Ratio and Proportion Handout 13

This handout will explain how to express simple ratios and solve proportion problems. After completion of the worksheet you should be able to:

A. Set up a ratio with like units
B. Set up a ratio with unlike units
C. Determine if two ratios are proportional
D. Solve for the missing number in a proportion
E. Solve word problems

Ratios

- Definition: A ratio is a comparison between two numbers. A ratio statement can be written three ways: $3/2$, 3 to 2, 3:2

  You want to bet on a horse race at the track and the odds are 2 to 1; this is a ratio that can be written 2 to 1, 2:1, $2/1$

  You bake cookies and the recipe calls for 4 parts (cups) flour to 2 parts (cups) sugar. The comparison of flour to sugar is a ratio: 4 to 2, 4:2, $4/2$.

Problem Set I: (answers to problem sets begin on page 10)

Express the following comparisons as ratios

Suppose a class has 14 redheads, 8 brunettes, and 6 blondes

a. What is the ratio of redheads to brunettes?
b. What is the ratio of redheads to blondes?
c. What is the ratio of blondes to brunettes?
d. What is ratio of blondes to total students?

Problem Set II

Express the following as ratios in fraction form and reduce

a. 3 to 12
b. 25 to 5
c. 6 to 30
d. 100 to 10
e. 42 to 4
f. 7 to 30

Finding common units:

Ratios should be written in the same units or measure whenever possible i.e. $^{4\text{ cups}}/_{2\text{ cups}}$ rather than $^{4\text{ quart}}/_{2\text{ cups}}$. This makes comparisons easier and accurate.

Note the problem "3 hours to 60 minutes" These units (hours and minutes) are not alike. You must convert one to the other's unit so that you have minutes to minutes or hours to hours. It is easier to convert the bigger unit (hours) to the smaller unit (minutes). Use dimensional analysis or proportions to make the conversion.

Dimensional Analysis method:

3 hours = _____ minutes

$$\frac{3\,\text{hours}}{1}\times\frac{60\,\text{min}}{1\,\text{hour}}=\frac{180}{1}=180\,\text{minutes}$$

Proportional method:

$$\frac{60\,\text{minutes}}{1\,\text{hour}}=\frac{x\,\text{min}}{3\,\text{hours}}$$ therefore x = 180 minutes

Note: Select the above method you like best and use it for all conversions.

Now you know that 3 hours is the same as 180 minutes so you can substitute 180 minutes in the ratio and have

$$\frac{180\,\text{minutes}}{60\,\text{min}}$$ then reduce to $\frac{3\,\text{minutes}}{1\,\text{min}}$

Summary:

1. State problem	3 hours to 60 minutes as a ratio
2. Analyze	Change one of the unlike units
3. Convert bigger unit to smaller	3 hours to 180 minutes
4. Substitute the converted number into the problem and write as a fraction	180 minutes to 60 minutes $\frac{180}{60}=\frac{18}{6}=\frac{3}{1}$

Example: Compare 2 quarters to 3 pennies. When comparing money, it is frequently easier to convert to pennies. Therefore, 2 quarters equal 50 pennies. Substitute 50 pennies for the 2 quarters. Now write as a ratio 50 to 3: $\frac{50}{3}$.

How about comparing a quarter to a dollar? $\frac{25\text{ pennies}}{100\text{ pennies}}=\frac{1}{4}$

How would you write the ratio "a dollar to a quarter?"
Remember, the first number goes on top: $\frac{100\text{ pennies}}{25\text{ pennies}}=\frac{4}{1}$

Example: Compare 4 yards to 3 feet. First analyze. Change (convert) 4 yards to equivalent in feet. Do either dimensional analysis or proportions to make the conversion.

Dimensional analysis: $\frac{4\text{ yards}}{1}\times\frac{3\text{ feet}}{1\text{ yard}}=\frac{12}{1}=12\text{feet}$

Proportions: $\frac{3\text{ feet}}{1\text{ yard}}=\frac{x\text{ feet}}{4\text{ yards}}$ therefore x = 12 feet

Problems Set III Express each of the following ratios in fractional form then simplify.

1. 5¢ to $2
2. 12 feet to 2 yards
3. 30 minutes to 2 hours
4. 5 days to 1 year
5. 1 dime to 1 quarter

Comparing unlike units (rates)

Sometimes measurable quantities of unlike are compared. These cannot be converted to a common unit because there is no equivalent for them.

Example: 80¢ for 2 lbs. of bananas

¢ or lbs. measure two different quantities, money and weight

$\frac{80¢}{2\,\text{lbs.}} = \frac{40¢}{1\,\text{lb.}}$ (40¢ per pound)

Example: 200 miles on 8 gallons of gas

Ratio = 200 miles: 8 gallons = $\frac{200\,\text{miles}}{8\,\text{gallons}} = \frac{25\,\text{mi.}}{1\,\text{gal.}}$ (25 miles per gallon)

Example: 200 miles: 240 minutes

$\frac{200\,\text{miles}}{240\,\text{minutes}}$ In comparing distance to time, the answer is always given in miles per hour (mph). Therefore, time must be converted to hours.

$\frac{200\,\text{miles}}{4\,\text{hours}} = \frac{50\,\text{mi.}}{1\,\text{hr.}}$ or 50 mph

<u>Problem Set IV</u> Express the following rates in fractional form and reduce to lowest terms.

1. 40¢ : 5 lbs
2. 60 benches for 180 people
3. 100 miles to 120 minutes (in miles per hour)
4. 84 miles on 2 gallons of gas

<u>Proportions</u>

- Definition: A proportion is a mathematical sentence that states that two ratios are equal. It is two ratios joined by an equal sign. The units <u>do not</u> have to be the same.

  Some examples: a) $\frac{2}{3}=\frac{6}{9}$ b) $\frac{5\text{ lbs.}}{\$2.00}=\frac{7\text{ lbs.}}{\$2.80}$

  c) $\frac{36\,3/4\text{ inches}}{2\text{ blouses}}=\frac{55\,1/8\text{ inches}}{3\text{ blouses}}$

  These can be written as 2:3::6:9; 5 lbs.:$2.00::7 lbs.:$2.80; $36\,3/4$ inches : 2 blouses :: $55\,1/8$: 3 blouses

Notice that when you cross-multiply the diagonal numbers, you get the same answers. This means the proportion is true.

Look at the first example a) $\frac{2}{3}=\frac{6}{9}$

Cross multiply: 2 x 9 = 18; 3 x 6 = 18

Look at the second example b) $\frac{5\text{ lbs.}}{\$2.00}=\frac{7\text{ lbs.}}{\$2.80}$

5 x $2.80 = $14.00; 7 x $2.00 = $14.00

Look at the third example c) $\frac{36\,3/4\text{ inches}}{2\text{ blouses}}=\frac{55\,1/8\text{ inches}}{3\text{ blouses}}$

3 x $36\,3/4$ = $441/4$ 2 x $55\,1/8$ = $441/4$

If the products of this cross multiplication are equal, then the ratios are a true proportion. This method can be used to see if you have done your math correctly in the following section.

Solving Proportion Problems

Sometimes you will be given two equivalent ratios but one number will be missing. You must find the number that goes where the x is placed. If your answer is correct then the cross multiplications will be equal.

You can find the missing number by using a formula called "Lonely Mate." This formula consists of two steps:

1) Cross multiply the two diagonal numbers
2) Divide that answer by the remaining number ("lonely mate")

Example: $\frac{3}{8}=\frac{x}{24}$ multiply 3 x 24 = 72

divide 72 by 8

X = 9

Example: $\frac{3}{2}=\frac{9}{x}$ multiply 2 x 9 = 18

divide 18 by 3

X = 6

Example: $\frac{2/5}{6}=\frac{4/5}{x}$ multiply $4/5 \times 6/1 = 24/5$

divide $24/5 \div 2/5$

X = 12

Example: 5:25::x:150 write in fraction form first:

$5/25 = x/150$

cross multiply 5 x 150 = 750

divide by 25

x = 30

Problem Set V Solve for the given variable

1. $\frac{3}{10} = \frac{x}{50}$

2. $\frac{5}{12} = \frac{80}{x}$

3. $\frac{\frac{2}{6}}{x} = \frac{5}{80}$

4. $\frac{1}{3} = \frac{x}{1.8}$

5. $\frac{\frac{1}{4}}{\frac{1}{2}} = \frac{3\frac{1}{2}}{x}$

6. x:81::27:2.43

Word Problems

In real life situations you will use ratios and proportions to solve problems. The hard part will be the set up of the equation.

Example: Sandra wants to give a party for 60 people. She has a punch recipe that makes 2 gallons of punch and serves 15 people. How many gallons of punch should she make for her party?

1) Set up a ratio from the recipe: gallons of punch to the number of people. $\frac{2 \text{ gallons}}{15 \text{ people}}$

2) Set up a ratio of gallons to the people coming to the party. (Place the gallons on top) $\frac{x \text{ gallons}}{60 \text{ people}}$

3) Set up a proportion $\frac{2 \text{ gallons}}{15 \text{ people}} = \frac{x \text{ gallons}}{60 \text{ people}}$

*Like units should be across from each other

4) Solve using the "lonely mate" formula

$\frac{2 \text{ gallons}}{15 \text{ people}} = \frac{x \text{ gallons}}{60 \text{ people}}$

2 X 60 = 120

120 ÷ 15 = 8

This means she should make 8 gallons of punch!

Example: Orlando stuffs envelopes for extra money. He makes a quarter for every dozen he stuffs. How many envelopes will he have to stuff to make $10.00?

1. Set up a ratio between the amount of money he makes and the dozen envelopes he stuffs $\frac{\$.25}{12 \text{ envelopes}}$

2. Set up a ratio between the amount of money he will make and the number of envelopes he will have to stuff $\frac{\$10}{x \text{ envelopes}}$

3. Set up a proportion between the two ratios

$$\frac{\$.25}{12 \text{ envelopes}} = \frac{\$10}{x \text{ envelopes}}$$

4. Solve using "lonely mate"

   a. $10 \times 12 = 120$

   b. $120 \div .25 = 480$

This means that he will have to stuff 480 envelopes!

Problem Set VI

1. A premature infant is gaining 2.5 ounces a day. At that rate, how many days will it take for him to gain 8 ounces?

2. A copy machine can duplicate 2400 copies in one hour. How many copies can it make per minute?

3. A pants factory pays its seamstresses by their production output. If the company paid a seamstress 5¢ for every 4 pockets she sewed on, how many pockets would she have to sew on to receive $60.00?

4. A five-kilogram sack of Beefy-Bones dog food weighs $11\frac{1}{4}$ lbs. How much would a 1-kilogram sack weigh?

5. A patient is receiving 1 liter of IV fluids every 8 hours. At that rate, how much will he receive in 3 hours?

Answers to Problem Sets

Problem Set I

(Note: the first number goes on top of the fraction bar)

a. $\frac{14}{8}$ or 14 to 8 or 14:8

b. $\frac{14}{6}$ or 14 to 6 or 14:6

c. $\frac{6}{8}$ or 6 to 8 or 6:8

d. $\frac{6}{28}$ or 6 to 28 or 6:28

Problem Set II

a. $\frac{3}{12}=\frac{1}{4}$

b. $\frac{25}{5}=\frac{5}{1}$

c. $\frac{6}{30}=\frac{1}{5}$

d. $\frac{100}{10}=\frac{10}{1}$

e. $\frac{42}{4}=\frac{21}{2}$

f. $\frac{7}{30}$

Problem Set III

1. $\frac{5¢}{\$2.00}=\frac{5}{200}=\frac{1}{40}$

2. $\frac{12\text{ ft.}}{2\text{ yds.}}=\frac{12\text{ ft.}}{6\text{ ft.}}=\frac{2}{1}$

3. $\frac{30\text{ min.}}{2\text{ hrs.}}=\frac{30\text{ min.}}{120\text{ min.}}=\frac{1}{4}$

4. $\frac{5\text{ days}}{1\text{ year}}=\frac{5\text{ days}}{365\text{ days}}=\frac{1}{73}$

5. $\frac{1\text{ dime}}{1\text{ quarter}}=\frac{10}{25}=\frac{2}{5}$

Problem Set IV

1. 40¢ : 5 lbs = $\frac{40¢}{5\text{ lbs.}}=\frac{8¢}{1\text{ lb.}}$
2. 60 benches for 180 people = $\frac{60}{180}=\frac{1\text{ bench}}{3\text{ people}}$
3. 100 miles : 120 minutes = $\frac{100\text{ miles}}{2\text{ hours}}=\frac{50\text{ miles}}{1\text{ hour}}$
4. 84 miles on 2 gallons = $\frac{84\text{ miles}}{2\text{ gallons}}=\frac{42\text{ miles}}{1\text{ gallon}}$

Problem Set V

1. x = 15
2. x = 192
3. x = $5\frac{1}{3}$ or 5.33
4. x = .6
5. x = 7
6. x = 900

Problem Set VI

1. x = 3.2 days
2. x = 40 copies
3. x = 4800 pockets
4. x = 2.25 lbs.
5. x = .375 L