Trigonometric Identities

Sum and Difference Formulas

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\sin(x-y) = \sin x \cos y - \cos x \sin y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

Half-Angle Formulas

$\sin \frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\cos \frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$\tan \frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$\tan \frac{\theta}{2} = \frac{1-\cos x}{\sin x}$

$\tan \frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$

Double-Angle Formulas

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta = 1 - 2\sin^2\theta$

Product-to-Sum Formulas

$\sin x \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$

$\cos x \cos y = \frac{1}{2}\left[\cos(x-y) + \cos(x+y)\right]$

$\sin x \cos y = \frac{1}{2}\left[\sin(x+y) + \sin(x-y)\right]$

Sum-to-Product Formulas

$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$\sin x - \sin y = 2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right)$

$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

The Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Suppose you are given two sides, a, b and the angle A opposite the side A. The height of the triangle is $h = b\sin A$. Then

1. If $a < h$, then a is too short to form a triangle, so there is no solution.
2. If $a = h$, then there is one triangle.
3. If $a > h$ and $a < b$, then there are two distinct triangles.
4. If $a \geq b$, then there is one triangle.

The Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$