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Chapter 8 Resource Masters

Algebra 1

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CONSUMABLE WORKBOOKS Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks in both English and Spanish.

	MHID	**ISBN**
Study Guide and Intervention Workbook	0-07-660292-3	978-0-07-660292-6
Homework Practice Workbook	0-07-660291-5	978-0-07-660291-9
Spanish Version		
Homework Practice Workbook	0-07-660294-X	978-0-07-660294-0

Answers For Workbooks The answers for Chapter 8 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

ConnectED All of the materials found in this booklet are included for viewing, printing, and editing at connected.mcgraw-hill.com.

Spanish Assessment Masters (MHID: 0-07-660289-3, ISBN: 978-0-07-660289-6) These masters contain a Spanish version of Chapter 8 Test Form 2A and Form 2C.

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Contents

Teacher's Guide to Using the *Chapter 8 Resource Masters*

The *Chapter 8 Resource Masters* includes the core materials needed for Chapter 8. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing, printing, and editing at connectED.mcgraw-hill.com.

Chapter Resources

Student-Built Glossary (pages 1–2) These masters are a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar. Give this to students before beginning Lesson 8-1. Encourage them to add these pages to their mathematics study notebooks. Remind them to complete the appropriate words as they study each lesson.

Anticipation Guide (pages 3–4) This master, presented in both English and Spanish, is a survey used before beginning the chapter to pinpoint what students may or may not know about the concepts in the chapter. Students will revisit this survey after they complete the chapter to see if their perceptions have changed.

Lesson Resources

Study Guide and Intervention These masters provide vocabulary, key concepts, additional worked-out examples and Check Your Progress exercises to use as a reteaching activity. It can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice This master focuses more on the computational nature of the lesson. Use as an additional practice option or as homework for second-day teaching of the lesson.

Practice This master closely follows the types of problems found in the Exercises section of the Student Edition and includes word problems. Use as an additional practice option or as homework for second-day teaching of the lesson.

Word Problem Practice This master includes additional practice in solving word problems that apply the concepts of the lesson. Use as an additional practice or as homework for second-day teaching of the lesson.

Enrichment These activities may extend the concepts of the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. They are written for use with all levels of students.

Graphing Calculator, TI-Nspire or Spreadsheet Activities
These activities present ways in which technology can be used with the concepts in some lessons of this chapter. Use as an alternative approach to some concepts or as an integral part of your lesson presentation.

Assessment Options

The assessment masters in the ***Chapter 8 Resource Masters*** offer a wide range of assessment tools for formative (monitoring) assessment and summative (final) assessment.

Student Recording Sheet This master corresponds with the standardized test practice at the end of the chapter.

Extended Response This master provides information for teachers and students on how to assess performance on open-ended questions.

Quizzes Four free-response quizzes offer assessment at appropriate intervals in the chapter.

Mid-Chapter Test This 1-page test provides an option to assess the first half of the chapter. It parallels the timing of the Mid-Chapter Quiz in the Student Edition and includes both multiple-choice and free-response questions.

Vocabulary Test This test is suitable for all students. It includes a list of vocabulary words and 9 questions to assess students' knowledge of those words. This can also be used in conjunction with one of the leveled chapter tests.

Leveled Chapter Tests

- ***Form 1*** contains multiple-choice questions and is intended for use with below grade level students.
- ***Forms 2A and 2B*** contain multiple-choice questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- ***Forms 2C and 2D*** contain free-response questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- ***Form 3*** is a free-response test for use with above grade level students.

All of the above mentioned tests include a free-response Bonus question.

Extended-Response Test Performance assessment tasks are suitable for all students. Sample answers and a scoring rubric are included for evaluation.

Standardized Test Practice These three pages are cumulative in nature. It includes three parts: multiple-choice questions with bubble-in answer format, griddable questions with answer grids, and short-answer free-response questions.

Answers

- The answers for the Anticipation Guide and Lesson Resources are provided as reduced pages.
- Full-size answer keys are provided for the assessment masters.

NAME ______________________ DATE ____________ PERIOD ____________

8 Student-Built Glossary

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 8. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
difference of two squares		
factoring		
factoring by grouping		
FOIL method		
perfect square trinomial try•NOH•mee•uhl		

(continued on the next page)

NAME ______________________________ DATE ____________ PERIOD ____________

8 Student-Built Glossary *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
prime polynomial		
quadratic equation		
quadratic expression		
Zero Product Property		

NAME ______________________________ DATE ____________ PERIOD ____________

8 Anticipation Guide

Quadratic Expressions and Equations

Step 1 *Before you begin Chapter 8*

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. When multiplying two powers that have the same base, multiply the exponents.	
	2. $(k^3)^4$ is equivalent to k^{12}.	
	3. To divide two powers that have the same base, subtract the exponents.	
	4. $\left(\frac{2}{5}\right)^3$ is the same as $\frac{2^3}{5}$.	
	5. A polynomial may contain one or more monomials.	
	6. The degree of the polynomial $3x^2y^3 - 5y^2 + 8x^3$ is 3 because the greatest exponent is 3.	
	7. The greatest common factor (GCF) of two or more monomials is the product of their unique factors when each monomial is written in factored form.	
	8. Any two numbers that have a greatest common factor of 1 are said to be relatively prime.	
	9. If the product of any two factors is 0, then at least one of the factors must equal 0.	
	10. A quadratic trinomial has a degree of 4.	
	11. To solve an equation such as $x^2 = 8 + 2x$, take the square root of both sides.	
	12. The polynomial $3r^2 - r - 2$ can not be factored because the coefficient of r^2 is not 1.	
	13. The polynomial $t^2 + 16$ is not factorable.	
	14. The numbers 16, 64, and 121 are perfect squares.	

Step 2 *After you complete Chapter 8*

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

NOMBRE ______________________ FECHA __________ PERÍODO __________

8 Ejercicios preparatorios

Expresiones y Ecuaciones Cuadráticos

Paso 1 *Antes de comenzar el Capítulo 8*

- Lee cada enunciado.
- Decide si estás de acuerdo (A) o en desacuerdo (D) con el enunciado.
- Escribe A o D en la primera columna O si no estás seguro(a) de la respuesta, escribe NS (No estoy seguro(a)).

Paso 1 A, D o NS	Enunciado	Paso 2 A o D
	1. Al multiplicar dos potencias con la misma base, multiplica los exponentes.	
	2. $(k^3)^4$ es equivalente a k^{12}.	
	3. Para dividir dos potencias que tienen la misma base, resta los exponentes.	
	4. $\left(\frac{2}{5}\right)^3$ es lo mismo que $\frac{2^3}{5}$.	
	5. Un polinomio puede contener uno o más monomios.	
	6. El grado del polinomio $3x^2y^3 - 5y^2 + 8x^3$ es 3 porque el exponente más grande es 3.	
	7. El máximo común divisor (MCD) de dos o más monomios es el producto de sus únicos factores cuando cada monomio se escribe en forma factorizada.	
	8. Se dice que dos números cualesquiera que tengan un máximo común divisor de 1 son relativamente primos.	
	9. Si el producto de dos factores cualesquiera es 0, entonces por lo menos uno de los factores debe ser igual a 0.	
	10. Un trinomio cuadrático tiene un grado de 4.	
	11. Para resolver una ecuación como $x^2 = 8 + 2x$, extrae la raíz cuadrada de ambos lados.	
	12. El polinomio $3s^2 - s - 2$ no se puede factorizar porque el coeficiente de s^2 no es 1.	
	13. El polinomio $t^2 + 16$ no es factorizable.	
	14. Los números 16, 64, y 121 son cuadrados perfectos.	

Paso 2 *Después de completar el Capítulo 8*

- Vuelve a leer cada enunciado y completa la última columna con una A o una D.
- ¿Cambió cualquiera de tus opiniones sobre los enunciados de la primera columna?
- En una hoja de papel aparte, escribe un ejemplo de por qué estás en desacuerdo con los enunciados que marcaste con una D.

NAME ______________________ DATE ____________ PERIOD ____________

8-1 Study Guide and Intervention

Adding and Subtracting Polynomials

Polynomials in Standard Form A **polynomial** is a monomial or a sum of monomials. A **binomial** is the sum of two monomials, and a **trinomial** is the sum of three monomials. Polynomials with more than three terms have no special name. The **degree** of a monomial is the sum of the exponents of all its variables. The **degree of the polynomial** is the same as the degree of the monomial term with the highest degree.

The terms of a polynomial are usually arranged so that the terms are in order from greatest degree to least degree. This is called the **standard form of a polynomial.**

Example **Determine whether each expression is a polynomial. If so, identify the polynomial as a *monomial*, *binomial*, or *trinomial*. Then find the degree of the polynomial.**

Expression	Polynomial?	Monomial, Binomial, or Trinomial?	Degree of the Polynomial
$3x - 7xyz$	Yes. $3x - 7xyz = 3x + (-7xyz)$, which is the sum of two monomials	binomial	3
-25	Yes. -25 is a real number.	monomial	0
$7n^3 + 3n^{-4}$	No. $3n^{-4} = \frac{3}{n^4}$, which is not a monomial	none of these	—
$9x^3 + 4x + x + 4 + 2x$	Yes. The expression simplifies to $9x^3 + 7x + 4$, which is the sum of three monomials	trinomial	3

Exercises

Determine whether each expression is a polynomial. If it is a polynomial, find the degree and determine whether it is a *monomial*, *binomial*, or *trinomial*.

1. 36

2. $\frac{3}{q^2} + 5$

3. $7x - x + 5$

4. $8g^2h - 7gh + 2$

5. $\frac{1}{4y^2} + 5y - 8$

6. $6x + x^2$

Write each polynomial in standard form. Identify the leading coefficient.

7. $x^3 + x^5 - x^2$

8. $x^4 + 4x^3 - 7x^5 + 1$

9. $-3x^6 - x^5 + 2x^8$

10. $2x^7 - x^8$

11. $3x + 5x^4 - 2 - x^2$

12. $-2x^4 + x - 4x^5 + 3$

8-1 Study Guide and Intervention *(continued)*

Adding and Subtracting Polynomials

Add and Subtract Polynomials To add polynomials, you can group like terms horizontally or write them in column form, aligning like terms vertically. **Like terms** are monomial terms that are either identical or differ only in their coefficients, such as $3p$ and $-5p$ or $2x^2y$ and $8x^2y$.

You can subtract a polynomial by adding its additive inverse. To find the additive inverse of a polynomial, replace each term with by adding its additive inverse. To find the additive inverse of a polynomial, replace each term with its additive inverse or opposite.

Example **Find $(3x^2 + 2x - 6) - (2x + x^2 + 3)$.**

Horizontal Method

Use additive inverses to rewrite as addition. Then group like terms.

$$\begin{aligned}&(3x^2 + 2x - 6) - (2x + x^2 + 3)\\ &= (3x^2 + 2x - 6) + [(-2x) + (-x^2) + (-3)]\\ &= [3x^2 + (-x^2)] + [2x + (-2x)] + [-6 + (-3)]\\ &= 2x^2 + (-9)\\ &= 2x^2 - 9\end{aligned}$$

The difference is $2x^2 - 9$.

Vertical Method

Align like terms in columns and subtract by adding the additive inverse.

$$\begin{array}{rrrr} & 3x^2 & +\ 2x & -\ 6 \\ \hline (-) & x^2 & +\ 2x & +\ 3 \\ & 3x^2 & +\ 2x & -\ 6 \\ \hline (+) & -x^2 & -\ 2x & -\ 3 \\ & 2x^2 & & -\ 9 \end{array}$$

The difference is $2x^2 - 9$.

Exercises

Find each sum or difference.

1. $(4a - 5) + (3a + 6)$

2. $(6x + 9) + (4x^2 - 7)$

3. $(6xy + 2y + 6x) + (4xy - x)$

4. $(x^2 + y^2) + (-x^2 + y^2)$

5. $(3p^2 - 2p + 3) + (p^2 - 7p + 7)$

6. $(2x^2 + 5xy + 4y^2) + (-xy - 6x^2 + 2y^2)$

7. $(8p - 5r) - (-6p^2 + 6r - 3)$

8. $(8x^2 - 4x - 3) - (-2x - x^2 + 5)$

9. $(3x^2 - 2x) - (3x^2 + 5x - 1)$

10. $(4x^2 + 6xy + 2y^2) - (-x^2 + 2xy - 5y^2)$

11. $(2h - 6j - 2k) - (-7h - 5j - 4k)$

12. $(9xy^2 + 5xy) - (-2xy - 8xy^2)$

NAME ______________________________ DATE ____________ PERIOD ____________

8-1 Skills Practice

Adding and Subtracting Polynomials

Find each sum or difference.

1. $(2x + 3y) + (4x + 9y)$

2. $(6s + 5t) + (4t + 8s)$

3. $(5a + 9b) - (2a + 4b)$

4. $(11m - 7n) - (2m + 6n)$

5. $(m^2 - m) + (2m + m^2)$

6. $(x^2 - 3x) - (2x^2 + 5x)$

7. $(d^2 - d + 5) - (2d + 5)$

8. $(2h^2 - 5h) + (7h - 3h^2)$

9. $(5f + g - 2) + (-2f + 3)$

10. $(6k^2 + 2k + 9) + (4k^2 - 5k)$

Determine whether each expression is a polynomial. If it is a polynomial, find the degree and determine whether it is a *monomial, binomial,* or *trinomial.*

11. $5mt + t^2$

12. $4by + 2b - by$

13. -32

14. $\frac{3x}{7}$

15. $5x^2 - 3x^{-4}$

16. $2c^2 + 8c + 9 - 3$

Write each polynomial in standard form. Identify the leading coefficient.

17. $3x + 1 + 2x^2$

18. $5x - 6 + 3x^2$

19. $9x^2 + 2 + x^3 + x$

20. $-3 + 3x^3 - x^2 + 4x$

21. $x^2 + 3x^3 + 27 - x$

22. $25 - x^3 + x$

23. $x - 3x^2 + 4 + 5x^3$

24. $x^2 + 64 - x + 7x^3$

NAME ______________________ DATE ____________ PERIOD ____________

8-1 Practice

Adding and Subtracting Polynomials

Find each sum or difference.

1. $(4y + 5) + (-7y - 1)$

2. $(-x^2 + 3x) - (5x + 2x^2)$

3. $(4k^2 + 8k + 2) - (2k + 3)$

4. $(2m^2 + 6m) + (m^2 - 5m + 7)$

5. $(5a^2 + 6a + 2) - (7a^2 - 7a + 5)$

6. $(-4p^2 - p + 9) + (p^2 + 3p - 1)$

7. $(x^3 - 3x + 1) - (x^3 + 7 - 12x)$

8. $(6x^2 - x + 1) - (-4 + 2x^2 + 8x)$

9. $(4y^2 + 2y - 8) - (7y^2 + 4 - y)$

10. $(w^2 - 4w - 1) + (-5 + 5w^2 - 3w)$

Determine whether each expression is a polynomial. If it is a polynomial, find the degree and determine whether it is a *monomial, binomial,* or *trinomial.*

11. $7a^2b + 3b^2 - a^2b$

12. $\frac{1}{5}y^3 + y^2 - 9$

13. $6g^2h^3k$

14. $\frac{x + 3x^4 - 21x^2}{x^3}$

Write each polynomial in standard form. Identify the leading coefficient.

15. $8x^2 - 15 + 5x^5$

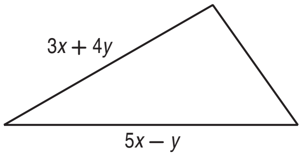
16. $10x - 7 + x^4 + 4x^3$

17. $13x^2 - 5 + 6x^3 - x$

18. $4x + 2x^5 - 6x^3 + 2$

19. BUSINESS The polynomial $s^3 - 70s^2 + 1500s - 10{,}800$ models the profit a company makes on selling an item at a price s. A second item sold at the same price brings in a profit of $s^3 - 30s^2 + 450s - 5000$. Write a polynomial that expresses the total profit from the sale of both items.

20. GEOMETRY The measures of two sides of a triangle are given. If P is the perimeter, and $P = 10x + 5y$, find the measure of the third side.



8-1 Word Problem Practice

Adding and Subtracting Polynomials

1. **PRIMES** Mei is trying to list as many prime numbers as she can for a challenge problem for her math class. She finds that the polynomial expression $n^2 - n + 41$ can be used to generate some, but not all, prime numbers. What is the degree of Mei's polynomial?

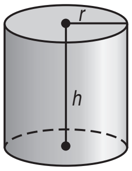
2. **PHONE CALLS** A long-distance telephone company charges a standard monthly service fee of $19.95 plus $0.05 per minute of long-distance use. Write a polynomial to express the monthly cost of the phone plan if x minutes of long-distance time are used per month. What is the degree of the polynomial?

3. **FIREWORKS** Two bottle rockets are launched straight up into the air. The height, in feet, of each rocket at t seconds after launch is given by the polynomial equations below. Write an equation to show how much higher Rocket A traveled.
Rocket A: $D_1 = -16t^2 + 122t$
Rocket B: $D_2 = -16t^2 + 84t$

4. **ENVELOPES** An office supply company produces yellow document envelopes. The envelopes come in a variety of sizes, but the length is always 4 centimeters more than double the width. Write a polynomial expression to give the perimeter of any of the envelopes.

5. **INDUSTRY** Two identical right cylindrical steel drums containing oil need to be covered with a fire-resistant sealant. In order to determine how much sealant to purchase, George must find the surface area of the two drums. The surface area including the top and bottom bases is given by the following formula.

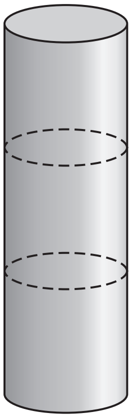
$$S = 2\pi rh + 2\pi r^2$$



   a. Write a polynomial to represent the total surface area of the two drums.

   b. Find the total surface area if the height of each drum is 2 meters and the radius of each is 0.5 meter. Let $\pi = 3.14$.

   c. The fire resistant sealant must be applied while they are stacked vertically in groups of three. If h is the height of each drum and r is the radius, write a polynomial to represent the exposed surface area.

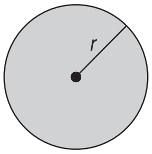


8-1 Enrichment

Circular Areas and Volumes

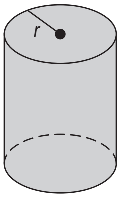
Area of Circle

$A = \pi r^2$



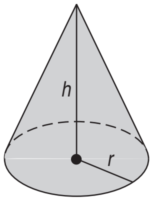
Volume of Cylinder

$V = \pi r^2 h$



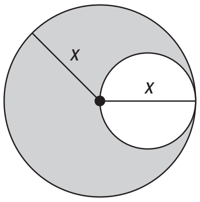
Volume of Cone

$V = \frac{1}{3}\pi r^2 h$

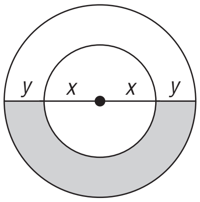


Write an algebraic expression for each shaded area. (Recall that the diameter of a circle is twice its radius.)

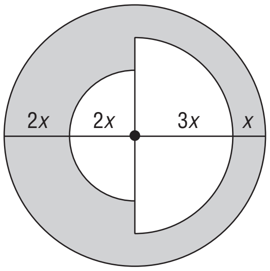
1.



2.

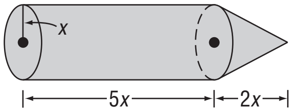


3.

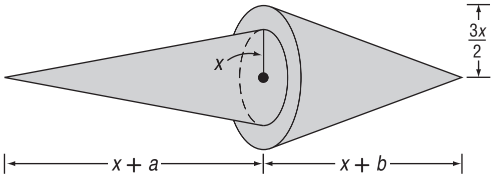


Write an algebraic expression for the total volume of each figure.

4.

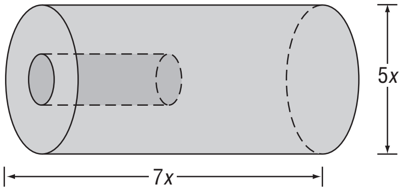


5.

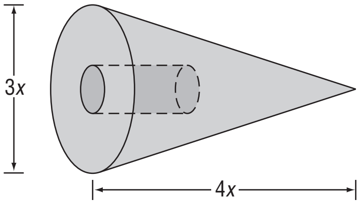


Each figure has a cylindrical hole with a radius of 2 inches and a height of 5 inches. Find each volume.

6.



7.



NAME ______________________________ DATE ____________ PERIOD ____________

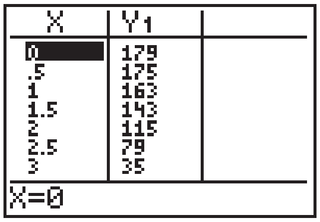
8-1 Graphing Calculator Activity

Second Degree Polynomial Functions

Many real world problems can be modeled using polynomial functions. The **TABLE** function can be used to evaluate a function for multiple values.

Example **An object is dropped from the top of a 179-foot cliff to the water below. The height of the object above the water can be modeled by $h(t) = -16t^2 + 179$ where t is time in seconds.**

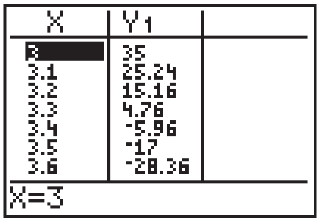
a. Determine the height of the object after 0.5 second, 1 second, 1.5 seconds, and 2 seconds.



```
  X    | Y1
 0     | 179
 .5    | 175
 1     | 163
 1.5   | 143
 2     | 115
 2.5   | 79
 3     | 35
X=0
```

Enter the function into **Y1**. Use 2nd **[TBLSET]** to set up the calculator to display values of t in 0.5 second intervals. Display the table and record the results.

Examine the table. When $x = 0.5$, $y = 175$. This means that $h(0.5) = 175$, or that after 0.5 second, the object is 175 feet above the water. Thus, $h(1) = 163$ feet, $h(1.5) = 143$ feet, and $h(2) = 115$ feet.

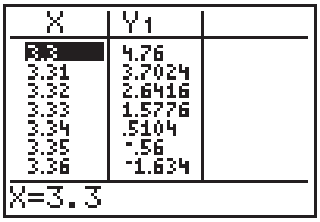


```
  X    | Y1
 3     | 35
 3.1   | 25.24
 3.2   | 15.16
 3.3   | 4.76
 3.4   | -5.96
 3.5   | -17
 3.6   | -28.36
X=3
```

b. After how many seconds does the object hit the water? Round to the nearest hundredth.

Scroll through the table. Notice that the y-values change from positive to negative between $x = 3$ and $x = 3.5$. Examine this interval more closely by resetting the table using **TblStart=** 3 and **ΔTbl=** 0.1. Look for the change in sign.

Further examine the interval from $x = 3.3$ to $x = 3.4$ using **TblStart=** 3.3 and **ΔTbl=** 0.01.



```
  X    | Y1
 3.3   | 4.76
 3.31  | 3.7024
 3.32  | 2.6416
 3.33  | 1.5776
 3.34  | .5104
 3.35  | -.56
 3.36  | -1.634
X=3.3
```

The y-value closest to zero occurs when $x = 3.34$. Thus, the object hits the water after about 3.34 seconds.

Exercises

1. An object is dropped from the top of a building that is 412 feet high. The distance, in feet, above the ground at x seconds is given by $P(x) = -16x^2 + 412$.

a. After how many seconds will the object be 100 feet above the ground?

b. How many seconds will it take the object to reach the ground?

2. A bungee jumper free falls from the Royal Gorge suspension bridge over the Arkansas River, 1053 feet above the river. The height h of the bungee jumper above the river, in feet, after t seconds can be represented by $h = -16t^2 + 1053$. Two seconds after the first bungee jumper falls, another person jumps down with an initial velocity of 80 feet per second. The position of the second jumper can be represented by the equation $h = -16(t - 2)^2 - 80(t - 2) + 1053$.

a. If the bungee cords are designed to stretch just enough so that the jumpers touch the water before springing back up, which jumper will touch the water first? How long does it take each jumper to touch the water?

b. Does the second jumper catch up to the first jumper? If so, how far are they above the river at this point and how long does it take each jumper to reach this point?

NAME ______________________________ DATE __________ PERIOD __________

8-2 Study Guide and Intervention

Multiplying a Polynomial by a Monomial

Polynomial Multiplied by Monomial The Distributive Property can be used to multiply a polynomial by a monomial. You can multiply horizontally or vertically. Sometimes multiplying results in like terms. The products can be simplified by combining like terms.

Example 1 **Find $-3x^2(4x^2 + 6x - 8)$.**

Horizontal Method

$-3x^2(4x^2 + 6x - 8)$

$= -3x^2(4x^2) + (-3x^2)(6x) - (-3x^2)(8)$

$= -12x^4 + (-18x^3) - (-24x^2)$

$= -12x^4 - 18x^3 + 24x^2$

Vertical Method

$$\begin{array}{lr} & 4x^2 + 6x - 8 \\ (\times) & -3x^2 \\ \hline & -12x^4 - 18x^3 + 24x^2 \end{array}$$

The product is $-12x^4 - 18x^3 + 24x^2$.

Example 2 **Simplify $-2(4x^2 + 5x) - x(x^2 + 6x)$.**

$-2(4x^2 + 5x) - x(x^2 + 6x)$

$= -2(4x^2) + (-2)(5x) + (-x)(x^2) + (-x)(6x)$

$= -8x^2 + (-10x) + (-x^3) + (-6x^2)$

$= (-x^3) + [-8x^2 + (-6x^2)] + (-10x)$

$= -x^3 - 14x^2 - 10x$

Exercises

Find each product.

1. $x(5x + x^2)$

2. $x(4x^2 + 3x + 2)$

3. $-2xy(2y + 4x^2)$

4. $-2g(g^2 - 2g + 2)$

5. $3x(x^4 + x^3 + x^2)$

6. $-4x(2x^3 - 2x + 3)$

7. $-4ax(10 + 3x)$

8. $3y(-4x - 6x^3 - 2y)$

9. $2x^2y^2(3xy + 2y + 5x)$

Simplify each expression.

10. $x(3x - 4) - 5x$

11. $-x(2x^2 - 4x) - 6x^2$

12. $6a(2a - b) + 2a(-4a + 5b)$

13. $4r(2r^2 - 3r + 5) + 6r(4r^2 + 2r + 8)$

14. $4n(3n^2 + n - 4) - n(3 - n)$

15. $2b(b^2 + 4b + 8) - 3b(3b^2 + 9b - 18)$

16. $-2z(4z^2 - 3z + 1) - z(3z^2 + 2z - 1)$

17. $2(4x^2 - 2x) - 3(-6x^2 + 4) + 2x(x - 1)$

NAME ______________________________ DATE ____________ PERIOD ____________

8-2 Study Guide and Intervention *(continued)*

Multiplying a Polynomial by a Monomial

Solve Equations with Polynomial Expressions Many equations contain polynomials that must be added, subtracted, or multiplied before the equation can be solved.

Example **Solve $4(n - 2) + 5n = 6(3 - n) + 19$.**

$4(n - 2) + 5n = 6(3 - n) + 19$	Original equation
$4n - 8 + 5n = 18 - 6n + 19$	Distributive Property
$9n - 8 = 37 - 6n$	Combine like terms.
$15n - 8 = 37$	Add 6*n* to both sides.
$15n = 45$	Add 8 to both sides.
$n = 3$	Divide each side by 15.

The solution is 3.

Exercises

Solve each equation.

1. $2(a - 3) = 3(-2a + 6)$

2. $3(x + 5) - 6 = 18$

3. $3x(x - 5) - 3x^2 = -30$

4. $6(x^2 + 2x) = 2(3x^2 + 12)$

5. $4(2p + 1) - 12p = 2(8p + 12)$

6. $2(6x + 4) + 2 = 4(x - 4)$

7. $-2(4y - 3) - 8y + 6 = 4(y - 2)$

8. $x(x + 2) - x(x - 6) = 10x - 12$

9. $3(x^2 - 2x) = 3x^2 + 5x - 11$

10. $2(4x + 3) + 2 = -4(x + 1)$

11. $3(2h - 6) - (2h + 1) = 9$

12. $3(y + 5) - (4y - 8) = -2y + 10$

13. $3(2a - 6) - (-3a - 1) = 4a - 2$

14. $5(2x^2 - 1) - (10x^2 - 6) = -(x + 2)$

15. $3(x + 2) + 2(x + 1) = -5(x - 3)$

16. $4(3p^2 + 2p) - 12p^2 = 2(8p + 6)$

NAME ______________________ DATE ____________ PERIOD ____________

8-2 Skills Practice

Multiplying a Polynomial by a Monomial

Find each product.

1. $a(4a + 3)$

2. $-c(11c + 4)$

3. $x(2x - 5)$

4. $2y(y - 4)$

5. $-3n(n^2 + 2n)$

6. $4h(3h - 5)$

7. $3x(5x^2 - x + 4)$

8. $7c(5 - 2c^2 + c^3)$

9. $-4b(1 - 9b - 2b^2)$

10. $6y(-5 - y + 4y^2)$

11. $2m^2(2m^2 + 3m - 5)$

12. $-3n^2(-2n^2 + 3n + 4)$

Simplify each expression.

13. $w(3w + 2) + 5w$

14. $f(5f - 3) - 2f$

15. $-p(2p - 8) - 5p$

16. $y^2(-4y + 5) - 6y^2$

17. $2x(3x^2 + 4) - 3x^3$

18. $4a(5a^2 - 4) + 9a$

19. $4b(-5b - 3) - 2(b^2 - 7b - 4)$

20. $3m(3m + 6) - 3(m^2 + 4m + 1)$

Solve each equation.

21. $3(a + 2) + 5 = 2a + 4$

22. $2(4x + 2) - 8 = 4(x + 3)$

23. $5(y + 1) + 2 = 4(y + 2) - 6$

24. $4(b + 6) = 2(b + 5) + 2$

25. $6(m - 2) + 14 = 3(m + 2) - 10$

26. $3(c + 5) - 2 = 2(c + 6) + 2$

NAME ______________________________ DATE ____________ PERIOD ____________

8-2 Practice

Multiplying a Polynomial by a Monomial

Find each product.

1. $2h(-7h^2 - 4h)$

2. $6pq(3p^2 + 4q)$

3. $5jk(3jk + 2k)$

4. $-3rt(-2t^2 + 3r)$

5. $-\frac{1}{4}m(8m^2 + m - 7)$

6. $-\frac{2}{3}n^2(-9n^2 + 3n + 6)$

Simplify each expression.

7. $-2\ell(3\ell - 4) + 7\ell$

8. $5w(-7w + 3) + 2w(-2w^2 + 19w + 2)$

9. $6t(2t - 3) - 5(2t^2 + 9t - 3)$

10. $-2(3m^3 + 5m + 6) + 3m(2m^2 + 3m + 1)$

11. $-3g(7g - 2) + 3(g^2 + 2g + 1) - 3g(-5g + 3)$

Solve each equation.

12. $5(2t - 1) + 3 = 3(3t + 2)$

13. $3(3u + 2) + 5 = 2(2u - 2)$

14. $4(8n + 3) - 5 = 2(6n + 8) + 1$

15. $8(3b + 1) = 4(b + 3) - 9$

16. $t(t + 4) - 1 = t(t + 2) + 2$

17. $u(u - 5) + 8u = u(u + 2) - 4$

18. NUMBER THEORY Let x be an integer. What is the product of twice the integer added to three times the next consecutive integer?

19. INVESTMENTS Kent invested \$5000 in a retirement plan. He allocated x dollars of the money to a bond account that earns 4% interest per year and the rest to a traditional account that earns 5% interest per year.

a. Write an expression that represents the amount of money invested in the traditional account.

b. Write a polynomial model in simplest form for the total amount of money T Kent has invested after one year. (*Hint:* Each account has $A + IA$ dollars, where A is the original amount in the account and I is its interest rate.)

c. If Kent put \$500 in the bond account, how much money does he have in his retirement plan after one year?

Lesson 8-2

NAME ______________________________ DATE ____________ PERIOD ____________

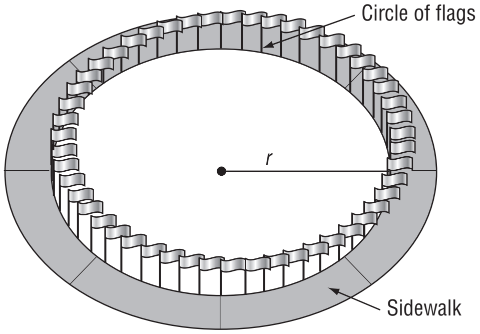
8-2 Word Problem Practice

Multiplying a Polynomial by a Monomial

1. **NUMBER THEORY** The sum of the first n whole numbers is given by the expression $\frac{1}{2}(n^2 + n)$. Expand the equation by multiplying, then find the sum of the first 12 whole numbers.

2. **COLLEGE** Troy's boss gave him \$700 to start his college savings account. Troy's boss also gives him \$40 each month to add to the account. Troy's mother gives him \$50 each month, but has been doing so for 4 fewer months than Troy's boss. Write a simplified expression for the amount of money Troy has received from his boss and mother after m months.

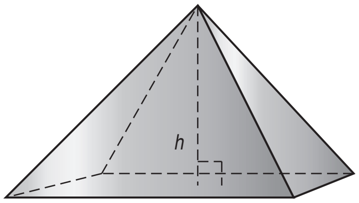
3. **LANDMARKS** A circle of 50 flags surrounds the Washington Monument. Suppose a new sidewalk 12 feet wide is installed just around the outside of the circle of flags. The outside circumference of the sidewalk is 1.10 times the circumference of the circle of flags.



Write an equation that equates the outside circumference of the sidewalk to 1.10 times the circumference of the circle of flags. Solve the equation for the radius of the circle of flags. Recall that circumference of a circle is $2\pi r$.

4. **MARKET** Sophia went to the farmers' market to purchase some vegetables. She bought peppers and potatoes. The peppers were \$0.39 each and the potatoes were \$0.29 each. She spent \$3.88 on vegetables, and bought 4 more potatoes than peppers. If x = the number of peppers, write and solve an equation to find out how many of each vegetable Sophia bought.

5. **GEOMETRY** Some monuments are constructed as rectangular pyramids. The volume of a pyramid can be found by multiplying the area of its base B by one third of its height. The area of the rectangular base of a monument in a local park is given by the polynomial equation $B = x^2 - 4x - 12$.



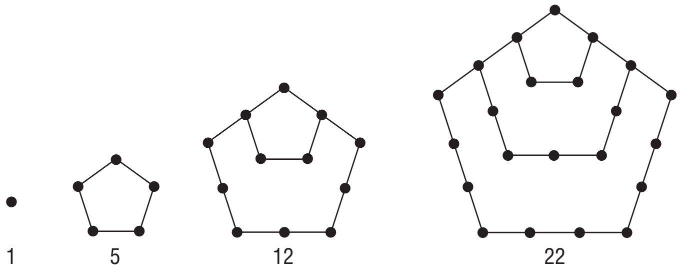
   a. Write a polynomial equation to represent V, the volume of a rectangular pyramid if its height is 10 centimeters.

   b. Find the volume of the pyramid if $x = 12$.

8-2 Enrichment

Figurate Numbers

The numbers below are called **pentagonal numbers**. They are the numbers of dots or disks that can be arranged as pentagons.



1. Find the product $\frac{1}{2}n(3n - 1)$.

2. Evaluate the product in Exercise 1 for values of n from 1 through 4.

3. What do you notice?

4. Find the next six pentagonal numbers.

5. Find the product $\frac{1}{2}n(n + 1)$.

6. Evaluate the product in Exercise 5 for values of n from 1 through 5. On another sheet of paper, make drawings to show why these numbers are called the triangular numbers.

7. Find the product $n(2n - 1)$.

8. Evaluate the product in Exercise 7 for values of n from 1 through 5. Draw these hexagonal numbers.

9. Find the first 5 square numbers. Also, write the general expression for any square number.

The numbers you have explored above are called the plane figurate numbers because they can be arranged to make geometric figures. You can also create solid figurate numbers.

10. If you pile 10 oranges into a pyramid with a triangle as a base, you get one of the tetrahedral numbers. How many layers are there in the pyramid? How many oranges are there in the bottom layers?

11. Evaluate the expression $\frac{1}{6}n^3 + \frac{1}{2}n^2 + \frac{1}{3}n$ for values of n from 1 through 5 to find the first five tetrahedral numbers.

8-3 Study Guide and Intervention

Multiplying Polynomials

Multiply Binomials To multiply two binomials, you can apply the Distributive Property twice. A useful way to keep track of terms in the product is to use the FOIL method as illustrated in Example 2.

Example 1 **Find $(x + 3)(x - 4)$.**

Horizontal Method

$$\begin{aligned}&(x + 3)(x - 4)\\ &= x(x - 4) + 3(x - 4)\\ &= (x)(x) + x(-4) + 3(x) + 3(-4)\\ &= x^2 - 4x + 3x - 12\\ &= x^2 - x - 12\end{aligned}$$

Vertical Method

$$\begin{array}{rrrr} & & x + & 3\\ (\times) & & x - & 4\\ \hline & & -4x - & 12\\ & x^2 + & 3x & \\ \hline & x^2 - & x - & 12\end{array}$$

The product is $x^2 - x - 12$.

Example 2 **Find $(x - 2)(x + 5)$ using the FOIL method.**

$(x - 2)(x + 5)$

$$\begin{aligned}&\quad\ \text{First}\qquad \text{Outer}\qquad \text{Inner}\qquad \text{Last}\\ &= (x)(x) + (x)(5) + (-2)(x) + (-2)(5)\\ &= x^2 + 5x + (-2x) - 10\\ &= x^2 + 3x - 10\end{aligned}$$

The product is $x^2 + 3x - 10$.

Exercises

Find each product.

1. $(x + 2)(x + 3)$

2. $(x - 4)(x + 1)$

3. $(x - 6)(x - 2)$

4. $(p - 4)(p + 2)$

5. $(y + 5)(y + 2)$

6. $(2x - 1)(x + 5)$

7. $(3n - 4)(3n - 4)$

8. $(8m - 2)(8m + 2)$

9. $(k + 4)(5k - 1)$

10. $(3x + 1)(4x + 3)$

11. $(x - 8)(-3x + 1)$

12. $(5t + 4)(2t - 6)$

13. $(5m - 3n)(4m - 2n)$

14. $(a - 3b)(2a - 5b)$

15. $(8x - 5)(8x + 5)$

16. $(2n - 4)(2n + 5)$

17. $(4m - 3)(5m - 5)$

18. $(7g - 4)(7g + 4)$

8-3 Study Guide and Intervention *(continued)*

Multiplying Polynomials

Multiply Polynomials The Distributive Property can be used to multiply any two polynomials.

Example **Find $(3x + 2)(2x^2 - 4x + 5)$.**

$(3x + 2)(2x^2 - 4x + 5)$

$= 3x(2x^2 - 4x + 5) + 2(2x^2 - 4x + 5)$ Distributive Property

$= 6x^3 - 12x^2 + 15x + 4x^2 - 8x + 10$ Distributive Property

$= 6x^3 - 8x^2 + 7x + 10$ Combine like terms.

The product is $6x^3 - 8x^2 + 7x + 10$.

Exercises

Find each product.

1. $(x + 2)(x^2 - 2x + 1)$

2. $(x + 3)(2x^2 + x - 3)$

3. $(2x - 1)(x^2 - x + 2)$

4. $(p - 3)(p^2 - 4p + 2)$

5. $(3k + 2)(k^2 + k - 4)$

6. $(2t + 1)(10t^2 - 2t - 4)$

7. $(3n - 4)(n^2 + 5n - 4)$

8. $(8x - 2)(3x^2 + 2x - 1)$

9. $(2a + 4)(2a^2 - 8a + 3)$

10. $(3x - 4)(2x^2 + 3x + 3)$

11. $(n^2 + 2n - 1)(n^2 + n + 2)$

12. $(t^2 + 4t - 1)(2t^2 - t - 3)$

13. $(y^2 - 5y + 3)(2y^2 + 7y - 4)$

14. $(3b^2 - 2b + 1)(2b^2 - 3b - 4)$

NAME ______________________ DATE ____________ PERIOD ____________

8-3 Skills Practice

Multiplying Polynomials

Find each product.

1. $(m + 4)(m + 1)$

2. $(x + 2)(x + 2)$

3. $(b + 3)(b + 4)$

4. $(t + 4)(t - 3)$

5. $(r + 1)(r - 2)$

6. $(n - 5)(n + 1)$

7. $(3c + 1)(c - 2)$

8. $(2x - 6)(x + 3)$

9. $(d - 1)(5d - 4)$

10. $(2\ell + 5)(\ell - 4)$

11. $(3n - 7)(n + 3)$

12. $(q + 5)(5q - 1)$

13. $(3b + 3)(3b - 2)$

14. $(2m + 2)(3m - 3)$

15. $(4c + 1)(2c + 1)$

16. $(5a - 2)(2a - 3)$

17. $(4h - 2)(4h - 1)$

18. $(x - y)(2x - y)$

19. $(w + 4)(w^2 + 3w - 6)$

20. $(t + 1)(t^2 + 2t + 4)$

21. $(k + 4)(k^2 + 3k - 6)$

22. $(m + 3)(m^2 + 3m + 5)$

NAME ______________________________ DATE ____________ PERIOD ____________

8-3 Practice

Multiplying Polynomials

Find each product.

1. $(q + 6)(q + 5)$

2. $(x + 7)(x + 4)$

3. $(n - 4)(n - 6)$

4. $(a + 5)(a - 6)$

5. $(4b + 6)(b - 4)$

6. $(2x - 9)(2x + 4)$

7. $(6a - 3)(7a - 4)$

8. $(2x - 2)(5x - 4)$

9. $(3a - b)(2a - b)$

10. $(4g + 3h)(2g + 3h)$

11. $(m + 5)(m^2 + 4m - 8)$

12. $(t + 3)(t^2 + 4t + 7)$

13. $(2h + 3)(2h^2 + 3h + 4)$

14. $(3d + 3)(2d^2 + 5d - 2)$

15. $(3q + 2)(9q^2 - 12q + 4)$

16. $(3r + 2)(9r^2 + 6r + 4)$

17. $(3n^2 + 2n - 1)(2n^2 + n + 9)$

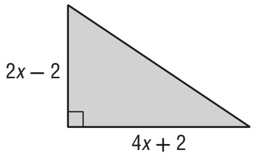
18. $(2t^2 + t + 3)(4t^2 + 2t - 2)$

19. $(2x^2 - 2x - 3)(2x^2 - 4x + 3)$

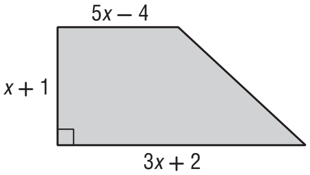
20. $(3y^2 + 2y + 2)(3y^2 - 4y - 5)$

GEOMETRY Write an expression to represent the area of each figure.

21.



22.



23. NUMBER THEORY Let x be an even integer. What is the product of the next two consecutive even integers?

24. GEOMETRY The volume of a rectangular pyramid is one third the product of the area of its base and its height. Find an expression for the volume of a rectangular pyramid whose base has an area of $3x^2 + 12x + 9$ square feet and whose height is $x + 3$ feet.

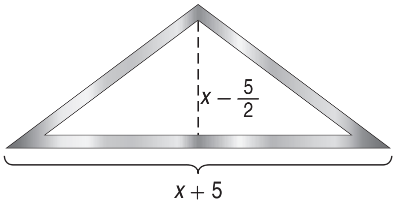
8-3 Word Problem Practice

Multiplying Polynomials

1. **THEATER** The Loft Theater has a center seating section with $3c + 8$ rows and $4c - 1$ seats in each row. Write an expression for the total number of seats in the center section.

2. **CRAFTS** Suppose a quilt made up of squares has a length-to-width ratio of 5 to 4. The length of the quilt is $5x$ inches. The quilt can be made slightly larger by adding a border of 1-inch squares all the way around the perimeter of the quilt. Write a polynomial expression for the area of the larger quilt.

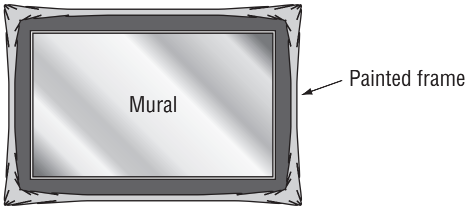
3. **SERVICE** A folded United States flag is sometimes presented to individuals in recognition of outstanding service to the country. The flag is presented folded in a triangle. Often the recipient purchases a case designed to display the folded flag to protect it from wear. One such display case has dimensions (in inches) shown below. Write a polynomial expression that represents the area of wall space covered by the display case.



Source: American Flag Store

4. **MATH FUN** Think of a whole number. Subtract 2. Write down this number. Take the original number and add 2. Write down this number. Find the product of the numbers you wrote down. Subtract the square of the original number. The result is always –4. Use polynomials to show how this number trick works.

5. **ART** The museum where Julia works plans to have a large wall mural replica of Vincent van Gogh's *The Starry Night* painted in its lobby. First, Julia wants to paint a large frame around where the mural will be. The mural's length will be 5 feet longer than its width, and the frame will be 2 feet wide on all sides. Julia has only enough paint to cover 100 square feet of wall surface. How large can the mural be?



a. Write an expression for the area of the mural.

b. Write an expression for the area of the frame.

c. Write and solve an equation to find how large the mural can be.

NAME ______________________ DATE __________ PERIOD __________

8-3 Enrichment

Pascal's Triangle

This arrangement of numbers is called Pascal's Triangle. It was first published in 1665, but was known hundreds of years earlier.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

1. Each number in the triangle is found by adding two numbers. What two numbers were added to get the 6 in the 5th row?

2. Describe how to create the 6th row of Pascal's Triangle.

3. Write the numbers for rows 6 through 10 of the triangle.
Row 6:
Row 7:
Row 8:
Row 9:
Row 10:

Multiply to find the expanded form of each product.

4. $(a + b)^2$

5. $(a + b)^3$

6. $(a + b)^4$

Now compare the coefficients of the three products in Exercises 4–6 with Pascal's Triangle.

7. Describe the relationship between the expanded form of $(a + b)^n$ and Pascal's Triangle.

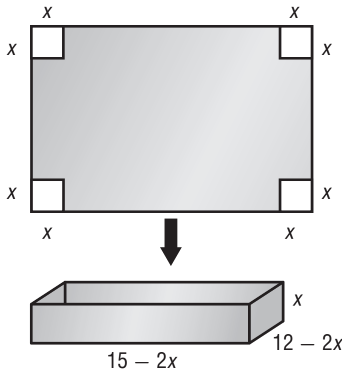
8. Use Pascal's Triangle to write the expanded form of $(a + b)^6$.

NAME ______________________________ DATE ____________ PERIOD ____________

8-3 Spreadsheet Activity

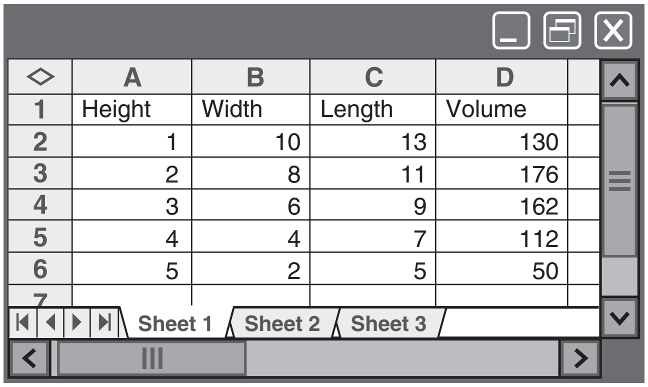
Multiplying Polynomials

Example **A box is made by cutting a square with sides x inches long from each corner of a piece of cardboard and folding up the sides. If the piece of cardboard is 15 inches long and 12 inches wide, what integer value of x allows you to make the box with the greatest volume? What is the volume?**



Step 1 The finished box will be x inches high, $12 - 2x$ inches wide, and $15 - 2x$ inches long. The volume of the box is $x(12 - 2x)(15 - 2x)$ cubic inches.

Step 2 Use Column A of the spreadsheet for the value of x. Enter the formulas for the width, length, and volume in Columns B, C, and D.



◇	A	B	C	D
1	Height	Width	Length	Volume
2	1	10	13	130
3	2	8	11	176
4	3	6	9	162
5	4	4	7	112
6	5	2	5	50
7				

Notice that x cannot be greater than 5 because $12 - 2x$ must be positive. In this case, the box with the greatest volume is 176 cubic inches when $x = 2$.

Exercises

Use the spreadsheet to find the value of x that allows the box with the greatest volume for each piece of cardboard. State the volume of the box.

1. 16 inches long and 10 inches wide

2. 24 inches long and 18 inches wide

3. 28 inches long and 16 inches wide

4. 36 inches long and 24 inches wide

5. 48 inches long and 48 inches wide

6. 108 inches long and 44 inches wide

7. Study the spreadsheet you created for Exercise 5. Suppose y is the volume of the box with a height of x inches. If you were to graph the ordered pairs (x, y) and connect them with a smooth curve, what would you expect the graph to look like? Use the graphing tool in the spreadsheet to verify your conjecture.

NAME ______________________________ DATE ______________ PERIOD _____________

8-4 Study Guide and Intervention

Special Products

Squares of Sums and Differences Some pairs of binomials have products that follow specific patterns. One such pattern is called the *square of a sum*. Another is called the *square of a difference*.

Square of a Sum	$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$
Square of a Difference	$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$

Example 1 **Find $(3a + 4)(3a + 4)$.**

Use the square of a sum pattern, with $a = 3a$ and $b = 4$.

$$(3a + 4)(3a + 4) = (3a)^2 + 2(3a)(4) + (4)^2$$
$$= 9a^2 + 24a + 16$$

The product is $9a^2 + 24a + 16$.

Example 2 **Find $(2z - 9)(2z - 9)$.**

Use the square of a difference pattern with $a = 2z$ and $b = 9$.

$$(2z - 9)(2z - 9) = (2z)^2 - 2(2z)(9) + (9)(9)$$
$$= 4z^2 - 36z + 81$$

The product is $4z^2 - 36z + 81$.

Exercises

Find each product.

1. $(x - 6)^2$

2. $(3p + 4)^2$

3. $(4x - 5)^2$

4. $(2x - 1)^2$

5. $(2h + 3)^2$

6. $(m + 5)^2$

7. $(a + 3)^2$

8. $(3 - p)^2$

9. $(x - 5y)^2$

10. $(8y + 4)^2$

11. $(8 + x)^2$

12. $(3a - 2b)^2$

13. $(2x - 8)^2$

14. $(x^2 + 1)^2$

15. $(m^2 - 2)^2$

16. $(x^3 - 1)^2$

17. $(2h^2 - k^2)^2$

18. $\left(\frac{1}{4}x + 3\right)^2$

19. $(x - 4y^2)^2$

20. $(2p + 4r)^2$

21. $\left(\frac{2}{3}x - 2\right)^2$

8-4 Study Guide and Intervention *(continued)*

Special Products

Product of a Sum and a Difference There is also a pattern for the product of a sum and a difference of the same two terms, $(a + b)(a - b)$. The product is called the **difference of squares**.

Product of a Sum and a Difference	$(a + b)(a - b) = a^2 - b^2$

Example **Find $(5x + 3y)(5x - 3y)$.**

$(a + b)(a - b) = a^2 - b^2$ Product of a Sum and a Difference

$(5x + 3y)(5x - 3y) = (5x)^2 - (3y)^2$ $a = 5x$ and $b = 3y$

$= 25x^2 - 9y^2$ Simplify.

The product is $25x^2 - 9y^2$.

Exercises

Find each product.

1. $(x - 4)(x + 4)$

2. $(p + 2)(p - 2)$

3. $(4x - 5)(4x + 5)$

4. $(2x - 1)(2x + 1)$

5. $(h + 7)(h - 7)$

6. $(m - 5)(m + 5)$

7. $(2d - 3)(2d + 3)$

8. $(3 - 5q)(3 + 5q)$

9. $(x - y)(x + y)$

10. $(y - 4x)(y + 4x)$

11. $(8 + 4x)(8 - 4x)$

12. $(3a - 2b)(3a + 2b)$

13. $(3y - 8)(3y + 8)$

14. $(x^2 - 1)(x^2 + 1)$

15. $(m^2 - 5)(m^2 + 5)$

16. $(x^3 - 2)(x^3 + 2)$

17. $(h^2 - k^2)(h^2 + k^2)$

18. $\left(\frac{1}{4}x + 2\right)\left(\frac{1}{4}x - 2\right)$

19. $(3x - 2y^2)(3x + 2y^2)$

20. $(2p - 5r)(2p + 5r)$

21. $\left(\frac{4}{3}x - 2y\right)\left(\frac{4}{3}x + 2y\right)$

8-4 Skills Practice

Special Products

Find each product.

1. $(n + 3)^2$

2. $(x + 4)(x + 4)$

31. $(y - 7)^2$

4. $(t - 3)(t - 3)$

5. $(b + 1)(b - 1)$

6. $(a - 5)(a + 5)$

7. $(p - 4)^2$

8. $(z + 3)(z - 3)$

9. $(\ell + 2)(\ell + 2)$

10. $(r - 1)(r - 1)$

11. $(3g + 2)(3g - 2)$

12. $(2m - 3)(2m + 3)$

13. $(6 + u)^2$

14. $(r + t)^2$

15. $(3q + 1)(3q - 1)$

16. $(c - d)^2$

17. $(2k - 2)^2$

18. $(w + 3h)^2$

19. $(3p - 4)(3p + 4)$

20. $(t + 2u)^2$

21. $(x - 4y)^2$

22. $(3b + 7)(3b - 7)$

23. $(3y - 3g)(3y + 3g)$

24. $(n^2 + r^2)^2$

25. $(2k + m^2)^2$

26. $(3t^2 - n)^2$

27. GEOMETRY The length of a rectangle is the sum of two whole numbers. The width of the rectangle is the difference of the same two whole numbers. Using these facts, write a verbal expression for the area of the rectangle.

Lesson 8-4

NAME ______________________ DATE ____________ PERIOD ____________

8-4 Practice

Special Products

Find each product.

1. $(n + 9)^2$

2. $(q + 8)^2$

3. $(x - 10)^2$

4. $(r - 11)^2$

5. $(p + 7)^2$

6. $(b + 6)(b - 6)$

7. $(z + 13)(z - 13)$

8. $(4j + 2)^2$

9. $(5w - 4)^2$

10. $(6h - 1)^2$

11. $(3m + 4)^2$

12. $(7v - 2)^2$

13. $(7k + 3)(7k - 3)$

14. $(4d - 7)(4d + 7)$

15. $(3g + 9h)(3g - 9h)$

16. $(4q + 5t)(4q - 5t)$

17. $(a + 6u)^2$

18. $(5r + p)^2$

19. $(6h - m)^2$

20. $(k - 6y)^2$

21. $(u - 7p)^2$

22. $(4b - 7v)^2$

23. $(6n + 4p)^2$

24. $(5q + 6t)^2$

25. $(6a - 7b)(6a + 7b)$

26. $(8h + 3d)(8h - 3d)$

27. $(9x + 2y^2)^2$

28. $(3p^3 + 2m)^2$

29. $(5a^2 - 2b)^2$

30. $(4m^3 - 2t)^2$

31. $(6b^3 - g)^2$

32. $(2b^2 - g)(2b^2 + g)$

33. $(2v^2 + 3x^2)(2v^2 + 3x^2)$

34. **GEOMETRY** Janelle wants to enlarge a square graph that she has made so that a side of the new graph will be 1 inch more than twice the original side g. What trinomial represents the area of the enlarged graph?

35. **GENETICS** In a guinea pig, pure black hair coloring B is dominant over pure white coloring b. Suppose two hybrid Bb guinea pigs, with black hair coloring, are bred.

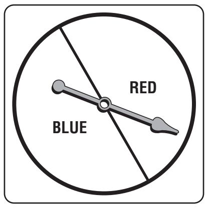
a. Find an expression for the genetic make-up of the guinea pig offspring.

b. What is the probability that two hybrid guinea pigs with black hair coloring will produce a guinea pig with white hair coloring?

8-4 Word Problem Practice

Special Products

1. **PROBABILITY** The spinner below is divided into 2 equal sections. If you spin the spinner 2 times in a row, the possible outcomes are shown in the table below.

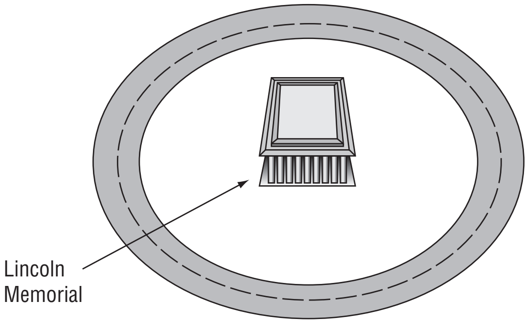


red red	blue red
red blue	blue blue

What is the probability of spinning a blue and a red in two spins?

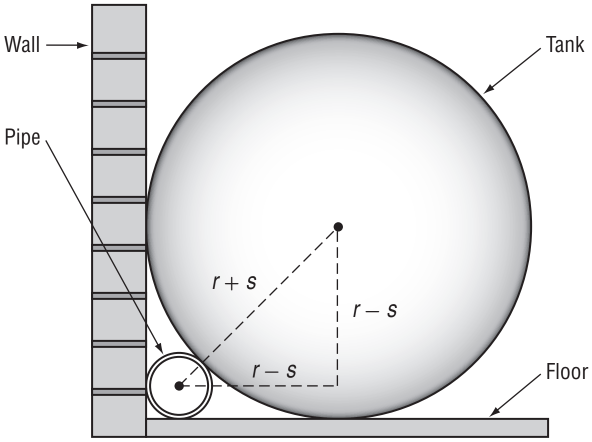
2. **GRAVITY** The height of a penny t seconds after being dropped down a well is given by the product of $(10 - 4t)$ and $(10 + 4t)$. Find the product and simplify. What type of special product does this represent?

3. **TRAFFIC PLANNING** The Lincoln Memorial in Washington, D.C., is surrounded by a circular drive called Lincoln Circle. Suppose the National Park Service wants to change the layout of Lincoln Circle so that there are two concentric circular roads. Write a polynomial equation for the area A of the space between the roads if the radius of the inside road is 10 meters less than the radius of the outside road.



4. **BUSINESS** The Combo Lock Company finds that its profit data from 2005 to the present can be modeled by the function $y = 4n^2 + 44n + 121$, where y is the profit n years since 2005. Which special product does this polynomial demonstrate? Explain.

5. **STORAGE** A cylindrical tank is placed along a wall. A cylindrical PVC pipe will be hidden in the corner behind the tank. See the side view diagram below. The radius of the tank is r inches and the radius of the PVC pipe is s inches.



a. Use the Pythagorean Theorem to write an equation for the relationship between the two radii. Simplify your equation so that there is a zero on one side of the equals sign.

b. Write a polynomial equation you could solve to find the radius s of the PVC pipe if the radius of the tank is 20 inches.

NAME ______________________________ DATE ____________ PERIOD ____________

8-4 Enrichment

Sums and Differences of Cubes

Recall the formulas for finding some special products:

Perfect-square trinomials: $(a + b)^2 = a^2 + 2ab + b^2$ or
$(a - b)^2 = a^2 - 2ab + b^2$
Difference of two squares: $(a + b)(a - b) = a^2 - b^2$

A pattern also exists for finding the cube of a sum $(a + b)^3$.

1. Find the product of $(a + b)(a + b)(a + b)$.

2. Use the pattern from Exercise 1 to evaluate $(x + 2)^3$.

3. Based on your answer to Exercise 1, predict the pattern for the cube of a difference $(a - b)^3$.

4. Find the product of $(a - b)(a - b)(a - b)$ and compare it to your answer for Exercise 3.

5. Use the pattern from Exercise 4 to evaluate $(x - 4)^3$.

Find each product.

6. $(x + 6)^3$

7. $(x - 10)^3$

8. $(3x - y)^3$

9. $(2x - y)^3$

10. $(4x + 3y)^3$

11. $(5x + 2)^3$

NAME ______________________________ DATE ____________ PERIOD ____________

8-5 Study Guide and Intervention

Using the Distributive Property

Use the Distributive Property to Factor The Distributive Property has been used to multiply a polynomial by a monomial. It can also be used to express a polynomial in factored form. Compare the two columns in the table below.

Multiplying	Factoring
$3(a + b) = 3a + 3b$	$3a + 3b = 3(a + b)$
$x(y - z) = xy - xz$	$xy - xz = x(y - z)$
$6y(2x + 1) = 6y(2x) + 6y(1)$ $= 12xy + 6y$	$12xy + 6y = 6y(2x) + 6y(1)$ $= 6y(2x + 1)$

Example 1 **Use the Distributive Property to factor $12mp + 80m^2$.**

Find the GCF of $12mp$ and $80m^2$.

$12mp = 2 \cdot 2 \cdot 3 \cdot m \cdot p$

$80m^2 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot m \cdot m$

GCF $= 2 \cdot 2 \cdot m$ or $4m$

Write each term as the product of the GCF and its remaining factors.

$$\begin{aligned}12mp + 80m^2 &= 4m(3 \cdot p) + 4m(2 \cdot 2 \cdot 5 \cdot m)\\ &= 4m(3p) + 4m(20m)\\ &= 4m(3p + 20m)\end{aligned}$$

Thus $12mp + 80m^2 = 4m(3p + 20m)$.

Example 2 **Factor $6ax + 3ay + 2bx + by$ by grouping.**

$$\begin{aligned}&6ax + 3ay + 2bx + by\\ &= (6ax + 3ay) + (2bx + by)\\ &= 3a(2x + y) + b(2x + y)\\ &= (3a + b)(2x + y)\end{aligned}$$

Check using the FOIL method.

$$\begin{aligned}&(3a + b)(2x + y)\\ &= 3a(2x) + (3a)(y) + (b)(2x) + (b)(y)\\ &= 6ax + 3ay + 2bx + by \checkmark\end{aligned}$$

Exercises

Factor each polynomial.

1. $24x + 48y$

2. $30mp^2 + m^2p - 6p$

3. $q^4 - 18q^3 + 22q$

4. $9x^2 - 3x$

5. $4m + 6p - 8mp$

6. $45r^3 - 15r^2$

7. $14t^3 - 42t^5 - 49t^4$

8. $55p^2 - 11p^4 + 44p^5$

9. $14y^3 - 28y^2 + y$

10. $4x + 12x^2 + 16x^3$

11. $4a^2b + 28ab^2 + 7ab$

12. $6y + 12x - 8z$

13. $x^2 + 2x + x + 2$

14. $6y^2 - 4y + 3y - 2$

15. $4m^2 + 4mp + 3mp + 3p^2$

16. $12ax + 3xz + 4ay + yz$

17. $12a^2 + 3a - 8a - 2$

18. $xa + ya + x + y$

NAME ______________________________ DATE ____________ PERIOD ________

8-5 Study Guide and Intervention *(continued)*

Using the Distributive Property

Solve Equations by Factoring The following property, along with factoring, can be used to solve certain equations.

Zero Product Property	For any real numbers a and b, if $ab = 0$, then either $a = 0$, $b = 0$, or both a and b equal 0.

Example **Solve $9x^2 + x = 0$. Then check the solutions.**

Write the equation so that it is of the form $ab = 0$.

$9x^2 + x = 0$	Original equation
$x(9x + 1) = 0$	Factor the GCF of $9x^2 + x$, which is x.
$x = 0$ or $9x + 1 = 0$	Zero Product Property
$x = 0$ $\quad x = -\frac{1}{9}$	Solve each equation.

The solution set is $\left\{0, -\frac{1}{9}\right\}$.

Check Substitute 0 and $-\frac{1}{9}$ for x in the original equation.

$$9x^2 + x = 0 \qquad\qquad 9x^2 + x = 0$$

$$9(0)^2 + 0 \stackrel{?}{=} 0 \qquad\qquad 9\left(-\frac{1}{9}\right)^2 + \left(-\frac{1}{9}\right) \stackrel{?}{=} 0$$

$$0 = 0 \checkmark \qquad\qquad \frac{1}{9} + \left(-\frac{1}{9}\right) \stackrel{?}{=} 0$$

$$0 = 0 \checkmark$$

Exercises

Solve each equation. Check your solutions.

1. $x(x + 3) = 0$

2. $3m(m - 4) = 0$

3. $(r - 3)(r + 2) = 0$

4. $3x(2x - 1) = 0$

5. $(4m + 8)(m - 3) = 0$

6. $5t^2 = 25t$

7. $(4c + 2)(2c - 7) = 0$

8. $5p - 15p^2 = 0$

9. $4y^2 = 28y$

10. $12x^2 = -6x$

11. $(4a + 3)(8a + 7) = 0$

12. $8y = 12y^2$

13. $x^2 = -2x$

14. $(6y - 4)(y + 3) = 0$

15. $4m^2 = 4m$

16. $12x = 3x^2$

17. $12a^2 = -3a$

18. $(12a + 4)(3a - 1) = 0$

NAME ______________________________ DATE ____________ PERIOD ____________

8-5 Skills Practice

Using the Distributive Property

Factor each polynomial.

1. $7x + 49$

2. $8m - 6$

3. $5a^2 - 15$

4. $10q - 25q^2$

5. $8ax - 56a$

6. $81r + 48rt$

7. $t^2h + 3t$

8. $a^2b^2 + a$

9. $x + x^2y + x^3y^2$

10. $3p^2r^2 + 6pr + p$

11. $4a^2b^2 + 16ab + 12a$

12. $10h^3n^3 - 2hn^2 + 14hn$

13. $x^2 + 3x + x + 3$

14. $b^2 - 2b + 3b - 6$

15. $2j^2 + 2j + 3j + 3$

16. $2a^2 - 4a + a - 2$

17. $6t^2 - 4t - 3t + 2$

18. $9x^2 - 3xy + 6x - 2y$

Solve each equation. Check your solutions.

19. $x(x - 8) = 0$

20. $b(b + 12) = 0$

21. $(m - 3)(m + 5) = 0$

22. $(a - 9)(2a + 1) = 0$

23. $x^2 - 5x = 0$

24. $y^2 + 3y = 0$

25. $3a^2 = 6a$

26. $2x^2 = 3x$

8-5 Practice

Using the Distributive Property

Factor each polynomial.

1. $64 - 40ab$

2. $4d^2 + 16$

3. $6r^2t - 3rt^2$

4. $15ad + 30a^2d^2$

5. $32a^2 + 24b^2$

6. $36xy^2 - 48x^2y$

7. $30x^3y + 35x^2y^2$

8. $9a^3d^2 - 6ad^3$

9. $75b^2g^3 + 60bg^3$

10. $8p^2r^2 - 24pr^3 + 16pr$

11. $5x^3y^2 + 10x^2y + 25x$

12. $9ax^3 + 18bx^2 + 24cx$

13. $x^2 + 4x + 2x + 8$

14. $2a^2 + 3a + 6a + 9$

15. $4b^2 - 12b + 2b - 6$

16. $6xy - 8x + 15y - 20$

17. $-6mp + 4m + 18p - 12$

18. $12a^2 - 15ab - 16a + 20b$

Solve each equation. Check your solutions.

19. $x(x - 32) = 0$

20. $4b(b + 4) = 0$

21. $(y - 3)(y + 2) = 0$

22. $(a + 6)(3a - 7) = 0$

23. $(2y + 5)(y - 4) = 0$

24. $(4y + 8)(3y - 4) = 0$

25. $2z^2 + 20z = 0$

26. $8p^2 - 4p = 0$

27. $9x^2 = 27x$

28. $18x^2 = 15x$

29. $14x^2 = -21x$

30. $8x^2 = -26x$

31. LANDSCAPING A landscaping company has been commissioned to design a triangular flower bed for a mall entrance. The final dimensions of the flower bed have not been determined, but the company knows that the height will be two feet less than the base. The area of the flower bed can be represented by the equation $A = \frac{1}{2}b^2 - b$.

a. Write this equation in factored form.

b. Suppose the base of the flower bed is 16 feet. What will be its area?

32. PHYSICAL SCIENCE Mr. Alim's science class launched a toy rocket from ground level with an initial upward velocity of 60 feet per second. The height h of the rocket in feet above the ground after t seconds is modeled by the equation $h = 60t - 16t^2$. How long was the rocket in the air before it returned to the ground?

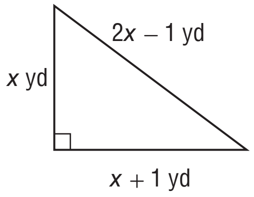
8-5 Word Problem Practice

Using the Distributive Property

1. **PHYSICS** According to legend, Galileo dropped objects of different weights from the so-called "leaning tower" of Pisa while developing his formula for free falling objects. The relationship that he discovered was that the distance d an object falls after t seconds is given by $d = 16t^2$ (ignoring air resistance). This relationship can be found in the equation $h = 4t - 16t^2$, where h is the height of an object thrown upward from ground level at a rate of 32 feet per second. Solve the equation for $h = 0$.

2. **SWIMMING POOL** The area A of a rectangular swimming pool is given by the equation $A = 12w - w^2$, where w is the width of one side. Write an expression for the other side of the pool.

3. **CONSTRUCTION** Unique Building Company is constructing a triangular roof truss for a building. The workers assemble the truss with the dimensions shown on the diagram below. Using the Pythagorean Theorem, find the length of the sides of the truss.



4. **VERTICAL JUMP** Your vertical jump height is measured by subtracting your standing reach height from the height of the highest point you can reach by jumping without taking a running start. Typically, NBA players have vertical jump heights of up to 34 inches. If an NBA player jumps this high, his height h in inches above his standing reach height after t seconds can be modeled by $h = 162t - 192t^2$. Solve the equation for $h = 0$ and interpret the solution. Round your answer to the nearest hundredth.

5. **PETS** Conner tosses a dog treat upward with an initial velocity of 13.7 meters per second. The height of the treat above the dog's mouth h in meters after t seconds is given by $h = 13.7t - 4.9t^2$.

   a. Assuming the dog doesn't jump, after how many seconds does the dog catch the treat?

   b. The dog treat reaches its maximum height halfway between when it was thrown and when it was caught. What is its maximum height?

   c. How fast would Connor have to throw the dog treat in order to make it fly through the air for 6 seconds?

NAME ______________________________ DATE ____________ PERIOD ____________

8-5 Enrichment

Linear Combinations

The greatest common factor, GCF, of two numbers can be written as a linear combination of the two numbers. A linear combination is an expression of the form $Ax + By$.

Example **Write the greatest common factor of 52 and 36 as a linear combination.**

First, use the Euclidean Algorithm to find the greatest common factor of the two numbers.

$\begin{array}{r} 1 \\ 36\overline{)52} \\ \underline{36} \end{array}$		Divide the greater number by the lesser number.
$\begin{array}{r} 2 \\ 16\overline{)36} \\ \underline{32} \end{array}$	original divisor;	Then divide using the remainder as the new divisor.
$\begin{array}{r} 4 \\ 4\overline{)16} \\ \underline{16} \end{array}$	second divisor;	Divide again.
0		Stop dividing.

Last divisor used is the GCF. In this case, 4 is the GCF for 36 and 52.

To write 4 as a linear combination of 36 and 52, it needs to be written as:

$4 = 36x + 52y$, where x and y are some integers.

Use trial and error to determine the two integers.

The two integers that work are $x = 3$ and $y = -2$. So, the linear combination for the greatest common factor of 52 and 36 is:

$4 = 36(3) + 52(-2)$

Exercises

Write the greatest common factor for each pair of numbers as a linear combination.

1. 16, 64

2. 21, 28

3. 3, 18

4. 15, 36

5. 6, 8

6. 18, 42

8-6 Study Guide and Intervention

Solving $x^2 + bx + c = 0$

Factor $x^2 + bx + c$ To factor a trinomial of the form $x^2 + bx + c$, find two integers, m and p, whose sum is equal to b and whose product is equal to c.

Factoring $x^2 + bx + c$	$x^2 + bx + c = (x + m)(x + p)$, where $m + p = b$ and $mp = c$.

Example 1 **Factor each polynomial.**

a. $x^2 + 7x + 10$

In this trinomial, $b = 7$ and $c = 10$.

Factors of 10	Sum of Factors
1, 10	11
2, 5	7

Since $2 + 5 = 7$ and $2 \cdot 5 = 10$, let $m = 2$ and $p = 5$.

$x^2 + 7x + 10 = (x + 5)(x + 2)$

b. $x^2 - 8x + 7$

In this trinomial, $b = -8$ and $c = 7$.

Notice that $m + p$ is negative and mp is positive, so m and p are both negative. Since $-7 + (-1) = -8$ and $(-7)(-1) = 7$, $m = -7$ and $p = -1$.

$x^2 - 8x + 7 = (x - 7)(x - 1)$

Example 2 **Factor $x^2 + 6x - 16$.**

In this trinomial, $b = 6$ and $c = -16$. This means $m + p$ is positive and mp is negative. Make a list of the factors of -16, where one factor of each pair is positive.

Factors of −16	Sum of Factors
1, −16	−15
−1, 16	15
2, −8	−6
−2, 8	6

Therefore, $m = -2$ and $p = 8$.

$x^2 + 6x - 16 = (x - 2)(x + 8)$

Exercises

Factor each polynomial.

1. $x^2 + 4x + 3$

2. $m^2 + 12m + 32$

3. $r^2 - 3r + 2$

4. $x^2 - x - 6$

5. $x^2 - 4x - 21$

6. $x^2 - 22x + 121$

7. $t^2 - 4t - 12$

8. $p^2 - 16p + 64$

9. $9 - 10x + x^2$

10. $x^2 + 6x + 5$

11. $a^2 + 8a - 9$

12. $y^2 - 7y - 8$

13. $x^2 - 2x - 3$

14. $y^2 + 14y + 13$

15. $m^2 + 9m + 20$

16. $x^2 + 12x + 20$

17. $a^2 - 14a + 24$

18. $18 + 11y + y^2$

19. $x^2 + 2xy + y^2$

20. $a^2 - 4ab + 4b^2$

21. $x^2 + 6xy - 7y^2$

NAME ______________________ DATE ____________ PERIOD ____________

8-6 Study Guide and Intervention *(continued)*

Solving $x^2 + bx + c = 0$

Solve Equations by Factoring Factoring and the Zero Product Property can be used to solve many equations of the form $x^2 + bx + c = 0$.

Example 1 **Solve $x^2 + 6x = 7$. Check your solutions.**

$x^2 + 6x = 7$	Original equation
$x^2 + 6x - 7 = 0$	Rewrite equation so that one side equals 0.
$(x - 1)(x + 7) = 0$	Factor.
$x - 1 = 0$ or $x + 7 = 0$	Zero Product Property
$x = 1$ $\quad$ $x = -7$	Solve each equation.

The solution set is {1, −7}. Since $1^2 + 6(1) = 7$ and $(-7)^2 + 6(-7) = 7$, the solutions check.

Example 2 **ROCKET LAUNCH The formula $h = vt - 16t^2$ gives the height h of a rocket after t seconds when the initial velocity v is given in feet per second. If a rocket is fired with initial velocity 2288 feet per second, how many seconds will it take for the rocket to reach a height of 6720 feet?**

$h = vt - 16t^2$	Formula
$6720 = 2288t - 16t^2$	Substitute.
$0 = -16t^2 + 2288t - 6720$	Rewrite equation so that one side equals 0.
$0 = -16(t - 143t + 420)$	Factor out GCF.
$0 = -16(t - 3)(t - 140)$	Factor
$t - 3 = 0$ or $t - 140 = 0$	Zero Product Property
$t = 3$ $\quad$ $t = 140$	Solve each equation.

The rocket reaches 6720 feet in 3 seconds and again in 140 seconds, or 2 minutes 20 seconds after launch.

Exercises

Solve each equation. Check the solutions.

1. $x^2 - 4x + 3 = 0$

2. $y^2 - 5y + 4 = 0$

3. $m^2 + 10m + 9 = 0$

4. $x^2 = x + 2$

5. $x^2 - 4x = 5$

6. $x^2 - 12x + 36 = 0$

7. $t^2 - 8 = -7t$

8. $p^2 = 9p - 14$

9. $-9 - 8x + x^2 = 0$

10. $x^2 + 6 = 5x$

11. $a^2 = 11a - 18$

12. $y^2 - 8y + 15 = 0$

13. $x^2 = 24 - 10x$

14. $a^2 - 18a = -72$

15. $b^2 = 10b - 16$

Use the formula $h = vt - 16t^2$ to solve each problem.

16. FOOTBALL A punter can kick a football with an initial velocity of 48 feet per second. How many seconds will it take for the ball to first reach a height of 32 feet?

17. ROCKET LAUNCH If a rocket is launched with an initial velocity of 1600 feet per second, when will the rocket be 14,400 feet high?

NAME ______________________ DATE ____________ PERIOD ____________

8-6 Skills Practice

Solving $x^2 + bx + c = 0$

Factor each polynomial.

1. $t^2 + 8t + 12$

2. $n^2 + 7n + 12$

3. $p^2 + 9p + 20$

4. $h^2 + 9h + 18$

5. $n^2 + 3n - 18$

6. $x^2 + 2x - 8$

7. $y^2 - 5y - 6$

8. $g^2 + 3g - 10$

9. $r^2 + 4r - 12$

10. $x^2 - x - 12$

11. $w^2 - w - 6$

12. $y^2 - 6y + 8$

13. $x^2 - 8x + 15$

14. $b^2 - 9b + 8$

15. $t^2 - 15t + 56$

16. $-4 - 3m + m^2$

Solve each equation. Check the solutions.

17. $x^2 - 6x + 8 = 0$

18. $b^2 - 7b + 12 = 0$

19. $m^2 + 5m + 6 = 0$

20. $d^2 + 7d + 10 = 0$

21. $y^2 - 2y - 24 = 0$

22. $p^2 - 3p = 18$

23. $h^2 + 2h = 35$

24. $a^2 + 14a = -45$

25. $n^2 - 36 = 5n$

26. $w^2 + 30 = 11w$

NAME ______________________ DATE ____________ PERIOD ____________

8-6 Practice

Solving $x^2 + bx + c = 0$

Factor each polynomial.

1. $a^2 + 10a + 24$

2. $h^2 + 12h + 27$

3. $x^2 + 14x + 33$

4. $g^2 - 2g - 63$

5. $w^2 + w - 56$

6. $y^2 + 4y - 60$

7. $b^2 + 4b - 32$

8. $n^2 - 3n - 28$

9. $t^2 + 4t - 45$

10. $z^2 - 11z + 30$

11. $d^2 - 16d + 63$

12. $x^2 - 11x + 24$

13. $q^2 - q - 56$

14. $x^2 - 6x - 55$

15. $32 + 18r + r^2$

16. $48 - 16g + g^2$

17. $j^2 - 9jk - 10k^2$

18. $m^2 - mv - 56v^2$

Solve each equation. Check the solutions.

19. $x^2 + 17x + 42 = 0$

20. $p^2 + 5p - 84 = 0$

21. $k^2 + 3k - 54 = 0$

22. $b^2 - 12b - 64 = 0$

23. $n^2 + 4n = 32$

24. $h^2 - 17h = -60$

25. $t^2 - 26t = 56$

26. $z^2 - 14z = 72$

27. $y^2 - 84 = 5y$

28. $80 + a^2 = 18a$

29. $u^2 = 16u + 36$

30. $17r + r^2 = -52$

31. Find all values of k so that the trinomial $x^2 + kx - 35$ can be factored using integers.

32. **CONSTRUCTION** A construction company is planning to pour concrete for a driveway. The length of the driveway is 16 feet longer than its width w.

a. Write an expression for the area of the driveway.

b. Find the dimensions of the driveway if it has an area of 260 square feet.

33. **WEB DESIGN** Janeel has a 10-inch by 12-inch photograph. She wants to scan the photograph, then reduce the result by the same amount in each dimension to post on her Web site. Janeel wants the area of the image to be one eighth that of the original photograph.

a. Write an equation to represent the area of the reduced image.

b. Find the dimensions of the reduced image.

8-6 Word Problem Practice

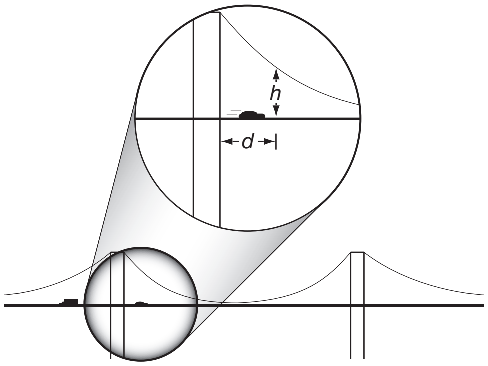
Solving $x^2 + bx + c = 0$

1. **COMPACT DISCS** A compact disc jewel case has a width 2 centimeters greater than its length. The area for the front cover is 168 square centimeters. The first two steps to finding the value of x are shown below. Solve the equation and find the length of the case.

$$Length \times width = area$$
$$x(x + 2) = 168$$
$$x^2 + 2x - 168 = 0$$

2. **MATH GAMES** Fiona and Greg play a number guessing game. Greg gives Fiona this hint about his two secret numbers, "The product of the two consecutive positive integers that I am thinking of is 11 more than their sum." What are Greg's numbers?

3. **BRIDGE ENGINEERING** A car driving over a suspension bridge is supported by a cable hanging between the ends of the bridge. Since its shape is parabolic, it can be modeled by a quadratic equation. The height above the road bed of a bridge's cable h in inches measured at distance d in yards from the first tower is given by $h = d^2 - 36d + 324$.



If the driver of a car looks out at a height of 49 inches above the roadbed, at what distance(s) from the tower will the driver's eyes be at the same height as the cable?

4. **PHYSICAL SCIENCE** The boiling point of water depends on altitude. The following equation approximates the number of degrees D below 212ºF at which water will boil at altitude h.

$$D^2 + 520D = h$$

In Denver, Colorado, the altitude is approximately 5300 feet above sea level. At approximately what temperature does water boil in Denver?

5. **MONUMENTS** Susan is designing a pyramidal stone monument for a local park. The design specifications tell her that the height needs to be 9 feet, the width of the base must be 5 feet less than the length, and the volume should be 150 cubic feet. Recall that the volume of a pyramid is given by $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height.

   a. Write and solve an equation to find the width of the base of the monument.

   b. Interpret each answer in terms of the situation.

NAME ______________________________ DATE ____________ PERIOD ____________

8-6 Enrichment

Puzzling Primes

A prime number has only two factors, itself and 1. The number 6 is not prime because it has 2 and 3 as factors; 5 and 7 are prime. The number 1 is not considered to be prime.

1. Use a calculator to help you find the 25 prime numbers less than 100.

Prime numbers have interested mathematicians for centuries. They have tried to find expressions that will give all the prime numbers, or only prime numbers. In the 1700s, Euler discovered that the trinomial $x^2 + x + 41$ will yield prime numbers for values of x from 0 through 39.

2. Find the prime numbers generated by Euler's formula for x from 0 through 7.

3. Show that the trinomial $x^2 + x + 31$ will not give prime numbers for very many values of x.

4. Find the greatest prime number generated by Euler's formula.

Goldbach's Conjecture is that every nonzero even number greater than 2 can be written as the sum of two primes. No one has ever proved that this is always true, but no one has found a counterexample, either.

5. Show that Goldbach's Conjecture is true for the first 5 even numbers greater than 2.

6. Give a way that someone could disprove Goldbach's Conjecture.

NAME ______________________________ DATE ____________ PERIOD ____________

8-7 Study Guide and Intervention

Solving $ax^2 + bx + c = 0$

Factor $ax^2 + bx + c$ To factor a trinomial of the form $ax^2 + bx + c$, find two integers, m and p whose product is equal to ac and whose sum is equal to b. If there are no integers that satisfy these requirements, the polynomial is called a **prime polynomial**.

Example 1 Factor $2x^2 + 15x + 18$.

In this example, $a = 2$, $b = 15$, and $c = 18$. You need to find two numbers that have a sum of 15 and a product of $2 \cdot 18$ or 36. Make a list of the factors of 36 and look for the pair of factors with a sum of 15.

Factors of 36	Sum of Factors
1, 36	37
2, 18	20
3, 12	15

Use the pattern $ax^2 + mx + px + c$, with $a = 2$, $m = 3$, $p = 12$, and $c = 18$.

$$\begin{aligned} 2x^2 + 15x + 18 &= 2x^2 + 3x + 12x + 18 \\ &= (2x^2 + 3x) + (12x + 18) \\ &= x(2x + 3) + 6(2x + 3) \\ &= (x + 6)(2x + 3) \end{aligned}$$

Therefore, $2x^2 + 15x + 18 = (x + 6)(2x + 3)$.

Example 2 Factor $3x^2 - 3x - 18$.

Note that the GCF of the terms $3x^2$, $3x$, and 18 is 3. First factor out this GCF.

$3x^2 - 3x - 18 = 3(x^2 - x - 6)$.

Now factor $x^2 - x - 6$. Since $a = 1$, find the two factors of -6 with a sum of -1.

Factors of –6	Sum of Factors
1, –6	–5
–1, 6	5
–2, 3	1
2, –3	–1

Now use the pattern $(x + m)(x + p)$ with $m = 2$ and $p = -3$.

$x^2 - x - 6 = (x + 2)(x - 3)$

The complete factorization is $3x^2 - 3x - 18 = 3(x + 2)(x - 3)$.

Exercises

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

1. $2x^2 - 3x - 2$

2. $3m^2 - 8m - 3$

3. $16r^2 - 8r + 1$

4. $6x^2 + 5x - 6$

5. $3x^2 + 2x - 8$

6. $18x^2 - 27x - 5$

7. $2a^2 + 5a + 3$

8. $18y^2 + 9y - 5$

9. $-4t^2 + 19t - 21$

10. $8x^2 - 4x - 24$

11. $28p^2 + 60p - 25$

12. $48x^2 + 22x - 15$

13. $3y^2 - 6y - 24$

14. $4x^2 + 26x - 48$

15. $8m^2 - 44m + 48$

16. $6x^2 - 7x + 18$

17. $2a^2 - 14a + 18$

18. $18 + 11y + 2y^2$

8-7 Study Guide and Intervention *(continued)*

Solving $ax^2 + bx + c = 0$

Solve Equations by Factoring Factoring and the Zero Product Property can be used to solve some equations of the form $ax^2 + bx + c = 0$.

Example **Solve $12x^2 + 3x = 2 - 2x$. Check your solutions.**

$12x^2 + 3x = 2 - 2x$ Original equation

$12x^2 + 5x - 2 = 0$ Rewrite equation so that one side equals 0.

$(3x + 2)(4x - 1) = 0$ Factor the left side.

$3x + 2 = 0$ or $4x - 1 = 0$ Zero Product Property

$x = -\frac{2}{3}$ $\quad x = \frac{1}{4}$ Solve each equation.

The solution set is $\left\{-\frac{2}{3}, \frac{1}{4}\right\}$.

Since $12\left(-\frac{2}{3}\right)^2 + 3\left(-\frac{2}{3}\right) = 2 - 2\left(-\frac{2}{3}\right)$ and $12\left(\frac{1}{4}\right)^2 + 3\left(\frac{1}{4}\right) = 2 - 2\left(\frac{1}{4}\right)$, the solutions check.

Exercises

Solve each equation. Check the solutions.

1. $8x^2 + 2x - 3 = 0$

2. $3n^2 - 2n - 5 = 0$

3. $2d^2 - 13d - 7 = 0$

4. $4x^2 = x + 3$

5. $3x^2 - 13x = 10$

6. $6x^2 - 11x - 10 = 0$

7. $2k^2 - 40 = -11k$

8. $2p^2 = -21p - 40$

9. $-7 - 18x + 9x^2 = 0$

10. $12x^2 - 15 = -8x$

11. $7a^2 = -65a - 18$

12. $16y^2 - 2y - 3 = 0$

13. $8x^2 + 5x = 3 + 7x$

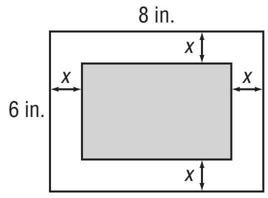
14. $4a^2 - 18a + 5 = 15$

15. $3b^2 - 18b = 10b - 49$

16. The difference of the squares of two consecutive odd integers is 24. Find the integers.

17. GEOMETRY The length of a Charlotte, North Carolina, conservatory garden is 20 yards greater than its width. The area is 300 square yards. What are the dimensions?

18. GEOMETRY A rectangle with an area of 24 square inches is formed by cutting strips of equal width from a rectangular piece of paper. Find the dimensions of the new rectangle if the original rectangle measures 8 inches by 6 inches.



NAME ______________________________ DATE ____________ PERIOD ____________

8-7 Skills Practice

Solving $ax^2 + bx + c = 0$

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

1. $2x^2 + 5x + 2$

2. $3n^2 + 5n + 2$

3. $2t^2 + 9t - 5$

4. $3g^2 - 7g + 2$

5. $2t^2 - 11t + 15$

6. $2x^2 + 3x - 6$

7. $2y^2 + y - 1$

8. $4h^2 + 8h - 5$

9. $4x^2 - 3x - 3$

10. $4b^2 + 15b - 4$

11. $9p^2 + 6p - 8$

12. $6q^2 - 13q + 6$

13. $3a^2 + 30a + 63$

14. $10w^2 - 19w - 15$

Solve each equation. Check the solutions.

15. $2x^2 + 7x + 3 = 0$

16. $3w^2 + 14w + 8 = 0$

17. $3n^2 - 7n + 2 = 0$

18. $5d^2 - 22d + 8 = 0$

19. $6h^2 + 8h + 2 = 0$

20. $8p^2 - 16p = 10$

21. $9y^2 + 18y - 12 = 6y$

22. $4a^2 - 16a = -15$

23. $10b^2 - 15b = 8b - 12$

24. $6d^2 + 21d = 10d + 35$

NAME ______________________________ DATE ____________ PERIOD ____________

8-7 Practice

Solving $ax^2 + bx + c = 0$

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

1. $2b^2 + 10b + 12$

2. $3g^2 + 8g + 4$

3. $4x^2 + 4x - 3$

4. $8b^2 - 5b - 10$

5. $6m^2 + 7m - 3$

6. $10d^2 + 17d - 20$

7. $6a^2 - 17a + 12$

8. $8w^2 - 18w + 9$

9. $10x^2 - 9x + 6$

10. $15n^2 - n - 28$

11. $10x^2 + 21x - 10$

12. $9r^2 + 15r + 6$

13. $12y^2 - 4y - 5$

14. $14k^2 - 9k - 18$

15. $8z^2 + 20z - 48$

16. $12q^2 + 34q - 28$

17. $18h^2 + 15h - 18$

18. $12p^2 - 22p - 20$

Solve each equation. Check the solutions.

19. $3h^2 + 2h - 16 = 0$

20. $15n^2 - n = 2$

21. $8q^2 - 10q + 3 = 0$

22. $6b^2 - 5b = 4$

23. $10r^2 - 21r = -4r + 6$

24. $10g^2 + 10 = 29g$

25. $6y^2 = -7y - 2$

26. $9z^2 = -6z + 15$

27. $12k^2 + 15k = 16k + 20$

28. $12x^2 - 1 = -x$

29. $8a^2 - 16a = 6a - 12$

30. $18a^2 + 10a = -11a + 4$

31. DIVING Lauren dove into a swimming pool from a 15-foot-high diving board with an initial upward velocity of 8 feet per second. Find the time t in seconds it took Lauren to enter the water. Use the model for vertical motion given by the equation $h = -16t^2 + vt + s$, where h is height in feet, t is time in seconds, v is the initial upward velocity in feet per second, and s is the initial height in feet. (*Hint:* Let $h = 0$ represent the surface of the pool.)

32. BASEBALL Brad tossed a baseball in the air from a height of 6 feet with an initial upward velocity of 14 feet per second. Enrique caught the ball on its way down at a point 4 feet above the ground. How long was the ball in the air before Enrique caught it? Use the model of vertical motion from Exercise 31.

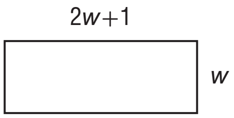
8-7 Word Problem Practice

Solving $ax^2 + bx + c = 0$

1. **BREAK EVEN** Breaking even occurs when the revenues for a business equal the cost. A local children's museum studied their costs and revenues from paid admission. They found that their break-even time is given by the equation $2h^2 - 2h - 24 = 0$, where h is the number of hours the museum is open per day. How many hours must the museum be open per day to reach the break even point?

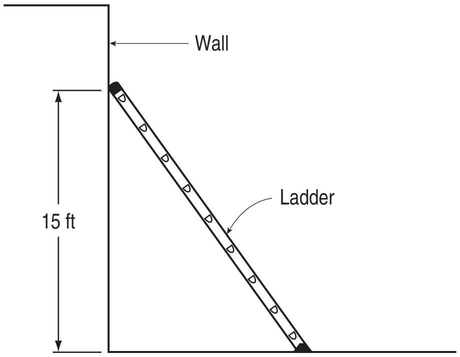
2. **CARPENTRY** Miko wants to build a toy box for her sister. It is to be 2 feet high, and the width is to be 3 feet less than its length. If it needs to hold a volume of 80 cubic feet, find the length and width of the box.

3. **FURNITURE** The student council wants to purchase a table for the school lobby. The table comes in a variety of dimensions, but for every table, the length is 1 meter greater than twice the width. The student council has budgeted for a table top with an area of exactly 3 square meters.



Find the width and length of the table they can purchase.

4. **LADDERS** A ladder is resting against a wall. The top of the ladder touches the wall at a height of 15 feet, and the length of the ladder is one foot more than twice its distance from the wall. Find the distance from the wall to the bottom of the ladder. (*Hint:* Use the Pythagorean Theorem to solve the problem.)



5. **FARMING** Mr. Hensley has a total of 480 square feet of sheet metal with which he would like to construct a cylindrical tank for storing grain. The local zoning law limits the height of the tank to 13.5 feet. Recall that a formula for the surface area of a bottomless cylinder with radius r and height h is $A = \pi r^2 + 2\pi rh$.

   a. Write a quadratic equation to represent the information.

   b. Using 3 as an approximation for π, solve the equation for r.

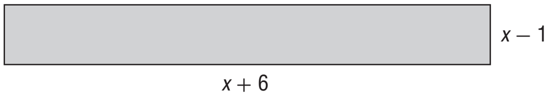
   c. What radius should Mr. Hensley use for his tank?

8-7 Enrichment

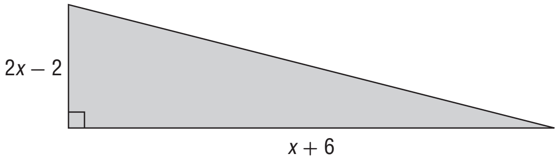
Area Models for Quadratic Trinomials

After you have factored a quadratic trinomial, you can use the factors to draw geometric models of the trinomial.

$x^2 + 5x - 6 = (x - 1)(x + 6)$



To draw a rectangular model, the value 2 was used for x so that the shorter side would have a length of 1. Then the drawing was done in centimeters. So, the area of the rectangle is $x^2 + 5x - 6$.



To draw a right triangle model, recall that the area of a triangle is one-half the base times the height. So, one of the sides must be twice as long as the shorter side of the rectangular model.

$$\begin{aligned} x^2 + 5x - 6 &= (x - 1)(x + 6) \\ &= \frac{1}{2}(2x - 2)(x + 6) \end{aligned}$$

The area of the right triangle is also $x^2 + 5x - 6$.

Factor each trinomial. Then follow the directions to draw each model of the trinomial.

1. $x^2 + 2x - 3$ Use $x = 2$. Draw a rectangle in centimeters.

2. $3x^2 + 5x - 2$ Use $x = 1$. Draw a rectangle in centimeters.

3. $x^2 - 4x + 3$ Use $x = 4$. Draw two different right triangles in centimeters.

4. $9x^2 - 9x + 2$ Use $x = 2$. Draw two different right triangles. Use 0.5 centimeter for each unit.

NAME ______________________________ DATE ____________ PERIOD ____________

8-7 Graphing Calculator Activity

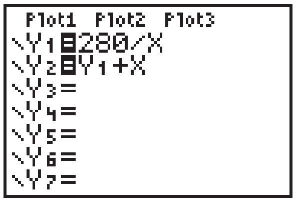
Using Tables in Factoring by Grouping

The **TABLE** feature can be used to help factor a polynomial by finding the factors of a certain product, which have a specific sum.

Example 1 Factor $10x^2 - 43x + 28$ by grouping.

Make a table of the negative factors of $10 \cdot 28$ or 280. Look for a pair of factors whose sum is -43.

Enter the equation $y = \frac{280}{x}$ in **Y1** to find the factors of 280. Then, find the sum of the factors using $y = \frac{280}{x} + x$ in **Y2**. Set up the table to display the negative factors of 280 by setting **ΔTbl** = to -1. Examine the results.



```
 Plot1  Plot2  Plot3
\Y1■280/X
\Y2■Y1+X
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
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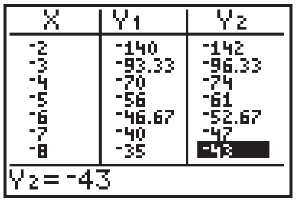


X	Y1	Y2
-2	-140	-142
-3	-93.33	-96.33
-4	-70	-74
-5	-56	-61
-6	-46.67	-52.67
-7	-40	-47
-8	-35	-43

Y2=-43

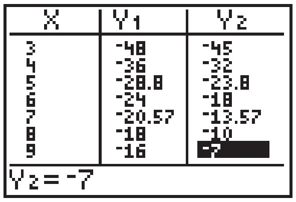
The last line of the table shows that $-43x$ may be replaced with $-8x + (-35x)$.

$$\begin{aligned} 10x^2 - 43x + 28 &= 10x^2 - 8x + (-35x) + 28 \\ &= 2x(5x - 4) + (-7)(5x - 4) \\ &= (5x - 4)(2x - 7) \end{aligned}$$

Thus, $10x^2 - 43x + 28 = (5x - 4)(2x - 7)$.

Example 2 Factor $12x^2 - 7x - 12$.

Look at the factors of $12 \cdot -12$ or -144 for a pair whose sum is -7. Enter an equation to determine the factors in **Y1** and an equation to find the sum of factors in **Y2.** Examine the table to find a sum of -7.



X	Y1	Y2
3	-48	-45
4	-36	-32
5	-28.8	-23.8
6	-24	-18
7	-20.57	-13.57
8	-18	-10
9	-16	-7

Y2=-7

$$\begin{aligned} 12x^2 - 7x - 12 &= 12x^2 + 9x + (-16x) - 12 \\ &= 3x(4x + 3) - 4(4x + 3) \\ &= (4x + 3)(3x - 4) \end{aligned}$$

Thus, $12x^2 - 7x - 12 = (4x + 3)(3x - 4)$.

Exercises

Factor each quadratic polynomial if possible.

1. $x^2 + 29x - 96$

2. $x^2 - 14x - 51$

3. $3z^2 + 16z - 35$

4. $4y^2 - 25y + 18$

5. $6a^2 - a - 15$

6. $6m^2 + 13m + 6$

7. $12z^2 - z - 6$

8. $16y^2 + 40y + 25$

9. $4b^2 + 24b - 493$

NAME ______________________ DATE ____________ PERIOD ____________

8-8 Study Guide and Intervention

Differences of Squares

Factor Differences of Squares The binomial expression $a^2 - b^2$ is called the **difference of two squares**. The following pattern shows how to factor the difference of squares.

Difference of Squares	$a^2 - b^2 = (a - b)(a + b) = (a + b)(a - b)$.

Example 1 **Factor each polynomial.**

a. $n^2 - 64$

$n^2 - 64$

$= n^2 - 8^2$ Write in the form $a^2 - b^2$.

$= (n + 8)(n - 8)$ Factor.

b. $4m^2 - 81n^2$

$4m^2 - 81n^2$

$= (2m)^2 - (9n)^2$ Write in the form $a^2 - b^2$.

$= (2m - 9n)(2m + 9n)$ Factor.

Example 2 **Factor each polynomial.**

a. $50a^2 - 72$

$50a^2 - 72$

$= 2(25a^2 - 36)$ Find the GCF.

$= 2[(5a)^2 - 6^2)]$ $25a^2 = 5a \cdot 5a$ and $36 = 6 \cdot 6$

$=2(5a + 6)(5a - 6)$ Factor the difference of squares.

b. $4x^4 + 8x^3 - 4x^2 - 8x$

$4x^4 + 8x^3 - 4x^2 - 8x$ Original polynomial

$= 4x(x^3 + 2x^2 - x - 2)$ Find the GCF.

$= 4x[(x^3 + 2x^2) - (x + 2)]$ Group terms.

$= 4x[x^2(x + 2) - 1(x + 2)]$ Find the GCF.

$= 4x[(x^2 - 1)(x + 2)]$ Factor by grouping.

$= 4x[(x - 1)(x + 1)(x + 2)]$ Factor the difference of squares.

Exercises

Factor each polynomial.

1. $x^2 - 81$

2. $m^2 - 100$

3. $16n^2 - 25$

4. $36x^2 - 100y^2$

5. $49x^2 - 36$

6. $16a^2 - 9b^2$

7. $225b^2 - a^2$

8. $72p^2 - 50$

9. $-2 + 2x^2$

10. $-81 + a^4$

11. $6 - 54a^2$

12. $8y^2 - 200$

13. $4x^3 - 100x$

14. $2y^4 - 32y^2$

15. $8m^3 - 128m$

16. $4x^2 - 25$

17. $2a^3 - 98ab^2$

18. $18y^2 - 72y^4$

19. $169x^3 - x$

20. $3a^4 - 3a^2$

21. $3x^4 + 6x^3 - 3x^2 - 6x$

8-8 Study Guide and Intervention *(continued)*

Differences of Squares

Solve Equations by Factoring Factoring and the Zero Product Property can be used to solve equations that can be written as the product of any number of factors set equal to 0.

Example **Solve each equation. Check your solutions.**

a. $\mathbf{x^2 - \frac{1}{25} = 0}$

$x^2 - \frac{1}{25} = 0$	Original equation
$x^2 - \left(\frac{1}{5}\right)^2 = 0$	$x^2 = x \cdot x$ and $\frac{1}{25} = \left(\frac{1}{5}\right)\left(\frac{1}{5}\right)$
$\left(x + \frac{1}{5}\right)\left(x - \frac{1}{5}\right) = 0$	Factor the difference of squares.
$x + \frac{1}{5} = 0$ or $x - \frac{1}{5} = 0$	Zero Product Property
$x = -\frac{1}{5}$ $\qquad$ $x = \frac{1}{5}$	Solve each equation.

The solution set is $\left\{-\frac{1}{5}, \frac{1}{5}\right\}$. Since $\left(-\frac{1}{5}\right)^2 - \frac{1}{25} = 0$ and $\left(\frac{1}{5}\right)^2 - \frac{1}{25} = 0$, the solutions check.

b. $\mathbf{4x^3 = 9x}$

$4x^3 = 9x$	Original equation
$4x^3 - 9x = 0$	Subtract 9x from each side.
$x(4x^2 - 9) = 0$	Factor out the GCF of x.
$x[(2x)^2 - 3^2] = 0$	$4x^2 = 2x \cdot 2x$ and $9 = 3 \cdot 3$
$x[(2x)^2 - 3^2] = x[(2x - 3)(2x + 3)]$	Factor the difference of squares.
$x = 0$ or $(2x - 3) = 0$ or $(2x + 3) = 0$	Zero Product Property
$x = 0$ $\qquad$ $x = \frac{3}{2}$ $\qquad$ $x = -\frac{3}{2}$	Solve each equation.

The solution set is $\left\{0, \frac{3}{2}, -\frac{3}{2}\right\}$.

Since $4(0)^3 = 9(0)$, $4\left(\frac{3}{2}\right)^3 = 9\left(\frac{3}{2}\right)$, and $4\left(-\frac{3}{2}\right)^3 = 9\left(-\frac{3}{2}\right)$, the solutions check.

Exercises

Solve each equation by factoring. Check the solutions.

1. $81x^2 = 49$

2. $36n^2 = 1$

3. $25d^2 - 100 = 0$

4. $\frac{1}{4}x^2 = 25$

5. $36 = \frac{1}{25}x^2$

6. $\frac{49}{100} - x^2 = 0$

7. $9x^3 = 25x$

8. $7a^3 = 175a$

9. $2m^3 = 32m$

10. $16y^3 = 25y$

11. $\frac{1}{64}x^2 = 49$

12. $4a^3 - 64a = 0$

13. $3b^3 - 27b = 0$

14. $\frac{9}{25}m^2 = 121$

15. $48n^3 = 147n$

NAME ______________________________ DATE ____________ PERIOD ____________

8-8 Skills Practice

Differences of Squares

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

1. $a^2 - 4$

2. $n^2 - 64$

3. $1 - 49d^2$

4. $-16 + p^2$

5. $k^2 + 25$

6. $36 - 100w^2$

7. $t^2 - 81u^2$

8. $4h^2 - 25g^2$

9. $64m^2 - 9y^2$

10. $4c^2 - 5d^2$

11. $-49r^2 + 4t^2$

12. $8x^2 - 72p^2$

13. $20q^2 - 5r^2$

14. $32a^2 - 50b^2$

Solve each equation by factoring. Check the solutions.

15. $16x^2 - 9 = 0$

16. $25p^2 - 16 = 0$

17. $36q^2 - 49 = 0$

18. $81 - 4b^2 = 0$

19. $16d^2 = 4$

20. $18a^2 = 8$

21. $n^2 - \frac{9}{25} = 0$

22. $k^2 - \frac{49}{64} = 0$

23. $\frac{1}{25}h^2 - 16 = 0$

24. $\frac{1}{16}y^2 = 81$

NAME ______________________________ DATE ____________ PERIOD ____________

8-8 Practice

Differences of Squares

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

1. $k^2 - 100$

2. $81 - r^2$

3. $16p^2 - 36$

4. $4x^2 + 25$

5. $144 - 9f^2$

6. $36g^2 - 49h^2$

7. $121m^2 - 144p^2$

8. $32 - 8y^2$

9. $24a^2 - 54b^2$

10. $32t^2 - 18u^2$

11. $9d^2 - 32$

12. $36z^3 - 9z$

13. $45q^3 - 20q$

14. $100b^3 - 36b$

15. $3t^4 - 48t^2$

Solve each equation by factoring. Check your solutions.

16. $4y^2 = 81$

17. $64p^2 = 9$

18. $98b^2 - 50 = 0$

19. $32 - 162k^2 = 0$

20. $t^2 - \frac{64}{121} = 0$

21. $\frac{16}{49} - v^2 = 0$

22. $\frac{1}{36}x^2 - 25 = 0$

23. $27h^3 = 48h$

24. $75g^3 = 147g$

25. EROSION A rock breaks loose from a cliff and plunges toward the ground 400 feet below. The distance d that the rock falls in t seconds is given by the equation $d = 16t^2$. How long does it take for the rock to hit the ground?

26. FORENSICS Mr. Cooper contested a speeding ticket given to him after he applied his brakes and skidded to a halt to avoid hitting another car. In traffic court, he argued that the length of the skid marks on the pavement, 150 feet, proved that he was driving under the posted speed limit of 65 miles per hour. The ticket cited his speed at 70 miles per hour. Use the formula $\frac{1}{24}s^2 = d$, where s is the speed of the car and d is the length of the skid marks, to determine Mr. Cooper's speed when he applied the brakes. Was Mr. Cooper correct in claiming that he was not speeding when he applied the brakes?

8-8 Word Problem Practice

Differences of Squares

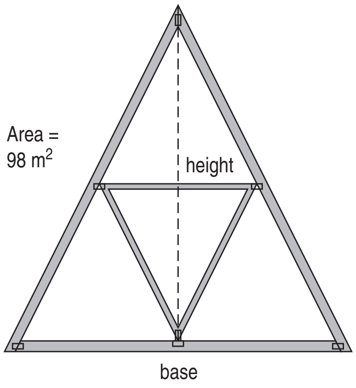
1. **LOTTERY** A state lottery commission analyzes the ticket purchasing patterns of its citizens. The following expression is developed to help officials calculate the likely number of people who will buy tickets for a certain size jackpot.

$$81a^2 - 36b^2$$

Factor the expression completely.

2. **OPTICS** A reflector on the inside of a certain flashlight is a parabola given by the equation $y = x^2 - 25$. Find the points where the reflector meets the lens by finding the values of x when $y = 0$.

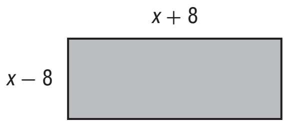
3. **ARCHITECTURE** The drawing shows a triangular roof truss with a base measuring the same as its height. The area of the truss is 98 square meters.



Find the height of the truss.

4. **BALLOONING** The function $f(t) = -16t^2 + 576$ represents the height of a freely falling ballast bag that starts from rest on a balloon 576 feet above the ground. After how many seconds t does the ballast bag hit the ground?

5. **DECORATING** Marvin wants to purchase a rectangular rug. It has an area of 80 square feet. He cannot remember the length and width, but he remembers that the length was 8 more than some number and the width was 8 less than that same number.



a. Write a quadratic equation using the information given.

b. What are the length and width of the rug?

NAME ______________________________ DATE ____________ PERIOD ____________

8-8 Enrichment

Factoring Trinomials of Fourth Degree

Some trinomials of the form $a^4 + a^2b^2 + b^4$ can be written as the difference of two squares and then factored.

Example **Factor $4x^4 - 37x^2y^2 + 9y^4$.**

Step 1 Find the square roots of the first and last terms.

$\sqrt{4x^4} = 2x^2$ $\sqrt{9y^4} = 3y^2$

Step 2 Find twice the product of the square roots.

$2(2x^2)(3y^2) = 12x^2y^2$

Step 3 Separate the middle term into two parts. One part is either your answer to Step 2 or its opposite. The other part should be the opposite of a perfect square.

$-37x^2y^2 = -12x^2y^2 - 25x^2y^2$

Step 4 Rewrite the trinomial as the difference of two squares and then factor.

$$\begin{aligned}4x^4 - 37x^2y^2 + 9y^4 &= (4x^4 - 12x^2y^2 + 9y^4) - 25x^2y^2\\ &= (2x^2 - 3y^2)^2 - 25x^2y^2\\ &= [(2x^2 - 3y^2) + 5xy][(2x^2 - 3y^2) - 5xy]\\ &= (2x^2 + 5xy - 3y^2)(2x^2 - 5xy - 3y^2)\end{aligned}$$

Factor each polynomial.

1. $x^4 + x^2y^2 + y^4$

2. $x^4 + x^2 + 1$

3. $9a^4 - 15a^2 + 1$

4. $16a^4 - 17a^2 + 1$

5. $4a^4 - 13a^2 + 1$

6. $9a^4 + 26a^2b^2 + 25b^4$

7. $4x^4 - 21x^2y^2 + 9y^4$

8. $4a^4 - 29a^2b^2 + 25b^4$

Lesson 8-8

NAME ______________________________ DATE ____________ PERIOD ____________

8-8 Spreadsheet Activity

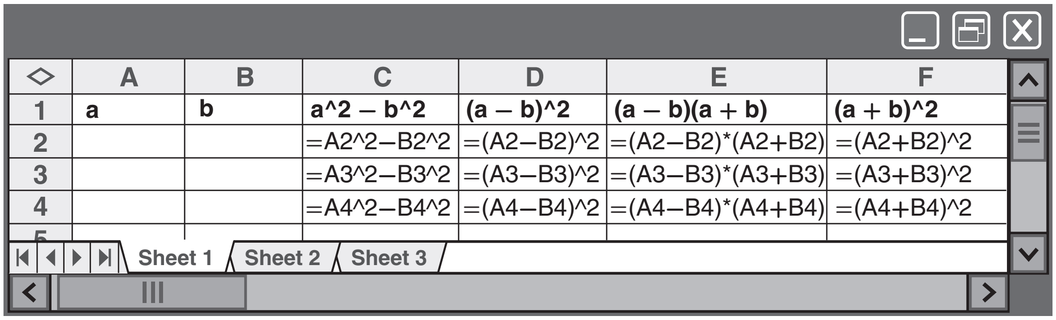
Differences of Squares

There is a special pattern you can use to factor binomials of the form $a^2 - b^2$. You can use a spreadsheet to discover this relationship.

Example **Use a spreadsheet to investigate the values of the expressions $(a^2 - b^2)$, $(a - b)^2$, $(a - b)(a + b)$, and $(a + b)^2$. What conjecture can you make about the expressions?**

Step 1 You will use Columns A and B to enter various values that you choose for a and b.

Step 2 Enter the formulas for $(a^2 - b^2)$, $(a - b)^2$, $(a - b)(a + b)$, and $(a + b)^2$ in Columns C, D, E, and F. To enter an exponent, use the symbol ^ followed by the exponent. For example, the square of the value in cell A2 is entered as A2^2.



◇	A	B	C	D	E	F
1	a	b	a^2 – b^2	(a – b)^2	(a – b)(a + b)	(a + b)^2
2			=A2^2–B2^2	=(A2–B2)^2	=(A2–B2)*(A2+B2)	=(A2+B2)^2
3			=A3^2–B3^2	=(A3–B3)^2	=(A3–B3)*(A3+B3)	=(A3+B3)^2
4			=A4^2–B4^2	=(A4–B4)^2	=(A4–B4)*(A4+B4)	=(A4+B4)^2
5						

Exercises

1. Enter various values of a and b in Columns A and B. Look for a pattern. What do you observe about the expressions?

2. Find the products of $(a - b)^2$, $(a - b)(a + b)$, and $(a + b)^2$. Do the results verify your conjecture?

Use the pattern you observed to factor each binomial.

3. $m^2 - t^2$

4. $x^2 - 4$

5. $y^2 - 16$

6. $q^2 - 121$

7. $r^2 - 169$

8. $b^2 - 1$

9. $4x^2 - 1$

10. $16t^2 - r^2$

11. $25a^2 - 81d^2$

NAME ______________________________ DATE ____________ PERIOD ____________

8-9 Study Guide and Intervention

Perfect Squares

Factor Perfect Square Trinomials

Perfect Square Trinomial	a trinomial of the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$

The patterns shown below can be used to factor perfect square trinomials.

Squaring a Binomial	Factoring a Perfect Square Trinomial
$(a + 4)^2 = a^2 + 2(a)(4) + 4^2$ $= a^2 + 8a + 16$	$a^2 + 8a + 16 = a^2 + 2(a)(4) + 4^2$ $= (a + 4)^2$
$(2x - 3)^2 = (2x)^2 - 2(2x)(3) + 3^2$ $= 4x^2 - 12x + 9$	$4x^2 - 12x + 9 = (2x)^2 - 2(2x)(3) + 3^2$ $= (2x - 3)^2$

Example 1 **Determine whether $16n^2 - 24n + 9$ is a perfect square trinomial. If so, factor it.**

Since $16n^2 = (4n)(4n)$, the first term is a perfect square.

Since $9 = 3 \cdot 3$, the last term is a perfect square.

The middle term is equal to $2(4n)(3)$.

Therefore, $16n^2 - 24n + 9$ is a perfect square trinomial.

$$16n^2 - 24n + 9 = (4n)^2 - 2(4n)(3) + 3^2$$
$$= (4n - 3)^2$$

Example 2 **Factor $16x^2 - 32x + 15$.**

Since 15 is not a perfect square, use a different factoring pattern.

$16x^2 - 32x + 15$	Original trinomial
$= 16x^2 + mx + px + 15$	Write the pattern.
$= 16x^2 - 12x - 20x + 15$	$m = -12$ and $p = -20$
$= (16x^2 - 12x) - (20x - 15)$	Group terms.
$= 4x(4x - 3) - 5(4x - 3)$	Find the GCF.
$= (4x - 5)(4x - 3)$	Factor by grouping.

Therefore $16x^2 - 32x + 15 = (4x - 5)(4x - 3)$.

Exercises

Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*. If so, factor it.

1. $x^2 - 16x + 64$

2. $m^2 + 10m + 25$

3. $p^2 + 8p + 64$

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

4. $98x^2 - 200y^2$

5. $x^2 + 22x + 121$

6. $81 + 18j + j^2$

7. $25c^2 - 10c - 1$

8. $169 - 26r + r^2$

9. $7x^2 - 9x + 2$

10. $16m^2 + 48m + 36$

11. $16 - 25a^2$

12. $b^2 - 16b + 256$

13. $36x^2 - 12x + 1$

14. $16a^2 - 40ab + 25b^2$

15. $8m^3 - 64m$

NAME ______________________________ DATE ____________ PERIOD ____________

8-9 Study Guide and Intervention *(continued)*

Perfect Squares

Solve Equations with Perfect Squares

Factoring and the Zero Product Property can be used to solve equations that involve repeated factors. The repeated factor gives just one solution to the equation. You may also be able to use the **Square Root Property** below to solve certain equations.

Square Root Property	For any number $n > 0$, if $x^2 = n$, then $x = \pm\sqrt{n}$.

Example **Solve each equation. Check your solutions.**

a. $\mathbf{x^2 - 6x + 9 = 0}$

$x^2 - 6x + 9 = 0$ Original equation

$x^2 - 2(3x) + 3^2 = 0$ Recognize a perfect square trinomial.

$(x - 3)(x - 3) = 0$ Factor the perfect square trinomial.

$x - 3 = 0$ Set repeated factor equal to 0.

$x = 3$ Solve.

The solution set is {3}. Since $3^2 - 6(3) + 9 = 0$, the solution checks.

b. $\mathbf{(a - 5)^2 = 64}$

$(a - 5)^2 = 64$ Original equation

$a - 5 = \pm\sqrt{64}$ Square Root Property

$a - 5 = \pm 8$ $64 = 8 \cdot 8$

$a = 5 \pm 8$ Add 5 to each side.

$a = 5 + 8$ or $a = 5 - 8$ Separate into 2 equations.

$a = 13$ $a = -3$ Solve each equation.

The solution set is {−3, 13}. Since $(-3 - 5)^2 = 64$ and $(13 - 5)^2 = 64$, the solutions check.

Exercises

Solve each equation. Check the solutions.

1. $x^2 + 4x + 4 = 0$

2. $16n^2 + 16n + 4 = 0$

3. $25d^2 - 10d + 1 = 0$

4. $x^2 + 10x + 25 = 0$

5. $9x^2 - 6x + 1 = 0$

6. $x^2 + x + \frac{1}{4} = 0$

7. $25k^2 + 20k + 4 = 0$

8. $p^2 + 2p + 1 = 49$

9. $x^2 + 4x + 4 = 64$

10. $x^2 - 6x + 9 = 25$

11. $a^2 + 8a + 16 = 1$

12. $16y^2 + 8y + 1 = 0$

13. $(x + 3)^2 = 49$

14. $(y + 6)^2 = 1$

15. $(m - 7)^2 = 49$

16. $(2x + 1)^2 = 1$

17. $(4x + 3)^2 = 25$

18. $(3h - 2)^2 = 4$

19. $(x + 1)^2 = 7$

20. $(y - 3)^2 = 6$

21. $(m - 2)^2 = 5$

NAME ______________________ DATE __________ PERIOD __________

8-9 Skills Practice

Perfect Squares

Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*. If so, factor it.

1. $m^2 - 6m + 9$

2. $r^2 + 4r + 4$

3. $g^2 - 14g + 49$

4. $2w^2 - 4w + 9$

5. $4d^2 - 4d + 1$

6. $9n^2 + 30n + 25$

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

7. $2x^2 - 72$

8. $6b^2 + 11b + 3$

9. $36t^2 - 24t + 4$

10. $4h^2 - 56$

11. $17a^2 - 24ab$

12. $q^2 - 14q + 36$

13. $y^2 + 24y + 144$

14. $6d^2 - 96$

Solve each equation. Check the solutions.

15. $x^2 - 18x + 81 = 0$

16. $4p^2 + 4p + 1 = 0$

17. $9g^2 - 12g + 4 = 0$

18. $y^2 - 16y + 64 = 81$

19. $4n^2 - 17 = 19$

20. $x^2 + 30x + 150 = -75$

21. $(k + 2)^2 = 16$

22. $(m - 4)^2 = 7$

NAME ______________________________ DATE ____________ PERIOD ____________

8-9 Practice

Perfect Squares

Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*. If so, factor it.

1. $m^2 + 16m + 64$

2. $9r^2 - 6r + 1$

3. $4y^2 - 20y + 25$

4. $16p^2 + 24p + 9$

5. $25b^2 - 4b + 16$

6. $49k^2 - 56k + 16$

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

7. $3p^2 - 147$

8. $6x^2 + 11x - 35$

9. $50q^2 - 60q + 18$

10. $6t^3 - 14t^2 - 12t$

11. $6d^2 - 18$

12. $30k^2 + 38k + 12$

13. $15b^2 - 24bf$

14. $12h^2 - 60h + 75$

15. $9n^2 - 30n - 25$

16. $7u^2 - 28m^2$

17. $w^4 - 8w^2 - 9$

18. $16a^2 + 72ad + 81d^2$

Solve each equation. Check the solutions.

19. $4k^2 - 28k = -49$

20. $50b^2 + 20b + 2 = 0$

21. $\left(\frac{1}{2}t - 1\right)^2 = 0$

22. $g^2 + \frac{2}{3}g + \frac{1}{9} = 0$

23. $p^2 - \frac{6}{5}p + \frac{9}{25} = 0$

24. $x^2 + 12x + 36 = 25$

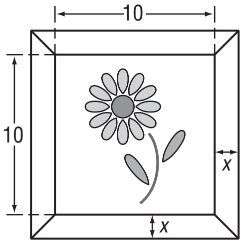
25. $y^2 - 8y + 16 = 64$

26. $(h + 9)^2 = 3$

27. $w^2 - 6w + 9 = 13$

28. GEOMETRY The area of a circle is given by the formula $A = \pi r^2$, where r is the radius. If increasing the radius of a circle by 1 inch gives the resulting circle an area of 100π square inches, what is the radius of the original circle?

29. PICTURE FRAMING Mikaela placed a frame around a print that measures 10 inches by 10 inches. The area of just the frame itself is 69 square inches. What is the width of the frame?



8-9 Word Problem Practice

Perfect Squares

1. **CONSTRUCTION** The area of Liberty Township's square playground is represented by the trinomial $x^2 - 10x + 25$. Write an expression using the variable x that represents the perimeter.

2. **AMUSEMENT PARKS** Funtown Downtown wants to build a vertical motion ride where the passengers are launched straight upward from ground level with an initial velocity of 96 feet per second. The ride car's height h in feet after t seconds is $h = 96t - 16t^2$. How many seconds after launch would the car reach 144 feet?

3. **BUSINESS** Saini Sprinkler Company installs irrigation systems. To track monthly costs C and revenues R, they use the following functions, where x is the number of systems they install.

$$R(x) = 8x^2 + 12x + 4$$

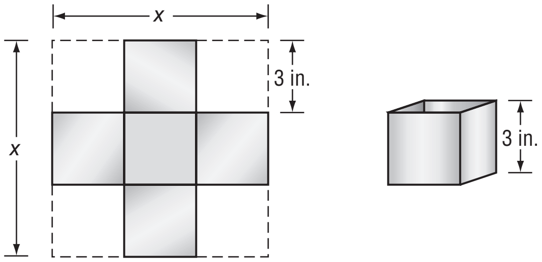
$$C(x) = 7x^2 + 20x - 12$$

The monthly profit can be found by subtracting cost from revenue.

$$P(x) = R(x) - C(x)$$

Find a function to project monthly profit and use it to find the break-even point where the profit is zero.

4. **GEOMETRY** Holly can make an open-topped box out of a square piece of cardboard by cutting 3-inch squares from the corners and folding up the sides to meet. The volume of the resulting box is $V = 3x^2 - 36x + 108$, where x is the original length and width of the cardboard.



a. Factor the polynomial expression from the volume equation.

b. What is the volume of the box if the original length of each side of the cardboard was 14 inches?

c. What is the original side length of the cardboard when the volume of the box is 27 in^3?

NAME ______________________ DATE __________ PERIOD __________

8-9 Enrichment

Squaring Numbers: A Shortcut

A shortcut helps you to square a positive two-digit number ending in 5. The method is developed using the idea that a two-digit number may be expressed as $10t + u$. Suppose $u = 5$.

$$\begin{aligned}(10t + 5)^2 &= (10t + 5)(10t + 5)\\ &= 100t^2 + 50t + 50t + 25\\ &= 100t^2 + 100t + 25\\ (10t + 5)^2 &= 100t(t + 1) + 25\end{aligned}$$

In words, this formula says that the square of a two-digit number ending in 5 has $t(t + 1)$ in the hundreds place. Then 2 is the tens digit and 5 is the units digit.

Example **Using the formula for $(10t + 5)^2$, find 85^2.**

$$\begin{aligned}85^2 &= 100 \cdot 8 \cdot (8 + 1) + 25\\ &= 7200 + 25\\ &= 7225\end{aligned}$$

Shortcut: First think $8 \cdot 9 = 72$. Then write 25.

Thus, to square a number, such as 85, you can write the product of the tens digit and the next consecutive integer $t + 1$. Then write 25.

Find each of the following using the shortcut.

1. 15^2

2. 25^2

3. 35^2

4. 45^2

5. 55^2

6. 65^2

7. What is the tens digit in the square of 95?

8. What are the first two digits in the square of 75?

9. Any three-digit number can be written as $100a + 10b + c$. Square this expression to show that if the last digit of a three-digit number is 5 then the last two digits of the square of the number are 2 and 5.

NAME ______________________________ DATE ____________ PERIOD ____________

8 Student Recording Sheet

Use this recording sheet with pages 538–539 of the Student Edition.

Multiple Choice

Read each question. Then fill in the correct answer.

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. Ⓕ Ⓖ Ⓗ Ⓙ

3. Ⓐ Ⓑ Ⓒ Ⓓ

4. Ⓕ Ⓖ Ⓗ Ⓙ

5. Ⓐ Ⓑ Ⓒ Ⓓ

6. Ⓕ Ⓖ Ⓗ Ⓙ

7. Ⓐ Ⓑ Ⓒ Ⓓ

8. Ⓕ Ⓖ Ⓗ Ⓙ

9. Ⓐ Ⓑ Ⓒ Ⓓ

Short Response/Gridded Response

Record your answer in the blank.

For gridded response questions, also enter your answer in the grid by writing each number or symbol in a box. Then fill in the corresponding circle for that number or symbol.

10. ____________ *(grid in)*

11. ____________

12. ____________

13. ____________

14. ____________

15. ____________

16. ____________ *(grid in)*

10.

	⊘	⊘	⊘	
⊙	⊙	⊙	⊙	⊙
⓪	⓪	⓪	⓪	⓪
①	①	①	①	①
②	②	②	②	②
③	③	③	③	③
④	④	④	④	④
⑤	⑤	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	⑦	⑦
⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨

16.

	⊘	⊘	⊘	
⊙	⊙	⊙	⊙	⊙
⓪	⓪	⓪	⓪	⓪
①	①	①	①	①
②	②	②	②	②
③	③	③	③	③
④	④	④	④	④
⑤	⑤	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	⑦	⑦
⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨

Extended Response

Record your answers for Question 17 on the back of this paper.

NAME ______________________________ DATE ____________ PERIOD ____________

8 Rubric for Scoring Extended Response

General Scoring Guidelines

- If a student gives only a correct numerical answer to a problem but does not show how he or she arrived at the answer, the student will be awarded only 1 credit. All extended-response questions require the student to show work.
- A fully correct answer for a multiple-part question requires correct responses for all parts of the question. For example, if a question has three parts, the correct response to one or two parts of the question that required work to be shown is *not* considered a fully correct response.
- Students who use trial and error to solve a problem must show their method. Merely showing that the answer checks or is correct is not considered a complete response for full credit.

Exercise 17 Rubric

Score	Specific Criteria
4	In part **a**, the student correctly factors out a t term to rewrite the function as $h(t) = t(-16t + 200)$. In part **b**, the function is solved for $h(0)$ to get $t = 0$, 12.5 seconds. The student shows understanding that this means the rocket is on the ground at take off and again 12.5 seconds later. In part **c**, the student finds the greatest height achieved by the rocket to be 625 feet after 6.25 seconds.
3	A generally correct solution, but may contain minor flaws in reasoning or computation.
2	A partially correct interpretation and/or solution to the problem.
1	A correct solution with no evidence or explanation.
0	An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given.

NAME ______________________________ DATE ____________ PERIOD ____________

8 Chapter 8 Quiz 1

SCORE ________

(Lessons 8-1 and 8-2)

Find the degree of each polynomial.

1. $5a - 2b^2 + 1$ **2.** $24xy - xy^3 + x^2$

Write each polynomial in standard form. Identify the leading coefficient.

3. $4x^2 - 3x^3 + 2x + 12$ **4.** $4x^2 - 7x^4 + x - 15$

Find each sum or difference.

5. $(-2x^2 + x + 6) + (5x^2 - 4x - 2)$

6. $(5a + 9b) - (2a + 4b)$

7. MULTIPLE CHOICE Simplify $5c^2(c + 10) - 4c(2c^2 - 6c + 1)$.

A $-3c^3 + 74c^2 - 4c$ **C** $-3c^3 - 6c + 11$

B $-3c^3 - 26c^2 + 4c$ **D** $3c^3 - 9c + 11$

8. Solve $4t(t - 5) - 2t(3t + 2) + 108 = 3t(2t - 5) - t(8t - 3)$.

1. ____________

2. ____________

3. ____________

4. ____________

5. ____________

6. ____________

7. ____________

8. ____________

NAME ______________________________ DATE ____________ PERIOD ____________

8 Chapter 8 Quiz 2

SCORE ________

(Lesson 8-3, 8-4, and 8-5)

Find each product.

1. $(2x + 1)(x - 4)$ **2.** $(3b + 4)(2b^2 - b + 4)$

3. $(4m - 1)(m + 2)$ **4.** $(2x + 6y)(2x - 6y)$

5. $(a - 3b)(a + 3b)$ **6.** $(x + 7)^2$

Factor each polynomial completely.

7. $48a^2b^2 - 12ab$ **8.** $6x^2y - 21y^2w + 24xw$

Solve each equation. Check your solutions.

9. $(y + 4)(3y - 5) = 0$ **10.** $y^2 = -11y$

11. MULTIPLE CHOICE Which is a factor of $2x^2 - 12x - 14$?

A $x - 7$ **C** $x + 7$

B $2x - 2$ **D** $2x$

1. ____________

2. ____________

3. ____________

4. ____________

5. ____________

6. ____________

7. ____________

8. ____________

9. ____________

10. ____________

11. ____________

NAME ______________________ DATE ________ PERIOD ________

8 Chapter 8 Quiz 3

SCORE ______

(Lessons 8-6 and 8-7)

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

1. $a^2 - 10a + 21$ **2.** $x^2 + 2x - 15$

3. $2x^2 + 7x + 3$ **4.** $6x^2 + x + 2$

5. MULTIPLE CHOICE Factor $x^2 + x - 20$.

A $(x + 4)(x + 5)$ **C** $(x - 4)(x + 5)$

B $(x + 4)(x - 5)$ **D** $(x - 4)(x - 5)$

Solve each equation. Check your solutions.

6. $x^2 - 6x - 27 = 0$ **7.** $y^2 + 23y = 24$

8. $3n^2 + 6 = 11n$ **9.** $10x^2 + 11x - 6 = 0$

10. Find two consecutive odd integers whose product is 195.

1. ______________
2. ______________
3. ______________
4. ______________
5. ______________
6. ______________
7. ______________
8. ______________
9. ______________
10. ______________

NAME ______________________ DATE ________ PERIOD ________

8 Chapter 8 Quiz 4

SCORE ______

(Lessons 8-8 and 8-9)

Factor each polynomial.

1. $a^2 - 25$ **2.** $49x^2 - 64y^2$ **3.** $x^3 + 3x^2 - 4x - 12$

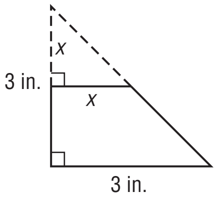
4. $a^2 + 14a + 49$ **5.** $9z^2 - 6z + 1$ **6.** $8m^3 - 24m^2 + 18m$

Solve each equation by factoring. Check your solutions.

7. $16x^2 = 81$ **8.** $\frac{4}{9}p^2 - 25 = 0$

9. $16r^2 - 8r + 1 = 0$ **10.** $(x - 5)^2 = 8$

11. MULTIPLE CHOICE A corner is cut off a right triangle whose legs each measure 3 inches. The cut is x inches from the vertex and parallel to the opposite leg. Write an equation in terms of x that represents the area A of the figure after the corner is removed.



A $9 + 3x$ **B** $\frac{3}{2}x - \frac{9}{2}$ **C** $\frac{9}{2} - \frac{1}{2}x^2$ **D** $\frac{9}{2} + \frac{3}{2}x$

1. ______________
2. ______________
3. ______________
4. ______________
5. ______________
6. ______________
7. ______________
8. ______________
9. ______________
10. ______________
11. ______________

NAME ______________________ DATE ____________ PERIOD ____________

8 Chapter 8 Mid-Chapter Test

SCORE ________

(Lessons 8-1 through 8-5)

Part I **Write the letter for the correct answer in the blank at the right of each question.**

1. Find $(x^3 - x + 1) - (3x - 1)$.

A $3x^3 - x$ **C** $x^3 - 4x + 2$

B $x^3 - 4x$ **D** $x^3 + 2x + 2$ **1.** ________

2. Simplify $3a(a^2 - 3a + 4) - 4(3a^3 - 2a^2)$.

F $-12a^3 + 11a^2 - 9a + 12$ **H** $-9a^3 - a^2 + 12a$

G $-15a^3 - 17a^2 + 12a$ **J** $-3a^3 + 5a^2 + 4a$ **2.** ________

3. Find $(2a - 3b)^2$.

A $4a^2 - 9b^2$ **C** $4a^2 - 6ab - 9b^2$

B $2a^2 - 6ab + 3b^2$ **D** $4a^2 - 12ab + 9b^2$ **3.** ________

4. Find $(2x + 11)(3x - 7)$.

F $6x^2 + 19x^2 - 77$ **H** $5x^2 + 47x - 4$

G $5x^2 - 47x + 4$ **J** $6x^2 - 19x + 77$ **4.** ________

5. Factor $15g^3h^2 - 35g^2h + 40g$.

A $5g(3g^2h^2 - 7gh + 8)$ **C** $5(3g^3h^2 - 7g^2h + 8g)$

B $15g^2h(gh - 3g + 4)$ **D** $g(15g^2h^2 - 35gh + 40)$ **5.** ________

6. Solve $4x^2 - 3x = 0$.

F $-\frac{3}{4}, 0$ **G** $0, 0$ **H** $\frac{3}{4}, 0$ **J** $\frac{4}{3}, 0$ **6.** ________

7. Factor $75b^2c^3 + 60bc^6 - 35b^2c^4$ completely.

A $5b^2c^6(15c^3 + 12b - 7c^2)$ **C** $15bc(5bc^2 + 4c^5 - 7bc^3)$

B $5bc^3(15b + 12c^3 - 7bc)$ **D** $bc^3(75b + 60c^3 - 35bc)$ **7.** ________

Part II

8. Factor the monomial $-70a^2b^2c$ completely. **8.** ________________

9. **FALL** Diego drops his camera as he climbs a hill and it falls to the ground 256 feet below. The distance d that the camera falls in t seconds is given by the equation $d = 16t^2$. How long does it take the camera to hit the ground? **9.** ________________

Factor each polynomial.

10. $36xy^2 - 48x^2y$ **11.** $t^2 - 16t + 48$ **12.** $2xy - x + 4y - 2$

10. ________________

11. ________________

12. ________________

Simplify each expression.

13. $(3g^3 - 2g^2 - 2) - (4g^2 - g - 3)$ **13.** ________________

14. $(3y - 4)(2y + 5)$ **14.** ________________

15. $(4m - 5n)^2$ **15.** ________________

NAME ______________________________ DATE ____________ PERIOD ____________

8 Chapter 8 Vocabulary Test

SCORE ________

difference of two squares	FOIL method	quadratic equation
factoring	perfect square trinomials	quadratic expression
factoring by grouping	prime polynomial	Zero Product Property

Write the letter of the term that best matches each statement or each expression.

_____ **1.** $x^2 + 8x + 16$

_____ **2.** $x^2 + 4x + 7 = 0$

_____ **3.** $x^2 - 4$

_____ **4.** $x^2 + 49$

_____ **5.** $(x - 2)(x + 3) = 0$

_____ **6.** factoring technique often used if a polynomial has 4 or more terms

_____ **7.** an expression in one variable with a degree of 2

a. difference of two squares
b. factoring by grouping
c. perfect square trinomial
d. prime polynomial
e. quadratic equation
f. quadratic expression
g. Zero Product Property

Define each term in your own words.

8. factoring

8. ____________________

9. FOIL method

9. ____________________

NAME ______________________________ DATE ____________ PERIOD ____________

8 Chapter 8 Test, Form 1

SCORE ________

Assessment

Write the letter for the correct answer in the blank at the right of each question.

1. Find $(2a - 5) - (3a + 1)$.

A $5a + 6$ **B** $a - 4$ **C** $-a - 6$ **D** $-a - 4$ **1.** ________

2. Find $3m^2(2m^2 - m)$.

F $5m^4 - 3m^3$ **G** $6m^4 - 3m^2$ **H** $5m^4 - 3m$ **J** $6m^4 - 3m^3$ **2.** ________

3. Simplify $3(x^2 + 2x) - x(x - 1)$.

A $4x^2 + x$ **B** $2x^2 + 7x$ **C** $2x^2 + 3x$ **D** $2x^2 + 5x$ **3.** ________

4. Find $(2n - 3)(n + 4)$.

F $3n + 1$ **H** $2n^2 - 12$

G $2n^2 + 5n - 12$ **J** $2n^2 + 11n + 1$ **4.** ________

5. Factor $xy + 3x - 2x^2$ completely.

A $x(y + 3 - 2x)$ **C** $x(y + 3) + 2x$

B $(2x - 3y)(y + x)$ **D** $y(x + 3x - 2x^2)$ **5.** ________

6. Solve $b(b + 17) = 0$.

F $\left\{0, \frac{1}{17}\right\}$ **G** $\{-17, 0\}$ **H** $\{0, 17\}$ **J** $\{17\}$ **6.** ________

7. Factor $m^2 + 13m + 42$.

A $(m + 1)(m + 13)$ **C** $(m + 10)(m + 3)$

B $(m + 6)(m + 7)$ **D** $(m - 6)(m - 7)$ **7.** ________

8. Find $(3y - 1)^2$.

F $6y^2 - 6y + 1$ **H** $9y^2 - 3y + 1$

G $9y^2 - 6y + 1$ **J** $9y^2 - 6y - 1$ **8.** ________

9. The area of a rectangle is $(y^2 - 8y + 15)$ square inches. Which expression represents a possible length for the rectangle?

A $(y + 5)$ **C** $(y - 15)$

B $(y - 2)$ **D** $(y - 3)$ **9.** ________

10. Solve $3(2n - 6) = -4(n - 3)$.

F 3 **G** $\frac{3}{5}$ **H** 6 **J** $1\frac{4}{5}$ **10.** ________

11. Solve $2x^2 - 5x - 3 = 0$.

A $\left\{-\frac{1}{2}, 3\right\}$ **B** $\left\{\frac{1}{2}, -3\right\}$ **C** $\left\{\frac{1}{2}, 3\right\}$ **D** $\left\{-\frac{1}{2}, -3\right\}$ **11.** ________

NAME ______________________ DATE ____________ PERIOD ____________

8 Chapter 8 Test, Form 1 *(continued)*

12. Factor $4m^2 - 25$.

F $(2m + 5)(2m + 5)$ **H** $(2m - 5)(2m - 5)$

G $(2m + 5)(2m - 5)$ **J** prime **12.** ______

13. A square can be changed into a rectangle by increasing the length of the square by 5 units and increasing the width by 3 units. Which expression represents the area of the rectangle in square units?

A $x^2 + 8x + 15$ **B** $x^2 + 15$ **C** $2x + 8$ **D** $2x + 15$ **13.** ______

14. Solve $64y^2 = 25$ by factoring.

F $\left\{\frac{8}{5}\right\}$ **G** $\left\{\frac{5}{8}\right\}$ **H** $\left\{-\frac{8}{5}, \frac{8}{5}\right\}$ **J** $\left\{-\frac{5}{8}, \frac{5}{8}\right\}$ **14.** ______

15. Which of the following polynomials shows the terms of $x^2 + 5x^3 - 4 - 2x$ arranged in standard form?

A $5x^3 - 2x + x^2 - 4$ **C** $5x^3 - 4 - 2x + x^2$

B $-4 - 2x + x^2 + 5x^3$ **D** $5x^3 + x^2 - 2x - 4$ **15.** ______

16. The area of a circle is given by $(\pi k^2 - 12\pi k + 36\pi)$ square inches. What is the radius of the circle?

F $k + 3$ **G** $k + 4$ **H** $k - 6$ **J** $k - 12$ **16.** ______

17. Find $(2x - 5)(2x + 5)$.

A $4x$ **B** $4x^2 - 25$ **C** $4x^2 - 20x - 25$ **D** $4x^2 + 25$ **17.** ______

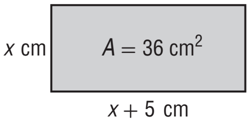
18. Solve $2x^2 + 12x = -18$.

F $\{-3\}$ **G** $\{3\}$ **H** $\{-3, 3\}$ **J** $\{-9\}$ **18.** ______

19. Find two different integers such that the square of the interger is 12 less than seven times the integer.

A 3 and 4 **B** −3 and 4 **C** −4 and 3 **D** −3 and −4 **19.** ______

20. GEOMETRY The length of a rectangle is 5 centimeters more than the width. The area of the rectangle is 36 square centimeters. What is the length?



F 4 cm **G** 9 cm **H** 14 cm **J** 26 cm **20.** ______

Bonus The sum of the squares of two consecutive odd integers is 74. Find the two integers. **B:** ____________________

NAME ______________________ DATE ____________ PERIOD ____________

8 Chapter 8 Test, Form 2A

SCORE ________

Write the letter for the correct answer in the blank at the right of each question.

1. Write $4x^3 - 6x + 2x^5 + 3$ in standard form.

A $3 - 6x + 4x^3 + 2x^5$ **C** $2x^5 + 4x^3 - 6x + 3$

B $4x^3 + 3 + 2x^5 - 6x$ **D** $-6x + 4x^3 + 3 + 2x^5$ **1.** ______

2. Find $(9t^2 + 4t - 6) - (t^2 - 2t + 4)$.

F $8t^2 + 6t - 10$ **G** $8t^2 + 2t - 2$ **H** $9t^2 + 6t - 2$ **J** $9t^2 + 6t - 10$ **2.** ______

3. Simplify $2a^2(5a - 6) - 5a(a^2 - 3a + 4) - 7(a - 5)$.

A $5a^3 + 3a^2 - 27a + 35$ **C** $5a^3 - 27a^2 + 13a - 35$

B $5a^3 - 10a - 7$ **D** none of these **3.** ______

4. Factor $24x^2y - 66xy^2 + 54x^2y^2$ completely.

F $2xy(12x - 33y + 27xy)$ **H** $(4x^2 + 6y)(6x - 9y^2)$

G $6x^2y^2(4y - 11x + 9)$ **J** $6xy(4x - 11y + 9xy)$ **4.** ______

5. Each side of a square x units long is decreased by 9 units. Which expression represents the area of the new square in square units?

A $x^2 - 81$ **C** $x^2 - 18x + 81$

B $x^2 - 18x + 18$ **D** $2x - 18$ **5.** ______

6. Solve $(3w + 4)(2w - 7) = 0$.

F $\left\{-\frac{3}{4}, \frac{2}{7}\right\}$ **G** $\left\{\frac{3}{4}, -\frac{2}{7}\right\}$ **H** $\left\{-\frac{4}{3}, \frac{7}{2}\right\}$ **J** $\left\{\frac{4}{3}, -\frac{7}{2}\right\}$ **6.** ______

7. Factor $x^2 - 10x + 9$.

A $(x - 1)(x - 9)$ **C** $(x - 1)(x + 9)$

B $(x + 1)(x + 9)$ **D** $(x + 1)(x - 9)$ **7.** ______

8. Find $(3y - 4)(2y^2 + y - 1)$.

F $6y^3 - 5y^2 - 7y - 4$ **H** $6y^3 - 5y^2 - 7y + 4$

G $6y^3 - 7y^2 - 7y + 4$ **J** $6y^3 - 5y^2 + 7y + 4$ **8.** ______

9. Solve $y^2 = 13y - 42$.

A $\{-6, -7\}$ **B** $\{6, 7\}$ **C** $\{-6, 7\}$ **D** $\{6, -7\}$ **9.** ______

10. Find $(4a^2 + b)^2$.

F $16a^4 + b^2$ **H** $8a^4 + b^2$

G $16a^4 + 8a^2b + b^2$ **J** $4a^4 + 8a^2b + b^2$ **10.** ______

11. Factor $5x^2 - 13x + 6$.

A $(x + 3)(5x - 2)$ **C** $(x + 2)(5x + 3)$

B $(x - 2)(5x - 3)$ **D** $(x - 3)(5x + 2)$ **11.** ______

NAME ______________________________ DATE ____________ PERIOD ____________

8 Chapter 8 Test, Form 2A *(continued)*

12. Solve $7x^2 - 20x = 3$.

F $\left\{-\frac{1}{7}, 3\right\}$ **G** $\left\{\frac{1}{7}, -3\right\}$ **H** $\left\{-\frac{1}{7}, -3\right\}$ **J** $\left\{\frac{1}{7}, 3\right\}$ **12.** ______

13. Factor $121r^2 - 64t^2$.

A $(11r + 8t)(11r - 8t)$ **C** $(11r + 8t)(11r + 8t)$

B $(11r - 8t)(11r - 8t)$ **D** prime **13.** ______

14. Solve $6(n - 11) = 12 + 4(2n - 3)$.

F -11 **G** 11 **H** -33 **J** 33 **14.** ______

15. Solve $5x^2 - 3x = (7x^2 + 5x) - (2x^2 + 16)$.

A 2 **B** -2 **C** 8 **D** -8 **15.** ______

16. Which binomial is a factor of $6x^2 + 48x + 96$?

F $x + 4$ **G** $3x + 8$ **H** $3x + 16$ **J** $6x + 16$ **16.** ______

17. If the area of a square is multiplied by nine, the area becomes 16 square inches. Find the length x of a side of the square.

A $\frac{16}{9}$ in. **B** $\frac{4}{3}$ in. **C** $\frac{3}{4}$ in. **D** $\frac{8}{3}$ in. **17.** ______

18. SOCCER Julian kicked a soccer ball into the air with an initial upward velocity of 40 feet per second. The height h in feet of the ball above the ground can be modeled by $h = -16t^2 + 40t$, where t is the time in seconds after Julian kicked the ball. Find the time it takes the ball to reach 25 feet above the ground.

F $2\frac{1}{2}$ s **G** $\frac{15}{16}$ s **H** $1\frac{1}{3}$ s **J** $1\frac{1}{4}$ s **18.** ______

19. The product of two consecutive odd integers is 143. Find their sum.

A -20 or 20 **B** -28 or 28 **C** -26 or 26 **D** -24 or 24 **19.** ______

20. The length of a rectangle is twice the width. The area is 72 square centimeters. What is the length?

F 48 cm **G** 24 cm **H** 12 cm **J** 6 cm **20.** ______

Bonus Find the value of c that will make $9x^2 + 30x + c$ a perfect square trinomial. **B:** ____________

NAME ______________________ DATE __________ PERIOD __________

8 Chapter 8 Test, Form 2B

SCORE ______

Assessment

Write the letter for the correct answer in the blank at the right of each question.

1. Write $x^2 + 4x - 3x^3 + 6$ in standard form.

A $-3x^3 + x^2 + 4x + 6$
C $6 + 4x - 3x^3 + x^2$
B $x^2 - 3x^3 + 4x + 6$
D $6 - 3x^3 + 4x + x^2$

1. ______

2. Find $(3c^2 - 8c + 5) + (c^2 - 8c - 6)$.

F $3c^2 - 1$
G $4c^2 + 11$
H $4c^2 - 16c - 1$
J $2c^2 - 16c - 1$

2. ______

3. Simplify $3b^2(4b + 7) - 2b(b^2 - 5b - 3) - 6(b - 2)$.

A $14b^3 + 11b^2 - 12b - 12$
C $41b^2 + 12$
B $14b^3 + 31b^2 + 12b + 12$
D $10b^3 + 31b^2 + 12$

3. ______

4. Factor $88a^2b^2 + 24a^2b - 32ab^2$ completely.

F $8a^2b^2(11 + 3b - 4a)$
H $8ab(11ab + 3a - 4b)$
G $2ab(44ab + 12a - 16b)$
J $(11a^2 + 8b)(8a - 4b^2)$

4. ______

5. Find $(3y + 4z)(3y - 4z)$.

A $9y^2 - 16z^2$
C $9y^2 - 24yz - 16z^2$
B $9y^2 + 16z^2$
D $9y^2 - 24yz + 16z^2$

5. ______

6. Solve $(3m - 2)(9m + 5) = 0$.

F $\left\{\frac{3}{2}, \frac{9}{5}\right\}$
G $\left\{-\frac{5}{9}, \frac{2}{3}\right\}$
H $\left\{-\frac{9}{5}, \frac{3}{2}\right\}$
J $\left\{-\frac{2}{3}, -\frac{5}{9}\right\}$

6. ______

7. Factor $x^2 - 11x + 18$.

A $(x - 2)(x - 9)$
C $(x - 2)(x + 9)$
B $(x + 2)(x - 9)$
D $(x + 2)(x + 9)$

7. ______

8. Find $(-2r^2 + n)^2$.

F $4r^4 + n^2$
H $4r^4 - 4r^2n + n^2$
G $-4r^4 + n^2$
J $-4r^4 - 4r^2n + n^2$

8. ______

9. Solve $y^2 = 15y - 56$.

A $\{7, 8\}$
B $\{-7, -8\}$
C $\{-7, 8\}$
D $\{7, -8\}$

9. ______

10. Find $(3x + 2)(4x^2 - 2x - 7)$.

F $12x^3 + 2x^2 - 25x - 14$
H $12x^3 + 14x^2 + 25x + 14$
G $7x^3 + 9x^2 - 25x - 14$
J $7x^3 + 7x^2 - 4x - 5$

10. ______

11. Factor $7x^2 - 16x + 4$.

A $(x - 3)(7x + 5)$
C $(x + 2)(7x - 4)$
B $(x - 4)(7x + 1)$
D $(x - 2)(7x - 2)$

11. ______

NAME ______________________ DATE __________ PERIOD __________

8 Chapter 8 Test, Form 2B *(continued)*

12. Solve $5x^2 + 13x = 6$.

F $\left\{3, -\frac{2}{5}\right\}$ **G** $\left\{3, \frac{2}{5}\right\}$ **H** $\left\{-3, \frac{2}{5}\right\}$ **J** $\left\{-3, -\frac{2}{5}\right\}$ **12.** ______

13. Factor $196w^2 - 81z^2$.

A $(14w + 9z)(14w + 9z)$ **C** $(14w + 9z)(14w - 9z)$

B $(14w - 9z)(14w - 9z)$ **D** prime **13.** ______

14. Solve $-4(5 - 2n) = 8(-6 - 5n)$.

F $-\frac{1}{9}$ **G** $-\frac{28}{3}$ **H** $-\frac{7}{8}$ **J** $-\frac{7}{12}$ **14.** ______

15. Solve $x(x + 3) - 2 = 2 + x(x + 1)$.

A 2 **B** −2 **C** 1 **D** 0 **15.** ______

16. Which binomial is a factor of $80y^2 - 120y + 45$?

F $4y - 3$ **G** $8y - 9$ **H** $16y - 9$ **J** $8y - 15$ **16.** ______

17. If the area of a square is multiplied by sixteen, the area becomes 25 square inches. Find the length x of a side of the square.

A $\frac{25}{16}$ in. **B** $\frac{5}{4}$ in. **C** $\frac{4}{5}$ in. **D** $\frac{5}{8}$ in. **17.** ______

18. GOLF Sisika hit a golf ball into the air with an initial upward velocity of 56 feet per second. The height h in feet of the ball above the ground can be modeled by $h = -16t^2 + 56t$, where t is the time in seconds after Sisika hit the ball. Find the time it takes the ball to reach 49 feet above the ground.

F $3\frac{1}{2}$ s **G** $\frac{7}{16}$ s **H** $1\frac{3}{5}$ s **J** $1\frac{3}{4}$ s **18.** ______

19. The product of two consecutive even integers is 224. Find their sum.

A −30 or 30 **B** −34 or 34 **C** −26 or 26 **D** −32 or 32 **19.** ______

20. The length of a rectangle is 5 times the width. The area is 125 square centimeters. What is the length?

F 5 cm **G** 25 cm **H** 10 cm **J** 30 cm **20.** ______

Bonus Factor $2n^4 - 10n^2 - 72$ completely. **B:** ______________

NAME ______________________ DATE ____________ PERIOD ____________

8 Chapter 8 Test, Form 2C

SCORE ________

Find each sum or difference.

1. $(5n^2 - 2ny + 3y^2) - (9n^2 - 8ny - 10y^2)$ **1.** ____________

2. $(11m^2 - 2mt + 8t^2) + (8m^2 + 4mt - 2t^2)$ **2.** ____________

Find each product.

3. $5hk^2(2h^2k - hk^3 + 4h^2k^2)$ **3.** ____________

4. $(4x^2 + 2y^2)(2x^2 - y^2)$ **4.** ____________

5. $(5c - 4)^2$ **5.** ____________

Factor each polynomial.

6. $35a^3bc^2 - 45a^2b^2c$ **6.** ____________

7. $3xy - 4x + 6y - 8$ **7.** ____________

8. $t^2 - 11t + 24$ **8.** ____________

Factor each polynomial, if possible. If the polynomial cannot be factored, write prime.

9. $10y^2 - 31y + 15$ **9.** ____________

10. $8n^2 - 36n + 40$ **10.** ____________

11. $2x^4 - 18x^2$ **11.** ____________

12. $9a^2 + 42a - 49$ **12.** ____________

Assessment

8 Chapter 8 Test, Form 2C *(continued)*

Solve each equation. Check the solutions.

13. $-6(3n - 2) = 4(-3 - 2n)$ **13.** ______________

14. $8n + 11 = 4 + 5(2n - 1)$ **14.** ______________

15. $(3n + 2)(n - 2) = 0$ **15.** ______________

16. $16y^2 - 8y = 0$ **16.** ______________

17. $8n^2 + 4 = 12n$ **17.** ______________

18. $4y^2 + 16y + 7 = 0$ **18.** ______________

19. $49w^2 - 25 = 0$ **19.** ______________

20. **GARDENING** The length of a rectangular garden is 8 feet longer than its width. The garden is surrounded by a sidewalk that is 4 feet wide and has an area of 320 square feet. Find the dimensions of the garden. **20.** ______________

21. A triangle has an area of 77 square inches. Find the length of the base if the base is 3 inches greater than the height. **21.** ______________

22. The volume of a box is 192 cubic inches. The height is 4 inches. The width is 2 inches more than the length. Find the missing dimensions. **22.** ______________

23. **BASEBALL** Tonisha hit a baseball into the air with an initial upward velocity of 48 feet per second. The height h in feet of the ball above the ground can be modeled by $h = -16t^2 + 48t + 2$, where t is the time in seconds after Tonisha hit the baseball. Find the time it takes the ball to reach 38 feet above the ground. **23.** ______________

24. The area of a rectangular room is 238 square feet. The width is 3 feet less than the length. Find the dimensions of the room. **24.** ______________

25. One number is 5 times another number. The product of the two numbers is 245. Find the two numbers. **25.** ______________

Bonus Factor $v^2x^2 - 9x^2 + v^2n^2 - 9n^2$ completely. **B:** ______________

NAME ______________________ DATE ____________ PERIOD ____________

8 Chapter 8 Test, Form 2D

SCORE ________

Find each sum or difference.

1. $(7m^2 + 3m - 4) - (3m^2 + 9m - 5)$ **1.** ____________

2. $(4y^2 + 3y - 7) + (4y^2 - 7y - 2)$ **2.** ____________

Find each product.

3. $3x^2y(2x^2y - 5xy^2 + 8y^3x^2)$ **3.** ____________

4. $(3r^2 + 5t^2)(3r^2 - 5t^2)$ **4.** ____________

5. $(5y + 6)^2$ **5.** ____________

Factor each polynomial.

6. $10x^2yz - 22x^3y^2z$ **6.** ____________

7. $2xy - 4x + 3y - 6$ **7.** ____________

8. $m^2 + 12m - 28$ **8.** ____________

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

9. $5t^2 + 17t - 12$ **9.** ____________

10. $6p^2 - 20p + 16$ **10.** ____________

11. $3x^5 - 75x^3$ **11.** ____________

12. $25x^2 + 70x - 49$ **12.** ____________

NAME ______________________________ DATE ____________ PERIOD ____________

8 Chapter 8 Test, Form 2D *(continued)*

Solve each equation. Check the solutions.

13. $5x + 8 = 3 + 2(3x - 4)$ **13.** ____________

14. $-5(2n - 3) = 7(3 - n)$ **14.** ____________

15. $(x + 5)(4x - 3) = 0$ **15.** ____________

16. $12b^2 - 8b = 0$ **16.** ____________

17. $9n^2 + 6n = 3$ **17.** ____________

18. $4b^2 - 8b - 5 = 0$ **18.** ____________

19. $64x^2 - 1 = 0$ **19.** ____________

20. **GARDENING** The length of a rectangular garden is 5 feet longer than its width. The garden is surrounded by a 2-foot-wide sidewalk. The sidewalk has an area of 76 square feet. Find the dimensions of the garden. **20.** ____________

21. A triangle has an area of 88 square inches. Find the length of the base if the base is 5 inches more than the height. **21.** ____________

22. The volume of a box is 231 cubic inches. The width is 7 inches. The length is 8 inches more than the height. Find the missing dimensions. **22.** ____________

23. **VOLLEYBALL** Lanu hit a volleyball into the air with an initial upward velocity of 24 feet per second. The height h in feet of the ball above the ground can be modeled by $h = -16t^2 + 24t + 3$, where t is the time in seconds after Lanu hit the volleyball. Find the time it takes the ball to reach 12 feet above the ground. **23.** ____________

24. The area of a rectangular room is 104 square feet. The length of the room is 5 feet longer than the width. Find the dimensions of the room. **24.** ____________

25. One number is 7 times another number. The product of the two numbers is 252. Find the two numbers. **25.** ____________

Bonus Find the value of c that will make $25x^2 - 40x + c$ a perfect square trinomial. **B:** ____________

NAME ______________________ DATE __________ PERIOD __________

8 Chapter 8 Test, Form 3

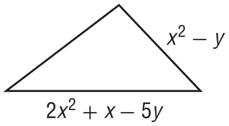
SCORE ________

Find each sum or difference.

1. $(8w^2 + 4w - 2) + (2w^2 - w + 6)$ **1.** __________

2. $(7u^2x - 3ux + 4ux^2) - (4ux - 3u^2x - 2ux^2)$ **2.** __________

3. **GEOMETRY** The measures of two sides of a triangle are given on the triangle at the right. If the perimeter of the triangle is $6x^2 + 8y$, find the measure of the third side.



3. __________

4. Simplify $5n^2(n - 6) - 2n(3n^2 + n - 6) + 7(n^2 - 3)$. **4.** __________

Factor each polynomial.

5. $12x^3y^2z - 24x^2y^3z + 16x^2y^3z^3$ **5.** __________

6. $4x^2y^2 - 9y^2 - 45 + 20x^2$ **6.** __________

7. $-x^2 + 5x + 24$ **7.** __________

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

8. $10x^2 + 29x - 21$ **8.** __________

9. $3p^2 - 14p + 12$ **9.** __________

10. $3x^3 - 24x^2y + 48xy^2$ **10.** __________

11. $3x^4 - 73x^2 - 50$ **11.** __________

12. If $a^2 + b^2 = 11$ and $ab = 3$, find the value of $(a - b)^2$. **12.** __________

13. Find all values of k so that $t^2 + kt - 8$ can be factored using integers. **13.** __________

14. Find an expression for c that will make $9x^2 + 12xy + c$ a perfect square trinomial. **14.** __________

Assessment

NAME ______________________________ DATE ____________ PERIOD ____________

8 Chapter 8 Test, Form 3 *(continued)*

Find each product.

15. $(2y - 7)(4y + 4)$ **15.** ____________

16. $\left(\frac{2}{3}m - 1\right)\left(\frac{1}{2}m - 2\right)$ **16.** ____________

Solve each equation.

17. $6x^2 = 22x$ **17.** ____________

18. $x^2 + \frac{8x}{3} = -\frac{7}{9}$ **18.** ____________

19. $a^2 - \frac{11}{2}a + \frac{121}{16} = 0$ **19.** ____________

20. Solve $(2x - 3)^2 - 25 = 0$ by factoring. Check your solution. **20.** ____________

21. The volume of a box is 96 cubic inches. The length is 8 inches more than the height. The width is 2 inches less than the height. Find the dimensions of the box. **21.** ____________

22. A rectangular rug 9 feet by 12 feet is placed in the center of a rectangular room covering three fifths of the floor. The rug leaves the same width of floor uncovered on each side. Find the dimensions of the room. **22.** ____________

23. One integer is 3 more than another integer. The difference in their squares is 6 more than 5 times the greater integer. Find the integers. **23.** ____________

24. TENNIS Josefina hit a tennis ball into the air with an initial upward velocity of 16 feet per second. The height h in feet of the ball above the ground can be modeled by $h = -16t^2 + 16t + 3$, where t is the time in seconds after Josefina hit the tennis ball. Find the time it takes the ball to reach 7 feet above the ground. **24.** ____________

25. FIBERS The basic breaking strength b in pounds for a natural fiber line is determined by the formula $900c^2 = b$, where c is the circumference of the line in inches. What circumference of natural line would have 100 pounds of breaking strength? **25.** ____________

Bonus Solve the equation, and check your solutions.
$9t^3 + 15t^2 + t - 6 = (t + 3)(t - 2) - 3t^3$ **B:** ____________

NAME ______________________________ DATE ____________ PERIOD ____________

8 Chapter 8 Extended-Response Test

SCORE ________

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. Theo wants to build a rectangular deck on the front of his family's café, and he wants the deck to have an area of A square feet. If the length of the deck is b feet longer than the width, then the equation $A = x(x + b)$ gives the area of the deck in terms of the width x.

a. Explain the differences and similarities between $A = x(x + b)$ and $x^2 + bx - A$.

b. What must be true about $-A$ and b so that $x^2 + bx - A$ is factorable?

c. Choose values for A and b so that $x^2 + bx - A$ is factorable. Then, substitute your values into $x^2 + bx - A$ and factor the resulting polynomial.

d. Substitute the values you chose for A and b in part **c** into $A = x(x + b)$, and determine the area of the deck corresponding to these values and the dimensions of the deck.

2. A ball thrown horizontally has an initial upward velocity of 0 feet per second. The height h, in feet, of the ball above the ground is modeled by the equation $h = c - 16t^2$ where c is the height at which the ball is thrown and t is the time in seconds after the ball is thrown.

a. Compare possible values of c and h, and determine whether h will ever be greater than c. Explain your reasoning.

b. Choose a value for c that is a perfect square, and determine how long it takes for the ball to hit the ground using your value for c.

c. Can a ball thrown horizontally at a height of 9 feet stay above the ground for more than 1 second? Explain your reasoning.

3. One way to factor a trinomial such as $x^2 - 2x - 3$ is to assign a value such as 10 to x and evaluate the expression.

$$\begin{aligned} x^2 - 2x - 3 &= 10^2 - 2(10) - 3 \\ &= 100 - 20 - 3 \\ &= 77 \end{aligned}$$

A factorization of 77 is 7×11. Since $x = 10$, $7 = x - 3$ and $11 = x + 1$. Multiply to see if $(x - 3)(x + 1) = x^2 - 2x - 3$.

a. Try the method above to factor $x^2 - 8x + 15$. Show your work and explain each step.

b. Try the method above to factor $2x^2 - 13x - 24$. (*Hint*: The factors will be of the form $(2x + a)(x - b)$. Show your work and explain each step.

c. Try the method above to factor $x^2 - 2x - 8$. Why is it difficult to find the correct factors for this trinomial using the method above?

d. Evaluate the expression in part **c** for $x = 5$. Then use the method above to find the factors. Explain each step. Remember in finding your factors that $x = 5$.

Assessment

NAME ______________________ DATE __________ PERIOD __________

8 Standardized Test Practice

SCORE ______

(Chapters 1–8)

Part 1: Multiple Choice

Instructions: Fill in the appropriate circle for the best answer.

1. Simplify $9a^2 + 7a + 4a^2 + 2a$. (Lesson 1-3)

A $22a^3$ **B** $13a^2 + 9a$ **C** $36a^2 + 14a$ **D** $16a^2 + 6a$

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. Which is an equation for the line that passes through $(-2, 7)$ and $(3, -8)$? (Lesson 4-2)

F $y = -3x + 1$ **H** $y = -\frac{1}{3}x + 8$

G $y = -\frac{1}{3}x + \frac{19}{3}$ **J** $y = -3x + 13$

2. Ⓕ Ⓖ Ⓗ Ⓙ

3. Solve $24 + x < 18$. (Lesson 5-1)

A $\{x \mid x < 42\}$ **C** $\left\{x \mid x < \frac{3}{4}\right\}$

B $\{x \mid x > 6\}$ **D** $\{x \mid x < -6\}$

3. Ⓐ Ⓑ Ⓒ Ⓓ

4. Half the perimeter of a garden is 18 feet. The garden is 8 feet longer than it is wide. How wide is the garden? (Lesson 6-3)

F 5 ft **G** 40 ft^2 **H** 36 ft **J** 8 ft

4. Ⓕ Ⓖ Ⓗ Ⓙ

5. Find $(3.2 \times 10^5) \div (8.0 \times 10^{-4})$. (Lesson 7-4)

A 4×10^0 **C** 4×10^1

B 4×10^8 **D** 2.56×10^{10}

5. Ⓐ Ⓑ Ⓒ Ⓓ

6. Find $(3g^2 - g + 2k) + (8k - 5g^2 + 7g)$. (Lesson 8-1)

F $11g^2 - 6g + 9k$ **H** $8g^2 - 8g - 6k$

G $-6g^2 + 9g + 11k$ **J** $-2g^2 + 6g + 10k$

6. Ⓕ Ⓖ Ⓗ Ⓙ

7. Find $(2x - 1)(3x + 2)$. (Lesson 8-3)

A $5x^2 + 4x - 2$ **C** $6x^2 - 3x + 2$

B $6x^2 + x - 2$ **D** $5x^2 + x + 1$

7. Ⓐ Ⓑ Ⓒ Ⓓ

8. Solve $8x^2 - 6x = 0$. (Lesson 8-5)

F $\left\{0, \frac{3}{4}\right\}$ **G** $\left\{0, \frac{4}{3}\right\}$ **H** $\{6, 8\}$ **J** $\left\{0, \frac{3}{4}, 2\right\}$

8. Ⓕ Ⓖ Ⓗ Ⓙ

9. Solve $p^2 - 10p = -21$. (Lesson 8-6)

A $\{-3, 10\}$ **B** $\{4, -7\}$ **C** $\{5\}$ **D** $\{3, 7\}$

9. Ⓐ Ⓑ Ⓒ Ⓓ

10. Each football game begins with a kickoff. The formula $h = -16t^2 + 64t$, where h is the height in feet of the football at t seconds, can be used to model a kickoff that is in the air for 4 seconds. At what times will the football be 48 feet above the ground? (Lesson 8-7)

F 1 s **H** 1 s, 3 s

G 2 s **J** 1.5 s, 2.5 s

10. Ⓕ Ⓖ Ⓗ Ⓙ

NAME ____________________ DATE __________ PERIOD __________

8 Standardized Test Practice *(continued)*

11. Solve $2(v + 3) - 26 = 7(1 - v)$. (Lesson 1-2)

A 10 **B** 3 **C** 9 **D** $\frac{10}{3}$ **11.** Ⓐ Ⓑ Ⓒ Ⓓ

12. Find the total price. (Lesson 2-7)

calculator: \$90 tax: 8%

F \$90.08 **G** \$98.00 **H** \$97.20 **J** \$90.80 **12.** Ⓕ Ⓖ Ⓗ Ⓙ

13. Solve $x - 2y = 6$ if the domain is $\{-2, 0, 2\}$. (Lesson 3-1)

A $\{(-2, -4), (0, -3), (2, -2)\}$ **C** $\{(-2, 5), (0, 3), (2, 5)\}$

B $\{(-2, -2), (3, 0), (2, 4)\}$ **D** $\{(2, -2), (3, 0), (4, 2)\}$ **13.** Ⓐ Ⓑ Ⓒ Ⓓ

14. A line passes through $(-1, 3)$ and $(1, -3)$. Which equation does *not* represent the line? (Lesson 4-2)

F $(y - 3) = -3(x + 1)$ **H** $(y + 3) = -3(x - 1)$

G $3x - y = 0$ **J** $y = -3x$ **14.** Ⓕ Ⓖ Ⓗ Ⓙ

15. Solve $\frac{n}{7} < -6$. (Lesson 5-2)

A $n > 42$ **B** $n < -42$ **C** $n < 42$ **D** $n > -42$ **15.** Ⓐ Ⓑ Ⓒ Ⓓ

16. Solve $8 \leq 2h + 6 \leq 22$. (Lesson 5-4)

F $\{h| 1 \leq h \leq 8\}$ **H** $\{h| 4 \leq h \leq 11\}$

G $\{h| 7 \leq h \leq 14\}$ **J** $\{h| 0 \leq h \leq 14\}$ **16.** Ⓕ Ⓖ Ⓗ Ⓙ

17. Solve $| 2u + 7 | = 13$. (Lesson 5-3)

A $\{-10, 3\}$ **B** $\{-10, -3\}$ **C** $\{3, 10\}$ **D** $\{-3, 10\}$ **17.** Ⓐ Ⓑ Ⓒ Ⓓ

18. Factor $2x^2 + 15x + 18$. (Lesson 8-8)

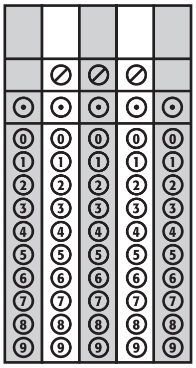
F $(2x - 3)(x + 6)$ **H** $(2x - 3)(x - 6)$

G $(2x + 2)(x + 9)$ **J** $(2x + 3)(x + 6)$ **18.** Ⓕ Ⓖ Ⓗ Ⓙ

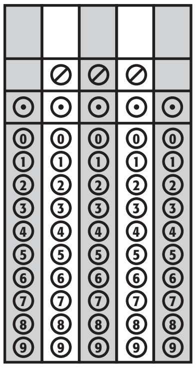
Part 2: Gridded Response

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate circle that corresponds to that entry.

19. Solve $3(x - 2) + 4x = 3(2x - 1)$.
(Lesson 8-2)



20. Solve $9x^2 + 16 = 24x$.
(Lesson 8-9)



Assessment

NAME ______________________ DATE ____________ PERIOD ____________

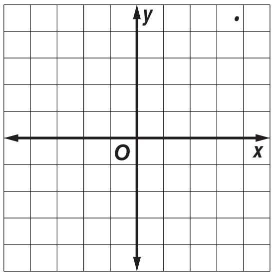
8 Standardized Test Practice *(continued)*

Part 3: Short Response

Instructions: Write your answers in the space provided.

21. Graph $y = 2x - 3$. (Lesson 3-1)

21.



22. Determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. (Lesson 6-1)

$3x - 4y = -12$

$x + 4 = \frac{4}{3}y$

22. ______________

23. Use elimination to solve the system of equations.

$6x - 3y = 11$ (Lesson 6-3)

$6x + 3y = 17$

23. ______________

24. Simplify $(-2x^2y^3)^4$. (Lesson 7-1)

24. ______________

25. Simplify $5(2y^2 + 3y - 2) + 8y(3y^2 + 4y - 2)$. (Lesson 8-2)

25. ______________

26. Simplify $(2x + 1)(x^2 - 3x - 4)$. (Lesson 8-3)

26. ______________

27. Factor $12a^2b^2 - 16a^2b^3$. (Lesson 8-5)

27. ______________

28. Solve $5x^2 - 6x + 1 = 0$. (Lesson 8-7)

28. ______________

29. Factor $4x^2 - 49y^2$. (Lesson 8-8)

29. ______________

30. The sum of two numbers is 18. The difference between three times the lesser number and the greater number is 10. (Lesson 2-1)

a. Define variables and formulate a system of equations from this situation.

30a. ______________

b. What is the greater number?

30b. ______________

NAME ______________________ DATE ________ PERIOD ________

8 Anticipation Guide

Quadratic Expressions and Equations

Step 1 ***Before you begin Chapter 8***

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. When multiplying two powers that have the same base, multiply the exponents.	**D**
	2. $(k^3)^4$ is equivalent to k^{12}.	**A**
	3. To divide two powers that have the same base, subtract the exponents.	**A**
	4. $\left(\frac{2}{5}\right)^3$ is the same as $\frac{2^3}{5}$.	**D**
	5. A polynomial may contain one or more monomials.	**A**
	6. The degree of the polynomial $3x^2y^3 - 5y^2 + 8x^3$ is 3 because the greatest exponent is 3.	**D**
	7. The greatest common factor (GCF) of two or more monomials is the product of their unique factors when each monomial is written in factored form.	**A**
	8. Any two numbers that have a greatest common factor of 1 are said to be relatively prime.	**A**
	9. If the product of any two factors is 0, then at least one of the factors must equal 0.	**A**
	10. A quadratic trinomial has a degree of 4.	**D**
	11. To solve an equation such as $x^2 = 8 + 2x$, take the square root of both sides.	**D**
	12. The polynomial $3r^2 - r - 2$ can not be factored because the coefficient of r^2 is not 1.	**D**
	13. The polynomial $t^2 + 16$ is not factorable.	**A**
	14. The numbers 16, 64, and 121 are perfect squares.	**A**

Step 2 ***After you complete Chapter 8***

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

NAME ______________________ DATE ________ PERIOD ________

8-1 Study Guide and Intervention

Adding and Subtracting Polynomials

Polynomials in Standard Form A **polynomial** is a monomial or a sum of monomials. A **binomial** is the sum of two monomials, and a **trinomial** is the sum of three monomials. Polynomials with more than three terms have no special name. The **degree** of a monomial is the sum of the exponents of all its variables. The **degree of the polynomial** is the same as the degree of the monomial term with the highest degree.

The terms of a polynomial are usually arranged so that the terms are in order from greatest degree to least degree. This is called the **standard form of a polynomial.**

Example **Determine whether each expression is a polynomial. If so, identify the polynomial as a *monomial*, *binomial*, or *trinomial*. Then find the degree of the polynomial.**

Expression	Polynomial?	Monomial, Binomial, or Trinomial?	Degree of the Polynomial
$3x - 7xyz$	Yes. $3x - 7xyz = 3x + (-7xyz)$, which is the sum of two monomials	binomial	3
-25	Yes. -25 is a real number.	monomial	0
$7n^3 + 3n^{-4}$	No. $3n^{-4} = \frac{3}{n^4}$, which is not a monomial	none of these	—
$9x^3 + 4x + x + 4 + 2x$	Yes. The expression simplifies to $9x^3 + 7x + 4$, which is the sum of three monomials	trinomial	3

Exercises

Determine whether each expression is a polynomial. If it is a polynomial, find the degree and determine whether it is a *monomial*, *binomial*, or *trinomial*.

1. 36 **yes; 0; monomial**

2. $\frac{3}{q^2} + 5$ **no**

3. $7x - x + 5$ **yes; 1; binomial**

4. $8g^2h - 7gh + 2$ **yes; 3; trinomial**

5. $\frac{1}{4y^2} + 5y - 8$ **no**

6. $6x + x^2$ **yes; 2; binomial**

Write each polynomial in standard form. Identify the leading coefficient.

7. $x^3 + x^5 - x^2$
$\mathbf{x^5 + x^3 - x^2; 1}$

8. $x^4 + 4x^3 - 7x^5 + 1$
$\mathbf{-7x^5 + x^4 + 4x^3 + 1; -7}$

9. $-3x^6 - x^5 + 2x^8$
$\mathbf{2x^8 - 3x^6 - x^5; 2}$

10. $2x^7 - x^8$
$\mathbf{-x^8 + 2x^7; -1}$

11. $3x + 5x^4 - 2 - x^2$
$\mathbf{5x^4 - x^2 + 3x - 2; 5}$

12. $-2x^4 + x - 4x^5 + 3$
$\mathbf{-4x^5 - 2x^4 + x + 3; -4}$

NAME ______________________ DATE __________ PERIOD ________

8-1 Study Guide and Intervention *(continued)*

Adding and Subtracting Polynomials

Add and Subtract Polynomials To add polynomials, you can group like terms horizontally or write them in column form, aligning like terms vertically. **Like terms** are monomial terms that are either identical or differ only in their coefficients, such as $3p$ and $-5p$ or $2x^2y$ and $8x^2y$.

You can subtract a polynomial by adding its additive inverse. To find the additive inverse of a polynomial, replace each term with by adding its additive inverse. To find the additive inverse of a polynomial, replace each term with its additive inverse or opposite.

Example **Find $(3x^2 + 2x - 6) - (2x + x^2 + 3)$.**

Horizontal Method

Use additive inverses to rewrite as addition. Then group like terms.

$$\begin{aligned}&(3x^2 + 2x - 6) - (2x + x^2 + 3)\\ &= (3x^2 + 2x - 6) + [(-2x) + (-x^2) + (-3)]\\ &= [3x^2 + (-x^2)] + [2x + (-2x)] + [-6 + (-3)]\\ &= 2x^2 + (-9)\\ &= 2x^2 - 9\end{aligned}$$

The difference is $2x^2 - 9$.

Vertical Method

Align like terms in columns and subtract by adding the additive inverse.

$$\begin{array}{rr} & 3x^2 + 2x - 6\\ (-) & x^2 + 2x + 3\\ \hline & 3x^2 + 2x - 6\\ (+) & -x^2 - 2x - 3\\ \hline & 2x^2 \qquad - 9\end{array}$$

The difference is $2x^2 - 9$.

Exercises

Find each sum or difference.

1. $(4a - 5) + (3a + 6)$ **$7a + 1$**

2. $(6x + 9) + (4x^2 - 7)$ **$4x^2 + 6x + 2$**

3. $(6xy + 2y + 6x) + (4xy - x)$ **$10xy + 5x + 2y$**

4. $(x^2 + y^2) + (-x^2 + y^2)$ **$2y^2$**

5. $(3p^2 - 2p + 3) + (p^2 - 7p + 7)$ **$4p^2 - 9p + 10$**

6. $(2x^2 + 5xy + 4y^2) + (-xy - 6x^2 + 2y^2)$ **$-4x^2 + 4xy + 6y^2$**

7. $(8p - 5r) - (-6p^2 + 6r - 3)$ **$6p^2 + 8p - 11r + 3$**

8. $(8x^2 - 4x - 3) - (-2x - x^2 + 5)$ **$9x^2 - 2x - 8$**

9. $(3x^2 - 2x) - (3x^2 + 5x - 1)$ **$-7x + 1$**

10. $(4x^2 + 6xy + 2y^2) - (-x^2 + 2xy - 5y^2)$ **$5x^2 + 4xy + 7y^2$**

11. $(2h - 6j - 2k) - (-7h - 5j - 4k)$ **$9h - j + 2k$**

12. $(9xy^2 + 5xy) - (-2xy - 8xy^2)$ **$17xy^2 + 7xy$**

NAME ______________________ DATE __________ PERIOD ________

8-1 Skills Practice

Adding and Subtracting Polynomials

Find each sum or difference.

1. $(2x + 3y) + (4x + 9y)$ **$6x + 12y$**

2. $(6s + 5t) + (4t + 8s)$ **$14s + 9t$**

3. $(5a + 9b) - (2a + 4b)$ **$3a + 5b$**

4. $(11m - 7n) - (2m + 6n)$ **$9m - 13n$**

5. $(m^2 - m) + (2m + m^2)$ **$2m^2 + m$**

6. $(x^2 - 3x) - (2x^2 + 5x)$ **$-x^2 - 8x$**

7. $(d^2 - d + 5) - (2d + 5)$ **$d^2 - 3d$**

8. $(2h^2 - 5h) + (7h - 3h^2)$ **$-h^2 + 2h$**

9. $(5f + g - 2) + (-2f + 3)$ **$3f + g + 1$**

10. $(6k^2 + 2k + 9) + (4k^2 - 5k)$ **$10k^2 - 3k + 9$**

Determine whether each expression is a polynomial. If it is a polynomial, find the degree and determine whether it is a *monomial*, *binomial*, or *trinomial*.

11. $5mt + t^2$ **yes; 2; binomial**

12. $4by + 2b - by$ **yes; 2; binomial**

13. -32 **yes; 0; monomial**

14. $\frac{3x}{7}$ **yes; 1; monomial**

15. $5x^2 - 3x^{-4}$ **no**

16. $2c^2 + 8c + 9 - 3$ **yes; 2; trinomial**

Write each polynomial in standard form. Identify the leading coefficient.

17. $3x + 1 + 2x^2$ **$2x^2 + 3x + 1; 2$**

18. $5x - 6 + 3x^2$ **$3x^2 + 5x - 6; 3$**

19. $9x^2 + 2 + x^3 + x$ **$x^3 + 9x^2 + x + 2; 1$**

20. $-3 + 3x^3 - x^2 + 4x$ **$3x^3 - x^2 + 4x - 3; 3$**

21. $x^2 + 3x^3 + 27 - x$ **$3x^3 + x^2 - x + 27; 3$**

22. $25 - x^3 + x$ **$-x^3 + x + 25; -1$**

23. $x - 3x^2 + 4 + 5x^3$ **$5x^3 - 3x^2 + x + 4; 5$**

24. $x^2 + 64 - x + 7x^3$ **$7x^3 + x^2 - x + 64; 7$**

NAME ______________ DATE ________ PERIOD ________

8-1 Practice

Adding and Subtracting Polynomials

Find each sum or difference.

1. $(4y + 5) + (-7y - 1)$ **$-3y + 4$**

2. $(-x^2 + 3x) - (5x + 2x^2)$ **$-3x^2 - 2x$**

3. $(4k^2 + 8k + 2) - (2k + 3)$ **$4k^2 + 6k - 1$**

4. $(2m^2 + 6m) + (m^2 - 5m + 7)$ **$3m^2 + m + 7$**

5. $(5a^2 + 6a + 2) - (7a^2 - 7a + 5)$ **$-2a^2 + 13a - 3$**

6. $(-4p^2 - p + 9) + (p^2 + 3p - 1)$ **$-3p^2 + 2p + 8$**

7. $(x^3 - 3x + 1) - (x^3 + 7 - 12x)$ **$9x - 6$**

8. $(6x^2 - x + 1) - (-4 + 2x^2 + 8x)$ **$4x^2 - 9x + 5$**

9. $(4y^2 + 2y - 8) - (7y^2 + 4 - y)$ **$-3y^2 + 3y - 12$**

10. $(w^2 - 4w - 1) + (-5 + 5w^2 - 3w)$ **$6w^2 - 7w - 6$**

Determine whether each expression is a polynomial. If it is a polynomial, find the degree and determine whether it is a *monomial*, *binomial*, or *trinomial*.

11. $7a^2b + 3b^2 - a^2b$ **yes; 3; binomial**

12. $\frac{1}{5}y^3 + y^2 - 9$ **yes; 3; trinomial**

13. $6g^2h^3k$ **yes; 6; monomial**

14. $\frac{x + 3x^4 - 21x^2}{x^3}$ **not a polynomial**

Write each polynomial in standard form. Identify the leading coefficient.

15. $8x^2 - 15 + 5x^5$ **$5x^5 + 8x^2 -15$; 5**

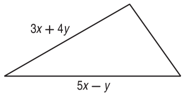
16. $10x - 7 + x^4 + 4x^3$ **$x^4 + 4x^3 + 10x -7$; 1**

17. $13x^2 - 5 + 6x^3 - x$ **$6x^3 + 13x^2 - x - 5$; 6**

18. $4x + 2x^5 - 6x^3 + 2$ **$2x^5 - 6x^3 + 4x + 2$; 2**

19. **BUSINESS** The polynomial $s^3 - 70s^2 + 1500s - 10{,}800$ models the profit a company makes on selling an item at a price s. A second item sold at the same price brings in a profit of $s^3 - 30s^2 + 450s - 5000$. Write a polynomial that expresses the total profit from the sale of both items. **$2s^3 - 100s^2 + 1950s - 15{,}800$**

20. **GEOMETRY** The measures of two sides of a triangle are given. If P is the perimeter, and $P = 10x + 5y$, find the measure of the third side. **$2x + 2y$**



NAME ______________ DATE ________ PERIOD ________

8-1 Word Problem Practice

Adding and Subtracting Polynomials

1. **PRIMES** Mei is trying to list as many prime numbers as she can for a challenge problem for her math class. She finds that the polynomial expression $n^2 - n + 41$ can be used to generate some, but not all, prime numbers. What is the degree of Mei's polynomial? **2**

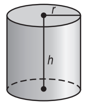
2. **PHONE CALLS** A long-distance telephone company charges a standard monthly service fee of \$19.95 plus \$0.05 per minute of long-distance use. Write a polynomial to express the monthly cost of the phone plan if x minutes of long-distance time are used per month. What is the degree of the polynomial? **\$0.05x + \$19.95; 1**

3. **FIREWORKS** Two bottle rockets are launched straight up into the air. The height, in feet, of each rocket at t seconds after launch is given by the polynomial equations below. Write an equation to show how much higher Rocket A traveled.
Rocket A: $D_1 = -16t^2 + 122t$
Rocket B: $D_2 = -16t^2 + 84t$
$D = 38t$

4. **ENVELOPES** An office supply company produces yellow document envelopes. The envelopes come in a variety of sizes, but the length is always 4 centimeters more than double the width. Write a polynomial expression to give the perimeter of any of the envelopes. **$6x + 8$**

5. **INDUSTRY** Two identical right cylindrical steel drums containing oil need to be covered with a fire-resistant sealant. In order to determine how much sealant to purchase, George must find the surface area of the two drums. The surface area including the top and bottom bases is given by the following formula.

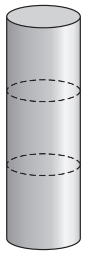
$$S = 2\pi rh + 2\pi r^2$$



a. Write a polynomial to represent the total surface area of the two drums. **$4\pi rh + 4\pi r^2$**

b. Find the total surface area if the height of each drum is 2 meters and the radius of each is 0.5 meter. Let $\pi = 3.14$. **15.7 m²**

c. The fire resistant sealant must be applied while they are stacked vertically in groups of three. If h is the height of each drum and r is the radius, write a polynomial to represent the exposed surface area. **$6\pi rh + \pi r^2$**



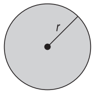
NAME ______________________ DATE ____________ PERIOD ________

8-1 Enrichment

Circular Areas and Volumes

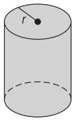
Area of Circle

$A = \pi r^2$



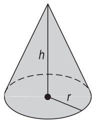
Volume of Cylinder

$V = \pi r^2 h$



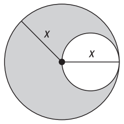
Volume of Cone

$V = \frac{1}{3}\pi r^2 h$



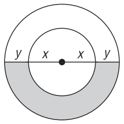
Write an algebraic expression for each shaded area. (Recall that the diameter of a circle is twice its radius.)

1.



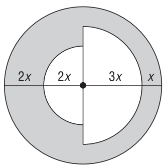
$\pi x^2 - \pi\left(\frac{x}{2}\right)^2 = \frac{3\pi x^2}{4}$

2.



$\frac{\pi}{2}(y^2 + 2xy)$

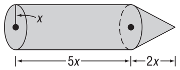
3.



$\frac{19}{2}\pi x^2$

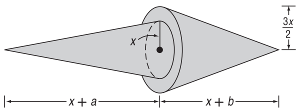
Write an algebraic expression for the total volume of each figure.

4.



$5\frac{2}{3}\pi x^3$

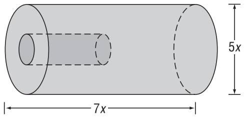
5.



$\frac{\pi}{12}[13x^3 + (4a + 9b)x^2]$

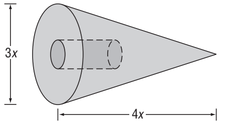
Each figure has a cylindrical hole with a radius of 2 inches and a height of 5 inches. Find each volume.

6.



$\frac{175\pi}{4}x^3 - 20\pi$ in^3

7.



$3\pi x^3 - 20\pi$ in^3

NAME ______________________ DATE ____________ PERIOD ________

8-1 Graphing Calculator Activity

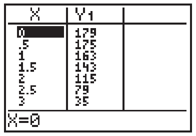
Second Degree Polynomial Functions

Many real world problems can be modeled using polynomial functions. The **TABLE** function can be used to evaluate a function for multiple values.

Example **An object is dropped from the top of a 179-foot cliff to the water below. The height of the object above the water can be modeled by $h(t) = -16t^2 + 179$ where t is time in seconds.**

a. Determine the height of the object after 0.5 second, 1 second, 1.5 seconds, and 2 seconds.

Enter the function into **Y1**. Use 2nd **[TBLSET]** to set up the calculator to display values of t in 0.5 second intervals. Display the table and record the results.

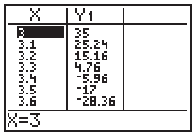


```
  X    | Y1   |
0      | 179  |
.5     | 175  |
1      | 163  |
1.5    | 143  |
2      | 115  |
2.5    | 79   |
3      | 35   |
X=0
```

Examine the table. When $x = 0.5$, $y = 175$. This means that $h(0.5) = 175$, or that after 0.5 second, the object is 175 feet above the water. Thus, $h(1) = 163$ feet, $h(1.5) = 143$ feet, and $h(2) = 115$ feet.

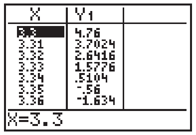
b. After how many seconds does the object hit the water? Round to the nearest hundredth.

Scroll through the table. Notice that the y-values change from positive to negative between $x = 3$ and $x = 3.5$. Examine this interval more closely by resetting the table using **TblStart=** 3 and **ΔTbl=** 0.1. Look for the change in sign.



```
  X    | Y1     |
3      | 35     |
3.1    | 25.24  |
3.2    | 15.16  |
3.3    | 4.76   |
3.4    | -5.96  |
3.5    | -17    |
3.6    | -28.36 |
X=3
```

Further examine the interval from $x = 3.3$ to $x = 3.4$ using **TblStart=** 3.3 and **ΔTbl=** 0.01.



```
  X    | Y1     |
3.3    | 4.76   |
3.31   | 3.7024 |
3.32   | 2.6416 |
3.33   | 1.5776 |
3.34   | .5104  |
3.35   | -.56   |
3.36   | -1.634 |
X=3.3
```

The y-value closest to zero occurs when $x = 3.34$. Thus, the object hits the water after about 3.34 seconds.

Exercises

1. An object is dropped from the top of a building that is 412 feet high. The distance, in feet, above the ground at x seconds is given by $P(x) = -16x^2 + 412$.

   a. After how many seconds will the object be 100 feet above the ground? **≈4.4 s**

   b. How many seconds will it take the object to reach the ground? **≈5.1 s**

2. A bungee jumper free falls from the Royal Gorge suspension bridge over the Arkansas River, 1053 feet above the river. The height h of the bungee jumper above the river, in feet, after t seconds can be represented by $h = -16t^2 + 1053$. Two seconds after the first bungee jumper falls, another person jumps down with an initial velocity of 80 feet per second. The position of the second jumper can be represented by the equation $h = -16(t - 2)^2 - 80(t - 2) + 1053$.

   a. If the bungee cords are designed to stretch just enough so that the jumpers touch the water before springing back up, which jumper will touch the water first? How long does it take each jumper to touch the water? **the second jumper; 8.1 s, 6.0 s**

   b. Does the second jumper catch up to the first jumper? If so, how far are they above the river at this point and how long does it take each jumper to reach this point? **yes; 477 ft; 6 s, 4 s**

NAME ______________________ DATE ___________ PERIOD _________

8-2 Study Guide and Intervention

Multiplying a Polynomial by a Monomial

Polynomial Multiplied by Monomial The Distributive Property can be used to multiply a polynomial by a monomial. You can multiply horizontally or vertically. Sometimes multiplying results in like terms. The products can be simplified by combining like terms.

Example 1 **Find $-3x^2(4x^2 + 6x - 8)$.**

Horizontal Method

$-3x^2(4x^2 + 6x - 8)$

$= -3x^2(4x^2) + (-3x^2)(6x) - (-3x^2)(8)$

$= -12x^4 + (-18x^3) - (-24x^2)$

$= -12x^4 - 18x^3 + 24x^2$

Vertical Method

$$\begin{array}{rr} & 4x^2 + 6x - 8 \\ (\times) & -3x^2 \\ \hline & -12x^4 - 18x^3 + 24x^2 \end{array}$$

The product is $-12x^4 - 18x^3 + 24x^2$.

Example 2 **Simplify $-2(4x^2 + 5x) - x(x^2 + 6x)$.**

$-2(4x^2 + 5x) - x(x^2 + 6x)$

$= -2(4x^2) + (-2)(5x) + (-x)(x^2) + (-x)(6x)$

$= -8x^2 + (-10x) + (-x^3) + (-6x^2)$

$= (-x^3) + [-8x^2 + (-6x^2)] + (-10x)$

$= -x^3 - 14x^2 - 10x$

Exercises

Find each product.

1. $x(5x + x^2)$ **$5x^2 + x^3$**

2. $x(4x^2 + 3x + 2)$ **$4x^3 + 3x^2 + 2x$**

3. $-2xy(2y + 4x^2)$ **$-4xy^2 - 8x^3y$**

4. $-2g(g^2 - 2g + 2)$ **$-2g^3 + 4g^2 - 4g$**

5. $3x(x^4 + x^3 + x^2)$ **$3x^5 + 3x^4 + 3x^3$**

6. $-4x(2x^3 - 2x + 3)$ **$-8x^4 + 8x^2 - 12x$**

7. $-4ax(10 + 3x)$ **$-40ax - 12ax^2$**

8. $3y(-4x - 6x^3 - 2y)$ **$-12xy - 18x^3y - 6y^2$**

9. $2x^2y^2(3xy + 2y + 5x)$ **$6x^3y^3 + 4x^2y^3 + 10x^3y^2$**

Simplify each expression.

10. $x(3x - 4) - 5x$ **$3x^2 - 9x$**

11. $-x(2x^2 - 4x) - 6x^2$ **$-2x^3 - 2x^2$**

12. $6a(2a - b) + 2a(-4a + 5b)$ **$4a^2 + 4ab$**

13. $4r(2r^2 - 3r + 5) + 6r(4r^2 + 2r + 8)$ **$32r^3 + 68r$**

14. $4n(3n^2 + n - 4) - n(3 - n)$ **$12n^3 + 5n^2 - 19n$**

15. $2b(b^2 + 4b + 8) - 3b(3b^2 + 9b - 18)$ **$-7b^3 - 19b^2 + 70b$**

16. $-2z(4z^2 - 3z + 1) - z(3z^2 + 2z - 1)$ **$-11z^3 + 4z^2 - z$**

17. $2(4x^2 - 2x) - 3(-6x^2 + 4) + 2x(x - 1)$ **$28x^2 - 6x - 12$**

NAME ______________________ DATE ___________ PERIOD _________

8-2 Study Guide and Intervention *(continued)*

Multiplying a Polynomial by a Monomial

Solve Equations with Polynomial Expressions Many equations contain polynomials that must be added, subtracted, or multiplied before the equation can be solved.

Example **Solve $4(n - 2) + 5n = 6(3 - n) + 19$.**

$4(n - 2) + 5n = 6(3 - n) + 19$	Original equation
$4n - 8 + 5n = 18 - 6n + 19$	Distributive Property
$9n - 8 = 37 - 6n$	Combine like terms.
$15n - 8 = 37$	Add 6*n* to both sides.
$15n = 45$	Add 8 to both sides.
$n = 3$	Divide each side by 15.

The solution is 3.

Exercises

Solve each equation.

1. $2(a - 3) = 3(-2a + 6)$ **3**

2. $3(x + 5) - 6 = 18$ **3**

3. $3x(x - 5) - 3x^2 = -30$ **2**

4. $6(x^2 + 2x) = 2(3x^2 + 12)$ **2**

5. $4(2p + 1) - 12p = 2(8p + 12)$ **-1**

6. $2(6x + 4) + 2 = 4(x - 4)$ **$-3\frac{1}{4}$**

7. $-2(4y - 3) - 8y + 6 = 4(y - 2)$ **1**

8. $x(x + 2) - x(x - 6) = 10x - 12$ **6**

9. $3(x^2 - 2x) = 3x^2 + 5x - 11$ **1**

10. $2(4x + 3) + 2 = -4(x + 1)$ **-1**

11. $3(2h - 6) - (2h + 1) = 9$ **7**

12. $3(y + 5) - (4y - 8) = -2y + 10$ **-13**

13. $3(2a - 6) - (-3a - 1) = 4a - 2$ **3**

14. $5(2x^2 - 1) - (10x^2 - 6) = -(x + 2)$ **-3**

15. $3(x + 2) + 2(x + 1) = -5(x - 3)$ **$\frac{7}{10}$**

16. $4(3p^2 + 2p) - 12p^2 = 2(8p + 6)$ **$-\frac{3}{2}$**

Lesson 8-2

Answers

NAME ______________ DATE ________ PERIOD ________

8-2 Skills Practice

Multiplying a Polynomial by a Monomial

Find each product.

1. $a(4a + 3)$ **$4a^2 + 3a$**

2. $-c(11c + 4)$ **$-11c^2 - 4c$**

3. $x(2x - 5)$ **$2x^2 - 5x$**

4. $2y(y - 4)$ **$2y^2 - 8y$**

5. $-3n(n^2 + 2n)$ **$-3n^3 - 6n^2$**

6. $4h(3h - 5)$ **$12h^2 - 20h$**

7. $3x(5x^2 - x + 4)$ **$15x^3 - 3x^2 + 12x$**

8. $7c(5 - 2c^2 + c^3)$ **$35c - 14c^3 + 7c^4$**

9. $-4b(1 - 9b - 2b^2)$ **$-4b + 36b^2 + 8b^3$**

10. $6y(-5 - y + 4y^2)$ **$-30y - 6y^2 + 24y^3$**

11. $2m^2(2m^2 + 3m - 5)$ **$4m^4 + 6m^3 - 10m^2$**

12. $-3n^2(-2n^2 + 3n + 4)$ **$6n^4 - 9n^3 - 12n^2$**

Simplify each expression.

13. $w(3w + 2) + 5w$ **$3w^2 + 7w$**

14. $f(5f - 3) - 2f$ **$5f^2 - 5f$**

15. $-p(2p - 8) - 5p$ **$-2p^2 + 3p$**

16. $y^2(-4y + 5) - 6y^2$ **$-4y^3 - y^2$**

17. $2x(3x^2 + 4) - 3x^3$ **$3x^3 + 8x$**

18. $4a(5a^2 - 4) + 9a$ **$20a^3 - 7a$**

19. $4b(-5b - 3) - 2(b^2 - 7b - 4)$ **$-22b^2 + 2b + 8$**

20. $3m(3m + 6) - 3(m^2 + 4m + 1)$ **$6m^2 + 6m - 3$**

Solve each equation.

21. $3(a + 2) + 5 = 2a + 4$ **-7**

22. $2(4x + 2) - 8 = 4(x + 3)$ **4**

23. $5(y + 1) + 2 = 4(y + 2) - 6$ **-5**

24. $4(b + 6) = 2(b + 5) + 2$ **-6**

25. $6(m - 2) + 14 = 3(m + 2) - 10$ **-2**

26. $3(c + 5) - 2 = 2(c + 6) + 2$ **1**

NAME ______________ DATE ________ PERIOD ________

8-2 Practice

Multiplying a Polynomial by a Monomial

Find each product.

1. $2h(-7h^2 - 4h)$ **$-14h^3 - 8h^2$**

2. $6pq(3p^2 + 4q)$ **$18p^3q + 24pq^2$**

3. $5jk(3jk + 2k)$ **$15j^2k^2 + 10jk^2$**

4. $-3rt(-2t^2 + 3r)$ **$6rt^3 - 9r^2t$**

5. $-\frac{1}{4}m(8m^2 + m - 7)$ **$-2m^3 - \frac{1}{4}m^2 + \frac{7}{4}m$**

6. $-\frac{2}{3}n^2(-9n^2 + 3n + 6)$ **$6n^4 - 2n^3 - 4n^2$**

Simplify each expression.

7. $-2\ell(3\ell - 4) + 7\ell$ **$-6\ell^2 + 15\ell$**

8. $5w(-7w + 3) + 2w(-2w^2 + 19w + 2)$ **$-4w^3 + 3w^2 + 19w$**

9. $6t(2t - 3) - 5(2t^2 + 9t - 3)$ **$2t^2 - 63t + 15$**

10. $-2(3m^3 + 5m + 6) + 3m(2m^2 + 3m + 1)$ **$9m^2 - 7m - 12$**

11. $-3g(7g - 2) + 3(g^2 + 2g + 1) - 3g(-5g + 3)$ **$-3g^2 + 3g + 3$**

Solve each equation.

12. $5(2t - 1) + 3 = 3(3t + 2)$ **8**

13. $3(3u + 2) + 5 = 2(2u - 2)$ **-3**

14. $4(8n + 3) - 5 = 2(6n + 8) + 1$ **$\frac{1}{2}$**

15. $8(3b + 1) = 4(b + 3) - 9$ **$-\frac{1}{4}$**

16. $t(t + 4) - 1 = t(t + 2) + 2$ **$\frac{3}{2}$**

17. $u(u - 5) + 8u = u(u + 2) - 4$ **-4**

18. NUMBER THEORY Let x be an integer. What is the product of twice the integer added to three times the next consecutive integer? **$5x + 3$**

19. INVESTMENTS Kent invested $5000 in a retirement plan. He allocated x dollars of the money to a bond account that earns 4% interest per year and the rest to a traditional account that earns 5% interest per year.

a. Write an expression that represents the amount of money invested in the traditional account. **$5000 - x$**

b. Write a polynomial model in simplest form for the total amount of money T Kent has invested after one year. (*Hint:* Each account has $A + IA$ dollars, where A is the original amount in the account and I is its interest rate.) **$T = 5250 - 0.01x$**

c. If Kent put $500 in the bond account, how much money does he have in his retirement plan after one year? **$5245**

NAME ______________________ DATE __________ PERIOD ________

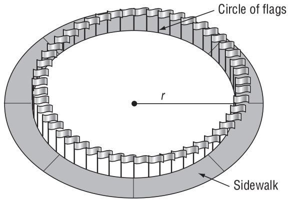
8-2 Word Problem Practice

Multiplying a Polynomial by a Monomial

1. **NUMBER THEORY** The sum of the first n whole numbers is given by the expression $\frac{1}{2}(n^2 + n)$. Expand the equation by multiplying, then find the sum of the first 12 whole numbers.
   $\mathbf{\frac{n^2}{2} + \frac{n}{2}}$; **78**

2. **COLLEGE** Troy's boss gave him \$700 to start his college savings account. Troy's boss also gives him \$40 each month to add to the account. Troy's mother gives him \$50 each month, but has been doing so for 4 fewer months than Troy's boss. Write a simplified expression for the amount of money Troy has received from his boss and mother after m months.
   \$90*m* + \$500

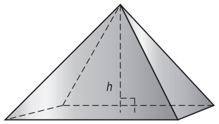
3. **LANDMARKS** A circle of 50 flags surrounds the Washington Monument. Suppose a new sidewalk 12 feet wide is installed just around the outside of the circle of flags. The outside circumference of the sidewalk is 1.10 times the circumference of the circle of flags.



Write an equation that equates the outside circumference of the sidewalk to 1.10 times the circumference of the circle of flags. Solve the equation for the radius of the circle of flags. Recall that circumference of a circle is $2\pi r$.
$\mathbf{1.10(2\pi r) = 2\pi(r + 12); r = 120}$ **ft**

4. **MARKET** Sophia went to the farmers' market to purchase some vegetables. She bought peppers and potatoes. The peppers were \$0.39 each and the potatoes were \$0.29 each. She spent \$3.88 on vegetables, and bought 4 more potatoes than peppers. If x = the number of peppers, write and solve an equation to find out how many of each vegetable Sophia bought.
   $\mathbf{\$3.88 = x(0.39) + (x + 4)(0.29)}$;
   4 peppers and 8 potatoes

5. **GEOMETRY** Some monuments are constructed as rectangular pyramids. The volume of a pyramid can be found by multiplying the area of its base B by one third of its height. The area of the rectangular base of a monument in a local park is given by the polynomial equation $B = x^2 - 4x - 12$.



   a. Write a polynomial equation to represent V, the volume of a rectangular pyramid if its height is 10 centimeters.
   $\mathbf{V = \frac{10}{3}x^2 - \frac{40}{3}x - 40}$

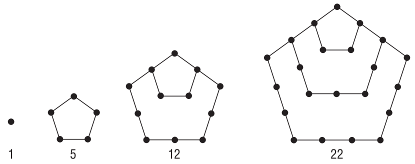
   b. Find the volume of the pyramid if $x = 12$.
   280 cm^3

NAME ______________________ DATE __________ PERIOD ________

8-2 Enrichment

Figurate Numbers

The numbers below are called **pentagonal numbers**. They are the numbers of dots or disks that can be arranged as pentagons.



1. Find the product $\frac{1}{2}n(3n - 1)$. $\mathbf{\frac{3n^2}{2} - \frac{n}{2}}$
2. Evaluate the product in Exercise 1 for values of n from 1 through 4. **1, 5, 12, 22**
3. What do you notice? **They are the first four pentagonal numbers.**
4. Find the next six pentagonal numbers. **35, 51, 70, 92, 117, 145**
5. Find the product $\frac{1}{2}n(n + 1)$. $\mathbf{\frac{n^2}{2} + \frac{n}{2}}$
6. Evaluate the product in Exercise 5 for values of n from 1 through 5. On another sheet of paper, make drawings to show why these numbers are called the triangular numbers.
   1, 3, 6, 10, 15; See students' drawings.
7. Find the product $n(2n - 1)$. $\mathbf{2n^2 - n}$
8. Evaluate the product in Exercise 7 for values of n from 1 through 5. Draw these hexagonal numbers. **1, 6, 15, 28, 45; See students' drawings.**
9. Find the first 5 square numbers. Also, write the general expression for any square number. **1, 4, 9, 16, 25;** $\mathbf{n^2}$

The numbers you have explored above are called the plane figurate numbers because they can be arranged to make geometric figures. You can also create solid figurate numbers.

10. If you pile 10 oranges into a pyramid with a triangle as a base, you get one of the tetrahedral numbers. How many layers are there in the pyramid? How many oranges are there in the bottom layers? **3 layers; 6**
11. Evaluate the expression $\frac{1}{6}n^3 + \frac{1}{2}n^2 + \frac{1}{3}n$ for values of n from 1 through 5 to find the first five tetrahedral numbers. **1, 4, 10, 20, 35**

NAME ______________________ DATE ____________ PERIOD ________

8-3 Study Guide and Intervention

Multiplying Polynomials

Multiply Binomials To multiply two binomials, you can apply the Distributive Property twice. A useful way to keep track of terms in the product is to use the FOIL method as illustrated in Example 2.

Example 1 **Find $(x + 3)(x - 4)$.**

Horizontal Method

$(x + 3)(x - 4)$

$= x(x - 4) + 3(x - 4)$

$= (x)(x) + x(-4) + 3(x) + 3(-4)$

$= x^2 - 4x + 3x - 12$

$= x^2 - x - 12$

Vertical Method

$$\begin{array}{rr} & x + 3 \\ (\times) & x - 4 \\ \hline & -4x - 12 \\ x^2 + 3x & \\ \hline x^2 - & x - 12 \end{array}$$

The product is $x^2 - x - 12$.

Example 2 **Find $(x - 2)(x + 5)$ using the FOIL method.**

$(x - 2)(x + 5)$

First Outer Inner Last

$= (x)(x) + (x)(5) + (-2)(x) + (-2)(5)$

$= x^2 + 5x + (-2x) - 10$

$= x^2 + 3x - 10$

The product is $x^2 + 3x - 10$.

Exercises

Find each product.

1. $(x + 2)(x + 3)$ **$x^2 + 5x + 6$**

2. $(x - 4)(x + 1)$ **$x^2 - 3x - 4$**

3. $(x - 6)(x - 2)$ **$x^2 - 8x + 12$**

4. $(p - 4)(p + 2)$ **$p^2 - 2p - 8$**

5. $(y + 5)(y + 2)$ **$y^2 + 7y + 10$**

6. $(2x - 1)(x + 5)$ **$2x^2 + 9x - 5$**

7. $(3n - 4)(3n - 4)$ **$9n^2 - 24n + 16$**

8. $(8m - 2)(8m + 2)$ **$64m^2 - 4$**

9. $(k + 4)(5k - 1)$ **$5k^2 + 19k - 4$**

10. $(3x + 1)(4x + 3)$ **$12x^2 + 13x + 3$**

11. $(x - 8)(-3x + 1)$ **$-3x^2 + 25x - 8$**

12. $(5t + 4)(2t - 6)$ **$10t^2 - 22t - 24$**

13. $(5m - 3n)(4m - 2n)$ **$20m^2 - 22mn + 6n^2$**

14. $(a - 3b)(2a - 5b)$ **$2a^2 - 11ab + 15b^2$**

15. $(8x - 5)(8x + 5)$ **$64x^2 - 25$**

16. $(2n - 4)(2n + 5)$ **$4n^2 + 2n - 20$**

17. $(4m - 3)(5m - 5)$ **$20m^2 - 35m + 15$**

18. $(7g - 4)(7g + 4)$ **$49g^2 - 16$**

NAME ______________________ DATE ____________ PERIOD ________

8-3 Study Guide and Intervention *(continued)*

Multiplying Polynomials

Multiply Polynomials The Distributive Property can be used to multiply any two polynomials.

Example **Find $(3x + 2)(2x^2 - 4x + 5)$.**

$(3x + 2)(2x^2 - 4x + 5)$

$= 3x(2x^2 - 4x + 5) + 2(2x^2 - 4x + 5)$ Distributive Property

$= 6x^3 - 12x^2 + 15x + 4x^2 - 8x + 10$ Distributive Property

$= 6x^3 - 8x^2 + 7x + 10$ Combine like terms.

The product is $6x^3 - 8x^2 + 7x + 10$.

Exercises

Find each product.

1. $(x + 2)(x^2 - 2x + 1)$ **$x^3 - 3x + 2$**

2. $(x + 3)(2x^2 + x - 3)$ **$2x^3 + 7x^2 - 9$**

3. $(2x - 1)(x^2 - x + 2)$ **$2x^3 - 3x^2 + 5x - 2$**

4. $(p - 3)(p^2 - 4p + 2)$ **$p^3 - 7p^2 + 14p - 6$**

5. $(3k + 2)(k^2 + k - 4)$ **$3k^3 + 5k^2 - 10k - 8$**

6. $(2t + 1)(10t^2 - 2t - 4)$ **$20t^3 + 6t^2 - 10t - 4$**

7. $(3n - 4)(n^2 + 5n - 4)$ **$3n^3 + 11n^2 - 32n + 16$**

8. $(8x - 2)(3x^2 + 2x - 1)$ **$24x^3 + 10x^2 - 12x + 2$**

9. $(2a + 4)(2a^2 - 8a + 3)$ **$4a^3 - 8a^2 - 26a + 12$**

10. $(3x - 4)(2x^2 + 3x + 3)$ **$6x^3 + x^2 - 3x - 12$**

11. $(n^2 + 2n - 1)(n^2 + n + 2)$ **$n^4 + 3n^3 + 3n^2 + 3n - 2$**

12. $(t^2 + 4t - 1)(2t^2 - t - 3)$ **$2t^4 + 7t^3 - 9t^2 - 11t + 3$**

13. $(y^2 - 5y + 3)(2y^2 + 7y - 4)$ **$2y^4 - 3y^3 - 33y^2 + 41y - 12$**

14. $(3b^2 - 2b + 1)(2b^2 - 3b - 4)$ **$6b^4 - 13b^3 - 4b^2 + 5b - 4$**

NAME ______________________ DATE ____________ PERIOD ____________

8-3 Skills Practice

Multiplying Polynomials

Find each product.

1. $(m + 4)(m + 1)$
$m^2 + 5m + 4$

2. $(x + 2)(x + 2)$
$x^2 + 4x + 4$

3. $(b + 3)(b + 4)$
$b^2 + 7b + 12$

4. $(t + 4)(t - 3)$
$t^2 + t - 12$

5. $(r + 1)(r - 2)$
$r^2 - r - 2$

6. $(n - 5)(n + 1)$
$n^2 - 4n - 5$

7. $(3c + 1)(c - 2)$
$3c^2 - 5c - 2$

8. $(2x - 6)(x + 3)$
$2x^2 - 18$

9. $(d - 1)(5d - 4)$
$5d^2 - 9d + 4$

10. $(2\ell + 5)(\ell - 4)$
$2\ell^2 - 3\ell - 20$

11. $(3n - 7)(n + 3)$
$3n^2 + 2n - 21$

12. $(q + 5)(5q - 1)$
$5q^2 + 24q - 5$

13. $(3b + 3)(3b - 2)$
$9b^2 + 3b - 6$

14. $(2m + 2)(3m - 3)$
$6m^2 - 6$

15. $(4c + 1)(2c + 1)$
$8c^2 + 6c + 1$

16. $(5a - 2)(2a - 3)$
$10a^2 - 19a + 6$

17. $(4h - 2)(4h - 1)$
$16h^2 - 12h + 2$

18. $(x - y)(2x - y)$
$2x^2 - 3xy + y^2$

19. $(w + 4)(w^2 + 3w - 6)$
$w^3 + 7w^2 + 6w - 24$

20. $(t + 1)(t^2 + 2t + 4)$
$t^3 + 3t^2 + 6t + 4$

21. $(k + 4)(k^2 + 3k - 6)$
$k^3 + 7k^2 + 6k - 24$

22. $(m + 3)(m^2 + 3m + 5)$
$m^3 + 6m^2 + 14m + 15$

NAME ______________________ DATE ____________ PERIOD ____________

8-3 Practice

Multiplying Polynomials

Find each product.

1. $(q + 6)(q + 5)$
$q^2 + 11q + 30$

2. $(x + 7)(x + 4)$
$x^2 + 11x + 28$

3. $(n - 4)(n - 6)$
$n^2 - 10n + 24$

4. $(a + 5)(a - 6)$
$a^2 - a - 30$

5. $(4b + 6)(b - 4)$
$4b^2 - 10b - 24$

6. $(2x - 9)(2x + 4)$
$4x^2 - 10x - 36$

7. $(6a - 3)(7a - 4)$
$42a^2 - 45a + 12$

8. $(2x - 2)(5x - 4)$
$10x^2 - 18x + 8$

9. $(3a - b)(2a - b)$
$6a^2 - 5ab + b^2$

10. $(4g + 3h)(2g + 3h)$
$8g^2 + 18gh + 9h^2$

11. $(m + 5)(m^2 + 4m - 8)$
$m^3 + 9m^2 + 12m - 40$

12. $(t + 3)(t^2 + 4t + 7)$
$t^3 + 7t^2 + 19t + 21$

13. $(2h + 3)(2h^2 + 3h + 4)$
$4h^3 + 12h^2 + 17h + 12$

14. $(3d + 3)(2d^2 + 5d - 2)$
$6d^3 + 21d^2 + 9d - 6$

15. $(3q + 2)(9q^2 - 12q + 4)$
$27q^3 - 18q^2 - 12q + 8$

16. $(3r + 2)(9r^2 + 6r + 4)$
$27r^3 + 36r^2 + 24r + 8$

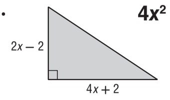
17. $(3n^2 + 2n - 1)(2n^2 + n + 9)$
$6n^4 + 7n^3 + 27n^2 + 17n - 9$

18. $(2t^2 + t + 3)(4t^2 + 2t - 2)$
$8t^4 + 8t^3 + 10t^2 + 4t - 6$

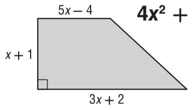
19. $(2x^2 - 2x - 3)(2x^2 - 4x + 3)$
$4x^4 - 12x^3 + 8x^2 + 6x - 9$

20. $(3y^2 + 2y + 2)(3y^2 - 4y - 5)$
$9y^4 - 6y^3 - 17y^2 - 18y - 10$

GEOMETRY Write an expression to represent the area of each figure.

21.


$4x^2 - 2x - 2$ units^2

22.


$4x^2 + 3x - 1$ units^2

23. NUMBER THEORY Let x be an even integer. What is the product of the next two consecutive even integers? **$x^2 + 6x + 8$**

24. GEOMETRY The volume of a rectangular pyramid is one third the product of the area of its base and its height. Find an expression for the volume of a rectangular pyramid whose base has an area of $3x^2 + 12x + 9$ square feet and whose height is $x + 3$ feet.
$x^3 + 7x^2 + 15x + 9$ ft^3

NAME ______________________ DATE __________ PERIOD __________

8-3 Word Problem Practice

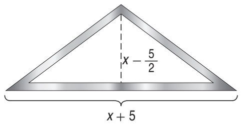
Multiplying Polynomials

1. **THEATER** The Loft Theater has a center seating section with $3c + 8$ rows and $4c - 1$ seats in each row. Write an expression for the total number of seats in the center section. $\mathbf{12c^2 + 29c - 8}$

2. **CRAFTS** Suppose a quilt made up of squares has a length-to-width ratio of 5 to 4. The length of the quilt is $5x$ inches. The quilt can be made slightly larger by adding a border of 1-inch squares all the way around the perimeter of the quilt. Write a polynomial expression for the area of the larger quilt. $\mathbf{20x^2 + 18x + 4}$

3. **SERVICE** A folded United States flag is sometimes presented to individuals in recognition of outstanding service to the country. The flag is presented folded in a triangle. Often the recipient purchases a case designed to display the folded flag to protect it from wear. One such display case has dimensions (in inches) shown below. Write a polynomial expression that represents the area of wall space covered by the display case.

$$\frac{x^2}{2} + \frac{5}{4}x - \frac{25}{4}$$

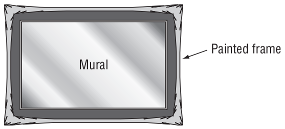


Source: American Flag Store

4. **MATH FUN** Think of a whole number. Subtract 2. Write down this number. Take the original number and add 2. Write down this number. Find the product of the numbers you wrote down. Subtract the square of the original number. The result is always –4. Use polynomials to show how this number trick works.

$$(x - 2)(x + 2) - x^2 = -4$$
$$x^2 - 2x + 2x - 4 - x^2 = -4$$
$$-4 = -4$$

5. **ART** The museum where Julia works plans to have a large wall mural replica of Vincent van Gogh's *The Starry Night* painted in its lobby. First, Julia wants to paint a large frame around where the mural will be. The mural's length will be 5 feet longer than its width, and the frame will be 2 feet wide on all sides. Julia has only enough paint to cover 100 square feet of wall surface. How large can the mural be?



a. Write an expression for the area of the mural. $\mathbf{w(w + 5)}$

b. Write an expression for the area of the frame.
$\mathbf{(w + 9)(w + 4) - (w + 5)w}$

c. Write and solve an equation to find how large the mural can be.
$\mathbf{(w + 9)(w + 4) - (w + 5)w = 100}$
8 ft by 13 ft

NAME ______________________ DATE __________ PERIOD __________

8-3 Enrichment

Pascal's Triangle

This arrangement of numbers is called Pascal's Triangle. It was first published in 1665, but was known hundreds of years earlier.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

1. Each number in the triangle is found by adding two numbers. What two numbers were added to get the 6 in the 5th row?
3 and 3

2. Describe how to create the 6th row of Pascal's Triangle.
The first and last numbers are 1. Evaluate 1 + 4, 4 + 6, 6 + 4, and 4 + 1 to find the other numbers.

3. Write the numbers for rows 6 through 10 of the triangle.

Row 6:	**1**	**5**	**10**	**10**	**5**	**1**					
Row 7:	**1**	**6**	**15**	**20**	**15**	**6**	**1**				
Row 8:	**1**	**7**	**21**	**35**	**35**	**21**	**7**	**1**			
Row 9:	**1**	**8**	**28**	**56**	**70**	**56**	**28**	**8**	**1**		
Row 10:	**1**	**9**	**36**	**84**	**126**	**126**	**84**	**36**	**9**	**1**	

Multiply to find the expanded form of each product.

4. $(a + b)^2$ $\mathbf{a^2 + 2ab + b^2}$

5. $(a + b)^3$ $\mathbf{a^3 + 3a^2b + 3ab^2 + b^3}$

6. $(a + b)^4$ $\mathbf{a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4}$

Now compare the coefficients of the three products in Exercises 4–6 with Pascal's Triangle.

7. Describe the relationship between the expanded form of $(a + b)^n$ and Pascal's Triangle.
The coefficients of the expanded form are found in row $n + 1$ of Pascal's Triangle.

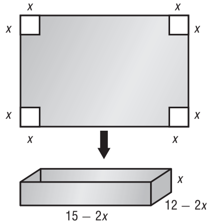
8. Use Pascal's Triangle to write the expanded form of $(a + b)^6$.
$\mathbf{a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6}$

NAME ______________________ DATE ____________ PERIOD ____________

8-3 Spreadsheet Activity

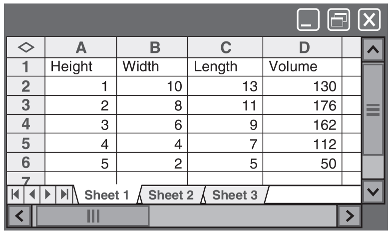
Multiplying Polynomials

Example **A box is made by cutting a square with sides x inches long from each corner of a piece of cardboard and folding up the sides. If the piece of cardboard is 15 inches long and 12 inches wide, what integer value of x allows you to make the box with the greatest volume? What is the volume?**



Step 1 The finished box will be x inches high, $12 - 2x$ inches wide, and $15 - 2x$ inches long. The volume of the box is $x(12 - 2x)(15 - 2x)$ cubic inches.

Step 2 Use Column A of the spreadsheet for the value of x. Enter the formulas for the width, length, and volume in Columns B, C, and D.



◇	A	B	C	D
1	Height	Width	Length	Volume
2	1	10	13	130
3	2	8	11	176
4	3	6	9	162
5	4	4	7	112
6	5	2	5	50
7				

Notice that x cannot be greater than 5 because $12 - 2x$ must be positive. In this case, the box with the greatest volume is 176 cubic inches when $x = 2$.

Exercises

Use the spreadsheet to find the value of x that allows the box with the greatest volume for each piece of cardboard. State the volume of the box.

1. 16 inches long and 10 inches wide
2; 144 in^3

2. 24 inches long and 18 inches wide
3; 648 in^3

3. 28 inches long and 16 inches wide
3; 660 in^3

4. 36 inches long and 24 inches wide
5; 1820 in^3

5. 48 inches long and 48 inches wide
8; 8192 in^3

6. 108 inches long and 44 inches wide
10; 21,120 in^3

7. Study the spreadsheet you created for Exercise 5. Suppose y is the volume of the box with a height of x inches. If you were to graph the ordered pairs (x, y) and connect them with a smooth curve, what would you expect the graph to look like? Use the graphing tool in the spreadsheet to verify your conjecture. **Sample answer: The graph would rise from (0, 0) to the point where x gives the greatest volume and then fall back down toward the x-axis.**

NAME ______________________ DATE ____________ PERIOD ____________

8-4 Study Guide and Intervention

Special Products

Squares of Sums and Differences Some pairs of binomials have products that follow specific patterns. One such pattern is called the *square of a sum*. Another is called the *square of a difference*.

Square of a Sum	$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$
Square of a Difference	$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$

Example 1 **Find $(3a + 4)(3a + 4)$.**

Use the square of a sum pattern, with $a = 3a$ and $b = 4$.

$$(3a + 4)(3a + 4) = (3a)^2 + 2(3a)(4) + (4)^2$$
$$= 9a^2 + 24a + 16$$

The product is $9a^2 + 24a + 16$.

Example 2 **Find $(2z - 9)(2z - 9)$.**

Use the square of a difference pattern with $a = 2z$ and $b = 9$.

$$(2z - 9)(2z - 9) = (2z)^2 - 2(2z)(9) + (9)(9)$$
$$= 4z^2 - 36z + 81$$

The product is $4z^2 - 36z + 81$.

Exercises

Find each product.

1. $(x - 6)^2$
$x^2 - 12x + 36$

2. $(3p + 4)^2$
$9p^2 + 24p + 16$

3. $(4x - 5)^2$
$16x^2 - 40x + 25$

4. $(2x - 1)^2$
$4x^2 - 4x + 1$

5. $(2h + 3)^2$
$4h^2 + 12h + 9$

6. $(m + 5)^2$
$m^2 + 10m + 25$

7. $(a + 3)^2$
$a^2 + 6a + 9$

8. $(3 - p)^2$
$9 - 6p + p^2$

9. $(x - 5y)^2$
$x^2 - 10xy + 25y^2$

10. $(8y + 4)^2$
$64y^2 + 64y + 16$

11. $(8 + x)^2$
$64 + 16x + x^2$

12. $(3a - 2b)^2$
$9a^2 - 12ab + 4b^2$

13. $(2x - 8)^2$
$4x^2 - 32x + 64$

14. $(x^2 + 1)^2$
$x^4 + 2x^2 + 1$

15. $(m^2 - 2)^2$
$m^4 - 4m^2 + 4$

16. $(x^3 - 1)^2$
$x^6 - 2x^3 + 1$

17. $(2h^2 - k^2)^2$
$4h^4 - 4h^2k^2 + k^4$

18. $\left(\frac{1}{4}x + 3\right)^2$
$\frac{1}{16}x^2 + \frac{3}{2}x + 9$

19. $(x - 4y^2)^2$
$x^2 - 8xy^2 + 16y^4$

20. $(2p + 4r)^2$
$4p^2 + 16pr + 16r^2$

21. $\left(\frac{2}{3}x - 2\right)^2$
$\frac{4}{9}x^2 - \frac{8}{3}x + 4$

NAME ______________ DATE ________ PERIOD ________

8-4 Study Guide and Intervention *(continued)*

Special Products

Product of a Sum and a Difference There is also a pattern for the product of a sum and a difference of the same two terms, $(a + b)(a - b)$. The product is called the **difference of squares**.

Product of a Sum and a Difference	$(a + b)(a - b) = a^2 - b^2$

Example **Find $(5x + 3y)(5x - 3y)$.**

$(a + b)(a - b) = a^2 - b^2$ Product of a Sum and a Difference

$(5x + 3y)(5x - 3y) = (5x)^2 - (3y)^2$ $a = 5x$ and $b = 3y$

$= 25x^2 - 9y^2$ Simplify.

The product is $25x^2 - 9y^2$.

Exercises

Find each product.

1. $(x - 4)(x + 4)$ **$x^2 - 16$**

2. $(p + 2)(p - 2)$ **$p^2 - 4$**

3. $(4x - 5)(4x + 5)$ **$16x^2 - 25$**

4. $(2x - 1)(2x + 1)$ **$4x^2 - 1$**

5. $(h + 7)(h - 7)$ **$h^2 - 49$**

6. $(m - 5)(m + 5)$ **$m^2 - 25$**

7. $(2d - 3)(2d + 3)$ **$4d^2 - 9$**

8. $(3 - 5q)(3 + 5q)$ **$9 - 25q^2$**

9. $(x - y)(x + y)$ **$x^2 - y^2$**

10. $(y - 4x)(y + 4x)$ **$y^2 - 16x^2$**

11. $(8 + 4x)(8 - 4x)$ **$64 - 16x^2$**

12. $(3a - 2b)(3a + 2b)$ **$9a^2 - 4b^2$**

13. $(3y - 8)(3y + 8)$ **$9y^2 - 64$**

14. $(x^2 - 1)(x^2 + 1)$ **$x^4 - 1$**

15. $(m^2 - 5)(m^2 + 5)$ **$m^4 - 25$**

16. $(x^3 - 2)(x^3 + 2)$ **$x^6 - 4$**

17. $(h^2 - k^2)(h^2 + k^2)$ **$h^4 - k^4$**

18. $\left(\frac{1}{4}x + 2\right)\left(\frac{1}{4}x - 2\right)$ **$\frac{1}{16}x^2 - 4$**

19. $(3x - 2y^2)(3x + 2y^2)$ **$9x^2 - 4y^4$**

20. $(2p - 5r)(2p + 5r)$ **$4p^2 - 25r^2$**

21. $\left(\frac{4}{3}x - 2y\right)\left(\frac{4}{3}x + 2y\right)$ **$\frac{16}{9}x^2 - 4y^2$**

Chapter 8 26 *Glencoe Algebra 1*

NAME ______________ DATE ________ PERIOD ________

8-4 Skills Practice

Special Products

Find each product.

1. $(n + 3)^2$ **$n^2 + 6n + 9$**

2. $(x + 4)(x + 4)$ **$x^2 + 8x + 16$**

31. $(y - 7)^2$ **$y^2 - 14y + 49$**

4. $(t - 3)(t - 3)$ **$t^2 - 6t + 9$**

5. $(b + 1)(b - 1)$ **$b^2 - 1$**

6. $(a - 5)(a + 5)$ **$a^2 - 25$**

7. $(p - 4)^2$ **$p^2 - 8p + 16$**

8. $(z + 3)(z - 3)$ **$z^2 - 9$**

9. $(\ell + 2)(\ell + 2)$ **$\ell^2 + 4\ell + 4$**

10. $(r - 1)(r - 1)$ **$r^2 - 2r + 1$**

11. $(3g + 2)(3g - 2)$ **$9g^2 - 4$**

12. $(2m - 3)(2m + 3)$ **$4m^2 - 9$**

13. $(6 + u)^2$ **$36 + 12u + u^2$**

14. $(r + t)^2$ **$r^2 + 2rt + t^2$**

15. $(3q + 1)(3q - 1)$ **$9q^2 - 1$**

16. $(c - d)^2$ **$c^2 - 2cd + d^2$**

17. $(2k - 2)^2$ **$4k^2 - 8k + 4$**

18. $(w + 3h)^2$ **$w^2 + 6wh + 9h^2$**

19. $(3p - 4)(3p + 4)$ **$9p^2 - 16$**

20. $(t + 2u)^2$ **$t^2 + 4tu + 4u^2$**

21. $(x - 4y)^2$ **$x^2 - 8xy + 16y^2$**

22. $(3b + 7)(3b - 7)$ **$9b^2 - 49$**

23. $(3y - 3g)(3y + 3g)$ **$9y^2 - 9g^2$**

24. $(n^2 + r^2)^2$ **$n^4 + 2n^2r^2 + r^4$**

25. $(2k + m^2)^2$ **$4k^2 + 4km^2 + m^4$**

26. $(3t^2 - n)^2$ **$9t^4 - 6t^2n + n^2$**

27. GEOMETRY The length of a rectangle is the sum of two whole numbers. The width of the rectangle is the difference of the same two whole numbers. Using these facts, write a verbal expression for the area of the rectangle. **The area is the square of the larger number minus the square of the smaller number.**

Chapter 8 27 *Glencoe Algebra 1*

Lesson 8-4

Answers (Lesson 8-4)

NAME ______________ DATE ________ PERIOD ________

8-4 Practice

Special Products

Find each product.

1. $(n + 9)^2$ **$n^2 + 18n + 81$**

2. $(q + 8)^2$ **$q^2 + 16q + 64$**

3. $(x - 10)^2$ **$x^2 - 20x + 100$**

4. $(r - 11)^2$ **$r^2 - 22r + 121$**

5. $(p + 7)^2$ **$p^2 + 14p + 49$**

6. $(b + 6)(b - 6)$ **$b^2 - 36$**

7. $(z + 13)(z - 13)$ **$z^2 - 169$**

8. $(4j + 2)^2$ **$16j^2 + 16j + 4$**

9. $(5w - 4)^2$ **$25w^2 - 40w + 16$**

10. $(6h - 1)^2$ **$36h^2 - 12h + 1$**

11. $(3m + 4)^2$ **$9m^2 + 24m + 16$**

12. $(7v - 2)^2$ **$49v^2 - 28v + 4$**

13. $(7k + 3)(7k - 3)$ **$49k^2 - 9$**

14. $(4d - 7)(4d + 7)$ **$16d^2 - 49$**

15. $(3g + 9h)(3g - 9h)$ **$9g^2 - 81h^2$**

16. $(4q + 5t)(4q - 5t)$ **$16q^2 - 25t^2$**

17. $(a + 6u)^2$ **$a^2 + 12au + 36u^2$**

18. $(5r + p)^2$ **$25r^2 + 10rp + p^2$**

19. $(6h - m)^2$ **$36h^2 - 12hm + m^2$**

20. $(k - 6y)^2$ **$k^2 - 12ky + 36y^2$**

21. $(u - 7p)^2$ **$u^2 - 14up + 49p^2$**

22. $(4b - 7v)^2$ **$16b^2 - 56bv + 49v^2$**

23. $(6n + 4p)^2$ **$36n^2 + 48np + 16p^2$**

24. $(5q + 6t)^2$ **$25q^2 + 60qt + 36t^2$**

25. $(6a - 7b)(6a + 7b)$ **$36a^2 - 49b^2$**

26. $(8h + 3d)(8h - 3d)$ **$64h^2 - 9d^2$**

27. $(9x + 2y^2)^2$ **$81x^2 + 36xy^2 + 4y^4$**

28. $(3p^3 + 2m)^2$ **$9p^6 + 12p^3m + 4m^2$**

29. $(5a^2 - 2b)^2$ **$25a^4 - 20a^2b + 4b^2$**

30. $(4m^3 - 2t)^2$ **$16m^6 - 16m^3t + 4t^2$**

31. $(6b^3 - g)^2$ **$36b^6 - 12b^3g + g^2$**

32. $(2b^2 - g)(2b^2 + g)$ **$4b^4 - g^2$**

33. $(2v^2 + 3x^2)(2v^2 + 3x^2)$ **$4v^4 + 12v^2x^2 + 9x^4$**

34. GEOMETRY Janelle wants to enlarge a square graph that she has made so that a side of the new graph will be 1 inch more than twice the original side g. What trinomial represents the area of the enlarged graph? **$4g^2 + 4g + 1$**

35. GENETICS In a guinea pig, pure black hair coloring B is dominant over pure white coloring b. Suppose two hybrid Bb guinea pigs, with black hair coloring, are bred.

a. Find an expression for the genetic make-up of the guinea pig offspring.
$0.25BB + 0.50Bb + 0.25bb$

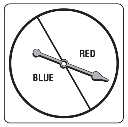
b. What is the probability that two hybrid guinea pigs with black hair coloring will produce a guinea pig with white hair coloring? **25%**

NAME ______________ DATE ________ PERIOD ________

8-4 Word Problem Practice

Special Products

1. PROBABILITY The spinner below is divided into 2 equal sections. If you spin the spinner 2 times in a row, the possible outcomes are shown in the table below.

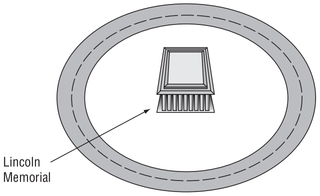


red red	blue red
red blue	blue blue

What is the probability of spinning a blue and a red in two spins? **50%**

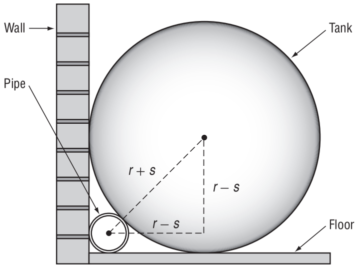
2. GRAVITY The height of a penny t seconds after being dropped down a well is given by the product of $(10 - 4t)$ and $(10 + 4t)$. Find the product and simplify. What type of special product does this represent?
$100 - 16t^2$; product of a sum and difference

3. TRAFFIC PLANNING The Lincoln Memorial in Washington, D.C., is surrounded by a circular drive called Lincoln Circle. Suppose the National Park Service wants to change the layout of Lincoln Circle so that there are two concentric circular roads. Write a polynomial equation for the area A of the space between the roads if the radius of the inside road is 10 meters less than the radius of the outside road.
$A = 20\pi r - 100\pi$



4. BUSINESS The Combo Lock Company finds that its profit data from 2005 to the present can be modeled by the function $y = 4n^2 + 44n + 121$, where y is the profit n years since 2005. Which special product does this polynomial demonstrate? Explain.
square of a sum; it can also be written as $(2n + 11)^2$

5. STORAGE A cylindrical tank is placed along a wall. A cylindrical PVC pipe will be hidden in the corner behind the tank. See the side view diagram below. The radius of the tank is r inches and the radius of the PVC pipe is s inches.



a. Use the Pythagorean Theorem to write an equation for the relationship between the two radii. Simplify your equation so that there is a zero on one side of the equals sign.
$0 = r^2 - 6rs + s^2$

b. Write a polynomial equation you could solve to find the radius s of the PVC pipe if the radius of the tank is 20 inches. **$0 = s^2 - 120s + 400$**

NAME ______________________ DATE ____________ PERIOD ____________

8-4 Enrichment

Sums and Differences of Cubes

Recall the formulas for finding some special products:

Perfect-square trinomials: $(a + b)^2 = a^2 + 2ab + b^2$ or
$(a - b)^2 = a^2 - 2ab + b^2$

Difference of two squares: $(a + b)(a - b) = a^2 - b^2$

A pattern also exists for finding the cube of a sum $(a + b)^3$.

1. Find the product of $(a + b)(a + b)(a + b)$.
$\mathbf{a^3 + 3a^2b + 3ab^2 + b^3}$

2. Use the pattern from Exercise 1 to evaluate $(x + 2)^3$.
$\mathbf{x^3 + 6x^2 + 12x + 8}$

3. Based on your answer to Exercise 1, predict the pattern for the cube of a difference $(a - b)^3$.
$\mathbf{a^3 - 3a^2b + 3ab^2 - b^3}$

4. Find the product of $(a - b)(a - b)(a - b)$ and compare it to your answer for Exercise 3.
$\mathbf{a^3 - 3a^2b + 3ab^2 - b^3}$

5. Use the pattern from Exercise 4 to evaluate $(x - 4)^3$.
$\mathbf{x^3 - 12x^2 + 48x - 64}$

Find each product.

6. $(x + 6)^3$
$\mathbf{x^3 + 18x^2 + 108x + 216}$

7. $(x - 10)^3$
$\mathbf{x^3 - 30x^2 + 300x - 1000}$

8. $(3x - y)^3$
$\mathbf{27x^3 - 27x^2y + 9xy^2 - y^3}$

9. $(2x - y)^3$
$\mathbf{8x^3 - 12x^2y + 6xy^2 - y^3}$

10. $(4x + 3y)^3$
$\mathbf{64x^3 + 144x^2y + 108xy^2 + 27y^3}$

11. $(5x + 2)^3$
$\mathbf{125x^3 + 150x^2 + 60x + 8}$

NAME ______________________ DATE ____________ PERIOD ____________

8-5 Study Guide and Intervention

Using the Distributive Property

Use the Distributive Property to Factor The Distributive Property has been used to multiply a polynomial by a monomial. It can also be used to express a polynomial in factored form. Compare the two columns in the table below.

Multiplying	Factoring
$3(a + b) = 3a + 3b$	$3a + 3b = 3(a + b)$
$x(y - z) = xy - xz$	$xy - xz = x(y - z)$
$6y(2x + 1) = 6y(2x) + 6y(1)$ $= 12xy + 6y$	$12xy + 6y = 6y(2x) + 6y(1)$ $= 6y(2x + 1)$

Example 1 **Use the Distributive Property to factor $12mp + 80m^2$.**

Find the GCF of $12mp$ and $80m^2$.

$12mp = 2 \cdot 2 \cdot 3 \cdot m \cdot p$

$80m^2 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot m \cdot m$

$\text{GCF} = 2 \cdot 2 \cdot m$ or $4m$

Write each term as the product of the GCF and its remaining factors.

$$\begin{aligned} 12mp + 80m^2 &= 4m(3 \cdot p) + 4m(2 \cdot 2 \cdot 5 \cdot m) \\ &= 4m(3p) + 4m(20m) \\ &= 4m(3p + 20m) \end{aligned}$$

Thus $12mp + 80m^2 = 4m(3p + 20m)$.

Example 2 **Factor $6ax + 3ay + 2bx + by$ by grouping.**

$$\begin{aligned} &6ax + 3ay + 2bx + by \\ &= (6ax + 3ay) + (2bx + by) \\ &= 3a(2x + y) + b(2x + y) \\ &= (3a + b)(2x + y) \end{aligned}$$

Check using the FOIL method.

$$\begin{aligned} &(3a + b)(2x + y) \\ &= 3a(2x) + (3a)(y) + (b)(2x) + (b)(y) \\ &= 6ax + 3ay + 2bx + by \checkmark \end{aligned}$$

Exercises

Factor each polynomial.

1. $24x + 48y$
$\mathbf{24(x + 2y)}$

2. $30mp^2 + m^2p - 6p$
$\mathbf{p(30mp + m^2 - 6)}$

3. $q^4 - 18q^3 + 22q$
$\mathbf{q(q^3 - 18q^2 + 22)}$

4. $9x^2 - 3x$
$\mathbf{3x(3x - 1)}$

5. $4m + 6p - 8mp$
$\mathbf{2(2m + 3p - 4mp)}$

6. $45r^3 - 15r^2$
$\mathbf{15r^2(3r - 1)}$

7. $14t^3 - 42t^5 - 49t^4$
$\mathbf{7t^3(2 - 6t^2 - 7t)}$

8. $55p^2 - 11p^4 + 44p^5$
$\mathbf{11p^2(5 - p^2 + 4p^3)}$

9. $14y^3 - 28y^2 + y$
$\mathbf{y(14y^2 - 28y + 1)}$

10. $4x + 12x^2 + 16x^3$
$\mathbf{4x(1 + 3x + 4x^2)}$

11. $4a^2b + 28ab^2 + 7ab$
$\mathbf{ab(4a + 28b + 7)}$

12. $6y + 12x - 8z$
$\mathbf{2(3y + 6x - 4z)}$

13. $x^2 + 2x + x + 2$
$\mathbf{(x + 1)(x + 2)}$

14. $6y^2 - 4y + 3y - 2$
$\mathbf{(2y + 1)(3y - 2)}$

15. $4m^2 + 4mp + 3mp + 3p^2$
$\mathbf{(4m + 3p)(m + p)}$

16. $12ax + 3xz + 4ay + yz$
$\mathbf{(3x + y)(4a + z)}$

17. $12a^2 + 3a - 8a - 2$
$\mathbf{(4a + 1)(3a - 2)}$

18. $xa + ya + x + y$
$\mathbf{(x + y)(a + 1)}$

NAME ______________ DATE ________ PERIOD ________

8-5 Study Guide and Intervention *(continued)*

Using the Distributive Property

Solve Equations by Factoring The following property, along with factoring, can be used to solve certain equations.

Zero Product Property	For any real numbers *a* and *b*, if $ab = 0$, then either $a = 0$, $b = 0$, or both *a* and *b* equal 0.

Example **Solve $9x^2 + x = 0$. Then check the solutions.**

Write the equation so that it is of the form $ab = 0$.

$9x^2 + x = 0$ Original equation

$x(9x + 1) = 0$ Factor the GCF of $9x^2 + x$, which is x.

$x = 0$ or $9x + 1 = 0$ Zero Product Property

$x = 0$ $\quad x = -\frac{1}{9}$ Solve each equation.

The solution set is $\left\{0, -\frac{1}{9}\right\}$.

Check Substitute 0 and $-\frac{1}{9}$ for x in the original equation.

$9x^2 + x = 0$

$9(0)^2 + 0 \stackrel{?}{=} 0$

$0 = 0$ ✓

$9x^2 + x = 0$

$9\left(-\frac{1}{9}\right)^2 + \left(-\frac{1}{9}\right) \stackrel{?}{=} 0$

$\frac{1}{9} + \left(-\frac{1}{9}\right) \stackrel{?}{=} 0$

$0 = 0$ ✓

Exercises

Solve each equation. Check your solutions.

1. $x(x + 3) = 0$ **{0, −3}**

2. $3m(m - 4) = 0$ **{0, 4}**

3. $(r - 3)(r + 2) = 0$ **{−2, 3}**

4. $3x(2x - 1) = 0$ $\left\{\mathbf{0, \frac{1}{2}}\right\}$

5. $(4m + 8)(m - 3) = 0$ **{−2, 3}**

6. $5t^2 = 25t$ **{0, 5}**

7. $(4c + 2)(2c - 7) = 0$ $\left\{\mathbf{-\frac{1}{2}, \frac{7}{2}}\right\}$

8. $5p - 15p^2 = 0$ $\left\{\mathbf{0, \frac{1}{3}}\right\}$

9. $4y^2 = 28y$ **{0, 7}**

10. $12x^2 = -6x$ $\left\{\mathbf{-\frac{1}{2}, 0}\right\}$

11. $(4a + 3)(8a + 7) = 0$ $\left\{\mathbf{-\frac{7}{8}, -\frac{3}{4}}\right\}$

12. $8y = 12y^2$ $\left\{\mathbf{0, \frac{2}{3}}\right\}$

13. $x^2 = -2x$ **{−2, 0}**

14. $(6y - 4)(y + 3) = 0$ $\left\{\mathbf{-3, \frac{2}{3}}\right\}$

15. $4m^2 = 4m$ **{0, 1}**

16. $12x = 3x^2$ **{0, 4}**

17. $12a^2 = -3a$ $\left\{\mathbf{-\frac{1}{4}, 0}\right\}$

18. $(12a + 4)(3a - 1) = 0$ $\left\{\mathbf{-\frac{1}{3}, \frac{1}{3}}\right\}$

NAME ______________ DATE ________ PERIOD ________

8-5 Skills Practice

Using the Distributive Property

Factor each polynomial.

1. $7x + 49$ **7(*x* + 7)**

2. $8m - 6$ **2(4*m* − 3)**

3. $5a^2 - 15$ $\mathbf{5(a^2 - 3)}$

4. $10q - 25q^2$ **5*q*(2 − 5*q*)**

5. $8ax - 56a$ **8*a*(*x* − 7)**

6. $81r + 48rt$ **3*r*(27 + 16*t*)**

7. $t^2h + 3t$ **t(th + 3)**

8. $a^2b^2 + a$ $\mathbf{a(ab^2 + 1)}$

9. $x + x^2y + x^3y^2$ $\mathbf{x(1 + xy + x^2y^2)}$

10. $3p^2r^2 + 6pr + p$ $\mathbf{p(3pr^2 + 6r + 1)}$

11. $4a^2b^2 + 16ab + 12a$ $\mathbf{4a(ab^2 + 4b + 3)}$

12. $10h^3n^3 - 2hn^2 + 14hn$ $\mathbf{2hn(5h^2n^2 - n + 7)}$

13. $x^2 + 3x + x + 3$ **(*x* + 1)(*x* + 3)**

14. $b^2 - 2b + 3b - 6$ **(*b* + 3)(*b* − 2)**

15. $2j^2 + 2j + 3j + 3$ **(2*j* + 3)(*j* + 1)**

16. $2a^2 - 4a + a - 2$ **(2*a* + 1)(*a* − 2)**

17. $6t^2 - 4t - 3t + 2$ **(2*t* − 1)(3*t* − 2)**

18. $9x^2 - 3xy + 6x - 2y$ **(3*x* + 2)(3*x* − *y*)**

Solve each equation. Check your solutions.

19. $x(x - 8) = 0$ **{0, 8}**

20. $b(b + 12) = 0$ **{−12, 0}**

21. $(m - 3)(m + 5) = 0$ **{−5, 3}**

22. $(a - 9)(2a + 1) = 0$ $\left\{\mathbf{-\frac{1}{2}, 9}\right\}$

23. $x^2 - 5x = 0$ **{0, 5}**

24. $y^2 + 3y = 0$ **{−3, 0}**

25. $3a^2 = 6a$ **{0, 2}**

26. $2x^2 = 3x$ $\left\{\mathbf{0, \frac{3}{2}}\right\}$

Lesson 8-5

Answers

NAME ________________ DATE ________ PERIOD ________

8-5 Practice

Using the Distributive Property

Factor each polynomial.

1. $64 - 40ab$ **$8(8 - 5ab)$**

2. $4d^2 + 16$ **$4(d^2 + 4)$**

3. $6r^2t - 3rt^2$ **$3rt(2r - t)$**

4. $15ad + 30a^2d^2$ **$15ad(1 + 2ad)$**

5. $32a^2 + 24b^2$ **$8(4a^2 + 3b^2)$**

6. $36xy^2 - 48x^2y$ **$12xy(3y - 4x)$**

7. $30x^3y + 35x^2y^2$ **$5x^2y(6x + 7y)$**

8. $9a^3d^2 - 6ad^3$ **$3ad^2(3a^2 - 2d)$**

9. $75b^2g^3 + 60bg^3$ **$15bg^3(5b + 4)$**

10. $8p^2r^2 - 24pr^3 + 16pr$ **$8pr(pr - 3r^2 + 2)$**

11. $5x^3y^2 + 10x^2y + 25x$ **$5x(x^2y^2 + 2xy + 5)$**

12. $9ax^3 + 18bx^2 + 24cx$ **$3x(3ax^2 + 6bx + 8c)$**

13. $x^2 + 4x + 2x + 8$ **$(x + 2)(x + 4)$**

14. $2a^2 + 3a + 6a + 9$ **$(a + 3)(2a + 3)$**

15. $4b^2 - 12b + 2b - 6$ **$(4b + 2)(b - 3)$**

16. $6xy - 8x + 15y - 20$ **$(2x + 5)(3y - 4)$**

17. $-6mp + 4m + 18p - 12$ **$(-2m + 6)(3p - 2)$**

18. $12a^2 - 15ab - 16a + 20b$ **$(3a - 4)(4a - 5b)$**

Solve each equation. Check your solutions.

19. $x(x - 32) = 0$ **$\{0, 32\}$**

20. $4b(b + 4) = 0$ **$\{-4, 0\}$**

21. $(y - 3)(y + 2) = 0$ **$\{-2, 3\}$**

22. $(a + 6)(3a - 7) = 0$ **$\left\{-6, \frac{7}{3}\right\}$**

23. $(2y + 5)(y - 4) = 0$ **$\left\{-\frac{5}{2}, 4\right\}$**

24. $(4y + 8)(3y - 4) = 0$ **$\left\{-2, \frac{4}{3}\right\}$**

25. $2z^2 + 20z = 0$ **$\{-10, 0\}$**

26. $8p^2 - 4p = 0$ **$\left\{0, \frac{1}{2}\right\}$**

27. $9x^2 = 27x$ **$\{0, 3\}$**

28. $18x^2 = 15x$ **$\left\{0, \frac{5}{6}\right\}$**

29. $14x^2 = -21x$ **$\left\{-\frac{3}{2}, 0\right\}$**

30. $8x^2 = -26x$ **$\left\{-\frac{13}{4}, 0\right\}$**

31. LANDSCAPING A landscaping company has been commissioned to design a triangular flower bed for a mall entrance. The final dimensions of the flower bed have not been determined, but the company knows that the height will be two feet less than the base. The area of the flower bed can be represented by the equation $A = \frac{1}{2}b^2 - b$.

a. Write this equation in factored form. **$A = b\left(\frac{1}{2}b - 1\right)$**

b. Suppose the base of the flower bed is 16 feet. What will be its area? **112 ft²**

32. PHYSICAL SCIENCE Mr. Alim's science class launched a toy rocket from ground level with an initial upward velocity of 60 feet per second. The height h of the rocket in feet above the ground after t seconds is modeled by the equation $h = 60t - 16t^2$. How long was the rocket in the air before it returned to the ground? **3.75 s**

NAME ________________ DATE ________ PERIOD ________

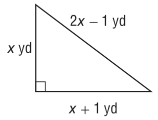
8-5 Word Problem Practice

Using the Distributive Property

1. PHYSICS According to legend, Galileo dropped objects of different weights from the so-called "leaning tower" of Pisa while developing his formula for free falling objects. The relationship that he discovered was that the distance d an object falls after t seconds is given by $d = 16t^2$ (ignoring air resistance). This relationship can be found in the equation $h = 4t - 16t^2$, where h is the height of an object thrown upward from ground level at a rate of 32 feet per second. Solve the equation for $h = 0$. **$t = 0.25$ and 0**

2. SWIMMING POOL The area A of a rectangular swimming pool is given by the equation $A = 12w - w^2$, where w is the width of one side. Write an expression for the other side of the pool. **$12 - w$**

3. CONSTRUCTION Unique Building Company is constructing a triangular roof truss for a building. The workers assemble the truss with the dimensions shown on the diagram below. Using the Pythagorean Theorem, find the length of the sides of the truss. **3 yd, 4 yd, 5 yd**



4. VERTICAL JUMP Your vertical jump height is measured by subtracting your standing reach height from the height of the highest point you can reach by jumping without taking a running start. Typically, NBA players have vertical jump heights of up to 34 inches. If an NBA player jumps this high, his height h in inches above his standing reach height after t seconds can be modeled by $h = 162t - 192t^2$. Solve the equation for $h = 0$ and interpret the solution. Round your answer to the nearest hundredth.

$t = 0$ and $t \approx 0.844$; The player lands after about 0.84 second.

5. PETS Conner tosses a dog treat upward with an initial velocity of 13.7 meters per second. The height of the treat above the dog's mouth h in meters after t seconds is given by $h = 13.7t - 4.9t^2$.

a. Assuming the dog doesn't jump, after how many seconds does the dog catch the treat? **2.795**

b. The dog treat reaches its maximum height halfway between when it was thrown and when it was caught. What is its maximum height? **9.6 m**

c. How fast would Connor have to throw the dog treat in order to make it fly through the air for 6 seconds? **at 29.4 m/s**

NAME ______________ DATE ________ PERIOD ________

8-5 Enrichment

Linear Combinations

The greatest common factor, GCF, of two numbers can be written as a linear combination of the two numbers. A linear combination is an expression of the form $Ax + By$.

Example **Write the greatest common factor of 52 and 36 as a linear combination.**

First, use the Euclidean Algorithm to find the greatest common factor of the two numbers.

$$36\overset{1}{\overline{)52}}\quad -36 \qquad \text{Divide the greater number by the lesser number.}$$

$$16\overset{2}{\overline{)36}}\ \text{original divisor;}\quad -32 \qquad \text{Then divide using the remainder as the new divisor.}$$

$$4\overset{4}{\overline{)16}}\ \text{second divisor;}\quad -16 \qquad \text{Divide again.}$$

$$0 \qquad \text{Stop dividing.}$$

Last divisor used is the GCF. In this case, 4 is the GCF for 36 and 52.

To write 4 as a linear combination of 36 and 52, it needs to be written as:

$4 = 36x + 52y$, where x and y are some integers.

Use trial and error to determine the two integers.

The two integers that work are $x = 3$ and $y = -2$. So, the linear combination for the greatest common factor of 52 and 36 is:

$4 = 36(3) + 52(-2)$

Exercises

Write the greatest common factor for each pair of numbers as a linear combination.

1. 16, 64
$16 = 16(1) + 64(0)$

2. 21, 28
$7 = 21(-1) + 28(1)$

3. 3, 18
$3 = 3(1) + 18(0)$

4. 15, 36
$3 = 15(5) - 36(2)$

5. 6, 8
$2 = 6(-1) + 8(1)$

6. 18, 42
$6 = 18(-2) + 42(1)$

NAME ______________ DATE ________ PERIOD ________

8-6 Study Guide and Intervention

Solving $x^2 + bx + c = 0$

Factor $x^2 + bx + c$ To factor a trinomial of the form $x^2 + bx + c$, find two integers, m and p, whose sum is equal to b and whose product is equal to c.

Factoring $x^2 + bx + c$	$x^2 + bx + c = (x + m)(x + p)$, where $m + p = b$ and $mp = c$.

Example 1 **Factor each polynomial.**

a. $x^2 + 7x + 10$

In this trinomial, $b = 7$ and $c = 10$.

Factors of 10	Sum of Factors
1, 10	11
2, 5	7

Since $2 + 5 = 7$ and $2 \cdot 5 = 10$, let $m = 2$ and $p = 5$.

$x^2 + 7x + 10 = (x + 5)(x + 2)$

b. $x^2 - 8x + 7$

In this trinomial, $b = -8$ and $c = 7$.

Notice that $m + p$ is negative and mp is positive, so m and p are both negative. Since $-7 + (-1) = -8$ and $(-7)(-1) = 7$, $m = -7$ and $p = -1$.

$x^2 - 8x + 7 = (x - 7)(x - 1)$

Example 2 **Factor $x^2 + 6x - 16$.**

In this trinomial, $b = 6$ and $c = -16$. This means $m + p$ is positive and mp is negative. Make a list of the factors of -16, where one factor of each pair is positive.

Factors of –16	Sum of Factors
1, –16	–15
–1, 16	15
2, –8	–6
–2, 8	6

Therefore, $m = -2$ and $p = 8$.

$x^2 + 6x - 16 = (x - 2)(x + 8)$

Exercises

Factor each polynomial.

1. $x^2 + 4x + 3$
$(x + 3)(x + 1)$

2. $m^2 + 12m + 32$
$(m + 4)(m + 8)$

3. $r^2 - 3r + 2$
$(r - 2)(r - 1)$

4. $x^2 - x - 6$
$(x - 3)(x + 2)$

5. $x^2 - 4x - 21$
$(x - 7)(x + 3)$

6. $x^2 - 22x + 121$
$(x - 11)(x - 11)$

7. $t^2 - 4t - 12$
$(t + 2)(t - 6)$

8. $p^2 - 16p + 64$
$(p - 8)(p - 8)$

9. $9 - 10x + x^2$
$(9 - x)(1 - x)$

10. $x^2 + 6x + 5$
$(x + 5)(x + 1)$

11. $a^2 + 8a - 9$
$(a - 1)(a + 9)$

12. $y^2 - 7y - 8$
$(y - 8)(y + 1)$

13. $x^2 - 2x - 3$
$(x - 3)(x + 1)$

14. $y^2 + 14y + 13$
$(y + 1)(y + 13)$

15. $m^2 + 9m + 20$
$(m + 4)(m + 5)$

16. $x^2 + 12x + 20$
$(x + 10)(x + 2)$

17. $a^2 - 14a + 24$
$(a - 2)(a - 12)$

18. $18 + 11y + y^2$
$(9 + y)(2 + y)$

19. $x^2 + 2xy + y^2$
$(x + y)(x + y)$

20. $a^2 - 4ab + 4b^2$
$(a - 2b)(a - 2b)$

21. $x^2 + 6xy - 7y^2$
$(x + 7y)(x - y)$

NAME ______________________ DATE ____________ PERIOD ____________

8-6 Study Guide and Intervention *(continued)*

Solving $x^2 + bx + c = 0$

Solve Equations by Factoring Factoring and the Zero Product Property can be used to solve many equations of the form $x^2 + bx + c = 0$.

Example 1 **Solve $x^2 + 6x = 7$. Check your solutions.**

$x^2 + 6x = 7$ Original equation

$x^2 + 6x - 7 = 0$ Rewrite equation so that one side equals 0.

$(x - 1)(x + 7) = 0$ Factor.

$x - 1 = 0$ or $x + 7 = 0$ Zero Product Property

$x = 1$ $x = -7$ Solve each equation.

The solution set is $\{1, -7\}$. Since $1^2 + 6(1) = 7$ and $(-7)^2 + 6(-7) = 7$, the solutions check.

Example 2 **ROCKET LAUNCH The formula $h = vt - 16t^2$ gives the height h of a rocket after t seconds when the initial velocity v is given in feet per second. If a rocket is fired with initial velocity 2288 feet per second, how many seconds will it take for the rocket to reach a height of 6720 feet?**

$h = vt - 16t^2$ Formula

$6720 = 2288t - 16t^2$ Substitute.

$0 = -16t^2 + 2288t - 6720$ Rewrite equation so that one side equals 0.

$0 = -16(t - 143t + 420)$ Factor out GCF.

$0 = -16(t - 3)(t - 140)$ Factor

$t - 3 = 0$ or $t - 140 = 0$ Zero Product Property

$t = 3$ $t = 140$ Solve each equation.

The rocket reaches 6720 feet in 3 seconds and again in 140 seconds, or 2 minutes 20 seconds after launch.

Exercises

Solve each equation. Check the solutions.

1. $x^2 - 4x + 3 = 0$ **{1, 3}**
2. $y^2 - 5y + 4 = 0$ **{1, 4}**
3. $m^2 + 10m + 9 = 0$ **{−1, −9}**
4. $x^2 = x + 2$ **{−1, 2}**
5. $x^2 - 4x = 5$ **{−1, 5}**
6. $x^2 - 12x + 36 = 0$ **{6}**
7. $t^2 - 8 = -7t$ **{−8, 1}**
8. $p^2 = 9p - 14$ **{2, 7}**
9. $-9 - 8x + x^2 = 0$ **{−1, 9}**
10. $x^2 + 6 = 5x$ **{2, 3}**
11. $a^2 = 11a - 18$ **{2, 9}**
12. $y^2 - 8y + 15 = 0$ **{3, 5}**
13. $x^2 = 24 - 10x$ **{−12, 2}**
14. $a^2 - 18a = -72$ **{6, 12}**
15. $b^2 = 10b - 16$ **{2, 8}**

Use the formula $h = vt - 16t^2$ to solve each problem.

16. FOOTBALL A punter can kick a football with an initial velocity of 48 feet per second. How many seconds will it take for the ball to first reach a height of 32 feet? **1 second**

17. ROCKET LAUNCH If a rocket is launched with an initial velocity of 1600 feet per second, when will the rocket be 14,400 feet high? **at 10 seconds and at 90 seconds**

NAME ______________________ DATE ____________ PERIOD ____________

8-6 Skills Practice

Solving $x^2 + bx + c = 0$

Factor each polynomial.

1. $t^2 + 8t + 12$ **(t + 2)(t + 6)**
2. $n^2 + 7n + 12$ **(n + 3)(n + 4)**
3. $p^2 + 9p + 20$ **(p + 5)(p + 4)**
4. $h^2 + 9h + 18$ **(h + 6)(h + 3)**
5. $n^2 + 3n - 18$ **(n + 6)(n − 3)**
6. $x^2 + 2x - 8$ **(x + 4)(x − 2)**
7. $y^2 - 5y - 6$ **(y + 1)(y − 6)**
8. $g^2 + 3g - 10$ **(g + 5)(g − 2)**
9. $r^2 + 4r - 12$ **(r − 2)(r + 6)**
10. $x^2 - x - 12$ **(x − 4)(x + 3)**
11. $w^2 - w - 6$ **(w − 3)(w + 2)**
12. $y^2 - 6y + 8$ **(y − 2)(y − 4)**
13. $x^2 - 8x + 15$ **(x − 5)(x − 3)**
14. $b^2 - 9b + 8$ **(b − 1)(b − 8)**
15. $t^2 - 15t + 56$ **(t − 7)(t − 8)**
16. $-4 - 3m + m^2$ **(m − 4)(m + 1)**

Solve each equation. Check the solutions.

17. $x^2 - 6x + 8 = 0$ **{2, 4}**
18. $b^2 - 7b + 12 = 0$ **{3, 4}**
19. $m^2 + 5m + 6 = 0$ **{−3, −2}**
20. $d^2 + 7d + 10 = 0$ **{−5, −2}**
21. $y^2 - 2y - 24 = 0$ **{−4, 6}**
22. $p^2 - 3p = 18$ **{−3, 6}**
23. $h^2 + 2h = 35$ **{−7, 5}**
24. $a^2 + 14a = -45$ **{−9, −5}**
25. $n^2 - 36 = 5n$ **{−4, 9}**
26. $w^2 + 30 = 11w$ **{5, 6}**

NAME ______________ DATE ________ PERIOD ________

8-6 Practice

Solving $x^2 + bx + c = 0$

Factor each polynomial.

1. $a^2 + 10a + 24$ **$(a + 4)(a + 6)$**

2. $h^2 + 12h + 27$ **$(h + 3)(h + 9)$**

3. $x^2 + 14x + 33$ **$(x + 11)(x + 3)$**

4. $g^2 - 2g - 63$ **$(g + 7)(g - 9)$**

5. $w^2 + w - 56$ **$(w + 8)(w - 7)$**

6. $y^2 + 4y - 60$ **$(y + 10)(y - 6)$**

7. $b^2 + 4b - 32$ **$(b - 4)(b + 8)$**

8. $n^2 - 3n - 28$ **$(n - 7)(n + 4)$**

9. $t^2 + 4t - 45$ **$(t - 5)(t + 9)$**

10. $z^2 - 11z + 30$ **$(z - 6)(z - 5)$**

11. $d^2 - 16d + 63$ **$(d - 9)(d - 7)$**

12. $x^2 - 11x + 24$ **$(x - 3)(x - 8)$**

13. $q^2 - q - 56$ **$(q - 8)(q + 7)$**

14. $x^2 - 6x - 55$ **$(x + 5)(x - 11)$**

15. $32 + 18r + r^2$ **$(r + 16)(r + 2)$**

16. $48 - 16g + g^2$ **$(g - 12)(g - 4)$**

17. $j^2 - 9jk - 10k^2$ **$(j - 10k)(j + k)$**

18. $m^2 - mv - 56v^2$ **$(m - 8v)(m + 7v)$**

Solve each equation. Check the solutions.

19. $x^2 + 17x + 42 = 0$ **$\{-14, -3\}$**

20. $p^2 + 5p - 84 = 0$ **$\{-12, 7\}$**

21. $k^2 + 3k - 54 = 0$ **$\{-9, 6\}$**

22. $b^2 - 12b - 64 = 0$ **$\{-4, 16\}$**

23. $n^2 + 4n = 32$ **$\{-8, 4\}$**

24. $h^2 - 17h = -60$ **$\{5, 12\}$**

25. $t^2 - 26t = 56$ **$\{-2, 28\}$**

26. $z^2 - 14z = 72$ **$\{-4, 18\}$**

27. $y^2 - 84 = 5y$ **$\{-7, 12\}$**

28. $80 + a^2 = 18a$ **$\{8, 10\}$**

29. $u^2 = 16u + 36$ **$\{-2, 18\}$**

30. $17r + r^2 = -52$ **$\{-13, -4\}$**

31. Find all values of k so that the trinomial $x^2 + kx - 35$ can be factored using integers.
−34, −2, 2, 34

32. **CONSTRUCTION** A construction company is planning to pour concrete for a driveway. The length of the driveway is 16 feet longer than its width w.

a. Write an expression for the area of the driveway. **$w(w + 16)$ ft^2**

b. Find the dimensions of the driveway if it has an area of 260 square feet.
10 ft by 26 ft

33. **WEB DESIGN** Janeel has a 10-inch by 12-inch photograph. She wants to scan the photograph, then reduce the result by the same amount in each dimension to post on her Web site. Janeel wants the area of the image to be one eighth that of the original photograph.

a. Write an equation to represent the area of the reduced image.
$(10 - x)(12 - x) = 15$, or $x^2 - 22x + 105 = 0$

b. Find the dimensions of the reduced image. **3 in. by 5 in.**

NAME ______________ DATE ________ PERIOD ________

8-6 Word Problem Practice

Solving $x^2 + bx + c = 0$

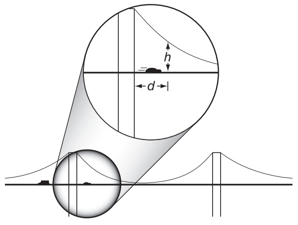
1. **COMPACT DISCS** A compact disc jewel case has a width 2 centimeters greater than its length. The area for the front cover is 168 square centimeters. The first two steps to finding the value of x are shown below. Solve the equation and find the length of the case.

$$\text{Length} \times \text{width} = \text{area}$$
$$x(x + 2) = 168$$
$$x^2 + 2x - 168 = 0$$

−14 or 12; 12 cm

2. **MATH GAMES** Fiona and Greg play a number guessing game. Greg gives Fiona this hint about his two secret numbers, "The product of the two consecutive positive integers that I am thinking of is 11 more than their sum." What are Greg's numbers? **4 and 5**

3. **BRIDGE ENGINEERING** A car driving over a suspension bridge is supported by a cable hanging between the ends of the bridge. Since its shape is parabolic, it can be modeled by a quadratic equation. The height above the road bed of a bridge's cable h in inches measured at distance d in yards from the first tower is given by $h = d^2 - 36d + 324$.



If the driver of a car looks out at a height of 49 inches above the roadbed, at what distance(s) from the tower will the driver's eyes be at the same height as the cable? **at 11 and 25 yds from the first tower**

4. **PHYSICAL SCIENCE** The boiling point of water depends on altitude. The following equation approximates the number of degrees D below 212ºF at which water will boil at altitude h.

$$D^2 + 520D = h$$

In Denver, Colorado, the altitude is approximately 5300 feet above sea level. At approximately what temperature does water boil in Denver?
$D = 10°$ drop
The boiling point is about 202°F.

5. **MONUMENTS** Susan is designing a pyramidal stone monument for a local park. The design specifications tell her that the height needs to be 9 feet, the width of the base must be 5 feet less than the length, and the volume should be 150 cubic feet. Recall that the volume of a pyramid is given by $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height.

a. Write and solve an equation to find the width of the base of the monument.
$150 = \frac{1}{3}w(w + 5) \cdot 9$ or
$3w^2 + 15w - 150 = 0$; $w = \{5, -10\}$

b. Interpret each answer in terms of the situation. **$w = 5$: the width of the pyramid is 5 feet; $w = -10$: negative length doesn't make sense in the situation.**

NAME ______________________ DATE ____________ PERIOD ____________

8-6 Enrichment

Puzzling Primes

A prime number has only two factors, itself and 1. The number 6 is not prime because it has 2 and 3 as factors; 5 and 7 are prime. The number 1 is not considered to be prime.

1. Use a calculator to help you find the 25 prime numbers less than 100.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Prime numbers have interested mathematicians for centuries. They have tried to find expressions that will give all the prime numbers, or only prime numbers. In the 1700s, Euler discovered that the trinomial $x^2 + x + 41$ will yield prime numbers for values of x from 0 through 39.

2. Find the prime numbers generated by Euler's formula for x from 0 through 7.

41, 43, 47, 53, 61, 71, 83, 97

3. Show that the trinomial $x^2 + x + 31$ will not give prime numbers for very many values of x.

It works for $x = 0, 2, 3, 5$, and 6 but not for $x = 1, 4$, and 7.

4. Find the greatest prime number generated by Euler's formula.

1601

Goldbach's Conjecture is that every nonzero even number greater than 2 can be written as the sum of two primes. No one has ever proved that this is always true, but no one has found a counterexample, either.

5. Show that Goldbach's Conjecture is true for the first 5 even numbers greater than 2.

$4 = 2 + 2, 6 = 3 + 3, 8 = 3 + 5, 10 = 3 + 7, 12 = 5 + 7$

6. Give a way that someone could disprove Goldbach's Conjecture.

Find an even number that cannot be written as the sum of two primes.

NAME ______________________ DATE ____________ PERIOD ____________

8-7 Study Guide and Intervention

Solving $ax^2 + bx + c = 0$

Factor $ax^2 + bx + c$ To factor a trinomial of the form $ax^2 + bx + c$, find two integers, m and p whose product is equal to ac and whose sum is equal to b. If there are no integers that satisfy these requirements, the polynomial is called a **prime polynomial**.

Example 1 **Factor $2x^2 + 15x + 18$.**

In this example, $a = 2$, $b = 15$, and $c = 18$. You need to find two numbers that have a sum of 15 and a product of $2 \cdot 18$ or 36. Make a list of the factors of 36 and look for the pair of factors with a sum of 15.

Factors of 36	Sum of Factors
1, 36	37
2, 18	20
3, 12	15

Use the pattern $ax^2 + mx + px + c$, with $a = 2$, $m = 3$, $p = 12$, and $c = 18$.

$$\begin{aligned} 2x^2 + 15x + 18 &= 2x^2 + 3x + 12x + 18 \\ &= (2x^2 + 3x) + (12x + 18) \\ &= x(2x + 3) + 6(2x + 3) \\ &= (x + 6)(2x + 3) \end{aligned}$$

Therefore, $2x^2 + 15x + 18 = (x + 6)(2x + 3)$.

Example 2 **Factor $3x^2 - 3x - 18$.**

Note that the GCF of the terms $3x^2$, $3x$, and 18 is 3. First factor out this GCF.

$3x^2 - 3x - 18 = 3(x^2 - x - 6)$.

Now factor $x^2 - x - 6$. Since $a = 1$, find the two factors of -6 with a sum of -1.

Factors of –6	Sum of Factors
1, –6	–5
–1, 6	5
–2, 3	1
2, –3	–1

Now use the pattern $(x + m)(x + p)$ with $m = 2$ and $p = -3$.

$x^2 - x - 6 = (x + 2)(x - 3)$

The complete factorization is

$3x^2 - 3x - 18 = 3(x + 2)(x - 3)$.

Exercises

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

1. $2x^2 - 3x - 2$ **(2x + 1)(x – 2)**

2. $3m^2 - 8m - 3$ **(3m + 1)(m – 3)**

3. $16r^2 - 8r + 1$ **(4r – 1)(4r – 1)**

4. $6x^2 + 5x - 6$ **(2x + 3)(3x – 2)**

5. $3x^2 + 2x - 8$ **(3x – 4)(x + 2)**

6. $18x^2 - 27x - 5$ **(3x – 5)(6x + 1)**

7. $2a^2 + 5a + 3$ **(2a + 3)(a + 1)**

8. $18y^2 + 9y - 5$ **(6y + 5)(3y – 1)**

9. $-4t^2 + 19t - 21$ **(4t – 7)(3 – t)**

10. $8x^2 - 4x - 24$ **2(2x – 4)(2x + 3)**

11. $28p^2 + 60p - 25$ **(2p + 5)(14p – 5)**

12. $48x^2 + 22x - 15$ **(6x + 5)(8x – 3)**

13. $3y^2 - 6y - 24$ **3(y + 2)(y – 4)**

14. $4x^2 + 26x - 48$ **2(x + 8)(2x – 3)**

15. $8m^2 - 44m + 48$ **4(2m – 3)(m – 4)**

16. $6x^2 - 7x + 18$ **prime**

17. $2a^2 - 14a + 18$ **$2(a^2 - 7a + 9)$**

18. $18 + 11y + 2y^2$ **prime**

NAME ______________________ DATE ____________ PERIOD ________

8-7 Study Guide and Intervention *(continued)*

Solving $ax^2 + bx + c = 0$

Solve Equations by Factoring Factoring and the Zero Product Property can be used to solve some equations of the form $ax^2 + bx + c = 0$.

Example **Solve $12x^2 + 3x = 2 - 2x$. Check your solutions.**

$12x^2 + 3x = 2 - 2x$ Original equation

$12x^2 + 5x - 2 = 0$ Rewrite equation so that one side equals 0.

$(3x + 2)(4x - 1) = 0$ Factor the left side.

$3x + 2 = 0$ or $4x - 1 = 0$ Zero Product Property

$x = -\frac{2}{3}$ $x = \frac{1}{4}$ Solve each equation.

The solution set is $\left\{-\frac{2}{3}, \frac{1}{4}\right\}$.

Since $12\left(-\frac{2}{3}\right)^2 + 3\left(-\frac{2}{3}\right) = 2 - 2\left(-\frac{2}{3}\right)$ and $12\left(\frac{1}{4}\right)^2 + 3\left(\frac{1}{4}\right) = 2 - 2\left(\frac{1}{4}\right)$, the solutions check.

Exercises

Solve each equation. Check the solutions.

1. $8x^2 + 2x - 3 = 0$
$\left\{\frac{1}{2}, -\frac{3}{4}\right\}$

2. $3n^2 - 2n - 5 = 0$
$\left\{-1, \frac{5}{3}\right\}$

3. $2d^2 - 13d - 7 = 0$
$\left\{-\frac{1}{2}, 7\right\}$

4. $4x^2 = x + 3$
$\left\{1, -\frac{3}{4}\right\}$

5. $3x^2 - 13x = 10$
$\left\{-\frac{2}{3}, 5\right\}$

6. $6x^2 - 11x - 10 = 0$
$\left\{-\frac{2}{3}, \frac{5}{2}\right\}$

7. $2k^2 - 40 = -11k$
$\left\{-8, \frac{5}{2}\right\}$

8. $2p^2 = -21p - 40$
$\left\{-\frac{5}{2}, -8\right\}$

9. $-7 - 18x + 9x^2 = 0$
$\left\{\frac{7}{3}, -\frac{1}{3}\right\}$

10. $12x^2 - 15 = -8x$
$\left\{-\frac{3}{2}, \frac{5}{6}\right\}$

11. $7a^2 = -65a - 18$
$\left\{-\frac{2}{7}, -9\right\}$

12. $16y^2 - 2y - 3 = 0$
$\left\{\frac{1}{2}, -\frac{3}{8}\right\}$

13. $8x^2 + 5x = 3 + 7x$
$\left\{\frac{3}{4}, -\frac{1}{2}\right\}$

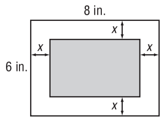
14. $4a^2 - 18a + 5 = 15$
$\left\{-\frac{1}{2}, 5\right\}$

15. $3b^2 - 18b = 10b - 49$
$\left\{\frac{7}{3}, 7\right\}$

16. The difference of the squares of two consecutive odd integers is 24. Find the integers.
−5, −7 and 5, 7

17. GEOMETRY The length of a Charlotte, North Carolina, conservatory garden is 20 yards greater than its width. The area is 300 square yards. What are the dimensions?
30 yd by 10 yd

18. GEOMETRY A rectangle with an area of 24 square inches is formed by cutting strips of equal width from a rectangular piece of paper. Find the dimensions of the new rectangle if the original rectangle measures 8 inches by 6 inches. **6 in. by 4 in.**



NAME ______________________ DATE ____________ PERIOD ________

8-7 Skills Practice

Solving $ax^2 + bx + c = 0$

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

1. $2x^2 + 5x + 2$
$(x + 2)(2x + 1)$

2. $3n^2 + 5n + 2$
$(3n + 2)(n + 1)$

3. $2t^2 + 9t - 5$
$(t + 5)(2t - 1)$

4. $3g^2 - 7g + 2$
$(3g - 1)(g - 2)$

5. $2t^2 - 11t + 15$
$(t - 3)(2t - 5)$

6. $2x^2 + 3x - 6$
prime

7. $2y^2 + y - 1$
$(y + 1)(2y - 1)$

8. $4h^2 + 8h - 5$
$(2h + 5)(2h - 1)$

9. $4x^2 - 3x - 3$
prime

10. $4b^2 + 15b - 4$
$(4b - 1)(b + 4)$

11. $9p^2 + 6p - 8$
$(3p - 2)(3p + 4)$

12. $6q^2 - 13q + 6$
$(3q - 2)(2q - 3)$

13. $3a^2 + 30a + 63$
$3(a + 7)(a + 3)$

14. $10w^2 - 19w - 15$
$(2w - 5)(5w + 3)$

Solve each equation. Check the solutions.

15. $2x^2 + 7x + 3 = 0$ $\left\{-3, -\frac{1}{2}\right\}$

16. $3w^2 + 14w + 8 = 0$ $\left\{-4, -\frac{2}{3}\right\}$

17. $3n^2 - 7n + 2 = 0$ $\left\{\frac{1}{3}, 2\right\}$

18. $5d^2 - 22d + 8 = 0$ $\left\{\frac{2}{5}, 4\right\}$

19. $6h^2 + 8h + 2 = 0$ $\left\{-1, -\frac{1}{3}\right\}$

20. $8p^2 - 16p = 10$ $\left\{-\frac{1}{2}, \frac{5}{2}\right\}$

21. $9y^2 + 18y - 12 = 6y$ $\left\{-2, \frac{2}{3}\right\}$

22. $4a^2 - 16a = -15$ $\left\{\frac{3}{2}, \frac{5}{2}\right\}$

23. $10b^2 - 15b = 8b - 12$ $\left\{\frac{4}{5}, \frac{3}{2}\right\}$

24. $6d^2 + 21d = 10d + 35$ $\left\{-\frac{7}{2}, \frac{5}{3}\right\}$

NAME ______________________ DATE ____________ PERIOD ____________

8-7 Practice

Solving $ax^2 + bx + c = 0$

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

1. $2b^2 + 10b + 12$ **$2(b + 2)(b + 3)$**

2. $3g^2 + 8g + 4$ **$(3g + 2)(g + 2)$**

3. $4x^2 + 4x - 3$ **$(2x + 3)(2x - 1)$**

4. $8b^2 - 5b - 10$ **prime**

5. $6m^2 + 7m - 3$ **$(3m - 1)(2m + 3)$**

6. $10d^2 + 17d - 20$ **$(5d - 4)(2d + 5)$**

7. $6a^2 - 17a + 12$ **$(3a - 4)(2a - 3)$**

8. $8w^2 - 18w + 9$ **$(4w - 3)(2w - 3)$**

9. $10x^2 - 9x + 6$ **prime**

10. $15n^2 - n - 28$ **$(5n - 7)(3n + 4)$**

11. $10x^2 + 21x - 10$ **$(2x + 5)(5x - 2)$**

12. $9r^2 + 15r + 6$ **$3(3r + 2)(r + 1)$**

13. $12y^2 - 4y - 5$ **$(2y + 1)(6y - 5)$**

14. $14k^2 - 9k - 18$ **$(2k - 3)(7k + 6)$**

15. $8z^2 + 20z - 48$ **$4(z + 4)(2z - 3)$**

16. $12q^2 + 34q - 28$ **$2(3q - 2)(2q + 7)$**

17. $18h^2 + 15h - 18$ **$3(2h + 3)(3h - 2)$**

18. $12p^2 - 22p - 20$ **$2(3p + 2)(2p - 5)$**

Solve each equation. Check the solutions.

19. $3h^2 + 2h - 16 = 0$ $\left\{-\frac{8}{3}, 2\right\}$

20. $15n^2 - n = 2$ $\left\{-\frac{1}{3}, \frac{2}{5}\right\}$

21. $8q^2 - 10q + 3 = 0$ $\left\{\frac{1}{2}, \frac{3}{4}\right\}$

22. $6b^2 - 5b = 4$ $\left\{-\frac{1}{2}, \frac{4}{3}\right\}$

23. $10r^2 - 21r = -4r + 6$ $\left\{-\frac{3}{10}, 2\right\}$

24. $10g^2 + 10 = 29g$ $\left\{\frac{2}{5}, \frac{5}{2}\right\}$

25. $6y^2 = -7y - 2$ $\left\{-\frac{2}{3}, -\frac{1}{2}\right\}$

26. $9z^2 = -6z + 15$ $\left\{-\frac{5}{3}, 1\right\}$

27. $12k^2 + 15k = 16k + 20$ $\left\{-\frac{5}{4}, \frac{4}{3}\right\}$

28. $12x^2 - 1 = -x$ $\left\{-\frac{1}{3}, \frac{1}{4}\right\}$

29. $8a^2 - 16a = 6a - 12$ $\left\{\frac{3}{4}, 2\right\}$

30. $18a^2 + 10a = -11a + 4$ $\left\{-\frac{4}{3}, \frac{1}{6}\right\}$

31. DIVING Lauren dove into a swimming pool from a 15-foot-high diving board with an initial upward velocity of 8 feet per second. Find the time t in seconds it took Lauren to enter the water. Use the model for vertical motion given by the equation $h = -16t^2 + vt + s$, where h is height in feet, t is time in seconds, v is the initial upward velocity in feet per second, and s is the initial height in feet. (*Hint:* Let $h = 0$ represent the surface of the pool.) **1.25 s**

32. BASEBALL Brad tossed a baseball in the air from a height of 6 feet with an initial upward velocity of 14 feet per second. Enrique caught the ball on its way down at a point 4 feet above the ground. How long was the ball in the air before Enrique caught it? Use the model of vertical motion from Exercise 31. **1 s**

NAME ______________________ DATE ____________ PERIOD ____________

8-7 Word Problem Practice

Solving $ax^2 + bx + c = 0$

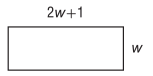
1. BREAK EVEN Breaking even occurs when the revenues for a business equal the cost. A local children's museum studied their costs and revenues from paid admission. They found that their break-even time is given by the equation $2h^2 - 2h - 24 = 0$, where h is the number of hours the museum is open per day. How many hours must the museum be open per day to reach the break even point?

4 hours

2. CARPENTRY Miko wants to build a toy box for her sister. It is to be 2 feet high, and the width is to be 3 feet less than its length. If it needs to hold a volume of 80 cubic feet, find the length and width of the box.

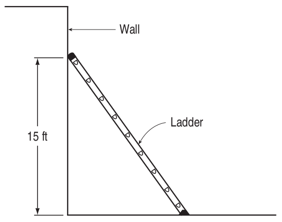
length = 8 ft; width = 5 ft

3. FURNITURE The student council wants to purchase a table for the school lobby. The table comes in a variety of dimensions, but for every table, the length is 1 meter greater than twice the width. The student council has budgeted for a table top with an area of exactly 3 square meters.



Find the width and length of the table they can purchase. **width = 1 m; length = 3 m**

4. LADDERS A ladder is resting against a wall. The top of the ladder touches the wall at a height of 15 feet, and the length of the ladder is one foot more than twice its distance from the wall. Find the distance from the wall to the bottom of the ladder. (*Hint:* Use the Pythagorean Theorem to solve the problem.) **8 ft**



5. FARMING Mr. Hensley has a total of 480 square feet of sheet metal with which he would like to construct a cylindrical tank for storing grain. The local zoning law limits the height of the tank to 13.5 feet. Recall that a formula for the surface area of a bottomless cylinder with radius r and height h is $A = \pi r^2 + 2\pi rh$.

a. Write a quadratic equation to represent the information. **$0 = \pi r^2 + 27\pi r - 480$**

b. Using 3 as an approximation for π, solve the equation for r. **$\{5, -32\}$**

c. What radius should Mr. Hensley use for his tank? **5 ft**

NAME ______________________________ DATE ____________ PERIOD ____________

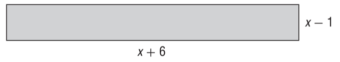
8-7 Enrichment

Area Models for Quadratic Trinomials

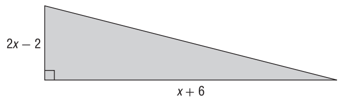
After you have factored a quadratic trinomial, you can use the factors to draw geometric models of the trinomial.

$x^2 + 5x - 6 = (x - 1)(x + 6)$

To draw a rectangular model, the value 2 was used for x so that the shorter side would have a length of 1. Then the drawing was done in centimeters. So, the area of the rectangle is $x^2 + 5x - 6$.



To draw a right triangle model, recall that the area of a triangle is one-half the base times the height. So, one of the sides must be twice as long as the shorter side of the rectangular model.

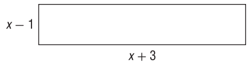


$$x^2 + 5x - 6 = (x - 1)(x + 6)$$
$$= \frac{1}{2}(2x - 2)(x + 6)$$

The area of the right triangle is also $x^2 + 5x - 6$.

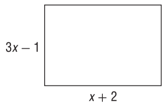
Factor each trinomial. Then follow the directions to draw each model of the trinomial.

1. $x^2 + 2x - 3$ Use $x = 2$. Draw a rectangle in centimeters.

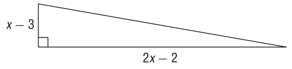


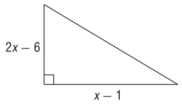
(x + 3)(x – 1)

2. $3x^2 + 5x - 2$ Use $x = 1$. Draw a rectangle in centimeters. **(x + 2)(3x – 1)**

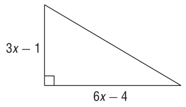


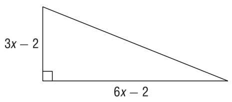
3. $x^2 - 4x + 3$ Use $x = 4$. Draw two different right triangles in centimeters. **(x – 1)(x – 3)**





4. $9x^2 - 9x + 2$ Use $x = 2$. Draw two different right triangles. Use 0.5 centimeter for each unit. **(3x – 2)(3x – 1)**





NAME ______________________________ DATE ____________ PERIOD ____________

8-7 Graphing Calculator Activity

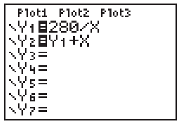
Using Tables in Factoring by Grouping

The **TABLE** feature can be used to help factor a polynomial by finding the factors of a certain product, which have a specific sum.

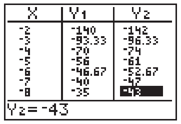
Example 1 **Factor $10x^2 - 43x + 28$ by grouping.**

Make a table of the negative factors of $10 \cdot 28$ or 280. Look for a pair of factors whose sum is –43.

Enter the equation $y = \frac{280}{x}$ in **Y1** to find the factors of 280. Then, find the sum of the factors using $y = \frac{280}{x} + x$ in **Y2**. Set up the table to display the negative factors of 280 by setting **ΔTbl** = to –1. Examine the results.



```
 Plot1  Plot2  Plot3
\Y1=280/X
\Y2=Y1+X
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



```
   X    |  Y1    |  Y2
 -2     | -140   | -142
 -3     | -93.33 | -96.33
 -4     | -70    | -74
 -5     | -56    | -61
 -6     | -46.67 | -52.67
 -7     | -40    | -47
 -8     | -35    | -43
Y2=-43
```

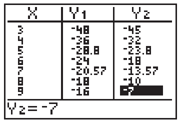
The last line of the table shows that $-43x$ may be replaced with $-8x + (-35x)$.

$$\begin{aligned} 10x^2 - 43x + 28 &= 10x^2 - 8x + (-35x) + 28 \\ &= 2x(5x - 4) + (-7)(5x - 4) \\ &= (5x - 4)(2x - 7) \end{aligned}$$

Thus, $10x^2 - 43x + 28 = (5x - 4)(2x - 7)$.

Example 2 **Factor $12x^2 - 7x - 12$.**

Look at the factors of $12 \cdot -12$ or –144 for a pair whose sum is –7. Enter an equation to determine the factors in **Y1** and an equation to find the sum of factors in **Y2**. Examine the table to find a sum of –7.



```
   X    |  Y1    |  Y2
 3      | -48    | -45
 4      | -36    | -32
 5      | -28.8  | -23.8
 6      | -24    | -18
 7      | -20.57 | -13.57
 8      | -18    | -10
 9      | -16    | -7
Y2=-7
```

$$\begin{aligned} 12x^2 - 7x - 12 &= 12x^2 + 9x + (-16x) - 12 \\ &= 3x(4x + 3) - 4(4x + 3) \\ &= (4x + 3)(3x - 4) \end{aligned}$$

Thus, $12x^2 - 7x - 12 = (4x + 3)(3x - 4)$.

Exercises

Factor each quadratic polynomial if possible.

1. $x^2 + 29x - 96$
(x + 32)(x – 3)

2. $x^2 - 14x - 51$
(x – 17)(x + 3)

3. $3z^2 + 16z - 35$
(3z – 5)(z + 7)

4. $4y^2 - 25y + 18$
prime

5. $6a^2 - a - 15$
(3a – 5)(2a + 3)

6. $6m^2 + 13m + 6$
(2m + 3)(3m + 2)

7. $12z^2 - z - 6$
(4z – 3)(3z + 2)

8. $16y^2 + 40y + 25$
$(4y + 5)^2$

9. $4b^2 + 24b - 493$
(2b + 29)(2b – 17)

NAME ______________________ DATE ____________ PERIOD ____________

8-8 Study Guide and Intervention

Differences of Squares

Factor Differences of Squares The binomial expression $a^2 - b^2$ is called the **difference of two squares**. The following pattern shows how to factor the difference of squares.

Difference of Squares	$a^2 - b^2 = (a - b)(a + b) = (a + b)(a - b)$.

Example 1 **Factor each polynomial.**

a. $\mathbf{n^2 - 64}$

$n^2 - 64$

$= n^2 - 8^2$ Write in the form $a^2 - b^2$.

$= (n + 8)(n - 8)$ Factor.

b. $\mathbf{4m^2 - 81n^2}$

$4m^2 - 81n^2$

$= (2m)^2 - (9n)^2$ Write in the form $a^2 - b^2$.

$= (2m - 9n)(2m + 9n)$ Factor.

Example 2 **Factor each polynomial.**

a. $\mathbf{50a^2 - 72}$

$50a^2 - 72$

$= 2(25a^2 - 36)$ Find the GCF.

$= 2[(5a)^2 - 6^2)]$ $25a^2 = 5a \cdot 5a$ and $36 = 6 \cdot 6$

$=2(5a + 6)(5a - 6)$ Factor the difference of squares.

b. $\mathbf{4x^4 + 8x^3 - 4x^2 - 8x}$

$4x^4 + 8x^3 - 4x^2 - 8x$ Original polynomial

$= 4x(x^3 + 2x^2 - x - 2)$ Find the GCF.

$= 4x[(x^3 + 2x^2) - (x + 2)]$ Group terms.

$= 4x[x^2(x + 2) - 1(x + 2)]$ Find the GCF.

$= 4x[(x^2 - 1)(x + 2)]$ Factor by grouping.

$= 4x[(x - 1)(x + 1)(x + 2)]$ Factor the difference of squares.

Exercises

Factor each polynomial.

1. $x^2 - 81$ **(x + 9)(x − 9)**

2. $m^2 - 100$ **(m + 10)(m − 10)**

3. $16n^2 - 25$ **(4n − 5)(4n + 5)**

4. $36x^2 - 100y^2$ **(6x + 10y)(6x − 10y)**

5. $49x^2 - 36$ **(7x + 6)(7x − 6)**

6. $16a^2 - 9b^2$ **(4a − 3b)(4a + 3b)**

7. $225b^2 - a^2$ **(15b − a)(15b + a)**

8. $72p^2 - 50$ **2(6p + 5)(6p − 5)**

9. $-2 + 2x^2$ **2(x − 1)(x + 1)**

10. $-81 + a^4$ **(a − 3)(a + 3)(a² + 9)**

11. $6 - 54a^2$ **6(1 + 3a)(1 − 3a)**

12. $8y^2 - 200$ **8(y + 5)(y − 5)**

13. $4x^3 - 100x$ **4x(x + 5)(x − 5)**

14. $2y^4 - 32y^2$ **2y²(y + 4)(y − 4)**

15. $8m^3 - 128m$ **8m(m + 4)(m − 4)**

16. $4x^2 - 25$ **(2x + 5)(2x − 5)**

17. $2a^3 - 98ab^2$ **2a(a − 7b)(a + 7b)**

18. $18y^2 - 72y^4$ **18y²(1 − 2y)(1 + 2y)**

19. $169x^3 - x$ **x(13x + 1)(13x − 1)**

20. $3a^4 - 3a^2$ **3a²(a + 1)(a − 1)**

21. $3x^4 + 6x^3 - 3x^2 - 6x$ **3x(x − 1)(x + 1)(x + 2)**

NAME ______________________ DATE ____________ PERIOD ____________

8-8 Study Guide and Intervention *(continued)*

Differences of Squares

Solve Equations by Factoring Factoring and the Zero Product Property can be used to solve equations that can be written as the product of any number of factors set equal to 0.

Example **Solve each equation. Check your solutions.**

a. $\mathbf{x^2 - \frac{1}{25} = 0}$

$x^2 - \frac{1}{25} = 0$ Original equation

$x^2 - \left(\frac{1}{5}\right)^2 = 0$ $x^2 = x \cdot x$ and $\frac{1}{25} = \left(\frac{1}{5}\right)\left(\frac{1}{5}\right)$

$\left(x + \frac{1}{5}\right)\left(x - \frac{1}{5}\right) = 0$ Factor the difference of squares.

$x + \frac{1}{5} = 0$ or $x - \frac{1}{5} = 0$ Zero Product Property

$x = -\frac{1}{5}$ $x = \frac{1}{5}$ Solve each equation.

The solution set is $\left\{-\frac{1}{5}, \frac{1}{5}\right\}$. Since $\left(-\frac{1}{5}\right)^2 - \frac{1}{25} = 0$ and $\left(\frac{1}{5}\right)^2 - \frac{1}{25} = 0$, the solutions check.

b. $\mathbf{4x^3 = 9x}$

$4x^3 = 9x$ Original equation

$4x^3 - 9x = 0$ Subtract 9x from each side.

$x(4x^2 - 9) = 0$ Factor out the GCF of x.

$x[(2x)^2 - 3^2] = 0$ $4x^2 = 2x \cdot 2x$ and $9 = 3 \cdot 3$

$x[(2x)^2 - 3^2] = x[(2x - 3)(2x + 3)]$ Factor the difference of squares.

$x = 0$ or $(2x - 3) = 0$ or $(2x + 3) = 0$ Zero Product Property

$x = 0$ $x = \frac{3}{2}$ $x = -\frac{3}{2}$ Solve each equation.

The solution set is $\left\{0, \frac{3}{2}, -\frac{3}{2}\right\}$.

Since $4(0)^3 = 9(0)$, $4\left(\frac{3}{2}\right)^3 = 9\left(\frac{3}{2}\right)$, and $4\left(-\frac{3}{2}\right)^3 = 9\left(-\frac{3}{2}\right)$, the solutions check.

Exercises

Solve each equation by factoring. Check the solutions.

1. $81x^2 = 49$ $\mathbf{\left\{\frac{7}{9}, -\frac{7}{9}\right\}}$

2. $36n^2 = 1$ $\mathbf{\left\{-\frac{1}{6}, \frac{1}{6}\right\}}$

3. $25d^2 - 100 = 0$ **{2, −2}**

4. $\frac{1}{4}x^2 = 25$ **{10, −10}**

5. $36 = \frac{1}{25}x^2$ **{−30, 30}**

6. $\frac{49}{100} - x^2 = 0$ $\mathbf{\left\{-\frac{7}{10}, \frac{7}{10}\right\}}$

7. $9x^3 = 25x$ $\mathbf{\left\{0, -\frac{5}{3}, \frac{5}{3}\right\}}$

8. $7a^3 = 175a$ **{0, −5, 5}**

9. $2m^3 = 32m$ **{0, −4, 4}**

10. $16y^3 = 25y$ $\mathbf{\left\{0, -\frac{5}{4}, \frac{5}{4}\right\}}$

11. $\frac{1}{64}x^2 = 49$ **{−56, 56}**

12. $4a^3 - 64a = 0$ **{0, −4, 4}**

13. $3b^3 - 27b = 0$ **{0, −3, 3}**

14. $\frac{9}{25}m^2 = 121$ $\mathbf{\left\{-\frac{55}{3}, \frac{55}{3}\right\}}$

15. $48n^3 = 147n$ $\mathbf{\left\{0, -\frac{7}{4}, \frac{7}{4}\right\}}$

NAME ______________ DATE ________ PERIOD ________

8-8 Skills Practice

Differences of Squares

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

1. $a^2 - 4$ **(a + 2)(a − 2)**

2. $n^2 - 64$ **(n + 8)(n − 8)**

3. $1 - 49d^2$ **(1 + 7d)(1 − 7d)**

4. $-16 + p^2$ **(p + 4)(p − 4)**

5. $k^2 + 25$ **prime**

6. $36 - 100w^2$ **(6 − 10w)(6 + 10w)**

7. $t^2 - 81u^2$ **(t + 9u)(t − 9u)**

8. $4h^2 - 25g^2$ **(2h + 5g)(2h − 5g)**

9. $64m^2 - 9y^2$ **(8m − 3y)(8m + 3y)**

10. $4c^2 - 5d^2$ **prime**

11. $-49r^2 + 4t^2$ **(2t + 7r)(2t − 7r)**

12. $8x^2 - 72p^2$ **8(x + 3p)(x − 3p)**

13. $20q^2 - 5r^2$ **5(2q + r)(2q − r)**

14. $32a^2 - 50b^2$ **2(4a + 5b)(4a − 5b)**

Solve each equation by factoring. Check the solutions.

15. $16x^2 - 9 = 0$ $\left\{\pm\frac{3}{4}\right\}$

16. $25p^2 - 16 = 0$ $\left\{\pm\frac{4}{5}\right\}$

17. $36q^2 - 49 = 0$ $\left\{\pm\frac{7}{6}\right\}$

18. $81 - 4b^2 = 0$ $\left\{\pm\frac{9}{2}\right\}$

19. $16d^2 = 4$ $\left\{\pm\frac{1}{2}\right\}$

20. $18a^2 = 8$ $\left\{\pm\frac{2}{3}\right\}$

21. $n^2 - \frac{9}{25} = 0$ $\left\{\pm\frac{3}{5}\right\}$

22. $k^2 - \frac{49}{64} = 0$ $\left\{\pm\frac{7}{8}\right\}$

23. $\frac{1}{25}h^2 - 16 = 0$ $\{\pm 20\}$

24. $\frac{1}{16}y^2 = 81$ $\{\pm 36\}$

NAME ______________ DATE ________ PERIOD ________

8-8 Practice

Differences of Squares

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

1. $k^2 - 100$ **(k + 10)(k − 10)**

2. $81 - r^2$ **(9 + r)(9 − r)**

3. $16p^2 - 36$ **(4p + 6)(4p − 6)**

4. $4x^2 + 25$ **prime**

5. $144 - 9f^2$ **(12 + 3f)(12 − 3f)**

6. $36g^2 - 49h^2$ **(6g + 7h)(6g − 7h)**

7. $121m^2 - 144p^2$ **(11m − 12p)(11m + 12p)**

8. $32 - 8y^2$ **8(2 − y)(2 + y)**

9. $24a^2 - 54b^2$ **6(2a − 3b)(2a + 3b)**

10. $32t^2 - 18u^2$ **2(4t − 3u)(4t + 3u)**

11. $9d^2 - 32$ **prime**

12. $36z^3 - 9z$ **9z(2z + 1)(2z − 1)**

13. $45q^3 - 20q$ **5q(3q + 2)(3q − 2)**

14. $100b^3 - 36b$ **4b(5b + 3)(5b − 3)**

15. $3t^4 - 48t^2$ **3t²(t + 4)(t − 4)**

Solve each equation by factoring. Check your solutions.

16. $4y^2 = 81$ $\left\{\pm\frac{9}{2}\right\}$

17. $64p^2 = 9$ $\left\{\pm\frac{3}{8}\right\}$

18. $98b^2 - 50 = 0$ $\left\{\pm\frac{5}{7}\right\}$

19. $32 - 162k^2 = 0$ $\left\{\pm\frac{4}{9}\right\}$

20. $t^2 - \frac{64}{121} = 0$ $\left\{\pm\frac{8}{11}\right\}$

21. $\frac{16}{49} - v^2 = 0$ $\left\{\pm\frac{4}{7}\right\}$

22. $\frac{1}{36}x^2 - 25 = 0$ $\{\pm 30\}$

23. $27h^3 = 48h$ $\left\{\pm\frac{4}{3}, 0\right\}$

24. $75g^3 = 147g$ $\left\{\pm\frac{7}{5}, 0\right\}$

25. EROSION A rock breaks loose from a cliff and plunges toward the ground 400 feet below. The distance d that the rock falls in t seconds is given by the equation $d = 16t^2$. How long does it take for the rock to hit the ground? **5 s**

26. FORENSICS Mr. Cooper contested a speeding ticket given to him after he applied his brakes and skidded to a halt to avoid hitting another car. In traffic court, he argued that the length of the skid marks on the pavement, 150 feet, proved that he was driving under the posted speed limit of 65 miles per hour. The ticket cited his speed at 70 miles per hour. Use the formula $\frac{1}{24}s^2 = d$, where s is the speed of the car and d is the length of the skid marks, to determine Mr. Cooper's speed when he applied the brakes. Was Mr. Cooper correct in claiming that he was not speeding when he applied the brakes?
60 mph; yes

NAME ______________ DATE ________ PERIOD ________

8-8 Word Problem Practice

Differences of Squares

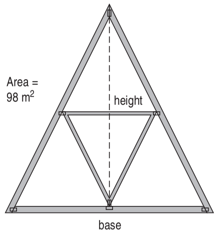
1. **LOTTERY** A state lottery commission analyzes the ticket purchasing patterns of its citizens. The following expression is developed to help officials calculate the likely number of people who will buy tickets for a certain size jackpot.

$$81a^2 - 36b^2$$

Factor the expression completely.
9(3*a* + 2*b*)(3*a* − 2*b*)

2. **OPTICS** A reflector on the inside of a certain flashlight is a parabola given by the equation $y = x^2 - 25$. Find the points where the reflector meets the lens by finding the values of x when $y = 0$.
5, −5

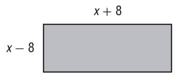
3. **ARCHITECTURE** The drawing shows a triangular roof truss with a base measuring the same as its height. The area of the truss is 98 square meters.



Find the height of the truss. **14 m**

4. **BALLOONING** The function $f(t) = -16t^2 + 576$ represents the height of a freely falling ballast bag that starts from rest on a balloon 576 feet above the ground. After how many seconds t does the ballast bag hit the ground?
after 6 seconds

5. **DECORATING** Marvin wants to purchase a rectangular rug. It has an area of 80 square feet. He cannot remember the length and width, but he remembers that the length was 8 more than some number and the width was 8 less than that same number.



a. Write a quadratic equation using the information given. **$x^2 - 64 = 80$ or $x^2 - 144 = 0$**

b. What are the length and width of the rug? **20 ft and 4 ft**

NAME ______________ DATE ________ PERIOD ________

8-8 Enrichment

Factoring Trinomials of Fourth Degree

Some trinomials of the form $a^4 + a^2b^2 + b^4$ can be written as the difference of two squares and then factored.

Example **Factor $4x^4 - 37x^2y^2 + 9y^4$.**

Step 1 Find the square roots of the first and last terms.

$\sqrt{4x^4} = 2x^2$ $\sqrt{9y^4} = 3y^2$

Step 2 Find twice the product of the square roots.

$2(2x^2)(3y^2) = 12x^2y^2$

Step 3 Separate the middle term into two parts. One part is either your answer to Step 2 or its opposite. The other part should be the opposite of a perfect square.

$-37x^2y^2 = -12x^2y^2 - 25x^2y^2$

Step 4 Rewrite the trinomial as the difference of two squares and then factor.

$$\begin{aligned}4x^4 - 37x^2y^2 + 9y^4 &= (4x^4 - 12x^2y^2 + 9y^4) - 25x^2y^2\\ &= (2x^2 - 3y^2)^2 - 25x^2y^2\\ &= [(2x^2 - 3y^2) + 5xy][(2x^2 - 3y^2) - 5xy]\\ &= (2x^2 + 5xy - 3y^2)(2x^2 - 5xy - 3y^2)\end{aligned}$$

Factor each polynomial.

1. $x^4 + x^2y^2 + y^4$
$(x^2 + xy + y^2)(x^2 - xy + y^2)$

2. $x^4 + x^2 + 1$
$(x^2 + x + 1)(x^2 - x + 1)$

3. $9a^4 - 15a^2 + 1$
$(3a^2 + 3a - 1)(3a^2 - 3a - 1)$

4. $16a^4 - 17a^2 + 1$
$(4a - 1)(a + 1)(4a + 1)(a - 1)$

5. $4a^4 - 13a^2 + 1$
$(2a^2 + 3a - 1)(2a^2 - 3a - 1)$

6. $9a^4 + 26a^2b^2 + 25b^4$
$(3a^2 + 2ab + 5b^2)(3a^2 - 2ab + 5b^2)$

7. $4x^4 - 21x^2y^2 + 9y^4$
$(2x^2 + 3xy - 3y^2)(2x^2 - 3xy - 3y^2)$

8. $4a^4 - 29a^2b^2 + 25b^4$
$(2a + 5b)(a - b)(2a - 5b)(a + b)$

NAME ______________________ DATE ____________ PERIOD ____________

8-8 Spreadsheet Activity

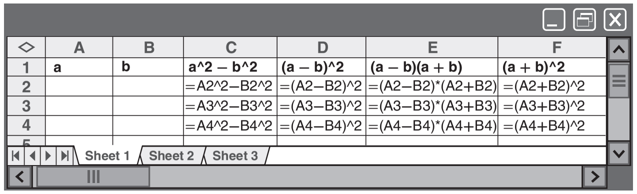
Differences of Squares

There is a special pattern you can use to factor binomials of the form $a^2 - b^2$. You can use a spreadsheet to discover this relationship.

Example **Use a spreadsheet to investigate the values of the expressions $(a^2 - b^2)$, $(a - b)^2$, $(a - b)(a + b)$, and $(a + b)^2$. What conjecture can you make about the expressions?**

Step 1 You will use Columns A and B to enter various values that you choose for a and b.

Step 2 Enter the formulas for $(a^2 - b^2)$, $(a - b)^2$, $(a - b)(a + b)$, and $(a + b)^2$ in Columns C, D, E, and F. To enter an exponent, use the symbol ^ followed by the exponent. For example, the square of the value in cell A2 is entered as A2^2.



◇	A	B	C	D	E	F
1	a	b	a^2 – b^2	(a – b)^2	(a – b)(a + b)	(a + b)^2
2			=A2^2–B2^2	=(A2–B2)^2	=(A2–B2)*(A2+B2)	=(A2+B2)^2
3			=A3^2–B3^2	=(A3–B3)^2	=(A3–B3)*(A3+B3)	=(A3+B3)^2
4			=A4^2–B4^2	=(A4–B4)^2	=(A4–B4)*(A4+B4)	=(A4+B4)^2

Exercises

1. Enter various values of a and b in Columns A and B. Look for a pattern. What do you observe about the expressions? **For any values of a and b, $(a^2 - b^2) = (a - b)(a + b)$.**

2. Find the products of $(a - b)^2$, $(a - b)(a + b)$, and $(a + b)^2$. Do the results verify your conjecture? **$(a - b)^2 = a^2 - 2ab + b^2$; $(a - b)(a + b) = a^2 - b^2$; and $(a + b)^2 = a^2 + 2ab + b^2$; yes**

Use the pattern you observed to factor each binomial.

3. $m^2 - t^2$ **$(m - t)(m + t)$**

4. $x^2 - 4$ **$(x - 2)(x + 2)$**

5. $y^2 - 16$ **$(y - 4)(y + 4)$**

6. $q^2 - 121$ **$(q - 11)(q + 11)$**

7. $r^2 - 169$ **$(r - 13)(r + 13)$**

8. $b^2 - 1$ **$(b - 1)(b + 1)$**

9. $4x^2 - 1$ **$(2x - 1)(2x + 1)$**

10. $16t^2 - r^2$ **$(4t - r)(4t + r)$**

11. $25a^2 - 81d^2$ **$(5a - 9d)(5a + 9d)$**

NAME ______________________ DATE ____________ PERIOD ____________

8-9 Study Guide and Intervention

Perfect Squares

Factor Perfect Square Trinomials

Perfect Square Trinomial	a trinomial of the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$

The patterns shown below can be used to factor perfect square trinomials.

Squaring a Binomial	Factoring a Perfect Square Trinomial
$(a + 4)^2 = a^2 + 2(a)(4) + 4^2$ $= a^2 + 8a + 16$	$a^2 + 8a + 16 = a^2 + 2(a)(4) + 4^2$ $= (a + 4)^2$
$(2x - 3)^2 = (2x)^2 - 2(2x)(3) + 3^2$ $= 4x^2 - 12x + 9$	$4x^2 - 12x + 9 = (2x)^2 - 2(2x)(3) + 3^2$ $= (2x - 3)^2$

Example 1 **Determine whether $16n^2 - 24n + 9$ is a perfect square trinomial. If so, factor it.**

Since $16n^2 = (4n)(4n)$, the first term is a perfect square.

Since $9 = 3 \cdot 3$, the last term is a perfect square.

The middle term is equal to $2(4n)(3)$.

Therefore, $16n^2 - 24n + 9$ is a perfect square trinomial.

$$16n^2 - 24n + 9 = (4n)^2 - 2(4n)(3) + 3^2 = (4n - 3)^2$$

Example 2 **Factor $16x^2 - 32x + 15$.**

Since 15 is not a perfect square, use a different factoring pattern.

$16x^2 - 32x + 15$ Original trinomial

$= 16x^2 + mx + px + 15$ Write the pattern.

$= 16x^2 - 12x - 20x + 15$ $m = -12$ and $p = -20$

$= (16x^2 - 12x) - (20x - 15)$ Group terms.

$= 4x(4x - 3) - 5(4x - 3)$ Find the GCF.

$= (4x - 5)(4x - 3)$ Factor by grouping.

Therefore $16x^2 - 32x + 15 = (4x - 5)(4x - 3)$.

Exercises

Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*. If so, factor it.

1. $x^2 - 16x + 64$ **yes; $(x - 8)(x - 8)$**

2. $m^2 + 10m + 25$ **yes; $(m + 5)(m + 5)$**

3. $p^2 + 8p + 64$ **no**

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

4. $98x^2 - 200y^2$ **$2(7x + 10y)(7x - 10y)$**

5. $x^2 + 22x + 121$ **$(x + 11)^2$**

6. $81 + 18j + j^2$ **$(9 + j)^2$**

7. $25c^2 - 10c - 1$ **prime**

8. $169 - 26r + r^2$ **$(13 - r)^2$**

9. $7x^2 - 9x + 2$ **$(7x - 2)(x - 1)$**

10. $16m^2 + 48m + 36$ **$4(2m + 3)^2$**

11. $16 - 25a^2$ **$(4 + 5a)(4 - 5a)$**

12. $b^2 - 16b + 256$ **prime**

13. $36x^2 - 12x + 1$ **$(6x - 1)^2$**

14. $16a^2 - 40ab + 25b^2$ **$(4a - 5b)^2$**

15. $8m^3 - 64m$ **$8m(m^2 - 8)$**

NAME ______________ DATE ________ PERIOD ________

8-9 Study Guide and Intervention *(continued)*

Perfect Squares

Solve Equations with Perfect Squares Factoring and the Zero Product Property can be used to solve equations that involve repeated factors. The repeated factor gives just one solution to the equation. You may also be able to use the **Square Root Property** below to solve certain equations.

Square Root Property	For any number $n > 0$, if $x^2 = n$, then $x = \pm\sqrt{n}$.

Example **Solve each equation. Check your solutions.**

a. $\mathbf{x^2 - 6x + 9 = 0}$

$x^2 - 6x + 9 = 0$	Original equation
$x^2 - 2(3x) + 3^2 = 0$	Recognize a perfect square trinomial.
$(x - 3)(x - 3) = 0$	Factor the perfect square trinomial.
$x - 3 = 0$	Set repeated factor equal to 0.
$x = 3$	Solve.

The solution set is {3}. Since $3^2 - 6(3) + 9 = 0$, the solution checks.

b. $\mathbf{(a - 5)^2 = 64}$

$(a - 5)^2 = 64$	Original equation
$a - 5 = \pm\sqrt{64}$	Square Root Property
$a - 5 = \pm 8$	$64 = 8 \cdot 8$
$a = 5 \pm 8$	Add 5 to each side.
$a = 5 + 8$ or $a = 5 - 8$	Separate into 2 equations.
$a = 13$ $\quad a = -3$	Solve each equation.

The solution set is {−3, 13}. Since $(-3 - 5)^2 = 64$ and $(13 - 5)^2 = 64$, the solutions check.

Exercises

Solve each equation. Check the solutions.

1. $x^2 + 4x + 4 = 0$ $\mathbf{\{-2\}}$

2. $16n^2 + 16n + 4 = 0$ $\mathbf{\left\{-\frac{1}{2}\right\}}$

3. $25d^2 - 10d + 1 = 0$ $\mathbf{\left\{\frac{1}{5}\right\}}$

4. $x^2 + 10x + 25 = 0$ $\mathbf{\{-5\}}$

5. $9x^2 - 6x + 1 = 0$ $\mathbf{\left\{\frac{1}{3}\right\}}$

6. $x^2 + x + \frac{1}{4} = 0$ $\mathbf{\left\{-\frac{1}{2}\right\}}$

7. $25k^2 + 20k + 4 = 0$ $\mathbf{\left\{-\frac{2}{5}\right\}}$

8. $p^2 + 2p + 1 = 49$ $\mathbf{\{-8, 6\}}$

9. $x^2 + 4x + 4 = 64$ $\mathbf{\{-10, 6\}}$

10. $x^2 - 6x + 9 = 25$ $\mathbf{\{-2, 8\}}$

11. $a^2 + 8a + 16 = 1$ $\mathbf{\{-3, -5\}}$

12. $16y^2 + 8y + 1 = 0$ $\mathbf{\left\{-\frac{1}{4}\right\}}$

13. $(x + 3)^2 = 49$ $\mathbf{\{-10, 4\}}$

14. $(y + 6)^2 = 1$ $\mathbf{\{-7, -5\}}$

15. $(m - 7)^2 = 49$ $\mathbf{\{0, 14\}}$

16. $(2x + 1)^2 = 1$ $\mathbf{\{-1, 0\}}$

17. $(4x + 3)^2 = 25$ $\mathbf{\left\{-2, \frac{1}{2}\right\}}$

18. $(3h - 2)^2 = 4$ $\mathbf{\left\{\frac{4}{3}, 0\right\}}$

19. $(x + 1)^2 = 7$ $\mathbf{\{-1 \pm \sqrt{7}\}}$

20. $(y - 3)^2 = 6$ $\mathbf{\{3 \pm \sqrt{6}\}}$

21. $(m - 2)^2 = 5$ $\mathbf{\{2 \pm \sqrt{5}\}}$

NAME ______________ DATE ________ PERIOD ________

8-9 Skills Practice

Perfect Squares

Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*. If so, factor it.

1. $m^2 - 6m + 9$ **yes; $(m - 3)^2$**

2. $r^2 + 4r + 4$ **yes; $(r + 2)^2$**

3. $g^2 - 14g + 49$ **yes; $(g - 7)^2$**

4. $2w^2 - 4w + 9$ **no**

5. $4d^2 - 4d + 1$ **yes; $(2d - 1)^2$**

6. $9n^2 + 30n + 25$ **yes; $(3n + 5)^2$**

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

7. $2x^2 - 72$ $\mathbf{2(x + 6)(x - 6)}$

8. $6b^2 + 11b + 3$ $\mathbf{(2b + 3)(3b + 1)}$

9. $36t^2 - 24t + 4$ $\mathbf{4(3t - 1)^2}$

10. $4h^2 - 56$ $\mathbf{4(h^2 - 14)}$

11. $17a^2 - 24ab$ $\mathbf{a(17a - 24b)}$

12. $q^2 - 14q + 36$ **prime**

13. $y^2 + 24y + 144$ $\mathbf{(y + 12)^2}$

14. $6d^2 - 96$ $\mathbf{6(d - 4)(d + 4)}$

Solve each equation. Check the solutions.

15. $x^2 - 18x + 81 = 0$ $\mathbf{\{9\}}$

16. $4p^2 + 4p + 1 = 0$ $\mathbf{\left\{-\frac{1}{2}\right\}}$

17. $9g^2 - 12g + 4 = 0$ $\mathbf{\left\{\frac{2}{3}\right\}}$

18. $y^2 - 16y + 64 = 81$ $\mathbf{\{-1, 17\}}$

19. $4n^2 - 17 = 19$ $\mathbf{\{\pm 3\}}$

20. $x^2 + 30x + 150 = -75$ $\mathbf{\{-15\}}$

21. $(k + 2)^2 = 16$ $\mathbf{\{-6, 2\}}$

22. $(m - 4)^2 = 7$ $\mathbf{\{4 \pm \sqrt{7}\}}$

NAME ______________________ DATE __________ PERIOD ________

8-9 Practice

Perfect Squares

Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*. If so, factor it.

1. $m^2 + 16m + 64$ **yes; $(m + 8)^2$**

2. $9r^2 - 6r + 1$ **yes; $(3r - 1)^2$**

3. $4y^2 - 20y + 25$ **yes; $(2y - 5)^2$**

4. $16p^2 + 24p + 9$ **yes; $(4p + 3)^2$**

5. $25b^2 - 4b + 16$ **no**

6. $49k^2 - 56k + 16$ **yes; $(7k - 4)^2$**

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

7. $3p^2 - 147$ **$3(p + 7)(p - 7)$**

8. $6x^2 + 11x - 35$ **$(2x + 7)(3x - 5)$**

9. $50q^2 - 60q + 18$ **$2(5q - 3)^2$**

10. $6t^3 - 14t^2 - 12t$ **$2t(3t + 2)(t - 3)$**

11. $6d^2 - 18$ **$6(d^2 - 3)$**

12. $30k^2 + 38k + 12$ **$2(5k + 3)(3k + 2)$**

13. $15b^2 - 24bf$ **$3b(5b - 8f)$**

14. $12h^2 - 60h + 75$ **$3(2h - 5)^2$**

15. $9n^2 - 30n - 25$ **prime**

16. $7u^2 - 28m^2$ **$7(u - 2m)(u + 2m)$**

17. $w^4 - 8w^2 - 9$ **$(w^2 + 1)(w + 3)(w - 3)$**

18. $16a^2 + 72ad + 81d^2$ **$(4a + 9d)^2$**

Solve each equation. Check the solutions.

19. $4k^2 - 28k = -49$ $\left\{\frac{7}{2}\right\}$

20. $50b^2 + 20b + 2 = 0$ $\left\{-\frac{1}{5}\right\}$

21. $\left(\frac{1}{2}t - 1\right)^2 = 0$ $\{2\}$

22. $g^2 + \frac{2}{3}g + \frac{1}{9} = 0$ $\left\{-\frac{1}{3}\right\}$

23. $p^2 - \frac{6}{5}p + \frac{9}{25} = 0$ $\left\{\frac{3}{5}\right\}$

24. $x^2 + 12x + 36 = 25$ $\{-11, -1\}$

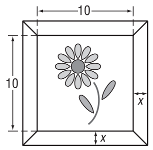
25. $y^2 - 8y + 16 = 64$ $\{-4, 12\}$

26. $(h + 9)^2 = 3$ $\{-9 \pm \sqrt{3}\}$

27. $w^2 - 6w + 9 = 13$ $\{3 \pm \sqrt{13}\}$

28. GEOMETRY The area of a circle is given by the formula $A = \pi r^2$, where r is the radius. If increasing the radius of a circle by 1 inch gives the resulting circle an area of 100π square inches, what is the radius of the original circle? **9 in.**

29. PICTURE FRAMING Mikaela placed a frame around a print that measures 10 inches by 10 inches. The area of just the frame itself is 69 square inches. What is the width of the frame? **1.5 in.**



NAME ______________________ DATE __________ PERIOD ________

8-9 Word Problem Practice

Perfect Squares

1. CONSTRUCTION The area of Liberty Township's square playground is represented by the trinomial $x^2 - 10x + 25$. Write an expression using the variable x that represents the perimeter. **$4x - 20$ or $4(x - 5)$**

2. AMUSEMENT PARKS Funtown Downtown wants to build a vertical motion ride where the passengers are launched straight upward from ground level with an initial velocity of 96 feet per second. The ride car's height h in feet after t seconds is $h = 96t - 16t^2$. How many seconds after launch would the car reach 144 feet? **3 seconds**

3. BUSINESS Saini Sprinkler Company installs irrigation systems. To track monthly costs C and revenues R, they use the following functions, where x is the number of systems they install.

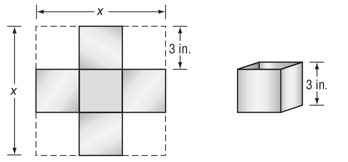
$$R(x) = 8x^2 + 12x + 4$$
$$C(x) = 7x^2 + 20x - 12$$

The monthly profit can be found by subtracting cost from revenue.

$$P(x) = R(x) - C(x)$$

Find a function to project monthly profit and use it to find the break-even point where the profit is zero.
$P(x) = x^2 - 8x + 16$; $x = 4$

4. GEOMETRY Holly can make an open-topped box out of a square piece of cardboard by cutting 3-inch squares from the corners and folding up the sides to meet. The volume of the resulting box is $V = 3x^2 - 36x + 108$, where x is the original length and width of the cardboard.



a. Factor the polynomial expression from the volume equation. **$3(x - 6)(x - 6)$**

b. What is the volume of the box if the original length of each side of the cardboard was 14 inches? **192 in^3**

c. What is the original side length of the cardboard when the volume of the box is 27 in^3? **9 in.**

NAME ______________ DATE ________ PERIOD ________

8-9 Enrichment

Squaring Numbers: A Shortcut

A shortcut helps you to square a positive two-digit number ending in 5. The method is developed using the idea that a two-digit number may be expressed as $10t + u$. Suppose $u = 5$.

$$\begin{aligned}(10t + 5)^2 &= (10t + 5)(10t + 5)\\ &= 100t^2 + 50t + 50t + 25\\ &= 100t^2 + 100t + 25\\ (10t + 5)^2 &= 100t(t + 1) + 25\end{aligned}$$

In words, this formula says that the square of a two-digit number ending in 5 has $t(t + 1)$ in the hundreds place. Then 2 is the tens digit and 5 is the units digit.

Example **Using the formula for $(10t + 5)^2$, find 85^2.**

$$\begin{aligned}85^2 &= 100 \cdot 8 \cdot (8 + 1) + 25\\ &= 7200 + 25\\ &= 7225\end{aligned}$$

Shortcut: First think $8 \cdot 9 = 72$. Then write 25.

Thus, to square a number, such as 85, you can write the product of the tens digit and the next consecutive integer $t + 1$. Then write 25.

Find each of the following using the shortcut.

1. 15^2 **225**

2. 25^2 **625**

3. 35^2 **1225**

4. 45^2 **2025**

5. 55^2 **3025**

6. 65^2 **4225**

7. What is the tens digit in the square of 95? **2**

8. What are the first two digits in the square of 75? **56**

9. Any three-digit number can be written as $100a + 10b + c$. Square this expression to show that if the last digit of a three-digit number is 5 then the last two digits of the square of the number are 2 and 5.

$10{,}000a^2 + 2000ab + 200ac + 100b^2 + 20bc + c^2 =$

$10{,}000a^2 + 2000ab + 1000a + 100b^2 + 100b + 25$

The last two digits are not affected by the first five terms.

Chapter 8 Assessment Answer Key

Quiz 1 (Lessons 8-1 and 8-2)
Page 65

1. 2
2. 4
3. $-3x^3 + 4x^2 + 2x + 12; -3$
4. $-7x^4 + 4x^2 + x - 15; -7$
5. $3x^2 - 3x + 4$
6. $3a + 5b$
7. A
8. 9

Quiz 2 (Lesson 8-3, 8-4, and 8-5)
Page 65

1. $2x^2 - 7x - 4$
2. $6b^3 + 5b^2 + 8b + 16$
3. $4m^2 + 7m - 2$
4. $4x^2 - 36y^2$
5. $a^2 - 9b^2$
6. $x^2 + 14x + 49$
7. $12ab(4ab - 1)$
8. $3(2x^2y - 7y^2w + 8xw)$
9. $\left\{-4, \frac{5}{3}\right\}$
10. $\{-11, 0\}$
11. A

Quiz 3 (Lessons 8-6 and 8-7)
Page 66

1. $(a - 3)(a - 7)$
2. $(x - 3)(x + 5)$
3. $(2x + 1)(x + 3)$
4. prime
5. C
6. $\{-3, 9\}$
7. $\{-24, 1\}$
8. $\left\{\frac{2}{3}, 3\right\}$
9. $\left\{-\frac{3}{2}, \frac{2}{5}\right\}$
10. -15 and -13 or 13 and 15

Quiz 4 (Lesson 8-8 and 8-9)
Page 66

1. $(a - 5)(a + 5)$
2. $(7x + 8y)(7x - 8y)$
3. $(x + 2)(x - 2)(x + 3)$
4. $(a + 7)^2$
5. $(3z - 1)^2$
6. $2m(2m - 3)^2$
7. $-\frac{9}{4}, \frac{9}{4}$
8. $-\frac{15}{2}, \frac{15}{2}$
9. $\left\{\frac{1}{4}\right\}$
10. $\{5 \pm \sqrt{8}\}$
11. C

Mid-Chapter Test
Page 67

1. C
2. H
3. D
4. F
5. A
6. H
7. B
8. $-1 \bullet 2 \bullet 5 \bullet 7 \bullet a \bullet a \bullet b \bullet b \bullet c$
9. 4 seconds
10. $12xy(3y - 4x)$
11. $(t - 4)(t - 12)$
12. $(2y - 1)(x + 2)$
13. $3g^3 - 6g^2 + g + 1$
14. $6y^2 + 7y - 20$
15. $16m^2 - 40mn + 25n^2$

Chapter 8 Assessment Answer Key

Vocabulary Test
Page 68

1. c
2. e
3. a
4. d
5. g
6. b
7. e
8. Sample answer: Factoring means writing a number or polynomial as a product of factors.
9. Sample answer: To multiply two binomials, find the sum of the products of the First terms, the Outer terms, the Inner terms, and the Last terms.

Form 1
Page 69

1. C
2. J
3. B
4. G
5. A
6. G
7. B
8. G
9. D
10. F
11. A

Page 70

12. G
13. A
14. J
15. D
16. H
17. B
18. F
19. A
20. G

B: 5 and 7 or –5 and –7

Chapter 8 Assessment Answer Key

Form 2A

Page 71

1. C
2. F
3. A
4. J
5. C
6. H
7. A
8. H
9. B
10. G
11. B

Page 72

12. F
13. A
14. H
15. A
16. F
17. B
18. J
19. D
20. H

B: 25

Form 2B

Page 73

1. A
2. H
3. D
4. H
5. A
6. G
7. A
8. H
9. A
10. F
11. D

Page 74

12. H
13. C
14. J
15. A
16. F
17. B
18. J
19. A
20. G

B: $2(n + 3)(n - 3)(n^2 + 4)$

Chapter 8 Assessment Answer Key

Form 2C

Page 75

1. $-4n^2 + 6ny + 13y^2$

2. $19m^2 + 2mt + 6t^2$

3. $10h^3k^3 - 5h^2k^5 + 20h^3k^4$

4. $8x^4 - 2y^4$

5. $25c^2 - 40c + 16$

6. $5a^2bc(7ac - 9b)$

7. $(3y - 4)(x + 2)$

8. $(t - 8)(t - 3)$

9. $(5y - 3)(2y - 5)$

10. $4(2n - 5)(n - 2)$

11. $2x^2(x + 3)(x - 3)$

12. prime

Page 76

13. $2\frac{2}{5}$

14. 6

15. $\left\{-\frac{2}{3}, 2\right\}$

16. $\left\{0, \frac{1}{2}\right\}$

17. $\left\{\frac{1}{2}, 1\right\}$

18. $\left\{-\frac{7}{2}, -\frac{1}{2}\right\}$

19. $\left\{-\frac{5}{7}, \frac{5}{7}\right\}$

20. length is 20 ft; width is 12 ft

21. 14 in.

22. length is 6 in.; width is 8 in.

23. $1\frac{1}{2}$ s

24. width is 14 ft; length is 17 ft

25. -7 and -35 or 7 and 35

B: $(v + 3)(v - 3)(x^2 + n^2)$

Chapter 8 Assessment Answer Key

Form 2D

Page 77

1. $4m^2 - 6m + 1$
2. $8y^2 - 4y - 9$
3. $6x^4y^2 - 15x^3y^3 + 24x^4y^4$
4. $9r^4 - 25t^4$
5. $25y^2 + 60y + 36$
6. $2x^2yz(5 - 11xy)$
7. $(y - 2)(2x + 3)$
8. $(m + 14)(m - 2)$
9. $(5t - 3)(t + 4)$
10. $2(3p - 4)(p - 2)$
11. $3x^3(x + 5)(x - 5)$
12. prime

Page 78

13. $\{13\}$
14. $\{-2\}$
15. $\left\{-5, \frac{3}{4}\right\}$
16. $\left\{0, \frac{2}{3}\right\}$
17. $\left\{-1, \frac{1}{3}\right\}$
18. $\left\{-\frac{1}{2}, \frac{5}{2}\right\}$
19. $\left\{-\frac{1}{8}, \frac{1}{8}\right\}$
20. length is 10 ft; width is 5 ft
21. 16 in.
22. length is 11 in.; height is 3 in.
23. $\frac{3}{4}$ s
24. width is 8 ft; length is 13 ft
25. −6 and −42 or 6 and 42

B: 16

Answers

Chapter 8 Assessment Answer Key

Form 3

Page 79

1. $10w^2 + 3w + 4$

2. $10u^2x - 7ux + 6ux^2$

3. $3x^2 - x + 14y$

4. $-n^3 - 25n^2 + 12n - 21$

5. $4x^2y^2z(3x - 6y + 4yz^2)$

6. $(2x + 3)(2x - 3) \cdot (y^2 + 5)$

7. $-1(x - 8)(x + 3)$

8. $(5x - 3)(2x + 7)$

9. prime

10. $3x(x - 4y)^2$

11. $(x + 5)(x - 5) \cdot (3x^2 + 2)$

12. 5

13. −7, −2, 2, 7

14. $4y^2$

Page 80

15. $8y^2 - 20y - 28$

16. $\frac{1}{3}m^2 - \frac{11}{6}m + 2$

17. $\left\{0, \frac{11}{3}\right\}$

18. $\left\{-\frac{7}{3}, -\frac{1}{3}\right\}$

19. $\left\{\frac{11}{4}\right\}$

20. $\{-1, 4\}$

21. 12 in. × 2 in. × 4 in.

22. 15 ft × 12 ft

23. 12, 15

24. $\frac{1}{2}$ s

25. $\frac{1}{3}$ in.

B: $\left\{-\frac{7}{6}, 0\right\}$

Chapter 8 Assessment Answer Key

Page 81, Extended-Response Test
Scoring Rubric

Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of the concepts of *finding factors of integers, factoring polynomials,* and *using the Zero Product Property to solve equations.* • Uses appropriate strategies to solve problems. • Computations are correct. • Written explanations are exemplary. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows an understanding of the concepts of *finding factors of integers, factoring polynomials,* and *using the Zero Product Property to solve equations.* • Uses appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are effective. • Satisfies all requirements of problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows an understanding of most of the concepts of *finding factors of integers, factoring polynomials,* and *using the Zero Product Property to solve equations.* • May not use appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are satisfactory. • Satisfies the requirements of most of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work is shown to substantiate the final computation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of most of the concepts of *finding factors of integers, factoring polynomials,* and *using the Zero Product Property to solve equations.* • Does not use appropriate strategies to solve problems. • Computations are incorrect. • Written explanations are unsatisfactory. • Does not satisfy requirements of problems. • No answer may be given.

Chapter 8 Assessment Answer Key

Page 81, Extended-Response Test
Sample Answers

In addition to the scoring rubric found on page A37, the following sample answers may be used as guidance in evaluating extended-response assessment items.

1a. The student should recognize that $A = x(x + b)$ is an equation and $x^2 + bx - A$ is a trinomial expression. When the equation is put into standard form, the trinomial expression is one side of the equation. For example, $A = x(x + b)$ is equivalent to $x^2 + bx - A = 0$.

1b. For $x^2 + bx - A$ to factor, there must be factors of $-A$ whose sum is b.

1c. Sample answer: $A = 144$, $b = 10$;
$x^2 + 10x - 144 = (x + 18)(x - 8)$

1d. Sample answer: $144 = x(x + 10)$;
144 sq ft; width is 8 ft, length is 18 ft

2a. The student should recognize that both c and h must be positive to be above ground, and since c is the starting height and h is equal to c minus $16t^2$ where t^2 is always positive, h will always be less than or equal to c.

2b. Sample answer: $c = 4$; $\frac{1}{2}$s

2c. The student should explain that $c = 9$ and $h = 0$ yields a solution set of $\left\{-\frac{3}{4}, \frac{3}{4}\right\}$. Since we are considering only the time after the ball is thrown, the ball is only in the air for $\frac{3}{4}$ second. Thus, a ball thrown horizontally at a height of 9 feet cannot stay above the ground for more than 1 second.

3a. Sample answer: Choose a value for x and evaluate $x^2 - 8x + 15$.
$x^2 - 8x + 15 = 10^2 - 8(10) + 15 = 35$
A factorization of 35 is 5×7. Since $x = 10$, $5 = x - 5$ and $7 = x - 3$.
Check the product $(x - 5)(x - 3)$.
$(x - 5)(x - 3) = x^2 - 3x - 5x + 15$
$= x^2 - 8x + 15$ ✓

3b. Sample answer: Choose a value for x and evaluate $2x^2 - 13x - 24$.
$2x^2 - 13x - 24 = 2(10)^2 - 13(10) - 24$
$= 46$

A factorization of 46 is 2×23.
Since $x = 10$, $2 = x - 8$ and $23 = 2x + 3$.
Check the product $(x - 8)(2x + 3)$.
$(x - 8)(2x + 3) = 2x^2 + 3x - 16x - 24$
$= 2x^2 - 13x - 24$ ✓

3c. Sample answer: Choose a value for x and evaluate $x^2 - 2x - 8$.
$x^2 - 2x - 8 = 10^2 - 2(10) - 8 = 72$
A factorization of 72 is 6×12. Since $x = 10$, $6 = x - 4$ and $12 = x + 2$.
Check the product $(x - 4)(x + 2)$.
$(x - 4)(x + 2) = x^2 + 2x - 4x - 8$
$= x^2 - 2x - 8$ ✓
The student should explain that there are more factors for 72 than for the numbers in the previous parts, thus making the process more difficult in finding the correct factors.

3d. Sample answer: Evaluate $x^2 - 2x - 8$ for $x = 5$.
$x^2 - 2x - 8 = 5^2 - 2(5) - 8 = 7$
The only factorization of 7 is 1×7.
Since $x = 5$, $1 = x - 4$ and $7 = (x + 2)$.
Check the product $(x - 4)(x + 2)$.
$(x - 4)(x + 2) = x^2 + 2x - 4x - 8$
$= x^2 - 2x - 8$ ✓

Chapter 8 Assessment Answer Key

Standardized Test Practice

Page 82

1. Ⓐ ● Ⓒ Ⓓ

2. ● Ⓖ Ⓗ Ⓙ

3. Ⓐ Ⓑ Ⓒ ●

4. ● Ⓖ Ⓗ Ⓙ

5. Ⓐ ● Ⓒ Ⓓ

6. Ⓕ Ⓖ Ⓗ ●

7. Ⓐ ● Ⓒ Ⓓ

8. ● Ⓖ Ⓗ Ⓙ

9. Ⓐ Ⓑ Ⓒ ●

10. Ⓕ Ⓖ ● Ⓙ

Page 83

11. Ⓐ ● Ⓒ Ⓓ

12. Ⓕ Ⓖ ● Ⓙ

13. ● Ⓑ Ⓒ Ⓓ

14. Ⓕ ● Ⓗ Ⓙ

15. Ⓐ ● Ⓒ Ⓓ

16. ● Ⓖ Ⓗ Ⓙ

17. ● Ⓑ Ⓒ Ⓓ

18. Ⓕ Ⓖ Ⓗ ●

19.

				3
	⊘	⊘	⊘	
⊙	⊙	⊙	⊙	⊙
⓪	⓪	⓪	⓪	⓪
①	①	①	①	①
②	②	②	②	②
③	③	③	③	●
④	④	④	④	④
⑤	⑤	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	⑦	⑦
⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨

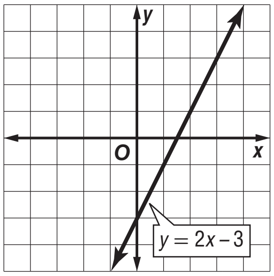
20.

		4	/	3
	⊘	⊘	●	
⊙	⊙	⊙	⊙	⊙
⓪	⓪	⓪	⓪	⓪
①	①	①	①	①
②	②	②	②	②
③	③	③	③	●
④	④	●	④	④
⑤	⑤	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	⑦	⑦
⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨

Chapter 8 Assessment Answer Key

Standardized Test Practice
Page 84

21.



22. infinitely many

23. $\left(2\frac{1}{3}, 1\right)$

24. $16x^8y^{12}$

25. $24y^3 + 42y^2 - y - 10$

26. $2x^3 - 5x^2 - 11x - 4$

27. $4a^2b^2(3 - 4b)$

28. $\left\{\frac{1}{5}, 1\right\}$

29. $(2x + 7y)(2x - 7y)$

30a. Sample answer:
x = lesser number,
y = greater number,
$x + y = 18$,
$3x - y = 10$

30b. 11