus, grade-specific standards outline the content topics to be taught and learned by students at particular grade levels, and a set of mathematical practice standards reflect NCTM's Process Standards and the reasoning habits of *Focus in High School Mathematics*. Choices for content standards and mathematical practices were based on research into how learning occurs, and learning progressions are provided for specific mathematics content (NGA Center and CCSSO 2010b). However, an ordering of topics within a grade level is not always provided, and the standards do not advocate a particular teaching methodology or theory of learning.

The focus of all mathematical standards is on developing deep mathematical understanding, not just on assessing whether or to what degree students can do the mathematics. Rather, the emphasis is on enabling students to justify their solutions and work through new problems on the basis of an understanding of central concepts.

Standards for Mathematical Practice (Processes and Proficiencies)

The main processes outlined in the CCSSM mathematical practices mirror NCTM's Process Standards for problem solving, reasoning and proof, and representation. Reasoning habits from *Focus in High School Mathematics* and proficiencies similar to those described in *Adding It Up* (Kilpatrick, Swafford, and Findell 2001) round out the mathematical practices. The eight standards for mathematical practice found in CCSSM are the following:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.

7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Standards for Mathematical Content

Content standards presented in CCSSM across grade levels show a progression of increasing sophistication as well as a change in the nature of key concepts. For example, at the kindergarten level, the content standards focus on whole numbers and shapes and space, with an emphasis on work with numbers in areas such as counting and cardinality, basics of adding and subtracting, and work with the numbers 10–20 to introduce place value. By the time that students reach second grade, the content expectations emphasize fluency in addition and subtraction and extending understanding of the base-ten numeration system up to 1000. By third grade, students are expected to encounter fractions as numbers on the number line and to use reasoning to compare fractions.

Sixth grade is characterized by emphasis on ratios, proportions, and related reasoning, as well as an introduction to algebraic expressions. Although number systems and data show up in earlier grades, sixth-grade content standards also focus on extending students' understanding of number systems by introducing negative rational numbers and on developing students' statistical thinking. Ratios, proportions, and proportional reasoning continue to be prominent in the middle school years.

High school content standards are listed by conceptual categories rather than by grade level and are organized by domains, clusters, and standards within each cluster. Coherence within and across categories is emphasized. The six conceptual categories include (with content domains listed in parentheses):

- Number and quantity (real number system, quantities, complex number system, vector and matrix quantities)
- Algebra (structure in expressions, polynomial and rational expressions, creating and reasoning with equations and inequalities, connections to functions and modeling)
- Functions (interpreting functions; linear, quadratic, and exponential models; trigonometric functions; building functions)
- Modeling
- Geometry (congruence, similarity, circles, trigonometry, connections to equations, measurement, dimensions, and modeling)
- Statistics and probability (interpreting data, making inferences and justifying conclusions, conditional probability, probability in decision making)

Proficiency clusters within each content domain provide more specificity, and each standard gives details that would be useful for assessing students' attainment of that standard. For example, within the domain of vector and matrix quantities (a domain of the number and quantity category), three proficiency clusters are defined. Students are to (1) represent and model with vector quantities, (2) perform operations on vectors, and (3) perform operations on matrices and use matrices in applications. Each proficiency cluster then lists a few specific standards, such as, within the "*perform operations on vectors"* cluster, a standard that reads, "Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum" (NGA Center and CCSSO 2010a, p. 61).

Additional Resources

In addition to the content standards for the high school level, CCSSM provides an appendix that offers sample "pathways" for implementing the high school standards (NGA Center and CCSSO 2010b). These model pathways illustrate methods for organizing the standards into courses to provide a strong foundation for further study of mathematics or for using mathematics in life and the workplace. An integrated pathway (similar to many international curricula) consists of three core courses, each of which explores number, algebra, geometry, data, and statistics. In contrast, a more traditional U.S. pathway includes separate courses for algebra (two levels), and geometry, with data and statistics integrated with these. "Compacted" or accelerated pathways are also outlined to serve student populations that might be considering four years of high school mathematics, including a course in calculus in the high school curriculum.

Connections and Future Pursuits

In all of its documents, the National Council of Teachers of Mathematics presents a vision for school mathematics that is intended to guide policy makers, school leaders, and teachers. The Standards, Focal Points, and reasoning and sense-making documents outline goals for any mathematics program. The intention of the Common Core State Standards Initiative, by comparison, is to provide a coherent set of standards for all grade levels that are ready for implementation. Thus, although there are some differences between the NCTM documents and CCSSM, their overall goals are closely aligned. In particular, CCSSM's description of career and college readiness through experience with mathematical practices correlates well with the treatment of reasoning and sense making in *Focus in High School Mathematics* (NCTM 2009).

PART VI: Exploring the Common Core State Standards

What is it about the Common Core State Standards for Mathematics (CCSSM) that has raised so much hope and, at the same time, so much apprehension in U.S. education circles? On the one hand, people are excited about the fact that the nation, for the first time in its history, is moving to the same set of grade-level expectations for student performance. The mathematics community speaks about more rigor and higher expectations being brought to the curriculum. Educators and learning experts point to the use of learning hierarchies or progressions in the design of curriculum. Others talk about the opportunities for better-sequenced education for students whose parents move during their K–8 school years. Still others talk about the fact that now educational programs can be compared, funded, and evaluated on an equal basis. On the other hand, curricular groups are concerned about the loss of emphasis on geometry and statistics. Others express concern about students with learning disabilities and their capability to stay with newer and more demanding expectations. Teachers note that current texts and materials are not correlated yet with the CCSSM outcomes, even though some commercial materials are making supplementary booklets to help bridge the gap and make the transition.

Resources for Change

Teachers and administrators are concerned about the actual mechanics of implementing a program requiring some phasing in of changes in content sequences in a period of high-stakes assessments and rising NCLB requirements for adequate yearly progress goals. Horizon Research, with funding from the NSF, has prepared a research agenda focused on many of the issues involved in the adapting and adopting of the Common Core State Standards for Mathematics in U.S. schools (Heck et al. 2011). In what follows, we briefly examine some of the resources coming with CCSSM, and we provide links to more expansive discussions of the issues surrounding the new standards.

Links from NCTM Standards to CCSSM: *Making It Happen*

Specific information on links between CCSSM and NCTM's *Principles and Standards for School Mathematics* (2000) can be found at www.nctm.org/mih. The tab "mih" at the end of the URL signifies the lead words in the NCTM's core guide to CCSSM, *Making It Happen: A Guide to Interpreting and Implementing Common Core State Standards for Mathematics* (2010). This publication identifies NCTM's array of books, instructional guides, and CCSSM-related materials designed to help teachers, administrators, and schools make the transition from their existing mathematics curriculum and instructional efforts to one centered on the Common Core State Standards for Mathematics. This work connects CCSSM to NCTM's *Principles and Standards for School Mathematics* (2000), *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (2006), *Focus in High School Mathematics: Reasoning and Sense Making* (2009), and NCTM's professional development materials targeted to topics and grade levels in the Essential Understanding Series, which will ultimately consist of sixteen volumes, eleven of which are currently in print. *Making It Happen* also points teachers to volumes in NCTM's *Principles and Standards for School Mathematics* Navigations Series, thirty-five books designed to support teaching aligned to NCTM's Content Standards and Process Standards in particular grade bands and grade levels.

Common Core State Standards Initiative Site

A second stop for teachers and administrators preparing to implement CCSSM might be the Common Core State Standards website, for a look at the actual standards and supporting documents developed by the Council of Chief State School Officers and the National Governors Association's Center for Best Practices writing team. The full standards document is at the site (http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf). The site also contains links to an appendix that illustrates ways in which the mathematics standards might be implemented in either a traditional course-based secondary curriculum or a school pursuing an integrated mathematics approach in its curriculum (http://www.corestandards.org/assets/CCSSI_Mathematics_Appendix_A.pdf).

After a look at these two sites, the next step might be to compare the standards and supporting documents at the site with the criteria that the writers and other developers were given as goals for their work. These begin with a preamble that opens in the following way:

> The Common Core State Standards define the rigorous skills and knowledge in English Language Arts and Mathematics that need to be effectively taught and learned for students to be ready to succeed academically in credit-bearing, college-entry courses and in workforce training programs. These standards have been developed to be:
>
> - Fewer, clearer, and higher, to best drive effective policy and practice;
> - Aligned with college and work expectations, so that all students are prepared for success upon graduating from high school;
> - Inclusive of rigorous content and applications of knowledge through higher-order skills, so that all students are prepared for the 21st century;
> - Internationally benchmarked, so that all students are prepared for succeeding in our global economy and society; and
> - Research and evidence-based. (NGA Center and CCSSO 2010c, p. 1)

Additional resources at this site include an international benchmarking report linking CCSS to practices in other countries, a document providing a listing of key points to the mathematics standards, and the allied English language arts standards and their supporting information.

Institute for Mathematics & Education

The Institute for Mathematics & Education at the University of Arizona hosts two important websites that are being developed in relation to CCSSM through funded projects housed in the Department of Mathematics:

1. The Illustrative Mathematics Project, under the leadership of William McCallum (University of Arizona) and Kristen Umland (University of New Mexico), will display the individual CCSSM standards sorted by various content dimensions and grade levels, with access to tasks associated with specific standards and cognitive outcomes. Work from this project can be accessed through http://illustrativemathematics.org. This project has funding from the Bill and Melinda Gates Foundation.
2. The Progressions Documents for the Common Core Math Standards Project,

under the leadership of Phil Daro (Strategic Education Research Partnership, San Francisco) and Jason Zimba (Bennington College), will focus on the development and display of the learning hierarchies undergirding CCSSM. At present, the team of writers, many of whom were involved in the writing of CCSSM, have developed the following progressions showing the flow of concepts, principles, and important landmarks that students encounter as they move upward through a CCSSM-based curriculum:

- Draft 3–5 progression on number and operations—fractions
- Data part of the K–5 progression on measurement and data
- Draft K–5 progression on number and operations in base ten
- Draft K–5 progression on counting and cardinality and operations and algebraic thinking
- Draft 6–8 progression on expressions and equations
- Draft 6–7 progression on ratios and proportional relationships

These materials, and others to come, can be accessed through http://ime.math.arizona.edu/progressions. This project has funding through the Brookhill Foundation.

Assessing CCSSM in Action

Part of the American Recovery and Reinvestment Act of 2009 was a segment of funding focused on the improvement of education, especially as it stimulated the economy and job creation. The $4.5 billion allocated for education included four targets, one of which was adopting standards and assessments that prepare students to succeed in college and the workplace and to compete in the global economy. The assessment portion of this funding, amounting to $350 million, was split between two consortia of states, totaling forty-four states and the District of Columbia. Some of the states belong to both of the consortia. These two consortia are the Partnership for Assessment of Readiness for College and Careers (PARCC) and the SMARTER Balanced Assessment Consortium (SBAC).

These consortia are presently developing draft frameworks, item specifications, and sample items for the states. With the assessments scheduled to go live in 2014–15, the consortia are now into the development and limited trialing of their formats and items for field-testing during the next school year. Both consortia plan to focus on the use of technology in the delivery of major portions of the assessments to provide schools with a quick turnaround and, in at least one case, make full use of adaptive testing. In addition to summative assessments, there will be formative and summative measures of students' progress toward college and career readiness. The consortia also have plans to develop group-testing measures that will produce individual scores. Further, there will be midyear assessments and other evaluative materials that teachers can use to check up on students' progress along the way during the year toward the final goals for the year.

The websites for the two consortia are http://www.parcconline.org for PARRC, and http://www.k12.wa.us/smarter for SBAC. These websites will provide the assessment frameworks as they become available and then, later, will give the item specifications as they are released, along with sample items. These will give the first look at the outcome expectations in reality.

PART VII: Programs for Special Populations of Students

A high school student who completes the standard college-bound curriculum concluding with precalculus before entering twelfth grade and who desires to continue the study of mathematics in twelfth grade has three potential paths to follow. First, if the school is very small and has no college nearby, then the student may be able to take an individualized course under teacher guidance or over the Internet. Second, if the school is near a college or university, the student may be able to take a college course and apply the credit toward high school graduation. Third, if enough students in the school are in the same position as this student, then the school may wish to offer Advanced Placement (AP) courses.

Advanced Placement Programs

In 1955, under the auspices of the College Board, the Educational Testing Service (ETS) created the Advanced Placement Program to enable students to take college-level work before graduating from high school (Handwerk et al. 2008). High schools participating in this program offer courses with syllabi designed to be in agreement with introductory college courses. Thirty-seven AP courses now exist in twenty-five different subject areas, and more than 16,000 high schools worldwide participate. In 2011, a total of 781,857 students took one or more AP examinations (College Board 2011d, 2011e, 2011f).

Most AP courses are a year in length. Many high schools, however, offer "block schedules" with longer class periods each day, and in these schedules, AP courses are compressed into one-semester configurations. In May of each year, ETS administers nationwide exams for each of the courses. Colleges have the option of offering college credit, placing students in more advanced classes (with or without credit), or ignoring the scores that students receive. Many colleges take scores on AP tests into account when placing students into courses.

When AP courses are taken in the eleventh grade or earlier, they can be considered along with a student's application to a college and may factor into admissions decisions. Although scores on AP tests in twelfth grade are not available to colleges before admissions decisions are made, enrollment in AP courses itself tends to signify that an applicant is a more serious student and the high school is more scholastically oriented and thus can increase the student's chances of admission to some colleges.

Scores on AP tests range from 1 to 5. The American Council on Education recommends that colleges give credit to students who score 3 or higher, but some colleges have higher cutoffs, and some give credit for part of the yearlong course, depending on the score (College Board 2011e, 2011c).

Advanced Placement Programs in Calculus

Two AP exams are offered in calculus: Calculus AB (since 1956) and Calculus BC (since 1969). Calculus BC is an extension of Calculus AB, not an enhancement; common topics require a similar depth of understanding. Three hours and 15 minutes are allotted for the completion of the examination for either course. Both course examinations consist of two parts: a multiple-choice portion and a constructed-response portion. Each of these portions is again split into two parts, one containing questions on which a student may not use a calculator and the other containing questions for which a graphing calculator is required.

The multiple-choice portion for each examination consists of 45 questions to be completed in 105 minutes. Part A consists of 28 questions to be completed in 55 minutes and does not allow the use of a calculator. Part B consists of 17 questions to be completed in 50 minutes and contains some questions for which a graphing calculator is required. The constructed-response section of each examination consists of 6 problems to be completed in 90 minutes. Part A consists of 2 problems to be completed in 30 minutes and requires the use of a graphing calculator. Part B consists of 4 problems to be completed in 60 minutes and does not allow the use of a calculator (College Board 2010a).

In 2011, 255,357 students took the Calculus AB examination, and 85,194 students took the Calculus BC examination. Both of these numbers of examinees were the highest on record for either examination. In 2011, 56.2% of students taking the Calculus AB test scored 3 or higher, and 80.2% of students taking the Calculus BC test scored 3 or higher (College Board 2011d, 2011e, 2011f). The scores 5 down to 3 relate to grades for the university courses that are tied to the respective tests from a content standpoint: 5 is equivalent to an A; 4 is equivalent to an A–, B+, or B; and 3 is equivalent to a B–, C+, or C (College Board 2011b). Lower scores are not suggested as constituting a basis for earned credit for advanced placement.

The syllabi for Calculus AB and BC are developed, and modified periodically, by a national committee of the College Board and might be said to represent a consensus regarding what a good calculus course should include. Both syllabi are primarily concerned with developing students' understanding of the concepts of calculus and providing experience with its methods and applications. The courses emphasize a multi-representational approach to calculus, with concepts, results, and problems expressed geometrically, numerically, analytically, and verbally. The connections among these representations also are important. The differentiation of the syllabi for Calculus AB topics and Calculus BC topics consists of the depth and breadth of coverage. The following listing provides an overview of the topic listing for the two courses with the topics covered only in Calculus BC identified with an asterisk (*) (College Board 2010a).

I. Functions, Graphs, and Limits

Analysis of graphs
Limits of functions (including one-sided limits)
Asymptotic and unbounded behavior
Continuity as a property of functions
*Parametric, polar, and vector functions

II. Derivatives

Concept of the derivative
Derivative at a point
Derivative as a function
Second derivatives
Applications of derivatives
Computation of derivatives

III. Integrals

Interpretations and properties of definite integrals
*Applications of integrals

Fundamental theorem of calculus
Applications of antidifferentiation
Numerical approximations to definite integrals

IV. *Polynomial Approximations and Series

*Concept of series
*Series of constants
*Taylor series

Advanced Placement Program in Statistics

A single Advanced Placement exam is offered in statistics. AP Statistics is meant to be equivalent to a one-semester, introductory, noncalculus-based college course in statistics. Graphing calculators with statistical capabilities are required for the exam, but the College Board emphasizes that they are not equivalent to computers in the teaching of statistics. In 2011, 142,910 students took the AP Statistics exam, and 58.8% of these scored 3 or higher (College Board 2011d, 2011e, 2011f). An outline of the areas covered by the AP Statistics examination (College Board 2010b) follows:

I. Exploring Data: Describing Patterns and Departures from Patterns

Constructing and interpreting graphical displays of distributions of univariate data (dot plot, stem plot, histogram, cumulative frequency plot)
Summarizing distributions of univariate data
Comparing distributions of univariate data (dot plots, back-to-back stem plots, parallel box plots)
Exploring bivariate data
Exploring categorical data

II. Sampling and Experimentation: Planning and Conducting a Study

Overview of methods of data collection
Planning and conducting surveys
Planning and conducting experiments
Generalizability of results and types of conclusions that can be drawn from observational studies, experiments, and surveys

III. Anticipating Patterns: Exploring Random Phenomena Using Probability & Simulation

Probability
Combining independent random variables
Normal distribution
Sampling distributions

IV. Statistical Inference: Estimating Population Parameters and Testing Hypotheses

Estimation (point estimators and intervals)
Tests of significance

Advanced Placement Program in Computer Science

In addition to the AP Calculus courses and the AP Statistics course, the College Board also offers an Advanced Placement examination for Advanced Placement Computer Science A. This course focuses on object-oriented programming methodology with a concentration on problem solving and algorithm development and is meant to be the equivalent of a first-semester college-level course in computer science. It also includes the study of data structures and algorithm design (College Board 2010c).

Special Schools and Programs

The National Consortium for Specialized Secondary Schools for Mathematics, Science, and Technology (NCSSSMST) includes more than one hundred institutional members with forty thousand students. The goal of the consortium, as its name indicates, is to foster, support, and advance the efforts of specialized schools to attract and academically prepare students for leadership in the subject areas of mathematics, science, and technology. Some members are boarding schools requiring state residence and highly competitive examinations for entrance; a few are local, specialized high schools; others are regional centers that students may attend for a half or full day for a single year (NCSSSMST 2011).

University-centered programs offer two types of summer programs in mathematics for very capable students. The first type follows a model initiated by the late Julian Stanley at Johns Hopkins University in the 1970s, identifying talent in the upper elementary or middle school grades and offering accelerated courses (usually in the summer but sometimes through the school year) to enable those students to study more advanced mathematics at a younger age (Johns Hopkins University 2011). The second type follows a model initiated by Arnold Ross at Notre Dame University around the same time, in which students are taught mathematics in a way that is different from the approach that they would normally be exposed to in school. Instead, they are expected to solve problems and deduce propositions in somewhat the same manner as professional mathematicians. These programs recruit either regionally or nationally, and opportunities are available for students across the entire nation (Ohio State University 2011).

The largest organization of mathematics clubs in the United States is Mu Alpha Theta, founded in 1957. Mu Alpha Theta has more than 1,800 high school and community college chapters and more than 88,000 student members across the United States. Its purpose is to stimulate interest in mathematics by providing recognition of superior mathematical scholarship in students. In addition to holding regional meetings and an annual national meeting, Mu Alpha Theta also publishes a newsletter and provides several other resources for its student members (Mu Alpha Theta 2011).

In the United States, 1,297 schools are authorized to offer some level of the program of the International Baccalaureate Organization (IBO); 3,290 are authorized worldwide in 141 countries. Of these 1,297 schools, 744 offer the Diploma Program, a demanding two-year, precollege program that leads to examinations and is designed for students who are sixteen to nineteen years of age. The remaining 553 schools offer either the Middle Years Program or the Primary Years Program, both of which are designed for younger students (IBO 2011).

Programs for Undergraduate and Graduate Students

The National Science Foundation (NSF) funds a large number of research opportunities for undergraduate students through its Research Experiences for Undergraduates (REU) program. An REU site consists of a group of ten or so undergraduates who work in the research programs of the host college or university. Each student is associated with a specific research project, on which the student works closely with the faculty and other researchers. Students are granted stipends and, in many instances, assistance with housing and travel. Undergraduate students supported with NSF funds must be citizens or permanent residents of the United States or its territories. In 2010–11, forty REU sites with research opportunities were available in mathematics. A list of the REU sites for 2012 can be found at NSF's website (NSF 2011).

Mathematics Competitions

Mathematics competitions in the United States are voluntary for both individuals and schools. Some middle schools and high schools have mathematics teams, often competing in events operated by local professional organizations. Descriptions follow of the larger competitions of national scope.

- **MATHCOUNTS.** The National Society of Professional Engineers, the CNA Foundation, and NCTM founded MATHCOUNTS in 1982 to increase interest and involvement in mathematics and to assist in developing a technologically literate population. The competition is now operated by the MATHCOUNTS Foundation; sponsors include the Raytheon Corporation, National Defense Education Programs, Northrup Grumman Foundation, Texas Instruments, 3M Foundation, and Think Fun. Participation is restricted to students in grades 7 and 8. In 2003, more than 250,000 students in 7,000 schools were exposed to MATHCOUNTS materials, and more than 125,000 participated in the national competition at some level (MATHCOUNTS 2010).
- **American Mathematics Competitions (AMC).** The AMC, centered at the University of Nebraska–Lincoln, involved more than 360,000 participants in 2010. These participants account for 20% of the high schools in the country each year. The AMC competitions began in 1950 under the sponsorship of the Mathematical Association of America (MAA) and the Society of Actuaries as the American High School Mathematics Examination (AHSME) for students in grades 9–12. This program, administered by the MAA and principally funded by the Akamai Foundation with the support of nineteen other mathematics organizations, has evolved into a series of examinations spanning the range from junior high school through grade 12. The original AHSME examination is now called the AMC 12. Over time, as other organizations became involved, new competitions were added. In 1985, an exam for students below grade 9, the American Junior High School Mathematics Examination (AJHSME), now called the AMC 8, was initiated. In 2000, the AMC 10, an exam for students below grade 11, was launched. In addition to being a freestanding competition, the AMC 12 is the first examination in a series of examinations that leads to the selection of the U.S. competitors for the Mathematical Olympiad. The highest scorers on the AMC 12 become eligible to participate in the United States of America Mathematical Olympiad

(USAMO), a six-question, six-hour exam that is used to determine the U.S. team members for the International Mathematical Olympiad (IMO). The AMC also operates a summer program for qualifying students (AMC 2011).

In 2010, the AMC-8 involved 153,211 students from 2,342 schools, the AMC 10 involved 100,345 students from 3,658 schools, and the AMC-12 involved 107,905 students from 4,562 schools. From the AMC-10 and the AMC-12, a total of 6,528 students were invited to participate in the American Invitational Mathematics Examination (AIME). From the AIME results, 329 students were selected to sit for the USAMO examination, from which 12 emerged to form the pool from which the final 6 students forming the U.S. team for the International Mathematical Olympiad were selected (AMC 2011).

- **The Math League.** The Math League, which was founded in 1977, specializes in math contests, books, and computer software designed to stimulate interest and confidence in mathematics for students from the fourth grade through high school. In recent years, more than one million students have participated in Math League contests each year. Contest problems are designed to cover a range of mathematical knowledge for each grade level and require no additional knowledge of mathematics beyond the grade level that they test (Math League 2011).
- **The American Regions Mathematics League (ARML).** ARML, begun in 1976 as the Atlantic Region Mathematics League, is a competition of teams of high school students who represent their school, local area, state, or country (outside the United States). This contest takes place during November and February of a school year. It pits teams of students from different schools in a contest to group-solve a set of honors-level problems in a 45-minute period of time. The papers are then mailed in and evaluated by a team of judges. A national competition, which takes place toward the end of the school year, occurs at three sites. In May 2011, more than 2,200 students from 170 teams representing schools or regions participated in the national competition (ARML 2011).
- **The Consortium for Mathematics and Its Applications (COMAP).** COMAP sponsors—in conjunction with the MAA, NCTM, the Information Science and Operations Research Society, and the Society for Industrial and Applied Mathematics—the High School Mathematical Contest in Modeling (HiMCM). Results from this contest are published in COMAP's publication for high school teachers and students, *Consortium*. COMAP also organizes the Mathematical Contest in Modeling for teams of college students and publishes the winning entries in the *UMAP Journal* (COMAP 2011).
- **The Student Mathematics League (SML).** The SML is a competition for students enrolled in two-year colleges. Originally founded in 1970 by Nassau Community College in New York, this twice-annual competition came under the sponsorship of AMATYC in 1981. The SML involves more than 8,000 two-year college students from 165 colleges in thirty-five states and Bermuda in its annual cycle of two examinations. The examinations are based on the standard syllabus in college algebra and trigonometry and may involve precalculus-level algebra, trigonometry, synthetic and analytic geometry, and probability. All questions are short-answer or multiple choice (AMATYC 2011).

- **The William Lowell Putnam Mathematical Competition.** The Putnam Examination, a competition for undergraduate mathematics students administered by the MAA, is perhaps the most rigorous and prestigious mathematics examination held annually. This examination can be entered by individuals or by three-person teams representing their college or university. Held annually in December, the Putnam Exam marked its seventy-second competition in December 2011. In December 2010, 4,296 individuals and 546 three-person teams competed from 442 colleges and universities in the United States and Canada (MAA 2011). Problems and solutions for the 2010 edition are available at www.maa.org/awards/putnam.html.

PART VIII: Teacher Education and Professional Development

A bachelor's degree and a teaching certificate are needed to teach in most public schools in the United States at any level, kindergarten–grade 12. The teaching certificate is generally obtained through a combination of courses taken at the college level and in-school experience (observations and work in schools, including supervised practice teaching) at or around the grade levels at which the teaching is to take place. Some states also require that the teacher pass a test, which usually consists of specific subject-matter knowledge and general knowledge about teaching and the education system. All states, with the exception of Alaska and Oregon, also provide some alternative route to teacher certification based on an individual's prior experiences, education, and, potentially, a bundled set of courses and internship experiences (National Center for Education Information [NCEI] 2010). The NCEI estimates, based on data submitted by the states, indicate that at least 59,000 individuals were issued certificates to teach through alternative routes in 2008–9. Also, in cases of teacher shortage or the movement of a teacher from one state to another, provisional certification is possible through state education officials until all the requirements for full certification have been met.

Most teachers earn certification before having had a full-time teaching position and gain tenure after two to four years of full-time teaching. With tenure comes job security; a tenured teacher cannot be removed from a teaching position without evidence of incompetence, breach of contract, or other wrongdoing. A teaching certificate, although not required to teach in private or parochial schools, is often desired because the agencies that accredit schools want schools to have certified teachers. (Accreditation is necessary for other schools to automatically recognize a school's graduates and students who transfer from that school.)

Teacher Preparation Programs

University mathematics departments typically offer the mathematics courses taken by preservice and in-service teachers as part of their training, although in some institutions education departments may offer those courses intended for preservice elementary school teachers. In these institutions, the methods courses for the teaching of mathematics may be taught in either the department of mathematics, if the mathematics educators are housed there, or in the college of education.

In 2010, the Conference Board of the Mathematical Sciences (CBMS) survey of undergraduate programs asked a special set of questions focused on the mathematical programs provided for prospective teachers for K–grade 12. These questions focused on where such programs were housed in the individual universities at which they existed and what courses and experiences were required of the students in these programs. The results of the survey indicated that 72% of the institutions had a K–8 teacher certification program in 2010. This was a decline from 84% in 2000 and 87% in 2005 (Kirkman, Blair, and Maxwell 2012). The reasons for the decline in the percentage of institutions offering such a program were not clear in an era of recommendations calling for more mathematics specialists in the K–8 years and movement in states to provide special certification of teachers at these levels. An examination of some subareas within the data groups indicate that the major sources of the decline were in universities with PhD programs in mathematics and four-year colleges offering the BA in mathematics as their highest mathematics degree. There was a slight increase in the percentage of K–8 programs in universities offering the MA in mathematics as their highest degree. This, perhaps, may indicate that these specialist programs are most often found in the universities that have grown out of what historically were the “normal schools.”

Two-year colleges have also become active in providing a major portion of the required coursework in mathematics for preservice teachers’ initial certification requirements and a significant amount of the coursework that can also be used to satisfy recertification requirements for practicing in-service teachers. Table 13 contains the data on the percentage of two-year college mathematics departments that provide organized programs allowing teachers to complete their *entire* mathematics course or licensure and certification requirements.

Table 13
Percentage of two-year colleges providing coursework supporting certification and licensure requirements

	Percent of two-year colleges providing coursework to satisfy entire mathematics initial course requirements for certification or relicensure/certification requirements
Preservice elementary teachers	41
Preservice middle school teachers	24
Preservice secondary teachers	13
In-service elementary teachers	25
In-service middle school teachers	12
In-service secondary teachers	10
Alternative certification for elementary school	30
Alternative certification for middle school	17
Alternative certification for secondary school	13

(Kirkman, Blair, and Maxwell 2012)

Examination of the data for four-year colleges that provide programs of preparation for “early grades mathematics specialization” reflect that, on average 42% require 2 courses, 14% require 3 courses, 14% require 4 courses, and 11% require 5 or more courses for such a distinction. Across all of these programs, the average course requirements are 2.7 mathematics content courses, 1.4 mathematics pedagogy courses taught either within or outside the mathematics department, and 0.5 courses in mathematics pedagogy taught within the mathematics department (Kirkman, Blair, and Maxwell 2012).

With respect to the full range of content and pedagogy courses offered to students by mathematics departments providing preparatory programs for preservice K–8 teachers, the data show that 74% of these departments offer courses in number and operations, 57%

offer courses in algebra, 69% offer courses in geometry/measurement, 56% offer courses in statistics or probability, and 31% offer courses dealing with teaching methods of elementary mathematics (Kirkman, Blair, and Maxwell 2012). Again, the entire survey, which will be available on the CBMS website (www.cbmsweb.org) later in 2012, will provide the complete set of data and break it down by size, type of institution, and level of faculty (tenure track or temporary).

Many colleges' teacher education programs are accredited by the National Council for Accreditation of Teacher Education (NCATE 2011b). NCATE has worked with NCTM to develop the standards that NCATE uses in the accreditation of programs, a process that involves documented satisfaction of the NCATE/NCTM guidelines for teacher education programs and a site visit by a team of experienced evaluators, usually including one mathematics educator (NCATE 2011a).

Data on the quantity and quality of the teacher workforce come from education agencies in individual states, a schools and staffing survey conducted by the National Center for Education Statistics (NCES), projects focusing on mathematics teacher education, and the NAEP assessments. The state data are not always complete, and some of the data raise questions about accuracy and completeness. Research conducted by Goldhaber and Brewer (1997) using NCES data from the National Education Longitudinal Study of 1988 found that students whose teachers held any certification in mathematics scored significantly higher on a twelfth-grade mathematics achievement test than did students who were taught by teachers with no certification or certification in another subject. Students who were taught by teachers with a mathematics certification recorded a 2-point increase (about three-quarters of a year of schooling) on the NELS:88 mathematics test. This was about twice the size of the association that Goldhaber and Brewer (1999, 2000) found in earlier studies among students whose teachers held a degree in mathematics.

Data from the *Education and Certification Qualifications of Departmentalized Public High School-Level Teachers of Core Subjects: Evidence from the 2007–08 Schools and Staffing Survey* (Hill and Gruber 2011) indicate that mathematics students in public schools in 2007–8 were disproportionately more likely to have a teacher with neither an undergraduate major in mathematics nor certification in mathematics (11.1% of students) as compared with students in English (7.3% of students), science (4.0% of students), or the social sciences (5.8% of students). At the same time, data from the National Center for Education Statistics indicate that the number of public and private school teachers with their highest degree in mathematics increased from 1999–2000 to 2003–4 to 2007–8 from 191,000 to 213,000 to 252,000, respectively. Many of these teachers teach other subjects because in these same years the numbers of high school teachers with a main assignment in mathematics fluctuated from to 33,000 to 31,000 to 36,000, respectively (Snyder and Dillow 2011). Many of these individuals with degrees in mathematics are in administration posts, teaching in special non-core curriculum programs, or teaching in the sciences.

The counting of teachers with a major in mathematics across the periods 1994–95, 1999–2000, and 2003–4 indicated that 72%, 67%, and 61% of middle and secondary school mathematics teachers, respectively, had a major in mathematics. The decline in teachers with mathematics degrees over the entire period was unique among core areas of mathematics, English, and science (Blank and Toye 2007). One hypothesis is that in a period of mathematics teacher candidate shortages, the need to ensure the filling of vacant positions created a market and resulted in schools settling for less qualified candidates. The overall increase in the number of teachers of mathematics at the middle and senior high school levels is partially due to population increases, but also to increased requirements for teaching mathematics and increased core curriculum requirements in mathematics for graduation.

The opposite trend held in the national data for teachers in prekindergarten through grade 8. Staffing data for these grades indicated that the numbers of teachers with full-time mathematics positions and a mathematics degree decreased from 26,000 in 1999–2000 to 19,000 in 2003–4 and rebounded to 28,000 in 2007–8 (Snyder & Dillow 2011). This finding is encouraging, given the added content and growth of the student population at this level of education. One of the factors responsible for the growth into 2007–8 was the increased emphasis placed on mathematics and changing teacher requirements for the teaching of mathematics at the middle grades and middle school levels. The nation still needs to consider addressing the inadequate number of highly qualified teachers of mathematics in the high schools.

Elementary School Mathematics Teachers and Teacher Education

The last fifty years of the twentieth century saw vast changes in the preparation of teachers for the nation's elementary schools. In 1952, nearly half of the nation's six hundred thousand public elementary school teachers did not hold college degrees (Lucas 1997). By the early 1990s, all states required an undergraduate degree for an individual to receive a teaching certificate. Even at present, however, the amount of mathematics included in the collegiate program for someone preparing for teaching K–grade 6 is minimal. Individual university programs vary, as do state requirements for certification. Most programs for students preparing to teach in K–6, however, consist of a major in education with only a modicum of coursework beyond the institution's general education requirements (Hawkins, Stancavage, and Dossey 1998; Smith, Arbaugh, and Fi 2007).

The 2009 national NAEP assessment asked grade 4 and grade 8 teachers to indicate their college majors, but they could provide multiple responses, which are somewhat difficult to interpret. The teachers' responses to these questions indicate that about 1% of the nation's grade 4 teachers had an undergraduate or graduate degree in mathematics, 1% had a degree in mathematics education, 62% had a degree in education, and the remaining 36% had a degree in some other major (NCES 2009a). Further, only 1% of the grade 4 students had a teacher reporting having a major in mathematics in graduate school, although 6% of the teachers reported a minor or special emphasis in mathematics at this level.

With the changes in NAEP during the early 2000s, much of the background data on teachers' content knowledge and pedagogical decision making ceased to be collected. Further, there has not been a large-scale study of elementary or middle school teachers' higher education backgrounds in mathematics since *The National Survey of Science and Mathematics Education* conducted in 2000 (Smith et al. 2002). Horizon Research has been awarded support to conduct the 2012 National Survey of Science and Mathematics Education. Future information about this project can be found on the company's website at http://www.horizon-research.com/projects/current.

Of the K–grade 4 teachers surveyed in *The National Survey*, 96% reported having completed a course in mathematics for elementary school teachers; 42%, a course in college algebra, trigonometry, or elementary functions; 33%, a course in probability or statistics; 21%, a course in applications of mathematics or problem solving; 21%, a course in geometry for teachers; and 12%, a course in calculus. These findings are not always consistent with the 2000 (1996) NAEP, in which 83% (84%) of students in grade 4 had teachers who had taken a course in the teaching of mathematics, 39% (43%), in number systems and numeration; 31% (37%), in measurement; 30% (34%), in geometry; 46% (45%), in college algebra; and 39% (36%), in probability and statistics (Malzahn 2002). Many articles in the public press have decried elementary school teachers' lack of depth of understanding of mathematics and their knowledge of relevant pedagogical practices in mathematics.

Middle School Mathematics Teachers and Teacher Education

In 2007, forty-six states plus the District of Columbia had either a middle school or junior high school certification or endorsement requirement (McEwin 2007). Many of these states also have special mathematics requirements for that certification or endorsement by the teachers' selected area of content expertise. In mathematics, these special requirements range from passing a test to completing the equivalent of an undergraduate minor in mathematics.

According to the results reported for middle-grades settings in *The National Survey of Science and Mathematics Education* in 2000, 16% of middle school teachers have a degree in mathematics and an additional 10% hold a degree in mathematics education. Given that these teachers are the ones who are undoubtedly teaching full time in mathematics, these data can triangulate with other data reported on students' teachers. Thirty-seven percent report having taken more than eleven semesters of mathematics. Examination of specific courses shows that 92% of teachers report a generic course on teaching methods, whereas only 78% report a course on the teaching of mathematics. Percentages of teachers who report other methods courses include the following: 45% mathematics for middle school teachers, 43% instructional uses of technology/other technology, and 36% geometry for K–8 teachers. In more closely related content areas, 66% have had a course in trigonometry/elementary functions, 56% a course in probability and statistics, 47% a course in geometry, 45% a course in computer programming/computer science, 43% a course (or courses) in calculus, 28% a course in linear algebra, 22% a course in abstract algebra, 21% a course in advanced calculus, 19% a course in differential equations, 12% a course in discrete mathematics, 11% a course in real analysis, and 16% a course in the history of mathematics (Whittington 2002a). These percentages would indicate that those with the degrees in mathematics have completed recommended coursework, but that serious questions persist about the mathematical readiness of the remaining teachers to provide high-quality instruction for their students in mathematics.

The 2003 national NAEP results (Smith, Arbaugh, and Fi 2007) support estimates that 85% of the nation's eighth graders are taught by teachers who are certified by their state. When examined by teachers' degrees, the data indicate that 30% of the nation's eighth graders had teachers with an undergraduate degree in mathematics; 26% had teachers with an undergraduate degree in mathematics education; and the remaining students were taught by a teacher with a degree in some other discipline. In a repeat of similar questions as part of NAEP 2009, grade 8 teachers' responses showed that 23% of the grade 8 students were taught by a teacher with a major in mathematics, 27% by a teacher with a major in mathematics education, 7% by a teacher with a degree in a related quantitative subject area (such as statistics), and 44% by a teacher with a degree in education. When asked about graduate studies, teachers provided responses indicating that 21% of grade 8 students were taught by a teacher with graduate coursework in mathematics or mathematics education (NCES 2009a). Under any interpretation, however, the fact that approximately half of the nation's grade 8 students are still being taught mathematics by teachers without substantial mathematics training is a matter of major concern.

Secondary School Mathematics Teachers and Teacher Education

For secondary school mathematics teacher certification, states require from eighteen (in South Dakota) to forty-five (in California) semester hours of mathematics, equivalent to six to fifteen semester courses, or they require a major in the subject. When states specify requirements in semester hours, almost half require the equivalent of ten three-hour courses. Eighty-two percent of the secondary school teachers report taking more than eleven such courses in mathematics. When tracked, these courses include three semesters of single-variable and one semester of multivariable calculus, as well as courses in linear algebra, geometry, abstract algebra, and other required courses (Whittington 2002b).

The National Survey of Science and Mathematics Education (2000) reports that 58% of mathematics teachers in grades 9–12 in its sample had an undergraduate major in mathematics, 21% had a degree in mathematics education, 10% had a degree in some other education field, and the remaining 10% had a degree in a field other than education or mathematics. In this sample, 96% of teachers had completed a course in calculus, 86% in probability and statistics, 82% in geometry, 81% linear algebra, 70% in advanced calculus, 68% in computer programming or other computer science, 65% in differential equations, 64% in abstract algebra, 56% in number theory, 41% in the history of mathematics, 38% in discrete mathematics, and 38% in real analysis (Whittington 2002b). NAEP does not collect data on grade 12 teachers, as the sampling process for grade 12 students does not link students to teachers. With respect to methods, and in particular the teachers' exposure to pedagogical content knowledge, the data show that 90% of the grade 9–12 teachers had a generic course on methods of teaching, 77% a course on methods of teaching mathematics, 43% a course in the instructional uses of computers/other technologies, 26% a course in mathematical methods for middle-school teachers, and 17% a course in geometry for elementary school teachers. The stricter certification rules at the secondary level may be to some degree responsible for the slightly stronger background levels of secondary teachers as compared with those of teachers at the other levels.

Previous initiatives of the mathematics community in recommending courses of study for preservice teachers of mathematics have been successful in helping shape state requirements for certification. NCTM's *Professional Standards for Teaching Mathematics* (1991), *Mathematics Teaching Today: Improving Practice, Improving Student Learning* (2007), and CBMS's *The Mathematical Education of Teachers* (2001) have provided recommendations for state certification. Although many states have significantly raised their certification requirements, some states still allow individuals to teach mathematics with less than the full requirements for certification if they are teaching mathematics for less than one-half of their teaching load or if they are hired in a region experiencing a shortage of mathematics teachers. These loopholes still subject many of the nation's students at the middle school and high school levels to learning mathematics from a teacher who fails to meet the stated certification requirements for teaching mathematics.

Comparison of Teachers' Mathematics Backgrounds with Classroom Needs

Liping Ma (1999) compared the mathematical knowledge of elementary school mathematics teachers in the United States with their counterparts in Shanghai, China, most of whom teach only mathematics. She found that U.S. teachers had far less depth of knowledge than their Chinese counterparts, and she argues for deep conceptual knowledge for teachers and for the importance of its role in the teachers' planning and guidance of lessons in their classrooms. Her study found U.S. teachers wanting in their content knowledge, but, even more so, in the depth of the mathematical understanding associated with the mathematics that they had studied. Similar findings exist in work done by Deborah Ball and her colleagues (Ball 1990; Ball, Thames, and Phelps 2008; Hill et al. 2008). Hill and Ball (2009) note that the skills that teachers need to see problems from others' perspectives and to understand what they are doing require mathematical knowledge, although not necessarily the mathematical knowledge required in the research laboratory or to prove a given theorem. This realization led to the subdividing and further defining of subject matter knowledge and pedagogical content knowledge—the two types of knowledge important to teaching made famous by Lee Shuman in his AERA Presidential Address in 1986 (Shuman 1986). Ball and her coworkers have identified a number of subcategories within each of the types of knowledge as it pertains to developing the deep understanding that Liping Ma and others speak of when they talk about depth of understanding and its relationship to quality teaching (Hill and Ball 2009).

A gap was evident between desired teacher knowledge, which spans deep knowledge of content and the ability to enact that knowledge in understanding students' conceptions and providing appropriate instruction, and the findings of research about teachers' knowledge of mathematics. This gap motivated the Conference Board of the Mathematical Sciences to create its guidelines for the development of teachers of mathematics: *The Mathematical Education of Teachers* (MET) (2001). MET recommends that programs for preservice elementary school teachers require at least nine semester hours (equivalent to three courses) of coursework in mathematics providing experiences in number and operations, in geometry and measurement, in algebra and functions, and in data analysis, statistics, and probability. Further, this training should be taught with the goal of developing teachers' in-depth understanding of the mathematics that they teach.

Such goals require significant effort to accomplish, as many prospective elementary school teachers take much of their general content coursework at community colleges and then transfer to a four-year college or university to complete their undergraduate degree program. In some states, these students must complete a fifth year before gaining certification. Although the program at each of these levels may be well-intentioned and staffed, the associated discontinuities in their implementation make pursuit of a carefully articulated sequence of courses and experiences in mathematics and mathematics education a formidable task for aspiring teachers. Further, it lessens the establishment of long-term professional relationships between faculty and preservice teachers at the point where the latter are preparing to enter their professional careers.

The CBMS (2001) recommendations call for the teaching of mathematics in grades 5–8 to be conducted by mathematics specialists, teachers specifically educated to teach mathematics to the students of the grade levels that they instruct. These teachers should have at least twenty-one semester hours in mathematics, including at least twelve semester hours on fundamental ideas of mathematics appropriate for middle school teachers. At the high school level, MET recommends that teachers of mathematics have a major in mathematics, including a six-hour capstone course connecting their college mathematics courses with high school mathematics. This recommendation stems from the view that teachers need to know the subjects that they will teach, need to understand the broad range of the mathematical sciences that their students will encounter in their careers (i.e., core subjects plus dynamical systems, graph theory, combinatorics, operations research, computer science, and so on), and need to develop the habits of mind and dispositions toward doing mathematics that characterize effective workers in the field.

In addition to specific courses, the CBMS (2001) report notes that teachers of the high school grades (9–12) need to develop understanding and skills associated with the use of technology in representing and exploring mathematical concepts and relationships in teaching. This includes experience in writing computer programs in a high-level language, such as C++, and experience with a computer algebra system, dynamic geometry software, and a statistical software package. These experiences should also be designed to enable teachers to become thoughtful and effective in using educational technology and to keep abreast of changes in the field. The entire CBMS MET document can be downloaded from the CBMS website: http://www.cbmsweb.org.

At present, the Conference Board of the Mathematical Sciences is working on a sequel to the 2001 MET to continue the discussion of the development of mathematics teachers who are highly qualified from both a mathematical content and a professional standpoint for the age of the Common Core State Standards for Mathematics. Given the broadened set of knowledge and applications that CCSSM expects and the course and time constraints that institutions place on academic undergraduate programs, the emerging MET2 draft is considering the needs of preservice teachers first and then outlining pathways to meet the needs of mid-career and master teachers. It is clear that professional development to meet the needs of practicing teachers of mathematics will have to be delivered both by institutions of higher education and by groups of experienced master teachers. These latter programs might take a form like that of the Teachers Teaching with Technology (T^3) program, developed by Texas Instruments and focusing on technology in the classroom (http://education.ti.com/calculators/pd/US/Community); National Fellowship Foundation's Teachers as Scholars initiative created by the Woodrow Wilson Programs (http://www.woodrow.org/school-initiatives/development/seminar_network/index.php); and related professional development programs instituted by NCTM, NCSM, and ASA.

From a coursework perspective, the fundamental content knowledge base at the undergraduate level needs to be expanded to at least twelve hours for K–grade 5, twenty-four hours for grades 6–8, and more than forty hours for grades 9–12. MET2 will be arguing that for prospective teachers at all levels this initial university-level coursework needs to meet a standard for preparing a really solid beginning teacher, equipped for future learning, over a preservice teacher with wide exposure but no real depth of understanding of the content that he or she will be teaching. Like the foundational MET document, drafts of the emerging MET2 document will be available on the CBMS website listed above.

Professional Development Programs

In 2008, forty-seven states reported policies defining requirements for continuing professional development of K–12 teachers to maintain certification by the state. Thirty-eight of the states reported that teachers must seek recertification after between four and six years, with a median time of five years between recertifications. Most of the variance outside the four-to-six-year interval appeared to be due to special programs. The median amount of coursework expected, for the nineteen states giving a coursework standard, is six semester credits (equivalent to two college courses) every five years; three require more. Sixteen states require the equivalent of 50 to 180 clock hours of professional development every five years. The remaining states allow a combination of credits, units, and contact hours, often varying by the area in which the teacher is certified (Stillman and Blank 2009).

In-service opportunities are widely offered within school districts, by professional organizations, by local colleges and universities, by regional education centers, and by commercial enterprises. State policies customarily give local school districts and often individual teachers the freedom to choose the kinds of in-service activities they desire. Unless a particular in-service program is given within the teacher's school district, an individual teacher is rarely is required to participate.

NCTM and its more than 230 affiliated national, state, and local organizations in mathematics education provide a number of professional development opportunities for teachers of mathematics. In addition to journals and publications, these organizations hold a number of regional and state-affiliated conferences with special sessions for teachers of all grade levels from kindergarten through undergraduate teacher preparation.

In recent years, the annual meeting of NCTM, held in the spring of each year, has been attended by 10,000–12,000 mathematics teachers and other mathematics educators. In addition, NCTM sponsors three regional meetings, geographically scattered across the United States and Canada, throughout the fall of the year to serve teachers on a regional basis, reaching a total of about 10,000 teachers. All of these meetings feature nationally known speakers, workshops, grade-level curriculum and teaching sessions, and displays of the most recent text materials, manipulatives, and technology for teaching mathematics.

In recent years, new methods of assessment have been a popular subject for professional development. Sometimes these sessions revolve around new tests that school systems and states have developed, often as a result of the NCLB legislation. Other professional development sessions emphasize assessment using open-ended questions, contextualized real-world tasks, and portfolios, as recommended in reports of the Mathematical Sciences Education Board (MSEB) and NCTM (MSEB 1991, 1993a, 1993b; Stenmark 1991; NCTM 2000).

Another popular subject for professional development programs is technology. The largest program in this area is Teachers Teaching with Technology. The T^3 group has established chapters in more than twenty-five countries and focuses on bringing teachers together to work with and learn from one another with the goal of increasing the appropriate use of educational technology in the teaching and learning of mathematics (Teachers Teaching with Technology 2012).

Professional Development in the Postsecondary Education Sector

In 2003, in response to a report of a commission headed by former senator John Glenn (National Commission on Mathematics and Science Teaching for the 21st Century 2000), the MAA initiated a program called Preparing Mathematicians to Educate Teachers (PMET) to help college and university mathematicians take a larger role in the training and support of classroom teachers (Katz and Tucker 2003, Tucker 2009). The PMET project had three major components: (1) summer workshops and minicourses for faculty training; (2) articles, websites, and other materials, as well as panels at meetings to support faculty instruction; and (3) mini-grants and regional networks to nurture and support grassroots innovation in teacher education on individual campuses. Although PMET funding has now expired, the project energized awareness throughout the mathematics community in the United States of the fact that the responsibility for high-quality mathematics education resides with mathematicians, mathematics educators, faculty in education departments, and classroom teachers of mathematics (Tucker 2009).

PART IX: Resources

Professional Organizations in Mathematics Education

Closed-Membership Organizations

Conference Board of the Mathematical Sciences (CBMS) (founded 1960)
e-mail: rosier@georgetown.edu; website: www.cbmsweb.org

CBMS is an umbrella organization consisting of the major professional societies in the mathematical sciences in the United States and composed of the CBMS Executive Committee and the presidents and executive directors of the member societies. Its purpose is to promote understanding and cooperation among the national professional organizations in mathematics so that they can work together, supporting one another in research, the improvement of education, and the expansion of the mathematical sciences. The following societies belong: American Mathematical Association of Two-Year Colleges (AMATYC), American Mathematical Society (AMS), Association of Mathematics Teacher Educators (AMTE), American Statistical Association (ASA), Association for Symbolic Logic (ASL), Association for Women in Mathematics (AWM), Association of State Supervisors of Mathematics (ASSM), Benjamin Banneker Association (BBA), Institute for Operations Research and the Management Sciences (INFORMS), Institute of Mathematical Statistics (IMS), Mathematical Association of America (MAA), National Association of Mathematicians (NAM), National Council of Supervisors of Mathematics (NCSM), National Council of Teachers of Mathematics (NCTM), Society for Industrial and Applied Mathematics (SIAM), the Society of Actuaries (SOA), and TODOS: Mathematics for All (TODOS).

Mathematical Sciences Education Board (MSEB) (founded 1986)
e-mail: mseb@nas.edu; website: www7.nationalacademies.org/mseb

MSEB is a standing board of the National Research Council (NRC) Center for Education with appointed members. The current mission of MSEB is to provide a continuing national leadership and guidance for policies, programs, and practices supporting the improvement of mathematics education of all students at all levels. MSEB is currently pursuing initiatives that focus on the learning, instruction, and assessment of mathematics; equity in mathematics; attracting and retaining students in mathematics majors and in mathematically intensive careers; capacity building and professionalization of mathematics education; evidence of effectiveness in mathematics education; and the public perception of mathematics education enterprise.

United States National Commission on Mathematics Instruction (USNC/MI) (founded 1978)
website: http://sites.nationalacademies.org/PGA/biso/ICMI/index.htm

The national adhering body to the International Commission on Mathematical Instruction (ICMI) is the U.S. National Academy of Sciences (NAS). The NAS, through the NRC, appoints the USNC/MI to conduct the work of the ICMI and foster other international collaborations in mathematics education. The NRC Board of Mathematical Sciences, MSEB, CBMS, and NCTM provide nominees for selection to the USNC/MI.

Open-Membership Organizations—Grades K–12

National Council of Supervisors of Mathematics (founded 1969)
e-mail: office@mathleadership.org; website: www.mathleadership.org
journal: *Journal of Mathematics Education Leadership*

National Council of Teachers of Mathematics (NCTM) (founded 1920)
e-mail: nctm@nctm.org; website: www.nctm.org
journals: *Teaching Children Mathematics, Mathematics Teaching in the Middle School*, *Mathematics Teacher, Journal for Research in Mathematics Education*

School Science and Mathematics Association (SSMA) (founded 1902)
e-mail: office@ssma.org; website: www.ssma.org
journal: *School Science and Mathematics*

Women and Mathematics Education (WME) (founded 1978)
e-mail: ipina@uwyo.edu; website: www.wme-usa.org

Open-Membership Organizations—Postsecondary Level

American Mathematical Association of Two-Year Colleges (AMATYC) (founded 1974)
e-mail: amatyc@amatyc.org; website: www.amatyc.org
journal: *MathAMATYC Educator*

American Mathematical Society (AMS) (founded 1888)
e-mail: ams@ams.org; website: www.ams.org
journals: *Bulletin of the American Mathematical Society; Notices of the American Mathematical Society*

American Statistical Association (ASA) (founded 1839)
e-mail: asainfo@amstat.org; website: www.amstat.org
journals: *The American Statistician, Chance, Significance* (and others devoted to research in statistics)

Mathematical Association of America (MAA) (founded 1915)
e-mail: maahq@maa.org; website: www.maa.org
journals: *The American Mathematical Monthly, College Mathematics Journal*, *Mathematics Magazine*

The National Association of Mathematicians (founded 1969)
e-mail: nd17@txstate.edu; website: www.nam-math.org
journal: *NAM Newsletter*

Open-Membership Organizations—Special Focus

Association of Mathematics Teacher Educators (AMTE) (founded 1993)
e-mail: nbezuk@mail.sdsu.edu; website: www.amte.net
journals: *AMTE Connections*, *Mathematics Teacher Educator*

Benjamin Banneker Association (founded 1986)
e-mail: director@bannekermath.org; website: www.bannekermath.org

National Association of Community College Teacher Education Programs (NACCTEP) (founded 2003)
e-mail: ray.ostos@domail.maricopa.edu; website: www.nacctep.org

Research Council on Mathematics Learning (founded 1974)
e-mail: kwohlhut@d.umn.edu; website: web.unlv.edu/RCML/
journal: *Investigations in Mathematics Learning*

TODOS: Mathematics for ALL (founded 2003)
e-mail: met@todos-math.org; website: www.todos-math.org/index.html
journal: *Noticias de TODOS*

K–Grade 12 Textbook Publishers

Textbooks for K–grade 12 are not listed in *Books in Print*, and currently available textbooks are not likely to be listed even in online bookstore catalogs. For this reason, this list of publishers, with their locations and URLs, is provided to assist those who might be interested in obtaining more information about textbooks for K–grade 12 and other curricular materials used in the United States. These publishers and their college publishing counterparts often have auxiliary materials available online. Additional information can be found at the website of the School Division of the Association of American Publishers: www.aapschool.org. This organization represents the nation's leading developers of instructional materials, technology-based curricula, and assessments. The National Association of School Textbook Administrators (www.nasta.org) offers another interesting website. NASTA is composed of the individuals responsible for the selection and administration of school textbook policies for the different states that have state adoption processes. These states control such a large percentage of the U.S. textbook purchases that their decisions shape, to a great extent, the actual contents and coverage sequences found in contemporary U.S. textbooks for K–grade 12.

Amsco School Publications, New York, NY 10013; www.amscopub.com

AnsMar Publishers, Inc., Poway, CA 92064; www.excelmath.com

Bates Publishing Company, Sandwich, MA 02563; www.batespub.com

Carnegie Learning, Pittsburgh, PA 15219; www.carnegielearning.com

CORD Communications, Waco, TX 76702; www.cord.org

CPM Educational Program, Sacramento, CA 95822; www.cpm.org

Curriculum Research and Development Group, Honolulu, HI 96822; www.hawaii.edu/crdg

Macmillan/McGraw Hill, Columbus, OH 53272; www.mheonline.com

Harcourt/Holt/McDougal/Houghton Mifflin, Boston, MA 02116; www.harcourtschool.com

It's About Time, Inc., Armonk, NY 10504; www.its-about-time.com

Kendall/Hunt Publishing Company, Dubuque, IA 52004; www.kendallhunt.com

Key Curriculum Press, Emeryville, CA 94608; www.keypress.com

Pearson/Prentice Hall/Addison Wesley/Dale Seymour/Scott-Foresman, Lebanon, IN 46052; www.pearsonschool.com

Sadlier-Oxford, New York, NY 10005; www.sadlier-oxford.com

W. H. Freeman & Co., New York, NY 10010; www.whfreeman.com

PART X: Bibliography

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The National Council of Teachers of Mathematics is a public voice of mathematics education, supporting teachers to ensure equitable mathematics learning of the highest quality for all students through vision, leadership, professional development, and research. With more than 80,000 members and 230 Affiliates, NCTM is the world's largest organization dedicated to improving mathematics education in prekindergarten through grade 12. The Council's *Principles and Standards for School Mathematics* includes guidelines for excellence in mathematics education and issues a call for all students to engage in more challenging mathematics. Its *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* identifies the most important mathematical topics for each grade level. *Focus in High School Mathematics: Reasoning and Sense Making* advocates practical changes to the high school mathematics curriculum to refocus learning on reasoning and sense making. NCTM is dedicated to ongoing dialogue and constructive discussion with all stakeholders about what is best for our nation's students. For more information on NCTM or the most up-to-date listing of NCTM resources on topics of interest to mathematics educators, as well as information on membership benefits, conferences, and workshops, visit the NCTM website at www.nctm.org, contact NCTM at inquiries@nctm.org, call (703) 620-9840, or follow NCTM on Twitter or Facebook.

United States National Commission on Mathematics Instruction

The U.S. National Commission on Mathematics Instruction (USNC/MI) is a committee of the U.S. National Academy of Sciences. The roles of the USNC/MI are to facilitate U.S. participation in the activities of the International Commission on Mathematical Instruction, and to engage the U.S. mathematics education community through the National Council of Teachers of Mathematics, the Conference Board of the Mathematical Sciences, and the National Research Council to advance mathematics education in the United States and throughout the world. Support to the USNC/MI is provided by the National Science Foundation. Further information is available by contacting usncmi@nas.edu.

Mathematics Education in the United States 2012

A Capsule Summary Fact Book

Written for the Twelfth International Congress on Mathematical Education (ICME-12)

Prepared for the Twelfth International Congress on Mathematical Education (ICME-12), this fact book brings together the latest information about mathematics education in the United States as of early 2012. This report updates the version prepared in 2008 for ICME-11.

The publication provides general information about education in the United States, describes the three kinds of curriculum identified in international mathematics studies (intended, implemented, and attained), and gives special focus to the emergence of a common K–grade 12 curriculum that has been adopted by forty-five states and the District of Columbia—the Common Core State Standards for Mathematics (CCSSM). The adoption of such a set of common outcomes, matching assessments, and similar instructional materials is expected to bring an unprecedented uniformity to U.S. mathematics education. The report also includes sections detailing programs for high-achieving students, programs for mathematics teacher education, and resources for additional information about mathematics education in the United States.

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ISBN 978-0-87353-721-6
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