

Chapter 11: Hypothesis Testing and the Wald Test

Chapter 11 Outline

- **No Money Illusion Theory: Taking Stock**
- **No Money Illusion Theory: Calculating Prob[Results IF H_0 True]**
 - Clever Algebraic Manipulation
 - Wald Test
 - Restricted Regression Reflects H_0
 - Unrestricted Regression Reflects H_1
 - Comparing the Restricted Sum of Squared Residuals and the Unrestricted Sum of Squared Residuals: The F -Statistic
 - Let Statistical Software Do the Work
- **Testing the Significance of the “Entire” Model**
- **Equivalence of Two-Tailed t -Tests and Wald Tests (F -Tests)**
 - Two-Tailed t -Test
 - Wald Test
- **Three Important Distributions: Normal, Student- t , and F**

Chapter 11 Prep Questions

1. Consider the log form of the constant elasticity demand model:
$$\log(Q_t) = \log(\beta_{Const}) + \beta_P \log(P_t) + \beta_I \log(I_t) + \beta_{CP} \log(ChickP_t) + e_t$$
Show that if $\beta_P + \beta_I + \beta_{CP} = 0$, then
$$\log(Q_t) = \log(\beta_{Const}) + \beta_P [\log(P_t) - \log(ChickP_t)] + \beta_I [\log(I_t) - \log(ChickP_t)] + e_t$$
Hint: If the coefficients sum to 0, solve for β_{CP} in terms β_P and β_I ; then, substitute this expression for β_{CP} into the log form of the constant elasticity demand model.
2. Review how the ordinary least squares (OLS) estimation procedure determines the value of the parameter estimates. What criterion does this procedure use to determine the value of the parameter estimates?
3. Recall that the presence of a random variable brings forth both bad news and good news.
 - a. What is the bad news?
 - b. What is the good news?
4. Focus on our beef consumption data:
Beef Consumption Data: Monthly time series data of beef consumption, beef prices, income, and chicken prices from 1985 and 1986.
 Q_t Quantity of beef demanded in month t (millions of pounds)
 P_t Price of beef in month t (cents per pound)

I_t Disposable income in month t (billions of chained 1985 dollars)

$ChickP_t$ Price of chicken in month t (cents per pound)

Consider the log form of the constant elasticity demand model:

$$\text{Model: } \log(Q_t) = \log(\beta_{\text{Const}}) + \beta_P \log(P_t) + \beta_I \log(I_t) + \beta_{CP} \log(ChickP_t) + e_t$$

- a. Use the ordinary least squares (OLS) estimation procedure to estimate the parameters of the constant elasticity demand model.

[Link to MIT-BeefDemand-1985-1986.wf1 goes here.]

- 1) What does the sum of squared residuals equal?
- 2) What is the ordinary least squares (OLS) estimate for
 - i) β_P ?
 - ii) β_I ?
 - iii) β_{CP} ?
 - iv) $\log(\beta_{\text{Const}})$?

What criterion do these estimates satisfy?

- b. Now, consider a different restriction. Restrict the value of β_{CP} to 0.
- 1) Incorporate this restriction into the constant elasticity demand model. What is the equation describing the restriction model?
 - 2) We wish to use the ordinary least squares (OLS) estimation procedure to estimate the remaining parameters.
 - i) What dependent variable should we use?
 - ii) What explanatory variables should we use?
 - iii) Run the regression.
 - iv) What does the sum of squared residuals equal?
 - 3) Compared to unrestricted regression, part a, has the sum of squared residuals risen or fallen? Explain why.
- c. Now, restrict the value of β_{CP} to 1.
- 1) Incorporate this restriction into the constant elasticity demand model. What is the equation describing the restriction model?
 - 2) We wish to use the ordinary least squares (OLS) estimation procedure to estimate the remaining parameters.
 - i) What dependent variable should we use?
 - ii) What explanatory variables should we use?
 - iii) Run the regression.
 - iv) What does the sum of squared residuals equal?

- 3) Compared to unrestricted regression, part a, has the sum of squared residuals risen or fallen? Explain why.

No Money Illusion Theory: Taking Stock

The money illusion theory contends that whenever all prices and income change by the same proportion the quantity demanded is unaffected. In terms of elasticities, this means that a good's elasticities (own price, income, and cross price) sum to 0. Let us briefly review the steps that we undertook in the last chapter to assess this theory.

Project: Assess the no money illusion theory.

Since the linear demand model is intrinsically inconsistent with the no money illusion theory, we cannot use it to assess the theory. The constant elasticity demand model can be used, however:

Constant Elasticity Demand Model: $Q = \beta_{const} P^{\beta_p} I^{\beta_I} ChickP^{\beta_{CP}}$

β_p = (Own) Price Elasticity of Demand

β_I = Income Elasticity of Demand

β_{CP} = Cross Price Elasticity of Demand

When the elasticities sum to 0, no money illusion exists:

No Money Illusion Theory: The elasticities sum to 0: $\beta_p + \beta_I + \beta_{CP} = 0$.

Next, we converted the constant elasticity demand model into a linear relationship by taking natural logarithms:

$$\log(Q_t) = \log(\beta_{const}) + \beta_p \log(P_t) + \beta_I \log(I_t) + \beta_{CP} \log(ChickP_t) + e_t$$

We then used the ordinary least squares (OLS) estimation procedure to estimate the elasticities:

[Link to MIT-BeefDemand-1985-1986.wf1 goes here.]

Ordinary Least Squares (OLS)

Dependent Variable: $\text{Log}Q$

Explanatory Variable(s):	Estimate	SE	t-Statistic	Prob
$\text{Log}P$	-0.411812	0.093532	-4.402905	0.0003
$\text{Log}I$	0.508061	0.266583	1.905829	0.0711
$\text{LogChick}P$	0.124724	0.071415	1.746465	0.0961
$Const$	9.499258	2.348619	4.044615	0.0006

Number of Observations 24 Degrees of Freedom 20

Estimated Equation: $\text{EstLog}Q = 9.50 - .41\text{Log}P + .51\text{Log}I + .12\text{LogChick}$

Interpretation of Estimates:

$b_p = -.41$: (Own) Price Elasticity of Demand = $-.41$

$b_I = .51$: Income Elasticity of Demand = $.51$

$b_{\text{Chick}P} = .12$: Cross Price Elasticity of Demand = $.12$

Critical Result: Sum of the elasticity estimates ($b_p + b_I + b_{CP} = -.41 + .51 + .12 = .22$) does not equal 0; the estimate is $.22$ from 0. This evidence suggests that money illusion is present and the no money illusion theory is incorrect.

Table 11.1: *Beef Demand Regression Results – Constant Elasticity Model*

If all prices and income increase by 1 percent, the quantity of beef demanded would increase by $.22$ percent. The sum of the elasticity estimates does not equal 0; more specifically, the sum lies $.22$ from 0. The nonzero sum suggests that money illusion exists.

However, as a consequence of random influences, we could never expect the sum of the elasticity estimates to equal exactly precisely 0, even if the sum of the actual elasticities did equal 0. Consequently, we followed the hypothesis testing procedure. We played the cynic in order to construct the null and alternative hypotheses. Finally, we needed to calculate the probability that the results would be like those we obtained (or even stronger), if the cynic is correct and null hypothesis is actually true; that is, we needed to calculate $\text{Prob}[\text{Results IF } H_0 \text{ True}]$.

No Money Illusion Theory: Calculating Prob[Results IF H_0 True]

Clever Algebraic Manipulation

In the last chapter, we explored one way to calculate this probability, the clever algebraic manipulation approach. First, we cleverly defined a new coefficient that equals 0 if and only if the null hypothesis is true:

$$\beta_{Clever} = \beta_P + \beta_I + \beta_{CP}$$

We then reformulated the null and alternative hypotheses in terms of the new coefficient, β_{Clever} :

$$H_0: \beta_P + \beta_I + \beta_{CP} = 0 \Rightarrow \beta_{Clever} = 0 \Rightarrow \text{Money illusion not present}$$

$$H_1: \beta_P + \beta_I + \beta_{CP} \neq 0 \Rightarrow \beta_{Clever} \neq 0 \Rightarrow \text{Money illusion present}$$

After incorporating the new coefficient into the model, we used the ordinary least squares (OLS) estimation procedure to estimate the value of the new coefficient. Since the null hypothesis is now expressed as the new, clever coefficient equaling 0, the new coefficient's tails probability reported in the regression printout is the probability that we need:

$$\text{Prob[Results IF } H_0 \text{ True]} = .4325$$

We shall now explore two other ways to calculate this probability:

- Wald (F -distribution) test
- Letting statistical software do the work

Wald (F-distribution) Test

The Wald test involves two different regressions:

- Restricted regression reflects H_0 ; the restricted regression “enforces” the theory.
- Unrestricted regression reflects H_1 ; the unrestricted regression does not “enforce” the theory.

We shall now discuss each regression.

Restricted Regression Reflects H_0

The restricted regression enforces the theory; that is, the restricted regression imposes the restriction specified by the null hypothesis. In this case, the null hypothesis requires the elasticities sum to equal 0:

$$\beta_P + \beta_I + \beta_{CP} = 0.$$

We now incorporate this restriction into the constant elasticity demand model:

$$\log(Q_t) = \log(\beta_{Const}) + \beta_P \log(P_t) + \beta_I \log(I_t) + \beta_{CP} \log(ChickP_t) + e_t$$

$$\beta_P + \beta_I + \beta_{CP} = 0.$$

Solving for β_{CP} :

$$\beta_{CP} = -(\beta_P + \beta_I).$$

Substitute for β_{CP} .

$$= \log(\beta_{Const}) + \beta_P \log(P_t) + \beta_I \log(I_t) + (\beta_P + \beta_I) \log(ChickP_t) + e_t$$

Multiplying $\log(ChickP_t)$ term.

$$= \log(\beta_{Const}) + \beta_P \log(P_t) + \beta_I \log(I_t) - \beta_P \log(ChickP_t) - \beta_I \log(ChickP_t) + e_t$$

Rearranging terms

$$= \log(\beta_{Const}) + \beta_P \log(P_t) - \beta_P \log(ChickP_t) + \beta_I \log(I_t) - \beta_I \log(ChickP_t) + e_t$$

Factoring the β_P and β_I terms.

$$= \log(\beta_{Const}) + \beta_P [\log(P_t) - \log(ChickP_t)] + \beta_I [\log(I_t) - \log(ChickP_t)] + e_t$$

Defining new variables.

$$LogQ_t = \log(\beta_{Const}) + \beta_P LogP_{Less}LogChickP_t + \beta_I LogI_{Less}LogChickP_t + e_t$$

where $LogQ_t = \log(Q_t)$

$$LogP_{Less}LogChickP_t = \log(P_t) - \log(ChickP_t)$$

$$LogI_{Less}LogChickP_t = \log(I_t) - \log(ChickP_t)$$

$$LogChickP_t = \log(ChickP_t)$$

Next, estimate the parameters of the restricted equation:

[Link to MIT-BeefDemand-1985-1986.wf1 goes here.]

Ordinary Least Squares (OLS)

Dependent Variable: $\text{Log}Q$

Explanatory Variable(s):	Estimate	SE	<i>t</i> -Statistic	Prob
$\text{Log}P\text{LessLogChick}P$	-0.467358	0.062229	-7.510284	0.0000
$\text{Log}I\text{LessLogChick}P$	0.301906	0.068903	4.381606	0.0003
<i>Const</i>	11.36876	0.260482	43.64516	0.0000

Number of Observations	24	Degrees of Freedom	21
Sum of Squared Residuals	.004825		

Estimated Equation: $\text{EstLog}Q = 11.4 - .47\text{Log}P\text{LessLogChick}P + .30\text{Log}I\text{LessLogChick}P$

Interpretation of Estimates:

$b_p = -.47$: (Own) Price Elasticity of Demand = $-.47$

$b_I = .30$: Income Elasticity of Demand = $.30$

Table 11.2: *Beef Demand Regression Results – Restricted Model*

To compute the cross price elasticity estimate we must remember that the restricted regression is based on the premise that the sum of the elasticities equals 0. Hence,

$$b_p + b_I + b_{CP} = 0$$

and

$$b_{CP} = -(b_p + b_I) = -(-.47 + .30) = .17$$

For future reference, note in the restricted regression the sum of squared residuals equals .004825 and the degrees of freedom equal 21:

$$SSR_R = .004825 \quad DF_R = 24 - 3 = 21$$

Unrestricted Regression Reflects H_1

The unrestricted regression does not force the model to enforce the theory; that is, the unrestricted regression considers the model that reflects the alternative hypothesis allowing the parameter estimates to take on any values. We have already run the unrestricted regression to estimate the coefficients of this model. The log of the quantity of beef demanded is the dependent variable; the logs of the price of beef (the good's own price), income, and the price of chicken are the explanatory variables:

$$\log(Q_t) = \log(\beta_{Const}) + \beta_P \log(P_t) + \beta_I \log(I_t) + \beta_{CP} \log(ChickP_t) + e_t$$

Let us review the regression printout:

Ordinary Least Squares (OLS)

Dependent Variable: $\text{Log}Q$

Explanatory Variable(s):	Estimate	SE	t-Statistic	Prob
$\text{Log}P$	-0.411812	0.093532	-4.402905	0.0003
$\text{Log}I$	0.508061	0.266583	1.905829	0.0711
$\text{LogChick}P$	0.124724	0.071415	1.746465	0.0961
$Const$	9.499258	2.348619	4.044615	0.0006

Number of Observations 24 Degrees of Freedom 20

Sum of Squared Residuals .004675

Estimated Equation: $\text{EstLog}Q = 9.50 - .41\text{Log}P + .51\text{Log}I + .12\text{LogChick}$

Table 11.3: *Beef Demand Regression Results – Unrestricted Model*

Record the sum of squared residuals and the degrees of freedom in the unrestricted regression:

$$SSR_U = .004675 \qquad DF_U = 24 - 4 = 20$$

Comparing the Restricted Sum of Squared Residuals and the Unrestricted Sum of Squared Residuals: The F -Statistic

Next, we compare the sum of squared residuals for the restricted and unrestricted regressions:

$$SSR_R = .004825 \quad SSR_U = .004675$$

The sum of squared residuals from the restricted equation is larger.

Question: Is this a coincidence?

Answer: No. Let us now explain why.

The parameter estimates of the restricted and unrestricted regressions differ:

	Restricted Regression	Unrestricted Regression
b_p	-.47	-.41
b_I	.30	.51
b_{ChickP}	.17	.12
b_{Const}	9.50	11.37
SSR	.004825	.004675

Table 11.4: *Comparison of Parameter Estimates*

Recall that the estimates of the constant and coefficients are chosen so as to minimize the sum of squared residuals; when b_p equals $-.47$, b_I equals $.30$, and b_{ChickP} equals $.17$, the sum of squared residuals is minimized. The estimates of the unrestricted regression minimize the sum of squared residuals. The estimates of the restricted regression do not equal the estimates of the unrestricted regression. Hence, the restricted sum of square residuals is greater than the unrestricted sum. More generally,

- The unrestricted equation places no restrictions on the estimates.
- Enforcing a restriction impedes our ability to make the sum of squared residuals as small as possible.
- A restriction can only increase the sum of squared residuals; a restriction cannot reduce the sum:

$$SSR_R \geq SSR_U$$

Econometrics Lab 11.1: The Restricted and Unrestricted Sums of Squared Residuals

[Link to MIT-Lab 11.1 goes here.]

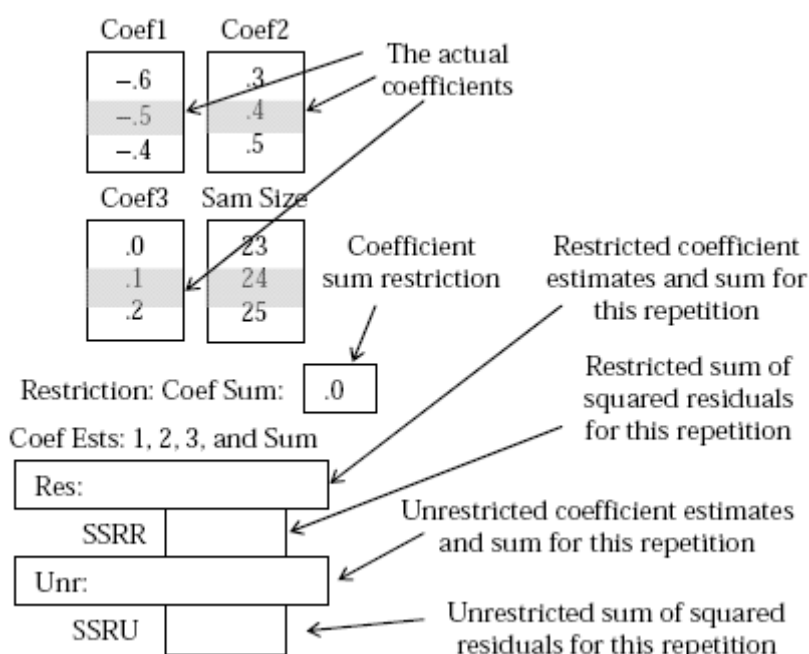


Figure 11.1: *Restricted and Unrestricted Sum of Square Residuals Simulation*

This simulation emphasizes the point. It mimics the problem at hand by including three explanatory variables whose coefficients are denoted as Coef1, Coef2, and Coef3. By default the actual values of the three coefficients are -0.5 , 0.4 , and 0.1 , respectively. The simulation allows us to specify a restriction on the coefficient sum. By default a coefficient sum of 0 is imposed.

Be certain the Pause checkbox is checked and click Start. The first repetition is now performed. The simulation calculates the parameter estimates for both the restricted and unrestricted equations. The sum of the restricted coefficient estimates equals 0; the sum of the unrestricted coefficient does not equal 0. If our logic is correct the restricted sum will be greater than the unrestricted sum. Check the two sums. Indeed, the restricted sum is greater. Click Continue a few times. Each time the restricted sum is always greater than the unrestricted sum confirming our logic.

Now let us consider a question:

Question: Since the imposition of a restriction can only make the sum of squared residuals larger, how much larger should we expect it to be?

The answer to this question depends on whether or not the restriction is actually true. If in reality the restriction is not true, we would expect the sum of squared residuals to increase by a large amount. On the other hand, if the restriction is actually true, we would expect the sum of squared residuals to increase only modestly.

Restriction not true

↓

SSR_R much larger than SSR_U

Restriction true

↓

SSR_R only a little larger than SSR_U

How do we decide if the restricted sum of squared residuals is much larger or just a little larger than the unrestricted sum? For reasons that we shall not delve into, we compare the magnitudes of the restricted and unrestricted sum of squared residuals by calculating what statisticians call the F -statistic:

$$F = \frac{(SSR_R - SSR_U) / (DF_R - DF_U)}{SSR_U / DF_U}$$

When the restricted sum is much larger than the unrestricted sum

- $SSR_R - SSR_U$ is large

and

- the F -statistic is large.

On the other hand, when the restricted sum is only a little larger than the unrestricted sum

- $SSR_R - SSR_U$ is small

and

- the F -statistic is small.

Note that since the restricted sum of squared residuals (SSR_R) cannot be less than the unrestricted sum (SSR_U), F -statistic can never be negative:

$$F \geq 0.$$

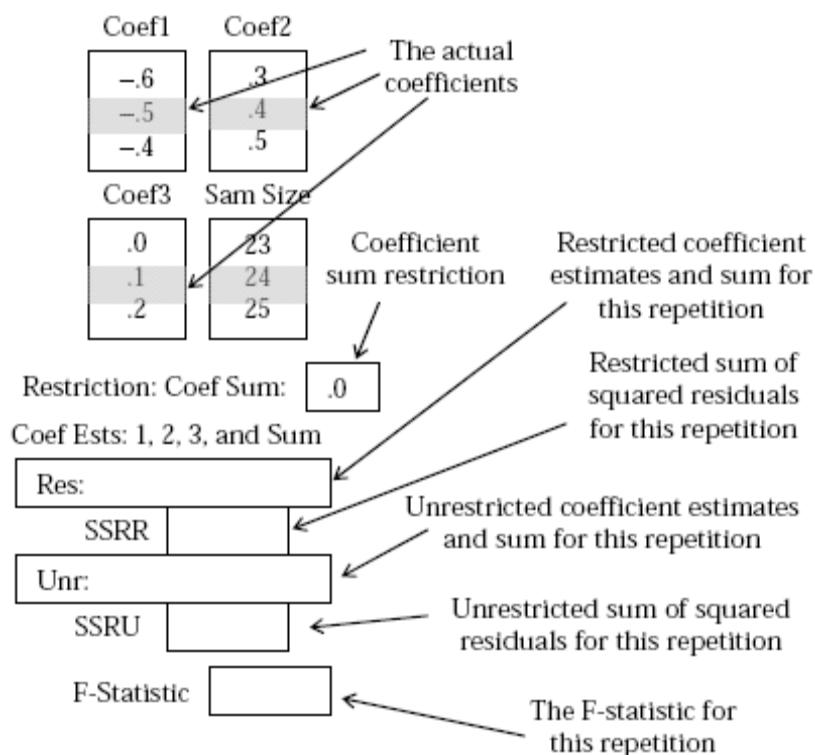


Figure 11.2: *Sums of Square Residuals and the F-Statistic Simulation*

Furthermore, the F -statistic is a random variable. The claim is based on the fact that:

- Since the parameter estimates for both the restricted and unrestricted equations are random variables both the restricted and unrestricted sums of squared residuals are random variables.
- Since both the restricted and unrestricted sums of squared residuals are random variables, the F -statistic is a random variable.

Econometrics Lab 11.2: The F-Statistic Is a Random Variable

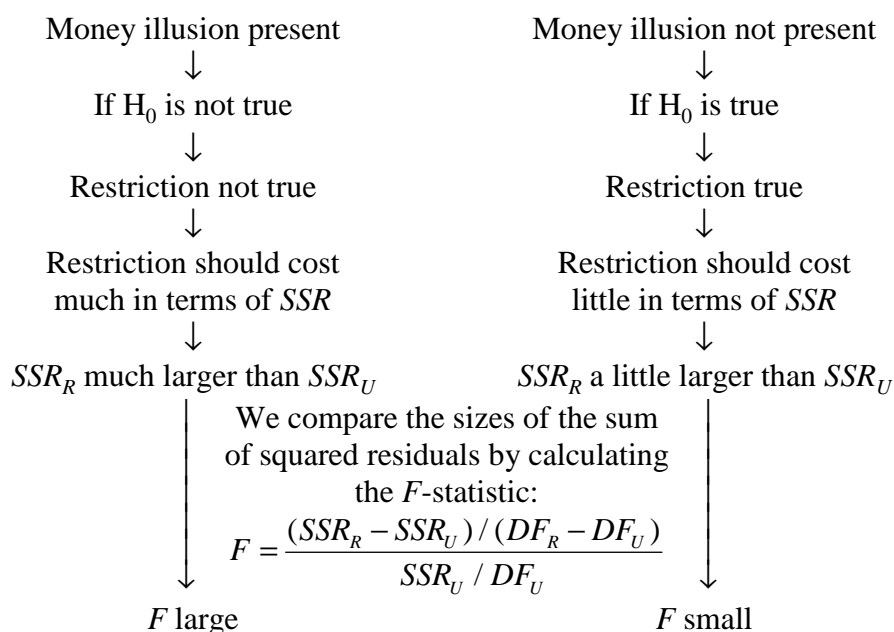
[Link to MIT-Lab 11.2 goes here.]

Again, let us use a simulation to illustrate that the F -statistic is a random variable. Be certain the Pause checkbox is checked and click Start. Then, click Continue a few times. We cannot predict the sums of squared residuals or the F -statistic beforehand. Clearly, the sums of squared residuals and the F -statistic are random variables.

Let us now put this all together:

$H_0: \beta_p + \beta_I + \beta_{CP} = 0 \Rightarrow$ Money illusion not present \Rightarrow Restriction true

$H_1: \beta_p + \beta_I + \beta_{CP} \neq 0 \Rightarrow$ Money illusion present \Rightarrow Restriction not true



Now, calculate the numerical value of our F -statistic:

$$\begin{aligned}
 SSR_R &= .004825 & DF_R &= 24 - 3 = 21 \\
 SSR_U &= \underline{.004675} & DF_U &= 24 - 4 = \underline{20} \\
 SSR_R - SSR_U &= .000150 & DF_R - DF_U &= 1 \\
 F &= \frac{(SSR_R - SSR_U) / (DF_R - DF_U)}{SSR_U / DF_U} = \frac{.000150 / 1}{.004675 / 20} = \frac{.000150}{.000234} = .64
 \end{aligned}$$

Next, consider the views of our cynic.

Cynic's View: Sure, the F -statistic is .64, but the F -statistic will always be positive because the restricted sum of squared residuals (SSR_R) will always be greater than the unrestricted sum (SSR_U). An F -statistic of .64 results from “the luck of the draw.”

We can characterize the cynic's view as follows:

- An F -statistic of .64 is small; it is not statistically different from 0.
- The restricted sum of squared errors is larger than the unrestricted sum only as a consequence of the luck of the draw.
- The restriction is true.
- The null hypothesis, H_0 , is actually true and money illusion is not present

Question Assess the Cynic's View

- **Generic Question:** What is the probability that the results would be like those we obtained (or even stronger), if the cynic is correct and H_0 is actually true?
- **Specific Question:** What is the probability that the F -statistic from one pair of regressions would be .64 or more, if H_0 were true, if the restriction is true and money illusion is not present?

Answer: Prob[Results IF H_0 True]

Prob[Results IF H_0 True] small



Unlikely that H_0 is true



Reject H_0



Reject the no money
illusion theory

Prob[Results IF H_0 True] large



Likely that H_0 is true



Do not reject H_0



Do not reject the no money
illusion theory

We must calculate $\text{Prob}[\text{Results IF } H_0 \text{ True}]$. Before doing so, however, recall that the F -statistic is a random variable. Recall what we have learned about random variables:

- The bad news is that we cannot predict the value of a random variable beforehand.
- The good news is that in this case we can describe its probability distribution.

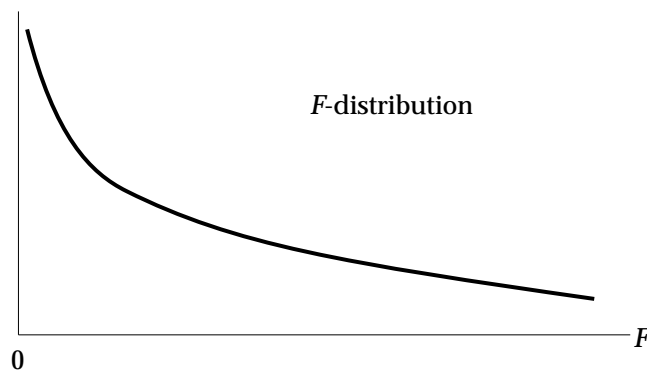


Figure 11.3: *The F-Distribution*

The F -distribution describes the probability distribution of the F -statistic. As Figure 11.3 shows, the F -distribution looks very different than the normal and Student t -distribution. The normal and Student t -distributions were symmetric bell shaped curves. The F -distribution is neither symmetric nor bell shaped. Since the F -statistic can never be negative, the F -distribution begins at F equals 0. Its precise shape depends on the numerator's and the denominator's degrees of freedom, the degrees of freedom of the restricted and unrestricted regressions.

Econometrics Lab 11.3: The Restricted and Unrestricted Sums of Squared Residuals and the F-Distribution

[Link to MIT-Lab 11.3 goes here.]

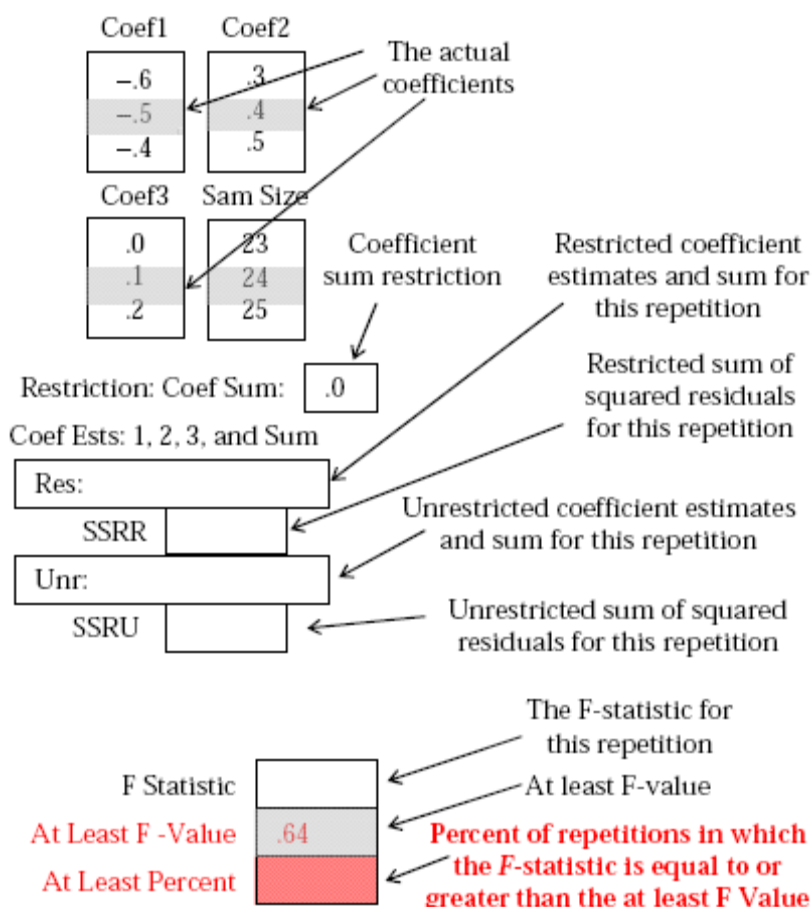


Figure 11.4: *F-Distribution Simulation*

Now, we shall use the simulation to calculate $\text{Prob}[\text{Results IF } H_0 \text{ True}]$:

- By default, the actual values of the three coefficients (Coef1, Coef2, and Coef3) are $-.5$, $.4$, and $.1$, respectively. The actual values of the coefficients sum to 0. Hence, the premise of the null hypothesis is met; that is, H_0 is true.
- Also, the At Least F-Value is set at $.64$; this is the value of the F -statistic that we just calculated for the restricted and unrestricted beef demand regressions. Click Continue a few more times. Sometimes the F -statistic is less than $.64$; other times it is greater than $.64$. Note the At Least Percent

line; the simulation is calculating the percent of repetitions in which the F -statistics is equal to or greater than .64.

- Clear the Pause checkbox, click Start, and then after many, many repetitions click Stop. The F -statistic equals .64 or more in about 43 percent of the repetitions.

We can now apply the relative frequency interpretation of probability; in one repetition of the experiment, the probability that the F -statistic would be .64 or more when the null hypothesis is true equals .43:

$$\text{Prob}[\text{Results IF } H_0 \text{ True}] = .43$$

There is another way to calculate this probability that does not involve a simulation. Just as there are tables that describe the normal and Student- t distributions, there are tables describing the F -distribution. Unfortunately, F -distribution tables are even more cumbersome than Student- t tables. Fortunately, we can use our Econometrics Lab to perform the calculation instead.

Econometrics Lab 11.4: F-Distribution Calculations

We wish to calculate the probability that the F -statistic from one pair of regressions would be .64 or more, if H_0 were true (if there is no money illusion, if actual elasticities sum to 0), $\text{Prob}[\text{Results IF } H_0 \text{ True}]$.

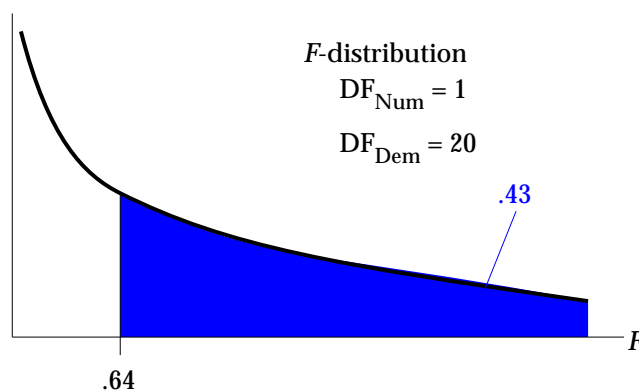


Figure 11.5: *Calculating Prob[Results IF H_0 True] – Using a Simulation*

Access the following link:

[Link to MIT-Lab 11.4 goes here.]

Recall our F -statistic calculation:

$$F = \frac{(SSR_R - SSR_U) / (DF_R - DF_U)}{SSR_U / DF_U} = \frac{.000150 / 1}{.004675 / 20} = \frac{.000150}{.000234} = .64$$

The following information has been entered:

Sum of Squares Numerator = .000150

Sum of Squares Denominator = .004675

Degrees of Freedom Numerator = 1

Degrees of Freedom Denominator = 20

Click Calculate:

Prob[Results IF H_0 True] = .43

Calculating Prob[Results IF H_0 True]: Let the Software Do the Work

Many statistical software packages can be used to conduct a Wald test automatically.

[Link to MIT-BeefDemand-1985-1986.wf1 goes here.]

First, the unrestricted regression:

Ordinary Least Squares (OLS)

Dependent Variable: $\text{Log}Q$

Explanatory Variable(s):	Estimate	SE	t -Statistic	Prob
$\text{Log}P$	-0.411812	0.093532	-4.402905	0.0003
$\text{Log}I$	0.508061	0.266583	1.905829	0.0711
$\text{LogChick}P$	0.124724	0.071415	1.746465	0.0961
Const	9.499258	2.348619	4.044615	0.0006

Number of Observations 24 Degrees of Freedom 20

Sum of Squared Residuals .004675

Estimated Equation: $\text{EstLog}Q = 9.50 - .41\text{Log}P + .51\text{Log}I + .12\text{LogChick}$

Table 11.5: *Beef Demand Regression Results – Unrestricted Model*

Then, choose the Wald test option and impose the appropriate restriction that the coefficients sum to 0.

Getting Started in EViews

After running the unrestricted regression:

- In the Equation window: Click **View, Coefficient Diagnostics, and Wald Test - Coefficient Restrictions**.
 - In the Wald Test window: Enter the restriction; in this case, $C(1) + C(2) + C(3) = 0$
 - Click OK.
-

Wald Test

	Value	Degrees of Freedom		Prob
		Num	Dem	
<i>F</i> -statistic	0.641644	1	20	0.4325

Table 11.6: *Beef Demand Regression Results – Wald Test of No Money Illusion Theory*

Prob[Results IF H_0 True]: The *F*-statistic equals .64. The probability that the *F*-statistic from one pair of regressions would be .64 or more, if H_0 were true (if there is no money illusion, if actual elasticities sum to 0) equals .43.

We have now described three ways to calculate Prob[Results IF H_0 True].

Let us compare the results:

Method	Prob[Results IF H_0 True]
<i>t</i> -Test Using Clever Definition	.43
Wald Test Using Restricted and Unrestricted Regressions	.43
Wald Test Using Statistical Software	.43

Table 11.7: *Comparison of the Methods to Calculate Prob[Results IF H_0 True]*

While the methods use different approaches, they produce identical conclusions. In fact, it can be shown rigorously that the methods are equivalent.

Testing the Significance of the “Entire” Model

Next, we shall consider a set of null and alternative hypotheses that assess the entire model:

$H_0: \beta_P = 0, \beta_I = 0, \text{ and } \beta_{CP} = 0$	No explanatory variables has an effect on the dependent variable
$H_1: \beta_P \neq 0 \text{ and/or } \beta_I \neq 0 \text{ and/or } \beta_{CP} \neq 0$ (at least one coefficient does not equal 0)	At least one explanatory variable has an effect on the dependent variable

If the null hypothesis were true, none of the explanatory variables would affect the dependent variable and consequently, the model would be seriously deficient. On the other hand, if the alternative hypothesis were true, at least one of the explanatory variables would be influencing the dependent variable.

We shall use the restricted and unrestricted regressions approach to calculate Prob[Results IF H_0 True]. We begin with estimating the restricted and unrestricted regressions.

[Link to MIT-BeefDemand-1985-1986.wf1 goes here.]

Restricted Equation – reflects $H_0: \beta_P = 0, \beta_I = 0, \text{ and } \beta_{CP} = 0$:

$$\text{Model: } \text{Log}Q_t = \log(\beta_{Const}) + e_t$$

Ordinary Least Squares (OLS)

Dependent Variable: $\text{Log}Q$

Explanatory Variable(s):	Estimate	SE	t-Statistic	Prob
<i>Const</i>	12.30576	0.005752	2139.539	0.0000

Number of Observations	24	Degrees of Freedom	23
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Sum of Squared Residuals	.018261
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Table 11.8: *Beef Demand Regression Results – No Explanatory Variables*

Unrestricted equation – reflects $H_1: \beta_P \neq 0 \text{ and/or } \beta_I \neq 0 \text{ and/or } \beta_{CP} \neq 0$:

$$\text{Model: } \text{Log}Q_t = \log(\beta_{Const}) + \beta_P \text{Log}P + \beta_I \text{Log}I_t + \beta_{CP} \text{Log}ChickP_t + e_t$$

We have estimated the unrestricted regression before:

Ordinary Least Squares (OLS)

Dependent Variable: $\text{Log}Q$

Explanatory Variable(s):	Estimate	SE	t-Statistic	Prob
$\text{Log}P$	-0.411812	0.093532	-4.402905	0.0003
$\text{Log}I$	0.508061	0.266583	1.905829	0.0711
$\text{LogChick}P$	0.124724	0.071415	1.746465	0.0961
Const	9.499258	2.348619	4.044615	0.0006

Number of Observations	24	Degrees of Freedom	20
Sum of Squared Residuals	.004675		

Estimated Equation: $\text{EstLog}Q = 9.50 - .41\text{Log}P + .51\text{Log}I + .12\text{LogChick}P$

Table 11.9: *Beef Demand Regression Results – Unrestricted Model*

Next, we compute the F -statistic and calculate Prob[Results IF H_0 True]:

$$\begin{aligned}
 SSR_R &= .018261 & DF_R &= 24 - 1 = 23 \\
 SSR_U &= \underline{.004675} & DF_U &= 24 - 4 = \underline{20} \\
 SSR_R - SSR_U &= .013586 & DF_R - DF_U &= 3 \\
 F &= \frac{(SSR_R - SSR_U) / (DF_R - DF_U)}{SSR_U / DF_U} = \frac{.013586 / 3}{.004675 / 20} = \frac{.004529}{.000234} = 19.4
 \end{aligned}$$

Prob[Results IF H_0 True]: What is the probability that the F -statistic from one pair of regressions would be 19.4 or more, if the H_0 were true (that is, if both prices and income have no effect on quantity of beef demanded, if each of the actual coefficients, β_p , β_I , and β_{CP} , equals 0)?

Prob[Results IF H_0 True] small	Prob[Results IF H_0 True] large
↓	↓
Unlikely that H_0 is true	Likely that H_0 is true
↓	↓
Reject H_0	Do not reject H_0

Econometrics Lab 11.5: Calculate Prob[Results IF H_0 True].

[Link to MIT-Lab 11.5 goes here.]

Using the Econometrics Lab we conclude that the probability of obtaining the results like we did if null hypothesis were true is less than .0001:

$$\text{Prob[Results IF } H_0 \text{ True]} < .0001$$

Also, we could let a statistical package do the work by using it to run the Wald test.

[Link to MIT-BeefDemand-1985-1986.wf1 goes here.]

After running the unrestricted regression, choose the Wald test option and impose the restriction that all the coefficients equal 0.

Getting Started in EViews

After running the unrestricted regression:

- Click View, Coefficient Diagnostics, and Wald Test - Coefficient Restrictions.
- Enter the restriction; in this case,
 $C(1) = C(2) = C(3) = 0$
- Click OK.

	Wald Test			
		Degrees of Freedom		
	Value	Num	Dem	Prob
<i>F</i> -statistic	19.37223	3	20	0.0000

Table 11.10: *Demand Regression Results – Wald Test of Entire Model*

$$\text{Prob[Results IF } H_0 \text{ True]} < .0001$$

Note that even though the Wald test printout reports the probability to be .0000, it is not precisely 0 because the printout reports the probability only to four decimals. To emphasize this fact, we report that Prob[Results IF H_0 True] is less than .0001.

In fact, most statistical packages automatically report this F -statistic and the probability when we estimate the unrestricted model:

Ordinary Least Squares (OLS)

Dependent Variable: $\text{Log}Q$

Explanatory Variable(s):	Estimate	SE	t -Statistic	Prob
$\text{Log}P$	-0.411812	0.093532	-4.402905	0.0003
$\text{Log}I$	0.508061	0.266583	1.905829	0.0711
$\text{LogChick}P$	0.124724	0.071415	1.746465	0.0961
Const	9.499258	2.348619	4.044615	0.0006
Number of Observations	24	Degrees of Freedom	20	
Sum of Squared Residuals	.004675			
F-Statistic	19.3722	Prob[F-Statistic]		.000004

Estimated Equation: $\text{EstLog}Q = 9.50 - .41\text{Log}P + .51\text{Log}I + .12\text{LogChick}$

Table 11.11: *Beef Demand Regression Results – Unrestricted Model*

The values appear in the F -statistic and Prob[F -statistic] rows:

$$F\text{-statistic} = 19.4$$

$$\text{Prob}[\text{Results IF } H_0 \text{ True}] = .000004$$

Using a significance level of 1 percent, we would conclude that Prob[Results IF H_0 True] is small. Consequently, we would reject the null hypothesis that none of the explanatory variables included in the model has an effect on the dependent variable.

Equivalence of Two-Tailed t -tests and Wald Tests (F -tests)

A two-tailed t -test is equivalent to a Wald test. We shall use the constant elasticity demand model to illustrate this:

$$\log(Q) = \log(\beta_{\text{Const}}) + \beta_p \log(P) + \beta_I \log(I) + \beta_{CP} \log(\text{Chick}P)$$

Focus on the coefficient of the price of chicken, β_{CP} . Consider the following two-tailed hypotheses:

$$H_0: \beta_{CP} = 0 \Rightarrow \text{Price of chicken has no effect on the quantity of beef demanded}$$

$$H_1: \beta_{CP} \neq 0 \Rightarrow \text{Price of chicken has an effect on the quantity of beef demanded}$$

We shall first calculate Prob[Results IF H_0 True] using a two-tailed t -test and then using a Wald test.

Two-Tailed t -Test

We begin by estimating the parameters of the model:

$$\text{Model: } \text{Log}Q_t = c + \beta_p \text{Log}P_t + \beta_I \text{Log}I_t + \beta_{CP} \text{LogChick}P_t + e_t$$

[Link to MIT-BeefDemand-1985-1986.wf1 goes here.]

Ordinary Least Squares (OLS)

Dependent Variable: $\text{Log}Q$

Explanatory Variable(s):	Estimate	SE	t -Statistic	Prob
$\text{Log}P$	-0.411812	0.093532	-4.402905	0.0003
$\text{Log}I$	0.508061	0.266583	1.905829	0.0711
$\text{LogChick}P$	0.124724	0.071415	1.746465	0.0961
Const	9.499258	2.348619	4.044615	0.0006

Number of Observations 24 Degrees of Freedom 20
 Sum of Squared Residuals .004675

Table 11.12: *Beef Demand Regression Results – Unrestricted Model*

Next, calculate $\text{Prob}[\text{Results IF } H_0 \text{ True}]$:

Prob[Results IF H_0 True]: What is the probability that the coefficient estimate in one regression, b_{CP} , would be at least .12 from 0, if H_0 were true (if the actual coefficient, β_{CP} , equals 0)?

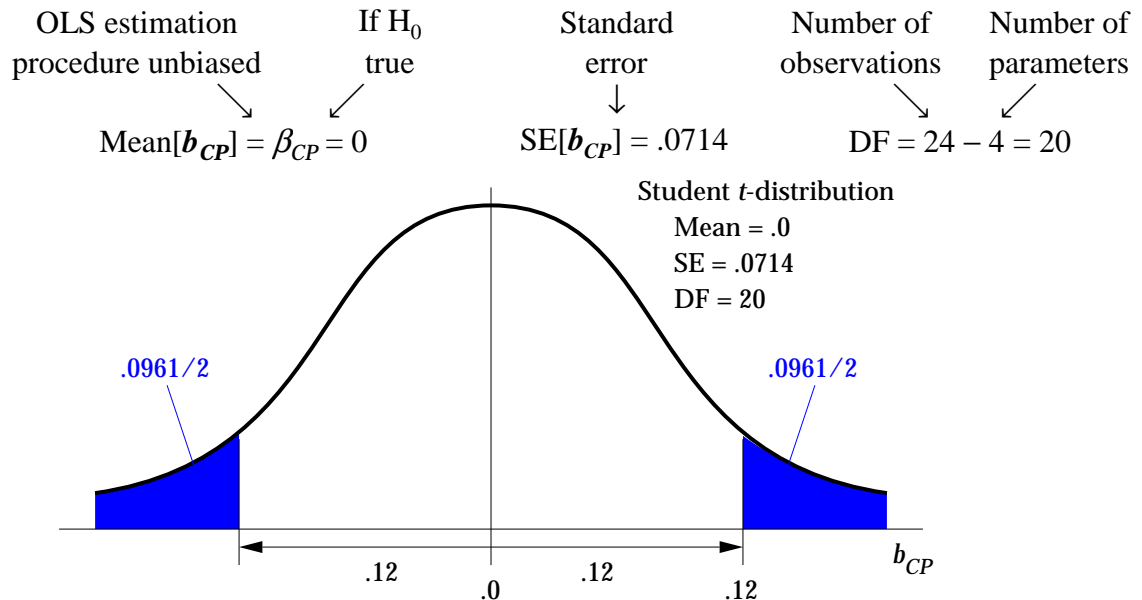


Figure 11.6: *Calculating Prob[Results IF H_0 True] – Using a t -Test*

Since the tails probability is based on the premise that the actual coefficient value equals 0, the tails probability reported in the regression printout is just what we are looking for:

$$\text{Prob}[\text{Results IF } H_0 \text{ True}] = .0961.$$

Wald Test

Next, we turn to a Wald test. Let us review the rationale behind the Wald test.

- The null hypothesis enforces the restriction and the alternative hypothesis does not:

H_0 : Restriction true

H_1 : Restriction not true

- We run two regressions: restricted and unrestricted. The restricted regression reflects the null hypothesis and the unrestricted regression the alternative hypothesis.

If H_0 is not true



Restriction is not true

If H_0 is true



Restriction is true

- SSR_R cannot be less than SSR_U : Since the least squares estimation procedure chooses the estimates so as to minimize the sum of squared residuals, any restriction can only increase, not decrease, the sum of squared residuals.
- So, the question becomes: By how much does the SSR_R exceed SSR_U ? The answer depends on whether or not the null hypothesis is actual true:

If H_0 is not true



Restriction not true



Restriction should cost much in terms of SSR



SSR_R much larger than SSR_U

If H_0 is true



Restriction true



Restriction should cost little in terms of SSR



SSR_R a little larger than SSR_U

We compare the sizes of the sum of squared residuals by calculating the F -statistic:

$$F = \frac{(SSR_R - SSR_U) / (DF_R - DF_U)}{SSR_U / DF_U}$$

F large

F small

Now, let us apply this to the problem at hand:

$$\text{Model: } \text{Log}Q_t = \log(\beta_{\text{Const}}) + \beta_p \text{Log}P_t + \beta_I \text{Log}I_t + \beta_{CP} \text{Log}ChickP_t + e_t$$

Recall the hypotheses:

H_0 : $\beta_{CP} = 0 \Rightarrow$ Price of chicken has no effect on the quantity of beef demanded

H_1 : $\beta_{CP} \neq 0 \Rightarrow$ Price of chicken has an effect on the quantity of beef demanded

Next, estimate the restricted and unrestricted regressions.

Restricted regression – Reflects the null hypothesis: $\beta_{CP} = 0$

- For the restricted regression, we just drop the price of chicken, $PChick$, as an explanatory variable because its coefficient is specified as 0:

$$\text{Model: } \text{Log}Q = \log(\beta_{Const}) + \beta_P \text{Log}P + \beta_I \text{Log}I + e_t$$

Ordinary Least Squares (OLS)

Dependent Variable: $\text{Log}Q$

Explanatory Variable(s):	Estimate	SE	t-Statistic	Prob
$\text{Log}P$	-0.305725	0.074513	-4.102963	0.0005
$\text{Log}I$	0.869706	0.175895	4.944466	0.0001
$Const$	6.407302	1.616841	3.962852	0.0007

Number of Observations	24	Degrees of Freedom	21
Sum of Squared Residuals	.005388		

Table 11.13: *Beef Demand Regression Results – Restricted Model*

- Unrestricted regression – Consistent with the alternative hypothesis: $\beta_{CP} \neq 0$

For the unrestricted regression, we include the price of chicken, $PChick$, as an explanatory variable. This is just the regression we have run many times before:

$$\text{Model: } \text{Log}Q_t = c + \beta_P \text{Log}P_t + \beta_I \text{Log}I_t + \beta_{CP} \text{Log}ChickP_t + e_t$$

Ordinary Least Squares (OLS)

Dependent Variable: $\text{Log}Q$

Explanatory Variable(s):	Estimate	SE	t-Statistic	Prob
$\text{Log}P$	-0.411812	0.093532	-4.402905	0.0003
$\text{Log}I$	0.508061	0.266583	1.905829	0.0711
$\text{Log}ChickP$	0.124724	0.071415	1.746465	0.0961
$Const$	9.499258	2.348619	4.044615	0.0006

Number of Observations	24	Degrees of Freedom	20
Sum of Squared Residuals	.004675		

Table 11.14: *Beef Demand Regression Results – Unrestricted Model*

Using these two regressions we can now calculate the F -statistic:

$$\begin{aligned}
 SSR_R &= .005388 & DF_R &= 24 - 3 = 21 \\
 SSR_U &= \underline{.004675} & DF_U &= 24 - 4 = \underline{20} \\
 SSR_R - SSR_U &= .000713 & DF_R - DF_U &= 1 \\
 F &= \frac{(SSR_R - SSR_U) / (DF_R - DF_U)}{SSR_U / DF_U} = \frac{.000713 / 1}{.004675 / 20} = \frac{.000713}{.000234} = 3.05
 \end{aligned}$$

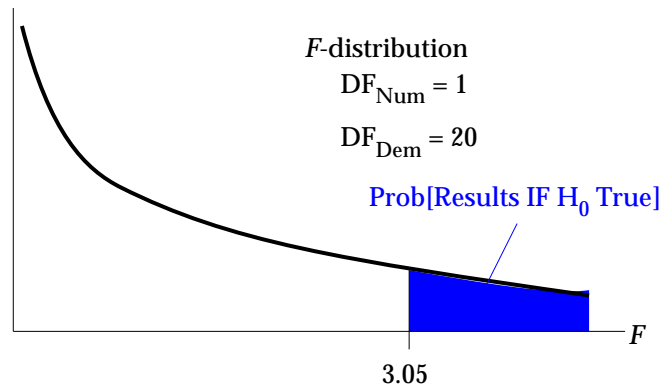


Figure 11.7: Calculating $\text{Prob}[\text{Results IF } H_0 \text{ True}]$ – Using an F -Test

Prob[Results IF H_0 True]: What is the probability that the F -statistic from one pair of regressions would be 3.05 or more, if the H_0 were true (if the actual coefficient, β_{CP} , equals 0; that is, if the price of chicken has no effect on the quantity of beef demanded)?

Econometrics Lab 11.6: Calculate $\text{Prob}[\text{Results IF } H_0 \text{ True}]$.

[Link to MIT-Lab 11.6 goes here.]

We can use the Econometrics Lab to calculate this probability.

$$\text{Prob}[\text{Results IF } H_0 \text{ True}] = .0961.$$

Alternatively we can use statistical software to calculate the probability. After running the unrestricted regression, choose the Wald test option and impose the restriction that all the coefficients equal 0.

Getting Started in EViews

After running the unrestricted regression:

- Click View, Coefficient Diagnostics, and Wald Test - Coefficient Restrictions.
 - Enter the restriction; in this case,
 $C(3) = 0$
 - Click OK.
-

	Wald Test			
		Degrees of Freedom		
	Value	Num	Dem	Prob
<i>F</i> -statistic	3.050139	1	20	.0961

Table 11.15: *Beef Demand Regression Results – Wald Test of LogChickP Coefficient*

Using either method, we conclude that based on a Wald test, the Prob[Results IF H_0 True] equals .0961:

Now, compare Prob[Results IF H_0 True]'s calculated for the two-tailed t -test and the Wald test:

$$t\text{-test: Prob[Results IF } H_0 \text{ True]} = .0961$$

$$\text{Wald test: Prob[Results IF } H_0 \text{ True]} = .0961$$

The probabilities are identical. This is not a coincidence. It can be shown rigorously that a two-tailed t -test is a special case of the Wald test.

Three Important Distributions: Normal, Student t , and F

We have introduced three distributions that are used to assess theories.

- **Theories involving a single variable:** Normal distribution and Student t -distribution
 - The normal distribution is used whenever we know the standard deviation of the distribution; the normal distribution is described by its mean and standard deviation.
 - Often, we do not know the standard deviation of the distribution, however. In these cases, we turn to the Student t -distribution; it is described by its mean, estimated standard deviation (standard error), and the degrees of freedom. The Student t -distribution is more “spread out” than the normal distribution because an additional element of uncertainty is added when the standard deviation is not known and must be estimated.
- **Theories involving several variables:** F -distribution
 - The F -distribution can be used to assess relationships among two or more estimates. We compute the F -statistic by using the sum of squared residuals and the degrees of freedom in the restricted and unrestricted regressions:

$$F = \frac{(SSR_R - SSR_U) / (DF_R - DF_U)}{SSR_U / DF_U}$$

The F -distribution is described by the degrees of freedom in the numerator and denominator, $DF_R - DF_U$ and DF_U .

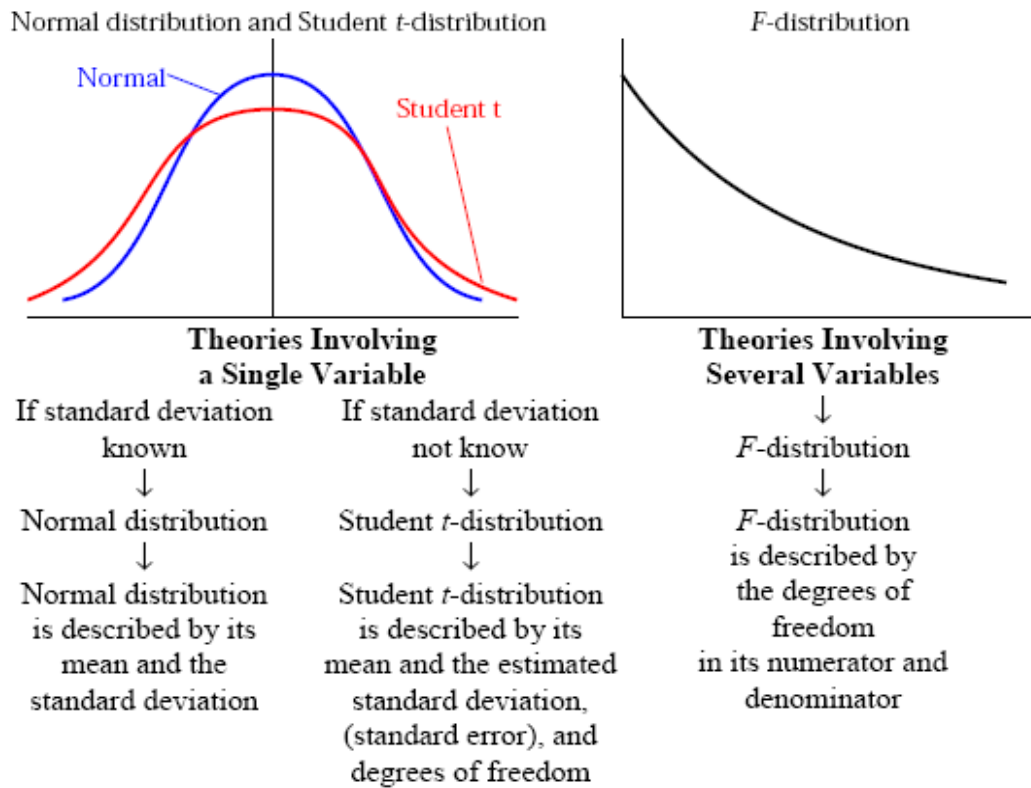


Figure 11.8: *Normal Distribution, Student t -distribution, and F -distribution*