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Ministry of Education  
Alain Educational Office



مدرسة صقر الإمارات الدولية الخاصة  
EMIRATES FALCON INT'L. PRIVATE SCHOOL

**MATH DEPARTMENT**

**GRADE 10**

**TERM 2**

**REVISION SHEET**

**FOR**

**FINAL EXAM**

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# Lesson (4-1) a

## Exponential Functions, Growth, and Decay

The **base** of an exponential function indicates whether the function shows growth or decay.

**Exponential function:**  $f(x) = ab^x$

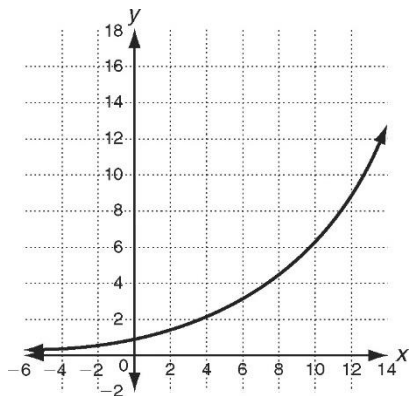
- $a$  is a constant.
- $b$  is the base. The base is a constant.  
If  $0 < b < 1$ , the function shows decay.  
If  $b > 1$ , the function shows growth.
- $x$  is an exponent.

$$f(x) = 1.2^x$$

$$a = 1$$

$$b = 1.2$$

$b > 1$ , so the function shows exponential growth.

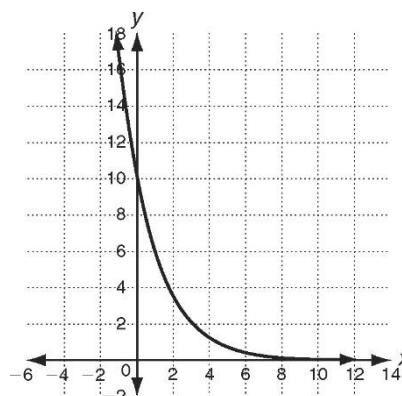


$$g(x) = 10(0.6)^x$$

$$a = 10$$

$$b = 0.6$$

$0 < b < 1$ , so the function shows exponential decay.



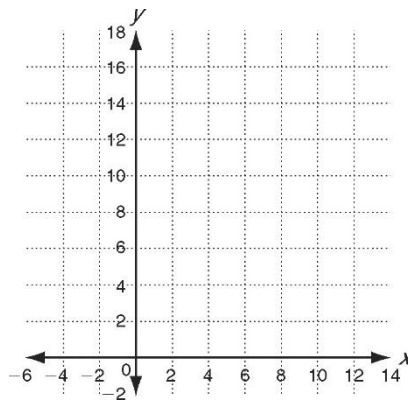
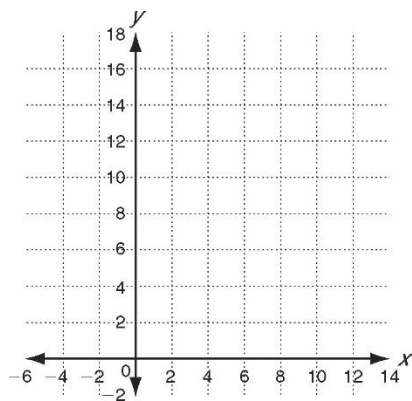
Tell whether each function shows growth or decay. Then graph.

1.  $h(x) = 0.8(1.6)^x$

$a =$  \_\_\_\_\_  $b =$  \_\_\_\_\_

2.  $p(x) = 12(0.7)^x$

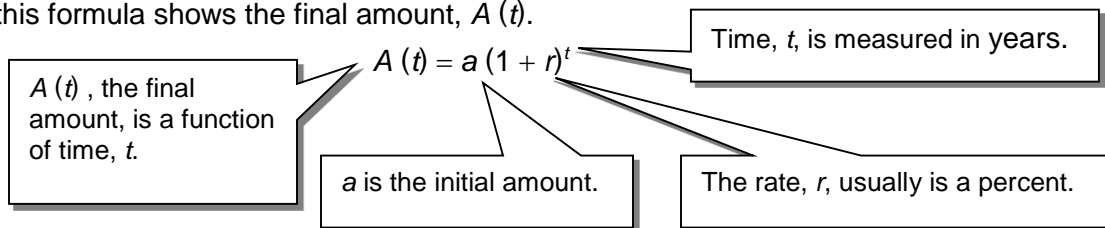
$a =$  \_\_\_\_\_  $b =$  \_\_\_\_\_



## Lesson (4-1) b

### Exponential Functions, Growth, and Decay (continued)

When an initial amount,  $a$ , increases or decreases by a constant rate,  $r$ , over a number of time periods,  $t$ , this formula shows the final amount,  $A(t)$ .



An initial amount of \$15,000 increases by 12% per year. In how many years will the amount reach \$25,000?

**Step 1** Identify values for  $a$  and  $r$ .

$$a = \$15,000 \quad r = 12\% = 0.12$$

**Step 2** Substitute values for  $a$  and  $r$  into the formula.

$$f(t) = a(1+r)^t$$

$$f(t) = 15,000(1+0.12)^t$$

$$f(t) = 15,000(1.12)^t \quad \text{Simplify.}$$

**Step 3** Graph the function using a graphing calculator. Modify the scales:  $[0, 10]$  and  $[0, 30,000]$ .

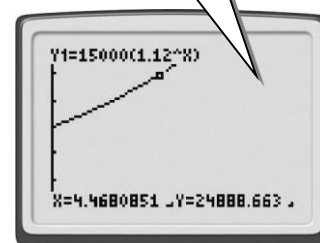
**Step 4** Use the graph and the [TRACE] feature on the calculator to find  $f(t) = 25,000$ .

**Step 5** Use the graph to approximate the value of  $t$  when  $f(t) = 25,000$ .

$$t \approx 4.5 \text{ when } f(t) = 25,000$$

The amount will reach \$25,000 in about 4.5 years.

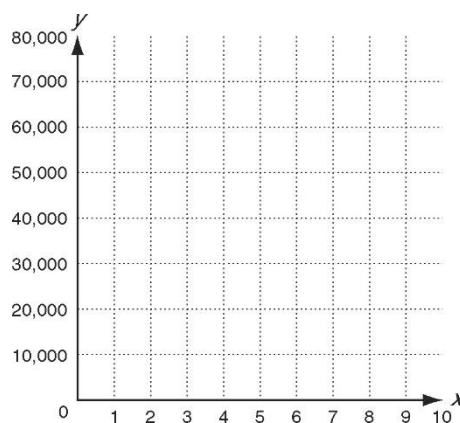
Remember:  
On the graph,  $x$  corresponds to  $t$  and  $y$  corresponds to  $f(t)$ .



**Write an exponential function and graph the function to solve.**

3. An initial amount of \$40,000 increases by 8% per year. In how many years will the amount reach \$60,000?

- $a =$  \_\_\_\_\_
- $r =$  \_\_\_\_\_
- $f(t) =$  \_\_\_\_\_
- Approximate  $t$  when  $f(t) = 60,000$   
 $t \approx$  \_\_\_\_\_



# Lesson (4-2) a

## Inverses of Relations and Functions

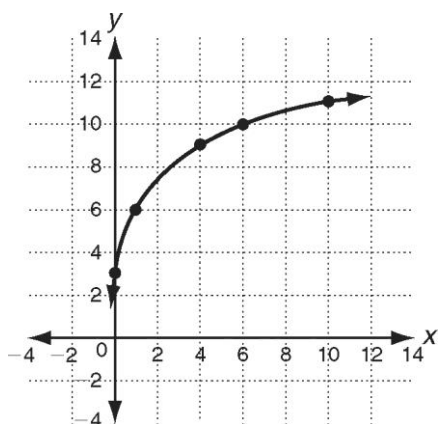
To graph an **inverse** relation, reflect each point across the line  $y = x$ . Or you can switch the  $x$ - and  $y$ -values in each ordered pair of the relation to find the ordered pairs of the inverse.

Remember, a relation is a set of ordered pairs.

<b>x</b>	0	1	4	6	10
<b>y</b>	3	6	9	10	11

The domain is all possible values of  $x$ :  $\{x \mid 0 \leq x \leq 10\}$ .

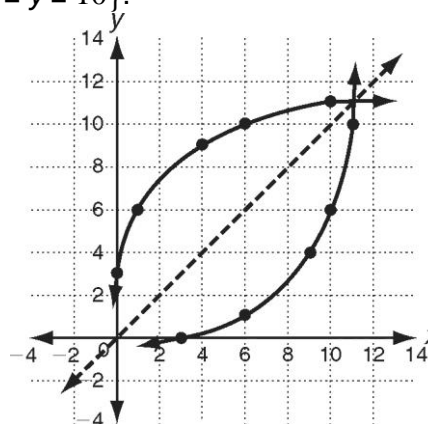
The range is all possible values of  $y$ :  $\{y \mid 3 \leq y \leq 11\}$ .



To write the **inverse** of the relation, switch the places of  $x$  and  $y$  in each ordered pair.

<b>x</b>	3	6	9	10	11
<b>y</b>	0	1	4	6	10

The domain of the inverse corresponds to the range of the original relation:  $\{x \mid 3 \leq x \leq 11\}$ . The range of the inverse corresponds to the domain of the original relation:  $\{y \mid 3 \leq y \leq 10\}$ .



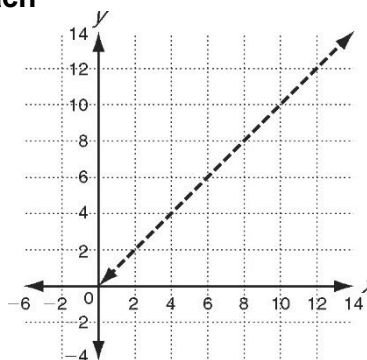
Complete the table to find the ordered pairs of the inverse. Graph the relation and its inverse. Identify the domain and range of each relation.

1. Relation

<b>x</b>	0	2	5	8	10
<b>y</b>	6	10	12	13	13

Inverse

<b>x</b>	6				
<b>y</b>	0				



Relation: Domain: \_\_\_\_\_

Range: \_\_\_\_\_

Inverse: Domain: \_\_\_\_\_

Range: \_\_\_\_\_

## Lesson (4-2) b

### ***Inverses of Relations and Functions*** (continued)

Inverse operations undo each other, like addition and subtraction, or multiplication and division.

In a similar way, **inverse functions** undo each other.

The inverse of a function  $f(x)$  is denoted  $f^{-1}(x)$ .

Use inverse operations to write inverse functions.

<b>Function: <math>f(x) = x + 8</math></b> ↑	<b>Function: <math>f(x) = 5x</math></b> ↑
Subtraction is the opposite of addition. Use subtraction to write the inverse.	Division is the opposite of multiplication. Use division to write the inverse.
<b>Inverse: <math>f^{-1}(x) = x - 8</math></b>	<b>Inverse: <math>f^{-1}(x) = \frac{x}{5}</math></b>
Choose a value for $x$ to check in the original function. Try $x = 1$ .	Choose a value for $x$ to check in the original function. Try $x = 2$ .
$f(x) = x + 8 \rightarrow f(1) = 1 + 8 = 9$	$f(x) = 5x \rightarrow f(2) = 5(2) = 10$
Substitute 9, into $f^{-1}(x)$ . The output of the inverse should be 1.	Substitute 10 into $f^{-1}(x)$ . The output of the inverse should be 2.
$f^{-1}(x) = x - 8 \rightarrow f^{-1}(9) = 9 - 8 = 1$	$f^{-1}(x) = \frac{x}{5} \rightarrow f^{-1}(10) = \frac{10}{5} = 2$
<i>Think:</i> (1, 9) in the original function should be (9, 1) in the inverse. ✓	<i>Think:</i> (2, 10) in the original function should be (10, 2) in the inverse. ✓

Use inverse operations to write the inverse of each function.

2.  $f(x) = x - 4$

\_\_\_\_\_

Use  $x = 5$  to check.

\_\_\_\_\_

\_\_\_\_\_

4.  $f(x) = x + 3$

\_\_\_\_\_

3.  $f(x) = \frac{x}{6}$

\_\_\_\_\_

Use  $x = 12$  to check.

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\_\_\_\_\_

5.  $f(x) = 14x$

\_\_\_\_\_

# Lesson (4-3) a

## Logarithmic Functions

A **logarithm** is another way to work with exponents in equations.

If  $b^x = a$ , then  $\log_b a = x$ .

If  $b$  to the  $x$  power equals  $a$ ,  
then  $x$  is the logarithm of  $a$  in base  $b$ .

Use the definition of the logarithm to write exponential equations in logarithmic form and to write logarithmic equations in exponential form.

### Exponential Form

$$3^4 = 81$$

base,  $b = 3$   
exponent,  $x = 4$   
value,  $a = 81$

### Logarithmic Form

$$\log_3 81 = 4$$

### Logarithmic Form

$$\log_5 125 = 3$$

base,  $b = 5$   
exponent,  $x = 3$   
value,  $a = 125$

### Exponential Form

$$5^3 = 125$$

If no base is written for a logarithm, the base is assumed to be 10.

Example:  $\log 100 = 2$  because  $10^2 = 100$ .

Assume the base is 10.

**Write each exponential equation in logarithmic form.**

1.  $7^2 = 49$

$b = 7, x = 2, a = 49$

\_\_\_\_\_

2.  $6^3 = 216$

$b = \underline{\quad}, x = \underline{\quad}, a = \underline{\quad}$

\_\_\_\_\_

3.  $2^5 = 32$

\_\_\_\_\_

\_\_\_\_\_

**Write each logarithmic equation in exponential form.**

4.  $\log_9 729 = 3$

$b = 9, x = 3, a = 729$

\_\_\_\_\_

5.  $\log_2 64 = 6$

$b = \underline{\quad}, x = \underline{\quad}, a = \underline{\quad}$

\_\_\_\_\_

6.  $\log 1000 = 3$

\_\_\_\_\_

\_\_\_\_\_

# Lesson (4-3) b

## Logarithmic Functions (continued)

The logarithmic function is the inverse of the exponential function. Use this fact to graph the logarithmic function.

Graph a function and its inverse.

Graph  $f(x) = 0.5^x$  using a table of values.

<b>x</b>	-2	-1	0	1	2
<b>f(x)</b>	4	2	1	0.5	0.25

Write the inverse function.

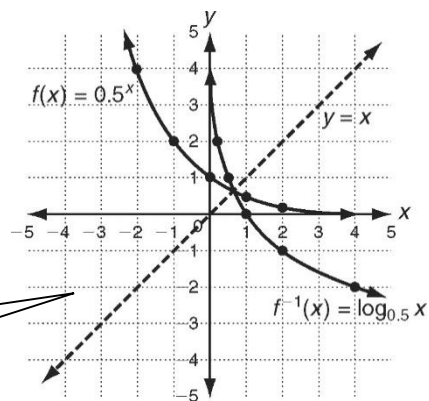
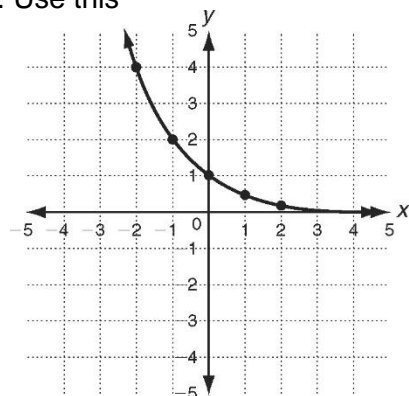
$$f^{-1}(x) = \log_{0.5} x$$

The base is 0.5.

x and f(x) switch places in the function and its inverse.

<b>x</b>	4	2	1	0.5	0.25
<b>f<sup>-1</sup>(x)</b>	-2	-1	0	1	2

Remember, the graph of the inverse is the reflection of the original function across the line  $y = x$ .



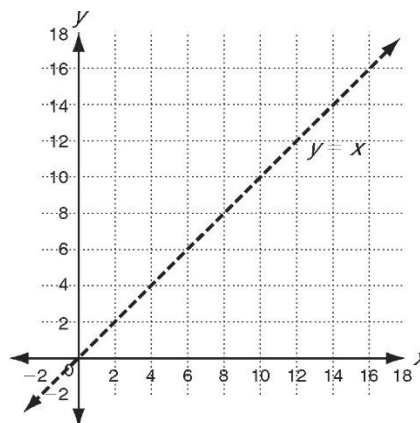
Complete the tables. Graph the functions.

7.  $f(x) = 4^x$

<b>x</b>	-2	-1	0	1	2
<b>f(x)</b>	$\frac{1}{16}$	$\frac{1}{4}$			

$$f^{-1}(x) = \log_4 x$$

<b>x</b>	$\frac{1}{16}$	$\frac{1}{4}$			
<b>f<sup>-1</sup>(x)</b>					



## Lesson (4-4) a

### Properties of Logarithms

Use properties of logarithms to simplify logarithms.

The Product Property uses addition instead of multiplication.

#### Product Property

The logarithm of a product can be written as the sum of the logarithm of the numbers.

$$\log_b mn = \log_b m + \log_b n$$

where  $m$ ,  $n$ , and  $b$  are all positive numbers and  $b \neq 1$

Simplify:  $\log_8 4 + \log_8 16 = \log_8 (4 \cdot 16) = \log_8 64 = 2$

The bases must be the same for both logarithms.

Think: 8 to what power is equal to 64, or  $8^? = 64$ .

The Quotient Property uses subtraction instead of division.

#### Quotient Property

The logarithm of a quotient can be written as the logarithm of the numerator minus the logarithm of the denominator.

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

where  $m$ ,  $n$ , and  $b$  are all positive numbers and  $b \neq 1$

Simplify:  $\log_3 243 - \log_3 9 = \log_3 \left( \frac{243}{9} \right) = \log_3 27 = 3$

The bases must be the same for both logarithms.

Think: 3 to what power is equal to 27, or  $3^? = 27$ .

Complete the steps to simplify each expression.

1.  $\log_6 54 + \log_6 4$

$\log_6 (54 \cdot 4)$

$\log_6 216$

\_\_\_\_\_

2.  $\log_2 128 - \log_2 8$

$\log_2 \left( \frac{128}{8} \right)$

\_\_\_\_\_

\_\_\_\_\_

3.  $\log_9 3 + \log_9 27$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_



## Lesson (4-4) b

### Properties of Logarithms (continued)

The Power Property uses multiplication instead of exponentiation.

#### Power Property

The logarithm of a power can be written as the product of the exponent and the logarithm of the base.

$$\log_b a^p = p \log_b a$$

for any real number  $p$

where  $a$  and  $b$  are positive numbers and  $b \neq 1$

Simplify:  $\log_4 64^5 = 5 \log_4 64 = 5(3) = 15$

“Bring down” the exponent to multiply.

Think: 4 to what power is equal to 64, or  $4^? = 64$ .

Logarithms and exponents undo each other when their bases are the same.

#### Inverse Properties

The logarithm of  $b^x$  to the base  $b$  is equal to  $x$ .

$$\log_b b^x = x$$



The logarithm undoes the exponent when the bases are the same.

Simplify:  $\log_7 7^{4x} = 4x$

The base of the log is 7 and the base of the exponent is 7.

$b$  raised to the logarithm of  $x$  to the base  $b$  is equal to  $x$ .

$$b^{\log_b x} = x$$



The exponent undoes the logarithm when the bases are the same.

Simplify:  $3^{\log_3 64} = 64$

The base of the exponent is 3 and the base of the log is 3.

Simplify each expression.

4.  $\log_5 125^2$

$2 \log_5 125$

\_\_\_\_\_

5.  $\log_2 16^4$

$4 \log_2 16$

\_\_\_\_\_

6.  $\log_9 81^3$

\_\_\_\_\_

7.  $\log_6 6^{5y}$

\_\_\_\_\_

8.  $4^{\log_4 75}$

\_\_\_\_\_

9.  $2^{\log_2 3x}$

\_\_\_\_\_

## Lesson (4-5) a

### Exponential and Logarithmic Equations and Inequalities

An **exponential equation** contains an expression that has a variable as an exponent.

$$5^x = 25 \text{ is an exponential equation.}$$

$$x = 2, \text{ since } 5(2) = 25.$$

Remember: You can take the logarithm of both sides of an exponential equation. Then use other properties of logarithms to solve.

$$\begin{aligned} \text{If } x = y, \text{ then} \\ \log x = \log y \\ (x > 0 \text{ and } y > 0). \end{aligned}$$

Solve  $6^{x+2} = 500$ .

**Step 1** Since the variable is in the exponent, take the log of both sides.

$$6^{x+2} = 500$$

$$\log 6^{x+2} = \log 500$$

**Step 2** Use the Power Property of Logarithms:  $\log a^p = p \log a$ .

$$\log 6^{x+2} = \log 500$$

$$(x + 2) \log 6 = \log 500$$

*"Bring down" the exponent to multiply.*

**Step 3** Isolate the variable. Divide both sides by  $\log 6$ .

$$(x + 2) \log 6 = \log 500$$

$$x + 2 = \frac{\log 500}{\log 6}$$

**Step 4** Solve for  $x$ . Subtract 2 from both sides.

$$x = \frac{\log 500}{\log 6} - 2$$

**Step 5** Use a calculator to approximate  $x$ .

$$x \approx 1.468$$

**Step 6** Use a calculator to check.

$$6^{1.468+2} \approx 499.607$$

#### Solve and check.

1.  $4^{-x} = 32$

$$\log 4^{-x} = \log 32$$

$$-x \log 4 = \log 32$$

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\_\_\_\_\_

2.  $3^{4x} = 90$

$$\log 3^{4x} = \log 90$$

$$4x \log 3 = \log 90$$

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3.  $5^{x-3} = 600$

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## Lesson (4-5) b

### Exponential and Logarithmic Equations and Inequalities (continued)

A **logarithmic equation** contains a logarithmic expression that has a variable.

$\log_5 x = 2$  is a logarithmic equation.

$x = 25$ , since  $5^2 = 25$ .

Combine and use properties of logarithms to solve logarithmic equations.

Solve:  $\log 80x - \log 4 = 1$

**Step 1** Use the Quotient Property of Logarithms.

$$\log 80x - \log 4 = 1$$

$$\log \frac{80x}{4} = 1$$

$$\log x - \log y = \log \frac{x}{y}$$

**Step 2** Simplify.

$$\log \frac{80x}{4} = 1$$

$$\log 20x = 1$$

**Step 3** Use the definition of the logarithm:

if  $b^x = a$ , then  $\log_b a = x$ .

$$\log_{10} 20x = 1$$

$$10^1 = 20x$$

Remember: Use 10 as the base when the base is not given.

**Step 4** Solve for  $x$ . Divide both sides by 20.

$$10 = 20x$$

$$\frac{1}{2} = x$$

**Solve and check.**

4.  $\log_3 x^4 = 8$

$$4 \log_3 x = 8$$

$$\log_3 x = \frac{8}{4}$$

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5.  $\log 4 + \log (x + 2) = 2$

$$\log 4 (x + 2) = 2$$

$$\log_{10} (4x + 8) = 2$$

$$4x + 8 = 10^2$$

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6.  $\log 75x - \log 3 = 1$

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# Lesson (4-6) a

## The Natural Base, e

The **natural logarithmic function**,  $f(x) = \ln x$ , is the inverse of the exponential function with the natural base e,  $f(x) = e^x$ .

The constant e is an irrational number.  $e \approx 2.71828\dots$

Properties of logarithms apply to the natural logarithm.

In particular:

- $\ln 1 = 0$                       The base is e and  $e^0 = 1$ .
- $\ln e = 1$                          Think:  $e^1 = e$ .
- $\ln e^x = x$                         The natural logarithm and the exponential function are inverses, so they undo each other.
- $e^{\ln x} = x$

Use properties of logarithms to simplify expressions with e or "ln."

<p>Simplify: <math>\ln e^{x+2}</math>  <b>Step 1</b> Use the Power Property. "Bring down" the exponent to multiply.  <math>\ln e^{x+2}</math>  <math>(x+2) \ln e</math></p> <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 10px auto;"> <math>\ln e = 1</math> </div> <p><b>Step 2</b> Simplify.  <math>(x+2) \ln e</math>  <math>x+2</math></p>	<p>Simplify: <math>e^{4 \ln x}</math>  <b>Step 1</b> Use the Power Property. Write the exponent.  <math>e^{4 \ln x}</math>  <math>e^{\ln x^4}</math></p> <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 10px auto;"> <math>e^{\ln x} = x</math> </div> <p><b>Step 2</b> Simplify.  <math>e^{\ln x^4}</math>  <math>x^4</math></p>
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**Simplify each expression.**

1.  $\ln e^{-6x}$

$-6x \ln e$

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2.  $\ln e^{t-3}$

$(t-3) \ln e$

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3.  $e^{2 \ln x}$

$e^{\ln x^2}$

\_\_\_\_\_

4.  $\ln e^{1.8}$

\_\_\_\_\_

\_\_\_\_\_

5.  $\ln e^{x+1}$

\_\_\_\_\_

\_\_\_\_\_

6.  $e^{7 \ln x}$

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\_\_\_\_\_

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## Lesson (4-6) b

### The Natural Base, $e$ (continued)

The natural base,  $e$ , appears in the formula for interest compounded continuously.

$$A = Pe^{rt}$$

$A$  = total amount

$P$  = principal, or initial amount

$r$  = annual interest rate

$t$  = time in years

What is the total amount for an investment of \$2000 invested at 3% and compounded continuously for 5 years?

**Step 1** Identify the values that correspond to the variables in the formula.

$$P = \text{initial investment} = \$2000$$

$$r = 3\% = 0.03$$

$$t = 5$$

**Step 2** Substitute the known values into the formula.

$$A = Pe^{rt}$$

$$A = 2000 e^{0.03(5)}$$

**Step 3** Use a calculator to solve for  $A$ , the total amount.

$$A = 2000 e^{0.03(5)}$$

$$A \approx 2323.67$$

Use the  $e^x$  key on a calculator:  
 $2000e^{(0.03 \cdot 5)} = 2323.668485$

The total amount is \$2323.67.

**Use the formula  $A = Pe^{rt}$  to solve.**

7. What is the total amount for an investment of \$500 invested at 4.5% and compounded continuously for 10 years?

$$P = \underline{\hspace{2cm}} \quad r = \underline{\hspace{2cm}} \quad t = \underline{\hspace{2cm}}$$

8. Randy deposited \$1000 into an account that paid 2.8% with continuous compounding. What was her balance after 6 years?

9. a. Martin borrows \$5500. The rate is set at 6% with continuous compounding. How much does he owe at the end of 2 years?

b. Martin found a bank with a better interest rate of 5.5%. How much less does he owe at the end of 2 years?

## Multiplying and Dividing Rational Expressions

Examples of rational expressions:  $\frac{3}{x}$ ,  $\frac{x+1}{x+2}$ , and  $\frac{x+3}{2x^2}$

Undefined at  $x = 0$

Undefined at  $x = -2$

Undefined at  $x = 0$

When simplifying a rational expression:

- Factor the numerator and the denominator completely.
- Divide out any common factors.
- Identify any  $x$ -values for which the expression is undefined.

Simplify:  $\frac{24x^6}{8x^2}$

$x \neq 0$ , because  $8x^2$  is undefined at  $x = 0$ .

$$\frac{24x^6}{8x^2} = \frac{\cancel{8} \cdot 3}{\cancel{8}} x^{6-2} = 3x^4$$

Use the Quotient of Powers Property.

Simplify:  $\frac{x^2 - 2x - 8}{x^2 + x - 2}$

First, factor the numerator and the denominator.

$$\frac{x^2 - 2x - 8}{x^2 + x - 2} = \frac{(x-4)(x+2)}{(x+2)(x-1)} = \frac{(x-4)\cancel{(x+2)}}{\cancel{(x+2)}(x-1)} = \frac{(x-4)}{(x-1)} = \frac{x-4}{x-1}$$

$x \neq -2$  and  $x \neq 1$

Divide out common factors.

Simplify.

1.  $\frac{x^2 - 2x - 3}{x^2 + 6x + 5}$

$$\frac{(x+1)(x-3)}{(x+1)(x+5)}$$

\_\_\_\_\_

$x \neq$  \_\_\_\_\_

2.  $\frac{20x^9}{4x^3}$

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\_\_\_\_\_

$x \neq$  \_\_\_\_\_

3.  $\frac{x^2 - 4x}{x^2 - 5x + 4}$

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\_\_\_\_\_

$x \neq$  \_\_\_\_\_

Multiplying rational expressions is similar to multiplying fractions.

Multiply:  $\frac{15x^2y^3}{4x^3y^5} \cdot \frac{2x^4y^3}{3xy^2}$ .

$$\frac{15x^2y^3}{4x^3y^5} \cdot \frac{2x^4y^3}{3xy^2} = \frac{15}{4} \cdot \frac{2}{3} \cdot \frac{x^2x^4}{x^3x} \cdot \frac{y^3y^3}{y^5y^2}$$

Group like factors.

$$= \frac{5}{2} \cdot \frac{x^6}{x^4} \cdot \frac{y^6}{y^7}$$

Simplify constants. Add exponents to multiply.

$$= \frac{5}{2} \cdot x^2 \cdot \frac{1}{y}$$

Subtract exponents to divide.

$$= \frac{5x^2}{2y}$$

Simplify.

Multiplying rational expressions is similar to simplifying rational expressions.

Multiply:  $\frac{x+3}{6x-6} \cdot \frac{x-1}{x^2-9}$ .

$$\frac{x+3}{6x-6} \cdot \frac{x-1}{x^2-9} = \frac{x+3}{6(x-1)} \cdot \frac{x-1}{(x+3)(x-3)}$$

Completely factor all numerators and denominators.

$$= \frac{\cancel{x+3}}{6(\cancel{x-1})} \cdot \frac{\cancel{x-1}}{(\cancel{x+3})(x-3)}$$

Divide out common factors.

$$= \frac{1}{6(x-3)}$$

Simplify.

To divide rational expressions, multiply by the reciprocal.

$$\frac{x+7}{x-2} \cdot \frac{x^2-49}{2x-4} = \frac{x+7}{x-2} \cdot \frac{2x-4}{x^2-49} = \frac{\cancel{x+7}}{\cancel{x-2}} \cdot \frac{2(x-2)}{(x-7)(x+7)} = \frac{2}{x-7}$$

**Multiply. Assume that all expressions are defined.**

4.  $\frac{12x^5y^2}{6x^2y^4} \cdot \frac{9x^3y}{3x^2y^3}$

5.  $\frac{2x-2}{x+4} \cdot \frac{x^2+4x}{x^2-3x+2}$

6.  $\frac{8x+16}{x^2-1} \cdot \frac{x+1}{4x+8}$

7.  $\frac{3x^3y}{5xy^2} \div \frac{9xy^3}{15y}$

8.  $\frac{4x-8}{x^2-4} \div \frac{3x}{x+2}$

9.  $\frac{x^2+2x-3}{x^2-9} \div \frac{x^2+3x-4}{x^2-2x-3}$

# Lesson (5-3) a

## Adding and Subtracting Rational Expressions

Use a common denominator to add or subtract rational expressions.

Add:  $\frac{6x+4}{x+5} + \frac{2x-8}{x+5}$ .

**Step 1** Add.

$$\frac{6x+4}{x+5} + \frac{2x-8}{x+5} = \frac{6x+4+2x-8}{x+5}$$

The denominators are the same.  
Add the numerators.

$$= \frac{6x+2x+4-8}{x+5}$$

Group like terms.

$$= \frac{8x-4}{x+5}$$

Combine like terms.

**Step 2** Identify x-values for which the expression is undefined.

$x \neq -5$  because  $-5$  makes the denominator equal 0.

Subtract:  $\frac{4x-3}{2x-1} - \frac{8x+2}{2x-1}$ .

**Step 1** Subtract.

$$\frac{4x-3}{2x-1} - \frac{8x+2}{2x-1} = \frac{(4x-3)-(8x+2)}{2x-1}$$

The denominators are the same.  
Subtract the numerators.

$$= \frac{4x-3-8x-2}{2x-1}$$

Use the Distributive Property.

$$= \frac{-4x-5}{2x-1}$$

Combine like terms.

**Step 2** Identify x-values for which the expression is undefined.

$x \neq \frac{1}{2}$  because  $\frac{1}{2}$  makes the denominator equal 0.

**Add or subtract.**

1.  $\frac{x-5}{x^2-4} + \frac{3x+2}{x^2-4}$

$$\frac{(x-5)+(3x+2)}{x^2-4}$$

x ≠ \_\_\_\_\_

2.  $\frac{7x-5}{x+3} - \frac{4x-1}{x+3}$

$$\frac{(7x-5)-(4x-1)}{x+3}$$

x ≠ \_\_\_\_\_

3.  $\frac{2x-1}{x-1} - \frac{5x+4}{x-1}$

\_\_\_\_\_

x ≠ \_\_\_\_\_

4.  $\frac{4x+1}{3x+7} + \frac{9-x}{3x+7}$

\_\_\_\_\_

x ≠ \_\_\_\_\_

5.  $\frac{8-x}{x-3} - \frac{5-x}{x-3}$

\_\_\_\_\_

x ≠ \_\_\_\_\_

6.  $\frac{5x+2}{x^2-1} - \frac{3x-7}{x^2-1}$

\_\_\_\_\_

x ≠ \_\_\_\_\_



## Lesson (5-3) b

### *Adding and Subtracting Rational Expressions (continued)*

Use the least common denominator (LCD) to add rational expressions with different denominators. The process is the same as adding fractions with different denominators.

$$\text{Add: } \frac{x-4}{x^2+2x-3} + \frac{2x}{x-1}$$

**Step 1** Factor denominators completely.

$$\frac{x-4}{x^2+2x-3} + \frac{2x}{x-1} = \frac{x-4}{(x+3)(x-1)} + \frac{2x}{x-1}$$

**Step 2** Find the LCD.

The LCD is the least common multiple of the denominators:  
 $(x+3)(x-1)$ .

**Step 3** Write each term of the expression using the LCD.

$$\frac{2x}{x-1} = \frac{2x}{x-1} \left( \frac{x+3}{x+3} \right) = \frac{2x^2+6x}{(x-1)(x+3)}$$

$$\text{So, } \frac{x-4}{(x+3)(x-1)} + \frac{2x}{x-1} = \frac{x-4}{(x+3)(x-1)} + \frac{2x^2+6x}{(x-1)(x+3)}$$

**Step 4** Add the numerators and simplify.

$$\frac{x-4+2x^2+6x}{(x+3)(x-1)} = \frac{2x^2+7x-4}{(x+3)(x-1)}$$

**Step 5** Identify  $x$ -values for which the expression is undefined.

$x \neq -3$  or  $1$  because both values make the denominator equal 0.

**Add.**

$$7. \frac{x-1}{x^2-4} + \frac{3x}{x+2}$$

$$\frac{x-1}{(x+2)(x-2)} + \frac{3x}{x+2}$$

$$\frac{x-1}{(x+2)(x-2)} + \frac{3x}{x+2} \left( \frac{x-2}{x-2} \right)$$

$$x \neq \underline{\hspace{2cm}}$$

$$8. \frac{4x-1}{x^2+3x+2} + \frac{3}{x+1}$$

$$\underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}}$$

$$x \neq \underline{\hspace{2cm}}$$

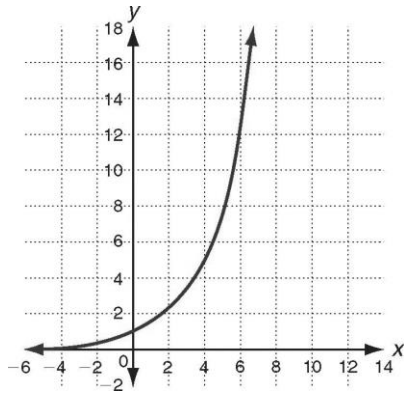
$$9. \text{ What is the LCD of } \frac{2x+1}{x^2-9} \text{ and } \frac{7}{x^2-x-6}?$$

$$\underline{\hspace{2cm}}$$

### Answer Key Lesson (4-1)

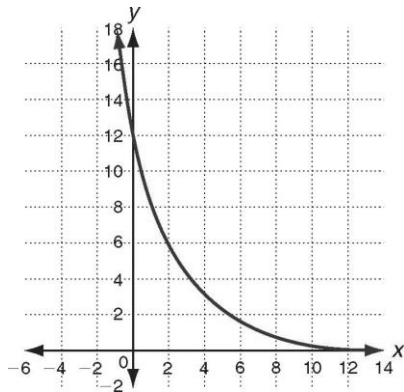
1. 0.8; 1.6

$h(x)$  shows exponential growth.



2.12; 0.7

$p(x)$  shows exponential decay.

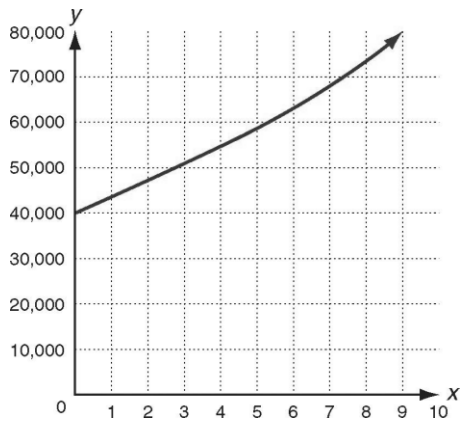


3. a. 40,000

b. 0.08

c.  $f(t) = 40,000(1.08)^t$

d. 5.25 yr



**Answer Key Lesson (4-2)**

1.

x	6	10	12	13	13
y	0	2	5	8	10

$\{x|0 \leq x \leq 10\}$

$\{y|6 \leq y \leq 13\}$

$\{x|6 \leq x \leq 13\}$

$\{y|0 \leq y \leq 10\}$

2.  $f^{-1}(x) = x + 4$

3.  $f^{-1}(x) = 6x$

$f(5) = 1; f^{-1}(1) = 5$

$f(12) = 2; f^{-1}(2) = 12$

4.  $f^{-1}(x) = x - 3$

5.  $f^{-1}(x) = \frac{x}{14}$

**Answer Key Lesson (4-3)**

1.  $\log_7 49 = 2$

2.  $b = 6, x = 3, a = 216$

$\log_6 216 = 3$

3.  $b = 2, x = 5, a = 32$

$\log_2 32 = 5$

4.  $9^3 = 729$

5.  $b = 2, x = 6, a = 64$

$2^6 = 64$

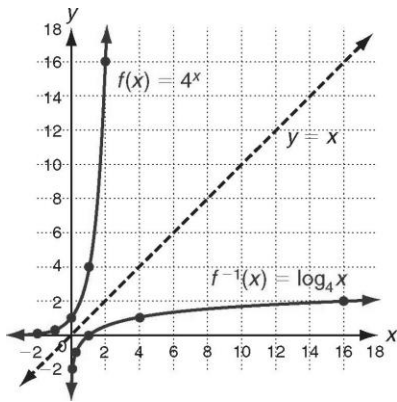
6.  $b = 10, x = 3, a = 1000$

$10^3 = 1000$

7.

x	-2	-1	0	1	2
f(x)	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16

x	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16
$f^{-1}(x)$	-2	-1	0	1	2



#### Answer Key Lesson (4-4)

1. 3
2.  $\log_2 16$ ; 4
3.  $\log_9 (3 \cdot 27)$ ;  $\log_9 81$ ; 2
4.  $2 \cdot 3 = 6$
5.  $4 \cdot 4 = 16$
6.  $3 \log_9 81$ ;  $3 \cdot 2 = 6$
7.  $5y$
8. 75
9.  $3x$

#### Answer Key Lesson (4-5)

1.  $x = -2.5$ ;  $4^{-(-2.5)} = 32$
2.  $x \approx 1.024$ ;  $3^{4(1.024)} \approx 90.01$
3.  $\log 5^{x-3} = \log 600$   
 $(x-3) \log 5 = \log 600$   
 $x \approx 6.975$   
 $5^{6.975-3} \approx 600.352$
4.  $3^2 = x$   
 $x = 9$
5.  $4x + 8 = 100$   
 $4x = 92$   
 $x = 23$
6.  $\log\left(\frac{75x}{3}\right) = 1$ ;  $\log 25x = 1$ ;  $10^1 = 25x$ ;  
 $10 = 25x$ ;  $x = \frac{2}{5}$

**Answer Key Lesson (4-6)**

1.  $-6x$
2.  $t - 3$
3.  $x^2$
4.  $1.8 \ln e$ ; 1.8
5.  $(x + 1) \ln e$ ;  $x + 1$
6.  $e^{\ln x^2}$ ;  $x^7$
7. 500; 0.045; 10; \$784.16
8. \$1182.94
9. a. \$6201.23  
b. \$61.70

**Answer Key Lesson (5-2)**

1.  $\frac{x-3}{x+5}$ , -1, -5
2.  $\frac{20}{4} \cdot \frac{x^9}{x^3}$ ;  $5x^6$ ; 0
3.  $\frac{x(x-4)}{(x-4)(x-1)}$ ;  $\frac{x}{x-1}$ ; 1, 4
4.  $\frac{6x^4}{y^4}$
5.  $\frac{2x}{x-2}$
6.  $\frac{2}{x-1}$
7.  $\frac{x}{y^4}$
8.  $\frac{4}{3x}$
9.  $\frac{x+1}{x+4}$

**Answer Key Lesson (5-3)**

1.  $\frac{4x-3}{x^2-4}$ ; -2, 2
2.  $\frac{3x-4}{x+3}$ ; -3
3.  $\frac{-3x-5}{x-1}$ ; 1
4.  $\frac{3x+10}{3x+7}$ ;  $-\frac{7}{3}$
5.  $\frac{3}{x-3}$ ; 3
6.  $\frac{2x+9}{x^2-1}$ ;  $\pm 1$
7.  $\frac{x-1+(3x^2-6x)}{(x+2)(x-2)} = \frac{3x^2-5x-1}{(x+2)(x-2)}$   
 $x \neq -2, 2$
8.  $\frac{4x-1}{(x+2)(x+1)} + \frac{3}{x+1} \left( \frac{x+2}{x+2} \right)$   
 $\frac{4x-1+3x+6}{(x+2)(x+1)}$   
 $\frac{7x+5}{(x+2)(x+1)}$   
 $x \neq -2, -1$
9.  $(x-3)(x+3)(x+2)$