Hypothesis-Testing-2-continued

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1. SIGNIFICANCE LEVEL

The significance level is a number α and is the chance of **Type I** error, that is, the probability of rejecting the null hypothesis when it is true.

2. DECISION RULE

A decision rule helps to reject or accept the null hypothesis (H_0).

3. REJECTION REGION

A rejection region is the set of values for which we would reject the null hypothesis H_0 at a certain level of significance.

4. ACCEPTANCE REGION

The acceptance region is the set of values for which we would accept the null hypothesis H_0 at a certain level of significance.

5. CRITICAL VALUE

A critical value is the starting point of a set of values that form the rejection region. It is generally denoted by Z_c or t_c etc.

6. LARGE SAMPLE TEST FOR SINGLE MEAN

Consider μ_0 to be population mean to be tested. There are generally six steps to apply any test.

Step I. Set the null hypothesis: The true population mean is equal to μ_0, thus:

$$H_0 : \mu = \mu_0 \text{, (where } \mu_0 \text{ is any fixed value)}$$

Step II. Set the alternative hypothesis: Under the alternative hypothesis, there are three possibilities. The true population mean, μ, may be:

(i) not equal to;

(ii) less than; or

(iii) more than the guessed value μ_0 as shown below:

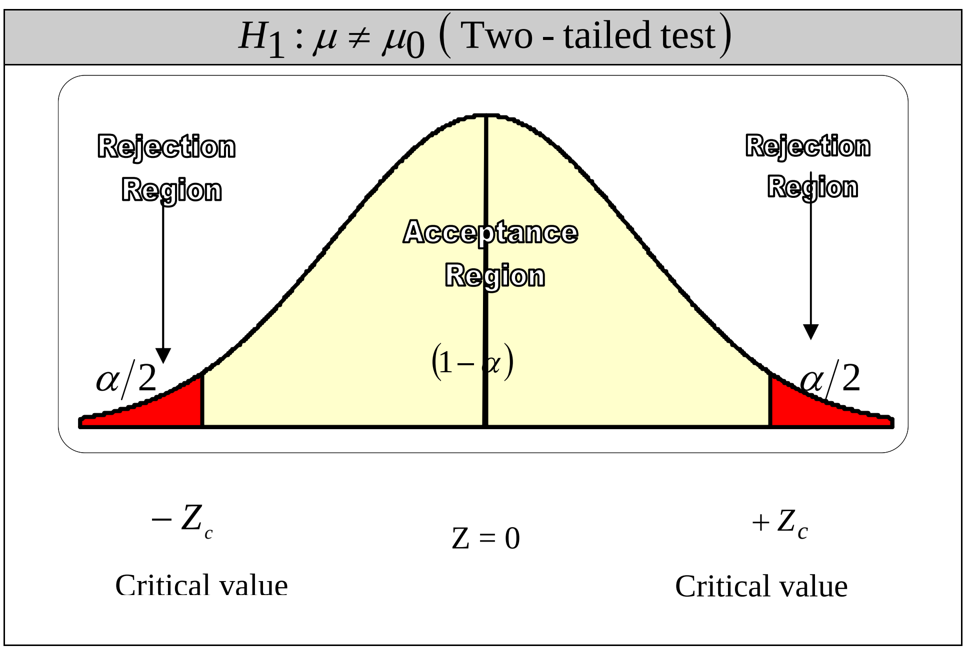

Fig. 1. Two-tailed test has two critical values.

Or

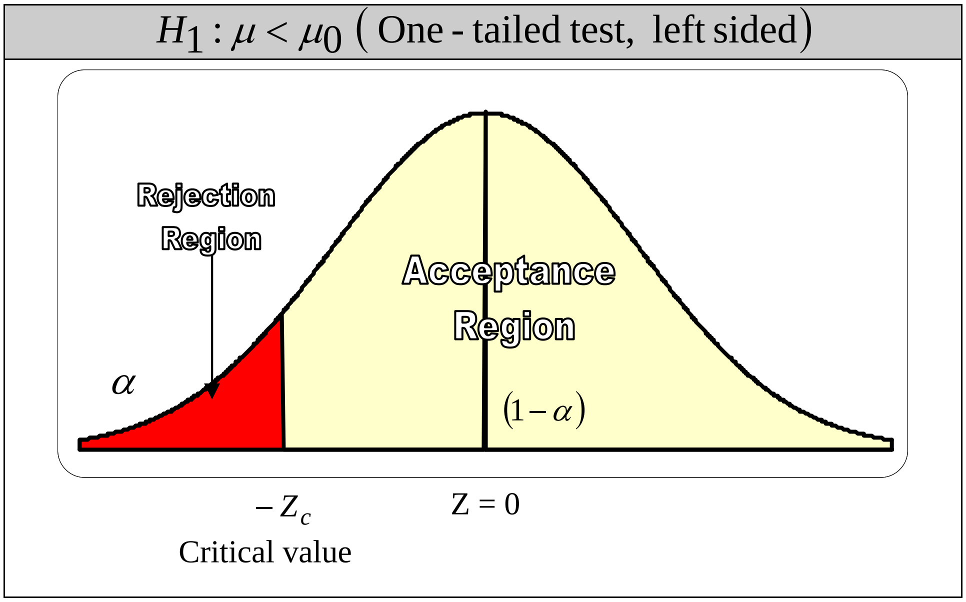

Fig. 2. Left-tailed test has only one critical value.

Or

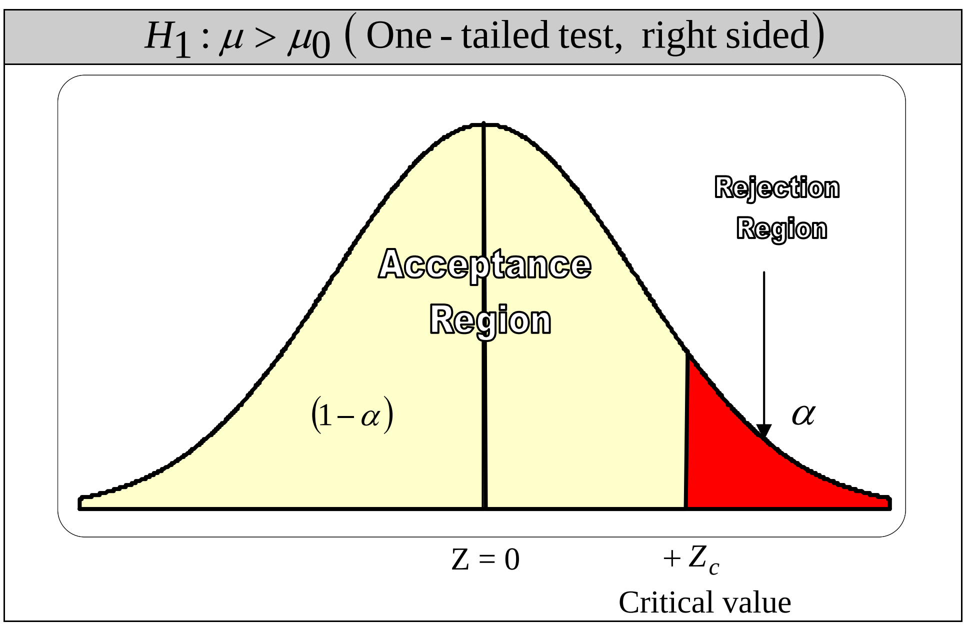

Fig. 3. Right-tailed test has only one critical value.

Note that the choice of alternative hypothesis depends upon the statement under the alternative hypothesis.

Step III. Set the level of significance: On the basis of the desired level of accuracy of the results, decide:

$\alpha = 0.05$, or $\alpha = 0.01$, … etc.

Step IV. Compute the Z-Statistic: Let $n \geq 30$ (large sample), then calculate the Z-statistic:

$$Z_{\text{cal}} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

where n is the sample size, $\bar{x}$ is the sample mean and s is the sample standard deviation.

Step V. Find critical value(s): For a large sample test, the critical value(s) are based on two things: (i) the level of significance α, and (ii) the type of the alternative hypothesis.

For example, (a) if $\alpha = 0.05$ and we are using a two-tailed (or non-directional) test, using **Z-Table**, there are two critical values given by: $Z_c = \pm 1.96$.

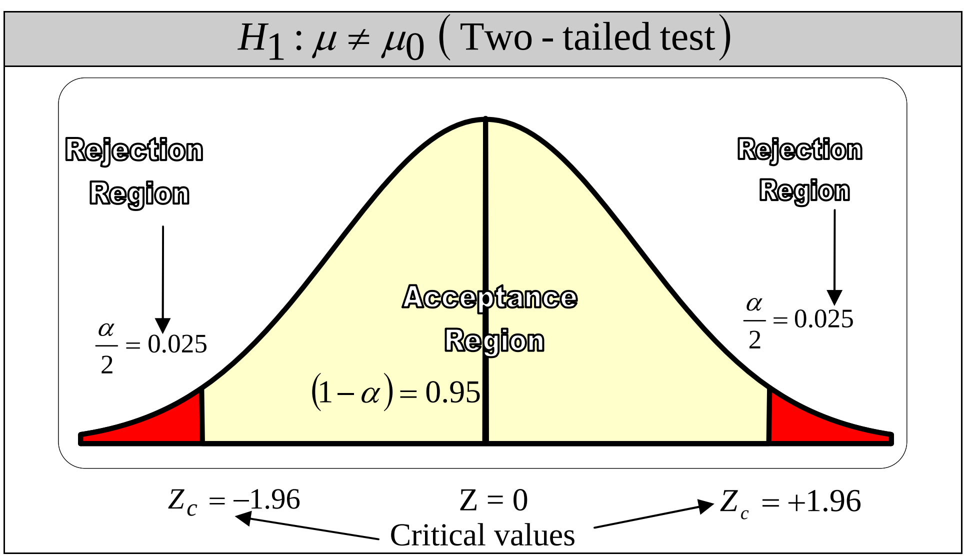

Fig. 4. Two-tailed test (or non-directional test).

(b) if $\alpha = 0.05$ and we are using a one-tailed (or left sided) test, using Z-**Table**, there is only one critical value given by: $Z_C = -1.645$.

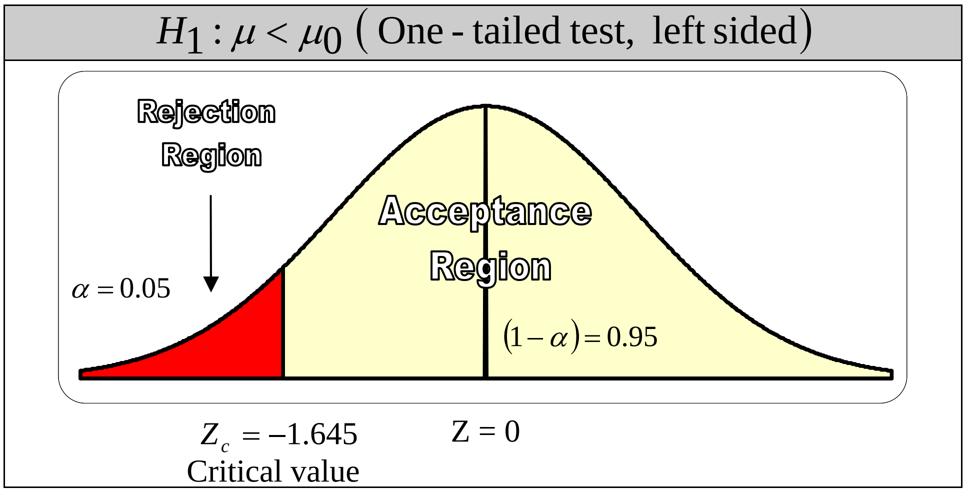

Fig. 5. Left-sided test.

(c) if $\alpha = 0.05$ and we are using a one-tailed (or right sided) test, using **Z-Table**, there is only one critical value given by: $Z_C = +1.645$.

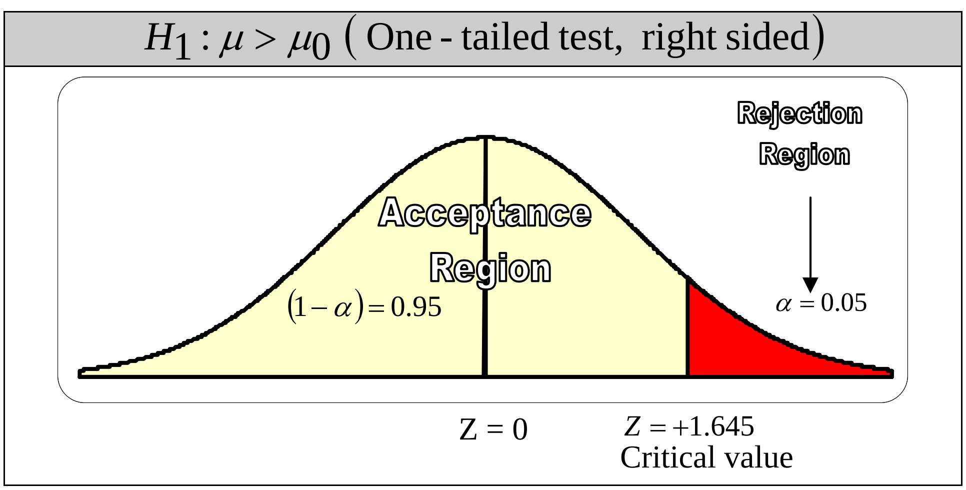

Fig. 6. Right sided test.

Step VI. Decision: If the value of Z_{cal} lies in the rejection region, we are $(1-\alpha)100\%$ sure that we should reject the null hypothesis H_0 at $\alpha\%$ level of significance.

Example 1. (LISTEN TO YOUR PARENTS) Amy went to the market with her mom and asked her to buy a swing.

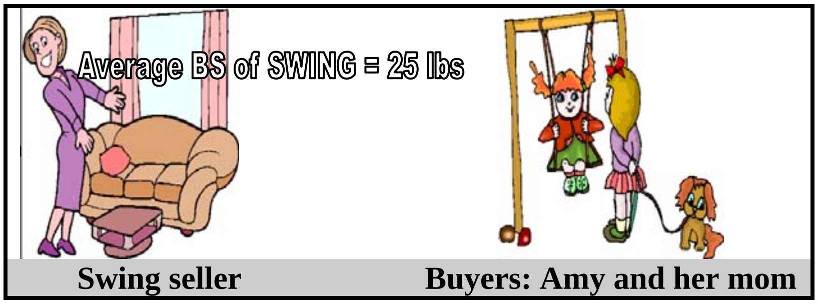

Fig. 7. Swing seller and Amy's mom.

Amy's mom talks to the seller and the seller, having 10 years of experience, says that these swings' ropes have an average breaking strength (BS) of 25 lbs with a standard deviation of 7 lbs and are great for kids. Amy's mom talked to 35 of her friends who bought these swings for their kids and found the average breaking strength to be 22 lbs. Now Amy's mom claims that the average breaking strength of these swings' ropes is less than 25 lbs. Test Amy's mom's claim at a 5% level of significance.

Solution. Step I. Set the null hypothesis:

$$H_0 : \mu = 25$$

Step II. Set the alternative hypothesis:

$H_1 : \mu < 25$ (left-tailed test)

Step III. Level of significance: Given $\alpha = 0.05$.

Step IV. Calculate the Z-statistic: Given $n = 35$, (large sample), $\bar{x} = 22$, and $\sigma = 7$.

Thus the calculated Z-statistic is given by:

$$Z_{\text{cal}} = \frac{(\bar{x} - \mu_0)}{\left(\frac{\sigma}{\sqrt{n}}\right)} = \frac{22 - 25}{\frac{7}{\sqrt{35}}} = -2.54.$$

Step V. Critical value(s): We are given $\alpha = 0.05$ and we are using a one-tailed test (left sided), so the critical value is given by:

$$Z_C = -1.645.$$

Step VI. Decision: The calculated Z-value falls in the rejection (or critical) region as shown below:

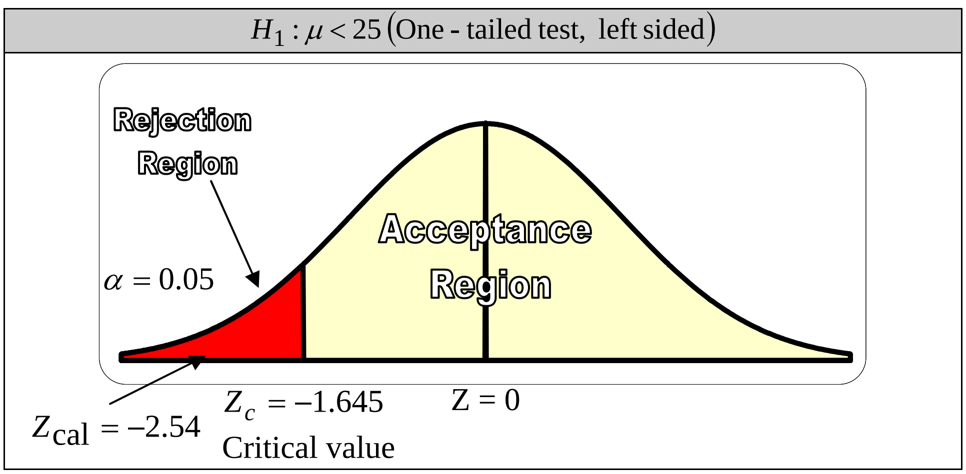

Fig. 8. Left sided test.

Thus, we are 95% sure that we should reject the null hypothesis H_0. In other words, the average breaking strength of the swings is less than 25 lbs at a 5% level of significance and Amy's mom is 95% correct. Just for fun we can say that we are 95% sure that kids should listen to their parents' advise while making any decision. Thus, Amy should look for a better swing instead of arguing with her mom.

p-Value: How much Amy's mom is sure?

The p-value is the minimum level of significance at which the null hypothesis can be rejected.

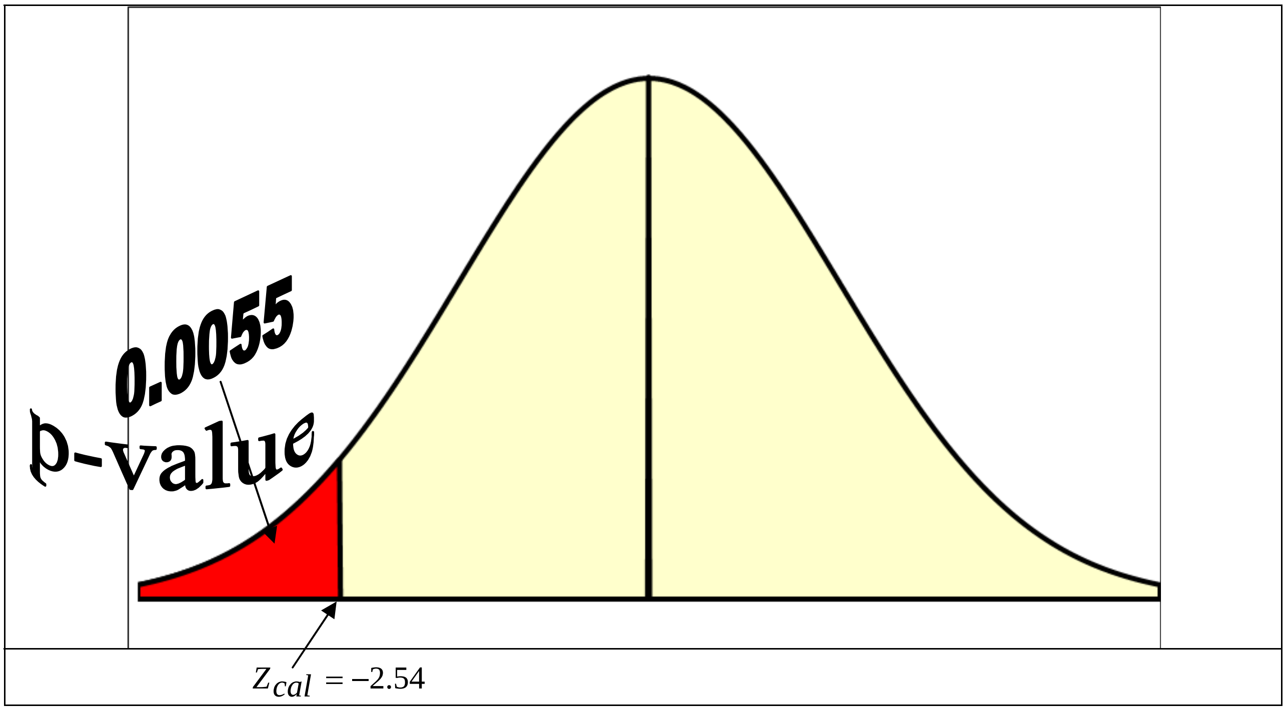

Fig. 9. Learning p-value.

From the Z-table, the area to the left of the $Z_{cal} = -2.54$ is 0.0055 which means the p-value is 0.0055, which is the exact level of significance to reject the null-hypothesis. In other words, Amy's mom is (1-0.0055)x100% = 99.45% sure to reject the null-hypothesis.

7 IDEA OF p-VALUE USING Z-SCORE

The p-value is the minimum level of significance at which the null hypothesis can be rejected.

8 p-VALUE FOR LEFT-TAILED TEST USING Z-SCORE

Consider a left-tailed test being applied to test any statement under the null hypothesis based on the Z-score such as:

H_0 : Donkeys and horses are the same height.
H_1 : Donkeys are shorter than horses (left-tailed test).

Let

$$Z_{\text{cal}} = -2.35$$

Fig. 10. Donkey and Horse.

Then, the p-value is the area to the left of the calculated Z-score as shown:

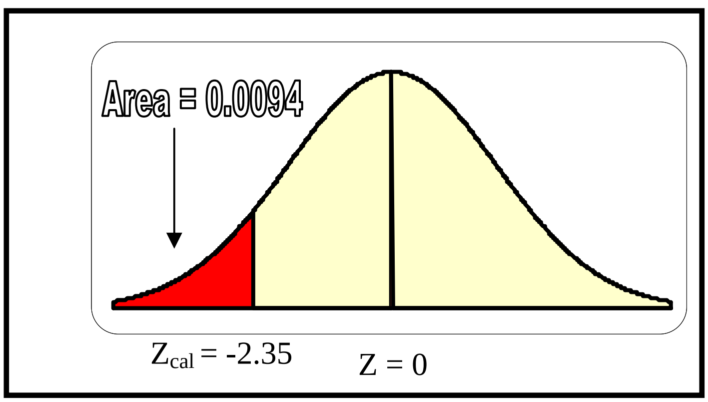

Fig. 11. p-value using a left-tailed test.

Thus, using Z-**Table**, the p-value is given by:

p-value = 0.0094

Here the p-value 0.0094 means we are (1-0.0094)100% = 99.06% sure that we should reject the null hypothesis. In other words, we are 99.06% sure we should accept the statement that donkeys are shorter than horses.

9 p-VALUE FOR RIGHT-TAILED TEST USING Z-SCORE

Assume that we are applying a right-tailed test to test any statement under the null hypothesis based on the Z-score such as:

H_0 : Dogs and cats are of the same height.
H_1 : Dogs are taller than cats (right-tailed test).

Let

$$Z_{\text{cal}} = +1.98$$

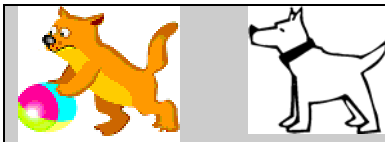

Fig. 12. Cat and Dog.

Then, the p-value is the area to the right of the calculated Z-score as shown:

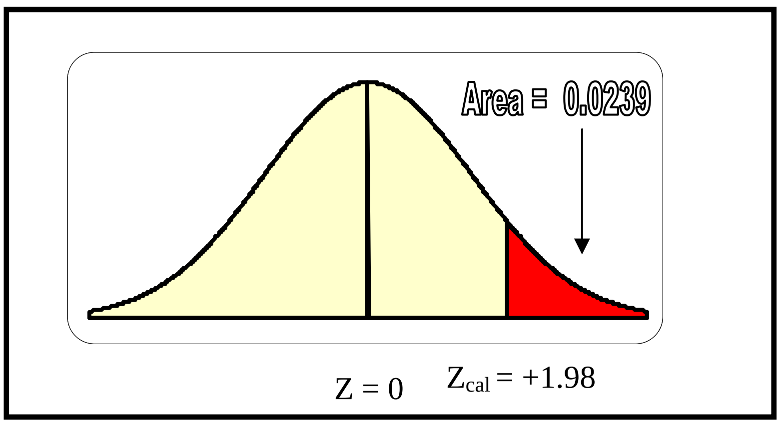

Fig. 13. p-value using a right-tailed test.

Thus, using Z-**Table**, the p-value is given by:

p-value = 0.0239

Here, the p-value 0.0239 means we are (1-0.0239)100% = 97.61% sure that we should reject the null hypothesis. In other words, we are 97.61% sure that we should accept the statement that dogs are taller than cats.

10 p-VALUE FOR TWO-TAILED TEST USING Z-SCORE

Consider a two-tailed test being applied to test any statement under the null hypothesis based on the Z-score such as:

H_0 : Elephants and camels are the same height.
H_1 : Elephants and camels are not the same height (two-tailed test).

Let

$$Z_{cal} = +1.32$$

Fig. 14. Camel and Elephant.

Then, the p-value is the total area to the right of the calculated Z-score and its image on the left side.

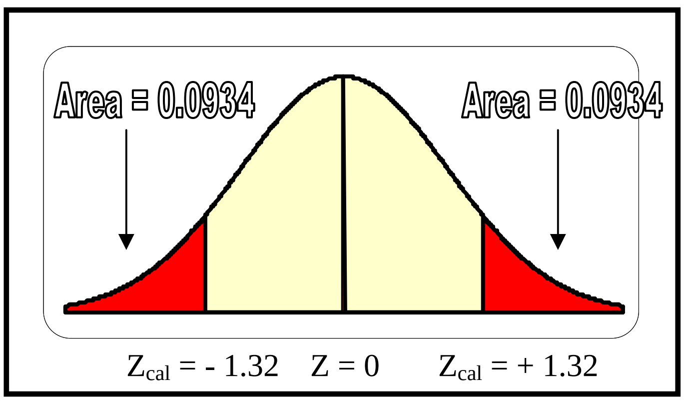

Fig. 15. p-value using a two-tailed test.

Thus, using Z-**Table**, the p-value is given by:

$$\text{p-value} = 0.0934 + 0.0934 = 0.1868$$

Here, the p-value 0.1868 means, we are (1-0.1868)100% = 81.32% sure that we should reject the null hypothesis. In other words, we are 81.32% sure that elephants and camels are not the same height.

Example 2 (THINK BEFORE YOU SPEAK)

In a court, a judge sets a null hypothesis:

H_0 : *Mr. Naïve* is not guilty

against the alternative hypothesis:

H_a : *Mr. Naïve* is guilty

Based on a statement given by *Mr. Naïve*, the judge is 95% sure to reject the null hypothesis. State which one of the following statements is true or false:

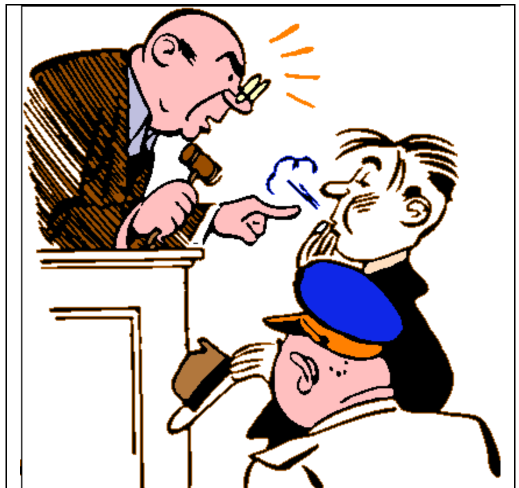

Fig.16. Court case

(a) The p-value is less than or equal to 0.05.

(b) The p-value is more than 0.05.

(c) The judge's decision is significant at 5% level of significance.

(d) The judge's decision is significant at 1% level of significance.

(e) The judge's decision is significant at 10% level of significance.