**72.** ***Overtime Pay*** A carpenter earns \$20 per hour when he works 40 hours or fewer per week, and time-and-a-half for the number of hours he works above 40. Let $x$ denote the number of hours he works in a given week and $y$ (dollars) the corresponding pay.

**(a)** Write a piecewise formula giving $y$ as a function of $x$ where $0 \le x \le 168$.

**(b)** If his pay for the week is \$1070, how many hours did he work?

**73.** ***Telephone Call Cost*** Suppose a telephone company charges 60 cents for a call up to one minute, and 50 cents for each additional minute (or fraction thereof). Let $x$ denote the number of minutes you talk and $y$ (dollars) the corresponding cost. Then $y$ is a function of $x$ given by

$$y = 0.60 + 0.50 Int(x).$$

**(a)** Check several values of $x$ to see that this formula gives what you would expect.

**(b)** Suppose you do not want to spend more than \$5.00 for a call. How long can you talk?
Use a graph in dot mode with a decimal window.

## 2.3 TRANSFORMATIONS OF GRAPHS

> Formal logic is an impoverished way of describing human thought, and the practice of mathematics goes far beyond a set of algorithmic rules.... Mathematics may indeed reflect the operations of the brain, but both brain and mind are far richer in their nature than is suggested by any structure of algorithms and logical operations.
>
> *F. David Peat*

> *[At thirteen] it was hard for me to imagine original mathematics, thinking of something that no one else had thought of before. When I went to college...I thought I might become a biologist. I was interested in many different things. I studied psychology and philosophy, for instance. We didn't have grades, but we did have written evaluations. And I kept getting the message that my true talents didn't lie in subject X but in mathematics.*
>
> William Thurston

Relationships among graphs will be used throughout precalculus and calculus. Whole families of graphs can be related to each other through a few transformations. When we understand the properties of the graph of one particular function $f$, we can immediately get information about domain and range, about intercepts and symmetry, for any function whose graph is the graph of $f$ shifted up or down, right or left, reflected, squeezed or stretched.

Sometimes we work with a family related to one of the *core graphs* shown in the catalog in Figure 2, but more generally we simply ask how the graphs of two functions are related to each other.

### Vertical Shifts

All of the transformations we consider can be justified algebraically. For example, the graph of a function $y = f(x)$ consists of all the points $(x, y)$ whose coordinates satisfy the equation. If $(x, y)$ is on the graph of $y = f(x)$, then the coordinates $(x, y - 1)$ satisfy the equation $y = f(x) - 1$. Each point $(x, y - 1)$ is one unit *below* the point $(x, y)$, so we have an observation that applies to any graph. The graph of $y = f(x) - 1$ is obtained by *shifting the graph of* $y = f(x)$ *down 1 unit*.

The same argument applies for any positive number $c$.

**Vertical shifts, $c > 0$**

From the graph of $y = f(x)$, the graph of

$$y = f(x) + c \text{ is } \textit{shifted up } c \textit{ units},$$
$$y = f(x) - c \text{ is } \textit{shifted down } c \textit{ units}.$$

Although we can give an argument to explain the effect of each transformation, we are more concerned with having you do enough examples to see for yourself what happens for each kind of transformation we examine. Accordingly, we will show lots of graphs, but for your benefit, we strongly encourage you to use your graphing calculator to draw each graph yourself.

The first example asks for graphs of vertical shifts of two core graphs. While it is good practice to graph such graphs on your calculator, you should be able to draw these graphs without technology. Look at the equation, recognize the graph as a vertical shift, and make a rough sketch.

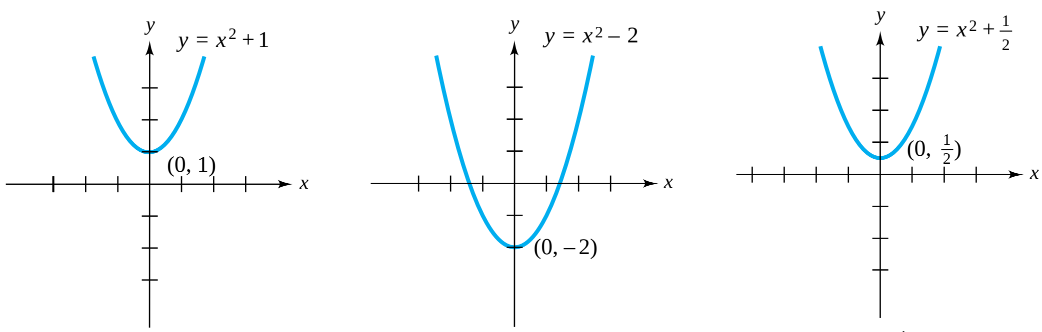
**▶*EXAMPLE 1*** ***Vertical shifts*** Identify the function as a vertical shift of a core graph (Figure 2 in Section 2.2) and sketch.

**(a)** $y = x^2 + 1$, $y = x^2 - 2$, $y = x^2 + \frac{1}{2}$
**(b)** $y = |x| - 2$, $y = |x| + 1$, $y = |x| - \frac{2}{3}$

***Solution***

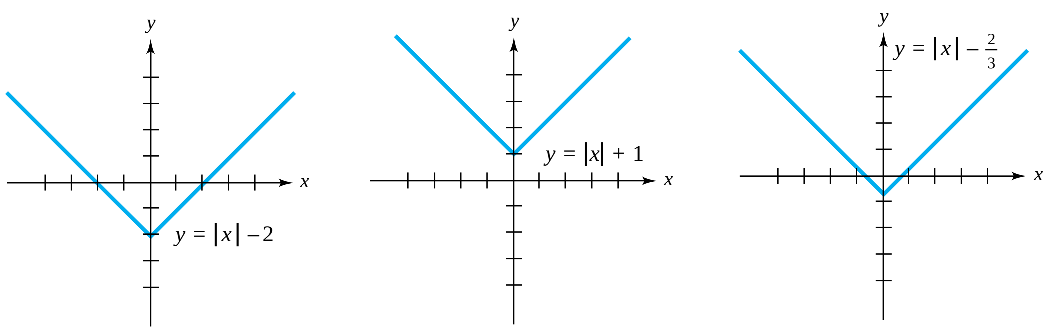
**(a)** Each graph is a vertical translation of the core parabola of Figure 2b. The first is shifted 1 unit up, the second 2 units down, and the third is $\frac{1}{2}$ up. The three graphs are labeled in Figure 9.

**(b)** Each is a vertical shift of the core absolute value graph of Figure 2d. The absolute value graphs are shown in Figure 10. ◀




**(a)** Up 1 unit **(b)** Down 2 units **(c)** Up $\frac{1}{2}$ unit

**FIGURE 9**
Vertical shifts of $f(x) = x^2$




**(a)** Down 2 units **(b)** Up 1 unit **(c)** Down $\frac{2}{3}$ unit

**FIGURE 10**
Vertical shifts of $f(x) = |x|$

## Horizontal Shifts

Some operations are applied to the "outside" of a function. For example,

$$y = f(x) + 3,\ y = -f(x),\ y = |f(x)|.$$

The effect of such operations is to change the graph *vertically*. Other operations apply to the "inside" of the function, as

$$y = f(x + 3),\ y = f(-x),\ y = f(|x|).$$

In the equation $y = f(u)$, $u$ is called the **argument** of the function. In contrast to operations that affect a graph vertically, we have the following useful observation.

> ***"Outside-inside operations"***
>
> Operations applied to the "outside" of a function affect the *vertical* aspects of the graph.
>
> Operations applied to the "inside" (argument) of a function affect the *horizontal* aspects of the graph.

**►EXAMPLE 2** ***Horizontal shifts*** Sketch graphs of

**(a)** $y = (x - 2)^2$, $y = (x + \frac{1}{2})^2$, $y = x^2 - 2x + 1$

**(b)** $y = \sqrt{x + 1}$, $y = \sqrt{x - \frac{2}{3}}$, $y = \sqrt{x + 3}$

### Solution

**(a)** The first two are obviously horizontal shifts of the core parabola $y = x^2$, 2 units right and $\frac{1}{2}$ unit left, respectively. For the third function, we must recognize that $x^2 - 2x + 1 = (x - 1)^2$, and so shift the graph of $y = x^2$ right 1 unit. We have the three graphs labeled in Figure 11. We should note that the calculator will provide the same graph, whether written $y = x^2 - 2x + 1$ or $y = (x - 1)^2$, and we might recognize the graph as a shifted parabola only after seeing the graph.

**(b)** Be careful with parentheses; note the difference between Y = √X + 1 (a vertical shift), and Y = √(X + 1) (a horizontal shift). Each graph in this part is a horizontal shift of the core square root function $y = \sqrt{x}$. See Figure 12. ◄

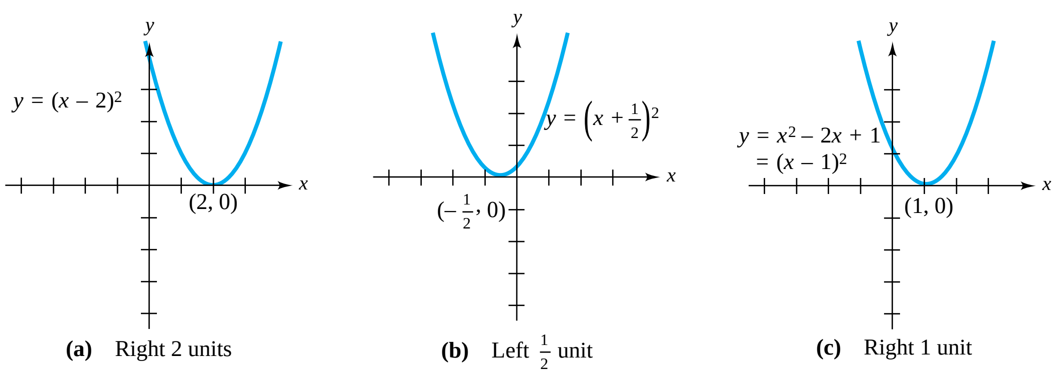
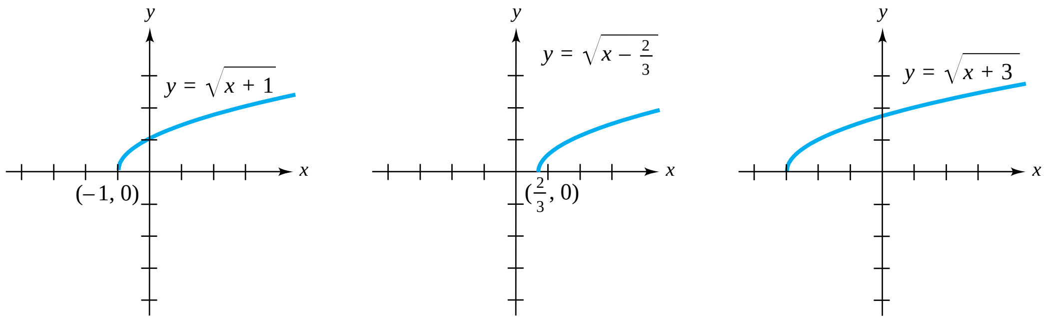
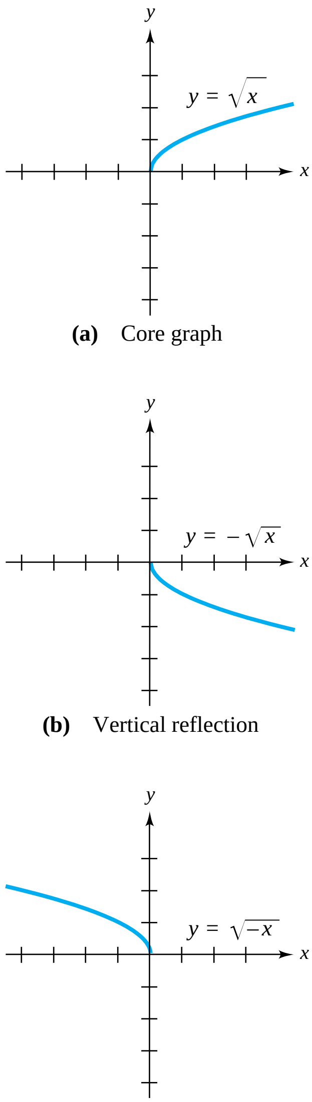



**FIGURE 11**
Horizontal shifts of $f(x) = x^2$




**(a)** Left 1 unit **(b)** Right $\frac{2}{3}$ unit **(c)** Left 3 units

**FIGURE 12**
Horizontal shifts of $f(x) = \sqrt{x}$




**(a)** Core graph
**(b)** Vertical reflection
**(c)** Horizontal reflection

**FIGURE 13**
Reflections of $f(x) = \sqrt{x}$

There are some important observations we must make in looking at the horizontal shifts in Example 2. While the graph of $y = x^2 - 2$ shifts *down* from the graph of $y = x^2$, the graph of $y = (x - 2)^2$ is shifted to the *right*, the opposite direction from what some people expect. It may help to remember that the low point on the parabola $y = x^2$ occurs when $x = 0$, and on the parabola $y = (x - 2)^2$, $y = 0$ when $x = 2$. However you choose to remember the relationships, we have the following.

**Horizontal shifts, $c > 0$**

From the graph of $y = f(x)$, the graph of

$y = f(x + c)$ is *shifted left c units,*
$y = f(x - c)$ is *shifted right c units.*

## Reflections

Comparing the graphs of $y = f(x)$ and $y = -f(x)$, it is clear that for any point $(x, y)$ on the graph of $y = f(x)$, the point $(x, -y)$ belongs to the graph of $y = -f(x)$. That is, the graph of $y = -f(x)$ is obtained from the graph of $y = f(x)$ by "tipping it upside down," or, in more mathematical terms, "reflecting in the $x$-axis." Since multiplying a function by $-1$ reflects the graph vertically, we would expect multiplication of the argument by $-1$ to reflect the graph horizontally, as the next example shows.

**▶EXAMPLE 3** ***Horizontal and vertical reflections*** Sketch graphs of

$$y = \sqrt{x}, \qquad y = -\sqrt{x}, \qquad y = \sqrt{-x}.$$

### Solution

With a graphing calculator we see essentially the graphs shown in Figure 13. The graph of $y = \sqrt{x}$ is the top half of a parabola. More important at the moment are the relations with the other graphs. From the graph $y = \sqrt{x}$, the graph of $y = -\sqrt{x}$ is a reflection in the $x$-axis, while the graph of $y = \sqrt{-x}$ is a reflection in the $y$-axis, as expected. ◀

**Horizontal and vertical reflections**

From the graph of $y = f(x)$, the graph of

$y = -f(x)$ is *a vertical reflection (in the x-axis),*
$y = f(-x)$ is *a horizontal reflection (in the y-axis).*

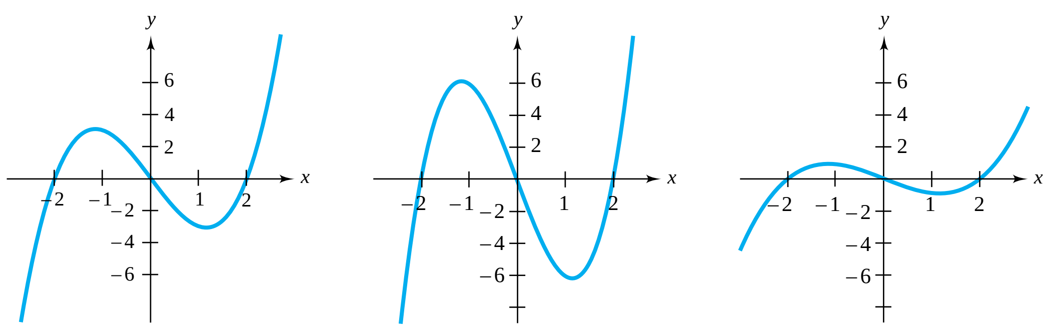
## Dilations: Stretching and Compressing Graphs

Multiplying a function by a constant greater than 1 has the effect of *stretching the graph vertically:* if the point $(x, y)$ belongs to the graph of $y = f(x)$, then the point $(x, cy)$ is on the graph of $y = cf(x)$. If the positive constant $c$ is smaller than 1, then the number $cy$ is smaller than $y$, so the graph of $y = cf(x)$ is a *vertical compression toward the x-axis.* In a similar fashion, it can be seen that multiplying the argument has the effect of compressing or stretching the graph horizontally, toward the $y$-axis. A stretching or compression is called a **dilation** of the graph.

**▶EXAMPLE 4** ***Vertical dilations*** For the function $f(x) = x^3 - 4x$, describe how the graphs of $y = 2f(x)$ and $y = 0.3f(x)$ are related to the graph of $y = f(x)$.

### Solution

Using a graphing calculator for $y = x^3 - 4x$, we get the graph shown in Figure 14(a), with $x$-intercept points where $x = -2, 0, 2$. Tracing along the curve, we see that the left "hump" is just a little higher than 3, where $x \approx -1.2$, and the low point is located symmetrically through the origin (the graph is clearly the graph of an odd function). For the graphs of the other two, the shape is similar, and the $x$-intercept points are the same, but the graph of $y = 2(x^3 - 4x)$ rises to a left hump well above 6, and the low point is below $-6$, *twice as far away from the x-axis* as the graph of $y = f(x)$. The graph of $y = 0.3(x^3 - 4x)$ is "squashed" vertically toward the $x$-axis, and the high and low points we see are less than 1 unit away from the axis. ◀




**(a)** $y = (x^3 - 4x)$ **(b)** $y = 2(x^3 - 4x)$ **(c)** $y = (0.3)(x^3 - 4x)$

**FIGURE 14**
Vertical dilations of $f(x) = x^3 - 4x$

**▶EXAMPLE 5** ***Horizontal dilations*** For the function $f(x) = x^3 - 4x$, sketch graphs of $y = f(x)$, $y = f(2x)$, and $y = f(0.5x)$.

### Solution

The graph of $y = f(x)$ is the one from the previous example, and is shown again in Figure 15(a). For the graph of $y = f(2x)$, we must replace each $x$ by $2x$, so we enter `Y = (2X)^3 - 4(2X)`, and similarly for $y = f(0.5x)$.

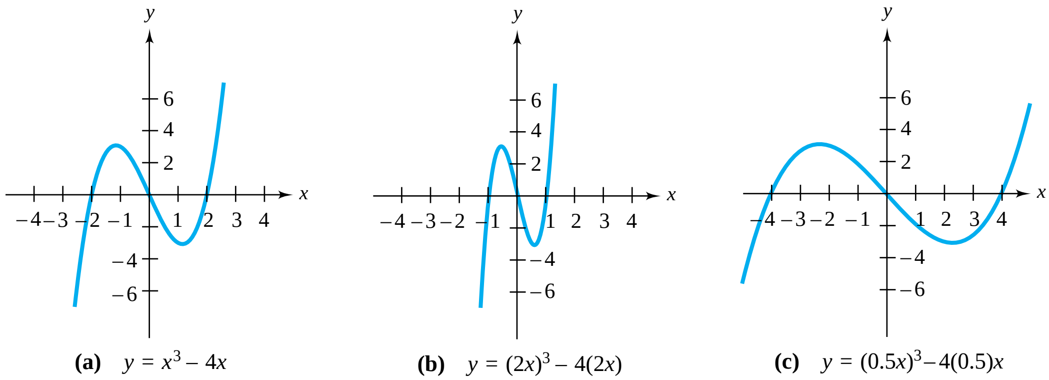



**FIGURE 15**
Horizontal dilations of $f(x) = x^3 - 4x$

The graph of $y = f(2x)$ has the same vertical rise and fall to the turning points, but the $x$-intercepts have been squeezed together; each is *twice as close to the origin* as for $y = f(x)$. The graph of $y = f(0.5x)$ is stretched horizontally. The $x$-intercept point that was at (2, 0) has been moved outward to where $x = 2/0.5 = 4$; the $x$-intercept points are $(\pm 4, 0)$. ◀

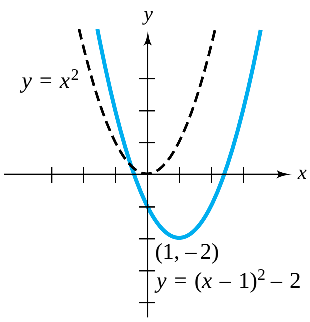



**FIGURE 16**
Translation of $f(x) = x^2$

***Dilations, $c > 0$***

From the graph of $y = f(x)$, the graph of

$y = cf(x)$ is a *vertical stretch if $c > 1$ (by a factor of $c$), vertical compression if $c < 1$ (by a factor of $c$);*
$y = f(cx)$ is a *horizontal compression if $c > 1$, horizontal stretch if $c < 1$.*

## Combining Transformations

All the transformations we have considered can be combined, and if we are careful, we can predict the effect of several transformations on a graph of a function. In most instances, we take the operations "from the inside out," looking first at anything that affects the argument of the function.

***▶EXAMPLE 6 Vertical and horizontal shifts*** Predict the effect on the graph of the function $f(x) = x^2$ in graphing

**(a)** $y = f(x - 1) - 2$ **(b)** $y = -f(x + 2)$.

Then check your prediction with a calculator graph.

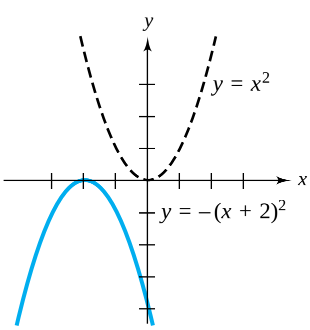



**FIGURE 17**
Translation and reflection of $f(x) = x^2$

***Solution***

**(a)** $y = f(x - 1) - 2 = (x - 1)^2 - 2$. From a parabola $y = x^2$, the graph of $y = (x - 1)^2$ is a shift 1 unit right. Then for $y = (x - 1)^2 - 2$, shift the graph down 2 units. The result is a parabola whose low point is at $(1, -2)$. A calculator graph shows the solid parabola in Figure 16.

**(b)** The graph of $y = (x + 2)^2$ is a parabola shifted 2 units left. Then multiplying by $-1$ reflects the graph in the $x$-axis, tipping it upside down. We have the solid parabola opening downward in Figure 17. ◀

**▶EXAMPLE 7** ***Identifying transformed graphs*** The graph of a function $f$ is given, together with three transformed graphs. Describe the transformations needed to get the given graph and write an equation for the function whose graph is shown. Check by graphing your function.

**(a)** $f(x) = |x| = abs(x)$ (Figure 18) **(b)** $f(x) = \sqrt{x}$ (Figure 19)

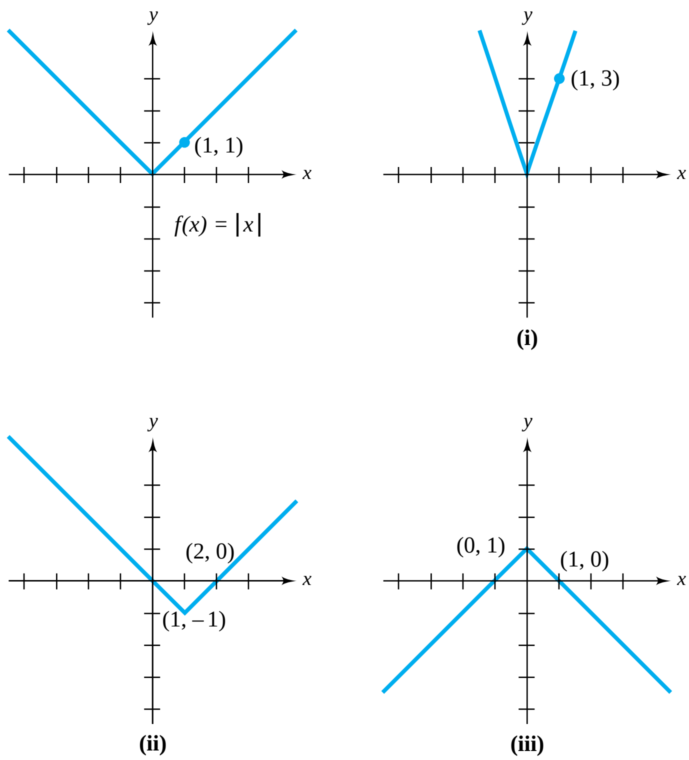



**FIGURE 18**
Graphs for Example 7a

## Solution

**(a)** The graph in Figure 18(i) is a vertical stretch by a factor of 3, since (1, 1) is sent to (1, 3), so an equation for the transformed graph is $y = 3|x|$. If we use a decimal window, we can trace on the graph of $y = 3|x|$ to see that (1, 3) is on our graph, as desired. We note that in this instance, we could just as easily have obtained the transformed graph by compressing toward the $y$-axis, for which an equation would be $y = |3x|$. Since $3|x| = |3x|$, the function can be described either way.

For the graph in Figure 18(ii), the absolute value graph is shifted 1 unit right (replace the argument $x$ by $x - 1$), and 1 unit down. An equation is $y = |x - 1| - 1$, which we graph to check.

In Figure 18(iii) the graph is tipped upside down (reflected in the $x$-axis) and shifted up 1 unit. An equation is $y = -|x| + 1$.

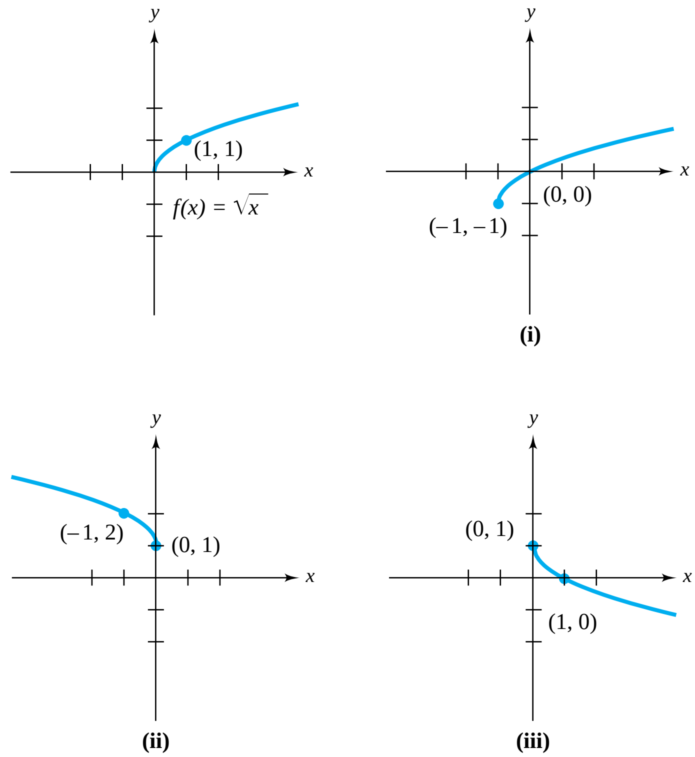



**FIGURE 19**
Graphs for Example 7b

**(b)** For panel (i) in Figure 19, shift the graph of $y = \sqrt{x}$ left 1 unit and down 1 unit, so an equation is $y = \sqrt{x + 1} - 1$. We trace on the graph to verify the location of the given points.

For the graph in panel (ii), reflect the graph of $y = \sqrt{x}$ in the $y$-axis, (replace $x$ by $-x$), and shift up 1 unit. An equation is $y = \sqrt{-x} + 1$.

For the third panel, reflect the graph of $y = \sqrt{x}$ in the $x$-axis and shift up 1 unit, so an equation is $y = -\sqrt{x} + 1$. Verify by graphing this equation. ◀

## Summary of Basic Transformations

We list here the basic transformations we have introduced in this section.

***Basic transformations of the graph of $y = f(x)$, $c > 0$***

The transformations that affect a graph vertically are applied "outside" the function; transformations that change horizontal aspects are applied "inside" the function (to the argument).

| *Vertical* | | *Horizontal* | |
|---|---|---|---|
| $y = f(x) + c$, | shift up | $y = f(x + c)$, | shift left |
| $y = f(x) - c$, | shift down | $y = f(x - c)$, | shift right |
| $y = -f(x)$, | reflect in $x$-axis | $y = f(-x)$, | reflect in $y$-axis |
| $y = cf(x)$, | dilate vertically | $y = f(cx)$, | dilate horizontally |

## EXERCISES 2.3

### Check Your Understanding

*Exercises 1–5 True or False. Give reasons.*

1. If the function $f$ has a positive zero and $g(x) = f(x - 2)$, then $g$ must have a positive zero.
2. If the function $f$ has a zero between 1 and 2 and $g(x) = f(x + 2)$, then $g$ must have a negative zero.
3. If the graphs of $y = f(x)$ and $y = g(x)$ intersect in Quadrants I and III, then the graphs of $y = f(-x)$ and $y = g(-x)$ must intersect in Quadrants II and IV.
4. If the graphs of $y = f(x)$ and $y = g(x)$ intersect in Quadrants II and IV, then the graphs of $y = -f(x)$ and $y = -g(x)$ must intersect in Quadrants I and III.
5. If the graph of $y = f(x)$ contains points in Quadrants III and IV, then the graph of $y = f(x) - 2$ must also contain points in Quadrants III and IV.

*Exercises 6–10 Fill in the blank so that the resulting statement is true. If calculator graphs of f and g are drawn using $[-10, 10] \times [-10, 10]$, then the display will show the graphs intersecting in Quadrant(s) ______.*

6. $f(x) = x^2 - 2x - 7$, $g(x) = -f(x) - 5$
7. $f(x) = x^2 - 4x - 4$, $g(x) = f(x - 4)$
8. $f(x) = x^2 - 2|x| - 3$, $g(x) = -f(x) + 3$
9. $f(x) = 2x - 5$, $g(x) = f(-x) + 15$
10. $f(x) = |x| - 2$, $g(x) = -f(x) + 3$

### Develop Mastery

*Exercises 1–6* ***Related Graphs*** *The graph of a function f contains the points $P(-2, 4)$ and $Q(4, -5)$. Give the coordinates of two points on the graph of the function* ***(a)*** *g,* ***(b)*** *h.*

1. $g(x) = f(x - 1)$; $h(x) = f(x + 2)$
2. $g(x) = f(x) - 3$; $h(x) = f(x) + 4$
3. $g(x) = f(2x)$; $h(x) = f(0.5x)$
4. $g(x) = -f(x)$; $h(x) = f(-x)$
5. $g(x) = f(x - 2) + 3$; $h(x) = 4 - f(x)$
6. $g(x) = -f(-x)$; $h(x) = 1 + f(-x)$

*Exercises 7–12* ***Related Graphs*** *For the function $f(x) = x^2 - x - 2$*

***(a)*** *Determine a formula for g and simplify.*
***(b)*** *Draw calculator graphs of f and g on the same screen.*
***(c)*** *Write a brief statement describing how the graphs of f and g are related.*

7. $g(x) = f(x + 2)$
8. $g(x) = f(-x)$
9. $g(x) = f(x - 3)$
10. $g(x) = f(x) + 2$
11. $g(x) = f(-x) + 2$
12. $g(x) = -f(x + 3)$

*Exercises 13–14* ***Related Line Graphs*** *The graph of f is a line through points P and Q. Draw graphs of the given function. It is not necessary to find an equation for any of the functions before drawing a graph.*
***(a)*** $y = f(x)$ ***(b)*** $y = f(x - 2)$ ***(c)*** $y = f(-x)$.

13. $P(1, -3)$ and $Q(3, 1)$
14. $P(-3, 4)$ and $Q(1, -2)$

*Exercises 15–16* ***Line Segment Graphs*** *The graph of f is a line segment joining P and Q. Draw a graph and give the domain and range of*
***(a)*** $y = f(x)$ ***(b)*** $y = f(x - 3)$ ***(c)*** $y = -f(x)$.

15. $P(2, -2)$ and $Q(4, 2)$
16. $P(-2, 3)$ and $Q(0, -3)$

*Exercises 17–18* ***Line Segment Graphs*** *The graph of f is two line segments PQ and QR. Draw a graph and give the domain and range of* ***(a)*** $y = f(x + 2)$ ***(b)*** $y = f(-x)$ ***(c)*** $y = -f(x)$.

17. $P(-3, -1)$, $Q(-2, 2)$, and $R(3, 0)$
18. $P(-4, -2)$, $Q(-1, 3)$, and $R(4, -1)$

*Exercises 19–22* ***Verbal Description*** *Give a verbal description of how you would draw a graph of g from the graph of f. Check by drawing the graphs.*

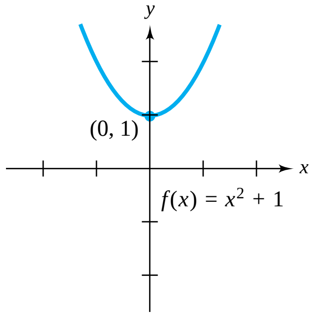
19. $f(x) = x^2 + 1$, $g(x) = (x + 2)^2 - 1$
20. $f(x) = \sqrt{x}$, $g(x) = -\sqrt{x + 1}$
21. $f(x) = x^2 - 3x$, $g(x) = 2(3x - x^2) + 1$
22. $f(x) = |x|$, $g(x) = |0.5x| + 2$

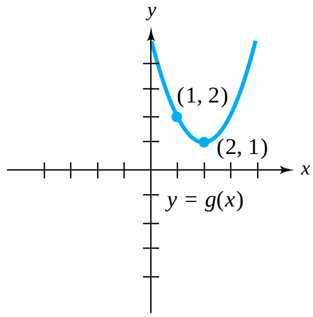
*Exercises 23–26* ***Verbal to Formula*** *A verbal description of transformations of the graph of $y = f(x)$ is given, resulting in a graph of function g. Give a formula that describes the function g. Confirm by drawing graphs of f and g on the same screen.*

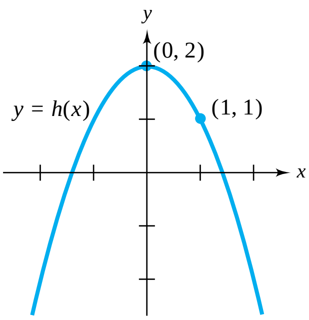
23. $f(x) = x^2 - 2x$. Translate the graph of $f$ to the left 2 units and then reflect about the $x$-axis.
24. $f(x) = 2x - 4$. Translate the graph of $f$ to the left 3 units and then reflect about the $y$-axis.
25. $f(x) = x^2 + 1$. Stretch the graph of $f$ vertically upward by a factor of 2, then translate downward 3 units.
26. $f(x) = x^2 + 1$. Compress the graph of $f$ vertically downward by a factor of 0.5, then reflect about the $y$-axis.

*Exercises 27–30* ***Graph to Verbal and Formula*** *The graph of function f is shown along with graphs of transformed functions g and h.* ***(a)*** *Give a verbal description of the transformations that will give the graphs of g and h from the graph of f.* ***(b)*** *Give a formula for g. Do the same for h.* ***(c)*** *As a check, draw graphs of your formulas in part* ***(b)*** *and see if they agree with the given graphs.*

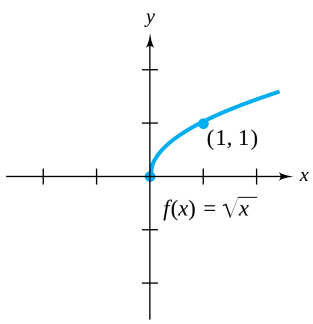
**27.**

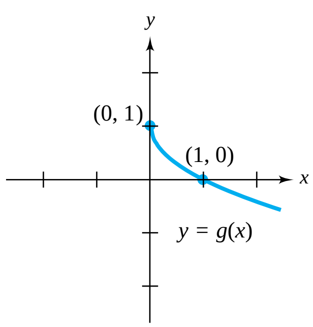

y
x
(0, 1)
f(x) = x2 + 1

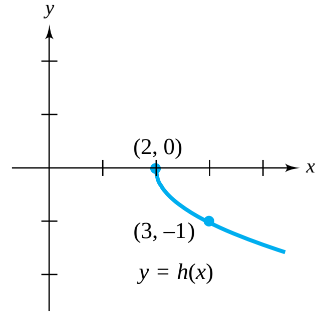

y
x
(1, 2)
(2, 1)
y = g(x)


y
x
(0, 2)
y = h(x)
(1, 1)

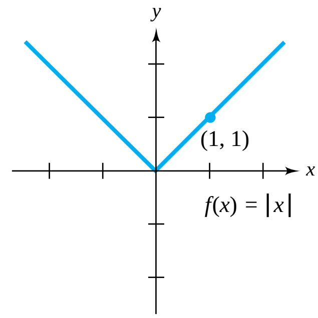
**28.**

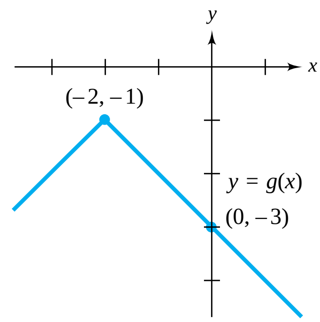

y
x
(1, 1)
f(x) = √x

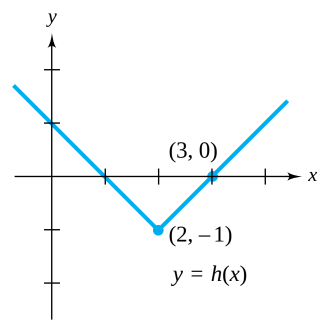

y
x
(0, 1)
(1, 0)
y = g(x)


y
x
(2, 0)
(3, –1)
y = h(x)

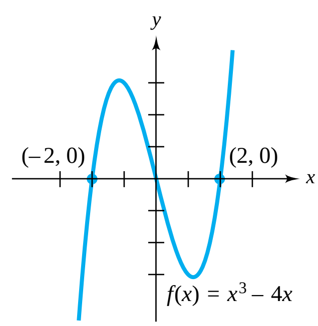
**29.**

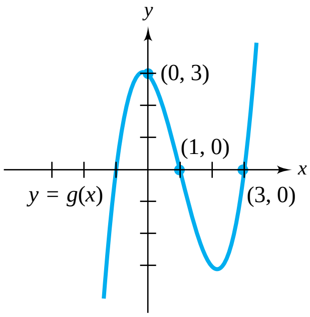

y
x
(1, 1)
f(x) = |x|

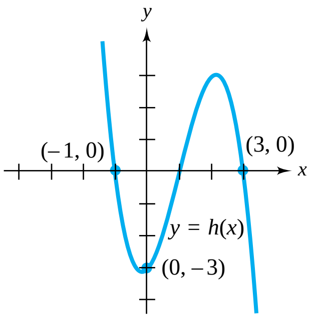

y
x
(–2, –1)
y = g(x)
(0, –3)


y
x
(3, 0)
(2, –1)
y = h(x)

**30.**


y
x
(–2, 0)
(2, 0)
f(x) = x3 – 4x




*Exercises 31–34* ***Domain and Range*** *The domain D and range R of function f are given in interval notation. Give the domain and range of the function **(a)** g, **(b)** h.*

**31.** $D = [-2, 6]$, $R = [4, 8]$;
$g(x) = f(x - 1)$, $h(x) = f(x + 2)$

**32.** $D = (-4, 5)$, $R = [-3, 5]$;
$g(x) = f(x) + 2$, $h(x) = f(x) - 3$

**33.** $D = (-\infty, 4]$, $R = [-4, 6]$;
$g(x) = -f(x)$, $h(x) = f(-x)$

**34.** $D = [-2, \infty)$, $R = (-4, 8)$;
$g(x) = -f(x)$, $h(x) = f(-x)$

*Exercises 35–38* ***Related Domain and Range*** *The domain D and range R of function g are given. Find the domain and range of function f.*

**35.** $g(x) = f(x + 2)$; $D = [-2, 4]$, $R = [-3, 5]$

**36.** $g(x) = f(x - 1) + 3$; $D = (-\infty, 4]$, $R = [-2, \infty)$

**37.** $g(x) = f(2x)$; $D = [3, 6]$, $R = [-1, 4]$

**38.** $g(x) = f(-x)$; $D = [-2, 8]$, $R = [4, \infty]$

*Exercises 39–42* ***Related Intercept Points*** ***(a)*** *Determine the coordinates of the x-intercept points of the graph of function f.* ***(b)*** *Find the x-intercept point(s) of the graph of function g.* ***(c)*** *What is the y-intercept point for g?* ***(d)*** *Draw graphs as a check.*

**39.** $f(x) = x^2 - 3x - 4$; $g(x) = f(x - 3)$

**40.** $f(x) = 4 + 3x - x^2$; $g(x) = f(x + 2)$

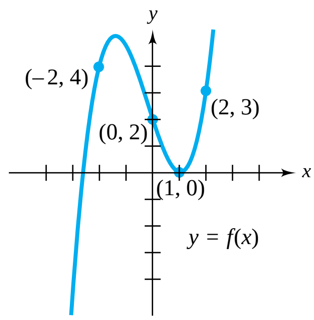
**41.** $f(x) = x^2 - 4$; $g(x) = f(2x)$

**42.** $f(x) = x^2 - 3x - 4$; $g(x) = f(2x)$

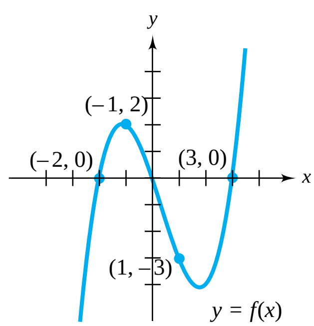
**43.** ***Explore*** For each number $k$ draw a calculator graph of $y = k\sqrt{4 - x^2}$ on the same screen. From what you observe write a brief paragraph comparing the graphs for different values of $k$.
**(a)** $k = 1$ **(b)** $k = 2$ **(c)** $k = 0.5$

**44.** Follow instructions of Exercise 43 for $y = k\sqrt{16 - x^2}$.

**45.** Use the graph of $f$ shown to sketch the graph of $y = -f(x - 1)$. Label the coordinates of four points that must be on your graph.




**46.** Use the graph of $f$ shown to sketch the graph of $y = f(-x) + 1$. Label the coordinates of four points that must be on your graph.




**47.** ***Points on Related Graphs*** Points $P(-4, 3)$ and $Q(2.4, 5.6)$ are on the graph of $y = f(x - 1)$. Give the coordinates of two points that must be on the graph of **(a)** $y = f(x)$ **(b)** $y = f(x) + 3$.