









## Algebra 3-4 Unit 1

### Absolute Value Functions and Equations

1.1	I can write domain and range in interval notation when given a graph or an equation.	 <input style="width: 100%; height: 20px;" type="text"/>
1.1	I can write a function given a real world situation and write an appropriate domain and range.	 <input style="width: 100%; height: 20px;" type="text"/>
1.2	I can identify intercepts and the slope of a linear equation.	 <input style="width: 100%; height: 20px;" type="text"/>
1.2	I can identify increasing, decreasing, and the average rate of change of a given or table of values.	 <input style="width: 100%; height: 20px;" type="text"/>
1.2	I can find a linear regression line and use it to predict values.	 <input style="width: 100%; height: 20px;" type="text"/>
1.3	I can graph absolute value equations, identifying transformations.	 <input style="width: 100%; height: 20px;" type="text"/>
1.4	I can identify min, max, vertex, end behavior, and compare absolute value graphs and tables.	 <input style="width: 100%; height: 20px;" type="text"/>
1.5	I can solve absolute value equations algebraically and graphically.	 <input style="width: 100%; height: 20px;" type="text"/>

My goal for this unit: \_\_\_\_\_

\_\_\_\_\_

What I need to do to reach my goal: \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

## GUIDED NOTES – 1.1

### Domain, Range & Notation

Name: \_\_\_\_\_ Period: \_\_\_\_\_

In this course we have the opportunity to explore a variety of functions, including quadratic, polynomial, rational, radical, exponential, logarithmic, and trigonometric. Before we get to all those functions, their graphs, and behaviors, the basic linear function is a good place to start.

**INTERVALS:** An interval is part of a function, in this case a line, without any breaks. A finite interval has two endpoints, which may or may not be included in the interval. An infinite interval is unbounded at one or both ends.

**NOTATION:** We have three ways to write the intervals of a function. We call these the notation.

Description	Type of Interval	Inequality	Set Notation	Interval notation
All real numbers from a to b, including a and b.	Finite			
All real numbers greater than a	Infinite			
All real numbers less than or equal to a	Infinite			

#### Example A

Write the interval notation for a set of all real numbers from -4 to 5, including -4 but not including 5.

#### Example B

Write the set notation for a set of all real numbers greater than or equal to 6.

**TALK ABOUT IT:** What can we conclude about the relationship between infinity and the use of brackets and parentheses in writing the notation for a function?

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### DOMAIN AND RANGE

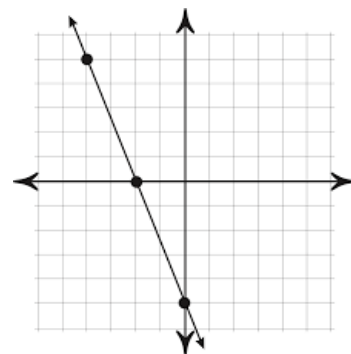
Unless otherwise stated, a function is assumed to have a **domain** (all the possible input or x-values) consisting of all real numbers for which the function is defined.

We can write it in interval notation as:

Another way to write the set of real numbers is:

The **range** consists of all the possible output or y-values the function, given the domain of the function.

**Example C** - Identify the domain and range for the graphed linear function shown here.



#### Example D

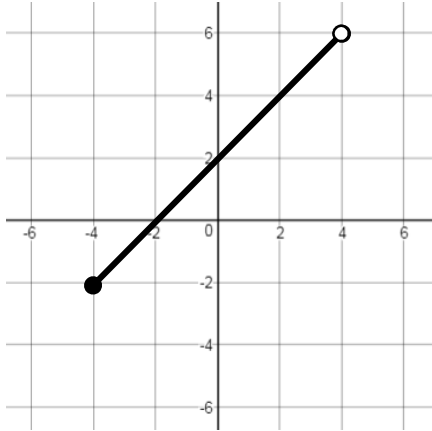
Given the function  $f(x) = 2x - 3$  with a domain of  $(-3, 5]$ , identify the range in the same notation.

**Example E**

Given the function  $f(x) = -\frac{1}{2}x + 5$  with a domain of  $\{x|x < 4\}$ , identify the range using the same notation.

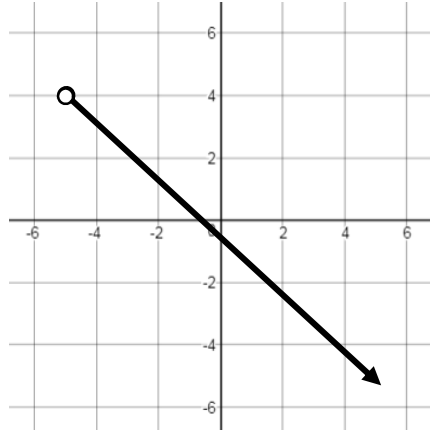
Domain and range will come into play with every type of function we encounter in future lessons. For now we will practice with some line segments and rays to get a feel for how these functions restrict the domain and range.

Write the domain and range in both set and interval notation for the following graphed functions.

**Example F**

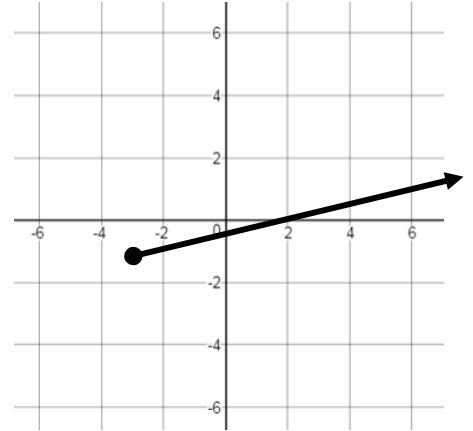
DOMAIN

RANGE

**Example G**

DOMAIN

RANGE

**Example H**

DOMAIN

RANGE

**TALK ABOUT IT:** If a student writes the domain of a function that has no x-values greater than 5 as  $(5, -\infty)$ , is that acceptable? Explain why or why not.

**LINEAR APPLICATIONS**

A 6 inch long candle burns at a rate of half an inch per hour. Write a function in terms of the candle's height  $h$  (in inches) at any time  $t$  (in hours).

Suppose the candle is lit and left burning for 5 hours. Identify the domain and range.

Write a domain and range to represent the time and height of the candle, should it be left burning until it reaches a height of 0 inches and can no longer burn.

**TALK ABOUT IT:** Will positive/negative infinity ever be part of the domain and range in a real world application problem? Explain why or why not.

## PRACTICE PROBLEMS – 1.1

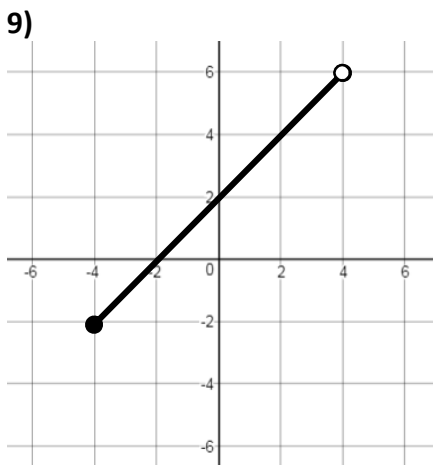
### Domain, Range & Notation

Name: \_\_\_\_\_ Period: \_\_\_\_\_

- 1) Write the interval notation for a set of all real numbers that are greater than 2 and less than or equal to 8.
- 2) Write the set notation for a set of all real numbers between  $-18$  and  $20$ , including  $-18$  but not including  $20$ .
- 3) Write the interval notation for a set of all real numbers that are greater than or equal to  $5$ .
- 4) Write the set notation for a set of all real numbers less than  $15$ .
- 5) Given the function  $f(x) = 4x - 6$  with a domain of  $(-3, 5]$ , identify the range in the same notation.
- 6) Given the function  $f(x) = \frac{1}{2}x + 8$  with a domain of  $\{x | 2 \leq x < 14\}$ , identify the range using the same notation.
- 7) Given the function  $f(x) = -3x - 12$  with a domain of  $[-5, 0]$ , identify the range in the same notation.
- 8) Given the function  $f(x) = -\frac{2}{3}x + 4$  with a domain of  $\{x | x > 6\}$ , identify the range using the same notation.

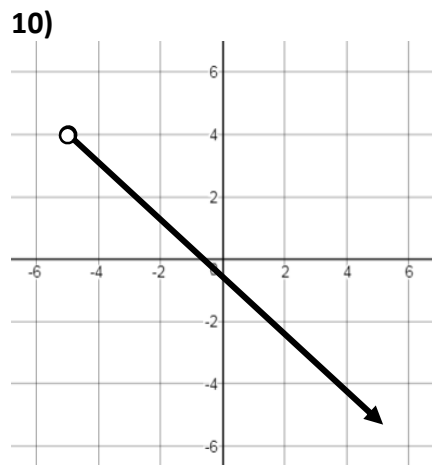
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Write the domain and range in INTERVAL NOTATION for the following graphed functions.



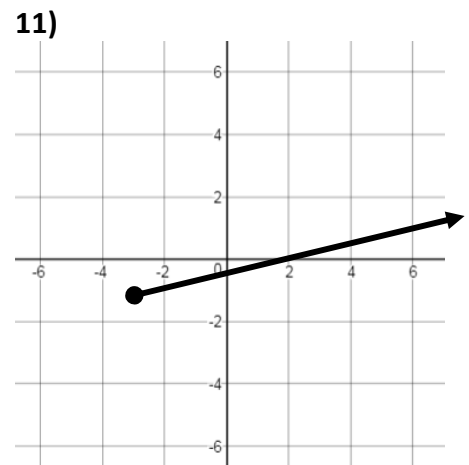
DOMAIN

RANGE



DOMAIN

RANGE

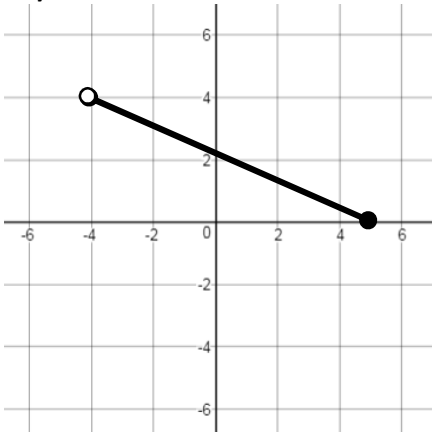


DOMAIN

RANGE

Write the domain and range in SET NOTATION for the following graphed functions.

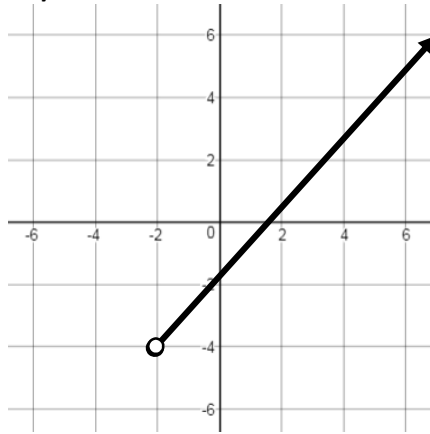
12)



DOMAIN

RANGE

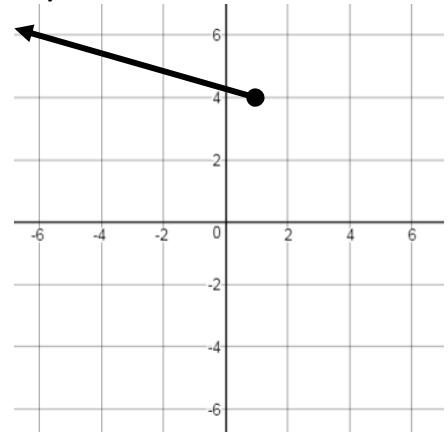
13)



DOMAIN

RANGE

14)



DOMAIN

RANGE

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15) It is estimated that the price of a book at a used book store increases by \$0.02 per page with a base cost of \$3. Write a function to represent this scenario in terms of  $C(p)$ , where  $C$  is the cost of the book (in dollars) and  $p$  represents the number of pages in the book.

Suppose you select a stack of books, where the largest book has 450 pages and the smallest book has 80 pages. Write the domain and range.

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16) Suppose you put a hot cup of coffee at 180 degrees out on the counter and it cools by 2.5 degrees per minute. Write a function to represent the temperature of the coffee as  $T(m)$ , where  $T$  represents the temperature and  $m$  represent the minutes the coffee is left out.

Write the domain and range of the function, should a person leave the coffee out for 20 minutes.

The coffee will not get cooler than the room temperature which is at 78 degrees. Write the domain and range to represent this.

## GUIDED NOTES – 1.2

Average Rate of Change & Linear Regression

Name: \_\_\_\_\_ Period: \_\_\_\_\_

### KEY FEATURES OF LINEAR FUNCTIONS

A linear function has some key features we want to review and focus on. The y-intercept tells us the y-value of the graph when  $x = \underline{\hspace{1cm}}$  and the x-intercept tells us the x-value when  $y = \underline{\hspace{1cm}}$ . The slope of the line indicates the rate at which the function is increasing or decreasing.

Identify the key features of the graphed linear function shown here.

y-intercept: \_\_\_\_\_ Slope: \_\_\_\_\_ x-intercept: \_\_\_\_\_

Write a function to represent the line:

How do we get the x-intercept out of that linear equation?

**Example A:** Given the linear function  $f(x) = -2x + 8$ , solve for the x-intercept. Confirm by graphing with technology.

**Example B:** Given the linear function  $f(x) = \frac{2}{3}x - 6$ , solve for the x-intercept. Confirm by graphing with technology.

The process of setting a function equal to 0 to get the x-intercept is important in future lessons as this is the process for 'solving' the function or getting the solutions.

### AVERAGE RATE OF CHANGE (GRAPHS)

In previous math courses you have used the 'slope formula' to find the slope between two points. In this course, we focus on the **average rate of change** between two points, which can be found using the same formula, but allows us to look at various points on a graph (or in time with application problems) to find the rate of change. Let's review and practice that skill now.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Over what intervals is the function increasing?

Over what intervals is the function decreasing?

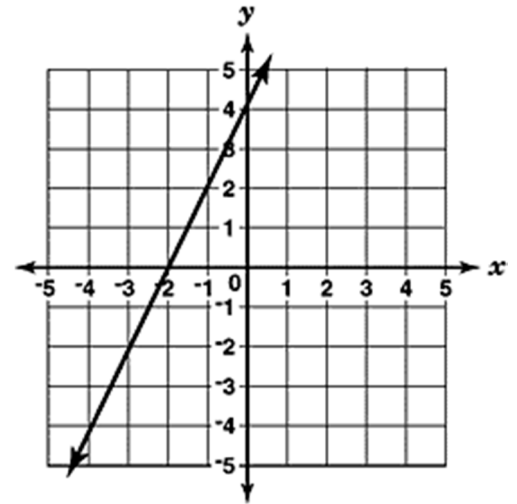
Find the average rate of change between  $x = -3$  and  $x = 0$ .

Find the average rate of change over the interval  $[0, 3]$ .

Find the average rate of change between  $x = -2$  and  $x = 1$ .

Find the average rate of change over the interval  $[-4, 2]$ .

**Extension/Spiral:** Suppose the domain of this function is  $[-5, 5]$ , what is the range?



## AVERAGE RATE OF CHANGE (TABLES)

When you can't see a function visually as a graph, the formula for average rate of change becomes helpful.

**Example:** The table shows the height (in feet) of a golf ball at various times (in seconds) after a golfer hits the ball into the air.

Time (s)	0	0.5	1	1.5	2	2.5	3	3.5	4
Height (ft)	0	28	48	60	64	60	48	28	0

What is the maximum height the golf ball reaches according to the data?

What is the average rate of change for the height of the golf ball between 0 and 2 seconds?

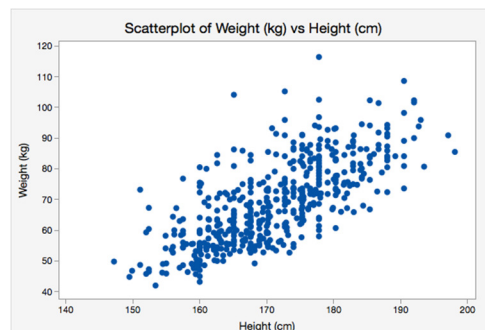
What is the average rate of change for the height of the golf ball between 1 and 3 seconds?

What is the average rate of change for the height of the golf ball between 2 and 3.5 seconds?

**TALK ABOUT IT:** What does the path of this golf ball look like? For a real life golf player, is this table realistic?

## LINEAR REGRESSION

Up to this point, we have dealt with pure linear equations, meaning they represent perfectly straight lines. We know in real life sets of data that pure, perfect data does not always exist. Were we to graph the relationship between the height and weight of a large group of people for example, we would end up with a graph like this. What do we call this type of graph?



Still, we can see there is a linear trend between height and weight:

The \_\_\_\_\_ people are, the \_\_\_\_\_ they tend to be.

**Linear regression** allows us to 'fit' a single line to the data, a line known as the **line of best fit**. We will use technology to generate this linear function and use it to make predictions about our data.

**Example:** As a science project, Shelley is studying the relationship of car mileage (in miles per gallon) and speed (in miles per hour). The table shows the data she gathered using her family's vehicle.

Speed (mi/h)	30	40	50	60	70
Mileage (mi/gal)	34.0	33.5	31.5	29.0	27.5

Use technology to write a function to represent the relationship between mileage  $m$ , as a function of speed  $s$ , that the vehicle is traveling.

Identify the domain and range for the function:

What does the y-intercept tell us in this function and the context?

What is the meaning of the slope of this function within the context?

Predict the miles per gallon her family's vehicle would get at a speed of 80 miles per hour.

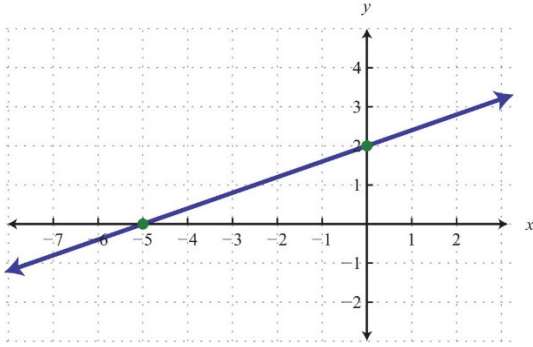
**TALK ABOUT IT:** If you were to get this data for your/your family's car, what would your domain restrictions be?

## PRACTICE PROBLEMS – 1.2

### Average Rate of Change & Linear Regression

Name: \_\_\_\_\_ Period: \_\_\_\_\_

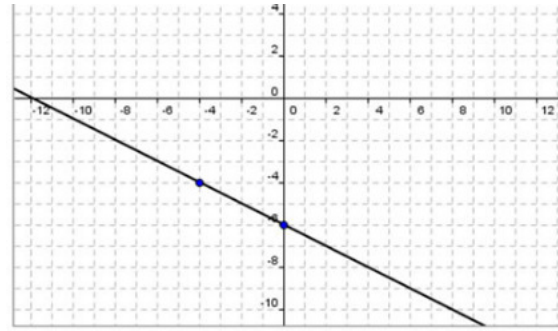
Identify the features of each graph below.



1) Slope:

y-Intercept:

x-Intercept:



2) Slope:

y-Intercept:

x-Intercept:

3) Given the linear function  $f(x) = -\frac{1}{2}x + 14$ , solve for the x-intercept. Confirm by graphing with technology.

4) Given the linear function  $f(x) = 3x - 15$ , solve for the x-intercept. Confirm by graphing with technology.

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5) Over what intervals is the function increasing?

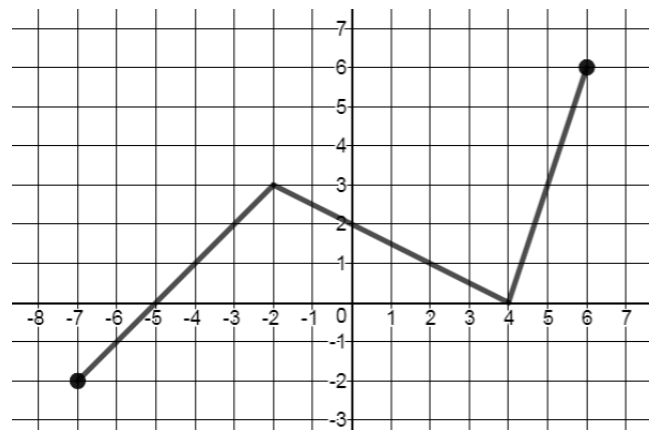
6) Over what intervals is the function decreasing?

7) Find the average rate of change between  $x = -6$  and  $x = 0$ .

8) Find the average rate of change over the interval  $[-2, 2]$ .

9) Find the average rate of change between  $x = 2$  and  $x = 6$ .

10) Find the average rate of change over the interval  $[-4, 2]$ .





Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Hits	5	10	21	24	28	36	33	21	27	40	46	50	31	38

Vern created a website for his school's sports teams. He has a hit counter on his site that lets him know how many people have visited the site. The table shows the number of hits the site received each day for the first two weeks.

- 11) What is the average rate of change from day 2 to day 10?
- 12) What is the average rate of change from day 5 to day 12?
- 13) Identify the domain and range of the data.
- 14) Use technology to find a regression equation in terms of  $H(d)$  (hits as a function of days) to fit the data.
- 15) Interpret the  $y$ -intercept in the context of the problem.
- 16) Interpret the slope in the context of the problem.
- 17) Use your regression equation to predict the number of hits on day 20.

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Mass (g)	3.1	2.0	3.2	4.0	3.7	1.9	4.5
Frequency of Wing Beats (beats per second)	60	85	50	45	55	90	40

The above table gives the mass of 7 different hummingbirds and their frequency of wing beats per second.

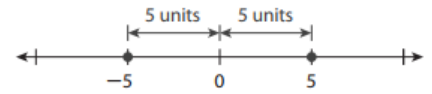
- 18) Identify the domain and range of the data.
- 19) Use technology to find a regression equation in terms of  $B(m)$ , beats as a function of mass.
- 20) Interpret the  $y$ -intercept in the context of the problem.
- 21) Interpret the slope in the context of the problem.
- 22) Use your regression equation to predict the number of wing beats for a bird that has a mass of 6.2 grams.

## GUIDED NOTES – 1.3

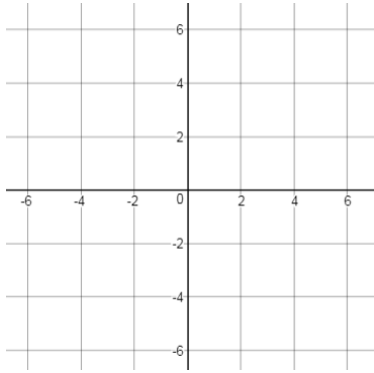
### Absolute Value Graphs & Transformations

Name: \_\_\_\_\_ Period: \_\_\_\_\_

**ABSOLUTE VALUE:** Recall that absolute value, written as \_\_\_\_\_ is the distance between a value and 0 on a number line.



$|4| =$              $|-4| =$              $-|4| =$              $-|-4| =$



Let's graph the parent function for absolute value:  $f(x) = |x|$

x	-3	-2	-1	0	1	2	3
f(x)							

The graph is symmetric about the \_\_\_\_\_, with a vertex at \_\_\_\_\_.

Domain:

Range:

**TALK ABOUT IT:** Why did we just call the function above a parent function?

**GRAPHING ABSOLUTE VALUE FUNCTIONS:** We will apply general transformations to help us graph absolute value functions by changing parameters in the equation:  $g(x) = a|x - h| + k$

- a** If **a** is greater than 1, the graph is vertically \_\_\_\_\_.
- If **a** is less than 1 (like a \_\_\_\_\_) the graph is vertically \_\_\_\_\_.
- If **a** is negative, the graph is vertically \_\_\_\_\_.
- 
- h** The value of **h** tells us whether the graph is **horizontally shifted left or right**.
- $|x - 3|$  for example, tells us that  $h =$  \_\_\_\_\_, so the graph is shifted 3 units \_\_\_\_\_.
- $|x + 3|$  for example, tells us that  $h =$  \_\_\_\_\_  $|x - (-3)|$  so the graph is shifted 3 units \_\_\_\_\_.
- 
- k** The value of **k** tells us whether the graph is **shifted vertically up or down**.  $+k$  is up and  $-k$  is down

**Example 1:** Identify the transformations and graph the function:

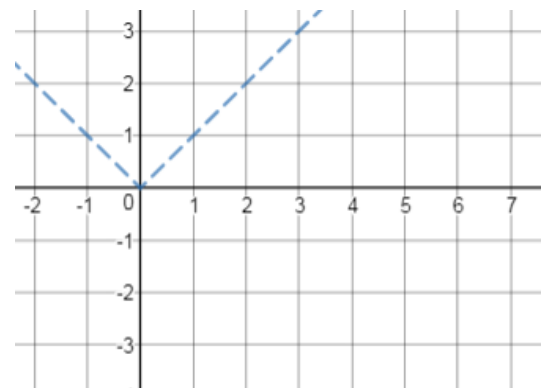
$$g(x) = 2|x - 5| - 3$$

The graph is shifted 5 units \_\_\_\_\_ and 3 units \_\_\_\_\_.

This means the new vertex is \_\_\_\_\_.

Now we look at the parameter  $a$ .

Since  $a =$  \_\_\_\_\_ we will be vertically \_\_\_\_\_ by a factor of \_\_\_\_\_.

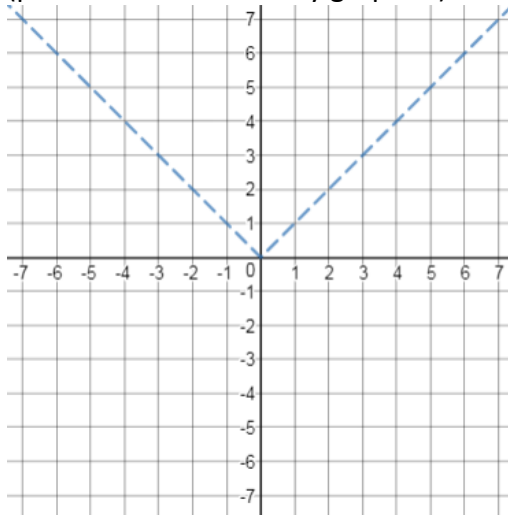


**Example 2:** Identify the transformations and graph the function  $a(x) = \frac{2}{3}|x + 1| - 3$

Transformations:

Vertex: \_\_\_\_\_

(parent function already graphed)



Domain:

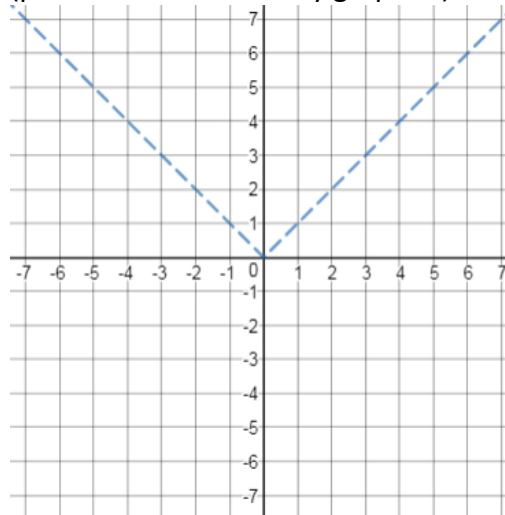
Range:

**Example 3:** Identify the transformations and graph the function  $b(x) = -2|x - 4| + 5$

Transformations:

Vertex: \_\_\_\_\_

(parent function already graphed)

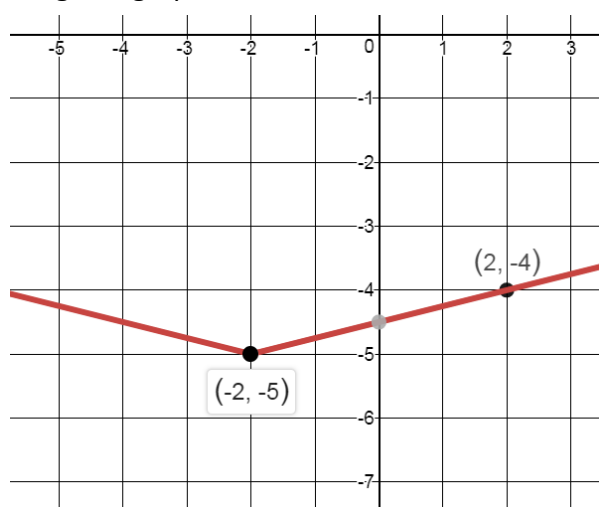
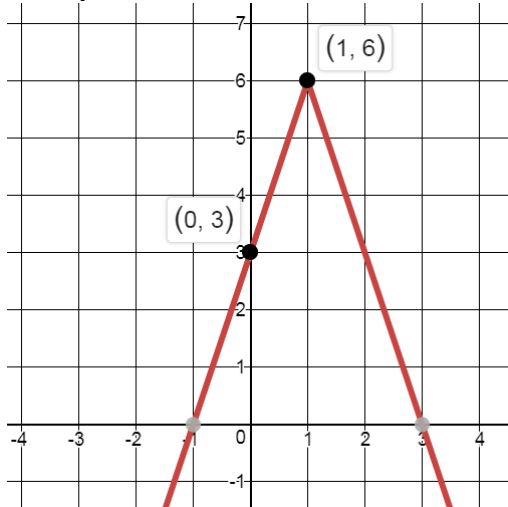


Domain:

Range:

**WRITING ABSOLUTE VALUE FUNCTIONS FROM A GRAPH:** We will use the same skills in reverse to create absolute value functions to represent given graphs. Remember that we can write two equivalent functions that look completely different using values for either  $a$  or  $b$ , so we will only use one parameter at a time and set the other to equal 1.

**Examples:** Write an absolute value function to represent each given graph.



### PRACTICE PROBLEMS – 1.3

#### Absolute Value Graphs & Transformations

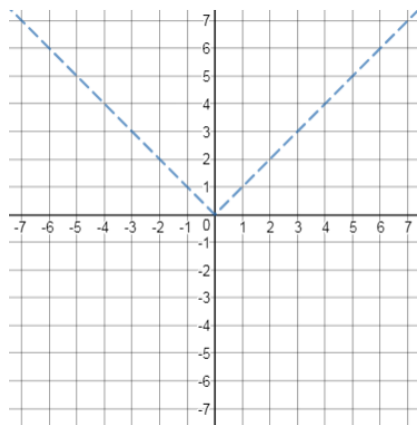
Name: \_\_\_\_\_ Period: \_\_\_\_\_

Identify the transformations, vertex, and domain/range, of each of the following absolute value functions and graph each function, given the parent function  $f(x) = |x|$  (which is graphed already).

1)  $g(x) = -|x + 3| - 2$

Transformations:

Vertex: \_\_\_\_\_



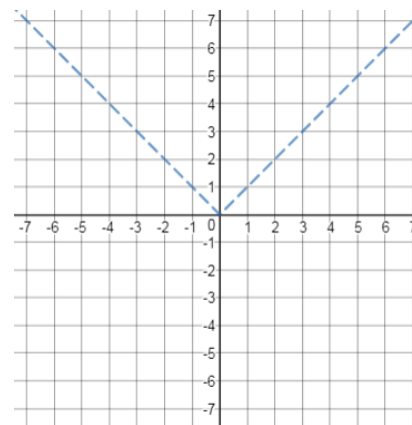
Domain:

Range:

2)  $a(x) = \frac{1}{2}|x - 2| + 1$

Transformations:

Vertex: \_\_\_\_\_



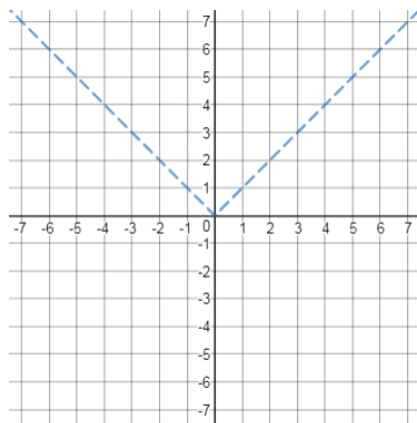
Domain:

Range:

3)  $g(x) = 4|x - 3| - 6$

Transformations:

Vertex: \_\_\_\_\_



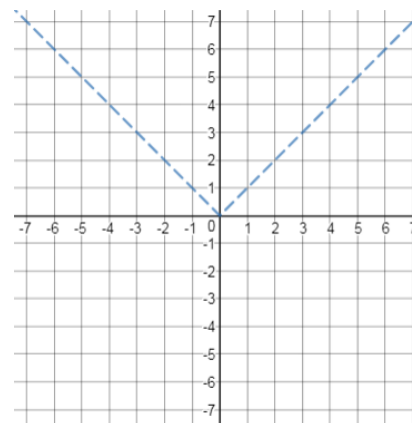
Domain:

Range:

4)  $a(x) = -\frac{3}{4}|x| + 5$

Transformations:

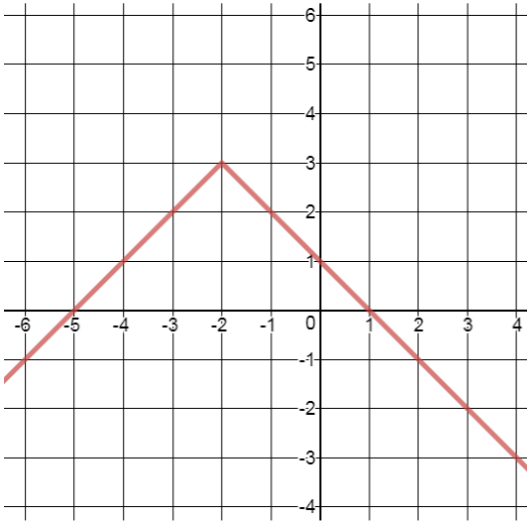
Vertex: \_\_\_\_\_



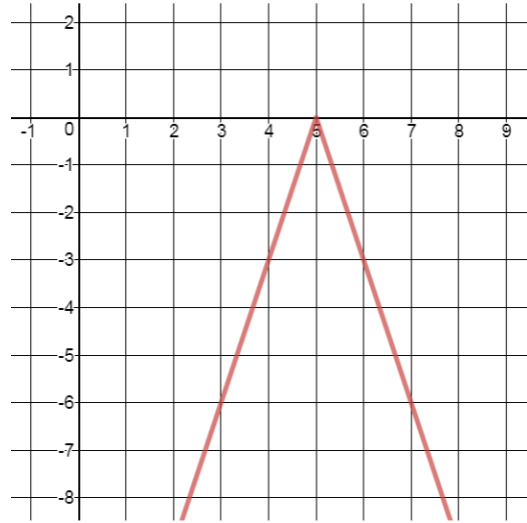
Domain:

Range:

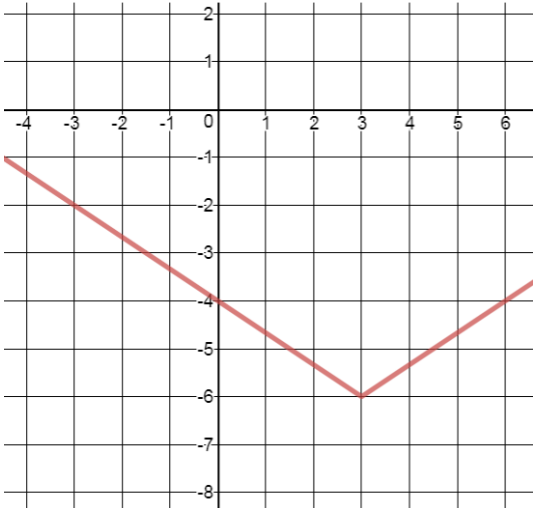
Write an absolute value function to represent each given graph.



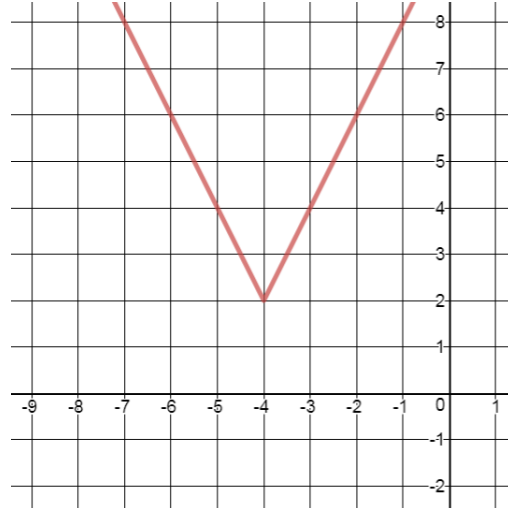
5) Function:



6) Function:



7) Function:



8) Function:

9) Compare the following absolute value equations and identify which function(s) meet the key features described below.

$$a(x) = -2|x + 2| - 1$$

$$b(x) = |x| - 6$$

$$c(x) = -\frac{3}{4}|x + 7|$$

$$d(x) = 3|x - 2| + 1$$

Which function(s) is/are...

...not shifted horizontally: \_\_\_\_\_

...not stretched or compressed: \_\_\_\_\_

...shifted 2 units to the left: \_\_\_\_\_

...not shifted vertically: \_\_\_\_\_

...shifted up by 1 unit: \_\_\_\_\_

...will never cross the parent function  $y = |x|$ : \_\_\_\_\_

...contains the point (5, 10): \_\_\_\_\_

...contains the point (-3, -3): \_\_\_\_\_

## GUIDED NOTES – 1.4

### Absolute Value Graph Features

Name: \_\_\_\_\_ Period: \_\_\_\_\_

#### MAXIMUM & MINIMUMS

We identified the vertex of absolute value functions in the previous lesson.

If the function is negative, the vertex is referred to as the **maximum**.

If the function is positive, the vertex will be the graph's **minimum**.

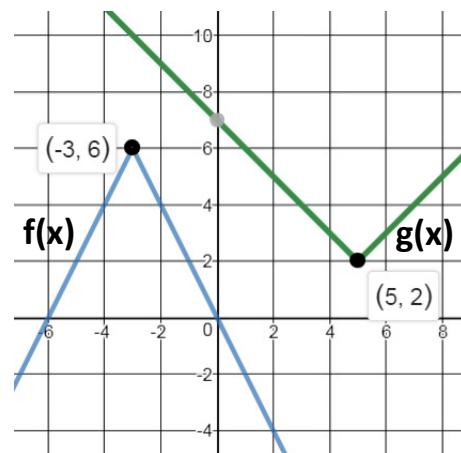
#### Example:

The function  $f(x)$  has a vertex at \_\_\_\_\_, which is a \_\_\_\_\_.

Domain: \_\_\_\_\_ Range: \_\_\_\_\_

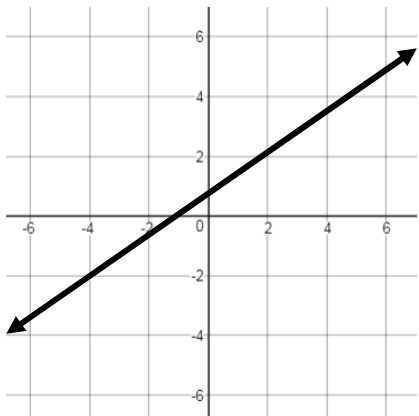
The function  $g(x)$  has a vertex at \_\_\_\_\_, which is a \_\_\_\_\_.

Domain: \_\_\_\_\_ Range: \_\_\_\_\_



#### END BEHAVIOR

A function's end behavior tells us what happens to the  $f(x)$ -values as the  $x$ -values either increase without bound (approaching positive infinity) or decrease without bound (approaching negative infinity).



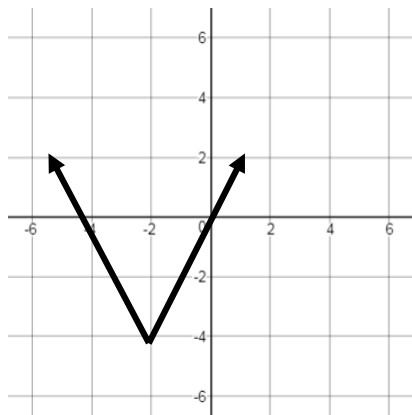
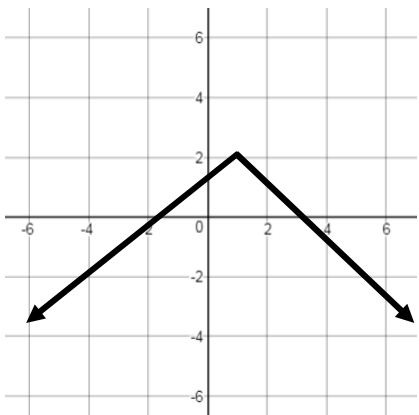
For this linear function for example, we would state the end behavior symbolically as follows:

$$\text{As } x \rightarrow +\infty, f(x) \rightarrow +\infty$$

$$\text{As } x \rightarrow -\infty, f(x) \rightarrow -\infty$$

**TALK ABOUT IT:** What would happen if the line was negative instead of positive?

**Examples:** Identify the end behavior for the following graphs.



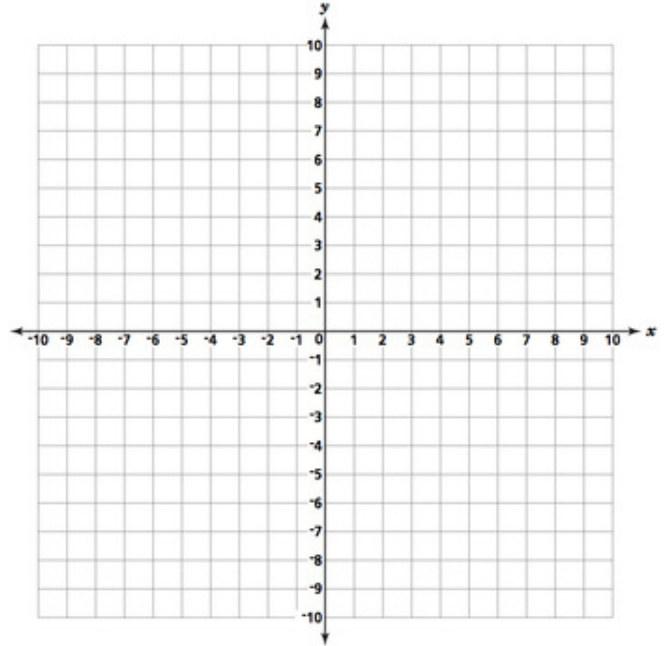
## COMPARING TABLES AND GRAPHS

It is important to be able to understand functions in various forms, whether given an explicit function, a table of values, or a graph. We will practice identifying the vertex from a table and comparing it with a given graph.

**Example A:** Given the table of values representing the function  $a(x)$  and the graphed function  $b(x)$ , perform the following analysis.

$x$	$a(x)$
-2	0
-1	1
0	2
1	3
2	4
3	3

$$b(x) = 2|x + 6| - 2$$

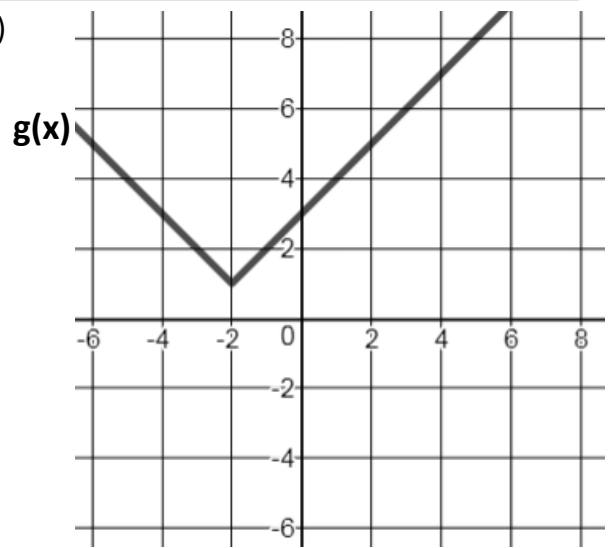


Write a function to represent  $a(x)$ :

- |  |      |      |      |         |
|--|------|------|------|---------|
| Which function has a vertex that is a maximum?       | a(x) | b(x) | both | neither |
| Which function has the greater y-intercept?          | a(x) | b(x) | both | neither |
| Which function contains the point $(-7, -5)$ ?       | a(x) | b(x) | both | neither |
| Which function has a domain of $(-\infty, \infty)$ ? | a(x) | b(x) | both | neither |
| Which function has a range of $\{y   y \leq 4\}$ ?   | a(x) | b(x) | both | neither |

**Example B:** Given the table of values representing the function  $f(x)$  and the graphed function  $g(x)$ , perform the following analysis.

$x$	1	4	7	10
$f(x)$	2	-6	2	10



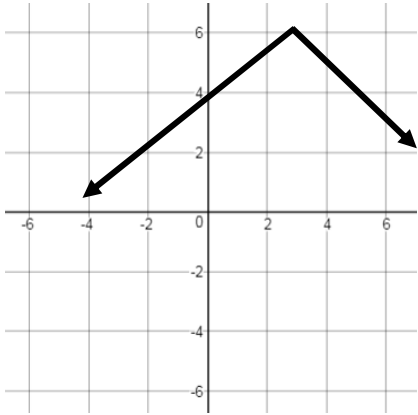
- |   |      |      |      |         |
|---|------|------|------|---------|
| Which function has a vertex that is a minimum?      | f(x) | g(x) | both | neither |
| Which function has the greater y-intercept?         | f(x) | g(x) | both | neither |
| Which function contains the point $(5, 8)$ ?        | f(x) | g(x) | both | neither |
| Which function has an end behavior of $-\infty$ ?   | f(x) | g(x) | both | neither |
| Which function has a range of $\{y   y \geq -6\}$ ? | f(x) | g(x) | both | neither |

# PRACTICE PROBLEMS – 1.4

## Absolute Value Graph Features

Name: \_\_\_\_\_ Period: \_\_\_\_

Identify the vertex, whether it is a max or a min point, the domain and range, and the end behavior for each of the graphed absolute value functions below.



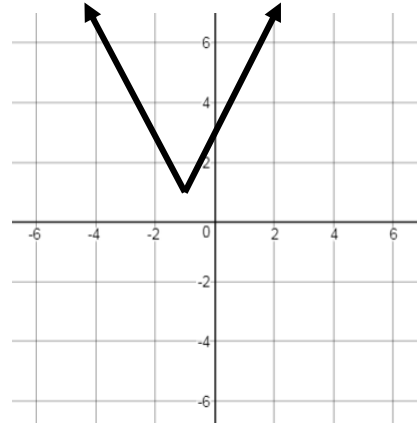
Vertex: \_\_\_\_\_ Max / Min

Domain: \_\_\_\_\_ Range: \_\_\_\_\_

End behavior:

As  $x \rightarrow +\infty, f(x) \rightarrow$

As  $x \rightarrow -\infty, f(x) \rightarrow$



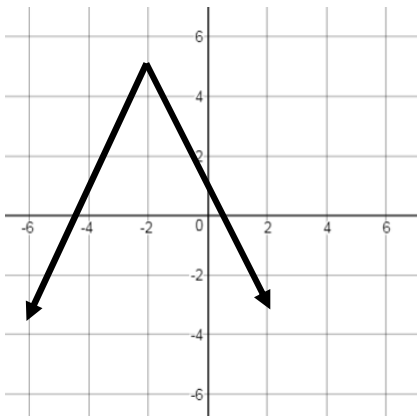
Vertex: \_\_\_\_\_ Max / Min

Domain: \_\_\_\_\_ Range: \_\_\_\_\_

End behavior:

As  $x \rightarrow +\infty, f(x) \rightarrow$

As  $x \rightarrow -\infty, f(x) \rightarrow$



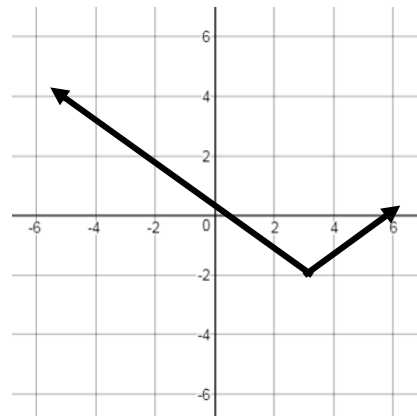
Vertex: \_\_\_\_\_ Max / Min

Domain: \_\_\_\_\_ Range: \_\_\_\_\_

End behavior:

As  $x \rightarrow +\infty, f(x) \rightarrow$

As  $x \rightarrow -\infty, f(x) \rightarrow$



Vertex: \_\_\_\_\_ Max / Min

Domain: \_\_\_\_\_ Range: \_\_\_\_\_

End behavior:

As  $x \rightarrow +\infty, f(x) \rightarrow$

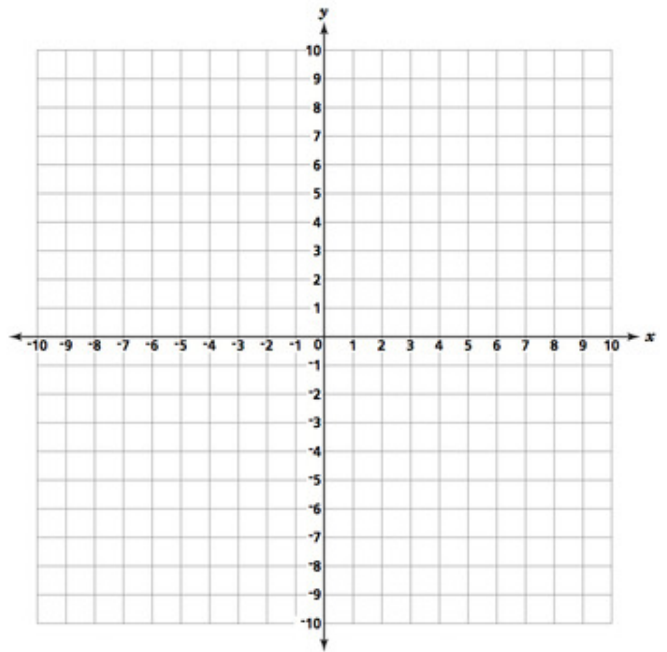
As  $x \rightarrow -\infty, f(x) \rightarrow$



Given the table of values representing the function  $a(x)$  and the equation  $b(x)$ , perform the following analysis.

$x$	$a(x)$
-1	1
-2	-1
-3	-3
-4	-1
-5	1
-6	3

$$b(x) = -|x + 5| + 6$$



Write a function to represent  $a(x)$ :

Which function has a vertex that is a maximum?  $a(x)$   $b(x)$  both neither

Which function has the greater y-intercept?  $a(x)$   $b(x)$  both neither

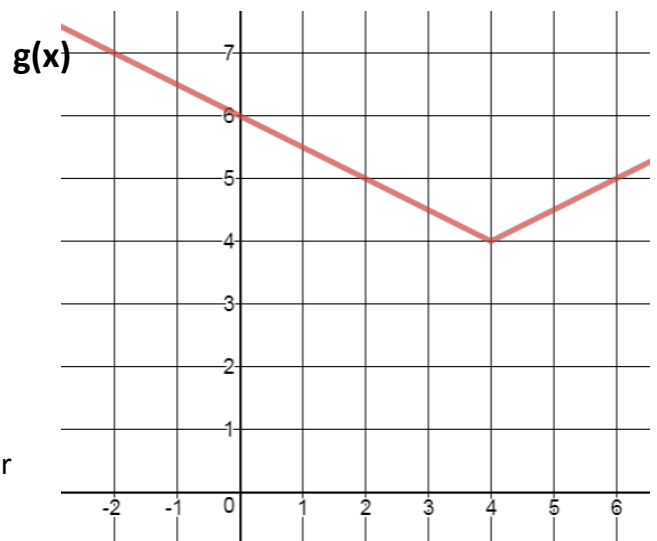
Which function contains the point  $(1, 5)$ ?  $a(x)$   $b(x)$  both neither

Which function has a domain of  $(-\infty, \infty)$ ?  $a(x)$   $b(x)$  both neither

Which function has a range of  $\{y | y \geq -3\}$ ?  $a(x)$   $b(x)$  both neither

Given the table of values representing the function  $f(x)$  and the graphed function  $g(x)$ , perform the following analysis.

$x$	1	0	-1	-2	-3
$f(x)$	10	7	4	7	10



Which function has a vertex that is a minimum?  
 $f(x)$   $g(x)$  both neither

Which function has the greater y-intercept?  
 $f(x)$   $g(x)$  both neither

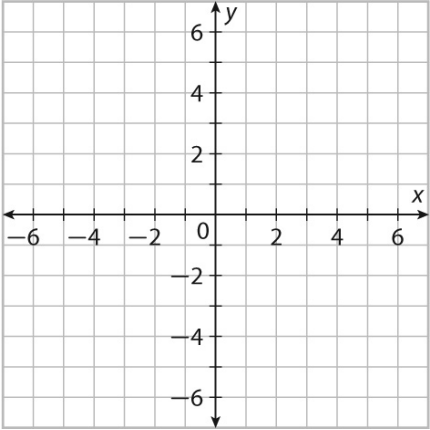
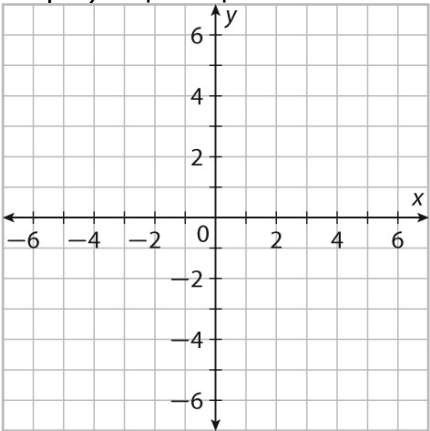
Which function contains the point  $(-2, 7)$ ?  $f(x)$   $g(x)$  both neither

Which function has an end behavior of  $-\infty$ ?  $f(x)$   $g(x)$  both neither

Which function has a range of  $\{y | y \geq 4\}$ ?  $f(x)$   $g(x)$  both neither

## Algebra 3-4                      1.5

### Solving Absolute Value Equations

<p>Graph <math>y = 2x - 2</math></p> 	<p>Name a point on the graph.</p> <p>What do you know about this ordered pair and the equation <math>y = 2x - 2</math>?</p>	<p>Once you have your graph, if you wanted to find the <math>x</math>-value when <math>y</math> is 6, instead of solving, how could you find this value?</p>
<p>Graph <math>y =  x - 3 </math></p> 	<p>Algebraically solve for <math>x</math> if <math>y = 2</math></p>	<p>Graphically solve by adding the line <math>y = 2</math> to your first graph.</p>

**Directions:** Answer each question or solve.

1. How many solutions does the equation $ x + 7  = 1$ have?	2. How many solutions does the equation $ x + 7  = 0$ have?	3. How many solutions does the equation $ x + 7  = -1$ have?
4. $ x  = 12$	5. $ x  - 6 = 4$	6. $ x + 3  = 10$
7. $ x - 2  - 3 = 5$	8. $ x + 7  + 2 = 10$	9. $4 x - 5  = 20$

10. $ 2x  + 1 = 7$	11. $6 +  x + 2  = 6$	12. $5 +  x - 1  = 0$
13. $ x - 1  = 2$	14. $4 x - 5  = 12$	15. $3 x - 1  = -15$
<p>16. Leticia sets the thermostat in her apartment to 68 degrees. The actual temperature in her apartment can vary from this by as much as 3.5 degrees.</p> <p>Write an absolute-value equation that you can use to find the minimum and maximum temperature.</p> <p>Solve the equation to find the minimum and maximum temperature.</p>		
<p>17. Troy's car can go 24 miles on one gallon of gas. However, his gas mileage can vary by 2 miles per gallon depending on where he drives.</p> <p>Write an absolute-value equation that you can use to find the minimum and maximum gas mileage.</p> <p>Solve the equation to find the minimum and maximum gas mileage.</p>		
<p>18. A carpenter cuts boards for a construction project. Each board must be 3 meters long, but the length is allowed to differ from this value by at most 0.5 centimeters. Write and solve an absolute-value equation to find the minimum and maximum acceptable lengths for a board.</p>		
<p>19. The owner of a butcher shop keeps the shop's freezer at <math>-5^{\circ}\text{C}</math>. It is acceptable for the temperature to differ from this value by <math>1.5^{\circ}</math>. Write and solve an absolute-value equation to find the minimum and maximum acceptable temperatures.</p>		

## GUIDED NOTES – 1.1

### Domain, Range & Notation

Name: Kay Period:     

In this course we have the opportunity to explore a variety of functions, including quadratic, polynomial, rational, radical, exponential, logarithmic, and trigonometric. Before we get to all those functions, their graphs, and behaviors, the basic linear function is a good place to start.

**INTERVALS:** An interval is part of a function, in this case a line, without any breaks. A finite interval has two endpoints, which may or may not be included in the interval. An infinite interval is unbounded at one or both ends.

**NOTATION:** We have three ways to write the intervals of a function. We call these the notation.

Description	Type of Interval	Inequality	Set Notation	Interval notation
All real numbers from a to b, including a and b.	Finite	$a \leq x \leq b$	$\{x \mid a \leq x \leq b\}$	$[a, b]$
All real numbers greater than a	Infinite	$x > a$	$\{x \mid x > a\}$	$(a, \infty)$
All real numbers less than or equal to a	Infinite	$x \leq a$	$\{x \mid x \leq a\}$	$(-\infty, a]$

#### Example A

Write the interval notation for a set of all real numbers from -4 to 5, including -4 but not including 5.

$$[-4, 5)$$

#### Example B

Write the set notation for a set of all real numbers greater than or equal to 6.

$$\{x \mid x \geq 6\}$$

**TALK ABOUT IT:** What can we conclude about the relationship between infinity and the use of brackets and parentheses in writing the notation for a function?

infinity uses parentheses instead of brackets

#### DOMAIN AND RANGE

Unless otherwise stated, a function is assumed to have a **domain** (all the possible input or x-values) consisting of all real numbers for which the function is defined.

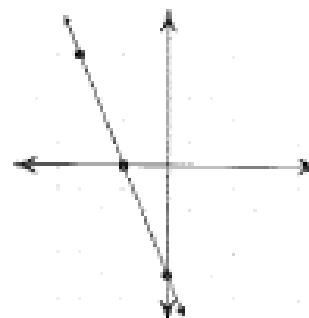
We can write it in interval notation as:  $(-\infty, \infty)$

Another way to write the set of real numbers is:  $\mathbb{R}$

The **range** consists of all the possible output or y-values the function, given the domain of the function.

**Example C** - Identify the domain and range for the graphed linear function shown here.

$$D (-\infty, \infty) \quad R (-\infty, \infty)$$



#### Example D

Given the function  $f(x) = 2x - 3$  with a domain of  $(-3, 5]$ , identify the range in the same notation.

$$\begin{aligned} 2(-3) - 3 &= -6 - 3 = -9 \\ 2(5) - 3 &= 10 - 3 = 7 \end{aligned} \quad (-9, 7]$$

**Example E**

Given the function  $f(x) = -\frac{1}{2}x + 5$  with a domain of  $\{x|x < 4\}$ , identify the range using the same notation.

$$-\frac{1}{2}(4) + 5 = -2 + 5 = 3 \quad \{y|y > 3\}$$

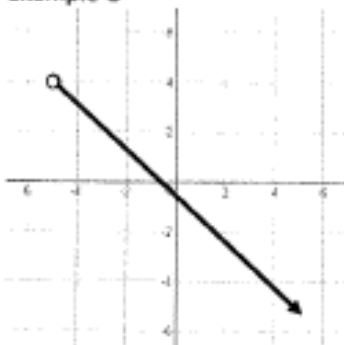
Domain and range will come into play with every type of function we encounter in future lessons. For now we will practice with some line segments and rays to get a feel for how these functions restrict the domain and range.

Write the domain and range in both set and interval notation for the following graphed functions.

**Example F**

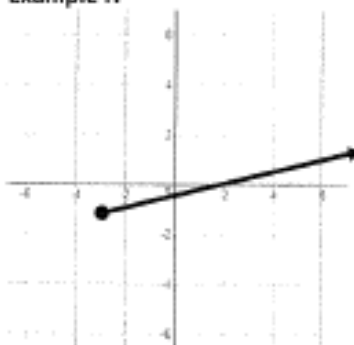
DOMAIN  $[-4, 4)$   
 $\{x|-4 \leq x < 4\}$

RANGE  $[-2, 6)$   
 $\{y|-2 \leq y < 6\}$

**Example G**

DOMAIN  $(-5, \infty)$   
 $\{x|x > -5\}$

RANGE  $(-\infty, 4)$   
 $\{y|y < 4\}$

**Example H**

DOMAIN  $[-3, \infty)$   
 $\{x|x \geq -3\}$

RANGE  $[-1, \infty)$   
 $\{y|y \geq -1\}$

**TALK ABOUT IT:** If a student writes the domain of a function that has no x-values greater than 5 as  $(5, -\infty)$ , is that acceptable? Explain why or why not.

smallest value listed first

**LINEAR APPLICATIONS**

A 6 inch long candle burns at a rate of half an inch per hour. Write a function in terms of the candle's height  $h$  (in inches) at any time  $t$  (in hours).

$$h(t) = 6 - 0.5t$$

Suppose the candle is lit and left burning for 5 hours. Identify the domain and range.

$$h(5) = 6 - 0.5(5) \\ = 6 - 2.5 = 3.5$$

Domain  $[0, 5]$   
 Range  $[3.5, 6]$

Write a domain and range to represent the time and height of the candle, should it be left burning until it reaches a height of 0 inches and can no longer burn.

$$0 = 6 - 0.5t \\ -6 = -0.5t \\ 12 = t$$

Domain  $[0, 12]$   
 Range  $[0, 6]$

**TALK ABOUT IT:** Will positive/negative infinity ever be part of the domain and range in a real world application problem? Explain why or why not.

depends on situation

**PRACTICE PROBLEMS – 1.1**

Domain, Range & Notation

Name: Key Period: \_\_\_\_\_

1) Write the interval notation for a set of all real numbers that are greater than 2 and less than or equal to 8.  $(2, 8]$

2) Write the set notation for a set of all real numbers between -18 and 20, including -18 but not including 20.  $\{x | -18 \leq x < 20\}$

3) Write the interval notation for a set of all real numbers that are greater than or equal to 5.  $[5, \infty)$

4) Write the set notation for a set of all real numbers less than 15.  $\{x | x < 15\}$

5) Given the function  $f(x) = 4x - 6$  with a domain of  $(-3, 5]$ , identify the range in the same notation.  $(-18, 14]$

$4(-3) - 6 = -12 - 6 = -18$        $4(5) - 6 = 20 - 6 = 14$

6) Given the function  $f(x) = \frac{1}{2}x + 8$  with a domain of  $\{x | 2 \leq x < 14\}$ , identify the range using the same notation.  $\{y | 9 \leq y < 15\}$

$\frac{1}{2}(2) + 8 = 1 + 8 = 9$        $\frac{1}{2}(14) + 8 = 7 + 8 = 15$

7) Given the function  $f(x) = -3x - 12$  with a domain of  $[-5, 0]$ , identify the range in the same notation.  $[-12, 3]$

$-3(-5) - 12 = 15 - 12 = 3$        $-3(0) - 12 = -12$

8) Given the function  $f(x) = -\frac{2}{3}x + 4$  with a domain of  $\{x | x > 6\}$ , identify the range using the same notation.  $\{y | y < 0\}$

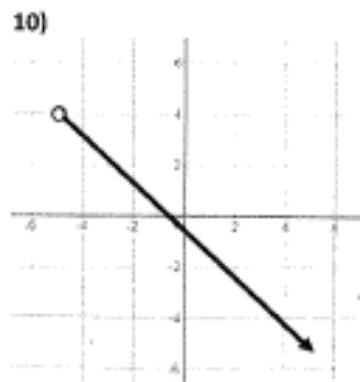
$-\frac{2}{3}(6) + 4 = -4 + 4 = 0$

Write the domain and range in INTERVAL NOTATION for the following graphed functions.



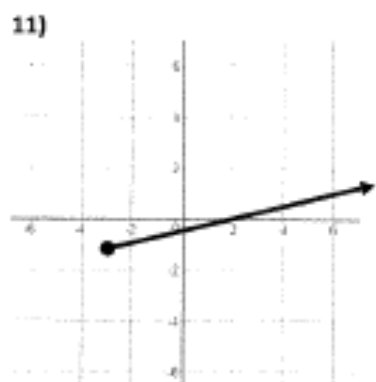
DOMAIN  $[-4, 4)$

RANGE  $[-2, 6)$



DOMAIN  $(-5, \infty)$

RANGE  $(-\infty, 4)$

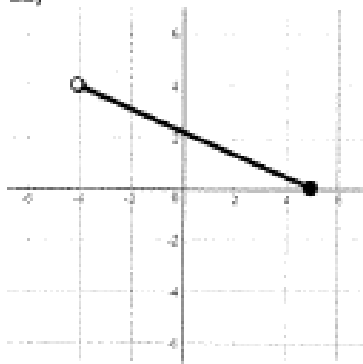


DOMAIN  $[-3, \infty)$

RANGE  $[-1, \infty)$

Write the domain and range in SET NOTATION for the following graphed functions.

12)



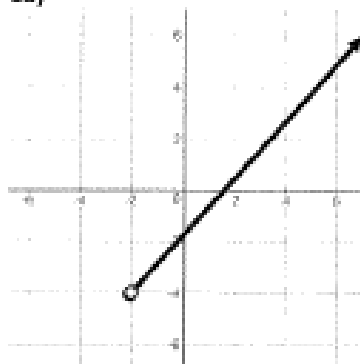
DOMAIN

$$\{x \mid -4 < x \leq 5\}$$

RANGE

$$\{y \mid 0 \leq y < 4\}$$

13)



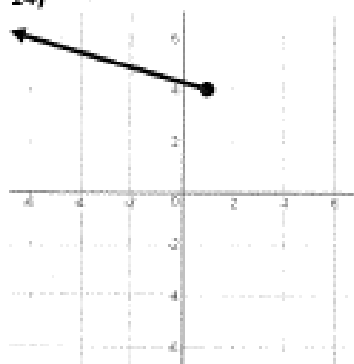
DOMAIN

$$\{x \mid x > -2\}$$

RANGE

$$\{y \mid y > -4\}$$

14)



DOMAIN

$$\{x \mid x \leq 1\}$$

RANGE

$$\{y \mid y \geq 4\}$$

15) It is estimated that the price of a book at a used book store increases by \$0.02 per page with a base cost of \$3. Write a function to represent this scenario in terms of  $C(p)$ , where  $C$  is the cost of the book (in dollars) and  $p$  represents the number of pages in the book.

$$C(p) = 3 + 0.02p$$

Suppose you select a stack of books, where the largest book has 450 pages and the smallest book has 80 pages. Write the domain and range.

$$C(450) = 3 + 0.02(450) = 12$$

$$C(80) = 3 + 0.02(80) = 4.60$$

$$\text{Domain } [80, 450]$$

$$\text{Range } [4.60, 12]$$

16) Suppose you put a hot cup of coffee at 180 degrees out on the counter and it cools by 2.5 degrees per minute. Write a function to represent the temperature of the coffee as  $T(m)$ , where  $T$  represents the temperature and  $m$  represent the minutes the coffee is left out.

$$T(m) = 180 - 2.5m$$

Write the domain and range of the function, should a person leave the coffee out for 20 minutes.

$$T(20) = 180 - 2.5(20) = 130$$

$$\text{Domain } [0, 20]$$

$$\text{Range } [130, 180]$$

The coffee will not get cooler than the room temperature which is at 78 degrees. Write the domain and range to represent this.

$$78 = 180 - 2.5(t)$$

$$-102 = -2.5t$$

$$40.8 = t$$

$$\text{Domain } [0, 40.8]$$

$$\text{Range } [78, 180]$$

## GUIDED NOTES – 1.2

Average Rate of Change & Linear Regression

Name: Key Period: \_\_\_\_\_

### KEY FEATURES OF LINEAR FUNCTIONS

A linear function has some key features we want to review and focus on. The y-intercept tells us the y-value of the graph when  $x = 0$  and the x-intercept tells us the x-value when  $y = 0$ . The slope of the line indicates the rate at which the function is increasing or decreasing.

Identify the key features of the graphed linear function shown here.

y-intercept:  $(0, 4)$       Slope:  $2$       x-intercept:  $(-2, 0)$

Write a function to represent the line:  $y = 2x + 4$

How do we get the x-intercept out of that linear equation?

plug in  $y = 0$  and solve

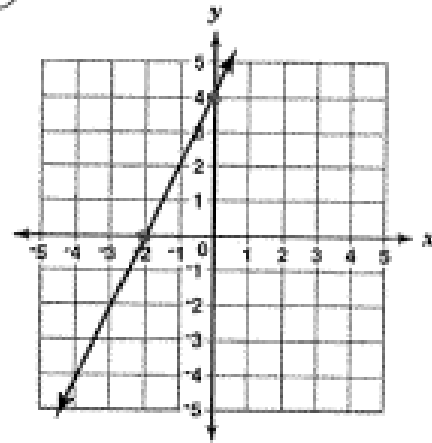
**Example A:** Given the linear function  $f(x) = -2x + 8$ , solve for the x-intercept. Confirm by graphing with technology.

$$\begin{aligned} 0 &= -2x + 8 \\ -8 &= -2x \\ 4 &= x \end{aligned} \quad (4, 0)$$

**Example B:** Given the linear function  $f(x) = \frac{2}{3}x - 6$ , solve for the x-intercept. Confirm by graphing with technology.

$$\begin{aligned} 0 &= \frac{2}{3}x - 6 \\ 6 &= \frac{2}{3}x \\ 9 &= x \end{aligned} \quad (9, 0)$$

The process of setting a function equal to 0 to get the x-intercept is important in future lessons as this is the process for 'solving' the function or getting the solutions.



### AVERAGE RATE OF CHANGE (GRAPHS)

In previous math courses you have used the 'slope formula' to find the slope between two points. In this course, we focus on the **average rate of change** between two points, which can be found using the same formula, but allows us to look at various points on a graph (or in time with application problems) to find the rate of change. Let's review and practice that skill now.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Over what intervals is the function increasing?  $(-3, 0) \cup (3, \infty)$

Over what intervals is the function decreasing?  $(-\infty, -3) \cup (0, 3)$

Find the average rate of change between  $x = -3$  and  $x = 0$ .

$$(-3, 0) \rightarrow (0, 3) \quad \frac{3-0}{0-(-3)} = \frac{3}{3} = 1$$

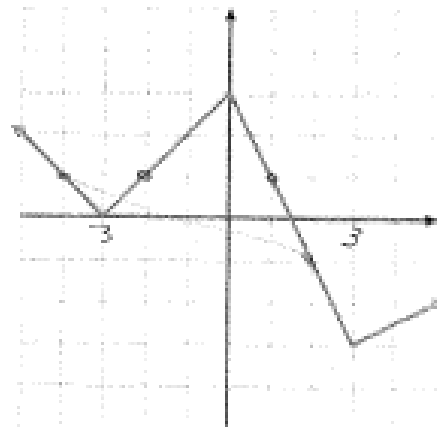
Find the average rate of change over the interval  $[0, 3]$ .  $-2$

Find the average rate of change between  $x = -2$  and  $x = 1$ .  $0$

Find the average rate of change over the interval  $[-4, 2]$ .  $-\frac{1}{3}$

**Extension/Spiral:** Suppose the domain of this function is  $[-5, 5]$ , what is the range?

$$[-3, 3]$$





## AVERAGE RATE OF CHANGE (TABLES)

When you can't see a function visually as a graph, the formula for average rate of change becomes helpful.

**Example:** The table shows the height (in feet) of a golf ball at various times (in seconds) after a golfer hits the ball into the air.

Time (s)	0	0.5	1	1.5	2	2.5	3	3.5	4
Height (ft)	0	28	48	60	64	60	48	28	0

What is the maximum height the golf ball reaches according to the data? 64 ft

What is the average rate of change for the height of the golf ball between 0 and 2 seconds?

$$(0,0) (2,64) \quad \frac{64-0}{2-0} = \frac{64}{2} = 32 \text{ ft/s}$$

What is the average rate of change for the height of the golf ball between 1 and 3 seconds?

$$(1,48) (3,48) \quad \frac{48-48}{3-1} = \frac{0}{2} = 0 \text{ ft/s}$$

What is the average rate of change for the height of the golf ball between 2 and 3.5 seconds?

$$(2,64) (3.5,28) \quad \frac{28-64}{3.5-2} = \frac{-36}{1.5} = -24 \text{ ft/s}$$

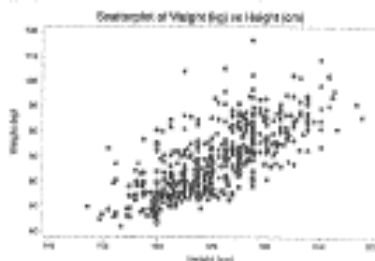
**TALK ABOUT IT:** What does the path of this golf ball look like? For a real life golfer, is this table realistic?

yes

## LINEAR REGRESSION

Up to this point, we have dealt with pure linear equations, meaning they represent perfectly straight lines. We know in real life sets of data that pure, perfect data does not always exist. Were we to graph the relationship between the height and weight of a large group of people for example, we would end up with a graph like this. What do we call this type of graph?

Scatter plot



Still, we can see there is a linear trend between height and weight:

The taller people are, the heavier they tend to be.

**Linear regression** allows us to 'fit' a single line to the data, a line known as the **line of best fit**. We will use technology to generate this linear function and use it to make predictions about our data.

**Example:** As a science project, Shelley is studying the relationship of car mileage (in miles per gallon) and speed (in miles per hour). The table shows the data she gathered using her family's vehicle.

Speed (mi/h)	30	40	50	60	70
Mileage (mi/gal)	34.0	33.5	31.5	29.0	27.5

Use technology to write a function to represent the relationship between mileage  $m$ , as a function of speed  $s$ , that the vehicle is traveling.

$$m(s) = -.175s + 39.85$$

Identify the domain and range for the function:

$$D: [0, 120] \quad R: [18.85, 39.85]$$

What does the y-intercept tell us in this function and the context?

(0, 39.85) mileage when going 0 mph

What is the meaning of the slope of this function within the context?

each mile faster, mileage drops .175

Predict the miles per gallon her family's vehicle would get at a speed of 80 miles per hour.

25.85 mph

**TALK ABOUT IT:** If you were to get this data for your/your family's car, what would your domain restrictions be?

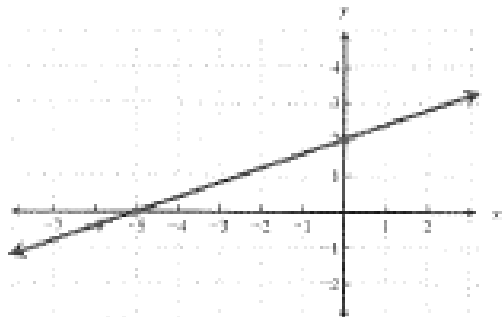
0 to how ever fast you plan to drive

## PRACTICE PROBLEMS – 1.2

### Average Rate of Change & Linear Regression

Name: Key Period:     

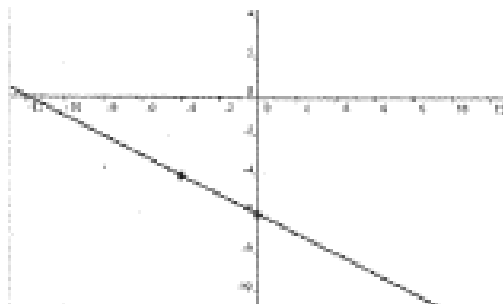
Identify the features of each graph below.



1) Slope:  $\frac{2}{5}$

y-Intercept:  $(0, 2)$

x-Intercept:  $(-5, 0)$



2) Slope:  $-\frac{1}{2}$

y-Intercept:  $(0, -6)$

x-Intercept:  $(-12, 0)$

3) Given the linear function  $f(x) = -\frac{1}{2}x + 14$ , solve for the x-intercept. Confirm by graphing with technology.

$$\begin{aligned} 0 &= -\frac{1}{2}x + 14 \\ -14 &= -\frac{1}{2}x \\ 28 &= x \end{aligned} \quad (28, 0)$$

4) Given the linear function  $f(x) = 3x - 15$ , solve for the x-intercept. Confirm by graphing with technology.

$$\begin{aligned} 0 &= 3x - 15 \\ 15 &= 3x \\ 5 &= x \end{aligned} \quad (5, 0)$$

5) Over what intervals is the function increasing?

$$[-7, -2) \cup (4, 6]$$

6) Over what intervals is the function decreasing?

$$(-2, 4)$$

7) Find the average rate of change between  $x = -6$  and  $x = 0$ .

$$\frac{2+1}{0+6} = \frac{3}{6} = \frac{1}{2}$$

8) Find the average rate of change over the interval  $[-2, 2]$ .

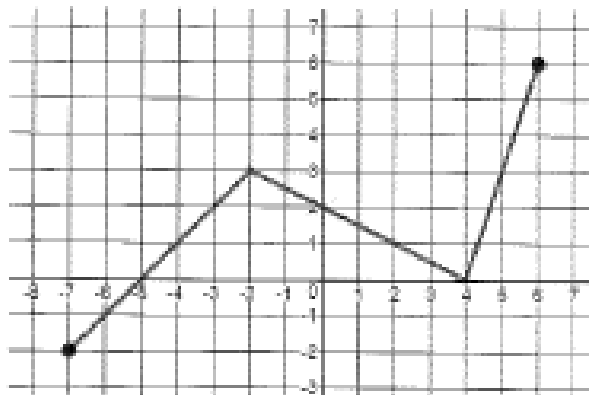
$$\begin{aligned} &(-2, 3) (2, 1) \\ &\frac{1-3}{2+2} = \frac{-2}{4} = -\frac{1}{2} \end{aligned}$$

9) Find the average rate of change between  $x = 2$  and  $x = 6$ .

$$\begin{aligned} &(2, 1) (6, 6) \\ &\frac{6-1}{6-2} = \frac{5}{4} \end{aligned}$$

10) Find the average rate of change over the interval  $[-4, 2]$ .

$$\begin{aligned} &(-4, 1) (2, 1) \\ &\frac{1-1}{2+4} = \frac{0}{6} = 0 \end{aligned}$$



Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Hits	5	10	21	24	28	36	33	21	27	40	46	50	31	38

Vern created a website for his school's sports teams. He has a hit counter on his site that lets him know how many people have visited the site. The table shows the number of hits the site received each day for the first two weeks.

11) What is the average rate of change from day 2 to day 10?

$$(2, 10) \quad (10, 40) \quad \frac{40-10}{10-2} = \frac{30}{8} = 3.75$$

12) What is the average rate of change from day 5 to day 12?

$$(5, 28) \quad (12, 50) \quad \frac{50-28}{12-5} = \frac{22}{7} \approx 3.14$$

13) Identify the domain and range of the data.

$$D: [0, 14] \quad R: [0, 50]$$

14) Use technology to find a regression equation in terms of  $H(d)$  (hits as a function of days) to fit the data.

$$H(d) = 2.41d + 11.22$$

15) Interpret the y-intercept in the context of the problem.

$(0, 11.22)$  at day 0 there were 11.22 hits

16) Interpret the slope in the context of the problem.

increase # hits by 2.41 / day

17) Use your regression equation to predict the number of hits on day 20.

$\approx 48.31$  hits

Mass (g)	3.1	2.0	3.2	4.0	3.7	1.9	4.5
Frequency of Wing Beats (beats per second)	60	85	50	45	55	90	40

The above table gives the mass of 7 different hummingbirds and their frequency of wing beats per second.

18) Identify the domain and range of the data.

$$D [1.9, 4.5] \quad R [40, 90]$$

19) Use technology to find a regression equation in terms of  $B(m)$ , beats as a function of mass.

$$B(m) = -19.14m + 121.97$$

20) Interpret the y-intercept in the context of the problem.

mass of 0, beats 121.97

21) Interpret the slope in the context of the problem.

beats decrease by 19.14 for each gram increase

22) Use your regression equation to predict the number of wing beats for a bird that has a mass of 6.2 grams.

3.3 beats/sec

## GUIDED NOTES – 1.3

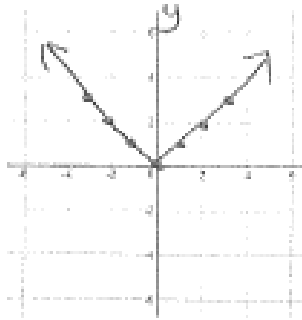
### Absolute Value Graphs & Transformations

Name: Key Period: \_\_\_\_\_

**ABSOLUTE VALUE:** Recall that absolute value, written as  $| |$  is the distance between a value and 0 on a number line.



$$|4| = 4 \quad |-4| = 4 \quad -|4| = -4 \quad -|-4| = -4$$



Let's graph the parent function for absolute value:  $f(x) = |x|$

x	-3	-2	-1	0	1	2	3
f(x)	3	2	1	0	1	2	3

The graph is symmetric about the y-axis, with a vertex at (0,0).

Domain:  $(-\infty, \infty)$  Range:  $[0, \infty)$

**TALK ABOUT IT:** Why did we just call the function above a parent function?

**GRAPHING ABSOLUTE VALUE FUNCTIONS:** We will apply general transformations to help us graph absolute value functions by changing parameters in the equation:  $g(x) = a|x - h| + k$

a

If  $a$  is greater than 1, the graph is vertically stretched.

If  $a$  is less than 1 (like a fraction) the graph is vertically shrunk.

If  $a$  is negative, the graph is vertically reflected.

h

The value of  $h$  tells us whether the graph is horizontally shifted left or right.

$|x - 3|$  for example, tells us that  $h = \underline{3}$ , so the graph is shifted 3 units right.

$|x + 3|$  for example, tells us that  $h = \underline{-3}$   $|x - (-3)|$  so the graph is shifted 3 units left.

k

The value of  $k$  tells us whether the graph is shifted vertically up or down.  $+k$  is up and  $-k$  is down

**Example 1:** Identify the transformations and graph the function:

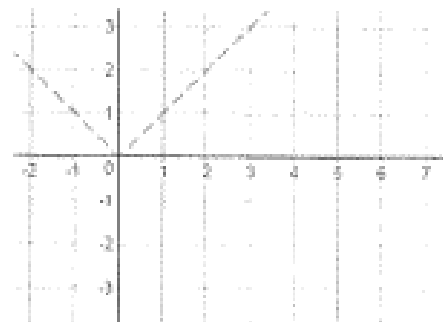
$$g(x) = 2|x - 5| - 3$$

The graph is shifted 5 units right and 3 units down.

This means the new vertex is (5, -3).

Now we look at the parameter  $a$ .

Since  $a = \underline{2}$  we will be vertically stretched by a factor of 2.

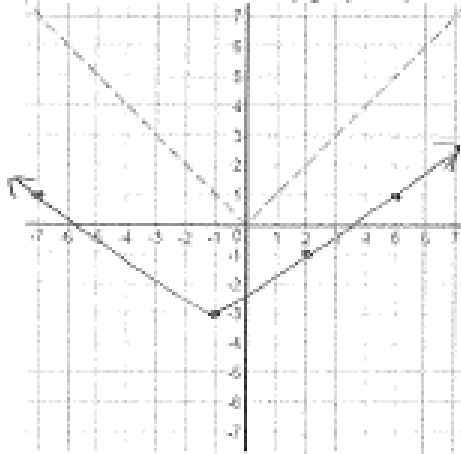


**Example 2:** Identify the transformations and graph the function  $a(x) = \frac{2}{3}|x + 1| - 3$

Transformations: left 1  
down 3  
vertical shrink

Vertex:  $(-1, -3)$

(parent function already graphed)



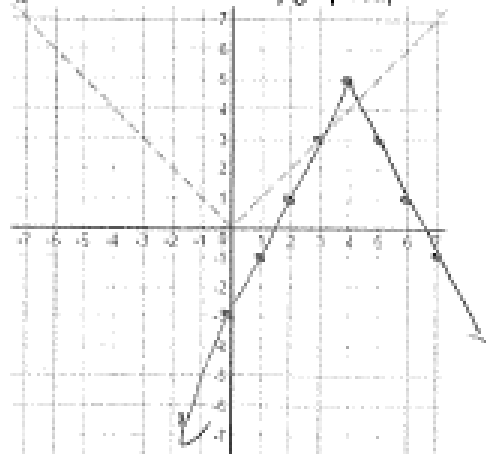
Domain:  $(-\infty, \infty)$  Range:  $[-3, \infty)$

**Example 3:** Identify the transformations and graph the function  $b(x) = -2|x - 4| + 5$

Transformations: reflect x-axis  
vertical stretch  
right 4  
up 5

Vertex:  $(4, 5)$

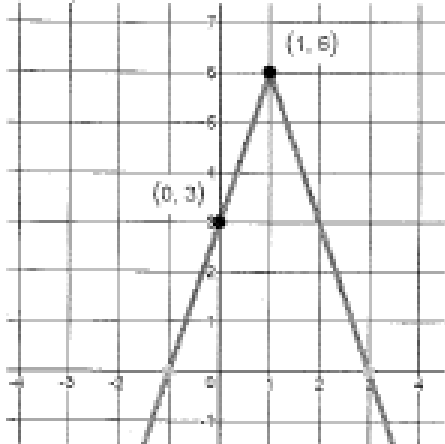
(parent function already graphed)



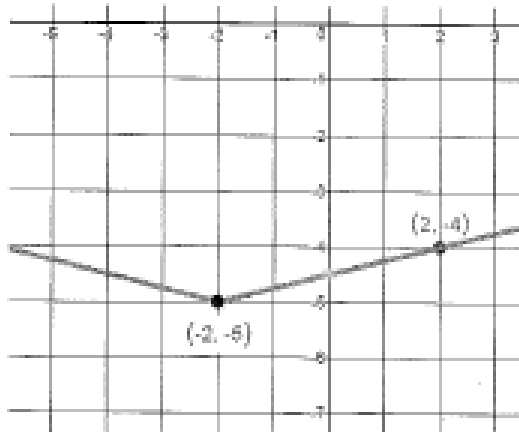
Domain:  $(-\infty, \infty)$  Range:  $(-\infty, 5]$

**WRITING ABSOLUTE VALUE FUNCTIONS FROM A GRAPH:** We will use the same skills in reverse to create absolute value functions to represent given graphs. Remember that we can write two equivalent functions that look completely different using values for either  $a$  or  $b$ , so we will only use one parameter at a time and set the other to equal 1.

**Examples:** Write an absolute value function to represent each given graph.



$$y = -3|x - 1| + 6$$



$$y = .25|x + 2| - 5$$

### PRACTICE PROBLEMS – 1.3

#### Absolute Value Graphs & Transformations

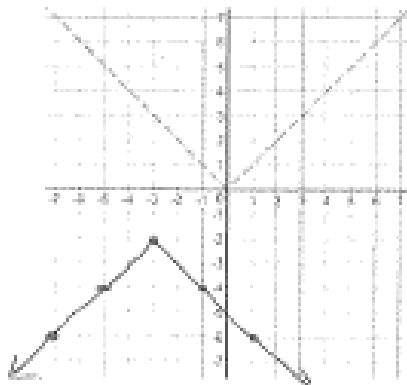
Name: Kley Period:     

Identify the transformations, vertex, and domain/range, of each of the following absolute value functions and graph each function, given the parent function  $f(x) = |x|$  (which is graphed already).

1)  $g(x) = -|x + 3| - 2$

Transformations: reflect x-axis  
left 3  
down 2

Vertex:  $(-3, -2)$

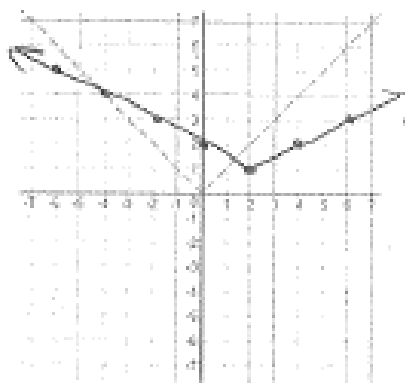


Domain:  $(-\infty, \infty)$  Range:  $(-\infty, -2]$

2)  $a(x) = \frac{1}{2}|x - 2| + 1$

Transformations: vertical shrink  
right 2  
up 1

Vertex:  $(2, 1)$

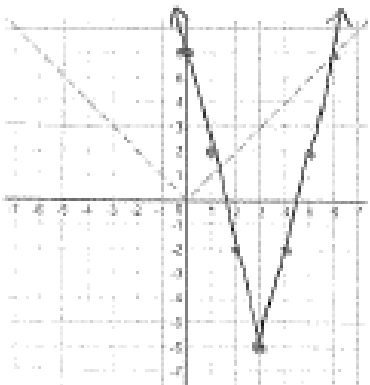


Domain:  $(-\infty, \infty)$  Range:  $[2, \infty)$

3)  $g(x) = 4|x - 3| - 6$

Transformations: vertical stretch  
right 3  
down 6

Vertex:  $(3, -6)$

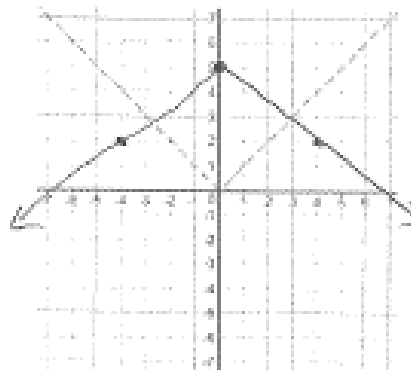


Domain:  $(-\infty, \infty)$  Range:  $[-6, \infty)$

4)  $a(x) = -\frac{3}{4}|x| + 5$

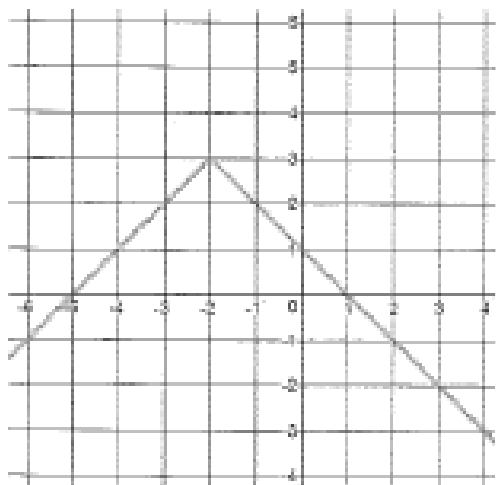
Transformations: reflect x-axis  
vertical shrink  
up 5

Vertex:  $(0, 5)$

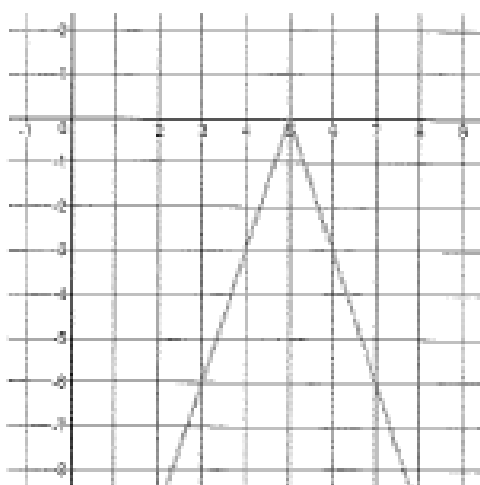


Domain:  $(-\infty, \infty)$  Range:  $(-\infty, 5]$

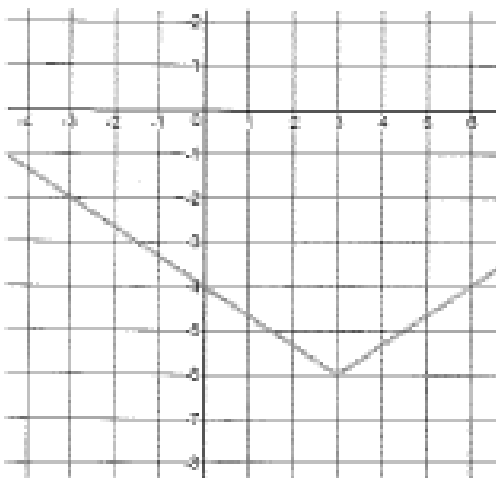
Write an absolute value function to represent each given graph.



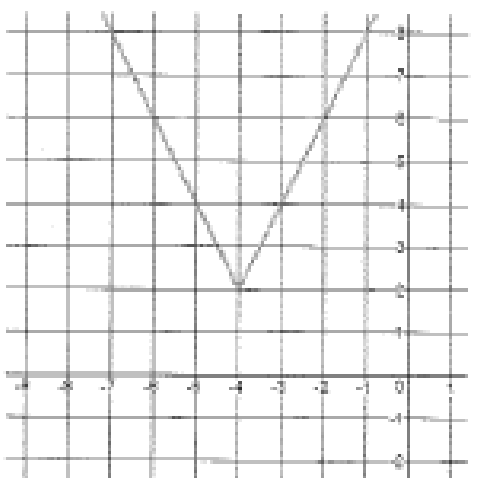
5) Function:  $y = -|x+2| + 3$



6) Function:  $y = -3|x-5|$



7) Function:  $y = \frac{2}{3}|x-3| - 6$



8) Function:  $y = 2|x+4| + 2$

9) Compare the following absolute value equations and identify which function(s) meet the key features described below.

$a(x) = -2|x + 2| - 1$

$b(x) = |x| - 6$

$c(x) = -\frac{3}{4}|x + 7|$

$d(x) = 3|x - 2| + 1$

Which function(s) is/are...

...not shifted horizontally: B

...not stretched or compressed: B

...shifted 2 units to the left: A

...not shifted vertically: C

...shifted up by 1 unit: D

...will never cross the parent function  $y = |x|$ : A, B, C

...contains the point (5, 10): D

...contains the point (-3, -3): A, B, C

## GUIDED NOTES – 1.4

### Absolute Value Graph Features

Name: Key Period:     

#### MAXIMUM & MINIMUMS

We identified the vertex of absolute value functions in the previous lesson.

If the function is negative, the vertex is referred to as the **maximum**.

If the function is positive, the vertex will be the graph's **minimum**.

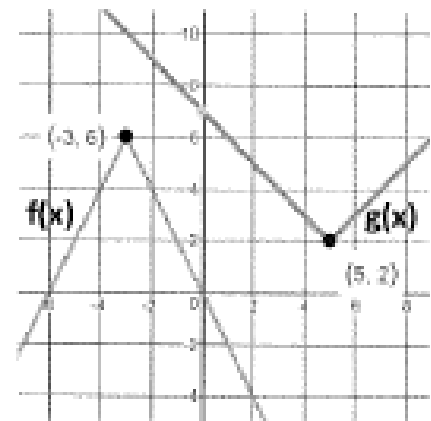
**Example:**

The function  $f(x)$  has a vertex at max, which is a  $(-3, 6)$

Domain:  $(-\infty, \infty)$  Range:  $(-\infty, 6]$

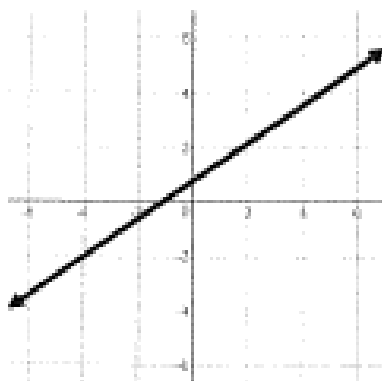
The function  $g(x)$  has a vertex at min, which is a  $(5, 2)$

Domain:  $(-\infty, \infty)$  Range:  $[2, \infty)$



#### END BEHAVIOR

A function's end behavior tells us what happens to the  $f(x)$ -values as the  $x$ -values either increase without bound (approaching positive infinity) or decrease without bound (approaching negative infinity).



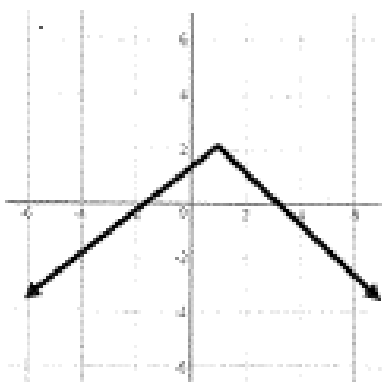
For this linear function for example, we would state the end behavior symbolically as follows:

$$\text{As } x \rightarrow +\infty, f(x) \rightarrow +\infty$$

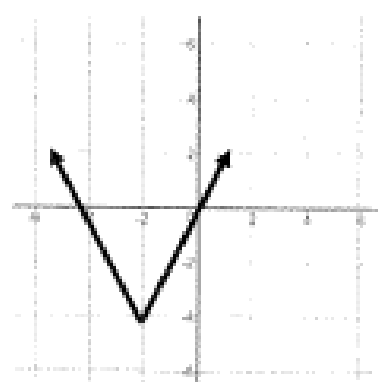
$$\text{As } x \rightarrow -\infty, f(x) \rightarrow -\infty$$

**TALK ABOUT IT:** What would happen if the line was negative instead of positive?

**Examples:** Identify the end behavior for the following graphs.



$$\begin{aligned} x \rightarrow +\infty, f(x) &\rightarrow -\infty \\ x \rightarrow -\infty, f(x) &\rightarrow -\infty \end{aligned}$$



$$\begin{aligned} x \rightarrow +\infty, f(x) &\rightarrow \infty \\ x \rightarrow -\infty, f(x) &\rightarrow \infty \end{aligned}$$



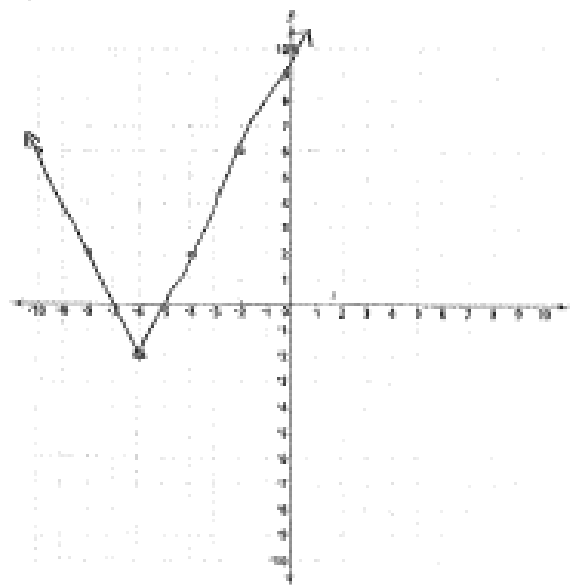
## COMPARING TABLES AND GRAPHS

It is important to be able to understand functions in various forms, whether given an explicit function, a table of values, or a graph. We will practice identifying the vertex from a table and comparing it with a given graph.

**Example A:** Given the table of values representing the function  $a(x)$  and the graphed function  $b(x)$ , perform the following analysis.

$x$	$a(x)$
-2	0
-1	1
0	2
1	3
2	4
3	3

$$b(x) = 2|x + 6| - 2$$



Write a function to represent  $a(x)$ :  $y = -|x - 2| + 4$

Which function has a vertex that is a maximum?   $a(x)$    $b(x)$   both  neither

Which function has the greater  $y$ -intercept?   $a(x)$    $b(x)$   both  neither

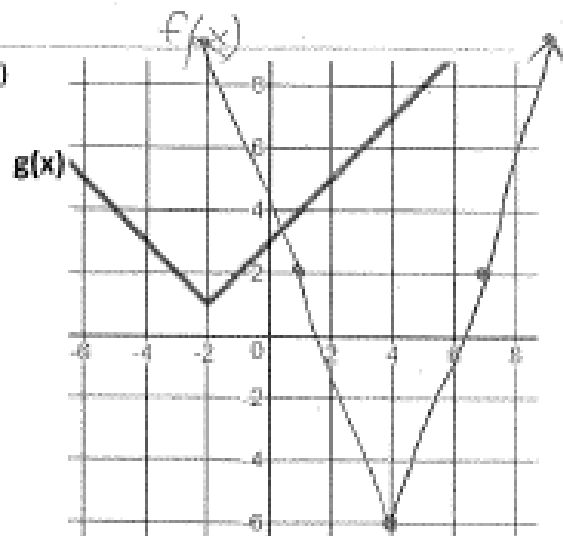
Which function contains the point  $(-7, -5)$ ?   $a(x)$    $b(x)$   both  neither

Which function has a domain of  $(-\infty, \infty)$ ?   $a(x)$    $b(x)$   both  neither

Which function has a range of  $\{y | y \leq 4\}$ ?   $a(x)$    $b(x)$   both  neither

**Example B:** Given the table of values representing the function  $f(x)$  and the graphed function  $g(x)$ , perform the following analysis.

$x$	1	4	7	8	10
$f(x)$	2	-6	2	5	10



Which function has a vertex that is a minimum?  
  $f(x)$    $g(x)$   both  neither

Which function has the greater  $y$ -intercept?  
  $f(x)$    $g(x)$   both  neither

Which function contains the point  $(5, 8)$ ?   $f(x)$    $g(x)$   both  neither

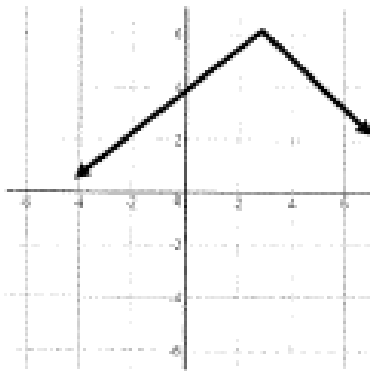
Which function has an end behavior of  $-\infty$ ?   $f(x)$    $g(x)$   both  neither

Which function has a range of  $\{y | y \geq -6\}$ ?   $f(x)$    $g(x)$   both  neither

**PRACTICE PROBLEMS – 1.4**  
**Absolute Value Graph Features**

Name: Key Period:     

Identify the vertex, whether it is a max or a min point, the domain and range, and the end behavior for each of the graphed absolute value functions below.



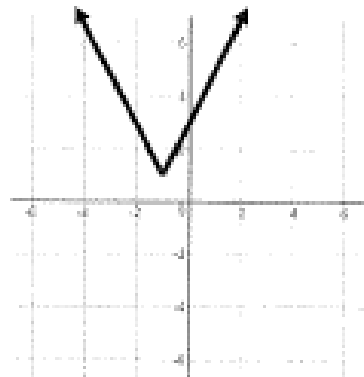
Vertex:  $(3, 6)$  Max / Min

Domain:  $(-\infty, \infty)$  Range:  $(-\infty, 6]$

End behavior:

As  $x \rightarrow +\infty, f(x) \rightarrow -\infty$

As  $x \rightarrow -\infty, f(x) \rightarrow -\infty$



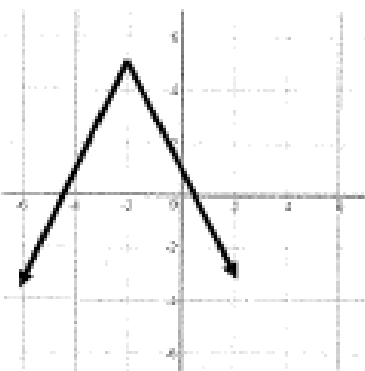
Vertex:  $(-1, 1)$  Max Min

Domain:  $(-\infty, \infty)$  Range:  $[1, \infty)$

End behavior:

As  $x \rightarrow +\infty, f(x) \rightarrow \infty$

As  $x \rightarrow -\infty, f(x) \rightarrow \infty$



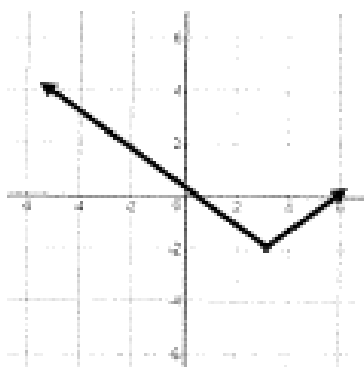
Vertex:  $(2, 5)$  Max / Min

Domain:  $(-\infty, \infty)$  Range:  $(-\infty, 5]$

End behavior:

As  $x \rightarrow +\infty, f(x) \rightarrow -\infty$

As  $x \rightarrow -\infty, f(x) \rightarrow -\infty$



Vertex:  $(3, -2)$  Max Min

Domain:  $(-\infty, \infty)$  Range:  $[-2, \infty)$

End behavior:

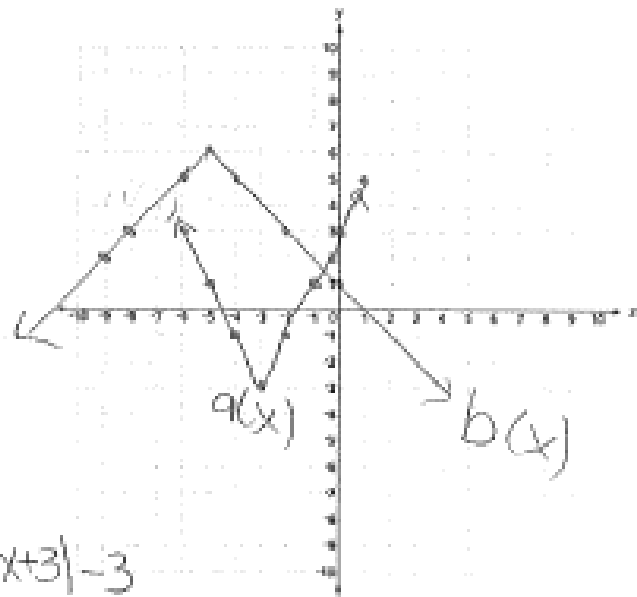
As  $x \rightarrow +\infty, f(x) \rightarrow \infty$

As  $x \rightarrow -\infty, f(x) \rightarrow \infty$

Given the table of values representing the function  $a(x)$  and the graphed function  $b(x)$ , perform the following analysis.

$x$	$a(x)$
-1	1
-2	-1
-3	-3
-4	-1
-5	1
-6	3

$$b(x) = -|x + 5| + 6$$

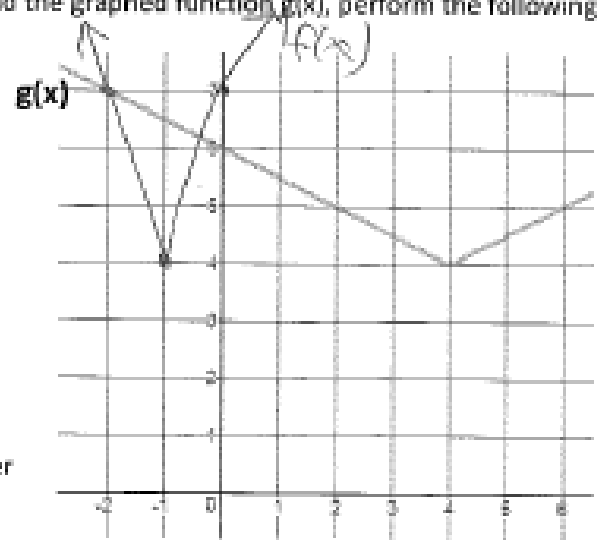


Write a function to represent  $a(x)$ :  $a(x) = 2|x+3| - 3$

- Which function has a vertex that is a maximum?  $a(x)$   $b(x)$  both neither
- Which function has the greater y-intercept?  $a(x)$   $b(x)$  both neither
- Which function contains the point (1, 5)?  $a(x)$   $b(x)$  both neither
- Which function has a domain of  $(-\infty, \infty)$ ?  $a(x)$   $b(x)$  both neither
- Which function has a range of  $\{y | y \geq -3\}$ ?  $a(x)$   $b(x)$  both neither

Given the table of values representing the function  $f(x)$  and the graphed function  $g(x)$ , perform the following analysis.

$x$	1	0	-1	-2	-3
$f(x)$	10	7	4	7	10



- Which function has a vertex that is a minimum?  $f(x)$   $g(x)$  both neither
- Which function has the greater y-intercept?  $f(x)$   $g(x)$  both neither
- Which function contains the point (-2, 7)?  $f(x)$   $g(x)$  both neither
- Which function has an end behavior of  $-\infty$ ?  $f(x)$   $g(x)$  both neither
- Which function has a range of  $\{y | y \geq 4\}$ ?  $f(x)$   $g(x)$  both neither

## Algebra 3-4      1.5

### Solving Absolute Value Equations

<p>Graph <math>y = 2x - 2</math></p>	<p>Name a point on the graph. <math>(3, 4)</math></p> <p>What do you know about this ordered pair and the equation <math>y = 2x - 2</math>?</p> <p><math>4 = 2(3) - 2</math> <math>4 = 6 - 2</math> it works in equation.</p>	<p>Once you have your graph, if you wanted to find the <math>x</math>-value when <math>y</math> is 6, instead of solving, how could you find this value?</p> <p>graph <math>y = 6</math> + find <math>x</math>-value</p>
<p>Graph <math>y =  x - 3 </math></p>	<p>Algebraically solve for <math>x</math> if <math>y = 2</math></p> <p><math>2 =  x - 3 </math></p> <p><math>2 = x - 3</math>      <math>-2 = x - 3</math> <math>x = 5</math>            <math>x = 1</math></p>	<p>Graphically solve by adding the line <math>y = 2</math> to your first graph.</p> <p><math>x = 1, 5</math></p>

Directions: Answer each question or solve.

<p>1. How many solutions does the equation <math> x + 7  = 1</math> have?</p> <p style="text-align: center;"><math>2</math></p>	<p>2. How many solutions does the equation <math> x + 7  = 0</math> have?</p> <p style="text-align: center;"><math>1</math></p>	<p>3. How many solutions does the equation <math> x + 7  = -1</math> have?</p> <p style="text-align: center;"><math>0</math></p>
<p>4. <math> x  = 12</math></p> <p><math>x = 12, -12</math></p>	<p>5. <math> x - 6  = 4</math></p> <p><math> x  = 10</math></p> <p><math>x = 10, -10</math></p>	<p>6. <math> x + 3  = 10</math></p> <p><math>x + 3 = 10</math>      <math>x + 3 = -10</math></p> <p><math>x = 7</math>            <math>x = -13</math></p>
<p>7. <math> x - 2  - 3 = 5</math></p> <p><math> x - 2  = 8</math></p> <p><math>x - 2 = 8</math>      <math>x - 2 = -8</math></p> <p><math>x = 10</math>            <math>x = -6</math></p>	<p>8. <math> x + 7  + 2 = 10</math></p> <p><math> x + 7  = 8</math></p> <p><math>x + 7 = 8</math>      <math>x + 7 = -8</math></p> <p><math>x = 1</math>            <math>x = -15</math></p>	<p>9. <math>4 x - 5  = 20</math></p> <p><math> x - 5  = 5</math></p> <p><math>x - 5 = 5</math>      <math>x - 5 = -5</math></p> <p><math>x = 10</math>            <math>x = 0</math></p>

<p>10. <math> 2x + 1  = 7</math>  <math> 2x  = 6</math>  <math>2x = 6</math>      <math>2x = -6</math>  <math>x = 3</math>      <math>x = -3</math></p>	<p>11. <math>6 =  x + 2  = 6</math>  <math> x + 2  = 0</math>  <math>x + 2 = 0</math>  <math>x = -2</math></p>	<p>12. <math>5 +  x - 1  = 0</math>  <math> x - 1  = -5</math>          No solution</p>
<p>13. <math> x - 1  = 2</math>  <math>x - 1 = 2</math>      <math>x - 1 = -2</math>  <math>x = 3</math>      <math>x = -3</math></p>	<p>14. <math>4 x - 5  = 12</math>  <math> x - 5  = 3</math>  <math>x - 5 = 3</math>      <math>x - 5 = -3</math>  <math>x = 8</math>      <math>x = 2</math></p>	<p>15. <math>3 x - 1  = -15</math>  <math> x - 1  = -5</math>          No solution</p>
<p>16. Leticia sets the thermostat in her apartment to 68 degrees. The actual temperature in her apartment can vary from this by as much as 3.5 degrees.          Write an absolute-value equation that you can use to find the minimum and maximum temperature.  <math> x - 68  = 3.5</math></p> <p>Solve the equation to find the minimum and maximum temperature.  <math>x - 68 = 3.5</math>      <math>x - 68 = -3.5</math>  <math>x = 71.5^\circ</math>      <math>x = 64.5^\circ</math></p>		
<p>17. Troy's car can go 24 miles on one gallon of gas. However, his gas mileage can vary by 2 miles per gallon depending on where he drives.          Write an absolute-value equation that you can use to find the minimum and maximum gas mileage.  <math> x - 24  = 2</math></p> <p>Solve the equation to find the minimum and maximum gas mileage.  <math>x - 24 = 2</math>      <math>x - 24 = -2</math>  <math>x = 26 \text{ mpg}</math>      <math>x = 22 \text{ mpg}</math></p>		
<p>18. A carpenter cuts boards for a construction project. Each board must be 3 meters long, but the length is allowed to differ from this value by at most 0.5 centimeters. Write and solve an absolute-value equation to find the minimum and maximum acceptable lengths for a board.  <math> x - 3  = 0.5</math>  <math>x - 3 = 0.5</math>      <math>x - 3 = -0.5</math>  <math>x = 3.5 \text{ m}</math>      <math>x = 2.5 \text{ m}</math></p>		
<p>19. The owner of a butcher shop keeps the shop's freezer at <math>-5^\circ\text{C}</math>. It is acceptable for the temperature to differ from this value by <math>1.5^\circ</math>. Write and solve an absolute-value equation to find the minimum and maximum acceptable temperatures.  <math> x + 5  = 1.5</math>  <math>x + 5 = 1.5</math>      <math>x + 5 = -1.5</math>  <math>x = -3.5^\circ\text{C}</math>      <math>x = -6.5^\circ\text{C}</math></p>		