

# Evaluating the Effect of Solar Panel Tilt:

Calculating Efficiencies of Williamstown  
Elementary School and Morley Science  
Laboratory Photovoltaic Arrays

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## Introduction

For millennia people have known about the sun's energy potential, using it in passive applications like heating homes and drying laundry. In the last century and a half, however, it was discovered that with photovoltaic cells, the sun's energy can be put to a more direct task: generation of electricity. Since electricity is useful in broad range of applications, greenhouse gas free solar electricity is a very appealing idea in our modern era of climate change. While still a small percentage, photovoltaic energy is one of the fastest growing sectors of the world's Total Primary Energy Supply, and is expected to be .4% by 2010<sup>1</sup>.

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<sup>1</sup> "Renewables In Global Energy Supply - An IEA Fact Sheet". International Energy Agency. 18 April 2008.  
<[http://www.iea.org/textbase/papers/2006/renewable\\_factsheet.pdf](http://www.iea.org/textbase/papers/2006/renewable_factsheet.pdf)>

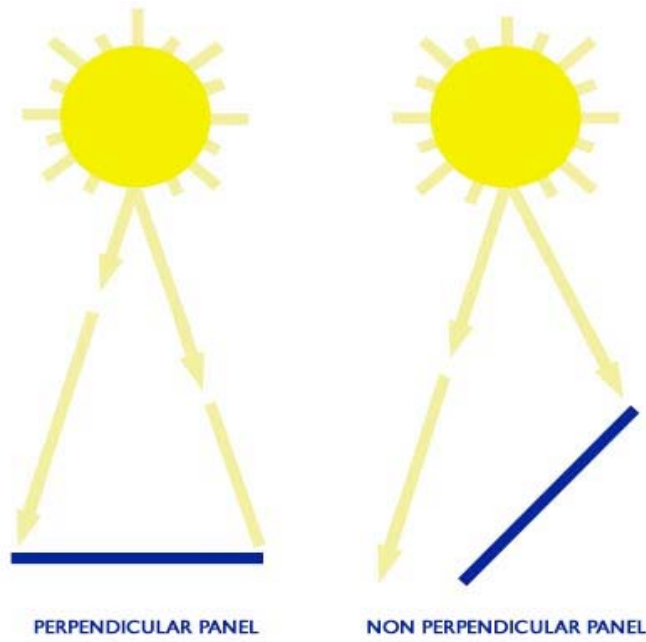
## **Solar Panel Tilt**

To derive the maximum amount of electricity from a photovoltaic panel, it is necessary to make sure that the panel is optimally oriented. A panel that meets incoming photons at a  $90^\circ$  angle has an effectively larger surface area, can capture more photons, and is thus most efficient at turning the sun's energy into electricity (*Figure 1*). As the figure shows, for a given amount of sunlight (shown by equiangular sets of arrows), a panel perpendicular to the sun's rays captures more energy than an obliquely oriented panel. It makes intuitive sense, therefore, that we should try to have solar panels perpendicular to the sun at all times. In areas of high insolation, like the American southwest, this is possible. Photovoltaic arrays are mechanized so that they follow the sun as it traverses the sky each day. In places like Williamstown, however, arrays like this are not economically feasible. At high latitude and with unpredictable weather, the energy required to move the panels exceeds the potential generation gains. Thus, around here, we tend to favor fixed panels. While fixed panels are easier and cheaper to operate, to realize their maximum energy output they must be tilted so that they capture the most sun possible.

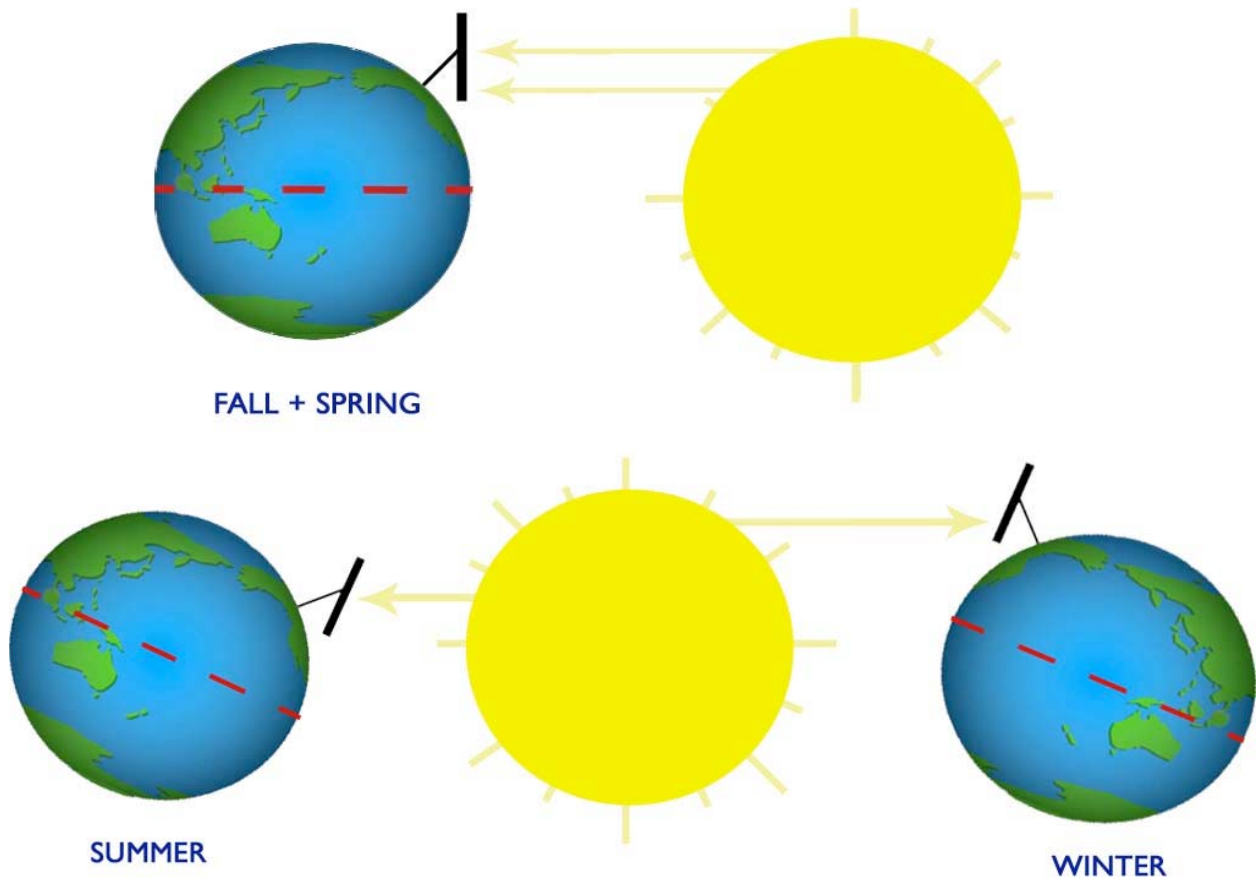
## **Latitude Tilt**

In a perfect solar energy world, one without weather, the ideal tilt for a fixed solar panel is equal to the latitude at which the panel is operating (*Figure 2*). As the figure shows, on the autumnal and vernal equinoxes, when the sun is directly above the equator, latitude tilt yields the perfect  $90^\circ$  angle between the sun's rays and the PV panel. Since, over the course of a year, this is sun's average position, latitude tilt will minimize yearly deviation from  $90^\circ$ . As *Figure 2* also shows, a panel fixed at latitude tilt will be steeper than ideal summer, and too flat in the winter.

While good in theory, latitude tilt is not optimal for Williamstown in practice. First, our weather tends to be sunnier and more conducive to solar electricity generation in the summer.



*Figure 1.* Perpendicular panel presents greater surface area, and receives more insolation than inclined panel.



*Figure 2.* Latitude tilt yields 90° tilt on equinoxes, too steep in summer, not steep enough in winter. For purpose of illustration, panel is situated at about 45° north latitude, tilted 45° south.

Second, PV panels are more efficient in the summer when the sun is higher in the sky because photons have to travel through less atmosphere to reach the panel. By contrast, in the winter, when the sun is low in the sky, photons must travel a longer path through the atmosphere, and are diffused along the way. For both of these reasons (mainly the first), the summer is the best time to generate solar electricity in Williamstown. Thus, for optimal productivity, the panels should be flatter than latitude tilt to take advantage of the sunniest season of the year.

### **Current Installations in Williamstown**

Currently in Williamstown there are PV arrays atop the Morley Science Laboratories (hereafter “MSL”) at Williams College, and on the roof of Williamstown Elementary School (“WES”). About half a mile apart as the crow flies, these two installations provide an excellent opportunity to examine the effect of array tilt on productivity to determine the optimal setup for this location. The arrays are essentially the same in two important respects: both are the same make and model panels, RWE Schott ASE-300 DGF/50, and both are oriented due south. While WES (24 kW) and MSL (7.2 kW) use different inverters, the important difference is their tilt,  $32^\circ$  and  $6^\circ$  respectively<sup>2</sup>. Because of their close proximity, it is reasonable to assume that the arrays are subject to the same weather and receive the same insolation. Since the main variable is the arrays’ tilt, by investigating their respective efficiencies we should be able to ascertain which setup is best suited for Williamstown. We would expect that the flatter array, MSL, will do relatively well in the summer but poorly the rest of the year; its low tilt angle provides very little surface area in the spring, fall, and especially winter. Meanwhile, we can predict that the steeper array, WES, will show relatively steady performance throughout the year since its tilt is within  $10^\circ$  of Williamstown’s latitude. The question is whether, over the course of the year, increased summer production at MSL will make up for its poor winter production.

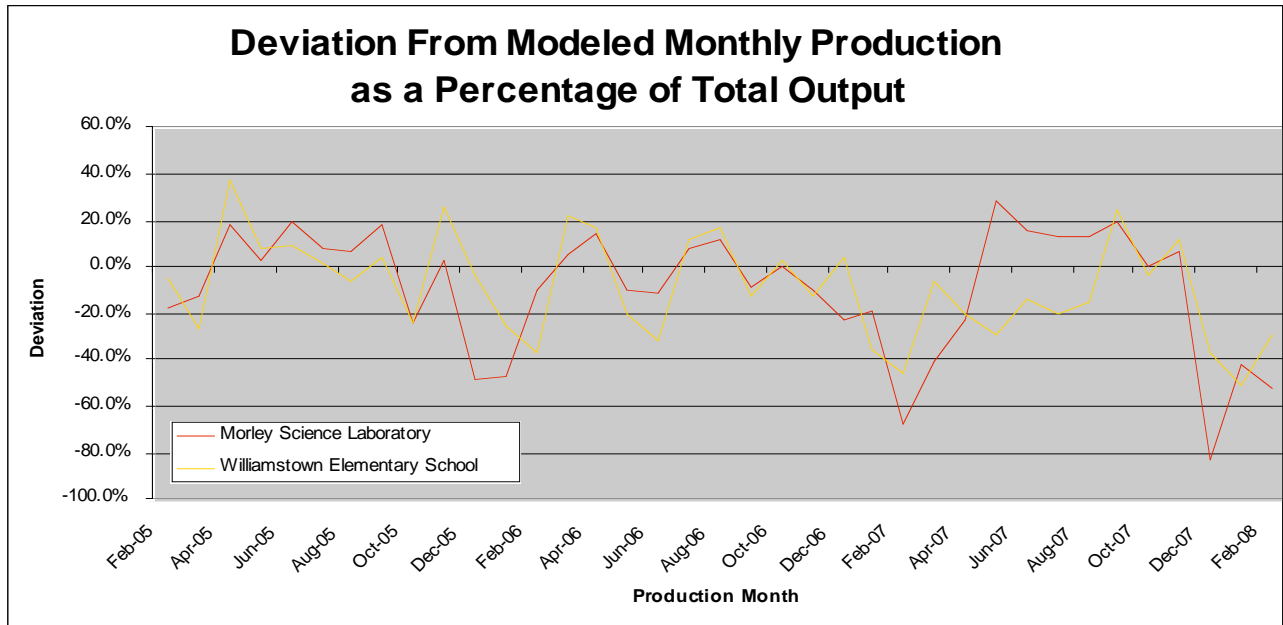
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<sup>2</sup> soltrex.com

## Deviation from Modeled Production

To test whether or not the arrays actually behave the way I predicted, my first step was to compare their actual production with modeled production. The model, from the National Renewable Energy Laboratory, calculates expected monthly electricity output for a given array by taking into account system size, module tilt and azimuth (direction of tilt), and latitude<sup>3</sup>. The model also corrects for local weather by using average monthly solar radiation as an input. By plotting actual production and modeled production together I hoped to see how each array performed throughout the year (*Figure 3*).

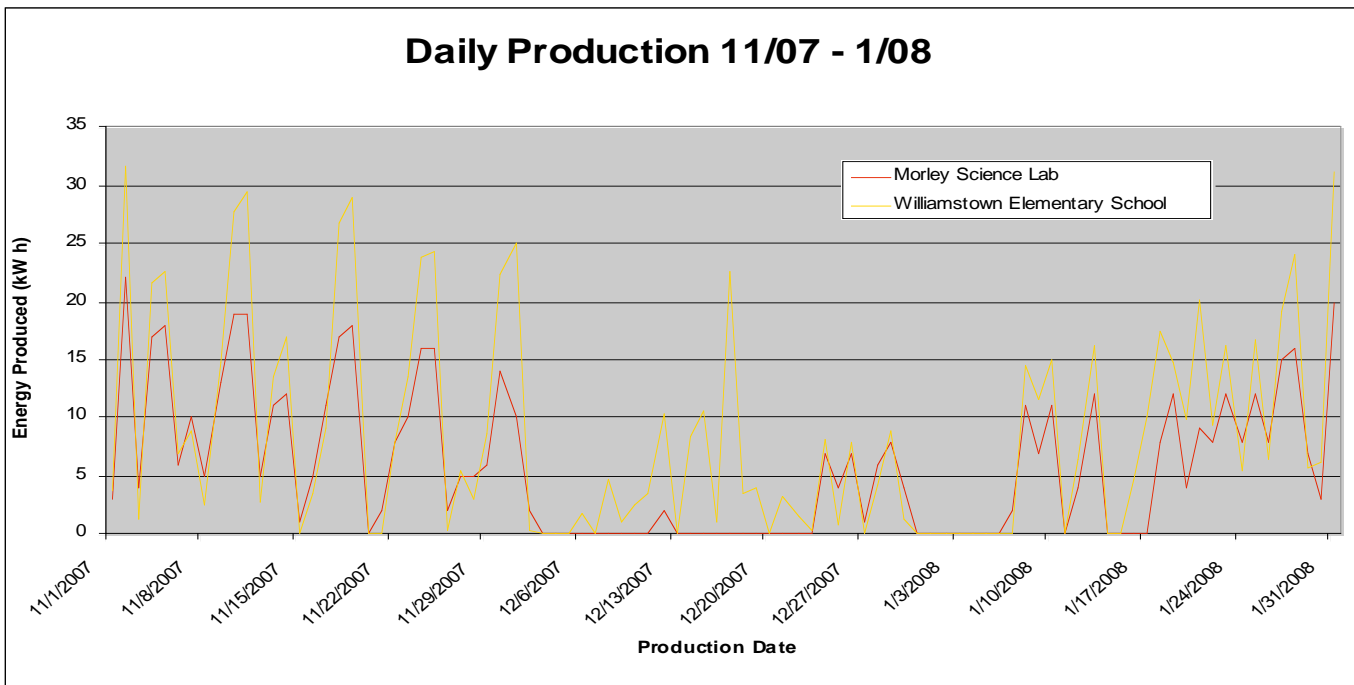
Based on data from the last three years, my prediction was correct. Whereas the WES array behaves unpredictably with respect to the model and is relatively steady all year, the MSL array has a wider range of deviations, and is consistently better than the model in the summer, and worse in the winter. This happens for two reasons. First, with a 6° tilt, MSL is not angled



**Figure 3.** Deviation from modeled monthly production as a percentage of total output. WES bounces above and below the model while MSL is consistently above modeled production in the summer and below in the winter.

<sup>3</sup> "A Performance Calculator for Grid-Connected PV Systems". NREL.gov. 13 May 2005. [http://rredc.nrel.gov/solar/codes\\_algs/PVWATTS/](http://rredc.nrel.gov/solar/codes_algs/PVWATTS/)

to receive much insolation during the winter. Second, because the panels are nearly flat, they have a tendency to gather snow, which covers the panels and renders them incapable of producing any electricity. At WES this effect is mitigated by the tilt of the panels. Because snow tends to slide off the tilted panels more easily, the WES panels remain covered for shorter periods of time. Over the course of a winter, this snow covering effect can have a significant impact on the amount of electricity generated at MSL (*Figure 4*). The figure shows three months of production from both MSL and WES this past winter, and includes two major snow events. The first, around December 2<sup>nd</sup>, initially covers both panels. This is evident by consecutive days of no production from both arrays. By about December 6<sup>th</sup>, the WES array has cleared itself and begun producing electricity again, but the MSL array is still covered. On December 17<sup>th</sup> it looks like the WES array is completely clear; it generated almost 23 kWh of electricity that day, but the MSL panels are still completely covered and generated nothing. As the figure shows, the net effect of this snowstorm was almost three weeks of lost production for MSL. On top of its



**Figure 4.** Effect of snow on daily production. A snow event on December 2<sup>nd</sup> stopped production at WES for only four days while it took the MSL array almost three weeks to clear off.

already low winter sun incidence, snow covering does additional work against MSL and creates a recipe for dismal winter production. MSL's only hope is that it can make up for lost time with outstanding summertime generation.

### **Years in Aggregate**

To answer the question of which fixed tilt is better in Williamstown, I needed to find out which setup is most efficient averaged over an entire year. It is clear that MSL does quite poorly in the winter, but it also does well in the summer when the sun is shining brightest and most frequently. If our goal is produce as much electricity as possible<sup>4</sup>, it might not matter that we get low gains in the winter if it can be made up for in the summer. I calculated the efficiency of both systems in two ways.

First, I examined each system's total yearly output as a fraction of its "potential output". Potential output is a value I defined as a module's instantaneous maximum capacity multiplied by the number of potentially sunny hours in a year ( $12 \text{ hr/day} * 365 \text{ days/year} = 4380$ ). Potential output represents the number of kilowatt hours the array would generate in a year if it produced its maximum rated capacity every hour the sun was out. Of course, this is not practically possible, and thus the absolute values of these ratios are somewhat arbitrary. They are useful only when comparing systems among which the only variable is efficiency, as is the case here. In other words, these numbers are only analytically meaningful for comparing arrays with all other variables controlled (i.e. weather, latitude, azimuth, hardware, etc). Since this is the case with WES and MSL; they have the same model panels, same azimuth, and are so close together we can assume that differences in latitude and weather are negligible, we can use these ratios to compare these two setups. Using data from the last three years I found that MSL had a three year

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<sup>4</sup> Instead of save as much money as possible which entails maximizing generation when energy prices are highest and is a topic for future work.

ratio of .226: with a potential output of 94,608 kWh, it produced 21,375 kWh of electricity<sup>5</sup>.

WES, meanwhile, had a ratio of .256: it produced 80,942 kWh of a possible 315,135 kWh<sup>6</sup>. With this analysis, WES generated a higher percentage of its potential capacity, and it looks, preliminarily, like MSL's good summers do not make up for its bad winters.

The second statistical analysis I applied to the data was calculating efficiency by dividing total energy output by total energy input. Using solar radiation data from MSL I calculated the total energy input by summing daily solar radiation for each month (in MJ/m<sup>2</sup>), dividing by 3.6 to convert to kWh<sup>7</sup>, and then multiplying by the area of the array. This gives the number of kWh of sunshine hitting the array each month. To calculate the area of the WES array, I relied on the known area of the MSL array: 59 ft<sup>2</sup>. Since the modules are the same make and model as the WES modules, I divided that area by the number of modules in the MSL array (24) and found that each module is 2.49 ft<sup>2</sup>. Since WES has 80 modules I estimated the area of that array to be 196.7 ft<sup>2</sup>. Using data from February 2006 to March 2008 (reported 2005 solar radiation data were suspect), I found WES to be 8.94% efficient, generating 52,012 kWh of electricity with 581,518 kWh total input<sup>8</sup>. MSL, meanwhile, was down a little less efficient over those two years, generating only 15,024 kWh with 174,461 kWh of input for an efficiency of 8.61%<sup>9</sup>. A plot of monthly efficiency for the two setups shows the familiar pattern of MSL doing well in the summer and very poorly in the winter while WES looks to actually be most efficient in the wintertime (*Figure 5*). Here again, good summers on the roof of MSL do not make up for bad winters, and throughout the year, WES generates more electricity per unit solar radiation.

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<sup>5</sup> Data in *Table 1* in Appendix

<sup>6</sup> Data in *Table 2* in Appendix

<sup>7</sup>  $(1 \text{ kW}\cdot\text{h})(1000 \text{ W/kW})(3600 \text{ s/h}) = 3,600,000 \text{ W}\cdot\text{s} = 3,600,000 \text{ J} = 3.6 \text{ MJ}$

<sup>8</sup> Data in *Table 3* in Appendix

<sup>9</sup> Data in *Table 4* in Appendix



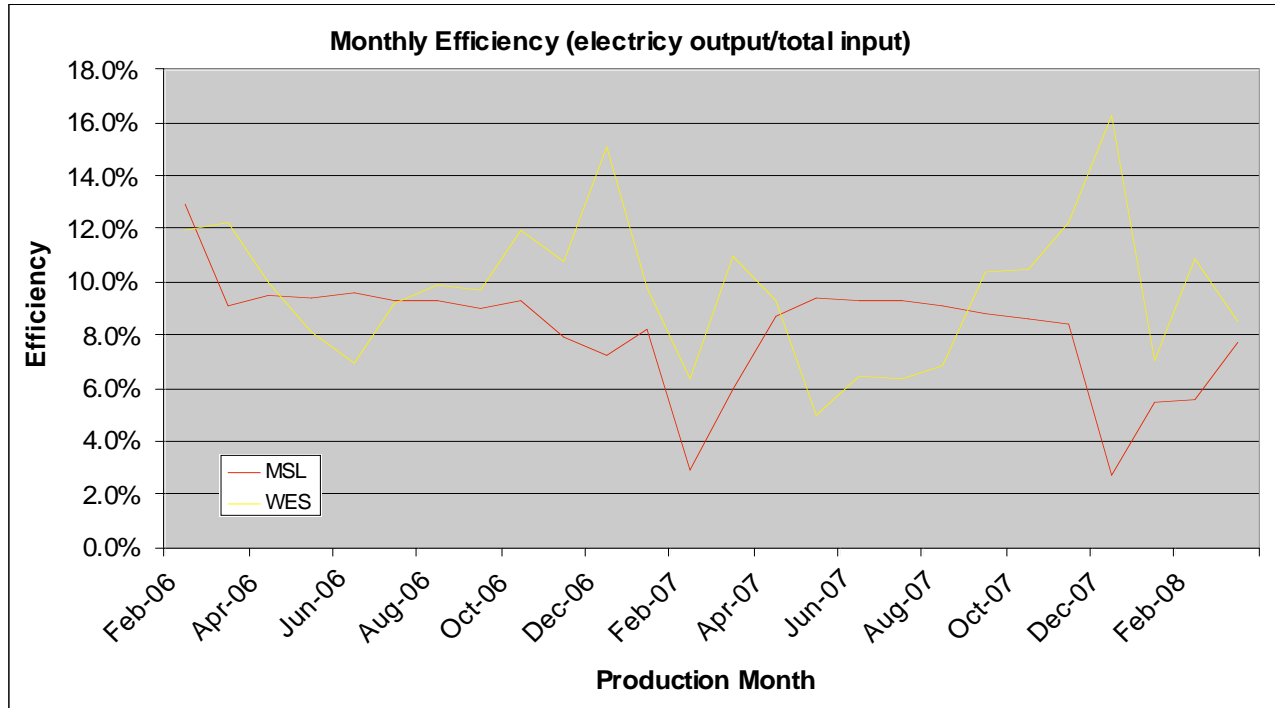


Figure 5. Monthly efficiencies of MSL and WES

## Conclusions

With both analyses, production as a fraction of potential output and production as a function of solar radiation, MSL's efficiency falls short of WES's. That means that in Williamstown, a fixed array inclined at  $32^\circ$  is more efficient than the same one tilted at  $6^\circ$ . That is not to say that  $32^\circ$  is the optimum, merely that it is better than  $6^\circ$ . In fact, the optimum fixed angle probably lies somewhere in the middle of those two values. The most efficient way to install photovoltaic panels, however, would be to engineer arrays that can be seasonally adjusted to an advantageous tilt. Neither the Morley Science Building nor the Williamstown Elementary School were designed with PV arrays in mind: solar panels were retrofit additions in both cases. As such, neither array was engineered as part of the roof of the structure it sits on. Thus, both arrays are necessarily fixed near the angle of the roof they rest on to avoid being ripped off by strong winds. If future arrays can be integrated with the building's roof so that seasonal tilting is

an option, aggregate efficiency could be improved. For example, taking the higher summer efficiencies of the MSL array from *Table 4* and the higher winter values of WES from *Table 3*, an imagined panel that alternates seasonally between  $32^{\circ}$  and  $6^{\circ}$  could potentially achieve an efficiency of 10.4%. This would represent a 16.8% increase in energy output over the 8.94% efficient array fixed at  $32^{\circ}$  tilt. For seasonally adjustable panels to be workable, however, they must be integrated into the designs of new buildings as they are built. Integrating adjustable panels into new architecture and improving efficiency has the potential to reduce payback times, and could make photovoltaics a more economically viable means of electricity cogeneration for the college. This is the direction Williams should go to improve the efficiency of solar arrays it builds in the future.

## Appendix

**Table 1. MSL Production 2005-2008**

Production Month	Production (kWh)
Feb-05	357
Mar-05	597
Apr-05	912
May-05	914
Jun-05	1034
Jul-05	968
Aug-05	871
Sep-05	778
Oct-05	370
Nov-05	294
Dec-05	128
Jan-06	172
Feb-06	391
Mar-06	718
Apr-06	887
May-06	798
Jun-06	767
Jul-06	976
Aug-06	913
Sep-06	599
Oct-06	488
Nov-06	258
Dec-06	193
Jan-07	264
Feb-07	143
Mar-07	401
Apr-07	595
May-07	1144
Jun-07	998
Jul-07	1021
Aug-07	925
Sep-07	787
Oct-07	487
Nov-07	307
Dec-07	41
Jan-08	189
Feb-08	209
Mar-08	525

**Table 2. WES Production 2005-2008**

Production Month	Production (kWh)
Feb-05	1823
Mar-05	1940
Apr-05	3651
May-05	3129
Jun-05	2967
Jul-05	2922
Aug-05	2574.5
Sep-05	2574.5
Oct-05	1558
Nov-05	1677
Dec-05	1238
Jan-06	1201.5
Feb-06	1201.5
Mar-06	3215
Apr-06	3120
May-06	2313
Jun-06	1848
Jul-06	3218
Aug-06	3218
Sep-06	2156
Oct-06	2097
Nov-06	1176
Dec-06	1331
Jan-07	1042
Feb-07	1042
Mar-07	2481
Apr-07	2120
May-07	2047
Jun-07	2321.67
Jul-07	2321.67
Aug-07	2321.67
Sep-07	3077
Oct-07	1971
Nov-07	1501
Dec-07	805
Jan-08	805
Feb-08	1354
Mar-08	1909

**Table 3. Monthly Efficiency of WES Array 2006-2008**

WES

Month	Production (kW h)	Radiation (mJ/m <sup>2</sup> )	Energy In (kW h)	Efficiency
Feb-06	1201.5	184.40	10073.4	11.93%
Mar-06	3215	479.10	26172.2	12.28%
Apr-06	3120	571.40	31214.3	10.00%
May-06	2313	520.90	28455.6	8.13%
Jun-06	1848	486.60	26581.9	6.95%
Jul-06	3218	643.20	35136.6	9.16%
Aug-06	3218	597.10	32618.2	9.87%
Sep-06	2156	407.10	22239.0	9.69%
Oct-06	2097	320.30	17497.3	11.98%
Nov-06	1176	199.80	10914.6	10.77%
Dec-06	1331	161.80	8838.8	15.06%
Jan-07	1042	195.8	10696.1	9.74%
Feb-07	1042	299.1	16339.2	6.38%
Mar-07	2481	413.0	22561.3	11.00%
Apr-07	2120	418.1	22839.9	9.28%
May-07	2047	746.6	40785.1	5.02%
Jun-07	2321.67	658.1	35950.5	6.46%
Jul-07	2321.67	667.3	36453.1	6.37%
Aug-07	2321.67	621.9	33973.0	6.83%
Sep-07	3077	544.1	29723.0	10.35%
Oct-07	1971	345.5	18873.9	10.44%
Nov-07	1501	223.7	12220.2	12.28%
Dec-07	805	90.6	4949.3	16.27%
Jan-08	805	209.40	11439.1	7.04%
Feb-08	1354	228.10	12460.6	10.87%
Mar-08	1909	412.10	22512.1	8.48%
TOTAL	52012.51	10645.1	581518.1572	8.94%

**Table 4. Monthly Efficiency of MSL Array 2006-2008**

Morley

Month	Production (kW h)	Radiation (mJ/m <sup>2</sup> )	Energy In (kW h)	Efficiency
Feb-06	391	184.40	3022.11	12.9%
Mar-06	718	479.10	7851.92	9.1%
Apr-06	887	571.40	9364.61	9.5%
May-06	798	520.90	8536.97	9.3%
Jun-06	767	486.60	7974.83	9.6%
Jul-06	976	643.20	10541.33	9.3%
Aug-06	913	597.10	9785.81	9.3%
Sep-06	599	407.10	6671.92	9.0%
Oct-06	488	320.30	5249.36	9.3%
Nov-06	258	199.80	3274.50	7.9%
Dec-06	193	161.80	2651.72	7.3%
Jan-07	264	195.8	3208.94	8.2%
Feb-07	143	299.1	4901.92	2.9%
Mar-07	401	413.0	6768.61	5.9%
Apr-07	595	418.1	6852.19	8.7%
May-07	1144	746.6	12235.94	9.3%
Jun-07	998	658.1	10785.53	9.3%
Jul-07	1021	667.3	10936.31	9.3%
Aug-07	925	621.9	10192.25	9.1%
Sep-07	787	544.1	8917.19	8.8%
Oct-07	487	345.5	5662.36	8.6%
Nov-07	307	223.7	3666.19	8.4%
Dec-07	41	90.6	1484.83	2.8%
Jan-08	189	209.40	3431.83	5.5%
Feb-08	209	228.10	3738.31	5.6%
Mar-08	525	412.10	6753.86	7.8%
TOTAL	15024	10645.1	174461.36	8.61%