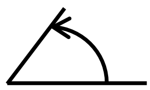
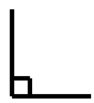
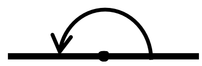
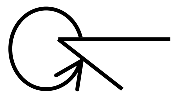
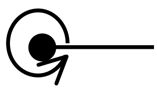
Geometry (Part 1)

Lines and angles

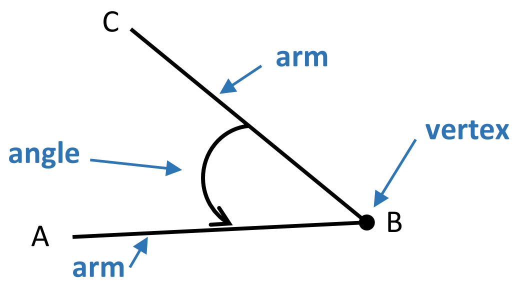
A **line** is an infinite number of points between two end points.

Where two lines meet or cross, they form an **angle**.

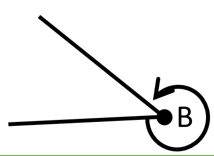
An **angle** is an amount of rotation. It is measured in **degrees**.

Types of angles		
Name of angle	**Example**	**Size of angle**
Acute angle		Between 0° and 90°
Right angle		Equal to 90°
Obtuse angle		Between 90° and 180°
Straight line		Equal to 180°
Reflex angle		Between 180° and 360°
Revolution/angles around a point		Equal to 360°

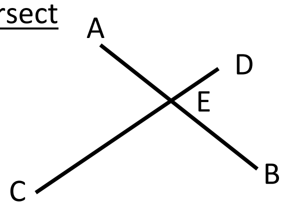
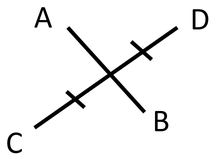
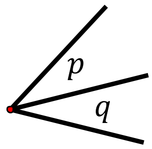
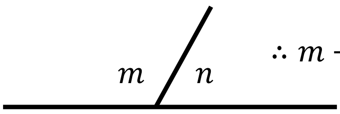
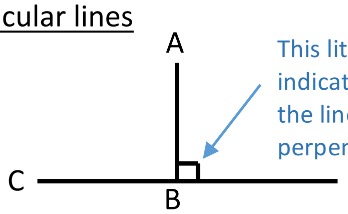
<u>Angle language:</u>



Labelling angles: $\hat{B}$ or $A\hat{B}C$
Also:



We refer to the reflex angle as **'reflex $\hat{B}$'**

Terminology	
<u>Intersect</u> 	AB and CD intersect (cross or cut) at E
<u>Bisect</u> 	AB bisect (cuts in half) CD
<u>Complementary angles</u> Angles that add up to $90°$ <u>Supplementary angles</u> Angles that add up to $180°$	E.g. the complement of $48°$ is $42°$ E.g. the supplement of $130°$ is $50°$
<u>Adjacent angles</u>   Adjacent angels on a straight line adds up to $180°$  $\therefore m + n = 180°$	Angles that have a common vertex and a common arm $\rightarrow$ p and q are adjacent angles.
<u>Perpendicular lines</u>  This little block indicates to us that the lines are perpendicular.	Lines that meet or cross at $90°$. $AB \perp CD$ Symbol for 'perpendicular'

Exercise 1:

(a) In the diagram below name:

(1) 5 acute angles

____

(2) 2 right angles

____

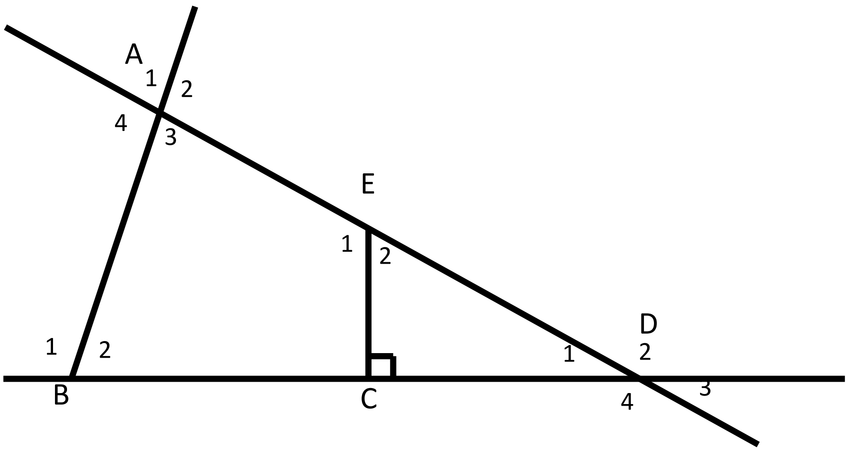
(3) 10 pairs of adjacent angles

____

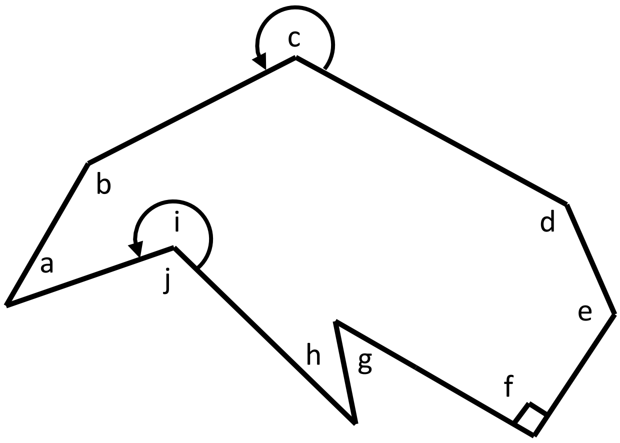
____

(4) 3 obtuse angles

____



(b) In the diagram below, classify the angles labelled a – j. The first one is done for you as an example:



a: Acute b: ______________________________

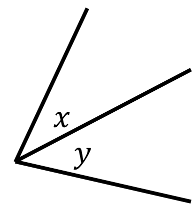
c: ______________________ d: ______________________

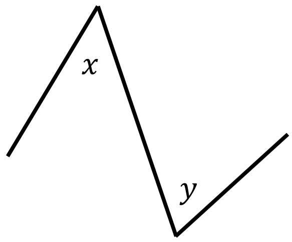
e: ______________________ f: ______________________

g: ______________________ h: ______________________

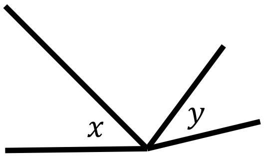
i: ______________________ j: ______________________

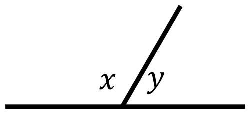
(c) Consider the angles marked x and y. State whether they are adjacent or not:



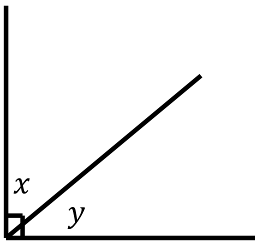


______________________ ______________________





______________________ ______________________



______________________ ______________________

(d) Complete the table by filling in the missing information:

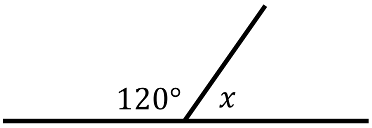
Measure of angle	**Complement**	**Supplement**
$37°$	$90° - 37° = 59°$	$180° - 37° = 143°$
$20°$		
$77°$		
$101°$		
$90°$		
$96°$		
x		
y		

REMEMBER: Adjacent angles on a straight line are supplementary.

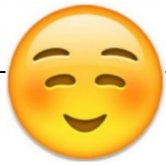
If they are adjacent angles on a straight line, then they add up to $180°$.

<u>Example:</u>

Determine, with reason, the value of x:



Statement	Reason
$x = 180° - 120°$	Adj $\angle$'s on a str line

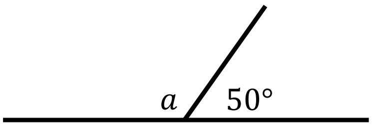
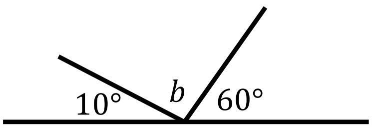
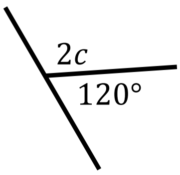
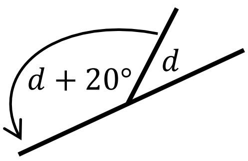


We use these abbreviations to make our lives a little bit easier! *there is a complete summary on page

In geometry we always need to provide **reasons** for 'why' we state something.

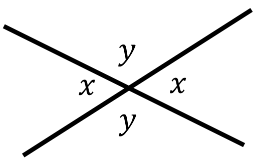
<u>Exercise 2:</u>

Calculate the size of the variables (a, b, c and d). Give a reason for your answer.

		Statement	Reason
(a)			
(b)			
(c)			
(d)			

<u>Vertically opposite angles:</u>

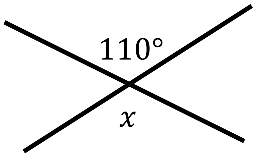
When two straight lines intersect the angles opposite each other are called **vertically opposite angles.**



Vertically opposite angles are equal to each other.

Example:

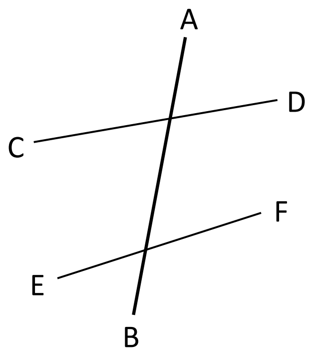
Determine, with reason, the value of x:



Statement	Reason
$x = 110°$	Vert opp ∠'s

Transversals

If a line **cuts or touches** another line, it is called a **transversal**.

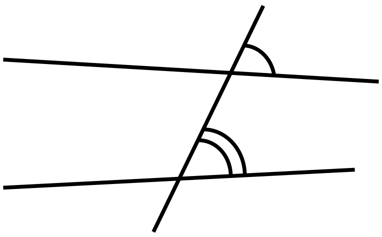


e.g. AB is a transversal because it cuts CD and EF, CD and EF are also transversals of AB.

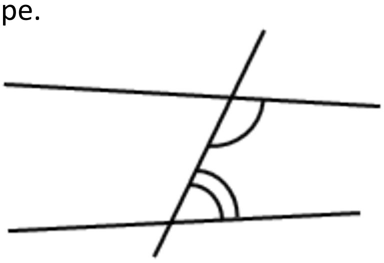
Transversals creates three important types of angles, namely:

1. Corresponding angles
2. Co-interior angles
3. Alternating angles

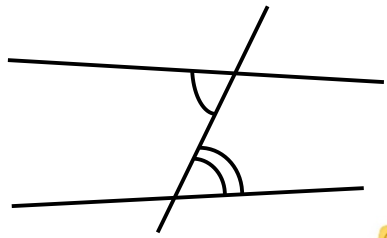
1. Corresponding angles are in the *same position* as each other. They make a F shape:



2. Co-interior angles are *between the lines* and *on the same side* of the transversal. They are "*inside together*". They make a C or U shape.



3. Alternate angles are *between the lines* and on *alternate (opposite) sides* of the transversal. They make a Z or N shape.



Remember the word FUN whenever you see a transversal!

Exercise 3:

Use the diagram below to find:

(a) 10 pairs of corresponding angles

____

____

(b) 8 pairs of vertically opposite angles

____

(c) 4 pairs of co-interior angles

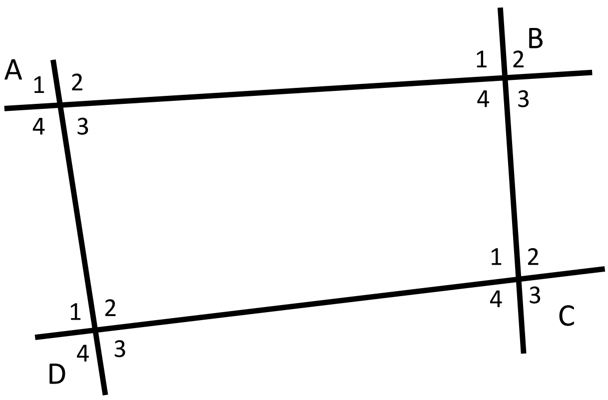
____

(d) 8 pairs of alternate angles

____

(e) 6 pairs of adjacent supplementary angles

____

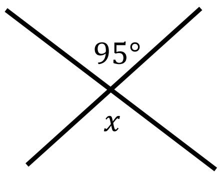
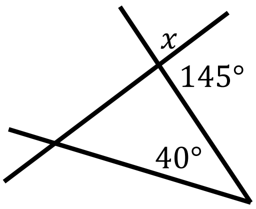
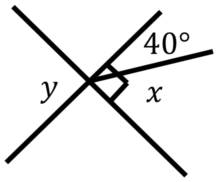
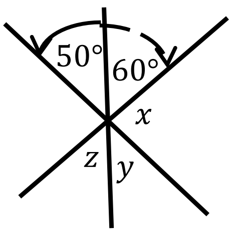
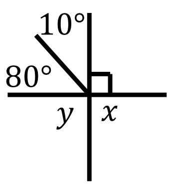


Use the following reasons to help you complete Ex 4 and 5

- Adj $\angle$'s on a str Line
- Adj comp $\angle$'s
- Vert opp $\angle$'s
- $\angle$'s at a pt

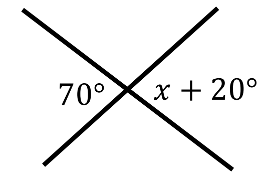
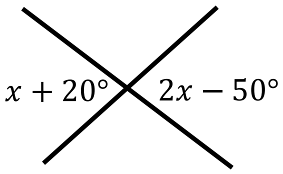
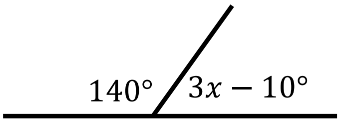
Exercise 4:

Find the value of each variable, in alphabetical order (where there is more than one variable), providing reasons for your statements:

		Statement	Reason
(a)			
(b)			
(c)			
(d)			
(e)			

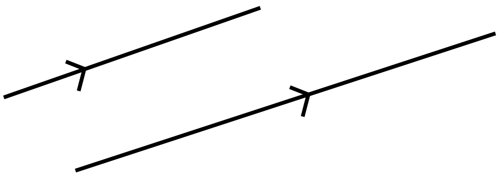
Exercise 5:

Use the diagram to write down an equation, with a reason, in order to calculate the value of x:

		Statement	Reason
(a)			
(b)			
(c)			

Parallel lines

Parallel lines are lines that stay the **same distance apart**, no matter how long the lines are (they are lines that never meet).



Arrows are used to indicate that lines are

NB: You have to mention the parallel lines

If lines are **parallel** then:

- The **corresponding angles are equal**
- The **alternate angles are equal**
- The **co-interior angles are supplementary**

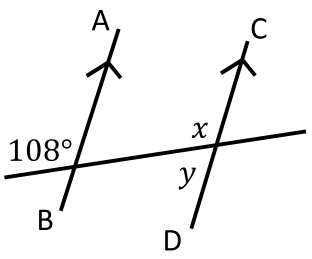
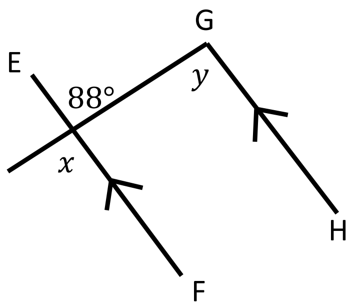
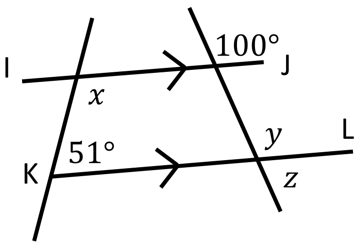
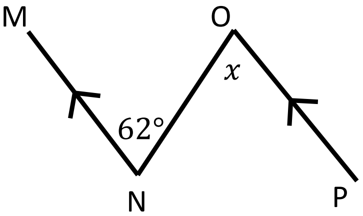
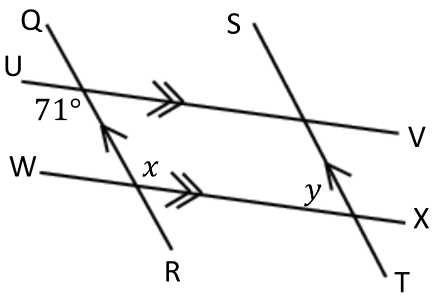
Reasons:
corr $\angle$'s ; ...//...
alt $\angle$'s ; ...//...
co-int $\angle$'s ; ...//...

To prove lines are parallel:
Prove the corresponding angles are equal — corr $\angle$'s =
Prove the alternate angles are equal — alt $\angle$'s =
Prove the co-interior angles are supplementary — co-int $\angle$'s =

Let's see in Exercise 6 how these parallel lines can help us determine the value of unknown angles…

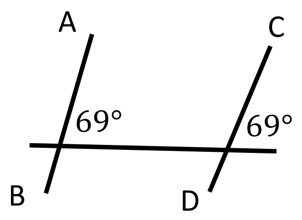
Exercise 6:

(a) Determine the sizes of the angles marked with variables, in alphabetical order. Give reasons for your answers. (The first one is done for you as an example)

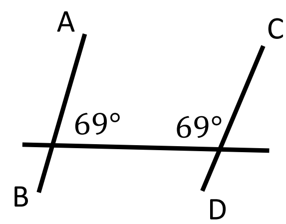
		Statement	Reason
(1)		$x = 108°$ $y = 180 - 108°$ $y = 72°$	Corr $\angle'$s ; AB//CD Adj $\angle'$s on a str line
(2)			
(3)			
(4)			
(5)			

(b) In each case, state whether AB is parallel to CD. Provide reasons for your statements.

(1)

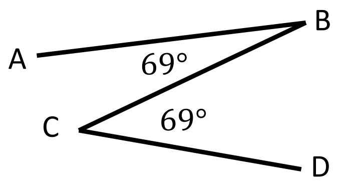


(2)

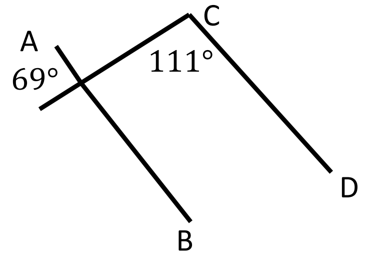


____

(3)



(4)



____

Summary of statements and reasons	
Statement	**Reason**
Angles on a straight line adds up to $180°$	Adj $\angle's$ on a str line
Complementary angles adds up to $90°$	Adj comp $\angle's$
Vertically opposite angles are equal	Vert opp $\angle's$
Angles around a point adds up to $360°$	$\angle's$ at a pt
Corresponding angles of parallel lines are equal	Corr $\angle's$; …//…
Co-interior angles between parallel lines add up to $180°$	Co-int $\angle'$; …//…
Alternating angles of parallel lines are equal	Alt $\angle's$; …//…

**Please note that none of the diagrams in this workbook are drawn according to scale.*

MEMO

Exercise 1:

(a.1) $\hat{A}_1$; $\hat{A}_3$; $\hat{E}_2$; $\hat{D}_1$; $\hat{D}_3$; $\hat{B}_2$ (any five)

(a.2) $E\hat{C}B$ and $E\hat{C}D$

(a.3) $\hat{A}_1$ and $\hat{A}_2$; $\hat{A}_2$ and $\hat{A}_3$; $\hat{A}_3$ and $\hat{A}_4$; $\hat{B}_1$ and $\hat{B}_2$; $E\hat{C}B$ and $E\hat{C}D$; $\hat{D}_1$ and $\hat{D}_2$; $\hat{D}_2$ and $\hat{D}_3$; $\hat{D}_3$ and $\hat{D}_4$
$\hat{A}_1$ and $\hat{A}_4$; $\hat{D}_1$ and $\hat{D}_4$; $\hat{E}_1$ and $\hat{E}_2$

(a.4) $\hat{A}_2$; $\hat{A}_4$; $\hat{E}_1$; $\hat{D}_2$; $\hat{D}_4$; $\hat{B}_1$ (any three)

(b) b: Obtuse
c: Reflex
d: Obtuse
e: Obtuse
f: Right
g: Acute
h: Acute
i: Reflex
j: Obtuse

(c.1) Adjacent

(c.2) Not adjacent (does not share a common point)

(c.3) Not adjacent (does not share a common arm)

(c.4) Adjacent

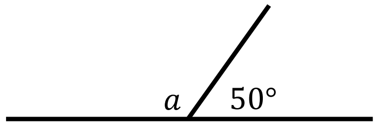
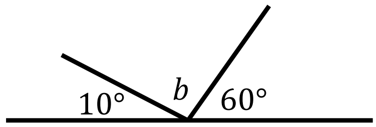
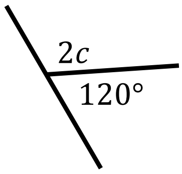
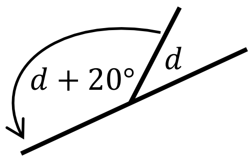
(c.5) Adjacent

(c.6) Not adjacent (does not share a common point)

(d)

Measure of angle	Complement	Supplement
$20°$	$70°$	$160°$
$77°$	$13°$	$103°$
$101°$	No complement	$79°$
$90°$	$0°$	$90°$
$96°$	No complement	$84°$
x	$90° - x$	$180° - x$
y	$90° - y$	$180° - y$

Exercise 2:

		Statement	Reason
(a)		$a = 180° - 150°$ $\therefore a = 130°$	Adj $\angle$'s on a str line
(b)		$b = 180° - 10° - 60°$ $\therefore b = 110°$	Adj $\angle$'s on a str line
(c)		$2c = 180° - 120°$ $2c = 60°$ $c = \frac{60°}{2}$ $\therefore c = 30°$	Adj $\angle$'s on a str line
(d)		$d + 20° + d = 180°$ $2d = 180° - 20°$ $2d = 160°$ $d = \frac{160°}{2}$ $\therefore d = 80°$	Adj $\angle$'s on a str line

Exercise 3:

(a) $\hat{A}_1$ and $\hat{B}_1$; $\hat{A}_2$ and $\hat{B}_2$; $\hat{A}_3$ and $\hat{B}_3$; $\hat{A}_4$ and $\hat{B}_4$; $\hat{A}_1$ and $\hat{D}_1$; $\hat{A}_2$ and $\hat{D}_2$; $\hat{A}_3$ and $\hat{D}_3$; $\hat{A}_4$ and $\hat{D}_4$ $\hat{B}_1$ and $\hat{C}_1$; $\hat{B}_2$ and $\hat{C}_2$; $\hat{B}_3$ and $\hat{C}_3$; $\hat{B}_4$ and $\hat{C}_4$; $\hat{C}_1$ and $\hat{D}_1$; $\hat{C}_2$ and $\hat{D}_2$; $\hat{C}_3$ and $\hat{D}_3$; $\hat{C}_4$ and $\hat{D}_4$
(any ten pairs)

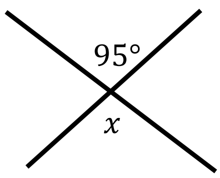
(b) $\hat{A}_1$ and $\hat{A}_3$; $\hat{A}_2$ and $\hat{A}_4$; $\hat{B}_1$ and $\hat{B}_3$; $\hat{B}_2$ and $\hat{B}_4$; $\hat{C}_1$ and $\hat{C}_3$; $\hat{C}_2$ and $\hat{C}_4$; $\hat{D}_1$ and $\hat{D}_3$; $\hat{D}_2$ and $\hat{D}_4$

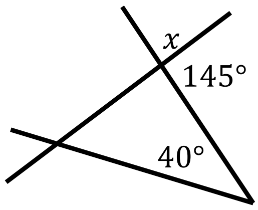
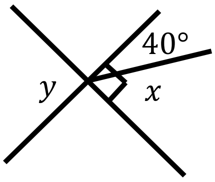
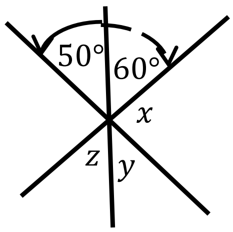
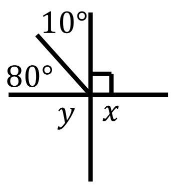
(c) $\hat{A}_3$ and $\hat{D}_2$; $\hat{A}_4$ and $\hat{D}_1$; $\hat{A}_2$ and $\hat{B}_1$; $\hat{B}_4$ and $\hat{C}_1$; $\hat{B}_3$ and $\hat{C}_2$; $\hat{C}_1$ and $\hat{D}_2$; $\hat{C}_4$ and $\hat{D}_3$
(any four)

(d) $\hat{A}_2$ and $\hat{B}_4$; $\hat{A}_4$ and $\hat{D}_2$; $\hat{A}_3$ and $\hat{D}_1$; $\hat{B}_1$ and $\hat{A}_3$; $\hat{B}_4$ and $\hat{C}_2$; $\hat{B}_3$ and $\hat{C}_1$; $\hat{C}_1$ and $\hat{D}_3$; $\hat{C}_4$ and $\hat{D}_2$

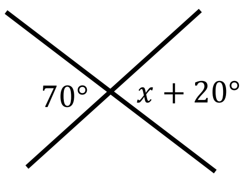
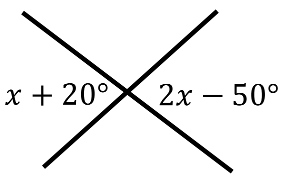
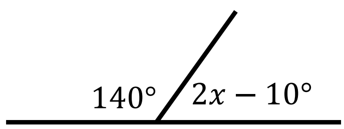
(e) Any two angles that are on a straight line and share the same point.

Exercise 4:

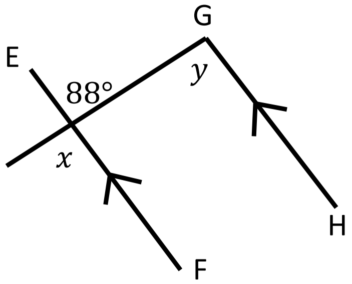
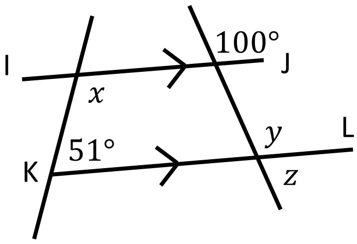
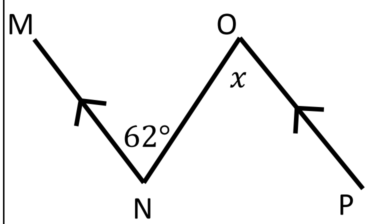
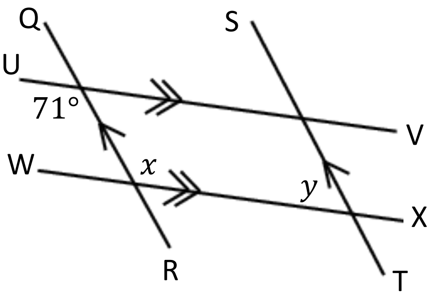
		Statement	Reason
(a)		$x = 95°$	Vert opp $\angle$'s

(b)		$x = 180° - 145°$ $\therefore x = 35°$	Adj $\angle$'s on a str line
(c)		$x = 90° - 40°$ $\therefore x = 50°$ $y = 90°$	Adj comp $\angle$'s
(d)		$x + 50° + 60° = 180°$ $x = 180° - 50° - 60°$ $\therefore x = 70°$ $y = 50°$ $z = 60°$	Adj $\angle$'s on a str line Vert opp $\angle$'s Vert opp $\angle$'s
(e)		$x = 90°$ $y = 90°$	Adj $\angle$'s on a str line Vert opp $\angle$'s

<u>Exercise 5:</u>

		Statement	Reason
(a)		$70° = x + 20°$ $\therefore x = 50°$	Vert opp $\angle$'s
(b)		$x + 20° = 2x - 50°$ $20° + 50° = x$ $70° = x$ $\therefore x = 70°$	Vert opp $\angle$'s
(c)		$2x - 10° + 140° = 180°$ $2x + 130° = 180°$ $2x = 50°$ $x = 25°$	Adj $\angle$'s on a str line

Exercise 6:

		Statement	Reason
(2)		$x = 88°$ $y = 88°$	Vert opp ∠'s Corr ∠'s ; EF // GH
(3)		$x + 51° = 180°$ $\therefore x = 129°$ $y = 100°$ $z = 180° - 100°$ $\therefore z = 80°$	Co-int ∠'s ; IJ // KL Corr ∠'s ; IJ // KL Adj ∠'s on a str line
(4)		$x = 62°$	Alt ∠'s ; MN // OP
(5)		$x = 71°$ $y + 71° = 180°$ $\therefore y = 109°$	Alt ∠'s ; UV // WX Co-int ∠'s ; QR // ST

(b.1) AB // DC because corresponding angles are equal.

(b.2) AB will not be parallel to DC because the co-interior angles are not supplementary.

(b.3) AB // DC because the alternating angles are equal.

(b.4) AB // DC because the co-interior angles will be supplementary.