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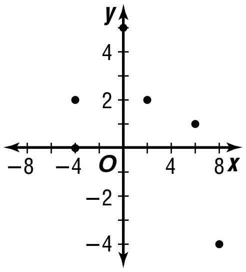
Chapter 1 Linear Relations and Functions

1-1 Relations and Functions

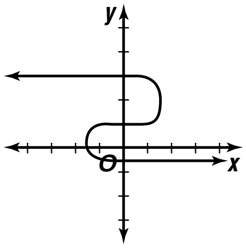
Pages 8–9 Check for Understanding

1.

x	y
-4	2
6	1
0	5
8	-4
2	2
-4	0



2. Sample answer:



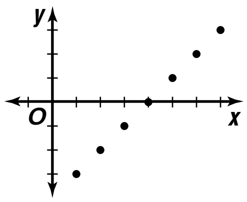
3. Determine whether a vertical line can be drawn through the graph so that it passes through more than one point on the graph. Since it does, the graph does not represent a function.

4. Keisha is correct. Since a function can be expressed as a set of ordered pairs, a function is always a relation. However, in a function, there is exactly one y-value for each x-value. Not all relations have this constraint.

5. Table:

x	y
1	-3
2	-2
3	-1
4	0
5	1
6	2
7	3

Graph:



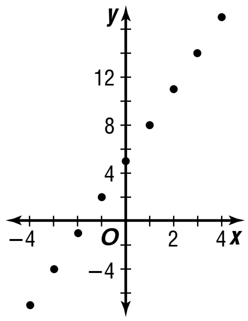
Equation: $y = x - 4$

6. $\{(-3, 4), (0, 0), (3, -4), (6, -8)\}$; $D = \{-3, 0, 3, 6\}$; $R = \{-8, -4, 0, 4\}$

7. $\{(-6, 1), (-4, 0), (-2, -4), (1, 3), (4, 3)\}$; $D = \{-6, -4, -2, 1, 4\}$; $R = \{-4, 0, 1, 3\}$

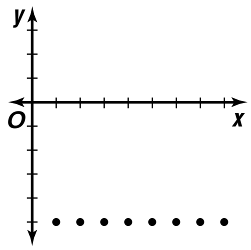
8.

x	y
-4	-7
-3	-4
-2	-1
-1	2
0	5
1	8
2	11
3	14
4	17



9.

x	y
1	-5
2	-5
3	-5
4	-5
5	-5
6	-5
7	-5
8	-5



10. $\{-3, 0, 1, 2\}$; $\{-6, 0, 2, 4\}$; yes; Each member of the domain is matched with exactly one member of the range.

11. $\{-3, 3, 6\}$; $\{-6, -2, 0, 4\}$; no; 6 is matched with two members of the range.

12a. domain: all reals; range: all reals

12b. Yes; the graph passes vertical line test.

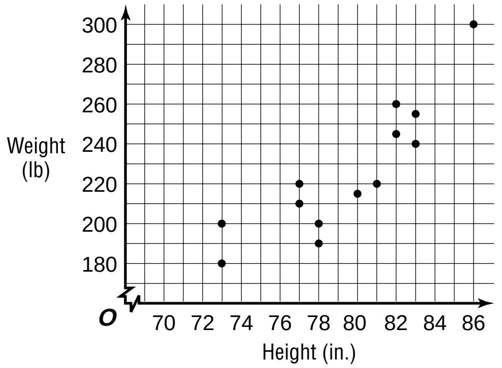
13. $f(-3) = 4(-3)^3 + (-3)^2 - 5(-3)$
$= -108 + 9 + 15 \quad \text{or} \quad -84$

14. $g(m + 1) = 2(m + 1)^2 - 4(m + 1) + 2$
$= 2(m^2 + 2m + 1) - 4m - 4 + 2$
$= 2m^2 + 4m + 2 - 4m - 4 + 2$
$= 2m^2$

15. $x + 1 < 0$
$x < -1$
The domain excludes numbers less than -1.
The domain is $\{x \mid x \geq -1\}$.

16a. $\{(83, 240), (81, 220), (82, 245), (78, 200), (83, 255), (73, 200), (80, 215), (77, 210), (78, 190), (73, 180), (86, 300), (77, 220), (82, 260)\}$; $\{73, 77, 78, 80, 81, 82, 83, 86\}$; $\{180, 190, 200, 210, 215, 220, 240, 245, 255, 260, 300\}$

16b.



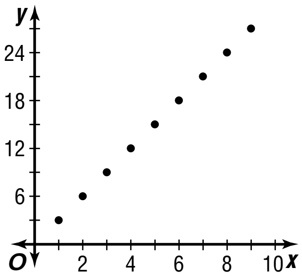
16c. No; a vertical line at $x = 77$, $x = 78$, $x = 82$, or $x = 83$ would pass through two points.

Pages 10–12 Exercises

17. Table

x	y
1	3
2	6
3	9
4	12
5	15
6	18
7	21
8	24
9	27

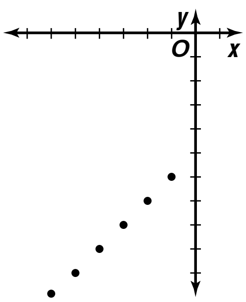
Graph:



Equation: $y = 3x$

18. Table:

x	y
−6	−11
−5	−10
−4	−9
−3	−8
−2	−7
−1	−6

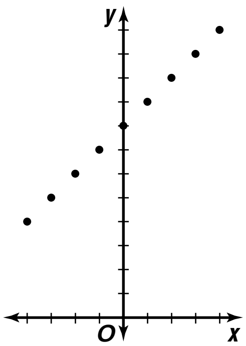


Equation: $y = x - 5$

19. Table:

x	y
−4	4
−3	5
−2	6
−1	7
0	8
1	9
2	10
3	11
4	12

Graph:



Equation: $y = 8 + x$

20. $\{(-5, -5), (-3, -3), (-1, -1), (1, 1)\}$; $D = \{-5, -3, -1, 1\}$; $R = \{-5, -3, -1, 1\}$

21. $\{(-10, 0), (-5, 0), \{0, 0), (5, 0)\}$; $D = \{-10, -5, 0, 5\}$; $R = \{0\}$

22. $\{(4, 0), (5, 1), (8, 0), (13, 1)\}$; $D = \{4, 5, 8, 13\}$; $R = \{0, 1\}$

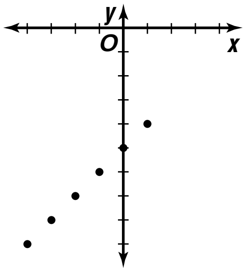
23. $\{(-3, -2), (-1, 1), (0, 0), (1, 1)\}$; $D = \{-3, -1, 0, 1\}$; $R = \{-2, 0, 1\}$

24. $\{(-5, 5), (-3, 3), (-1, 1), (2, -2), (4, -4)\}$; $D = \{-5, -3, -1, 2, 4\}$; $R = \{-4, -2, 1, 3, 5\}$

25. $\{(3, -4), (3, -2), (3, 0), (3, 1), (3, 3)\}$; $D = \{3\}$; $R = \{-4, -2, 0, 1, 3\}$

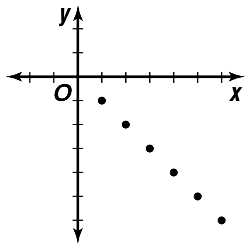
26.

x	y
−4	−9
−3	−8
−2	−7
−1	−6
0	−5
1	−4



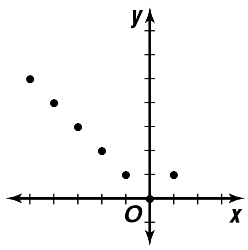
27.

x	y
1	−1
2	−2
3	−3
4	−4
5	−5
6	−6



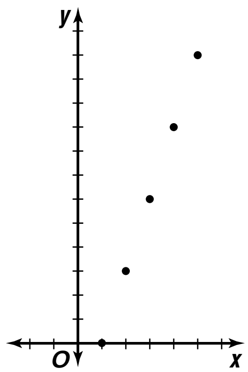
28.

x	y
−5	5
−4	4
−3	3
−2	2
−1	1
0	0
1	1



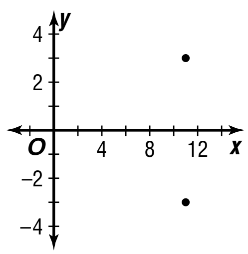
29.

x	y
1	0
2	3
3	6
4	9
5	12



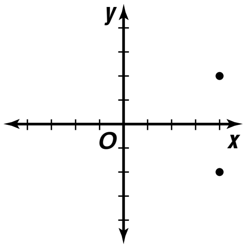
30.

x	y
11	3
11	−3



31.

x	y
4	2
4	-2



32. {4, 5, 6}; {4}; yes; Each x-value is paired with exactly one y-value.

33. {1}; {−6, −2, 0, 4}; no; The x-value 1 is paired with more than one y-value.

34. {0, 1, 4); {−2, −1, 0, 1, 2}; no; The x-values 1 and 4 are paired with more than one y-value.

35. {0, 2, 5}; {−8, −2, 0, 2, 8}; no; The x-values 2 and 5 are paired with more than one y-value.

36. {−1.1, −0.4, −0.1}; {−2, −1}; yes; Each x-value is paired with exactly one y-value.

37. {−9, 2, 8, 9}; {−3, 0, 8}; yes; Each x-value is paired with exactly one y-value.

38. domain: all reals; range: all reals; Not a function because it fails the vertical line test.

39. domain: {−3, −2, −1, 1, 2, 3}; range: {−1, 1, 2, 3}; A function because each x-value is paired with exactly one y-value.

40. domain: $\{x \mid -8 \le x \le 8\}$; range: $\{y \mid -8 \le y \le 8\}$; Not a function because it fails the vertical line test.

41. $f(3) = 2(3) + 3$
$= 6 + 3$ or 9

42. $g(-2) = 5(-2)^2 + 3(-2) - 2$
$= 20 - 6 - 2$ or 12

43. $h(0.5) = \frac{1}{0.5}$
$= 2$

44. $j(2a) = 1 - 4(2a)^3$
$= 1 - 4(8a^3)$
$= 1 - 32a^3$

45. $f(n - 1) = 2(n - 1)^2 - (n - 1) + 9$
$= 2(n^2 - 2n + 1) - n + 1 + 9$
$= 2n^2 - 4n + 2 - n + 1 + 9$
$= 2n^2 - 5n + 12$

46. $g(b^2 + 1) = \frac{3 - (b^2 + 1)}{5 + (b^2 + 1)}$
$= \frac{3 - b^2 - 1}{6 + b^2}$ or $\frac{2 - b^2}{6 + b^2}$

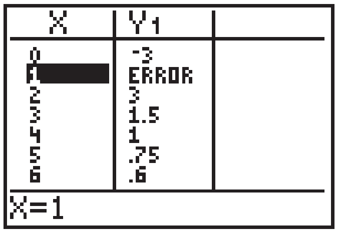
47. $f(5m) = |(5m)^2 - 13|$
$= |25m^2 - 13|$

48. $x^2 - 5 = 0$
$x^2 = 5$
$x = \pm\sqrt{5}$; $x \ne \pm\sqrt{5}$

49. $x^2 - 9 < 0$
$x^2 < 9$
$-3 < x < 3$; $x < -3$ or $x \ge 3$

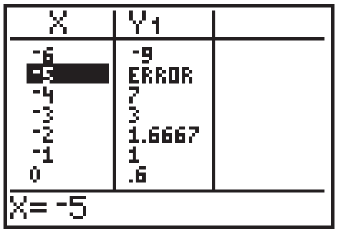
50. $x^2 - 7 \le 0$
$x^2 \le 7$
$-\sqrt{7} \le x \le \sqrt{7}$; $x < -\sqrt{7}$ or $x > \sqrt{7}$

51a.



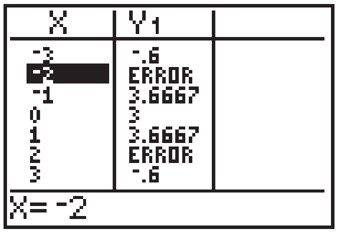
$x \ne 1$

51b.



$x \ne -5$

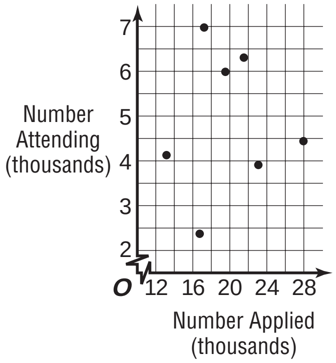
51c.



$x \ne -2, 2$

52a. {(13,264, 4184), (27,954, 4412), (21,484, 6366), (23,117, 3912), (16,849, 2415), (19,563, 5982), (17,284, 6949)}; {13,264, 16,849, 17,284, 19,563, 21,484, 23,117, 27,954}; {2415, 3912, 4184, 4412, 5982, 6366, 6949}

52b.



52c. Yes; no member of the domain is paired with more than one member of the range.

53. $x = 2m + 1$, so $\frac{x-1}{2} = m$.
Substitute $\frac{x-1}{2}$ for m in $f(2m + 1)$ to solve for $f(x)$,
$24m^3 + 36m^2 + 26m$
$= 24\left(\frac{x-1}{2}\right)^3 + 36\left(\frac{x-1}{2}\right)^2 + 26\left(\frac{x-1}{2}\right)$
$= 24\left(\frac{x^3 - 3x^2 + 3x - 1}{8}\right) + 36\left(\frac{x^2 - 2x + 1}{4}\right) + 26\left(\frac{x-1}{2}\right)$
$= 3x^3 - 9x^2 + 9x - 3 + 9x^2 - 18x + 9 + 13x - 13$
$= 3x^3 + 4x - 7$

54a. $t(500) = 95 - 0.005(500)$
$= 92.5°F$

54b. $t(750) = 95 - 0.005(750)$
$= 91.25°F$

54c. $t(1000) = 95 - 0.005(1000)$
$= 90°F$

54d. $t(5000) = 95 - 0.005(5000)$
$= 70°F$

54e. $t(30{,}000) = 95 - 0.005(30{,}000)$
$= -55°F$

55a. $d(0.05) = 299{,}792{,}458(0.05)$
$= 14{,}989{,}622.9$ m

$d(0.02) = 299{,}792{,}458(0.2)$
$= 59{,}958{,}491.6$ m

$d(1.4) = 299{,}792{,}458(1.4)$
$= 419{,}709{,}441.2$ m

$d(5.9) = 299{,}792{,}458(5.9)$
$= 1{,}768{,}775{,}502$ m

55b. $d(0.008) = 299{,}792{,}458(0.08)$
$= 23{,}983{,}396.64$ m

56. $P(4) = \frac{(1)(2) + 1}{3} = 1$

$P(5) = \frac{(2)(3) + 1}{1} = 7$

$P(6) = \frac{(3)(1) + 1}{7} = \frac{4}{7}$

57. $7^2 - (3^2 + 4^2) = 49 - (9 + 16)$
$= 49 - 25$ or 24

The correct choice is B.

1-2 Composition of Functions

Page 13 Graphing Calculator Exploration

1.

X	Y1	Y2
0	-1	2
1	1	5
2	3	8
3	5	11
4	7	14
5	9	17
6	11	20

Y1=-1

X	Y2	Y3
0	2	-3
1	5	-4
2	8	-5
3	11	-6
4	14	-7
5	17	-8
6	20	-9

Y3=-3

2.

X	Y1	Y2
0	-1	2
1	1	5
2	3	8
3	5	11
4	7	14
5	9	17
6	11	20

X=0

X	Y2	Y3
0	2	-2
1	5	5
2	8	24
3	11	55
4	14	98
5	17	153
6	20	220

Y3=-2

3.

X	Y1	Y2
0	-1	2
1	1	5
2	3	8
3	5	11
4	7	14
5	9	17
6	11	20

X=0

X	Y2	Y3
0	2	-.5
1	5	.2
2	8	.375
3	11	.45455
4	14	.5
5	17	.52941
6	20	.55

Y3=-.5

4. Sample answer: The (sum/difference/product/quotient) of the function values is the function values of the (sum/difference/product/quotient) of the functions.

5. Sample answer: For functions $f(x)$ and $g(x)$, $(f + g)(x) = f(x) + g(x)$; $(f - g)(x) = f(x) - g(x)$; $(f \cdot g)(x) = f(x) \cdot g(x)$; and $\left(\frac{f}{g}\right)(x) = \left(\frac{f(x)}{g(x)}\right)$, $g(x) \neq 0$

Page 17 Check for Understanding

1. Sample answer: $f(x) = 2x - 1$ and $g(x) = x + 6$; Sample explanation: Factor $2x^2 + 11x - 6$.

2. Iteration is composing a function on itself by evaluating the function for a value and then evaluating the function on that function value.

3. No; $[f \circ g](x)$ is the function $f(x)$ performed on $g(x)$ and $[g \circ f](x)$ is the function $g(x)$ performed on $f(x)$. See students' counter examples.

4. Sample answer: Composition of functions is performing one function after another. An everyday example is putting on socks and then putting shoes on top of the socks. Buying an item on sale is an example of when a composition of functions is used in a real-world situation.

5. $f(x) + g(x) = 3x^2 + 4x - 5 + 2x + 9$
$= 3x^2 + 6x + 4$

$f(x) - g(x) = 3x^2 + 4x - 5 - (2x + 9)$
$= 3x^2 + 2x - 14$

$f(x) \cdot g(x) = (3x^2 + 4x - 5)(2x + 9)$
$= 6x^3 + 35x^2 + 26x - 45$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$
$= \frac{3x^2 + 4x - 5}{2x + 9}, x \neq -\frac{9}{2}$

6. $[f \circ g](x) = f(g(x))$
$= f(3 + x)$
$= 2(3 + x) + 5$
$= 2x + 11$

$[g \circ f](x) = g(f(x))$
$= g(2x + 5)$
$= 3 + (2x + 5)$
$= 2x + 8$

7. $[f \circ g](x) = f(g(x))$
$= f(x^2 - 2x)$
$= 2(x^2 - 2x) - 3$
$= 2x^2 - 4x - 3$

$[g \circ f](x) = g(f(x))$
$= g(2x - 3)$
$= (2x - 3)^2 - 2(2x - 3)$
$= (4x^2 - 12x + 9) - 4x + 6$
$= 4x^2 - 16x + 15$

8. Domain of $f(x)$: $x \neq 1$
Domain of $g(x)$: all reals
$g(x) = 1$
$x + 3 = 1$
$x = -2$
Domain of $[f \circ g](x)$ is $x \neq -2$.

9. $x_1 = f(x_0) = f(2)$
$= 2(2) + 1$ or 5
$x_2 = f(x_1) = f(5)$
$= 2(5) + 1$ or 11
$x_3 = f(x_2) = f(11)$
$= 2(11) + 1$ or 23
5, 11, 23

10a. $[K \circ C](F) = K(C(F))$
$= K\left(\frac{5}{9}(F - 32)\right)$
$= \frac{5}{9}(F - 32) + 273.15$

10b. $K(-40) = \frac{5}{9}(-40 - 32) + 273.15$
$= -40 + 273.15$ or 233.15
$K(-12) = \frac{5}{9}(-12 - 32) + 273.15$
$= -24.44 + 273.15$ or 248.71
$K(0) = \frac{5}{9}(0 - 32) + 273.15$
$= -17.78 + 273.15$ or 255.37
$K(32) = \frac{5}{9}(32 - 32) + 273.15$
$= 0 + 273.15$ or 273.15
$K(212) = \frac{5}{9}(212 - 32) + 273.15$
$= 100 + 273.15$ or 373.15

Pages 17–19 Exercises

11. $f(x) + g(x) = x^2 - 2x + x + 9$
$= x^2 - x + 9$
$f(x) - g(x) = x^2 - 2x - (x + 9)$
$= x^2 - 3x - 9$
$f(x) \cdot g(x) = (x^2 - 2x)(x + 9)$
$= x^3 + 7x^2 - 18x$
$\left(\frac{f}{g}\right)(x) = \frac{x^2 - 2x}{x + 9}, x \neq 9$

12. $f(x) + g(x) = \frac{x}{x + 1} + x^2 - 1$
$= \frac{x}{x + 1} + \frac{(x^2 - 1)(x + 1)}{x + 1}$
$= \frac{x}{x + 1} + \frac{x^3 + x^2 - x - 1}{x + 1}$
$= \frac{x^3 + x^2 - 1}{x + 1}, x \neq -1$

$f(x) - g(x) = \frac{x}{x + 1} - (x^2 - 1)$
$= \frac{x}{x + 1} - \frac{(x^2 - 1)(x + 1)}{x + 1}$
$= \frac{x}{x + 1} - \frac{x^3 + x^2 - x - 1}{x + 1}$
$= \frac{-x^3 - x^2 + 2x + 1}{x + 1}, x \neq -1$

$f(x) + g(x) = \frac{x}{x + 1} \cdot (x^2 - 1)$
$= \frac{x(x + 1)(x - 1)}{x + 1}$
$= x^2 - x, x \neq -1$

$\left(\frac{f}{g}\right)(x) = \frac{\frac{x}{x + 1}}{x^2 - 1}$
$= \frac{x}{x + 1} \cdot \frac{1}{x^2 - 1}$
$= \frac{x}{x^3 + x^2 - x - 1}, x \neq -1$ or 1

13. $f(x) + g(x) = \frac{3}{x - 7} + x^2 + 5x$
$= \frac{3}{x - 7} + \frac{(x^2 + 5x)(x - 7)}{x - 7}$
$= \frac{3}{x - 7} + \frac{x^3 - 7x^2 + 5x^2 - 35x}{x - 7}$
$= \frac{x^3 - 2x^2 - 35x + 3}{x - 7}, x \neq 7$

$f(x) - g(x) = \frac{3}{x - 7} - (x^2 + 5x)$
$= \frac{3}{x - 7} - \frac{(x^2 + 5x)(x - 7)}{x - 7}$
$= \frac{3}{x - 7} - \frac{x^3 - 7x^2 + 5x^2 - 35x}{x - 7}$
$= -\frac{x^3 - 2x^2 - 35x - 3}{x - 7}, x \neq 7$

$f(x) \cdot g(x) = \frac{3}{x - 7} \cdot (x^2 + 5x)$
$= \frac{3x^2 + 15x}{x - 7}, x \neq 7$

$\left(\frac{f}{g}\right)(x) = \frac{\frac{3}{x - 7}}{x^2 + 5x}$
$= \frac{3}{x - 7} \cdot \frac{1}{x^2 + 5x}$
$= \frac{3}{x^3 - 2x^2 - 35x}, x \neq -5, 0, 7$

14. $f(x) + g(x) = x + 3 + \frac{2x}{x - 5}$
$= \frac{(x + 3)(x - 5)}{x - 5} + \frac{2x}{x - 5}$
$= \frac{x^2 - 2x - 15}{x - 5} + \frac{2x}{x - 5}$
$= \frac{x^2 - 15}{x - 5}, x \neq 5$

$f(x) - g(x) = x + 3 - \left(\frac{2x}{x - 5}\right)$
$= \frac{(x + 3)(x - 5)}{x - 5} - \frac{2x}{x - 5}$
$= \frac{x^2 - 2x - 15}{x - 5} - \frac{2x}{x - 5}$
$= \frac{x^2 - 4x - 15}{x - 5}, x \neq 5$

$f(x) \cdot g(x) = (x + 3) - \left(\frac{2x}{x - 5}\right)$
$= \frac{2x^2 + 6x}{x - 5}, x \neq 5$

$\left(\frac{f}{g}\right)(x) = \frac{x + 3}{\frac{2x}{x - 5}}$
$= x + 3 \cdot \frac{x - 5}{2x}$
$= \frac{x^2 - 2x - 15}{2x}, x \neq 0$ or 5

15. $[f \circ g](x) = f(g(x))$
$= f(x + 4)$
$= (x + 4)^2 - 9$
$= x^2 + 8x + 16 - 9$
$= x^2 + 8x + 7$

$[g \circ f](x) = g(f(x))$
$= g(x^2 - 9)$
$= x^2 - 9 + 4$
$= x^2 - 5$

16. $[f \circ g](x) = f(g(x))$
$= f(x + 6)$
$= \frac{1}{2}(x + 6) - 7$
$= \frac{1}{2}x + 3 - 7$
$= \frac{1}{2}x - 4$

$[g \circ f](x) = g(f(x))$
$= g(\frac{1}{2}x - 7)$
$= \frac{1}{2}x - 7 + 6$
$= \frac{1}{2}x - 1$

17. $[f \circ g](x) = f(g(x))$
$= f(3x^2)$
$= 3x^2 - 4$

$[g \circ f](x) = g(f(x))$
$= g(x - 4)$
$= 3(x - 4)^2$
$= 3(x^2 - 8x + 16)$
$= 3x^2 - 24x + 48$

18. $[f \circ g](x) = f(g(x))$
$= f(5x^2)$
$= (5x^2)^2 - 1$
$= 25x^4 - 1$

$[g \circ f](x) = g(f(x))$
$= g(x^2 - 1)$
$= 5(x^2 - 1)^2$
$= 5(x^4 - 2x^2 + 1)$
$= 5x^4 - 10x^2 + 5$

19. $[f \circ g](x) = f(g(x))$
$= f(x^3 + x^2 + 1)$
$= 2(x^3 + x^2 + 1)$
$= 2x^3 + 2x^2 + 2$

$[g \circ f](x) = g(f(x))$
$= g(2x)$
$= (2x)^3 + (2x)^2 + 1$
$= 8x^3 + 4x^2 + 1$

20. $[f \circ g](x) = f(g(x))$
$= f(x^2 + 5x + 6)$
$= 1 + x^2 + 5x + 6$
$= x^2 + 5x + 7$

$[g \circ f](x) = g(f(x))$
$= g(1 + x)$
$= (x + 1)^2 + 5(x + 1) + 6$
$= x^2 + 2x + 1 + 5x + 5 + 6$
$= x^2 + 7x + 12$

21. $[f \circ g](x) = f\big(g(x)\big)$
$= f\left(\frac{1}{x - 1}\right)$
$= \frac{1}{x - 1} + 1$
$= \frac{1}{x - 1} + \frac{x - 1}{x - 1}$
$= \frac{x}{x - 1}, x \neq 1$

$[g \circ f](x) = g(f(x))$
$= g(x + 1)$
$= \frac{1}{x + 1 - 1}$
$= \frac{1}{x}, x \neq 0$

22. Domain of $f(x)$: all reals
Domain of $g(x)$: all reals
Domain of $[f \circ g](x)$: all reals

23. Domain of $f(x)$: $x \neq 0$
Domain of $g(x)$: all reals
$g(x) = 0$
$7 - x = 0$
$7 = x$
Domain of $[f \circ g](x)$ is $x \neq 7$.

24. Domain of $f(x)$: $x \geq 2$
Domain of $g(x)$: $x \neq 0$
$g(x) \geq 2$
$\frac{1}{4}x \geq 2$
$1 \geq 8x$
$\frac{1}{8} \geq x$
Domain of $[f \circ g](x)$ is $x \leq \frac{1}{8}, x \neq 0$.

25. $x_1 = f(x_0) = f(2)$
$= 9 - 2$ or 7
$x_2 = f(x_1) = f(7)$
$= 9 - 7$ or 2
$x_3 = f(x_2) = f(2)$
$= 9 - 2$ or 7
7, 2, 7

26. $x_1 = f(x_0) = f(1)$
$= (1)^2 + 1$ or 2
$x_2 = f(x_1) = f(2)$
$= (2)^2 + 1$ or 5
$x_3 = f(x_2) = f(5)$
$= (5)^2 + 1$ or 26
2, 5, 26

27. $x_1 = f(x_0) = f(1)$
$= 1(3 - 1)$ or 2
$x_2 = f(x_1) = f(2)$
$= 2(3 - 2)$ or 2
$x_3 = f(x_2) = f(2)$
$= 2(3 - 2)$ or 2
2, 2, 2

28. \$43.98 + \$38.59 + \$31.99 = \$114.56

Let x = the original price of the clothes, or \$114.56.

Let $T(x) = 1.0825x$. (The cost with 8.25% tax rate)

Let $S(x) = 0.75x$. (The cost with 25% discount)

The cost of clothes is $[T \circ S](x)$.

$$\begin{aligned}[T \circ S](x) &= T(S(x)) \\ &= T(0.75x) \\ &= T(0.75(114.56)) \\ &= T(85.92) \\ &= 1.0825(85.92) \\ &= 93.0084\end{aligned}$$

Yes; the total with the discount and tax is \$93.01.

29. Yes; If $f(x)$ and $g(x)$ are both lines, they can be represented as $f(x) = m_1x + b_1$ and $g(x) = m_2x + b_2$. Then $[f \circ g](x) = m_1(m_2x + b_2) + b_1$
$= m_1m_2x + m_1b_2 + b_1$

Since m_1 and m_2 are constants, m_1m_2 is a constant. Similarly, m_1, b_2, and b_1 are constants, so $m_1b_2 + b_1$ is a constant. Thus, $[f \circ g](x)$ is a linear function if $f(x)$ and $g(x)$ are both linear.

30a. $$\begin{aligned}W_n &= W_p - W_f \\ &= F_pd - F_fd \\ &= d(F_p - F_f)\end{aligned}$$

30b. $$\begin{aligned}W_n &= d(F_p - F_f) \\ &= 50(95 - 55) \\ &= 2000 \text{ J}\end{aligned}$$

31a. $h[f(x)]$, because you must subtract before figuring the bonus

31b. $$\begin{aligned}h[f(x)] &= h[f(400{,}000)] \\ &= h(400{,}000 - 275{,}000) \\ &= h(125{,}000) \\ &= 0.03(125{,}000) \\ &= \$3750\end{aligned}$$

32. $$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(1 - x^2) \\ &= \frac{x^2(x^2 + 1)}{1 + x^2} \\ &= x^2 \\ &= -(1 - x^2) + 1\end{aligned}$$

So, $f(x) = -x + 1$ and $f\left(\frac{1}{2}\right) = -\frac{1}{2} + 1 = \frac{1}{2}$.

33a. $v(p) = \frac{7p}{47}$

33b. $r(v) = 0.84v$

33c. $$\begin{aligned}r(p) &= r(v(p)) \\ &= r\left(\frac{7p}{47}\right) \\ &= 0.84\left(\frac{7p}{47}\right) \\ &= \frac{5.88p}{47} \text{ or } \frac{147p}{1175}\end{aligned}$$

33d. $$\begin{aligned}r(423.18) &= \frac{147(423.18)}{1175} \\ &= \$52.94 \\ r(225.64) &= \frac{147(225.64)}{1175} \\ &= \$28.23 \\ r(797.05) &= \frac{147(797.05)}{1175} \\ &= \$99.72\end{aligned}$$

34a. $$\begin{aligned}I &= prt \\ &= 5000(0.08)(1) \\ &= 400\end{aligned}$$
$$\begin{aligned}I &= prt \\ &= 5400(0.08)(1) \\ &= 432\end{aligned}$$
$$\begin{aligned}I &= prt \\ &= 5832(0.08)(1) \\ &= 466.56\end{aligned}$$
$$\begin{aligned}I &= prt \\ &= 6298.56(0.08)(1) \\ &= 503.88\end{aligned}$$
$$\begin{aligned}I &= prt \\ &= 6802.44(0.08)(1) \\ &= 544.20\end{aligned}$$

(year, interest): (1, \$400), (2,\$432), (3, \$466.56), (4, \$503.88), (5, \$544.20)

34b. {1, 2, 3, 4, 5}; {\$400, \$432, \$466.56, \$503.88, \$544.20}

34c. Yes; for each element of the domain there is exactly one corresponding element of the range.

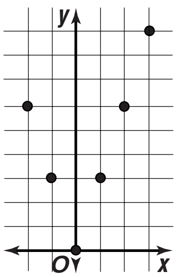
35. $\{(-1, 8), (0, 4), (2, -6), (5, -9)\}$; $D = \{-1, 0, 2, 5\}$; $R = \{-9, -6, 4, 8\}$

36. $D = \{1, 2, 3, 4\}$; $R = \{5, 6, 7, 8\}$; Yes, every element in the domain is paired with exactly one element of the range.

37. $$\begin{aligned}g(-4) &= \frac{(-4)^3 + 5}{4(-4)} \\ &= \frac{-64 + 5}{-16} \\ &= \frac{-59}{-16} \text{ or } 3\tfrac{11}{16}\end{aligned}$$

38.

x	y
−2	6
−1	3
0	0
1	3
2	6
3	9



39. $$\begin{aligned}f(n - 1) &= 2(n - 1)^2 - (n - 1) + 9 \\ &= 2(n^2 - 2n + 1) - n + 1 + 9 \\ &= 2n^2 - 5n + 12\end{aligned}$$

The correct choice is C.

1-3 Graphing Linear Equations

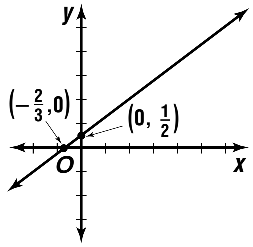
Page 23 Check for Understanding

1. m represents the slope of the graph and b represents the y-intercept
2. 7; the line intercepts the x-axis at (7, 0)
3. Sample answer: Graph the y-intercept at (0, 2). Then move down 4 units and right 1 unit to graph a second point. Draw a line to connect the points.

4. Sample answer: Both graphs are lines. Both lines have a y-intercept of 8. The graph of $y = 5x + 8$ slopes upward as you move from left to right on the graph and the graph of $y = -5y + 8$ slopes downward as you move from left to right on the graph.

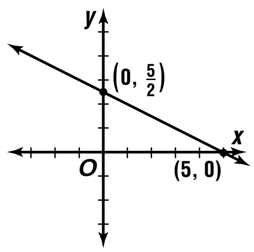
5. $3x - 4(0) + 2 = 0$ $\quad$ $3(0) - 4y + 2 = 0$

$$\begin{aligned} 3x + 2 &= 0 & -4y + 2 &= 0 \\ 3x &= -2 & -4y &= -2 \\ x &= -\tfrac{2}{3} & y &= \tfrac{1}{2} \end{aligned}$$

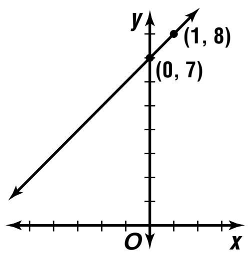


6. $x + 2(0) - 5 = 0$ $\quad$ $0 + 2y - 5 = 0$

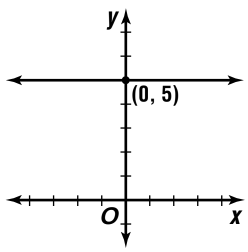
$$\begin{aligned} x - 5 &= 0 & 2y - 5 &= 0 \\ x &= 5 & 2y &= 5 \\ & & y &= \tfrac{5}{2} \end{aligned}$$



7. The y-intercept is 7. Graph (0, 7).
The slope is 1.

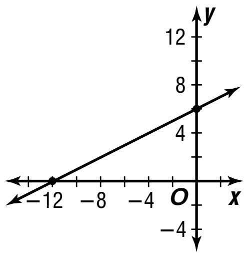


8. The y-intercept is 5. Graph (0, 5).
The slope is 0.

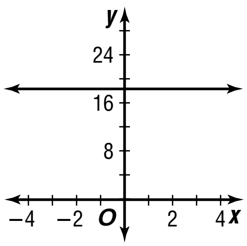


9. $\frac{1}{2}x + 6 = 0$

$$\begin{aligned} \tfrac{1}{2}x &= -6 \\ x &= -12 \end{aligned}$$



10. Since $m = 0$ and $b = 19$, this function has no x-intercept, and therefore no zeros.



11a. (38.500, 173), (44.125, 188)

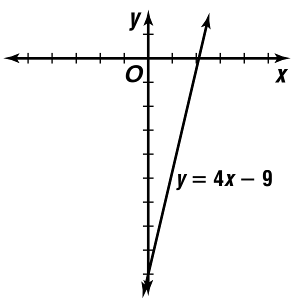
11b. $m = \frac{188 - 173}{44.125 - 38.500}$

$= \frac{15}{5.625}$ or about 2.667

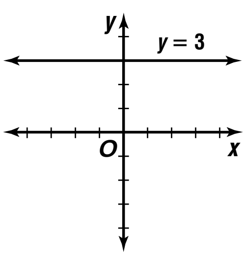
11c. For each 1 centimeter increase in the length of a man's tibia, there is an 2.667-centimeter increase in the man's height.

Pages 24–25 Exercises

12. The y-intercept is -9. The slope is 4.



13. The y-intercept is 3. The slope is 0.

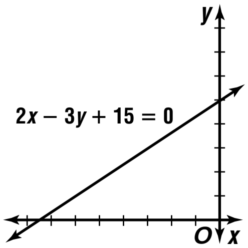


14. $2x - 3y + 15 = 0$

$$-3y = -2x - 15$$

$$y = \frac{2}{3}x + 5$$

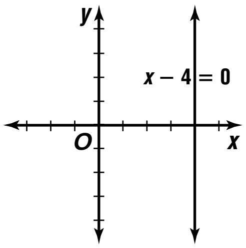
The y-intercept is 5. The slope is $\frac{2}{3}$.



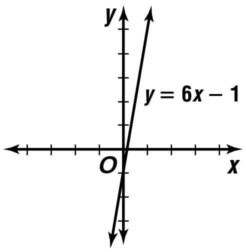
15. $x - 4 = 0$

$$x = 4$$

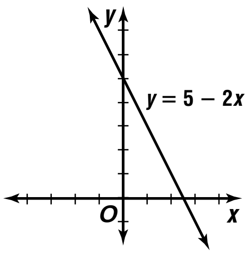
There is no slope. The x-intercept is 4.



16. The y-intercept is -1. The slope is 6.



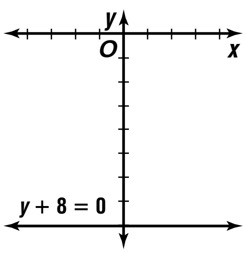
17. The y-intercept is 5. The slope is -2.



18. $y + 8 = 0$

$$y = -8$$

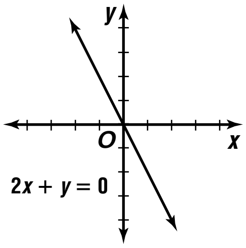
The y-intercept is -8. The slope is 0.



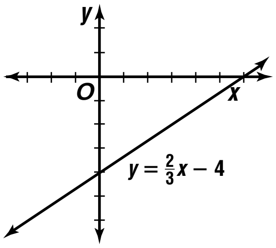
19. $2x + y = 0$

$$y = -2x$$

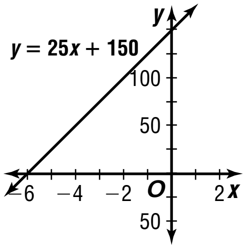
The y-intercept is 0. The slope is -2.



20. The y-intercept is -4. The slope is $\frac{2}{3}$.



21. The y-intercept is 150. The slope is 25.

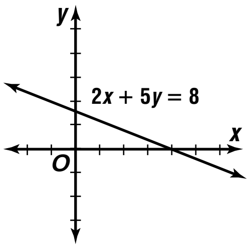


22. $2x + 5y = 8$

$$5y = -2x + 8$$

$$y = -\frac{2}{5}x + \frac{8}{5}$$

The y-intercept is $\frac{8}{5}$. The slope is $-\frac{2}{5}$.

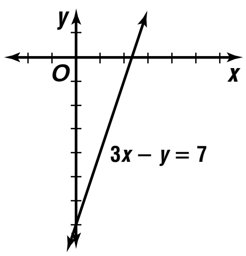


23. $3x - y = 7$

$$-y = -3x + 7$$

$$y = 3x - 7$$

The y-intercept is -7. The slope is 3.

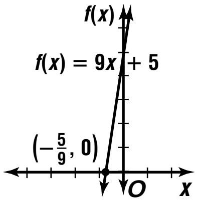


24. $9x + 5 = 0$

$9x = -5$

$x = -\frac{5}{9}$

The y-intercept is 5.

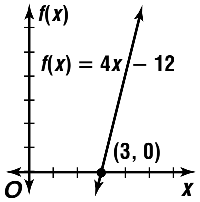


25. $4x - 12 = 0$

$4x = 12$

$x = 3$

The y-intercept is -12.

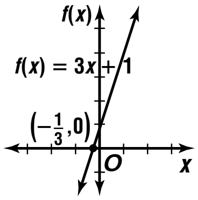


26. $3x + 1 = 0$

$3x = -1$

$x = -\frac{1}{3}$

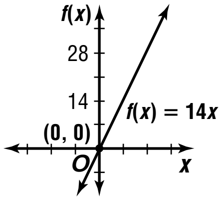
The y-intercept is 1.



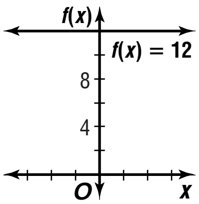
27. $14x = 0$

$x = 0$

The slope is 14.



28. None; since $m = 0$ and $b = 12$, this function has no x-intercepts, and therefore no zeros.

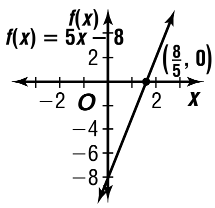


29. $5x - 8 = 0$

$5x = 8$

$x = \frac{8}{5}$

The y-intercept is -8.



30. $5x - 2 = 0$

$5x = 2$

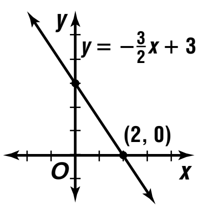
$x = \frac{2}{5}$

31. The y-intercept is 3. The slope is $-\frac{3}{2}$.

$-\frac{3}{2}x + 3 = 0$

$-\frac{3}{2}x = -3$

$x = 2$



32. Sample answer: $f(x) = 5$; $f(x) = 0$

33a. (1.0, 12.0), (10.0, 8.4)

$m = \frac{8.4 - 12.0}{10.0 - 1.0}$

$= \frac{-3.6}{9}$ or -0.4

$-(-0.4) = 0.4$ ohms

33b. $-0.4 = \frac{12 - v}{1.0 - 25.0}$

$-0.4 = \frac{12 - v}{-24}$

$9.6 = 12 - v$

$v = 2.4$ volts

34. $m = \frac{9 - 7}{-4 - 3}$

$= \frac{2}{-7}$

$-\frac{2}{7} = \frac{1 - 9}{a - (-4)}$

$-\frac{2}{7} = \frac{-8}{a + 4}$

$-2(a + 4) = -56$

$-2a - 8 = -56$

$-2a = -48$

$a = 24$

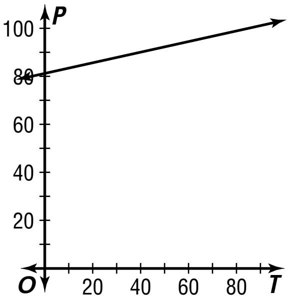
35a. (86.85, 90), (126.85, 100)

$m = \frac{100 - 90}{126.85 - 86.85}$

$= \frac{10}{40}$ or $\frac{1}{4}$

35b. For each 1 degree increase in the temperature, there is a $\frac{1}{4}$-pascal increase in the pressure.

35c.



36. No; the product of two positives is positive, so for the product of the slopes to be -1, one of the slopes must be positive and the other must be negative.

37a. $10{,}440 - 290t = 0$

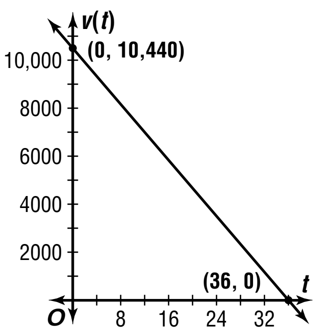
$-290t = -10{,}440$

$t = 36$

The software has no monetary value after 36 months.

37b. -290; For every 1-month change in the number of months, there is a \$290 decrease in the value of the software.

37c.



38. A function with a slope of 0 has no zeros if its y-intercept is not 0; a function with a slope of 0 has an infinite number of zeros if its y-intercept is 0; a function with any slope other than 0 has exactly 1 zero.

39a. (56, 50), (76, 67.2)

$$m = \frac{67.2 - 50}{76 - 56} = \frac{17.2}{20} \text{ or } 0.86$$

39b. $1805(0.86) = \$1552.30$

39c. $1 - \text{MPC} = 1 - 0.86 = 0.14$

39d. $1805(0.14) = \$252.70$

40. $(f + g)(x) = 2x + x^2 - 4$ or $x^2 + 2x - 4$

$$(f - g)(x) = 2x - (x^2 - 4) = -x^2 + 2x + 4$$

41a. $1 - 0.12 = 0.88$

$d(p) = 0.88p$

41b. $r(d) = d - 100$

41c. $r(d(p)) = r(0.88p) = 0.88p - 100$

41d.

$$r(799.99) = 0.88(799.99) - 100 = 603.9912 \text{ or about } \$603.99$$

$$r(999.99) = 0.88(999.99) - 100 = 779.9912 \text{ or about } \$779.99$$

$$r(1499.99) = 0.88(1499.99) - 100 = 1219.9912 \text{ or about } \$1219.99$$

42.

$$\begin{aligned}[f \circ g](-3) &= f(g(-3)) \\ &= f(-3 - 2) \\ &= f(-5) \\ &= (-5)^2 - 4(-5) + 5 \\ &= 25 + 20 + 5 \text{ or } 50\end{aligned}$$

$$\begin{aligned}[g \circ f](-3) &= g(f(-3)) \\ &= g((-3)^2 - 4(-3) + 5) \\ &= g(9 + 12 + 5) \\ &= g(26) \\ &= 26 - 2 \text{ or } 24\end{aligned}$$

43. $f(9) = 4 + 6(9) - (9)^3 = 4 + 54 - 729$ or -671

44. No; the graph fails the vertical line test.

45.

x	y
-3	14
-2	13
-1	12
0	11

{(−3, 14), (−2, 13), (−1, 12), (0, 11)}, yes

46. Let s = sum.

$\frac{s}{4} = 15$

$s = 60$

The correct choice is D.

1-3B Graphing Calculator Exploration: Analyzing Families of Linear Graphs

Page 26

1. See students' graphs. All of the graphs are lines with y-intercept at $(0, -2)$. Each line has a different slope.
2. A line parallel to the ones graphed in the Example and passing through $(0, -2)$.
3. See students' sketches. Sample answer: The graphs of lines with the same value of m are parallel. The graphs of lines with the same value for b have the same y-intercept.

1-4 Writing Linear Equations

Page 29 Check for Understanding

1. slope and y-intercept; slope and any point; two points

2. Sample answer:

Use point-slope form:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-4) &= \tfrac{1}{4}(x - 3) \\ y + 4 &= \tfrac{1}{4}x - \tfrac{3}{4} \end{aligned}$$

$x - 4y - 19 = 0$

Use slope-intercept form.

$$\begin{aligned} y &= mx + b \\ -4 &= \tfrac{1}{4}(3) + b \\ -\tfrac{19}{4} &= b \end{aligned}$$

Substitute the slope and intercept into the general form.

$y = \frac{1}{4}x - \frac{19}{4}$

Write in standard form.

$x - 4y - 19 = 0$

3. 55 represents the hourly rate and 49 represents the fee for coming to the house.

4. $m = \frac{-3 - 0}{0 - 6} = \frac{-3}{-6}$ or $\frac{1}{2}$ $\qquad y = \frac{1}{2}x - 3$

5. Sample answer: When given the slope and the y-intercept, use slope-intercept form. When given the slope and a point, use point-slope form. When given two points, find the slope, then use point-slope form.

6. $y = mx + b \rightarrow y = -\frac{1}{4}x - 10$

7.

$$\begin{aligned} y - 2 &= 4(x - 3) \\ y - 2 &= 4x - 12 \\ y &= 4x - 10 \end{aligned}$$

8.

$$\begin{aligned} m &= \frac{9 - 2}{7 - 5} = \frac{7}{2} \\ y - 2 &= \tfrac{7}{2}(x - 5) \\ y - 2 &= \tfrac{7}{2}x - \tfrac{35}{2} \\ y &= \tfrac{7}{2}x - \tfrac{31}{2} \end{aligned}$$

9.

$$\begin{aligned} y - 2 &= 0(x - (-9)) \\ y - 2 &= 0 \\ y &= 2 \end{aligned}$$

10a. $y = 5.9x + 2$

10b. $y = 5.9(7) + 2$
$= 41.3 + 2$ or 43.3 in.

10c. Sample answer: No; the grass could not support it own weight if it grew that tall.

Pages 30–31 Exercises

11. $y = mx + b \rightarrow y = 5x - 2$

12. $y - 5 = 8(x - (-7))$
$y - 5 = 8x + 56$
$y = 8x + 61$

13. $y = mx + b \rightarrow y = -\frac{3}{4}x$

14. $y = mx + b \rightarrow y = -12x + \frac{1}{2}$

15. $y - 5 = 6(x - 4)$
$y - 5 = 6x - 24$
$y = 6x - 19$

16. $x = 12$

17. $m = \frac{9-5}{-8-1}$
$= \frac{4}{-9}$
$y - 5 = -\frac{4}{9}(x - 1)$
$y - 5 = -\frac{4}{9}x + \frac{4}{9}$
$y = -\frac{4}{9}x + \frac{49}{9}$

18. $(-8, 0), (0, 5)$
$m = \frac{5-0}{0-(-8)}$
$= \frac{5}{8}$
$y - 0 = \frac{5}{8}(x - (-8))$
$y = \frac{5}{8}x + 5$

19. $m = \frac{1-1}{-3-8}$
$= \frac{0}{-11}$ or 0
$y - 1 = 0(x - 8)$
$y - 1 = 0$
$y = 1$

20. $x = -4$

21. $x = 0$

22. $y - 0 = 0.25(x - 24)$
$y = 0.25x - 6$

23. $y - (-4) = -\frac{1}{2}(x - (-2))$
$y + 4 = -\frac{1}{2}x - 1$
$\frac{1}{2}x + y + 5 = 0$
$x + 2y + 10 = 0$
$x + 2y = -10$

24. $m = \frac{-3-0}{1-(-2)}$
$= \frac{-3}{3}$
$= -1$
$y - 0 = -1(x + 2)$
$y = -x - 2$
$x + y = -2$

25a. $t = 2 + \frac{x - 7000}{2000}$

25b. $t = 2 + \frac{14{,}494 - 7000}{2000}$
$= 2 + 3.747$ or 5.747; about 5.7 weeks

26. $Ax + By + C = 0$
$By = -Ax - C$
$y = -\frac{A}{B}x - \frac{C}{B}$; $m = -\frac{A}{B}$

27a. Sample answer: using (20, 28) and (27, 37),
$m = \frac{37-28}{27-20}$
$= \frac{9}{7}$
$y - 28 = \frac{9}{7}(x - 20)$
$y - 28 = \frac{9}{7}x - \frac{180}{7}$
$y = \frac{9}{7}x + \frac{16}{7}$

27b. Using sample answer from part a,
$y = \frac{9}{7}(19) + \frac{16}{7}$
$= \frac{171}{7} + \frac{16}{7}$ or $\frac{187}{7}$ or about 26.7 mpg

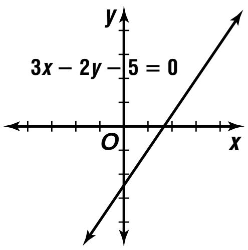
27c. Sample answer: The estimate is close but not exact since only two points were used to write the equation.

28a. See students' work.

28b. Sample answer: Only two points were used to make the prediction equation, so many points lie off of the line.

29. Yes; the slope of the line through (5, 9) and (−3, 3) is $\frac{3-9}{-3-5}$ or $\frac{3}{4}$. The slope of the line through (−3, 3), and (1, 6) is $\frac{6-3}{1-(-3)}$ or $\frac{3}{4}$. Since these two lines would have the same slope and would share a point, their equations would be the same. Thus, they are the same line and all three points are collinear.

30. $3x - 2y - 5 = 0$
$-2y = -3x + 5$
$y = \frac{3}{2}x - \frac{5}{2}$



31a. (1995, 70,583), (1997, 82,805)
$m = \frac{82{,}805 - 70{,}583}{1997–1999}$
$= \frac{12{,}222}{2}$ or 6111; \$6111 billion

31b. The rate is the slope.

32. $g[f(-2)] = g(f(-2))$
$= g((-2)^3)$
$= g(-8)$
$= 3(-8)$ or -24

33. $(f \cdot g)(x) = f(x) \cdot g(x)$
$= x^3(x^2 - 3x + 7)$
$= x^5 - 3x^4 + 7x^3$
$\left(\frac{f}{g}\right)(x) = \frac{x^3}{x^2 - 3x + 7}$, where $g(x) \neq 0$

34.

x	y
−4	16
−3	9
−2	4

{(−4, 16), (−3, 9), (−2, 4)}, yes

35. $x = \frac{1}{y}$
$\frac{y+1}{y} = \frac{\frac{y^2}{y} + \frac{1}{y}}{2}$
$= \frac{\frac{y^2+1}{y}}{2}$
$= \frac{y^2+1}{y} \cdot \frac{1}{2}$
$= \frac{y^2+1}{2y}$

The correct choice is A.

Page 31 Mid-Chapter Quiz

1. $\{-2, 2, 4\}, \{-8, -3, 3, 7\}$; No, -2 in the domain is paired with more than one element of the range.

2. $f(4) = 7 - 4^2$
$= 7 - 16$ or 9

3. $g(n + 2) = \frac{3}{n + 2 - 1}$
$= \frac{3}{n + 1}$

4. Let x = original price of jacket
Let $T(x) = 1.055x$. (The cost with 5.5% tax rate)
Let $S(x) = 0.67x$. (The cost with 33% discount)
The cost of the jacket is $[T \circ S](x)$.

$$[T \circ S](x) = T(S(x)) = T(0.67x) = 1.055(0.67x)$$

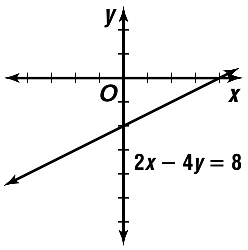
The amount paid was \$49.95.

$$45.95 = 1.055(0.67x)$$
$$43.55 \approx 0.67x$$
$$65 \approx x; \$65$$

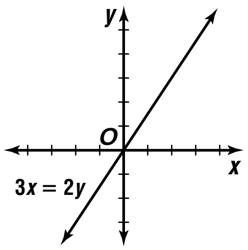
5. $[f \circ g](x) = f(g(x))$
$= f(x + 1)$
$= \frac{1}{x + 1 - 1}$
$= \frac{1}{x}, x \neq 0$

$[g \circ f](x) = g(f(x))$
$= g(\frac{1}{x - 1})$
$= \frac{1}{x - 1} + 1$
$= \frac{1}{x - 1} + \frac{x - 1}{x - 1}$
$= \frac{x}{x - 1}, x \neq 1$

6. $2x - 4y = 8$
$-4y = -2x + 8$
$y = \frac{1}{2}x - 2$



7. $3x = 2y$
$\frac{3}{2}x = y$



8. $5x - 3 = 0$
$5x = 3$
$x = \frac{3}{5}$

9. $m = \frac{8 - 5}{7 - 2}$
$= \frac{3}{5}$

$y - 5 = \frac{3}{5}(x - 2)$
$y - 5 = \frac{3}{5}x - \frac{6}{5}$
$-\frac{3}{5}x + y - \frac{19}{5} = 0$
$3x - 5y + 19 = 0$

10a. (1990, 6,478,216), (2000, 8,186,453)
$m = \frac{8{,}186{,}453 - 6{,}478{,}216}{2000 - 1990}$
$= \frac{1{,}708{,}237}{10}$ or about 170,823.7

10b. $y - 6{,}478{,}216 = 170{,}823.7(x - 1990)$
$y - 6{,}478{,}216 = 170{,}823.7x - 339{,}939{,}163$
$y = 170{,}823.7x - 333{,}460{,}947$

1-5 Writing Equations of Parallel and Perpendicular Lines

Pages 35–36 Check for Understanding

1. If A, B, and C are the same or the ratios of the As and the Bs and the Cs are proportional, then the lines are coinciding. If A and B are the same and C is different, or the ratios of the As and the Bs are proportional, but the ratio of the Cs is not, then the lines are parallel.

2. They have no slope.

3. $4x + 3y + 19 = 0$
$y = -\frac{4}{3}x - \frac{19}{3}$
$-\frac{4}{3}; \frac{3}{4}$

4. All vertical lines have undefined slope and only horizontal lines are perpendicular to them. The slope of a horizontal line is 0.

5. none of these

6. perpendicular

7. $y = x - 6$ $\quad x - y + 8 = 0$
$y = x + 8$
parallel

8. $y = 2x - 8$ $\quad 4x - 2y - 16 = 0$
$y = 2x - 8$
coinciding

9. $y - 9 = 5(x - 5)$
$y - 9 = 5x - 25$
$5x - y - 16 = 0$

10. $6x - 5y = 24$
$y = \frac{6}{5}x - \frac{24}{5}$
$y - (-5) = -\frac{5}{6}(x - (-10))$
$y + 5 = -\frac{5}{6}x - \frac{25}{3}$
$6y + 30 = -5x - 50$
$5x + 6y + 80 = 0$

11. m of EF: $m = \frac{8 - 4}{4 - 3}$
$= \frac{4}{1}$ or 4

m of EH: $m = \frac{2 - 4}{6 - 3}$
$= -\frac{2}{3}$

m of GH: $m = \frac{2 - 6}{6 - 7}$
$= \frac{-4}{-1}$ or 4

m of FG: $m = \frac{6 - 8}{7 - 4}$
$= -\frac{2}{3}$

parallelogram

Pages 36–37 Exercises

12. $y = 5x - 18$ $\quad 2x + 10y + 10 = 0$
$y = -\frac{1}{5}x + 1$
slopes are opposite reciprocals; perpendicular

13. $y - 7x + 5 = 0$ $\quad$ $y - 7x - 9 = 0$

$y = 7x - 5$ $\quad$ $y = 7x + 9$

same slopes, different y-intercepts; parallel

14. different slopes, not reciprocals; none of these

15. horizontal line, vertical line; perpendicular

16. $y = 4x - 3$ $\quad$ $4.8x - 1.2y = 3.6$

$y = 4x - 3$

same slopes, same y-intercepts; coinciding

17. $4x - 6y = 11$ $\quad$ $3x + 2y = 9$

$y = \frac{2}{3}x - \frac{11}{6}$ $\quad$ $y = -\frac{3}{2}x + \frac{9}{2}$

Slopes are opposite reciprocals; perpendicular.

18. $y = 3x + 2$ $\quad$ $3x + y = 2$

$y = -3x + 2$

different slopes, not reciprocals; none of these

19. $5x + 9y = 14$ $\quad$ $y = -\frac{5}{9}x + \frac{14}{9}$

$y = -\frac{5}{9}x + \frac{14}{9}$

same slopes, same y-intercepts; coinciding

20. $y + 4x - 2 = 0$ $\quad$ $y + 4x + 1 = 0$

$y = -4x + 2$ $\quad$ $y = -4x - 1$

same slopes, different y-intercepts; parallel

21. None of these; the slopes are not the same nor opposite reciprocals.

22. $y - (-8) = 2(x - 0)$

$y + 8 = 2x$

$2x - y - 8 = 0$

23. $m = -\frac{4}{(-9)}$ or $\frac{4}{9}$

$y - (-15) = \frac{4}{9}(x - 12)$

$y + 15 = \frac{4}{9}x - \frac{16}{3}$

$9y + 135 = 4x - 48$

$4x - 9y - 183 = 0$

24. $y - (-11) = 0(x - 4)$

$y + 11 = 0$

25. $y - (-3) = -\frac{1}{5}(x - 0)$

$y + 3 = -\frac{1}{5}x$

$5y + 15 = -x$

$x + 5y + 15 = 0$

26. $m = -\frac{6}{(-1)}$ or 6; perpendicular slope is $-\frac{1}{6}$

$y - (-2) = -\frac{1}{6}(x - 7)$

$y + 2 = -\frac{1}{6}x + \frac{7}{6}$

$6y + 12 = -x + 7$

$x + 6y + 5 = 0$

27. $x = 12$ is a vertical line; perpendicular slope is 0.

$y - (-13) = 0(x - 6)$

$y + 13 = 0$

28a. $5y - 4x = 10$

$4x - 5y + 10 = 0$ $\quad$ $m = -\frac{4}{(-5)}$ or $\frac{4}{5}$

$y - 8 = \frac{4}{5}(x - (-15))$

$y - 8 = \frac{4}{5}x + 12$

$5y - 40 = 4x + 60$

$4x - 5y + 100 = 0$

28b. perpendicular slope: $-\frac{5}{4}$

$y - 8 = -\frac{5}{4}(x - (-15))$

$y - 8 = -\frac{5}{4}x - \frac{75}{4}$

$4y - 32 = -5x - 75$

$5x + 4y + 43 = 0$

29a. $8x - 14y + 3 = 0$ $\quad$ $kx - 7y + 10 = 0$

$m = -\frac{8}{(-14)}$ or $\frac{4}{7}$ $\quad$ $m = -\frac{k}{(-7)}$ or $\frac{k}{7}$

$\frac{4}{7} = \frac{k}{7} \rightarrow k = 4$

29b. $\frac{k}{7} = -\frac{7}{4}$

$4k = -49$

$k = -\frac{49}{4}$

30a. Sample answer: $y - 1 = 0$, $x - 1 = 0$

30b. Sample answer: $x - 7 = 0$, $x - 9 = 0$

31. altitude from A to BC:

m of $BC = \frac{-5 - (-5)}{10 - 4}$

$= \frac{0}{6}$ or 0

m of altitude is undefined; $x = 7$

altitude from B to AC:

m of $AC = \frac{-5 - 10}{4 - 7}$

$= \frac{-15}{-3}$ or 5

m of altitude $= -\frac{1}{5}$

$y - (-5) = -\frac{1}{5}(x - 10)$

$y + 5 = -\frac{1}{5}x + 2$

$5y + 25 = -x + 10$

$x + 5y + 15 = 0$

altitude from C to AB:

m of $AB = \frac{-5 - 10}{10 - 7}$

$= \frac{-15}{3}$ or -5

m of altitude $= \frac{1}{5}$

$y - (-5) = \frac{1}{5}(x - 4)$

$y + 5 = \frac{1}{5}x - \frac{4}{5}$

$5y + 25 = x - 4$

$x - 5y - 29 = 0$

32. We are given $y = m_1x + b_1$ and $y = m_2x + b_2$ with $m_1 = m_2$ and $b_1 \neq b_2$. Assume that the lines intersect at point (x_1, y_1). Then $y_1 = m_1x_1 + b_1$ and $y_1 = m_2x_1 + b_2$. Substitute $m_1x_1 + b_1$ for y_1 in $y_1 = m_2x_1 + b_2$. Then $m_1x_1 + b_1 = m_2x_1 + b_2$. Since $m_1 = m_2$, substitute m_1 for m_2. The result is $m_1x_1 + b_1 = m_1x_1 + b_2$. Subtract m_1x_1 from each side to find $b_1 = b_2$. However, this contradicts the given information that $b_1 \neq b_2$. Thus, the assumption is incorrect and the lines do not share any points.

33a. Let x = regular espressos.
Let y = large espressos.
$216x + 162y = 783 \qquad 248x + 186y = 914$
$y = -\frac{4}{3}x + \frac{29}{6} \qquad y = -\frac{4}{3}x + \frac{457}{93}$
No; the lines that represent the situation do not coincide.

33b. Let x = regular espressos.
Let y = large espressos.
$216x + 162y = 783 \qquad 344x + 258y = 1247$
$y = -\frac{4}{3}x + \frac{29}{6} \qquad y = -\frac{4}{3}x + \frac{29}{6}$
Yes; the lines that represent the situation coincide.

34a. (15, 1939.20), (16, 1943.09)
$m = \frac{1943.09 - 1939.20}{16 - 15}$
$= \frac{3.89}{1}$ or 3.89
$y - 1943.09 = 3.89(x - 16)$
$y - 1943.09 = 3.89x - 62.24$
$y = 3.89x + 1880.85$
(16, 1943.09), (17, 1976.76)
$m = \frac{1976.76 - 1943.09}{17 - 16}$
$= \frac{33.67}{1}$ or 33.67
$y - 1976.76 = 33.67(x - 17)$
$y - 1976.76 = 33.67x - 572.39$
$y = 33.67x + 1404.37$
(17, 1976.76), (18, 1962.44)
$m = \frac{1962.44 - 1976.76}{18 - 17}$
$= \frac{-14.32}{1}$ or -14.32
$y - 1962.44 = -14.32(x - 18)$
$y - 1962.44 = -14.32x + 257.76$
$y = -14.32 + 2220.2$
(18, 1962.44), (19, 1940.47)
$m = \frac{1940.47 - 1962.44}{19 - 18}$
$= \frac{-21.97}{1}$ or -21.97
$y - 1940.47 = -21.97(x - 19)$
$y - 1940.47 = -21.97x + 417.43$
$y = -21.97x + 2357.9$

34b. parallel lines or the same line; no

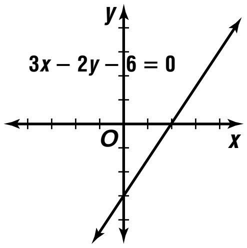
34c. $y = 3.89(22) + 1880.85$
$= 1966.43$
$y = 33.67(22) + 1404.37$
$= 2145.11$
$y = -14.32(22) + 2220.2$
$= 1905.16$
$y = -21.97(22) + 2357.9$
$= 1874.56$
No; the equations take only one pair of days into account.

35. $y - 5 = -2(x - 1)$
$y - 5 = -2x + 2$
$y = -2x + 7$

36a. (40, 295), (80, 565)
$m = \frac{565 - 295}{80 - 40}$
$= \frac{270}{40}$ or 6.75
$y - 565 = 6.75(x - 80)$
$y - 565 = 6.75x - 540$
$y = 6.75x + 25$

36b. \$6.75 **36c.** \$25

37. $3x - 2y - 6 = 0$
$y = \frac{3}{2}x - 3$



38. $[g \circ h](x) = g(h(x))$
$= g(x^2)$
$= x^2 - 1$

39. Sample answer: {(2, 4), (2, −4), (1, 2), (1, −2), (0, 0)}; because the x-values 1 and 2 are paired with more than one y-value.

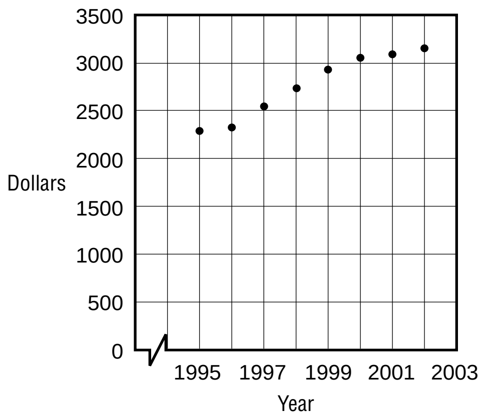
40. $2x + y = 12$
$y = -2x + 12$
$x + 2y = -6$
$x + 2(-2x + 12) = -6$
$x - 4x + 24 = -6$
$-3x = -30$
$x = 10$
$2(10) + y = 12$
$y = -8$
$2x + 2y = 2(10) + 2(-8)$
$= 20 + (-16)$ or 4

1-6 Modeling Real-World Data with Linear Functions

Pages 41–42 Check for Understanding

1. the rate of change
2. Choose two ordered pairs of data and find the equation of the line that contains their graphs. Find a median-fit line by separating the data into three sets and using the medians to locate a line. Use a graphing calculator to find a regression equation.
3. Sample answer: age of a car and its value

4a. **Personal Consumption on Durable Goods**



4b. Sample answer: using (1995, 2294) and (2002, 3158)

$m = \frac{3158 - 2294}{2002 - 1995}$

$= \frac{864}{7}$ or 123.4

$y - 3158 = 123.4(x - 2002)$

$y = 123.4x - 243{,}888.8$

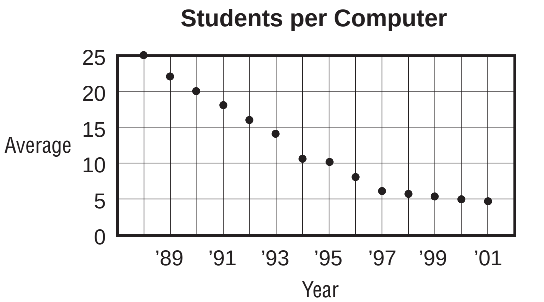
4c. $y = 132.8x - 262{,}621.2$; $r \approx 0.98$

4d. $y = 132.8(2010) - 262{,}621.2$

$= 4306.8$

\$4306.80; yes, the correlation coefficient shows a strong correlation.

5a.



5b. Sample answer: using (1997, 6.1) and (2001, 4.9)

$m = \frac{4.9 - 6.1}{2001 - 1997}$

$= \frac{-1.2}{4}$ or -0.3

$y - 6.1 = -0.3(x - 1997)$

$y = -0.3x + 605.2$

5c. $y = -1.61x + 3231.43$; $r \approx -0.97$

5d. $1 = -1.61x + 3231.43$

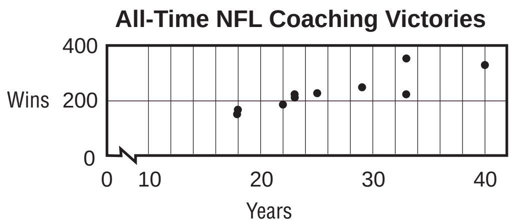
$-3230.43 = -1.61x$

$2006.47 = x$

2006; yes, the number of students per computer is decreasing steadily.

Pages 42–44 Exercises

6a.



6b. Sample answer: using (18, 170) and (40, 324)

$m = \frac{324 - 170}{40 - 18}$

$= \frac{154}{22}$ or 7

$y - 324 = 7(x - 40)$

$y = 7x + 44$

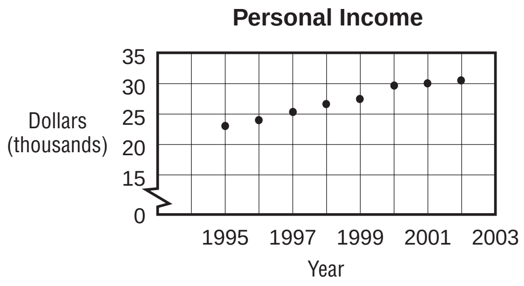
6c. $y = 7.57x + 33.38$; $r \approx 0.88$

6d. $y = 7.57(16) + 33.38$

$= 154.5$

155; yes, r is fairly close to 1. (Actual data is 159.)

7a.



7b. Sample answer: using (1995, 23,255) and (2002, 30,832)

$m = \frac{30{,}832 - 23{,}255}{2002 - 1995}$

$= \frac{7577}{7}$ or 1082.43

$y - 23{,}255 = 1082.43(x - 1995)$

$y = 1082.43x - 2{,}136{,}192.85$

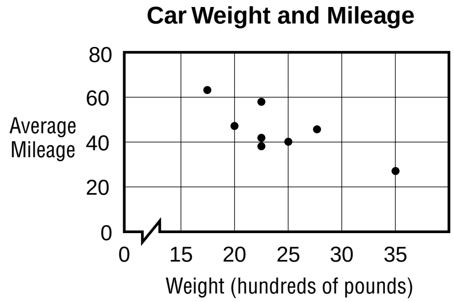
7c. $y = 1164.11x - 2{,}299{,}128.75$; $r \approx 0.99$

7d. $y = 1164.11(2010) - 2{,}299{,}128.75$

$= 40{,}732.35$

\$40,732.35; yes, r shows a strong relationship.

8a.



8b. Sample answer: using (17.5, 65.4) and (35.0, 27.7)

$m = \frac{27.7 - 65.4}{35.0 - 17.5}$

$= \frac{-37.7}{17.5}$ or -2.15

$y - 65.4 = -2.15(x - 17.5)$

$y = -2.15x + 103.0$

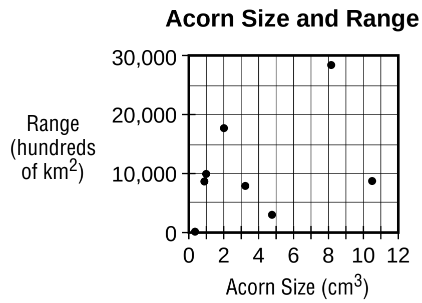
8c. $y = -1.72x + 87.59$; $r \approx -0.77$

8d. $y = -1.72(45.0) + 87.59$

$y = 10.19$

10.19; no, r doesn't show a particularly strong relationship.

9a.



9b. Sample answer: using (0.3, 233) and (3.4, 7900)

$$m = \frac{7900 - 233}{3.4 - 0.3} = \frac{7667}{3.1} \text{ or } 2473.23$$

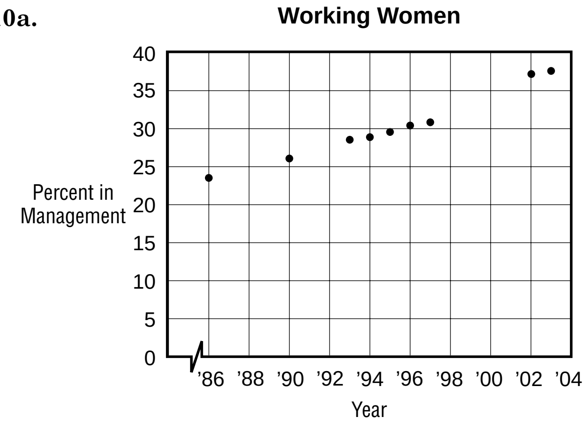
$$y - 7900 = 2473.23(x - 3.4)$$

$$y = 2473.23x - 508.97$$

9c. $y = 885.82 + 6973.14$; $r \approx 0.38$

9d. The correlation value does not show a strong or moderate relationship.

10a.



10b. Sample answer: using (1990, 26.2) and (2003, 37.6)

$$m = \frac{37.6 - 26.2}{2003 - 1990} = \frac{11.4}{13} \text{ or } 0.88$$

$$y - 26.2 = 0.88(x - 1990)$$

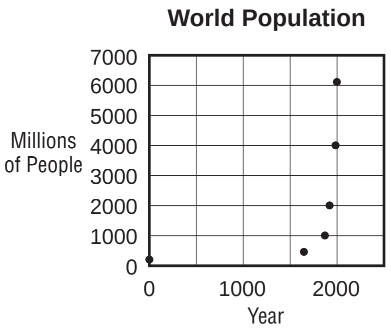
$$y = 0.88x - 1725$$

10c. $y = 0.84x - 1648.27$; $r \approx 0.984$

10d. $y = 0.84(2008) - 1648.27$ or 38.45

38.45%; yes, r is very close to 1.

11a.



11b. Sample answer: using (1, 200) and (2000, 6050)

$$m = \frac{6050 - 200}{2000 - 1} = \frac{5850}{1999} \text{ or } 2.93$$

$$y - 200 = 2.93(x - 1)$$

$$y = 2.93x + 197.07$$

11c. $y = 1.65x - 289.00$; $r \approx 0.56$

11d. $y = 1.65(2010) - 289.00 = 3027.5$

3028 million; no, the correlation value is not showing a very strong relationship.

12a. Sample answer: the space shuttle; because anything less than perfect could endanger the lives of the astronauts.

12b. Sample answer: a medication that proves to help delay the progress of a disease; because any positive correlation is better than none or a negative correlation.

12c. Sample answer: comparing a dosage of medicine to the growth factor of cancer cells; because the greater the dosage the fewer cells that are produced.

13.

Men's Median Salary	Women's Median Salary
LinReg	LinReg
$y = ax + b$	$y = ax + b$
$a = 885.2867133$	$a = 625.041958$
$b = -1{,}742{,}768.136$	$b = -1{,}234{,}368.061$
$r = .9716662959$	$r = .9869509009$

The rate of growth, which is the slope of the graphs of the regression equations, for the women is less than that of the men's rate of growth. If that trend continues, the men's median salary will always be more than the women's.

14a. Let x = computers.

Let y = printers.

$24x + 40y = 38{,}736$

$y = -0.6x + 968.4$

$30x + 50y = 51{,}470$

$y = -0.6x + 1029.4$

No; the lines do not coincide.

14b. Let x = computers.

Let y = printers.

$24x + 40y = 38{,}736$

$y = -0.6x + 968.4$

$30x + 50y = 48{,}420$

$y = -0.6x + 968.4$

Yes; the lines coincide.

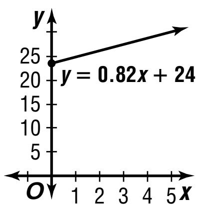
15.

$$y - (-4) = -6(x - (-3))$$

$$y + 4 = -6x - 18$$

$$6x + y + 22 = 0$$

16a.



16b. \$24 billion

16c. If the nation had no disposable income, personal consumption expenditures would be \$24 billion. For each 1 billion increase in disposable income, there is a 0.82 billion dollar increase in personal consumption expenditures.

17.

$$[f \circ g](x) = f(g(x)) = f(x + 1) = (x + 1)^3 = x^3 + 3x^2 + 3x + 1$$

$$[g \circ f](x) = g(f(x)) = g(x^3) = x^3 + 1$$

18. Yes; each domain value is paired with exactly one range value.

19. The y-intercept is 1.
The slope is -3 (move down 3 and right 1).
The correct choice is C.

1-7 Piecewise Functions

Pages 48–49 Check for Understanding

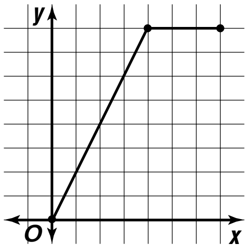
1. $f(x) = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$

2. reals, even integers

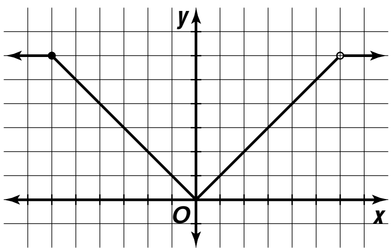
3. $f(x) = \begin{cases} x + 2 \text{ if } x \leq 0 \\ x \text{ if } 0 < x \leq 4 \\ x - 2 \text{ if } x > 4 \end{cases}$

4. Alex is correct because he is applying the definition of a function.

5.

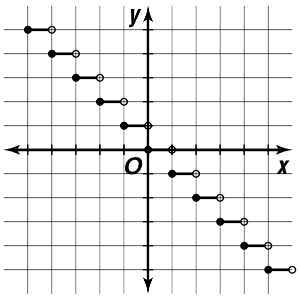


6.



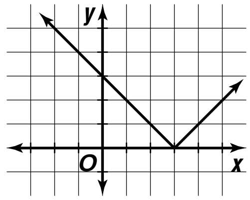
7.

x	$f(x)$
$-3 \leq x < -2$	3
$-2 \leq x < -1$	2
$-1 \leq x < 0$	1
$0 \leq x < 1$	0
$1 \leq x < 2$	-1
$2 \leq x < 3$	-2
$3 \leq x < 4$	-3
$4 \leq x < 5$	-4



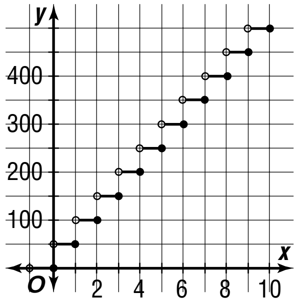
8.

x	$f(x)$
-1	4
0	3
1	2
2	1
3	0
4	1



9. greatest integer function; h is hours, $c(h)$ is the cost, $c(h) = \begin{cases} 50h \text{ if } [[h]] = h \\ 50[[h + 1]] \text{ if } [[h]] < h \end{cases}$

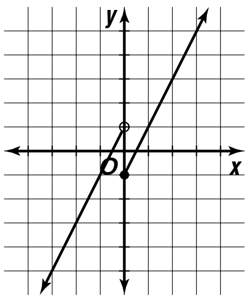
x	$f(x)$
$0 < x \leq 1$	50
$1 < x \leq 2$	100
$2 < x \leq 3$	150
$3 < x \leq 4$	200



10. long term lot:
$2(6) + 3(1) = 12 + 3$ or 15
shuttle facility:
$4(3) = 12$
shuttle facility

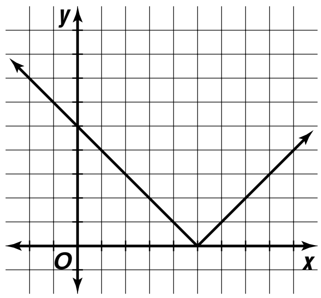
Pages 49–51 Exercises

11.



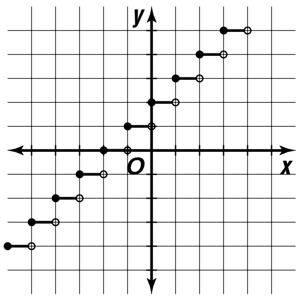
12.

x	$f(x)$
1	4
3	2
5	0
7	2
9	4



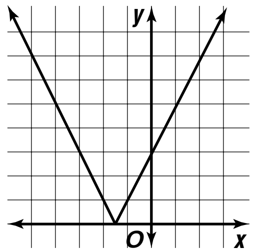
13.

x	$f(x)$
$-2 \leq x < -1$	0
$-1 \leq x < 0$	1
$0 \leq x < 1$	2
$1 \leq x < 2$	3
$2 \leq x < 3$	4



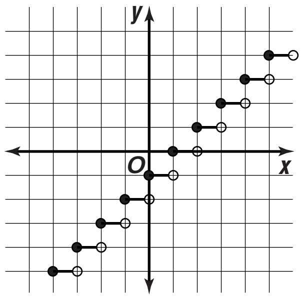
14.

x	$f(x)$
-5	7
-3	3
-1.5	0
0	3
2	7

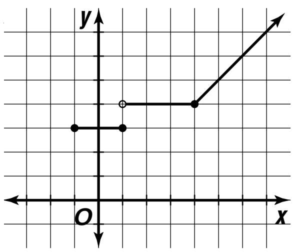


15.

x	$f(x)$
$-2 \le x < -1$	-3
$-1 \le x < 0$	-2
$0 \le x < 1$	-1
$1 \le x < 2$	0
$2 \le x < 3$	1

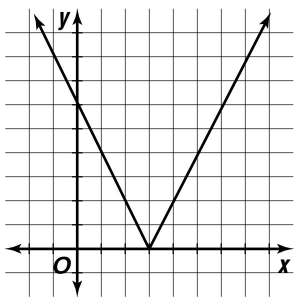


16.



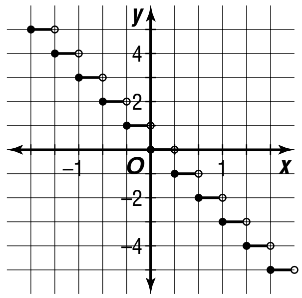
17.

x	$f(x)$
0	6
1	4
2	2
3	0
4	2

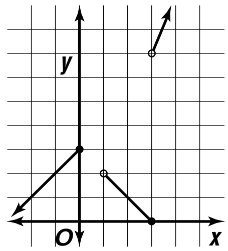


18.

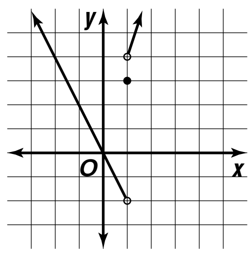
x	$f(x)$
$-1 \le x < -\frac{2}{3}$	3
$-\frac{2}{3} \le x < -\frac{1}{3}$	2
$-\frac{1}{3} \le x < 0$	1
$0 \le x < \frac{1}{3}$	0
$\frac{1}{3} \le x < \frac{2}{3}$	-1
$\frac{2}{3} \le x < 1$	-2



19.

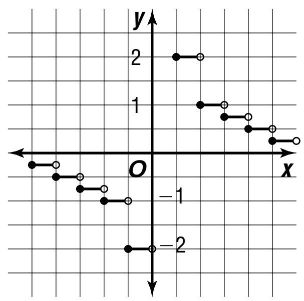


20.



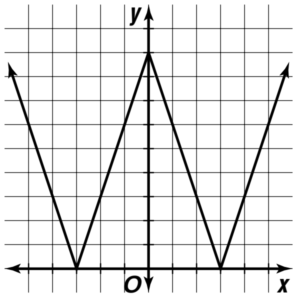
21.

x	$f(x)$
$-5 \le x < -4$	$-\frac{2}{5}$
$-4 \le x < -3$	$-\frac{1}{2}$
$-3 \le x < -2$	$-\frac{2}{3}$
$-2 \le x < -1$	-1
$-1 \le x < 0$	-2
$1 \le x < 2$	2
$2 \le x < 3$	1
$3 \le x < 4$	$\frac{2}{3}$
$4 \le x < 5$	$\frac{1}{2}$
$5 \le x < 6$	$\frac{2}{5}$



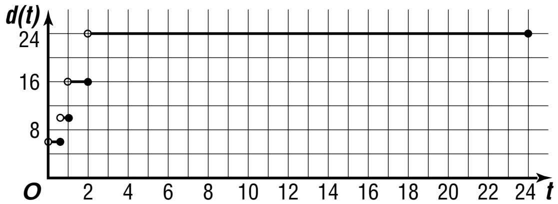
22.

x	$f(x)$
-5	6
-3	0
-1	6
0	9
1	6
3	0
5	6



23. Step; t is the time in hours, $c(t)$ is the cost in dollars,

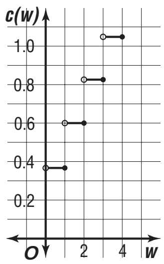
$$c(t) = \begin{cases} 6 \text{ if } t \leq \frac{1}{2} \\ 10 \text{ if } \frac{1}{2} < t \leq 1 \\ 16 \text{ if } 1 < t \leq 2 \\ 24 \text{ if } 2 < t \leq 24 \end{cases}$$



24. Greatest integer; w is the weight in ounces, $c(w)$ is the cost in dollars,

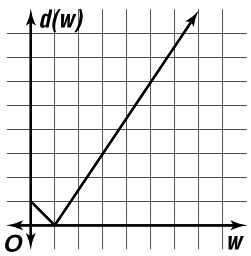
$$c(w) = \begin{cases} 0.37 + 0.23(w - 1) \text{ if } [[w]] = w \\ 0.37 + 0.23[[w]] \text{ if } [[w]] < w \end{cases}$$

x	$f(x)$
$0 < x < 1$	0.37
1	0.37
$1 < x < 2$	0.60
2	0.60
$2 < x < 3$	0.83
3	0.83



25. Absolute value; w is the weight in pounds, $d(w)$ is the discrepancy, $d(w) = |1 - w|$

x	$f(x)$
0	1
1	0
2	1
3	2

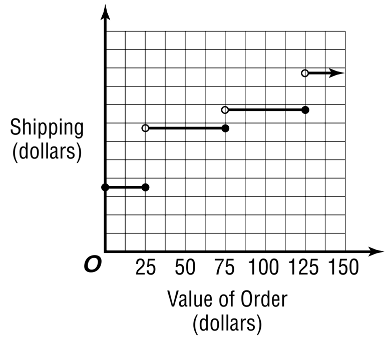


26a. step

26b. v is the value of the order, $s(v)$ is the shipping,

$$s(v) = \begin{cases} 3.50 \text{ if } 0.00 \leq v \leq 25.00 \\ 5.95 \text{ if } 25.01 \leq v \leq 75.00 \\ 7.95 \text{ if } 75.01 \leq v \leq 125.00 \\ 9.95 \text{ if } 125.01 \leq v \end{cases}$$

26c.

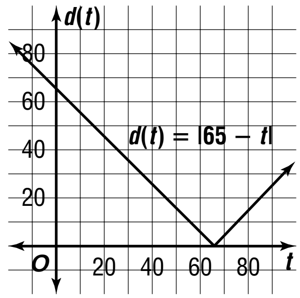


27. If n is any integer, then all ordered pairs (x, y) where x and y are both in the interval $[n, n + 1)$ are solutions.

28a. absolute value

28b. $d(t) = |65 - t|$

28c.



28d. $d(63) = |65 - 63|$ or 2

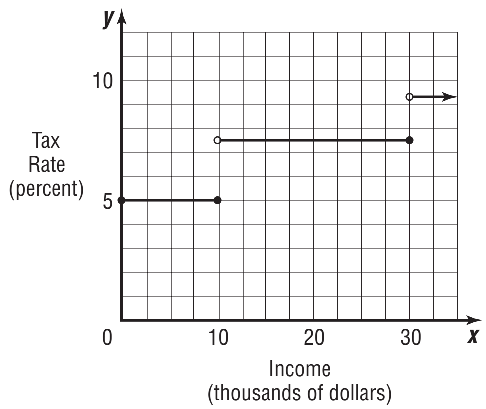
$d(28) = |65 - 28|$ or 37

$\frac{2 + 37}{2} = 19.5$ heating degree days

29a. step

29b. $t(x) = \begin{cases} 5\% \text{ if } x \leq \$10{,}000 \\ 7.5\% \text{ if } \$10{,}000 < x \leq \$30{,}000 \\ 9.3\% \text{ if } x > \$30{,}000 \end{cases}$

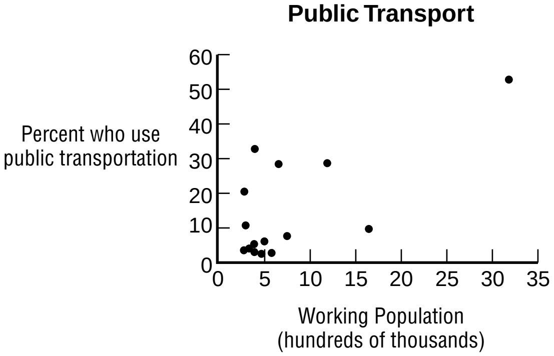
29c.



29d. 9.5%

30. No; the functions are the same if x is positive. However, if x is negative, the functions yield different values. For example, $[g \circ f](1.5) = 1$ and $[f \circ g](1.5) = 1$; $[g \circ f](-1.5) = 2$ and $[f \circ g](-1.5) = 1$.

31a.



31b. Sample answer: using (3,183,088, 53.4) and (362,777, 3.3)

$m = \frac{3.3 - 53.4}{362{,}777 - 3{,}183{,}088}$

$= \frac{-50.1}{-2{,}820{,}311}$ or 0.0000178

$y - 53.4 = 0.0000178(x - 3{,}183{,}088)$

$y = 0.0000178x - 3.26$

31c. $y = 0.0000136x + 4.55$, $r \approx 0.68$

31d. $y = 0.0000136(307{,}679) + 4.55$

$y = 8.73$

8.73%; No, the actual value is 22%.

32. $y - 2 = 2(x - 4)$

$y - 2 = 2x - 8$

$2x - y - 6 = 0$

33a. (39, 29), (32, 15)

33b. $m = \frac{15 - 29}{32 - 39}$

$= \frac{-14}{-7}$ or 2

33c. The average number of points scored each minute.

34. $p(x) = (r - c)(x)$

$= (400x - 0.2x^2) - (0.1x + 200)$

$= 399.9x - 0.2x^2 - 200$

35. Let x = the original price, or \$59.99.

Let $T(x) = 1.065x$. (The cost with 6.5% tax rate)

Let $S(x) = 0.75x$. (The cost with 25% discount)

$[T \circ S](x) = (T(S(x))$

$= (T(0.75x))$

$= (T(0.75(59.99))$

$= (T(44.9925)$

$= 1.065(44.9925)$

$= \$47.92$

36. {−7, −2, 0, 4, 9}; {−2, 0, 2, 3, 11}; Yes; no element of the domain is paired with more than one element of the range.

37. $5 \times 6^{12} = 10{,}883{,}911{,}680$

$5 + 6^{12} = 2{,}176{,}782{,}341$

So, $5 + 6^{12}$ is not greater than 5×6^{12}.

The correct choice is A.

1-8 Graphing Linear Inequalities

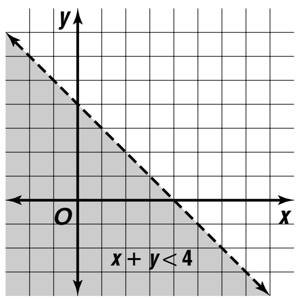
Pages 54–55 Check for Understanding

1. $y \geq 2x - 6$

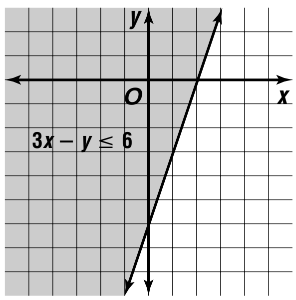
2. Graph the lines $-3 = 2x + y$ and $2x + y = 7$. The graph of $-3 = 2x + y$ is solid and the graph of $2x + y = 7$ is dashed. Test points to determine which region(s) should be shaded. Then shade the correct region(s).

3. Sample answer: The boundaries separate the plane into regions. Each point in a region either does or does not satisfy the inequality. Using a test point allows you to determine whether all of the points in a region satisfy the inequality.

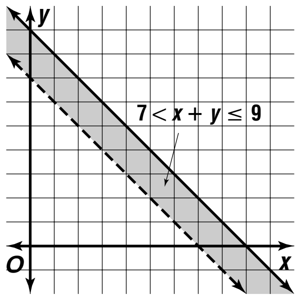
4.



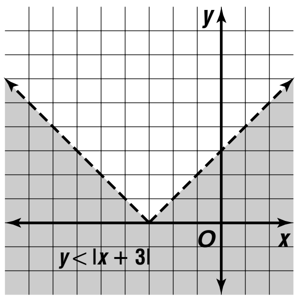
5.



6.

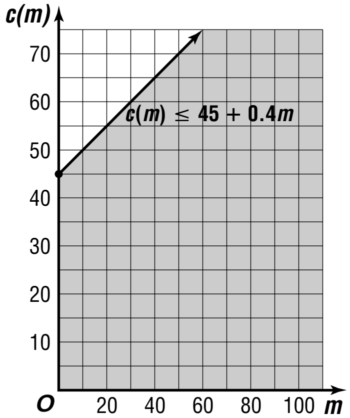


7.



8a. $c(m) \leq 45 + 0.4m$

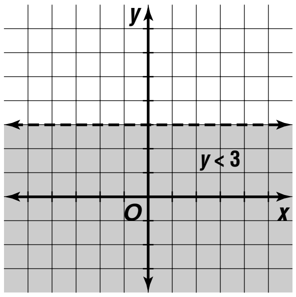
8b.



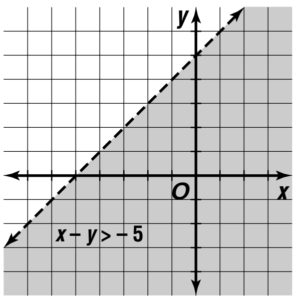
8c. Sample answer: (0, 45), (10, 49), (20, 50)

Pages 55–56 Exercises

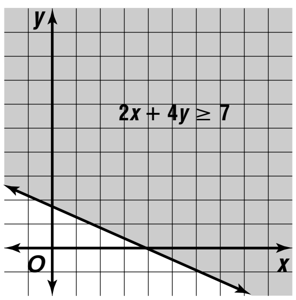
9.



10.



11.



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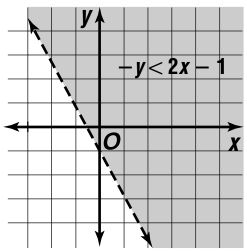

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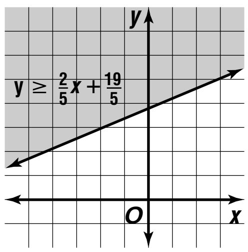

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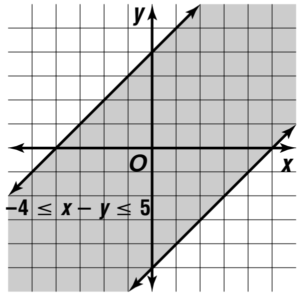

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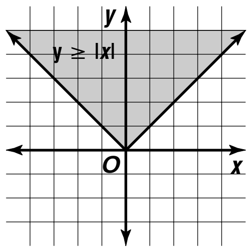

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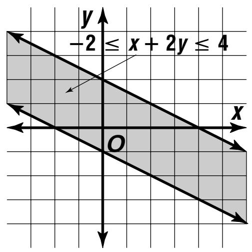

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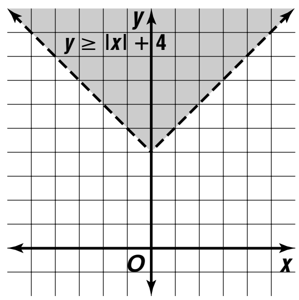

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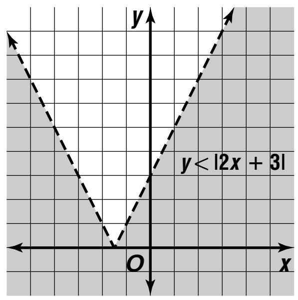

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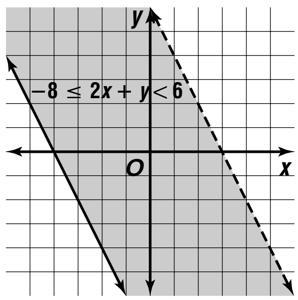

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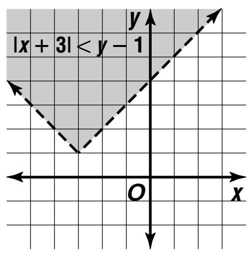

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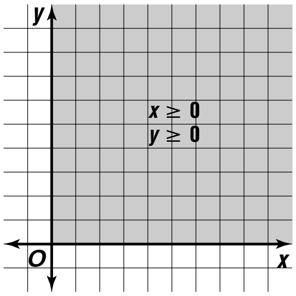

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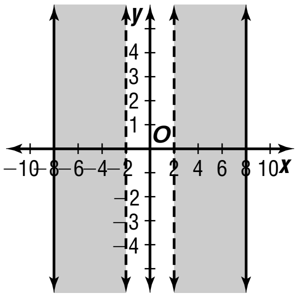
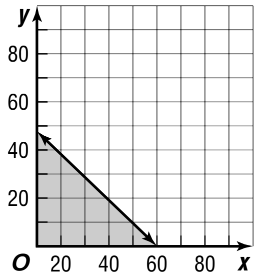


23a. $8x + 10y \le 8(60)$

$8x + 10y \le 480$

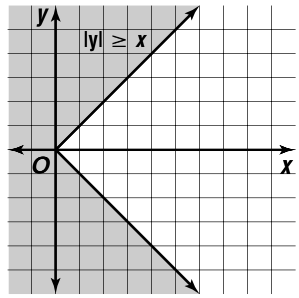
23b.



23c. Sample answer: (0, 48) (60, 0), (45, 6)

23d. Sample answer: using complex computer programs and systems of inequalities.

24.



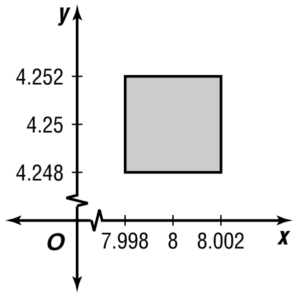
25a. points in the first and third quadrants

25b. If x and y satisfy the inequality, then either $x \ge 0$ and $y \ge 0$ or $x \le 0$ and $y \le 0$. If $x \ge 0$ and $y \ge 0$, then $|x| = x$ and $|y| = y$. Thus, $|x| + |y| = x + y$. Since $x + y$ is positive, $|x + y| = x + y$.

If $x \le 0$ and $y \le 0$, then $|x| = -x$ and $|y| = -y$. Then $|x| + |y| = -x + (-y)$ or $-(x + y)$. Since both x and y are negative, $(x + y)$ is negative, and $|x + y| = -(x + y)$.

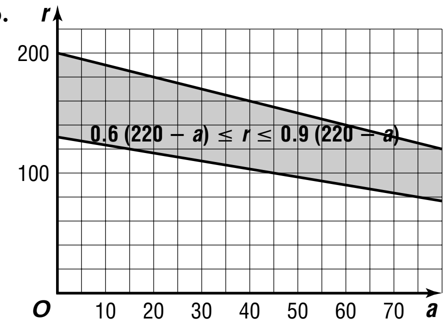
26a. $|8 - x| \le \frac{1}{500}$; $|4\frac{1}{4} - y| \le \frac{1}{500}$

26b.



27a. $0.6(220 - a) \le r \le 0.9(220 - a)$

27b.

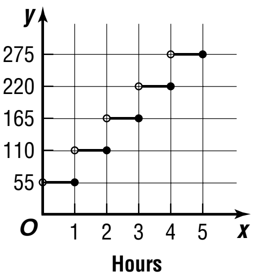


28a. step

28b. Let $c(h)$ represent the cost for h hours.

Then $c(h) = \begin{cases} 55h \text{ if } [[h]] = h \\ 55[[h + 1]] \text{ if } [[h]] < h \end{cases}$

28c.



29a. $3x - y = 10$

$y = 3x - 10$

$y - (-2) = 3(x - 0)$

$3x - y - 2 = 0$

29b. perpendicular slope: $-\frac{1}{3}$

$y - (-2) = -\frac{1}{3}(x - 0)$

$y + 2 = -\frac{1}{3}x$

$3y + 6 = -x$

$x + 3y + 6 = 0$

30. $m = \frac{7 - 4}{5 - 1}$ $\qquad y - 7 = \frac{3}{4}(x - 5)$

$= \frac{3}{4}$ $\qquad y = \frac{3}{4}x + 3\frac{1}{4}$

31a. (0, 23), (16, 48);

$m = \frac{48 - 23}{16 - 0}$

$= \frac{25}{16}$ or 1.5625

31b. the average change in the temperature per hour

32. $\frac{9^5 - 9^4}{8} = \frac{59{,}049 - 6561}{8}$

$= \frac{52{,}488}{8}$ or 6561 or 9^4

The correct choice is E.

Chapter 1 Study Guide and Assessment

Page 57 Understanding the Vocabulary

1. c
2. f
3. d
4. g
5. i
6. a
7. h
8. j
9. e
10. b

Pages 58–60 Skills and Concepts

11. $f(4) = 5(4) - 10$
$= 20 - 10 \text{ or } 10$

12. $g(2) = 7 - (2)^2$
$= 7 - 4 \text{ or } 3$

13. $f(-3) = 4(-3)^2 - 4(-3) + 9$
$= 36 + 12 + 9 \text{ or } 57$

14. $h(0.2) = 6 - 2(0.2)^3$
$= 6 - 0.016 \text{ or } 5.984$

15. $g\left(\frac{1}{3}\right) = \dfrac{2}{5\left(\frac{1}{3}\right)}$
$= \dfrac{2}{\frac{5}{3}} \text{ or } \frac{6}{5}$

16. $k(4c) = (4c)^2 + 2(4c) - 4$
$= 16c^2 + 8c - 4$

17. $f(m + 1) = \left|(m + 1)^2 + 3(m + 1)\right|$
$= \left|m^2 + 2m + 1 + 3m + 3\right|$
$= \left|m^2 + 5m + 4\right|$

18. $(f + g)(x) = f(x) + g(x)$
$= 6x - 4 + 2$
$= 6x - 2$

$(f - g)(x) = f(x) - g(x)$
$= 6x - 4 - (2)$
$= 6x - 6$

$(f \cdot g)(x) = f(x) \cdot g(x)$
$= (6x - 4)(2)$
$= 12x - 8$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$
$= \frac{6x - 4}{2}$
$= 3x - 2$

19. $(f + g)(x) = f(x) + g(x)$
$= x^2 + 4x + x - 2$
$= x^2 + 5x - 2$

$(f - g)(x) = f(x) - g(x)$
$= x^2 + 4x - (x - 2)$
$= x^2 + 3x + 2$

$(f \cdot g)(x) = f(x) \cdot g(x)$
$= (x^2 + 4x)(x - 2)$
$= x^3 + 2x^2 - 8x$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)})$
$= \frac{x^2 + 4x}{x - 2}, x \neq 2$

20. $(f + g)(x) = f(x) + g(x)$
$= 4 - x^2 + 3x \text{ or } 4 + 3x - x^2$

$(f - g)(x) = f(x) - g(x)$
$= 4 - x^2 - (3x)$
$= 4 - 3x - x^2$

$(f \cdot g)(x) = f(x) \cdot g(x)$
$= (4 - x^2)(3x)$
$= 12x - 3x^3$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$
$= \frac{4 - x^2}{3x}, x \neq 0$

21. $(f + g)(x) = f(x) + g(x)$
$= x^2 + 7x + 12 + x + 4$
$= x^2 + 8x + 16$

$(f - g)(x) = f(x) - g(x)$
$= x^2 + 7x + 12 - (x + 4)$
$= x^2 + 6x + 8$

$(f \cdot g)(x) = f(x) \cdot g(x)$
$= (x^2 + 7x + 12)(x + 4)$
$= x^3 + 11x^2 + 40x + 48$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$
$= \frac{x^2 + 7x + 12}{x + 4}$
$= \frac{(x + 4)(x + 3)}{x + 4}$
$= x + 3, x \neq -4$

22. $(f + g)(x) = f(x) + g(x)$
$= x^2 - 1 + x + 1$
$= x^2 + x$

$(f - g)(x) = f(x) - g(x)$
$= x^2 - 1 - (x + 1)$
$= x^2 - x - 2$

$(f \cdot g)(x) = f(x) \cdot g(x)$
$= (x^2 - 1)(x + 1)$
$= x^3 + x^2 - x - 1$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$
$= \frac{x^2 - 1}{x + 1}$
$= \frac{(x - 1)(x + 1)}{x + 1}$
$= x + 1, x \neq -1$

23. $(f + g)(x) = f(x) + g(x)$
$= x^2 - 4x + \frac{4}{x - 4}$
$= \frac{x^3 - 8x^2 + 16x + 4}{x - 4}$
$= x^2 - 4x + \frac{4}{x - 4}, x \neq 4$

$(f - g)(x) = f(x) - g(x)$
$= x^2 - 4x - \left(\frac{4}{x - 4}\right)$
$= \frac{x^3 - 8x^2 + 16x - 4}{x - 4}$
$= x^2 - 4x - \frac{4}{x - 4}, x \neq 4$

$(f \cdot g)(x) = f(x) \cdot g(x)$
$= (x^2 - 4x)\left(\frac{4}{x - 4}\right)$
$= 4x, x \neq 4$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$
$= \dfrac{x^2 - 4x}{\frac{4}{x - 4}} \text{ or } \dfrac{x^3 - 8x^2 + 16x}{4}, x \neq 4$

24. $[f \circ g](x) = f(g(x))$
$= f(2x)$
$= (2x)^2 - 4$
$= 4x^2 - 4$

$[g \circ f](x) = g(f(x))$
$= g(x^2 - 4)$
$= 2(x^2 - 4)$
$= 2x^2 - 8$

25. $[f \circ g](x) = f(g(x))$
$= f(3x^2)$
$= 0.5(3x^2) + 5$
$= 1.5x^2 + 5$

$[g \circ f](x) = g(\mathrm{f}(\mathrm{x}))$
$= g(0.5x + 5)$
$= 3(0.5x + 5)^2$
$= 0.75x^2 + 15x + 75$

26. $[f \circ g](x) = f(g(x))$
$= f(3x)$
$= 2(3x)^2 + 6$
$= 18x^2 + 6$

$[g \circ f](x) = g(f(x))$
$= g(2x^2 + 6)$
$= 3(2x^2 + 6)$
$= 6x^2 + 18$

27. $[f \circ g](x) = f(g(x))$
$= f(x^2 - x + 1)$
$= 6 + (x^2 - x + 1)$
$= x^2 - x + 7$

$[g \circ f](x) = g(f(x))$
$= g(6 + x)$
$= (6 + x)^2 - (6 + x) + 1$
$= x^2 + 11x + 31$

28. $[f \circ g](x) = f(g(x))$
$= f(x + 1)$
$= (x + 1)^2 - 5$
$= x^2 + 2x - 4$

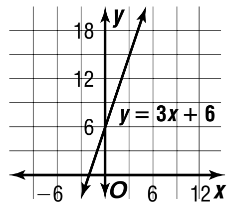
$[g \circ f](x) = g(f(x))$
$= g(x^2 - 5)$
$= x^2 - 5 + 1$
$= x^2 - 4$

29. $[f \circ g](x) = f(g(x))$
$= f(2x^2 + 10)$
$= 3 - (2x^2 + 10)$
$= -2x^2 - 7$

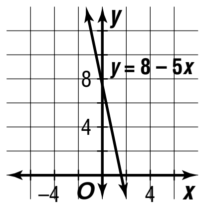
$[g \circ f](x) = g(f(x))$
$= g(3 - x)$
$= 2(3 - x)^2 + 10$
$= 2x^2 - 12x + 28$

30. Domain of $f(x)$: $x \geq 16$
Domain of $g(x)$: all reals
$g(x) \geq 16$
$5 - x \geq 16$
$x \leq -11$
Domain of $[f \circ g](x)$ is $x \leq -11$.

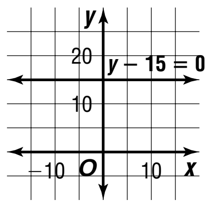
31. The y-intercept is 6. The slope is 3.



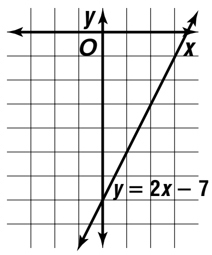
32. The y-intercept is 8. The slope is -5.



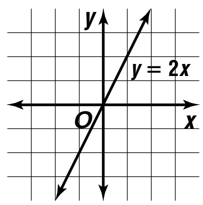
33. $y - 15 = 0$
$y = 15$
The y-intercept is 15. The slope is 0.



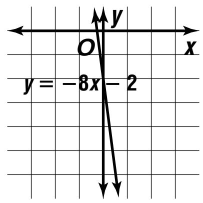
34. $0 = 2x - y - 7$
$y = 2x - 7$
The y-intercept is -7. The slope is 2.



35. The y-intercept is 0. The slope is 2.



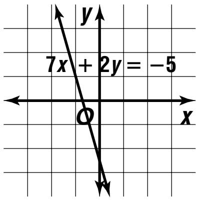
36. The y-intercept is -2. The slope is -8.



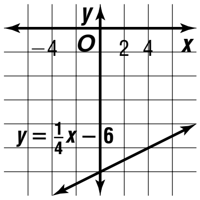
37. $7x + 2y = -5$

$$y = -\frac{7}{2}x - \frac{5}{2}$$

The y-intercept is $-\frac{5}{2}$. The slope is $-\frac{7}{2}$.



38. The y-intercept is -6. The slope is $\frac{1}{4}$.



39. $y = 2x - 3$

40. $y = -x + 1$

41. $y - 2 = \frac{1}{2}(x - (-5))$

$$y - 2 = \frac{1}{2}x + \frac{5}{2}$$
$$y = \frac{1}{2}x + \frac{9}{2}$$

42. $m = \frac{5 - 2}{2 - (-4)}$

$$= \frac{3}{6} \text{ or } \frac{1}{2}$$
$$y - 5 = \frac{1}{2}(x - 2)$$
$$y - 5 = \frac{1}{2}x - 1$$
$$y = \frac{1}{2}x + 4$$

43. $(1, 0), (0, -4)$

$$m = \frac{-4 - 0}{0 - 1}$$
$$= \frac{-4}{-1} \text{ or } 4$$
$$y - (-4) = 4(x - 0)$$
$$y + 4 = 4x$$
$$y = 4x - 4$$

44. $y = -1$

45. $y = 0$

46. $y - 0 = 0.1(x - 1)$

$$y = 0.1x - 0.1$$

47. $y - 1 = 1(x - 1)$

$$y - 1 = x - 1$$
$$x - y = 0$$

48. $y - 6 = \frac{1}{3}(x - (-1))$

$$y - 6 = \frac{1}{3}x + \frac{1}{3}$$
$$3y - 18 = x + 1$$
$$x - 3y + 19 = 0$$

49. $m = -\frac{2}{1}$ or -2

$$y - 2 = -2(x - (-3))$$
$$y - 2 = -2x - 6$$
$$2x + y + 4 = 0$$

50. $y - (-8) = \frac{1}{2}(x - 4)$

$$y + 8 = \frac{1}{2}x - 2$$
$$2y + 16 = x - 4$$
$$x - 2y - 20 = 0$$

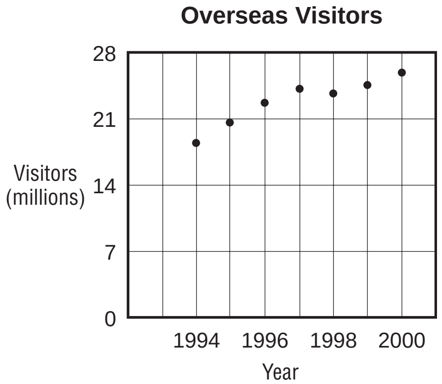
51. $m = -\frac{4}{(-2)}$ or 2, perpendicular slope is $-\frac{1}{2}$

$$y - 4 = -\frac{1}{2}(x - 1)$$
$$y - 4 = -\frac{1}{2}x + \frac{1}{2}$$
$$2y - 8 = -x + 1$$
$$x + 2y - 9 = 0$$

52. $x = -8$ is a vertical line; perpendicular slope is 0.

$$y - (-6) = 0(x - 4)$$
$$y + 6 = 0$$

53a.



53b. Sample answer: using (1994, 18,458) and (2000, 25,975)

$$m = \frac{25{,}975 - 18{,}458}{2000 - 1994}$$
$$= \frac{7517}{6} \text{ or } 1252.8$$
$$y - 25{,}975 = 1252.8(x - 2000)$$
$$y = 1252.8x - 2{,}479{,}625$$

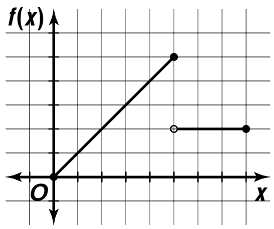
53c. $y = 1115.9x - 2{,}205{,}568$; $r = 0.9441275744$

53d. $y = 1115.9(2008) - 2{,}205{,}568$

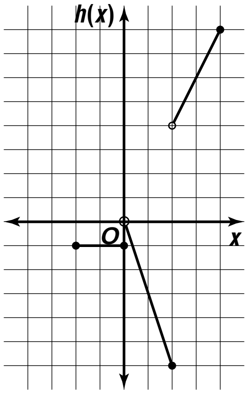
$$= 35{,}159.2$$

35,159,200 visitors; Sample answer: This is a good prediction, because the r value indicates a strong relationship.

54.

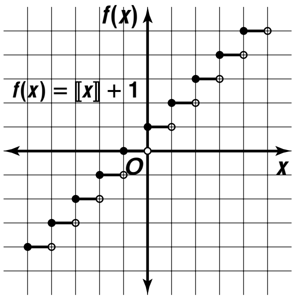


55.



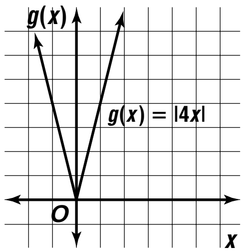
56.

x	$f(x)$
$-2 \le x < -1$	-1
$-1 \le x < 0$	0
$0 \le x < 1$	1
$1 \le x < 2$	2
$2 \le x < 3$	3



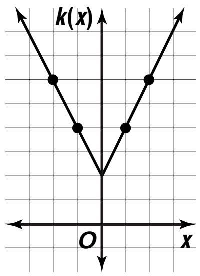
57.

x	$f(x)$
-2	8
-1	4
0	0
1	4
2	8



58.

x	$f(x)$
-2	6
-1	4
0	2
1	4
2	6



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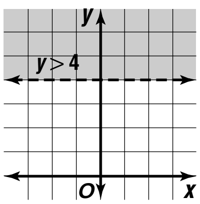

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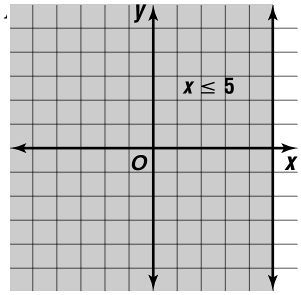
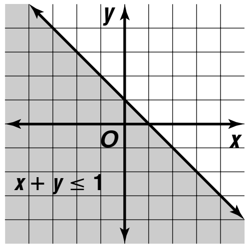
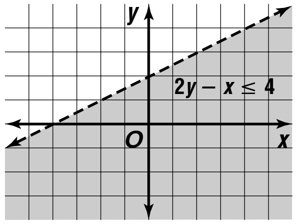


61.



62.



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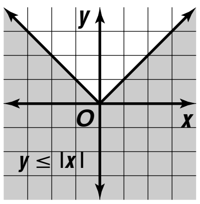

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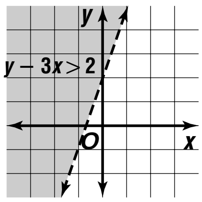

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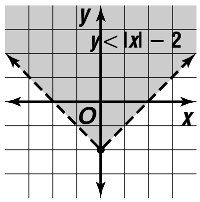

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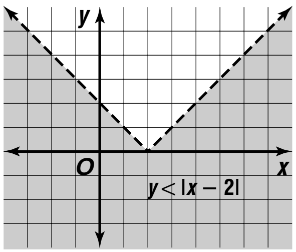


Page 61 Applications and Problem Solving

67a. $d = \frac{1}{2}(20)(1)^2$
$= 10$
$d = \frac{1}{2}(20)(2)^2$
$= 40$
$d = \frac{1}{2}(20)(3)^2$
$= 90$
$d = \frac{1}{2}(20)(4)^2$
$= 160$
$d = \frac{1}{2}(20)(5)^2$
$= 250$
10 m, 40 m, 90 m, 160 m, 250 m

67b. Yes; each element of the domain is paired with exactly one element of the range.

68a. (1999, 500) and (2004, 636)
$m = \frac{636 - 500}{2004 - 1999}$
$= \frac{136}{5}$ or 27.2; about $27.2 billion

68b. $y - 500 = 27.2(x - 1999)$
$y = 27.2x - 53{,}872.8$

69. $y = -0.284x + 12.964$; The correlation is moderately negative, so the regression line is somewhat representative of the data.

Page 61 Open-Ended Assessment

1. Possible answer: $f(x) = 4x - 4$, $g(x) = x^2$; $[f \circ g](x) = f(g(x)) = 4(x^2) - 4 = 4x^2 - 4$

2a. No; Possible explanation: If the lines have the same x-intercept, then they either intersect on the x-axis or they are the same line. In either case, they cannot be parallel.

2b. Yes; Possible explanation: If the lines have the same x-intercept, they can intersect on the x-axis. If they have slopes that are negative reciprocals, then they are perpendicular.

3a. $y = \begin{cases} 4 \text{ if } x < 4 \\ 2x - 4 \text{ if } x \geq 4 \end{cases}$

3b. $y = \begin{cases} x - 1 \text{ if } x < -1 \\ 3x + 1 \text{ if } x \geq -1 \end{cases}$

Chapter 1 SAT & ACT Preparation

Page 65 SAT and ACT Practice

1. Prime factorization of a number is a product of prime numbers that equals the number. Choices A, B, and E contain numbers that are not prime. Choice C does not equal 54. Choice D, $3 \times 3 \times 3 \times 2$, is the prime factorization of 54. The correct choice is D.

2. Since this is a multiple-choice question, you can try each choice. Choice A, 16, is not divisible by 12, so eliminate it. Choice B, 24, is divisible by both 8 and 12. Choice C, 48, is also divisible by both 8 and 12. Choice D, 96, is also divisible by both 8 and 12. It cannot be determined from the information given. The correct choice is E.

3. Write the mixed numbers as fractions.
$4\frac{1}{3} = \frac{13}{3}$ $\quad 2\frac{3}{5} = \frac{13}{5}$
Remember that dividing by a fraction is equivalent to multiplying by its reciprocal
$$\frac{4\frac{1}{3}}{2\frac{3}{5}} = \frac{\frac{13}{3}}{\frac{13}{5}} = \frac{13}{3} \times \frac{5}{13} = \frac{5}{3}$$
The correct choice is B.

4. Since this is a multiple-choice question, try each choice to see if it answers the question. Start with 10, because it is easy to calculate with tens. If 10 adult tickets are sold, then 20 student tickets must be sold. Check to see if the total sales exceeds $90.
Students sales + Adult sales > $90
20($2.00) + 10($5.00) = 40 + 50 = $90
So 10 is too low a number for adult tickets. This eliminates answer choices A, B, C, and D. Check choice E. Eleven is the minimum number of adult tickets.
19($2.00) + 11($5.00) = 38 + 55 = $93
The correct choice is E.

5. Recall the definition of absolute value: the number of units a number is from zero on the number line. Simplify the expression by writing it without the absolute value symbols.
$|-7| = 7$
$-|-7| = -7$
$-|-7| - |-5| - 3|-4| = -7 - 5 - 12 = -24$
The correct choice is A.

6. Write each part of the expression without exponents.
$(-4)^2 = 16$
$(2)^{-4} = \frac{1}{2^4} = \frac{1}{16}$
$16 + \frac{1}{16} + \frac{3}{4} = 16 + \frac{13}{16} = 16\frac{13}{16}$
The correct choice is A.

7. Use your calculator. First find the total amount per year by adding.
$12.90 + $16.00 + $18.00 + $21.90 = $68.80
Then find one half of this, which is the amount paid in equal payments.
$68.80 ÷ 2 = $34.40
Then divide this amount by 4 to get each of 4 monthly payments.
$34.40 ÷ 4 = $8.60.
The correct choice is A.

8. First combine the numbers inside the square root symbol. Then find the square root of the result.

$\sqrt{64 + 36} = \sqrt{100} = 10$

The correct choice is A.

9. $60 = 2 \times 2 \times 3 \times 5$

$= 2^2 \times 3 \times 5$

The number of distinct prime factors of 60 is 3.
The correct choice is C.

10. First find the number of fish that are not tetras.

$\left(\frac{1}{8}\right)(24)$ or 3 are tetras. $24 - 3$ or 21 are not tetras. Then $\frac{2}{3}$ of these are guppies.

$\left(\frac{2}{3}\right)(21) = 14$

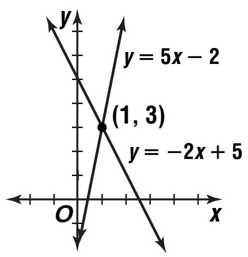
The answer is 14.

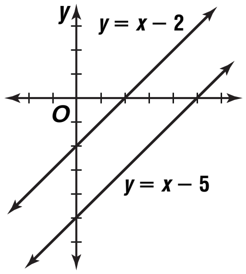
2-1 Solving Systems of Equations in Two Variables

Page 69 Graphing Calculator Exploration

1. (1, 480)
2. $3x - 4y = 320 \quad \rightarrow \quad y = \frac{3}{4}x - 80$
   $5x + 2y = 340 \quad\quad\;\; y = -\frac{5}{2}x + 170$
   (76.923077, −22.30769)
3. accurate to a maximum of 8 digits
4. $5x - 7y = 70 \quad \rightarrow \quad y = \frac{5}{7}x - 10$
   $10x - 14y = 120 \quad\quad y = \frac{5}{7}x - \frac{60}{7}$
   Inconsistent; error message occurs.
5. See students' systems and graphs; any point in TRACE mode will be the intersection point since the two lines intersect everywhere.

Page 70 Check for Understanding

1. Sample answer:
   $4x + 7x = 21$
   $y = 2x - 1$
   The substitution method is usually easier to use whenever one or both of the equations are already solved for one variable in terms of the other.
2. Sample answer: Madison might consider whether the large down-payment would strap her financially; if she wants to buy the car at the end of the lease, then she might also consider which lease would offer the best buyout.
3. Sample answer: consistent systems of equations have at least one solution. A consistent, independent system has exactly one solution; a consistent, dependent system has an infinite number of solutions. An inconsistent system has no solution. See students' work for examples and solutions.
4. $2y + 3x = 6 \quad \rightarrow \quad y = -\frac{3}{2}x + 3$
   $4y = 16 - 6x \quad\quad y = -\frac{3}{2}x + 4$
   Inconsistent; Sample answer: The graphs of the equations are lines with slope $-\frac{3}{2}$, but each equation has a different y-intercept. Therefore, the graphs of the two equations do not intersect and the system has no solution.
5. 

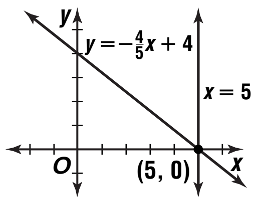
   (1, 3)
6. 

   no solution
7. $$\begin{aligned} 7x + y &= 9 \\ 5x - y &= 15 \\ \hline 12x \quad &= 24 \\ x &= 2 \end{aligned} \qquad \begin{aligned} 7x + y &= 9 \\ 7(2) + y &= 9 \\ y &= -5 \end{aligned}$$
   (2, −5)
8. $3x + 4y = -1 \quad\quad\quad\; 3x + 4y = -1$
   $2(6x - 2y) = 2(3) \rightarrow 12x - 4y = 6$
   $$\begin{aligned} 15x &= 5 \\ x &= \frac{1}{3} \end{aligned}$$
   $$\begin{aligned} 3x + 4y &= -1 \\ 3\left(\frac{1}{3}\right) + 4y &= -1 \\ 4y &= -2 \\ y &= -\frac{1}{2} \end{aligned} \qquad \left(\frac{1}{3}, -\frac{1}{2}\right)$$
9. $30\left(\frac{1}{3}x - \frac{3}{2}y\right) = 30(-4) \quad\quad 10x - 45y = -120$
   $-2(5x - 4y) = -2(14) \rightarrow -10x + 8y = -28$
   $$\begin{aligned} -37y &= -148 \\ y &= 4 \end{aligned}$$
   $$\begin{aligned} 5x - 4y &= 14 \\ 5x - 4(4) &= 14 \\ 5x &= 30 \\ x &= 6 \end{aligned} \qquad (6, 4)$$
10. Let b represent the number of baseball racks and k represent the number of karate-belt racks.
    $$\begin{aligned} b &= 6k \\ 3b + 5k &= 46{,}000 \\ 3(6k) + 5k &= 46{,}000 \\ 23k &= 46{,}000 \\ k &= 2000 \end{aligned} \qquad \begin{aligned} b &= 6k \\ &= 6(2000) \\ &= 12{,}000 \end{aligned}$$
    12,000 baseball, 2000 karate

Pages 71–72 Exercises

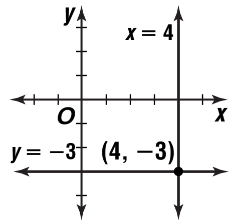
11. $x + 3y = 18 \quad \rightarrow \quad y = -\frac{1}{3}x + 6$
    $-x + 2y = 7 \quad\quad y = \frac{1}{2}x + \frac{7}{2}$
    consistent and independent
12. $y = 0.5x \quad \rightarrow \quad y = 0.5x$
    $2y = x + 4 \quad\quad y = \frac{1}{2}x + 2$
    inconsistent
13. $-35x + 40y = 55 \quad \rightarrow \quad y = \frac{7}{8}x + \frac{11}{8}$
    $7x = 8y - 11 \quad\quad y = \frac{7}{8}x + \frac{11}{8}$
    consistent and dependent

14.



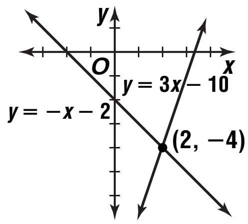
(5, 0)

15.



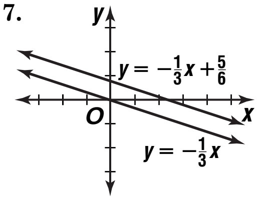
(4, −3)

16.



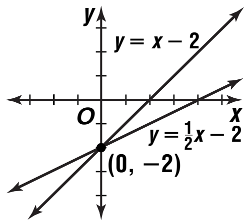
(2, −4)

17.



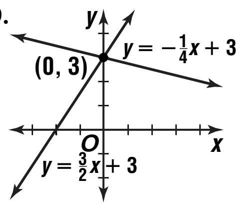
no solution

18.



(0, −2)

19.



(0, 3)

20. $3x - 8y = 10 \quad \rightarrow \quad y = \frac{3}{8}x - \frac{5}{2}$

$16x - 32y = 75 \quad\quad y = \frac{1}{2}x - \frac{75}{32}$

Consistent and independent; if each equation is written in slope-intercept form, they have different slopes, which means they will intersect at some point.

21. $3(5x - y) = 3(16) \rightarrow 15x - 3y = 48$

$2x + 3y = 3 \quad\quad 2x + 3y = 3$

$17x = 51$

$x = 3$

$5x - y = 16$

$5(3) - y = 16$

$-y = 1$

$y = -1 \quad (3, -1)$

22. $3x - 5y = -8 \rightarrow 3x - 5y = -8$

$-3(x + 2y) = -3(1) \quad\quad -3x - 6y = -3$

$-11y = -11$

$y = 1$

$x + 2y = 1$

$x + 2(1) = 1$

$x = -1 \quad (-1, 1)$

23. $x = 4.5 + y \quad\quad y = 6 - x$

$x = 4.5 + 6 - x \quad\quad y = 6 - 5.25$

$2x = 10.5 \quad\quad y = 0.75$

$x = 5.25 \quad\quad (5.25, 0.75)$

24. $5(2x + 3y) = 5(3) \rightarrow 10x + 15y = 15$

$12x - 15y = -4 \quad\quad 12x - 15y = -4$

$22x = 11$

$x = \frac{1}{2}$

$2x + 3y = 3$

$2\left(\frac{1}{2}\right) + 3y = 3$

$3y = 2$

$y = \frac{2}{3} \quad \left(\frac{1}{2}, \frac{2}{3}\right)$

25. $2(-3x + 10y = 5) \rightarrow -6x + 20y = 10$

$3(2x + 7y = 24) \quad\quad 6x + 21y = 72$

$41y = 82$

$y = 2$

$2x + 7y = 24$

$2x + 7(2) = 24$

$2x = 10$

$x = 5 \quad (5, 2)$

26. $2x - y = -7 \quad\quad x = 2y - 8$

$2(2y - 8) - y = -7 \quad\quad x = 2(3) - 8$

$3y = 9 \quad\quad x = -2$

$y = 3 \quad (-2, 3)$

27. $3(2x + 5y) = 3(4) \rightarrow 6x + 15y = 12$

$-2(3x + 6y) = -2(5) \quad\quad -6x - 12y = -10$

$3y = 2$

$y = \frac{2}{3}$

$2x + 5y = 4$

$2x + 5\left(\frac{2}{3}\right) = 4$

$2x = \frac{2}{3}$

$x = \frac{1}{3} \quad \left(\frac{1}{3}, \frac{2}{3}\right)$

28. $5\left(\frac{3}{5}x - \frac{1}{6}y\right) = 5(1) \rightarrow 3x - \frac{5}{6}y = 5$

$\frac{1}{5}x + \frac{5}{6}y = 1 \quad\quad \frac{1}{5}x + \frac{5}{6}y = 11$

$\frac{16}{5}x = 16$

$x = 5$

$\frac{1}{5}x + \frac{5}{6}y = 11$

$\frac{1}{5}(5) + \frac{5}{6}y = 11$

$\frac{5}{6}y = 10$

$y = 12 \quad (5, 12)$

29. $7(4x + 5y) = 7(-8) \rightarrow 28x + 35y = -56$

$5(3x - 7y) = 5(10) \quad\quad 15x - 35y = 50$

$43x = -6$

$x = -\frac{6}{43}$

$4x + 5y = -8$

$4\left(-\frac{6}{43}\right) + 5y = -8$

$5y = -\frac{320}{43}$

$y = -\frac{64}{43} \quad \left(-\frac{6}{43}, -\frac{64}{43}\right)$

30. $-2(3x - y) = -2(-9) \rightarrow -6x + 2y = 18$

$4x - 2y = -8 \quad\quad 4x - 2y = -8$

$-2x = 10$

$x = -5$

$3x - y = -9$

$3(-5) - y = -9$

$-y = 6$

$y = -6 \quad (-5, -6)$

31. Sample answer: Elimination could be considered easiest since the first equation multiplied by 2 added to the second equation eliminates b; substitution could also be considered easiest since the first equation can be written as $a = b$, making substitution very easy.

$a - b = 0 \quad\quad 3a + 2b = -15$

$a = b \quad\quad 3(b) + 2b = -15$

$5b = -15$

$b = -3$

$a - b = 0$

$a - (-3) = 0$

$a = -3 \quad (-3, -3)$

32a. B

32b. $S - 4V = 0$ $\quad$ $S + V = 30{,}000$

$S = 4V$ $\quad$ $4V + V = 30{,}000$

$5V = 30{,}000$

$V = 6000$

$S - 4V = 0$

$S - 4(6000) = 0$

$S - 24{,}000 = 0$

$S = 24{,}000$

Spartans: 24,000; visitors: 6000

33a. Let b represent the base and ℓ represent the leg.

Perimeter of first triangle: $b + 2\ell = 20$

Perimeter of second triangle: $6 + b + \ell = 20$

$b + 2\ell = 20$

$b = 20 - 2\ell$

$6 + b + \ell = 20$ $\quad$ $b + 2\ell = 20$

$6 + 20 - 2\ell + \ell = 20$ $\quad$ $b + 2(6) = 20$

$-\ell = -6$ $\quad$ $b = 8$

$\ell = 6$

6, 6, 8; 6, 6, 8

33b. isosceles

34. $y - (-3) = 4(x - 4)$

$y = 4x - 19$

$y - (-3) = -\frac{1}{4}(x - 4)$

$y = -\frac{1}{4}x - 2$

35a. Let x represent the number of refills. Then, $x + 1$ = number of drinks purchased.

$C = 2.95 + 0.50x$

$C = 0.85 + 0.85x$

$C = 2.95 + 0.50x$

$0.85 + 0.85x = 2.95 + 0.50x$

$0.35x = 2.1$

$x = 6$

$x + 1 = 7$ $\quad$ $C = 2.95 + 0.50x$

$C = 2.95 + 0.50(7)$ or 5.95

(7, 5.95)

35b. If you drink 7 servings of soft drink, the price for each option is the same. If you drink fewer than 7 servings of soft drink during that week, the disposable cup price is better. If you drink more than 7 servings of soft drink, the refillable mug price is better. See students' choices.

35c. Over a year's time, the refillable mug would be more economical.

36a. $\frac{a}{b} \neq \frac{d}{e}$

36b. $\frac{a}{b} = \frac{d}{e}$, $c = f$

36c. $\frac{a}{b} = \frac{d}{e}$, $c \neq f$

37. Let x represent the full incentive.

Let y represent the value of the computer.

$x = 516 + y$

$\frac{3.5}{4}x = 264 + y$

$\frac{3.5}{4}x = 264 + y$

$\frac{3.5}{4}(516 + y) = 264 + y$

$451.5 + 0.875y = 264 + y$

$y = \$1500$

38. Let x represent the number of people in line behind you. $200 + x$ represents the number in front of you. Let ℓ represent the whole line.

$200 + x + 1 + x = \ell$

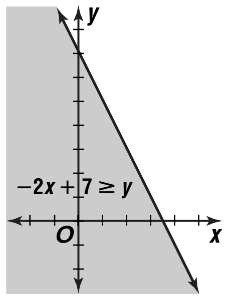
$\ell = 3x$

$200 + x + 1 + x = 3x$ $\quad$ $\ell = 3x$

$201 = x$ $\quad$ $\ell = 3(201)$

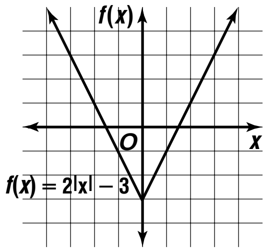
603 people $\quad$ $= 603$

39.



40.

x	$f(x)$
−2	1
−1	−1
0	−3
1	−1
2	1



41. $y - 6 = 2(x - 0)$

$y = 2x + 6$

42. \$12,500

43. $[f \circ g](x) = f(g(x))$

$= f(x + 2)$

$= 3(x + 2) - 5$

$= 3x + 1$

44. {18}, {−3, 3}; no, because there are two range values paired with a single domain value.

45. $\sqrt{\frac{\sqrt{25}}{5}} = \sqrt{\frac{5}{5}}$

$= \sqrt{1}$

$= 1$

The correct choice is A.

2-2 Solving Systems of Equations in Three Variables

Page 76 Check for Understanding

1. Solving a system of three equations involves eliminating one variable to form two systems of two equations. Then solving is the same.

2. The solution would be an equation in two variables. Sample example: the system $2x + 4y + 6z = 12$, $x + 2y + 3z = 6$, and $3x - 5y - 6z = 27$ has a solution of all values of x and y that satisfy $5x - y = 39$.

$$\begin{array}{rcl} 2x + 4y + 6z &=& 12 \\ 3x - 5y - 6z &=& 27 \\ \hline 5x - y \phantom{{}+6z} &=& 39 \end{array} \qquad \begin{array}{rcl} -2(x + 2y + 3z) &=& -2(6) \\ -2x - 4y - 6z &=& -12 \end{array}$$

$$\downarrow$$

$$\begin{array}{rcl} -2x - 4y - 6z &=& -12 \\ 2x + 4y + 6z &=& 12 \\ \hline 0 &=& 0 \end{array}$$

all reals

3. Sample answer: Use one equation to eliminate one of the variables from the other two equations. Then eliminate one of the remaining variables from the resulting equations. Solve for a variable and substitute to find the values of the other variables.

4. $$\begin{array}{rcl} 4(4x + 2y + z) &=& 4(7) \\ 2x + 2y - 4z &=& -4 \end{array} \qquad \begin{array}{rcl} 2(4x + 2y + z) &=& 2(7) \\ x + 3y - 2z &=& -8 \end{array}$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$\begin{array}{rcl} 16x + 8y + 4z &=& 28 \\ 2x + 2y - 4z &=& -4 \\ \hline 18x + 10y &=& 24 \end{array} \qquad \begin{array}{rcl} 8x + 4y + 2z &=& 14 \\ x + 3y - 2z &=& -8 \\ \hline 9x + 7y &=& 6 \end{array}$$

$$\begin{array}{rcl} 18x + 10y &=& 24 \\ -2(9x + 7y) &=& -2(6) \end{array} \rightarrow \begin{array}{rcl} 18x + 10y &=& 24 \\ -18x - 14y &=& -12 \\ \hline -4y &=& 12 \\ y &=& -3 \end{array}$$

$$\begin{array}{rcl} 9x + 7y &=& 6 \\ 9x + 7(-3) &=& 6 \\ x &=& 3 \end{array} \qquad \begin{array}{rcl} 4x + 2y + z &=& 7 \\ 4(3) + 2(-3) + z &=& 7 \\ z &=& 1 \end{array}$$

$(3, -3, 1)$

5. $$\begin{array}{rcl} 2(x - y - z) &=& 2(7) \\ -x + 2y - 3z &=& -12 \end{array} \qquad \begin{array}{rcl} -x + 2y - 3z &=& -12 \\ 3x - 2y + 7z &=& 30 \\ \hline 2x \phantom{{}-2y} + 4z &=& 18 \end{array}$$

$$\downarrow$$

$$\begin{array}{rcl} 2x - 2y - 2z &=& 14 \\ -x + 2y - 3z &=& -12 \\ \hline x \phantom{{}+2y} - 5z &=& 2 \end{array}$$

$$\begin{array}{rcl} -2(x - 5z) &=& -2(2) \\ 2x + 4z &=& 18 \end{array} \rightarrow \begin{array}{rcl} -2x + 10z &=& -4 \\ 2x + 4z &=& 18 \\ \hline 14z &=& 14 \\ z &=& 1 \end{array}$$

$$\begin{array}{rcl} x - 5z &=& 2 \\ x - 5(1) &=& 2 \\ x &=& 7 \end{array} \qquad \begin{array}{rcl} x - y - z &=& 7 \\ 7 - y - 1 &=& 7 \\ y &=& -1 \end{array} \qquad (7, -1, 1)$$

6. $$\begin{array}{rcl} -2(2x - 2y + 3z) && \\ &=& -2(6) \\ 4x - 3y + 2z &=& 0 \end{array} \qquad \begin{array}{rcl} -2(2x - 3y + 7z) && \\ &=& -2(-1) \\ 4x - 3y + 2z &=& 0 \end{array}$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$\begin{array}{rcl} -4x + 4y - 6z &=& -12 \\ 4x - 3y + 2z &=& 0 \\ \hline y - 4z &=& -12 \end{array} \qquad \begin{array}{rcl} -4x + 6y - 14z &=& 2 \\ 4x - 3y + 2z &=& 0 \\ \hline 3y - 12z &=& 2 \end{array}$$

$$\begin{array}{rcl} -3(y - 4z) &=& -3(-12) \\ 3y - 12z &=& 2 \end{array} \rightarrow \begin{array}{rcl} -3y + 12z &=& 36 \\ 3y - 12z &=& 2 \\ \hline 0 &=& 38 \end{array}$$

no solution

7. $$\begin{aligned} 75 &= \tfrac{1}{2}a(1)^2 + v_0(1) + s_0 \\ &= 0.5a + v_0 + s_0 \\ 75 &= \tfrac{1}{2}a(2.5)^2 + v_0(2.5) + s_0 \\ &= 3.125a + 2.5v_0 + s_0 \\ 3 &= \tfrac{1}{2}a(4)^2 + v_0(4) + s_0 \\ &= 8a + 4v_0 + s_0 \end{aligned}$$

$$\begin{array}{rcl} -2.5(75) &=& -2.5(0.5a + v_0 + s_0) \\ 75 &=& 3.125a + 2.5v_0 + s_0 \end{array}$$

$$\downarrow$$

$$\begin{array}{rcl} -187.5 &=& -1.25a - 2.5v_0 - 2.5s_0 \\ 75 &=& 3.125a + 2.5v_0 + s_0 \\ \hline -112.5 &=& 1.875a \phantom{{}+2.5v_0} - 1.5s_0 \end{array}$$

$$\begin{array}{rcl} -4(75) &=& -4(0.5a + v_0 + s_0) \\ 3 &=& 8a + 4v_0 + s_0 \end{array}$$

$$\downarrow$$

$$\begin{array}{rcl} -300 &=& -2a - 4v_0 - 4s_0 \\ 3 &=& 8a + 4v_0 + s_0 \\ \hline -297 &=& 6a \phantom{{}+4v_0} - 3s_0 \end{array}$$

$$\begin{array}{rcl} -2(-112.5) &=& -2(1.875a - 1.5s_0) \\ -297 &=& 6a - 3s_0 \end{array}$$

$$\downarrow$$

$$\begin{array}{rcl} 225 &=& -3.75a + 3s_0 \\ -297 &=& 6a - 3s_0 \\ \hline -72 &=& 2.25a \\ -32 &=& a \end{array}$$

$$\begin{array}{rcl} -297 &=& 6a - 3s_0 \\ -297 &=& 6(-32) - 3s_0 \\ 35 &=& s_0 \end{array} \qquad \begin{array}{rcl} 3 &=& 8a + 4v_0 + s_0 \\ 3 &=& 8(-32) + 4v_0 + s_0 \\ 56 &=& v_0 \end{array}$$

acceleration -32 ft/s^2, initial velocity: 56 ft/s, initial height: 35 ft

Pages 76–77 Exercises

8. $$\begin{array}{rcl} 3(5x + 3y - z) &=& 3(-11) \\ x + 2y + 3z &=& 5 \end{array}$$

$$\downarrow$$

$$\begin{array}{rcl} 15x + 9y - 3z &=& -33 \\ x + 2y + 3z &=& 5 \\ \hline 16x + 11y &=& -28 \end{array}$$

$$\begin{array}{rcl} -2(5x + 3y - z) &=& -2(-11) \\ 3x + 2y - 2z &=& -13 \end{array}$$

$$\downarrow$$

$$\begin{array}{rcl} -10x - 6y + 2z &=& 22 \\ 3x + 2y - 2z &=& -13 \\ \hline -7x - 4y &=& 9 \end{array}$$

$$\begin{array}{rcl} 4(16x + 11y) &=& 4(-28) \\ 11(-7x - 4y) &=& 11(9) \end{array} \rightarrow \begin{array}{rcl} 64x + 44y &=& -112 \\ -77x - 44y &=& 99 \\ \hline -13x &=& -13 \\ x &=& 1 \end{array}$$

$$\begin{array}{rcl} 16x + 11y &=& -28 \\ 16(1) + 11y &=& -28 \\ y &=& -4 \end{array} \qquad \begin{array}{rcl} x + 2y + 3z &=& 5 \\ 1 + 2(-4) + 3z &=& 5 \\ z &=& 4 \end{array}$$

$(1, -4, 4)$

9.

$$\begin{array}{l} 7(-x+3y+2z)=7(16) \\ 7x+5y+z=0 \\ \downarrow \\ -7x+21y+14z=112 \\ 7x+5y+z=0 \\ \hline 26y+15z=112 \end{array} \qquad \begin{array}{r} -x+3y+2z=16 \\ x-6y-z=-18 \\ \hline -3y+z=-2 \end{array}$$

$$\begin{array}{l} -15(-3y+z)=-15(-2) \\ 26y+15z=112 \end{array} \rightarrow \begin{array}{r} 45y-15z=30 \\ 26y+15z=112 \\ \hline 71y=142 \\ y=2 \end{array}$$

$$\begin{array}{r} -3y+z=-2 \\ -3(2)+z=-2 \\ z=4 \end{array} \qquad \begin{array}{r} x-6y-z=-18 \\ x-6(2)-4=-18 \\ x=-2 \end{array}$$

$(-2, 2, 4)$

10.

$$\begin{array}{l} 2(2x+y-2z)=2(11) \\ -x-2y+9z=13 \\ \downarrow \\ 4x+2y-4z=22 \\ -x-2y+9z=13 \\ \hline 3x+5z=35 \end{array}$$

$$\begin{array}{l} -3(x-3z)=-3(7) \\ 3x+5z=35 \end{array} \rightarrow \begin{array}{r} -3x+9z=-21 \\ 3x+5z=35 \\ \hline 14z=14 \\ z=1 \end{array}$$

$$\begin{array}{r} x-3z=7 \\ x-3(1)=7 \\ x=10 \end{array} \qquad \begin{array}{r} 2x+y-2z=11 \\ 2(10)+y-2(1)=11 \\ y=-7 \end{array}$$

$(10, -7, 1)$

11.

$$\begin{array}{l} -3(x-3y-2z)=-3(-8) \\ 3x-5y+z=9 \\ \downarrow \\ -3x+9y+6z=24 \\ 3x-5y+z=9 \\ \hline 4y+7z=33 \end{array} \qquad \begin{array}{l} -5(x-3y-2z)=-5(-8) \\ 5x-6y+3z=15 \\ \downarrow \\ -5x+15y+10z=40 \\ 5x-6y+3z=15 \\ \hline 9y+13z=55 \end{array}$$

$$\begin{array}{l} 9(4y+7z)=9(33) \\ -4(9y+13z)=-4(55) \end{array} \rightarrow \begin{array}{r} 36y+63z=297 \\ -36y-52z=-220 \\ \hline 11z=77 \\ z=7 \end{array}$$

$$\begin{array}{r} 4y+7z=33 \\ 4y+7(7)=33 \\ y=-4 \end{array} \qquad \begin{array}{r} x-3y-2z=-8 \\ x-3(-4)-2(7)=-8 \\ x=-6 \end{array}$$

$(-6, -4, 7)$

12.

$$\begin{array}{l} 8x-z=4 \\ y+z=5 \\ \hline 8x+y=9 \end{array}$$

$$\begin{array}{l} -1(8x+y)=-1(9) \\ 11x+y=15 \end{array} \rightarrow \begin{array}{r} -8x-y=-9 \\ 11x+y=15 \\ \hline 3x=6 \\ x=2 \end{array}$$

$$\begin{array}{r} 8x-z=4 \\ 8(2)-z=4 \\ z=12 \end{array} \qquad \begin{array}{r} y+z=5 \\ y+12=5 \\ y=-7 \end{array}$$

$(2, -7, 12)$

13.

$$\begin{array}{l} 3(x+y-z)=3(3) \\ 4x-3y+2z=12 \\ \downarrow \\ 3x+3y-3z=9 \\ 4x-3y+2z=12 \\ \hline 7x+z=21 \end{array} \qquad \begin{array}{l} 2(x+y-z)=2(3) \\ -2x-2y+2z=5 \\ \downarrow \\ 2x+2y-2z=6 \\ -2x-2y+2z=5 \\ \hline 0=11 \end{array}$$

no solution

14.

$$\begin{array}{l} 3(36x-15y+50z)=3(-10) \\ -5(54x-5y+30z)=-5(-160) \\ \downarrow \\ 108x-45y+150z=-30 \\ -270x+25y-150z=800 \\ \hline -162x-20y=770 \end{array}$$

$$\begin{array}{l} 81(2x+25y)=81(40) \\ -162x-20y=770 \end{array} \rightarrow \begin{array}{r} 162x+2025y=3240 \\ -162x-20y=770 \\ \hline 2005y=4010 \\ y=2 \end{array}$$

$$\begin{array}{r} 2x+25y=40 \\ 2x+25(2)=40 \\ x=-5 \end{array} \qquad \begin{array}{r} 36x-15y+50z=-10 \\ 36(-5)-15(2)+50z=-10 \\ z=4 \end{array}$$

$(-5, 2, 4)$

15.

$$\begin{array}{l} 4(-x-3y+z)=4(54) \\ 4x+2y-3z=-32 \\ \downarrow \\ -4x-12y+4z=216 \\ 4x+2y-3z=-32 \\ \hline -10y+z=184 \end{array}$$

$$\begin{array}{l} 5(2y+8z)=5(78) \\ -10y+z=184 \end{array} \rightarrow \begin{array}{r} 10y+40z=390 \\ -10y+z=184 \\ \hline 41z=574 \\ z=14 \end{array}$$

$$\begin{array}{r} 2y+8z=78 \\ 2y+8(14)=78 \\ y=-17 \end{array} \qquad \begin{array}{r} -x-3y+z=54 \\ -x-3(-17)+14=54 \\ x=11 \end{array}$$

$(11, -17, 14)$

16.

$$\begin{array}{l} 1.8x-z=0.7 \\ 1.2y+z=0.7 \\ \hline 1.8x+1.2y=0 \end{array}$$

$$\begin{array}{l} 3(1.8x+1.2y)=3(0) \\ 1.2(1.5x-3y)=1.2(3) \end{array} \rightarrow \begin{array}{r} 5.4x+3.6y=0 \\ 1.8x-3.6y=3.6 \\ \hline 7.2x=3.6 \\ x=0.5 \end{array}$$

$$\begin{array}{r} 1.5x-3y=3 \\ 1.5(0.5)-3y=3 \\ y=-0.75 \end{array} \qquad \begin{array}{r} 1.8x-z=0.7 \\ 1.8(0.5)-z=0.7 \\ z=0.2 \end{array}$$

$(0.5, -0.75, 0.2)$

17.

$$\begin{array}{l} y=x+2z \\ y=(y-14)+2z \\ 7=z \\ x=y-14 \\ -4=y-14 \\ 10=y \end{array} \qquad \begin{array}{l} z=-1-2x \\ 7=-1-2x \\ -4=x \end{array}$$

$(-4, 10, 7)$

18. $\frac{5}{2}\left(\frac{3}{4}x + \frac{1}{6}y - \frac{1}{3}z\right) = \frac{5}{2}(-12)$

$\frac{1}{8}x - \frac{2}{3}y + \frac{5}{6}z = -8$

$$\downarrow$$

$$\begin{array}{r}
\frac{15}{8}x + \frac{5}{12}y - \frac{5}{6}z = -30 \\
\frac{1}{8}x - \frac{2}{3}y + \frac{5}{6}z = -8 \\
\hline
2x - \frac{1}{4}y \qquad = -38
\end{array}$$

$-\frac{7}{4}\left(\frac{3}{4}x + \frac{1}{6}y - \frac{1}{3}z\right) = -\frac{7}{4}(-12)$

$\frac{3}{16}x - \frac{5}{8}y - \frac{7}{12}z = -25$

$$\downarrow$$

$$\begin{array}{r}
-\frac{21}{16}x - \frac{7}{24}y + \frac{7}{12}z = 21 \\
\frac{3}{16}x - \frac{5}{8}y - \frac{7}{12}z = -25 \\
\hline
-\frac{9}{8}x - \frac{11}{12}y \qquad = -4
\end{array}$$

$\frac{9}{16}\left(2x - \frac{1}{4}y\right) = \frac{9}{16}(-38)$

$-\frac{9}{8}x - \frac{11}{12}y = -4$

$$\rightarrow \quad \begin{array}{r}
\frac{9}{8}x - \frac{9}{64}y = -\frac{171}{8} \\
-\frac{9}{8}x - \frac{11}{12}y = -4 \\
\hline
-\frac{203}{192}y = -\frac{203}{8} \\
y = 24
\end{array}$$

$2x - \frac{1}{4}y = -38$

$2x - \frac{1}{4}(24) = -38$

$x = -16$

$\frac{3}{4}x + \frac{1}{6}y - \frac{1}{3}z = -12$

$\frac{3}{4}(-16) + \frac{1}{6}(24) - \frac{1}{3}z = -12$

$z = 12$

$(-16, 24, 12)$

19. Let x represent amount in International Fund, y represent amount in Fixed Assets Fund, and z represent amount in company stock.

$x + y + z = 2000$
$x = 2z$
$0.045x + 0.026y - 0.002z = 58$

$x + y + z = 2000$
$2z + y + z = 2000$
$y + 3z = 2000$
$y = 2000 - 3z$

$0.045x + 0.026y - 0.002z = 58$
$0.045(2z) + 0.026y - 0.002z = 58$
$0.026y + 0.088z = 58$

$0.026y + 0.088z = 58$
$0.026(2000 - 3z) + 0.088z = 58$
$z = 600$

$x = 2z$
$x = 2(600)$
$x = 1200$

$x + y + z = 2000$
$1200 + y + 600 = 2000$
$y = 200$

International Fund = \$1200; Fixed Assets Fund = \$200; company stock = \$600

20a. Sample answer: $x + y + z = 15$; $2x + z = 1$; $2y - z = 7$

20b. Sample answer: $4x + y + z = 12$; $-4x - y - z = -10$; $5y - z = 9$

20c. Sample answer: $x + y + z = 6$; $2x - y - 2z = 8$; $x - 2y - 3z = 2$

21. $124 = \frac{1}{2}a(1)^2 + v_0(1) + s_0$

$272 = \frac{1}{2}a(3)^2 + v_0(3) + s_0$

$82 = \frac{1}{2}a(8)^2 + v_0(8) + s_0$

$$\downarrow$$

$124 = \frac{1}{2}a + v_0 + s_0$

$272 = \frac{9}{2}a + 3v_0 + s_0$

$82 = 32a + 8v_0 + s_0$

$-1(124) = -1\left(\frac{1}{2}a + v_0 + s_0\right)$

$272 = \frac{9}{2}a + 3v_0 + s_0$

$$\downarrow$$

$$\begin{array}{r}
-124 = -\frac{1}{2}a - v_0 - s_0 \\
272 = \frac{9}{2}a + 3v_0 + s_0 \\
\hline
148 = 4a + 2v_0
\end{array}$$

$-1(124) = -1\left(\frac{1}{2}a + v_0 + s_0\right)$

$82 = 32a + 8v_0 + s_0$

$$\downarrow$$

$$\begin{array}{r}
-124 = -\frac{1}{2}a - v_0 - s_0 \\
82 = 32a + 8v_0 + s_0 \\
\hline
-42 = 31.5a + 7v_0
\end{array}$$

$7(148) = 7(4a + 2v_0)$

$-2(-42) = -2(31.5a + 7v_0)$

$$\downarrow$$

$$\begin{array}{r}
1036 = 28a + 14v_0 \\
84 = -63a - 14v_0 \\
\hline
1120 = -35a
\end{array}$$

$-32 = a$

$148 = 4a + 2v_0$

$148 = 4(-32) + 2v_0$

$138 = v_0$

$124 = \frac{1}{2}a + v_0 + s_0$

$124 = \frac{1}{2}(-32) + 138 + s_0$

$2 = s_0$

$(-32, 138, 2)$

22a. Sample answer: A system has no solution when you reach a contradiction, such as $1 = 0$, as you solve.

22b. Sample answer: A system has an infinite number of solutions when you reduce the system to two equivalent equations such as $x + y = 1$ and $2x + 2y = 2$.

23. $x + yz = 2$
$y + xz = 2$
$z + xy = 2$

$$\begin{aligned} x + yz &= 2 \\ y + xz &= 2 \end{aligned} \quad \rightarrow \quad \begin{array}{r} x + \quad yz = \ \ 2 \\ -y - \quad xz = -2 \\ \hline (x - y) + (yz - xz) = 0 \\ (x - y) - z(x - y) = 0 \\ (1 - z)(x - y) = 0 \\ 1 - z = 0 \quad \text{or} \quad x - y = 0 \\ z = 1 \qquad\qquad x = y \end{array}$$

$$\begin{aligned} y + xz &= 2 \\ z + xy &= 2 \end{aligned} \quad \rightarrow \quad \begin{array}{r} y + \quad xz = \ \ 2 \\ -z - \quad xy = -2 \\ \hline (y - z) - (xz - xy) = 0 \\ (y - z) - x(y - z) = 0 \\ (1 - x)(y - z) = 0 \\ 1 - x = 0 \quad \text{or} \quad y - z = 0 \\ x = 1 \qquad\qquad y = z \end{array}$$

If $z = 1$ and $x = 1$, $1 + 1y = 2$ and $y = 1$.
If $z = 1$ and $y = z$, $x + 1 \cdot 1 = 2$ and $x = 1$.
If $x = y$ and $x = 1$, $1 + 1z = 2$ and $z = 1$.
If $x = y$ and $y = z$, $x = y = z$.

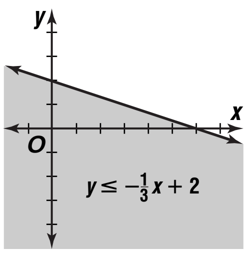
$$\begin{aligned} x + x \cdot x &= 2 \\ x + x^2 &= 2 \\ x^2 + x - 2 &= 0 \\ (x + 2)(x - 1) &= 0 \end{aligned}$$
$$x + 2 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x^2 = -2 \qquad\qquad x = 1$$

If $x = -2$, $y = -2$ and $z = -2$.
The answers are $(1, 1, 1)$ and $(-2, -2, -2)$.

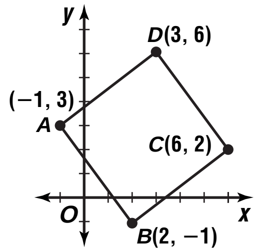
24. $$\begin{aligned} 3x + 4y &= 375 \\ -2(5x + 2y) &= -2(345) \end{aligned} \quad \rightarrow \quad \begin{array}{r} 3x + 4y = \ \ 375 \\ -10x - 4y = -690 \\ \hline -7x \qquad = -315 \\ x = 45 \end{array}$$

$$\begin{aligned} 5x + 2y &= 345 \\ 5(45) + 2y &= 345 \\ y &= 60 \qquad (45, 60) \end{aligned}$$

25.



26.



AB: $d = \sqrt{(-1 - 3)^2 + (2 - (-1))^2} = 5$
BC: $d = \sqrt{(2 - (-1))^2 + (6 - 2)^2} = 5$
CD: $d = \sqrt{(6 - 2)^2 + (3 - 6)^2} = 5$
AD: $d = \sqrt{(6 - 3)^2 + (3 - (-1))^2} = 5$

AB: $m = \frac{-1 - 3}{2 - (-1)} = -\frac{4}{3}$ $\qquad$ $\overline{BC}$: $m = \frac{2 - (-1)}{6 - 2} = \frac{3}{4}$

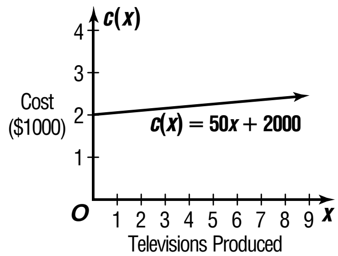
$AB = BC = CD = AD = 5$ units; $ABCD$ is rhombus. Slope of $\overline{AB} = -\frac{4}{3}$ and slope of $\overline{BC} = \frac{3}{4}$, so $\overline{AB} \perp \overline{BC}$. A rhombus with a right angle is a square.

27a. (20, 3000), (60, 5000)

$$\begin{aligned} m &= \frac{5000 - 3000}{60 - 20} \\ &= \frac{2000}{40} \quad \text{or} \quad 50 \end{aligned}$$
$$\begin{aligned} y - 5000 &= 50(x - 60) \\ y &= 50x + 2000 \\ C(x) &= 50x + 2000 \end{aligned}$$

27b. $2000; $50

27c.



28. $$\begin{aligned} A &= s^2 \\ 2 &= s^2 \\ \sqrt{2} &= s \end{aligned} \qquad\qquad \begin{aligned} A &= \pi r^2 \\ &= \pi(\sqrt{2})^2 \\ &= 2\pi \end{aligned}$$

The correct choice is C.

2-3 Modeling Real-World Data with Matrices

Pages 82–83 Check for Understanding

1. Sample answer:

$$\begin{array}{l} \\ \\ \text{Atlanta} \\ \text{Los Angeles} \\ \text{Mexico City} \\ \text{Tokyo} \end{array} \begin{array}{c} \begin{array}{ccc} \text{film} & \text{pain reliever} & \text{blow dryer} \\ \text{(24 exp.)} & \text{(100 ct)} & \end{array} \\ \begin{bmatrix} \$4.03 & \$6.78 & \$18.98 \\ \$4.21 & \$7.41 & \$20.49 \\ \$3.97 & \$7.43 & \$32.25 \\ \$7.08 & \$36.57 & \$63.71 \end{bmatrix} \end{array}$$

2. 2×4

3. The sum of two matrices exists if the matrices have the same dimensions.

4. Anthony is correct. A third order matrix has 3 rows and 3 columns. This matrix has 4 rows and 3 columns.

5. $2y = x - 3$ $\quad$ $2y = y + 5 - 3$
$x = y + 5$ $\quad$ $y = 2$
$x = y + 5$
$x = 2 + 5$
$x = 7$ $\quad$ $(7, 2)$

6. $18 = 4x - y$ $\quad$ $24 = 12y$
$24 = 12y$ $\quad$ $2 = y$
$18 = 4x - y$
$18 = 4x - 2$
$5 = x$ $\quad$ $(5, 2)$

7. $16 = 4x$ $\quad$ $16 = 4x$
$0 = y$ $\quad$ $4 = x$
$2x = 8 - y$ $\quad$ $(4, 0)$

8. $$X + Z = \begin{bmatrix} 4 + (-1) & 1 + 3 \\ -2 + 0 & 6 + (-2) \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -2 & 4 \end{bmatrix}$$

9. impossible

10. $$Z - X = \begin{bmatrix} -1 - 4 & 3 - 1 \\ 0 - (-2) & -2 - 6 \end{bmatrix} = \begin{bmatrix} -5 & 2 \\ 2 & -8 \end{bmatrix}$$

11. $$4X = \begin{bmatrix} 4(4) & 4(1) \\ 4(-2) & 4(6) \end{bmatrix} = \begin{bmatrix} 16 & 4 \\ -8 & 24 \end{bmatrix}$$

12. impossible

13. $$YX = [0 \quad -3] \cdot \begin{bmatrix} 4 & 1 \\ -2 & 6 \end{bmatrix}$$
$$= [0(4) + (-3)(-2) \quad 0(1) + (-3)(6)]$$
or $[6 \quad -18]$

14.

$$\begin{array}{l} \\ \\ \text{soft-drink} \\ \text{package delivery} \\ \text{telecommunications} \end{array} \begin{array}{c} \text{Budget} \quad\quad \text{Viewers} \\ (\$\text{ million}) \quad (\text{million}) \\ \begin{bmatrix} 40.1 & 78.6 \\ 22.9 & 21.9 \\ 154.9 & 88.9 \end{bmatrix} \end{array}$$

Pages 83–86 Exercises

15. $y = 2x - 1$ $\quad$ $y = 2(y - 5) - 1$
$x = y - 5$ $\quad$ $y = 11$

$x = y - 5$
$x = 11 - 5$
$x = 6$ $\quad$ $(6, 11)$

16. $9 = x + 2y$ $\quad$ $13 = 4x + 1$
$13 = 4x + 1$ $\quad$ $3 = x$
$9 = x + 2y$
$9 = 3 + 2y$
$3 = y$ $\quad$ $(3, 3)$

17. $4x = 15 + x$ $\quad$ $4x = 15 + x$
$5 = 2y$ $\quad$ $x = 5$

$5 = 2y$
$2.5 = y$ $\quad$ $(5, 2.5)$

18. $x = 2y$ $\quad$ $y = 2(2y) - 6$
$y = 2x - 6$ $\quad$ $y = 2$

$x = 2y$
$x = 2(2)$
$x = 4$ $\quad$ $(4, 2)$

19. $27 = 3y$ $\quad$ $27 = 3y$
$8 = 5x - 3y$ $\quad$ $9 = y$

$8 = 5x - 3y$
$8 = 5x - 3(9)$
$7 = x$ $\quad$ $(7, 9)$

20. $4x - 3y = 11$
$x + y = 1$

$$\begin{array}{l} 4x - 3y = 11 \\ 3(x + y) = 1 \end{array} \rightarrow \begin{array}{r} 4x - 3y = 11 \\ \underline{3x + 3y = 3} \\ 7x = 14 \\ x = 2 \end{array}$$

$x + y = 1$
$2 + y = 1$
$y = -1$ $\quad$ $(2, -1)$

21. $2x = -10$ $\quad$ $2x = -10$ $\quad$ $y = 3x$
$y = 3x$ $\quad$ $x = -5$ $\quad$ $y = 3(-5)$
$-y = 15$ $\quad$ $y = -15$
$(-5, -15)$

22. $-12 = 6x$ $\quad$ $-12 = 6x$ $\quad$ $2 = y + 1$
$2 = y + 1$ $\quad$ $-2 = x$ $\quad$ $1 = y$
$12y = 10 - x$ $\quad$ $(-2, 1)$

23. $x + y = 0$
$y = y^2$
$3 = 2y - x$
$6 = 4 - 2x$
$6 = 4 - 2x$ $\quad$ $x + y = 0$
$-1 = x$ $\quad$ $-1 + y = 0$
$y = 1$ $\quad$ $(-1, 1)$

24. $x^2 + 1 = 2$
$x + y = 5$
$5 - y = x$
$y - 4 = 2$
$y - 4 = 2$ $\quad$ $x + y = 5$
$y = 6$ $\quad$ $x + 6 = 5$
$x = -1$ $\quad$ $(-1, 6)$

25. $$3\begin{bmatrix} x & y - 1 \\ 4 & 3z \end{bmatrix} = \begin{bmatrix} 15 & 6 \\ 6z & 3x + y \end{bmatrix}$$
$$\begin{bmatrix} 3x & 3y - 3 \\ 12 & 9z \end{bmatrix} = \begin{bmatrix} 15 & 6 \\ 6z & 3x + y \end{bmatrix}$$
$3x = 15$
$12 = 6z$
$3y - 3 = 6$
$9z = 3x + y$
$3x = 15$ $\quad$ $12 = 6z$ $\quad$ $3y - 3 = 6$
$x = 5$ $\quad$ $2 = z$ $\quad$ $y = 3$
$(5, 3, 2)$

26. $$-2\begin{bmatrix} w+5 & x-z \\ 3y & 8 \end{bmatrix} = \begin{bmatrix} -16 & -4 \\ 6 & 2x+8z \end{bmatrix}$$

$$\begin{bmatrix} -2w-10 & -2x+2z \\ -6y & -16 \end{bmatrix} = \begin{bmatrix} -16 & -4 \\ 6 & 2x+8z \end{bmatrix}$$

$-2w - 10 = -16 \quad -2w - 10 = -16 \quad -6y = 6$

$-6y = 6 \quad w = 3 \quad y = -1$

$-2x + 2z = -4$

$-16 = 2x + 8z$

$$\begin{array}{r} -2x + 2z = -4 \\ 2x + 8z = -16 \\ \hline 10z = -20 \\ z = -2 \end{array} \qquad \begin{array}{r} -2x + 2z = -4 \\ -2x + 2(-2) = -4 \\ x = 0 \\ (3, 0, -1, -2) \end{array}$$

27. $A + B = \begin{bmatrix} 5+3 & 7+5 \\ -6+(-1) & 1+8 \end{bmatrix} = \begin{bmatrix} 8 & 12 \\ -7 & 9 \end{bmatrix}$

28. impossible

29. impossible

30. $D + C = \begin{bmatrix} 0+4 & 1+(2) & 2+3 \\ -2+5 & 3+0 & 0+(-1) \\ 4+9 & 4+0 & -2+1 \end{bmatrix} = \begin{bmatrix} 4 & -1 & 5 \\ 3 & 3 & -1 \\ 13 & 4 & -1 \end{bmatrix}$

31. $B - A = B + (-A)$

$= \begin{bmatrix} 3 & 5 \\ -1 & 8 \end{bmatrix} + \begin{bmatrix} -5 & -7 \\ 6 & -1 \end{bmatrix}$

$= \begin{bmatrix} 3+(-5) & 5+(-7) \\ -1+6 & 8+(-1) \end{bmatrix}$ or $\begin{bmatrix} -2 & -2 \\ 5 & 7 \end{bmatrix}$

32. $C - D = C + (-D)$

$= \begin{bmatrix} 4 & -2 & 3 \\ 5 & 0 & -1 \\ 9 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 & -2 \\ 2 & -3 & 0 \\ -4 & -4 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4+0 & -2+(-1) & 3+(-2) \\ 5+2 & 0+(-3) & -1+0 \\ 9+(-4) & 0+(-4) & 1+2 \end{bmatrix}$

or $\begin{bmatrix} 4 & -3 & 1 \\ 7 & -3 & -1 \\ 5 & -4 & 3 \end{bmatrix}$

33. $4D = \begin{bmatrix} 4(0) & 4(1) & 4(2) \\ 4(-2) & 4(3) & 4(0) \\ 4(4) & 4(4) & 4(-2) \end{bmatrix} = \begin{bmatrix} 0 & 4 & 8 \\ -8 & 12 & 0 \\ 16 & 16 & -8 \end{bmatrix}$

34. $-2F = \begin{bmatrix} -2(-6) & -2(-1) & -2(0) \\ -2(1) & -2(4) & -2(0) \end{bmatrix} = \begin{bmatrix} 12 & 2 & 0 \\ -2 & -8 & 0 \end{bmatrix}$

35. $F - E = F + (-E)$

$= \begin{bmatrix} -6 & -1 & 0 \\ 1 & 4 & 0 \end{bmatrix} + \begin{bmatrix} -8 & 4 & -2 \\ -3 & -1 & 5 \end{bmatrix}$

$= \begin{bmatrix} -14 & 3 & -2 \\ -2 & 3 & 5 \end{bmatrix}$

36. $E - F = E + (-F)$

$= \begin{bmatrix} 8 & -4 & 2 \\ 3 & 1 & -5 \end{bmatrix} + \begin{bmatrix} 6 & 1 & 0 \\ -1 & -4 & 0 \end{bmatrix}$

$= \begin{bmatrix} 14 & -3 & 2 \\ 2 & -3 & -5 \end{bmatrix}$

37. $5A = \begin{bmatrix} 5(5) & 5(7) \\ 5(-6) & 5(1) \end{bmatrix} = \begin{bmatrix} 25 & 35 \\ -30 & 5 \end{bmatrix}$

38. $BA = \begin{bmatrix} 3 & 5 \\ -1 & 8 \end{bmatrix} \cdot \begin{bmatrix} 5 & 7 \\ -6 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3(5)+5(-6) & 3(7)+5(1) \\ -1(5)+8(-6) & -1(7)+8(1) \end{bmatrix}$

or $\begin{bmatrix} -15 & 26 \\ -53 & 1 \end{bmatrix}$

39. impossible

40. $FC = \begin{bmatrix} -6 & -1 & 0 \\ 1 & 4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 4 & -2 & 3 \\ 5 & 0 & -1 \\ 9 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -6(4)+(-1)(5)+0(9) & -6(-2)+(-1)(0)+0(0) & -6(3)+(-1)(-1)+0(1) \\ 1(4)+4(5)+0(9) & 1(-2)+4(0)+0(0) & 1(3)+4(-1)+0(1) \end{bmatrix}$

$= \begin{bmatrix} -29 & 12 & -17 \\ 24 & -2 & -1 \end{bmatrix}$

41. $ED = \begin{bmatrix} 8 & -4 & 2 \\ 3 & 1 & -5 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 2 \\ -2 & 3 & 0 \\ 4 & 4 & -2 \end{bmatrix}$

$= \begin{bmatrix} 8(0)+(-4)(-2)+2(4) & 8(1)+(-4)(3)+2(4) & 8(2)+(-4)(0)+2(-2) \\ 3(0)+1(-2)+(-5)(4) & 3(1)+1(3)+(-5)(4) & 3(2)+1(0)+(-5)(-2) \end{bmatrix}$

$= \begin{bmatrix} 16 & 4 & 12 \\ -22 & -14 & 16 \end{bmatrix}$

42. $AA = \begin{bmatrix} 5 & 7 \\ -6 & 1 \end{bmatrix} \cdot \begin{bmatrix} 5 & 7 \\ -6 & 1 \end{bmatrix}$

$= \begin{bmatrix} 5(5)+7(-6) & 5(7)+7(1) \\ -6(5)+1(-6) & -6(7)+1(1) \end{bmatrix}$

or $\begin{bmatrix} -17 & 42 \\ -36 & -41 \end{bmatrix}$

43. $FD = \begin{bmatrix} -6 & -1 & 0 \\ 1 & 4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 2 \\ -2 & 3 & 0 \\ 4 & 4 & -2 \end{bmatrix}$

$= \begin{bmatrix} -6(0)+(-1)(-2)+0(4) & -6(1)+(-1)(3)+0(4) & -6(2)+(-1)(0)+0(-2) \\ 1(0)+4(-2)+0(4) & 1(1)+4(3)+0(4) & 1(2)+4(0)+0(-2) \end{bmatrix}$

$= \begin{bmatrix} 2 & -9 & -12 \\ -8 & 13 & 2 \end{bmatrix}$

$E + FD = \begin{bmatrix} 8+2 & -4+(-9) & 2+(-12) \\ 3+(-8) & 1+13 & -5+2 \end{bmatrix}$

$= \begin{bmatrix} 10 & -13 & -10 \\ -5 & 14 & -3 \end{bmatrix}$

44. $-3AB = -3\begin{bmatrix}5&7\\-6&1\end{bmatrix}\cdot\begin{bmatrix}3&5\\-1&8\end{bmatrix}$

$$= \begin{bmatrix}(-3)(5)&-3(7)\\(-3)(-6)&-3(1)\end{bmatrix}\cdot\begin{bmatrix}3&5\\-1&8\end{bmatrix}$$

$$= \begin{bmatrix}-15&-21\\18&-3\end{bmatrix}\cdot\begin{bmatrix}3&5\\-1&8\end{bmatrix}$$

$$= \begin{bmatrix}-15(3)+(-21)(-1)&-15(5)+(-21)(8)\\18(3)+(-3)(-1)&18(5)+(-3)(8)\end{bmatrix}$$

$$= \begin{bmatrix}-24&-243\\57&66\end{bmatrix}$$

45. $(BA)E = \left(\begin{bmatrix}3&5\\-1&8\end{bmatrix}\cdot\begin{bmatrix}5&7\\-6&1\end{bmatrix}\right)\left(\begin{bmatrix}8&-4&2\\3&1&-5\end{bmatrix}\right)$

$$= \begin{bmatrix}3(5)+5(-6)&3(7)+5(1)\\-1(5)+8(-6)&-1(7)+8(1)\end{bmatrix}\cdot\begin{bmatrix}8&-4&2\\3&1&-5\end{bmatrix}$$

$$= \begin{bmatrix}-15&26\\-53&1\end{bmatrix}\cdot\begin{bmatrix}8&-4&2\\3&1&-5\end{bmatrix}$$

$$= \begin{bmatrix}-15(8)+26(3)&-15(-4)+26(1)&-15(2)+26(-5)\\-53(8)+1(3)&-53(-4)+1(1)&-53(2)+1(-5)\end{bmatrix}$$

$$= \begin{bmatrix}-42&86&-160\\-421&213&-111\end{bmatrix}$$

46. $F - 2EC = F + (-2EC)$

$$-2EC = -2\begin{bmatrix}8&-4&2\\3&1&-5\end{bmatrix}\cdot\begin{bmatrix}4&-2&3\\5&0&-1\\9&0&1\end{bmatrix}$$

$$= \begin{bmatrix}-2(8)&-2(-4)&-2(2)\\-2(3)&-2(1)&-2(-5)\end{bmatrix}\cdot\begin{bmatrix}4&-2&3\\5&0&-1\\9&0&1\end{bmatrix}$$

$$= \begin{bmatrix}-16&8&-4\\-6&-2&10\end{bmatrix}\cdot\begin{bmatrix}4&-2&3\\5&0&-1\\9&0&1\end{bmatrix}$$

$$= \begin{bmatrix}-16(4)+8(5)+(-4)(9)&-16(-2)+8(0)+(-4)(0)&-16(3)+8(-1)+(-4)(1)\\-6(4)+(-2)(5)+10(9)&-6(-2)+(-2)(0)+10(0)&-6(3)+(-2)(-1)+10(1)\end{bmatrix}$$

$$= \begin{bmatrix}-60&32&-60\\56&12&-6\end{bmatrix}$$

$$F + (-2EC) = \begin{bmatrix}-6+(-60)&-1+32&0+(-60)\\1+56&4+12&0+(-6)\end{bmatrix}$$

$$= \begin{bmatrix}-66&31&-60\\57&16&-6\end{bmatrix}$$

47. $3XY = 3\begin{bmatrix}2&4\\8&-4\\-2&6\end{bmatrix}\cdot\begin{bmatrix}3&-3&6\\5&4&-2\end{bmatrix}$

$$= \begin{bmatrix}3(2)&3(4)\\3(8)&3(-4)\\3(-2)&3(6)\end{bmatrix}\cdot\begin{bmatrix}3&-3&6\\5&4&-2\end{bmatrix}$$

$$= \begin{bmatrix}6&12\\24&-12\\-6&18\end{bmatrix}\cdot\begin{bmatrix}3&-3&6\\5&4&-2\end{bmatrix}$$

$$= \begin{bmatrix}6(3)+12(5)&6(-3)+12(4)&6(6)+12(-2)\\24(3)+(-12)(5)&24(-3)+(-12)(4)&24(6)+(-12)(-2)\\-6(3)+18(5)&-6(-3)+18(4)&-6(6)+18(-2)\end{bmatrix}$$

$$= \begin{bmatrix}78&30&12\\12&-120&168\\72&90&-72\end{bmatrix}$$

48. $2K - 3J = 2\begin{bmatrix}1&-7\\3&2\end{bmatrix} + (-3)\begin{bmatrix}-4&5\\1&-1\end{bmatrix}$

$$= \begin{bmatrix}2(1)&2(-7)\\2(3)&2(2)\end{bmatrix} + \begin{bmatrix}(-3)(-4)&-3(5)\\(-3)(1)&-3(-1)\end{bmatrix}$$

$$= \begin{bmatrix}2&-14\\6&4\end{bmatrix} + \begin{bmatrix}12&-15\\-3&3\end{bmatrix}$$

$$= \begin{bmatrix}2+12&-14+(-15)\\6+(-3)&4+3\end{bmatrix}$$

$$= \begin{bmatrix}14&-29\\3&7\end{bmatrix}$$

49. Sample answer:

	1996	2000	2006
18 to 24	8485	8526	8695
25 to 34	10,102	9316	9078
35 to 44	8766	9036	8433
45 to 54	6045	6921	7900
55 to 64	2444	2741	3521
65 and older	2381	2440	2572

50a. $\begin{bmatrix}18&24&19\\16&24&17\\6&6&6\\12&2&4\end{bmatrix} - \begin{bmatrix}26&31&24\\22&28&21\\12&9&7\\17&4&6\end{bmatrix}$

$$= \begin{bmatrix}18&24&19\\26&24&17\\6&6&6\\12&2&4\end{bmatrix} + \begin{bmatrix}-26&-31&-24\\-22&-28&-21\\-12&-9&-7\\-17&-4&-6\end{bmatrix}$$

$$= \begin{bmatrix}18+(-26)&24+(-31)&19+(-24)\\16+(-22)&24+(-28)&17+(-21)\\6+(-12)&6+(-9)&6+(-7)\\12+(-17)&2+(-4)&4+(-6)\end{bmatrix}$$

or $\begin{bmatrix}-8&-7&-5\\-6&-4&-4\\-6&-3&-1\\-5&-2&-2\end{bmatrix}$

	TV	Radio	Recording
Classical	−8	−7	−5
Jazz	−6	−4	−4
Opera	−6	−3	−1
Musicals	−5	−2	−2

50b. classical performances on TV

51a. $\begin{bmatrix} -2 & 3 \\ 4 & -5 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ 4 & -5 \end{bmatrix}$

$\begin{bmatrix} -2(a) + 3(c) & -2(b) + 3(d) \\ 4(a) + (-5)(c) & 4(b) + (-5)(d) \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ 4 & -5 \end{bmatrix}$

$-2a + 3c = -2 \qquad -2b + 3d = 3$
$4a - 5c = 4 \qquad 4b - 5d = -5$

$2(-2a + 3c) = 2(-2) \rightarrow -4a + 6c = -4$
$4a - 5c = 4 \qquad\qquad\quad\; \underline{4a - 5c = 4}$
$c = 0$

$4a - 5c = 4$
$4a - 5(0) = 4$
$a = 1$

$2(-2b + 3d) = 2(3) \rightarrow -4b + 6d = 6$
$4b - 5d = -5 \qquad\qquad \underline{4b - 5d = -5}$
$d = 1$

$4b - 5d = -5$
$4b - 5(1) = -5$
$b = 0$

51b. a matrix equal to the original matrix

52a. [42 59 21 18]

52b.

$$[42 \quad 59 \quad 21 \quad 18] \cdot \begin{bmatrix} 33.81 & 30.94 & 27.25 \\ 15.06 & 13.25 & 8.75 \\ 54 & 54 & 46.44 \\ 52.06 & 44.69 & 34.38 \end{bmatrix}$$

$$= [42(33.81) + 59(15.06) + 21(54) + 18(52.06) \quad 42(30.94) + 59(13.25) + 21(54) + 18(44.69) \quad 42(27.25) + 59(8.75) + 21(46.44) + 18(34.38)]$$

$= [4379.64 \quad 4019.65 \quad 3254.83]$

July, \$4379.64; Aug, \$4019.65; Sep, \$3254.83

53. The numbers in the first row are the triangular numbers. If you look at the diagonals in the matrix, the triangular numbers are the end numbers. To find the diagonal that contains 2001, find the smallest triangular number that is greater than or equal to 2001. The formula for the nth triangular number is $\frac{n(n+1)}{2}$. Solve $\frac{n(n+1)}{2} \geq 2001$. The solution is 63. So the 63rd entry in the first row is $\frac{63(63+1)}{2} = 2016$. Since $2016 - 2001 = 15$, we must count 15 places backward along the diagonal to locate 2001 in the matrix. This movement takes us from the position (row, column) = (1, 63) to (1 + 15, 63 − 15) = (16, 48).

54a.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

54b. No; since the matrix shows the number of nodes and the numbers of edges between each pair of nodes, only equivalent graphs will have the same matrix.

55.
$2x + 6y + 8z = 5$
$\underline{-2x + 9y + 12z = 5}$
$15y + 20z = 10$

$2(-2x + 9y + 12z) = 2(5) \rightarrow -4x + 18y + 24z = 10$
$4x + 6y - 4z = 3 \qquad\qquad\qquad \underline{4x + 6y - 4z = 3}$
$24y + 20z = 13$

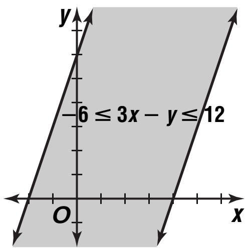
$15y + 20z = 10 \qquad\qquad \rightarrow 15y + 20z = 10$
$-1(24y + 20z) = -1(13) \quad \underline{-24y - 20z = -13}$
$-9y = -3$
$y = \frac{1}{3}$

$15y + 20z = 10 \qquad 2x + 6y + 8z = 5$
$15\left(\frac{1}{3}\right) + 20z = 10 \qquad 2x + 6\left(\frac{1}{3}\right) + 8\left(\frac{1}{4}\right) = 5$
$z = \frac{1}{4} \qquad x = \frac{1}{2}$

$\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right)$

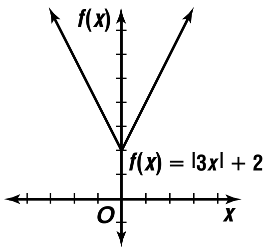
56. $4x - 2y = 7 \qquad \rightarrow \qquad y = 2x - \frac{7}{2}$
$-12x + 6y = -21 \qquad\quad y = 2x - \frac{7}{2}$
consistent and dependent

57.



58.

x	$f(x)$
−2	8
−1	5
0	2
1	5
2	8



59. Sample answer: using (60, 83) and (10, 65),
$m = \frac{65 - 83}{10 - 60}$
$= \frac{-18}{-50}$ or 0.36
$y - 65 = 0.36(x - 10)$
$y = 0.36x + 61.4$

60. $m = \frac{7 - 4}{5 - 1} \qquad y - 7 = \frac{3}{4}(x - 5)$
$= \frac{3}{4} \qquad y = \frac{3}{4}x + 3\frac{1}{4}$

61. $f(x) = 5x - 3$
$0 = 5x - 3$
$\frac{3}{5} = x$

62. $[f \cdot g](x) = f(x) \cdot g(x)$
$= \left(\frac{2}{5}x\right)(40x - 10)$
$= 16x^2 - 4x$

63. $f(x) = 4 + 6x - x^3$
$f(14) = 4 + 6(14) - (14)^3$
$= -2656$

64.
$$\frac{2x-3}{x} = \frac{3-x}{2}$$
$$2(2x-3) = x(3-x)$$
$$4x - 6 = 3x - x^2$$
$$x^2 + x - 6 = 0$$
$$(x+3)(x-2) = 0$$
$$x + 3 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -3 \qquad\qquad x = 2$$
The correct choice is A.

Page 86 Graphing Calculator Exploration

1. All of the properties except for the Commutative Property of Multiplication hold true. When multiplying matrices, the order of the multiplication produces different results. However, in addition of matrices, order is not important.

2a. Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$.

$$A + B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$
$$= \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix}$$
$$= \begin{bmatrix} b_{11} + a_{11} & b_{12} + a_{12} \\ b_{21} + a_{21} & b_{22} + a_{22} \end{bmatrix}$$
$$= \begin{bmatrix} b_{11} & b_{21} \\ b_{21} & b_{12} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

2b. Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$,

and $C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$

$$(A + B) + C = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$
$$= \begin{bmatrix} (a_{11} + b_{11}) + c_{11} & (a_{12} + b_{12}) + c_{12} \\ (a_{21} + b_{21}) + c_{21} & (a_{22} + b_{22}) + c_{22} \end{bmatrix}$$
$$= \begin{bmatrix} a_{11} + (b_{11} + c_{11}) & a_{12} + (b_{12} + c_{12}) \\ a_{21} + (b_{21} + c_{21}) & a_{22} + (b_{22} + c_{22}) \end{bmatrix}$$
$$= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} + c_{11} & b_{12} + c_{12} \\ b_{21} + c_{21} & b_{22} + c_{22} \end{bmatrix}$$
$$= A + (B + C)$$

2c. Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{23} \end{bmatrix}$ and $B \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$.

$$AB = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{23} \end{bmatrix}$$
$$BA = \begin{bmatrix} b_{11}a_{11} + b_{12}a_{21} & b_{11}a_{12} + b_{12}a_{22} \\ b_{21}a_{11} + b_{22}a_{21} & b_{21}a_{12} + b_{22}a_{22} \end{bmatrix}$$

Thus, $AB \neq BA$, since $a_{12}b_{21} \neq b_{12}a_{21}$.

2d. Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$,

and $C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$.

$$(AB)C = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$
$$= \begin{bmatrix} a_{11}b_{11}c_{11} + a_{11}b_{12}c_{21} + a_{12}b_{21}c_{11} + a_{21}b_{22}c_{21} & a_{11}b_{11}c_{12} + a_{11}b_{12}c_{22} + a_{12}b_{21}c_{12} + a_{12}b_{22}c_{22} \\ a_{21}b_{11}c_{11} + a_{21}b_{12}c_{21} + a_{22}b_{21}c_{11} + a_{22}b_{22}c_{21} & a_{21}b_{11}c_{12} + a_{21}b_{12}c_{22} + a_{22}b_{21}c_{12} + a_{22}b_{22}c_{22} \end{bmatrix}$$
$$A(BC) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11}c_{11} + b_{12}c_{21} & b_{11}c_{12} + b_{12}c_{22} \\ b_{21}c_{11} + b_{22}c_{21} & b_{21}c_{12} + b_{22}c_{22} \end{bmatrix}$$
$$= \begin{bmatrix} a_{11}b_{11}c_{11} + a_{11}b_{12}c_{21} + a_{12}b_{21}c_{11} + a_{21}b_{22}c_{21} & a_{11}b_{11}c_{12} + a_{11}b_{12}c_{22} + a_{12}b_{21}c_{12} + a_{12}b_{22}c_{22} \\ a_{21}b_{11}c_{11} + a_{21}b_{12}c_{21} + a_{22}b_{21}c_{11} + a_{22}b_{22}c_{21} & a_{21}b_{11}c_{12} + a_{21}b_{12}c_{22} + a_{22}b_{21}c_{12} + a_{22}b_{22}c_{22} \end{bmatrix}$$

Therefore $(AB)C = A(BC)$.

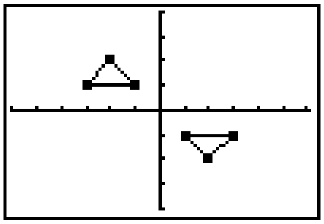
3. All properties except the Commutative Property of Multiplication will hold for square matrices. A proof similar to the ones in Exercises 2a-2d can be used to verify this conjecture.

2-4A Transformation Matrices

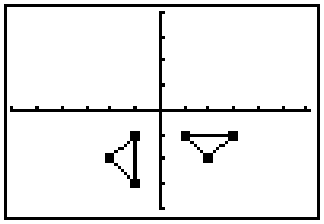
Page 87

1. The new figure is a 90° counterclockwise rotation of $\triangle LMN$.

2. The new figure is an 180° counterclockwise rotation of $\triangle LMN$.



3. The new figure is an 270° counterclockwise rotation of $\triangle LMN$.



4. See students' work for graphs. Multiplying a vertex matrix by $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ results in a vertex matrix for a figure that is a 90° counterclockwise rotation of the original figure. Multiplying a vertex matrix by $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ results in a vertex matrix for a figure that is a 180° counterclockwise rotation of the original figure. Multiplying a vertex matrix by $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ results in a vertex matrix for a figure that is a 270° counterclockwise rotation of the original figure.

2-4 Modeling Motion with Matrices

Pages 92–93 Check for Understanding

1. Translation, reflection, rotation, dilation; translations do not affect the shape, size, or orientation of figures; reflections and rotations do not change the shape or size of figures; dilations do not change the shape, but do change the size of figures.

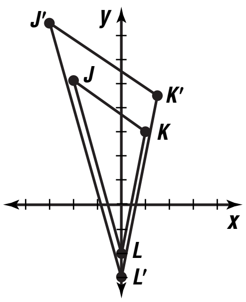
2. 90° counterclockwise = (360 − 90)° or 270° clockwise; 180° counterclockwise = (360 − 180)° or 180° clockwise; 270° counterclockwise = (360 − 270)° or 90° clockwise.

3. Sample answer: the first row of the reflection matrix affects the x-coordinates and the second row affects the y-coordinates. A reflection over the x-axis changes $(x, y) + 0\ (x, -y)$, so the first row needs to be [1 0] so the x is unchanged and the second row needs to be [0 −1] so the y-coordinates are the opposite. Similar reasoning can be used for a reflection over the y-axis, which changes (x, y) to $(-x, y)$ and a reflection over the line $y = x$, which interchanges the values for x and y.

4a. 6 **4b.** 2 **4c.** 3 **4d.** 4

5. $1.5\begin{bmatrix} -2 & 1 & 0 \\ 5 & 3 & -2 \end{bmatrix} = \begin{bmatrix} -3 & 1.5 & 0 \\ 7.5 & 4.5 & -3 \end{bmatrix}$

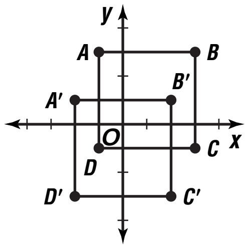
$J'(-3, 7.5)$, $K'(1.5, 4.5)$, $L'(0, -3)$



6. $\begin{bmatrix} -1 & 3 & 3 & -1 \\ 3 & 3 & -1 & -1 \end{bmatrix} + \begin{bmatrix} -1 & -1 & -1 & -1 \\ -2 & -2 & -2 & -2 \end{bmatrix}$

$= \begin{bmatrix} -2 & 2 & 2 & -2 \\ 1 & 1 & -3 & -3 \end{bmatrix}$

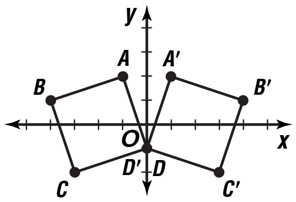
$A'(-2, 1)$, $B'(2, 1)$, $C'(2, -3)$, $D'(-2, -3)$



7. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & -4 & -3 & 0 \\ 2 & 1 & -2 & -1 \end{bmatrix}$

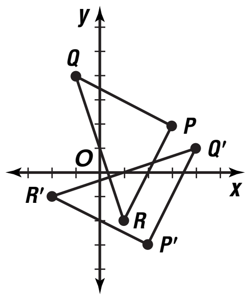
$= \begin{bmatrix} 1 & 4 & 3 & 0 \\ 2 & 1 & -2 & -1 \end{bmatrix}$

$A'(1, 2)$, $B'(4, 1)$, $C'(3, -2)$, $D'(0, -1)$



8. $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 & 1 \\ 2 & 4 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & -2 \\ -3 & 1 & -1 \end{bmatrix}$

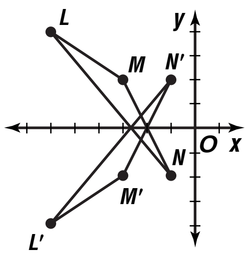
$P'(2, -3)$, $Q'(4, 1)$, $R'(-2, -1)$



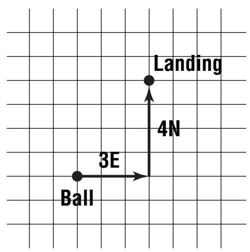
9. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -6 & -3 & -1 \\ 4 & 2 & -2 \end{bmatrix} = \begin{bmatrix} -6 & -3 & -1 \\ -4 & -2 & 2 \end{bmatrix}$

$L'(-6, -4)$, $M'(-3, -2)$, $N'(-1, 2)$



10a.

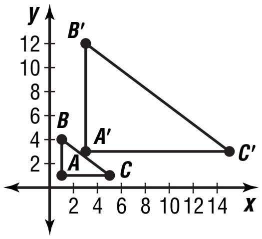


10b. $\begin{bmatrix} x + 3 \\ y + 4 \end{bmatrix}$

Pages 93–96 Exercises

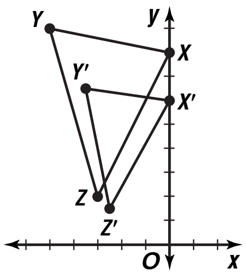
11. $3\begin{bmatrix}1 & 1 & 5\\ 1 & 4 & 1\end{bmatrix} = \begin{bmatrix}3 & 3 & 15\\ 3 & 12 & 3\end{bmatrix}$

$A'(3, 3)$, $B'(3, 12)$, $C'(15, 3)$



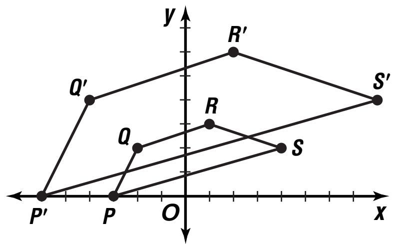
12. $\frac{3}{4}\begin{bmatrix}0 & -5 & -3\\ 8 & 9 & 2\end{bmatrix} = \begin{bmatrix}0 & \frac{-15}{4} & \frac{-9}{4}\\ 6 & \frac{27}{4} & \frac{3}{2}\end{bmatrix}$

$X'(0, 6)$, $Y'\left(-3\frac{3}{4}, 6\frac{3}{4}\right)$, $Z'\left(-2\frac{1}{4}, 1\frac{1}{2}\right)$



13. $2\begin{bmatrix}-3 & -2 & 1 & 4\\ 0 & 2 & 3 & 2\end{bmatrix} = \begin{bmatrix}-6 & -4 & 2 & 8\\ 0 & 4 & 6 & 4\end{bmatrix}$

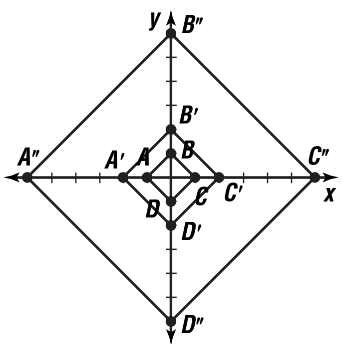
$P'(-6, 0)$, $Q'(-4, 4)$, $R'(2, 6)$, S' (8,4)



14a. $2\begin{bmatrix}-1 & 0 & 1 & 0\\ 0 & 1 & 0 & -1\end{bmatrix} = \begin{bmatrix}-2 & 0 & 2 & 0\\ 0 & 2 & 0 & -2\end{bmatrix}$

$3\begin{bmatrix}-2 & 0 & 2 & 0\\ 0 & 2 & 0 & -2\end{bmatrix} = \begin{bmatrix}-6 & 0 & 6 & 0\\ 0 & 6 & 0 & -6\end{bmatrix}$

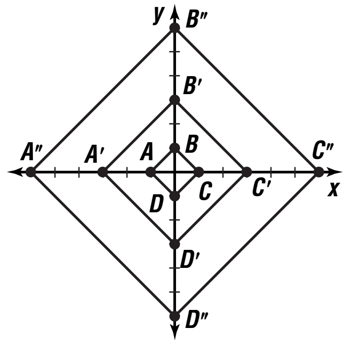
$A'(-2, 0)$, $B'(0, 2)$, $C'(2, 0)$, $D'(0, -2)$; $A''(-6, 0)$, $B''(0, 6)$, $C'(6, 0)$, $D''(0, -6)$



14b. $3\begin{bmatrix}-1 & 0 & 1 & 0\\ 0 & 1 & 0 & -1\end{bmatrix} = \begin{bmatrix}-3 & 0 & 3 & 0\\ 0 & 3 & 0 & -3\end{bmatrix}$

$2\begin{bmatrix}-3 & 0 & 3 & 0\\ 0 & 3 & 0 & -3\end{bmatrix} = \begin{bmatrix}-6 & 0 & 6 & 0\\ 0 & 6 & 0 & -6\end{bmatrix}$

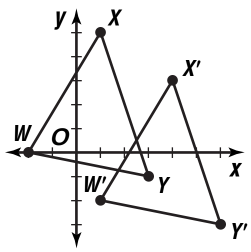
$A'(-3, 0)$, $B'(0, 3)$, $O'(3, 0)$, $D'(0, -3)$; $A''(-6, 0)$, $B''(0, 6)$, $C'(6, 0)$, $D''(0, -6)$



14c. The final results are the same image.

15. $\begin{bmatrix}-2 & 1 & 3\\ 0 & 5 & -1\end{bmatrix} + \begin{bmatrix}3 & 3 & 3\\ -2 & -2 & -2\end{bmatrix} = \begin{bmatrix}1 & 4 & 6\\ -2 & 3 & -3\end{bmatrix}$

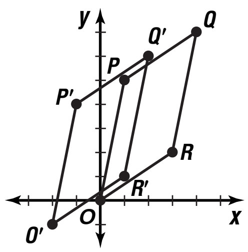
$W'(1, -2)$, $X'(4, 3)$, $Y'(6, -3)$



16. $\begin{bmatrix}0 & 1 & 4 & 3\\ 0 & 5 & 7 & 2\end{bmatrix} + \begin{bmatrix}-2 & -2 & -2 & -2\\ -1 & -1 & -1 & -1\end{bmatrix}$

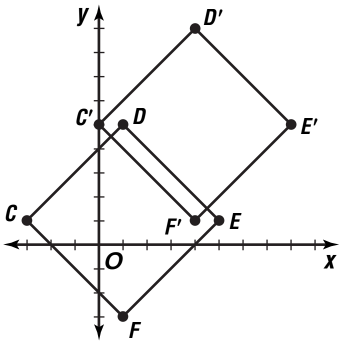
$= \begin{bmatrix}-2 & -1 & 2 & 1\\ -1 & 4 & 6 & 1\end{bmatrix}$

$Q'(-2, -1)$, $P'(-1, 4)$, $Q'(2, 6)$, $R'(1, 1)$



17. $\begin{bmatrix}-3 & 1 & 5 & 1\\ 1 & 5 & 1 & -3\end{bmatrix} + \begin{bmatrix}3 & 3 & 3 & 3\\ 4 & 4 & 4 & 4\end{bmatrix} = \begin{bmatrix}0 & 4 & 8 & 4\\ 5 & 9 & 5 & 1\end{bmatrix}$

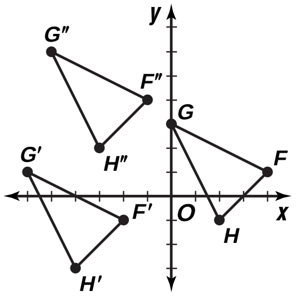
$C'(0, 5)$, $D'(4, 9)$, $E'(8, 5)$, $F'(4, 1)$



18a. $\begin{bmatrix} 4 & 0 & 2 \\ 1 & 3 & -1 \end{bmatrix} + \begin{bmatrix} -6 & -6 & -6 \\ -2 & -2 & -2 \end{bmatrix} = \begin{bmatrix} -2 & -6 & -4 \\ -1 & 1 & -3 \end{bmatrix}$

$F'(-2, -1)$, $G'(-6, 1)$, $H'(-4, -3)$

18a-b.



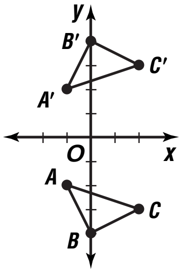
18b. $\begin{bmatrix} -2 & -6 & -4 \\ -1 & 1 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\ 5 & 5 & 5 \end{bmatrix} = \begin{bmatrix} -1 & -5 & -3 \\ 4 & 6 & 2 \end{bmatrix}$

$F''(-1, 4)$, $G''(-5, 6)$, $H''(-3, 2)$

18c. translation of 5 units left and 3 units up

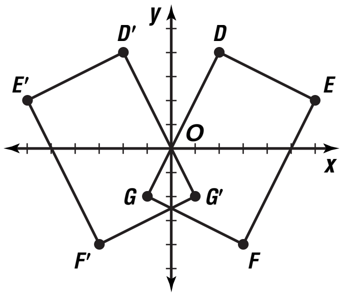
19. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 & 2 \\ -2 & -4 & -3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 2 \\ 2 & 4 & 3 \end{bmatrix}$

$A'(-1, 2)$, $B'(0, 4)$, $C'(2, 3)$



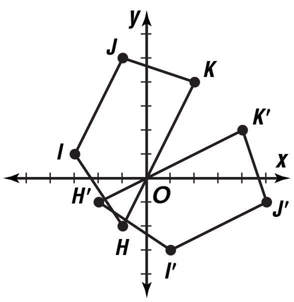
20. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 6 & 3 & -1 \\ 4 & 2 & -4 & -2 \end{bmatrix} = \begin{bmatrix} -2 & -6 & -3 & 1 \\ 4 & 2 & -4 & -2 \end{bmatrix}$

$D'(-2, 4)$, $E'(-6, 2)$, $F'(-3, -4)$, $G'(1, -2)$



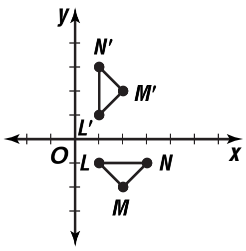
21. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -1 & -3 & -1 & 2 \\ -2 & 1 & 5 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 5 & 4 \\ -1 & -3 & -1 & 2 \end{bmatrix}$

$H'(-2, -1)$, $I'(1, -3)$, $J'(5, -1)$, $K'(4, 2)$



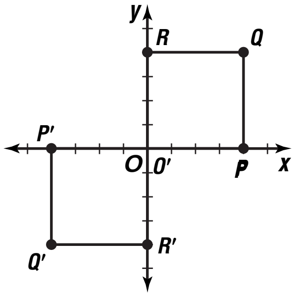
22. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ -1 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

$L'(1, 1)$, $M'(2, 2)$, $N'(1, 3)$



23. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 4 & 4 & 0 \\ 0 & 0 & 4 & 4 \end{bmatrix} = \begin{bmatrix} 0 & -4 & -4 & 0 \\ 0 & 0 & -4 & -4 \end{bmatrix}$

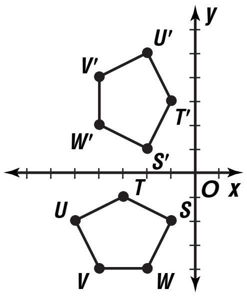
$O'(0, 0)$, $P'(-4, 0)$, $Q'(-4, -4)$, $R'(0, -4)$



24. $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -1 & -3 & -5 & -4 & -2 \\ -2 & -1 & -2 & -4 & -4 \end{bmatrix}$

$= \begin{bmatrix} -2 & -1 & -2 & -4 & -4 \\ 1 & 3 & 5 & 4 & 2 \end{bmatrix}$

$S'(-2, 1)$, $T'(-1, 3)$, $U'(-2, 5)$, $V'(-4, 4)$, $W'(-4, 2)$



25a. Let $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = R_{x\text{-axis}}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 & -1 \\ 3 & -1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -1 \\ -3 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} a + 3b & -2a - b & -a - 3b \\ c + 3d & -2c - d & -c - 3d \end{bmatrix} = \begin{bmatrix} 1 & -2 & -1 \\ -3 & 1 & 3 \end{bmatrix}$$

$a + 3b = 1 \quad -2a - b = -2 \quad -a - 3b = -1$

$c + 3d = -3 \quad -2c - d = 1 \quad -c - 3d = 3$

Thus, $a = 1$, $b = 0$, $c = 0$, and $d = -1$. By substitution, $R_{x\text{-axis}} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

25b. Let $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = R_{y\text{-axis}}$.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 & -1 \\ 3 & -1 & -3 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 3 & -1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} a+3b & -2a-b & -a-3b \\ c+3d & -2c-d & -c-3d \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 3 & -1 & -3 \end{bmatrix}$$

$a + 3b = -1 \quad -2a - b = 2 \quad -a - 3b = 1$
$c + 3d = 3 \quad -2c - d = -1 \quad -c - 3d = -3$

Thus, $a = -1$, $b = 0$, $c = 0$, and $d = 1$. By substitution, $R_{y\text{-axis}} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$.

25c. Let $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = R_{y=x}$.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 & -1 \\ 3 & -1 & -3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & -3 \\ 1 & -2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} a+3b & -2a-b & -a-3b \\ c+3d & -2c-d & -c-3d \end{bmatrix} = \begin{bmatrix} 3 & -1 & -3 \\ 1 & -2 & -1 \end{bmatrix}$$

$a + 3b = 3 \quad -2a - b = -1 \quad -a - 3b = -3$
$c + 3d = 1 \quad -2c - d = -2 \quad -c - 3d = -1$

Thus, $a = 0$, $b = 1$, $c = 1$, and $d = 0$. By substitution, $R_{y=x} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

25d. Let $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = Rot_{90}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 & -1 \\ 3 & -1 & -3 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 3 \\ 1 & -2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} a+3b & -2a-b & -a-3b \\ c+3d & -2c-d & -c-3d \end{bmatrix} = \begin{bmatrix} -3 & 1 & 3 \\ 1 & -2 & -1 \end{bmatrix}$$

$a + 3b = -3 \quad -2a - b = 1 \quad -a - 3b = 3$
$c + 3d = 1 \quad -2c - d = -2 \quad -c - 3d = -1$

Thus, $a = 0$, $b = -1$, $c = 1$, and $d = 0$. By substitution, $Rot_{90} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

25e. Let $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = Rot_{180}$.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 & -1 \\ 3 & -1 & -3 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ -3 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} a+3b & -2a-b & -a-3b \\ c+3d & -2c-d & -c-3d \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ -3 & 1 & 3 \end{bmatrix}$$

$a + 3b = -1 \quad -2a - b = 2 \quad -a - 3b = 1$
$c + 3d = -3 \quad -2c - d = 1 \quad -c - 3d = 3$

Thus, $a = -1$, $b = 0$, $c = 0$, and $d = -1$. By substitution, $Rot_{180} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.

25f. Let $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = Rot_{270}$.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 & -1 \\ 3 & -1 & -3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & -3 \\ -1 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a+3b & -2a-b & -a-3b \\ c+3d & -2c-d & -c-3d \end{bmatrix} = \begin{bmatrix} 3 & -1 & -3 \\ -1 & 2 & 1 \end{bmatrix}$$

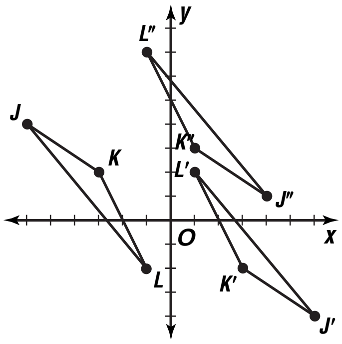
$a + 3b = 3 \quad -2a - b = -1 \quad -a - 3b = -3$
$c + 3d = -1 \quad -2c - d = 2 \quad -c - 3d = 1$

Thus, $a = 0$, $b = 1$, $c = -1$, and $d = 0$. By substitution, $Rot_{270} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

26. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -6 & -3 & -1 \\ 4 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 6 & 3 & 1 \\ -4 & -2 & 2 \end{bmatrix}$

$\begin{bmatrix} 6 & 3 & 1 \\ -4 & -2 & 2 \end{bmatrix} + \begin{bmatrix} -2 & -2 & -2 \\ 5 & 5 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 1 & -1 \\ 1 & 3 & 7 \end{bmatrix}$

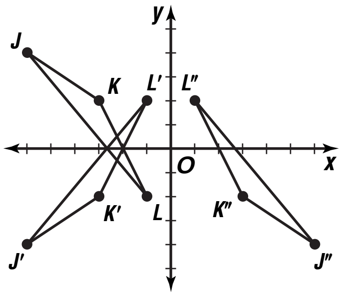
$J''(4, 1)$, $K''(1, 3)$, $L''(-1, 7)$



27. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -6 & -3 & -1 \\ 4 & 2 & -2 \end{bmatrix} = \begin{bmatrix} -6 & -3 & -1 \\ -4 & -2 & 2 \end{bmatrix}$

$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -6 & -3 & -1 \\ -4 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 3 & 1 \\ -4 & -2 & 2 \end{bmatrix}$

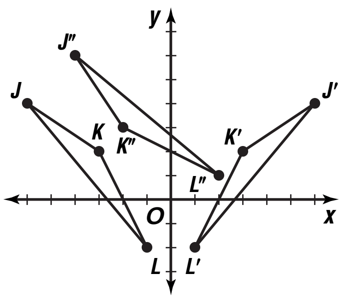
$J''(6, -4)$, $K''(3, -2)$, $L''(1, 2)$



28. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -6 & -3 & -1 \\ 4 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 6 & 3 & 1 \\ 4 & 2 & -2 \end{bmatrix}$

$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 6 & 3 & 1 \\ 4 & 2 & -2 \end{bmatrix} = \begin{bmatrix} -4 & -2 & 2 \\ 6 & 3 & 1 \end{bmatrix}$

$J''(-4, 6)$, $K''(-2, 3)$, $L''(2, 1)$



29a. The bishop moves along a diagonal until it encounters the edge of the board or another piece. The line along which it moves changes vertically and horizontally by 1 unit with each square moved, so the translation matrices are scalars. Sample matrices are $c\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $c\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$, $c\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$, and

$c\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$, where c is the number of squares moved.

29b. The knight moves in combinations of 2 vertical-1 horizontal or 1 vertical-2 horizontal squares. These can be either up or down, left or right. Sample matrices are $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, $\begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix}$, $\begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix}$, $\begin{bmatrix} -1 & -1 \\ -2 & -2 \end{bmatrix}$, $\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$,

$\begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}$, $\begin{bmatrix} -2 & -2 \\ 1 & 1 \end{bmatrix}$, and $\begin{bmatrix} -2 & -2 \\ -1 & -1 \end{bmatrix}$.

29c. The king can move 1 unit in any direction. The matrices describing this are $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$,
$\begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ -1 & -1 \end{bmatrix}$, $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$,
$\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$, and $\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$.

30. Consider $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Dilation with scale factor -1

$-1\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix}$

Rotation of 180°

$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix}$

The vertex matrices for the images of a dilation with scale factor -1 and a rotation of 180° are the same, so the images are the same.

31. $(0, -125)$; $(125, 0)$, $(0, 125)$, $(-125, 0)$

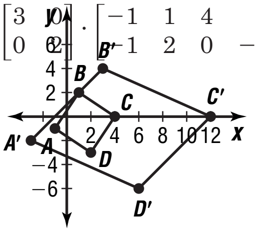
32. Sample answer: There is no single matrix to achieve this. You could reflect over the x-axis and then translate 2(4) or 8 units upward.

33. See students' work; the repeated dilations animate the growth of something from small to large, similar to a lens zooming into the origin.

34. $\begin{bmatrix} -1 & 1 & 4 & 2 \\ -1 & 2 & 0 & -3 \end{bmatrix}$

34a. Sample answer: the figure would be enlarged disproportionally.

34b. $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 & 1 & 4 & 2 \\ -1 & 2 & 0 & -3 \end{bmatrix} = \begin{bmatrix} -3 & 3 & 12 & 6 \\ -2 & 4 & 0 & -6 \end{bmatrix}$



34c. See students' work; the figure appears as if blown out of proportion.

35. $\begin{bmatrix} 3 & 8 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 5 \\ -2 & 8 \end{bmatrix} = \begin{bmatrix} 3+1 & 8+5 \\ -2+(-2) & 4+8 \end{bmatrix}$

$= \begin{bmatrix} 4 & 13 \\ -4 & 12 \end{bmatrix}$

36.
$$\begin{aligned} x + y + z &= 1.8 \\ y - z &= -5.6 \\ \hline x + 2y \quad &= -3.8 \end{aligned} \qquad \begin{aligned} x - 2y &= 4.6 \\ x + 2y &= -3.8 \\ \hline 2x \quad &= 0.8 \\ x &= 0.4 \end{aligned}$$

$x + 2y = -3.8$ $\qquad$ $y - z = -5.6$

37. Let x represent hardback books and y represent paperback books.

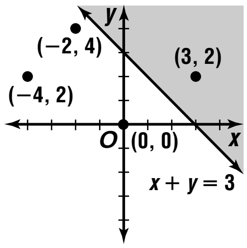
$4x + 7y = 5.75$
$3x + 5y = 4.25$

$$\begin{aligned} 3(4x + 7y) &= 3(5.75) \\ -4(3x + 5y) &= -4(4.25) \end{aligned} \rightarrow \begin{aligned} 12x + 21y &= 17.25 \\ -12x - 20y &= -17 \\ \hline y &= 0.25 \end{aligned}$$

$$\begin{aligned} 4x + 7y &= 5.75 \\ 4x + 7(0.25) &= 5.75 \\ x &= 1 \end{aligned}$$

hardbacks \$1, paperbacks \$0.25

38. $x + y \geq 3$
$y \leq -x + 3$



(3, 2)

39.
$$\begin{aligned} y - 1 &= 4(x - 2)) \\ y - 1 &= 4x + 8 \\ 4x - y + 9 &= 0 \end{aligned}$$

40. $y - 6 = 2(x - 1)$
$y = 2x + 4$

41.
$$\begin{aligned} (f \cdot g)(x) &= f(x) \cdot g(x) \\ &= x^3(x^2 - 3x + 7) \\ &= x^5 - 3x^4 + 7x^3 \end{aligned}$$
$$\begin{aligned} \left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)} \\ &= \frac{x^3}{x^2 - 3x + 7} \end{aligned}$$

42.
$$\begin{aligned} 2x + y &= 12 \\ -2(x + 2y) &= -2(-6) \end{aligned} \rightarrow \begin{aligned} 2x + \quad y &= 12 \\ -2x - 4y &= 12 \\ \hline -3y &= 24 \\ y &= -8 \end{aligned}$$

$$\begin{aligned} 2x + y &= 12 \\ 2x + (-8) &= 12 \\ x &= 10 \end{aligned}$$

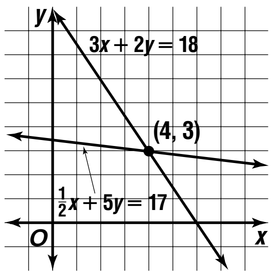
$$\begin{aligned} 2x + 2y &= 2(10) + 2(-8) \\ &= 4 \end{aligned}$$

The correct choice is B.

Page 96 Mid-Chapter Quiz

1. $\frac{1}{2}x + 5y = 17$ $\rightarrow$ $y = -\frac{1}{10}x + 3\frac{2}{5}$

$3x + 2y = 18$ $\qquad$ $y = -\frac{3}{2}x + 9$



2. $4x + y = 8$ $\quad 6x - 2y = -9$

$y = 8 - 4x$ $\quad 6x - 2(8 - 4x) = -9$

$x = \frac{1}{2}$

$4x + y = 8$

$4\left(\frac{1}{2}\right) + y = 8$

$y = 6$ $\quad \left(\frac{1}{2}, 6\right)$

3. Let x represent trucks and y represent cars.

$x = 4y$

$6x + 5y = 29{,}000$ $\quad x = 4y$

$6(4y) + 5y = 29{,}000$ $\quad x = 4(1000)$

$y = 1000$ $\quad = 4000$

4000 trucks, 1000 cars

4. $$\begin{array}{r} 2x + y + 4z = 13 \\ 3x - y - 2z = -1 \\ \hline 5x \quad\quad + 2z = 12 \end{array}$$

$$\begin{array}{l} 2(3x - y - 2z) = 2(-1) \\ 4x + 2y + z = 19 \end{array} \rightarrow \begin{array}{r} 6x - 2y - 4z = -2 \\ 4x + 2y + z = 19 \\ \hline 10x \quad - 3z = 17 \end{array}$$

$$\begin{array}{l} -2(5x + 2z) = -2(12) \\ 10x - 3z = 17 \end{array} \rightarrow \begin{array}{r} -10x - 4z = -24 \\ 10x - 3z = 17 \\ \hline -7z = -7 \\ z = 1 \end{array}$$

$5x + 2z = 12$ $\quad 2x + y + 4z = 13$

$5x + 2(1) = 12$ $\quad 2(2) + y + 4(1) = 13$

$x = 2$ $\quad y = 5$

$(2, 5, 1)$

5. $$\begin{array}{r} x + y = 1 \\ 2x - y = -2 \\ \hline 3x \quad = -1 \\ x = -\frac{1}{3} \end{array} \qquad \begin{array}{r} x + y = 1 \\ -\frac{1}{3} + y = 1 \\ y = 1\frac{1}{3} \end{array}$$

$4x + y + z = 8$

$4\left(-\frac{1}{3}\right) + 1\frac{1}{3} + z = 8$

$z = 8$ $\quad \left(-\frac{1}{3}, 1\frac{1}{3}, 8\right)$

6. $y - 3 = x$ $\quad y - 3 = x$

$y = 2x$ $\quad 2x - 3 = x$

$x = 3$

$y = 2x$

$y = 2(3)$

$= 6$ $\quad (3, 6)$

7. $$A + B = \begin{bmatrix} 3 + (-2) & 5 + 8 & -7 + 6 \\ -1 + 5 & 0 + (-9) & 4 + 10 \end{bmatrix} = \begin{bmatrix} 1 & 13 & -1 \\ 4 & -9 & 14 \end{bmatrix}$$

8. impossible

9. $B - 3A = B + (-3A)$

$$-3A = -3\begin{bmatrix} 3 & 5 & -7 \\ -1 & 0 & 4 \end{bmatrix} = \begin{bmatrix} -3(3) & -3(5) & -3(-7) \\ -3(-1) & -3(0) & -3(4) \end{bmatrix} \text{ or } \begin{bmatrix} -9 & -15 & 21 \\ 3 & 0 & -12 \end{bmatrix}$$

$$B + (-3A) = \begin{bmatrix} -2 & 8 & 6 \\ 5 & -9 & 10 \end{bmatrix} + \begin{bmatrix} -9 & -15 & 21 \\ 3 & 0 & -12 \end{bmatrix} = \begin{bmatrix} -2 + (-9) & 8 + (-15) & 6 + 21 \\ 5 + 3 & -9 + 0 & 10 + (-12) \end{bmatrix} = \begin{bmatrix} -11 & -7 & 27 \\ 8 & -9 & -2 \end{bmatrix}$$

10. The result is the original figure. The original figure is represented by $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$. The reflection over the x-axis is found by $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} a & b & c \\ -d & -e & -f \end{bmatrix}$. The reflection of the image over the x-axis is found by $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} a & b & c \\ -d & -e & -f \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$. The matrix for the final image is the same as that of the original figure.

2-5 Determinants and Multiplicative Inverses of Matrices

Page 102 Check for Understanding

1. Sample answer: a matrix with a nonzero determinant

2. Sample answer: $\begin{bmatrix} 3 & 2 & 0 \\ 4 & -3 & 5 \end{bmatrix}$ is not a square matrix. $\begin{bmatrix} 2 & -5 \\ 1 & -1 \\ 0 & 9 \end{bmatrix}$ also has no determinant.

3. Sample answer: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

4. Sample answer: The system has a solution if $ad - bc \neq 0$, since you can use the inverse of the matrix $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$ to find the solution.

5. $\begin{vmatrix} 4 & -1 \\ -2 & 3 \end{vmatrix} = 4(3) - (-2)(-1)$ or 10

6. $\begin{vmatrix} 12 & -26 \\ -15 & 32 \end{vmatrix} = 12(32) - (-15)(-26)$ or -6

7. $$\begin{vmatrix} 4 & 1 & 0 \\ 5 & -15 & -1 \\ -2 & 10 & 7 \end{vmatrix} = 4\begin{vmatrix} -15 & -1 \\ 10 & 7 \end{vmatrix} - 1\begin{vmatrix} 5 & -1 \\ -2 & 7 \end{vmatrix} + 0\begin{vmatrix} 5 & -15 \\ -2 & 10 \end{vmatrix} = 4(-95) - 1(33) + 0(20) = -413$$

8. $$\begin{vmatrix} 6 & 4 & -1 \\ 0 & 3 & 3 \\ -9 & 0 & 0 \end{vmatrix} = 6\begin{vmatrix} 3 & 3 \\ 0 & 0 \end{vmatrix} - 4\begin{vmatrix} 0 & 3 \\ -9 & 0 \end{vmatrix} + (-1)\begin{vmatrix} 0 & 3 \\ -9 & 0 \end{vmatrix} = 6(0) - 4(27) + (-1)(27) = -135$$

9. $\begin{vmatrix} -2 & 3 \\ 5 & 7 \end{vmatrix} = -2(7) - 5(3)$ or -29

$-\frac{1}{29}\begin{bmatrix} 7 & -3 \\ -5 & -2 \end{bmatrix}$

10. $\begin{vmatrix} 4 & 6 \\ 6 & 9 \end{vmatrix} = 4(9) - 6(6)$ or 0

does not exist

11. $$\begin{bmatrix} 5 & 4 \\ -3 & -5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ -24 \end{bmatrix}$$

$$\frac{1}{\begin{vmatrix} 5 & 4 \\ -3 & -5 \end{vmatrix}} \begin{bmatrix} -5 & -4 \\ 3 & 5 \end{bmatrix} = -\frac{1}{13}\begin{bmatrix} -5 & -4 \\ 3 & 5 \end{bmatrix}$$

$$-\frac{1}{13}\begin{bmatrix} -5 & -4 \\ 3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 5 & 4 \\ -3 & -5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{13}\begin{bmatrix} -5 & -4 \\ 3 & 5 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ -24 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{111}{13} \\ \frac{129}{13} \end{bmatrix}$$

$\left(-\frac{111}{13}, \frac{129}{13}\right)$

12. $$\begin{bmatrix} 6 & -3 \\ 5 & 9 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 63 \\ 85 \end{bmatrix}$$

$$\frac{1}{\begin{vmatrix} 6 & -3 \\ 5 & -9 \end{vmatrix}} \begin{bmatrix} -9 & 3 \\ -5 & 6 \end{bmatrix} = -\frac{1}{39}\begin{bmatrix} -9 & 3 \\ -5 & 6 \end{bmatrix}$$

$$-\frac{1}{39}\begin{bmatrix} -9 & 3 \\ -5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 6 & -3 \\ 5 & -9 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{39}\begin{bmatrix} -9 & 3 \\ -5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 63 \\ 85 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \end{bmatrix}$$

(8, −5)

13. Let x represent the amount of metal with 55% aluminum content, and let y represent the amount of metal with 80% aluminum content.

$x + y = 20$
$0.55x + 0.8y = 0.7(x + y)$
$0.55x + 0.8y = 0.7x + 0.7y$
$0.15x - 0.1y = 0$
$15x - 10y = 0$

$$\begin{aligned} x + y &= 20 \\ 15x - 10y &= 0 \end{aligned} \quad \rightarrow \quad \begin{bmatrix} 1 & 1 \\ 15 & -10 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$\frac{1}{\begin{vmatrix} 1 & 1 \\ 15 & -10 \end{vmatrix}} \begin{bmatrix} -10 & -1 \\ -15 & 1 \end{bmatrix} = -\frac{1}{25}\begin{bmatrix} -10 & -1 \\ -15 & 1 \end{bmatrix}$$

$$-\frac{1}{25}\begin{bmatrix} -10 & -1 \\ -15 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 15 & -10 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{25}\begin{bmatrix} -10 & -1 \\ -15 & 1 \end{bmatrix} \cdot \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 12 \end{bmatrix}$$

8 kg of the metal with 55% aluminum and 12 kg of the metal with 80% aluminum

Pages 102–105 Exercises

14. $\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix} = 3(5) - 2(4)$ or 7

15. $\begin{vmatrix} -4 & -1 \\ 0 & -1 \end{vmatrix} = -4(-1) - 0(-1)$ or 4

16. $\begin{vmatrix} 9 & 12 \\ 12 & 16 \end{vmatrix} = 9(16) - 12(12)$ or 0

17. $\begin{vmatrix} -2 & 3 \\ -2 & 1 \end{vmatrix} = -2(1) - (-2)(3)$ or 4

18. $\begin{vmatrix} 13 & 7 \\ -5 & -8 \end{vmatrix} = 13(-8) - (-5)(7)$ or −69

19. $\begin{vmatrix} -6 & 5 \\ 0 & -8 \end{vmatrix} = -6(-8) - 0(5)$ or 48

20. $$\begin{vmatrix} 4 & -1 & -2 \\ 0 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 4\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} - (-1)\begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} + (-2)\begin{vmatrix} 0 & 2 \\ 2 & 1 \end{vmatrix}$$
$= 4(5) + 1(-2) - 2(-4)$
$= 26$

21. $$\begin{vmatrix} 2 & -1 & 3 \\ 3 & 0 & -2 \\ 1 & -3 & 0 \end{vmatrix} = 2\begin{vmatrix} 0 & -2 \\ -3 & 0 \end{vmatrix} - (-1)\begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix} + 3\begin{vmatrix} 3 & 0 \\ 1 & -3 \end{vmatrix}$$
$= 2(-6) + 1(2) + 3(-9)$
$= -37$

22. $$\begin{vmatrix} 8 & 9 & 3 \\ 3 & 5 & 7 \\ -1 & 2 & 4 \end{vmatrix} = 8\begin{vmatrix} 5 & 7 \\ 2 & 4 \end{vmatrix} - 9\begin{vmatrix} 3 & 7 \\ -1 & 4 \end{vmatrix} + 3\begin{vmatrix} 3 & 5 \\ -1 & 2 \end{vmatrix}$$
$= 8(6) - 9(19) + 3(11)$
$= -90$

23. $$\begin{vmatrix} 4 & 6 & 7 \\ 3 & -2 & -4 \\ 1 & 1 & 1 \end{vmatrix} = 4\begin{vmatrix} -2 & -4 \\ 1 & 1 \end{vmatrix} - 6\begin{vmatrix} 3 & -4 \\ 1 & 1 \end{vmatrix} + 7\begin{vmatrix} 3 & -2 \\ 1 & 1 \end{vmatrix}$$
$= 4(2) - 6(7) + 7(5)$
$= 1$

24. $$\begin{vmatrix} 25 & 36 & 15 \\ 31 & -12 & -2 \\ 17 & 15 & 9 \end{vmatrix} = 25\begin{vmatrix} -12 & -2 \\ 15 & 9 \end{vmatrix} - 36\begin{vmatrix} 31 & -2 \\ 17 & 9 \end{vmatrix} + 15\begin{vmatrix} 31 & -12 \\ 17 & 15 \end{vmatrix}$$
$= 25(-78) - 36(313) + 15(669)$
$= -3183$

25. $$\begin{vmatrix} 1.5 & -3.6 & 2.3 \\ 4.3 & 0.5 & 2.2 \\ -1.6 & 8.2 & 6.6 \end{vmatrix} = 1.5\begin{vmatrix} 0.5 & 2.2 \\ 8.2 & 6.6 \end{vmatrix} - (-36)\begin{vmatrix} 4.3 & 2.2 \\ -1.6 & 6.6 \end{vmatrix} + 23\begin{vmatrix} 4.3 & 0.5 \\ -1.6 & 8.2 \end{vmatrix}$$
$= 1.5(-14.74) + 3.6(31.9) + 2.3(36.06)$
$= 175.668$

26. $$\begin{vmatrix} 0 & 1 & -4 \\ 3 & 2 & 3 \\ 8 & -3 & 4 \end{vmatrix} = 0\begin{vmatrix} 2 & 3 \\ -3 & 4 \end{vmatrix} - 1\begin{vmatrix} 3 & 3 \\ 8 & 4 \end{vmatrix} + (-4)\begin{vmatrix} 3 & 2 \\ 8 & -3 \end{vmatrix}$$
$= 0(17) - 1(-12) - 4(-25)$
$= 112$

27. $\begin{vmatrix} 2 & -3 \\ -2 & -2 \end{vmatrix} = 2(-2) - (-2)(-3)$ or −10

$-\frac{1}{10}\begin{bmatrix} -2 & 3 \\ 2 & 2 \end{bmatrix}$

28. $\begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} = 2(0) - 1(0)$ or 0

does not exist

29. $\begin{vmatrix} 4 & 2 \\ 1 & 2 \end{vmatrix} = 4(2) - 1(2)$ or 6

$\frac{1}{6}\begin{bmatrix} 2 & -2 \\ -1 & 4 \end{bmatrix}$

30. $\begin{vmatrix} 6 & 7 \\ -6 & 7 \end{vmatrix} = 6(7) - (-6)(7)$ or 84

$\frac{1}{84}\begin{bmatrix} 7 & -7 \\ 6 & 6 \end{bmatrix}$

31. $\begin{vmatrix} -4 & 6 \\ 8 & -12 \end{vmatrix} = -4(-12) - 8(6)$ or 0

does not exist

32. $\begin{vmatrix} 9 & 13 \\ 27 & 36 \end{vmatrix} = 9(36) - 27(13)$ or -27

$-\frac{1}{27}\begin{bmatrix} 36 & -13 \\ -27 & 9 \end{bmatrix}$

33. $\begin{vmatrix} \frac{3}{4} & -\frac{1}{8} \\ -5 & \frac{1}{2} \end{vmatrix} = \frac{3}{4}\left(\frac{1}{2}\right) - 5\left(-\frac{1}{8}\right)$ or 1

$\begin{bmatrix} \frac{1}{2} & \frac{1}{8} \\ -5 & \frac{3}{4} \end{bmatrix}$

34.
$$\begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$
$$\frac{1}{\begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix}} \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$$
$$\frac{1}{9}\begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$
(1, 3)

35.
$$\begin{bmatrix} 9 & -6 \\ 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ -12 \end{bmatrix}$$
$$\frac{1}{\begin{vmatrix} 9 & -6 \\ 4 & 6 \end{vmatrix}} \begin{bmatrix} 6 & 6 \\ -4 & 9 \end{bmatrix} = \frac{1}{78}\begin{bmatrix} 6 & 6 \\ -4 & 9 \end{bmatrix}$$
$$\frac{1}{78}\begin{bmatrix} 6 & 6 \\ -4 & 9 \end{bmatrix} - \begin{bmatrix} 9 & -6 \\ 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{78}\begin{bmatrix} 6 & 6 \\ -4 & 9 \end{bmatrix} \cdot \begin{bmatrix} 12 \\ -12 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$
(0, −2)

36.
$$\begin{bmatrix} 1 & 5 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 26 \\ -41 \end{bmatrix}$$
$$\frac{1}{\begin{vmatrix} 1 & 5 \\ 3 & -2 \end{vmatrix}} \begin{bmatrix} -2 & -5 \\ -3 & 1 \end{bmatrix} = -\frac{1}{17}\begin{bmatrix} -2 & -5 \\ -3 & 1 \end{bmatrix}$$
$$-\frac{1}{17}\begin{bmatrix} -2 & -5 \\ -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 5 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{17}\begin{bmatrix} -2 & -5 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 26 \\ -41 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -9 \\ 7 \end{bmatrix}$$
(−9, 7)

37.
$$\begin{bmatrix} 4 & 8 \\ 3 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \end{bmatrix}$$
$$\frac{1}{\begin{vmatrix} 4 & 8 \\ 3 & -3 \end{vmatrix}} \begin{bmatrix} -3 & -8 \\ -3 & 4 \end{bmatrix} = -\frac{1}{36}\begin{bmatrix} -3 & -8 \\ -3 & 4 \end{bmatrix}$$
$$-\frac{1}{36}\begin{bmatrix} -3 & -8 \\ -3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 & 8 \\ 3 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{36}\begin{bmatrix} -3 & -8 \\ -3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{7}{12} \\ \frac{7}{12} \end{bmatrix}$$
$\left(\frac{7}{12}, \frac{7}{12}\right)$

38.
$$\begin{bmatrix} 3 & -5 \\ 5 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -24 \\ -3 \end{bmatrix}$$
$$\frac{1}{\begin{vmatrix} 3 & -5 \\ 5 & 4 \end{vmatrix}} \begin{bmatrix} 4 & 5 \\ -5 & 3 \end{bmatrix} = \frac{1}{37}\begin{bmatrix} 4 & 5 \\ -5 & 3 \end{bmatrix}$$
$$\frac{1}{37}\begin{bmatrix} 4 & 5 \\ -5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -5 \\ 5 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{37}\begin{bmatrix} 4 & 5 \\ -5 & 3 \end{bmatrix} \cdot \begin{bmatrix} -24 \\ -3 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$$
(−3, 3)

39.
$$\begin{bmatrix} 9 & 3 \\ 5 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$\frac{1}{\begin{vmatrix} 9 & 3 \\ 5 & 1 \end{vmatrix}} \begin{bmatrix} 1 & -3 \\ -5 & 9 \end{bmatrix} = -\frac{1}{6}\begin{bmatrix} 1 & -3 \\ -5 & 9 \end{bmatrix}$$
$$-\frac{1}{6}\begin{bmatrix} 1 & -3 \\ -5 & 9 \end{bmatrix} \cdot \begin{bmatrix} 9 & 3 \\ 5 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{6}\begin{bmatrix} 1 & -3 \\ -5 & 9 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ -\frac{2}{3} \end{bmatrix}$$
$\left(\frac{1}{3}, -\frac{2}{3}\right)$

40.
$$\begin{bmatrix} 3 & -2 & 3 \\ 1 & 2 & 2 \\ -2 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix}$$
$$\frac{1}{9}\begin{bmatrix} -4 & 1 & -10 \\ -3 & 3 & -3 \\ 5 & 1 & 8 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 & 3 \\ 1 & 2 & 2 \\ -2 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$
$$= \frac{1}{9}\begin{bmatrix} -4 & 1 & -10 \\ -3 & 3 & -3 \\ 5 & 1 & 8 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} -4(-4) + 1(0) + (-10)(1) \\ -3(-4) + 3(0) + (-3)(1) \\ 5(-4) + 1(0) + 8(1) \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 6 \\ 9 \\ -12 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ 1 \\ -\frac{4}{3} \end{bmatrix}$$
$\left(\frac{2}{3}, 1, -\frac{4}{3}\right)$

41.
$$\begin{bmatrix} -6 & 5 & 3 \\ 9 & -2 & -1 \\ 3 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -9 \\ 5 \\ -1 \end{bmatrix}$$
$$-\frac{1}{9}\begin{bmatrix} -1 & -2 & 1 \\ -12 & -15 & 21 \\ 15 & 21 & -33 \end{bmatrix} \cdot \begin{bmatrix} -6 & 5 & 3 \\ 9 & -2 & -1 \\ 3 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$
$$= -\frac{1}{9}\begin{bmatrix} -1 & -2 & 1 \\ -12 & -15 & 21 \\ 15 & 21 & -33 \end{bmatrix} \cdot \begin{bmatrix} -9 \\ 5 \\ -1 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{9}\begin{bmatrix} -1(-9) + (-2)(5) + 1(-1) \\ -12(-9) + (-15)(5) + 21(-1) \\ 15(-9) + 21(5) + (-33)(-1) \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{9}\begin{bmatrix} -2 \\ 12 \\ 3 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{2}{9} \\ -\frac{4}{3} \\ -\frac{1}{3} \end{bmatrix}$$
$\left(\frac{2}{9}, -\frac{4}{3}, -\frac{1}{3}\right)$

42. −216

43. 30,143

44. $\begin{bmatrix} 0.3 & 0.5 \\ 12 & -6.5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4.74 \\ -1.2 \end{bmatrix}$

$\dfrac{1}{\begin{vmatrix} 0.3 & 0.5 \\ 12 & -6.5 \end{vmatrix}} \begin{bmatrix} -6.5 & -0.5 \\ -12 & 0.3 \end{bmatrix} = -\frac{1}{7.95}\begin{bmatrix} -6.5 & -0.5 \\ -12 & 0.3 \end{bmatrix}$

$-\frac{1}{7.95}\begin{bmatrix} -6.5 & -0.5 \\ -12 & 0.3 \end{bmatrix} \cdot \begin{bmatrix} 0.3 & 0.5 \\ 12 & -65 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$

$= -\frac{1}{7.95}\begin{bmatrix} -6.5 & -0.5 \\ -12 & 0.3 \end{bmatrix} \cdot \begin{bmatrix} 4.74 \\ -1.2 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3.8 \\ 7.2 \end{bmatrix}$

(3.8, 7.2)

45. $2(x - 2y + z) = 2(7)$ → $2x - 4y + 2z = 14$
$6x + 2y - 2z = 4$ $\quad$ $\underline{6x + 2y - 2z = 4}$
$8x - 2y = 18$

$2(6x + 2y - 2z) = 2(4)$ → $12x + 4y - 4z = 8$
$4x + 6y + 4z = 14$ $\quad$ $\underline{4x + 6y + 4z = 14}$
$16x + 10y = 22$

$\begin{bmatrix} 8 & -2 \\ 16 & 10 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 18 \\ 22 \end{bmatrix}$

$\dfrac{1}{\begin{vmatrix} 8 & -2 \\ 16 & 10 \end{vmatrix}} \begin{bmatrix} 10 & 2 \\ -16 & 8 \end{bmatrix} = \frac{1}{112}\begin{bmatrix} 10 & 2 \\ -16 & 8 \end{bmatrix}$

$\frac{1}{112}\begin{bmatrix} 10 & 2 \\ -16 & 8 \end{bmatrix} \cdot \begin{bmatrix} 8 & -2 \\ 16 & 10 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{112}\begin{bmatrix} 10 & 2 \\ -16 & 8 \end{bmatrix} \cdot \begin{bmatrix} 18 \\ 22 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$x - 2y + z = 7$
$2 - 2(-1) + z = 7$
$z = 3$

(2, −1, 3)

46. Let x represent the number of cars produced in the first year, and let y represent the number of cars in the second year.

$x + y = 390{,}000$ → $x + y = 390{,}000$
$x = y + 90{,}000$ $\quad$ $x - y = 90{,}000$

$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 390{,}000 \\ 90{,}000 \end{bmatrix}$

$\dfrac{1}{\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$

$-\frac{1}{2}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$

$= -\frac{1}{2}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 390{,}000 \\ 90{,}000 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 240{,}000 \\ 150{,}000 \end{bmatrix}$

150,000 in the second year and 240,000 in the first year

47. Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

$A^{-1} = \begin{bmatrix} \frac{a_{22}}{a_{11}a_{22} - a_{21}a_{12}} & \frac{-a_{12}}{a_{11}a_{22} - a_{21}a_{12}} \\ \frac{-a_{21}}{a_{11}a_{22} - a_{21}a_{12}} & \frac{a_{11}}{a_{11}a_{22} - a_{21}a_{12}} \end{bmatrix}$

$AA^{-1} = \begin{bmatrix} \frac{a_{11}a_{22} - a_{21}a_{12}}{a_{11}a_{22} - a_{21}a_{12}} & \frac{-a_{11}a_{12} + a_{12}a_{11}}{a_{11}a_{22} - a_{21}a_{12}} \\ \frac{a_{21}a_{22} - a_{21}a_{22}}{a_{11}a_{22} - a_{21}a_{12}} & \frac{a_{11}a_{22} - a_{21}a_{12}}{a_{11}a_{22} - a_{21}a_{12}} \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Thus, $AA^{-1} = I$.

48. Let x represent the number of gallons of 10% alcohol solution, and let y represent the number of gallons of 25% alcohol solution.

$x + y = 12$
$0.10x + 0.25y = 0.15(x + y)$
$0.10x + 0.25y = 0.15x + 0.15y$
$-0.05x + 0.10y = 0$

$x + y = 12$
$-0.05x + 0.10y = 0$ → $\begin{bmatrix} 1 & 1 \\ -0.05 & 0.10 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix}$

$\dfrac{1}{\begin{vmatrix} 1 & 1 \\ -0.05 & 0.10 \end{vmatrix}} \begin{bmatrix} 0.10 & -1 \\ 0.05 & 1 \end{bmatrix} = \frac{1}{0.15}\begin{bmatrix} 0.10 & -1 \\ 0.05 & 1 \end{bmatrix}$

$\frac{1}{0.15}\begin{bmatrix} 0.10 & -1 \\ 0.05 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ -0.05 & 0.10 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$

$= \frac{1}{0.15}\begin{bmatrix} 0.10 & -1 \\ 0.05 & 1 \end{bmatrix} \cdot \begin{bmatrix} 12 \\ 0 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$

8 gal of 10% and 4 gal of 25%

49. Yes

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Does $(A^2)^{-1} = (A^{-1})^2$

$A^2 = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix}$

$(A^2)^{-1} = \frac{1}{a^2d^2 - 2abcd + b^2c^2}\begin{bmatrix} bc + d^2 & -ab - bd \\ -ac - cd & a^2 + bc \end{bmatrix}$

$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} \frac{d}{ad - bc} & \frac{-b}{ad - bc} \\ \frac{-c}{ad - bc} & \frac{a}{ad - bc} \end{bmatrix}$

$(A^{-1})^2 = \frac{1}{a^2d^2 - 2abcd + b^2c^2}\begin{bmatrix} bc + d^2 & -ab - bd \\ -ac - cd & a^2 + bc \end{bmatrix}$

Thus, $(A^2)^{-1} = (A^{-1})^2$.

50. $A = \frac{1}{2}\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 1 & -3 & 1 \\ 0 & 4 & 1 \\ 3 & 0 & 1 \end{vmatrix}$

$= \frac{1}{2}\left|\left(1\begin{vmatrix} 4 & 1 \\ 0 & 1 \end{vmatrix} - (-3)\begin{vmatrix} 0 & 1 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} 0 & 4 \\ 3 & 0 \end{vmatrix}\right)\right|$

$= \frac{1}{2}\left|(1(4) + 3(-3) + 1(-12))\right|$

$= \frac{1}{2}\left|-17\right|$ or 8.5

$8\frac{1}{2}$ or 8.5 square units

51. Let x represent the cost of complete computer systems, and let y represent the cost of printers.

day 1: $38x + 53y = 49{,}109$
day 2: $22x + 44y = 31{,}614$
day 3: $21x + 26y = 26{,}353$
using day 1 and day 2:

$\begin{bmatrix} 38 & 53 \\ 22 & 44 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 49{,}109 \\ 31{,}614 \end{bmatrix}$

$\dfrac{1}{\begin{vmatrix} 38 & 53 \\ 22 & 44 \end{vmatrix}} \begin{bmatrix} 44 & -53 \\ -22 & 38 \end{bmatrix} = \frac{1}{506}\begin{bmatrix} 44 & -53 \\ -22 & 38 \end{bmatrix}$

$\frac{1}{506}\begin{bmatrix} 44 & -53 \\ -22 & 38 \end{bmatrix} \cdot \begin{bmatrix} 38 & 53 \\ 22 & 44 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$

$= \frac{1}{506}\begin{bmatrix} 44 & -53 \\ -22 & 38 \end{bmatrix} \cdot \begin{bmatrix} 49{,}109 \\ 31{,}614 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 959 \\ 239 \end{bmatrix}$

computer system: \$959, printer: \$239

52. Let x represent Jessi's first test score, and let y represent Jessi's second test score.

$$\begin{aligned} x + y &= 179 \\ y &= x + 7 \end{aligned} \quad \rightarrow \quad \begin{aligned} x + y &= 179 \\ x - y &= -7 \end{aligned}$$

$$\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}179\\-7\end{bmatrix}$$

$$\frac{1}{\begin{vmatrix}1 & 1\\1 & -1\end{vmatrix}}\begin{bmatrix}-1 & -1\\-1 & 1\end{bmatrix} = -\frac{1}{2}\begin{bmatrix}-1 & -1\\-1 & 1\end{bmatrix}$$

$$-\frac{1}{2}\begin{bmatrix}-1 & -1\\-1 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix} = -\frac{1}{2}\begin{bmatrix}-1 & -1\\-1 & 1\end{bmatrix}\cdot\begin{bmatrix}179\\-7\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}86\\93\end{bmatrix}$$

first test: 86, second test: 93

53. $\begin{bmatrix}8 & 4 & 0 & 4\\5 & 1 & 5 & 9\end{bmatrix} + \begin{bmatrix}-3 & -3 & -3 & -3\\4 & 4 & 4 & 4\end{bmatrix}$

$= \begin{bmatrix}8 + (-3) & 4 + (-3) & 0 + (-3) & 4 + (-3)\\5 + 4 & 1 + 4 & 5 + 4 & 9 + 4\end{bmatrix}$

$= \begin{bmatrix}5 & 1 & -3 & 1\\9 & 5 & 9 & 13\end{bmatrix}$

$H'(5, 9)$, $I'(1, 5)$, $J'(-3, 9)$, $K'(1, 13)$

54. $\frac{3}{4}\begin{bmatrix}8 & -7\\-4 & 0\end{bmatrix} = \begin{bmatrix}6 & -\frac{21}{4}\\-3 & 0\end{bmatrix}$

55.
$$\begin{aligned} x - 3y + 2z &= 6 \\ 2(4x + y - z) &= 2(8) \end{aligned} \quad \rightarrow \quad \begin{aligned} x - 3y + 2z &= 6 \\ 8x + 2y - 2z &= 16 \\ \hline 9x - y &= 22 \end{aligned}$$

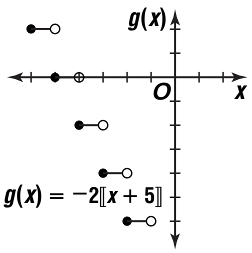
$$\begin{aligned} 4(4x + y - z) &= 4(8) \\ -7x - 5y + 4z &= -10 \end{aligned} \quad \rightarrow \quad \begin{aligned} 16x + 4y - 4z &= 32 \\ -7x - 5y + 4z &= -10 \\ \hline 9x - y &= 22 \end{aligned}$$

$$\begin{aligned} -1(9x - y) &= -1(22) \\ 9x - y &= 22 \end{aligned} \quad \rightarrow \quad \begin{aligned} -9x + y &= -22 \\ 9x - y &= 22 \\ \hline 0 &= 0 \end{aligned}$$

infinitely many solutions

56.

x	$f(x)$
$-6 \le x < -5$	2
$-5 \le x < -4$	0
$-4 \le x < -3$	-2
$-3 \le x < -2$	-4
$-2 \le x < -1$	-6



57.
$$\begin{aligned} y - 5 &= \tfrac{1}{2}(x - 2) \\ y - 5 &= \tfrac{1}{2}x - 1 \\ 2y - 10 &= x - 2 \\ x - 2y + 8 &= 0 \end{aligned}$$

58.
$$\begin{aligned} m &= \tfrac{3 - 5}{2 - 1} \\ &= \tfrac{-2}{1} \quad \text{or} \quad -2 \end{aligned}$$

$(y - 5) = -2(x - 1)$ or $(y - 3) = -2(x - 2)$

$$\begin{aligned} y - 5 &= -2(x - 1) \\ y - 5 &= -2x + 2 \\ y &= -2x + 7 \end{aligned}$$

59a. $\frac{1}{12}$ or approximately 0.0833

59b. $\frac{1}{12} = \frac{x}{18}$

$18 = 12x$

$1.5 = x$; 1.5 ft

60.
$$\begin{aligned} [f \circ g](x) &= f(g(x)) \\ &= f(x - 1) \\ &= (x - 1)^2 + 3(x - 1) + 2 \\ &= x^2 - 2x + 1 + 3x - 3 + 2 \\ &= x^2 + x \\ [g \circ f](x) &= g(f(x)) \\ &= g(x^2 + 3x + 2) \\ &= x^2 + 3x + 2 - 1 \\ &= x^2 + 3x + 1 \end{aligned}$$

61. No, more than one element of the range is paired with the same element of the domain.

62. The radius of circle E is 3, so the circumference is $2\pi(3)$ or about 18.85. The diagonal of the square has length 6, so each side has length $\frac{6}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$. The perimeter of the square is $4(3\sqrt{2})$ or $12\sqrt{2} \approx 16.92$.

The difference between the circumference of the circle and the perimeter of the square is approximately $18.85 - 16.92 = 1.93$.

The correct choice is B.

2-5B Graphing Calculator Exploration: Augmented Matrices and Reduced Row-Echelon Form

Page 106

1. $\left[\begin{array}{rrr:r}2 & 1 & -2 & 7\\1 & -2 & -5 & -1\\4 & 1 & 1 & -1\end{array}\right]$, $\left[\begin{array}{rrr:r}1 & 0 & 0 & -1\\0 & 1 & 0 & 5\\0 & 0 & 1 & -2\end{array}\right]$,

$(-1, 5, -2)$

2. $\left[\begin{array}{rrr:r}1 & 1 & 1 & 6\\2 & -3 & 4 & 3\\4 & -8 & 4 & 12\end{array}\right]$, $\left[\begin{array}{rrr:r}1 & 0 & 0 & 7\\0 & 1 & 0 & 1\\0 & 0 & 1 & -2\end{array}\right]$,

$(7, 1, -2)$

3. $\left[\begin{array}{rrrr:r}1 & 1 & 1 & 1 & 0\\2 & 1 & -1 & -1 & 1\\-1 & -1 & 1 & 1 & 0\\0 & 2 & 1 & 0 & 0\end{array}\right]\left[\begin{array}{rrrr:r}1 & 0 & 0 & 0 & 1\\0 & 1 & 0 & 0 & -1\\0 & 0 & 1 & 0 & 2\\0 & 0 & 0 & 1 & -2\end{array}\right]$,

$(1, -1, 2, -2)$

4. Exercise 1: $x = -1$, $y = 5$, $z = -2$; Exercise 2: $x = 7$, $y = 1$, $z = -2$; Exercise 3: $w = 1$, $x = -1$, $y = 2$, $z = -2$; They are the solutions for the system.

5. The calculator would show the first part of the number and follow it by ....

2-6 Solving Systems of Linear Inequalities

Page 109–110 Check for Understanding

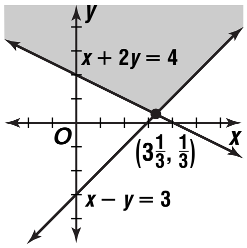
1a. the sum of twice the width and twice the height

1b. Sample answer: skis, fishing rods

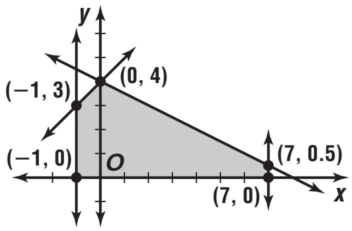
2. Tomas is correct. There are functions in which the coordinates of more than one vertex will yield the same value for the function.

3. You might expect five vertices; however, if the equations were dependent or if they did not intersect to form the sides of a convex polygon, there would be fewer vertices.

4.

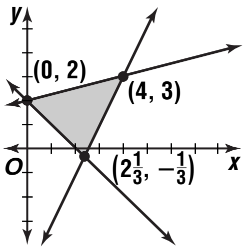


5.



$(-1, 0), (-1, 3), (0, 4), (7, 0.5), (7, 0)$

6.



$f(x, y) = 4x + 3y$

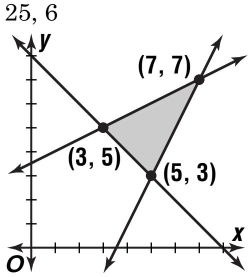
$f(0, 2) = 4(0) + 3(2) \text{ or } 6$

$f(4, 3) = 4(4) + 3(3) \text{ or } 25$

$f\left(2\frac{1}{3}, -\frac{1}{3}\right) = 4\left(2\frac{1}{3}\right) + 3\left(-\frac{1}{3}\right) \text{ or } 8\frac{1}{3}$

25, 6

7.



$f(x, y) = 3x - 4y$

$f(3, 5) = 3(3) - 4(5) \text{ or } -11$

$f(7, 7) = 3(7) - 4(7) \text{ or } -7$

$f(5, 3) = 3(5) - 4(3) \text{ or } 3$

$3, -11$

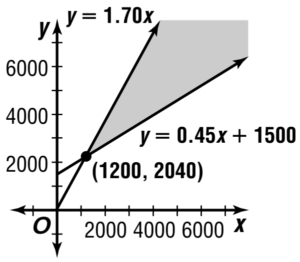
8. Let x represent the number of greeting cards sold, and let y represent the income in dollars.

$x \geq 0$

$y \geq 0$

$y \geq 0.45x + 1500$

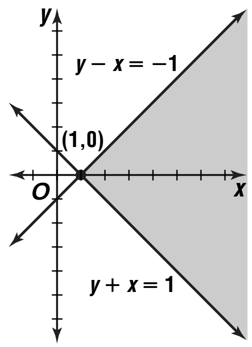
$y \leq 1.70x$



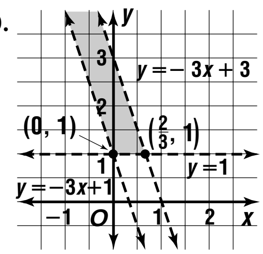
at least 1200 cards

Pages 110–111 Exercises

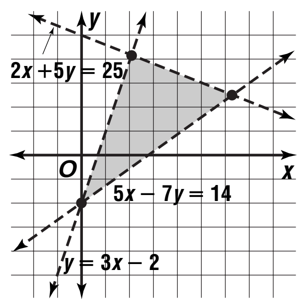
9.



10.



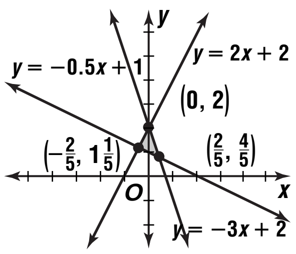
11.



12. yes, it is true for both inequalities:

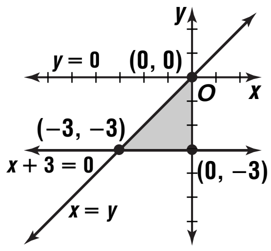
$y < \frac{1}{3}x + 5$	$y \leq 2x + 1$
$-2 \stackrel{?}{<} \frac{1}{3}(3) + 5$	$-2 \stackrel{?}{<} 2(3) + 1$
$-2 < 4$ true	$-2 < 7$ true

13.



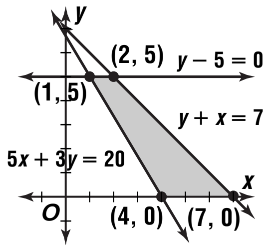
$\left(\frac{3}{5}, 3\frac{1}{5}\right), \left(-\frac{2}{5}, 1\frac{1}{5}\right), \left(1\frac{3}{5}, \frac{1}{5}\right)$

14.



(0, 0), (0, −3), (−3, −3)

15.



(2, 5), (7, 0), (4, 0), (1, 5)

16. $f(x, y) = 8x + y$

$f(0, 0) = 8(0) + 0$ or 0

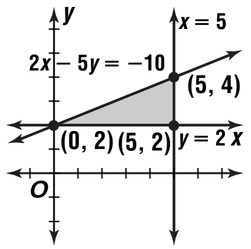
$f(4, 0) = 8(4) + 0$ or 32

$f(3, 5) = 8(3) + 5$ or 29

$f(0, 5) = 8(0) + 5$ or 5

32, 0

17.



$f(x, y) = 3x + y$

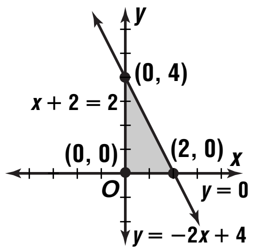
$f(0, 2) = 3(0) + 2$ or 2

$f(5, 4) = 3(5) + 4$ or 19

$f(5, 2) = 3(5) + 2$ or 17

19, 2

18.



$f(x, y) = y - x$

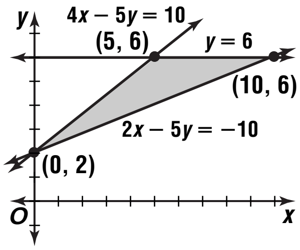
$f(0, 0) = 0 - 0$ or 0

$f(0, 4) = 4 - 0$ or 4

$f(2, 0) = 0 - 2$ or -2

4, −2

19.



$f(x, y) = x + y$

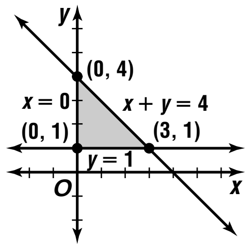
$f(0, 2) = 0 + 2$ or 2

$f(5, 6) = 5 + 6$ or 11

$f(10, 6) = 10 + 6$ or 16

16, 2

20.



$f(x, y) = 4x + 2y + 7$

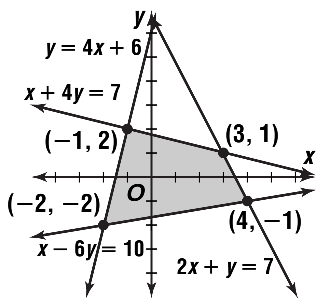
$f(0, 1) = 4(0) + 2(1) + 7$ or 9

$f(0, 4) = 4(0) + 2(4) + 7$ or 15

$f(3, 1) = 4(3) + 2(1) + 7$ or 21

21, 9

21.



$f(x, y) = 2x - y$

$f(-2, -2) = 2(-2) - (-2)$ or 0

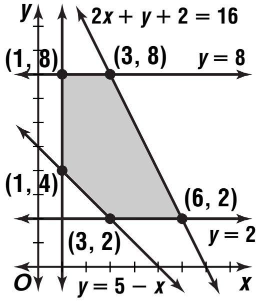
$f(-1, 2) = 2(-1) - 2$ or -4

$f(3, 1) = 2(3) - 1$ or 5

$f(4, -1) = 2(4) - (-1)$ or 9

9, −4

22.



$f(x, y) = -2x + y + 5$
$f(1, 4) = -2(1) + 4 + 5$ or 7
$f(1, 8) = -2(1) + 8 + 5$ or 11
$f(3, 8) = -2(3) + 8 + 5$ or 7
$f(6, 2) = -2(6) + 2 + 5$ or -5
$f(3, 2) = -2(3) + 2 + 5$ or 1
$11, -5$

23. $x \le 4, x \ge -4, y \le 4, y \ge -4$

24. Sample answer: $y \ge 3, x \ge 4, 4x + 3y \ge 12$

25a.

$3y = -2x + 11$
$y = 0$

$3y = -2x + 11$
$3(0) = -2x + 11$
$5\frac{1}{2} = x$ $\left(5\frac{1}{2}, 0\right)$

$y = 2x - 13$
$y = 0$

$y = 2x - 13$
$0 = 2x - 13$
$6\frac{1}{2} = x$ $\left(6\frac{1}{2}, 0\right)$

$y = 16 - x$
$y = 2x - 13$

$y = 16 - x$
$2x - 13 = 16 - x$
$x = \frac{29}{3}$
$y = 16 - x$
$y = 16 - \frac{29}{3}$
$= \frac{19}{3}$ $\left(\frac{29}{3}, \frac{19}{3}\right)$

$y = 16 - x$
$2y = 17$

$2y = 17$
$2(16 - x) = 17$
$x = 7\frac{1}{2}$
$2y = 17$
$y = 8\frac{1}{2}$ $\left(7\frac{1}{2}, 8\frac{1}{2}\right)$

$2y = 17$
$y = 3x + 1$

$2y = 17$
$y = 8\frac{1}{2}$
$y = 3x + 1$
$8\frac{1}{2} = 3x + 1$
$2\frac{1}{2} = x$ $\left(2\frac{1}{2}, 8\frac{1}{2}\right)$

$y = 7 - 2x$
$y = 3x + 1$

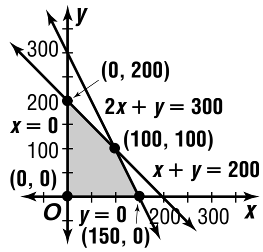
$y = 7 - 2x$
$3x + 1 = 7 - 2x$
$x = \frac{6}{5}$
$y = 7 - 2x$
$y = 7 - 2\left(\frac{6}{5}\right)$
$= \frac{23}{5}$ $\left(\frac{6}{5}, \frac{23}{5}\right)$

$3y = -2x + 11$
$y = 7 - 2x$

$3y = -2x + 11$
$3(7 - 2x) = -2x + 11$
$2\frac{1}{2} = x$
$y = 7 - 2x$
$y = 7 - 2\left(2\frac{1}{2}\right)$
$= 2$ $\left(2\frac{1}{2}, 2\right)$

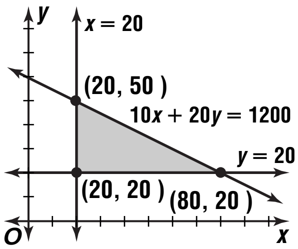
25b. $f(x, y) = 5x + 6y$
$f\left(5\frac{1}{2}, 0\right) = 5\left(5\frac{1}{2}\right) + 6(0)$ or $27\frac{1}{2}$
$f\left(6\frac{1}{2}, 0\right) = 5\left(6\frac{1}{2}\right) + 6(0)$ or $32\frac{1}{2}$
$f\left(\frac{29}{3}, \frac{19}{3}\right) = 5\left(\frac{29}{3}\right) + 6\left(\frac{19}{3}\right)$ or $86\frac{1}{3}$
$f\left(7\frac{1}{2}, 8\frac{1}{2}\right) = 5\left(7\frac{1}{2}\right) + 6\left(8\frac{1}{2}\right)$ or $88\frac{1}{2}$
$f\left(2\frac{1}{2}, 8\frac{1}{2}\right) = 5\left(2\frac{1}{2}\right) + 6\left(8\frac{1}{2}\right)$ or $63\frac{1}{2}$
$f\left(\frac{6}{5}, \frac{23}{5}\right) = 5\left(\frac{6}{5}\right) + 6\left(\frac{23}{5}\right)$ or $33\frac{1}{5}$
$f\left(2\frac{1}{2}, 2\right) = 5\left(2\frac{1}{2}\right) + 6(2)$ or $24\frac{1}{2}$
max at $\left(7\frac{1}{2}, 8\frac{1}{2}\right) = 88\frac{1}{2}$; min at $\left(2\frac{1}{2}, 2\right) = 24\frac{1}{2}$

26. $x + y \le 200$
$2x + y \le 300$
$x \ge 0$
$y \ge 0$



$f(x, y) = \$6.00x + \$4.80y$
$f(0, 0) = \$6.00(0) + \$4.80(0)$ or 0
$f(0, 200) = \$6.00(0) + \$4.80(200)$ or $\$960$
$f(100, 100) = \$6.00(100) + \$4.80(100)$ or $\$1080$
$f(150, 0) = \$6.00(150) + \$4.80(0)$ or $\$900$
$\$1080$

27a. Let x represent the Main Street site, and let y represent the High Street site.
$x \ge 20$
$y \ge 20$
$10x + 20y \le 1200$



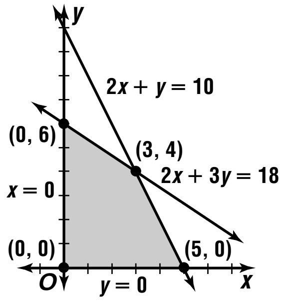
27b. $f(x, y) = 30x + 40y$

27c. $f(x, y) = 30x + 40y$
$f(20, 20) = 30(20) + 40(20)$ or 1400
$f(20, 50) = 30(20) + 40(50)$ or 2600
$f(80, 20) = 30(80) + 40(20)$ or 3200
80 ft^2 at the Main St. site and 20 ft^2 at the High St. site

27d. Main Street: $\$1200 \div \$10 = 120$ ft^2
$120 \times 30 = 3600$ customers
High Street: $\$1200 \div 20 = 60$ ft^2
$60 \times 40 = 240$ customers
The maximum number of customers can be reached by renting 120 ft^2 at Main St.

28a. 3 is \$3 profit on each batch of garlic dressing and 2 is \$2 profit on each batch of raspberry dressing.

28b. $2x + 3y \le 18$
$2x + y \le 10$
$x \ge 0$
$y \ge 0$

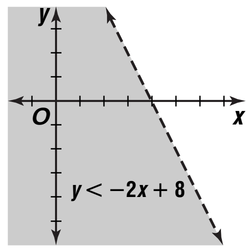


$f(x, y) = 3x + 2y$
$f(0, 0) = 3(0) + 2(0)$ or 0
$f(0, 6) = 3(0) + 2(6)$ or 12
$f(3, 4) = 3(3) + 2(4)$ or 17
$f(5, 0) = 3(5) + 2(0)$ or 15
3 batches garlic dressing, 4 batches raspberry dressing

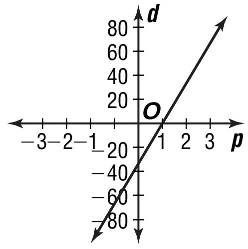
29. $\begin{vmatrix} 2 & 1 \\ -3 & 2 \end{vmatrix} = 2(2) - (-3)(1) \quad \text{or} \quad 7$

$\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

30.



31.



32. {16}, {−4, 4}; no, two y-values for one x-value

33. $\frac{w + x + y + z}{4} = 15$

$w + x + y + z = 60$

2-7 Linear Programming

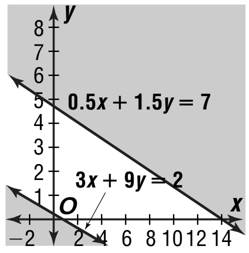
Pages 115–116 Check for Understanding

1. Sample answer: These inequalities are usually included because in real life, you cannot make less than 0 of something.

2. Sample answer: In an infeasible problem, the region defined by the constraints contains no points. An unbounded region contains an infinite number of points.

3. Sample answer: First define variables. Then write the constraints as a system of inequalities. Graph the system and find the coordinates of the vertices of the polygon formed. Then write an expression to be maximized or minimized. Finally, substitute values from the coordinates of the vertices into the expression and select the greatest or least result.

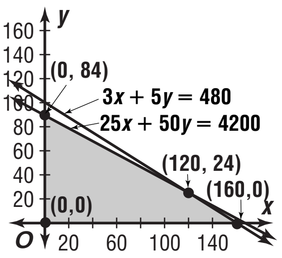
4.



5a. $25x + 50y \le 4200$

5b. $3x + 5y \le 480$

5c.



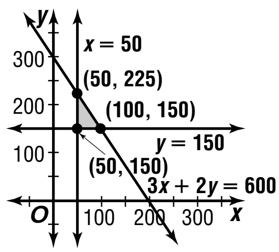
5d. $P(x, y) = 5x + 8y$

5e. $P(x, y) = 5x + 8y$
$P(0, 0) = 5(0) + 8(0)$ or 0
$P(0, 84) = 5(0) + 8(84)$ or 672
$P(120, 24) = 5(120) + 8(24)$ or 792
$P(160, 0) = 5(160) + 8(0)$ or 800
160 small packages, 0 large packages

5f. \$800

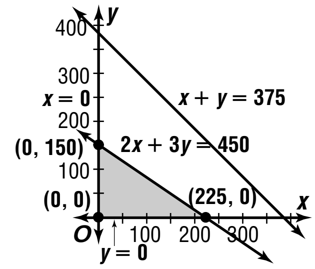
5g. No; if revenue is maximized, the company will not deliver any large packages, and customers with large packages to ship will probably choose another carrier for all of their business.

6. Let x = the number of brochures.
Let y = the number of fliers.
$3x + 2y \le 600$
$x \ge 50$
$y \ge 150$



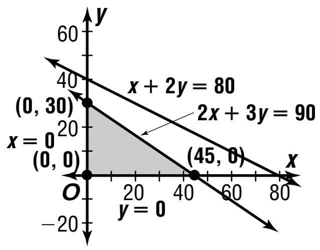
$C(x, y) = 8x + 4y$
$C(50, 150) = 8(50) + 4(150)$ or 1000¢
$C(50, 225) = 8(50) + 4(225)$ or 1300¢
$C(100, 150) = 8(100) + 4(150)$ or 1400¢
50 brochures, 150 fliers

7. Let x = the number of Explorers.
Let y = the number of Grande Expeditions.
$x + y \leq 375$
$2x + 3y \leq 450$
$x \geq 0$
$y \geq 0$



$R(x, y) = 250x + 350y$
$R(0, 0) = 250(0) + 350(0)$ or 0
$R(0, 150) = 250(0) + 350(150)$ or 52,500
$R(225, 0) = 250(225) + 350(0)$ or 56,250
225 Explorers, 0 Grande Expeditions

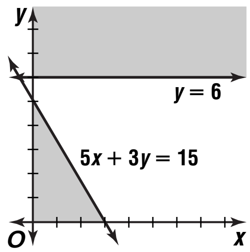
8. Let x = the number of loaves of light whole wheat.
Let y = the number of loaves of regular whole wheat.
$2x + 3y \leq 90$
$x + 2y \leq 80$
$x \geq 0$
$y \geq 0$



$P(x, y) = 1x + 1.50y$
$P(0, 0) = 1(0) + 1.50(0)$ or 0
$P(0, 30) = 1(0) + 1.50(30)$ or 45
$P(45, 0) = 1(45) + 1.50(0)$ or 45
alternate optimal solutions

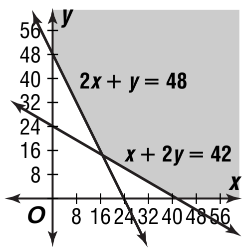
Pages 116–118 Exercises

9.



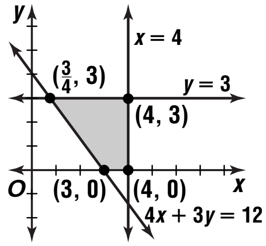
infeasible

10.



unbounded

11.

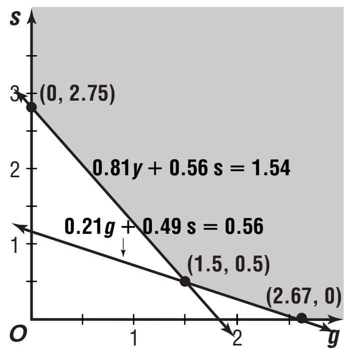


$f(x, y) = 3 + 3y$
$f\left(\frac{3}{4}, 3\right) = 3 + 3(3)$ or 12
$f(4, 3) = 3 + 3(3)$ or 12
$f(4, 0) = 3 + 3(0)$ or 3
$f(3, 0) = 3 + 3(0)$ or 3
alternate optimal solutions

12a. Let g = the number of cups of Good Start food and s = the number of cups of Sirius food.
$0.84g + 0.56s \geq 1.54$

12b. $0.21g + 0.49s \geq 0.56$

12c.



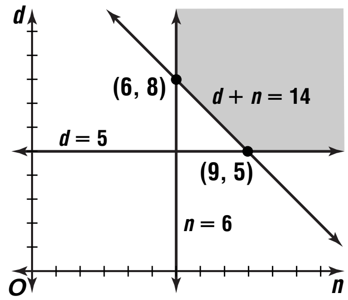
12d. $C(g, s) = 36g + 22s$

12e. $C(g, s) = 36g + 22s$
$C(0, 2.75) = 36(0) + 22(2.75)$ or 60.5
$C(1.5, 0.5) = 36(1.5) + 22(0.5)$ or 65
$C(2.66, 0) = 36(2.66) + 22(0)$ or 95.76
0 cups of *Good Start* and 2.75 cups of *Sirius*

12f. 60.5¢

13a. Let d = the number of day-shift workers and n = the number of night-shift workers.
$d \geq 5$
$n \geq 6$
$d + n \geq 14$

13b.

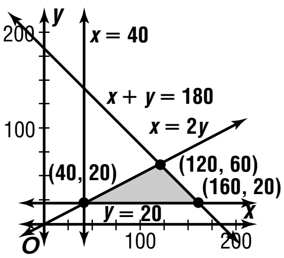


13c. $\$5.50 \cdot 4 + \$7.50 \cdot 4 = \$52$
$\$7.50 \cdot 8 = \$60$
$C(n, d) = 52d + 60n$

13d. $C(n, d) = 52d + 60n$
$C(6, 8) = 52(8) + 60(6)$ or 776
$C(9, 5) = 52(5) + 60(9)$ or 800
8 day-shift and 6 night-shift workers

13e. \$776

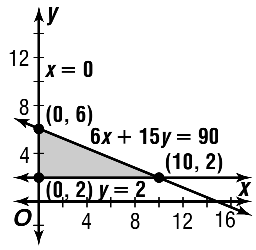
14a. Let x = the number of acres of corn.
Let y = the number of acres of soybeans.
$x + y \le 180$
$x \ge 40$
$y \ge 20$
$x \ge 2y$



$P(x, y) = 150x + 250y$
$P(40, 20) = 150(40) + 250(20)$ or 11,000
$P(120, 60) = 150(120) + 250(60)$ or 33,000
$P(160, 20) = 150(160) + 250(20)$ or 29,000
120 acres of corn, 60 acres of soybeans

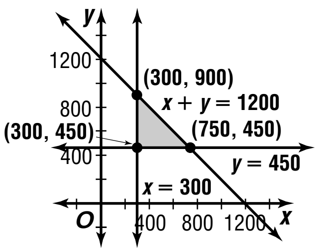
14b. \$33,000

15. Let x = the questions from section I.
Let y = the questions from section II.
$6x + 15y \le 90$
$y \ge 2$
$x \ge 0$



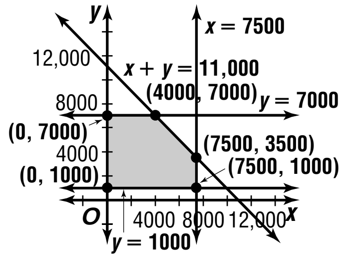
$S(x, y) = 10x + 15y$
$S(0, 2) = 10(0) + 15(2)$ or 30
$S(0, 6) = 10(0) + 15(6)$ or 90
$S(10, 2) = 10(10) + 15(2)$ or 130
10 section I questions, 2 section II questions

16. Let x = the number of food containers.
Let y = the number of drink containers.
$x + y \le 1200$
$x \ge 300$
$y \ge 450$



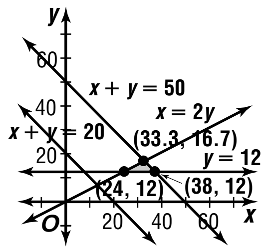
$P(x, y) = 17.50x + 20y$
$P(300, 450) = 17.50(300) + 20(450)$ or 14,250
$P(300, 900) = 17.50(300) + 20(900)$ or 23,250
$P(750, 450) = 17.50(750) + 20(450)$ or 22,125
\$23,250

17. Let x = amount to deposit at First Bank.
Let y = amount to deposit at City Bank.
$x + y \le 11{,}000$
$0 \le x \le 7500$
$1000 \le y \le 7000$



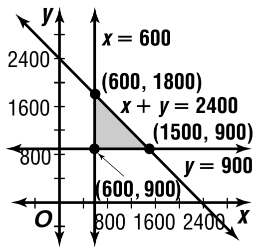
$I(x, y) = 0.06x + 0.065y$
$I(0, 1{,}000) = 0.06(0) + 0.065(1{,}000)$ or 65
$I(0, 7000) = 0.06(0) + 0.065(7{,}000)$ or 455
$I(4000, 7000) = 0.06(4000) + 0.065(7000)$ or 695
$I(7500, 3500) = 0.06(7500) + 0.065(3500)$ or 677.5
$I(7500, 1000) = 0.06(7500) + 0.065(1000)$ or 515
\$4000 in First Bank, \$7000 in City Bank

18. Let x = the number of nurses.
Let y = the number of nurse's aides.
$x + y \le 50$
$x + y \ge 20$
$y \ge 12$
$x \ge 2y$



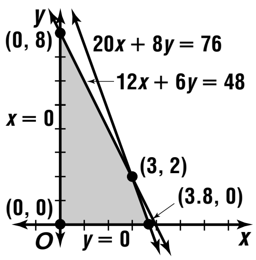
$C(x, y) = 35{,}000x + 18{,}000y$
$C(24,12) = 35{,}000(24) + 18{,}000(12)$ or 1,056,000
$C(33.3, 16.7) = 35{,}000(33.3) + 18{,}000(16.7)$ or 1,466,100
$C(38, 12) = 35{,}000(38) + 18{,}000(12)$ or 1,546,000
24 nurses, 12 nurse's aides

19. Let x = units of snack-size bags.
Let y = units of family-size bags.
$x + y \le 2400$
$x \ge 600$
$y \ge 900$



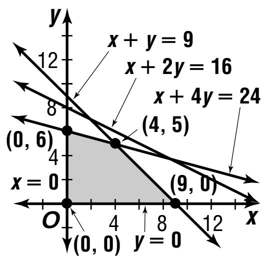
$P(x, y) = 12x + 18y$
$P(600, 900) = 12(600) + 18(900)$ or 23,400
$P(600, 1800) = 12(600) + 18(1800)$ or 39,600
$P(1500, 900) = 12(1500) + 18(900)$ or 34,200
600 units of snack-size, 1800 units of family-size

20. Let x = batches of soap.
Let y = batches of shampoo.
$12x + 6y \le 48$
$20x + 8y \le 76$
$x \ge 0$
$y \ge 0$



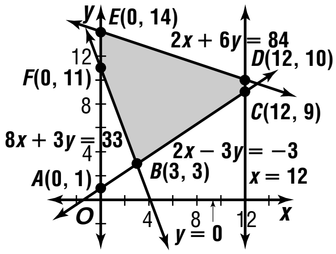
3 batches of soap and 2 batches of shampoo

21. Let x = the number of small monitors.
Let y = the number of large monitors.
$x + 2y \le 16$
$x + y \le 9$
$x + 4y \le 24$
$x \ge 0$
$y \ge 0$



$P(x, y) = 40x + 40y$
$P(0, 0) = 40(0) + 40(0)$ or 0
$P(0, 6) = 40(0) + 40(6)$ or 240
$P(4, 5) = 40(4) + 40(5)$ or 360
$P(9, 0) = 40(9) + 40(0)$ or 360
alternate optimal solutions

22.

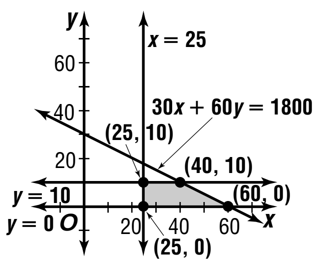


Area of trapezoid $ACDE = \frac{1}{2}(12)(13 + 1)$
$= 84$

Area of $\triangle ABF = \frac{1}{2}(10)(3)$
$= 15$

Area of shaded origin $= 84 - 15$
$= 69$ square units

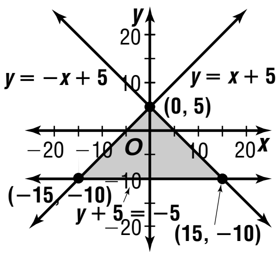
23a. Let x = oil changes.
Let y = tune-ups.
$x \ge 25$
$0 \le y \le 10$
$30x + 60y \le 30(60)$



$P(x, y) = 12x + 20y$
$P(25, 0) = 12(25) + 20(0)$ or 300
$P(25, 10) = 12(25) + 20(10)$ or 500
$P(40, 10) = 12(40) + 20(10)$ or 680
$P(60, 0) = 12(60) + 20(0)$ or 720
\$720

23b. Sample answer: Spend more than 30 hours per week on these services.

24.



$f(x, y) = \frac{1}{3}x - \frac{1}{2}y$
$f(-15, -10) = \frac{1}{3}(-15) - \frac{1}{2}(-10)$ or 0
$f(0, 5) = \frac{1}{3}(0) - \frac{1}{2}(5)$ or $-2\frac{1}{2}$
$f(15, -10) = \frac{1}{3}(15) - \frac{1}{2}(-10)$ or 10
minimum: $-2\frac{1}{2}$, maximum: 10

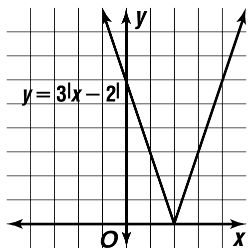
25. $4x + y = 6$
$x = 2y - 12$

$4x + y = 6$
$4(2y - 12) + y = 6$
$8y - 48 + y = 6$
$y = 6$

$x = 12y - 12$
$x = 2(6) - 12$ or 0 (0, 6)

26.

x	y
−1	9
0	6
1	3
2	0
3	3



27. Sample answer: $C = \$13.65 + \$0.15(n - 30)$;
$C = \$13.65 + \$0.15(n - 30)$
$C = \$13.65 + \$0.15(42 - 30)$
$= \$15.45$

28. $\frac{2x - 3}{x} = \frac{3 - x}{2}$
$2(2x - 3) = x(3 - x)$
$4x - 6 = 3x - x^2$
$x^2 + x - 6 = 0$
$(x + 3)(x - 2) = 0$
$x + 3 = 0$ or $x - 2 = 0$
$x = -3$ $x = 2$
The correct choice is A.

Chapter 2 Study Guide and Assessment

Page 119 Understanding and Using the Vocabulary

1. translation
2. added
3. determinant
4. inconsistent
5. scalar multiplication
6. equal matrices
7. polygonal convex set
8. reflections
9. element
10. multiplied

11.
$$\begin{aligned} 2y &= -4x \\ 2(-x-2) &= -4x \\ -2x - 4 &= -4x \\ x &= 2 \end{aligned} \qquad \begin{aligned} y &= -x-2 \\ y &= -2-2 \\ y &= -4 \end{aligned}$$
$(2, -4)$

12.
$$\begin{aligned} 6y - x &= 0 \\ 6(x-5) - x &= 0 \\ 6x - 30 - x &= 0 \\ x &= 6 \end{aligned} \qquad \begin{aligned} y &= x-5 \\ y &= 6-5 \\ y &= 1 \end{aligned}$$
$(6, 1)$

13.
$$\begin{aligned} 3y + x &= -1 \\ x &= -1 - 3y \end{aligned} \qquad \begin{aligned} 2x &= 5y \\ 2(-1-3y) &= 5y \\ -2 - 6y &= 5y \\ -\tfrac{2}{11} &= y \end{aligned}$$
$$\begin{aligned} 2x &= 5y \\ 2x &= 5\left(-\tfrac{2}{11}\right) \\ x &= -\tfrac{5}{11} \end{aligned} \qquad \left(-\tfrac{5}{11}, -\tfrac{2}{11}\right)$$

14.
$$\begin{aligned} 2y - 15x &= -4 \\ 2(6x+1) - 15x &= -4 \\ 12x + 2 - 15x &= -4 \\ x &= 2 \end{aligned} \qquad \begin{aligned} y &= 6x+1 \\ y &= 6(2)+1 \\ y &= 13 \end{aligned} \qquad (2, 13)$$

15.
$$\begin{aligned} 5(3x-2y) &= 5(-1) \\ 2(2x+5y) &= 2(12) \end{aligned} \rightarrow \begin{aligned} 15x - 10y &= -5 \\ 4x + 10y &= 24 \\ \hline 19x &= 19 \\ x &= 1 \end{aligned}$$
$$\begin{aligned} 2x + 5y &= 12 \\ 2(1) + 5y &= 12 \\ y &= 2 \end{aligned} \qquad (1, 2)$$

16.
$$\begin{aligned} x + 5y &= 20.5 \\ 3y - x &= 13.5 \end{aligned} \rightarrow \begin{aligned} x + 5y &= 20.5 \\ -x + 3y &= 13.5 \\ \hline 8y &= 34 \\ y &= 4.25 \end{aligned}$$
$$\begin{aligned} x + 5y &= 20.5 \\ x + 5(4.25) &= 20.5 \\ x &= -0.75 \end{aligned} \qquad (-0.75, 4.25)$$

17.
$$\begin{aligned} 3(x-2y-3z) &= 3(2) \\ -3x + 5y + 4z &= 0 \end{aligned} \qquad \begin{aligned} 3(x-4y+3z) &= 3(14) \\ -3x + 5y + 4z &= 0 \end{aligned}$$
$$\downarrow \qquad\qquad \downarrow$$
$$\begin{aligned} 3x - 6y - 9z &= 6 \\ -3x + 5y + 4z &= 0 \\ \hline -y - 5z &= 6 \end{aligned} \qquad \begin{aligned} 3x - 12y + 9z &= 42 \\ -3x + 5y + 4z &= 0 \\ \hline -7y + 13z &= 42 \end{aligned}$$
$$\begin{aligned} -7(-y-5z) &= -7(16) \\ -7y + 13z &= 42 \end{aligned} \rightarrow \begin{aligned} 7y + 35z &= -42 \\ -7y + 13z &= 42 \\ \hline 48z &= 0 \\ z &= 0 \end{aligned}$$
$$\begin{aligned} -y - 5z &= 6 \\ -y - 5(0) &= 6 \\ y &= -6 \end{aligned} \qquad \begin{aligned} x - 2y - 3z &= 2 \\ x - 2(-6) - 3(0) &= 2 \\ x &= -10 \end{aligned}$$
$(-10, -6, 0)$

18.
$$\begin{aligned} -x + 2y - 6z &= 4 \\ x + y + 2z &= 3 \\ \hline 3y - 4z &= 7 \end{aligned} \qquad \begin{aligned} 2(-x+2y-6z) &= 2(4) \\ 2x + 3y - 4z &= 5 \end{aligned}$$
$$\downarrow$$
$$\begin{aligned} -2x + 4y - 12z &= 8 \\ 2x + 3y - 4z &= 5 \\ \hline 7y - 16z &= 13 \end{aligned}$$
$$\begin{aligned} -4(3y-4z) &= -4(7) \\ 7y - 16z &= 13 \end{aligned} \rightarrow \begin{aligned} -12y + 16z &= -28 \\ 7y - 16z &= 13 \\ \hline -5y &= -15 \\ y &= 3 \end{aligned}$$
$$\begin{aligned} 3y - 4z &= 7 \\ 3(3) - 4z &= 7 \\ z &= 0.5 \end{aligned} \qquad \begin{aligned} x + y + 2z &= 3 \\ x + 3 + 2(0.5) &= 3 \\ x &= -1 \end{aligned}$$
$(-1, 3, 0.5)$

19.
$$\begin{aligned} x - 2y + z &= 7 \\ 3x + y - z &= 2 \\ \hline 4x - y &= 9 \end{aligned} \qquad \begin{aligned} 2(3x+y-z) &= 2(2) \\ 2x + 3y + 2z &= 7 \end{aligned}$$
$$\downarrow$$
$$\begin{aligned} 6x + 2y - 2z &= 4 \\ 2x + 3y + 2z &= 7 \\ \hline 8x + 5y &= 11 \end{aligned}$$
$$\begin{aligned} 5(4x-y) &= 5(9) \\ 8x + 5y &= 11 \end{aligned} \rightarrow \begin{aligned} 20x - 5y &= 45 \\ 8x + 5y &= 11 \\ \hline 28x &= 56 \\ x &= 2 \end{aligned}$$
$$\begin{aligned} 4x - y &= 9 \\ 4(2) - y &= 9 \\ y &= -1 \end{aligned} \qquad \begin{aligned} x - 2y + z &= 7 \\ 2 - 2(-1) + z &= 7 \\ z &= 3 \end{aligned}$$
$(2, -1, 3)$

20. $A + B = \begin{bmatrix} 7+(-3) & 8+(-5) \\ 0+2 & -4+(-2) \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 2 & -6 \end{bmatrix}$

21. $B - A = \begin{bmatrix} -3-7 & -5-8 \\ 2-0 & -2-(-4) \end{bmatrix} = \begin{bmatrix} -10 & -13 \\ 2 & 2 \end{bmatrix}$

22. $3B = \begin{bmatrix} 3(-3) & 3(-5) \\ 3(2) & 3(-2) \end{bmatrix} = \begin{bmatrix} -9 & -15 \\ 6 & -6 \end{bmatrix}$

23. $-4C = \begin{bmatrix} -4(2) \\ -4(-5) \end{bmatrix} = \begin{bmatrix} -8 \\ 20 \end{bmatrix}$

24. $AB = \begin{bmatrix} 7 & 8 \\ 0 & -4 \end{bmatrix} \cdot \begin{bmatrix} -3 & -5 \\ 2 & -2 \end{bmatrix}$
$= \begin{bmatrix} 7(-3)+8(2) & 7(-5)+8(-2) \\ 0(-3)+(-4)(2) & 0(2)+(-4)(-2) \end{bmatrix}$
or $\begin{bmatrix} -5 & -51 \\ -8 & 8 \end{bmatrix}$

25. impossible

26. $4A + 4B = 4A + (-4B)$

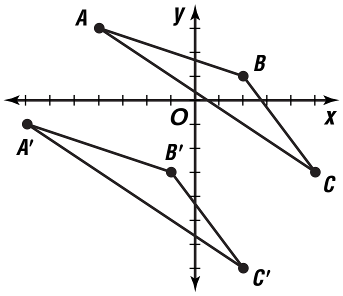
$4A = \begin{bmatrix} 4(7) & 4(8) \\ 4(0) & 4(-4) \end{bmatrix} = \begin{bmatrix} 28 & 32 \\ 0 & -16 \end{bmatrix}$ $\qquad -4B = \begin{bmatrix} -4(-3) & -4(-5) \\ -4(2) & -4(-2) \end{bmatrix} = \begin{bmatrix} 12 & 20 \\ -8 & 8 \end{bmatrix}$

$4A + (-4B) = \begin{bmatrix} 28+12 & 32+20 \\ 0+(-8) & -16+8 \end{bmatrix} = \begin{bmatrix} 40 & 52 \\ -8 & -8 \end{bmatrix}$

27. impossible

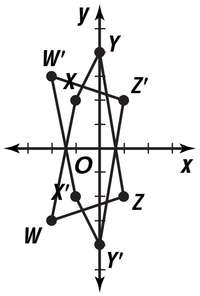
28. $\begin{bmatrix} -4 & 2 & 5 \\ 3 & 1 & -3 \end{bmatrix} + \begin{bmatrix} -3 & -3 & -3 \\ -4 & -4 & -4 \end{bmatrix} = \begin{bmatrix} -7 & -1 & 2 \\ -1 & -3 & -7 \end{bmatrix}$

$A'(-7, -1)$, $B'(-1, -3)$, $C'(2, -7)$



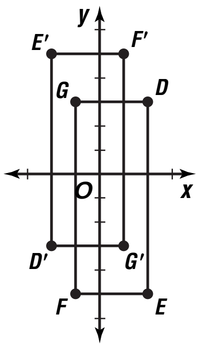
29. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -2 & -1 & 0 & 1 \\ -3 & 2 & 4 & -2 \end{bmatrix} = \begin{bmatrix} -2 & -1 & 0 & 1 \\ 3 & -2 & -4 & 2 \end{bmatrix}$

$W'(-2, 3)$, $X'(-1, -2)$, $Y'(0, -4)$, $Z'(1, 2)$



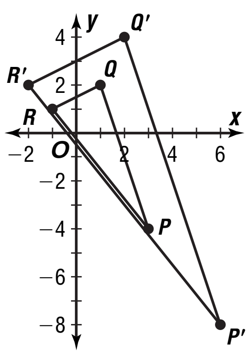
30. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2 & -1 & -1 \\ 3 & -5 & -5 & 3 \end{bmatrix} = \begin{bmatrix} -2 & -2 & 1 & 1 \\ -3 & 5 & 5 & -3 \end{bmatrix}$

$D'(-2, -3)$, $E'(-2, 5)$, $F'(1, 5)$, $G'(1, -3)$



31. $0.5\begin{bmatrix} 3 & 1 & -1 \\ -4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1.5 & 0.5 & -0.5 \\ -2 & 1 & 0.5 \end{bmatrix}$

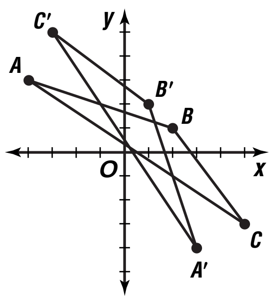
$P'(6, -8)$, $Q'(2, 4)$, $R'(-2, 2)$



32. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -4 & 2 & 5 \\ 3 & 1 & -3 \end{bmatrix} = \begin{bmatrix} -4 & 2 & 5 \\ -3 & 1 & 3 \end{bmatrix}$

$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -4 & 2 & 5 \\ -3 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & -3 \\ -4 & -2 & 5 \end{bmatrix}$

$A'(3, -4)$, $B'(1, 2)$, $C'(-3, 5)$



33. $\begin{bmatrix} -4 & -4 & -4 \\ 6 & 6 & 6 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ -3 & -3 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}$

34. $\begin{vmatrix} -3 & 5 \\ -4 & 7 \end{vmatrix} = -3(7) - (-4)(5)$ or -1

35. $\begin{vmatrix} 8 & -4 \\ -6 & 3 \end{vmatrix} = 8(3) - (-6)(-4)$ or 0

36. $\begin{vmatrix} 3 & -1 & 4 \\ 5 & -2 & 6 \\ 7 & 3 & -4 \end{vmatrix}$

$= 3\begin{vmatrix} -2 & 6 \\ 3 & -4 \end{vmatrix} - (-1)\begin{vmatrix} 5 & 6 \\ 7 & -4 \end{vmatrix} + 4\begin{vmatrix} 5 & -2 \\ 7 & 3 \end{vmatrix}$

$= 3(-10) + 1(-62) + 4(29)$

$= 24$

37. $\begin{vmatrix} 5 & 0 & -4 \\ 7 & 3 & -1 \\ 2 & -2 & 6 \end{vmatrix}$

$= 5\begin{vmatrix} 3 & -1 \\ -2 & 6 \end{vmatrix} - 0\begin{vmatrix} 7 & -1 \\ 2 & 6 \end{vmatrix} + (-4)\begin{vmatrix} 7 & 3 \\ 2 & -2 \end{vmatrix}$

$= 5(16) - 0(44) - 4(-20)$

$= 160$

38. no, not a square matrix

39. $\begin{vmatrix} 3 & 8 \\ -1 & 5 \end{vmatrix} = 3(5) - (-1)(8)$ or 23

$\frac{1}{23}\begin{bmatrix} 5 & -8 \\ 1 & 3 \end{bmatrix}$

40. $\begin{vmatrix} 5 & 2 \\ 10 & 4 \end{vmatrix} = 5(4) - 10(2)$ or 0

no inverse

41. $\begin{vmatrix} -3 & 5 \\ 1 & -4 \end{vmatrix} = -3(-4) - 1(5)$ or 7

$\frac{1}{7}\begin{bmatrix} -4 & -5 \\ -1 & -3 \end{bmatrix}$

42. $\begin{vmatrix} 3 & 2 \\ 5 & 7 \end{vmatrix} = 3(7) - 5(2)$ or 11

$\frac{1}{11}\begin{bmatrix} 7 & -2 \\ -5 & 3 \end{bmatrix}$

43. $\begin{vmatrix} 2 & -5 \\ 6 & 1 \end{vmatrix} = 2(1) - 6(-5)$ or 32

$\frac{1}{32}\begin{bmatrix} 1 & 5 \\ -6 & 2 \end{bmatrix}$

44. $\begin{vmatrix} 2 & -4 \\ -1 & 2 \end{vmatrix} = 2(2) - (-1)(-4)$ or 0

no inverse

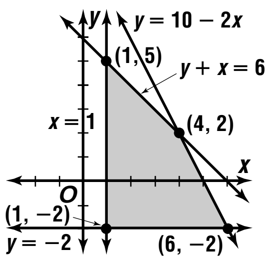
45.
$$\begin{bmatrix}2&5\\-1&-3\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}1\\2\end{bmatrix}$$
$$\frac{1}{\begin{vmatrix}2&5\\-1&-1\end{vmatrix}}\begin{bmatrix}-3&-5\\1&2\end{bmatrix}=-1\begin{bmatrix}-3&-5\\1&2\end{bmatrix}$$
$$-1\begin{bmatrix}-3&-5\\1&2\end{bmatrix}\cdot\begin{bmatrix}2&5\\-1&-3\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix}=-1\begin{bmatrix}-3&-5\\1&2\end{bmatrix}\cdot\begin{bmatrix}1\\2\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}13\\-5\end{bmatrix}$$
$(13, -5)$

46.
$$\begin{bmatrix}3&2\\-6&4\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-3\\6\end{bmatrix}$$
$$\frac{1}{\begin{vmatrix}3&2\\-6&4\end{vmatrix}}\begin{bmatrix}4&-2\\6&3\end{bmatrix}=\frac{1}{24}\begin{bmatrix}4&-2\\6&3\end{bmatrix}$$
$$\frac{1}{24}\begin{bmatrix}4&-2\\6&3\end{bmatrix}\cdot\begin{bmatrix}3&2\\-6&4\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{24}\begin{bmatrix}4&-2\\6&3\end{bmatrix}\cdot\begin{bmatrix}-3\\6\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-1\\0\end{bmatrix}$$
$(-1, 0)$

47.
$$\begin{bmatrix}-3&5\\-2&4\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}1\\-2\end{bmatrix}$$
$$\frac{1}{\begin{vmatrix}-3&5\\-2&4\end{vmatrix}}\begin{bmatrix}4&-5\\2&-3\end{bmatrix}=-\frac{1}{2}\begin{bmatrix}4&-5\\2&-3\end{bmatrix}$$
$$-\frac{1}{2}\begin{bmatrix}4&-5\\2&-3\end{bmatrix}\cdot\begin{bmatrix}-3&5\\-2&4\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix}=-\frac{1}{2}\begin{bmatrix}4&-5\\2&-3\end{bmatrix}\cdot\begin{bmatrix}1\\-2\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-7\\-4\end{bmatrix}$$
$(-7, -4)$

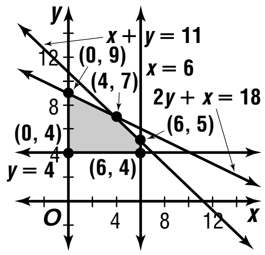
48.
$$\frac{1}{\begin{vmatrix}4.6&-2.7\\2.9&8.8\end{vmatrix}}\begin{bmatrix}8.8&2.7\\-2.9&4.6\end{bmatrix}=\frac{1}{48.31}\begin{bmatrix}8.8&2.7\\-2.9&4.6\end{bmatrix}$$
$$\frac{1}{48.31}\begin{bmatrix}8.8&2.7\\-2.9&4.6\end{bmatrix}\cdot\begin{bmatrix}4.6&-2.7\\2.9&8.8\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix}$$
$$=\frac{1}{48.31}\begin{bmatrix}8.8&2.7\\-2.9&4.6\end{bmatrix}\cdot\begin{bmatrix}8.4\\74.61\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}5.7\\6.6\end{bmatrix}$$
$(5.7, 6.6)$

49.



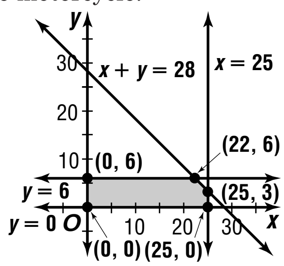
$f(x, y) = 2x + 3y$
$f(1, -2) = 2(1) + 3(-2) \text{ or } -4$
$f(1, 5) = 2(1) + 3(5) \text{ or } 17$
$f(4, 2) = 2(4) + 3(2) \text{ or } 14$
$f(6, -2) = 2(6) + 3(-2) \text{ or } 6$
17, -4

50.



$f(x, y) = 3x + 2y + 1$
$f(0, 4) = 3(0) + 2(4) + 1 \text{ or } 9$
$f(0, 9) = 3(0) + 2(9) + 1 \text{ or } 19$
$f(4, 7) = 3(4) + 2(7) + 1 \text{ or } 27$
$f(6, 5) = 3(6) + 2(5) + 1 \text{ or } 29$
$f(6, 4) = 3(6) + 2(4) + 1 \text{ or } 27$
29, 9

51. Let x = gallons in the truck.
Let y = gallons in the motorcycle.
$x + y \le 28$
$0 \le x \le 25$
$0 \le y \le 6$



$m(x, y) = 22x + 42y$
$m(0, 0) = 22(0) + 42(0) \text{ or } 0$
$m(0, 6) = 22(0) + 42(6) \text{ or } 252$
$m(22, 6) = 22(22) + 42(6) \text{ or } 746$
$m(25, 3) = 22(25) + 42(3) \text{ or } 676$
$m(25, 0) = 22(25) + 42(0) \text{ or } 550$
22 gallons in the truck and 6 gallons in the motorcycle

Page 123 Applications and Problem Solving

52.
$$\begin{bmatrix}2&5&5\\8&2&3\\6&4&1\end{bmatrix}\begin{bmatrix}5\\3\\1\end{bmatrix}=\begin{bmatrix}2(5)+5(3)+5(1)\\8(5)+2(3)+3(1)\\6(5)+4(3)+1(1)\end{bmatrix}=\begin{bmatrix}30\\49\\43\end{bmatrix}$$
Broadman 30; Girard 49; Niles 43

53. Let x = the shortest side.
Let y = the middle-length side.
Let z = the longest side.

$x + y + z = 83 \qquad\qquad x + y + z = 83$
$z = 3x \qquad\qquad x + y + 3x = 83$
$z = \frac{1}{2}(x + y) + 17 \qquad\qquad 4x + y = 83$

$z = \frac{1}{2}(x + y) + 17$
$3x = \frac{1}{2}(x + y) + 17$
$5x - y = 34$

$$\begin{bmatrix} 4 & 1 \\ 5 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 83 \\ 34 \end{bmatrix}$$

$$\frac{1}{\begin{vmatrix} 4 & 1 \\ 5 & -1 \end{vmatrix}} \begin{bmatrix} -1 & -1 \\ -5 & 4 \end{bmatrix} = -\frac{1}{9}\begin{bmatrix} -1 & -1 \\ -5 & 4 \end{bmatrix}$$

$$-\frac{1}{9}\begin{bmatrix} -1 & -1 \\ -5 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 \\ 5 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{9}\begin{bmatrix} -1 & -1 \\ -5 & 4 \end{bmatrix} \cdot \begin{bmatrix} 83 \\ 34 \end{bmatrix}$$

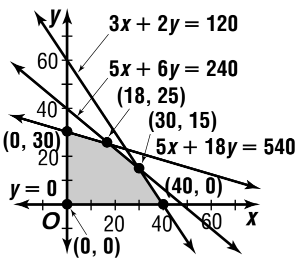
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ 31 \end{bmatrix}$$

$z = 3x$
$z = 3(13)$
$z = 39$

13 in., 31 in., 39 in.

54a. Let x = number of Voyagers.
Let y = number of Explorers.

$5x + 6y = 240$
$3x + 2y \le 120$
$5x + 18y \le 540$
$x \ge 0$
$y \ge 0$



$P(x, y) = 2.40x + 5.00y$
$P(0, 0) = 240(0) + 5.00(0)$ or 0
$P(0, 30) = 2.40(0) + 5.00(30)$ or 150
$P(18, 25) = 2.40(18) + 5.00(25)$ or 168.20
$P(30, 15) = 2.40(30) + 5.00(15)$ or 147
$P(40, 0) = 2.40(40) + 5.00(0)$ or 96

18 Voyagers and 25 Explorers

54b. \$168.20

Page 123 Open-Ended Assessment

1a. $A(-2, -2)$, $B(1, -2)$, $C(2, 1)$, and $D(-3, 0)$
Sample answer: Two consecutive 90° rotations is the same as one 180° rotation. An additional 180° rotation will return the image to its original position.

1b. Two consecutive 90° rotations is the same as one 180° rotation.

2. No; such a coefficient matrix will not have an inverse. Consider the matrix equation $\begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 24 \end{bmatrix}$. The coefficient $\begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix}$ has a determinant of 0, so it has no inverse.

Chapter 2 SAT & ACT Preparation

Page 125 SAT and ACT Practice

1. Translate the information from words into an equation. Then solve the equation for x. Use the correct order of operations.

$(1 + 2)(2 + 3)(3 + 4) = \frac{1}{2}(20 + x)$
$(3)(5)(7) = \frac{1}{2}(20 + x)$
$105 = \frac{1}{2}(20 + x)$
$210 = 20 + x$
$x = 190$

The correct choice is D.

2. First convert the numbers to improper fractions.

$5\frac{1}{3} - 6\frac{1}{4} = \frac{16}{3} - \frac{25}{4}$

Express both fractions with a common denominator. Then subtract.

$5\frac{1}{3} - 6\frac{1}{4} = \frac{16}{3} - \frac{25}{4}$
$= \frac{64}{12} - \frac{75}{12}$
$= -\frac{11}{12}$

The correct choice is A.

3. You can solve this problem by writing algebraic expressions.

Amount of root beer at start: x
Amount poured into each glass: y
Number of glasses: z
Total amount poured out: yz
Amount remaining: $x - yz$

The correct choice is D.

4. $2|x - 2| - 1 = 5$
$2|x - 2| = 6$
$|x - 2| = 3$

$x - 2 = 3$ or $x - 2 = -3$
$x = 5 \qquad x = -1$

The correct choice is D.

5. The total amount charged is \$113. Of that, \$75 is for the first 30 minutes. The rest (113 − \$75 = \$38) is the cost of the additional minutes. At \$2 per minutes, \$38 represents 19 minutes. (19 × \$2 = \$38). The plumber worked 30 minutes plus 19 minutes, for a total of 49 minutes.

The correct choice is C.

6. Start by simplifying the fraction expression on the right side of the equation.

$\frac{2+x}{5+x} = \frac{2}{5} + \frac{2}{5}$

$\frac{2+x}{5+x} = \frac{4}{5}$

To finish solving the equation, treat it as a proportion and write the cross products.

$\frac{2+x}{5+x} = \frac{4}{5}$

$5(2 + x) = 4(5 + x)$

$10 + 5x = 20 + 4x$

$x = 10$

The correct choice is E.

7. Notice that the question asks what *must* be true.

There are two ways to solve this problem. The first is by choosing specific integers that meet the criteria and finding their sums.

I. $2 + 3 = 5$, $3 + 4 = 7$
Choose consecutive integers where the first one is even and where the first one is odd. In either case, the result is odd. So statement I is true. Eliminate answer choice B.

II. $2 + 3 + 4 = 9$, $3 + 4 + 5 = 12$
One sum is odd, and the other is even. So statement II is *not* always true. Eliminate answer choices B, C, and E.

III. $2 + 3 + 4 = 9$, $3 + 4 + 5 = 12$, $10 + 11 + 12 = 33$
Statement III is true for these examples and seems to be true in general. Eliminate answer choice A.

Another method is to use algebra. Represent consecutive integers by n and $n + 1$. Represent even integers by $2k$, and odd integers by $2k + 1$.

I. $n + (n + 1) = 2n + 1$
$2n + 1$ is odd for any value of n. So statement I is always true.

II. $n + (n + 1) + (n + 2) = 3n + 3$
If n is even, then $3n + 3 = 3(2k) + 3$ is odd. If n is odd, then $3n + 3 = 3(2k + 1) + 3 = 6k + 3 + 3 = 6k + 6$ is even. So statement II is *not* always true.

III. By the same reasoning as in II, the sum is a multiple of 3, so statement III is always true.
$n + (n + 1) + (n + 2) = 3n + 3 = 3(n + 1)$
The correct choice is D.

8. Start by representing the relationships that are given in the problem. Let P represent the number of pennies; N the number of nickels; D the number of dimes; and Q the number of quarters. He has twice as many pennies as nickels.

$P = 2N$

Similarly, $N = 2D$ and $D = 2Q$. You know he has at least one quarter. Since you need to find the *least* amount of money he could have, he must have exactly one quarter.

Since he has 1 quarter, he must have 2 dimes, because $D = 2Q$. Since he has 2 dimes, he must have 4 nickels. Since he has 4 nickels, he must have 8 pennies.

Now calculate the total amount of money.

1 quarter = \$0.25
2 dimes = \$0.20
4 nickels = \$0.20
8 pennies = \$0.08

The total amount is \$0.73. The correct choice is D.

9. $\dfrac{\frac{3}{2}}{\left(\frac{3}{2}\right)^2} = \dfrac{\frac{3}{2}}{\frac{9}{4}} = \frac{3}{2} \times \frac{4}{9} = \frac{12}{18} = \frac{2}{3}$

The correct choice is C.

10. There are two products, CDs and tapes. You need to find the number of tapes sold. You also have information about the total sales and CD sales. You might want to arrange the information in a table. Let t be the number of tapes sold.

	Price each	Number Sold	Total Sales
CDs		40	\$480
Tapes		t	
Total			\$600

You can calculate the price of each CD. Since 40 CDs sold for \$480, each CD must cost \$12 (\$480 ÷ 40 = \$12). You know that the price of a CD is three times the price of a tape. So a CD costs $\frac{1}{3}$ of \$12 or \$4. You can calculate the total sales of CDs by subtracting \$480 from \$600 to get \$120.

	Price each	Number Sold	Total Sales
CDs	\$12	40	\$480
Tapes	\$4	t	\$120
Total			\$600

Now you can find t using an equation.

$4t = 120$

$t = 30$

Thirty cassette tapes were sold. The answer is 30.

Chapter 3 The Nature of Graphs

3-1 Symmetry and Coordinate Graphs

Page 133 Graphing Calculator Exploration

1. $f(-x) = f(x)$ **2.** $f(-x) = -f(x)$

3. even; odd

4. $f(x) = x^8 - 3x^4 + 2x^2 + 2$
$f(-x) = (-x)^8 - 3(-x)^4 + 2(-x)^2 + 2$
$= x^8 - 3x^4 + 2x^2 + 2$
$= f(x)$
$f(x) = x^7 + 4x^5 - x^3$
$f(-x) = (-x)^7 + 4(-x)^5 - (-x)^3$
$= -x^7 - 4x^5 + x^3$
$= -(x^7 + 4x^5 - x^3)$
$= -f(x)$

5. First find a few points of the graph in either the first or fourth quadrants. For an even function, a few other points of the graph are found by using the same y-values as those points, but with opposite x-coordinates. For an odd function, a few other points are found by using the opposite of both the x- and y-coordinates as those original points.

6. By setting the INDPNT menu option in TBLSET to ASK instead of AUTO, you can then go to TABLE and input x-values and determine their corresponding y-values on the graph. By inputting several sets of opposite pairs, you can observe whether $f(-x) = f(x)$, $f(-x) = -f(x)$, or neither of these relationships is apparent.

Pages 133–134 Check for Understanding

1. The graph of $y = -x^2 + 12$ is an even function. The graph of $xy = 6$ is an odd function. The graphs of $x = y^2 - 4$ and $17x^2 + 16xy + 17y^2 = 225$ are neither.

2. The graph of an odd function is symmetric with respect to the origin. Therefore, rotating the graph 180° will have no effect on its appearance. See student's work for example.

3a. Sample answer: $y = 0$, $x = 0$, $y = x$, $y = -x$

3b. infinitely many

3c. point symmetry about the origin

4. Substitute (a, b) into the equation. Substitute $(-b, -a)$ into the equation. Check to see whether both substitutions result in equivalent equations.

5. Alicia
Graphically: If a graph has origin symmetry, then any portion of the graph in Quadrant I has an image in Quadrant III. If the graph is then symmetric with respect to the y-axis, the portion in Quadrants I and II have reflections in Quadrants II and IV, respectively. Therefore, any piece in Quadrant I has a reflection in Quadrant IV and the same is true for Quadrants II and III. Therefore, the graph is symmetric with respect to the x-axis.
Algebraically: Substituting $(-x, -y)$ into the equation followed by substituting $(-x, y)$ is the same as substituting $(x, -y)$.

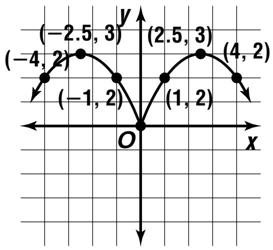
6. $f(x) = x^6 + 9x$
$f(-x) = (-x)^6 + 9(-x)$ $\quad -f(x) = -(x^6 + 9x)$
$f(-x) = x^6 - 9x$ $\quad -f(x) = -x^6 - 9x$
no

7. $f(x) = \frac{1}{5x} - x^{19}$
$f(-x) = \frac{1}{5(-x)} - (-x)^{19}$ $\quad -f(x) = -\left(\frac{1}{5x} - x^{19}\right)$
$f(-x) = -\frac{1}{5x} + x^{19}$ $\quad -f(x) = -\frac{1}{5x} + x^{19}$
yes

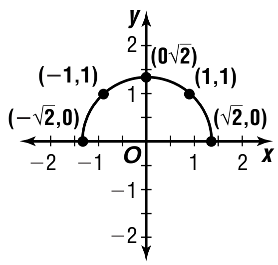
8. $6x^2 = y - 1 \quad \to \quad 6a^2 = b - 1$
x-axis $\quad 6a^2 = (-b) - 1$
$6a^2 = -b - 1$ no
y-axis $\quad 6(-a)^2 = b - 1$
$6a^2 = b - 1$ yes
$y = x$ $\quad 6(b)^2 = a - 1$
$6b^2 = a - 1$ no
$y = -x$ $\quad 6(-b)^2 = (-a) - 1$
$6b^2 = -a - 1$ no
y-axis

9. $x^3 + y^3 = 4 \quad \to \quad a^3 + b^3 = 4$
x-axis $\quad a^3 + (-b)^3 = 4$
$a^3 - b^3 = 4$ no
y-axis $\quad (-a)^3 + b^3 = 4$
$-a^3 + b^3 = 4$ no
$y = x$ $\quad (b)^3 + (a)^3 = 4$
$a^3 + b^3 = 4$ yes
$y = -x$ $\quad (-b)^3 + (-a)^3 = 4$
$-a^3 - b^3 = 4$ no
$y = x$

10.



11. $y = \sqrt{2 - x^2} \quad \to \quad b = \sqrt{2 - a^2}$
x-axis $\quad -b = \sqrt{2 - a^2}$ no
y-axis $\quad b = \sqrt{2 - (-a)^2}$
$b = \sqrt{2 - a^2}$ yes



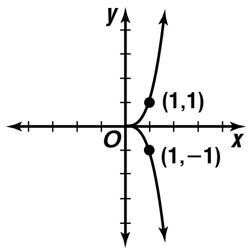
12. $|y| = x^3 \quad \rightarrow \quad |b| = a^3$

x-axis $\quad |-b| = a^3$

$|b| = a^3$ yes

y-axis $\quad |b| = (-a)^3$

$|b| = -a^3$ no



13. x-intercept:

$$\frac{x^2}{25} - \frac{y^2}{9} = 1$$
$$\frac{x^2}{25} - \frac{0^2}{9} = 1$$
$$\frac{x^2}{25} = 1$$
$$x^2 = 25$$
$$x = \pm 5 \quad (5, 0)$$

other points:

when $x = 6$

$$\frac{36}{25} - \frac{y^2}{9} = 1$$
$$-\frac{y^2}{9} = -\frac{11}{25}$$
$$y^2 = \frac{99}{25}$$
$$y = \pm\frac{3\sqrt{11}}{5}$$

when $x = -6$

$$\frac{36}{25} - \frac{y^2}{9} = 1$$
$$-\frac{y^2}{9} = -\frac{11}{25}$$
$$y^2 = \frac{99}{25}$$
$$y = \pm\frac{3\sqrt{11}}{5}$$

$\left(6, -\frac{3\sqrt{11}}{5}\right), \left(-6, -\frac{3\sqrt{11}}{5}\right), \left(-6, \frac{3\sqrt{11}}{5}\right)$

Pages 134–136 Exercises

14. $f(x) = 3x$

$f(-x) = 3(-x) \qquad -f(x) = -(3x)$

$f(-x) = -3x \qquad -f(x) = -3x$

yes

15. $f(x) = x^3 - 1$

$f(-x) = (-x)^3 - 1 \qquad -f(x) = -(x^3 - 1)$

$f(-x) = -x^3 - 1 \qquad -f(x) = -x^3 + 1$

no

16. $f(x) = 5x^2 + 6x + 9$

$f(-x) = 5(-x)^2 + 6(-x) + 9$

$f(-x) = 5x^2 - 6x + 9$

$-f(x) = -(5x^2 + 6x + 9)$

$-f(x) = -5x^2 - 6x - 9$

no

17. $f(x) = \frac{1}{4x^7}$

$f(-x) = \frac{1}{4(-x)^7} \qquad -f(x) = -\left(\frac{1}{4x^7}\right)$

$f(-x) = -\frac{1}{4x^7} \qquad -f(x) = -\frac{1}{4x^7}$

yes

18. $f(x) = -7x^5 + 8x$

$f(-x) = -7(-x)^5 + 8(-x)$

$f(-x) = 7x^5 - 8x$

$-f(x) = -(-7x^5 + 8x)$

$-f(x) = 7x^5 - 8x$

yes

19. $f(x) = \frac{1}{x} - x^{100}$

$f(-x) = \frac{1}{(-x)} - (-x)^{100} \qquad -f(x) = -\left(\frac{1}{x} - x^{100}\right)$

$f(-x) = -\frac{1}{x} - x^{100} \qquad -f(x) = -\frac{1}{x} + x^{100}$

no

20. yes;

$g(x) = \frac{x^2 - 1}{x}$

$g(-x) = \frac{(-x)^2 - 1}{(-x)}$ *Replace x with $-x$.*

$g(-x) = -\frac{x^2 - 1}{x}$ $\quad (-x)^2 = x^2$

$-g(x) = -\left(\frac{x^2 - 1}{x}\right)$ *Determine the opposite of the function.*

$g(-x) = -g(x)$

21. $xy = 5 \quad \rightarrow \quad ab = -5$

x-axis $\quad a(-b) = -5$

$-ab = -5$

$ab = 5$ no

y-axis $\quad (-a)b = -5$

$-ab = -5$

$ab = 5$ no

$y = x \quad (b)(a) = -5$

$ab = -5$ yes

$y = -x \quad (-b)(-a) = -5$

$ab = -5$ yes

$y = x$ and $y = -x$

22. $x + y^2 = 1 \quad \rightarrow \quad a + b^2 = 1$

x-axis $\quad a + (-b)^2 = 1$

$a + b^2 = 1$ yes

y-axis $\quad (-a) + b^2 = 1$

$-a + b^2 = 1$ no

$y = x \quad (b) + (a)^2 = 1$

$b + a^2 = 1$ no

$y = -x \quad (-b) + (-a)^2 = 1$

$-b + a^2 = 1$ no

x-axis

23. $y = -8x \quad \rightarrow \quad b = -8a$

x-axis $\quad (-b) = -8a$

$-b = -8a$ no

y-axis $\quad b = -8(-a)$

$b = 8a$ no

$y = x \quad (a) = -8(b)$

$a = -8b$ no

$y = -x \quad (-a) = -8(-b)$

$-a = 8b$ no

none of these

24. $y = \frac{1}{x^2}$ $\rightarrow$ $b = \frac{1}{a^2}$

x-axis $(-b) = \frac{1}{a^2}$

$-b = \frac{1}{a^2}$ no

y-axis $b = \frac{1}{(-a)^2}$

$b = \frac{1}{a^2}$ yes

$y = x$ $(a) = \frac{1}{(b)^2}$

$a = \frac{1}{b^2}$ no

$y = -x$ $(-a) = \frac{1}{(-b)^2}$

$-a = \frac{1}{b^2}$ no; y-axis

25. $x^2 + y^2 = 4$ $\rightarrow$ $a^2 + b^2 = 4$

x-axis $a^2 + (-b)^2 = 4$

$a^2 + b^2 = 4$ yes

y-axis $(-a)^2 + b^2 = 4$

$a^2 + b^2 = 4$ yes

$y = x$ $(b)^2 + (a)^2 = 4$

$a^2 + b^2 = 4$ yes

$y = -x$ $(-b)^2 + (-a)^2 = 4$

$a^2 + b^2 = 4$ yes

all

26. $y^2 = \frac{4x^2}{9} - 4$ $\rightarrow$ $b^2 = \frac{4a^2}{9} - 4$

x-axis $(-b)^2 = \frac{4a^2}{9} - 4$

$b^2 = \frac{4a^2}{9} - 4$ yes

y-axis $b^2 = \frac{4(-a)^2}{9} - 4$

$b^2 = \frac{4a^2}{9} - 4$ yes

$y = x$ $(a)^2 = \frac{4(b)^2}{9} - 4$

$a^2 = \frac{4b^2}{9} - 4$ no

$y = -x$ $(-a)^2 = \frac{4(-b)^2}{9} - 4$

$a^2 = \frac{4b^2}{9} - 4$ no

x-axis and y-axis

27. $x^2 = \frac{1}{y^2}$ $\rightarrow$ $a^2 = \frac{1}{b^2}$

x-axis $a^2 = \frac{1}{(-b)^2}$

$a^2 = \frac{1}{b^2}$ yes

y-axis $(-a)^2 = \frac{1}{b^2}$

$a^2 = \frac{1}{b^2}$ yes

$y = x$ $(b)^2 = \frac{1}{(a)^2}$

$b^2 = \frac{1}{a^2}$

$a^2 = \frac{1}{b^2}$ yes

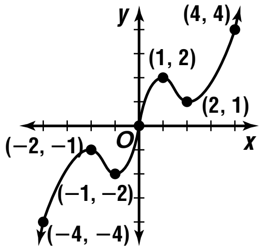
$y = -x$ $(-b)^2 = \frac{1}{(-a)^2}$

$b^2 = \frac{1}{a^2}$

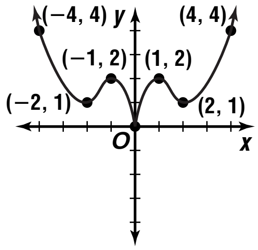
$a^2 = \frac{1}{b^2}$ yes

x-axis, y-axis, $y = x$, $y = -x$

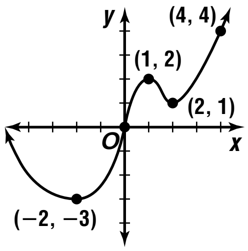
28.



29.



30. Sample answer:



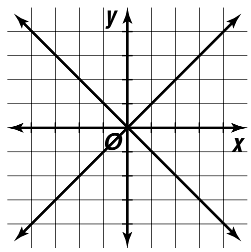
31. $y^2 = x^2$ $\rightarrow$ $b^2 = a^2$

x-axis $(-b)^2 = a^2$

$b^2 = a^2$

y-axis $b^2 = (-a)^2$

$b^2 = a^2$ yes; both



32. $|x| = -3y$ $\rightarrow$ $|a| = -3b$

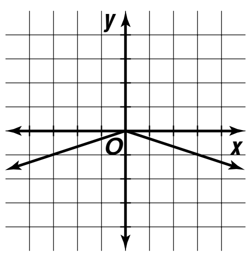
x-axis $|a| = -3(-b)$

$|a| = 3b$ no

y-axis $|(-a)| = -3b$

$|a| = -3b$ yes

y-axis



33. $y^2 + 3x = 0 \rightarrow b^2 + 3a = 0$

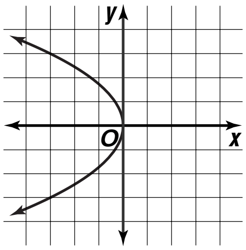
x-axis: $(-b)^2 + 3a = 0$

$b^2 + 3a = 0$ yes

y-axis: $b^2 + 3(-a) = 0$

$b^2 - 3a = 0$ no

x-axis



34. $|y| = 2x^2 \rightarrow |b| = 2a^2$

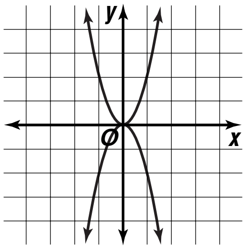
x-axis: $|(-b)| = 2a^2$

$|b| = 2a^2$ yes

y-axis: $|b| = 2(-a)^2$

$|b| = 2a^2$

yes; both



35. $x = \pm\sqrt{12 - 8y^2} \rightarrow a = \pm\sqrt{12 - 8b^2}$

x-axis: $a = \pm\sqrt{12 - 8(-b)^2}$

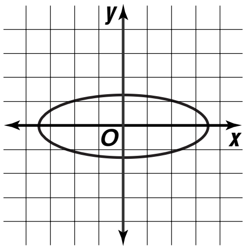
$a = \pm\sqrt{12 - 8b^2}$

yes

y-axis: $(-a) = \pm\sqrt{12 - 8b^2}$

$a = \pm\sqrt{12 - 8b^2}$

yes; both



36. $|y| = xy \rightarrow |b| = ab$

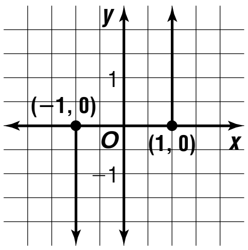
x-axis: $|(-b)| = a(-b)$

$|b| = -ab$ no

y-axis: $|b| = (-a)b$

$|b| = -ab$ no

neither



37. $|y| = x^3 - x \rightarrow |b| = a^3 - a$

x-axis: $|(-b)| = a^3 - a$

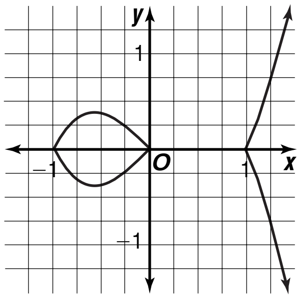
$|b| = a^3 - a$ yes

y-axis: $|b| = (-a)^3 - (-a)$

$|b| = -a^3 + a$ no

x-axis

The equation $|y| = x^3 - x$ is symmetric about the x-axis.



38a. $\frac{x^2}{8} + \frac{y^2}{10} = 1 \rightarrow \frac{a^2}{8} + \frac{b^2}{10} = 1$

x-axis: $\frac{a^2}{8} + \frac{(-b)^2}{10} = 1$

$\frac{a^2}{8} + \frac{b^2}{10} = 1$ yes

y-axis: $\frac{(-a)^2}{8} + \frac{b^2}{10} = 1$

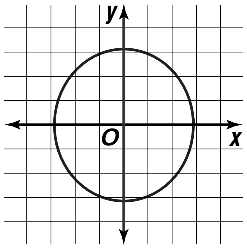
$\frac{a^2}{8} + \frac{b^2}{10} = 1$ yes

origin: $\frac{(-a)^2}{8} + \frac{(-b)^2}{10} = 1$

$\frac{a^2}{8} + \frac{b^2}{10} = 1$ yes

x- and y-axis symmetry

38b.



38c. $(2, -\sqrt{5})$, $(-2, \sqrt{5})$, $(-2, -\sqrt{5})$

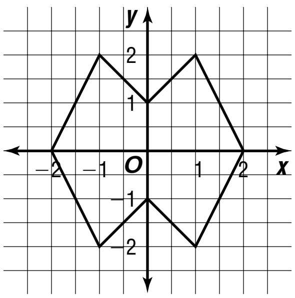
39. Sample answer: $y = 0$

40. Sample answer:

$y = x + 1$ $\quad y = -x + 1$ $\quad y = x - 1$

$y = -x - 1$ $\quad y = -2x + 4$ $\quad y = 2x + 4$

$y = -2x - 4$ $\quad y = 2x - 4$



41. $\frac{y^2}{12} - \frac{x^2}{16} = 1$

$\frac{(6)^2}{12} - \frac{x^2}{16} = 1$

$3 - \frac{x^2}{16} = 1$

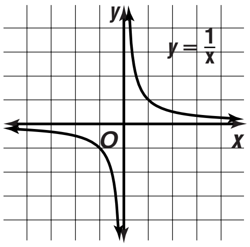
$-\frac{x^2}{16} = -2$

$x^2 = 32$

$x = \pm 4\sqrt{2}$

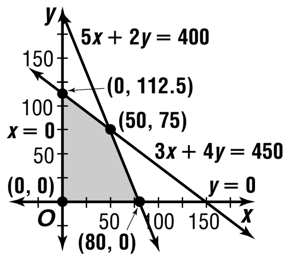
$(4\sqrt{2}, 6)$ or $(-4\sqrt{2}, 6)$

42. No; if an odd function has a y-intercept, then it must be the origin. If it were not, say it were (0, 1), then the graph would have to contain $(-1, 0)$. This would cause the relation to fail the vertical line test and would therefore not be a function. But, not all odd functions have a y-intercept. Consider the graph of $y = \frac{1}{x}$.



43. Let x = number of bicycles.
Let y = number of tricycles.

$3x + 4y \leq 450$
$5x + 2y \leq 400$
$x \geq 0$
$y \geq 0$



$P(x, y) = 6x + 4y$
$P(0, 0) = 6(0) + 4(0)$ or 0
$P(0, 112.5) = 6(0) + 4(112.5)$ or 450
$P(50, 75) = 6(50) + 4(75)$ or 600
$P(80, 0) = 6(80) + 4(0)$ or 480

50 bicycles, 75 tricycles

44. $\begin{bmatrix} 4 & 3 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 8 & 5 \\ 9 & 6 \end{bmatrix} = \begin{bmatrix} 4(8) + 3(9) & 4(5) + 3(6) \\ 7(8) + 2(9) & 7(5) + 2(6) \end{bmatrix}$

$= \begin{bmatrix} 59 & 38 \\ 74 & 47 \end{bmatrix}$

45. $3(2x + y + z) = 3(0)$ → $6x + 3y + 3z = 0$
$3x - 2y - 3z = -21$ $\underline{3x - 2y - 3z = -21}$
$9x + y = -21$

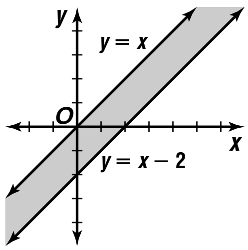
$3x - 2y - 3z = -21$
$\underline{4x + 5y + 3z = -2}$
$7x + 3y = -23$

$-3(9x + y) = -3(-21)$ → $-27x - 3y = 63$
$7x + 3y = -23$ $\underline{7x + 3y = -23}$
$-20x = 40$
$x = -2$

$9x + y = -21$ $2x + y + z = 0$
$9(-2) + y = -21$ $2(-2) + (-3) + z = 0$
$y = -3$ $z = 7$
$(-2, -3, 7)$

46. $4x - 2y = 7$ → $y = 2x - \frac{7}{2}$
$-12x + 6y = -21$ $y = 2x - \frac{7}{2}$
consistent and dependent

47.



48. $m = \frac{16 - 2}{-2 - 0}$
$= \frac{14}{-2}$ or -7

$y - 2 = -7(x - 0)$
$y = -7x + 2$

49. $[f \circ g](x) = f(g(x))$
$= f(x - 6)$
$= -2(x - 6) + 11$
$= -2x + 23$

$[g \circ f](x) = g(f(x))$
$= g(-2x + 11)$
$= (-2x + 11) - 6$
$= -2x + 5$

50. $75^3 \cdot 75^7 = 75^{3+7}$
$= 75^{10}$

The correct choice is B.

3-2 Families of Graphs

Page 142 Check for Understanding

1. $y = (x + 4)^3 - 7$

2. The graph of $y = (x + 3)^2$ is a translation of $y = x^2$ three units to the left. The graph of $y = x^2 + 3$ is a translation of $y = x^2$ three units up.

3. reflections and translations

4. When $c > 1$, the graph of $y = f(x)$ is compressed horizontally by a factor of c.
When $c = 1$, the graph of $y = f(x)$ is unchanged.
When $0 < c < 1$, the graph is expanded horizontally by a factor of $\frac{1}{c}$.

5a. $g(x) = \sqrt[3]{x} + 1$

5b. $h(x) = -\sqrt[3]{x - 1}$

5c. $k(x) = \sqrt[3]{x + 2} + 1$

6. The graph of $g(x)$ is the graph of $f(x)$ translated left 4 units.

7. The graph of $g(x)$ is the graph of $f(x)$ compressed horizontally by a factor of $\frac{1}{3}$, and then reflected over the x-axis.

8a. expanded horizontally by a factor of 5

8b. translated right 5 units and down 2 units

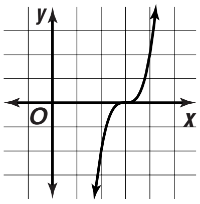
8c. expanded vertically by a factor of 3, translated up 6 units

9a. translated up 3 units, portion of graph below x-axis reflected over the x-axis

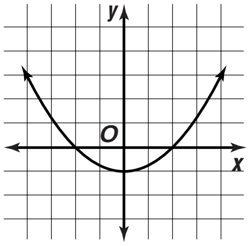
9b. reflected over the x-axis, compressed horizontally by a factor of $\frac{1}{2}$

9c. translated left 1 unit, compressed vertically by a factor of 0.75

10.

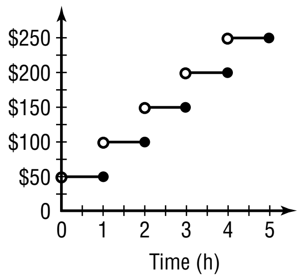


11.

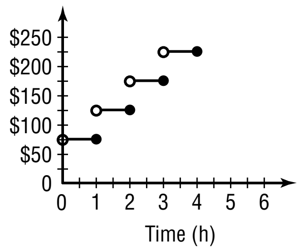


12a.

x	$f(x)$
$0 < x \leq 1$	50
$1 < x \leq 2$	100
$2 < x \leq 3$	150
$3 < x \leq 4$	200
$4 < x \leq 5$	250



12b.



12c. $225

Pages 143–145 Exercises

13. The graph of $g(x)$ is a translation of the graph of $f(x)$ up 6 units.

14. The graph of g(x) is the graph of f(x) compressed vertically by a factor of $\frac{3}{4}$.

15. The graph of $g(x)$ is the graph of $f(x)$ compressed horizontally by a factor of $\frac{1}{5}$.

16. The graph of $g(x)$ is a translation of $f(x)$ right 5 units.

17. The graph of $g(x)$ is the graph of $f(x)$ expanded vertically by a factor of 3.

18. The graph of $g(x)$ is the graph of $f(x)$ reflected over the x-axis.

19. The graph of $g(x)$ is the graph of $f(x)$ reflected over the x-axis, expanded horizontally by a factor of 2.5, and translated up 3 units.

20a. reflected over the x-axis, compressed horizontally by a factor of $0.\overline{6}$

20b. translated right 3 units, expanded vertically by a factor of 4

20c. compressed vertically by a factor of $\frac{1}{2}$, translated down 5 units

21a. expanded horizontally by a factor of 5

21b. expanded vertically by a factor of 7, translated down 0.4 units

21c. reflected across the x-axis, translated left 1 unit, expanded vertically by a factor of 9

22a. translated left 2 units and down 5 units

22b. expanded horizontally by a factor of 1.25, reflected over the x-axis

22c. compressed horizontally by a factor of $\frac{3}{5}$, translated up 2 units

23a. translated left 2 units, compressed vertically by a factor of $\frac{1}{3}$

23b. reflected over the y-axis, translated down 7 units

23c. expanded vertically by a factor of 2, translated right 3 units and up 4 units

24a. expanded horizontally by a factor of 2

24b. compressed horizontally by a factor of $\frac{1}{6}$, translated 8 units up

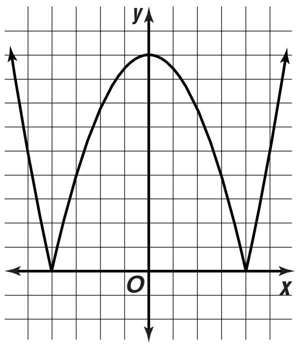
24c. The portion of parent graph on the left of the y-axis is replaced by a reflection of the portion on the right of the y-axis.

25a. compressed horizontally by a factor of $\frac{2}{5}$, translated down 3 units

25b. reflected over the y-axis, compressed vertically by a factor of 0.75

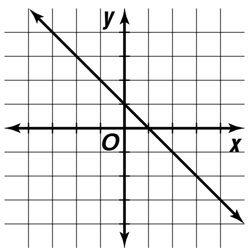
25c. The portion of the parent graph on the left of the y-axis is replaced by a reflection of the portion on the right of the y-axis. The new image is then translated 4 units right.

26. $y = x^2$

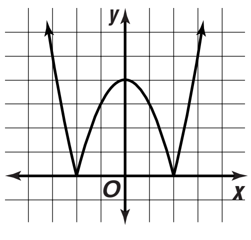


27. $y = \frac{0.25}{x - 4} + 3$

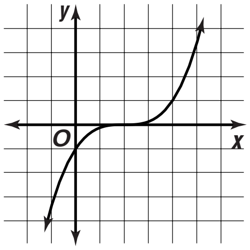
28.



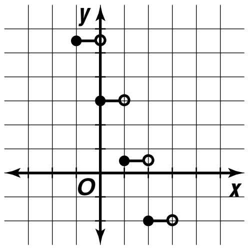
29.



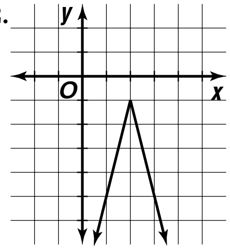
30.



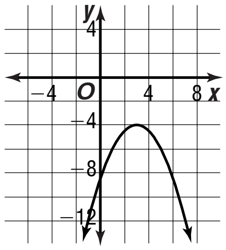
31.



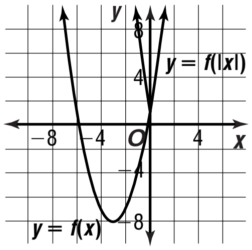
32.



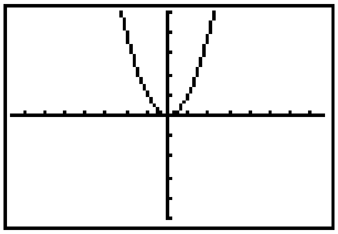
33.



34.

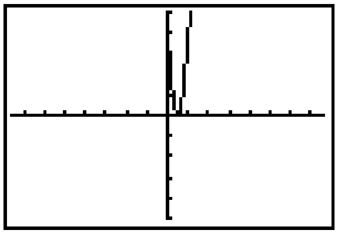


35a. 0



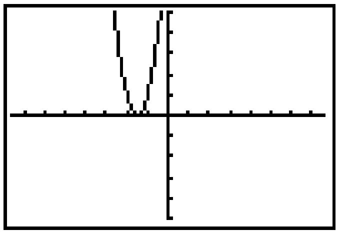
[−7.6, 7.6] scl:1 by [−5, 5] scl:1

35b. 0.5



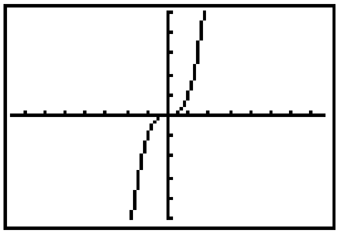
[−7.6, 7.6] scl:1 by [−5, 5] scl:1

35c. −1.5



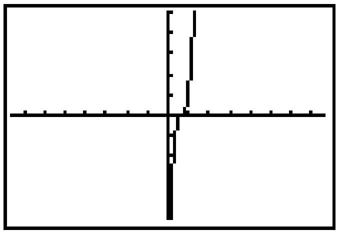
[−7.6, 7.6] scl:1 by [−5, 5] scl:1

36a. 0



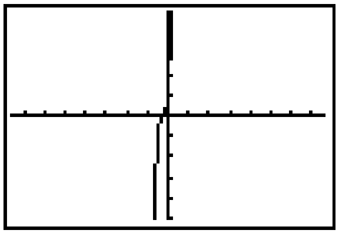
[−7.6, 7.6] scl:1 by [−5, 5] scl:1

36b. $0.6\overline{6}$



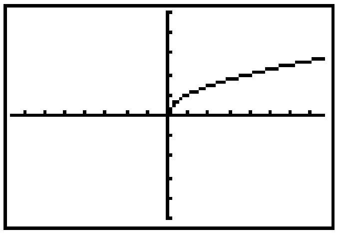
[−7.6, 7.6] scl:1 by [−5, 5] scl:1

36c. −0.25



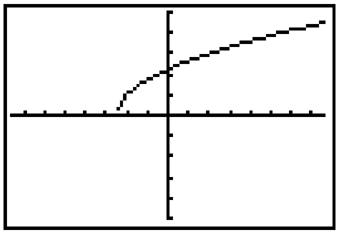
[−7.6, 7.6] scl:1 by [−5, 5] scl:1

37a. 0



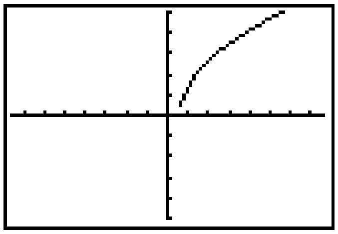
[−7.6, 7.6] scl:1 by [−5, 5] scl:1

37b. −2.5



[−7.6, 7.6] scl:1 by [−5.5] scl:1

37c. 0.6



$[-7.6, 7.6]$ scl:1 by $[-5, 5]$ scl:1

38a. The graph would continually move left 2 units and down 3 units.

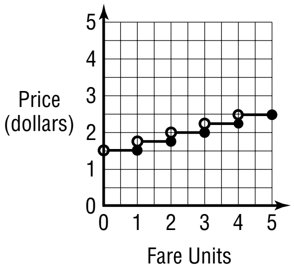
38b. The graph would continually be reflected over the x-axis and moved right 1 unit.

39. The x-intercept will be $-\frac{b}{a}$.

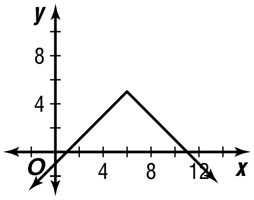
40a. $y = \begin{cases} 0.25[[x-1]] + 1.50 \text{ if } [[x]] = x \\ 0.25[[x]] + 1.50 \text{ if } [[x]] < x \end{cases}$

40b.

x	y
$0 < x \le 1$	1.50
$1 < x \le 2$	1.75
$2 < x \le 3$	2.00
$3 < x \le 4$	2.25
$4 < x \le 5$	2.50

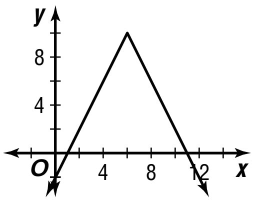


41a.

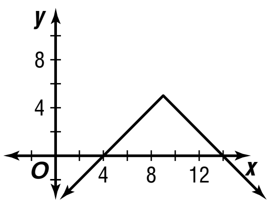


$$A = \frac{1}{2}bh = \frac{1}{2}(10)(5) = 25 \text{ units}^2$$

41b. The area of the triangle is $A = \frac{1}{2}(10)(10)$ or 50 units^2. Its area is twice as large as that of the original triangle. The area of the triangle formed by $y = c \cdot f(x)$ would be $25c$ units^2.

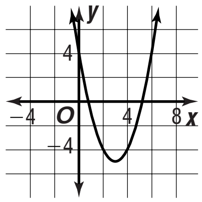


41c. The area of the triangle is $A = \frac{1}{2}(10)(5) =$ 25 units^2. Its area is the same as that of the original triangle. The area of the triangle formed by $y = f(x + c)$ would be 25 units^2.

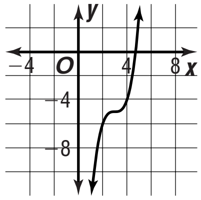


42a. (1) $y = x^2$ (2) $y = x^3$ (3) $y = -x^2$ (4) $y = -x^3$

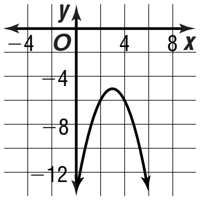
42b. (1)



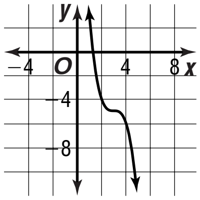
(2)



(3)



(4)



42c. (1) $y = (x-3)^2 - 5$ (2) $y = (x-3)^3 - 5$ (3) $y = -(x-3)^2 - 5$ (4) $y = -(x-3)^3 - 5$

43a. reflection over the x-axis, reflection over the y-axis, vertical translation, horizontal compression or expansion, and vertical expansion or compression

43b. horizontal translation

44. $f(x) = x^{17} - x^{15}$

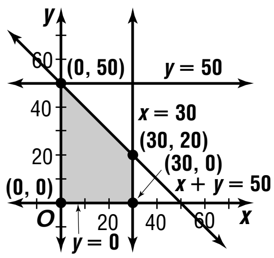
$f(-x) = (-x)^{17} - (-x)^{15}$ $\quad -f(x) = -(x^{17} - x^{15})$

$f(-x) = -x^{17} + x^{15}$ $\quad -f(x) = -x^{17} + x^{15}$

yes; $f(-x) = -f(x)$

45. Let x = number of preschoolers.
Let y = number of school-age children.

$x + y \le 50$
$x \le 3(10)$
$y \le 5(10)$
$x \ge 0$
$y \ge 0$



$I(x, y) = 18x + 6y$
$I(0, 0) = 18(0) + 6(0)$ or 0
$I(0, 50) = 18(0) + 6(50)$ or 300
$I(30, 20) = 18(30) + 6(20)$ or 660
$I(30, 0) = 18(30) + 6(0)$ or 540
30 preschoolers and 20 school-age

46. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 5 & 1 & -2 \\ -4 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -3 & 1 \\ 5 & 1 & -2 \end{bmatrix}$

$A'(4, 5)$, $B'(-3, 1)$, $C'(1, -2)$

47. $x^2 = 25$ $\quad 9 = y$ $\quad 12 = 2z$
$x = \pm 5$ $\quad\quad 6 = z$

48. $-5(6x + 5y) = -5(-14)$ $\rightarrow$ $-30x - 25y = 70$
$6(5x + 2y) = 6(-3)$ $\quad 30x + 12y = -18$
$-13y = 52$
$y = -4$

$5x + 2y = -3$
$5x + 2(-4) = -3$
$x = 1$ $\quad (1, -4)$

49. The graph implies a negative linear relationship.

50. $3x - 4y = 0 \quad \rightarrow \quad y = \frac{3}{4}x$
perpendicular slope: $-\frac{4}{3}$

51. $5d - 2p = 500 \quad \rightarrow \quad p = \frac{5}{2}d - 250$
-250

52. $[f \circ g](x) = f(g(x))$
$= f(x^2 - 6x + 9)$
$= \frac{2}{3}(x^2 - 6x + 9) - 2$
$= \frac{2}{3}x^2 - 4x + 4$
$[g \circ f](x) = g(f(x))$
$= g\left(\frac{2}{3}x - 2\right)$
$= \left(\frac{2}{3}x - 2\right)^2 - 6\left(\frac{2}{3}x - 2\right) + 9$
$= \frac{4}{9}x^2 - \frac{8}{3}x + 4 - 4x + 12 + 9$
$= \frac{4}{9}x^2 - \frac{20}{3}x + 25$

53. If $m = 1$; $d = 1 - \frac{50}{1}$ or -49.
If $m = 10$; $d = 10 - \frac{50}{10}$ or 5.
If $m = 50$; $d = 50 - \frac{50}{50}$ or 49.
If $m = 100$; $d = 100 - \frac{50}{100}$ or 99.5
If $m = 1000$; $d = 1000 - \frac{50}{1000}$ or 999.95.
The correct choice is A.

3-3 Graphs of Nonlinear Inequalities

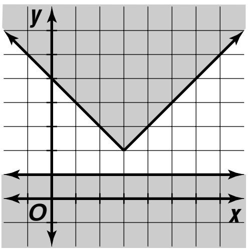
Page 149 Check for Understanding

1. A knowledge of transformations can help determine the graph of the boundary of the shaded region, $y = 5 + \sqrt{x - 2}$.

2. When solving a one variable inequality algebraically, you must consider the case where the quantity inside the absolute value is non-negative and the case where the quantity inside the absolute value is negative.

3. Sample answer: Pick a point not on the boundary of the inequality, and test to see whether it is a solution to the inequality. If that point is a solution, shade all points in that region. If it is not a solution to the inequality, test a point on the other side of the boundary and shade accordingly.

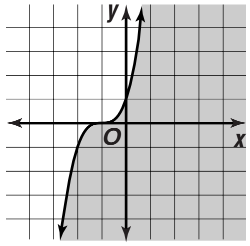
4. This inequality has no solution since the two graphs do not intersect



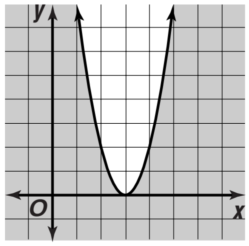
5. $y \geq -5x^4 + 7x^3 + 8$
$-3 \overset{?}{\geq} -5(-1)^4 + 7(-1)^3 + 8$
$-3 \geq -4$; yes

6. $y < |3x - 4| - 1$
$3 \overset{?}{<} |3(0) - 4| - 1$
$3 \not< 3$; no

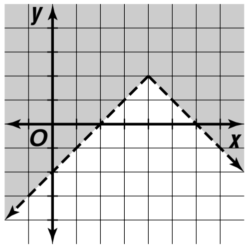
7.



8.



9.



10. Case 1
$|x + 6| > 4$
$-(x + 6) > 4$
$-x - 6 > 4$
$-x > 10$
$x < -10$

Case 2
$|x + 6| > 4$
$x + 6 > 4$
$x > -2$

$\{x \mid x < -10 \text{ or } x > -2\}$

11. Case 1
$|3x - 4| \leq x$
$-(3x - 4) \leq x$
$-3x + 4 \leq x$
$-4x \leq -4$
$x \geq 1$

Case 2
$|3x - 4| \leq x$
$3x - 4 \leq x$
$2x \leq 4$
$x \leq 2$

$\{x \mid 1 \leq x \leq 2\}$

12a. $|x - 12| \leq 0.005$

12b. Case 1
$|x - 12| \leq 0.005$
$-(x - 12) \leq 0.005$
$-x + 12 \leq 0.005$
$-x \leq -11.995$
$x \geq 11.995$

Case 2
$|x - 12| \leq 0.005$
$x - 12 \leq 0.005$
$x \leq 12.005$

12.005 cm, 11.995 cm

Pages 150–151 Exercises

13. $y < x^3 - 4x^2 + 2$
$0 \overset{?}{<} (1)^3 - 4(1)^2 + 2$
$0 \not< -1$; no

14. $y < |x - 2| + 7$
$8 \overset{?}{<} |3 - 2| + 7$
$8 \not< 8$; no

15. $y > -\sqrt{x + 11} + 1$
$-1 \overset{?}{>} -\sqrt{(-2) + 11} + 1$
$-1 > -2$; yes

16. $y < -0.2x^2 + 9x - 7$
$63 \overset{?}{<} -0.2(10)^2 + 9(10) - 7$
$63 \not< 63$; no

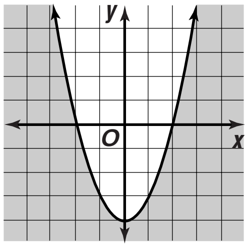
17. $y \leq \frac{x^2 - 6}{x}$
$-9 \overset{?}{\leq} \frac{(-6)^2 - 6}{-6}$
$-9 \leq -5$; yes

18. $y \geq 2|x|^3 - 7$
$0 \overset{?}{\geq} 2|0|^3 - 7$
$0 \geq -7$; yes

19. $y \le \sqrt{x} + 2$
$0 \overset{?}{\le} \sqrt{0} + 2$
$0 \le 2$; yes
$y \le \sqrt{x} + 2$
$1 \overset{?}{\le} \sqrt{1} + 2$
$1 \le 3$; yes
$y \le \sqrt{x} + 2$
$-1 \overset{?}{\le} \sqrt{1} + 2$
$-1 \le 3$; yes

$y \le \sqrt{x} + 2$
$4 \overset{?}{\le} \sqrt{1} + 2$
$4 \not\le 3$; no
$y \le \sqrt{x} + 2$
$0 \overset{?}{\le} \sqrt{-1} + 2$
$0 \not\le \boldsymbol{i} + 2$; no

(0, 0) (1, 1) and (1, −1); if these points are in the shaded region and the other points are not, then the graph is correct.

20.


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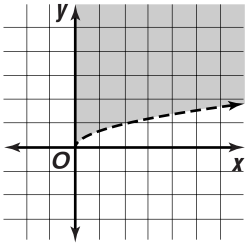

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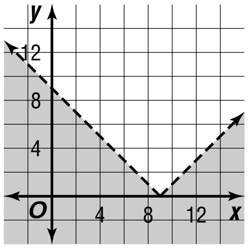

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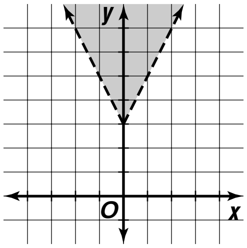

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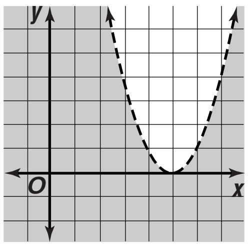

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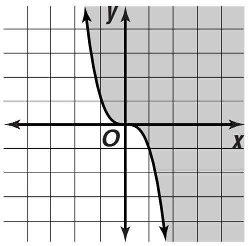

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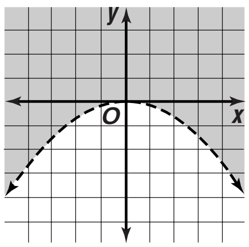

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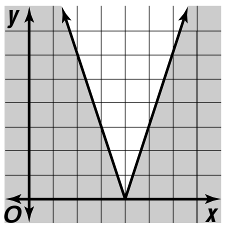

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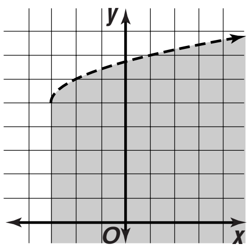

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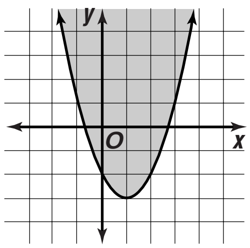

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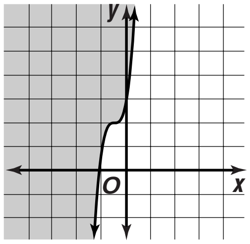

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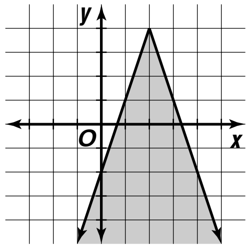

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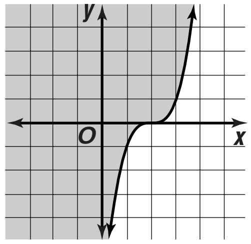


33. Case 1
$|x + 4| > 5$
$-(x + 4) > 5$
$-x - 4 > 5$
$-x > 9$
$x < -9$

Case 2
$|x + 4| > 5$
$x + 4 > 5$
$x > 1$

$\{x \mid x < -9 \text{ or } x > 1\}$

34. Case 1
$|3x + 12| \ge 42$
$-(3x + 12) \ge 42$
$-3x - 12 \ge 42$
$-3x \ge 54$
$x \le -18$

Case 2
$|3x + 12| \ge 42$
$3x + 12 \ge 42$
$3x \ge 30$
$x \ge 10$

$\{x \mid x \le -18 \text{ or } x \ge 10\}$

35. Case 1
$|7 - 2x| - 8 < 3$
$-(7 - 2x) - 8 < 3$
$-7 + 2x - 8 < 3$
$2x < 18$
$x < 9$

Case 2
$|7 - 2x| - 8 < 3$
$7 - 2x - 8 < 3$
$-2x < 4$
$x > -2$

$\{x \mid -2 < x < 9\}$

36. Case 1
$|5 - x| \le x$
$-(5 - x) \le x$
$-5 + x \le x$
$-5 \le 0$; true

Case 2
$|5 - x| \le x$
$5 - x \le x$
$-2x \le -5$
$x \ge 2.5$

$\{x \mid x > 2.5\}$

37. Case 1
$|5x - 8| < 0$
$-(5x - 8) < 0$
$-5x + 8 < 0$
$-5x < -8$
$x > \frac{8}{5}$

Case 2
$|5x - 8| < 0$
$5x - 8 < 0$
$5x < 8$
$x > \frac{8}{5}$

no solution

38. Case 1
$|2x + 9| - 2x \ge 0$
$-(2x + 9) - 2x \ge 0$
$-2x - 9 - 2x \ge 0$
$-4x \ge 9$
$x \le -\frac{9}{4}$

Case 2
$|2x + 9| - 2x \ge 0$
$2x + 9 - 2x \ge 0$
$9 \ge 0$; true

all real numbers

39. Case 1

$$\begin{aligned} -\tfrac{2}{3}|x+5| &\geq -8 \\ -\tfrac{2}{3}(-(x+5)) &\geq -8 \\ -\tfrac{2}{3}(-x-5) &\geq -8 \\ \tfrac{2}{3}x + \tfrac{10}{3} &\geq -8 \\ \tfrac{2}{3}x &\geq -\tfrac{34}{3} \\ x &\geq -17 \end{aligned}$$

Case 2

$$\begin{aligned} -\tfrac{2}{3}|x+5| &\geq -8 \\ -\tfrac{2}{3}(x+5) &\geq -8 \\ -\tfrac{2}{3}x - \tfrac{10}{3} &\geq -8 \\ -\tfrac{2}{3}x &\geq -\tfrac{14}{3} \\ x &\leq 7 \end{aligned}$$

$\{x \mid -17 \leq x \leq 7\}$

40. $|x - 37.5| \leq 1.2$

Case 1

$$\begin{aligned} |x - 37.5| &\leq 1.2 \\ -(x - 37.5) &\leq 1.2 \\ -x + 37.5 &\leq 1.2 \\ -x &\leq -36.3 \\ x &\geq 36.3 \end{aligned}$$

Case 2

$$\begin{aligned} |x - 37.5| &\leq 1.2 \\ x - 37.5 &\leq 1.2 \\ x &\leq 38.7 \end{aligned}$$

$36.3 \leq x \leq 38.7$

41. Case 1

$$\begin{aligned} 3|x-7| &< x - 1 \\ 3(-(x-7)) &< x - 1 \\ 3(-x+7) &< x - 1 \\ -3x + 21 &< x - 1 \\ -4x &< -22 \\ x &> 5.5 \end{aligned}$$

Case 2

$$\begin{aligned} 3|x-7| &< |x-1| \\ 3(x-7) &< x - 1 \\ 3x - 21 &< x - 1 \\ 2x &< 20 \\ x &< 10 \end{aligned}$$

$\{x \mid 5.5 < x < 10\}$

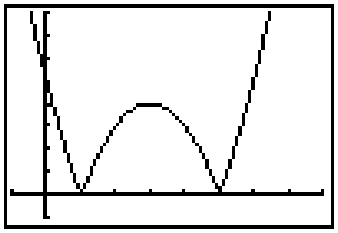
42. 30 units^2

The triangular region has vertices $A(0, 10)$, $B(3, 4)$, and $C(8, 14)$. The slope of side AB is -2. The slope of side AC is 0.5, therefore AB is perpendicular to AC. The length of side AB is $3\sqrt{5}$. The length of side AC is $4\sqrt{5}$ the area of the triangle is $0.5(3\sqrt{5})(4\sqrt{5})$ or 30.

43.

$$\begin{aligned} 0.10(90) + 0.15(75) + 0.20(76) & \\ + 0.40(80) + 0.15(x) &\geq 80 \\ 0.15x &\geq 12.55 \\ x &\geq 83\tfrac{2}{3} \end{aligned}$$

44.



$[-1, 8]$ scl:1 by $[-1, 8]$ scl:1

44a. $b < 0$

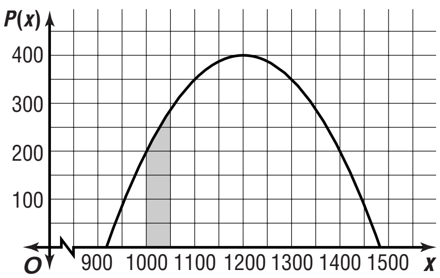
44b. none

44c. $b = 0$ or $b > 4$

44d. $b = 4$

44e. $0 < b < 4$

45a.



45b. The shaded region shows all points (x, y) where x represents the number of cookies sold and y represents the possible profit made for a given week.

46. The graph of $g(x)$ is the graph of $f(x)$ reflected over the x-axis and expanded vertically by a factor of 2.

47. $y = -\frac{1}{a^4}$ $\rightarrow$ $b = -\frac{1}{a^4}$

x-axis

$$\begin{aligned} (-b) &= -\tfrac{1}{a^4} \\ -b &= -\tfrac{1}{a^4} \quad \text{no} \end{aligned}$$

y-axis

$$\begin{aligned} b &= -\tfrac{1}{(-a)^4} \\ b &= -\tfrac{1}{a^4} \quad \text{yes} \end{aligned}$$

$y = x$

$$\begin{aligned} (a) &= -\tfrac{1}{(b)^4} \\ a &= -\tfrac{1}{(b)^4} \quad \text{no} \end{aligned}$$

$y = -x$

$$\begin{aligned} (-a) &= -\tfrac{1}{(-b)^4} \\ -a &= -\tfrac{1}{(b)^4} \quad \text{no} \end{aligned}$$

y-axis

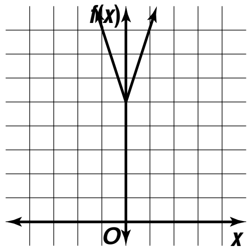
48. $\dfrac{1}{\begin{vmatrix} 8 & -3 \\ 4 & -5 \end{vmatrix}} \begin{bmatrix} -5 & 3 \\ -4 & 8 \end{bmatrix} = -\dfrac{1}{28}\begin{bmatrix} -5 & 3 \\ -4 & 8 \end{bmatrix}$

49. $\dfrac{3}{4}\begin{bmatrix} 8 & -7 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} \frac{3}{4}(8) & \frac{3}{4}(-7) \\ \frac{3}{4}(-4) & \frac{3}{4}(0) \end{bmatrix}$

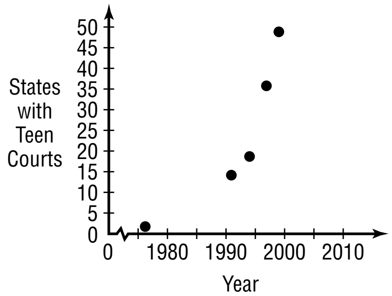
$$= \begin{bmatrix} 6 & \frac{-21}{4} \\ -3 & 0 \end{bmatrix}$$

50.

x	$f(x)$
-2	11
-1	8
0	5
1	8
2	11



51.



52.

$$\begin{aligned} [f \circ g](4) &= f(g(4)) \\ &= f(0.5(4) - 1) \\ &= f(1) \\ &= 5(1) + 9 \\ &= 14 \end{aligned}$$

$$\begin{aligned} [g \circ f](4) &= g(f(4)) \\ &= g(5(4) + 9) \\ &= g(29) \\ &= 0.5(29) - 1 \\ &= 13.5 \end{aligned}$$

53. Student A = 15

Student B = $\frac{1}{3}(15) + 15$ or 20

Let x = number of years past.

$20 - x = 2(15 - x)$

$20 - x = 30 - 2x$

$x = 10$

Page 151 Mid-Chapter Quiz

1. $x^2 + y^2 - 9 = 0 \rightarrow a^2 + b^2 - 9 = 0$

x-axis: $a^2 + (-b)^2 - 9 = 0$

$a^2 + b^2 - 9 = 0$ yes

y-axis: $(-a)^2 + b^2 - 9 = 0$

$a^2 + b^2 - 9 = 0$ yes

$y = x$: $(b)^2 + (a)^2 - 9 = 0$

$a^2 + b^2 - 9 = 0$ yes

$y = -x$: $(-b)^2 + (-a)^2 - 9 = 0$

$a^2 + b^2 - 9 = 0$ yes

$x^2 + y^2 - 9 = 0 \rightarrow f(x) = \pm\sqrt{-x^2 + 9}$

$f(-x) = \pm\sqrt{-(-x)^2 + 9}$ $\quad -f(x) = -(\pm\sqrt{-x^2 + 9})$

$f(-x) = \pm\sqrt{-x^2 + 9}$ $\quad -f(x) = \pm\sqrt{-x^2 + 9}$

yes

x-axis, y-axis, $y = x$, $y = -x$, origin

2. $5x^2 + 6x - 9 = y \rightarrow 5a^2 + 6a - 9 = b$

x-axis: $5a^2 + 6a - 9 = (-b)$

$5a^2 + 6a - 9 = -b$ no

y-axis: $5(-a)^2 + 6(-a) - 9 = b$

$5a^2 - 6a - 9 = b$ no

$y = x$: $5(b)^2 + 6(b) - 9 = (a)$

$5b^2 + 6b - 9 = a$ no

$y = -x$: $5(-b)^2 + 6(-b) - 9 = (-a)$

$5b^2 - 6b - 9 = -a$ no

$5x^2 + 6x - 9 = y \rightarrow f(x) = 5x^2 + 6x - 9$

$f(-x) = 5(-x)^2 + 6(-x) - 9$

$= 5x^2 - 6x - 9$

$-f(x) = -(5x^2 + 6x - 9)$

$-f(x) = -5x^2 - 6x + 9$ no

none of these

3. $x = \frac{7}{y} \rightarrow a = \frac{7}{b}$

x-axis: $a = \frac{7}{(-b)}$

$a = -\frac{7}{b}$ no

y-axis: $(-a) = \frac{7}{b}$

$-a = \frac{7}{b}$ no

$y = x$: $(b) = \frac{7}{(a)}$

$a = \frac{7}{b}$ yes

$y = -x$: $(-b) = \frac{7}{(-a)}$

$a = \frac{7}{b}$ yes

$x = \frac{7}{y} \rightarrow f(x) = \frac{7}{x}$

$f(-x) = \frac{7}{(-x)}$ $\quad -f(x) = -\left(\frac{7}{x}\right)$

$f(-x) = -\frac{7}{x}$ $\quad -f(x) = -\frac{7}{x}$ yes

$y = x$, $y = -x$, origin

4. $y = |x| + 1 \rightarrow b = |a| + 1$

x-axis: $(-b) = |a| + 1$

$-b = |a| + 1$ no

y-axis: $b = |(-a)| + 1$

$b = |a| + 1$ yes

$y = x$: $(a) = |(b)| + 1$

$a = |b| + 1$ no

$y = -x$: $(-a) = |(-b)| + 1$

$-a = |b| + 1$ no

$y = |x| + 1 \rightarrow f(x) = |x| + 1$

$f(-x) = |-x| + 1$ $\quad -f(x) = -(|x| + 1)$

$f(-x) = |x| + 1$ $\quad -f(x) = -|x| - 1$ no

y-axis

5a. translated down 2 units

5b. reflected over the x-axis, translated right 3 units

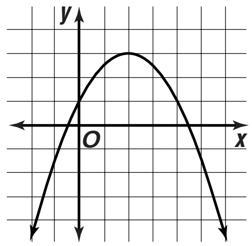
5c. compressed vertically by a factor of $\frac{1}{4}$, translated up 1 unit

6a. expanded vertically by a factor of 3

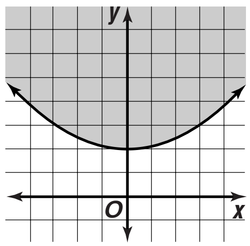
6b. expanded horizontally by a factor of 2 and translated down 1 unit

6c. translated left 1 unit and up 4 units

7.



8.



9. Case 1

$|2x - 7| < 15$

$-(2x - 7) < 15$

$-2x + 7 < 15$

$-2x < 8$

$x > -4$

$-4 < x < 11$

Case 2

$|2x - 7| < 15$

$2x - 7 < 15$

$2x < 22$

$x < 11$

10. $|x - 64| < 3$

Case 1

$|x - 64| < 3$

$-(x - 64) < 3$

$-x + 64 < 3$

$-x < -61$

$x > 61$

$61 < x < 67$

Case 2

$|x - 64| < 3$

$x - 64 < 3$

$x < 67$

3-4 Inverse Functions and Relations

Pages 155–156 Check for Understanding

1. Sample answer: First, let $y = f(x)$. Then interchange x and y. Finally, solve the resulting equation for y.

2. n is odd

3. Sample answer: $f(x) = x^2$

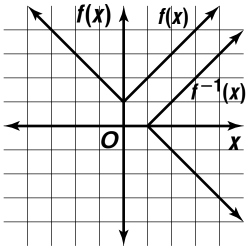
4. Sample answer: If you draw a horizontal line through the graph of the function and it intersects the graph more than once, then the inverse is not a function.

5. She is wrong. The inverse is $f^{-1}(x) = (x - 3)^2 - 2$, which is a function.

6.

$f(x) = \lvert x \rvert + 1$	
x	$f(x)$
-2	3
-1	2
0	1
1	2
2	3

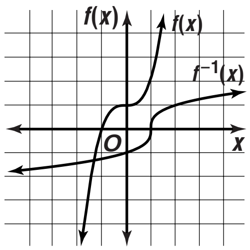
$f^{-1}(x)$	
x	$f^{-1}(x)$
3	-2
2	-1
1	0
2	1
3	2



7.

$f(x) = x^3 + 1$	
x	$f(x)$
-2	-7
-1	0
0	1
1	2
2	9

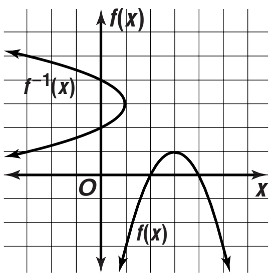
$f^{-1}(x)$	
x	$f^{-1}(x)$
-7	-2
0	-1
1	0
2	1
9	2



8.

$f(x) = -(x - 3)^2 + 1$	
x	$f(x)$
1	-3
2	0
3	1
4	0
5	-3

$f^{-1}(x)$	
x	$f^{-1}(x)$
-3	1
0	2
1	3
0	4
-3	5

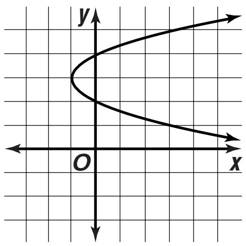


9.
$$f(x) = -3x + 2$$
$$y = -3x + 2$$
$$x = -3y + 2$$
$$x - 2 = -3y$$
$$y = -\tfrac{1}{3}x + \tfrac{2}{3}$$
$f^{-1}(x) = -\frac{1}{3}x + \frac{2}{3}$; $f^{-1}(x)$ is a function.

10.
$$f(x) = \frac{1}{x^3}$$
$$y = \frac{1}{x^3}$$
$$x = \frac{1}{y^3}$$
$$y^3 = \frac{1}{x}$$
$$y = \sqrt[3]{\frac{1}{x}} \text{ or } \frac{1}{\sqrt[3]{x}}$$
$f^{-1}(x) = \frac{1}{\sqrt[3]{x}}$; $f^{-1}(x)$ is a function.

11.
$$f(x) = (x + 2)^2 + 6$$
$$y = (x + 2)^2 + 6$$
$$x = (y + 2)^2 + 6$$
$$x - 6 = (y + 2)^2$$
$$\pm\sqrt{x - 6} = y + 2$$
$$y = -2 \pm \sqrt{x - 6}$$
$f^{-1}(x) = -2 \pm \sqrt{x - 6}$; $f^{-1}(x)$ is not a function.

12. Reflect the graph of $y = x^2$ over the line $y = x$. Then, translate the new graph 1 unit to the left and up 3 units.



13.
$$f(x) = \tfrac{1}{2}x - 5$$
$$y = \tfrac{1}{2}x - 5$$
$$x = \tfrac{1}{2}y - 5$$
$$x + 5 = \tfrac{1}{2}y$$
$$y = 2x + 10$$
$$f^{-1}(x) = 2x + 10$$
$$[f \circ f^{-1}](x) = f(2x + 10)$$
$$= \tfrac{1}{2}(2x + 10) - 5$$
$$= x$$
$$[f^{-1} \circ f](x) = f^{-1}\left(\tfrac{1}{2}x - 5\right)$$
$$= 2\left(\tfrac{1}{2}x - 5\right) + 10$$
$$= x$$
Since $[f \circ f^{-1}](x) = [f^{-1} \circ f](x) = x$, f and f^{-1} are inverse functions.

14a.
$$B(r) = 1000(1 + r)^3$$
$$B = 1000(1 + r)^3$$
$$\frac{B}{1000} = (1 + r)^3$$
$$\sqrt[3]{\frac{B}{1000}} = 1 + r$$
$$r = -1 + \frac{\sqrt[3]{B}}{10}$$

14b. $r = -1 + \frac{\sqrt[3]{B}}{10}$

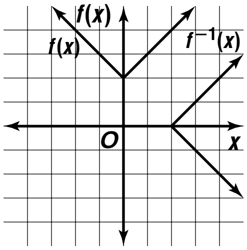
$= -1 + \frac{\sqrt[3]{1100}}{10}$ or 0.0323; 3.23%

Pages 156–158 Exercises

15.

$f(x) = \lvert x \rvert + 2$	
x	$f(x)$
−2	4
−1	3
0	2
1	3
2	4

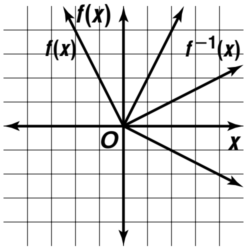
$f^{-1}(x)$	
x	$f^{-1}(x)$
4	−2
3	−1
2	0
3	1
4	2



16.

$f(x) = \lvert 2x \rvert$	
x	$f(x)$
−2	4
−1	2
0	0
1	2
2	4

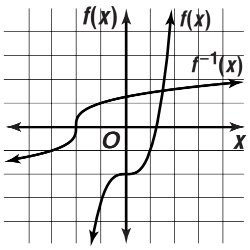
$f^{-1}(x)$	
x	$f^{-1}(x)$
4	−2
2	−1
0	0
2	1
4	2



17.

$f(x) = x^3 - 2$	
x	$f(x)$
−2	−10
−1	−3
0	−2
1	−1
2	6

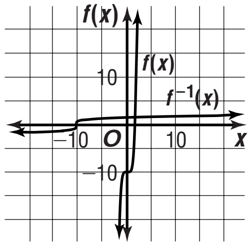
$f^{-1}(x)$	
x	$f^{-1}(x)$
−10	−2
−3	−1
−2	0
−1	1
6	2



18.

$f(x) = x^5 - 10$	
x	$f(x)$
−2	−42
−1	−11
0	−10
1	−9
2	22

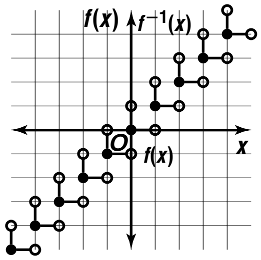
$f^{-1}(x)$	
x	$f^{-1}(x)$
−42	−2
−11	−1
−10	0
−9	1
22	2



19.

$f(x) = [x]$	
x	$f(x)$
$-2 \le x < -1$	−2
$-1 \le x < 0$	−1
$0 \le x < 1$	0
$1 \le x < 2$	1
$2 \le x < 3$	2

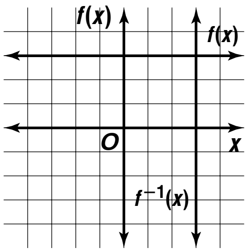
$f^{-1}(x)$	
x	$f^{-1}(x)$
−2	$-2 \le x < -1$
−1	$-1 \le x < 0$
0	$0 < x < 1$
1	$1 \le x < 2$
2	$2 \le x < 3$



20.

$f(x) = 3$	
x	$f(x)$
−2	3
−1	3
0	3
1	3
2	3

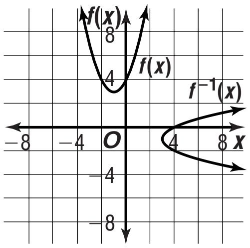
$f^{-1}(x)$	
x	$f^{-1}(x)$
3	−2
3	−1
3	0
3	1
3	2



21.

$f(x) = x^2 + 2x + 4$	
x	$f(x)$
-3	7
-2	4
-1	3
0	4
1	7

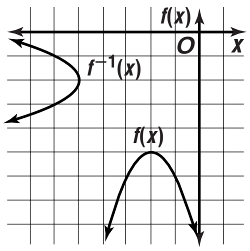
$f^{-1}(x)$	
x	$f^{-1}(x)$
7	-3
4	-2
3	-1
4	0
7	1



22.

$f(x) = -(x + 2)^2 - 5$	
x	$f(x)$
-4	-9
-3	-6
-2	-5
-1	-6
0	-9

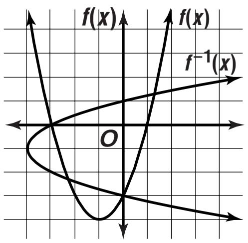
$f^{-1}(x)$	
x	$f^{-1}(x)$
-9	-4
-6	-3
-5	-2
-6	-1
-9	0



23.

$f(x) = (x + 1)^2 - 4$	
x	$f(x)$
-4	5
-2	-3
-1	-4
0	-3
2	5

$f^{-1}(x)$	
x	$f^{-1}(x)$
5	-4
-3	-2
-4	-1
-3	0
5	2



24. $f(x) = x^2 + 4$

$y = x^2 + 4$

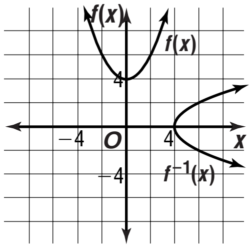
$x = y^2 + 4$

$x - 4 = y^2$

$y = \pm\sqrt{x - 4}$; $f^{-1}(x) = \pm\sqrt{x - 4}$

$f(x) = x^2 + 4$	
x	$f(x)$
-2	8
-1	5
0	4
1	5
2	8

$f^{-1}(x) = \pm\sqrt{x - 4}$	
x	$f^{-1}(x)$
8	± 2
5	± 1
4	0



25. $f(x) = 2x + 7$

$y = 2x + 7$

$x = 2y + 7$

$x - 7 = 2y$

$y = \frac{x - 7}{2}$

$f^{-1}(x) = \frac{x - 7}{2}$; $f^{-1}(x)$ is a function.

26. $f(x) = -x - 2$

$y = -x - 2$

$x = -y - 2$

$y = -x - 2$

$f^{-1}(x) = -x - 2$; $f^{-1}(x)$ is a function.

27. $f(x) = \frac{1}{x}$

$y = \frac{1}{x}$

$x = \frac{1}{y}$

$y = \frac{1}{x}$

$f^{-1}(x) = \frac{1}{x}$; $f^{-1}(x)$ is a function.

28. $f(x) = -\frac{1}{x^2}$

$y = -\frac{1}{x^2}$

$x = -\frac{1}{y^2}$

$y^2 = -\frac{1}{x}$

$y = \pm\sqrt{-\frac{1}{x}}$

$f^{-1}(x) = \pm\sqrt{-\frac{1}{x}}$; $f^{-1}(x)$ is not a function.

29. $f(x) = (x - 3)^2 + 7$

$y = (x - 3)^2 + 7$

$x = (y - 3)^2 + 7$

$x - 7 = (y - 3)^2$

$\pm\sqrt{x - 7} = y - 3$

$y = 3 \pm \sqrt{x - 7}$

$f^{-1}(x) = 3 \pm \sqrt{x - 7}$; $f^{-1}(x)$ is not a function.

30. $$\begin{aligned} f(x) &= x^2 - 4x + 3 \\ y &= x^2 - 4x + 3 \\ x &= y^2 - 4y + 3 \\ x + 1 &= y^2 - 4y + 4 \\ x + 1 &= (y - 2)^2 \\ \pm\sqrt{x + 1} &= y - 2 \\ y &= 2 \pm \sqrt{x + 1} \\ f^{-1}(x) &= 2 \pm \sqrt{x + 1};\ f^{-1}(x) \text{ is not a function.} \end{aligned}$$

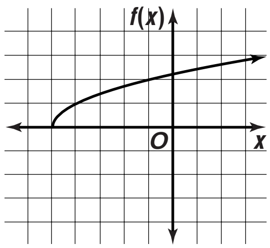
31. $$\begin{aligned} f(x) &= \frac{1}{x + 2} \\ y &= \frac{1}{x + 2} \\ x &= \frac{1}{y + 2} \\ y + 2 &= \frac{1}{x} \\ y &= \frac{1}{x} - 2 \\ f^{-1}(x) &= \frac{1}{x} - 2;\ f^{-1}(x) \text{ is not a function.} \end{aligned}$$

32. $$\begin{aligned} f(x) &= \frac{1}{(x - 1)^2} \\ y &= \frac{1}{(x - 1)^2} \\ x &= \frac{1}{(y - 1)^2} \\ (y - 1)^2 &= \frac{1}{x} \\ y - 1 &= \pm\sqrt{\frac{1}{x}} \\ y &= 1 \pm \frac{1}{\sqrt{x}} \\ f^{-1}(x) &= 1 \pm \frac{1}{\sqrt{x}};\ f^{-1}(x) \text{ is not a function.} \end{aligned}$$

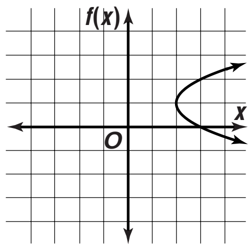
33. $$\begin{aligned} f(x) &= -\frac{2}{(x - 2)^3} \\ y &= -\frac{2}{(x - 2)^3} \\ x &= -\frac{2}{(y - 2)^3} \\ (y - 2)^3 &= -\frac{2}{x} \\ y - 2 &= -\sqrt[3]{\frac{2}{x}} \\ y &= 2 - \sqrt[3]{\frac{2}{x}} \\ f^{-1}(x) &= 2 - \sqrt[3]{\frac{2}{x}};\ f^{-1}(x) \text{ is not a function.} \end{aligned}$$

34. $$\begin{aligned} g(x) &= \frac{3}{x^2 + 2x} \\ y &= \frac{3}{x^2 + 2x} \\ x &= \frac{3}{y^2 + 2y} \\ y^2 + 2y &= \frac{3}{x} \\ y^2 + 2y + 1 &= \frac{3}{x} + 1 \\ (y + 1)^2 &= \frac{3}{x} + 1 \\ y + 1 &= \pm\sqrt{\frac{3}{x} + 1} \\ y &= -1 \pm \sqrt{\frac{3}{x} + 1} \\ g^{-1}(x) &= -1 \pm \sqrt{\frac{3}{x} + 1} \end{aligned}$$

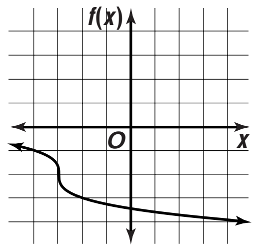
35. Reflect the part of the graph of x^2 that lies in the first quadrant about $y = x$. Then, translate 5 units to the left.



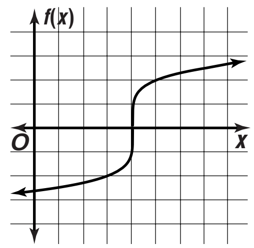
36. Reflect the graph of x^2 about $y = x$. Then, translate 2 units to the right and up 1 unit.



37. Reflect the graph of x^3 about $y = x$ to obtain the graph of $\sqrt[3]{x}$. Reflect the graph of $\sqrt[3]{x}$ about the x-axis. Then, translate 3 units to the left and down 2 units.



38. Reflect the graph of x^5 about $y = x$. Then, translate 4 units to the right. Finally, stretch the translated graph vertically by a factor of 2.



39. $f(x) = -\frac{2}{3}x + \frac{1}{6}$

$$\begin{aligned}
y &= -\tfrac{2}{3}x + \tfrac{1}{6} \\
x &= -\tfrac{2}{3}y + \tfrac{1}{6} \\
x - \tfrac{1}{6} &= -\tfrac{2}{3}y \\
y &= -\tfrac{3}{2}x + \tfrac{1}{4} \\
f^{-1}(x) &= -\tfrac{3}{2}x + \tfrac{1}{4}
\end{aligned}$$

$$\begin{aligned}
[f \circ f^{-1}](x) &= f\left(-\tfrac{3}{2}x + \tfrac{1}{4}\right) \\
&= -\tfrac{2}{3}\left(-\tfrac{3}{2}x + \tfrac{1}{4}\right) + \tfrac{1}{6} \\
&= x - \tfrac{1}{6} + \tfrac{1}{6} \\
&= x
\end{aligned}$$

$$\begin{aligned}
[f^{-1} \circ f](x) &= f^{-1}\left(-\tfrac{2}{3}x + \tfrac{1}{6}\right) \\
&= -\tfrac{3}{2}\left(-\tfrac{2}{3}x + \tfrac{1}{6}\right) + \tfrac{1}{4} \\
&= x - \tfrac{1}{4} + \tfrac{1}{4} \\
&= x
\end{aligned}$$

Since $[f \circ f^{-1}](x) = [f^{-1} \circ f](x) = x$, f and f^{-1} are inverse functions.

40. $f(x) = (x - 3)^3 + 4$

$$\begin{aligned}
y &= (x - 3)^3 + 4 \\
x &= (y - 3)^3 + 4 \\
x - 4 &= (y - 3)^3 \\
\sqrt[3]{x - 4} &= y - 3 \\
y &= 3 + \sqrt[3]{x - 4} \\
f^{-1}(x) &= 3 + \sqrt[3]{x - 4}
\end{aligned}$$

$$\begin{aligned}
[f \circ f^{-1}](x) &= f(3 + \sqrt[3]{x - 4}) \\
&= [(3 + \sqrt[3]{x - 4}) - 3]^3 + 4 \\
&= x - 4 + 4 \\
&= x
\end{aligned}$$

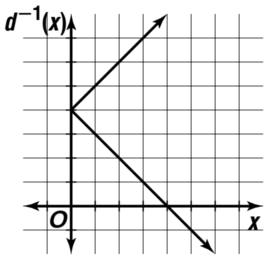
$$\begin{aligned}
[f^{-1} \circ f](x) &= f^{-1}[(x - 3)^3 + 4] \\
&= 3 + \sqrt[3]{[(x - 3)^3 + 4] - 4} \\
&= 3 + x - 3 \\
&= x
\end{aligned}$$

Since $[f \circ f^{-1}](x) = [f^{-1} \circ f](x) = x$, f and f^{-1} are inverse functions.

41a.

$d(x) = \lvert x - 4 \rvert$	
x	$d(x)$
6	2
5	1
4	0
3	1
2	2

$d^{-1}(x)$	
x	$d^{-1}(x)$
2	6
1	5
0	4
1	3
2	2



41b. No; the graph of $d(x)$ fails the horizontal line test.

41c. $d^{-1}(x)$ gives the numbers that are 4 units from x on the number line. There are always two such numbers, so d^{-1} associates two values with each x-value. Hence, $d^{-1}(x)$ is not a function.

42a.

$$\begin{aligned}
v &= \sqrt{2gh} \\
v^2 &= 2gh \\
h &= \frac{v^2}{2g} \\
h &= \frac{v^2}{2(32)} \\
h &= \frac{v^2}{64}
\end{aligned}$$

42b.

$$\begin{aligned}
h &= \frac{v^2}{64} \\
h &= \frac{(75)^2}{64} \\
h &= 87.89
\end{aligned}$$

Yes. The pump can propel water to a height of about 88 ft.

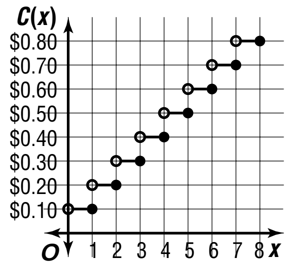
43a. Sample answer: $y = -x$

43b. The graph of the function must be symmetric about the line $y = x$.

43c. Yes, because the line $y = x$ is the axis of symmetry *and* the reflection line.

44a.

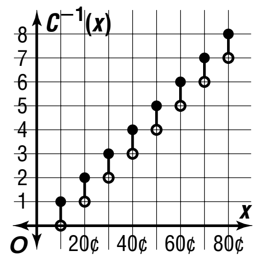
x	$C(x)$
$0 < x \le 1$	\$0.10
$1 < x \le 2$	\$0.20
$2 < x \le 3$	\$0.30
$3 < x \le 4$	\$0.40
$4 < x \le 5$	\$0.50



44b. positive real numbers; positive multiples of 10

44c.

x	$C^{-1}(x)$
\$0.10	$0 < x \le 1$
\$0.20	$1 < x \le 2$
\$0.30	$2 < x \le 3$
\$0.40	$3 < x \le 4$
\$0.50	$4 < x \le 5$



44d. positive multiples of 10; positive real numbers

44e. $C^{-1}(x)$ gives the possible lengths of phone calls that cost x.

45. It must be translated up 6 units and 5 units to the left; $y = (x - 6)^2 - 5$; $y = 6 \pm \sqrt{x + 5}$.

46a.

$$\begin{aligned}
KE &= \tfrac{1}{2}mv^2 \\
2KE &= mv^2 \\
\frac{2KE}{m} &= v^2 \\
v &= \pm\sqrt{\frac{2KE}{m}}
\end{aligned}$$

46b.

$$\begin{aligned}
v &= \pm\sqrt{\frac{2KE}{m}} \\
v &= \pm\sqrt{\frac{2(15)}{1}} \\
v &\approx \pm 5.477;\ \pm 5.5 \text{ m/sec}
\end{aligned}$$

46c. There are always two velocities.

47a. Yes; if the encoded message is not unique, it may not be decoded properly.

47b. The inverse of the encoding function must be a function so that the encoded message may be decoded.

47c.

$$\begin{aligned}
C(x) &= -2 + \sqrt{x + 3} \\
y &= -2 + \sqrt{x + 3} \\
x &= -2 + \sqrt{y + 3} \\
x + 2 &= \sqrt{y + 3} \\
(x + 2)^2 &= y + 3 \\
y &= (x + 2)^2 - 3 \\
C^{-1}(x) &= (x + 2)^2 - 3
\end{aligned}$$

47d. $C^{-1}(x) = (x + 2)^2 - 3$
$C^{-1}(1) = (1 + 2)^2 - 3$ or 6, F
$C^{-1}(2.899) = (2.899 + 2)^2 - 3$ or 21, U
$C^{-1}(2.123) = (2.123 + 2)^2 - 3$ or 14, N
$C^{-1}(0.449) = (0.449 + 2)^2 - 3$ or 3, C
$C^{-1}(2.796) = (2.796 + 2)^2 - 3$ or 20, T
$C^{-1}(1.464) = (1.464 + 2)^2 - 3$ or 9, I
$C^{-1}(2.243) = (2.243 + 2)^2 - 3$ or 15, O
$C^{-1}(2.123) = (2.123 + 2)^2 - 3$ or 14, N
$C^{-1}(2.690) = (2.690 + 2)^2 - 3$ or 19, S
$C^{-1}(0) = (0 + 2)^2 - 3$ or 1, A
$C^{-1}(2.583) = (2.583 + 2)^2 - 3$ or 18, R
$C^{-1}(0.828) = (0.828 + 2)^2 - 3$ or 5, E
$C^{-1}(1) = (1 + 2)^2 - 3$ or 6, F
$C^{-1}(2.899) = (2.899 + 2)^2 - 3$ or 21, U
$C^{-1}(2.123) = (2.123 + 2)^2 - 3$ or 14, N
FUNCTIONS ARE FUN

48. Case 1
$|2x + 4| \le 6$
$-(2x + 4) \le 6$
$-2x - 4 \le 6$
$-2x \le 10$
$x \ge -5$

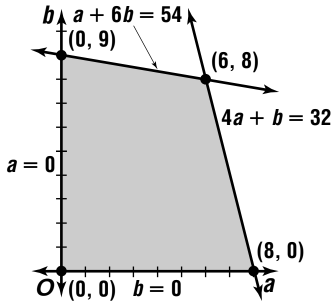
Case 2
$|2x + 4| \le 6$
$2x + 4 \le 6$
$2x \le 2$
$x \le 1$

$\{x \mid -5 \le x \le 1\}$

49. both

50a. $a \ge 0$, $b \ge 0$, $4a + b \le 32$, $a + 6b \le 54$

50b.



$G(a, b) = a + b$
$G(0, 0) = 0 + 0$ or 0
$G(0, 9) = 0 + 9$ or 9
$G(6, 8) = 6 + 8$ or 14
$G(8, 0) = 8 + 0$ or 8
14 gallons

51. $4x + 2y = 10$, $y = 6 - x$ $\rightarrow$ $4x + 2y = 10$, $x + y = 6$

$$\begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \end{bmatrix}$$

$$\frac{1}{\begin{vmatrix} 4 & 2 \\ 1 & 1 \end{vmatrix}} \begin{bmatrix} 1 & -2 \\ -1 & 4 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & -2 \\ -1 & 4 \end{bmatrix}$$

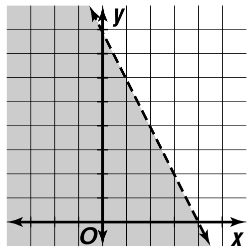
$$\frac{1}{2}\begin{bmatrix} 1 & -2 \\ -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & -2 \\ -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$$

$(-1, 7)$

52. $$\frac{1}{2}\begin{bmatrix} 9 & -3 \\ -6 & 6 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(9) & \frac{1}{2}(-3) \\ \frac{1}{2}(-6) & \frac{1}{2}(6) \end{bmatrix} = \begin{bmatrix} \frac{9}{2} & -\frac{3}{2} \\ -3 & 3 \end{bmatrix}$$

53.



54. $\frac{1}{4} \ne 4$; $\frac{1}{4} \cdot 4 \ne -1$; neither

55. $m = \frac{2 - 7}{5 - 0}$
$= \frac{-5}{5}$ or -1

$y - y_1 = m(x - x_1)$
$y - 7 = -1(x - 0)$
$y = -x + 7$

56. $b + c = 180$
If $\overline{PQ}$ is perpendicular to $\overline{QR}$, then $m\angle PQR = 90$.
Since the angles of a triangle total 180,
$a + d + 90 = 180$.
$a + d = 90$
$a + b + c + d = 180 + 90$ or 270
The correct choice is C.

3-5 Continuity and End Behavior

Page 165 Check for Understanding

1. Sample answer: The function approaches 1 as x approaches 2 from the left, but the function approaches -4 as x approaches 2 from the right. This means the function fails the second condition in the continuity test.

2.

a_n	n	$x \rightarrow$	$p(x) \rightarrow$
positive	even	∞	∞
positive	even	$-\infty$	∞
positive	odd	∞	∞
positive	odd	$-\infty$	$-\infty$

a_n	n	$x \rightarrow$	$p(x) \rightarrow$
negative	even	∞	$-\infty$
negative	even	$-\infty$	$-\infty$
negative	odd	∞	$-\infty$
negative	odd	$-\infty$	∞

3. Infinite discontinuity; $f(x) \rightarrow \infty$ as $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$ as $x \rightarrow \infty$.

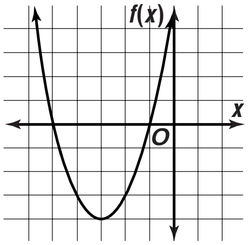
4. $f(x) = x^2$ is decreasing for $x < 0$ and increasing for $x > 0$, $g(x) = -x^2$ is increasing for $x < 0$ and decreasing for $x < 0$. Reflecting a graph switches the monotonicity. In other words, if $f(x)$ is increasing, the reflection will be decreasing and vice versa.

5. No; y is undefined when $x = -3$.

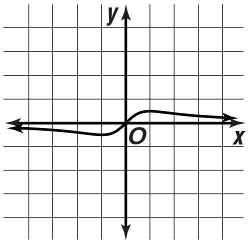
6. No; $f(x)$ approaches 6 as x approaches -2 from the left but $f(x)$ approaches -6 as x approaches -2 from the right.

7. a_n: positive, n: odd
$y \to \infty$ as $x \to \infty$, $y \to -\infty$ as $x \to -\infty$.

8. a_n: negative, n: even
$y \to -\infty$ as $x \to \infty$, $y \to -\infty$ as $x \to -\infty$.

9. 

decreasing for $x < -3$; increasing for $x > -3$

10. 

decreasing for $x < -1$ and $x > 1$; increasing for $-1 < x < 1$

11a. $t = 4$ **11b.** when $t = 4$ **11c.** 10 amps

Pages 166–168 Exercises

12. Yes; the function is defined when $x = 1$; the function approaches -3 as x approaches 1 from both sides; and $y = -3$ when $x = 1$.
13. No; the function is undefined when $x = 2$.
14. Yes; the function is defined when $x = -3$; the function approaches 0 as x approaches -3 from both sides; and $f(-3) = 0$.
15. Yes; the function is defined when $x = 3$; the function approaches 1 (in fact is equal to 1) as x approaches 3 from both sides; and $y = 1$ when $x = 3$.
16. No; $f(x)$ approaches -7 as x approaches -4 from the left, but $f(x)$ approaches 6 as x approaches -4 from the right.
17. Yes; the function is defined when $x = 1$; $f(x)$ approaches 3 as x approaches 1 from both sides; and $f(1) = 3$.
18. jump discontinuity
19. Sample answer: $x = 0$; $g(x)$ is undefined when $x = 0$.
20. a_n: positive, n: odd
$y \to \infty$ as $x \to \infty$, $y \to -\infty$ as $x \to -\infty$.
21. a_n: negative, n: even
$y \to -\infty$ as $x \to \infty$, $y \to -\infty$ as $x \to -\infty$.
22. a_n: positive, n: even
$y \to \infty$ as $x \to \infty$, $y \to \infty$ as $x \to -\infty$.
23. a_n: positive, n: even
$y \to \infty$ as $x \to \infty$, $y \to \infty$ as $x \to -\infty$.

24.

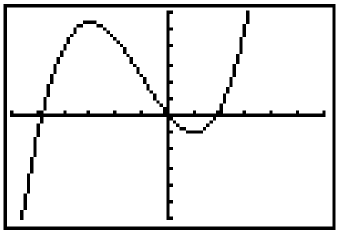
$y = \frac{1}{x^2}$	
x	y
$-10{,}000$	1×10^{-8}
-1000	1×10^{-6}
-100	1×10^{-4}
-10	0.01
0	undefined
10	0.01
100	1×10^{-4}
1000	1×10^{-6}
10,000	1×10^{-8}

$y \to 0$ as $x \to \infty$, $y \to 0$ as $x \to -\infty$.

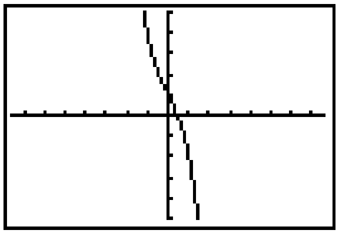
25.

$f(x) = -\frac{1}{x^3} + 2$	
x	$f(x)$
$-10{,}000$	2
-1000	2.000000001
-100	2.000001
-10	2.001
0	undefined
10	1.999
100	1.999999
1000	1.999999999
10,000	2

$f(x) \to 2$ as $x \to \infty$, $f(x) \to 2$ as $x \to -\infty$.

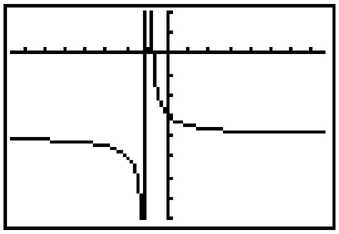
26. 

$[-6, 6]$ scl:1 by $[-30, 30]$ scl:5
increasing for $x < -3$ and $x > 1$; decreasing for $-3 < x < 1$

27. 

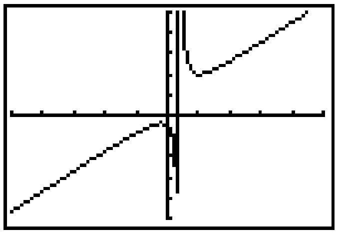
$[-7.6, 7.6]$ scl:1 by $[-5, 5]$ scl:1
decreasing for all x

28.



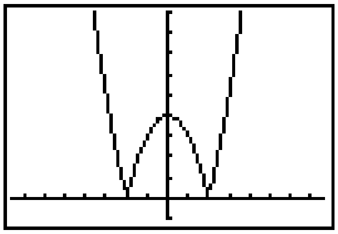
$[-7.6, 7.6]$ scl:1 by $[-8, 2]$ scl:1
decreasing for $x < -1$ and $x > -1$

29.



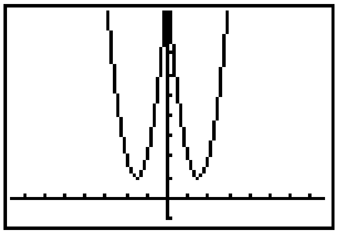
$[-25, 25]$ scl:5 by $[-25, 25]$ scl:5
increasing for $x < -1$ and $x > 5$; decreasing for $-1 < x < 2$ and $2 < x < 5$

30.



$[-7.6, 7.6]$ scl:1 by $[-1, 9]$ scl:1
decreasing for $x < -2$ and $0 < x < 2$; increasing for $-2 < x < 0$ and $x > 2$

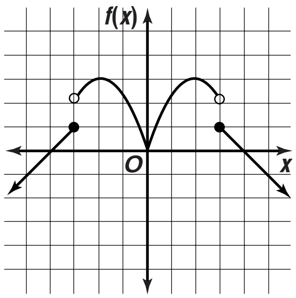
31.



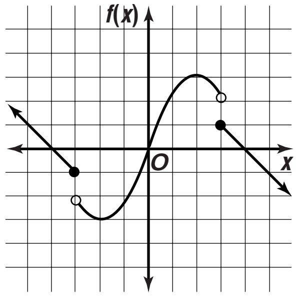
$[-7.6, 7.6]$ scl:1 by $[-1, 9]$ scl:1
decreasing for $x < -\frac{3}{2}$ and $0 < x < \frac{3}{2}$; increasing for $-\frac{3}{2} < x < 0$ and $x > \frac{3}{2}$

32. As the denominator, r, gets larger, the value of $U(r)$ gets smaller. $U(r)$ approaches 0.

33a. Since f is even, its graph must be symmetric with respect to the y-axis. Therefore, f is decreasing for $-2 < x < 0$ and increasing for $x < -2$. f must have a jump discontinuity when $x = -3$ and $f(x) \to -\infty$ as $x \to -\infty$.

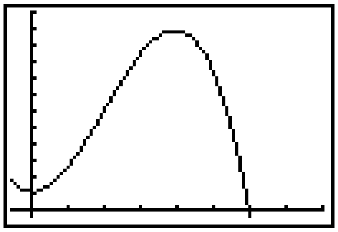


33b. Since f is odd, its graph must be symmetric with respect to the origin. Therefore, f is increasing for $-2 < x < 0$ and decreasing for $x < -2$. f must have a jump discontinuity when $x = -3$ and $f(x) \to \infty$ as $x \to -\infty$.



34a. polynomial

34b.



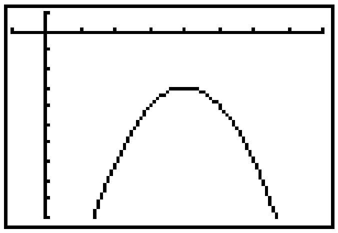
$[-5, 80]$ scl:10 by $[-500, 12000]$ scl:1000
$0.5 < t < 39.5$

34c. $0 < t < 0.5$ and $t > 39.5$

35a. 1954-1956, 1960-1961, 1962-1963, 1966-1968, 1973-1974, 1975-1976, 1977-1978, 1989-1991, 1995-1997

35b. 1956-1960, 1961-1962, 1963-1966, 1968-1973, 1974-1975, 1976-1977, 1978-1989, 1991-1995, 1997-2004

36a.



$[-1, 8]$ scl:1 by $[-10, 1]$ scl:1
$x < 4$

36b. Answers will vary.

36c. The slope is positive. In an interval where a function is increasing, for any two points on the graph, the x- and y-coordinates of one point will be greater than that of the other point, ensuring that the slope of the line through the two points will be positive.

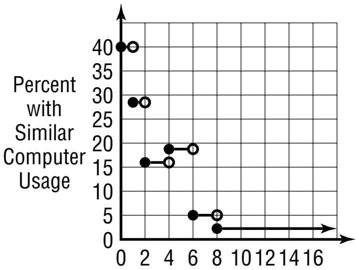
36d. See graph in 36a. $x > 4$

36e. The slope is negative; see students' work.

37a. The function has to be monotonic. If the function were increasing on one interval and decreasing on another interval, the function could not pass the horizontal line test.

37b. The inverse must be monotonic. If the inverse were increasing on one interval and decreasing on another interval, the inverse would fail the horizontal line test. That would mean the function fails the vertical line test, which is impossible.

38a.



38b. $0 < x < 1$, $1 < x < 2$, $2 < x < 4$, $4 < x < 6$, $6 < x < 8$, $x > 8$

39. For the function to be continuous at 2, $bx + a$ and $x^2 + a$ must approach the same value as x approaches 2 from the left and right, respectively. Plugging in $x = 2$ to find that common value gives $2b + a = 4 + a$. Solving for b gives $b = 2$. For the function to be continuous at -2, $\sqrt{-b - x}$ and $bx + a$ must approach the same value as x approaches -2 from the left and right, respectively. Plugging in $x = -2$ gives $\sqrt{-b + 2} = -2b + a$. We already know $b = 2$, so the equation becomes $0 = -4 + a$. Hence, $a = 4$.

40.
$$\begin{aligned} f(x) &= (x + 5)^2 \\ y &= (x + 5)^2 \\ x &= (y + 5)^2 \\ \pm\sqrt{x} &= y + 5 \\ y &= -5 \pm \sqrt{x} \\ f^{-1}(x) &= -5 \pm \sqrt{x} \end{aligned}$$

41. The graph of $g(x)$ is the graph of $f(x)$ translated left 2 units and down 4 units.

42. $f(x, y) = x + 2y$
$f(0, 0) = 0 + 2(0)$ or 0
$f(4, 0) = 4 + 2(0)$ or 4
$f(3, 5) = 3 + 2(5)$ or 13
$f(0, 5) = 0 + 2(5)$ or 10
13, 0

43. $\begin{vmatrix} 5 & -4 \\ 8 & 2 \end{vmatrix} = 5(2) - 8(-4)$ or 42

44a. $c = 47.5h + 35$

44b.
$$\begin{aligned} c &= 47.5h + 35 \\ c &= 47.5\left(2\tfrac{1}{4}\right) + 35 \\ c &= \$141.875 \end{aligned}$$

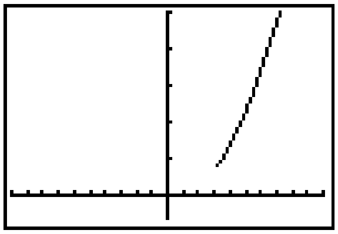
45.
$$\begin{aligned} f(x) &= 2x^2 - 2x + 8 \\ f(-2) &= 2(-2)^2 - 2(-2) + 8 \\ &= 8 + 4 + 8 \text{ or } 20 \end{aligned}$$

46. The volume of the cube is x^3.
The volume of the other box is $x(x - 1)(x + 1) = x(x^2 - 1)$ or $x^3 - x$.
The difference between the volumes of the two boxes is $x^3 - (x^3 - x)$ or x.
The correct choice is A.

3-5B Gap Discontinuities

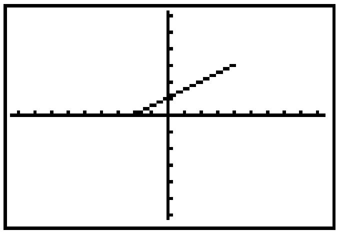
Page 170

1. {all real numbers $x \mid x > 3$}



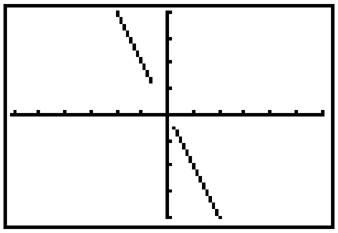
$[-10, 10]$ scl:1 by $[-6, 50]$ scl:10

2. {all real numbers $x \mid -2 \leq x \leq 4$}



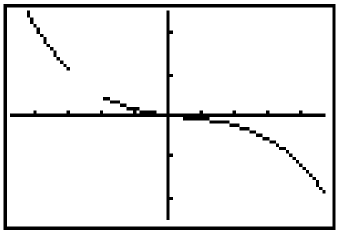
$[-9.4, 9.4]$ scl:1 by $[-6.2, 6.2]$ scl:1

3. {all real numbers $x \mid x < -3$ or $x \geq 1$}



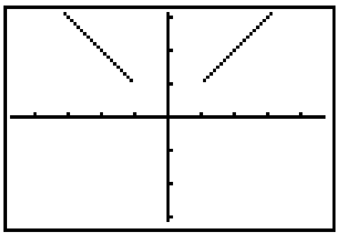
$[-18.8, 18.8]$ scl:1 by $[-12.4, 12.4]$ scl:1

4. {all real numbers $x \mid x \leq -3$ or $x > -2$}



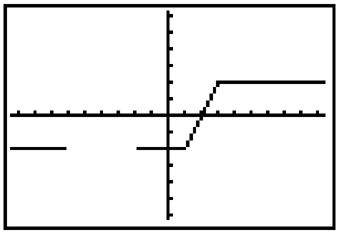
$[-4.7, 4.7]$ scl:1 by $[-25, 25]$ scl:10

5. {all real numbers $x \mid x < -1$ or $x > 1$}



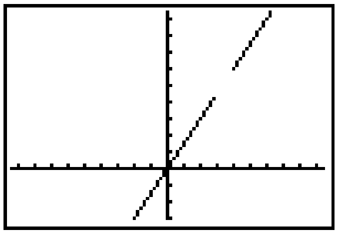
$[-4.7, 4.7]$ scl:1 by $[-3.1, 3.1]$ scl:1

6. {all real numbers $x \mid x < -6$ or $x > -2$}



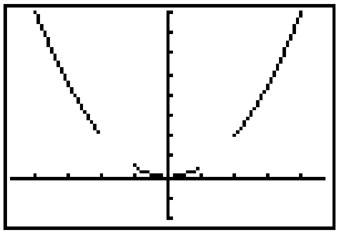
$[-9.4, 9.4]$ scl:1 by $[-6.2, 6.2]$ scl:1

7. {all real numbers $x \mid x < 3$ or $x \geq 4$}



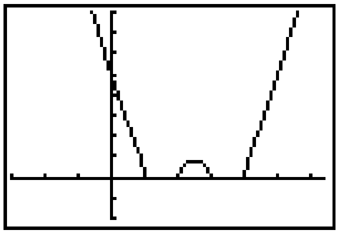
$[-9.4, 9.4]$ scl:1 by $[-3, 9.4]$ scl:1

8. {all real numbers $x \mid x < -2$ or $-1 \leq x < 1$ or $x \geq 2$}



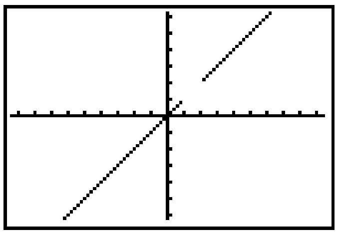
$[-4.7, 4.7]$ scl:1 by $[-2, 8]$ scl:1

9. {all real numbers $x \mid x \leq 1$ or $2 \leq x \leq 3$ or $x \geq 4$}



$[-3, 6.4]$ scl:1 by $[-2, 8]$ scl:1

10. {all real numbers $x \mid x < 1$ or $x > 2$}



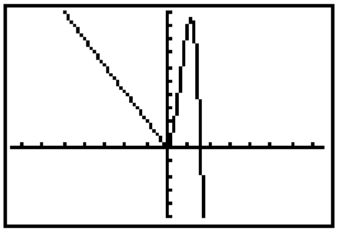
$[-9.4, 9.4]$ scl:1 by $[-6.2, 6.2]$ scl:1

11. Sample answer: $y = \dfrac{x^2}{((x \leq 2) \text{ or } ((x \geq 5) \text{ and } (x \leq 7)) \text{ or } x \geq 8))}$

12. Yes; sample justification: if $f(x)$ is a polynomial function, then the graph of $y = \dfrac{f(x)}{(|x - [[x]]| < 0.25)}$ is like the graph of $f(x)$, but with an infinite number of "interval bites" removed.

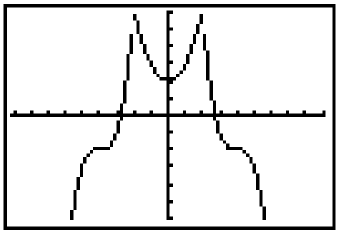
13. Yes; sample justification: the equation $y = \dfrac{x^2(x \leq -2) + (2x - 4)(x \geq 4)}{((x \leq -2) \text{ or } (x \geq 4))}$ is a possible equation for the function described.

14a.



$[-15, 15]$ scl:2 by $[-10, 20]$ scl:2

14b.



$[-9.1, 9.1]$ scl:1 by $[-6, 6]$ scl:1

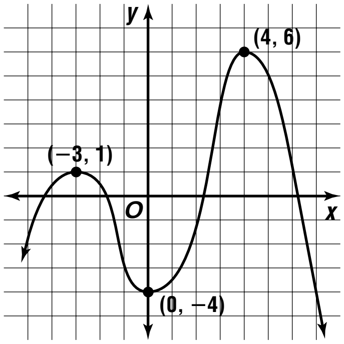
3-6 Critical Points and Extrema

Page 176 Check for Understanding

1. Check values of the function at x-values very close to the critical point. Be sure to check values on both sides. If the function values change from increasing to decreasing, the critical point is a maximum. If the function values change from decreasing to increasing, the critical point is a minimum. If the function values continue to increase or to decrease, the critical point is a point of inflection.

2. rel. min.;
$f(0.99) \approx -3.9997$
$f(1) = -4$
$f(1.01) \approx -3.9997$
By testing points on either side of the critical point, it is evident that the graph of the function is decreasing as x approaches 1 from the left and increasing as x moves away from 1 to the right. Therefore, on the interval $0.99 < x < 1.01$, $(1, 4)$ is a relative minimum.

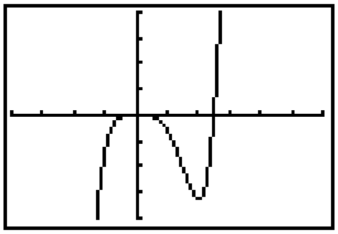
3. Sample answer:



4. rel. min.: (−3, −2); rel. max.: (1, 6)

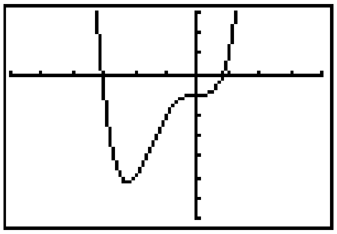
5. rel. min.: (−1, −3); rel. max.: (3, 3)

6. rel. max.: (0, 0); rel. min.: (2, −16)



[−4, 6] scl:1 by [−20, 20] scl:5

7. rel. min.: (−2.25, −10.54)



[−6, 4] scl:1 by [−14, 6] scl:2

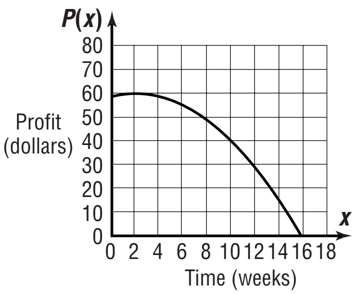
8. $f(-1.1) = 0.907$
$f(-1) = 1$
$f(-0.9) = 0.913$ max.

9. $f(-2.6) = -12.24$
$f(-2.5) = -12.25$
$f(-2.4) = -12.24$ min.

10. $f(-0.1) = -0.00199$
$f(0) = 0$
$f(0.1) = 0.00199$ pt. of inflection

11. $f(-0.1) \approx -0.97$
$f(0) = -1$
$f(0.1) \approx -0.97$ min.

12a. $P(x) = (120 + 10x)(0.48 - 0.03x)$



12b. 2 weeks

12c. \$58.80 per acre

12d. Rain or other bad weather could delay harvest and/or destroy part of the crop.

Pages 177–179 Exercises

13. abs. max.: (−4, 1)

14. abs. max.: (−1, 3); rel. min.: (0.5; 0.5); rel. max.: (1.5, 2)

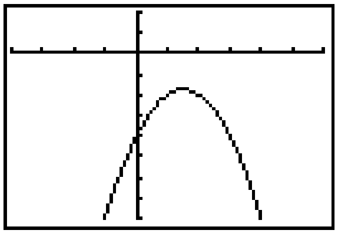
15. rel. max.: (−2, 7): abs. min.: (3, −3)

16. rel. max.: (−6, 4), rel. min.: (−2, −3)

17. abs. min.: (3, −8); rel. max.: (5, −2); rel. min.: (8, −5)

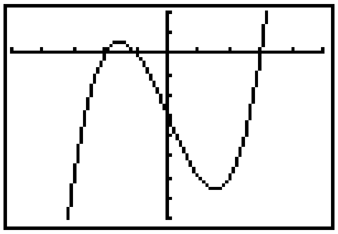
18. no extrema

19. abs. max.: (1.5, −1.75)



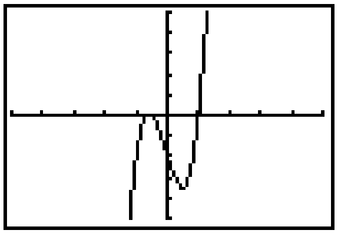
[−5, 5] scl:1 by [−8, 2] scl:1

20. rel. max.: (−1.53, 1.13); rel. min.: (1.53, −13.13)



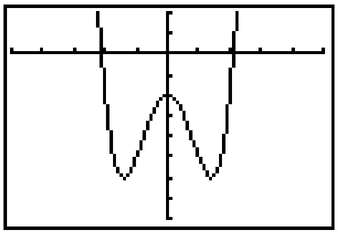
[−5, 5] scl:1 by [−16, 4] scl:2

21. rel. max.: (−0.59, 0.07), rel. min.: (0.47, −3.51)



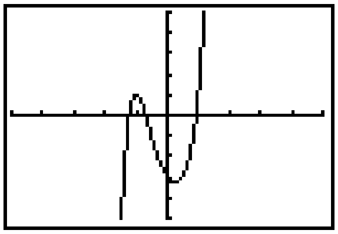
[−5, 5] scl:1 by [−5, 5] scl:1

22. abs. min.: (−1.41, −6), (1.41, −6); rel. max.: (0, −2)



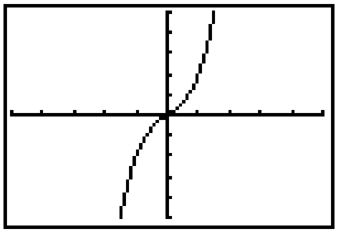
[−5, 5] scl:1 by [−8, 2] scl:1

23. rel. max.: $(-1, 1)$; rel. min.: $(0.25, -3.25)$



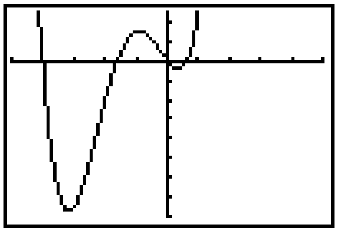
$[-5, 5]$ scl:1 by $[-5, 5]$ scl:1

24. no extrema



$[-5, 5]$ scl:1 by $[-5, 5]$ scl:1

25. abs. min.: $(-3.18, -15.47)$; rel. min. $(0.34, -0.80)$; rel. max.: $(-0.91, 3.04)$



$[-5, 5]$ scl:1 by $[-16, 5]$ scl:2

26. $f(-0.1) = -0.001$
$f(0) = 0$
$f(0.1) = 0.001$ pt of inflection

27. $f(3.9) = 5.99$
$f(4) = 6$
$f(4.1) = 5.99$ max.

28. $f(-2.6) = -19.48$
$f(-2.5) = -19.5$
$f(-2.4) = -19.48$ min.

29. $f(-0.1) \approx 6.98$
$f(0) = 7$
$f(0.1) \approx 6.98$ max.

30. $f(1.9) \approx -3.96$
$f(2) = -4.82$
$f(2.1) \approx -3.96$ min.

31. $f(2.9) = -0.001$
$f(3) = 0$
$f(3.1) = 0.001$ pt. of inflection

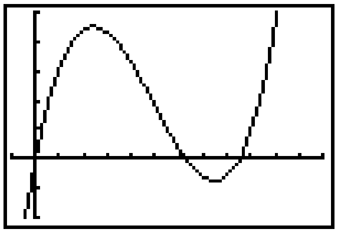
32. $f(-2.1) \approx 4.32$
$f(-2) \approx 4.53$
$f(-1.9) \approx 4.32$ max.

33. $f(0.57) \approx 2.86$
$f\left(\frac{2}{3}\right) \approx 2.85$
$f(0.77) \approx 2.86$ min.

34. The point of inflection is now at $x = -6$ and there is now a minimum at $x = -3$.

35a. $V(x) = 2x(12.5 - 2x)(17 - 2x)$

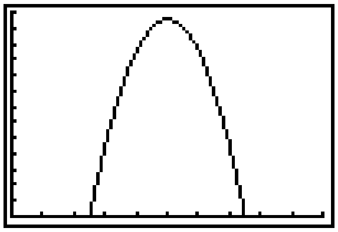
35b.



$[-1, 12]$ scl:1 by $[-200, 500]$ scl:100
2.37 cm by 2.37 cm

35c. See students' work.

36a. $P = sd - 25d$
$= s(-200s + 15{,}000) - 25(-200s + 15{,}000)$
$= -200s^2 + 20{,}000s - 375{,}000$

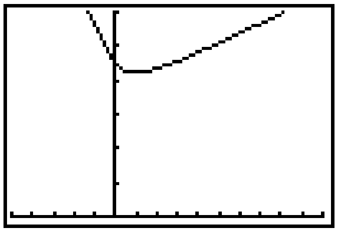


$[0, 100]$ scl:10 by $[0, 130{,}000]$ scl:10,000
abs. max.: $(50, 125{,}000)$
\$50

36b. Sample answer: The company's competition might offer a similar product at a lower cost.

37a. $AM^2 = MB^2 + AB^2$
$AM^2 = x^2 + 2^2$
$AM = \sqrt{x^2 + 4}$
$f(x) = 5000(\sqrt{x^2 + 4}) + 3500(10 - x)$

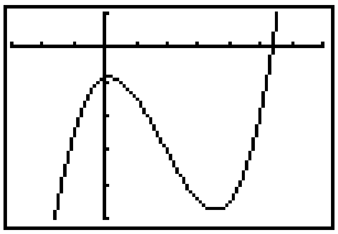
37b.



$[-10, 20]$ scl:2 by $[0, 60{,}000]$ scl:10,000
abs. min.: $(1.96, 42{,}141.4)$
1.96 km from point B

38. equations of the form $y = x^n$ or $y = \sqrt[n]{x}$, where n is odd

39.

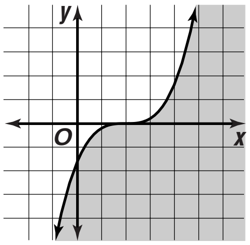


$[-3, 7]$ scl:1 by $[-50, 10]$ scl:10
The particle is at rest when $t \approx 0.14$ and when $t \approx 3.52$. Its positions at these times are $s(0.14) \approx -8.79$ and $s(3.52) \approx -47.51$.

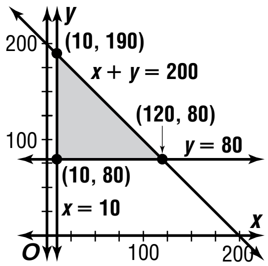
40. If a cubicle has one critical point, then it must be a point of inflection. If it were a relative maximum or minimum, then the end behavior for a cubic would not be satisfied. If a cubic has three critical points, then one must be a maximum, another a minimum, and the third a point of inflection.

41. No; the function is undefined when $x = 5$.

42.

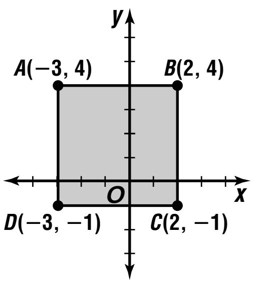


43. Let x = units of notebook paper.
Let y = units of newsprint.
$x + y \leq 200$
$x \geq 10$
$y \geq 80$



$P(x, y) = 400x + 350y$
$P(10, 80) = 400(10) + 350(80)$ or $32{,}000$
$P(10, 190) = 400(10) + 350(190)$ or $70{,}500$
$P(120, 80) = 400(120) + 350(80)$ or $76{,}000$
120 units of notebook and 80 units of newsprint

44.



$-3 \leq x \leq 2,\ -1 \leq y \leq 4$

45. $\begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix} = 1(5) - 2(3)$ or -1; yes

46.
$$3A = 3\begin{bmatrix} 4 & -2 \\ 5 & 7 \end{bmatrix}$$
$$= \begin{bmatrix} 3(4) & 3(-2) \\ 3(5) & 3(7) \end{bmatrix} \text{ or } \begin{bmatrix} 12 & -6 \\ 15 & 21 \end{bmatrix}$$
$$2B = 2\begin{bmatrix} -3 & 5 \\ -4 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 2(-3) & 2(5) \\ 2(-4) & 2(3) \end{bmatrix} \text{ or } \begin{bmatrix} -6 & 10 \\ -8 & 6 \end{bmatrix}$$
$$3A + 2B = \begin{bmatrix} 12 & -6 \\ 15 & 21 \end{bmatrix} + \begin{bmatrix} -6 & 10 \\ -8 & 6 \end{bmatrix}$$
$$= \begin{bmatrix} 12 + (-6) & -6 + 10 \\ 15 + (-8) & 21 + 6 \end{bmatrix}$$
$$= \begin{bmatrix} 6 & 4 \\ 7 & 27 \end{bmatrix}$$

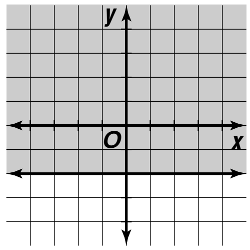
47. Let x = number of 1-point free throws.
Let y = number of 2-point field goals.
Let z = number of 3-point field goals.
$1x + 2y + 3z = 32$
$x + y + z = 17$
$y = 0.50(18)$

$$\begin{array}{lcrcl} 1x + 2y + 3z = 32 & \rightarrow & 1x + 2y + 3z &=& 32 \\ -1(x + y + z = 17) & & -x - y - z &=& -17 \\ \hline & & y + 2z &=& 15 \end{array}$$

$y = 0.50(18)$ $\quad$ $y + 2z = 15$
$y = 9$ $\quad$ $9 + 2z = 15$
$z = 3$

$x + y + z = 17$
$x + 9 + 3 = 17$
$x = 5$ $\quad$ 5 free throws, 9 2-point field goals, 3 3-point field goals

48. $y + 6 \geq 4$
$y \geq -2$



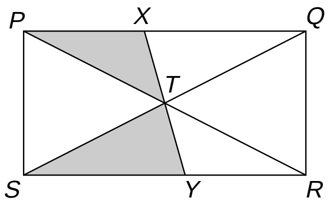
49. $2x + 3y = 15 \rightarrow y = -\frac{2}{3}x + 5$

$6x = 4y + 16 \rightarrow y = \frac{3}{2}x - 4$

$-\frac{2}{3} \cdot \frac{3}{2} = -1$; perpendicular

50. A relation relates members of a set called the domain to members of a set called the range. In a function, the relation must be such that each member of the domain is related to one and only one member of the range. You can use the vertical line test to determine whether a graph is the graph of a function.

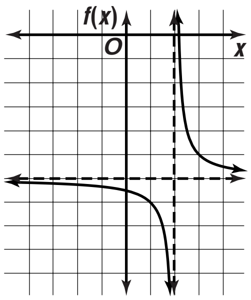
51. The area of $\triangle PTX$ is equal to the area of $\triangle RTY$. The area of $\triangle STR$ is 25% of the area of rectangle $PQRS$. The correct choice is D.



3-7 Graphs of Rational Functions

Pages 185–186 Check for Understanding

1.

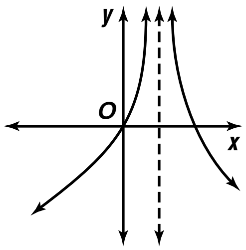


1a. $x = 2, y = -6$

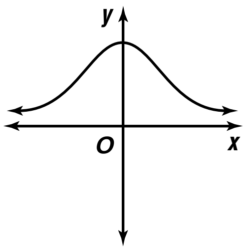
1b. $y = \frac{1}{x-2} - 6$

2. Sample graphs:

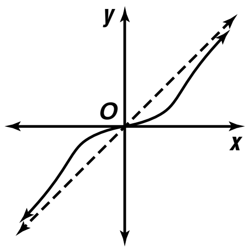
Vertical Asymptote



Horizontal Asymptote



Slant Asymptote



3. Sample answer: $f(x) = \frac{x(x+1)}{x+1}$

4. False; sample explanation: if that x-value also causes the numerator to be 0, there could be a hole instead of a vertical asymptote.

5. $x = 5$

$$
\begin{aligned}
f(x) &= \frac{x}{x-5}\\
y &= \frac{x}{x-5}\\
y(x-5) &= x\\
xy - 5y &= x\\
xy - x &= 5y\\
x(y-1) &= 5y\\
x &= \frac{5y}{y-1};\ y = 1
\end{aligned}
$$

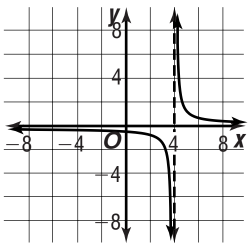
6. $x = 2, x = -1$

$$
\begin{aligned}
y &= \frac{x^3}{(x-2)(x+1)}\\
y &= \frac{x^3}{x^2 - x - 2}\\
y &= \frac{\frac{x^3}{x^3}}{\frac{x^2}{x^3} - \frac{x}{x^3} - \frac{2}{x^3}}\\
y &= \frac{1}{\frac{1}{x} - \frac{1}{x^2} - \frac{2}{x^3}}
\end{aligned}
$$

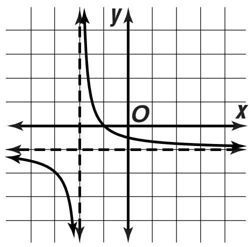
no horizontal asymptotes

7. $f(x) = \frac{1}{x+1} - 2$

8. The parent graph is translated 4 units right. The vertical asymptote is now at $x = 4$. The horizontal asymptote, $y = 0$, is unchanged.



9. The parent graph is translated 2 units left and down 1 unit. The vertical asymptote is now at $x = -2$ and the horizontal asymptote is now $y = -1$.

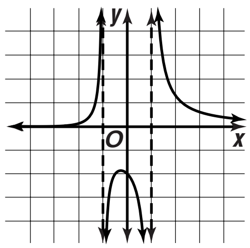


10.

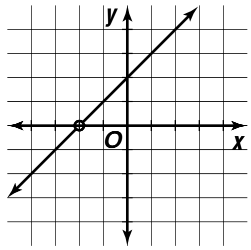
$$
\begin{array}{r}
3x + 5 \\
x - 3\overline{)\,3x^2 - 4x + 5} \\
\underline{3x^2 - 9x} \\
5x + 5 \\
\underline{5x - 15} \\
20
\end{array}
\quad \rightarrow \quad 3x + 5 + \frac{20}{x-3}
$$

$y = 3x + 5$

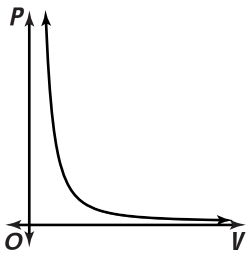
11.



12.



13a.



13b. $P = 0, V = 0$

13c. The pressure approaches 0.

Pages 186–188 Exercises

14. $x = -4$

$$f(x) = \frac{2x}{x+4}$$
$$y = \frac{2x}{x+4}$$
$$y(x+4) = 2x$$
$$xy + 4y = 2x$$
$$xy - 2x = -4y$$
$$x(y-2) = -4y$$
$$x = \frac{-4y}{y-2};\ y = 2$$

15. $x = -6$

$$y = \frac{x^2}{x+6}$$
$$y = \frac{\frac{x^2}{x^2}}{\frac{x^2}{x^2} + \frac{6}{x^2}}$$
$$y = \frac{1}{\frac{1}{x} + \frac{6}{x^2}}$$

no horizontal asymptote

16. $x = -\frac{1}{2}, x = 5$

$$y = \frac{x-1}{(2x+1)(x-5)}$$
$$y = \frac{x-1}{2x^2 - 9x - 5}$$
$$y = \frac{\frac{x}{x^2} - \frac{1}{x^2}}{\frac{2x^2}{x^2} - \frac{9x}{x^2} - \frac{5}{x^2}}$$
$$y = \frac{\frac{1}{x} - \frac{1}{x^2}}{2 - \frac{9}{x} - \frac{5}{x^2}};\ y = 0$$

17. $x = -1, x = -3$

$$y = \frac{x-2}{x^2 - 4x + 3}$$
$$y = \frac{\frac{x}{x^2} + \frac{2}{x^2}}{\frac{x^2}{x^2} + \frac{4x}{x^2} + \frac{3}{x^2}}$$
$$y = \frac{\frac{1}{x} + \frac{2}{x^2}}{1 + \frac{4}{x} + \frac{3}{x^2}};\ y = 0$$

18. no vertical asymptote,

$$y = \frac{x^2}{x^2+1}$$
$$y = \frac{\frac{x^2}{x^2}}{\frac{x^2}{x^2} + \frac{1}{x^2}}$$
$$y = \frac{1}{1 + \frac{1}{x^2}};\ y = 1$$

19. $x = 1$

$$y = \frac{(x+1)^2}{x^2-1}$$
$$y = \frac{x^2+2x+1}{x^2-1}$$
$$y = \frac{\frac{x^2}{x^2} + \frac{2x}{x^2} + \frac{1}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}}$$
$$y = \frac{1 + \frac{2}{x} + \frac{1}{x^2}}{1 - \frac{1}{x^2}};\ y = 1$$

20. $x = 2$

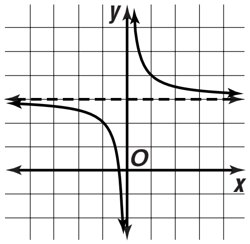
$$y = \frac{x^3}{(x-2)^4}$$
$$y = \frac{x^3}{x^4 - 8x^3 + 24x^2 - 32x + 16}$$
$$y = \frac{\frac{x^3}{x^4}}{\frac{x^4}{x^4} - \frac{8x^3}{x^4} + \frac{24x^2}{x^4} - \frac{32x}{x^4} + \frac{16}{x^4}}$$
$$y = \frac{\frac{1}{x}}{1 - \frac{8}{x} + \frac{24}{x^2} - \frac{32}{x^3} + \frac{16}{x^4}};\ y = 0$$

21. $f(x) = \frac{1}{x+3} + 1$

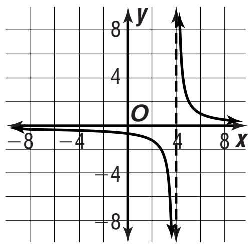
22. $f(x) = \frac{1}{x-2} - 3$

23. $f(x) = -\frac{1}{x} + 1$

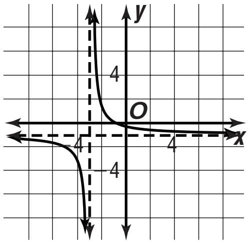
24. The parent graph is translated 3 units up. The vertical asymptote, $x = 0$, is unchanged. The horizontal asymptote is now $y = 3$.



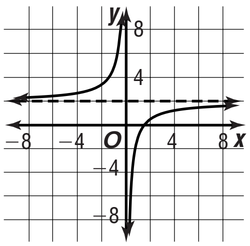
25. The parent graph is translated 4 units right and expanded vertically by a factor of 2. The vertical asymptote is now $x = 4$. The horizontal asymptote, $y = 0$, is unchanged.



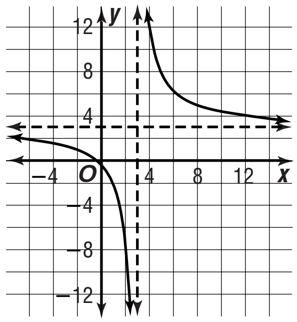
26. The parent graph is translated 3 units left. The translated graph is then expanded vertically by a factor of 2 and translated 1 unit down. The vertical asymptote is now $x = -3$ and the horizontal asymptote is now $y = -1$.



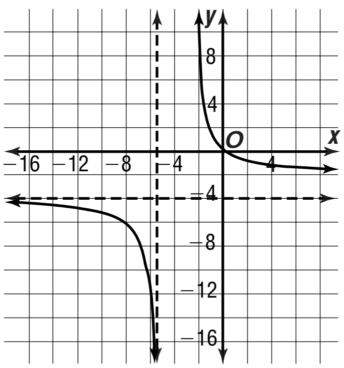
27. The parent graph is expanded vertically by a factor of 3, reflected about the x-axis, and translated 2 units up. The vertical asymptote, $x = 0$, is unchanged. The horizontal asymptote is now $y = 2$.



28. The parent graph is translated 3 units right. The translated graph is expanded vertically by a factor of 10 and then translated 3 units up. The vertical asymptote is $x = 3$ and the horizontal asymptote is $y = 3$.



29. The parent graph is translated 5 units left. The translated graph is expanded vertically by a factor of 22 and then translated 4 units down. The vertical asymptote is $x = -5$ and the horizontal asymptote is $y = -4$.



30.
$$\begin{array}{r} x - 1 \\ x + 4\overline{\big)\,x^2 + 3x - 3} \\ \underline{x^2 + 4x} \\ -x - 3 \\ \underline{-x - 4} \\ 1 \end{array} \quad \rightarrow \quad x - 1 + \frac{1}{x+4}$$

$y = x - 1$

31.
$$\begin{array}{r} x + 3 \\ x\overline{\big)\,x^2 + 3x - 4} \\ \underline{x^2 \qquad\quad} \\ 3x - 4 \\ \underline{3x \quad\;} \\ -4 \end{array} \quad \rightarrow \quad x + 3 - \frac{4}{x}$$

$y = x + 3$

32.
$$\begin{array}{r} x - 2 \\ x^2 + 1\overline{\big)\,x^3 - 2x^2 + x - 4} \\ \underline{x^3 \qquad\quad + x \quad} \\ -2x^2 \qquad - 4 \\ \underline{-2x^2 \qquad - 2} \\ -2 \end{array} \quad \rightarrow \quad x - 2 - \frac{2}{x^2+1}$$

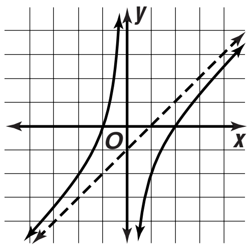
$y = x - 2$

33.
$$\begin{array}{r} \frac{1}{2}x - \frac{5}{4} \\ 2x - 3\overline{\big)\,x^2 - 4x + 1} \\ \underline{x^2 - \frac{3}{2}x \qquad} \\ -\frac{5}{2}x + 1 \\ \underline{-\frac{5}{2}x + \frac{15}{4}} \\ -\frac{11}{4} \end{array} \quad \rightarrow \quad \frac{1}{2}x - \frac{5}{4} - \frac{\frac{11}{4}}{2x-3}$$

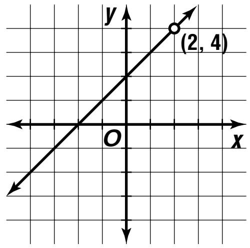
$y = \frac{1}{2}x - \frac{5}{4}$

34. No; the degree of the numerator is 2 more than that of the denominator.

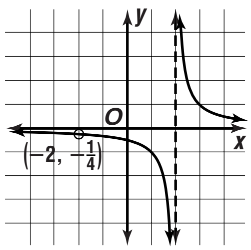
35.



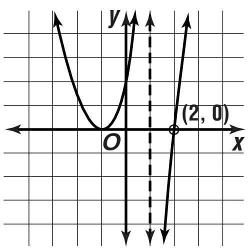
36.



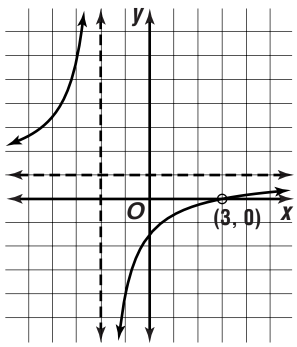
37.



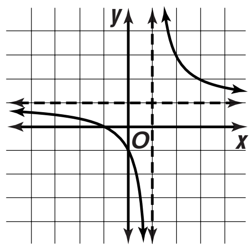
38.



39.



40.



41a. $C(t) = \frac{480 + 3t}{40 + t}$

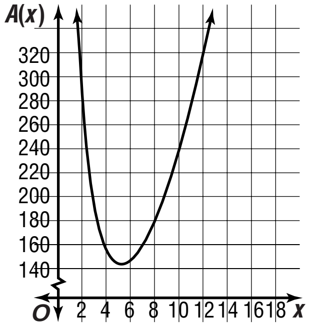
41b.
$$\begin{aligned} C(t) &= \frac{480 + 3t}{40 + t} \\ 10 &= \frac{480 + 3t}{40 + t} \\ 400 + 10t &= 480 + 3t \\ 7t &= 80 \\ t &\approx 11.43 \text{ L} \end{aligned}$$

42. Sample answer: The circuit melts or one of the components burns up.

43. To get the proper x-intercepts, $x - 2$ and $x + 3$ should be factors of the numerator. The vertical asymptote indicates that $x - 4$ should be a factor of the denominator. To get point discontinuity at $(-5, 0)$, make $x + 5$ a factor of both the numerator and denominator with a bigger exponent in the numerator. Thus, a sample answer is $f(x) = \frac{(x - 2)(x + 3)(x + 5)^2}{(x - 4)(x + 5)}$.

44a.
$$\begin{aligned} V &= x^2 \cdot h & A(x) &= 4x \cdot h + 2x^2 \\ 120 &= x^2 \cdot h & A(x) &= 4x\left(\frac{120}{x^2}\right) + 2x^2 \\ \frac{120}{x^2} &= h & A(x) &= \frac{480}{x} + 2x^2 \end{aligned}$$

44b.



44c. The surface area approaches infinity.

45. If the degree of the denominator is larger than that of the numerator, then $y = 0$ will be a horizontal asymptote. To make the graph intersect the x-axis, the simplest numerator to use is x. Thus, a sample answer is $f(x) = \frac{x}{x^2 + 1}$.

46a. A vertical asymptote at $r = 0$ and a horizontal asymptote at $F = 0$.

46b. The force of repulsion increases without bound as the charges are moved closer and closer together. The force of repulsion approaches 0 as the charges are moved farther and farther apart.

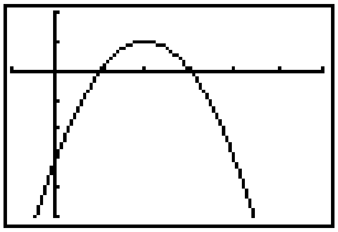
47a. $\frac{a^2 - 9}{a - 3}$

47b.

x	2.9	2.99	3	3.01	3.1
m	5.9	5.99	—	6.01	6.1

The slope approaches 6.

48. abs. max.: (2, 1)



$[-1, 6]$ scl:1 by $[-5, 2]$ scl:1

49.
$$\begin{aligned} x^2 - 9 &= y \\ y^2 - 9 &= x \\ y^2 &= x + 9 \\ y &= \pm\sqrt{x + 9} \end{aligned}$$

50. $f(x, y) = y - x$
$f(0, 0) = 0 - 0$ or 0
$f(4, 0) = 0 - 4$ or -4
$f(3, 5) = 5 - 3$ or 2
$f(0, 5) = 5 - 0$ or 5
5; -4

51.
$$-4\begin{bmatrix} -6 & 5 \\ 8 & -4 \end{bmatrix} = \begin{bmatrix} -4(-6) & -4(5) \\ -4(8) & -4(-4) \end{bmatrix} = \begin{bmatrix} 24 & -20 \\ -32 & 16 \end{bmatrix}$$

52. Let x = price of film and y = price of sunscreen.
$8x + 2y = 35.10$
$3x + y = 14.30$
$y = 14.30 - 3x$

$8x + 2y = 35.10$
$8x + 2(14.30 - 3x) = 35.10$
$2x + 28.60 = 35.10$
$x = 3.25$

$y = 14.30 - 3x$
$y = 14.30 - 3(3.25)$
$y = 4.55$
\$3.25; \$4.55

53.
$x + y \geq 3$
$0 + 0 \overset{?}{\geq} 3$
$0 \ngeq 3$ no

$x + y \geq 3$
$3 + 2 \overset{?}{\geq} 3$
$5 \geq 3$ yes

$x + y \geq 3$
$-4 + 2 \overset{?}{\geq} 3$
$-2 \ngeq 3$ no

$x + y \geq 3$
$-2 + 4 \overset{?}{\geq} 3$
$2 \ngeq 3$ no

(3, 2)

54. $15y - x = 1 \rightarrow y = \frac{1}{15}x + \frac{1}{15}$

55.
$$\begin{aligned} [f \circ g](x) &= f(g(x)) \\ &= f(2 - x^2) \\ &= 8(2 - x^2) \\ &= 16 - 8x^2 \end{aligned}$$
$$\begin{aligned} [g \circ f](x) &= g(f(x)) \\ &= g(8x) \\ &= 2 - (8x)^2 \\ &= 2 - 64x^2 \end{aligned}$$

56. Let x = the width of each card and y = the height of each card. The rectangle has a base of $4x$ or $5y$. The rectangle has a height of $x + y$.

$A = bh$ | $4x = 5y$
$180 = 4x(x + y)$ | $y = \frac{4x}{5}$
$180 = 4x\left(x + \frac{4x}{5}\right)$ | $y = \frac{4(5)}{5}$
$180 = \frac{36x^2}{5}$ | $y = 4$
$25 = x^2$
$5 = x$

Perimeter $= 2(4x) + 2(x + y)$
$P = 2(4 \cdot 5) + 2(5 + 4)$
$P = 58$ in.

The correct choice is B.

3-8 Direct, Inverse, and Joint Variation

Pages 193–194 Check for Understanding

1a. inverse

1b. neither

1c. direct

2. Sample answer:
Suppose y varies directly as x^n.
Then $y_1 = kx_1{}^n$ and $y_2 = kx_2{}^n$

$y_1 = kx_1{}^n$

$\frac{y_1}{y_2} = \frac{kx_1}{kx_2}$ *Division property of equality.*

$\frac{y_1}{y_2} = \frac{x_1}{x_2}$ *Simplify.*

3. The line does not go through the origin, therefore its equation is not of the form $y = kx^n$.

4a. Sample answer: The amount of money earned varies directly with the number of hours worked.

4b. Sample answer: The distance traveled by a car varies inversely as the amount of gas in the car.

4c. Sample answer: The volume of a cylinder varies jointly as its height and the radius of its base.

5. $xy = k$; $4(3) = k$; $12 = k$ | $xy = 12$; $15y = 12$; $y = \frac{4}{5}$

6. $y = kx^2$; $-54 = k(9)^2$; $-\frac{2}{3} = k$ | $y = -\frac{2}{3}x^2$; $y = -\frac{2}{3}(6)^2$; $y = -24$

7. $y = kxz^3$; $16 = k(4)(2)^3$; $0.5 = k$ | $y = 0.5xz^3$; $y = (0.5)(-8)(-3)^3$; $y = 108$

8. $y = \frac{kxz}{w^2}$; $3 = \frac{k(3)(10)}{2^2}$; $0.4 = k$ | $y = \frac{0.4xz}{w^2}$; $y = \frac{0.4(4)(20)}{4^2}$; $y = 2$

9. y varies directly as x^4; $\frac{1}{7}$.

10. A varies jointly as ℓ and w; 1

11. y varies inversely as x; -3.

12a. $V = khg^2$
$288 = k(40)(1.5)^2$
$3.2 = k$
$V = 3.2hg^2$

12b. $V = 3.2hg^2$
$V = 3.2(75)(2)^2$
$V = 960$
$50 \cdot 960 = 48{,}000 \text{ m}^3$

Pages 194–196 Exercises

13. $y = kx$; $0.3 = k(1.5)$; $0.2 = k$ | $y = 0.2x$; $y = 0.2(6)$; $y = 1.2$

14. $xy = k$; $25(-2) = k$; $-50 = k$ | $xy = -50$; $x(-40) = -50$; $x = 1.25$

15. $y = kxz$; $36 = k(1.2)(2)$; $15 = k$ | $y = 15xz$; $y = 15(0.4)(3)$; $y = 18$

16. $x^2y = k$; $(2)^2(9) = k$; $36 = k$ | $x^2y = 36$; $3^2y = 36$; $y = 4$

17. $r = kt^2$; $4 = k\left(\frac{1}{2}\right)^2$; $16 = k$ | $r = 16t^2$; $r = 16\left(\frac{1}{4}\right)^2$; $r = 1$

18. $\sqrt{x}y = k$
$\sqrt{1.21}(0.44) = k$
$0.484 = k$
$\sqrt{x}y = 0.484$ or $y = 0.484 \cdot \frac{1}{\sqrt{x}}$
$y = 0.484 \cdot \frac{1}{\sqrt{0.16}}$
$y = 1.21$

19. $y = kx^3z^2$; $-9 = k(-3)^3(2)^2$; $\frac{1}{12} = k$ | $y = \frac{1}{12}x^3z^2$; $y = \frac{1}{12}(-4)^3(-3)^2$; $y = -48$

20. $y = \frac{kx}{z^2}$; $\frac{1}{6} = \frac{k(20)}{6^2}$; $0.3 = k$ | $y = \frac{0.3x}{z^2}$; $y = \frac{0.3(14)}{5^2}$; $y = 0.168$

21. $y = \frac{kxz}{w}$; $-3 = \frac{k(2)(-3)}{4}$; $2 = k$ | $y = \frac{2xz}{w}$; $y = \frac{2(4)(-7)}{-4}$; $y = 14$

22. $y = \frac{kz^2}{x^3}$; $-6 = \frac{k(9)^2}{3^3}$; $-2 = k$ | $y = \frac{-2z^2}{x^2}$; $y = \frac{-2(-4)^2}{6^3}$; $y = -\frac{4}{27}$

23. $a = \frac{kb^2}{c}$; $45 = \frac{k(6)^2}{12}$; $15 = k$ | $a = \frac{15b^2}{c}$; $96 = \frac{15b^2}{10}$; $\pm 8 = b$

24. $x^2y = k$; $(4)^2(2) = k$; $32 = k$ | $yx^2 = 32$; $8x^2 = 32$; $x = \pm 2$

25. C varies directly as d; π.

26. y varies directly as x; $\frac{1}{4}$.

27. y varies jointly as x and the square of z; $\frac{4}{3}$.

28. V varies directly as the cube of r; $\frac{4}{3}\pi$.

29. y varies inversely as the square of x; $\frac{5}{4}$.

30. y varies inversely as the square root of x; 2.

31. A varies jointly as h and the quantity $b_1 + b_2$; 0.5.

32. y varies directly as x and inversely as the square of z; $\frac{1}{3}$.

33. y varies directly as x^2 and inversely as the cube of z; 7.

34. y varies jointly as the product of the cube of x and z and inversely as the square of w.

35a. Joint variation; to reduce torque one must either reduce the distance or reduce the mass on the end of the fulcrum. Thus, torque varies directly as the mass and the distance from the fulcrum. Since there is more than one quantity in direct variation with the torque on the seesaw, the variation is joint.

35b.
$$T_1 = km_1d_1 \text{ and } T_2 = km_2d_2$$
$$T_1 = T_2$$
$$km_1d_1 = km_2d_2 \qquad \textit{Substitution property}$$
$$m_1d_1 = m_2d_2 \qquad \textit{of equality}$$

35c.
$$m_1d_1 = m_2d_2$$
$$75(3.3) = (125)d_2$$
$$1.98 = d_2\text{; 1.98 meters}$$

36a. $tr = k$

36b.
$$tr = k \qquad tr = 36{,}000$$
$$45(800) = k \qquad t(1000) = 36{,}000$$
$$36{,}000 = k \qquad t = 36 \text{ minutes}$$

37. If y varies directly as x then there is a nonzero constant k such that $y = kx$. Solving for x, we find $x = \frac{1}{k}y \cdot \frac{1}{k}$ is a nonzero constant, so x varies directly as y.

38a. $I = \frac{k}{d^2}$

38b.
$$I = \frac{k}{d^2} \qquad a^2 + b^2 = c^2 \qquad I = \frac{576}{d^2}$$
$$16 = \frac{k}{6^2} \qquad (6)^2 + (25)^2 = c^2 \qquad I = \frac{576}{(6.5)^2}$$
$$576 = k \qquad 6.5 = c \qquad I \approx 13.6 \text{ lux}$$

39. a is doubled
$$a = \frac{kb^2}{c^3}$$
$$a = \frac{k\left(\frac{1}{2}b\right)^2}{\left(\frac{1}{2}c\right)^3}$$
$$a = \frac{\frac{1}{4}kb^2}{\frac{1}{8}c^3}$$
$$a = 2\frac{kb^2}{c^3}$$

40a. $F = G\frac{m_1 \cdot m_2}{d^2}$

40b.
$$F = G\frac{m_1 \cdot m_2}{d^2}$$
$$1.99 \times 10^{20} = G\frac{(5.98 \times 10^{24})(7.36 \times 10^{22})}{(3.84 \times 10^{8})^2}$$
$$6.67 \times 10^{-11} = G;\ 6.67 \times 10^{-11}\ \frac{\text{N} \cdot m^2}{\text{kg}^2}$$

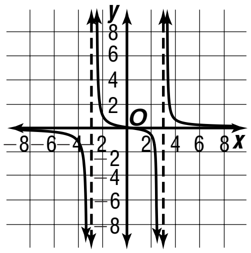
40c.
$$F = G\frac{m_1 \cdot m_2}{d^2}$$
$$= (6.67 \times 10^{-11})\ \frac{(5.98 \times 10^{24})(1.99 \times 10^{30})}{(1.50 \times 10^{11})^2}$$
$$\approx 3.53 \times 10^{22} \text{ N}$$

40d. $3.53 \times 10^{22} = (1.99 \times 10^{20})x$

$178 \approx x$; about 178 times greater

41.
$$R = \frac{kL}{\pi r^2} \qquad R = \frac{1.68 \times 10^{-8}(3)}{\pi(0.003)^2}$$
$$1.07 \times 10^{-2} = \frac{k \cdot 2}{\pi(0.001)^2} \qquad R = 1.78 \times 10^{-3}\ \Omega$$
$$1.68 \times 10^{-8} \approx k$$

42.



43.
$$f(x) = (x - 3)^3 + 6$$
$$y = (x - 3)^3 + 6$$
$$x = (y - 3)^3 + 6$$
$$x - 6 = (y - 3)^3$$
$$\sqrt[3]{x - 6} = y - 3$$
$$y = \sqrt[3]{x - 6} + 3$$
$$f^{-1}(x) = \sqrt[3]{x - 6} + 3;\ f^{-1}(x) \text{ is a function.}$$

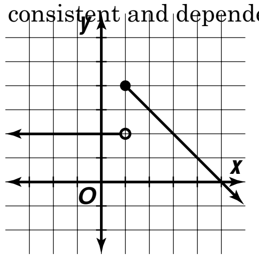
44.
$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 & -1 & -3 \\ 2 & -2 & -4 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -3 & 1 & 3 \\ 2 & -2 & -4 & 0 \end{bmatrix}$$

$A'(-1, 2)$, $B'(-$ $-2)$, $C'(1, -4)$, $D'(3, 0)$

45.
$$4x - 2y = 7 \quad \rightarrow \quad y = 2x - \frac{7}{2}$$
$$-12x + 6y = -21 \quad \rightarrow \quad y = 2x - \frac{7}{2}$$

consistent and dependent

46.



47.
$$m = \frac{18.6 - 23.2}{2000 - 1995} \qquad y - 18.6 = -0.92(x - 2000)$$
$$= \frac{-4.6}{5} \text{ or } -0.92 \qquad y = -0.92x + 1858.60$$

48. $144 = 4^2 \cdot 9$ or $12^2 \cdot 1$

12 is divisible by 3, 4, 6, and 12.

The correct choice is D.

Chapter 3 Study Guide and Assessment

Page 197 Understanding and Using the Vocabulary

1. even **2.** continuous **3.** point
4. decreasing **5.** maximum **6.** rational
7. inverse **8.** monotonic **9.** slant
10. Joint

Pages 198–200 Skills and Concepts

11. $f(-x) = -2(-x)$ $\quad -f(x) = -(-2x)$
$f(-x) = 2x$ $\quad -f(x) = 2x$ yes

12. $f(-x) = (-x)^2 + 2$ $\quad -f(x) = -(x^2 + 2)$
$f(-x) = x^2 + 2$ $\quad -f(x) = -x^2 - 2$ no

13. $f(-x) = (-x)^2 - (-x) + 3$
$f(-x) = x^2 + x + 3$
$-f(x) = -(x^2 - x + 3)$
$-f(x) = -x^2 + x - 3$ no

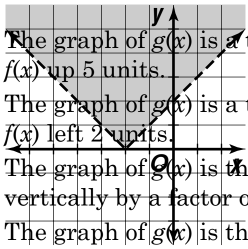
14. $f(-x) = (-x)^3 - 6(-x) + 1$
$f(-x) = -x^3 + 6x + 1$
$-f(x) = -(x^3 - 6x + 1)$
$-f(x) = -x^3 + 6x - 1$ no

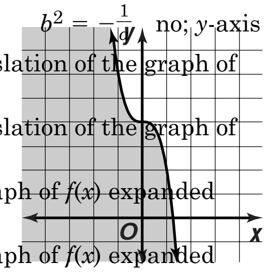
15. $xy = 4 \rightarrow ab = 4$
x-axis $\quad a(-b) = 4$
$-ab = 4$ no
y-axis $\quad (-a)b = 4$
$-ab = 4$ no
$y = x$ $\quad (b)(a) = 4$
$ab = 4$ yes
$y = -x$ $\quad (-b)(-a) = 4$
$ab = 4$ yes
$y = x$ and $y = -x$

16. $x + y^2 = 4 \rightarrow a + b^2 = 4$
x-axis $\quad a + (-b)^2 = 4$
$a + b^2 = 4$ yes
y-axis $\quad (-a) + b^2 = 4$
$-a + b^2 = 4$ no
$y = x$ $\quad (b) + (a)^2 = 4$
$a^2 + b = 4$ no
$y = -x$ $\quad (-b) + (-a)^2 = 4$
$a^2 - b = 4$ no
x-axis

17. $x = -2y \rightarrow a = -2b$
x-axis $\quad a = -2(-b)$
$a = 2b$ no
y-axis $\quad (-a) = -2b$
$a = 2b$ no
$y = x$ $\quad (b) = -2(a)$
$b = -2a$ no
$y = -x$ $\quad (-b) = -2(-a)$
$b = -2a$ no; none

18. $x^2 = \frac{1}{y} \rightarrow a^2 = \frac{1}{b}$
x-axis $\quad a^2 = \frac{1}{(-b)}$
$a^2 = -\frac{1}{b}$ no
y-axis $\quad (-a)^2 = \frac{1}{b}$
$a^2 = \frac{1}{b}$ yes
$y = x$ $\quad (b)^2 = \frac{1}{(a)}$
$b^2 = \frac{1}{a}$ no
$y = -x$ $\quad (-b)^2 = \frac{1}{(-a)}$
$b^2 = -\frac{1}{a}$ no; y-axis





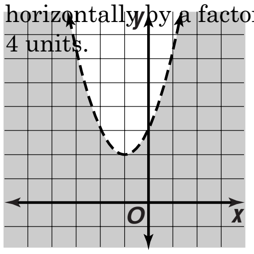
19. The graph of $g(x)$ is a translation of the graph of $f(x)$ up 5 units.

20. The graph of $g(x)$ is a translation of the graph of $f(x)$ left 2 units.

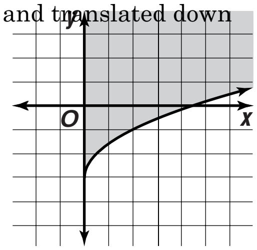
21. The graph of $g(x)$ is the graph of $f(x)$ expanded vertically by a factor of 6.

22. The graph of $g(x)$ is the graph of $f(x)$ expanded horizontally by a factor of $\frac{4}{3}$ and translated down 4 units.

23.



24.



25. **26.**

27. Case 1
$|4x + 5| > 7$
$-(4x + 5) > 7$
$-4x - 5 > 7$

Case 2
$|4x + 5| > 7$
$4x + 5 > 7$
$4x > 2$

$-4x > 12$
$x < -3$

$x > 0.5$

$\{x \mid x < -3 \text{ or } x > 0.5\}$

28. Case 1

$|x - 3| + 2 \le 11$
$-(x - 3) + 2 \le 11$
$-x + 5 \le 11$
$-x \le 6$
$x \ge -6$

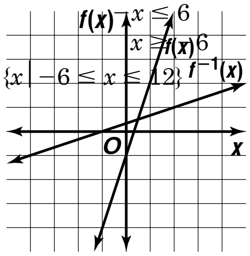
Case 2

$|x - 3| + 2 \le 11$
$x - 3 + 2 \le 11$
$x \le 12$

$\{x \mid -6 \le x \le 12\}$

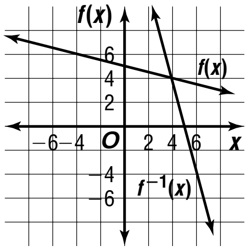
$f(x) = 3x - 1$	
x	$f(x)$
-2	-7
-1	-4
0	-1
1	2

$f^{-1}(x)$	
x	$f^{-1}(x)$
-7	-2
-4	-1
-1	0
2	1

29. 

$f(x) = -\frac{1}{4}x + 5$	
x	$f(x)$
-2	5.5
-1	5.25
0	5
1	4.75
2	4.5

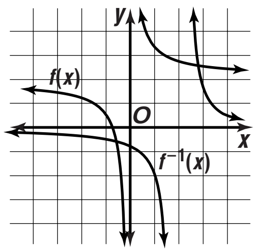
$f^{-1}(x)$	
x	$f^{-1}(x)$
5.5	-2
5.25	-1
5	0
4.75	1
4.5	2

30. 

31.

$f(x) = \frac{2}{x} + 3$	
x	$f(x)$
-3	2.3
-2	2
-1	1
$-\frac{1}{2}$	-1
0	—
$\frac{1}{2}$	7
1	5
2	4
3	3.7

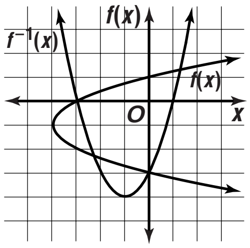
$f^{-1}(x)$	
x	$f^{-1}(x)$
2.3	-3
2	-2
1	-1
-1	$-\frac{1}{2}$
7	$\frac{1}{2}$
5	1
4	2
3.7	3



32.

$f(x) = (x + 1)^2 - 4$	
x	$f(x)$
-3	0
-2	-3
-1	-4
0	-3
1	0

$f^{-1}(x)$	
x	$f^{-1}(x)$
0	-3
-3	-2
-4	-1
-3	0
0	1



33.

$$\begin{aligned} f(x) &= (x - 2)^3 - 8 \\ y &= (x - 2)^3 - 8 \\ x &= (y - 2)^3 - 8 \\ x + 8 &= (y - 2)^3 \\ \sqrt[3]{x + 8} &= y - 2 \\ y &= \sqrt[3]{x + 8} + 2 \\ f^{-1}(x) &= \sqrt[3]{x + 8} + 2\text{; yes} \end{aligned}$$

34.

$$\begin{aligned} f(x) &= 3(x + 7)^4 \\ y &= 3(x + 7)^4 \\ x &= 3(y + 7)^4 \\ \frac{x}{3} &= (y + 7)^4 \\ \pm\sqrt[4]{\frac{x}{3}} &= y + 7 \\ y &= -7 \pm \sqrt[4]{\frac{x}{3}} \\ f^{-1}(x) &= -7 \pm \sqrt[4]{\frac{x}{3}}\text{; no} \end{aligned}$$

35. Yes; the function is defined when $x = 2$; the function approaches 6 as x approaches 2 from both sides; and $y = 6$ when $x = 2$.

36. No; the function is undefined when $x = -1$.

37. Yes; the function is defined when $x = 1$; the function approaches 2 as x approaches 1 from both sides; and $y = 2$ when $x = 1$.

38. a_n: negative, n: odd
$y \to -\infty$ as $x \to \infty$, $y \to \infty$ as $x \to -\infty$.

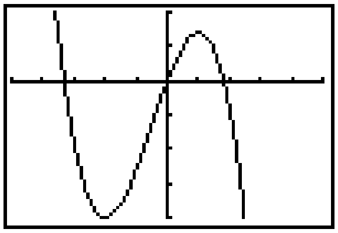
39. a_n: positive, n: odd
$y \to \infty$ as $x \to \infty$, $y \to -\infty$ as $x \to -\infty$.

40.

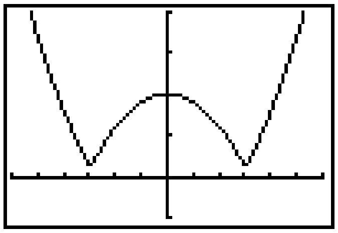
$y = \frac{1}{x^2} + 1$	
x	y
-1000	1.000001
-100	1.0001
-10	1.01
1	2
10	1.01
100	1.0001
1000	1.000001

$y \to 1$ as $x \to \infty$, $y \to 1$ as $x \to -\infty$.

41. a_n: positive, n: odd
$y \to \infty$ as $x \to \infty$, $y \to -\infty$ as $x \to -\infty$.

42. 

$[-5, 5]$ scl:1 by $[-20, 10]$ scl:5
decreasing for $x < -2$ and $x > 1$;
increasing for $-2 < x < 1$

43. 

$[-6, 6]$ scl:1 by $[-5, 20]$ scl:5
decreasing for $x < -3$ and $0 < x < 3$;
increasing for $-3 < x < 0$ and $x > 3$

44. abs. max.: $(-2, 1)$

45. rel. max.: $(0, 4)$, rel. min.: $(2, 0)$

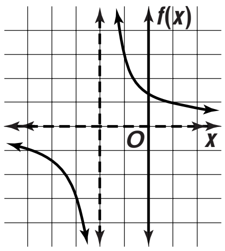
46. $f(2.9) = 0.029$
$f(3) = 0$
$f(3.1) = 0.031$ min.

47. $f(-0.1) = 6.996$
$f(0) = 7$
$f(0.1) = 7.004$ pt of inflection

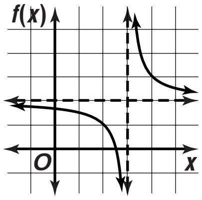
48. $f(x) = \frac{1}{x} + 1$

49. $f(x) = -\frac{2}{x}$

50. The parent graph is translated 2 units left and expanded vertically by a factor of 3. The vertical asymptote is now $x = -3$. The horizontal asymptote, $f(x) = 0$, is unchanged.



51. The parent graph is translated 3 units right and then translated 2 units up. The vertical asymptote is now $x = 3$ and the horizontal asymptote is $f(x) = 2$.



52. $x = 1$

$$y = \frac{x}{x-1}$$
$$y = \frac{\frac{x}{x}}{\frac{x}{x} - \frac{1}{x}}$$
$$y = \frac{1}{1 - \frac{1}{x}}; \quad y = 1$$

53. $x = -2$

$$y = \frac{x^2+1}{x+2}$$
$$y = \frac{\frac{x^2}{x^2} + \frac{1}{x^2}}{\frac{x}{x^2} + \frac{2}{x^2}}$$
$$y = \frac{1 + \frac{1}{x^2}}{\frac{1}{x} + \frac{2}{x^2}}$$

no horizontal asymptotes

54. $x = -3$,

$$y = \frac{(x-3)^2}{x^2-9}$$
$$y = \frac{x^2 - 6x + 9}{x^2 - 9}$$
$$y = \frac{\frac{x^2}{x^2} - \frac{6x}{x^2} + \frac{9}{x^2}}{\frac{x^2}{x^2} - \frac{9}{x^2}}$$
$$y = \frac{1 - \frac{6}{x} + \frac{9}{x^2}}{1 - \frac{9}{x^2}}; \; y = 1$$

55.

$$\begin{array}{r} x + 2 \\ x\overline{)x^2 + 2x + 1} \\ \underline{x^2 } \\ 2x \\ \underline{2x } \\ 1 \end{array} \quad \rightarrow \quad x + 2 + \frac{1}{x}$$

yes; $y = x + 2$

56.
$y = kxz$
$5 = k(-4)(-2)$
$0.625 = k$

$y = 0.625xz$
$y = 0.625(-6)(-3)$
$y = 11.25$

57.
$y = \frac{k}{\sqrt{x}}$
$20 = \frac{k}{\sqrt{49}}$
$140 = k$

$y = \frac{140}{\sqrt{x}}$
$10 = \frac{140}{\sqrt{x}}$
$\sqrt{x} = 14$
$x = 196$

58.
$y = \frac{kx^2}{z}$
$7.2 = \frac{k(0.3)^2}{4}$
$320 = k$

$y = \frac{320x^2}{z}$
$y = \frac{320(1)^2}{40}$
$y = 8$

Page 201 Applications and Problem Solving

59. $|x - 6.5| \leq 0.2$;

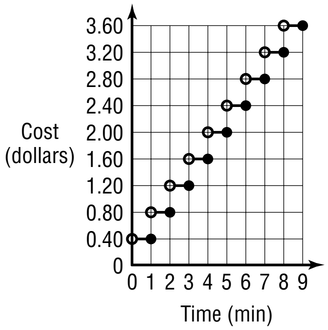
Case 1
$|x - 6.5| \leq 0.2$
$-(x - 6.5) \leq 0.2$
$-x + 6.5 \leq 0.2$
$-x \leq -6.3$
$x \geq 6.3$

Case 2
$|x - 6.5| \leq 0.2$
$x - 6.5 \leq 0.2$
$x \leq 6.7$

$6.3 \leq x \leq 6.7$

60a.

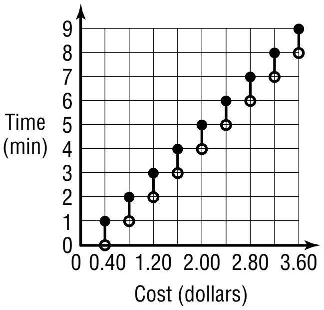
x	$C(x)$
$0 < x \le 1$	0.40
$1 < x \le 2$	0.80
$2 < x \le 3$	1.20
$3 < x \le 4$	1.60
$4 < x \le 5$	2.00
$5 < x \le 6$	2.40



60b. positive real numbers;
positive multiples of \$0.40

60c.

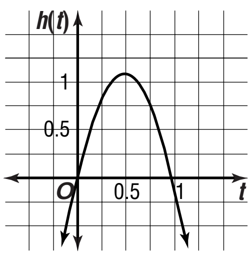
x	$C^{-1}(x)$
0.40	$0 < x \le 1$
0.80	$1 < x \le 2$
1.20	$2 < x \le 3$
1.60	$3 < x \le 4$
2.00	$4 < x \le 5$
2.40	$5 < x \le 6$



60d. positive multiples of \$0.40;
positive real numbers

60e. $C^{-1}(x)$ gives the possible number of minutes spent using the scanner that cost x dollars.

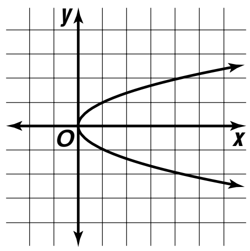
61a.



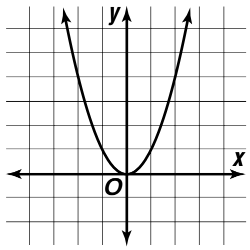
61b. 1.08 m

Page 201 Open-Ended Assessment

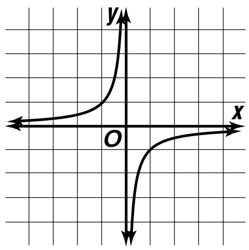
1a. Sample answer: $x = y^2$



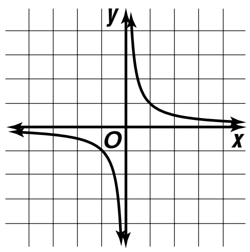
1b. Sample answer: $y = x^2$



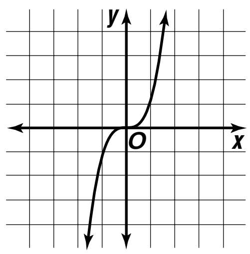
1c. Sample answer: $-xy = 1$



1d. Sample answer: $xy = 1$

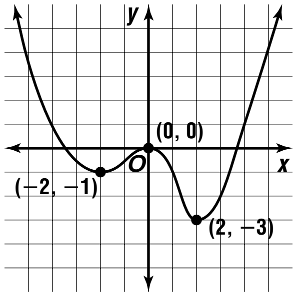


1e. Sample answer: $y = x^3$



2. Sample answer: $-2(x - 4)^2 + 1$

3a. Sample answer:



3b. abs. min.: $(2, -3)$; rel. max.: $(0, 0)$; rel. min.: $(-2, -1)$

Chapter 3 SAT & ACT Preparation

Page 203 SAT & ACT Practice

1. Always factor or simplify algebraic expressions when possible. Notice that the numerator in the problem is the difference of two squares, $a^2 - b^2$. Factor it.

$\frac{y^2 - 9}{3y - 9} = \frac{(y + 3)(y - 3)}{3y - 9}$

Factor the denominator. Both the numerator and denominator contain the factor $(y - 3)$. Simplify the fraction.

$\frac{(y + 3)(y - 3)}{3y - 9} = \frac{(y + 3)(y - 3)}{3(y - 3)} = \frac{y + 3}{3}$

The correct answer is E.

2. You need to find the statement that is *not* true. Compare the given information with each answer choice. Choice A looks like $x + y = z$, except for the numbers. Multiply both sides of the equation $x + y = z$ by 2.

$2(x + y) = 2z$ or $2x + 2y = 2z$

So choice A is true. For choice B, start with $x = y$ and subtract y from each side.

$x - y = y - y = 0$

So choice B is true. For choice C, start with $x = y$ and subtract z from each side.

$x - z = y - z$

So choice C is true. For choice D, substitute y for x and $x + y$ for z.

$x = \frac{z}{2}$

$y = \frac{x + y}{2} = \frac{2y}{2}y$

So choice D is also true. For choice E, write each side of the equation in terms of y.

$z - y = (x + y) - x = y$

$2x = 2y$

$y \neq 2y$

So choice E is *not* true. The correct choice is E.

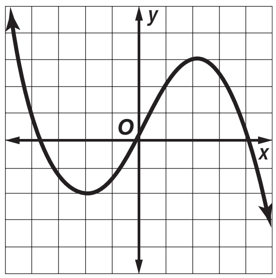
3. Notice that 450 miles is the distance to Grandmother's house, not the round trip. This is a multiple-step problem. First calculate the number of gallons of gasoline used in each direction of the trip.

$\frac{\text{miles}}{\text{miles per gallon}} = \text{gallons}$

$\frac{450}{25} = 18 \text{ gallons}$

On the trip to Grandmother's, the cost of gasoline is 18 gallons × \$1.25 per gallon or \$22.50.
On the trip back, the gasoline cost is 18 gallons × \$1.50 per gallon or \$27.00. The difference between the costs is \$4.50.
A faster way to find the cost difference is to reason that each gallon cost \$0.25 more on the trip back. So the total amount more that was paid was 18 gallons × \$0.25 or \$4.50. The correct choice is B.

4.



The portion of the graph of $f(x)$ which is shown crosses the x-axis 3 times.
The correct choice is D.

5. Notice that the denominators are all powers of ten. Carefully convert each fraction to a decimal. Then add the three decimals.

$\frac{900}{10} + \frac{90}{100} + \frac{9}{1000} = 90 + 0.9 + 0.009 = 90.909$

The correct choice is C. You could also use your calculator on this problem.

6. Combine like terms.

$(10x^4 - x^2 + 2x - 8) - (3x^4 + 3x^3 + 2x + 9)$
$= (10x^4 - 3x^4) + (-3x^3) + (-x^2) + (2x - 2x) + (-8 - 9)$
$= 7x^4 - 3x^3 - x^2 + 0 - 17$

The correct choice is A.

7. One method of solving this problem is to "plug in" a number in place of n. Choose a number that when divided by 8, has a remainder of 5. For example, choose 21.

$21 = 2(8) + 5$

Then use this value for n in the answer choices. Find the expression that has a remainder of 7.

Choice A: $\frac{n + 1}{8} = \frac{21 + 1}{8} = \frac{22}{8} = 2\text{R}6$

The remainder is 6.

Choice B: $\frac{n + 2}{8} = \frac{21 + 2}{8} = \frac{23}{8} = 2\text{R}7$

You could also reason that since n divided by 8 has a remainder of 5, then $(n + 2)$ divided by 8 will have a remainder of $(5 + 2)$ or 7. The correct choice is B.

8. Simplify the expression inside the square root symbol. Factor 100 from each term. Then factor the trinomial.

$$\frac{\sqrt{100x^2 + 600x + 900}}{x + 3} = \frac{\sqrt{100(x^2 + 6x + 9)}}{x + 3}$$
$$= \frac{10\sqrt{x^2 + 6x + 9}}{x + 3}$$
$$= \frac{10\sqrt{(x + 3)(x + 3)}}{x + 3}$$
$$= \frac{10\sqrt{(x + 3)^2}}{x + 3}$$
$$= \frac{10(x + 3)}{x + 3}$$
$$= 10$$

The correct choice is B.

9. Since $a + b = c$, substitute $a + b$ for c in $a - c = 5$. So, $a - (a + b) = 5$. Then $-b = 5$ or $b = -5$. Substitute -5 for b in $b - c = 3$. So, $-5 - c = 3$. Then $-c = 8$ or $c = -8$.
The correct choice is B.

10. There are two equations and two variables, so this is a system of equations. First simplify the equations. Start with the first equation. Divide both sides by 2.

$4x + 2y = 24$
$2x + y = 12$

Now simplify the second equation. Multiply both sides by $2x$.

$\frac{7y}{2x} = 7$
$7y = 7(2x)$
$7y = 14x$

Divide both sides by 7.

$y = 2x$

You need to find the value of x. Substitute $2x$ for y in the first equation.

$2x + y = 12$
$2x + (2x) = 12$
$4x = 12$
$x = 3$

The answer is 3.

Chapter 4 Polynomial and Rational Functions

4-1 Polynomial Functions

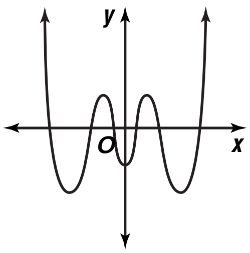
Pages 209–210 Check for Understanding

1. A zero is the value of the variable for which a polynomial function in one variable equals zero. A root is a solution of a polynomial equation in one variable. When a polynomial function is the related function to the polynomial equation, the zeros of the function are the same as the roots of the equation.

2. The ordered pair $(x, 0)$ represents the points on the x-axis. Therefore, the x-intercept of a graph of a function represents the point where $f(x) = 0$.

3. A complex number is any number in the form $a + b\boldsymbol{i}$, where a and b are real numbers and $\boldsymbol{i}$ is the imaginary unit. In a pure imaginary number, $a = 0$ and $b \neq 0$. Examples: $-2\boldsymbol{i}$, $3\boldsymbol{i}$; Nonexamples: 5, $1 + \boldsymbol{i}$

4.



5. 3; 1 **6.** 5; 8

7. no; $f(x) = x^3 - 5x^2 - 3x - 18$

$$f(5) = (5)^3 - 5(5)^2 - 3(5) - 18$$
$$f(5) = 125 - 125 - 15 - 18$$
$$f(5) = -33$$

8. yes; $f(x) = x^3 - 5x^2 - 3x - 18$

$$f(6) = (6)^3 - 5(6)^2 - 3(6) - 18$$
$$f(6) = 216 - 180 - 18 - 18$$
$$f(6) = 0$$

9. $(x - (-5))(x - 7) = 0$

$$(x + 5)(x - 7) = 0$$
$$x^2 - 2x - 35 = 0\text{; even; 2}$$

10. $(x - 6)(x - 2\boldsymbol{i})(x - (-2\boldsymbol{i}))(x - \boldsymbol{i})\,(x - (-\boldsymbol{i})) = 0$

$$(x - 6)(x - 2\boldsymbol{i})(x + 2\boldsymbol{i})(x - \boldsymbol{i})(x + \boldsymbol{i}) = 0$$
$$(x - 6)(x^2 - 4\boldsymbol{i}^2)(x^2 - \boldsymbol{i}^2) = 0$$
$$(x - 6)(x^2 + 4)(x^2 + 1) = 0$$
$$(x^3 - 6x^2 + 4x - 24)(x^2 + 1) = 0$$
$$x^5 - 6x^4 + 5x^3 - 30x^2 + 4x - 24 = 0;$$

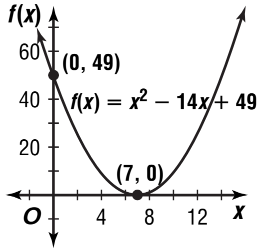
odd; 1

11. 2; $x^2 - 14x + 49 = 0$

$$(x - 7)(x - 7) = 0$$

$x - 7 = 0$ $\qquad$ $x - 7 = 0$

$x = 7$ $\qquad$ $x = 7$

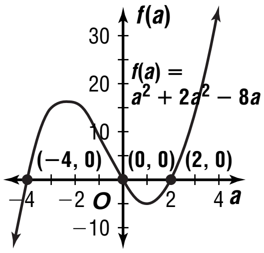


12. 3; $a^3 + 2a^2 - 8a = 0$

$$a(a^2 + 2a - 8) = 0$$
$$a(a + 4)(a - 2) = 0$$

$a = 0$ $\qquad$ $a + 4 = 0$ $\qquad$ $a - 2 = 0$

$a = -4$ $\qquad$ $a = 2$



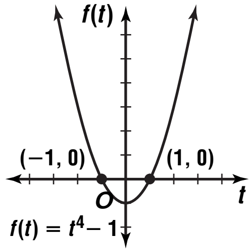
13. 4; $t^4 - 1 = 0$

$$(t^2 - 1)(t^2 + 1) = 0$$
$$(t - 1)(t + 1)(t^2 + 1) = 0$$

$t - 1 = 0$ $\qquad$ $t + 1 = 0$ $\qquad$ $t^2 + 1 = 0$

$t = 1$ $\qquad$ $t = -1$ $\qquad$ $t^2 = -1$

$t = \pm\boldsymbol{i}$



14a. $x^2 + r^2 = 6^2$ $\qquad$ $V(x) = Bh$

$r^2 = 36 - x^2$ $\qquad$ $V(x) = \pi(36 - x^2)(2x)$

14b. $V(x) = \pi(36 - x^2)(2x)$

$$V(x) = (36\pi - \pi x^2)(2x)$$
$$V(x) = 72\pi x - 2\pi x^3$$

14c. $V(x) = 72\pi x - 2\pi x^3$

$$V(4) = 72\pi(4) - 2\pi(4)^3$$
$$V(4) \approx 502.65 \text{ units}^3$$

Pages 210–212 Exercises

15. 4; 5 **16.** 7; 3 **17.** 3; 5 **18.** 5; −25

19. 6; −1 **20.** 2; 1

21. Yes; the coefficients are complex numbers and the exponents of the variable are nonnegative integers.

22. No; $\frac{1}{a} = a^{-1}$, which is a negative exponent.

23. yes; $f(a) = a^4 - 13a^2 + 12a$
$f(0) = (0)^4 - 13(0)^2 + 12(0)$
$f(0) = 0$

24. no; $f(a) = a^4 - 13a^2 + 12a$
$f(-1) = (-1)^4 - 13(-1)^2 + 12(-1)$
$f(-1) = 1 - 13 - 12$
$f(-1) = -24$

25. yes; $f(a) = a^4 - 13a^2 + 12a$
$f(1) = (1)^4 - 13(1)^2 + 12(1)$
$f(1) = 1 - 13 + 12$
$f(1) = 0$

26. yes; $f(a) = a^4 - 13a^2 + 12a$
$f(-4) = (-4)^4 - 13(-4)^2 + 12(-4)$
$f(-4) = 256 - 208 - 48$
$f(-4) = 0$

27. no; $f(a) = a^4 - 13a^2 + 12a$
$f(-3) = (-3)^4 - 13(-3)^2 + 12(-3)$
$f(-3) = 81 - 117 - 36$
$f(-3) = -72$

28. yes; $f(a) = a^4 - 13a^2 + 12a$
$f(3) = (3)^4 - 13(3)^2 + 12(3)$
$f(3) = 81 - 117 + 36$
$f(3) = 0$

29. $f(b) = b^4 - 3b^2 - 2b + 4$
$f(-2) = (-2)^4 - 3(-2)^2 - 2(-2) + 4$
$f(-2) = 16 - 12 + 4 + 4$
$f(-2) = 12$; no

30. $f(x) = x^4 - 4x^3 - x^2 + 4x$
$f(-1) = (-1)^4 - 4(-1)^3 - (-1)^2 + 4(-1)$
$f(-1) = 1 + 4 - 1 - 4$
$f(-1) = 0$; yes

31a. 3; 1 **31b.** 2; 2 **31c.** 4; 2

32. $(x - (-2))(x - 3) = 0$
$(x + 2)(x - 3) = 0$
$x^2 - x - 6 = 0$; even; 2

33. $(x - (-1))(x - 1)(x - 5) = 0$
$(x + 1)(x - 1)(x - 5) = 0$
$(x^2 - 1)(x - 5) = 0$
$x^3 - 5x^2 - x + 5 = 0$; odd; 3

34. $(x - (-2))(x - (-0.5))(x - 4) = 0$
$(x + 2)(x + 0.5)(x - 4) = 0$
$(x^2 + 2.5x + 1)(x - 4) = 0$
$x^3 - 4x^2 + 2.5x^2 - 10x + x - 4 = 0$
$x^3 - 1.5x^2 - 9x - 4 = 0$
$2x^3 - 3x^2 - 18x - 8 = 0$; odd; 3

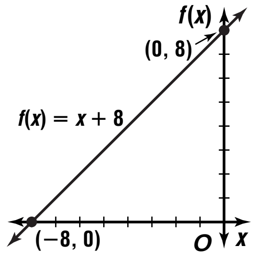
35. $(x - (-3))(x - (-2\boldsymbol{i}))(x - 2\boldsymbol{i}) = 0$
$(x + 3)(x + 2\boldsymbol{i})(x - 2\boldsymbol{i}) = 0$
$(x + 3)(x^2 - 4\boldsymbol{i}^2) = 0$
$(x + 3)(x^2 + 4) = 0$
$x^3 + 3x^2 + 4x + 12 = 0$; odd; 1

36. $(x - (-5\boldsymbol{i}))(x - (-\boldsymbol{i}))(x - \boldsymbol{i})(x - 5\boldsymbol{i}) = 0$
$(x + 5\boldsymbol{i})(x + \boldsymbol{i})(x - \boldsymbol{i})(x - 5\boldsymbol{i}) = 0$
$(x + 5\boldsymbol{i})(x - 5\boldsymbol{i})(x - \boldsymbol{i})(x + \boldsymbol{i}) = 0$
$(x^2 + 25)(x^2 + 1) = 0$
$x^4 + 26x^2 + 25 = 0$; even; 0

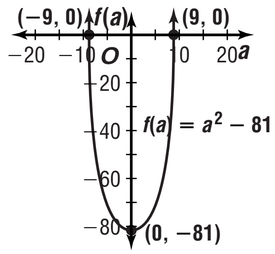
37. $(x - (-1))(x - 1)(x - 4)(x - (-4)(x - 5) = 0$
$(x + 1)(x - 1)(x - 4)(x + 4)(x - 5) = 0$
$(x^2 - 1)(x^2 - 16)(x - 5) = 0$
$(x^4 - 17x^2 + 16)(x - 5) = 0$
$x^5 - 5x^4 - 17x^3 + 85x^2 + 16x - 80 = 0$; odd; 5

38. $(x - (-1))(x - 1)(x - 3)(x - (-3)) = 0$
$(x + 1)(x - 1)(x - 3)(x + 3) = 0$
$(x^2 - 1)(x^2 - 9) = 0$
$x^4 - 10x^2 + 9 = 0$

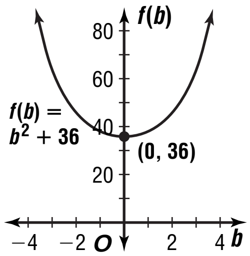
39. 1; $x + 8 = 0$
$x = -8$



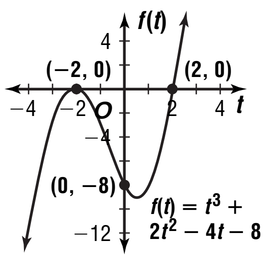
40. 2; $a^2 - 81 = 0$
$(a - 9)(a + 9) = 0$
$a - 9 = 0$ $a + 9 = 0$
$a = 9$ $a = -9$



41. 2; $b^2 + 36 = 0$
$b^2 = -36$
$b = \pm 6\boldsymbol{i}$



42. 3; $t^3 + 2t^2 - 4t - 8 = 0$
$t^2(t + 2) - 4(t + 2) = 0$
$(t + 2)(t^2 - 4) = 0$
$(t + 2)(t + 2)(t - 2) = 0$
$t + 2 = 0$ $t + 2 = 0$ $t - 2 = 0$
$t = -2$ $t = -2$ $t = 2$



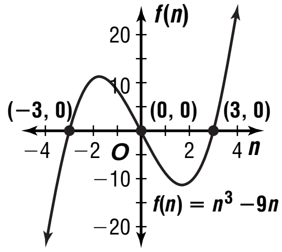
43. 3; $n^3 - 9n = 0$

$n(n^2 - 9) = 0$

$n(n - 3)(n + 3) = 0$

$n = 0 \quad n - 3 = 0 \quad n + 3 = 0$

$n = 3 \quad n = -3$



44. 3; $6c^3 - 3c^2 - 45c = 0$

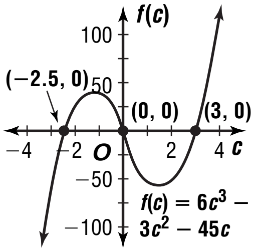
$c(6c^2 - 3c - 45) = 0$

$c(c - 3)(6c + 15) = 0$

$c = 0 \quad c - 3 = 0 \quad 6c + 15 = 0$

$c = 3 \quad c = -\frac{15}{6}$

$c = -2.5$



45. 4; $a^4 + a^2 - 2 = 0$

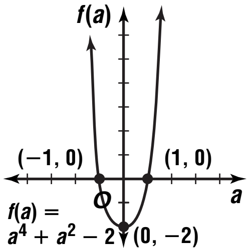
$(a^2 + 2)(a^2 - 1) = 0$

$(a^2 + 2)(a - 1)(a + 1) = 0$

$a^2 + 2 = 0 \quad a - 1 = 0 \quad a + 1 = 0$

$a^2 = -2 \quad a = 1 \quad a = -1$

$a = \pm\sqrt{2}i$



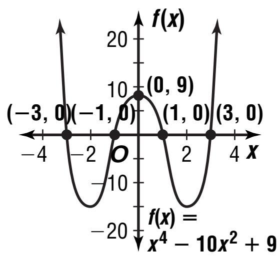
46. 4; $x^4 - 10x^2 + 9 = 0$

$(x^2 - 9)(x^2 - 1) = 0$

$(x - 3)(x + 3)(x - 1)(x + 1) = 0$

$x - 3 = 0 \quad x + 3 = 0 \quad x - 1 = 0 \quad x + 1 = 0$

$x = 3 \quad x = -3 \quad x = 1 \quad x = -1$



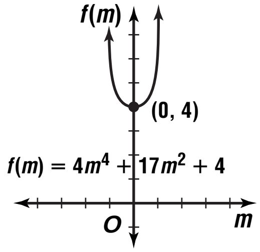
47. 4; $4m^4 + 17m^2 + 4 = 0$

$(4m^2 + 1)(m^2 + 4) = 0$

$4m + 1 = 0 \quad m^2 + 4 = 0$

$m = \pm\sqrt{\frac{-1}{4}} \quad m = \pm\sqrt{-4}$

$m = \pm 0.5i \quad m = \pm 2i$

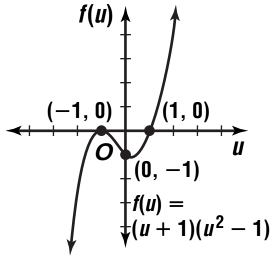


48. $(u + 1)(u^2 - 1) = 0$

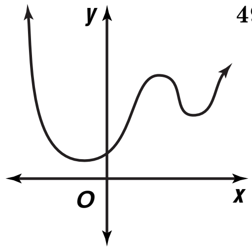
$(u + 1)(u + 1)(u - 1) = 0$

$u + 1 = 0 \quad u + 1 = 0 \quad u - 1 = 0$

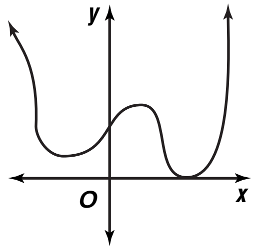
$u = -1 \quad u = -1 \quad u = 1$



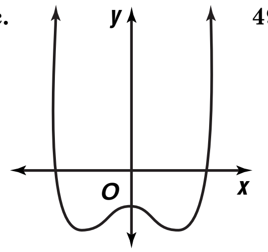
49a.



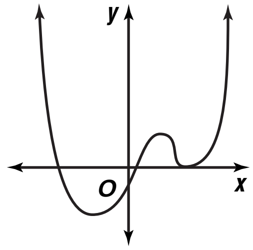
49b.



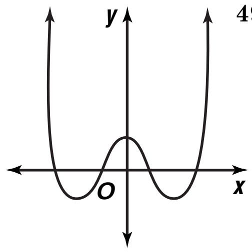
49c.



49d.

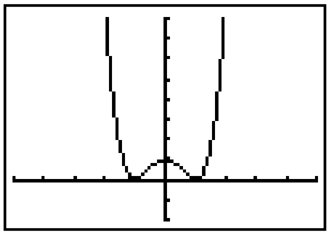


49e.



49f. not possible

50.



$[-5, 5]$ sc1:1 by $[-2, 8]$ sc1:1

50a. 4

50b. 2; -1, 1

50c. There are 4 real roots. However, there is a double root at -1 and a double root at 1.

51a. $V(x) = 99{,}000x^3 + 55{,}000x^2 + 65{,}000x$

51b. $r = 0.15$

$x = 1 + r$
$x = 1 + 0.15$
$x = 1.15$

$$V(x) = 99{,}000x^3 + 55{,}000x^2 + 65{,}000x$$
$$V(1.15) = 99{,}000(1.15)^3 + 55{,}000(1.15)^2 + 65{,}000(1.15)$$
$$V(1.15) = 150{,}566.625 + 72{,}737.5 + 74{,}750$$
$$V(1.15) = 298{,}054.125;\ \text{about } \$298{,}054.13$$

52. 1 and 3 are two of its zeros.

53a.
$$d(t) = \tfrac{1}{2}at^2 \qquad d(t) = \tfrac{1}{2}at^2$$
$$d(30) = \tfrac{1}{2}(16.4)(30)^2 \qquad d(60) = \tfrac{1}{2}(16.4)(60)^2$$
$$d(30) = 7380 \text{ ft} \qquad d(60) = 29{,}520 \text{ ft}$$
$$d(t) = \tfrac{1}{2}at^2$$
$$d(120) = \tfrac{1}{2}(16.4)(120)^2$$
$$d(120) = 118{,}080 \text{ ft}$$

53b. It quadruples; $(2t)^2 = 4t^2$.

54. Let x = the width of the sidewalk.
The length of the pool would be $70 - 2x$ feet.
The width of the pool would be $50 - 2x$ feet.
$$A = \ell w$$
$$2400 = (70 - 2x)(50 - 2x)$$
$$2400 = 3500 - 240x + 4x^2$$
$$0 = 4x^2 - 240x + 1100$$
$$0 = x^2 - 60x + 275$$
$$0 = (x - 55)(x - 5)$$
$$x - 55 = 0 \qquad x - 5 = 0$$
$$x = 55 \qquad x = 5$$
Use $x = 5$ since 55 is an unreasonable solution.
5 ft

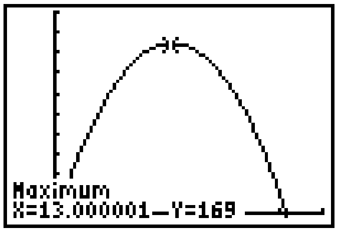
55. Let x = the number of pizzas.
$$(160 + 16x)(16 - 0.40x) = 4000$$
$$-6.4x^2 + 192x + 2560 = 4000$$
$$-6.4x^2 + 192x - 1440 = 0$$
$$x^2 - 30x - 225 = 0$$
$$(x - 15)(x - 15) = 0$$
$$x - 15 = 0 \qquad x - 15 = 0$$
$$x = 15 \qquad x = 15$$
$$16 - 0.40x = 16 - 0.40(15) = \$10$$

56.
$$(x - B)(x - C) = 0$$
$$x^2 - Cx - Bx + BC = 0$$
$$x^2 - (C + B)x + BC = 0$$
$$\left.\begin{array}{l} -C - B = B \\ BC = C \end{array}\right\} \text{from } x^2 + Bx + C = 0$$
$$B = 1$$
$$-C - 1 = 1$$
$$-C = 2$$
$$C = -2$$
Sample answer: 1; -2

57. $y = \frac{x - 2}{x(x + 2)(x - 2)}$

58a. Let x = the width. The length $= \frac{1}{2}(52 - 2x)$ or $26 - x$. $A(x) = x(26 - x)$

58b. $A(x) = x(26 - x)$
$A(x) = 26x - x^2$



$[-5, 30]$ sc1:5 by $[-2, 200]$ sc1:20
$x = 13$
$26 - x = 26 - 13$
$= 13$
13 yd by 13 yd

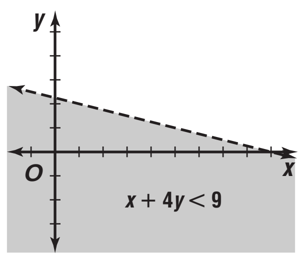
59. The graph of $y = 2x^3 + 1$ is the graph of $y = 2x^3$ shifted 1 unit up.

60. $(-6, 9)$

61. $\begin{vmatrix} -15 & 5 \\ -9 & 3 \end{vmatrix} = -15(3) - (-9)(5) = -45 + 45$ or 0; no

62.
$$AB = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & -9 & 2 \\ 5 & 7 & -6 \end{bmatrix}$$
$$= \begin{bmatrix} 2(3) + (-1)(5) & 2(-9) + (-1)(7) & 2(2) + (-1)(-6) \\ 3(3) + 4(5) & 3(-9) + 4(7) & 3(2) + 4(-6) \end{bmatrix}$$
$$= \begin{bmatrix} 1 & -25 & 10 \\ 29 & 1 & -18 \end{bmatrix}$$

63. $x + 4y < 9$
$4y < -x + 9$
$y < -\frac{1}{4}x + \frac{9}{4}$



64. Parallel; the lines have the same slope.

65. $[f \circ g](x) = f(g(x))$

$$\begin{aligned} &= f\left(\tfrac{1}{2}x + 6\right) \\ &= \left(\tfrac{1}{2}x + 6\right)^2 - 4 \\ &= \tfrac{1}{4}x^2 + 6x + 36 - 4 \\ &= \tfrac{1}{4}x^2 + 6x + 32 \end{aligned}$$

$[g \circ f](x) = g(f(x))$

$$\begin{aligned} &= g(x^2 - 4) \\ &= \tfrac{1}{2}(x^2 - 4) + 6 \\ &= \tfrac{1}{2}x^2 - 2 + 6 \\ &= \tfrac{1}{2}x^2 + 4 \end{aligned}$$

66. The pictograph shows two more small car symbols in the row for 1999 than it does for 2000. These two small cars represent the 270 additional cars that were sold in 1999 compared to 2000. Since the two small cars represent 270 real cars, each small car symbol must represent $\frac{270}{2}$ or 135 real cars.
The correct choice is A.

4-2 Quadratic Equations

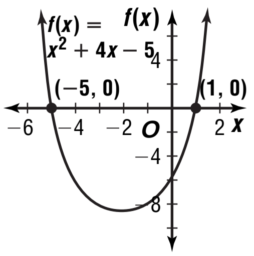
Pages 218–219 Check for Understanding

1. Add 4 to each side of the equation to get $t^2 - 6t = 4$. Determine the value needed to make $t^2 - 6t$ a perfect square trinomial. Add this value (9) to each side. Take the square root of each side of the equation and solve the two resulting equations. $t = 3 \pm \sqrt{13}$
2. Quadratic Formula; Since the leading coefficient does not equal 1 and the discriminant equals 185 which is not a perfect square, the Quadratic Formula would be the best way to get an exact answer. Completing the square can also be used, but errors in arithmetic are more likely. A graph will give only approximate solutions.
$p = \frac{13 \pm \sqrt{(-13)^2 - 4(5)(7)}}{2(5)}$ or $\frac{13 \pm \sqrt{29}}{10}$

3a. equals 0 **3b.** negative number

3c. positive number

4. Graphing



Factoring

$x^2 + 4x - 5 = 0$

$(x + 5)(x - 1) = 0$

$x + 5 = 0 \qquad x - 1 = 0$

$x = -5 \qquad x = 1$

Completing the Square

$x^2 + 4x - 5 = 0$

$x^2 + 4x = 5$

$x^2 + 4x + 4 = 5 + 4$

$(x + 2)^2 = 9$

$x + 2 = \pm 3$

$x + 2 = 3 \qquad x + 2 = -3$

$x = 1 \qquad x = -5$

Quadratic Formula

$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-5)}}{2(1)}$

$x = \frac{-4 \pm \sqrt{36}}{2}$

$x = \frac{-4 \pm 6}{2}$

$x = -2 \pm 3$

$x = -2 + 3 \qquad x = -2 - 3$

$x = 1 \qquad x = -5$

See students' work.

5. $x^2 + 8x - 20 = 0$

$x^2 + 8x = 20$

$x^2 + 8x + 16 = 20 + 16$

$(x + 4)^2 = 36$

$x + 4 = \pm 6$

$x + 4 = 6 \qquad x + 4 = -6$

$x = 2 \qquad x = -10$

6. $2a^2 + 11a - 21 = 0$

$a^2 + \frac{11}{2}a - \frac{21}{2} = 0$

$a^2 + \frac{11}{2}a = \frac{21}{2}$

$a^2 + \frac{11}{2}a + \frac{121}{16} = \frac{21}{2} + \frac{121}{16}$

$\left(a + \frac{11}{4}\right)^2 = \frac{289}{16}$

$a + \frac{11}{4} = \pm\frac{17}{4}$

$a + \frac{11}{4} = \frac{17}{4} \qquad a + \frac{11}{4} = -\frac{17}{4}$

$a = \frac{3}{2} \qquad a = -7$

7. $b^2 - 4ac = 12^2 - 4(1)(36)$

$= 0$; 1 real

$m = -12 \pm \frac{\sqrt{0}}{2(1)}$

$m = -\frac{12}{2}$ or -6

8. $b^2 - 4ac = (-6)^2 - 4(1)(13)$

$= -16$; 2 imaginary

$t = \frac{-(-6) \pm \sqrt{-16}}{2(1)}$

$= \frac{6 \pm 4\boldsymbol{i}}{2}$

$= 3 \pm 2\boldsymbol{i}$

9. $p^2 - 6p + 5 = 0$

$(p - 5)(p - 1) = 0$

$p - 5 = 0 \qquad p - 1 = 0$

$p = 5 \qquad p = 1$

10. $r^2 - 4r + 10 = 0$

$r = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(10)}}{2(1)}$

$r = \frac{4 \pm \sqrt{-24}}{2}$

$r = \frac{4 \pm 2\boldsymbol{i}\sqrt{6}}{2}$

$r = 2 \pm \boldsymbol{i}\sqrt{6}$

11.
$$P = 12I - 0.02I^2$$
$$1600 = 12I - 0.02I^2$$
$$0.02I^2 - 12I + 1600 = 0$$
$$I^2 - 600I + 80{,}000 = 0$$
$$(I - 200)(I - 400) = 0$$
$I - 200 = 0$ $\quad$ $I - 400 = 0$

$I = 200$ amps $\quad$ $I = 400$ amps

Pages 219–221 Exercises

12. $z^2 - 2z - 24 = 0$
$$z^2 - 2z = 24$$
$$z^2 - 2z + 1 = 24 + 1$$
$$(z - 1)^2 = 25$$
$$z - 1 = \pm 5$$
$z - 1 = 5$ $\quad$ $z - 1 = -5$

$z = 6$ $\quad$ $z = -4$

13. $p^2 - 3p - 88 = 0$
$$p^2 - 3p = 88$$
$$p^2 - 3p + \frac{9}{4} = 88 + \frac{9}{4}$$
$$\left(p - \frac{3}{2}\right)^2 = \frac{361}{4}$$
$p - \frac{3}{2} = \frac{19}{2}$ $\quad$ $p - \frac{3}{2} = -\frac{19}{2}$

$p = 11$ $\quad$ $p = -8$

14. $x^2 - 10x + 21 = 0$
$$x^2 - 10x = -21$$
$$x^2 - 10x + 25 = -21 + 25$$
$$(x - 5)^2 = 4$$
$$x - 5 = \pm 2$$
$x - 5 = 2$ $\quad$ $x - 5 = -2$

$x = 7$ $\quad$ $x = 3$

15. $d^2 - \frac{3}{4}d + \frac{1}{8} = 0$
$$d^2 - \frac{3}{4}d = -\frac{1}{8}$$
$$d^2 - \frac{3}{4}d + \frac{9}{64} = -\frac{1}{8} + \frac{9}{64}$$
$$\left(d - \frac{3}{8}\right)^2 = \frac{1}{64}$$
$$d - \frac{3}{8} = \pm\frac{1}{8}$$
$d - \frac{3}{8} = \frac{1}{8}$ $\quad$ $d - \frac{3}{8} = -\frac{1}{8}$

$d = \frac{1}{2}$ $\quad$ $d = \frac{1}{4}$

16. $3g^2 - 12g = -4$
$$g^2 - 4g = -\frac{4}{3}$$
$$g^2 - 4g + 4 = -\frac{4}{3} + 4$$
$$(g - 2)^2 = \frac{8}{3}$$
$$g - 2 = \pm\frac{2\sqrt{6}}{3}$$
$$g = 2 \pm \frac{2\sqrt{6}}{3}$$

17. $t^2 - 3t - 7 = 0$
$$t^2 - 3t = 7$$
$$t^2 - 3t + \frac{9}{4} = 7 + \frac{9}{4}$$
$$\left(t - \frac{3}{2}\right)^2 = \frac{37}{4}$$
$$t - \frac{3}{2} = \pm\frac{\sqrt{37}}{2}$$
$$t = \frac{3}{2} \pm \frac{\sqrt{37}}{2}$$

18. $\left(-\frac{1}{2}\right)^2 = \frac{1}{4}$

19. $b^2 - 4ac = (6)^2 - 4(4)(25)$
$$= -364$$
2 imaginary; the discriminant is negative.

20. $b^2 - 4ac = 7^2 - 4(6)(-3)$ or 121; 2 real
$$m = \frac{-7 \pm \sqrt{121}}{2(6)}$$
$$m = \frac{-7 \pm 11}{12}$$
$$m = -\frac{3}{2}, \frac{1}{3}$$

21. $b^2 - 4ac = (-5)^2 - 4(1)(9)$ or -11; 2 imaginary
$$s = \frac{5 \pm \sqrt{-11}}{2(1)}$$
$$s = \frac{5 \pm i\sqrt{11}}{2}$$

22. $b^2 - 4ac = (-84)^2 - 4(36)(49)$ or 0; 1 real
$$d = \frac{84 \pm \sqrt{0}}{2(36)}$$
$$d = \frac{84}{72} \text{ or } \frac{7}{6}$$

23. $b^2 - 4ac = (-2)^2 - 4(4)(9)$ or -140; 2 imaginary
$$x = \frac{2 \pm \sqrt{-140}}{2(4)}$$
$$x = \frac{2 \pm 2i\sqrt{35}}{8}$$
$$x = \frac{1 \pm i\sqrt{35}}{4}$$

24. $3p^2 + 4p = 8$
$$3p^2 + 4p - 8 = 0$$
$b^2 - 4ac = 4^2 - 4(3)(-8)$ or 112; 2 real
$$p = \frac{-4 \pm \sqrt{112}}{2(3)}$$
$$p = \frac{-4 \pm 4\sqrt{7}}{6}$$
$$p = \frac{-2 \pm 2\sqrt{7}}{3}$$

25. $2k^2 + 5k = 9$
$$2k^2 + 5k - 9 = 0$$
$b^2 - 4ac = 5^2 - 4(2)(-9)$ or 97; 2 real
$$k = \frac{-5 \pm \sqrt{97}}{2(2)}$$
$$k = \frac{-5 \pm \sqrt{97}}{4}$$

26. $-7 + i\sqrt{5}$ $\quad$ **27.** $5 + 2i$

28. $s = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$
$$s = \frac{5 \pm \sqrt{(-5)^2 - 4(3)(9)}}{2(3)}$$
$$s = \frac{5 \pm \sqrt{-83}}{6}$$
$$s = \frac{5 \pm i\sqrt{83}}{6}$$

29. $x^2 - 3x - 28 = 0$

$(x - 7)(x + 4) = 0$

$x - 7 = 0 \qquad x + 4 = 0$

$x = 7 \qquad x = -4$

30. $4w^2 + 19w - 5 = 0$

$(4w - 1)(w - 5) = 0$

$4w - 1 = 0 \qquad w + 5 = 0$

$4w = 1 \qquad w = -5$

$w = \frac{1}{4}$

31. $4r^2 - r = 5$

$4r^2 - r - 5 = 0$

$(4r - 5)(r + 1) = 0$

$4r - 5 = 0 \qquad r + 1 = 0$

$4r = 5 \qquad r = -1$

$r = \frac{5}{4}$

32. $p = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$

$p = \frac{-2 \pm \sqrt{2^2 - 4(1)(8)}}{2(1)}$

$p = \frac{-2 \pm \sqrt{-28}}{2}$

$p = \frac{-2 \pm 2\boldsymbol{i}\sqrt{7}}{2}$

$p = -1 \pm \boldsymbol{i}\sqrt{7}$

33. $x = \frac{2\sqrt{6} \pm \sqrt{(2\sqrt{6})^2 - 4(1)(-2)}}{2(1)}$

$x = \frac{2\sqrt{6} \pm \sqrt{32}}{2}$

$x = \frac{2\sqrt{6} \pm 4\sqrt{2}}{2}$

$x = \sqrt{6} + 2\sqrt{2}$

34a. $P = 0.01A^2 + 0.05A + 107$

$P = 0.01(25)^2 + 0.05(25) + 107$

$P = 6.25 + 1.25 + 107$

$P = 114.5$ mm Hg

34b. $P = 0.01A^2 + 0.05A + 107$

$125 = 0.01A^2 + 0.05A + 107$

$0 = 0.01A^2 + 0.05A - 18$

$A = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$A = \frac{-0.05 \pm \sqrt{0.05^2 - 4(0.01)(-18)}}{2(0.01)}$

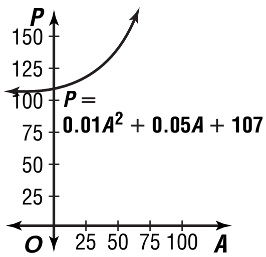
$A = \frac{-0.05 \pm \sqrt{0.7225}}{0.02}$

$A = \frac{-0.05 + \sqrt{0.7225}}{0.02}$ or $A = \frac{-0.05 - \sqrt{0.7225}}{0.02}$

$A = 40 \qquad A = -45$

40 years old

34c.



As a woman gets older, the normal systolic pressure increases.

35. $b^2 - 4ac < 0$

$8^2 - 4(1)(c) < 0$

$64 - 4c < 0$

$-4c < -64$

$c > 16$

36a. $A = bh$

$A = 12(16)$

$A = 192$

$(12 - 2x)(16 - 2x) = \frac{1}{2}(192)$

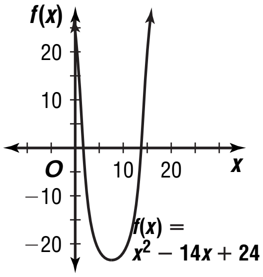
$(12 - 2x)(16 - 2x) = 96$

36b. $(12 - 2x)(16 - 2x) = 96$

$192 - 56x + 4x^2 = 96$

$4x^2 - 56x + 96 = 0$

$x^2 - 14x + 24 = 0$



36c. roots: 2, 12

$12 - 2x = 12 - 2(2)$ or 8

$16 - 2x = 16 - 2(2)$ or 12

8 ft by 12 ft

$12 - 2x = 12 - 2(12)$ or -12

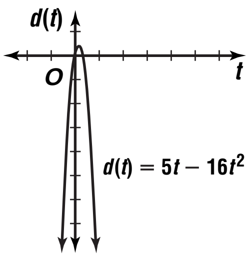
$16 - 2x = 16 - 2(12)$ or -8

$\varnothing$

37a. $d(t) = v_0 t - \frac{1}{2}gt^2$

$d(t) = 5t - \frac{1}{2}(32)t^2$

$d(t) = 5t - 16t^2$



37b. 0 and about 0.3

37c. The x-intercepts indicate when the woman is at the same height as the beginning of the jump.

37d. $d(t) = 5t - 16t^2$

$-50 = 5t - 16t^2$

37e. $-50 = 5t - 16t^2$

$16t^2 - 5t - 50 = 0$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{5 \pm \sqrt{(-5)^2 - 4(16)(-50)}}{2(16)}$

$t = \frac{5 \pm \sqrt{3225}}{32}$

$t = \frac{5 + \sqrt{3225}}{32} \qquad t = \frac{5 - \sqrt{3225}}{32}$

$t \approx 1.93$ s $\qquad t \approx -1.62$

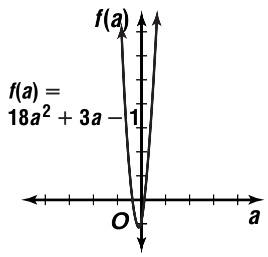
about 1.93 s

38.
$$\begin{aligned}
ax^2 + bx + c &= 0\\
x^2 + \frac{b}{a}x + \frac{c}{a} &= 0\\
x^2 + \frac{b}{a}x &= -\frac{c}{a}\\
x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 &= -\frac{c}{a} + \left(\frac{b}{2a}\right)^2\\
\left(x + \frac{b}{2a}\right)^2 &= -\frac{c}{a} + \frac{b^2}{4a^2}\\
\left(x + \frac{b}{2a}\right)^2 &= \frac{-4ac + b^2}{4a^2}\\
x + \frac{b}{2a} &= \pm\frac{\sqrt{b^2 - 4ac}}{2a}\\
x &= -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}\\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{aligned}$$

39. 2; $18a^2 + 3a - 1 = 0$

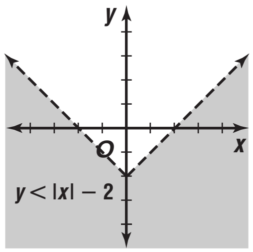
$(3a + 1)(6a - 1) = 0$

$3a + 1 = 0$	$6a - 1 = 0$
$3a = -1$	$6a = 1$
$a = -\frac{1}{3}$	$a = \frac{1}{6}$



40.

x	y
−2	0
−1	−1
0	−2
1	−1
2	0



41.
$$\begin{aligned}
f(x) &= (x - 9)^2\\
y &= (x - 9)^2\\
x &= (y - 9)^2\\
\pm\sqrt{x} &= y - 9\\
y &= \pm\sqrt{x} + 9\\
f^{-1}(x) &= \pm\sqrt{x} + 9
\end{aligned}$$

42. $3x + 4y = 375$

$-2(5x + 2y) = -2(345)$ $\rightarrow$

$$\begin{array}{rcr}
3x + 4y &=& 375\\
-10x - 4y &=& -690\\ \hline
-7x &=& -315\\
x &=& 45
\end{array}$$

$3x + 4y = 375$

$3(45) + 4y = 375$

$y = 60$ (45, 60)

43.

$m = \frac{619 - 595}{2.8 - 2.4}$	$60 = \frac{619 - x}{2.8 - 3.2}$
$m = \frac{24}{0.4}$	$619 - x = -24$
$m = 60$	$x = \$643$

44. $3y + 8x = 12$

$3y = -8x + 12$

$y = 3x + 4;\ -\frac{8}{3}$

45. $x^2 + x - 20 = (x + 5)(x - 4)$

The correct choice is A.

4-3 The Remainder and Factor Theorems

Page 226 Check for Understanding

1. The Remainder Theorem states that if a polynomial $P(x)$ is divided by $x - r$, the remainder is $P(r)$. If a division problem has a remainder of 0, then the divisor is a factor of the dividend. This leads to the Factor Theorem which states that the binomial $x - r$ is a factor if and only if $P(r) = 0$.
2. $(x^3 - 4x^2 - 7x + 8) \div (x - 5)$; $x^2 + x - 2$; -2
3. The degree of a polynomial is one more than the degree of its depressed polynomial.
4. Isabel; if $f(-3) = 0$, then $(x - (-3))$ or $(x + 3)$ is a factor.
5. $$\begin{array}{r|rrr} 2 & 1 & -1 & 4\\ & & 2 & 2\\ \hline & 1 & 1 & 6 \end{array}$$
$x + 1$, R6
6. $$\begin{array}{r|rrrr} -5 & 1 & 1 & -17 & 15\\ & & -5 & 20 & -15\\ \hline & 1 & -4 & 3 & 0 \end{array}$$
$x^2 - 4x + 3$
7. $f(x) = x^2 + 2x - 15$
$f(3) = (3)^2 + 2(3) - 15$
$= 9 + 6 - 15$ or 0; yes
8. $f(x) = x^4 + x^2 + 2$
$f(3) = (3)^4 + (3)^2 + 2$
$= 81 + 9 + 2$ or 92; no
9. $f(x) = x^3 - 5x^2 - x + 5$
$f(1) = (1)^3 - 5(1)^2 - 1 + 5$
$= 1 - 5 - 1 + 5$ or 0
$$\begin{array}{r|rrrr} 1 & 1 & -5 & -1 & 5\\ & & 1 & -4 & -5\\ \hline & 1 & -4 & -5 & 0 \end{array}$$
$x - 1$ is a factor $x^2 - 4x - 5 = (x - 5)(x + 1)$
$(x - 5), (x + 1), (x - 1)$
10. $f(x) = x^3 - 6x^2 + 11x - 6$
$f(1) = (1)^3 - 6(1)^2 + 11(1) - 6$
$= 1 - 6 + 11 - 6$ or 0
$$\begin{array}{r|rrrr} 1 & 1 & -6 & 11 & -6\\ & & 1 & -5 & 6\\ \hline & 1 & -5 & 6 & 0 \end{array}$$
$x - 1$ is a factor $x^2 - 5x + 6 = (x - 2)(x - 3)$
$(x - 1), (x - 2), (x - 3)$
11. $$\begin{array}{r|rrrr} -1 & 1 & 0 & -7 & k\\ & & -1 & 1 & 6\\ \hline & 1 & -1 & -6 & 6 + k \end{array}$$
$6 + k = 2$
$k = -4$

12a. 12 **12b.** 12 **12c.** 11

12d.
$$\begin{aligned}
f(x) &= x^7 + x^9 + x^{12} - 2x^2\\
&= x^{12} + x^9 + x^7 - 2x^2\\
&= x(x^{11} + x^8 + x^6 - 2x)\\
&= x^2(x^{10} + x^7 + x^5 - 2)
\end{aligned}$$
x, x^2, $x^{11} + x^8 + x^6 - 2x$, or $x^{10} + x^7 + x^5 - 2$

13. $h = r + 4$
$$\begin{aligned}
V &= \pi r^2 h\\
V &= \pi r^2(r + 4)\\
5\pi &= \pi r^2(r + 4)\\
5\pi &= \pi r^3 + 4\pi r^2\\
0 &= \pi r^3 + 4\pi r^2 - 5\pi\\
0 &= \pi(r^3 + 4r^2 - 5)
\end{aligned}$$
$$\begin{array}{r|rrrr} 1 & 1 & 4 & 0 & -5\\ & & 1 & 5 & 5\\ \hline & 1 & 5 & 5 & 0 \end{array}$$

$r - 1 = 0$	$h = r + 4$
$r = 1$ in.	$h = 1 + 4$ or 5 in.

Pages 226–228 Exercises

14. $$\begin{array}{r|rrr} -7 & 1 & 20 & 91 \\ & & -7 & -91 \\ \hline & 1 & 13 & 0 \end{array}$$
$x + 13$

15. $$\begin{array}{r|rrrr} 3 & 1 & -9 & 27 & -28 \\ & & 3 & -18 & 27 \\ \hline & 1 & -6 & 9 & -1 \end{array}$$
$x^2 - 6x + 9$, R -1

16. $$\begin{array}{r|rrrrr} 2 & 1 & 1 & 0 & 0 & -1 \\ & & 2 & 6 & 12 & 24 \\ \hline & 1 & 3 & 6 & 12 & 23 \end{array}$$
$x^3 + 3x^2 + 6x + 12$, R23

17. $$\begin{array}{r|rrrrr} -2 & 1 & 0 & -8 & 0 & 16 \\ & & -2 & 4 & 8 & -16 \\ \hline & 1 & -2 & -4 & 8 & 0 \end{array}$$
$x^3 - 2x^2 - 4x + 8$

18. $$\begin{array}{r|rrrrr} -1 & 3 & -2 & 5 & -4 & -2 \\ & & -3 & 5 & -10 & 14 \\ \hline & 3 & -5 & 10 & -14 & 12 \end{array}$$
$3x^3 - 5x^2 + 10x - 14$, R12

19. $$\begin{array}{r|rrrr} 1 & 2 & 0 & -2 & -3 \\ & & 2 & 2 & 0 \\ \hline & 2 & 2 & 0 & -3 \end{array}$$
$2x^2 + 2x$, R -3

20. $f(x) = x^2 - 2$
$f(1) = (1)^2 - 2$
$= 1 - 2$ or -1; no

21. $f(x) = x^5 + 32$
$f(-2) = (-2)^5 + 32$
$= -32 + 32$ or 0; yes

22. $f(x) = x^4 - 6x^2 + 8$
$f(\sqrt{2}) = (\sqrt{2})^4 - 6(\sqrt{2})^2 + 8$
$= 4 - 12 + 8$ or 0; yes

23. $f(x) = x^3 - x + 6$
$f(2) = (2)^3 - 2 + 6$
$= 8 - 2 + 6$ or 12; no

24. $f(x) = 4x^3 + 4x^2 + 2x + 3$
$f(1) = 4(1)^3 + 4(1)^2 + 2(1) + 3$
$= 4 + 4 + 2 + 3$ or 13; no

25. $f(x) = 2x^3 - 3x^2 + x$
$f(1) = 2(1)^3 - 3(1)^2 + 1$
$= 2 - 3 + 1$ or 0; yes

26a-d.

r	1	3	-2	-8
1	1	4	2	-6
-1	1	2	-4	-4
2	1	5	8	8
-2	1	1	-4	0

d

27. $(\sqrt{6})^4 - 36 = 36 - 36$ or 0

28. $$\begin{array}{r|rrrr} -1 & 1 & 7 & -1 & -7 \\ & & -1 & -6 & 7 \\ \hline & 1 & 6 & -7 & 0 \end{array}$$
$x^2 + 6x - 7 = (x - 1)(x + 7)$
$(x - 1)(x + 1)(x + 7)$

29. $$\begin{array}{r|rrrr} 2 & 1 & 1 & -4 & -4 \\ & & 2 & 6 & 4 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$
$x^2 + 3x + 2 = (x + 1)(x + 2)$
$(x - 2), (x + 1), (x + 2)$

30. $$\begin{array}{r|rrrr} 1 & 1 & -1 & -49 & 49 \\ & & 1 & 0 & -49 \\ \hline & 1 & 0 & -49 & 0 \end{array}$$
$x^2 - 49 = (x - 7)(x + 7)$
$(x - 1), (x - 7), (x + 7)$

31. $$\begin{array}{r|rrrr} 4 & 1 & -5 & 2 & 8 \\ & & 4 & -4 & -8 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$
$x^2 - x - 2 = (x - 2)(x + 1)$
$(x - 4), (x - 2), (x + 1)$

32. $$\begin{array}{r|rrrr} 2 & 1 & -2 & -4 & 8 \\ & & 2 & 0 & -8 \\ \hline & 1 & 0 & -4 & 0 \end{array}$$
$x^2 - 4 = (x - 2)(x + 2)$
$(x - 2), (x - 2), (x + 2)$

33. $$\begin{array}{r|rrrr} 1 & 1 & 4 & -1 & -4 \\ & & 1 & 5 & 4 \\ \hline & 1 & 5 & 4 & 0 \end{array}$$
$x^2 + 5x + 4 = (x + 1)(x + 4)$
$(x - 1), (x + 1), (x + 4)$

34. $$\begin{array}{r|rrrr} -1 & 1 & 3 & 3 & 1 \\ & & -1 & -2 & -1 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$
$x^2 + 2x + 1 = (x + 1)(x + 1)$
$(x + 1), (x + 1), (x + 1)$

35. $$\begin{array}{r|rrrrrrr} 2 & 1 & 0 & -9 & 0 & 24 & 0 & -16 \\ & & 2 & 4 & -10 & -20 & 8 & 16 \\ \hline & 1 & 2 & -5 & -10 & 4 & 8 & 0 \end{array}$$
$x^5 + 2x^4 - 5x^3 - 10x^2 + 4x + 8$
$$\begin{array}{r|rrrrrr} 2 & 1 & 2 & -5 & -10 & 4 & 8 \\ & & 2 & 8 & 6 & -8 & -8 \\ \hline & 1 & 4 & 3 & -4 & -4 & 0 \end{array}$$
$x^4 + 4x^3 + 3x^2 - 4x - 4$
$$\begin{array}{r|rrrrr} 2 & 1 & 4 & 3 & -4 & -4 \\ & & 2 & 12 & 30 & -52 \\ \hline & 1 & 6 & 15 & -26 & -56 \end{array}$$
2 times

36. $$\begin{array}{r|rrrr} -1 & 1 & 2 & -1 & -2 \\ & & -1 & -1 & 2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$
$x^2 + x - 2 = (x + 2)(x - 1)$
1 time; -2, 1

37. $f(x) = 2x^3 - x^2 + x + k$
$f(1) = 2(1)^3 - (1)^2 + 1 + k$
$0 = 2 - 1 + 1 + k$
$-2 = k$

38. $f(x) = x^3 - kx^2 + 2x - 4$
$f(2) = (2)^3 - k(2)^2 + 2(2) - 4$
$0 = 8 - 4k + 4 - 4$
$0 = -4k + 8$
$2 = k$

39. $f(x) = x^3 + 18x^2 + kx + 4$
$f(-2) = (-2)^3 + 18(-2)^2 + k(-2) + 4$
$0 = -8 + 72 - 2k + 4$
$0 = -2k + 68$
$34 = k$

40. $f(x) = x^3 + 4x^2 - kx + 1$
$f(-1) = (-1)^3 + 4(-1)^2 - k(-1) + 1$
$0 = -1 + 4 + k + 1$
$0 = k + 4$
$-4 = k$

41. $d(t) = v_0t + \frac{1}{2}at^2$

$25 = 4t + \frac{1}{2}(0.4)t^2$

$0 = 0.2t^2 + 4t - 25$

$$\begin{array}{r|rrr} 5 & 0.2 & 4 & -25 \\ & & 1 & 25 \\ \hline & 0.2 & 5 & 0 \end{array}$$

$t - 5 = 0$

$t = 5$ s

42.

$$\begin{array}{r|rrrrr} 1 & 1 & 1 & -7 & a & b \\ & & 1 & 2 & -5 & -5 + a \\ \hline & 1 & 2 & -5 & -5 + a & -5 + a + b \end{array}$$

$$\begin{array}{r|rrrrr} -2 & 1 & 2 & -5 & -5 + a & -5 + a + b \\ & & -2 & 0 & 10 & -10 - 2a \\ \hline & 1 & 0 & -5 & 5 + a & -15 - a + b \end{array}$$

$-5 + a + b = 0$ $\qquad$ $-5 + a + b = 0$

$-15 - a + b = 0$ $\qquad$ $-5 + a + 10 = 0$

$-20 \quad + 2b = 0$ $\qquad$ $a + 5 = 0$

$2b = 20$ $\qquad$ $a = -5$

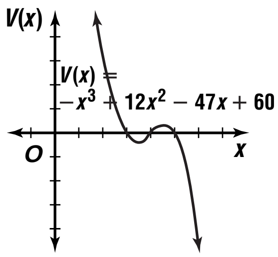
$b = 10$

43a. $V(x) = (3 - x)(4 - x)(5 - x)$

$V(x) = (12 - 7x + x^2)(5 - x)$

$V(x) = -x^3 + 12x^2 - 47x + 60$

43b.



43c. $V = \ell \cdot w \cdot h$

$V = 3 \cdot 4 \cdot 5$ or 60

$\frac{3}{5}V = \frac{3}{5}(60)$

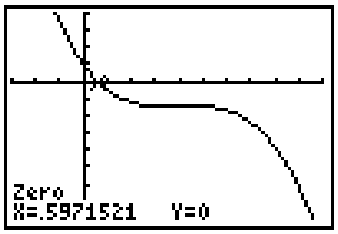
$= 36$

$V(x) = -x^3 + 12x^2 - 47x + 60$

$36 = -x^3 + 12x^2 - 47x + 60$

43d. $36 = -x^3 + 12x^2 - 47x + 60$

$0 = -x^3 + 12x^2 - 47x + 24$



$[-3, 10]$ sc1:1 by $[-200, 100]$ sc1:25

about 0.60 ft

44a. $\ell = \frac{1}{2}(20 - 2x)$ or $10 - x$

$w = 18 - 2x$

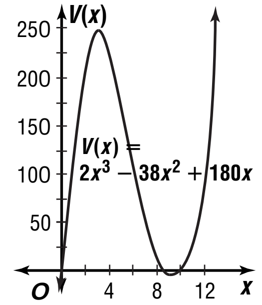
$h = x$

$V(x) = (10 - x)(18 - 2x)(x)$

$V(x) = (180 - 38x + 2x^2)(x)$

$V(x) = 2x^3 - 38x^2 + 180x$

44b.



44c. $V(x) = 2x^3 - 38x^2 + 180x$

$224 = 2x^3 - 38x^2 + 180x$

44d. $224 = 2x^3 - 38x^2 + 180x$

$0 = 2x^3 - 38x^2 + 180x + 224$

$0 = x^3 - 19x^2 + 90x + 112$

$$\begin{array}{r|rrrr} 2 & 1 & -19 & 90 & 112 \\ & & 2 & -34 & -112 \\ \hline & 1 & -17 & 56 & 0 \end{array}$$

2 in.

45. $P(3 + 4\boldsymbol{i}) = 0$ and $P(3 - 4\boldsymbol{i}) = 0$ implies that these are both roots of $ax^2 + bx + c$. Since this polynomial is of degree 2 it has only these two roots.

$x = 3 \pm 4\boldsymbol{i}$

$x - 3 = \pm 4\boldsymbol{i}$

$(x - 3)^2 = -16$

$x^2 - 6x + 9 = 16$

$x^2 - 6x + 25 = 0$

$a = 1, b = -6, c = 25$

46. $r^2 + 5r - 8 = 0$

$r^2 + 5r = 8$

$r^2 + 5r + \frac{25}{4} = 8 + \frac{25}{4}$

$\left(r + \frac{5}{2}\right)^2 = \frac{57}{4}$

$r + \frac{5}{2} = \pm\frac{\sqrt{57}}{2}$

$r = -\frac{5}{2} \pm \frac{\sqrt{57}}{2}$

47a. $f(x) = x^4 - 4x^3 - x^2 + 4x$

$f(2) = (2)^4 - 4(2)^3 - (2)^2 + 4(2)$

$f(2) = 16 - 32 - 4 + 8$ or -12; no

47b. $f(0) = (0)^4 - 4(0)^3 - (0)^2 + 4(0)$

$f(0) = 0 - 0 - 0 + 0$ or 0; yes

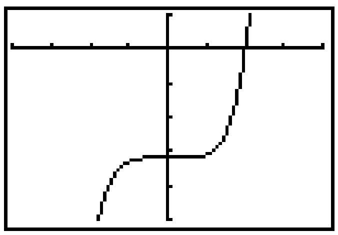
47c. $f(-2) = (-2)^4 - 4(-2)^3 - (-2)^2 + 4(-2)$

$f(-2) = 16 + 32 - 4 - 8$ or 36; no

47d. $f(4) = (4)^4 - 4(4)^3 - (4)^2 + 4(4)$

$f(4) = 256 - 256 - 16 + 16$ or 0; yes

48. $f(x) = x^5 - 32$



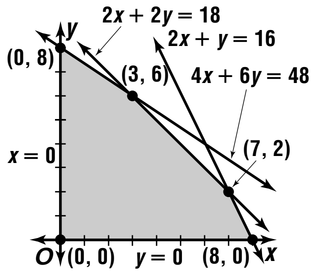
$(0, -32)$; point of inflection

$[-4, 4]$ sc1:1 by $[-50, 10]$ sc1:10

49. wider than parent graph and moved 1 unit left

50. Let x = number of 100 foot units of Pipe A and y = number of 100 foot units of Pipe B.

$4x + 6y \le 48$
$2x + 2y \le 18$
$2x + y \le 16$
$x \ge 0$
$y \ge 0$



$P(x, y) = 34x + 40y$
$P(0, 0) = 34(0) + 40(0)$ or 0
$P(0, 8) = 34(0) + 40(8)$ or 320
$P(3, 6) = 34(3) + 40(6)$ or 342
$P(7, 2) = 34(7) + 40(2)$ or 318
$P(8, 0) = 34(8) + 40(0)$ or 272
3 − 100 foot units of A, or 300 ft of A
6 − 100 foot units of B, or 600 ft of B

51.
$$\begin{aligned} 4x + 2y + 3z &= 6 \\ 2x + 7y &= 3z \\ -3x - 9y + 13 &= -2z \end{aligned} \quad \rightarrow \quad \begin{aligned} 4x + 2y + 3z &= 6 \\ 2x + 7y - 3z &= 0 \\ -3x - 9y + 2z &= -13 \end{aligned}$$

$$\begin{array}{r} 4x + 2y + 3z = 6 \\ \underline{2x + 7y - 3z = 0} \\ 6x + 9y \qquad = 6 \end{array}$$

$2(2x + 7y - 3z) = 2(0)$
$3(-3x - 9y + 2z) = 3(-13)$
$\downarrow$

$$\begin{array}{r} 4x + 14y - 6z = 0 \\ \underline{-9x - 27y + 6z = -39} \\ -5x - 13y \qquad = -39 \end{array}$$

$5(6x + 9y) = 5(6)$
$6(-5x - 13y) = 6(-39)$
$\downarrow$

$$\begin{array}{r} 30x + 45y = 30 \\ \underline{-30x - 78y = -234} \\ -33y = -204 \\ y = \frac{68}{11} \end{array}$$

$$\begin{aligned} 6x + 9y &= 6 \\ 6x + 9\left(\tfrac{68}{11}\right) &= 6 \\ 6x &= -\tfrac{546}{11} \\ x &= -\tfrac{91}{11} \end{aligned} \qquad \begin{aligned} 4x + 2y + 3z &= 6 \\ 4\left(-\tfrac{91}{11}\right) + 2\left(\tfrac{68}{11}\right) + 3z &= 6 \\ 3z &= \tfrac{294}{11} \\ z &= \tfrac{98}{11} \end{aligned}$$

$\left(-\frac{91}{11}, \frac{68}{11}, \frac{98}{11}\right)$

52. $M\left(\frac{-7-2}{2}, \frac{2+6}{2}\right)$ or $(-4.5, 4)$
$N\left(\frac{-2-2}{2}, \frac{6-3}{2}\right)$ or $(-2, 1.5)$
slope of $\overline{MN} = \frac{4 - 1.5}{-4.5 - (-2)}$ or -1
slope of $\overline{RI} = \frac{2 - (-3)}{-7 - (-2)}$ or -1
Since the slopes are the same, $\overline{MN} \parallel \overline{RI}$.

53.

$a > b$	$a > b$	$a > b$
$ac < bc$	$a + c > b + c$	$a - c > b - c$
I. true	II. true	III. false

The correct choice is D.

4-4 The Rational Root Theorem

Page 232 Graphing Calculator Exploration

1. $3; 1, -1, -2$
2. $2; -1, 2$
3. (1) 1 positive;
$f(-x) = (-x)^4 + 4(-x)^3 + 3(-x)^2 - 4(-x) - 4$
$f(-x) = x^4 - 4x^3 + 3x^2 + 4x - 4$; 3 or 1
(2) 1 positive; $f(-x) = (-x)^3 - 3(-x) - 2$
$f(-x) = -x^3 + 3x - 2$; 2 or 0
4. In the first function, there are 2 negative zeros, but according to Descartes' Rule of Signs, there should be 3 or 1 negative zeros. This is because the -2 is a double zero. In the second function, there is one negative zero, but according to Descartes' Rule of Signs, there should be 2 or 0 zeros. This is because -1 is a double root.
5. One number represents two zeros of the function.

Page 233 Check for Understanding

1. possible values of p: $\pm1, \pm2, \pm3, \pm6$
possible values of q: ±1
possible rational roots: $\pm1, \pm2, \pm3, \pm6$
2. If the leading coefficient is 1, then q must equal 1. Therefore, $\frac{p}{q}$ becomes $\frac{p}{1}$ or p, and p is defined as a factor of a_n.
3. Sample answer: $f(x) = x^3 - x^2 + x - 3$;
$f(-x) = (-x)^3 - (-x)^2 + (-x) - 3$
$f(-x) = -x^3 - x^2 - x - 3$; 0
3 or 1 possible positive zeros and no possible negative zeros
4. Sample answer: You can factor the polynomial, graph the function, complete the square, or use the Quadratic Formula if it is a second-degree function, or use the Factor Theorem and the Rational Root Theorem. I would factor the polynomial if it can be factored easily. If not and it is a second-degree function, I would use the Quadratic Formula. Otherwise, I would graph the function on a graphing utility and use the Rational Root Theorem to find the exact zeros.
5. $\frac{p}{q}$: $\pm1, \pm2$

r	1	−4	1	2
1	1	−3	−2	0
−1	1	−5	6	−4
2	1	−2	−3	−4
−2	1	−6	13	−24

rational root: 1

6. p: $\pm1, \pm3$
q: $\pm1, \pm2$
$\frac{p}{q}$: $\pm1, \pm3, \pm\frac{1}{2}, \pm\frac{3}{2}$

r	2	3	−8	3
1	2	5	−3	0
−1	2	1	−9	12
$\frac{1}{2}$	2	4	−6	0
$-\frac{1}{2}$	2	2	−6	−1.5
3	2	9	19	60
−3	2	−3	1	0

rational roots: $-3, \frac{1}{2}, 1$

7. 2 or 0; $f(-x) = 8(-x)^3 - 6(-x)^2 - 23(-x) + 6$
$f(-x) = -8x^3 - 6x^2 + 23x + 6$; 1
$\frac{p}{q}$: $\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{1}{8}, \pm\frac{3}{8}$

r	8	−6	−23	6
1	8	2	−21	−15
2	8	10	−3	0

$8x^2 + 10x - 3 = 0$
$(4x - 1)(2x + 3) = 0$
$4x - 1 = 0$ $\quad$ $2x + 3 = 0$
$4x = 1$ $\quad$ $2x = -3$
$x = \frac{1}{4}$ $\quad$ $x = -\frac{3}{2}$ or $-1\frac{1}{2}$
$-1\frac{1}{2}, \frac{1}{4}, 2$

8. 1; $f(-x) = (-x)^3 + 7(-x)^2 + 7(-x) - 15$
$f(-x) = -x^3 + 7x^2 - 7x - 15$; 2 or 0
$\frac{p}{q}$: $\pm1, \pm3, \pm5, \pm15$

r	1	7	7	−15
1	1	8	15	0
−1	1	6	1	−16
−3	1	4	−5	0
−5	1	2	−3	0

$-5, -3, 1$

9. $r^2 = 15^2 - x^2$

$$V = \frac{1}{3}\pi r^2 h$$
$$1152\pi = \frac{1}{3}\pi(15^2 - x^2)(15 + x)$$
$$3456 = (15^2 - x^2)(15 + x)$$
$$3456 = 3375 + 225x - 15x^2 - x^3$$

$x^3 + 15x^2 - 225x + 81 = 0$
Possible rational roots: $\pm1, \pm9, \pm81$
$f(x) = x^3 + 15x^2 - 225x + 81 = 0$
$f(1) = -128$ $\quad$ $f(-1) = 320$
$f(9) = 0$ $\quad$ $f(-9) = 2592$
$f(81) = 611{,}712$ $\quad$ $f(-81) = -414{,}720$
x represents 9 cm.

Pages 234–235 Exercises

10. $\frac{p}{q}$: $\pm1, \pm2, \pm3, \pm6$

r	1	2	−5	−6
1	1	3	−2	−8
2	1	4	3	0

$x^2 + 4x + 3 = 0$
$(x + 3)(x + 1) = 0$
$x = -3, x = -1$
rational roots: $-3, -1, 2$

11. $\frac{p}{q}$: $\pm1, \pm2, \pm3, \pm6, \pm9, \pm18$

r	1	−2	1	18
−1	1	−3	4	14
−2	1	−4	9	0

$x^2 - 4x + 9 = 0$
does not factor
rational root: -2

12. $\frac{p}{q}$: $\pm1, \pm2$

r	1	−5	9	−7	2
1	1	−4	5	−2	0

$x^3 - 4x^2 + 5x - 2$

r	1	−4	5	−2
2	1	−2	1	0

$x^2 - 2x + 1 = 0$
$(x - 1)(x - 1) = 0$
$x = 1, x = 1$
rational roots: 1, 2

13. $\frac{p}{q}$: $\pm1, \pm2, \pm4, \pm5, \pm10, \pm20$

r	1	−5	−4	20
1	1	−4	−8	12
−1	1	−6	2	18
2	1	−3	−10	0

$x^2 - 3x - 10 = 0$
$(x - 5)(x + 2) = 0$
$x = 5, x = -2$
rational roots: $-2, 2, 5$

14. p: $\pm1, \pm3$
q: $\pm1, \pm2$
$\frac{p}{q}$: $\pm1, \pm3, \pm\frac{1}{2}, \pm\frac{3}{2}$

r	2	−1	0	−6	3
$\frac{1}{2}$	2	0	0	−6	0

$2x^3 - 6 = 0$
$x^3 = 3$
$x = \sqrt[3]{3}$
rational root: $\frac{1}{2}$

15. p: ± 1

q: $\pm 1, \pm 2, \pm 3, \pm 6$

$\frac{p}{q}$: $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}$

r	6	35	−1	−7	−1
$\frac{1}{2}$	6	38	18	2	0

$6x^3 + 38x^2 + 18x + 2$

r	6	38	18	2
$-\frac{1}{3}$	6	36	6	0

$6x^2 + 36x + 6 = 0$

$x^2 + 6x + 1 = 0$

does not factor

rational roots: $-\frac{1}{3}, \frac{1}{2}$

16. 4; 3 or 1;

$f(-x) = (-x)^4 - 2(-x)^3 + 7(-x) + 4(-x) - 15$

$f(-x) = x^4 + 2x^3 - 7x - 4x - 15$; 1 negative

1 positive

17. $f(-x) = -x^3 + 7x - 6$

0 or 2 negative

r	1	0	−7	−6
1	1	1	−6	−12
−1	1	−1	−6	0

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3, x = -2$

rational zeros: $-2, -1, 3$

18. 1 positive

$f(-x) = -x^3 - 2x^2 + 8$

1 negative

$f(x) = x^3 - 2x^2 - 8x$

$0 = x(x^2 - 2x - 8)$

$= x(x - 4)(x + 2)$

$x = 0, x = 4, x = -2$

rational zeros: $-2, 0, 4$

19. 1 positive

$f(-x) = -x^3 + 3x^2 + 10x - 24$

2 or 0 negative

r	1	3	−10	−24
3	1	6	8	14

$x^2 + 6x + 8 = 0$

$(x + 4)(x + 2) = 0$

$x = -4, x = -2$

rational zeros: $-4, -2, 3$

20. 2 or 0 positive

$f(-x) = -10x^3 - 17x^2 + 7x + 2$

1 negative

r	10	−17	−7	2
$-\frac{1}{2}$	10	−22	4	0

$10x^2 - 22x + 4 = 0$

$5x^2 - 11x + 2 = 0$

$(5x - 1)(x - 2) = 0$

$x = \frac{1}{5}, x = 2$

rational zeros: $-\frac{1}{2}, \frac{1}{5}, 2$

21. 2 or 0 positive

$f(-x) = x^4 - 2x^3 - 9x^2 + 2x + 8$

2 or 0 negative

r	1	2	−9	−2	8
1	1	3	−6	−8	0

$x^3 + 3x^2 - 6x - 8$

r	1	3	−6	−8
−1	1	2	−8	0

$x^2 + 2x - 8 = 0$

$(x + 4)(x - 2) = 0$

$x = -4, x = 2$

rational zeros: $-4, -1, 1, 2$

22. 2 or 0 positive

$f(-x) = x^4 - 5x^2 + 4$

2 or 0 negative

r	1	0	−5	0	4
1	1	1	−4	−4	0

$x^3 + x^2 - 4x - 4$

r	1	1	−4	−4
−1	1	0	−4	0

$x^2 - 4 = 0$

$(x - 2)(x + 2) = 0$

$x = 2, x = -2$

rational zeros: $-2, -1, 1, 2$

23a. $f(x) = (x - 2)(x + 2)(x + 1)^2$

$0 = (x - 2)(x + 2)(x + 1)^2$

$x - 2 = 0$ $\quad$ $x + 2 = 0$ $\quad$ $(x + 1)^2 = 0$

$x = 2$ $\quad$ $x = -2$ $\quad$ $x + 1 = 0$

$x = -1$

23b. $f(x) = (x - 2)(x + 2)(x + 1)^2$

$f(x) = (x^2 - 4)(x^2 + 2x + 1)$

$f(x) = x^4 + 2x^3 - 3x^2 - 8x - 4$

23c. 1 positive

$f(-x) = x^4 - 2x^3 - 3x^2 + 8x - 4$

3 or 1 negative

23d. There are 2 negative zeros, but according to Descartes' Rule of Signs, there should be 3 or 1. This is because -1 is actually a zero twice.

24a. Let ℓ = the length.

$w = \ell - 4$

$h = 2\ell - 1$

$V(\ell) = \ell \cdot w \cdot h$

$V(\ell) = \ell(\ell - 4)(2\ell - 1)$

$V(\ell) = (\ell^2 - 4\ell)(2\ell - 1)$

$V(\ell) = 2\ell^3 - 9\ell^2 + 4\ell$

24b. $V(\ell) = 2\ell^3 - 9\ell^2 + 4\ell$
$2208 = 2\ell^3 - 9\ell^2 + 4\ell$

24c. $2208 = 2\ell^3 - 9\ell^2 + 4\ell$
$0 = 2\ell^3 - 9\ell^2 + 4\ell - 2208$

r	2	−9	4	−2208
12	2	15	184	0

$\ell = 12$ $\quad w = \ell - 4$ $\quad h = 2\ell - 1$
$w = 12 - 4$ or 8 $\quad h = 2(12) - 1$ or 23
12 in. × 8 in. × 23 in.

25a. Sample answer: $x^4 + x^3 + x^2 + x + 3 = 0$

25b. Sample answer: $x^3 - x^2 - 2 = 0$

25c. Sample answer: $x^3 - x = 0$

26a. Let ℓ = the length.
$h = \ell - 9$
$V(\ell) = \frac{1}{3}Bh$
$V(\ell) = \frac{1}{3}(\ell^2)(\ell - 9)$
$V(\ell) = \frac{1}{3}\ell^3 - 3\ell^2$

26b. $V(\ell) = \frac{1}{3}\ell^3 - 3\ell^2$
$6300 = \frac{1}{3}\ell^3 - 3\ell^2$

26c. $6300 = \frac{1}{3}\ell^3 - 3\ell^2$
$0 = \frac{1}{3}\ell^3 - 3\ell^2 - 6300$
$0 = \ell^3 - 9\ell^2 - 18{,}900$

r	1	−9	0	−18,900
30	1	21	630	0

$\ell = 30$ $\quad h = \ell - 9$
$h = 30 - 9$ or 21
base: 30 in. by 30 in., height: 21 in.

27. $d = 0.0000008x^2(200 - x)$
$0.8 = 0.0000008x^2(200 - x)$
$0 = 0.00016x^2 - 0.0000008x^3 - 0.8$
$0 = 8x^3 - 1600x^2 - 8{,}000{,}000$
$0 = x^3 - 200x^2 - 1{,}000{,}000$

r	1	−200	0	1,000,000
100	1	−100	−10,000	0

$x = 100$ ft

28. The graphs are reflections of each other over the x-axis. The zeros are the same.

29.
$$\begin{array}{r|rrr} -7 & 1 & -1 & -56 \\ & & -7 & 56 \\ \hline & 1 & -8 & 0 \end{array}$$
$x - 8$

30. $b^2 - 4ac = 6^2 - 4(4)(25)$
$= -364$; 2 imaginary

31. $(x - 1)(x - (-1))(x - 2)(x - (-2)) = 0$
$(x - 1)(x + 1)(x - 2)(x + 2) = 0$
$(x^2 - 1)(x^2 - 4) = 0$
$x^4 - 5x^2 + 4 = 0$

32. $y = 4.3x - 8424.3$
$y = 4.3(2008) - 8424.3$
$y = \$210.10$

33. $\frac{2x - 3}{x} = \frac{3 - x}{2}$
$2(2x - 3) = x(3 - x)$
$4x - 6 = 3x - x^2$
$x^2 + x - 6 = 0$
$(x + 3)(x - 2) = 0$
$x + 3 = 0 \quad x - 2 = 0$
$x = -3 \quad x = 2$
The correct choice is A.

Page 235 Mid-Chapter Quiz

1. $(x - 1)(x - (-1))(x - 2\boldsymbol{i})(x - (-2\boldsymbol{i})) = 0$
$(x - 1)(x + 1)(x - 2\boldsymbol{i})(x + 2\boldsymbol{i}) = 0$
$(x^2 - 1)(x^2 + 4) = 0$
$x^4 + 3x^2 - 4 = 0$

2. 3; $x^3 - 11x^2 + 30x = 0$
$x(x^2 - 11x + 30) = 0$
$x(x - 6)(x - 5) = 0$
$x = 0 \quad x - 6 = 0 \quad x - 5 = 0$
$x = 6 \quad x = 5$

3. $x^2 + 5x = 150$
$x^2 + 5x + \frac{25}{4} = 150 + \frac{25}{4}$
$\left(x + \frac{5}{2}\right)^2 = \frac{625}{4}$
$x + \frac{5}{2} = \pm\frac{25}{2}$
$x = \frac{-5 \pm 25}{2}$
$x = \frac{-5 + 25}{2} \quad x = \frac{-5 - 25}{2}$
$x = 10 \quad x = -15$

4. $b^2 - 4ac = (-39)^2 - 4(6)(45)$
$= 441$; 2 real roots
$b = \frac{39 \pm \sqrt{441}}{2(6)}$
$b = \frac{39 \pm 21}{12}$
$b = \frac{39 + 21}{12}$ or $b = \frac{39 - 21}{12}$
$b = 5 \quad b = \frac{3}{2}$

5.
$$\begin{array}{r|rrrr} -2 & 1 & 3 & -2 & -8 \\ & & -2 & -2 & 8 \\ \hline & 1 & 1 & -4 & 0 \end{array}$$
$x^2 + x - 4$

6.
$$\begin{array}{r|rrrr} 4 & 1 & -4 & 2 & -6 \\ & & 4 & 0 & 8 \\ \hline & 1 & 0 & 2 & 2 \end{array}$$
2; no

7.
$$\begin{array}{r|rrrr} 1 & 1 & -2 & -5 & 6 \\ & & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$
$x^2 - x - 6 = (x - 3)(x + 2)$
$(x - 3)(x - 1)(x + 2)$

8. $\frac{p}{q}$: ±1, ±3

r	1	6	10	3
−3	1	3	1	0

$x^2 + 3x + 1 = 0$
does not factor
rational root: −3

9. 1 positive
$F(-x) = x^4 - 4x^3 + 3x^2 + 4x - 4$
3 or 1 negative

r	1	4	3	−4	−4
1	1	5	8	4	0

$x^3 + 5x^2 + 8x + 4 = 0$

r	1	5	8	4
−1	1	4	4	0

$x^2 + 4x + 4 = 0$
$(x + 2)(x + 2) = 0$
$x = -2, x = -2$
rational zeros: −2, −1, 1

10. Let r = radius.
$h = r + 6$
$V = \frac{1}{3}\pi r^2 h$
$27\pi = \frac{1}{3}\pi r^2(r + 6)$
$0 = \frac{1}{3}\pi r^3 + 2\pi r^2 - 27\pi$
$0 = r^3 + 6r^2 - 81$

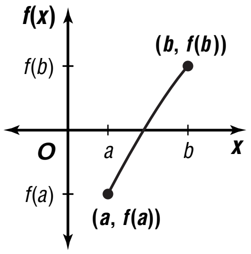
r	1	6	0	−81
3	1	9	27	0

$r = 3$ $\quad$ $h = r + 6$
$h = 3 + 6$ or 9
$r = 3$ cm, $h = 9$ cm

4-5 Locating Zeros of a Polynomial Function

Pages 239–240 Check for Understanding

1. If the function is negative for one value and positive for another value, the function must cross the x-axis in at least one point between the two values.



2. Use synthetic division to find the values of the polynomial function for consecutive integers. When the values of the function change from positive to negative or from negative to positive, there is a zero between the integers.

3. Use synthetic division to find the values of the polynomial function for consecutive integers. An integer that produces no sign change in the quotient and the remainder is an upper bound. To find a lower bound of a function, find an upper bound for the function of $-x$. The lower bound is the negative of the upper bound for the function of $-x$.

4. Nikki; the sign changes between −2 and −1.

5.

r	1	−4	−2
−2	1	−6	10
−1	1	−5	3
0	1	−4	−2
1	1	−3	−5
2	1	−2	−6
3	1	−1	−5
4	1	0	−2
5	1	1	3

4 and 5, −1 and 0

6.

r	1	−3	−2	4
−2	1	−5	8	−12
−1	1	−4	2	2
0	1	−3	−2	4
1	1	−2	−4	0 ←
2	1	−1	−4	−4
3	1	0	−2	−2
4	1	1	2	12

−2 and −1, at 1, 3 and 4

7.

r	2	−4	0	−3
0	2	−4	0	−3
1	2	−2	−2	−5
2	2	0	0	−3
3	2	2	6	15

approximate zero: 2.3

8.

r	1	3	2
−2	1	1	0
−1	1	2	0

zeros: −2, −1

9. Sample answer:

r	1	0	0	−8	2
1	1	1	1	−7	−5
2	1	2	4	0	2

upper bound: 2
$f(-x) = x^4 + 8x + 2$

r	1	0	0	8	2
0	1	0	0	8	2

lower bound: 0

10. Sample answer:

r	1	0	1	0	−3
1	1	1	2	2	−1
2	1	2	5	10	17

upper bound: 2
$f(-x) = x^4 + x^2 - 3$

r	1	0	1	0	−3
1	1	1	2	2	−1
2	1	2	5	10	17

lower bound: −2

11a. Let x = amount of increase.
$V(x) = (25 + x)(30 + x)(5 + x)$
$V(x) = (750 + 55x + x^2)(5 + x)$
$V(x) = x^3 + 60x^2 + 1025x + 3750$

11b. $V = \ell \cdot w \cdot h$ $\qquad$ $1.5V = 1.5(3750)$

$V = 25(30)(5)$ $\qquad$ $1.5V = 5625$

$V = 3750$

$V(x) = x^3 + 60x^2 + 1025x + 3750$

$5625 = x^3 + 60x^2 + 1025x + 3750$

11c. $5625 = x^3 + 60x^2 + 1025x + 3750$

$0 = x^3 + 60x^2 + 1025x - 1875$

r	1	60	1025	−1875
1	1	61	1086	−789
2	1	62	1149	423

$x = 1.7$

$25 + x = 25 + 1.7$ $\qquad$ $30 + x = 30 + 1.7$

$= 26.7$ $\qquad$ $= 31.7$

$5 + x = 5 + 1.7$

$= 6.7$

about 26.7 cm by 31.7 cm by 6.7 cm

Pages 240–242 Exercises

12.

r	1	0	0	−2
−1	1	−1	1	−3
0	1	0	0	−2
1	1	1	1	−1
2	1	2	4	6
3	1	3	9	25

1 and 2

13.

r	2	−5	1
−1	2	−7	8
0	2	−5	1
1	2	−3	−2
2	2	−1	−1
3	2	1	4

0 and 1, 2 and 3

14.

r	1	−2	0	1	−2
−2	1	−4	8	−15	28
−1	1	−3	3	−2	0
0	1	−2	0	1	−2
1	1	−1	−1	0	−1
2	1	0	0	1	0

at −1, at 2

15.

r	1	0	−8	0	10
−3	1	−3	1	−3	19
−2	1	−2	−4	8	−6
−1	1	−1	−7	7	3
0	1	0	−8	0	10
1	1	1	−7	−7	3
2	1	2	−4	−8	−6
3	1	3	1	3	19

−3 and −2, −2 and −1, 1 and 2, 2 and 3

16.

r	1	0	−3	1
−2	1	−2	1	−1
−1	1	−1	−2	3
0	1	0	−3	1
1	1	1	−2	−1
2	1	2	1	3

−2 and −1, 0 and 1, 1 and 2

17.

r	2	0	1	−3	3
−3	2	−6	19	−60	183
−2	2	−4	9	−21	45
−1	2	−2	3	−6	9
0	2	0	1	−3	3
1	2	2	3	0	3
2	2	4	9	16	35

no real zeros

18.

r	6	24	−54	−3
−6	6	−12	18	−111
−5	6	−6	−24	117

yes; $f(-6) = -111$, $f(-5) = 117$

19–25. Use the TABLE feature of a graphing calculator.

19. −0.7, 0.7

20. −2.6, −0.4

21. −2.5

22. −0.4, 3.4

23. −1, 1

24. −1.3, 0.9, 7.4

25. −1.24

26. Sample answers:

r	3	−2	5	−1
1	3	1	6	5

upper bound: 1

$f(-x) = -3x^3 - 2x^2 - 5x - 1$

r	−3	−2	−5	−1
0	−3	−2	−5	−1

lower bound: 0

27. Sample answers:

r	1	−1	−1
1	1	0	−1
2	1	1	1

upper bound: 2

$f(-x) = x^2 + x - 1$

r	1	1	−1
1	1	2	1

lower bound: −1

28. Sample answers:

r	1	−6	2	6	−13
1	1	−5	−3	3	−10
2	1	−4	−6	−6	−25
3	1	−3	−7	−15	−58
4	1	−2	−6	−18	−85
5	1	−1	−3	−9	−58
6	1	0	2	18	95

upper bound: 6

$f(-x) = x^4 + 6x^3 + 2x^2 - 6x - 13$

r	1	6	2	−6	−13
1	1	7	9	3	−10
2	1	8	18	30	47

lower bound: −2

29. Sample answers:

r	1	5	-3	-20
1	1	6	3	-17
2	1	7	11	2

upper bound: 2

$f(-x) = -x^3 + 5x^2 + 3x - 20$

r	-1	5	3	-20
1	-1	4	7	-13
2	-1	3	9	-2
3	-1	2	9	7
4	-1	1	7	8
5	-1	0	3	-5
6	-1	-1	-3	-38

lower bound: -6

30. Sample answers:

r	1	-3	-2	3	-5
1	1	-2	-4	-1	-6
2	1	-1	-4	-5	-15
3	1	0	-2	-3	-14
4	1	1	2	11	39

upper bound: 4

$f(-x) = x^4 + 3x^3 - 2x^2 - 3x - 5$

r	1	3	-2	-3	-5
1	1	4	2	-1	-6
2	1	5	8	13	21

lower bound: -2

31. Sample answers:

r	1	5	-3	20	0	-15
1	1	6	3	23	23	8

upper bound: 1

$f(-x) = -x^5 + 5x^4 + 3x^3 + 20x^2 - 15$

r	-1	5	3	20	0	-15
1	-1	4	7	27	27	8
2	-1	3	9	38	76	137
3	-1	2	9	47	141	408
4	-1	1	7	48	192	753
5	-1	0	3	35	175	860
6	-1	-1	-3	2	12	57
7	-1	-2	-11	-57	-399	-2808

lower bound: -7

32a. 4

32b. $\pm 1, \pm 5$

32c. 3 or 1; $f(-x) = x^4 + 3x^3 - 2x^2 - 3x - 5$
1 negative real zero

32d.

r	1	-3	-2	3	-5
5	1	2	8	43	210
4	1	1	2	11	39
3	1	0	-2	-3	-14
2	1	-1	-4	-5	-15
1	1	-2	-4	-1	-6
0	1	-3	-2	3	-5
-1	1	-4	2	1	-6
-2	1	-5	8	-13	21
-3	1	-6	16	-45	130

-2 and -1, 3 and 4

32e. Sample answers:
upper bound: 4 (See table in 32d.)
$f(-x) = x^4 + 3x^3 - 2x^2 - 3x - 5$

r	1	3	-2	-3	-5
1	1	4	2	-1	-6
2	1	5	8	13	21

lower bound: -2

32f. -1.4, 3.4 (Use TABLE feature of a graphing calculator.)

33a. 1890: $P(0) = -0.78(0)^4 + 133(0)^3 - 7500(0)^2 + 147{,}500(0) + 1{,}440{,}000 = 1{,}440{,}000$

1910: $P(20) = -0.78(20)^4 + 133(20)^3 - 7500(20)^2 + 147{,}500(20) + 1{,}440{,}000 = 2{,}329{,}200$

1930: $P(40) = -0.78(40)^4 + 133(40)^3 - 7500(40)^2 + 147{,}500(40) + 1{,}440{,}000 = 1{,}855{,}200$

1950: $P(60) = -0.78(60)^4 + 133(60)^3 - 7500(60)^2 + 147{,}500(60) + 1{,}440{,}000 = 1{,}909{,}200$

1970: $P(80) = -0.78(80)^4 + 133(80)^3 - 7500(80)^2 + 147{,}500(80) + 1{,}440{,}000 = 1{,}387{,}200$

The model is fairly close, although it is less accurate at for 1950 and 1970.

33b. $1980 - 1890 = 90$

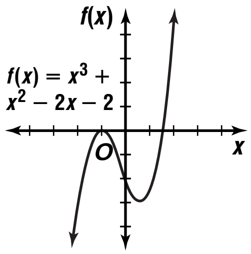
$P(90) = -0.78(90)^4 + 133(90)^3 - 7500(90)^2 + 147{,}500(90) + 1{,}440{,}000$

$P(90) = -253{,}800$

33c. The population becomes 0.

33d. No; there are still many people living in Manhattan.

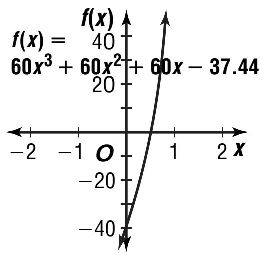
34. Sample answer:
$f(x) = (x - \sqrt{2})(x + \sqrt{2})(x + 1)$
$f(x) = (x^2 - 2)(x + 1)$
$f(x) = x^3 + x^2 - 2x - 2$; $\sqrt{2}$, -1



35a. $37.44 = 60x^3 + 60x^2 + 60x$

35b. $f(x) = 60x^3 + 60x^2 + 60x - 37.44$

35c.

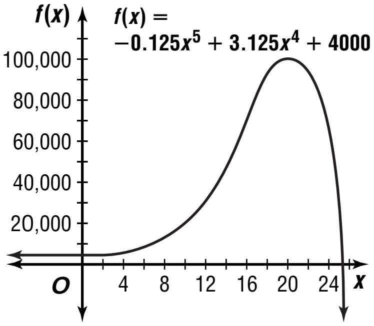


about $\frac{1}{2}$

35d. 0.4 (Use TABLE feature of a graphing calculator.)

36. Sample answer: $f(x) = x^2 - 1$

37a.



37b. $f(0) = -0.125(0)^5 + 3.125(0)^4 + 4000$
$f(0) = 4000$ deer

37c. $1920 - 1905 = 15$
$f(15) = -0.125(15)^5 + 3.125(15)^4 + 4000$
$f(15) = 67{,}281.25$
about 67,281 deer

37d. in 1930

38a. $81.58 = 6x^4 + 18x^3 + 24x^2 + 18x$

38b. $81.58 = 6x^4 + 18x^3 + 24x^2 + 18x$
$0 = 6x^4 + 18x^3 + 24x^2 + 18x - 81.58$
about 1.1 (Use TABLE feature of a graphing calculator.)

38c. $x = \text{rate} + 1$
$1.1 = \text{rate} + 1$
$0.1 = \text{rate}$
about 10%

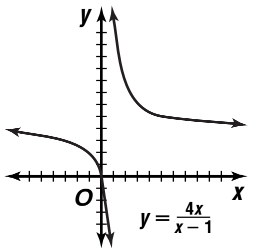
39. 2 or 0; $f(-x) = -2x^3 - 5x^2 + 28x + 15$
1 negative zero

r	2	−5	−28	15
−3	2	−11	5	0

$2x^2 - 11x + 5 = 0$
$(2x + 1)(x - 5) = 0$
$x = 0.5$ $\quad x = 5$
rational zeros: −3, 0.5, 5

40.
$$d(t) = v_0t - \frac{1}{2}gt^2$$
$$-1750 = 4t - \frac{1}{2}(9.8)t^2$$
$4.9t^2 - 4t - 1750 = 0$
$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$t = \frac{4 \pm \sqrt{(-4)^2 - 4(4.9)(-1750)}}{2(4.9)}$
$t = \frac{4 \pm \sqrt{34{,}316}}{9.8}$
$t = \frac{4 + \sqrt{34{,}316}}{9.8}$ $\quad t = \frac{4 - \sqrt{34{,}316}}{9.8}$
$t \approx 19.3$ $\quad t \approx -18.5$
about 19.3 s

41.

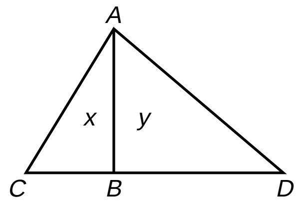


42. $\begin{vmatrix} 7 & 9 \\ 3 & 6 \end{vmatrix} = 7(6) - 3(9)$ or 15

43. $(x, y) = \left(\frac{-3 + 8}{2}, \frac{-2 + 4}{2}\right)$
$= (2.5, 1)$

44. $x - 2y - 4 = 0$
$y = \frac{1}{2}x - 2$; $\frac{1}{2}$; -2

45.



$y < x$ cannot be true.
The correct choice is B.

4-6 Rational Equations and Partial Fractions

Page 247 Check for Understanding

1. Multiply by the LCD, $6(b - 2)$. Then, solve the resulting equation.

2. If a possible solution causes a denominator to equal 0, it is not a solution of the equation.

3. Decomposing a fraction means to find two fractions whose sum or difference equals the original fraction.

4.
$$x + \frac{2}{x-2} = 2 + \frac{2}{x-2}$$
$$\left(x + \frac{2}{x-2}\right)(x - 2) = \left(2 + \frac{2}{x-2}\right)(x - 2)$$
$$x(x - 2) + 2 = 2(x - 2) + 2$$
$$x^2 - 2x + 2 = 2x - 4 + 2$$
$$x^2 - 4x + 4 = 0$$
$$(x - 2)(x - 2) = 0$$
$x - 2 = 0$ $\quad x - 2 = 0$
$x = 2$ $\quad x = 2$

If you solve the equation, you will get $x = 2$. However, if $x = 2$, the denominators will equal 0.

5.
$$b - \frac{5}{b} = 4$$
$$\left(b - \frac{5}{b}\right)b = (4)b$$
$$b^2 - 5 = 4b$$
$$b^2 - 4b - 5 = 0$$
$$(b - 5)(b + 1) = 0$$
$b - 5 = 0$ $\quad b + 1 = 0$
$b = 5$ $\quad b = -1$

6.
$$\frac{9}{b+5} = \frac{3}{b-3}$$
$$\left(\frac{9}{b+5}\right)(b + 5)(b - 3) = \left(\frac{3}{b-3}\right)(b + 5)(b - 3)$$
$$9(b - 3) = 3(b + 5)$$
$$9b - 27 = 3b + 15$$
$$6b - 42 = 0$$
$$b = 7$$

7. $$\frac{t+4}{t} + \frac{3}{t-4} = \frac{-16}{t^2-4t}$$
$$\left(\frac{t+4}{t} + \frac{3}{t-4}\right)(t)(t-4) = \left(\frac{-16}{t^2-4t}\right)(t)(t-4)$$
$$(t+4)(t-4) + 3(t) = -16$$
$$t^2 - 16 + 3t = -16$$
$$t^2 + 3t = 0$$
$$t(t+3) = 0$$
$t = 0$ $\quad$ $t + 3 = 0$

$t = -3$

But $t \neq 0$, so $t = -3$.

8. $$\frac{3p-1}{p^2-1} = \frac{3p-1}{(p+1)(p-1)}$$
$$\frac{3p-1}{p^2-1} = \frac{A}{p+1} + \frac{B}{p-1}$$
$3p - 1 = A(p-1) + B(p+1)$

Let $p = 1$.

$3(1) - 1 = A(1-1) + B(1+1)$

$2 = 2B$

$1 = B$

Let $p = -1$.

$3(-1) - 1 = A(-1-1) + B(-1+1)$

$-4 = -2A$

$2 = A$

$$\frac{3p-1}{p^2-1} = \frac{2}{p+1} + \frac{1}{p-1}$$

9. $5 + \frac{1}{x} > \frac{16}{x}$; exclude: 0

$5x + 1 = 16$

$5x = 15$

$x = 3$

Test $x = -1$: $5 + \frac{1}{(-1)} > \frac{16}{(-1)}$

$4 > -16$ true

Test $x = 1$: $5 + \frac{1}{1} > \frac{16}{1}$

$6 > 16$ false

Test $x = 4$: $5 + \frac{1}{4} > \frac{16}{4}$

$5\frac{1}{4} > 4$ true

Solution: $x < 0$, $x > 3$

10. $1 + \frac{5}{a-1} \leq \frac{7}{6}$; exclude: 1

$6(a-1) + 30 = 7(a-1)$

$6a - 6 + 30 = 7a - 7$

$31 = a$

Test $a = -1$: $1 + \frac{5}{-1-1} \leq \frac{7}{6}$

$-\frac{3}{2} \leq \frac{7}{6}$ true

Test $a = 2$: $1 + \frac{5}{2-1} \leq \frac{7}{6}$

$6 \leq \frac{7}{6}$ false

Test $a = 36$: $1 + \frac{5}{36-1} \leq \frac{7}{6}$

$1 + \frac{1}{7} \leq \frac{7}{6}$

$\frac{48}{42} \leq \frac{49}{42}$ true

Solution: $a < 1$, $a \geq 31$

11a. $\frac{3 \times 60 + 20}{3 + x} = 57.14$

11b. $\frac{3 \times 60 + 20}{3 + x} = 57.14$

$3 \times 60 + 20 = 57.14(3 + x)$

$200 = 171.42 + 57.14x$

$0.50 \approx x$; 0.50 h

Pages 247–250 Exercises

12. $\frac{12}{t} + t - 8 = 0$

$\left(\frac{12}{t} + t - 8\right)t = (0)t$

$12 + t^2 - 8t = 0$

$t^2 - 8t + 12 = 0$

$(t-6)(t-2) = 0$

$t - 6 = 0$ $\quad$ $t - 2 = 0$

$t = 6$ $\quad$ $t = 2$

13. $$\frac{1}{m} = \frac{m-34}{2m^2}$$
$\left(\frac{1}{m}\right)2m^2 = \left(\frac{m-34}{2m^2}\right)2m^2$

$m^2 - 34m = 2m^2$

$0 = m^2 + 34m$

$0 = m(m + 34)$

$m = 0$ $\quad$ $m + 34 = 0$

$m = -34$

But $m \neq 0$, so $m = -34$.

14. $$\frac{2}{y+2} + \frac{3}{y} = \frac{-y}{y+2}$$
$$\left(\frac{2}{y+2} + \frac{3}{y}\right)(y)(y+2) = \left(\frac{-y}{y+2}\right)(y)(y+2)$$
$$2y + 3(y+2) = -y^2$$
$$5y + 6 = -y^2$$
$$y^2 + 5y + 6 = 0$$
$$(y+3)(y+2) = 0$$
$y + 3 = 0$ $\quad$ $y + 2 = 0$

$y = -3$ $\quad$ $y = -2$

But $y \neq -2$, so $y = -3$.

15. $$\frac{10}{n^2-1} + \frac{2n-5}{n-1} = \frac{2n+5}{n+1}$$
$$\left(\frac{10}{n^2-1} + \frac{2n-5}{n-1}\right)(n-1)(n+1) = \left(\frac{2n+5}{n+1}\right)(n-1)(n+1)$$
$$10 + (2n-5)(n+1) = (2n+5)(n-1)$$
$$2n^2 - 3n + 5 = 2n^2 + 3n - 5$$
$$-6n = -10$$
$$n = \frac{5}{3}$$

16. $$\frac{1}{b+2} + \frac{1}{b+2} = \frac{3}{b+1}$$
$$\left(\frac{1}{b+2} + \frac{1}{b+2}\right)(b+2)(b+1) = \left(\frac{3}{b+1}\right)(b+2)(b+1)$$
$$b + 1 + b + 1 = 3(b+2)$$
$$2b + 2 = 3b + 6$$
$$-4 = b$$

17. $$\frac{7a}{3a+3} - \frac{5}{4a-4} = \frac{3a}{2a+2}$$
$$\frac{7a}{3(a+1)} - \frac{5}{4(a-1)} = \frac{3a}{2(a+1)}$$
$$\left(\frac{7a}{3(a+1)} - \frac{5}{4(a-1)}\right)(12)(a-1)(a+1) =$$
$$\left(\frac{3a}{2(a+1)}\right)(12)(a-1)(a+1)$$
$$4(a-1)7a - 3(a+1)5 = 6(a-1)3a$$
$$28a^2 - 28a - 15a - 15 = 18a^2 - 18a$$
$$10a^2 - 25a - 15 = 0$$
$$2a^2 - 5a - 3 = 0$$
$$(2a+1)(a-3) = 0$$
$2a + 1 = 0$ $\quad$ $a - 3 = 0$

$a = -\frac{1}{2}$ $\quad$ $a = 3$

18. $1 = \frac{1}{1-a} + \frac{a}{a-1}$

$1(1-a)(a-1) = \left(\frac{1}{1-a} + \frac{a}{a-1}\right)(1-a)(a-1)$

$a - 1 - a^2 + a = a - 1 + a(1-a)$

$-a^2 + 2a - 1 = -a^2 + 2a - 1$

$0 = 0$

all reals except 1

19. $\frac{2q}{2q+3} - \frac{2q}{2q-3} = 1$

$\left(\frac{2q}{2q+3} - \frac{2q}{2q-3}\right)(2q+3)(2q-3) = 1(2q+3)(2q-3)$

$2q(2q-3) - 2q(2q+3) = (2q+3)(2q-3)$

$4q^2 - 6q - 4q^2 - 6q = 4q^2 - 9$

$0 = 4q^2 + 12q - 9$

$q = \frac{-12 \pm \sqrt{144 - 4(4)(-9)}}{2 \cdot 4}$

$= \frac{-12 \pm \sqrt{288}}{8}$

$= \frac{-12 \pm 12\sqrt{2}}{8}$

$= \frac{-3 \pm 3\sqrt{2}}{2}$

20. $\frac{1}{3m} + \frac{6m-9}{3m} = \frac{3m-3}{4m}$

$\left(\frac{1}{3m} + \frac{6m-9}{3m}\right)(12m) = \left(\frac{3m-3}{4m}\right)(12m)$

$4 + 4(6m-9) = 3(3m-3)$

$4 + 24m - 36 = 9m - 9$

$15m = 23$

$m = \frac{23}{15}$

21. $\frac{-4}{x-1} = \frac{7}{2-x} + \frac{3}{x+1}$

$\left(\frac{-4}{x-1}\right)(x-1)(2-x)(x+1) = \left(\frac{7}{2-x} + \frac{3}{x+1}\right)$
$(x-1)(2-x)(x+1)$

$-4(2-x)(x+1) = 7(x-1)(x+1)$
$+ 3(x-1)(2-x)$

$-4(-x^2 + x + 2) = 7(x^2 - 1)$
$+ 3(-x^2 + 3x - 2)$

$4x^2 - 4x - 8 = 4x^2 + 9x - 13$

$5 = 13x$

$\frac{5}{13} = x$

22a. $(n+1)(n-2)$ **22b.** $-1, 2$

22c. $1 + \frac{n+6}{n+1} = \frac{4}{n-2}$

$\left(1 + \frac{n+6}{n+1}\right)(n+1)(n-2) = \left(\frac{4}{n-2}\right)(n+1)(n-2)$

$(n+1)(n-2) + (n-2)(n+6) = 4(n+1)$

$n^2 - n - 2 + n^2 + 4n - 12 = 4n + 4$

$2n^2 - n - 18 = 0$

$n = \frac{1 \pm \sqrt{1 - 4(2)(-18)}}{2 \cdot 2}$

$= \frac{1 \pm \sqrt{145}}{4}$

23. $\frac{x-6}{x^2-2x} = \frac{x-6}{x(x-2)}$

$\frac{x-6}{x^2-2x} = \frac{A}{x} + \frac{B}{x-2}$

$x - 6 = A(x-2) + B(x)$

Let $x = 2$.

$2 - 6 = A(2-2) + B(2)$

$-4 = 2B$

$-2 = B$

Let $x = 0$.

$0 - 6 = A(0-2) + B(0)$

$-6 = -2A$

$3 = A$

$\frac{x-6}{x^2-2x} = \frac{3}{x} + \frac{-2}{x-2}$

24. $\frac{5m-4}{m^2-4} = \frac{5m-4}{(m+2)(m-2)}$

$\frac{5m-4}{m^2-4} = \frac{A}{m+2} + \frac{B}{m-2}$

$5m - 4 = A(m-2) + B(m+2)$

Let $m = 2$.

$5(2) - 4 = A(2-2) + B(2+2)$

$6 = 4B$

$1.5 = B$

Let $m = -2$.

$5(-2) - 4 = A(-2-2) + B(-2+2)$

$-14 = -4A$

$3.5 = A$

$\frac{5m-4}{m^2-4} = \frac{3.5}{m+2} + \frac{1.5}{m-2}$

25. $\frac{-4y}{3y^2-4y+1} = \frac{-4y}{(3y-1)(y-1)}$

$\frac{-4y}{3y^2-4y+1} = \frac{A}{3y-1} + \frac{B}{y-1}$

$-4y = A(y-1) + B(3y-1)$

Let $y = 1$.

$-4(1) = A(1-1) + B(3(1)-1)$

$-4 = 2B$

$-2 = B$

Let $y = \frac{1}{3}$.

$-4\left(\frac{1}{3}\right) = A\left(\frac{1}{3} - 1\right) + B\left(3\left(\frac{1}{3}\right) - 1\right)$

$-\frac{4}{3} = -\frac{2}{3}A$

$2 = A$

$\frac{-4y}{3y^2-4y+1} = \frac{2}{3y-1} + \frac{-2}{y-1}$

26. $\frac{9-9x}{x^2-9} = \frac{9-9x}{(x+3)(x-3)}$

$\frac{9-9x}{x^2-9} = \frac{A}{(x+3)} + \frac{B}{(x-3)}$

Let $x = 3$.

$9 - 9(3) = A(3-3) + B(3+3)$

$-18 = 6B$

$-3 = B$

Let $x = -3$.

$9 - 9(-3) = A(-3-3) + B(-3+3)$

$36 = -6A$

$-6 = A$

$\frac{9-9x}{x^2-9} = \frac{-6}{x+3} + \frac{-3}{x-3}$

$\frac{-6}{x+3}, \frac{-3}{x-3}$

27a. $a(a-6)$

27b. $\frac{a-2}{a} = \frac{a-4}{a-6}$

$\left(\frac{a-2}{a}\right)(a)(a-6) = \left(\frac{a-4}{a-6}\right)(a)(a-6)$

$(a-2)(a-6) = (a-4)(a)$

$a^2 - 8a + 12 = a^2 - 4a$

$12 = 4a$

$3 = a$

27c. 0, 6

27d. Test $a = -1$: $\frac{-1-2}{-1} < \frac{-1-4}{-1-6}$

$3 < \frac{5}{7}$ false

Test $a = 1$: $\frac{1-2}{1} < \frac{1-4}{1-6}$

$-1 < \frac{3}{5}$ true

Test $a = 4$: $\frac{4-2}{4} < \frac{4-4}{4-6}$

$\frac{1}{2} < 0$ false

Test $a = 7$: $\frac{7-2}{7} < \frac{7-4}{7-6}$

$\frac{5}{7} < 3$ true

Solution: $0 < a < 3$, $6 < a$

28. $\frac{2}{w} + 3 > \frac{29}{w}$; exclude: 0

$2 + 3w = 29$

$w = 9$

Test $w = -1$: $\frac{2}{-1} + 3 > \frac{29}{-1}$

$1 > -29$ true

Test $w = 1$: $\frac{2}{1} + 3 > \frac{29}{1}$

$5 > 29$ false

Test $w = 10$: $\frac{2}{(10)} + 3 > \frac{29}{10}$

$\frac{32}{10} > \frac{29}{10}$ true

Solution: $w < 0$, $w > 9$

29. $\frac{(x-3)(x-4)}{(x-5)(x-6)^2} \le 0$; exclude 5, 6

$(x-3)(x-4) = 0$

$x - 3 = 0$ $\quad$ $x - 4 = 0$

$x = 3$ $\quad$ $x = 4$

Test $x = 0$: $\frac{(0-3)(0-4)}{(0-5)(0-6)^2} \le 0$

$\frac{12}{-180} \le 0$ true

Test $x = 3.5$: $\frac{(3.5-3)(3.5-4)}{(3.5-5)(3.5-6)^2} \le 0$

$\frac{-0.25}{-9.375} \le 0$ false

Test $x = 4.5$: $\frac{(4.5-3)(4.5-4)}{(4.5-5)(4.5-6)^2} \le 0$

$\frac{0.75}{-1.125} \le 0$ true

Test $x = 5.5$: $\frac{(5.5-3)(5.5-4)}{(5.5-5)(5.5-6)^2} \le 0$

$\frac{3.75}{0.125} \le 0$ false

Test $x = 6.5$: $(\frac{6.5-3)(6.5-4)}{(6.5-5)(6.5-6)^2} \le 0$

$\frac{8.75}{0.375} \le 0$ false

Solution: $x \le 3$, $4 \le x < 5$

30. $\frac{x^2-16}{x^2-4x-5} \ge 0$

$\frac{x^2-16}{(x-5)(x+1)} = 0$; exclude 5, -1

$x^2 - 16 = 0$

$x^2 = 16$

$x = \pm 4$

Test $x = -5$: $\frac{(-5)^2-16}{(-5-5)(-5+1)} \ge 0$

$\frac{9}{40} \ge 0$ true

Test $x = -2$: $\frac{(-2)^2-16}{(-2-5)(-2+1)} \ge 0$

$\frac{-12}{7} \ge 0$ false

Test $x = 0$: $\frac{0-16}{0^2-4(0)-5} \ge 0$

$\frac{-16}{-5} \ge 0$ true

Test $x = 4.5$: $\frac{4.5^2-16}{(4.5-5)(4.5+1)} \ge 0$

$\frac{4.25}{-2.75} \ge 0$ false

Test $x = 6$: $\frac{6^2-16}{6^2-24-5} \ge 0$

$\frac{20}{7} \ge 0$ true

Solution: $x \le -4$, $-1 < x \le 4$, $x > 5$

31. $\frac{1}{4a} + \frac{5}{8a} > \frac{1}{2}$; exclude: 0

$\frac{1}{4a} + \frac{5}{8a} = \frac{1}{2}$

$2 + 5 = 4a$

$\frac{7}{4} = a$

Test $a = -1$: $\frac{1}{4(-1)} + \frac{5}{8(-1)} > \frac{1}{2}$

$-\frac{7}{8} > \frac{1}{2}$ false

Test $a = 1$: $\frac{1}{4(1)} + \frac{5}{8(1)} > \frac{1}{2}$

$\frac{7}{8} > \frac{1}{2}$ true

Test $a = 2$: $\frac{1}{4(2)} + \frac{5}{8(2)} > \frac{1}{2}$

$\frac{7}{16} > \frac{1}{2}$ false

Solution: $0 < a < \frac{7}{4}$

32. $\frac{1}{2b+1} + \frac{1}{b+1} > \frac{8}{15}$; exclude: $-\frac{1}{2}, -1$

$\frac{1}{2b+1} + \frac{1}{b+1} = \frac{8}{15}$

$15(b+1) + 15(2b+1) = 8(2b+1)(b+1)$

$45b + 30 = 16b^2 + 24b + 8$

$0 = 16b^2 - 21b - 22$

$0 = (16b+11)(b-2)$

$16b + 11 = 0 \qquad b - 2 = 0$

$b = -\frac{11}{16} \qquad b = 2$

Test $b = -2$: $\frac{1}{2(-2)+1} + \frac{1}{-2+1} > \frac{8}{15}$

$-\frac{4}{3} > \frac{8}{15}$ false

Test $b = -0.8$: $\frac{1}{2(-0.8)+1} + \frac{1}{(-0.8)+1} > \frac{8}{15}$

$\frac{10}{3} > \frac{8}{15}$ true

Test $b = -0.6$: $\frac{1}{2(-0.6)+1} + \frac{1}{(-0.6)+1} > \frac{8}{15}$

$-\frac{5}{2} > \frac{8}{15}$ false

Test $b = 0$: $\frac{1}{2(0)+1} + \frac{1}{0+1} > \frac{8}{15}$

$2 > \frac{8}{15}$ true

Test $b = 3$: $\frac{1}{2(3)+1} + \frac{1}{3+1} > \frac{8}{15}$

$\frac{11}{28} > \frac{8}{15}$ false

Solution: $-1 < b < -\frac{11}{16}$, $-\frac{1}{2} < b < 2$

33. $\frac{7}{y+1} > 7$; exclude -1

$7 = 7(y+1)$

$1 = y + 1$

$0 = y$

Test $y = -2$: $\frac{7}{-2+1} > 7$

$-7 > 7$ false

Test $y = -0.5$: $\frac{7}{-0.5+1} > 7$

$14 > 7$ true

Test $y = 1$: $\frac{7}{1+1} > 7$

$\frac{7}{2} > 7$ false

Solution: $-1 < y < 0$

34. Let x = the number.

$4\left(\frac{1}{x}\right) + x = 10\frac{2}{5}$

$20 + 5x^2 = 52x$

$5x^2 - 52x + 20 = 0$

$(5x-2)(x-10) = 0$

$5x - 2 = 0 \qquad x - 10 = 0$

$x = \frac{2}{5} \qquad x = 10$

35. $\frac{x+2}{x-5} > 0.30$

$\frac{x+2}{x-5} = 0.30$; exclude 5

$x + 2 = 0.30(x-5)$

$x + 2 = 0.30x - 1.5$

$0.7x = -3.5$

$x = -5$

Test $x = -6$: $\frac{-6+2}{-6-5} > 0.30$

$0.36 > 0.30$ true

Test $x = 0$: $\frac{0+2}{0-5} > 0.30$

$-0.4 > 0.30$ false

Test $x = 6$: $\frac{6+2}{6-5} > 0.30$

$8 > 0.30$ true

Solution: $x < -5$ or $x > 5$

36a. $\frac{1}{8} = \frac{1}{d_i} + \frac{1}{32}$

36b. $\frac{1}{8} = \frac{1}{d_i} + \frac{1}{32}$

$\left(\frac{1}{8}\right)(32d_i) = \left(\frac{1}{d_i} + \frac{1}{32}\right)(32d_i)$

$4d_i = 32 + d_i$

$3d_i = 32$

$d_i = 10\frac{2}{3}$ cm

37. Sample answer: $\frac{x}{x-3} = \frac{1}{x+2}$

38. Let x = capacity of larger truck.

$\frac{5}{2} = \frac{x}{x-3}$

$5(x-3) = 2x$

$5x - 15 = 2x$

$3x = 15$

$x = 5$ tons

39a. $\frac{1}{10} = \frac{1}{2r} + \frac{1}{r} + \frac{1}{20}$

39b. $\frac{1}{10} = \frac{1}{2r} + \frac{1}{r} + \frac{1}{20}$

$\left(\frac{1}{10}\right)(20r) = \left(\frac{1}{2r} + \frac{1}{r} + \frac{1}{20}\right)(20r)$

$2r = 10 + 20 + r$

$r = 30$

$2r = 2(30)$ or 60; 60 ohms, 30 ohms

40. Let x = the number of quiz questions to be answered.

$\frac{11+x}{20+x} = 0.70$

$11 + x = 0.70(20+x)$

$11 + x = 14 + 0.70x$

$0.3x = 3$

$x = 10$ questions

41. Let x = the speed of the wind.

$\frac{1062}{200+x} = \frac{738}{200-x}$

$1062(200-x) = 738(200+x)$

$212{,}400 - 1062x = 147{,}600 + 738x$

$64{,}800 = 1800x$

$36 = x$; 36 mph

42. $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$

$$\left(\frac{1}{a} + \frac{1}{b}\right)(a)(b)(c) = \left(\frac{1}{c}\right)(a)(b)(c)$$
$$bc + ac = ab$$
$$bc = ab - ac$$
$$bc = a(b - c)$$
$$\frac{bc}{b - c} = a$$

43a. $\frac{1}{x} = \frac{1}{2}\left(\frac{1}{y} + \frac{1}{z}\right)$

$\frac{1}{x} = \frac{1}{2}\left(\frac{1}{30} + \frac{1}{45}\right)$

43b.
$$\frac{1}{x} = \frac{1}{2}\left(\frac{1}{30} + \frac{1}{45}\right)$$
$$\frac{1}{x} = \frac{1}{60} + \frac{1}{90}$$
$$\left(\frac{1}{x}\right)(360x) = \left(\frac{1}{60} + \frac{1}{90}\right)(360x)$$
$$360 = 6x + 4x$$
$$360 = 10x$$
$$36 = x$$

44. Let x = number of gallons of gasoline.

$\frac{20 \text{ m}}{\text{g}} = \frac{15{,}000 \text{ m}}{x \text{ g}}$

$20x = 15{,}000$

$x = 750$ gallons

$750 \times \$1.20 = \$900$

$x \cdot \$1.20 = \$900 - \$200$

$x = 583\frac{1}{3}$ gallons

Let y = number of miles per gallon.

$\frac{y \text{ m}}{\text{g}} = \frac{15{,}000 \text{ m}}{583\frac{1}{3} \text{ g}}$

$583\frac{1}{3}y = 15{,}000$

$y = 25.7$; about 25.7 mpg

45.
$$T = \frac{d}{s}$$
$$10\frac{2}{3} = \frac{26}{s + 5} + \frac{26}{s - 5}$$
$$\left(10\frac{2}{3}\right)(3)(s + 5)(s - 5) = \left(\frac{26}{s + 5} + \frac{26}{s - 5}\right)(3)(s + 5)(s - 5)$$
$$32(s + 5)(s - 5) = 26(3)(s - 5) + 26(3)(s + 5)$$
$$32s^2 - 800 = 78s - 390 + 78s + 390$$
$$32s^2 - 156s - 800 = 0$$
$$8s^2 - 39s - 200 = 0$$
$$(8s + 25)(s - 8) = 0$$

$8s + 25 = 0$ $\qquad$ $s - 8 = 0$

$s = -\frac{25}{8}$ $\qquad$ $s = 8$

8 mph

46. $\frac{3x - 5y}{5y} = \frac{3x}{5y} - \frac{5y}{5y}$

$= 11 - 1$

$= 10$

47.

r	1	2	−3	−5
−3	1	−1	0	−5
−2	1	0	−3	1
−1	1	1	−4	−1
0	1	2	−3	−5
1	1	3	0	−5
2	1	4	5	5

−3 and −2, −2 and −1, 1 and 2

48.
$$\begin{array}{r|rrrr} -5 & 1 & 0 & -30 & 0 \\ & & -5 & 25 & 25 \\ \hline & 1 & -5 & -5 & \boxed{25} \end{array}$$

no

49. 2; $12x^2 + 8x - 15 = 0$

$(6x - 5)(2x + 3) = 0$

$6x - 5 = 0$ $\qquad$ $2x + 3 = 0$

$x = \frac{5}{6}$ $\qquad$ $x = -\frac{3}{2}$

50. $|3x| - 12 = 0$

$|3x| = 12$

$3x = 12$ $\qquad$ $-3x = 12$

$x = 4$ $\qquad$ $x = -4$

51. $y \leq \frac{2x + 3}{x}$

$3 \overset{?}{\leq} \frac{2(6) + 3}{6}$

$3 \leq \frac{5}{2}$ $\quad$ false

no

52. $y^2 = 121x^2 \rightarrow b^2 = 121a^2$

52a. $(-b)^2 = 121a^2$

$b^2 = 121a^2$ $\quad$ yes

52b. $b^2 = 121(-a)^2$

$b^2 = 121a^2$ $\quad$ yes

52c. $(a)^2 = 121(b)^2$

$a^2 = 121b^2$ $\quad$ no

52d. $(-a)^2 = 121(-b)^2$

$a^2 = 121b^2$ $\quad$ no

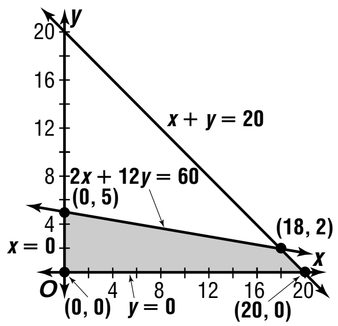
53a. Let x = short answer questions and y = essay questions.

$x + y \leq 20$

$2x + 12y \leq 60$

$x \geq 0$

$y \geq 0$



$S(x, y) = 5x + 15y$

$S(0, 0) = 5(0) + 15(0)$ or 0

$S(0, 5) = 5(0) + 15(5)$ or 75

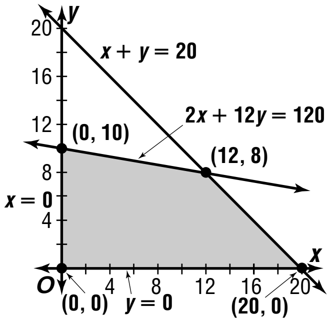
$S(18, 2) = 5(18) + 15(2)$ or 120

$S(20, 0) = 5(20) + 15(0)$ or 100

18 short answer and 2 essay for a score of 120 points

53b. Let x = short answer questions and y = essay questions.

$x + y \le 20$
$2x + 12y \le 120$
$x \ge 0$
$y \ge 0$



$S(x, y) = 5x + 15y$
$S(0, 0) = 5(0) + 15(0)$ or 0
$S(0, 10) = 5(0) + 15(10)$ or 150
$S(12, 8) = 5(12) + 15(8)$ or 180
$S(20, 0) = 5(20) + 15(0)$ or 100
12 short answer and 8 essay for a score of 180 points

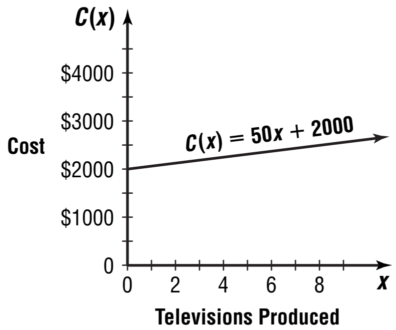
54.
$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ -3 & -5 \end{bmatrix} = x$$
$$\begin{bmatrix} 1(3) + 1(-3) & 1(5) + 1(-5) \\ 1(3) + 1(-3) & 1(5) + 1(-5) \end{bmatrix} = x$$
$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = x$$

55.
$$y - y_1 = m(x - x_1)$$
$$y - 1 = 2(x - (-3))$$
$$y - 1 = 2x + 6$$
$$2x - y + 7 = 0$$

56a. $m = \frac{3000 - 5000}{20 - 60}$
$m = 50$
$y - 3000 = 50(x - 20)$
$y = 50x + 2000$
$C(x) = 50x + 2000$

56b. \$2000; \$50

56c.



57. A of $\triangle JKL = \frac{1}{2}(9)(7)$ or 31.5

A of small triangle $= \frac{1}{2}(5)(3)$ or 7.5

A of shaded region $= 31.5 - 7.5$ or 24
The answer is 24.

4-7 Radical Equations and Inequalities

Pages 254–255 Check for Understanding

1. To solve the equation, you need to get rid of the radical by squaring both sides of the equation. If the radical is not isolated first, a radical will remain in the equation.

2. The process of raising to a power sometimes creates a new equation with more solutions than the original equation. These extra or extraneous solutions do not solve the original equation.

3. When solving an equation with one radical, you isolate the radical on one side and then square each side. When there is more than one radical expression in an equation, you isolate one of the radicals and then square each side. Then you isolate the other radical and square each side. In both cases, once you have eliminated all radical signs, you solve for the variable.

4.
$$\sqrt{1 - 4t} = 2$$
$$1 - 4t = 4$$
$$-4t = 3$$
$$t = -\tfrac{3}{4}$$

Check:
$$\sqrt{1 - 4t} = 2$$
$$\sqrt{1 - 4\left(-\tfrac{3}{4}\right)} \stackrel{?}{=} 2$$
$$\sqrt{1 + 3} \stackrel{?}{=} 2$$
$$2 = 2 \checkmark$$

5.
$$\sqrt[3]{x + 4} + 12 = 3$$
$$\sqrt[3]{x + 4} = -9$$
$$x + 4 = -729$$
$$x = -733$$

Check:
$$\sqrt[3]{x + 4} + 12 = 3$$
$$\sqrt[3]{-733 + 4} + 12 \stackrel{?}{=} 3$$
$$\sqrt[3]{-729} + 12 \stackrel{?}{=} 3$$
$$-9 + 12 \stackrel{?}{=} 3$$
$$3 = 3 \checkmark$$

6.
$$5 + \sqrt{x - 4} = 2$$
$$\sqrt{x - 4} = -3$$
$$x - 4 = 9$$
$$x = 13$$

Check:
$$5 + \sqrt{x - 4} = 2$$
$$5 + \sqrt{13 - 4} \stackrel{?}{=} 2$$
$$5 + \sqrt{9} \stackrel{?}{=} 2$$
$$5 + 3 \neq 2$$
no real solution

7.
$$\sqrt{6x - 4} = \sqrt{2x + 10}$$
$$6x - 4 = 2x + 10$$
$$4x = 14$$
$$x = 3.5$$

Check:
$$\sqrt{6x - 4} = \sqrt{2x + 10}$$
$$\sqrt{6\left(\tfrac{7}{2}\right) - 4} = \sqrt{2\left(\tfrac{7}{2}\right) + 10}$$
$$\sqrt{21 - 4} = \sqrt{7 + 10}$$
$$\sqrt{17} = \sqrt{17} \checkmark$$

8.
$$\sqrt{a + 4} + \sqrt{a - 3} = 7$$
$$\sqrt{a - 4} = 7 - \sqrt{a - 3}$$
$$a - 4 = 49 - 14\sqrt{a - 3} + a - 3$$
$$-42 = -14\sqrt{a - 3}$$
$$1764 = 196(a - 3)$$
$$9 = a - 3$$
$$12 = a$$

Check:
$$\sqrt{a + 4} + \sqrt{a - 3} = 7$$
$$\sqrt{12 + 4} + \sqrt{12 - 3} \stackrel{?}{=} 7$$
$$\sqrt{16} + \sqrt{9} \stackrel{?}{=} 7$$
$$4 + 3 = 7 \checkmark$$

9. $\sqrt{5x + 4} \le 8$ $\qquad$ $5x + 4 \ge 0$

$5x + 4 \le 64$ $\qquad$ $5x \ge -4$

$5x \le 60$ $\qquad$ $x \ge -0.8$

$x \le 12$

Test $x = -1$: $\sqrt{5(-1) + 4} \le 8$

$\sqrt{-1} \le 8$ meaningless

Test $x = 0$: $\sqrt{5(0) + 4} \le 8$

$\sqrt{4} \le 8$ true

Test $x = 13$: $\sqrt{5(13) + 4} \le 8$

$\sqrt{69} \le 8$ false

Solution: $-0.8 \le x \le 12$

10. $3 + \sqrt{4a - 5} \le 10$ $\qquad$ $4a - 5 \ge 0$

$\sqrt{4a - 5} \le 7$ $\qquad$ $4a \ge 5$

$4a - 5 \le 49$ $\qquad$ $a \ge 1.25$

$4a \le 54$

$a \le 13.5$

Test $a = 0$: $3 + \sqrt{4(0) - 5} \le 10$

$3 + \sqrt{-5} \le 10$ meaningless

Test $a = 2$: $3 + \sqrt{4(2) - 5} \le 10$

$4 + \sqrt{3} \le 10$ true

Test $a = 14$: $3 + \sqrt{4(14) - 5} \le 10$

$3 + \sqrt{51} \le 10$ false

Solution: $1.25 \le a \le 13.5$

11a. $v = \sqrt{{v_0}^2 + 64h}$

$90 = \sqrt{10^2 + 64h}$

$90 = \sqrt{100 + 64h}$

11b. $90 = \sqrt{100 + 64h}$ $\qquad$ Check: $90 = \sqrt{100 + 64h}$

$8100 = 100 + 64h$ $\qquad$ $90 \stackrel{?}{=} \sqrt{100 + 64(125)}$

$8000 = 64h$ $\qquad$ $90 \stackrel{?}{=} \sqrt{8100}$

$1125 = h$; 125 ft $\qquad$ $90 = 90 \checkmark$

Pages 255–257 Exercises

12. $\sqrt{x + 8} = 5$ $\qquad$ Check: $\sqrt{x + 8} = 5$

$\sqrt{x + 8} = 25$ $\qquad$ $\sqrt{17 + 8} \stackrel{?}{=} 5$

$x = 17$ $\qquad$ $\sqrt{25} \stackrel{?}{=} 5$

$5 = 5 \checkmark$

13. $\sqrt[3]{y - 7} = 4$ $\qquad$ Check: $\sqrt[3]{y - 7} = 4$

$y - 7 = 64$ $\qquad$ $\sqrt[3]{71 - 7} \stackrel{?}{=} 4$

$y = 71$ $\qquad$ $\sqrt[3]{64} \stackrel{?}{=} 4$

$4 = 4 \checkmark$

14. $\sqrt{8n - 5} - 1 = 2$ $\qquad$ Check: $\sqrt{8n - 5} - 1 = 2$

$\sqrt{8n - 5} = 3$ $\qquad$ $\sqrt{8\left(\frac{7}{4}\right) - 5} - 1 \stackrel{?}{=} 2$

$8n - 5 = 9$ $\qquad$ $\sqrt{14 - 5} - 1 \stackrel{?}{=} 2$

$8n = 14$ $\qquad$ $3 - 1 \stackrel{?}{=} 2$

$n = \frac{7}{4}$ $\qquad$ $2 = 2 \checkmark$

15. $\sqrt{x + 16} = \sqrt{x} + 4$

$x + 16 = x + 8\sqrt{x} + 16$

$0 = 8\sqrt{x}$

$0 = \sqrt{x}$

$0 = x$

Check: $\sqrt{x + 16} = \sqrt{x} + 4$

$\sqrt{0 + 16} \stackrel{?}{=} \sqrt{0} + 4$

$\sqrt{16} \stackrel{?}{=} 4$

$4 = 4 \checkmark$

16. $4\sqrt{3m^2 - 15} = 4$

$\sqrt{3m^2 - 15} = 1$

$3m^2 - 15 = 1$

$3m^2 = 16$

$m^2 = \frac{16}{3}$

$m = \pm\frac{4}{3}\sqrt{3}$

Check: $4\sqrt{3m^2 - 15} = 4$

$4\sqrt{3\left(\frac{4}{3}\sqrt{3}\right)^2 - 15} \stackrel{?}{=} 4$

$4\sqrt{1} \stackrel{?}{=} 4$

$4 = 4 \checkmark$

Check: $4\sqrt{3m^2 - 15} = 4$

$4\sqrt{3\left(-\frac{4}{3}\sqrt{3}\right)^2 - 15} \stackrel{?}{=} 4$

$4\sqrt{1} \stackrel{?}{=} 4$

$4 = 4 \checkmark$

17. $\sqrt{9u - 4} = \sqrt{7u - 20}$

$9u - 4 = 7u - 20$

$2u = -16$

$u = -8$

Check: $\sqrt{9u - 4} = \sqrt{7u - 20}$

$\sqrt{9(-8) - 4} \stackrel{?}{=} \sqrt{7(-8) - 20}$

$\sqrt{-76} = \sqrt{-76}$

no real solution

18. $\sqrt[3]{6u - 5} + 2 = -3$

$\sqrt[3]{6u - 5} = -5$

$6u - 5 = -125$

$6u = -120$

$u = -20$

Check: $\sqrt[3]{6u - 5} + 2 = -3$

$\sqrt[3]{6(-20) - 5} + 2 \stackrel{?}{=} -3$

$\sqrt[3]{-125} + 2 \stackrel{?}{=} -3$

$-5 + 2 \stackrel{?}{=} -3$

$-3 = -3 \checkmark$

19. $\sqrt{4m^2 - 3m + 2} - 2m - 5 = 0$

$\sqrt{4m^2 - 3m + 2} = 2m + 5$

$4m^2 - 3m + 2 = 4m^2 + 20m + 25$

$-23 = 23m$

$-1 = m$

Check: $\sqrt{4m^2 - 3m + 2} - 2m - 5 = 0$

$\sqrt{4(-1)^2 - 3(-1) + 2} - 2(-1) - 5 \stackrel{?}{=} 0$

$\sqrt{9} + 2 - 5 \stackrel{?}{=} 0$

$3 - 3 = 0$

$0 = 0 \checkmark$

20. $\sqrt{k + 9} - \sqrt{k} = \sqrt{3}$

$\sqrt{k + 9} = \sqrt{3} + \sqrt{k}$

$k + 9 = 3 + 2\sqrt{3k} + k$

$6 = 2\sqrt{3k}$

$36 = 4(3k)$

$36 = 12k$

$3 = k$

Check: $\sqrt{k + 9} - \sqrt{k} = \sqrt{3}$

$\sqrt{3 + 9} - \sqrt{3} \stackrel{?}{=} \sqrt{3}$

$\sqrt{12} - \sqrt{3} \stackrel{?}{=} \sqrt{3}$

$2\sqrt{3} - \sqrt{3} \stackrel{?}{=} \sqrt{3}$

$\sqrt{3} = \sqrt{3} \checkmark$

21.
$$\begin{aligned}\sqrt{a+21}-1&=\sqrt{a+12}\\ a+21-2\sqrt{a+21}+1&=a+12\\ -2\sqrt{a+21}&=-10\\ 4(a+21)&=100\\ a+21&=25\\ a&=4\end{aligned}$$
Check: $\sqrt{a+21}-1=\sqrt{a+12}$
$$\begin{aligned}\sqrt{4+21}-1&\stackrel{?}{=}\sqrt{4+12}\\ \sqrt{25}-1&\stackrel{?}{=}\sqrt{16}\\ 5-1&\stackrel{?}{=}4\\ 4&=4\ \checkmark\end{aligned}$$

22.
$$\begin{aligned}\sqrt{3x+4}-\sqrt{2x-7}&=3\\ \sqrt{3x+4}&=3+\sqrt{2x-7}\\ 3x+4&=9+6\sqrt{2x-7}+2x-7\\ x+2&=6\sqrt{2x-7}\\ x^2+4x+4&=36(2x-7)\\ x^2-68x+256&=0\\ (x-4)(x-64)&=0\end{aligned}$$
$x-4=0$ $\quad$ $x-64=0$
$x=4$ $\quad$ $x=64$
Check: $\sqrt{3x+4}-\sqrt{2x-7}=3$
$$\begin{aligned}\sqrt{3(4)+4}-\sqrt{2(4)-7}&\stackrel{?}{=}3\\ \sqrt{16}-\sqrt{1}&\stackrel{?}{=}3\\ 4-1&\stackrel{?}{=}3\\ 3&=3\ \checkmark\end{aligned}$$
Check: $\sqrt{3x+4}-\sqrt{2x-7}=3$
$$\begin{aligned}\sqrt{3(64)+4}-\sqrt{2(64)-7}&\stackrel{?}{=}3\\ \sqrt{196}-\sqrt{121}&\stackrel{?}{=}3\\ 14-11&\stackrel{?}{=}3\\ 3&=3\ \checkmark\end{aligned}$$

23.
$$\begin{aligned}2\sqrt[3]{7b-1}-4&=0\\ 2\sqrt[3]{7b-1}&=4\\ 8(7b-1)&=64\\ 7b-1&=8\\ 7b&=9\\ b&=\tfrac{9}{7}\end{aligned}$$
Check: $2\sqrt[3]{7b-1}-4=0$
$$\begin{aligned}2\sqrt[3]{7\left(\tfrac{9}{7}\right)-1}-4&\stackrel{?}{=}0\\ 2\sqrt[3]{8}-4&\stackrel{?}{=}0\\ 4-4&\stackrel{?}{=}0\\ 0&=0\ \checkmark\end{aligned}$$

24.
$$\begin{aligned}\sqrt[4]{3t}-2&=0\\ \sqrt[4]{3t}&=2\\ 3t&=16\\ t&=\tfrac{16}{3}\end{aligned}$$
Check: $\sqrt[4]{3t}-2=0$
$$\begin{aligned}\sqrt[4]{3\left(\tfrac{16}{3}\right)}-2&\stackrel{?}{=}0\\ \sqrt[4]{16}-2&\stackrel{?}{=}0\\ 2-2&\stackrel{?}{=}0\\ 0&=0\ \checkmark\end{aligned}$$

25.
$$\begin{aligned}\sqrt{x+2}-7&=\sqrt{x+9}\\ x+2-14\sqrt{x+2}+49&=x+9\\ -14\sqrt{x+2}&=-42\\ 196(x+2)&=1764\\ x+2&=9\\ x&=7\end{aligned}$$
Check: $\sqrt{x+2}-7=\sqrt{x+9}$
$$\begin{aligned}\sqrt{7+2}-7&\stackrel{?}{=}\sqrt{7+9}\\ 3-7&\stackrel{?}{=}4\\ -4&\neq 4\end{aligned}$$
no real solution

26.
$$\begin{aligned}\sqrt{2x+1}+\sqrt{2x+6}&=5\\ \sqrt{2x+1}&=5-\sqrt{2x+6}\\ 2x+1&=25-10\sqrt{2x+6}+2x+6\\ -30&=-10\sqrt{2x+6}\\ 3&=\sqrt{2x+6}\\ 9&=2x+6\\ 3&=2x\\ \tfrac{3}{2}&=x\end{aligned}$$
Check: $\sqrt{2x+1}+\sqrt{2x+6}=5$
$$\begin{aligned}\sqrt{2\left(\tfrac{3}{2}\right)+1}+\sqrt{2\left(\tfrac{3}{2}\right)+6}&\stackrel{?}{=}5\\ \sqrt{4}+\sqrt{9}&\stackrel{?}{=}5\\ 2+3&\stackrel{?}{=}5\\ 5&=5\ \checkmark\end{aligned}$$

27.
$$\begin{aligned}\sqrt{3x+10}&=\sqrt{x+11}-1\\ 3x+10&=x+11-2\sqrt{x+11}+1\\ 2x-2&=-2\sqrt{x+11}\\ -x+1&=\sqrt{x+11}\\ x^2-2x+1&=x+11\\ x^2-3x-10&=0\\ (x-5)(x+2)&=0\end{aligned}$$
$x-5=0$ $\quad$ $x+2=0$
$x=5$ $\quad$ $x=-2$
Check: $\sqrt{3x+10}=\sqrt{x+11}-1$
$$\begin{aligned}\sqrt{3(5)+10}&\stackrel{?}{=}\sqrt{5+11}-1\\ \sqrt{25}&\stackrel{?}{=}\sqrt{16}-1\\ 5&\stackrel{?}{=}4-1\\ 5&\neq 3\end{aligned}$$
Check: $\sqrt{3x+10}=\sqrt{x+11}-1$
$$\begin{aligned}\sqrt{3(-2)+10}&\stackrel{?}{=}\sqrt{-2+11}-1\\ \sqrt{4}&\stackrel{?}{=}\sqrt{9}-1\\ 2&=3-1\\ 2&=2\ \checkmark\end{aligned}$$
Solution: $x=-2$

28a.
$$\begin{aligned}\sqrt{3t-14}+t&=6\\ \sqrt{3t-14}&=6-t\\ 3t-14&=36-12t+t^2\\ 0&=t^2-15t+50\\ 0&=(t-5)(t-10)\end{aligned}$$
$t-5=0$ $\quad$ $t-10=0$
$t=5$ $\quad$ $t=10$
Check: $\sqrt{3t-14}+t=6$
$$\begin{aligned}\sqrt{3(5)-14}+5&\stackrel{?}{=}6\\ \sqrt{1}+5&\stackrel{?}{=}6\\ 1+5&\stackrel{?}{=}6\\ 6&=6\ \checkmark\end{aligned}$$
Check: $\sqrt{3t-14}+t=6$
$$\begin{aligned}\sqrt{3(10)-14}+10&\stackrel{?}{=}6\\ \sqrt{16}+10&\stackrel{?}{=}6\\ 4+10&\stackrel{?}{=}6\\ 14&\neq 6\end{aligned}$$
10

28b. 5

29. $\sqrt{2x-7} \geq 5$
$2x - 7 \geq 25$
$2x \geq 32$
$x \geq 16$
$2x - 7 \geq 0$
$2x \geq 7$
$x \geq \frac{7}{2}$

Test $x = 0$: $\sqrt{2(0)-7} \geq 5$
$\sqrt{-7} \geq 5$
meaningless
Test $x = 4$: $\sqrt{2(4)-7} \geq 5$
$\sqrt{8-7} \geq 5$
$1 \geq 5$
false
Test $x = 17$: $\sqrt{2(17)-7} \geq 5$
$\sqrt{27} \geq 5$
true

Solution: $x \geq 16$

30. $\sqrt{b+4} \leq 6$
$b + 4 \leq 36$
$b \leq 32$
$b + 4 \geq 0$
$b \geq -4$

Test $b = -5$: $\sqrt{-5+4} \leq 6$
$\sqrt{-1} \leq 6$
meaningless
Test $b = 0$: $\sqrt{0+4} \leq 6$
$\sqrt{4} \leq 6$
$2 \leq 6$
true
Test $b = 33$: $\sqrt{33+4} \leq 6$
$\sqrt{37} \leq 6$
false

Solution: $-4 \leq b \leq 32$

31. $\sqrt{a-5} \leq 4$
$a - 5 \leq 16$
$a \leq 21$
$a - 5 \geq 0$
$a \geq 5$

Test $a = 0$: $\sqrt{0-5} \leq 4$
$\sqrt{-5} \leq 4$
meaningless
Test $a = 6$: $\sqrt{6-5} \leq 4$
$\sqrt{1} \leq 4$
$1 \leq 4$
true
Test $a = 22$: $\sqrt{22-5} \leq 4$
$\sqrt{17} \leq 4$
false

Solution: $5 \leq a \leq 21$

32. $\sqrt{2x-5} \leq 6$
$2x - 5 \leq 36$
$2x \leq 41$
$x \leq 20.5$
$2x - 5 \geq 0$
$2x \geq 5$
$x \geq 2.5$

Test $x = 0$: $\sqrt{2(0)-5} \leq 6$
$\sqrt{-5} \leq 6$
meaningless
Test $x = 5$: $\sqrt{2(5)-5} \leq 6$
$\sqrt{5} \leq 6$
true
Test $x = 22$: $\sqrt{2(22)-5} \leq 6$
$\sqrt{39} \leq 6$
false

Solution: $2.5 \leq x \leq 20.5$

33. $\sqrt[4]{5y-9} \leq 2$
$5y - 9 \leq 16$
$5y \leq 25$
$y \leq 5$
$5y - 9 \geq 0$
$5y \geq 9$
$y \geq 1.8$

Test $y = 0$: $\sqrt[4]{5(0)-9} \leq 2$
$\sqrt[4]{-9} \leq 2$
meaningless
Test $y = 2$: $\sqrt[4]{5(0)-9} \leq 2$
$\sqrt[4]{1} \leq 2$
true
Test $y = 6$: $\sqrt[4]{5(6)-9} \leq 2$
$\sqrt[4]{21} \leq 2$
false

Solution: $1.8 \leq y \leq 5$

34. $\sqrt{m+2} \leq \sqrt{3m+4}$
$m + 2 \leq 3m + 4$
$-2m \leq 2$
$m \geq -1$
$m + 2 \geq 0$
$m \geq -2$
$3m + 4 \geq 0$
$3m \geq -4$
$m \geq -\frac{4}{3}$
Test $m = -3$: $\sqrt{-3+2} \leq \sqrt{3(-3)+4}$
$\sqrt{-1} \leq \sqrt{-5}$ meaningless
Test $m = -1.6$: $\sqrt{-1.6+2} \leq \sqrt{3(-1.6)+4}$
$\sqrt{0.4} \leq \sqrt{-0.8}$
meaningless
Test $m = -1.2$: $\sqrt{-1.2+2} \leq \sqrt{3(-1.2)+4}$
$\sqrt{0.8} \leq \sqrt{0.4}$ false
Test $m = 0$: $\sqrt{0+2} \leq \sqrt{3(0)+4}$
$\sqrt{2} \leq \sqrt{4}$ true
Solution: $m \geq -1$

35. $\sqrt{2c-5} > 7$
$2c - 5 > 49$
$2c > 54$
$c > 27$
$2c - 5 \geq 0$
$2c \geq 5$
$c \geq 2.5$

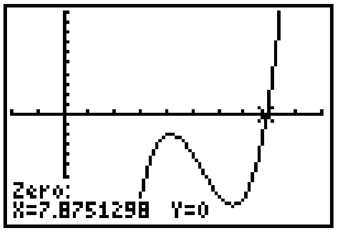
Test $c = 0$: $\sqrt{2(0)-5} \geq 7$
$\sqrt{-5} \geq 7$
meaningless
Test $c = 5$: $\sqrt{2(5)-5} \geq 7$
$\sqrt{5} \geq 7$
false
Test $c = 28$: $\sqrt{2(28)-5} \geq 7$
$\sqrt{51} \geq 7$
true

Solution: $c > 27$

36a. $t = \sqrt{\frac{2s}{g}}$
$3 = \sqrt{\frac{2(7.2)}{g}}$
$3 = \sqrt{\frac{14.4}{g}}$

36b. $3 = \sqrt{\frac{14.4}{g}}$
$9 = \frac{14.4}{g}$
$9g = 14.4$
$g = 1.6 \text{ m/s}^2$

37. $\sqrt{x-5} = \sqrt[3]{x-3}$
$x - 5 = \sqrt[3]{(x-3)^2}$
$x - 5 = \sqrt[3]{x^2 - 6x + 9}$
$(x-5)^3 = x^2 - 6x + 9$
$x^3 - 15x^2 + 75x - 125 = x^2 - 6x + 9$
$x^3 - 16x^2 + 81x - 134 = 0$
Use a graphing calculator to find the zero.



$[-2, 10]$ scl:1 by $[-10, 10]$ scl:1
about 7.88

38a. $s = \sqrt{30fd}$
$s = \sqrt{30(0.6)(25)}$
$s = \sqrt{450}$
$s \approx 21.2$ mph

38b. $s = \sqrt{30fd}$
$35 = \sqrt{30(0.6)d}$
$1225 = 18d$
$68.06 \approx d$; about 68 ft

38c. No; it is not a linear function.

39a. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$T = 2\pi\sqrt{\frac{1}{9.8}}$

$T \approx 2.01$ s

39b. $t = 2\pi\sqrt{\frac{\ell}{g}}$

$T = 2\ \pi\sqrt{\frac{1}{8.9}}$

$T \approx 2.11$ s

39c. Let x = the new length of the pendulum.

$$2\left(2\pi\sqrt{\frac{\ell}{g}}\right) = 2\pi\sqrt{\frac{x}{g}}$$
$$4\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{x}{g}}$$
$$2\sqrt{\frac{\ell}{g}} = \sqrt{\frac{x}{g}}$$
$$4\frac{\ell}{g} = \frac{x}{g}$$
$$4\ell = x$$

It must be multiplied by 4.

40.
$$\frac{T_a}{T_b} = \sqrt{\left(\frac{r_a}{r_b}\right)^3}$$
$$\frac{225}{687} = \sqrt{\left(\frac{67{,}200{,}000}{r_b}\right)^3}$$
$$\frac{50{,}625}{471{,}969} = \frac{3.03 \times 10^{23}}{{r_b}^3}$$
$$50{,}625{r_b}^3 = 1.43 \times 10^{29}$$
$${r_b}^3 = 2.83 \times 10^{24}$$
$$r = 141{,}433{,}433.8; \text{ about } 141{,}433{,}434 \text{ mi}$$

41. $\sqrt{2x - 9} - a = b$

$\sqrt{2x - 9} = a + b$

$2x + 9 \geq 0$, so $a + b \geq 0$

no real solution when $a + b < 0$

42.
$$T = \frac{t + c}{2} + \sqrt{\left(\frac{t - c}{2}\right)^2 + p^2}$$
$$108 = \frac{t + (-200)}{2} + \sqrt{\left(\frac{t - (-200)}{2}\right)^2 + 50^2}$$
$$108 - \left(\frac{t - 200}{2}\right) = \sqrt{\left(\frac{t + 200}{2}\right)^2 + 2500}$$
$$\frac{-t + 416}{2} = \sqrt{\frac{t^2 - 400t + 40{,}000}{4} + 2500}$$
$$\frac{t^2 - 832t + 173{,}056}{4} = \frac{t^2 + 400t + 50{,}000}{4}$$
$$t^2 - 832t + 173{,}056 = t^2 + 400t + 50{,}000$$
$$123{,}056 = 1232t$$
$$99.88 = t; \text{ about } 99.88 \text{ psi}$$

43.
$$\frac{a + 2}{2a + 1} = \frac{a}{3} + \frac{3}{4a + 2}; \text{ exclude: } -\frac{1}{2}$$
$$\left(\frac{a + 2}{2a + 1}\right)(6)(2a + 1) = \left(\frac{a}{3} + \frac{3}{2(2a + 1)}\right)(6)(2a + 1)$$
$$6(a + 2) = a(2)(2a + 1) + 3(3)$$
$$6a + 12 = 4a^2 + 2a + 9$$
$$0 = 4a^2 - 4a - 3$$
$$0 = (2a - 3)(2a + 1)$$

$2a - 3 = 0$ $\qquad$ $2a + 1 = 0$

$a = \frac{3}{2}$ $\qquad$ $a = -\frac{1}{2}$

$\frac{3}{2}$

44. $\frac{p}{q}$: $\pm 6, \pm 3, \pm 2, \pm 1$

r	1	5	5	-5	-6
1	1	6	11	6	0

$x^3 + 6x^2 + 11x + 6 = 0$

r	1	6	11	6
-1	1	5	6	0

$x^2 + 5x + 6 = 0$

$(x + 3)(x + 2) = 0$

$x + 3 = 0$ $\qquad$ $x + 2 = 0$

$x = -3$ $\qquad$ $x = -2$

$-3, -2, -1, 1$

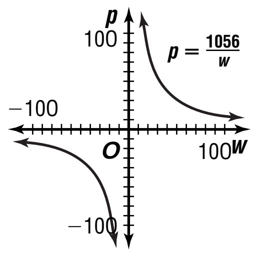
45a. point discontinuity

45b. jump discontinuity

45c. infinite discontinuity

46a. $p = \frac{v}{w}$

$p = \frac{1056}{w}$



46b. x- and y-axes

46c. It increases.

46d. It is halved.

47.
$$\begin{bmatrix} 4 & -1 & 6 \\ 4 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & 3 \\ 2 & -2 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 4(0) + (-1)(2) + 6(5) & 4(3)(-1)(-2) + 6(1) \\ 4(0) + \quad 0(2) + 2(5) & 4(3) + 0(-2) + 2(1) \end{bmatrix}$$
$$= \begin{bmatrix} 28 & 20 \\ 10 & 14 \end{bmatrix}$$

48. $-4(a + b + c) = -4(6)$ $\quad\rightarrow\quad$ $-4a - 4b - 4c = -24$

$2a - 3b + 4c = 3$ $\qquad\qquad\qquad$ $2a - 3b + 4c = 3$

$\qquad\qquad\qquad\qquad\qquad\qquad$ $-2a - 7b = -21$

$-4(a + b + c) = -4(6)$ $\quad\rightarrow\quad$ $-4a - 4b - 4c = -24$

$4a - 8b + 4c = 12$ $\qquad\qquad\qquad$ $4a - 8b + 4c = 12$

$\qquad\qquad\qquad\qquad\qquad\qquad$ $-12b = -12$

$\qquad\qquad\qquad\qquad\qquad\qquad$ $b = 1$

$-2a - 7b = -21$ $\qquad$ $a + b + c = 6$

$-2a - 7(1) = -21$ $\qquad$ $7 + 1 + c = 6$

$a = 7$ $\qquad$ $c = -2$

$(7, 1, -2)$

49. $y = -3.54x + 7125.4$

$y = -3.54(2010) + 7125.4$

$y = 10$ students

50. $7y + 4x - 3 = 0$

$y = -\frac{4}{7}x + \frac{3}{7}$

perpendicular slope: $\frac{7}{4}$

$y - 5 = \frac{7}{4}(x - 2)$

$y = \frac{7}{4}x + \frac{3}{2}$

51. $A = \pi r^2$ $\qquad$ $A = \pi r^2$

$= \pi\left(\frac{1}{2}\right)^2$ $\qquad$ $= \pi(1)^2$

$= \frac{1}{4}\pi$ $\qquad$ $= 1\pi$

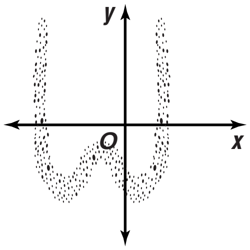
$\frac{1}{4}\pi + 1\pi = \frac{5}{4}\pi$

The correct choice is C.

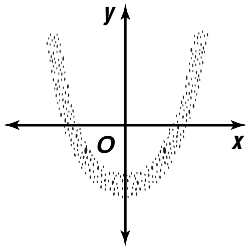
4-8 Modeling Real-World Data with Polynomial Functions

Pages 261-262 Check for Understanding

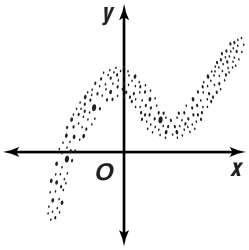
1a. Sample answer:



1b. Sample answer:



1c. Sample answer:



2. You need to recognize the general shape so that you can tell the graphing calculator which type of polynomial function to use as a model.

3. Sample answer: If companies use less packaging materials, consumers keep items longer, and old buildings are restored instead of demolished, the amount of waste will decrease more rapidly. If consumers buy more products, companies package items in larger containers, and many old buildings are destroyed, the amount of waste will increase instead of decrease.

4. quartic

5. Sample answer:
$f(x) = 1.98x^4 + 2.95x^3 - 5.91x^2 + 0.22x + 4.89$

6. Sample answer: $f(x) = 3.007x^2 + 0.001x - 7.896$

7a. Sample answer: $f(x) = 0.48x + 58.0$

7b. Sample answer: $2010 - 1950 = 60$
$f(x) = 0.48x + 58.0$
$= 0.48(60) + 58.0$
$= 86.8\%$

7c. Sample answer: $f(x) = 0.48x + 58.0$
$89 = 0.48x + 58.0$
$64 \approx x$
$1950 + 64 = 2014$

Pages 262-264 Exercises

8. cubic

9. quadratic

10. linear

11. quadratic

12. $f(x) = -1.25x + 5$

13. $f(x) = 8x^2 - 3x - 9$

14. Sample answer:
$f(x) = 1.03x^4 - 5.16x^3 + 6.08x^2 + 0.23x + 0.94$

15. Sample answer:
$f(x) = 0.09x^3 - 2.70x^2 + 24.63x - 65.21$

16. Sample answer:
$f(x) = 4.05x^4 - 0.09x^3 + 6.69x^2 - 222.03x + 2697.74$

17. Sample answer:
$f(x) = -0.02x^3 + 8.79x^2 + 3.35x + 27.43$

18a. Sample answer: $f(x) = 1.99x^2 - 1.74x + 2.76$

18b. Sample answer:
$f(x) = -0.96x^3 + 0.56x^2 + 0.36x + 4.05$

18c. Sample answer: Cubic; the value of r^2 for the cubic function is closer to 1.

19a. Sample answer: $f(x) = 0.126x + 22.732$

19b. Sample answer:
$2010 - 1900 = 110$
$f(x) = 0.126x + 22.732$
$f(110) = 0.126(110) + 22.732$
$f(110) = 36.592$
37

19c. Sample answer:
$2025 - 1900 = 125$
$f(x) = 0.126x + 22.732$
$f(125) = 0.126(125) + 22.732$
$f(125) = 38.482$
38

20. Sample answer:

x	1	2	3	4	5	6	7	8	9
$f(x)$	1	3	6	3	–13	–49	–112	–209	–347

21a. Sample answer:
$f(x) = 0.008x^4 - 0.138x^3 + 0.621x^2 + 0.097x + 18.961$

21b. Sample answer: $1994 - 1992 = 2$
$f(x) = 0.008x^4 - 0.138x^3 + 0.621x^2 + 0.097x + 18.961$
$f(2) = 0.008(2)^4 - 0.138(2)^3 + 0.621(2)^2 + 0.097(2) + 18.961$
$f(2) = 20.663$
about 21%

22. A sixth-degree polynomial; there are 5 changes in direction.

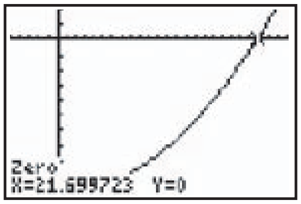
23a. Sample answer:
$f(x) = 0.109x^2 - 0.001x + 48.696$

23b. Sample answer:

$f(x) = 0.109x^2 - 0.001x + 48.696$

$100 = 0.109x^2 - 0.001x + 48.696$

$0 = 0.109x^2 - 0.001x - 51.304$



$[-5, 25]$ sc1:1 by $[-50, 10]$ sc1:5

root: (21.7, 0)

1985 + 22 = 2007

23c. Sample answer: 1998 − 1985 = 13

$f(x) = 0.109x^2 - 0.001x + 48.696$

$f(13) = 0.109(13)^2 - 0.001(13) + 48.696$

$f(13) = 67.104$

No; according to the model, there should have been an attendance of only about 67 million. Since the actual attendance was much higher than the projected number, it is likely that the race to break the homerun record increased the attendance.

24a. Sample answer:

$f(x) = -0.033x^3 + 1.471x^2 - 1.368x + 5.563$

24b. Sample answer: 1996 − 1990 = 6

$f(x) = -0.033x^3 + 1.471x^2 - 1.368x + 5.563$

$f(6) = -0.033(6)^3 + 1.471(6)^2 - 1.368(6) + 5.563$

$f(6) = 43.183$

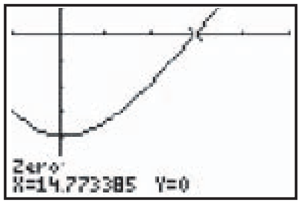
about 43.18 million

24c. Sample answer:

$f(x) = -0.033x^3 + 1.471x^2 - 1.368x + 5.563$

$200 = -0.033x^3 + 1.471x^2 - 1.368x + 5.563$

$0 = -0.033x^3 + 1.471x^2 - 1.368x - 194.437$



$[-5, 25]$ sc1:5 by $[-300, 50]$ sc1:50

root: (14.8, 0)

14 + 1990 = 2004

about 2004

25.
$$\begin{aligned} 5 - \sqrt{b+2} &= 0 \\ 5 &= \sqrt{b+2} \\ 25 &= b + 2 \\ 23 &= b \end{aligned}$$

Check:
$$\begin{aligned} 5 - \sqrt{b+2} &= 0 \\ 5 - \sqrt{23+2} &\stackrel{?}{=} 0 \\ 5 - \sqrt{25} &\stackrel{?}{=} 0 \\ 5 - 5 &\stackrel{?}{=} 0 \\ 0 &= 0 \ \checkmark \end{aligned}$$

26.
$$\begin{aligned} \frac{6}{p+3} + \frac{p}{p-3} &= 1 \\ \left(\frac{6}{p+3} + \frac{p}{p-3}\right)(p+3)(p-3) &= 1(p+3)(p-3) \\ 6(p-3) + p(p+3) &= (p+3)(p-3) \\ 6p - 18 + p^2 + 3p &= p^2 - 9 \\ 9p &= 9 \\ p &= 1 \end{aligned}$$

27.

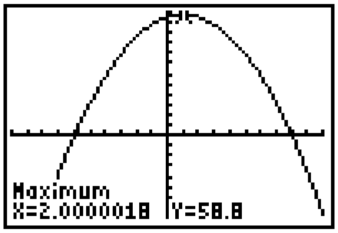
r	2	−1	0	1	−2
−1	2	−3	3	−2	0
0	2	−1	0	1	−2
1	2	1	1	2	0

−1, 1

28a. Let x = number of weeks.

$P = (120 + 10x)(0.48 - 0.03x)$

$P = 57.6 + 1.2x - 0.3x^2$



$[-20, 20]$ sc1:2 by $[-40, 60]$ sc1:5

maximum: (2, 58.8)

2 weeks

28b. $58.80 per tree

29. $x - 0.10x = 0.90x$

$$\begin{aligned} 0.90x + 0.10(0.90x) &= 0.90x + 0.09x \\ &= 0.99x \end{aligned}$$

The correct choice is B.

4-8B Fitting a Polynomial Function to a Set of Points

Page 266

1. $y = 7x^3 - 4x^2 - 17x + 15$
2. $y = 7x^3 - 4x^2 - 17x + 15$; yes
3. Sample answer: $y = -5x^6 - 2x^5 + 40x^4 - 2x^3 + x^2 + 8x - 4$
4. Infinitely many; suppose that you are given a set of n points in a coordinate plane, no two of which are on the same vertical line. You can pick an infinite number of other points with different x-coordinates. You could find polynomial functions that went through the original n points and any number of the other points.
5. There is no problem with using L1^0 with list L_1 for the example. However, if you are using a different list which happens to have 0 as one of its elements, using L1^0 will result in an error message, since 0^0 is undefined.

Chapter 4 Study Guide and Assessment

Page 267 Understanding and Using the Vocabulary

1. Quadratic Formula
2. Integral Root Theorem
3. zero
4. Factor Theorem
5. polynomial function
6. lower bound

7. Extraneous **8.** complex roots

9. complex numbers **10.** quadratic equation

Pages 268-270 Skills and Concepts

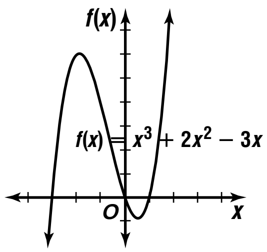
11. no; $f(a) = a^3 - 3a^2 - 3a - 4$
$f(0) = (0)^3 - 3(0)^2 - 3(0) - 4$
$f(0) = -4$

12. yes; $f(a) = a^3 - 3a^2 - 3a - 4$
$f(4) = (4)^3 - 3(4)^2 - 3(4) - 4$
$f(4) = 0$

13. no; $f(a) = a^3 - 3a^2 - 3a - 4$
$f(-2) = (-2)^3 - 3(-2)^2 - 3(-2) - 4$
$f(-2) = -18$

14. $f(t) = t^4 - 2t^2 - 3t + 1$
$f(-3) = (-3)^4 - 2(-3)^2 - 3(-3) + 1$
$f(-3) = 73$
no

15. 3; $x^3 + 2x^2 - 3x = 0$
$x(x^2 + 2x - 3) = 0$
$x(x + 3)(x - 1) = 0$
$x = 0 \qquad x + 3 = 0 \qquad x - 1 = 0$
$\qquad\qquad x = -3 \qquad x = 1$



16. $b^2 - 4ac = (-7)^2 - 4(2)(-4)$
$= 81$; 2 real
$x = \frac{7 \pm \sqrt{81}}{2(2)}$
$x = \frac{7 \pm 9}{4}$
$x = \frac{7 + 9}{4} \qquad x = \frac{7 - 9}{4}$
$x = 4 \qquad x = -\frac{1}{2}$

17. $b^2 - 4ac = (-10)^2 - 4(3)(5)$
$= 40$; 2 real
$m = \frac{10 \pm \sqrt{40}}{2(3)}$
$m = \frac{10 \pm 2\sqrt{10}}{6}$
$m = \frac{5 \pm \sqrt{10}}{3}$

18. $b^2 - 4ac = (-1)^2 - 4(1)(6)$
$= -23$; 2 imaginary
$x = \frac{1 \pm \sqrt{-23}}{2(1)}$
$x = \frac{1 \pm i\sqrt{23}}{2}$

19. $b^2 - 4ac = 3^2 - 4(-2)(8)$
$= 73$; 2 real
$y = \frac{-3 \pm \sqrt{73}}{2(2)}$
$y = \frac{3 \pm \sqrt{73}}{4}$

20. $b^2 - 4ac = 4^2 - 4(1)(4)$
$= 0$; 1 real
$a = \frac{-4 \pm \sqrt{0}}{2(1)}$
$a = -\frac{4}{2}$
$a = -2$

21. $b^2 - 4ac = (-1)^2 - 4(5)(10)$
$= -199$; 2 imaginary
$r = \frac{1 \pm \sqrt{-199}}{2(5)}$
$r = \frac{1 \pm i\sqrt{199}}{10}$

22. $f(x) = x^3 - x^2 - 10x - 8$
$f(-2) = (-2)^3 - (-2)^2 - 10(-2) - 8$
$= -8 - 4 + 20 - 8$ or 0; yes

23. $f(x) = 2x^3 - 5x^2 + 7x + 1$
$f(5) = 2(5)^3 - 5(5)^2 + 7(5) + 1$
$= 250 - 125 + 35 + 1$ or 161; no

24. $f(x) = 4x^3 - 7x + 1$
$f\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{2}\right)^3 - 7\left(-\frac{1}{2}\right) + 1$
$= -\frac{4}{8} + \frac{7}{2} + 1$ or 4; no

25. $f(x) = x^4 - 10x^2 + 9$
$f(3) = (3)^4 - 10(3)^2 + 9$
$= 81 - 90 + 9$ or 0; yes

26. $\frac{p}{q}$: $\pm1, \pm2$

r	1	−2	−1	2
1	1	−1	−2	0

$x^2 - x - 2 = 0$
$(x - 2)(x + 1) = 0$
$x - 2 = 0 \qquad x + 1 = 0$
$x = 2 \qquad x = -1$
rational roots: −1, 1, 2

27. $\frac{p}{q}$: ±1

r	1	0	−1	1	−1
−1	1	−1	0	1	−2
1	1	1	0	1	0

rational root: 1

28. p: $\pm1, \pm2, \pm4$
q: $\pm1, \pm2$
$\frac{p}{q}$: $\pm1, \pm2, \pm4; \pm\frac{1}{2}$

r	2	−2	−2	−4
1	2	0	−2	−6
2	2	2	2	0

$2x^2 + 2x + 2 = 0$
$x^2 + x + 1 = 0$
does not factor

rational root: 2

29. p: $\pm 1, \pm 3$

q: $\pm 1, \pm 2$

$\frac{p}{q}$: $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}$

r	2	3	-6	-11	-3
1	2	5	-1	-12	-15
-1	2	1	-7	-4	1
3	2	9	21	52	153
-3	2	-3	3	-20	57
$\frac{1}{2}$	2	4	-4	-13	$-\frac{19}{2}$
$-\frac{1}{2}$	2	2	-7	$-\frac{15}{2}$	$\frac{3}{4}$
$\frac{3}{2}$	2	6	-3	$-\frac{31}{2}$	$-\frac{105}{4}$
$-\frac{3}{2}$	2	0	-6	-2	0

rational root: $-\frac{3}{2}$

30. $\frac{p}{q}$: $\pm 1, \pm 2, \pm 4$

r	1	0	-7	1	12	-4
1	1	1	-6	-5	7	3
2	1	2	-3	-5	2	0

$x^4 + 2x^3 - 3x^2 - 5x + 2 = 0$

r	1	2	-3	-5	2
-2	1	0	-3	1	0

$x^3 - 3x + 1 = 0$

r	1	0	-3	1
-2	1	-2	1	-1
-1	1	-1	-2	3
4	1	4	13	53
-4	1	-4	13	-51

rational roots: $-2, 2$

31. p: $\pm 1, \pm 2, \pm 4, \pm 8$

q: $\pm 1, \pm 3$

$\frac{p}{q}$: $\pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$

r	3	7	-2	-8
1	3	10	8	0

$3x^2 + 10x + 8 = 0$

$(3x + 4)(x + 2) = 0$

$3x + 4 = 0 \qquad x + 2 = 0$

$x = -\frac{4}{3} \qquad x = -2$

rational roots: $-2, -\frac{4}{3}, 1$

32. p: $\pm 1, \pm 2$

q: $\pm 1, \pm 2, \pm 4$

$\frac{p}{q}$: $\pm 1, \pm 2, \pm\frac{1}{4}, \pm\frac{1}{2}$

r	4	1	8	2
$-\frac{1}{4}$	4	0	8	0

$4x^2 + 8 = 0$

$x^2 + 2 = 0$

does not factor

rational root: $-\frac{1}{4}$

33. $\frac{p}{q}$: $\pm 1, \pm 5$

r	1	0	4	0	-5
1	1	1	5	5	0

$x^3 + x^2 + 5x + 5 = 0$

r	1	1	5	5
-1	1	0	5	0

$x^2 + 5 = 0$

does not factor

rational roots: $-1, 1$

34. 1 positive

$f(-x) = -x^3 - x^2 + 34x - 56$

2 or 0 negative

r	1	-1	-34	-56
7	1	6	8	0

$x^2 + 6x + 8 = 0$

$(x + 4)(x + 2) = 0$

$x + 4 = 0 \qquad x + 2 = 0$

$x = -4 \qquad x = -2$

rational zeros: $-4, -2, 7$

35. 2 or 0 positive

$f(-x) = -2x^3 - 11x^2 - 12x + 9$

1 negative

r	2	-11	12	9
$-\frac{1}{2}$	2	-12	18	0

$2x^2 - 12x + 18 = 0$

$x^2 - 6x + 9 = 0$

$(x - 3)(x - 3) = 0$

$x - 3 = 0 \qquad x - 3 = 0$

$x = 3 \qquad x = 3$

rational zeros: $-\frac{1}{2}, 3$

36. 2 or 0 positive

$f(-x) = x^4 - 13x^2 + 36$

2 or 0 negative

r	1	0	-13	0	36
2	1	2	-9	-18	0

$x^3 + 2x^2 - 9x - 18 = 0$

r	1	2	-9	-18
3	1	5	6	0

$x^2 + 5x + 6 = 0$

$(x + 3)(x + 2) = 0$

$x + 3 = 0 \qquad x + 2 = 0$

$x = -3 \qquad x = -2$

rational zeros: $-3, -2, 2, 3$

37.

r	3	0	0	1
−2	3	−6	12	−23
−1	3	−3	3	−2
0	3	0	0	1

−1 and 0

38.

r	1	−4	2
0	1	−4	2
1	1	−3	−1
2	1	−2	−2
3	1	−1	−1
4	1	0	2

0 and 1, 3 and 4

39.

r	1	−3	−3
−1	1	−4	1
0	1	−3	−3
1	1	−2	−5
2	1	−1	−5
3	1	0	−3
4	1	1	1

−1 and 0, 3 and 4

40.

r	1	−1	0	1
−2	1	−3	6	−11
−1	1	−2	2	−1
0	1	−1	0	1
1	1	0	0	1

−1 and 0

41.

r	4	2	−11	3
−2	4	−7	3	−3
−1	4	−3	−8	11
0	4	1	−11	3
1	4	5	−6	−3
2	4	9	7	17

−2 and −1, 0 and 1, 1 and 2

42.

r	−9	25	−24	6
−1	−9	34	−58	64
0	−9	25	−24	6
1	−9	16	−8	−2
2	−9	7	−10	−14

0 and 1

43. Use the TABLE feature of a graphing calculator.
−4.9, −1.8, 2.2

44.
$$n - \frac{6}{n} + 5 = 0$$
$$\left(n - \frac{6}{n} + 5\right)(n) = 0(n)$$
$$n^2 + 5n - 6 = 0$$
$$(n + 6)(n - 1) = 0$$
$$n + 6 = 0 \qquad n - 1 = 0$$
$$n = -6 \qquad n = 1$$

45.
$$\frac{1}{x} = \frac{x+3}{2x^2}$$
$$\left(\frac{1}{x}\right)(2x^2) = \left(\frac{x+3}{2x^2}\right)(2x^2)$$
$$2x = x + 3$$
$$x = 3$$

46.
$$\frac{5}{6} = \frac{2m}{2m+2} - \frac{1}{3m-3}$$
$$\left(\frac{5}{6}\right)6(m+1)(m-1) = \left(\frac{2m}{2(m+1)} - \frac{1}{3(m-1)}\right)6(m+1)(m-1)$$
$$5(m+1)(m-1) = (2m)(3)(m-1) - 2(m+1)$$
$$5m^2 - 5 = 6m^2 - 6m - 2m - 2$$
$$0 = m^2 - 8m + 3$$
$$m = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(3)}}{2(1)}$$
$$m = \frac{8 \pm \sqrt{52}}{2}$$
$$m = 4 \pm \sqrt{13}$$

47.
$$\frac{3}{y} - 2 < \frac{5}{y}\text{; exclude: } 0$$
$$\left(\frac{3}{y} - 2\right)y = \left(\frac{5}{y}\right)y$$
$$3 - 2y = 5$$
$$y = -1$$
Test $y = -2$: $\frac{3}{-2} - 2 < \frac{5}{-2}$

$-\frac{7}{2} < -\frac{5}{2}$ true

Test $y = -0.5$: $\frac{3}{-0.5} - 2 < \frac{5}{-0.5}$

$-8 < -10$ false

Test $y = 1$: $\frac{3}{1} - 2 < \frac{5}{1}$

$1 < 5$ true

Solution: $y < -1, y > 0$

48.
$$\frac{2}{x+1} < 1 - \frac{1}{x-1}\text{; exclude } -1, 1$$
$$\left(\frac{2}{x+1}\right)(x+1)(x-1) = \left(1 - \frac{1}{x-1}\right)(x+1)(x-1)$$
$$2(x-1) = (x+1)(x-1) - (x+1)$$
$$2x - 2 = x^2 - x - 2$$
$$0 = x^2 - 3x$$
$$0 = x(x-3)$$
$$x = 0 \qquad x - 3 = 0$$
$$x = 3$$
Test $x = -2$: $\frac{2}{-2+1} < 1 - \frac{1}{-2-1}$

$-2 < \frac{4}{3}$ true

Test $x = -0.5$: $\frac{2}{-0.5+1} < 1 - \frac{1}{-0.5+1}$

$4 < -1$ false

Test $x = 0.5$: $\frac{2}{0.5+1} < 1 - \frac{1}{0.5-1}$

$\frac{4}{3} < 3$ true

Test $x = 2$: $\frac{2}{2+1} < 1 - \frac{1}{2-1}$

$\frac{2}{3} < 0$ false

Test $x = 4$: $\frac{2}{4+1} < 1 - \frac{1}{4-1}$

$\frac{2}{5} < \frac{2}{3}$ true

Solution: $x < -1, 0 < x < 1, x > 3$

49.
$$5 - \sqrt{x+2} = 0$$
$$5 = \sqrt{x+2}$$
$$25 = x + 2$$
$$23 = x$$
Check:
$$5 - \sqrt{x+2} = 0$$
$$5 - \sqrt{23+2} \stackrel{?}{=} 0$$
$$5 - \sqrt{25} \stackrel{?}{=} 0$$
$$5 - 5 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

50. $\sqrt[3]{4a-1}+8=5$

$\sqrt[3]{4a-1}=-3$

$4a-1=-27$

$4a=-26$

$a=-6.5$

Check: $\sqrt[3]{4a-1}+8=5$

$\sqrt[3]{4(-6.5)-1}+8 \stackrel{?}{=} 5$

$\sqrt[3]{-27}+8 \stackrel{?}{=} 5$

$-3+8 \stackrel{?}{=} 5$

$5=5$ ✓

51. $3+\sqrt{x+8}=\sqrt{x+35}$

$9+6\sqrt{x+8}+x+8=x+35$

$6\sqrt{x+8}=18$

$\sqrt{x+8}=3$

$x+8=9$

$x=1$

Check: $3+\sqrt{x+8}=\sqrt{x+35}$

$3+\sqrt{1+8} \stackrel{?}{=} \sqrt{1+35}$

$3+\sqrt{9} \stackrel{?}{=} \sqrt{36}$

$3+3=6$

$6=6$ ✓

52. $\sqrt{x-5}<7$ $\quad$ $x-5>0$

$x-5<49$ $\quad$ $x>5$

$x<54$

Test $x=0$: $\sqrt{0-5}<7$

$\sqrt{-5}<7$ meaningless

Test $x=10$: $\sqrt{10-5}<7$

$\sqrt{5}<7$ true

Test $x=60$: $\sqrt{60-5}<7$

$\sqrt{55}<7$ false

Solution: $5<x<54$

53. $4+\sqrt{2a+7}\ge 6$ $\quad$ $2a+7>0$

$\sqrt{2a+7}\ge 2$ $\quad$ $2a>-7$

$2a+7\ge 4$ $\quad$ $a>-3.5$

$2a\ge -3$

$a\ge -1.5$

Test $a=-5$: $4+\sqrt{2(-5)+7}\ge 6$

$4+\sqrt{-3}\ge 6$ meaningless

Test $a=-2$: $4+\sqrt{2(-2)+7}\ge 6$

$4+\sqrt{3}\ge 6$ false

Test $a=0$: $4+\sqrt{2(0)+7}\ge 6$

$4+\sqrt{7}\ge 6$ true

Solution: $a\ge -1.5$

54. cubic

55. $f(x)=2x^2-x+3$

Page 271 Applications and Problem Solving

56. Let x = width of window.

Let $x+6$ = height of window.

$A=\ell w$

$315=x(x+6)$

$315=x^2+6x$

$0=x^2+6x-315$

$0=(x+21)(x-15)$

$x+21=0$ $\quad$ $x-15=0$

$x=-21$ $\quad$ $x=15$

Since distance cannot be negative, $x=15$ and $x+6=21$. the window should be 15 in. by 21 in.

57. Let x = width.

Let $x+6$ = length.

$(x+12)(x+6)-(x+6)(x)=288$

$x^2+18x+72-x^2-6x=288$

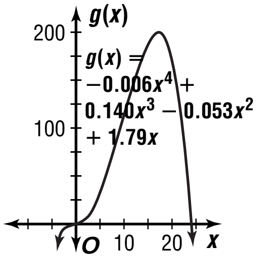
$12x+72=288$

$x=18$

$x+6=24$

18 ft by 24 ft

58a.



58b. $g(x)=-0.006x^4+0.140x^3-0.053x^2+1.79x$

$=x(-0.006x^3+0.140x^2-0.053x+1.79)$

$=x(x^3-23.\overline{3}x^2+8.8\overline{3}x-298.\overline{3})$

r	1	−23.333	8.833	−298.333
1	1	−22.333	−13.503	−311.836
5	1	−18.333	−82.835	−712.508
23.5	1	0.167	12.758	≈ 0

rational zeros: 0, about 23.5

59. $T=2\pi\sqrt{\frac{\ell}{g}}$

$1.6=2\pi\sqrt{\frac{\ell}{9.8}}$

$0.25=\sqrt{\frac{\ell}{9.8}}$

$0.06=\frac{\ell}{9.8}$

$0.64\approx \ell$; about 0.64 m

Page 271 Open-Ended Assessment

1. Sample answer: $\frac{x}{x+3}=\frac{2}{2x+1}$

$\left(\frac{x}{x+3}\right)(x+3)(2x+1)=\left(\frac{2}{2x+1}\right)(x+3)(2x+1)$

$x(2x+1)=2(x+3)$

$2x^2+x=2x+6$

$2x^2-x-6=0$

$(2x+3)(x-2)=0$

$2x+3=0$ $\quad$ $x-2=0$

$x=-\frac{3}{2}$ $\quad$ $x=2$

2a. Sample answer: $x-4=\sqrt{x-2}$

2b. Sample answer: $x-4=\sqrt{x-2}$

$(x-4)^2=x-2$

$x^2-8x+16=x-2$

$x^2-9x-18=0$

$(x-6)(x-3)=0$

$x-6=0$ $\quad$ $x-3=0$

$x=6$ $\quad$ $x=3$

Check: $x-4=\sqrt{x-2}$ $\quad$ $x-4=\sqrt{x-2}$

$6-4 \stackrel{?}{=} \sqrt{6-2}$ $\quad$ $3-4 \stackrel{?}{=} \sqrt{3-2}$

$2 \stackrel{?}{=} \sqrt{4}$ $\quad$ $-1 \stackrel{?}{=} \sqrt{1}$

$2=2$ ✓ $\quad$ $-1\ne 1$

The solution is 6. Since $-1\ne 1$, 3 is an extraneous root.

3a. Sample answer:

x	−3	−2	−1	−0.5
$f(x)$	−12	0	2	1.125
x	0	0.5	1	2
$f(x)$	0	−0.625	0	8

3b. Sample answer: $f(x)=x^3+x^2-2x$

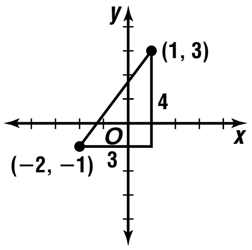
3c. Sample answer: −2, 0, 1

Chapter 4 SAT & ACT Preparation

Page 273 SAT and ACT Practice

1. There are two ways to solve this problem. You can use the distance formula or you can sketch a graph.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(1 - (-2))^2 + (3 - (-1))^2}$$
$$= \sqrt{3^2 + 4^2}$$
$$= \sqrt{9 + 16}$$
$$= \sqrt{25} \text{ or } 5$$



When you sketch the points and draw a right triangle as shown above, you can see that this is a 3-4-5 right triangle. Using the Pythagorean Theorem, you can calculate that the length of the hypotenuse is 5.

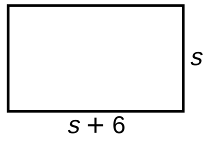
$5^2 = 4^2 + 3^2$ The correct choice is C.

2. Points on the graph of $f(x)$ are of the form $(x, f(x))$. To move the entire graph of $f(x)$ up 2 units, 2 must be added to each of the second coordinates. Points on the translated graph are of the form $(x, f(x) + 2)$. The function which represents the translation of the graph up 2 units is $f(x) + 2$. The correct choice is E.

3. You need to find both the x- and y-coordinates of point C. Use the properties of a parallelogram. First find the y-coordinate. Since opposite sides of a parallelogram are parallel and side AD is on the x-axis, point C must have the same y-coordinate as point B. So the y-coordinate is b. This means you can eliminate answer choices A and B

   Now find the x-coordinate. Since opposite sides of a parallelogram have equal length and side AD has length d, side BC must also have length d. Point B is a units from the y-axis, so point C must be $a + d$ units from the y-axis. The x-coordinate of point C is $a + d$. So point C has coordinates $(a + d, b)$. The correct choice is E.

4. You may want to draw a diagram.



Use the formula for the perimeter of a rectangle, where ℓ represents the length and w represents the width.

$2\ell + 2w = P$

Replace w with s. Replace ℓ with $s + 6$.

$2(s + 6) + 2s = 60$

The correct choice is E.

5. First find the slope of the given line. Write the equation in the form $y = mx + b$.

$$3x - 6y = 12$$
$$6y = 3x - 12$$
$$y = \frac{1}{2}x - 2 \qquad \text{The slope is } \tfrac{1}{2}.$$

So the slope of the line perpendicular to this line is the negative reciprocal of this slope. The slope of the perpendicular line is -2. The correct choice is A.

6. Be sure to notice the small piece of given information: x is an integer. You need to find the number, written in scientific notation, that could be x^3. This means that the cube root of the number is an integer.

   Take the cube root of each of the answer choices and see which one is an integer. You can use your calculator or do the calculations by hand. Notice that 2.7 is one-tenth of 27, which is 3^3.

$2.7 \times 10^{13} = 27 \times 10^{12}$

$\sqrt[3]{27 \times 10^{12}} = 3 \times 10^4$ or 30,000. 30,000 is an integer.

When you try the same calculation with each of the other answer choices, the resulting power of 10 has a fractional exponent. So the number cannot be an integer. The correct choice is C.

7. This is a system of equations, but you do not need to solve for x or y. You need to find the value of $6x + 6y$.

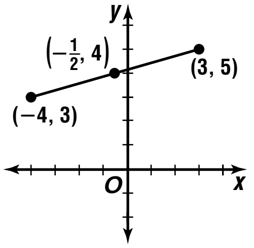
   Notice that the first equation contains $5y$ and the second contains $-1y$. If you subtract the second from the first, you have $6y$. Similarly, subtraction of the x values gives a result of $6x$. Use the same strategy that you would for solving a system. Subtract the second equation from the first.

$$\begin{array}{r} 10x + 5y = 14 \\ \underline{-4x + y = -2} \\ 6x + 6y = 12 \end{array}$$

The correct choice is C.

8. You can solve this problem using the midpoint formula or by sketching a graph.

The midpoint formula:
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3 + (-4)}{2}, \frac{5 + 3}{2}\right) = \left(\frac{-1}{2}, \frac{8}{2}\right) = \left(-\frac{1}{2}, 4\right)$



The correct choice is B.

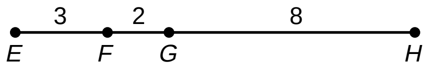
9. The expression $x^2 - 2ax + a^2$ is a perfect square trinomial and can be factored as $(x - a)^2$. The square of a real quantity is never negative.

The correct choice is A.

$(x - a^2 = x^2 - 2ax + a^2$
$= (x^2 + a^2) - 2ax$

So the quantity in Column A equals the quantity in Column B plus the sum of the squares of x and a. Since neither x nor a equal 0, their squares must be greater than 0. So the quantity in Column A is always greater than the quantity in Column B. The correct choice is A.

10. Since the problem does not include a figure, draw one. Label the four points.



One method of solving this problem is to “plug-in” numbers for the segment lengths. Since $EG = \frac{5}{3}\,EF$, let $EF = 3$. Then $EG = 5$. This means that FG must equal 2, since $EF + FG = EG$.

$HF = 5FG = 5(2) = 10$

$HG = HF - FG = 10 - 2 = 8$

$\frac{EF}{HG} = \frac{3}{8}$

The answer is .375 or 3/8.

Chapter 5 The Trigonometric Functions

5-1 Angles and Degree Measure

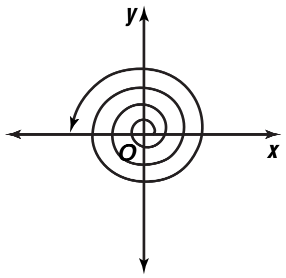
Pages 280–281 Check for Understanding

1. If an angle has a positive measure, the rotation is in a counterclockwise direction. If an angle has a negative measure, the rotation is in a clockwise direction.

2. Add 29, $\frac{45}{60}$, and $\frac{26}{3600}$.

3. $270° + 360k°$ where k is an integer

4.



1260°

5. $34.95° = 34° + (0.95 \cdot 60)'$
$= 34° + 57'$
$34°\ 57'$

6. $-72.775° = -(72° + (0.775 \cdot 60)')$
$= -(72° + 46.5')$
$= -(72° + 46' + (0.5 \cdot 60)'')$
$= -(72° + 46' + 30'')$
$-72°\ 46'\ 30''$

7. $-128°\ 30'\ 45' = -\left(128° + 30'\left(\frac{1°}{60'}\right) + 45''\left(\frac{1°}{3600''}\right)\right)$
$= -128.513°$

8. $29°\ 6'\ 6'' = 29° + 6'\left(\frac{1°}{60'}\right) + 6''\left(\frac{1°}{3600''}\right)$
$= 29.102°$

9. $2 \times (-360°) = -720°$

10. $4.5 \times 360° = 1620°$

11. $22° + 360k°$; Sample answers:
$22° + 360k° = 22° + 360(1)°$ or $382°$
$22° + 360k° = 22° + 360(-1)°$ or $-338°$

12. $-170° + 360k°$; Sample answers:
$-170° + 360k° = -170° + 360(1)°$ or $190°$
$-170° + 360k° = -170° + 360(-1)°$ or $-530°$

13. $\frac{453}{360} \approx 1.26$
$\propto + 360(1)° = 453°$
$\propto + 360 = 453°$
$\propto = 93°$; II

14. $-\frac{798}{360} \approx -2.22$
$-2.22 + 2 = -0.22$
$-0.22 \times 360° = -78°$
$360° - 78° = 282°$; IV

15. $\propto - 180° = 227° - 180°$
$= 47°$

16. $360° - 210° = 150°$
$180° - \propto = 180° - 150°$
$= 30°$

17. $\frac{1}{24}(360°) = 15°$
$\frac{1}{60}\left(\frac{1}{24}\left(360°\right)\right) = 0.25°$, or $0.25(60') = 15'$
$\frac{1}{60}\left(\frac{1}{60}\left(\frac{1}{24}\left(360°\right)\right)\right) \approx 0.0042°$,
or $0.0042(60)(60) = 15''$

Pages 281–283 Exercises

18. $-16.75° = -(16° + (0.75 \cdot 60)')$
$= -(16° + 45')$
$-16°\ 45'$

19. $168.35° = 168° + (0.35 \cdot 60)'$
$= 168° + 21'$
$168°\ 21'$

20. $-183.47° = -(183° + (0.47 \cdot 60)')$
$= -(183° + 28.2')$
$= -(183° + 28' + (0.2 \cdot 60''))$
$= -(183° + 28' + 12'')$
$-183°\ 28'\ 12''$

21. $286.88° = 286° + (0.88 \cdot 60)'$
$= 286° + 52.8'$
$= 286° + 52' + (0.8 \cdot 60)''$
$= 286° + 52' + 48''$
$286°\ 52'\ 48''$

22. $27.465° = 27° + (0.465 \cdot 60)'$
$= 27° + 27.9'$
$= 27° + 27' + (0.9 \cdot 60)''$
$= 27° + 27' + 54''$
$27°\ 27'\ 54''$

23. $246.876° = 246° + (0.876 \cdot 60)'$
$= 246° + 52.56'$
$= 246° + 52' + (0.56 \cdot 60)''$
$= 246° + 52' - 33.6''$
$246°\ 52'\ 33.6''$

24. $23°\ 14'\ 30'' = 23° + 14'\left(\frac{1°}{60'}\right) + 30''\left(\frac{1°}{3600''}\right)$
$= 23.242°$

25. $-14°\ 5'\ 20'' = -\left(14° + 5'\left(\frac{1°}{60'}\right) + 20''\left(\frac{1°}{3600''}\right)\right)$
$= -14.089°$

26. $233°\ 25'\ 15'' = 233° + 25'\left(\frac{1°}{60'}\right) + 15''\left(\frac{1°}{3600''}\right)$
$= 233.421°$

27. $173°\ 24'\ 35'' = 173° + 24'\left(\frac{1°}{60'}\right) + 35''\left(\frac{1°}{3600''}\right)$
$= 173.410°$

28. $-405°\ 16'\ 18'' = -\left(405° + 16'\left(\frac{1°}{60'}\right) + 18''\left(\frac{1°}{3600''}\right)\right)$
$= -405.272°$

29. $1002°\ 30'\ 30'' = 1002° + 30'\left(\frac{1°}{60'}\right) + 30''\left(\frac{1°}{3600''}\right)$
$= 1002.508°$

30. $3 \times -360° = -1080°$

31. $2 \times 360° = 720°$

32. $1.5 \times 360° = 540°$

33. $7.5 \times (-360°) = -2700°$

34. $2.25 \times 360° = 810°$

35. $5.75 \times (-360°) = -2070°$

36. $4 \times 360° = 1440°$

37. $30° + 360k°$; Sample answers:
$30° + 360k° = 30° + 360(1)°$ or $390°$
$30° + 360k° = 30° + 360(-1)°$ or $-330°$

38. $-45° + 360k°$; Sample answers:
$-45° + 360k° = -45° + 360(1)°$ or $315°$
$-45° + 360k° = -45° + 360(-1)°$ or $-405°$

39. $113° + 360k°$; Sample answers:
$113° + 360k° = 113° + 360(1)°$ or $473°$
$113° + 360k° = 113° + 360(-1)°$ or $-247°$

40. $217° + 360k$; Sample answers:
$217° + 360k° = 217° + 360(1)°$ or $577°$
$217° + 360k° = 217° + 360(-1)°$ or $-143°$

41. $-199° + 360k°$; Sample answers:
$-199° + 360k° = -199° + 360(1)°$ or $161°$
$-199° + 360k° = -199° + 360(-1)°$ or $-559°$

42. $-305° + 360k°$; Sample answers:
$-305° + 360k° = -305° + 360(1)°$ or $55°$
$-305° + 360k° = -305° + 360(-1)°$ or $-665°$

43. $310° + 360k° = 310° + 360(0)°$ or $310°$

44. $60° + 360k° = 60° + 360(2)°$ or $780°$
$60° + 360k° = 60° + 360(-3)$ or $-1020°$

45. $\frac{400°}{360°} \approx 1.11$
$\propto + 360(1)° = 400°$
$\propto + 360° = 400°$
$\propto = 40°$; I

46. $\frac{-280°}{360°} \approx -0.78$
$\propto + 360(-1)° = -280°$
$\propto - 360° = -280°$
$\propto = 80°$; I

47. $\frac{940°}{360°} \approx 2.61$
$\propto + 360(2)° = 940°$
$\propto + 720° = 940°$
$\propto = 220°$; III

48. $\frac{1059°}{360°} \approx 2.94$
$\propto + 360(2)° = 1059°$
$\propto + 720° = 1059°$
$\propto = 339°$; IV

49. $\frac{-624°}{360°} \approx -1.73$
$-1.73 + 1 = -0.73$
$-0.73 \times 360° = -264°$
$360° - 264° = 96°$; II

50. $\frac{-989°}{360°} \approx -2.75$
$-2.75 + 2 = -0.75$
$-0.75 \times 360° = -269°$
$360° - 269° = 91°$; II

51. $\frac{1275°}{360°} \approx 3.54$
$\propto + 360(3)° = 1275°$
$\propto + 1080° = 1275°$
$\propto = 195°$; III

52. $360° - \propto = 360° - 327°$ or $33°$

53. $180° - \propto = 180° - 148°$ or $32°$

54. $563° - 360° = 203°$
$\propto - 180° = 203° - 180°$ or $23°$

55. $-420° + 360° = -60°$
$360° - 60° = 300°$
$360° - 300° = 60°$

56. $360° - 197° = 163°$
$180° - 163° = 17°$

57. $\frac{1045°}{360°} \approx 2.90$
$\propto + 360(2)° = 1045°$
$\propto + 720° = 1045°$
$\propto = 325°$
$360° - \propto = 360° - 325°$ or $35°$

58. $20°$
$180° - \propto = 180° - 20°$ or $160°$
$180° + \propto = 180° + 20°$ or $200°$
$360° - \propto = 360° - 20°$ or $340°$

59. $\frac{90 \text{ revolutions}}{\text{second}} \cdot \frac{360°}{\text{revolution}} = 32{,}400°/\text{second}$
$\frac{90 \text{ revolutions}}{\text{second}} \cdot \frac{60 \text{ seconds}}{1 \text{ minute}} \cdot \frac{360°}{\text{revolution}} =$
$1{,}944{,}000°/\text{minute}$

60. $90k$, where k is an integer

61. $\frac{95 \text{ revolutions}}{\text{minute}} \cdot \frac{360°}{\text{revolution}} = 34{,}200°/\text{minute}$
30 seconds $= \frac{1}{2}$ minute
$34{,}200° \cdot \frac{1}{2} = 17{,}100°$

62. $\frac{30{,}000 \text{ revolutions}}{\text{minute}} \cdot \frac{360°}{\text{revolution}} =$
$10{,}800{,}000$ or 1.08×10^7
$\frac{100{,}000 \text{ revolutions}}{\text{minute}} \cdot \frac{360°}{\text{revolution}} =$
$36{,}000{,}000$ or 3.6×10^7
1.08×10^7 to 3.6×10^7 degrees

63. $\frac{62 \text{ rotations}}{\text{second}} \cdot \frac{360°}{\text{rotation}} = 22{,}320°$ second
$\frac{62 \text{ rotations}}{\text{second}} \cdot \frac{360°}{\text{rotation}} \cdot \frac{60 \text{ seconds}}{\text{minute}} =$
$1{,}339{,}200°/\text{minute}$
$\frac{62 \text{ rotations}}{\text{second}} \cdot \frac{360°}{\text{rotation}} \cdot \frac{60 \text{ seconds}}{\text{minute}} \cdot \frac{60 \text{ minutes}}{\text{hour}} =$
$80{,}352{,}000°/\text{hour}$
$\frac{62 \text{ rotations}}{\text{second}} \cdot \frac{360°}{\text{rotation}} \cdot \frac{60 \text{ seconds}}{\text{minute}} \cdot \frac{60 \text{ minutes}}{\text{hour}} \cdot$
$24 \frac{\text{hours}}{\text{day}} = 1{,}928{,}448{,}000°/\text{day}$

64. $25° + 120k°$, where k is an integer

65a. $44.4499° = 44° + (0.4499 \cdot 60)'$
$= 44° + 26.994'$
$= 44° + 26' + (0.994 \cdot 60)''$
$= 44° + 26' + 59.64''$
$44°\ 26'\ 59.64''$
$68.2616° = 68° + (0.2616 \cdot 60)'$
$= 68° + 15.696'$
$= 68° + 15' + (0.696 \cdot 60)''$
$= 68° + 15' + 41.76''$
$68°\ 15'\ 41.76''$

65b. $24°\ 33'\ 32'' = 24° + 33'\left(\frac{1°}{60'}\right) + 32''\left(\frac{1°}{3600''}\right)$
$\approx 24.559°$
$81°\ 45'\ 34.4'' = 81° + 45'\left(\frac{1°}{60'}\right) + 34.4''\left(\frac{1°}{3600''}\right)$
$\approx 81.760°$

66a. Sydney:
$\frac{1 \text{ rotation}}{70 \text{ minutes}} \cdot \frac{60 \text{ minutes}}{\text{hour}} \cdot \frac{24 \text{ hours}}{\text{day}} \approx \frac{20.6 \text{ rotations}}{\text{day}}$
San Antonio: $\frac{1 \text{ revolution}}{1 \text{ hour}} \cdot \frac{24 \text{ hours}}{\text{day}} = \frac{24 \text{ revolutions}}{\text{day}}$
$24 - 20.6 = 3.4$ revolutions
about 3.4 revolutions

66b. Sydney:
$\frac{20.6 \text{ rotations}}{\text{day}} \cdot \frac{7 \text{ days}}{\text{week}} \cdot \frac{360°}{\text{rotation}} = 51{,}840°$
San Antonio: $\frac{24 \text{ revolutions}}{\text{day}} \cdot \frac{7 \text{ days}}{\text{week}} \cdot \frac{360°}{\text{revolution}} =$
$60{,}480°$
$60{,}480° - 51{,}840° = 8640°$

67a. Use graphing calculator to find cubic regression.
Sample answer: $f(x) = -0.00055x^3 + 0.0797x^2$
$- 3.7242x + 76.2147$

67b. $2010 - 1950 = 60$
$f(x) = -0.00055x^3 - 0.0797x^2 - 3.7242x$
$+ 76.2147$
$f(60) = -0.00055(60)^3 + 0.0797(60)^2$
$- 3.7242(60) + 76.2147$
$= 20.8827$
Sample answer: about 20.9%

68. $\sqrt[3]{6n+5} - 15 = -10$
$$\sqrt[3]{6n+5} = 5$$
$$6n + 5 = 125$$
$$6n = 120$$
$$n = 20$$
Check: $\sqrt[3]{6n+5} - 15 = -10$
$$\sqrt[3]{6(20)+5} - 15 \stackrel{?}{=} -10$$
$$\sqrt[3]{125} - 15 \stackrel{?}{=} -10$$
$$5 - 15 = -10$$
$$-10 = -10 \quad \checkmark$$

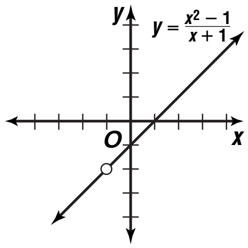
69. $$\frac{x+3}{x+2} = 2 - \frac{3}{x^2+5x+6}$$
$$\frac{x+3}{x+2} = 2 - \frac{3}{(x+2)(x+3)}$$
$$(x+2)(x+3)\left(\frac{x+3}{x+2}\right) = (x+2)(x+3)(2) - (x+2)(x+3)\left(\frac{3}{(x+2)(x+3)}\right)$$
$$(x+3)(x+3) = (x+2)(x+3)(2) - 3$$
$$x^2 + 6x + 9 = 2x^2 + 10x + 12 - 3$$
$$0 = x^2 + 4x$$
$$0 = x(x+4)$$
$x = 0$ or $x + 4 = 0$
$$x = -4$$

70.
$$\begin{array}{r|rrrrr} 2 & 1 & 1 & 0 & 8 & 1 \\ & & 2 & 4 & 24 & \\ \hline & 1 & 2 & 12 & 25 & \end{array}$$
25

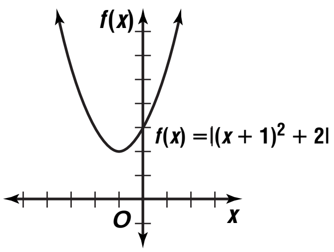
71. $\big(x - (-5)\big)\big(x - (-6)\big)(x - 10) = 0$
$$(x+5)(x+6)(x-10) = 0$$
$$(x^2 + 11x + 30)(x - 10) = 0$$
$$x^3 + x^2 - 80x - 300 = 0$$

72. $r_1 t_1 = r_2 t_2$
$$18(-3) = r_2(-11)$$
$$\frac{18(-3)}{-11} = r_2$$
$$4.91 \approx r_2$$
about 4.91

73. $x + 1 = 0$
$x \neq -1$
Point discontinuity



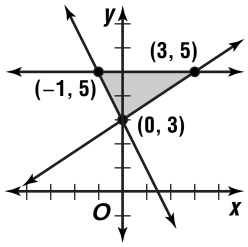
74.



decreasing for $x < -1$, increasing for $x > -1$

75. expanded vertically by a factor of 3, translated down 2 units

76.



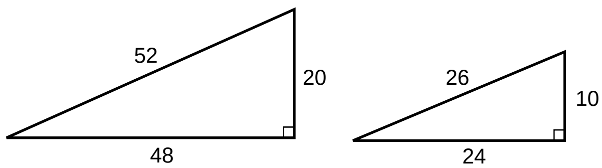
77. $[f \cdot g](x) = f(g(x))$
$$= f(x - 0.3x)$$
$$= (x - 0.3x) - 0.2(x - 0.3x)$$
$$= x - 0.3x - 0.2x + 0.06x$$
$$= 0.56x$$

78. $m\angle EOD = 180° - m\angle EOA - m\angle BOD$
$$= 180° - 85° - 15°$$
$$= 80°$$
$m\angle OED = m\angle EDO$
$m\angle OED = \frac{1}{2}(180° - m\angle EOD)$
$$= \frac{1}{2}(180° - 80°)$$
$$= 50°$$
$m\angle ECA = 180° - m\angle EOC - m\angle OED$
$$= 180° - (80° + 15°) - 50°$$
$$= 35°$$
The correct choice is D.

5-2 Trigonometric Ratios in Right Triangles

Page 284 Graphing Calculator Exploration

1. Sample answers:



2. $R_1 = \frac{5}{13}$ or about 0.3846
$R_1 = \frac{15}{39}$ or about 0.3846
$R_2 = \frac{12}{13}$ or about 0.9231
$R_2 = \frac{36}{39}$ or about 0.9231
$R_3 = \frac{5}{12}$ or about 0.4167
$R_3 = \frac{15}{36}$ or about 0.4167

3. $R_1 = \frac{12}{13}$ or about 0.9231
$R_1 = \frac{36}{39}$ or about 0.9231
$R_2 = \frac{5}{13}$ or about 0.3846
$R_2 = \frac{15}{39}$ or about 0.3846
$R_3 = \frac{12}{5}$ or 2.4
$R_3 = \frac{36}{15}$ or 2.4

4. Each ratio has the same value for all 22.6° angles.

5. yes

6. Yes; the triangles are similar.

Pages 287–288 Check for Understanding

1. The side opposite the acute angle of a right triangle is the side that is not part of either side of the angle. The side adjacent to the acute angle is the side of the triangle that is part of the side of the angle, but is not the hypotenuse.

2. cosecant; secant; cotangent

3. $\sin A = \frac{a}{c}$, $\cos A = \frac{b}{c}$, $\tan A = \frac{a}{b}$,
$\csc A = \frac{c}{a}$, $\sec A = \frac{c}{b}$, $\cot A = \frac{b}{a}$

4. $\sin A = \cos B$, $\csc A = \sec B$, $\tan A = \cot B$

5. $(TV)^2 + (VU)^2 = (TU)^2$
$17^2 + 15^2 = (TU)^2$
$514 = (TU)^2$
$\sqrt{514} = TU$

$\sin T = \frac{\text{side opposite}}{\text{hypotenuse}}$ $\quad\cos T = \frac{\text{side adjacent}}{\text{hypotenuse}}$

$\sin T = \frac{15}{\sqrt{514}}$ or $\frac{15\sqrt{514}}{514}$ $\quad\cos T = \frac{17}{\sqrt{514}}$ or $\frac{17\sqrt{514}}{514}$

$\tan T = \frac{\text{side opposite}}{\text{side adjacent}}$

$\tan T = \frac{15}{17}$

6. $\csc\theta = \frac{1}{\sin\theta}$
$\csc\theta = \frac{1}{\frac{2}{5}}$ or $\frac{5}{2}$

7. $\tan\theta = \frac{1}{\cot\theta}$
$\tan\theta = \frac{1}{1.5}$ or about 0.6667

8. $(PS)^2 + (QS)^2 = (QP)^2$
$(PS)^2 + 6^2 = 20^2$
$(PS)^2 = 364$
$PS = \sqrt{364}$ or $2\sqrt{91}$

$\sin P = \frac{\text{side opposite}}{\text{hypotenuse}}$ $\quad\cos P = \frac{\text{side adjacent}}{\text{hypotenuse}}$

$\sin P = \frac{6}{20}$ or $\frac{3}{10}$ $\quad\cos P = \frac{2\sqrt{91}}{20}$ or $\frac{\sqrt{91}}{10}$

$\tan P = \frac{\text{side opposite}}{\text{side adjacent}}$ $\quad\csc P = \frac{\text{hypotenuse}}{\text{side opposite}}$

$\tan P = \frac{6}{2\sqrt{91}}$ or $\frac{3\sqrt{91}}{91}$ $\quad\csc P = \frac{20}{6}$ or $\frac{10}{3}$

$\sec P = \frac{\text{hypotenuse}}{\text{side adjacent}}$ $\quad\cot P = \frac{\text{side adjacent}}{\text{side opposite}}$

$\sec P = \frac{20}{2\sqrt{91}}$ or $\frac{10\sqrt{91}}{91}$ $\quad\cot P = \frac{2\sqrt{91}}{6}$ or $\frac{\sqrt{91}}{3}$

9. $\cos\theta = \sqrt{\frac{I_t}{I_o}}$

$\cos 45° = \frac{\sqrt{I_t}}{I_o}$

$\frac{\sqrt{2}}{2} = \frac{\sqrt{I_t}}{\sqrt{I_o}}$

$\frac{2}{4} = \frac{I_t}{I_o}$

$0.5I_0 = I_t$

Pages 288–290 Exercises

10. $(AC)^2 + (CB)^2 = (AB)^2$
$80^2 + 60^2 = (AB)^2$
$10{,}000 = (AB)^2$
$100 = AB$

$\sin A = \frac{\text{side opposite}}{\text{hypotenuse}}$ $\quad\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}}$

$\sin A = \frac{60}{100}$ or $\frac{3}{5}$ $\quad\cos A = \frac{80}{100}$ or $\frac{4}{5}$

$\tan A = \frac{\text{side opposite}}{\text{side adjacent}}$

$\tan A = \frac{60}{80}$ or $\frac{3}{4}$

11. $(AC)^2 + (CB)^2 = (AB)^2$
$8^2 + 5^2 = (AB)^2$
$89 = (AB)^2$
$\sqrt{89} = AB$

$\sin A = \frac{\text{side opposite}}{\text{hypotenuse}}$ $\quad\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}}$

$\sin A = \frac{5}{\sqrt{89}}$ or $\frac{5\sqrt{89}}{89}$ $\quad\cos A = \frac{8}{\sqrt{89}}$ or $\frac{8\sqrt{89}}{89}$

$\tan A = \frac{\text{side opposite}}{\text{side adjacent}}$

$\tan A = \frac{5}{8}$

12. $(AC)^2 + (BC)^2 = (AB)^2$
$(AC)^2 + 12^2 = 40^2$
$(AC)^2 = 1456$
$AC = \sqrt{1456}$ or $4\sqrt{91}$

$\sin A = \frac{\text{side opposite}}{\text{hypotenuse}}$ $\quad\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}}$

$\sin A = \frac{12}{40}$ or $\frac{3}{10}$ $\quad\cos A = \frac{4\sqrt{91}}{40}$ or $\frac{\sqrt{91}}{10}$

$\tan A = \frac{\text{side opposite}}{\text{side adjacent}}$

$\tan A = \frac{12}{4\sqrt{91}}$ or $\frac{3\sqrt{91}}{91}$

13. tangent

14. $\cot\theta = \frac{1}{\tan\theta}$
$\cot\theta = \frac{1}{\frac{1}{3}}$ or 3

15. $\csc\theta = \frac{1}{\sin\theta}$
$\csc\theta = \frac{1}{\frac{3}{7}}$ or $\frac{7}{3}$

16. $\cos\theta = \frac{1}{\sec\theta}$
$\cos\theta = \frac{1}{\frac{5}{9}}$ or $\frac{9}{5}$

17. $\sin\theta = \frac{1}{\csc\theta}$
$\sin\theta = \frac{1}{2.5}$ or 0.4

18. $\tan\theta = \frac{1}{\cot\theta}$
$\tan\theta = \frac{1}{0.75}$ or about 1.3333

19. $\sec\theta = \frac{1}{\cos\theta}$
$\sec\theta = \frac{1}{0.125}$ or 8

20. $(RT)^2 + (TS)^2 = (RS)^2$
$14^2 + (TS)^2 = 48^2$
$(TS)^2 = 2108$
$TS = \sqrt{2108}$ or $2\sqrt{527}$

$\sin R = \frac{\text{side opposite}}{\text{hypotenuse}}$ $\quad\cos R = \frac{\text{side adjacent}}{\text{hypotenuse}}$

$\sin R = \frac{2\sqrt{527}}{48}$ or $\frac{\sqrt{527}}{24}$ $\quad\cos R = \frac{14}{48}$ or $\frac{7}{24}$

$\tan R = \frac{\text{side opposite}}{\text{side adjacent}}$ $\quad\csc R = \frac{\text{hypotenuse}}{\text{side opposite}}$

$\tan R = \frac{2\sqrt{527}}{14}$ or $\frac{\sqrt{527}}{7}$ $\quad\csc R = \frac{48}{2\sqrt{527}}$ or $\frac{24\sqrt{527}}{527}$

$\sec R = \frac{\text{hypotenuse}}{\text{side adjacent}}$ $\quad\cot R = \frac{\text{side adjacent}}{\text{side opposite}}$

$\sec R = \frac{48}{14}$ or $\frac{24}{7}$ $\quad\cot R = \frac{14}{2\sqrt{527}}$ or $\frac{7\sqrt{527}}{527}$

21. $(ST)^2 + (TR)^2 = (SR)^2$
$38^2 + (TR)^2 = 40^2$
$(TR)^2 = 156$
$TR = \sqrt{156}$ or $2\sqrt{39}$

$\sin R = \dfrac{\text{side opposite}}{\text{hypotenuse}}$ $\qquad$ $\cos R = \dfrac{\text{side adjacent}}{\text{hypotenuse}}$

$\sin R = \dfrac{38}{40}$ or $\dfrac{19}{20}$ $\qquad$ $\cos R = \dfrac{2\sqrt{39}}{40}$ or $\dfrac{\sqrt{39}}{20}$

$\tan R = \dfrac{\text{side opposite}}{\text{side adjacent}}$ $\qquad$ $\csc R = \dfrac{\text{hypotenuse}}{\text{side opposite}}$

$\tan R = \dfrac{38}{2\sqrt{39}}$ or $\dfrac{19\sqrt{39}}{39}$ $\qquad$ $\csc R = \dfrac{40}{38}$ or $\dfrac{20}{19}$

$\sec R = \dfrac{\text{hypotenuse}}{\text{side adjacent}}$ $\qquad$ $\cot R = \dfrac{\text{side adjacent}}{\text{side opposite}}$

$\sec R = \dfrac{40}{2\sqrt{39}}$ or $\dfrac{20\sqrt{39}}{39}$ $\qquad$ $\cot R = \dfrac{2\sqrt{39}}{38}$ or $\dfrac{\sqrt{39}}{19}$

22. $(ST)^2 + (TR)^2 = (SR)^2$
$(\sqrt{7})^2 - 9^2 = (SR)^2$
$88 = (SR)^2$
$\sqrt{88} = SR$; $\sqrt{88}$ or $2\sqrt{22}$

$\sin R = \dfrac{\text{side opposite}}{\text{hypotenuse}}$ $\qquad$ $\cos R = \dfrac{\text{side adjacent}}{\text{hypotenuse}}$

$\sin R = \dfrac{\sqrt{7}}{2\sqrt{22}}$ or $\dfrac{\sqrt{154}}{44}$ $\qquad$ $\cos R = \dfrac{9}{2\sqrt{22}}$ or $\dfrac{9\sqrt{22}}{44}$

$\tan R = \dfrac{\text{side opposite}}{\text{side adjacent}}$ $\qquad$ $\csc R = \dfrac{\text{hypotenuse}}{\text{side opposite}}$

$\tan R = \dfrac{\sqrt{7}}{9}$ $\qquad$ $\csc R = \dfrac{2\sqrt{22}}{\sqrt{7}}$ or $\dfrac{2\sqrt{154}}{7}$

$\sec R = \dfrac{\text{hypotenuse}}{\text{side adjacent}}$ $\qquad$ $\cot R = \dfrac{\text{side adjacent}}{\text{side opposite}}$

$\sec R = \dfrac{2\sqrt{22}}{9}$ $\qquad$ $\cot R = \dfrac{9}{\sqrt{7}}$ or $\dfrac{9\sqrt{7}}{7}$

23. $\cot(90° - \theta) = \tan\theta$
$\cot(90° - \theta) = 1.3$

24a. 0.7963540136

24b. 0.186524036

24c. 35.34015106

24d. 1.37638192

25.

θ	**72°**	**74°**	**76°**	**78°**	**80°**
sin	0.951	0.961	0.970	0.978	0.985
cos	0.309	0.276	0.242	0.208	0.174

θ	**82°**	**84°**	**86°**	**88°**
sin	0.990	0.995	0.998	0.999
cos	0.139	0.105	0.070	0.035

25a. 1

25b. 0

26.

θ	**18°**	**16°**	**14°**	**12°**	**10°**
sin	0.309	0.276	0.242	0.208	0.174
cos	0.951	0.961	0.970	0.978	0.985
tan	0.325	0.287	0.249	0.213	0.176

θ	**8°**	**6°**	**4°**	**2°**
sin	0.139	0.105	0.070	0.035
cos	0.990	0.995	0.998	0.999
tan	0.141	0.105	0.070	0.035

26a. 0

26b. 1

26c. 0

27. $\dfrac{\sin\theta_i}{\sin\theta_r} = n$
$\dfrac{\sin 45°}{\sin 27°\,55'} = n$
$1.5103 \approx n$

28. $\sin R = \dfrac{\text{side opposite}}{\text{hypotenuse}}$
$\sin R = \dfrac{3}{7}$
$a^2 + b^2 = c^2$
$3^2 + b^2 = 7^2$
$b^2 = 40$
$b = \sqrt{40}$ or $2\sqrt{10}$

$\cos R = \dfrac{\text{side adjacent}}{\text{hypotenuse}}$ $\qquad$ $\tan R = \dfrac{\text{side opposite}}{\text{side adjacent}}$

$\cos R = \dfrac{2\sqrt{10}}{7}$ $\qquad$ $\tan R = \dfrac{3}{2\sqrt{10}}$ or $\dfrac{3\sqrt{10}}{20}$

$\csc R = \dfrac{\text{hypotenuse}}{\text{side opposite}}$ $\qquad$ $\sec R = \dfrac{\text{hypotenuse}}{\text{side adjacent}}$

$\csc R = \dfrac{7}{3}$ $\qquad$ $\sec R = \dfrac{7}{2\sqrt{10}}$ or $\dfrac{7\sqrt{10}}{20}$

$\cot R = \dfrac{\text{side adjacent}}{\text{side opposite}}$

$\cot R = \dfrac{2\sqrt{10}}{3}$

29a. $\tan\theta = \dfrac{v^2}{gr}$
$\tan 11° = \dfrac{v^2}{9.8(15.5)}$
$29.53 \approx v^2$
$5.4 \approx v$
about 5.4 m/s

29b. $\tan\theta = \dfrac{v^2}{gr}$
$\tan 13° = \dfrac{v^2}{9.8(15.5)}$
$35.07 \approx v^2$
$5.9 \approx v$
about 5.9 m/s

29c. $\tan\theta = \dfrac{v^2}{gr}$
$\tan 15° = \dfrac{v^2}{9.8(15.5)}$
$40.70 \approx v^2$
$6.4 \approx v$
about 6.4 m/s

29d. increase

30. $\sin\theta = \dfrac{\text{side opposite}}{\text{hypotenuse}}$ $\qquad$ $\cos\theta = \dfrac{\text{side adjacent}}{\text{hypotenuse}}$

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{\text{side opposite}}{\text{hypotenuse}}}{\frac{\text{side adjacent}}{\text{hypotenuse}}}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{\text{side opposite}}{\text{hypotenuse}} \div \frac{\text{side adjacent}}{\text{hypotenuse}}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{\text{side opposite}}{\text{hypotenuse}} \cdot \frac{\text{hypotenuse}}{\text{side adjacent}}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{\text{side opposite}}{\text{side adjacent}}$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

31a. $\angle = 90° - L - 23.5° \times \cos\left[\dfrac{(N + 10)360}{365}\right]$
$\angle = 90° - 26° - 23.5° \times \cos\left[\dfrac{(172 + 10)360}{365}\right]$
$\angle \approx 90° - 26° - 23.5° \times (-0.99997)$
$\angle \approx 87.5°$
$\angle = 90° - L - 23.5° \times \cos\left[\dfrac{(N + 10)360}{365}\right]$
$\angle = 90° - 26° - 23.5° \times \cos\left[\dfrac{(355 + 10)360}{365}\right]$
$\angle = 90° - 26° - 23.5° \times 1$
$\angle = 40.5°$

31b. $\angle = 90° - L - 23.5° \times \cos\left[\dfrac{(N + 10)360}{365}\right]$
$\angle = 90° - 64° - 23.5° \times \cos\left[\dfrac{(172 + 10)360}{365}\right]$
$\angle \approx 90° - 64° - 23.5° \times -0.99997$
$\angle \approx 49.5°$
$\angle = 90° - L - 23.5° \times \cos\left[\dfrac{(N + 10)360}{365}\right]$
$\angle = 90° - 64° - 23.5° \times \cos\left[\dfrac{(355 + 10)360}{365}\right]$
$\angle = 90° - 64° - 23.5° \times 1$
$\angle = 2.5°$

31c. $87.5° - 40.5° = 47°$
$49.5° - 25° = 47°$
neither

32. $x = t\left(\frac{\sin(B - A)}{\cos A}\right)$

$x = 10\left(\frac{\sin(60° - 41°)}{\cos 41°}\right)$

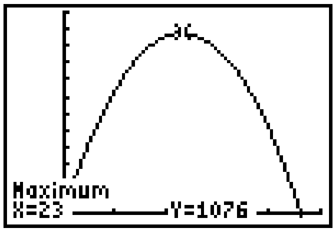
$x \approx 10(0.4314)$

$x \approx 4.31$; about 4.31 cm

33. $88.37° = 88° + (0.37 \cdot 60)'$
$= 88° + 22.2'$
$= 88° + 22' + (0.2 \cdot 60)''$
$= 88° + 22' + 12''$
$88°\ 22'\ 12''$

34. positive: 1
$f(-x) = x^4 - 2x^3 + 6x - 1$
negative: 3 or 1

35.



35a. 23 employees
35b. $1076

$[-10, 50]$ scl:10 by $[-10, 1200]$ scl:100

36. $\begin{vmatrix} 7 & -3 & 5 \\ 4 & 0 & -1 \\ 8 & 2 & 0 \end{vmatrix} = 7\begin{vmatrix} 0 & -1 \\ 2 & 0 \end{vmatrix} - (-3)\begin{vmatrix} 4 & -1 \\ 8 & 0 \end{vmatrix} + 5\begin{vmatrix} 4 & 0 \\ 8 & 2 \end{vmatrix}$

$= 7(2) - (-3)(8) + 5(8)$
$= 78$

37. $m = \frac{3 - 5}{6 - 2}$

$m = \frac{-2}{4}$ or $-\frac{1}{2}$

$y - y_1 = m(x - x_1)$

$y - 3 = -\frac{1}{2}(x - 6)$

$y = -\frac{1}{2}x + 6$

38. $A = \frac{1}{2}bh$

$12 = \frac{1}{2}(2x)(3x)$

$12 = 3x^2$
$4 = x^2$
$2 = x$

$2x = 2(2)$ or 4

$3x = 3(2)$ or 6

$a^2 + b^2 = c^2$
$4^2 + 6^2 = c^2$
$52 = c^2$
$\sqrt{52} = c$; $\sqrt{52}$ or $2\sqrt{13}$

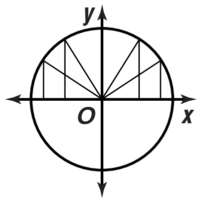
The correct choice is C.

5-3 Trigonometric Functions on the Unit Circle

Page 296 Check for Understanding

1. Terminal side of a 180° angle in standard position is the negative x-axis which intersects the unit circle at $(-1, 0)$. Since $\csc \theta = \frac{1}{y}$, $\csc 180° = \frac{1}{0}$ which is undefined.

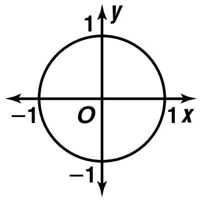
2.



As θ goes from 0° to 90°, the y-coordinate increases. As θ goes from 90° to 180°, the y-coordinate decreases.

3. $\cot \theta = \frac{x}{y} = \frac{\cos \theta}{\sin \theta}$

4.



Function	Quadrant			
	I	II	III	IV
sin ∝ or cos ∝	+	+	−	−
cos ∝ or sec ∝	+	−	−	+
tan ∝ or cot ∝	+	−	+	−

5. $(-1, 0)$; $\tan 180° = \frac{y}{x}$ or $\frac{0}{-1}$; 0

6. $(0, -1)$; $\sec(-90°) = \frac{1}{x}$ or $\frac{1}{0}$; undefined

7. $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\sin 30° = y$

$\sin 30° = \frac{1}{2}$

$\cos 30° = x$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{y}{x}$

$\tan 30° = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$\tan 30° = \frac{1}{\sqrt{3}}$

$\tan 30° = \frac{\sqrt{3}}{3}$

$\csc 30° = \frac{1}{y}$

$\csc 30° = \frac{1}{\frac{1}{2}}$

$\csc 30° = 2$

$\sec 30° = \frac{1}{x}$

$\sec 30° = \frac{1}{\frac{\sqrt{3}}{2}}$

$\sec 30° = \frac{2}{\sqrt{3}}$

$\sec 30° = \frac{2\sqrt{3}}{3}$

$\cot 30° = \frac{x}{y}$

$\cot 30° = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$\cot 30° = \sqrt{3}$

8. terminal side — Quadrant III
reference angle: $225° - 180° = 45°$
$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\sin 225° = y$ $\cos 225° = x$
$\sin 225° = -\frac{\sqrt{2}}{2}$ $\cos 225° = -\frac{\sqrt{2}}{2}$
$\tan 225° = \frac{y}{x}$ $\csc 225° = \frac{1}{y}$
$\tan 225° = \frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}$ $\csc 225° = \frac{1}{-\frac{\sqrt{2}}{2}}$
$\tan 225° = 1$ $\csc 225° = -\frac{2}{\sqrt{2}}$
$\csc 225° = -\sqrt{2}$
$\sec 225° = \frac{1}{x}$ $\cot 225° = \frac{x}{y}$
$\sec 225° = \frac{1}{-\frac{\sqrt{2}}{2}}$ $\cot 225° = \frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}$
$\sec 225° = -\frac{2}{\sqrt{2}}$ $\cot 225° = 1$
$\sec 225° = -\sqrt{2}$

9. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{3^2 + 4^2}$
$t = \sqrt{25}$ or 5

$\sin \theta = \frac{y}{r}$ $\cos \theta = \frac{x}{r}$ $\tan \theta = \frac{y}{x}$
$\sin \theta = \frac{4}{5}$ $\cos \theta = \frac{3}{5}$ $\tan \theta = \frac{4}{3}$
$\csc \theta = \frac{r}{y}$ $\sec \theta = \frac{r}{x}$ $\cot \theta = \frac{x}{y}$
$\csc \theta = \frac{5}{4}$ $\sec \theta = \frac{5}{3}$ $\cot \theta = \frac{3}{4}$

10. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{(-6)^2 + 6^2}$
$r = \sqrt{72}$ or $6\sqrt{2}$

$\sin \theta = \frac{y}{r}$ $\cos \theta = \frac{x}{r}$ $\tan \theta = \frac{y}{x}$
$\sin \theta = \frac{6}{6\sqrt{2}}$ $\cos \theta = \frac{-6}{6\sqrt{2}}$ $\tan \theta = \frac{6}{-6}$ or -1
$\sin \theta = \frac{\sqrt{2}}{2}$ $\cos \theta = -\frac{\sqrt{2}}{2}$
$\csc \theta = \frac{r}{y}$ $\sec \theta = \frac{r}{x}$
$\csc \theta = \frac{6\sqrt{2}}{6}$ or $\sqrt{2}$ $\sec \theta = \frac{6\sqrt{2}}{-6}$ or $-\sqrt{2}$
$\cot \theta = \frac{x}{y}$
$\cot \theta = \frac{-6}{6}$ or -1

11. $\tan \theta = \frac{y}{x}$ $r^2 = x^2 + y^2$
$\tan \theta = -1$ $r^2 = 1^2 + (-1)^2$
$x = 1, y = -1$ $r^2 = 2$
$r = \sqrt{2}$

$\sin \theta = \frac{y}{r}$ $\cos \theta = \frac{x}{r}$
$\sin \theta = \frac{-1}{\sqrt{2}}$ $\cos \theta = \frac{1}{\sqrt{2}}$
$\sin \theta = -\frac{\sqrt{2}}{2}$ $\cos \theta = \frac{\sqrt{2}}{2}$
$\csc \theta = \frac{r}{y}$ $\sec \theta = \frac{r}{x}$
$\csc \theta = \frac{\sqrt{2}}{-1}$ or $-\sqrt{2}$ $\sec \theta = \frac{\sqrt{2}}{1}$ or $\sqrt{2}$
$\cot \theta = \frac{x}{y}$
$\cot \theta = \frac{1}{-1}$ or -1

12. $\cos \theta = \frac{x}{r}$ $r^2 = x^2 + y^2$
$\cos \theta = -\frac{1}{2}$ $2^2 = (-1)^2 + y^2$
$x = -1, r = 2$ $3 = y^2$
$\pm\sqrt{3} = y$
Quadrant II, so y = $\sqrt{3}$

$\sin \theta = \frac{y}{r}$ $\tan \theta = \frac{y}{x}$ $\csc \theta = \frac{r}{y}$
$\sin \theta = \frac{\sqrt{3}}{2}$ $\tan \theta = \frac{\sqrt{3}}{-1}$ or $-\sqrt{3}$ $\csc \theta = \frac{2}{\sqrt{3}}$
$\csc \theta = \frac{2\sqrt{3}}{3}$
$\sec \theta = \frac{r}{x}$ $\cot \theta = \frac{x}{y}$
$\sec \theta = \frac{2}{-1}$ or -2 $\cot \theta = \frac{-1}{\sqrt{3}}$
$\cot \theta = \frac{-\sqrt{3}}{3}$

13. $C = 2\pi r \cos L$ $C = 2\pi r \cos L$
$C = 2\pi(3960) \cos 0°$ $C = 2\pi(3960) \cos 90°$
$C \approx 24{,}881.41$ $C = 0$
The circumference goes from about 24,881 miles to 0 miles.

Pages 296–298 Exercises

14. (0, 1); $\sin 90° = y$ or 1
15. (1, 0); $\tan 360° = \frac{y}{x}$ or $\frac{0}{1}$; 0
16. (−1, 0); $\cot(-180°) = \frac{x}{y}$ or $\frac{-1}{0}$; undefined
17. (0, −1); $\csc 270° = \frac{1}{y}$ or $\frac{1}{-1}$; −1
18. (0, 1); $\cos(-270°) = x$ or 0
19. (−1, 0); $\sec 180° = \frac{1}{x}$ or $\frac{1}{-1}$; −1
20. Sample answers: 0°, 180° **21.** undefined
22. $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\sin 45° = y$ $\cos 45° = x$
$\sin 45° = \frac{\sqrt{2}}{2}$ $\cos 45° = \frac{\sqrt{2}}{2}$
$\tan 45° = \frac{y}{x}$ $\csc 45° = \frac{1}{y}$
$\tan 45° = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$ or 1 $\csc 45° = \frac{1}{\frac{\sqrt{2}}{2}}$
$\csc 45° = \frac{2}{\sqrt{2}}$
$\csc 45° = \sqrt{2}$
$\sec 45° = \frac{1}{x}$ $\cot 45° = \frac{x}{y}$
$\sec 45° = \frac{1}{\frac{\sqrt{2}}{2}}$ $\cot 45° = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$ or 1
$\sec 45° = \frac{2}{\sqrt{2}}$
$\sec 45° = \sqrt{2}$

23. terminal side — Quadrant II
reference angle: 180° − 150° = 30°
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\sin 150° = y$ $\qquad$ $\cos 150° = x$

$\sin 150° = \frac{1}{2}$ $\qquad$ $\cos 150° = -\frac{\sqrt{3}}{2}$

$\tan 150° = \frac{y}{x}$ $\qquad$ $\csc 150° = \frac{1}{y}$

$\tan 150° = \dfrac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$ $\qquad$ $\csc 150° = \dfrac{1}{\frac{1}{2}}$

$\tan 150° = -\frac{1}{\sqrt{3}}$ $\qquad$ $\csc 150° = 2$

$\tan 150° = -\frac{\sqrt{3}}{3}$

$\sec 150° = \frac{1}{x}$ $\qquad$ $\cot 150° = \frac{x}{y}$

$\sec 150° = \dfrac{1}{-\frac{\sqrt{3}}{2}}$ $\qquad$ $\cot 150° = \dfrac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$\sec 150° = -\frac{2}{\sqrt{3}}$ $\qquad$ $\cot 150° = -\sqrt{3}$

$\sec 150° = -\frac{2\sqrt{3}}{3}$

24. terminal side — Quadrant IV
reference angle: 360° − 315° = 45°
$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\sin 315° = y$ $\qquad$ $\cos 315° = x$

$\sin 315° = -\frac{\sqrt{2}}{2}$ $\qquad$ $\cos 315° = \frac{\sqrt{2}}{2}$

$\tan 315° = \frac{y}{x}$ $\qquad$ $\csc 315° = \frac{1}{y}$

$\tan 315° = \dfrac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$ or -1 $\qquad$ $\csc 315° = \dfrac{1}{-\frac{\sqrt{2}}{2}}$

$\csc 315° = -\frac{2}{\sqrt{2}}$

$\csc 315° = -\sqrt{2}$

$\sec 315° = \frac{1}{x}$ $\qquad$ $\cot 315° = \frac{x}{y}$

$\sec 315° = \dfrac{1}{\frac{\sqrt{2}}{2}}$ $\qquad$ $\cot 315° = \dfrac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}$ or -1

$\sec 315° = \frac{2}{\sqrt{2}}$

$\sec 315° = \sqrt{2}$

25. terminal side — Quadrant III
reference angle: 210° − 180° = 30°
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\sin 210° = y$ $\qquad$ $\cos 210° = x$

$\sin 210° = -\frac{1}{2}$ $\qquad$ $\cos 210° = -\frac{\sqrt{3}}{2}$

$\tan 210° = \frac{y}{x}$ $\qquad$ $\csc 210° = \frac{1}{y}$

$\tan 210° = \dfrac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$ $\qquad$ $\csc 210° = \dfrac{1}{-\frac{1}{2}}$

$\tan 210° = \frac{1}{\sqrt{3}}$ $\qquad$ $\csc 210° = -2$

$\tan 210° = \frac{\sqrt{3}}{3}$

$\sec 210° = \frac{1}{x}$ $\qquad$ $\cot 210° = \frac{x}{y}$

$\sec 210° = \dfrac{1}{-\frac{\sqrt{3}}{2}}$ $\qquad$ $\cot 210° = -\dfrac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$

$\sec 210° = -\frac{2}{\sqrt{3}}$ $\qquad$ $\cot 210° = \sqrt{3}$

$\sec 210° = -\frac{2\sqrt{3}}{3}$

26. terminal side — Quadrant IV
reference angle: 360° − 330° = 30°
$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\sin 330° = y$ $\qquad$ $\cos 330° = x$

$\sin 330° = -\frac{1}{2}$ $\qquad$ $\cos 330° = \frac{\sqrt{3}}{2}$

$\tan 330° = \frac{y}{x}$ $\qquad$ $\csc 330° = \frac{1}{y}$

$\tan 330° = \dfrac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$ $\qquad$ $\csc 330° = \dfrac{1}{-\frac{1}{2}}$

$\tan 330° = -\frac{1}{\sqrt{3}}$ $\qquad$ $\csc 330° = -2$

$\tan 330° = -\frac{\sqrt{3}}{3}$

$\sec 330° = \frac{1}{x}$ $\qquad$ $\cot 330° = \frac{x}{y}$

$\sec 330° = \dfrac{1}{\frac{\sqrt{3}}{2}}$ $\qquad$ $\cot 330° = \dfrac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$

$\sec 330° = \frac{2}{\sqrt{3}}$ $\qquad$ $\cot 330° = -\sqrt{3}$

$\sec 330° = \frac{2\sqrt{3}}{3}$

27. terminal side — Quadrant I
reference angle: $420° - 360° = 60°$
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\sin 420° = y$ $\quad$ $\cos 420° = x$

$\sin 420° = \frac{\sqrt{3}}{2}$ $\quad$ $\cos 420° = \frac{1}{2}$

$\tan 420° = \frac{y}{x}$ $\quad$ $\csc 420° = \frac{1}{y}$

$\tan 420° = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ $\quad$ $\csc 420° = \frac{1}{\frac{\sqrt{3}}{2}}$

$\tan 420° = \sqrt{3}$ $\quad$ $\csc 420° = \frac{2}{\sqrt{3}}$

$\csc 420° = \frac{2\sqrt{3}}{3}$

$\sec 420° = \frac{1}{x}$ $\quad$ $\cot 420° = \frac{x}{y}$

$\sec 420° = \frac{1}{\frac{1}{2}}$ $\quad$ $\cot 420° = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$\sec 420° = 2$ $\quad$ $\cot 420° = \frac{1}{\sqrt{3}}$

$\cot 420° = \frac{\sqrt{3}}{3}$

28. terminal side — Quadrant IV
reference angle: $45°$
$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\cot(-45°) = \frac{x}{y}$

$\cot(-45°) = -\frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$ or -1

29. terminal side — Quadrant 1
reference angle: $390° - 360° = 30°$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\csc 390° = \frac{1}{y}$

$\csc 390° = \frac{1}{\frac{1}{2}}$ or 2

30. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{(-4)^2 + (-3)^2}$
$r = \sqrt{25}$ or 5

$\sin\theta = \frac{y}{r}$ $\quad$ $\cos\theta = \frac{x}{r}$ $\quad$ $\tan\theta = \frac{y}{x}$

$\sin\theta = -\frac{3}{5}$ $\quad$ $\cos\theta = -\frac{4}{5}$ $\quad$ $\tan\theta = \frac{-3}{-4}$ or $\frac{3}{4}$

$\csc\theta = \frac{r}{y}$ $\quad$ $\sec\theta = \frac{r}{x}$ $\quad$ $\cot\theta = \frac{x}{y}$

$\csc\theta = -\frac{5}{3}$ $\quad$ $\sec\theta = -\frac{5}{4}$ $\quad$ $\cot\theta = \frac{-4}{-3}$ or $\frac{4}{3}$

31. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{(-6)^2 + 6^2}$
$r = \sqrt{72}$ or $6\sqrt{2}$

$\sin\theta = \frac{y}{r}$ $\quad$ $\cos\theta = \frac{x}{r}$ $\quad$ $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{6}{6\sqrt{2}}$ $\quad$ $\cos\theta = \frac{-6}{6\sqrt{2}}$ $\quad$ $\tan\theta = \frac{6}{-6}$ or -1

$\sin\theta = \frac{\sqrt{2}}{2}$ $\quad$ $\cos\theta = -\frac{\sqrt{2}}{2}$

$\csc\theta = \frac{r}{y}$ $\quad$ $\sec\theta = \frac{r}{x}$ $\quad$ $\cot\theta = \frac{x}{y}$

$\csc\theta = \frac{6\sqrt{2}}{6}$ or $\sqrt{2}$ $\quad$ $\sec\theta = \frac{6\sqrt{2}}{-6}$ or $-\sqrt{2}$ $\quad$ $\cot\theta = \frac{-6}{6}$ or -1

32. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{2^2 + 0^2}$
$r = \sqrt{4}$ or 2

$\sin\theta = \frac{y}{r}$ $\quad$ $\cos\theta = \frac{x}{r}$ $\quad$ $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{0}{2}$ or 0 $\quad$ $\cos\theta = \frac{2}{2}$ or 1 $\quad$ $\tan\theta = \frac{0}{2}$ or 0

$\csc\theta = \frac{r}{y}$ $\quad$ $\sec\theta = \frac{r}{x}$ $\quad$ $\cot\theta = \frac{x}{y}$

$\csc\theta = \frac{2}{0}$ undefined $\quad$ $\sec\theta = \frac{2}{2}$ or 1 $\quad$ $\cot\theta = \frac{2}{0}$ undefined

33. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{1^2 + (-8)^2}$
$r = \sqrt{65}$

$\sin\theta = \frac{y}{r}$ $\quad$ $\cos\theta = \frac{x}{r}$ $\quad$ $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{-8}{\sqrt{65}}$ $\quad$ $\cos\theta = \frac{1}{\sqrt{65}}$ $\quad$ $\tan\theta = \frac{-8}{1}$ or -8

$\sin\theta = -\frac{8\sqrt{65}}{65}$ $\quad$ $\cos\theta = \frac{\sqrt{65}}{65}$

$\csc\theta = \frac{r}{y}$ $\quad$ $\sec\theta = \frac{r}{x}$

$\csc\theta = \frac{\sqrt{65}}{-8}$ or $-\frac{\sqrt{65}}{8}$ $\quad$ $\sec\theta = \frac{\sqrt{65}}{1}$ or $\sqrt{65}$

$\cot\theta = \frac{x}{y}$

$\cot\theta = \frac{1}{-8}$ or $-\frac{1}{8}$

34. $r = \sqrt{x^2 - y^2}$
$r = \sqrt{5^2 + (-3)^2}$
$r = \sqrt{34}$

$\sin\theta = \frac{y}{r}$ $\quad$ $\cos\theta = \frac{x}{r}$ $\quad$ $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{-3}{\sqrt{34}}$ $\quad$ $\cos\theta = \frac{5}{\sqrt{34}}$ $\quad$ $\tan\theta = \frac{-3}{5}$ or $-\frac{3}{5}$

$\sin\theta = -\frac{3\sqrt{34}}{34}$ $\quad$ $\cos\theta = \frac{5\sqrt{34}}{34}$

$\csc\theta = \frac{r}{y}$ $\quad$ $\sec\theta = \frac{r}{x}$

$\csc\theta = \frac{\sqrt{34}}{-3}$ or $-\frac{\sqrt{34}}{3}$ $\quad$ $\sec\theta = \frac{\sqrt{34}}{5}$

$\cot\theta = \frac{x}{y}$

$\cot\theta = \frac{5}{-3}$ or $-\frac{5}{3}$

35. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{(-8)^2 + 15^2}$
$r = \sqrt{289}$ or 17

$\sin\theta = \frac{y}{r}$ $\quad$ $\cos\theta = \frac{x}{r}$ $\quad$ $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{15}{17}$ $\quad$ $\cos\theta = \frac{-8}{17}$ or $-\frac{8}{17}$ $\quad$ $\tan\theta = \frac{15}{-8}$ or $-\frac{15}{8}$

$\csc\theta = \frac{r}{y}$ $\quad$ $\sec\theta = \frac{r}{x}$ $\quad$ $\cot\theta = \frac{x}{y}$

$\csc\theta = \frac{17}{15}$ $\quad$ $\sec\theta = \frac{17}{-8}$ or $-\frac{17}{8}$ $\quad$ $\cot\theta = \frac{-8}{15}$ or $-\frac{8}{15}$

36. $r = \sqrt{x^2 + y^2}$ $\qquad$ $r = \sqrt{x^2 + y^2}$

$r = \sqrt{5^2 + (-6)^2}$ $\qquad$ $r = \sqrt{(-5)^2 + 6^2}$

$r = \sqrt{61}$ $\qquad$ $r = \sqrt{61}$

$\sin\theta = \frac{y}{r}$ $\qquad$ $\sin\theta = \frac{y}{r}$

$\sin\theta = \frac{-6}{\sqrt{61}}$ $\qquad$ $\sin\theta = \frac{6}{\sqrt{61}}$

$\sin\theta = -\frac{6\sqrt{61}}{61}$ $\qquad$ $\sin\theta = \frac{6\sqrt{61}}{61}$

The sine of one angle is the negative of the sine of the other angle.

37. If $\sin\theta < 0$, y must be negative, so the terminal side is located in Quadrant III or IV

38. $\cos\theta = \frac{x}{r}$ $\qquad$ $r^2 = x^2 + y^2$

$\cos\theta = -\frac{12}{13}$ $\qquad$ $13^2 = (-12)^2 + y^2$

$x = -12, r = 13$ $\qquad$ $25 = y^2$

$\pm 5 = y$

Quadrant III, so $y = -5$

$\sin\theta = \frac{y}{r}$ $\qquad$ $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{-5}{13}$ or $-\frac{5}{13}$ $\qquad$ $\tan\theta = \frac{-5}{-12}$ or $\frac{5}{12}$

$\csc\theta = \frac{r}{y}$ $\qquad$ $\sec\theta = \frac{r}{x}$

$\csc\theta = \frac{13}{-5}$ or $-\frac{13}{5}$ $\qquad$ $\sec\theta = \frac{13}{-12}$ or $-\frac{13}{12}$

$\cot\theta = \frac{r}{y}$

$\cot\theta = \frac{-12}{-5}$ or $\frac{12}{5}$

39. $\csc\theta = \frac{r}{y}$ $\qquad$ $r^2 = x^2 + y^2$

$\csc\theta = 2$ $\qquad$ $2^2 = x^2 + 1^2$

$r = 2, y = 1$ $\qquad$ $3 = x^2$

$\pm\sqrt{3} = x$

Quadrant II, so $x = -\sqrt{3}$

$\sin\theta = \frac{y}{r}$ $\qquad$ $\cos\theta = \frac{x}{r}$ $\qquad$ $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{1}{2}$ $\qquad$ $\cos\theta = \frac{-\sqrt{3}}{2}$ or $-\frac{\sqrt{3}}{2}$ $\qquad$ $\tan\theta = \frac{1}{-\sqrt{3}}$

$\tan\theta = -\frac{\sqrt{3}}{3}$

$\sec\theta = \frac{r}{x}$ $\qquad$ $\cot\theta = \frac{x}{y}$

$\sec\theta = \frac{2}{-\sqrt{3}}$ $\qquad$ $\cot\theta = \frac{-\sqrt{3}}{1}$ or $-\sqrt{3}$

$\sec\theta = -\frac{2\sqrt{3}}{3}$

40. $\sin\theta = \frac{y}{r}$ $\qquad$ $r^2 = x^2 + y^2$

$\sin\theta = -\frac{1}{5}$ $\qquad$ $5^2 = x^2 + (-1)^2$

$y = -1, r = 5$ $\qquad$ $24 = x^2$

$\pm 2\sqrt{6} = x$

Quadrant IV, so $x = 2\sqrt{6}$

$\cos\theta = \frac{x}{r}$ $\qquad$ $\tan\theta = \frac{y}{x}$ $\qquad$ $\csc\theta = \frac{r}{y}$

$\cos\theta = \frac{2\sqrt{6}}{5}$ $\qquad$ $\tan\theta = \frac{-1}{2\sqrt{6}}$ $\qquad$ $\csc\theta = \frac{5}{-1}$ or -5

$\tan\theta = -\frac{\sqrt{6}}{12}$

$\sec\theta = \frac{r}{x}$ $\qquad$ $\cot\theta = \frac{x}{y}$

$\sec\theta = \frac{5}{2\sqrt{6}}$ $\qquad$ $\cot\theta = \frac{2\sqrt{6}}{-1}$ or $-2\sqrt{6}$

$\sec\theta = \frac{5\sqrt{6}}{12}$

41. $\tan\theta = \frac{y}{x}$ $\qquad$ $r^2 = x^2 + y^2$

$\tan\theta = 2$ $\qquad$ $r^2 = 1^2 + 2^2$

$y = 2, x = 1$ $\qquad$ $r^2 = 5$

$r = \sqrt{5}$

$\sin\theta = \frac{y}{r}$ $\qquad$ $\cos\theta = \frac{x}{r}$ $\qquad$ $\csc\theta = \frac{r}{y}$

$\sin\theta = \frac{2}{\sqrt{5}}$ $\qquad$ $\cos\theta = \frac{1}{\sqrt{5}}$ $\qquad$ $\csc\theta = \frac{\sqrt{5}}{2}$

$\sin\theta = \frac{2\sqrt{5}}{5}$ $\qquad$ $\cos\theta = \frac{\sqrt{5}}{5}$

$\sec\theta = \frac{r}{x}$ $\qquad$ $\cot\theta = \frac{x}{y}$

$\sec\theta = \frac{\sqrt{5}}{1}$ or $\sqrt{5}$ $\qquad$ $\cot\theta = \frac{1}{2}$

42. $\sec\theta = \frac{r}{x}$ $\qquad$ $r^2 = x^2 + y^2$

$\sec\theta = \sqrt{3}$ $\qquad$ $(\sqrt{3})^2 = 1^2 + y^2$

$r = \sqrt{3}, x = 1$ $\qquad$ $2 = y^2$

$\pm\sqrt{2} = y$

Quadrant IV, so $y = -\sqrt{2}$

$\sin\theta = \frac{y}{r}$ $\qquad$ $\cos\theta = \frac{x}{r}$ $\qquad$ $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{-\sqrt{2}}{\sqrt{3}}$ $\qquad$ $\cos\theta = \frac{1}{\sqrt{3}}$ $\qquad$ $\tan\theta = \frac{-\sqrt{2}}{1}$ or $-\sqrt{2}$

$\sin\theta = -\frac{\sqrt{6}}{3}$ $\qquad$ $\cos\theta = \frac{\sqrt{3}}{3}$

$\csc\theta = \frac{r}{y}$ $\qquad$ $\cot\theta = \frac{x}{y}$

$\csc\theta = \frac{\sqrt{3}}{-\sqrt{2}}$ $\qquad$ $\cot\theta = \frac{1}{-\sqrt{2}}$

$\csc\theta = -\frac{\sqrt{6}}{2}$ $\qquad$ $\cot\theta = -\frac{\sqrt{2}}{2}$

43. $\cot\theta = \frac{x}{y}$ $\qquad$ $r^2 = x^2 + y^2$

$\cot\theta = 1$ (Quadrant III) $\qquad$ $r^2 = (-1)^2 + (-1)^2$

$x = -1, y = -1$ $\qquad$ $r^2 = 2$

$r = \sqrt{2}$

$\sin\theta = \frac{y}{r}$ $\qquad$ $\cos\theta = \frac{x}{r}$ $\qquad$ $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{-1}{\sqrt{2}}$ $\qquad$ $\cos\theta = \frac{-1}{\sqrt{2}}$ $\qquad$ $\tan\theta = \frac{-1}{-1}$ or 1

$\sin\theta = -\frac{\sqrt{2}}{2}$ $\qquad$ $\cos\theta = -\frac{\sqrt{2}}{2}$

$\csc\theta = \frac{r}{y}$ $\qquad$ $\sec\theta = \frac{r}{x}$

$\csc\theta = \frac{\sqrt{2}}{-1}$ or $-\sqrt{2}$ $\qquad$ $\sec\theta = \frac{\sqrt{2}}{-1}$ or $-\sqrt{2}$

44. $\csc\theta = \frac{r}{y}$ $\qquad$ $r^2 = x^2 + y^2$

$\csc\theta = -2$ $\qquad$ $2^2 = x^2 + (-1)^2$

$r = 2, y = -1$ $\qquad$ $3 = x^2$

$\pm\sqrt{3} = x$

Quadrant III, so $x = -\sqrt{3}$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{-1}{-\sqrt{3}}$

$\tan\theta = \frac{\sqrt{3}}{3}$

45. $g\sin\theta\cos\theta = 0$

$\sin\theta = 0$ or $\cos\theta = 0$

$\theta = 0°$ $\qquad$ $\theta = 90°$

46a. k is an even integer. $\qquad$ **46b.** k is an odd integer.

47. $\cos\theta = \sqrt{\frac{I_t}{I_o}}$

$\cos\theta = \sqrt{1}$ $\qquad$ $I_t = I_o$

$\cos\theta = 1$

$\theta = 0°$

48. Let $x = -1$. $y = -3(-1)$

$$y = 3$$
$$r^2 = x^2 + y^2$$
$$r^2 = (-1)^2 + (3)^2$$
$$r^2 = 10$$
$$r = \sqrt{10}$$

$\sin\theta = \frac{y}{r}$ $\quad\cos\theta = \frac{x}{r}$ $\quad\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{3}{\sqrt{10}}$ $\quad\cos\theta = \frac{-1}{\sqrt{10}}$ $\quad\tan\theta = \frac{3}{-1}$ or -3

$\sin\theta = \frac{3\sqrt{10}}{10}$ $\quad\cos\theta = -\frac{\sqrt{10}}{10}$

$\csc\theta = \frac{r}{y}$ $\quad\sec\theta = \frac{r}{x}$

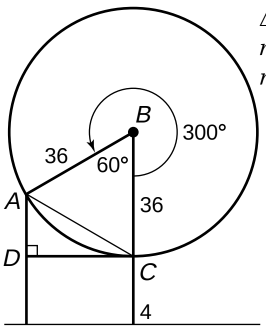
$\csc\theta = \frac{\sqrt{10}}{3}$ $\quad\sec\theta = \frac{\sqrt{10}}{-1}$ or $-\sqrt{10}$

$\cot\theta = \frac{x}{y}$

$\cot\theta = \frac{-1}{3}$ or $-\frac{1}{3}$

49a. $4 + 2(36) = 76$ ft

49b.



$\triangle ABC$ is equilateral.

$$m\angle BCA = 60°$$
$$m\angle ACD + m\angle BCA = 90°$$
$$m\angle ACD + 60° = 90°$$
$$m\angle ACD = 30°$$

Since $AC = 36$, $AD = 18$.
$18 + 4 = 22$ ft

49c. Refer to 49b for diagram and reasoning.
Since $AC = 30$, $AD = 15$.
$15 + 4 = 19$ ft

49d. $\frac{1}{2}r + 4$

50. $\sin\theta = \frac{1}{\csc\theta}$

$$\sin\theta = \frac{1}{\frac{7}{5}}$$
$$\sin\theta = \frac{5}{7}$$

51. $\frac{-840}{360} \approx -2.33$ $\qquad \alpha + 360(-2)° = -840°$

$$\alpha - 720° = -840°$$
$$\alpha = -120°$$

$360° - 120° = 240°$; III

52. $5 - \sqrt{b+2} = 0$

$$5 = \sqrt{b+2}$$
$$25 = b + 2$$
$$23 = b$$

53. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{9 \pm \sqrt{(-9)^2 - 4(4)(5)}}{2(4)}$$
$$x = \frac{9 \pm \sqrt{1}}{8}$$
$$x = \frac{9+1}{8} \quad\text{or}\quad x = \frac{9-1}{8}$$
$$x = \frac{10}{8} \text{ or } 1.25 \quad x = \frac{8}{8} \text{ or } 1$$

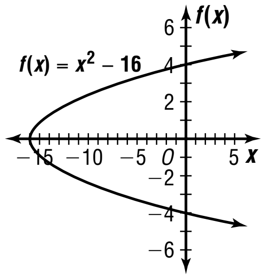
54. $k = \frac{y}{x}$ $\qquad y = kx$

$k = \frac{9}{-15}$ $\qquad y = (-0.6)(21)$

$k = -0.6$ $\qquad y = -12.6$

55.

$$f(x) = x^2 - 16$$
$$y = x^2 - 16$$
$$x = y^2 - 16$$
$$x + 16 = y^2$$
$$\pm\sqrt{x+16} = y$$



56. $\begin{vmatrix} 2 & 1 \\ -3 & 2 \end{vmatrix} = 2(2) - (-3)1$

$$= 7$$

$$\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$$

57. $3(8m - 3n - 4p) = 3(6)$ $\quad$ $4m + 9n - 2p = -4$ $\rightarrow$

$$\begin{array}{rcrcrcr} 24m & - & 9n & - & 12p & = & 18 \\ 4m & + & 9n & - & 2p & = & -4 \\ \hline 28m & & & - & 14p & = & 14 \end{array}$$

$4(8m - 3n - 4p) = 4(6)$ $\quad$ $6m + 12n + 5p = -1$ $\rightarrow$

$$\begin{array}{rcrcrcr} 32m & - & 12n & - & 16p & = & 24 \\ 6m & + & 12n & + & 5p & = & -1 \\ \hline 38m & & & - & 11p & = & 23 \end{array}$$

$11(28m - 14p) = 11(14)$
$-14(38m - 11p) = -14(23)$
$\downarrow$

$$\begin{array}{rcrcr} 308m & - & 154p & = & 154 \\ -532m & + & 154p & = & -322 \\ \hline -224m & & & = & -168 \end{array}$$

$$m = \frac{3}{4}$$

$38m - 11p = 23$ $\qquad 4m + 9n - 2p = -4$

$38\left(\frac{3}{4}\right) - 11p = 23$ $\qquad 4\left(\frac{3}{4}\right) + 9n - 2\left(\frac{1}{2}\right) = -4$

$p = \frac{1}{2}$ $\qquad n = -\frac{2}{3}$

$$\left(\frac{3}{4}, -\frac{2}{3}, \frac{1}{2}\right)$$

58. $2x - 4y \le 7$ $\qquad 2x - 4y \le 7$

$2(9) - 4(3) \overset{?}{\le} 7$ $\qquad 2(-1) - 4(2) \overset{?}{\le} 7$

$6 \le 7$; yes $\qquad -10 \le 7$; yes

$2x - 4y \le 7$

$2(2) - 4(-2) \overset{?}{\le} 7$

$12 \not\le 7$; no

59. absolute value; $f(x) = \left|2\frac{1}{2} - x\right|$

60. A of square $-$ A of circle $= A$

$$s^2 - \pi r^2 = A$$
$$2^2 - \pi(1)^2 = A$$
$$0.86 \approx A$$

The correct choice is C.

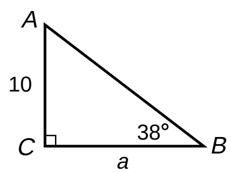
5-4 Applying Trigonometric Functions

Pages 301–302 Check for Understanding

1a. cos or sec $\qquad$ **1b.** tan or cot

1c. sin or csc

2. Sample answer: Find a.



3. $\angle DCB$; $\angle ABC$; the measures are equal; if parallel lines are cut by a transversal, the alternate interior angles are congruent.

4. Sample answer: If you know the angle of elevation of the sun at noon on a particular day, you can measure the length of the shadow of the building at noon on that day. The height of the building equals the length of the shadow times the tangent of the angle of elevation of the sun.

5.
$$\tan A = \frac{a}{b}$$
$$\tan 76° = \frac{a}{113}$$
$$13 \tan 76° = a$$
$$52.1 \approx a$$

6.
$$\sin B = \frac{b}{c}$$
$$\sin 26° = \frac{18}{c}$$
$$c \sin 26° = 18$$
$$c = \frac{18}{\sin 26°}$$
$$c \approx 41.1$$

7.
$$\cos B = \frac{a}{c}$$
$$\cos 16° \, 45' = \frac{a}{13}$$
$$13 \cos 16° \, 45' = a$$
$$12.4 \approx a$$

8a. Let x = altitude.
$$\sin 55° \, 30' = \frac{x}{10}$$
$$10 \sin 55° \, 30' = x$$
$$8.2 \approx x$$
about 8.2 cm

8b. Let $x = \frac{1}{2}$ of the base.
$$\cos 55° \, 30' = \frac{x}{10}$$
$$10 \cos 55° \, 30' = x$$
$$5.66 \approx x$$
base $= 2x$
base $\approx 2(5.66)$
base ≈ 11.3 cm

8c. $A = \frac{1}{2}bh$
$A \approx \frac{1}{2}(11.3)(8.2)$
$A \approx 46.7 \text{ cm}^2$

9.
$$\tan 13° \, 15' = \frac{175}{x}$$
$$x \tan 13° \, 15' = 175$$
$$x = \frac{175}{\tan 13° \, 15'}$$
$x \approx 743.2$ ft

Pages 302–304 Exercises

10.
$$\tan A = \frac{a}{b}$$
$$\tan 37° = \frac{a}{6}$$
$$6 \tan 37° = a$$
$$4.5 \approx a$$

11.
$$\cos B = \frac{a}{c}$$
$$\cos 67° = \frac{a}{16}$$
$$16 \cos 67° = a$$
$$6.3 \approx a$$

12.
$$\sin B = \frac{b}{c}$$
$$\sin 62° = \frac{b}{24}$$
$$24 \sin 62° = b$$
$$21.2 \approx b$$

13.
$$\sin A = \frac{a}{c}$$
$$\sin 29° = \frac{4.6}{c}$$
$$c \sin 29° = 4.6$$
$$c = \frac{4.6}{\sin 29°}$$
$$c \approx 9.5$$

14.
$$\cos B = \frac{a}{c}$$
$$\cos 77° = \frac{17.3}{c}$$
$$c \cos 77° = 17.3$$
$$c = \frac{17.3}{\cos 77°}$$
$$c \approx 76.9$$

15.
$$\tan B = \frac{b}{a}$$
$$\tan 61° = \frac{33.2}{a}$$
$$a \tan 61° = 33.2$$
$$a = \frac{33.2}{\tan 61°}$$
$$a \approx 18.4$$

16.
$$\tan B = \frac{b}{a}$$
$$\tan 49° \, 13' = \frac{10}{a}$$
$$a \tan 49° \, 13' = 10$$
$$a = \frac{10}{\tan 49° \, 13'}$$
$$a \approx 8.6$$

17.
$$\sin A = \frac{a}{c}$$
$$\sin 16° \, 55' = \frac{a}{13.7}$$
$$13.7 \sin 16° \, 55' = a$$
$$4.0 \approx a$$

18.
$$\cos B = \frac{a}{c}$$
$$\cos 47° \, 18' = \frac{22.3}{c}$$
$$c \cos 47° \, 18' = 22.3$$
$$c = \frac{22.3}{\cos 47° \, 18'}$$
$$c \approx 32.9$$

19.
$$\sin 30° = \frac{h}{12}$$
$$12 \sin 30° = h$$
$$6 = h$$
$$\tan 45° = \frac{6}{m}$$
$$m \tan 45° = 6$$
$$m = \frac{6}{\tan 45°}$$
$$m = 6$$

$$\cos 30° = \frac{n}{12}$$
$$12 \cos 30° = n$$
$$10.4 \approx n$$
$$\sin 45° = \frac{6}{p}$$
$$p \sin 45° = 6$$
$$p = \frac{6}{\sin 45°}$$
$$p \approx 8.5$$

20a.
$$\cos 36° = \frac{10.8}{x}$$
$$x \cos 36° = 10.8$$
$$x = \frac{10.8}{\cos 36°}$$
$x \approx 13.3$ cm

20b.
$$\tan 36° = \frac{\frac{1}{2}s}{10.8}$$
$$10.8 \tan 36° = \frac{1}{2}s$$
$$2 \cdot 10.8 \tan 36° = s$$
$$15.7 \approx s$$
about 15.7 cm

20c. $P = 5s$
$P \approx 5(15.7)$
$P \approx 78.5$ cm

21a.
$$\cos 42° \, 30' = \frac{\frac{1}{2}(14.6)}{x}$$
$$x \cos 42° \, 30' = \frac{1}{2}(14.6)$$
$$x = \frac{\frac{1}{2}(14.6)}{\cos 42° \, 30'}$$
$x \approx 9.9$ m

21b.
$$\tan 42° \, 30' = \frac{x}{\frac{1}{2}(14.6)}$$
$$\frac{1}{2}(14.6) \tan 42° \, 30' = x$$
$$6.7 \approx x$$
about 6.7 m

21c. $A = \frac{1}{2}bh$

$A \approx \frac{1}{2}(14.6)(6.7)$

$A \approx 48.8\ \text{m}^2$

22a. $r = \frac{1}{2}(6.4)$ or 3.2

$\cos 30° = \frac{a}{3.2}$

$3.2 \cos 30° = a$

$2.771281292 = a$

about 2.8 cm

22b. Let x = side of hexagon.

$\sin 30° = \frac{\frac{1}{2}x}{3.2}$

$32 \sin 30° = \frac{1}{2}x$

$2 \cdot 3.2 \sin 30° = x$

$3.2 = x$; 32 cm

22c. $P = 6s$

$P = 6(3.2)$

$P = 19.2$ cm

22d. $A = \frac{1}{2}pa$

$A \approx \frac{1}{2}(19.2)(2.771281292)$

$A \approx 26.6\ \text{cm}^2$

23. $\sin 10° 21' 36'' = \frac{195.8}{x}$

$x \sin 10° 21' 36'' = 195.8$

$x = \frac{195.8}{\sin 10° 21' 36''}$

$x \approx 1088.8$ ft

24. height:

$\tan \propto = \frac{x}{\frac{1}{2}s}$

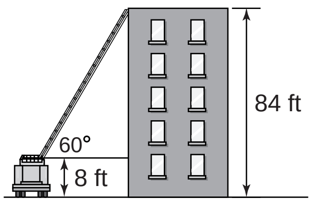
$\frac{1}{2}s \tan \propto = x$

$V = \frac{1}{3}$ area of base $\cdot$ height

$V = \frac{1}{3}(s^2)\left(\frac{1}{2} s \tan \propto\right)$

$V = \frac{1}{6} s^3 \tan \propto$

25a.



25b. $84 - 8 = 76$

$\tan 60° = \frac{76}{x}$

$x \tan 60° = 76$

$x = \frac{76}{\tan 60°}$

$x \approx 43.9$ ft

25c. $\sin 60° = \frac{76}{x}$

$x \sin 60° = 76$

$x = \frac{76}{\sin 60°}$

$x \approx 87.8$ ft

26a. $\tan 6° = \frac{3900}{x}$

$x \tan 6° = 3900$

$x = \frac{3900}{\tan 6°}$

$x \approx 37{,}106.0$ ft

26b. $\sin 6° = \frac{3900}{x}$

$x \sin 6° = 3900$

$x = \frac{3900}{\sin 6°}$

$x \approx 37{,}310.4$ ft

27. Yacht:

$\tan 20° = \frac{208}{x}$

$x \tan 20° = 208$

$x = \frac{208}{\tan 20°}$

$x \approx 571.5$

Barge:

$\tan 12° 30' = \frac{208}{x}$

$x \tan 12° 30' = 208$

$x = \frac{208}{\tan 12° 30'}$

$x \approx 938.2$

$938.2 - 571.5 \approx 366.8$ ft; no

28. Let M represent the point of intersection of the altitude and $\overline{EF}$. Since $\triangle GEF$ is isosceles, the altitude bisects $\overline{EF}$. $\triangle EMG$ is a right triangle. Therefore, $\sin \theta = \frac{a}{s}$ or $s \sin \theta = a$ and $\tan \theta = \frac{a}{0.5b}$ or $0.5b \tan \theta = a$.

29. Latasha:

$\sin 35° = \frac{x}{250}$

$250 \sin 35° = x$

$143.4 \approx x$

Markisha:

$\sin 42° = \frac{x}{225}$

$225 \sin 42° = x$

$150.6 \approx x$

$1506 - 143.4 = 7.2$

Markisha's; about 7.2 ft

30. Let x = the height of the building.
Let y = the distance between the buildings.

$\tan 47° 30' = \frac{x}{y}$

$y \tan 47° 30' = x$

$y = \frac{x}{\tan 47° 30'}$

$\tan 54° 54' = \frac{40 + x}{y}$

$y \tan 54° 54' = 40 + x$

$y = \frac{40 + x}{\tan 54° 54'}$

$\frac{x}{\tan 47° 30'} = \frac{40 + x}{\tan 54° 54'}$

$\tan 54° 54'(x) = \tan 47° 30'(40 + x)$

$x \tan 54° 54' = 40 \tan 47° 30' +$
$x \tan 47° 30'$

$x(\tan 54° 54' - \tan 47° 30') = 40 \tan 47° 30'$

$x = \frac{40 \tan 47° 30'}{\tan 54° 54' - \tan 47° 30'}$

$x \approx 131.7$ ft

31. terminal side — Quadrant II
reference angle: $180° - 120° = 60°$

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\sin 120° = y$	$\cos 120° = x$
$\sin 120° = \frac{\sqrt{3}}{2}$	$\cos 120° = -\frac{1}{2}$
$\tan 120° = \frac{y}{x}$	$\csc 120° = \frac{1}{y}$
$\tan 120° = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$	$\csc 120° = \frac{1}{\frac{\sqrt{3}}{2}}$
$\tan 120° = -\sqrt{3}$	$\csc 120° = \frac{2\sqrt{3}}{3}$
$\sec 120° = \frac{1}{x}$	$\cot 120° = \frac{x}{y}$
$\sec 120° = \frac{1}{-\frac{1}{2}}$	$\cot 120° = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$
$\sec 120° = -2$	$\cot 120° = -\frac{\sqrt{3}}{3}$

32. $(PR)^2 + (RQ)^2 = (PQ)^2$

$7^2 + 2^2 = (PQ)^2$

$53 = (PQ)^2$

$\sqrt{53} = PQ$

$\sin P = \frac{p}{r}$	$\cos P = \frac{q}{r}$
$\sin P = \frac{2}{\sqrt{53}}$	$\cos P = \frac{7}{\sqrt{53}}$
$\sin P = \frac{2\sqrt{53}}{53}$	$\cos P = \frac{7\sqrt{53}}{53}$

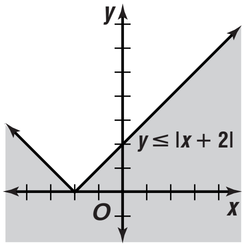
$\tan P = \frac{p}{q}$

$\tan P = \frac{2}{7}$

33. $43° 15' 35'' = 43° + 15°\left(\frac{1°}{60'}\right) + 35''\left(\frac{1°}{3600''}\right)$

$\approx 43.260°$

34.



35. Let x = the cost of notebooks and y = the cost of pencils.

$3x + 2y = 5.89$
$4x + y = 6.20$

$$\begin{array}{lcrcl} 3x + 2y = 5.80 & \rightarrow & 3x + 2y & = & 5.80 \\ -2(4x + y) = -2(6.20) & & -8x - 2y & = & -12.40 \\ \hline & & -5x & = & -6.60 \\ & & x & = & \$1.32 \end{array}$$

$4x + y = 6.20$
$4(1.32) + y = 6.20$
$y = \$0.92$

36. $\frac{m \text{ miles}}{h \text{ hours}} \cdot x \text{ hours} = \frac{mx}{h} \text{ miles}$

The correct choice is E.

Page 304 Mid-Chapter Quiz

1. $34.605° = 34° + (0.605 \cdot 60)'$
$= 34° + 36.3'$
$= 34° + 36' + (0.3 \cdot 60)''$
$= 34° + 36' + 18''$
$34° \, 36' \, 18''$

2. $\frac{-400°}{360°} \approx -1.11$
$-1.11 + 1 = -0.11$
$-0.11 \times 360° = -40°$
$360° - 40° = 320°$; IV

3. $(GI)^2 + (IH)^2 = (GH)^2$
$(GI)^2 + 10^2 = 12^2$
$(GI)^2 = 44$
$GI = \sqrt{44}$ or $2\sqrt{11}$

$\sin G = \frac{\text{side opposite}}{\text{hypotenuse}}$ $\quad$ $\cos G = \frac{\text{side adjacent}}{\text{hypotenuse}}$

$\sin G = \frac{10}{12}$ or $\frac{5}{6}$ $\quad$ $\cos G = \frac{2\sqrt{11}}{12}$ or $\frac{\sqrt{11}}{6}$

$\tan G = \frac{\text{side opposite}}{\text{side adjacent}}$ $\quad$ $\csc G = \frac{\text{hypotenuse}}{\text{side opposite}}$

$\tan G = \frac{10}{2\sqrt{11}}$ $\quad$ $\csc G = \frac{12}{10}$ or $\frac{6}{5}$

$\tan G = \frac{5\sqrt{11}}{11}$

$\sec G = \frac{\text{hypotenuse}}{\text{side adjacent}}$ $\quad$ $\cot G = \frac{\text{side adjacent}}{\text{side opposite}}$

$\sec G = \frac{12}{2\sqrt{11}}$ $\quad$ $\cot G = \frac{2\sqrt{11}}{10}$ or $\frac{\sqrt{11}}{5}$

$\sec G = \frac{6\sqrt{11}}{11}$

4. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{2^2 + (-5)^2}$
$r = \sqrt{29}$

$\sin \theta = \frac{y}{r}$ $\quad$ $\cos \theta = \frac{x}{r}$ $\quad$ $\tan \theta = \frac{y}{x}$

$\sin \theta = \frac{-5}{\sqrt{29}}$ $\quad$ $\cos \theta = \frac{2}{\sqrt{29}}$ $\quad$ $\tan \theta = \frac{-5}{2}$ or $-\frac{5}{2}$

$\sin \theta = -\frac{5\sqrt{29}}{29}$ $\quad$ $\cos \theta = \frac{2\sqrt{29}}{29}$

$\csc \theta = \frac{r}{y}$ $\quad$ $\sec \theta = \frac{r}{x}$

$\csc \theta = \frac{\sqrt{29}}{-5}$ or $-\frac{\sqrt{29}}{5}$ $\quad$ $\sec \theta = \frac{\sqrt{29}}{2}$

$\cot \theta = \frac{x}{y}$

$\cot \theta = \frac{2}{-5}$ or $-\frac{2}{5}$

5. $\tan 27.8° = \frac{550}{x}$
$x \tan 27.8° = 550$
$x = \frac{550}{\tan 27.8°}$
$x \approx 1043.2$ ft

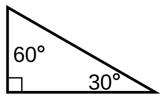
5-5 Solving Right Triangles

Pages 308–309 Check for Understanding

1a. linear $\quad$ **1b.** angle

2. They are complementary.

3. Sample answer:



4. Marta; they need to find the inverse of the cosine, not $\frac{1}{\cos}$.

5. 60°, 300° $\quad$ **6.** 150°, 330°

7. $\sin\left(\sin^{-1} \frac{\sqrt{3}}{2}\right)$
Let $A = \sin^{-1} \frac{\sqrt{3}}{2}$. Then $\sin A = \frac{\sqrt{3}}{2}$.
$\sin\left(\sin^{-1} \frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$

8. Let $A = \cos^{-1} \frac{3}{5}$. Then $\cos A = \frac{3}{5}$
$r^2 = x^2 + y^2$
$5^2 = 3^2 + y^2$
$16 = y^2$
$4 = y$
$\tan A = \frac{4}{3}$; $\tan\left(\cos^{-1} \frac{3}{5}\right) = \frac{4}{3}$

9. $\tan R = \frac{r}{s}$

$\tan R = \frac{7}{10}$

$R = \tan^{-1} \frac{7}{10}$

$R \approx 35.0°$

10. $\cos S = \frac{r}{t}$

$\cos S = \frac{12}{20}$

$S = \cos^{-1} \frac{12}{20}$

$S \approx 53.1°$

11. $A + 78° = 90°$

$A = 12°$

Find b.

$\tan B = \frac{b}{a}$

$\tan 78° = \frac{b}{41}$

$41 \tan 78° = b$

$192.9 \approx b$

Find c.

$\cos B = \frac{a}{c}$

$\cos 78° = \frac{41}{c}$

$c \cos 78° = 41$

$c = \frac{41}{\cos 78°}$

$c \approx 197.2$

$A = 12°, b = 192.9, c = 197.2$

12. $a^2 + b^2 = c^2$

$11^2 + 21^2 = c^2$

$\sqrt{562} = c$

$23.7 \approx c$

Find B.

$\tan B = \frac{b}{a}$

$\tan B = \frac{21}{11}$

$B = \tan^{-1} \frac{21}{11}$

$B \approx 62.4°$

Find A.

$A + 62.35402464 \approx 90$

$A \approx 27.64597536$

$c = 23.7, A = 27.6°, B = 62.4°$

13. $3.2° + B = 90°$

$B = 58°$

Find a.

$\sin A = \frac{a}{c}$

$\sin 32° = \frac{a}{13}$

$13 \sin 32° = a$

$6.9 \approx a$

Find b.

$\cos A = \frac{b}{c}$

$\cos 32° = \frac{b}{13}$

$13 \cos 32° = b$

$11.0 \approx b$

$B = 58°, a = 6.9, b = 11.0$

14a. $\tan x = \frac{1280}{2100}$

$x = \tan^{-1} \frac{1280}{2100}$

$x \approx 31.4°$

14b. $\tan 38° = \frac{1280}{x}$

$x \tan 38° = 1280$

$x = \frac{1280}{\tan 38°}$

$x \approx 1638.3$ ft

14c. $\tan 65° = \frac{1280}{x}$

$x \tan 65° = 1280$

$x = \frac{1280}{\tan 65°}$

$x \approx 596.9$ ft

Pages 309–312 Exercises

15. 90°

16. 120°, 300°

17. 30°, 330°

18. 90°, 270°

19. 225°, 315°

20. 135°, 315°

21. Sample answers: 30°, 150°, 390°, 510°

22. Let $A = \arccos \frac{4}{5}$. Then $\cos A = \frac{4}{5}$.

$\cos\left(\arccos \frac{4}{5}\right) = \frac{4}{5}$

23. Let $A = \tan^{-1} \frac{2}{3}$. Then $\tan A = \frac{2}{3}$.

$\tan\left(\tan^{-1} \frac{2}{3}\right) = \frac{2}{3}$

24. Let $A \cos^{-1} \frac{2}{5}$. Then $\cos A = \frac{2}{5}$.

$\sec A = \frac{1}{\cos A}$

$\sec A = \frac{1}{\frac{2}{5}}$

$\sec A = \frac{5}{2}$

$\sec\left(\cos^{-1} \frac{2}{5}\right) = \frac{5}{2}$

25. Let $A = \arcsin 1$. Then $\sin A = 1$.

$\csc A = \frac{1}{\sin A}$

$\csc A = \frac{1}{1}$ or 1

$\csc(\arcsin 1) = 1$

26. Let $A = \cos^{-1} \frac{5}{13}$. Then $\cos A = \frac{5}{13}$

$r^2 = x^2 + y^2$

$13^2 = 5^2 + y^2$

$144 = y^2$

$12 = y$

$\tan A = \frac{12}{5}$; $\tan\left(\cos^{-1} \frac{5}{13}\right) = \frac{12}{5}$

27. Let $A = \sin^{-1} \frac{2}{5}$. Then $\sin A = \frac{2}{5}$

$r^2 = x^2 + y^2$

$5^2 = x^2 + 2^2$

$21 = x^2$

$\sqrt{21} = x$

$\cos A = \frac{\sqrt{21}}{5}$; $\cos\left(\sin^{-1} \frac{2}{5}\right) = \frac{\sqrt{21}}{5}$

28. $\tan N = \frac{n}{m}$

$\tan N = \frac{15}{9}$

$N = \tan^{-1} \frac{15}{9}$

$N \approx 59.0°$

29. $\sin M = \frac{m}{p}$

$\sin M = \frac{8}{14}$

$M = \sin^{-1} \frac{8}{14}$

$M \approx 34.8°$

30. $\cos M = \frac{n}{p}$

$\cos M = \frac{22}{30}$

$M = \cos^{-1} \frac{22}{30}$

$M \approx 42.8°$

31. $\tan N = \frac{n}{m}$

$\tan N = \frac{18.8}{14.3}$

$N = \tan^{-1} \frac{18.8}{14.3}$

$N \approx 52.7°$

32. $\cos N = \frac{m}{p}$

$\cos N = \frac{7.2}{17.1}$

$N = \cos^{-1} \frac{7.2}{17.1}$

$N \approx 65.1°$

33. $\sin M = \frac{m}{p}$

$\sin M = \frac{32.5}{54.7}$

$M = \sin^{-1} \frac{32.5}{54.7}$

$M \approx 36.5°$

34. $\tan A = \frac{18}{24}$

$A = \tan^{-1} \frac{18}{24}$

$A \approx 36.9°$

$\tan B = \frac{24}{18}$

$B = \tan^{-1} \frac{24}{18}$

$B \approx 53.1°$

35. $\frac{1}{2}(14) = 7$

base angles: $\tan A = \frac{8}{7}$, $A = \tan^{-1}\frac{8}{7}$, $A \approx 48.8°$

vertex angle $= 2m\angle B$: $\tan B = \frac{7}{8}$, $B = \tan^{-1}\frac{7}{8}$, $B \approx 41.2°$, $2m\angle B \approx 82.4°$

about 48.8°, 48.8°, and 82.4°

36. $a^2 + b^2 = c^2$
$21^2 + b^2 = 30^2$
$b = \sqrt{459}$
$b \approx 21.4$
$44.427004 + B \approx 90$
$B \approx 45.6°$

$\sin A = \frac{a}{c}$
$\sin A = \frac{21}{30}$
$A = \sin^{-1}\frac{21}{30}$
$A \approx 44.4°$

37. $35° + B = 90°$
$B = 55°$

$\tan A = \frac{a}{b}$
$\tan 35° = \frac{a}{8}$
$8 \tan 35° = a$
$5.6 \approx a$

$\cos A = \frac{b}{c}$
$\cos 35° = \frac{8}{c}$
$c \cos 35° = 8$
$c = \frac{8}{\cos 35°}$
$c \approx 9.8$

38. $A + 47° = 90°$
$A = 43°$

$\tan B = \frac{b}{a}$
$\tan 47° = \frac{12.5}{a}$
$a \tan 47° = 12.5$
$a = \frac{12.5}{\tan 47°}$
$a \approx 11.7$

$\sin B = \frac{b}{c}$
$\sin 47° = \frac{12.5}{c}$
$c \sin 47° = 12.5$
$c = \frac{12.5}{\text{sim } 47°}$
$c \approx 17.1$

39. $a^2 + b^2 = c^2$
$3.8^2 + 4.2^2 = c^2$
$\sqrt{32.08} = c$
$5.7 \approx c$
$42.13759477 + B \approx 90$
$B \approx 47.9°$

$\tan A = \frac{a}{b}$
$\tan A = \frac{3.8}{4.2}$
$A = \tan^{-1}\frac{3.8}{4.2}$
$A \approx 42.1°$

40. $a^2 + b^2 = c^2$
$a^2 + 3.7^2 = 9.5^2$
$a = \sqrt{76.56}$
$a \approx 8.7$
$A + 22.92175446 \approx 90$
$A \approx 67.1°$

$\sin B = \frac{b}{c}$
$\sin B = \frac{3.7}{9.5}$
$B = \sin^{-1}\frac{3.7}{9.5}$
$B \approx 22.9°$

41. $51.5° + B = 90°$
$B = 38.5°$

$\tan A = \frac{a}{b}$
$\tan 51.5° = \frac{13.3}{b}$
$b \tan 51.5° = 13.3$
$b = \frac{13.3}{\tan 51.5°}$
$b \approx 10.6$

$\sin A = \frac{a}{c}$
$\sin 51.5° = \frac{13.3}{c}$
$c \sin 51.5° = 13.3$
$c = \frac{13.3}{\sin 51.5°}$
$c \approx 17.0$

42. $A + 33° = 90°$
$A = 57°$

$\sin B = \frac{b}{c}$
$\sin 33° = \frac{b}{15.2}$
$15.2 \sin 33° = b$
$8.3 \approx b$

$\cos B = \frac{a}{c}$
$\cos 33° = \frac{a}{15.2}$
$15.2 \cos 33° = a$
$12.7 \approx a$

43. $14° + B = 90°$
$B = 76°$

$\sin A = \frac{a}{c}$
$\sin 14° = \frac{a}{9.8}$
$9.8 \sin 14° = a$
$2.4 \approx a$

$\cos A = \frac{b}{c}$
$\cos 14° = \frac{b}{9.8}$
$9.8 \cos 14° = b$
$9.5 \approx b$

44a. $\sin \theta = \frac{647}{1020}$
$\theta = \sin^{-1}\frac{647}{1020}$
$\theta \approx 39.4°$

44b. $\tan 39.4° \approx \frac{647}{x}$
$x \tan 39.4° \approx 647$
$x \approx \frac{647}{\tan 39.4°}$
$x \approx 788.5$ ft

45a. Since the sine function is the side opposite divided by the hypotenuse, the sine cannot be greater than 1.

45b. Since the secant function is the hypotenuse divided by the side opposite, the secant cannot be between 1 and −1.

45c. Since cosine function is the side adjacent divided by the hypotenuse, the cosine cannot be less than −1.

46. $10 - 6 = 4$
$\tan \theta = \frac{4}{15}$
$\theta = \tan^{-1}\frac{4}{15}$
$\theta \approx 14.9°$

47a. $\tan \theta = \frac{8}{100}$
$\theta = \tan^{-1}\frac{8}{100}$
$\theta \approx 4.6°$

47b. $\tan \theta = \frac{5}{100}$
$\theta = \tan^{-1}\frac{5}{100}$
$\theta \approx 2.9°$

48. $\tan \theta = \frac{45}{2200}$
$\theta = \tan^{-1}\frac{45}{2200}$
$\theta \approx 1.2°$

49. $\frac{65 \text{ miles}}{\text{hour}} \cdot \frac{5280 \text{ feet}}{\text{mile}} \cdot \frac{1 \text{ hour}}{3600 \text{ seconds}} \approx 95.3 \frac{\text{feet}}{\text{second}}$
$\tan \theta = \frac{v^2}{gr}$
$\tan \theta = \frac{95.3^2}{32\ (1200)}$
$\theta = \tan^{-1}\frac{95.3^2}{32(1200)}$
$\theta \approx 13.3°$

50. $\frac{\sin \theta_i}{\sin \theta_r} = n$

$\frac{\sin 60°}{\sin \theta_r} = 2.42$

$\frac{\sin 60°}{2.42} = \sin \theta_r$

$0.3579 \approx \sin \theta_r$

$\sin^{-1} 0.3579 \approx \theta_r$

$21.0° \approx \theta_r$

51. Draw the altitude from Y to XZ. Call the point of intersection W.

$m\angle X + m\angle XYW = 90°$

$30° + m\angle XYW = 90°$

$m\angle XYW = 60°$

In $\triangle XYW$:

$\cos 30° = \frac{XW}{16}$ $\qquad$ $\sin 30° = \frac{WY}{16}$

$16 \cos 30° = XW$ $\qquad$ $16 \sin 30° = WY$

$13.9 \approx XW$ $\qquad$ $8 = \text{WY}$

In $\triangle ZYW$:

$\sin Z = \frac{8}{24}$ $\qquad$ $\tan 19.5° = \frac{8}{WZ}$

$Z = \sin^{-1} \frac{8}{24}$ $\qquad$ $WZ \tan 19.5° = 8$

$Z \approx 19.5°$ $\qquad$ $WZ = \frac{8}{\tan 19.5°}$

$WZ \approx 22.6$

$\cos m\angle WYZ = \frac{8}{24}$

$m\angle WYZ = \cos^{-1} \frac{8}{24}$

$m\angle WYZ \approx 70.5°$

$Y = m\angle XYW + m\angle WYZ$ $\qquad$ $y = XW + WZ$

$Y \approx 60° + 70.5°$ $\qquad$ $y \approx 13.9 + 22.6$

$\text{Y} \approx 130.5°$ $\qquad$ $y \approx 36.5$

52. baseball stadium: $\qquad$ football stadium:

$\tan 63° = \frac{1000}{x}$ $\qquad$ $\tan 18° = \frac{1000}{y}$

$x \tan 63° = 1000$ $\qquad$ $y \tan 18° = 1000$

$x = \frac{1000}{\tan 63°}$ $\qquad$ $y = \frac{1000}{\tan 18°}$

$x \approx 509.5$ $\qquad$ $y \approx 3077.7$

distance $= x + y$

distance $\approx 509.5 + 3077.7$

distance ≈ 3587.2 ft

53. $(FD)^2 + (DE)^2 = (FE)^2$

$7^2 + (DE)^2 = 15^2$

$(DE)^2 = 176$

$DE = \sqrt{176}$ or $4\sqrt{11}$

$\sin F = \frac{\text{side opposite}}{\text{hypotenuse}}$ $\qquad$ $\cos F = \frac{\text{side adjacent}}{\text{hypotenuse}}$

$\sin F = \frac{4\sqrt{11}}{15}$ $\qquad$ $\cos F = \frac{7}{15}$

$\tan F = \frac{\text{side opposite}}{\text{side adjacent}}$ $\qquad$ $\csc F = \frac{\text{hypotenuse}}{\text{side opposite}}$

$\tan F = \frac{4\sqrt{11}}{7}$ $\qquad$ $\csc F = \frac{15}{4\sqrt{11}}$ or $\frac{15\sqrt{11}}{44}$

$\sec F = \frac{\text{hypotenuse}}{\text{side adjacent}}$ $\qquad$ $\cot F = \frac{\text{side adjacent}}{\text{side opposite}}$

$\sec F = \frac{15}{7}$ $\qquad$ $\cot F = \frac{7}{4\sqrt{11}}$ or $\frac{7\sqrt{11}}{44}$

54. Use TABLE feature of a graphing calculator.
$-0.3, 1.4, 4.3$

55. x-axis
$y^3 - x^2 = 2$
$(-y)^3 - x^2 = 2$
$-y^3 - x^2 = 2$; no

y-axis
$y^3 - x^2 = 2$
$y^3 - (-x)^2 = 2$
$y^3 - x^2 = 2$; yes

$y = x$
$y^3 - x^2 = 2$
$(x)^3 - (y)^2 = 2$
$x^3 - y^2 = 2$; no

$y = -x$
$y^3 - x^2 = 2$
$(-x)^3 - (-y)^2 = 2$
$-x^3 - y^2 = 2$; no

y-axis

56. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -5 & -5 & -3 & -1 & -2 \\ -3 & 4 & 6 & 3 & -2 \end{bmatrix}$

$= \begin{bmatrix} -1(-5)+0(-3) & -1(-5)+0(4) & -1(-3)+0(6) & -1(-1)+0(3) & -1(-2)+0(-2) \\ 0(-5)+1(-3) & 0(-5)+1(4) & 0(-3)+1(6) & 0(-1)+1(3) & 0(-2)+1(-2) \end{bmatrix}$

$= \begin{bmatrix} 5 & 5 & 3 & 1 & 2 \\ -3 & 4 & 6 & 3 & -2 \end{bmatrix}$

$(5, -3), (5, 4), (3, 6), (1, 3), (2, -2)$

57. $\begin{bmatrix} 4 & -3 & 2 \\ 8 & -2 & 0 \\ 9 & 6 & -3 \end{bmatrix} + \begin{bmatrix} -2 & 2 & -2 \\ -5 & 1 & 1 \\ -7 & 2 & -2 \end{bmatrix}$

$= \begin{bmatrix} 4+(-2) & -3+2 & 2+(-2) \\ 8+(-5) & -2+1 & 0+1 \\ 9+(-7) & 6+2 & -3+(-2) \end{bmatrix}$

$= \begin{bmatrix} 2 & -1 & 0 \\ 3 & -1 & 1 \\ 2 & 8 & -5 \end{bmatrix}$

58. $m = \frac{22.2 - 42.5}{1950 - 1880}$

$m = \frac{-20.3}{70}$ or -0.29

$y - 22.2 = -0.29(x - 1950)$

$y = -0.29x + 587.7$

59. $2x + 5y - 10 = 0$

$5y = -2x + 10$

$y = -\frac{2}{5}x + 2$

$-\frac{2}{5}$; 2

60. $\frac{a}{a+c} \cdot b + \frac{c}{a+c} \cdot d + 10 = \frac{ab + cd}{a + c} + 10$

The correct choice is A.

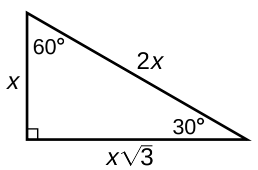
5-6 The Law of Sines

Page 316 Check for Understanding

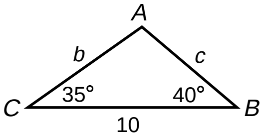
1. $\frac{x}{\sin 30°} \stackrel{?}{=} \frac{x\sqrt{3}}{\sin 60°} \stackrel{?}{=} \frac{2x}{\sin 90°}$

$\frac{x}{\frac{1}{2}} \stackrel{?}{=} \frac{x\sqrt{3}}{\frac{\sqrt{3}}{2}} \stackrel{?}{=} \frac{2x}{1}$

$2x = 2x = 2x$



2. Sample answer:



3. Area of $WXYZ$ = Area of triangle ZWY + Area of triangle XYW.
$m\angle X = m\angle Z$

triangle ZWY: triangle XYW:

$K = \frac{1}{2ab}\sin Z$ $\quad k = \frac{1}{2}ab\sin X$

$K = \frac{1}{2ab}\sin X$

$K = \frac{1}{2}ab\sin X + \frac{1}{2}ab\sin X$

$K = ab\sin X$

4. Both; if the measures of two angles and a non-included side are known or if the measures of two angles and the included side are known, the triangle is unqiue.

5. $C = 180° - (40° + 59°)$ or $81°$

$\frac{a}{\sin A} = \frac{c}{\sin C}$ $\quad \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{a}{\sin 40°} = \frac{14}{\sin 81°}$ $\quad \frac{b}{\sin 59°} = \frac{14}{\sin 81°}$

$a = \frac{14\sin 40°}{\sin 81°}$ $\quad b = \frac{14\sin 59°}{\sin 81°}$

$a \approx 9.111200533$ $\quad b \approx 12.14992798$

$C = 81°, a = 9.1, b = 12.1$

6. $C = 180° - (27.3° + 55.9°)$ or $96.8°$

$\frac{b}{\sin B} = \frac{a}{\sin A}$ $\quad \frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{b}{\sin 55.9°} = \frac{8.6}{\sin 27.3°}$ $\quad \frac{c}{\sin 96.8°} = \frac{8.6}{\sin 27.3°}$

$b = \frac{8.6\sin 55.9°}{\sin 27.3°}$ $\quad c = \frac{8.6\sin 96.8°}{\sin 27.3°}$

$b \approx 15.52671055$ $\quad c \approx 18.61879792$

$C = 96.8°, b = 15.5, c = 18.6$

7. $A = 180° - (17°\,55' + 98°\,15')$ or $63°\,50'$

$\frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{c}{\sin 98°\,15'} = \frac{17}{\sin 63°\,50'}$

$c = \frac{17\sin 98°\,15'}{\sin 63°\,50'}$

$c \approx 18.7$

8. $K = \frac{1}{2}bc\sin A$

$K = \frac{1}{2}(14)(12)\sin 78°$

$K \approx 82.2$ units^2

9. $C = 180° - (22° + 105°)$ or $53°$

$K = \frac{1}{2}b^2\frac{\sin A\sin C}{\sin B}$

$K = \frac{1}{2}(14)^2\frac{\sin 22°\sin 53°}{\sin 105°}$

$K \approx 30.4$ units^2

10. Let d = the distance from the fan to the pitcher's mound.

$\theta = 180° - (24°\,12' + 5°\,42')$ or $150°\,6'$

$\frac{d}{\sin 150°\,6'} = \frac{60.5}{\sin 5°\,42'}$

$d = \frac{60.5\sin 150°\,6'}{\sin 5°\,42'}$

$d \approx 303.7$ ft

Pages 316–318 Exercises

11. $B = 180° - (40° + 70°)$ or $70°$

$\frac{b}{\sin B} = \frac{a}{\sin A}$ $\quad \frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{b}{\sin 70°} = \frac{20}{\sin 40°}$ $\quad \frac{c}{\sin 70°} = \frac{20}{\sin 40°}$

$b = \frac{20\sin 70°}{\sin 40°}$ $\quad c = \frac{20\sin 70°}{\sin 40°}$

$b \approx 29.238044$ $\quad c \approx 29.238044$

$B = 70°, b = 29.2, c = 29.2$

12. $A = 180° - (100° + 50°)$ or $30°$

$\frac{a}{\sin A} = \frac{c}{\sin C}$ $\quad \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{a}{\sin 30°} = \frac{30}{\sin 50°}$ $\quad \frac{b}{\sin 100°} = \frac{30}{\sin 50'}$

$a = \frac{30\sin 30°}{\sin 50°}$ $\quad b = \frac{30\sin 100°}{\sin 50°}$

$a \approx 19.58110934$ $\quad b \approx 38.56725658$

$A = 30°, a = 19.6, b = 38.6$

13. $C = 180° - (25° + 35°)$ or $120°$

$\frac{a}{\sin A} = \frac{b}{\sin B}$ $\quad \frac{c}{\sin C} = \frac{b}{\sin B}$

$\frac{a}{\sin 25°} = \frac{12}{\sin 35°}$ $\quad \frac{c}{\sin 120°} = \frac{12}{\sin 35°}$

$a = \frac{12\sin 25°}{\sin 35°}$ $\quad c = \frac{12\sin 120°}{\sin 35°}$

$a \approx 8.84174945$ $\quad c \approx 18.11843058$

$C = 120°, a = 8.8, c = 18.1$

14. $C = 180° - (65° + 50°)$ or $65°$

$\frac{a}{\sin A} = \frac{c}{\sin C}$ $\quad \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{a}{\sin 65°} = \frac{12}{\sin 65°}$ $\quad \frac{b}{\sin 50°} = \frac{12}{\sin 65°}$

$a = \frac{12\sin 65°}{\sin 65°}$ $\quad b = \frac{12\sin 50°}{\sin 65°}$

$a = 12$ $\quad b \approx 10.14283828$

$C = 65°, a = 12, b = 10.1$

15. $A = 180° - (24.8° + 61.3°)$ or $93.9°$

$\frac{b}{\sin B} = \frac{a}{\sin A}$ $\quad \frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{b}{\sin 24.8°} = \frac{8.2}{\sin 93.9°}$ $\quad \frac{c}{\sin 61.3°} = \frac{8.2}{\sin 93.9°}$

$b = \frac{8.2\sin 24.8°}{\sin 93.9°}$ $\quad c = \frac{8.2\sin 61.3°}{\sin 93.9°}$

$b \approx 3.447490503$ $\quad c \approx 7.209293255$

$A = 93.9°, b = 3.4, c = 7.2$

16. $B = 180° - (39°\,15' + 64°\,45')$ or $76°$

$\frac{a}{\sin A} = \frac{c}{\sin C}$ $\quad \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{a}{\sin 39'\,51'} = \frac{19.3}{\sin 64°\,45'}$ $\quad \frac{b}{\sin 76°} = \frac{19.3}{\sin 64°\,45'}$

$a = \frac{19.3\sin 39°\,15'}{\sin 64°\,45'}$ $\quad b = \frac{19.3\sin 76°}{\sin 64°\,45'}$

$a \approx 13.50118124$ $\quad b \approx 20.7049599$

$B = 76°, a = 13.5, b = 20.7$

17. $C = 180° - (37°\,20' + 51°\,30')$ or $91°\,10'$

$\frac{a}{\sin B} = \frac{c}{\sin C}$

$\frac{b}{\sin 51°\,30'} = \frac{125}{\sin 91°\,10'}$

$b = \frac{125\sin 51°\,31'}{\sin 91°\,10'}$

$b \approx 97.8$

18. A = $180° - (29°\,34' + 23°\,48')$ or $126°\,38'$

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{a}{\sin 126°\,38'} = \frac{11}{\sin 29°\,34'}$

$a = \frac{11\sin 126°\,38'}{\sin 29°\,34'}$

$a \approx 17.9$

19. $K = \frac{1}{2}\, bc \sin A$

$K = \frac{1}{2}(14)(9) \sin 28°$

$K \approx 29.6 \text{ units}^2$

20. $A = 180° - (37° + 84°)$ or $59°$

$K = \frac{1}{2}a^2 \frac{\sin B \sin C}{\sin A}$

$K = \frac{1}{2}(5)^2 \frac{\sin 37° \sin 84°}{\sin 59°}$

$K \approx 8.7 \text{ units}^2$

21. $C = 180° - (15° + 113°)$ or $52°$

$K = \frac{1}{2}\, b^2 \frac{\sin A \sin C}{\sin B}$

$K = \frac{1}{2}(7)^2 \frac{\sin 15° \sin 52°}{\sin 113°}$

$K \approx 5.4 \text{ units}^2$

22. $K = \frac{1}{2}bc \sin A$

$K = \frac{1}{2}(146.2)(209.3) \sin 62.2°$

$K \approx 13{,}533.9 \text{ units}^2$

23. $K = \frac{1}{2}ac \sin B$

$K = \frac{1}{2}(12.7)(5.8) \sin 42.8°$

$K \approx 25.0 \text{ units}^2$

24. $B = 180° - (53.8° + 65.4°)$ or $60.8°$

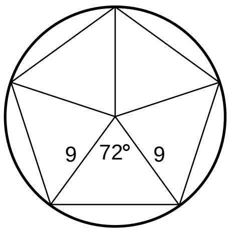
$K = \frac{1}{2}a^2 \frac{\sin B \sin C}{\sin A}$

$K = \frac{1}{2}(19.2)^2 \frac{\sin 60.8° \sin 65.4°}{\sin 53.8°}$

$K \approx 181.3 \text{ units}^2$

25. $K = ab \sin X$ (formula from Exercise 3)
$K = (14)(20) \sin 57°$
$K \approx 234.8 \text{ cm}^2$

26.

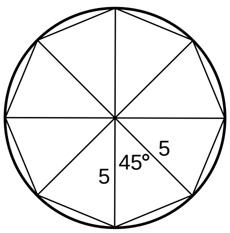


Area of pentagon =
5 · Area of triangle
360° ÷ 5 = 72°

$K = \frac{1}{2}(9)(9) \sin 72°$ $\quad 5K \approx 5(38.51778891)$

$K \approx 38.51778891$ $\quad 5K \approx 192.6 \text{ in}^2$

27.



Area of octagon =
8 · Area of triangle
360° ÷ 8 = 45°

$K = \frac{1}{2}(5)(5) \sin 45°$ $\quad 8K \approx 8(8.838834765)$

$K \approx 8.838834765$ $\quad 8K \approx 70.7 \text{ ft}^2$

28a. $180° - (95° + 40°) = 45°$

28b. $\frac{x}{\sin 95°} = \frac{80}{\sin 45°}$ $\quad \frac{y}{\sin 40°} = \frac{80}{\sin 45°}$

$x = \frac{80 \sin 95°}{\sin 45°}$ $\quad y = \frac{80 \sin 40°}{\sin 45°}$

$x \approx 112.7065642$ $\quad y \approx 72.72311643$

about 112.7 ft and 72.7 ft

28c. $P \approx 112.7 + 72.7 + 80$
$P \approx 265.4$ ft

29. Applying the Law of Sines, $\frac{m}{\sin M} = \frac{n}{\sin N}$ and $\frac{r}{\sin R} = \frac{s}{\sin S}$. Thus $\sin M = \frac{n \sin N}{n}$ and $\sin R = \frac{r \sin S}{s}$. Since $\angle M \cong \angle R$, $\sin M = \sin R$ and $\frac{m \sin N}{n} = \frac{r \sin S}{s}$. However, $\angle N \cong \angle S$ and $\sin N = \sin S$, so $\frac{m}{n} = \frac{r}{s}$ and $\frac{m}{r} = \frac{n}{s}$. Similar proportions can be derived for p and t. Therefore, $\triangle MNP \cong \triangle RST$.

30. $360° \div 5 = 72°$

triangle: $K = \frac{1}{2}(300)(300)\sin 72°$

$K \approx 42{,}797.54323$

pentagon: $5K \approx 5(42{,}797.54323)$

$5K \approx 213{,}987.7$ ft

31a. $\theta = 180° - (20°\,15' + 62°\,30')$ or $97°\,15'$
Let x = the distance from the balloon to the soccer fields.

$\frac{x}{\sin 62°\,30'} = \frac{4}{\sin 97°\,15'}$

$x = \frac{4 \sin 62°\,30'}{\sin 97°\,15'}$

$x \approx 3.6$ mi

31b. $\theta = 180° - (20°\,15' + 62°\,30')$ or $97°\,15'$
Let y = the distance from the balloon to the football field.

$\frac{4}{\sin 20°\,15'} = \frac{4}{\sin 97°\,15'}$

$y = \frac{4 \sin 20°\,15'}{\sin 97°\,15'}$

$y \approx 1.4$ mi

32. $180° - 30° = 150°$
$\theta = 180° - (26.8° + 150°)$ or $3.2°$
Let x = the length of the track.

$\frac{x}{\sin 26.8°} = \frac{100}{\sin 3.2°}$

$x = \frac{100 \sin 26.8°}{\sin 3.2°}$

$x \approx 807.7$ ft

33a. Let x = the distance of the second part of the flight.
$\theta = 180° - (13° + 160°)$ or $7°$

$\frac{x}{\sin 13°} = \frac{80}{\sin 7°}$

$x = \frac{80 \sin 13°}{\sin 7°}$

$x \approx 147.6670329$

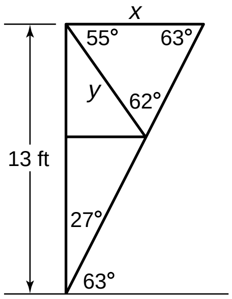
distance of flight ≈ 80 + 147.7 or about 227.7 mi

33b. Let y = the distance of a direct flight.

$\frac{y}{\sin 160°} = \frac{80}{\sin 7°}$

$y = \frac{80 \sin 160°}{\sin 7°}$

$y \approx 224.5$ mi

34. 

$90° - 63° = 27°$
$180° - (55° + 63°) = 62°$
Let x = the vertical distance.
Let y = the length of the overhang.

$$\frac{x}{\sin 27°} = \frac{13}{\sin 63°} \qquad \frac{y}{\sin 63°} = \frac{66}{\sin 62°}$$
$$x = \frac{13 \sin 27°}{\sin 63°} \qquad y = \frac{6.6 \sin 63°}{\sin 62°}$$
$$x \approx 6.623830843 \qquad y \approx 6.684288563$$
about 6.7 ft

35a. $\frac{a}{\sin A} = \frac{b}{\sin B}$
$\frac{a}{b} = \frac{\sin A}{\sin B}$

35b. $\frac{a}{\sin A} = \frac{c}{\sin C}$
$\frac{a}{c} = \frac{\sin A}{\sin C}$
$\frac{a}{c} - 1 = \frac{\sin A}{\sin C} - 1$
$\frac{a}{c} - \frac{c}{c} = \frac{\sin A}{\sin C} - \frac{\sin C}{\sin C}$
$\frac{a - c}{c} = \frac{\sin A - \sin C}{\sin C}$

35c. From Exercise 34b, $\frac{a - c}{c} = \frac{\sin A - \sin C}{\sin C}$
or $\frac{\sin A - \sin C}{a - c} = \frac{\sin C}{c}$.
$\frac{a}{\sin A} = \frac{c}{\sin C}$
$\frac{a}{c} = \frac{\sin A}{\sin C}$
$\frac{a}{c} + 1 = \frac{\sin A}{\sin C} + 1$
$\frac{a}{c} + \frac{c}{c} = \frac{\sin A}{\sin C} + \frac{\sin C}{\sin C}$
$\frac{a + c}{c} = \frac{\sin A + \sin C}{\sin C}$
$\frac{\sin C}{c} = \frac{\sin A + \sin C}{a + c}$
Therefore, $\frac{\sin A - \sin C}{a - c} = \frac{\sin A + \sin C}{a + c}$
or $\frac{a + c}{a - c} = \frac{\sin A + \sin C}{\sin A - \sin C}$.

35d. $\frac{a}{\sin A} = \frac{b}{\sin B}$
$\frac{a}{b} = \frac{\sin A}{\sin B}$
$\frac{a}{b} + 1 = \frac{\sin A}{\sin B} + 1$
$\frac{a}{b} + \frac{b}{b} = \frac{\sin A}{\sin B} + \frac{\sin B}{\sin B}$
$\frac{a + b}{b} = \frac{\sin A + \sin B}{\sin B}$
$\frac{b}{a + b} = \frac{\sin B}{\sin A + \sin B}$

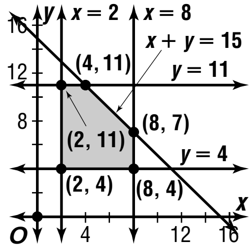
36. $\tan \theta = \frac{45}{20}$
$\theta = \tan^{-1} \frac{45}{20}$
$\theta \approx 66.0°$

37. $\sin \theta = \frac{y}{r}$ $\qquad r^2 = x^2 + y^2$
$\sin \theta = -\frac{1}{6}$ $\qquad 6^2 = x^2 + (-1)^2$
$y = -1,\ r = 6$ $\qquad 35 = x^2$
$\pm\sqrt{35} = x$
Quadrant IV, so $x = \sqrt{35}$

$\cos \theta = \frac{x}{r}$ $\qquad \tan \theta = \frac{y}{x}$ $\qquad \csc \theta = \frac{r}{y}$
$\cos \theta = \frac{\sqrt{35}}{6}$ $\qquad \tan \theta = \frac{-1}{\sqrt{35}}$ $\qquad \csc \theta = \frac{6}{-1}$ or -6
$\tan \theta = -\frac{\sqrt{35}}{35}$
$\sec \theta = \frac{r}{x}$ $\qquad \cot \theta = \frac{x}{y}$
$\sec \theta = \frac{6}{\sqrt{35}}$ $\qquad \cot \theta = \frac{\sqrt{35}}{-1}$ or $-\sqrt{35}$
$\sec \theta = \frac{6\sqrt{35}}{35}$

38. $83° + 360k°$

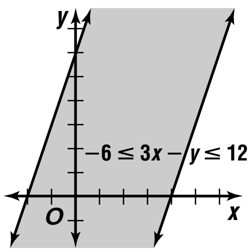
39. Let x = standard carts and let y = deluxe carts.
$2 \le x \le 8$
$4 \le y \le 11$
$x + y \le 15$



$M(x, y) = 100x + 250y$
$M(2, 4) = 100(2) + 250(4)$ or 1200
$M(2, 11) = 100(2) + 250(11)$ or 2950
$M(4, 11) = 100(4) + 250(11)$ or 3150
$M(8, 7) = 100(8) + 250(7)$ or 2550
$M(8, 4) = 100(8) + 250(4)$ or 1800
4 standard carts, 11 deluxe carts

40. $4x + y + 2z = 0$
$\underline{3x + 4y - 2z = 20}$
$7x + 5y = 20$
$3(3x + 4y - 2z) = 3(20)$
$2(-2x + 5y + 3z) = 2(14)$
$\downarrow$
$9x + 12y - 6z = 60$
$\underline{-4x + 10y + 6z = 28}$
$5x + 22y = 88$
$-5(7x + 5y) = -5(20) \quad \rightarrow \quad -35x - 25y = -100$
$7(5x + 22y) = 7(88) \qquad\qquad \underline{35x + 154y = 616}$
$129y = 516$
$y = 4$
$7x + 5y = 20$ $\qquad 4x + y + 2z = 0$
$7x + 5(4) = 20$ $\qquad 4(0) + 4 + 2z = 0$
$x = 0$ $\qquad z = -2$
$(0, 4, -2)$

41. $-6 \le 3x - y$ $\qquad 3x - y \le 12$
$y \le 3x + 6$ $\qquad y \ge 3x - 12$



42. Area of one face of the small cube = 1^2 or 1 in^2.
Surface area of the small cube = $6 \cdot 1$ or 6 in^2.
Area of one face of large cube = 2^2 or 4 in^2.
Surface area of large cube = $6 \cdot 4$ or 24 in^2.
Surface area of all small cubes = $8 \cdot 6$ or 48 in^2.
The difference in surface areas is 48 in^2 − 24 in^2 or 24 in^2.
The correct choice is A.

Page 319 History of Mathematics

1. See students' work; the sum is greater than 180°. In spherical geometry, the sum of the angles of a triangle can exceed 180°.
2. See students' work. Sample answer: Postulate 4 states that all right angles are equal to one another.
3. See students' work.

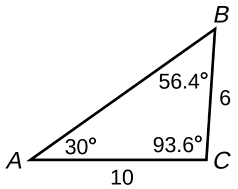
5-7 The Ambiguous Case for the Law of Sines

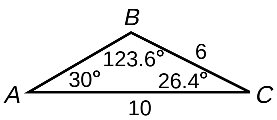
Page 323 Graphing Calculator Exploration

1. $B = 44.1°$, $C = 23.9°$, $c = 1.8$
2. $B = 52.7°$, $C = 76.3°$, $b = 41.0$; $B = 25.3°$, $C = 103.7°$, $b = 22.0$
3. The answers are slightly different.
4. Answers will vary if rounded numbers are used to find some values.

Page 324 Check for Understanding

1. A triangle cannot exist if $m\angle A < 90°$ and $a < b \sin A$ or if $m\angle A \geq 90°$ and $a \leq b$.
2.





$\frac{6}{\sin 30°} = \frac{10}{\sin B}$

$\sin B = \frac{10 \sin 30°}{6}$

$B = \sin^{-1}\left(\frac{10 \sin 30°}{6}\right)$

$B \approx 56.44269024$

$C \approx 180° - (30° + 56.4°)$
$\approx 93.6°$

$180° - \propto \approx 180° - 56.4°$
$\approx 123.6°$

$C \approx 180° - (30° + 123.6°)$
$\approx 26.4°$

3. Step 1: Determine that there is one solution for the triangle.
Step 2: Use the Law of Sines to solve for B.
Step 3: Subtract the sum of 120 and B from 180 to find C.
Step 4: Use the Law of Sines to solve for c.
4. Since $113° \geq 90°$, consider Case II.
$15 \geq 8$; 1 solution
5. Since $44° < 90°$, consider Case I.
$a \sin B = 23 \sin 44°$
$a \sin B \approx 23\ (0.6947)$
$a \sin B \approx 15.97714252$
$12 < 16.0$; 0 solutions
6. Since $17° < 90°$, consider Case I.
$a \sin C = 10 \sin 17°$
≈ 2.923717047
$2.9 < 10 < 11$; 1 solution

$\frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{11}{\sin 17°} = \frac{10}{\sin A}$

$\sin A = \frac{10 \sin 17°}{11}$

$A = \sin^{-1}\left(\frac{10 \sin 17°}{11}\right)$

$A \approx 15.41404614$

$B \approx 180° - (15.4° + 17°)$ or about 147.6°

$\frac{c}{\sin C} = \frac{b}{\sin B}$

$\frac{11}{\sin 17°} \approx \frac{b}{\sin 147.6°}$

$b \approx \frac{11 \sin 147.6°}{\sin 17°}$

$b \approx 20.16738057$

$A = 15.4°$, $B = 147.6°$, $b = 20.2$
7. Since $140° \geq 90°$, consider Case II.
$3 \leq 10$; no solutions
8. Since $38° < 90°$, consider Case I.
$b \sin A = 10 \sin 38°$
$b \sin A \approx 6.156614753$
$6.2 < 8 < 10$; 2 solutions

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{8}{\sin 38°} = \frac{10}{\sin B}$

$\sin B = \frac{10 \sin 38°}{8}$

$B = \sin^{-1}\left(\frac{10 \sin 38°}{8}\right)$

$B \approx 50.31590502$

$180° - \propto \approx 180° - 50.3°$ or 129.7°

Solution 1

$C \approx 180° - (50.3° + 38°)$ or 91.7°

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{8}{\sin 38°} \approx \frac{c}{\sin 91.7°}$

$c \approx \frac{8 \sin 91.7°}{\sin 38°}$

$c \approx 12.98843472$

Solution 2

$C \approx 180° - (129.7° + 38°)$ or 12.3°

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{8}{\sin 38°} \approx \frac{c}{\sin 12.3°}$

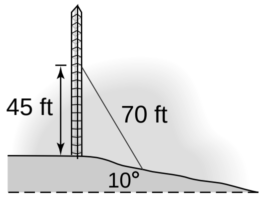
$c \approx \frac{8 \sin 12.3°}{\sin 38°}$

$c \approx 2.768149638$

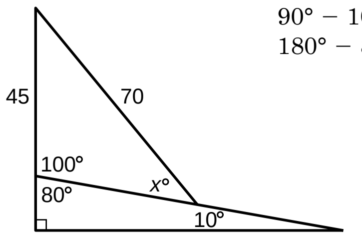
$B = 50.3°$, $C = 91.7°$, $c = 13.0$; $B = 129.7°$, $C = 12.3°$, $c = 2.8$

9. Since $130° \geq 90°$, consider Case II.
$17 > 5$; 1 solution
$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{17}{\sin 130°} = \frac{5}{\sin B}$$
$$\sin B = \frac{5 \sin 130°}{17}$$
$$B = \sin^{-1}\left(\frac{5 \sin 130°}{17}\right)$$
$$B \approx 13.02094264$$
$A \approx 180° - (13.0 + 130°)$ or $37.0°$
$$\frac{c}{\sin C} = \frac{a}{\sin A}$$
$$\frac{17}{\sin 130°} \approx \frac{a}{\sin 37.0°}$$
$$a \approx \frac{17 \sin 37.0°}{\sin 130°}$$
$$a \approx 13.35543321$$
$A = 37.0°$, $B = 13.0°$, $a = 13.4$

10a.



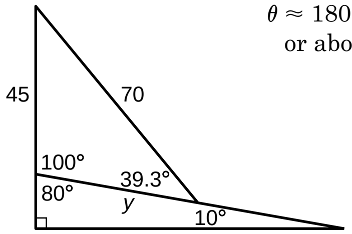
10b.



$90° - 10° = 80°$
$180° - 80° = 100°$

$$\frac{70}{\sin 100°} = \frac{45}{\sin x}$$
$$\sin x = \frac{45 \sin 100°}{70}$$
$$x = \sin^{-1}\left(\frac{45 \sin 100°}{70}\right)$$
$$x \approx 39.3°$$

10c.



$\theta \approx 180° - (100° - 39.3°)$ or about $40.7°$

$$\frac{y}{\sin 40.7°} = \frac{70}{\sin 100°}$$
$$y = \frac{70 \sin 40.7°}{\sin 100°}$$
$$y \approx 46.4 \text{ ft}$$

Pages 324–326 Exercises

11. Since $57° < 90°$, consider Case I.
$b \sin A = 19 \sin 57°$
$b \sin A \approx 15.93474079$
$11 < 15.9$; 0 solutions

12. Since $30° < 90°$, consider Case I.
$c \sin A = 26 \sin 30°$
$c \sin A = 13$
$13 = 13$; 1 solution.

13. Since $61° < 90°$, consider Case I.
$a \sin B = 12 \sin 61°$
$a \sin B \approx 10.49543649$
$8 < 10.5$; 0 solutions

14. two angles are given; 1 solution

15. Since $100° \geq 90°$, consider Case II.
$15 < 18$; 0 solutions

16. Since $37° < 90°$, consider Case I.
$a \sin B = 32 \sin 37°$
$a \sin B \approx 19.25808074$
$27 > 19.3$; 2 solutions

17. Since $65° < 90°$, consider Case I.
$b \sin A = 57 \sin 65°$
$b \sin A \approx 51.65954386$
$55 > 51.7$; 2 solutions

18. Since $150° \geq 90°$, consider Case II.
$6 \leq 8$; no solution

19. Since $58° < 90°$, consider Case I.
$b \sin A = 29 \sin 58°$
$b \sin A \approx 24.59339479$
$26 > 24.6$; 2 solutions
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{26}{\sin 58°} = \frac{29}{\sin B}$$
$$\sin B = \frac{29 \sin 58°}{26}$$
$$B = \sin^{-1}\left(\frac{29 \sin 58°}{26}\right)$$
$$B \approx 71.06720496$$
$180° - \propto \approx 180° - 71.1°$ or $108.9°$
Solution 1
$C \approx 180° - (58° + 71.1°)$ or $50.9°$
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{26}{\sin 58°} \approx \frac{c}{\sin 50.9°}$$
$$c \approx \frac{26 \sin 50.9°}{\sin 58°}$$
$$c \approx 23.80359004$$
Solution 2
$C \approx 180° - (58° + 108.9)°$ or $13.1°$
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{26}{\sin 58°} \approx \frac{c}{\sin 13.1°}$$
$$c \approx \frac{26 \sin 13.1°}{\sin 58°}$$
$$c \approx 6.931727606$$
$B = 71.1°$, $C = 50.9°$, $c = 23.8$; $B = 108.9°$, $C = 13.1°$, $c = 6.9$

20. Since $30° < 90°$, consider Case I.
$b \sin A = 8 \sin 30°$
$b \sin A = 4$
$4 = 4$; 1 solution

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{4}{\sin 30°} = \frac{8}{\sin B}$$
$$\sin B = \frac{8 \sin 30°}{4}$$
$$B = \sin^{-1}\left(\frac{8 \sin 30°}{4}\right)$$
$$B = 90$$

$C = 180° - (30° - 90°)$ or $60°$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{4}{\sin 30°} = \frac{c}{\sin 60°}$$
$$C = \frac{4 \sin 60°}{\sin 30°}$$
$$C \approx 6.92820323$$

$B = 90°$; $C = 60°$, $c = 6.9$

21. Since $70° < 90°$, consider Case I.
$a \sin C = 25 \sin 70°$
$a \sin C \approx 23.49231552$
$24 > 23.5$; 2 solutions

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$
$$\frac{24}{\sin 70°} = \frac{25}{\sin A}$$
$$\sin A = \frac{25 \sin 70°}{24}$$
$$A = \sin^{-1}\left(\frac{25 \sin 70°}{24}\right)$$
$$A \approx 78.1941432$$

$180° - \propto \approx 180° - 79.2°$ or $101.8°$

Solution 1

$B \approx 180° - (70° + 79.2°)$ or $31.8°$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{24}{\sin 70°} \approx \frac{b}{\sin 31.8°}$$
$$b \approx \frac{24 \sin 31.8°}{\sin 70°}$$
$$b \approx 13.46081025$$

Solution 2

$B \approx 180° - (70° + 101.8°)$ or $8.2°$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{24}{\sin 70°} \approx \frac{b}{\sin 8.2°}$$
$$b \approx \frac{24 \sin 8.2°}{\sin 70°}$$
$$b \approx 3.640196918$$

$A = 78.2°$, $B = 31.8°$, $b = 13.5$; $A = 101.8°$, $B = 8.2°$, $b = 3.6$

22. $C = 180° - (40° + 60°)$ or $80°$

$$\frac{c}{\sin C} = \frac{a}{\sin A} \qquad \frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{20}{\sin 80°} = \frac{a}{\sin 40°} \qquad \frac{20°}{\sin 80°} = \frac{b}{\sin 60°}$$
$$a = \frac{20 \sin 40°}{\sin 80°} \qquad b = \frac{20 \sin 60°}{\sin 80°}$$
$$a \approx 13.05407289 \qquad b \approx 17.58770483$$

$C = 80°$, $a = 13.1$, $b = 17.6$

23. Since $90° \geq 90°$; consider Case II.
$12 \leq 14$; no solution

24. Since $36° < 90°$, consider Case I.
$c \sin B = 30 \sin 36°$
$c \sin B \approx 17.63355757$
$19 > 17.6$; 2 solutions

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{19}{\sin 36°} = \frac{30}{\sin C}$$
$$\sin C = \frac{30 \sin 36°}{19}$$
$$C = \sin^{-1}\left(\frac{30 \sin 36°}{19}\right)$$
$$C \approx 68.1377773$$

$180° - \propto \approx 180° - 68.1°$ or $111.9°$

Solution 1

$A \approx 180° - (36° + 68.1°)$ or $75.9°$

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{19}{\sin 36°} \approx \frac{a}{\sin 75.9°}$$
$$a \approx \frac{19 \sin 75.9°}{\sin 36°}$$
$$a \approx 31.34565276$$

Solution 2

$A \approx 180° - (36° + 111.9°)$ or $32.1°$

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{19}{\sin 36°} \approx \frac{a}{\sin 32.1°}$$
$$a \approx \frac{19 \sin 32.1°}{\sin 36}$$
$$a \approx 17.1953669$$

$A = 75.9°$, $C = 68.1°$, $a = 31.3$; $A = 32.1°$, $C = 111.9°$, $a = 17.2$

25. Since $107.2° \geq 90°$, consider Case II.
$17.2 > 12.2$; 1 solution

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{17.2}{\sin 107.2°} = \frac{12.2}{\sin C}$$
$$\sin C = \frac{12.2 \sin 107.2°}{17.2}$$
$$C = \sin^{-1}\left(\frac{12.2 \sin 107.2°}{17.2}\right)$$
$$C \approx 42.65491459$$

$B \approx 180° - (107.2° + 42.7°)$ or $30.1°$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{17.2}{\sin 107.2°} \approx \frac{b}{\sin 30.1°}$$
$$b \approx \frac{17.2 \sin 30.1°}{\sin 107.2°}$$
$$b \approx 9.042067456$$

$B = 30.1°$, $C = 42.7°$, $b = 9.0$

26. Since $76° < 90°$, consider Case I.
$b \sin A = 20 \sin 76°$
$b \sin A \approx 19.40591453$
$5 < 19.4$; no solution

27. Since $47° < 90°$, consider Case I.
$16 \geq 10$; 1 solution

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$
$$\frac{16}{\sin 47°} = \frac{10}{\sin A}$$
$$\sin A = \frac{10 \sin 47°}{16}$$
$$A = \sin^{-1}\left(\frac{10 \sin 47°}{16}\right)$$
$$A \approx 27.19987995$$
$$B \approx 180° - (47° - 27.2) \text{ or } 105.8°$$
$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{16}{\sin 47°} = \frac{b}{\sin 105.8°}$$
$$b = \frac{16 \sin 105.8°}{\sin 47°}$$
$$b \approx 21{,}0506609$$

$A = 27.2°$, $B = 105.8°$, $b = 21.1$

28. Since $40° < 90°$, consider Case I.
$c \sin B = 60 \sin 40°$
$c \sin B \approx 38.56725658$
$42 > 38.6$; 2 solutions

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{42}{\sin 40°} = \frac{60}{\sin C}$$
$$\sin C = \frac{60 \sin 40°}{42}$$
$$C = \sin^{-1}\left(\frac{60 \sin 40°}{42}\right)$$
$$C \approx 66.67417652$$
$$180° - \propto \approx 180° - 66.7° \text{ or } 113.3°$$

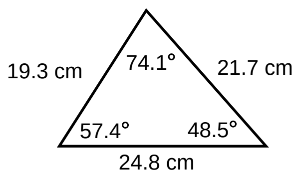
Solution 1
$$A \approx 180° - (40° + 66.7°) \text{ or } 73.3°$$
$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{42}{\sin 40°} \approx \frac{a}{\sin 73.3°}$$
$$a \approx \frac{42 \sin 73.3°}{\sin 40°}$$
$$a \approx 62.58450564$$

Solution 2
$$A \approx 180° - (40° + 113.3°) \text{ or } 26.7°$$
$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{42}{\sin 40°} = \frac{a}{\sin 26.7°}$$
$$a = \frac{42 \sin 26.7°}{\sin 40°}$$
$$a \approx 29.33237132$$

$A = 73.3°$, $C = 66.7°$, $a = 62.6$; $A = 26.7°$, $C = 113.3°$, $a = 29.3$

29. Since $125.3° \geq 90°$, consider Case II.
$32 \leq 40$; no solution

30. $$\frac{21.7}{\sin 57.4°} = \frac{19.3}{\sin x}$$
$$\sin x = \frac{19.3 \sin 57.4°}{21.7}$$
$$x = \sin^{-1}\left(\frac{19.3 \sin 57.4°}{21.7}\right)$$
$$x \approx 48.52786934$$
$$\theta \approx 180 - (57.4° + 48.5°) \text{ or } 74.1°$$
$$\frac{21.7}{\sin 57.4°} \approx \frac{y}{\sin 74.1°}$$
$$y \approx \frac{21.7 \sin 74.1°}{\sin 57.4°}$$
$$y \approx 24.76922417$$



31. $$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{15}{\sin 29°} = \frac{20}{\sin B}$$
$$\sin B = \frac{20 \sin 29°}{15}$$
$$B = \sin^{-1}\left(\frac{20 \sin 29°}{15}\right)$$
$$B \approx 40.27168721$$
$$180° - \propto \approx 180° - 40.3° \text{ or } 139.7°$$

Solution 1
$$C \approx 180° - (29° - 40.3°) \text{ or } 110.7°$$
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{15}{\sin 29°} \approx \frac{c}{\sin 110.7°}$$
$$c \approx \frac{15 \sin 110.7°}{\sin 29°}$$
$$c \approx 28.93721187$$
$$\text{Perimeter} = a + b + c$$
$$\approx 15 + 20 + 28.9 \quad \text{or about } 63.9 \text{ units}$$

Solution 2
$$C \approx 180° - (29° - 139.7°) \text{ or } 11.3°$$
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{15}{\sin 29°} \approx \frac{c}{\sin 11.3°}$$
$$c \approx \frac{15 \sin 11.3°}{\sin 29°}$$
$$c \approx 6.047576406$$
$$\text{Perimeter} = a + b + c$$
$$\approx 15 + 20 + 6.0 \text{ or about } 41.0 \text{ units}$$

32. $$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{13}{\sin 55°} = \frac{15}{\sin A}$$
$$\sin A = \frac{15 \sin 55°}{13}$$
$$A = \sin^{-1}\left(\frac{15 \sin 55°}{13}\right)$$
$$A \approx 70.93970395$$
$$180° - \propto \approx 180° - 70.9° \text{ or } 109.1°$$

Solution 1
$$C \approx 180° - (70.9° + 55°) \text{ or } 54.1°$$
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{13}{\sin 55°} \approx \frac{c}{\sin 54.1°}$$
$$c \approx \frac{13 \sin 54.1°}{\sin 55°}$$
$$c \approx 12.8489656$$
$$\text{Perimeter} = a + b + c$$
$$\approx 15 + 13 + 12.8 \text{ or about } 40.8$$

Solution 2
$$c \approx 180° - (109.1° + 55°) \text{ or } 15.9°$$
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{13}{\sin 55°} \approx \frac{c}{\sin 15.9°}$$
$$c \approx \frac{13 \sin 15.9°}{\sin 55°}$$
$$c \approx 4.35832749$$
$$\text{Perimeter} = a + b + c$$
$$\approx 15 + 13 + 4.4 \text{ or about } 32.4$$

$A \approx 70.9°$, $B = 55°$, $C \approx 54.1°$

33. side opposite 37° = 15 + 18 or 33
side between θ and 37° = 15 + 22 or 37
Let x = the measure of the third angle.

$\frac{33}{\sin 37°} = \frac{37}{\sin x}$

$\sin x = \frac{37 \sin 37°}{33}$

$x = \sin^{-1}\left(\frac{37 \sin 37°}{33}\right)$

$x \approx 42.43569405$

$\theta \approx 180 - (37° + 42.4°)$ or about 100.6°

34a. $a < b \sin A$
$a < 14 \sin 30°$
$a < 7$

34b. $a = b \sin A$
$a = 14 \sin 30°$
$a = 7$ or $a \geq 14$

34c. $a > b \sin A$
$a > 14 \sin 30°$
$a > 7$ and $a < 14$
$7 < a < 14$

35. $\frac{184.5}{\sin 59°} = \frac{140}{\sin x}$

$\sin x = \frac{140 \sin 59°}{184.5}$

$x = \sin^{-1}\left(\frac{140 \sin 59°}{184.5}\right)$

$x \approx 40.57365664$

$\theta \approx 180° - (59° + 40.6°)$ or about 80.4°
$90° - 80.4° \approx 9.6°$

36a. $12° < 90°$ and $316 > 450 \sin 12°$; 2 solutions

$\frac{316}{\sin 12°} = \frac{450}{\sin \theta}$

$\sin \theta = \frac{450 \sin 12°}{316}$

$\theta = \sin^{-1}\left(\frac{450 \sin 12°}{316}\right)$

$\theta \approx 17.22211674$

$180° - \propto \approx 180° - 17.2°$ or 162.8°
turn angle $\approx 180° - 162.8°$ or 17.2°
about 17.2° east of north

36b. $\theta \approx 180° - (162.8° + 12°)$ or about 5.2°

$\frac{x}{\sin 5.2°} = \frac{316}{\sin 12°}$

$x = \frac{316 \sin 5.2°}{\sin 12°}$

$x \approx 138.3094714$

$d = rt$

$138.3 \approx 23t$

$6.013455278 \approx t$; about 6 hr

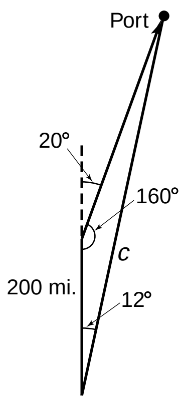
36c. $180° - 20° = 160°$
$180° - (160° + 12°) = 8°$

$\frac{200}{\sin 8°} = \frac{c}{\sin 160°}$

$c = \frac{200 \sin 160°}{\sin 8°}$

$c \approx 491.5032301$

Since $491.5 \neq 450$, the ship will not reach port.



37. Distance from satellite to center of earth is 3960 + 1240 or 5200 miles.
angle across from 5200 mi side = 45° + 90° or 135°

$\frac{5200}{\sin 135°} = \frac{3960}{\sin x}$

$\sin x = \frac{3960 \sin 135°}{5200}$

$x = \sin^{-1}\left(\frac{3960 \sin 135°}{5200}\right)$

$x \approx 32.58083835$

$\theta \approx 180° - (135° + 32.6°)$ or about 21.4°

$\frac{21.4°}{360°}$ (2 hours) ≈ 0.0689953425 hours or about 4.1 minutes

38. P turns 20(360°) or 7200° every second which equals 72° every 0.01 second.

$\frac{PQ}{\sin O} = \frac{OP}{\sin Q}$

$\frac{15}{\sin 72°} = \frac{5}{\sin Q}$

$\sin Q = \frac{5 \sin 72°}{15}$

$Q = \sin^{-1}\left(\frac{5 \sin 72°}{15}\right)$

$Q \approx 18.48273235$

$m\angle P \approx 180° - (72° + 18.5°)$ or about 89.5°

$\frac{QO}{\sin P} = \frac{PQ}{\sin O}$

$\frac{QO}{\sin 89.5°} \approx \frac{15}{\sin 72°}$

$QO \approx \frac{15 \sin 89.5°}{\sin 72°}$

$QO \approx 15.77133282$

$QO - 5 \approx 15.8 - 5$ or about 10.8 cm

39a. $b < c \sin B$
$12 < 17 \sin B$
$\frac{12}{17} < \sin B$
$\sin^{-1} \frac{12}{17} < B$
$44.90087216 < B$
$B > 44.9°$

39b. $b = c \sin B$
$12 = 17 \sin B$
$\frac{12}{17} = \sin B$
$\sin^{-1} \frac{12}{17} = B$
$44.90087216 \approx B$
$B \approx 44.9°$

39c. $b > c \sin B$
$12 > 17 \sin B$
$\frac{12}{17} > \sin B$
$\sin^{-1} \frac{12}{17} > B$
$44.90087216 > B$
$B < 44.9°$

40. Area of rhombus = 2(Area of triangle)
triangle:

$K = \frac{1}{2}bc \sin A$

$K = \frac{1}{2}(24)(24) \sin 32°$

$K \approx 152.6167481$

rhombus:
$A \approx 2(152.6)$ or about 305.2 in^2

41. $\tan 22° = \frac{75}{x}$

$x = \frac{75}{\tan 22°}$

$x \approx 185.6$ m

42. $3; \frac{1}{2}$

$$\begin{array}{r|rrrr} \frac{1}{2} & 4 & -4 & 13 & -6 \\ & & 2 & -1 & 6 \\ \hline & 4 & -2 & 12 & \multicolumn{1}{|r}{0} \end{array}$$

$4x^2 - 2x + 12 = 0$

$x = \frac{2 \pm \sqrt{(-2)^2 - 4(4)(12)}}{2(4)}$

$x = \frac{2 \pm \sqrt{-188}}{8}$

$x = \frac{2 \pm 2i\sqrt{47}}{8}$

$x = \frac{1 \pm i\sqrt{47}}{4}$

43. no

$$\frac{\frac{3x}{x-1} + 1}{3\left(\frac{3x}{x-1}\right)} = \frac{\frac{3x}{x-1} + \frac{x-1}{x-1}}{\frac{9x}{x-1}} = \frac{\frac{4x-1}{x-1}}{\frac{9x}{x-1}} = \frac{4x-1}{9x} \neq x$$

44.

$5x - 2y = 9$ $\quad y = 3x - 1$

$5x - 2(3x - 1) = 9$ $\quad y = 3(-7) - 1$

$5x - 6x + 2 = 9$ $\quad y = -22$

$x = -7$

$(-7, -22)$

45. $-2x + 5y = 7$

$y = \frac{2}{5}x + \frac{7}{5}$

perpendicular slope: $-\frac{5}{2}$

$y - 4 = -\frac{5}{2}(x - (-6))$

$y - 4 = -\frac{5}{2}x - 15$

$2y - 8 = -5x - 30$

$5x + 2y = -22$

46. Perimeter of $XYZ = 4 + 8 + 9$ or 21

length of $\overline{AB} = \frac{1}{3}$ of perimeter

$= \frac{1}{3}(21)$ or 7

The answer is 7.

5-8 The Law of Cosines

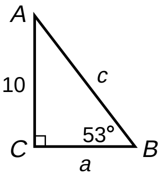
Pages 330–331 Check for Understanding

1. The Law of Cosines is needed to solve a triangle if the measures of all three sides or the measures of two sides and the included angle are given.

2. Sample answer: 1 in., 2 in., 4 in.

3. If the included angle measures 90°, the equation becomes $c^2 = a^2 + b^2 - 2ab \cos C$. Since $\cos 90° = 0$, $c^2 = a^2 + b^2 - 2ab(0)$ or $c^2 = a^2 + b^2$.

4. Sample answers:



$A = 180° - (90° + 53°)$ or $37°$

$\sin B = \frac{b}{c}$

$\sin 53° = \frac{10}{c}$

$c = \frac{10}{\sin 53°}$

$c \approx 12.52135658$

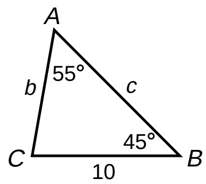
$\tan B = \frac{b}{a}$

$\tan 53° = \frac{10}{a}$

$a = \frac{10}{\tan 53°}$

$a \approx 7.535540501$

$A = 37°, a \approx 7.5, c \approx 12.5$



$C = 180° - (5.5° + 45°)$ or $80°$

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{b}{\sin 45°} = \frac{10}{\sin 55°}$

$b = \frac{10 \sin 45°}{\sin 55°}$

$b \approx 8.6321799$

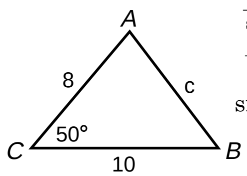
$\frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{c}{\sin 80°} = \frac{10}{\sin 55°}$

$c = \frac{10 \sin 80°}{\sin 55°}$

$c \approx 12.0222828$

$C = 80°, b \approx 8.6, c \approx 12.0$



$c^2 = a^2 + b^2 - 2ab \cos C$

$c^2 = 10^2 + 8^2 - 2(10)(8) \cos 50°$

$c^2 \approx 61.15398245$

$c \approx 7.820101179$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{10}{\sin A} \approx \frac{7.8}{\sin 50°}$

$\sin A \approx \frac{10 \sin 50°}{7.8}$

$A \approx \sin^{-1}\left(\frac{10 \sin 50°}{7.8}\right)$

$A \approx 78.4024367$

$B \approx 180° - (78.4° + 50°)$ or $51.6°$

$A \approx 78.4°, B \approx 51.6°, c \approx 7.8$

5. $a^2 = b^2 + c^2 - 2bc \cos A$

$32^2 = 38^2 + 46^2 - 2(38)(46) \cos A$

$\frac{32^2 - 38^2 - 46^2}{-2(38)(46)} = \cos A$

$\cos^{-1}\left(\frac{32^2 - 38^2 - 46^2}{-2(38)(46)}\right) = A$

$43.49782861 \approx A$

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{32}{\sin 43.5°} \approx \frac{38}{\sin B}$

$\sin B \approx \frac{38 \sin 43.5°}{32}$

$B \approx \sin^{-1}\left(\frac{38 \sin 43.5°}{32}\right)$

$B \approx 54.8$

$C \approx 180° - (43.5° + 54.8°)$ or $81.7°$

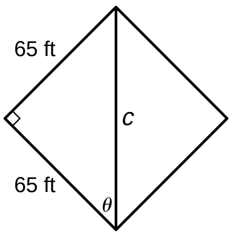
$A = 43.5°, B = 54.8°, C = 81.7°$

6. $c^2 = a^2 + b^2 - 2ab\cos C$
$c^2 = 25^2 + 30^2 - 2(25)(30)\cos 160°$
$c^2 \approx 2934.538931$
$c \approx 54.1713848$
$\frac{c}{\sin C} = \frac{a}{\sin A}$
$\frac{54.2}{\sin 160°} \approx \frac{25}{\sin A}$
$\sin A \approx \frac{25 \sin 160°}{54.2}$
$A \approx \sin^{-1}\left(\frac{25 \sin 160°}{54.2}\right)$
$A \approx 9.1$
$B \approx 180° - (9.1° + 160°)$ or $10.9°$
$A = 9.1°, B = 10.9°, c = 54.2$

7. The angle with greatest measure is across from the longest side.
$21^2 = 18^2 + 14^2 - 2(18)(14)\cos\theta$
$\frac{21^2 - 18^2 - 14^2}{-2(18)(14)} = \cos\theta$
$\cos^{-1}\left(\frac{21^2 - 18^2 - 14^2}{-2(18)(14)}\right) = \theta$
$81.0 \approx \theta$
about $81.0°$

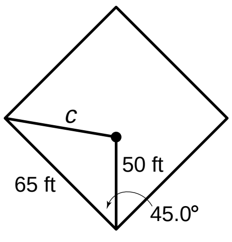
8. $s = \frac{1}{2}(2 + 7 + 8) = 8.5$
$K = \sqrt{8.5(8.5 - 2)(8.5 - 7)(8.5 - 8)}$
≈ 6.4 units^2

9. $s = \frac{1}{2}(25 + 13 + 17) = 27.5$
$K = \sqrt{27.5(27.5 - 25)(27.5 - 13)(27.5 - 17)}$
≈ 102.3 units^2

10.


$a^2 + b^2 = c^2$
$65^2 + 65^2 = c^2$
$8450 = c^2$
$91.92388155 \approx c$

$a^2 = b^2 + c^2 - 2bc\cos\theta$
$65^2 \approx 65^2 + 91.9^2 - 2(65)(91.9)\cos\theta$
$\frac{65^2 - 65^2 - 91.9^2}{-2(65)(91.9)} \approx \cos\theta$
$\cos^{-1}\left(\frac{65^2 - 65^2 - 91.9^2}{-2(65)(91.9)}\right) \approx \theta$
$45.01488334 \approx \theta$



$c^2 = a^2 + b^2 - 2ab\cos C$
$c^2 \approx 65^2 + 50^2 - 2(65)(50)\cos 45.0°$
$c^2 \approx 2128.805922$
$c \approx 46.1$ ft

Pages 331–332 Exercises

11. $a^2 = b^2 + c^2 - 2bc\cos A$
$a^2 = 7^2 + 10^2 - 2(7)(10)\cos 51°$
$a^2 \approx 60.89514525$
$a \approx 7.803534151$
$\frac{a}{\sin A} = \frac{b}{\sin B}$
$\frac{7.8}{\sin 51°} \approx \frac{7}{\sin B}$
$\sin B \approx \frac{7 \sin 51°}{7.8}$
$B \approx \sin^{-1}\left(\frac{7 \sin 51°}{7.8}\right)$
$B \approx 44.22186872$
$C \approx 180° - (51° + 44.2°)$ or $84.8°$
$B = 44.2°, C = 84.8°, a = 7.8$

12. $c^2 = a^2 + b^2 - 2ab\cos C$
$7^2 = 5^2 + 6^2 - 2(5)(6)\cos C$
$\frac{7^2 - 5^2 - 6^2}{-2(5)(6)} = \cos C$
$\cos^{-1}\left(\frac{7^2 - 5^2 - 6^2}{-2(5)(6)}\right) = C$
$78.46304097 \approx C$
$\frac{a}{\sin A} = \frac{c}{\sin C}$
$\frac{5}{\sin A} \approx \frac{7}{\sin 78.5°}$
$\sin A \approx \frac{5 \sin 78.5°}{7}$
$A \approx \sin^{-1}\left(\frac{5 \sin 78.5°}{7}\right)$
$A \approx 44.42268919$
$B \approx 180° - (44.4° + 78.5°)$ or $57.1°$
$A = 44.4°, B = 57.1°, C = 78.5°$

13. $c^2 = a^2 + b^2 - 2ab\cos C$
$7^2 = 4^2 + 5^2 - 2(4)(5)\cos C$
$\frac{7^2 - 4^2 - 5^2}{-2(4)(5)} = \cos C$
$\cos^{-1}\left(\frac{7^2 - 4^2 - 5^2}{-2(4)(5)}\right) = C$
$101.536959 \approx C$
$\frac{a}{\sin A} = \frac{c}{\sin C}$
$\frac{4}{\sin A} \approx \frac{7}{\sin 101.5°}$
$\sin A \approx \frac{4 \sin 101.5°}{7}$
$A \approx \sin^{-1}\left(\frac{4 \sin 101.5°}{7}\right)$
$A \approx 34.05282227$
$B \approx 180° - (34.1° + 101.5°)$ or $44.4°$
$A = 34.1°, B = 44.4°, C = 101.5°$

14. $b^2 = a^2 + c^2 - 2ac\cos B$
$b^2 = 16^2 + 12^2 - 2(16)(12)\cos 63°$
$b^2 \approx 225.6676481$
$b \approx 15.02223845$
$\frac{a}{\sin A} = \frac{b}{\sin B}$
$\frac{16}{\sin A} \approx \frac{15.0}{\sin 63°}$
$\sin A \approx \frac{16 \sin 63°}{15.0}$
$A \approx \sin^{-1}\left(\frac{16 \sin 63°}{15.0}\right)$
$A \approx 71.62084388$
$C \approx 180° - (71.6° + 63°)$ or $45.4°$
$A = 71.6°, C = 45.4°, b = 15.0$

15. $b^2 = a^2 + c^2 - 2ac\cos B$

$13.7^2 = 11.4^2 + 12.2^2 - 2(11.4)(12.2)\cos B$

$\frac{13.7^2 - 11.4^2 - 12.2^2}{-2(11.4)(12.2)} = \cos B$

$\cos^{-1}\left(\frac{13.7^2 - 11.4^2 - 12.2^2}{-2(11.4)(12.2)}\right) = B$

$70.8801474 \approx B$

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{11.4}{\sin A} \approx \frac{13.7}{\sin 70.9°}$

$\sin A \approx \frac{11.4 \sin 70.9°}{13.7}$

$A \approx \sin^{-1}\left(\frac{11.4 \sin 70.9°}{13.7}\right)$

$A \approx 51.84180107$

$C \approx 180° - (51.8° + 70.9°)$ or $57.3°$

$A = 51.8°, B = 70.9°, C = 57.3°$

16. $c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 21.5^2 + 13^2 - 2(21.5)(13)\cos 79.3°$

$c^2 \approx 527.462362$

$c \approx 22.96654876$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{21.5}{\sin A} \approx \frac{23.0}{\sin 79.3°}$

$\sin A \approx \frac{21.5 \sin 79.3°}{23.0}$

$A \approx \sin^{-1}\left(\frac{21.5 \sin 79.3°}{23.0}\right)$

$A \approx 66.90667662$

$B \approx 180° - (66.9° - 79.3°)$ or $33.8°$

$A = 66.9°, B = 33.8°, c = 23.0$

17. $14.9^2 = 23.8^2 + 36.9^2 - 2(23.8)(36.9)\cos\theta$

$\frac{14.9^2 - 23.8^2 - 36.9^2}{-2(23.8)(36.9)} = \cos\theta$

$\cos^{-1}\left(\frac{14.9^2 - 23.8^2 - 36.9^2}{-2(23.8)(36.9)}\right) = \theta$

$13.75878964 \approx \theta$

about 13.8°

18. ${d_1}^2 = 40^2 + 60^2 - 2(40)(60)\cos 132°$

${d_1}^2 \approx 8411.826911$

$d_1 \approx 91.71601229$

$180° - 132° = 48°$

${d_2}^2 = 40^2 + 60^2 - 2(40)(60)\cos 48°$

${d_2}^2 \approx 1988.173089$

$d_2 \approx 44.58893461$

about 91.7 cm and 44.6 cm

19. $s = \frac{1}{2}(4 + 6 + 8) = 9$

$K = \sqrt{9(9-4)(9-6)(9-8)}$

≈ 11.6 units^2

20. $s = \frac{1}{2}(17 + 13 + 19) = 24.5$

$K = \sqrt{24.5(24.5-17)(24.5-13)(24.5-19)}$

≈ 107.8 units^2

21. $s = \frac{1}{2}(20 + 30 + 40) = 45$

$K = \sqrt{45(45-20)(45-30)(45-40)}$

≈ 290.5 units^2

22. $s = \frac{1}{2}(33 + 51 + 42) = 63$

$K = \sqrt{63(63-33)(63-51)(63-42)}$

≈ 690.1 units^2

23. $s = \frac{1}{2}(174 + 138 + 188) = 250$

$K = \sqrt{250 \cdot 76 \cdot 112 \cdot 62}$

$\approx 11{,}486.3$ units^2

24. $s = \frac{1}{2}(11.5 + 13.7 + 12.2) = 18.7$

$K = \sqrt{187(18.7 - 11.5)(18.7 - 13.7)(18.7 - 12.2)}$

≈ 66.1 units^2

25a. $d^2 = 30^2 + 48^2 - 2(30)(48)\cos 120°$

$d^2 = 4644$

$d \approx 68.1$ in.

25b. Area of parallelogram = 2(Area of triangle)

$K = \frac{1}{2}(30)(48)\sin 120°$

$K \approx 623.5382907$

$2K \approx 2(623.5382907)$ or about 1247.1 in^2

26a. $s = \frac{1}{2}(15 + 15 + 24.6) = 27.3$

$K = \sqrt{27.3(27.3-15)(27.3-15)(27.3-24.6)}$

≈ 105.6

Area of rhombus $\approx 2(105.6) \approx 211.2$ cm^2

26b. $24.6^2 = 15^2 + 15^2 - 2(15)(15)\cos\theta$

$\frac{24.6^2 - 15^2 - 15^2}{-2(15)(15)} = \cos\theta$

$\cos^{-1}\left(\frac{24.6^2 - 15^2 - 15^2}{-2(15)(15)}\right) = \theta$

$110.1695875 \approx \theta$

$180° - 110.2° = 69.8°$

about 110.2°, 69.8°, 110.2°, 69.8°

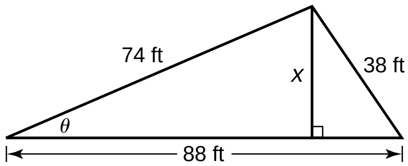
27. The angle opposite the missing side = 45°.

$x^2 = 400^2 + 90^2 - 2(400)(90)\cos 45°$

$x^2 \approx 117{,}188.3118$

$x \approx 342.3$ ft

28.



$38^2 = 74^2 + 88^2 - 2(74)(88)\cos\theta$

$\frac{38^2 - 74^2 - 88^2}{-2(74)(88)} = \cos\theta$

$\cos^{-1}\left(\frac{38^2 - 74^2 - 88^2}{-2(74)(88)}\right) = \theta$

$25.28734695 \approx \theta$

$\sin\theta = \frac{\text{side opposite}}{\text{hypotenuse}}$

$\sin 25.3° \approx \frac{x}{74}$

$31.60970664 \approx x$

about 31.6 ft

29a. $x^2 = 100^2 + 220^2 - 2(100)(220)\cos 10°$

$x^2 \approx 15{,}068.45887$

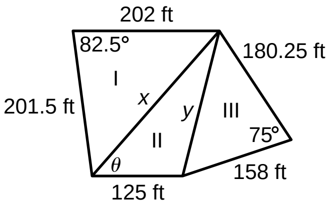
$x \approx 122.7536511$

about 122.8 mi

29b. $(100 + 122.7536511) - 220 \approx 2.7536511$

about 2.8 mi

30.



I: $K = \frac{1}{2}(201.5)(202)\sin 82.5°$

$K \approx 20{,}177.3901$

II: $x^2 = 201.5^2 + 202^2 - 2(201.5)(202)\cos 82.5°$

$x^2 \approx 70{,}780.6348$

$x \approx 266.046302$

$y^2 = 158^2 + 180.25^2 - 2(158)(180.25)\cos 75°$

$y^2 \approx 42.711.98851$

$y \approx 206.6687894$

$206.7^2 \approx 266.0^2 + 125^2 - 2(266.0)(125)\cos\theta$

$\frac{206.7^2 - 266.0^2 - 125^2}{-2(266.0)(125)} \approx \cos\theta$

$\cos^{-1}\left(\frac{206.7^2 - 266.0^2 - 125^2}{-2(266.0)(125)}\right) \approx \theta$

$48.93361962 \approx \theta$

$K \approx \frac{1}{2}(266.0)(125)\sin 48.9°$

$K \approx 12{,}536.58384$

III: $K = \frac{1}{2}(180.25)(158)\sin 75°$

$K \approx 13{,}754.54228$

Area of pentagon = I + II + III

$\approx 20{,}177.4 + 12{,}536.6 + 13{,}754.5$

$\approx 46{,}468.5$ ft

31. I: $24^2 = 35^2 + 40^2 - 2(35)(40)\cos\theta$

$\frac{24^2 - 35^2 - 40^2}{-2(35)(40)} = \cos\theta$

$\cos^{-1}\left(\frac{24^2 - 35^2 - 40^2}{-2(35)(40)}\right) = \theta$

$36.56185036 \approx \theta$

II: $24^2 = 30^2 + 20^2 - 2(30)(20)\cos\theta$

$\frac{24^2 - 30^2 - 20^2}{-2(30)(20)} = \cos\theta$

$\cos^{-1}\left(\frac{24^2 - 30^2 - 20^2}{-2(30)(20)}\right) = \theta$

$52.89099505 \approx \theta$

the player 30 ft and 20 ft from the posts

32a. $\sin 6° = \frac{20{,}000}{x}$

$x = \frac{20{,}000}{\sin 6°}$

$x \approx 191{,}335.4$ ft

32b. $\sin 3° = \frac{15{,}000}{y}$

$y = \frac{15{,}000}{\sin 3°}$

$y \approx 286{,}609.8$ ft

32c. $6° - 3° = 3°$

$d^2 \approx 191{,}335.4^2 + 286{,}609.8^2 - 2(191{,}335.4)(286{,}609.8)\cos 3°$

$d^2 \approx 9{,}227{,}519{,}077$

$d \approx 96{,}060.0$ ft

33. Since $63.2° < 90°$, consider Case I.

$b \sin A = 18 \sin 63.2°$

$b \sin A \approx 16.06654473$

$17 > 16.1$; 2 solutions

34. $\tan\theta = \frac{\text{side opposite}}{\text{side adjacent}}$

$\tan\theta = \frac{570}{700}$

$\theta = \tan^{-1}\frac{570}{700}$

$\theta \approx 39.2°$

35. $-775° + 2(360°) = -55°$

reference angle = $|-55°|$ or $55°$

36.

$$\begin{array}{r|rrrr} 3 & 1 & -7 & -k & 6 \\ & & 3 & -12 & -36 - 3k \\ \hline & 1 & -4 & -12 - k & -30 - 3k \end{array}$$

$-30 - 3k = 0$

$k = -10$

37. $m = \frac{5t - t}{5t - 2t}$

$m = \frac{4t}{3t}$ or $\frac{4}{3}$

38. $\left(\frac{2x^2}{y}\right)^3 = \frac{2^3x^6}{y^3}$

$= \frac{8x^6}{y^3}$

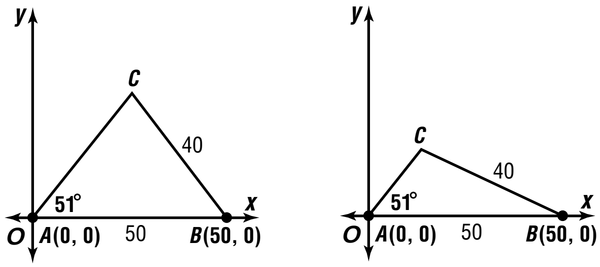
The correct choice is A.

5-8B Graphing Calculator Exploration: Solving Triangles

Page 334

1. AB ≈ 12.1, $B \approx 25.5°$, $C \approx 119.5°$

2.



Find y using $\overline{AC}$.

$y = (\tan 51°)x$

Find y using $\overline{BC}$.

$\frac{40}{\sin 51°} = \frac{50}{\sin C}$

$\sin C = \frac{50 \sin 51°}{40}$

$C = \sin^{-1}\left(\frac{50 \sin 51°}{40}\right)$

$C \approx 76.27180414$

$B \approx 80 - 51 - 76.27180414$

≈ 52.72819586

$\tan(180 - 52.72819586) \approx \frac{y}{x - 50}$

$(x - 50)\tan(127.2718041) \approx y$

Set the two values of y equal to each other.

$(\tan 51°)x \approx (x - 50)$, $\tan 127.2718041°$

$(\tan 51°)x \approx x(\tan 127.2718041° - 50(\tan 127.2718041°)$

$x \approx \frac{-50(\tan 127.2718041°)}{\tan 51° - \tan 127.271804°}$

$x \approx 25.77612538$

$y \approx (\tan 51°)(25.77612538) \approx 31.83086394$

C could also equal $180 - 76.27180414$ or $103.728195°$

$B \approx 180 - 51 - 103.7281959°$

≈ 25.2718041

$\tan(180 - 25.2718041) \approx \frac{y}{x - 50}$

$(x - 50) \tan 154.7281959° \approx y$

Set the two values of y equal to each other.

$(\tan 51°)\, x \approx (x - 50)\tan 154.7281959°$

$(\tan 51°)\, x \approx x(\tan 154.7281959°) - 50(\tan 154.7281959°)$

$x \approx \frac{-50(\tan 154.7281959°)}{\tan 51° - \tan 154.7281959°}$

$x \approx 13.82829048$

$y \approx (\tan 51°)(13.82828048)$

≈ 17.07651659

$B \approx 52.7°$, $C \approx 76.3°$, $b \approx 40.9$; $B \approx 25.3°$, $C \approx 103.7°$, $b \approx 220$

3. Law of Cosines

4. Sample answer: put vertex A at the origin and vertex C at (3, 0).

Chapter 5 Study Guide and Assessment

Page 335 Understanding and Using the Vocabulary

1. false; depression
2. false; arcosine
3. true
4. false; adjacent to
5. true
6. false; coterminal
7. true
8. false; Law of Cosines
9. false; terminal side
10. true

Pages 336–338 Skills and Concepts

11. $57.15° = 57° + (0.15 \cdot 60)'$
$= 57° + 9'$

$57°\ 9'$

12. $-17.125° = -(17° + (0.125 \cdot 60)')$
$= -(17° + 7.5')$
$= -(17° + 7' + (0.5 \cdot 60)'')$
$= -(17° + 7' + 30'')$

$-17°\ 7'\ 30''$

13. $\frac{860°}{360°} \approx 2.39$

$\propto + 360(2)° = 860°$
$\propto + 720° = 860°$
$\propto = 140°$; II

14. $\frac{1146°}{360°} \approx 3.18$

$\propto + 360(3)° = 1146°$
$\propto + 1080° = 1146°$
$\propto = 66°$; I

15. $\frac{-156°}{360°} \approx -0.43$

$\propto + 360(-1)° = -156°$
$\propto - 360° = -156°$
$\propto = 204°$; III

16. $\frac{998°}{360°} \approx 2.77$

$\propto + 360(2)° = 998°$
$\propto + 720° = 998°$
$\propto = 278°$; IV

17. $\frac{-300°}{360°} \approx -0.83$

$\propto + 360(-1)° = -300°$
$\propto - 360° = -300°$
$\propto = 60°$; I

18. $\frac{1072°}{360°} \approx 2.98$

$\propto + 360(2)° = 1072°$
$\propto + 720° = 1072°$
$\propto = 352°$; IV

19. $\frac{654°}{360°} \approx 1.82$

$\propto + 360(1)° = 654°$
$\propto + 360° = 654°$
$\propto = 294°$; IV

20. $\frac{-832°}{360°} \approx -2.31$

$\propto + 360(-2)° = -832°$
$\propto - 720° = -832°$
$\propto = -112°$

$360° - 112° = 248°$; III

21. $-284°$ has terminal side in first quadrant.
$360° - 284° = 76°$

22. $\frac{592°}{360°} \approx 1.64$

$\propto + 360(1)° = 592°$
$\propto + 360° = 592°$
$\propto = 232°$

terminal side in third quadrant

$232° - 180° = 52°$

23. $(BC)^2 + (AC)^2 = (AB)^2$
$15^2 + 9^2 = (AB)^2$
$306 = (AB)^2$
$\sqrt{306} = AB$
$3\sqrt{34} = AB$

$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}}$ $\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}}$

$\sin A = \frac{15}{3\sqrt{34}}$ or $\frac{5\sqrt{34}}{34}$ $\cos A = \frac{9}{3\sqrt{34}}$ or $\frac{3\sqrt{34}}{34}$

$\tan A = \frac{\text{side opposite}}{\text{side adjacent}}$

$\tan A = \frac{15}{9}$ or $\frac{5}{3}$

24. $(PM)^2 + (PN)^2 = (MN)^2$

$8^2 + 12^2 = (MN)^2$

$208 = (MN)^2$

$\sqrt{208} = MN$

$4\sqrt{13} = MN$

$\sin M = \frac{\text{opposite side}}{\text{hypotenuse}}$ $\quad$ $\cos M = \frac{\text{adjacent side}}{\text{hypotenuse}}$

$\sin M = \frac{12}{4\sqrt{13}}$ or $\frac{3\sqrt{13}}{13}$ $\quad$ $\cos M = \frac{8}{4\sqrt{13}}$ or $\frac{2\sqrt{13}}{13}$

$\tan M = \frac{\text{side opposite}}{\text{side adjacent}}$ $\quad$ $\csc M = \frac{\text{hypotenuse}}{\text{side opposite}}$

$\tan M = \frac{12}{8}$ or $\frac{3}{2}$ $\quad$ $\csc M = \frac{4\sqrt{13}}{12}$ or $\frac{\sqrt{13}}{3}$

$\sec M = \frac{\text{hypotenuse}}{\text{side adjacent}}$ $\quad$ $\cot M = \frac{\text{side adjacent}}{\text{side opposite}}$

$\sec M = \frac{4\sqrt{13}}{8}$ or $\frac{\sqrt{13}}{2}$ $\quad$ $\cot M = \frac{8}{12}$ or $\frac{2}{3}$

25. $(MP)^2 + (PN)^2 = (MN)^2$

$(MP)^2 + 10^2 = 12^2$

$(MP)^2 = 44$

$MP = \sqrt{44}$ or $2\sqrt{11}$

$\sin M = \frac{\text{opposite side}}{\text{hypotenuse}}$ $\quad$ $\cos M = \frac{\text{adjacent side}}{\text{hypotenuse}}$

$\sin M = \frac{10}{12}$ or $\frac{5}{6}$ $\quad$ $\cos M = \frac{2\sqrt{11}}{12}$ or $\frac{\sqrt{11}}{6}$

$\tan M = \frac{\text{side opposite}}{\text{side adjacent}}$ $\quad$ $\csc M = \frac{\text{hypotenuse}}{\text{side opposite}}$

$\tan M = \frac{10}{2\sqrt{11}}$ or $\frac{5\sqrt{11}}{11}$ $\quad$ $\csc M = \frac{12}{10}$ or $\frac{6}{5}$

$\sec M = \frac{\text{hypotenuse}}{\text{side adjacent}}$ $\quad$ $\cot M = \frac{\text{side adjacent}}{\text{side opposite}}$

$\sec M = \frac{12}{2\sqrt{11}}$ or $\frac{6\sqrt{11}}{11}$ $\quad$ $\cot M = \frac{2\sqrt{11}}{10}$ or $\frac{\sqrt{11}}{5}$

26. $\sec \theta = \frac{1}{\cos \theta}$

$\cos \theta = \frac{1}{\sec\theta}$

$\cos \theta = \frac{1}{\frac{7}{5}}$ or $\frac{5}{7}$

27. $r = \sqrt{x^2 + y^2}$

$r = \sqrt{3^2 + 3^2}$

$r = \sqrt{18}$ or $3\sqrt{2}$

$\sin \theta = \frac{y}{r}$ $\quad$ $\cos \theta = \frac{x}{r}$ $\quad$ $\tan \theta = \frac{y}{x}$

$\sin \theta = \frac{3}{3\sqrt{2}}$ $\quad$ $\cos \theta = \frac{3}{3\sqrt{2}}$ $\quad$ $\tan \theta = \frac{3}{3}$ or 1

$\sin \theta = \frac{\sqrt{2}}{2}$ $\quad$ $\cos \theta = \frac{\sqrt{2}}{2}$

$\csc \theta = \frac{r}{y}$ $\quad$ $\sec \theta = \frac{r}{x}$

$\csc \theta = \frac{3\sqrt{2}}{3}$ or $\sqrt{2}$ $\quad$ $\sec \theta = \frac{3\sqrt{2}}{3}$ or $\sqrt{2}$

$\cot \theta = \frac{x}{y}$

$\cot \theta = \frac{3}{3}$ or 1

28. $r = \sqrt{x^2 + y^2}$

$r = \sqrt{(-5)^2 + 12^2}$

$r = \sqrt{169}$ or 13

$\sin \theta = \frac{y}{r}$ $\quad$ $\cos \theta = \frac{x}{r}$ $\quad$ $\tan \theta = \frac{y}{x}$

$\sin \theta = \frac{12}{13}$ $\quad$ $\cos \theta = \frac{-5}{13}$ or $-\frac{5}{13}$ $\quad$ $\tan \theta = \frac{12}{-5}$ or $-\frac{12}{5}$

$\csc \theta = \frac{r}{y}$ $\quad$ $\sec \theta = \frac{r}{x}$ $\quad$ $\cot \theta = \frac{x}{y}$

$\csc \theta = \frac{13}{12}$ $\quad$ $\sec \theta = \frac{13}{-5}$ or $-\frac{13}{5}$ $\quad$ $\cot \theta = \frac{-5}{12}$ or $-\frac{5}{12}$

29. r $= \sqrt{x^2 + y^2}$

$r = \sqrt{8^2 + (-2)^2}$

$r = \sqrt{68}$ or $2\sqrt{17}$

$\sin \theta = \frac{y}{x}$ $\quad$ $\cos \theta = \frac{x}{y}$ $\quad$ $\tan \theta = \frac{y}{x}$

$\sin \theta = \frac{-2}{2\sqrt{17}}$ $\quad$ $\cos \theta = \frac{8}{2\sqrt{17}}$ $\quad$ $\tan \theta = \frac{-2}{8}$ or $-\frac{1}{4}$

$\sin \theta = -\frac{\sqrt{17}}{17}$ $\quad$ $\cos \theta = \frac{4\sqrt{17}}{17}$

$\csc \theta = \frac{r}{y}$ $\quad$ $\sec \theta = \frac{r}{x}$

$\csc \theta = \frac{2\sqrt{17}}{-2}$ or $-\sqrt{17}$ $\quad$ $\sec \theta = \frac{2\sqrt{17}}{8}$ or $\frac{\sqrt{17}}{4}$

$\cot \theta = \frac{x}{y}$

$\cot \theta = \frac{8}{-2}$ or -4

30. $r = \sqrt{x^2 + y^2}$

$r = \sqrt{(-2)^2 + 0^2}$

$r = \sqrt{4}$ or 2

$\sin \theta = \frac{y}{r}$ $\quad$ $\cos \theta = \frac{x}{r}$ $\quad$ $\tan \theta = \frac{y}{x}$

$\sin \theta = \frac{0}{2}$ or 0 $\quad$ $\cos \theta = \frac{-2}{2}$ or -1 $\quad$ $\tan \theta = \frac{0}{-2}$ or 0

$\csc \theta = \frac{r}{y}$ $\quad$ $\sec \theta = \frac{r}{x}$ $\quad$ $\cot \theta = \frac{x}{y}$

$\csc \theta = \frac{2}{0}$ $\quad$ $\sec \theta = \frac{2}{-2}$ or -1 $\quad$ $\cot \theta = \frac{-2}{0}$

undefined $\quad$ undefined

31. $r = \sqrt{x^2 + y^2}$

$r = \sqrt{4^2 + 5^2}$

$r = \sqrt{41}$

$\sin \theta = \frac{y}{r}$ $\quad$ $\cos \theta = \frac{x}{r}$ $\quad$ $\tan \theta = \frac{y}{x}$

$\sin \theta = \frac{5}{\sqrt{41}}$ $\quad$ $\cos \theta = \frac{4}{\sqrt{41}}$ $\quad$ $\tan \theta = \frac{5}{4}$

$\sin \theta = \frac{5\sqrt{41}}{41}$ $\quad$ $\cos \theta = \frac{4\sqrt{41}}{41}$

$\csc \theta = \frac{r}{y}$ $\quad$ $\sec \theta = \frac{r}{x}$ $\quad$ $\cot \theta = \frac{x}{y}$

$\csc \theta = \frac{\sqrt{41}}{5}$ $\quad$ $\sec \theta = \frac{\sqrt{41}}{4}$ $\quad$ $\cot \theta = \frac{4}{5}$

32. $r = \sqrt{x^2 + y^2}$

$r = \sqrt{(-5)^2 + (-9)^2}$

$r = \sqrt{106}$

$\sin \theta = \frac{y}{r}$ $\quad$ $\cos \theta = \frac{x}{r}$ $\quad$ $\tan \theta = \frac{y}{x}$

$\sin \theta = \frac{-9}{\sqrt{106}}$ $\quad$ $\cos \theta = \frac{-5}{\sqrt{106}}$ $\quad$ $\tan \theta = \frac{-9}{-5}$ or $\frac{9}{5}$

$\sin \theta = -\frac{9\sqrt{106}}{106}$ $\quad$ $\cos \theta = -\frac{5\sqrt{106}}{106}$

$\csc \theta = \frac{r}{y}$ $\quad$ $\sec \theta = \frac{r}{x}$

$\csc \theta = \frac{\sqrt{106}}{-9}$ or $-\frac{\sqrt{106}}{9}$ $\quad$ $\sec \theta = \frac{\sqrt{106}}{-5}$ or $-\frac{\sqrt{106}}{5}$

$\cot \theta = \frac{x}{y}$

$\cot \theta = \frac{-5}{-9}$ or $\frac{5}{9}$

33. $r = \sqrt{x^2 + y^2}$

$r = \sqrt{(-4)^2 + 4^2}$

$r = \sqrt{32}$ or $4\sqrt{2}$

$\sin \theta = \frac{y}{r}$ $\quad$ $\cos \theta = \frac{x}{r}$ $\quad$ $\tan \theta = \frac{y}{x}$

$\sin \theta = \frac{4}{4\sqrt{2}}$ $\quad$ $\cos \theta = \frac{-4}{4\sqrt{2}}$ $\quad$ $\tan \theta = \frac{4}{-4}$ or -1

$\sin \theta = \frac{\sqrt{2}}{2}$ $\quad$ $\cos \theta = -\frac{\sqrt{2}}{2}$

$\csc \theta = \frac{r}{y}$ $\quad$ $\sec \theta = \frac{r}{x}$ $\quad$ $\cot \theta = \frac{x}{y}$

$\csc \theta = \frac{4\sqrt{2}}{4}$ $\quad$ $\sec \theta = \frac{4\sqrt{2}}{-4}$ $\quad$ $\cot \theta = \frac{-4}{4}$ or -1

$\csc \theta = \sqrt{2}$ $\quad$ $\sec \theta = -\sqrt{2}$

34. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{5^2 + 0^2}$
$r = \sqrt{25}$ or 5

$\sin\theta = \frac{y}{r}$ $\quad\cos\theta = \frac{x}{r}$ $\quad\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{0}{5}$ or 0 $\quad\cos\theta = \frac{5}{5}$ or 1 $\quad\tan\theta = \frac{0}{5}$ or 0

$\csc\theta = \frac{r}{y}$ $\quad\sec\theta = \frac{r}{x}$ $\quad\cot\theta = \frac{x}{y}$

$\csc\theta = \frac{5}{0}$ $\quad\sec\theta = \frac{5}{5}$ or 1 $\quad\cot\theta = \frac{5}{0}$

undefined $\quad$ undefined

35. $\cos\theta = \frac{x}{r}$ $\quad r^2 = x^2 + y^2$

$\cos\theta = -\frac{3}{8}$ $\quad 8^2 = (-3)^2 + y^2$

$x = -3, r = 8$ $\quad 55 = y^2$

$\pm\sqrt{55} = y$

Quadrant 11, so $y = \sqrt{55}$

$\sin\theta = \frac{y}{r}$ $\quad\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{\sqrt{55}}{8}$ $\quad\tan\theta = \frac{\sqrt{55}}{-3}$ or $-\frac{\sqrt{55}}{3}$

$\csc\theta = \frac{r}{y}$ $\quad\sec\theta = \frac{r}{x}$ $\quad\cot\theta = \frac{x}{y}$

$\csc\theta = \frac{8}{\sqrt{55}}$ $\quad\sec\theta = \frac{8}{-3}$ or $-\frac{8}{3}$ $\quad\cot\theta = \frac{-3}{\sqrt{55}}$

$\csc\theta = \frac{8\sqrt{55}}{55}$ $\quad\cot\theta = -\frac{3\sqrt{55}}{55}$

36. $\tan\theta = \frac{y}{x}$ $\quad r^2 = x^2 + y^2$

$\tan\theta = 3$; Quadrant III $\quad r^2 = (-1)^2 + (-3)^2$

$y = -3, x = -1$ $\quad r^2 = 10$

$r = \sqrt{10}$

$\sin\theta = \frac{y}{r}$ $\quad\cos\theta = \frac{x}{r}$ $\quad\csc\theta = \frac{r}{x}$

$\sin\theta = \frac{-3}{\sqrt{10}}$ $\quad\cos\theta = \frac{-1}{\sqrt{10}}$ $\quad\csc\theta = \frac{\sqrt{10}}{-3}$ or $-\frac{\sqrt{10}}{3}$

$\sin\theta = -\frac{3\sqrt{10}}{10}$ $\quad\cos\theta = -\frac{\sqrt{10}}{10}$

$\sec\theta = \frac{r}{x}$ $\quad\cot\theta = \frac{x}{y}$

$\sec\theta = \frac{\sqrt{10}}{-1}$ or $-\sqrt{10}$ $\quad\cot\theta = \frac{-1}{-3}$ or $\frac{1}{3}$

37. $\sin B = \frac{b}{c}$
$\sin 42° = \frac{b}{15}$
$15 \sin 42° = b$
$10.0 \approx b$

38. $\sin A = \frac{a}{c}$
$\sin 38° = \frac{24}{c}$
$c \sin 38° = 24$
$c = \frac{24}{\sin 38°}$
$c \approx 39.0$

39. $\tan B = \frac{b}{a}$
$\tan 67° = \frac{24}{a}$
$a \tan 67° = 24$
$a = \frac{24}{\tan 67°}$
$a \approx 10.2$

40. 30°, 210°

41. 180°

42. $A + 49° = 90°$
$A = 41°$

$\tan B = \frac{b}{a}$ $\quad\cos B = \frac{a}{c}$

$\tan 49° = \frac{b}{16}$ $\quad\cos 49° = \frac{16}{c}$

$16 \tan 49° = b$ $\quad c\cos 49° = 16$

$18.4 \approx b$ $\quad c = \frac{16}{\cos 49°}$

$c \approx 24.4$

$A = 41°, b = 18.4, c = 24.4$

43. $a^2 + b^2 = c^2$ $\quad\cos A = \frac{b}{c}$
$a^2 + 15^2 = 20^2$ $\quad\cos A = \frac{15}{20}$
$a = \sqrt{175}$ $\quad A = \cos^{-1}\frac{15}{20}$
$a \approx 13.2$ $\quad A \approx 41.4°$

$41.40962211° + B \approx 90°$
$B \approx 48.6°$
$a = 13.2, A = 41.4°, B = 48.6°$

44. $64° + B = 90°$
$B = 26°$

$\sin A = \frac{a}{c}$ $\quad\cos A = \frac{b}{c}$

$\sin 64° = \frac{a}{28}$ $\quad\cos 64° = \frac{b}{28}$

$28 \sin 64° = a$ $\quad 28\cos 64° = b$

$25.2 \approx a$ $\quad 12.3 \approx b$

$B = 26°, a = 25.2, b = 12.3$

45. $A = 180° - (70° + 58°)$ or 52°

$\frac{b}{\sin B} = \frac{a}{\sin A}$ $\quad\frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{b}{\sin 70°} = \frac{84}{\sin 52°}$ $\quad\frac{c}{\sin 58°} = \frac{84}{\sin 52°}$

$b = \frac{84 \sin 70°}{\sin 52°}$ $\quad c = \frac{84\sin 58°}{\sin 52°}$

$b \approx 100.1689124$ $\quad c \approx 90.39983243$

$A = 52°, b = 100.2, c = 90.4$

46. $A = 180° - (57° + 49°)$ or 74°

$\frac{a}{\sin A} = \frac{c}{\sin C}$ $\quad\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{a}{\sin 74°} = \frac{8}{\sin 49°}$ $\quad\frac{b}{\sin 57°} = \frac{8}{\sin 49°}$

$a = \frac{8\sin 74°}{\sin 49°}$ $\quad b = \frac{8 \sin 57°}{\sin 49°}$

$a \approx 10.1891739$ $\quad b \approx 8.889995197$

$A = 74°, a = 10.2, b = 8.9$

47. $B = 180° - (20° + 64°)$ or 96°

$K = \frac{1}{2}a^2 \frac{\sin B \sin C}{\sin A}$

$K = \frac{1}{2}(19)^2 \frac{\sin 96° \sin 64°}{\sin 20°}$

$K \approx 471.7$ units^2

48. $C = 180° - (56° + 78°)$ or 46°

$K = \frac{1}{2}b^2 \frac{\sin A \sin C}{\sin B}$

$K = \frac{1}{2}(24)^2 \frac{\sin 56° \sin 46°}{\sin 78°}$

$K \approx 175.6$ units^2

49. $K = \frac{1}{2}\, bc \sin A$

$K = \frac{1}{2}(65.5)(89.4) \sin 58.2°$

$K \approx 2488.4$ units^2

50. $K = \frac{1}{2}\, ac \sin B$

$K = \frac{1}{2}(18.4)(6.7) \sin 22.6°$

$K \approx 23.7$ units^2

51. Since $38.7° < 90°$, consider Case I.

$c \sin A = 203 \sin 38.7°$

$c \sin A \approx 126.9242592$

$172 > 126.9$; 2 solutions

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{172}{\sin 38.7°} = \frac{203}{\sin C}$$
$$\sin C = \frac{203 \sin 38.7°}{172}$$
$$C = \sin^{-1}\left(\frac{203 \sin 38.7°}{172}\right)$$
$$C \approx 47.55552829$$

$180° - \propto \approx 180° - 47.6°$ or $132.4°$

Solution 1

$B \approx 180° - (38.7° + 47.6°)$ or $93.7°$

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{b}{\sin 937°} \approx \frac{172}{\sin 38.7°}$$
$$b \approx \frac{172 \sin 93.7°}{\sin 38.7°}$$
$$b \approx 274.5059341$$

Solution 2

$B \approx (180° - (38.7° + 132.4°)$ or $8.9°$

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{b}{\sin 8.9°} \approx \frac{172}{\sin 38.7°}$$
$$b \approx \frac{172 \sin 8.9°}{\sin 38.7°}$$
$$b \approx 42.34881128$$

$B = 93.7°, C = 47.6°, b = 274.5; B = 8.9°, C = 132.4°, b = 42.3$

52. Since $57° < 90°$, consider Case I.

$b \sin A = 19 \sin 57°$

$b \sin A \approx 15.93474074$

$12 < 15.9$; no solution

53. Since $29° < 90°$, consider Case I.

$c \sin A = 15 \sin 29°$

$c \sin A \approx 7.272144304$

$12 > 7.3$; 2 solutions

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{12}{\sin 29°} = \frac{15}{\sin C}$$
$$\sin C = \frac{15 \sin 29°}{12}$$
$$C = \sin^{-1}\left(\frac{15 \sin 29°}{12}\right)$$
$$C \approx 37.30170167$$

$180 - \propto \approx 180° - 37.3°$ or $142.7°$

Solution 1

$B \approx 180° - (29° + 37.3°)$ or $113.7°$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{12}{\sin 29°} \approx \frac{b}{\sin 113.7°}$$
$$b \approx \frac{12 \sin 113.7°}{\sin 29°}$$
$$b \approx 22.6647614$$

Solution 2

$B \approx 180° - (29° + 142.7°)$ or $8.3°$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{12}{\sin 29°} \approx \frac{b}{\sin 8.3°}$$
$$b \approx \frac{12 \sin 8.3°}{\sin 29°}$$
$$b \approx 3.573829815$$

$B = 113.7°, C = 37.3°, b = 22.7;$
$B = 8.3°, C = 142.7°, b = 3.6$

54. Since $45° < 90°$, consider Case I.

$83 > 79$; 1 solution

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{83}{\sin 45°} = \frac{79}{\sin B}$$
$$\sin B = \frac{79 \sin 45°}{83}$$
$$B = \sin^{-1}\left(\frac{79 \sin 45°}{83}\right)$$
$$B \approx 42.30130394$$
$$C \approx 180° - (45° + 42.3°) \text{ or } 92.7°$$
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{83}{\sin 45°} \approx \frac{c}{\sin 92.7°}$$
$$c \approx \frac{83 \sin 92.7°}{\sin 45°}$$
$$c \approx 117.2495453$$

$B = 42.3°, C = 92.7°, c = 117.2$

55. $a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 40^2 + 45^2 - 2(40)(45) \cos 51°$

$a^2 \approx 1359.446592$

$a \approx 36.87067388$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{36.9}{\sin 51°} \approx \frac{40}{\sin B}$$
$$\sin B \approx \frac{40 \sin 51°}{36.9}$$
$$B \approx \sin^{-1}\left(\frac{40 \sin 51°}{36.9}\right)$$
$$B \approx 57.39811237$$

$C \approx 180° - (51° + 57.4)$ or $71.6°$

$a = 36.9, B = 57.4°, C = 71.6°$

56. $b^2 = a^2 + c^2 - 2ac \cos B$

$b^2 = 51^2 + 61^2 - 2(51)(61) \cos 19°$

$b^2 \approx 438.9834226$

$b \approx 20.95193124$

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{21.0}{\sin 19°} \approx \frac{51}{\sin A}$$
$$\sin A \approx \frac{51 \sin 19°}{21.0}$$
$$A \approx \sin^{-1}\left(\frac{51 \sin 19°}{21.0}\right)$$
$$A \approx 52.4178316$$

$C \approx 180° - (52.4° + 19°)$ or $108.6°$

$b = 21.0, A = 52.4°, C = 108.6°$

57.
$$c^2 = a^2 + b^2 - 2ab \cos C$$
$$20^2 = 11^2 + 13^2 - 2(11)(13) \cos C$$
$$\frac{20^2 - 11^2 - 13^2}{-2(11)(13)} = \cos C$$
$$\cos^{-1}\left(\frac{20^2 - 11^2 - 13^2}{-2(11)(13)}\right) = C$$
$$112.6198649 \approx C$$
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{11}{\sin A} \approx \frac{20}{\sin 112.6°}$$
$$\sin A \approx \frac{11 \sin 112.6°}{20}$$
$$A \approx \sin^{-1}\left(\frac{11 \sin 112.6°}{20}\right)$$
$$A \approx 30.51023741$$

$B \approx 180° - (30.5° + 112.6°)$ or $36.9°$

$A = 30.5, B = 36.9°, C = 112.6°$

58. $b^2 = a^2 + c^2 - 2ac \cos B$

$b^2 = 42^2 + 6.5^2 - 2(42)(6.5) \cos 24°$

$b^2 \approx 1307.45418$

$b \approx 36.15873588$

$\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{36.2}{\sin 24°} \approx \frac{6.5}{\sin C}$

$\sin C \approx \frac{6.5 \sin 24°}{36.2}$

$C \approx \sin^{-1}\left(\frac{6.5 \sin 24°}{36.2}\right)$

$C \approx 4.192989407$

$A \approx 180° - (24° + 4.2°)$ or $151.8°$

$b = 36.2, A = 151.8°, C = 4.2°$

Page 339 Applications and Problem Solving

59a. $\sin \theta = \frac{8}{12}$

$\theta = \sin^{-1} \frac{8}{12}$

$\theta \approx 41.8°$

59b. $\cos \theta = \frac{x}{12}$

$\cos 41.8° \approx \frac{x}{12}$

$12 \cos 41.8° \approx x$

$8.94427191 \approx x$

about 8.9 ft

60a. $x^2 = 4.5^2 + 8.2^2 - 2(4.5)(8.2) \cos 32°$

$x^2 \approx 24.9040505$

$x \approx 5.0$ mi

60b. $\frac{8.2}{\sin \theta} \approx \frac{5.0}{\sin 32°}$

$\sin \theta \approx \frac{8.2 \sin 32°}{5.0}$

$\theta \approx \sin^{-1}\left(\frac{8.2 \sin 32°}{5.0}\right)$

$\theta \approx 60.54476292$

$180 - \theta \approx 180 - 60.5$ or about $119.5°$

Page 339 Open-Ended Assessment

1. $K = \frac{1}{2} ab \sin C$

$125 = \frac{1}{2} ab \sin 35°$

$435.86 \approx ab$

Sample answer: about 40 cm and 10.9 cm

2a. Sample answer: $a = 10$, $b = 24$, $A = 30°$; $10 < 24$, $10 < 24 \sin 30°$

2b. Sample answer: $b = 18$; $10 < 18$, $10 > 18 \sin 30°$

Chapter 5 SAT & ACT Preparation

Page 341 SAT and ACT Practice

1. There are several ways to solve this problem. Use the Pythagorean Theorem on the large triangle.

$(2y + 3y)^2 = 4^2 + 3^2$

$(5y)^2 = 16 + 9$

$25y^2 = 25$

$\frac{25y^2}{25} = \frac{25}{25}$

$y^2 = 1$

$y = \pm 1$

Since y is a length, use only the positive root. Another method is to use the Triangle Inequality Theorem. The hypotenuse must be shorter than the sum of the lengths of the other two sides.

$5y < 3 + 4$

$5y < 7$

Which of the answer choices make this inequality true?

$5(1) = 5 < 7$

$5(2) = 10 > 7$

The correct choice is *A*.

2. If you recall the general form of the equation of a circle, you can immediately see that this equation represents a circle with its center at the origin.

$(x - h)^2 + (y - k)^2 = r^2$

If you don't recall the equation, you can try to eliminate some of the answer choices. Since the equation contains squared variables, it cannot represent a straight line. Eliminate choice D. Similarly, eliminate choice E. Since both the x and y variables are squared, it cannot represent a parabola. Eliminate choice C. The choices remaining are circle and ellipse. This is a good time to make an educated guess, since you have a 50% chance of guessing correctly. It represents a circle. The correct choice is A.

3. Use factoring and the associative property.

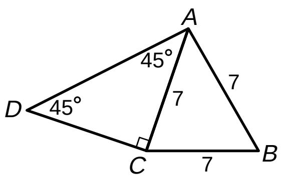
$999 \times 111 = 3 \times 3 \times n^2$

$(9 \times 111) \times 111 = 3 \times 3 \times n^2,$

$3 \times 3 \times (111)^2 = 3 \times 3 \times n^2.$

So n must equal 111. The correct choice is C.

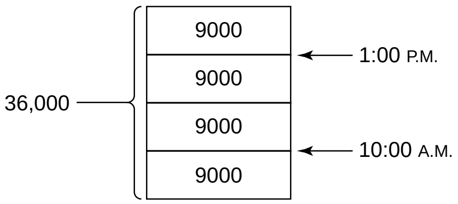
4.



Since $\triangle ABC$ is an equilateral triangle and one side is 7 units long, each side is 7 units long. so $AC = 7$. $\overline{AD}$ is the hypotenuse of right triangle ACD. One leg is 7 units long. One angle is 45°, so the other angle must also be 45°. A 45°–45°–90° triangle is a special right triangle. Its hypotenuse is $\sqrt{2}$ times the length of a leg. (The SAT includes this triangle in the Reference Information at the beginning of the mathematics sections.) The hypotenuse is $7\sqrt{2}$. The correct choice is B.

5. You need to find the fraction's range of values, from the minimum to the maximum. The minimum value of the fraction occurs when a is as small as possible and b is as large as possible. Since the smallest value of a must be slightly greater than 4, and the largest value of b must be slightly less than 9, this minimum value of the fraction must be larger than $\frac{4}{9}$. The maximum value of the fraction occurs when a is as large as possible and b is as small as possible. This maximum must be smaller than $\frac{7}{7}$ or 1. The correct choice is A.

6. Start by making a sketch of the situation.



By 1:00 P.M. the pool is three-fourths full. Three fourths of 36,000 gallons is 27,000 gallons. The pool contained 9,000 gallons at the start. So 27,000 − 9,000 or 18,000 gallons were added in 3 hours. The constant rate of flow is 18,000 gallons ÷ 3 hours or 6,000 gallons per hour. To fill the remaining 9,000 gallons at this same rate will take 9,000 gallons ÷ 6,000 gallons per hour or 1.5 hours. One and a half hours from 1:00 P.M., is 2:30 P.M. The correct choice is C.

7. There are two right triangles in the figure. You need to find the length of one leg of the larger triangle, but you don't know the length of the other leg. Use the Pythagorean Theorem twice—once for each triangle. Let y represent the length of side AC.

In the smaller right triangle,

$y^2 = 4^2 + 6^2$

$y^2 = 16 + 36$

$x^2 = 52$

You do *not* need to solve for y.

In the larger triangle,

$10^2 = x^2 + y^2$

$100 = x^2 + 52$

$x^2 = 48$

$x = \sqrt{48}$

$x = 4\sqrt{3}$

The correct choice is B.

8. Factor the polynomial in the numerator of the fraction. Simplify the fraction. Solve for x.

$$\frac{x^2 + 7x + 12}{x + 4} = 5$$

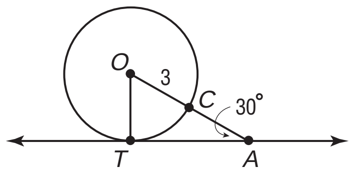
$$\frac{(x + 3)(x + 4)}{x + 4} = 5$$

$$x + 3 = 5$$

$$x = 2$$

The correct choice is B.

9.



T is on the circle, so $\overline{OT}$ is a radius of the circle. The length of $\overline{OT}$ is 3. Since $\overleftrightarrow{TA}$ is tangent to the circle, $\angle OTA$ is a right angle, and ΔOTA is a right triangle. In particular, ΔOTA is a 30°-60°-90° right triangle. In a 30°-60°-90° right triangle, the length of the hypotenuse is 2 times the length of the shorter leg.

$OA = 2(OT)$

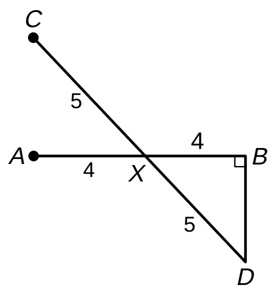
$OC + CA = 2(OT)$

$3 + CA = 2(3)$ or 6

$CA = 3$

The correct choice is B.

10. Draw a diagram from the information given in the problem. Drawing a valid diagram is the most difficult part of solving this problem. Your diagram could be different from the one below and still be valid.



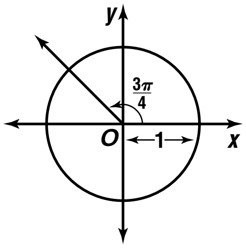
Since two segments bisect each other, you know the length of each half of the segment. Notice that $\overline{BD}$ is a side of a right triangle. It is a 3-4-5 right triangle. So $BD = 3$. The answer is 3.

Chapter 6 Graphs of Trigonometric Functions

6-1 Angles and Radian Measure

Pages 347–348 Check for Understanding

1.



2. $90°$; $\frac{\pi}{4}$

3. Divide 10 by 8.

4. Let $R = 2r$. For the circle with radius R, $s' = R\theta$ or $2r\theta$ which is $2(r\theta)$. Thus, $s' = 2s$. For the circle with radius R, $A' = \frac{1}{2}R^2\theta$ or $\frac{1}{2}(2r)^2\theta$ which is $\frac{1}{2}(4r^2)\theta$ or $4\left(\frac{1}{2}r^2\theta\right)$. Thus, $A' = 4A$.

5. $240° = 240° \times \frac{\pi}{180°}$
$= \frac{4\pi}{3}$

6. $570° = 570° \times \frac{\pi}{180°}$
$= \frac{19\pi}{6}$

7. $\frac{3\pi}{2} = \frac{3\pi}{2} \times \frac{180°}{\pi}$
$= 270°$

8. $-1.75 = -1.75 \times \frac{180°}{\pi}$
$= 100.3°$

9. reference angle: $\frac{3\pi}{4} - \pi$ or $\frac{\pi}{4}$; Quadrant 2
$\sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$

10. reference angle: $\frac{11\pi}{6} - \pi$ or $\frac{5\pi}{6}$; Quadrant 3
$\tan \frac{11\pi}{6} = -\frac{\sqrt{3}}{3}$

11. $s = r\theta$
$s = 15\left(\frac{5\pi}{6}\right)$
$s \approx 39.3$ in.

12. $77° = 77° \times \frac{\pi}{180°}$
$= \frac{77\pi}{180}$
$s = r\theta$
$s = 15\left(\frac{77\pi}{180}\right)$
$s \approx 20.2$ in.

13. $A = \frac{1}{2}r^2\theta$
$A = \frac{1}{2}(1.4^2)\left(\frac{2\pi}{3}\right)$
$A \approx 2.1$ units^2

14. $54° = 54° \times \frac{\pi}{180°}$
$= \frac{3\pi}{10}$
$A = \frac{1}{2}r^2\theta$
$A = \frac{1}{2}(6^2)\left(\frac{3\pi}{10}\right)$
$A \approx 17.0$ units^2

15. $30° = 30° \times \frac{\pi}{180°}$
$= \frac{\pi}{6}$
$s = r\theta$
$s = 1.4\left(\frac{\pi}{6}\right)$
$s \approx 0.7$ m

Pages 348–351 Exercises

16. $135° = 135° \times \frac{\pi}{180°}$
$= \frac{3\pi}{4}$

17. $210° = 210° \times \frac{\pi}{180°}$
$= \frac{7\pi}{6}$

18. $300° = 300° \times \frac{\pi}{180°}$
$= \frac{5\pi}{3}$

19. $-450° = -450° \times \frac{\pi}{180°}$
$= -\frac{5\pi}{2}$

20. $-75° = -75° \times \frac{\pi}{180°}$
$= -\frac{5\pi}{12}$

21. $1250° = 1250° \times \frac{\pi}{180°}$
$= \frac{125\pi}{18}$

22. $\frac{7\pi}{12} = \frac{7\pi}{12} \times \frac{180°}{\pi}$
$= 105°$

23. $\frac{11\pi}{3} = \frac{11\pi}{3} \times \frac{180°}{\pi}$
$= 660°$

24. $17 = 17 \times \frac{180°}{\pi}$
$= 974.0°$

25. $-3.5 = -3.5 \times \frac{180°}{\pi}$
$= -200.5°$

26. $-\frac{\pi}{6.2} = -\frac{\pi}{6.2} \times \frac{180°}{\pi}$
$= -29.0°$

27. $17.5 = 17.5 \times \frac{180°}{\pi}$
$= 1002.7°$

28. reference angle: $2\pi - \frac{5\pi}{3}$ or $\frac{\pi}{3}$; Quadrant 4
$\sin \frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$

29. reference angle: $\frac{7\pi}{6} - \pi$ or $\frac{\pi}{6}$; Quadrant 3
$\tan \frac{7\pi}{6} = \frac{\sqrt{3}}{3}$

30. reference angle: $\frac{5\pi}{4} - \pi$ or $\frac{\pi}{4}$; Quadrant 3
$\cos \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$

31. reference angle: $\frac{7\pi}{6} - \pi$ or $\frac{\pi}{6}$; Quadrant 3
$\sin \frac{7\pi}{6} = -\frac{1}{2}$

32. $\frac{14\pi}{3}$ is coterminal with $\frac{2\pi}{3}$
reference angle: $\pi - \frac{2\pi}{3}$ or $\frac{\pi}{3}$; Quadrant 2
$\tan \frac{14\pi}{3} = -\sqrt{3}$

33. $-\frac{19\pi}{6}$ is coterminal with $\frac{5\pi}{6}$
reference angle: $\pi - \frac{5\pi}{6}$ or $\frac{\pi}{6}$; Quadrant 2
$\cos\left(-\frac{19\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

34. $s = r\theta$
$s = 14\left(\frac{2\pi}{3}\right)$
$s \approx 29.3$ cm

35. $s = r\theta$
$s = 14\left(\frac{5\pi}{12}\right)$
$s \approx 18.3$ cm

36. $150° = 150° \times \frac{\pi}{180°}$
$= \frac{5\pi}{6}$
$s = r\theta$
$s = 14\left(\frac{5\pi}{6}\right)$
$s \approx 36.7$ cm

37. $282° = 282° \times \frac{\pi}{180°}$
$= \frac{47\pi}{30}$
$s = r\theta$
$s = 14\left(\frac{47\pi}{30}\right)$
$s \approx 68.9$ cm

38. $s = r\theta$
$s = 14\left(\frac{3\pi}{11}\right)$
$s \approx 12.0$ cm

39. $320° = 320° \times \frac{\pi}{180°}$
$= \frac{16\pi}{9}$
$s = r\theta$
$s = 14\left(\frac{16\pi}{9}\right)$
$s \approx 78.2$ cm

40. $r = \frac{1}{2}d$

$r = \frac{1}{2}(22)$

$r = 11$

$78° = 78° \times \frac{\pi}{180°}$

$= \frac{13\pi}{30}$

$s = r\theta$

$s = 11\left(\frac{13\pi}{30}\right)$

$s \approx 15.0$ in.

41. $s = r\theta$

$70.7 = r\left(\frac{5\pi}{4}\right)$

$18.00360716 \approx r$

$d = 2r$

$d \approx 2(18.0)$

$d \approx 36.0$ m

42. $60° = 60° \times \frac{\pi}{180°}$

$= \frac{\pi}{3}$

$s = r\theta$

$14.2 = r\left(\frac{\pi}{3}\right)$

$13.56 \approx r$; about 13.6 cm

43. $A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(10^2)\left(\frac{5\pi}{12}\right)$

$A \approx 65.4$ units^2

44. $90° = 90° \times \frac{\pi}{180°}$

$= \frac{\pi}{2}$

$A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(22^2)\left(\frac{\pi}{2}\right)$

$A \approx 380.1$ units^2

45. $A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(7^2)\left(\frac{\pi}{8}\right)$

$A \approx 9.6$ units^2

46. $A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(12.5^2)\left(\frac{4\pi}{7}\right)$

$A \approx 140.2$ units^2

47. $225° = 225° \times \frac{\pi}{180°}$

$= \frac{5\pi}{4}$

$A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(6^2)\left(\frac{5\pi}{4}\right)$

$A \approx 70.7$ units^2

48. $82° = 82° \times \frac{\pi}{180°}$

$= \frac{41\pi}{90}$

$A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(7.3^2)\left(\frac{41\pi}{90}\right)$

$A \approx 38.1$ units^2

49a. $s = r\theta$

$6 = r(1.2)$

$5 = r$; 5 ft

49b. $A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(5^2)(1.2)$

$A = 15$ ft^2

50a. $135° = 135° \times \frac{\pi}{180°}$

$= \frac{3\pi}{4}$

$s = r\theta$

$114 = r\left(\frac{3\pi}{4}\right)$

$48.38 \approx r$; about 48.4 mm

50b. $A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(48.4^2)\left(\frac{3\pi}{4}\right)$

$A \approx 2757.8$ mm^2

51a. $A = \frac{1}{2}r^2\theta$

$15 = \frac{1}{2}r^2(0.2)$

$150 = r^2$

$12.247 \approx r$

about 12.2 in.

51b. $s = r\theta$

$s \approx 12.2(0.2)$

$s \approx 2.4$ in.

52a. $A = \frac{1}{2}r^2\theta$

$15.3 = \frac{1}{2}(3^2)\theta$

$3.4 = \theta$; 3.4 radians

52b. $3.4 = 3.4 \times \frac{180°}{\pi}$

$\approx 194.8°$

52c. $s = r\theta$

$s = 3(3.4)$

$s = 10.2$ m

53a. $225° = 225° \times \frac{\pi}{180°}$

$= \frac{5\pi}{4}$

$s = r\theta$

$s = 2\left(\frac{5\pi}{4}\right)$

$s \approx 7.9$ ft

53b. $s = r\theta$

$5 = 2\theta$

$2.5 = \theta$

$2.5 = 2.5 \times \frac{180°}{\pi}$

$\approx 143.2°$

54. $330° = 330° \times \frac{\pi}{180°}$

$= \frac{11\pi}{6}$

$s = r\theta$

$s = 2\left(\frac{11\pi}{6}\right)$

$s \approx 11.5$ in.

$s = r\theta$

$11.5 \approx 8\theta$

$1.44 \approx \theta$; about 1.4 radians

55. $s = r\theta$

$10.5 = 22.9\theta$

$0.46 \approx \theta$; about 0.5

$0.5 \approx 0.5 \times \frac{180°}{\pi}$

$\approx 26.3°$

56a. $45° - 34° = 11°$

$11° = 11° \times \frac{\pi}{180°}$

$= \frac{11\pi}{180}$

$s = r\theta$

$s = 3960\left(\frac{11\pi}{180}\right)$

$s \approx 760.3$ mi

56b. $45° - 31° = 14°$

$14° = 14° \times \frac{\pi}{180°}$

$= \frac{7\pi}{90}$

$s = r\theta$

$s = 3960\left(\frac{7\pi}{90}\right)$

$s \approx 967.6$ mi

56c. $34° - 31° = 3°$

$3° = 3° \times \frac{\pi}{180°}$

$= \frac{\pi}{60}$

$s = r\theta$

$s = 3960\left(\frac{\pi}{60}\right)$

$s \approx 207.3$ mi

57. $84.5° = 84.5° \times \frac{\pi}{180°}$

$= \frac{169\pi}{360}$

$s = r\theta$

$s = 0.70\left(\frac{169\pi}{360}\right)$

$s \approx 1.03$ mi

$80° = 80° \times \frac{\pi}{180°}$

$= \frac{4\pi}{9}$

$s = r\theta$

$s = 0.67\left(\frac{4\pi}{9}\right)$

$s \approx 0.94$ mi

$1.03 + 1.46 + 0.94 + 1.8 \approx 5.23$ mi

58a. $r = \frac{1}{2}d$

$r = \frac{1}{2}\left(2\frac{1}{2}\right)$

$r = 1.25$

$s = r\theta$

$s = 1.25(3\pi)$

$s \approx 11.8$ ft

1.5 rotations $= 1.5 \times 2\pi$ radians

$= 3\pi$ radians

58b. $s = r\theta$

$4\frac{1}{2} = 1.25\theta$

$3.6 = \theta$

$3.6 = 3.6 \times \frac{180°}{\pi}$

$\approx 206.3°$

59a. $\theta = 2\pi - \frac{\pi}{2}$ or $\frac{3\pi}{2}$

$A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(15^2)\left(\frac{3\pi}{2}\right)$

$A \approx 530.1$ ft^2

59b. $A = \frac{1}{2}r^2\theta$

$750 = \frac{1}{2}r^2\left(\frac{3\pi}{2}\right)$

$318.3098862 \approx r^2$

$17.84124116 \approx r$; about 17.8 ft

60. 3.5 km = 350,000 cm

$$s = r\theta$$
$$350{,}000 = 32\theta$$
$$10{,}937.5 = \theta;\ 10{,}937.5 \text{ radians}$$

61. Area of segment = Area of sector − Area of triangle

$$A = \tfrac{1}{2}r^2\alpha - \tfrac{1}{2}r \cdot r \cdot \sin\alpha$$
$$A = \tfrac{1}{2}r^2(\alpha - \sin\alpha)$$

62.
$$\begin{aligned} s &= \tfrac{1}{2}(6 + 8 + 12)\\ &= 13\\ K &= \sqrt{s(s-a)(s-b)(s-c)}\\ K &= \sqrt{13(13-6)(13-8)(13-12)}\\ K &= \sqrt{455}\\ K &\approx 21.3 \text{ in}^2 \end{aligned}$$

63. Since 152° ≥ 90°, consider Case II.
10.2 ≤ 12, so there is no solution.

64. $C = 180° - 38° - 27° = 115°$

$$\frac{560}{\sin 115°} = \frac{a}{\sin 27°}$$
$$a \approx 280.52$$
$$\sin 38° \approx \frac{x}{280.52}$$
$$x \approx 172.7 \text{ yd}$$

65. I, III

66a. Find a quadratic regression line using a graphing calculator. Sample answer: $y = 102x^2 - 505x + 18{,}430$

66b. $2020 - 1970 = 50$
$y = 102x^2 - 505x + 18{,}430$
$y = 102(50)^2 - 505(50) + 18{,}430$
$y = 248{,}180$
Sample answer: about 248,180

67.

r	1	−3	−2	6	10
1	1	−2	−4	2	12
2	1	−1	−4	−2	6
3	1	0	−2	0	10
4	1	1	2	14	66

$f(-x) = (-x)^4 - 3(-x^3 - 2(-x)^2 + 6(-x) + 10$
$f(-x) = x^4 + 3x^3 - 2x^2 - 6x + 10$

r	1	3	−2	−6	10
1	1	4	2	−4	6
2	1	5	8	10	30

Sample answers: 4; −2

68.
$$\begin{array}{r|rrrr} -2 & 1 & 6 & 12 & 12\\ & & -2 & -8 & -8\\ \hline & 1 & 4 & 4 & 4 \end{array}$$

No; there is a remainder of 4.

69. $x^2 + y^2 = 16 \rightarrow a^2 + b^2 = 16$

$$\begin{aligned} x\text{-axis}\quad & a^2 + b^2 = 16\\ & a^2 + (-b)^2 = 16\\ & a^2 + b^2 = 16;\ \text{yes}\\ y\text{-axis}\quad & a^2 + b^2 = 16\\ & (-a)^2 + b^2 = 16\\ & a^2 + b^2 = 16;\ \text{yes}\\ y = x\quad & a^2 + b^2 = 16\\ & (b)^2 + (a)^2 = 16\\ & a^2 + b^2 = 16;\ \text{yes}\\ y = -x\quad & a^2 + b^2 = 16\\ & (-b)^2 + (-a)^2 = 16\\ & a^2 + b^2 = 16;\ \text{yes} \end{aligned}$$

all

70.
$$\begin{aligned} 4x - 2y + 3z &= -6\\ 5x - 4y - 3z &= -75\\ \hline 9x - 6y \quad &= -81 \end{aligned}$$

$$\begin{aligned} 2(4x - 2y + 3z) &= 2(-6)\\ 3(3x + 3y - 2z) &= 3(2) \end{aligned} \rightarrow \begin{aligned} 8x - 4y + 6z &= -12\\ 9x + 9y - 6z &= 6\\ \hline 17x + 5y \quad &= -6 \end{aligned}$$

$$\begin{aligned} 5(9x - 6y) &= 5(-81)\\ 6(17x + 5y) &= 6(-6) \end{aligned} \rightarrow \begin{aligned} 45x - 30y &= -405\\ 102x + 30y &= -36\\ \hline 147x \quad &= -441\\ x &= -3 \end{aligned}$$

$$\begin{aligned} 9x - 6y &= -81\\ 9(-3) - 6y &= -81\\ y &= 9 \end{aligned} \qquad \begin{aligned} 4x - 2y + 3z &= -6\\ 4(-3) - 2(9) + 3z &= -6\\ z &= 8 \end{aligned}$$

$(-3, 9, 8)$

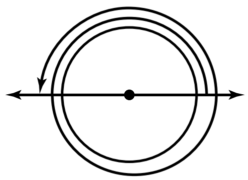
71. b

72. Since $q < 0$, $-q > 0$. Given that $p > 0$, $p - q = p + |q|$ and $p + |q| > 0$. So the expression $p - q$ is nonnegative.
The correct choice is B.

6-2 Linear and Angular Velocity

Page 355 Check for Understanding

1.



2. $\frac{5 \text{ rev}}{1 \text{ min}} \times \frac{2\pi \text{ radians}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ s}}$

3. Linear velocity is the movement along the arc with respect to time while angular velocity is the change in the angle with respect to time.

4. Both individuals would have the same change in angle during the same amount of time. However, an individual on the outside of the carousel would travel farther than an individual on the inside during the same amount of time.

5. Since angular velocity is $\frac{\theta}{t}$, the radius has no effect on the angular velocity. Let $R = 2r$. For a circle with radius R, $v' = R\frac{\theta}{t}$ or $(2r)\frac{\theta}{t}$ which is $2\left(r\frac{\theta}{t}\right)$. Thus $v' = 2v$.

6. $5.8 \times 2\pi = 11.6\pi$ or about 36.4 radians

7. $710 \times 2\pi = 1420\ \pi$ or about 4461.1 radians

8. $3.2 \times 2\pi = 6.4\pi$
$$\omega = \frac{\theta}{t}$$
$$\omega = \frac{6.4\pi}{7}$$
$$\omega \approx 2.9 \text{ radians/s}$$

9. $700 \times 2\pi = 1400\pi$
$$\omega = \frac{\theta}{t}$$
$$\omega = \frac{1400\pi}{15}$$
$$\omega \approx 293.2 \text{ radians/min}$$

10. $v = r\omega$
$v = 12(36)$
$v = 432$ in./s

11. $v = r\omega$
$v = 7(5\pi)$
$v \approx 110.0$ m/min

12a. $r = 3960 + 22{,}300$ or $26{,}260$ mi
$s = r\theta$
$s = 26{,}260(2\pi)$
$s \approx 164{,}996.4$ mi

12b. $v = r\frac{\theta}{t}$
$v = 26{,}260\left(\frac{2\pi}{24}\right)$
$v \approx 6874.9$ mph

Pages 355–358 Exercises

13. $3 \times 2\pi = 6\pi$ or about 18.8 radians

14. $2.7 \times 2\pi = 5.4\pi$ or about 17.0 radians

15. $13.2 \times 2\pi = 26.4\pi$ or about 82.9 radians

16. $15.4 \times 2\pi = 30.8\pi$ or about 96.8 radians

17. $60.7 \times 2\pi = 121.4\pi$ or about 381.4 radians

18. $3900 \times 2\pi = 7800\pi$ or about 24,504.4 radians

19. $1.8 \times 2\pi = 3.6\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{3.6\pi}{9}$
$\omega \approx 1.3$ radians/s

20. $3.5 \times 2\pi = 7\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{7\pi}{3}$
$\omega \approx 7.3$ radians/min

21. $17.2 \times 2\pi = 34.4\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{34.4\pi}{12}$
$\omega \approx 9.0$ radians/s

22. $28.4 \times 2\pi = 56.8\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{56.8\pi}{19}$
$\omega \approx 9.4$ radians/s

23. $100 \times 2\pi = 200\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{200\pi}{16}$
$\omega \approx 39.3$ radians/min

24. $122.6 \times 2\pi = 245.2\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{245.2\pi}{27}$
$\omega \approx 28.5$ radians/min

25. $\frac{1 \text{ revolution}}{50 \text{ seconds}} \times \frac{2\pi \text{ radians}}{1 \text{ revolution}} \approx 0.1$ radian/s

26. $\frac{500 \text{ revolutions}}{1 \text{ minute}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} \times \frac{2\pi \text{ radians}}{1 \text{ revolution}} \approx 52.4$ radians/s

27. $\frac{85 \text{ radians}}{1 \text{ second}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} \times \frac{1 \text{ revolution}}{2\pi \text{ radians}} \approx 811.7$ rpm

28. $v = r\omega$
$v = 8(16.6)$
$v = 132.8$ cm/s

29. $v = r\omega$
$v = 4(27.4)$
$v = 109.6$ ft/s

30. $v = r\omega$
$v = 1.8(6.1\pi)$
$v \approx 34.5$ m/min

31. $v = r\omega$
$v = 17(75.3\pi)$
$v \approx 4021.6$ in./s

32. $v = r\omega$
$v = 39(805.6)$
$v = 31{,}418.4$ in./min

33. $v = r\omega$
$v = 88.9(64.5\pi)$
$v \approx 18{,}014.0$ mm/min

34a. $\frac{120°}{1 \text{ second}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} \times \frac{1 \text{ revolution}}{360°} = 20$ rpm

34b. $120° = 120° \times \frac{\pi}{180°}$
$= \frac{2\pi}{3}$

$\omega = \frac{\theta}{t}$
$\omega = \frac{\frac{2\pi}{3}}{1}$
$\omega = \frac{2\pi}{3}$

$v = r\omega$
$v = 5\left(\frac{2\pi}{3}\right)$
$v \approx 10.5$ in./s

35a. In 1 second, the second hand moves $\frac{1}{60}(360°)$ or 6°.
$6° = 6° \times \frac{\pi}{180°}$ or $\frac{\pi}{30}$
$v = r\theta$
$v = 30\left(\frac{\pi}{30}\right)$
$v \approx 3.1$ mm/s

35b. In 1 second, the minute hand moves $\frac{1}{60}\left(\frac{1}{60}\right)$ (360°) or 0.1°.
$0.1° = 0.1° \times \frac{\pi}{180°}$ or $\frac{0.1\pi}{180}$
$v = r\theta$
$v = 27\left(\frac{0.1\pi}{180}\right)$
$v \approx 0.05$ mm/s

35c. In 1 second, the hour hand moves $\frac{1}{12}\left(\frac{1}{60}\right)\left(\frac{1}{60}\right)$ (360°) or about 0.008°.
$0.008° = 0.008° \times \frac{\pi}{180°}$ or $\frac{0.008\pi}{180}$
$v = r\theta$
$v = 18\left(\frac{0.008\pi}{180}\right)$
$v \approx 0.003$ mm/s

36a. $r = \frac{1}{2}d$
$r = \frac{1}{2}(80)$ or 40

$v = r\frac{\theta}{t}$
$v = 40\left(\frac{2\pi}{45}\right)$
$v \approx 5.6$ ft/s

36b. $v = r\frac{\theta}{t}$
$8 = 40\left(\frac{2\pi}{t}\right)$
$t \approx 31$ s

37a. $3 \times 2\pi = 6\pi$ radians 1 minute = 60 seconds
$v = r\frac{\theta}{t}$
$v = 22\frac{1}{2}\left(\frac{6\pi}{60}\right)$
$v \approx 7.1$ ft/s

37b. $v = r\frac{\theta}{t}$
$3.1 = r\left(\frac{6\pi}{60}\right)$
$9.87 \approx r$; about 9.9 ft

37c. $7.1 - 3.1 \approx 4$ ft/s

38a. $35° = 35° \times \frac{\pi}{180°}$
$= \frac{7\pi}{36}$

lighter child: $\omega = \frac{\theta}{t}$
$\omega = \frac{\frac{7\pi}{36}}{\frac{1}{2}}$
$\omega \approx 1.2$ radians/s

heavier child: $\omega = \frac{\theta}{t}$
$\omega = \frac{\frac{7\pi}{36}}{\frac{1}{2}}$
$\omega \approx 1.2$ radians/s

38b. lighter child: $v = r\omega$

$v \approx 9(1.2)$

$v \approx 11.0$ ft/s

heavier child: $v = r\omega$

$v \approx 6(1.2)$

$v \approx 7.3$ ft/s

39a. 3 miles = 190,080 inches

$r = \frac{1}{2}d$ $\qquad s = r\theta$

$r = \frac{1}{2}(30)$ $\qquad 190{,}080 = 15\theta$

$r = 15$ $\qquad 12{,}672 = \theta$

$12{,}672 \times \frac{1 \text{ revolution}}{2\pi} \approx 2017$ revolutions

39b. $\frac{2.75 \text{ revolutions}}{\text{second}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{2\pi \text{ radians}}{1 \text{ revolution}}$

$= 19{,}800\pi$ radians/hour

$v = r\omega$

$v = 15(19{,}800\pi)$

$v \approx 933{,}053.0181$

$\frac{933{,}053.0181 \text{ inches}}{} \times \frac{1 \text{ mile}}{} \approx 14.7$ mph

40a. Mercury: $\qquad$ Venus:

$v = r\frac{\theta}{t}$ $\qquad v = r\frac{\theta}{t}$

$v = 2440\left(\frac{2\pi}{1407.6}\right)$ $\qquad v = 6052\left(\frac{2\pi}{5832.5}\right)$

$v \approx 10.9$ km/h $\qquad v \approx 6.5$ km/h

Earth: $\qquad$ Mars:

$v = r\frac{\theta}{t}$ $\qquad v = r\frac{\theta}{t}$

$v = 6356\left(\frac{2\pi}{23.935}\right)$ $\qquad v = 3375\left(\frac{2\pi}{24.623}\right)$

$v \approx 1668.5$ km/h $\qquad v \approx 861.2$ km/h

40b. The linear velocity of Earth is about twice that of Mars.

41a. $\theta = \theta_m \cos \omega t$

$\theta = \frac{\pi}{4} \cos \pi t$

41b. $\theta = \frac{\pi}{4} \cos\pi t$

$0 = \frac{\pi}{4} \cos \pi t$

$0 = \cos \pi t$

$\pi t = \frac{\pi}{2}$ $\quad$ or $\quad$ $\pi t = \frac{3\pi}{2}$

$t = \frac{1}{2}$ or 0.5 s $\qquad t = \frac{3}{2}$ or 1.5 s

42a. 3960 + 200 = 4160 miles

$C = 2\pi r$ $\qquad t = C \div \text{speed}$

$C = 2\pi(4160)$ $\qquad t \approx 26{,}138.05088 \div 17{,}000$

$C \approx 26138.05088$ $\qquad t \approx 1.537532405$

$\omega = \frac{\theta}{t}$

$\omega \approx \frac{2\pi}{1.54}$

$\omega \approx 4.1$ radians/h

42b. $\omega = \frac{\theta}{t}$ $\qquad t = C \div \text{speed}$

$4 = \frac{2\pi}{t}$ $\qquad \frac{\pi}{2} = 2\pi r \div 17{,}000$

$\frac{\pi}{2} = t$ $\qquad \frac{\pi}{2}(17{,}000) = 2\pi r$

$\frac{\frac{\pi}{2}(17{,}000)}{2\pi} = r$

$4250 = r$

$4250 - 3960 = 290$; about 290 mi

42c. $3960 + 500 = 4460$; $C = 2\pi(4460)$ or 28023.00647

$t = 28{,}023.00647 \div 17{,}000$ or 1.648412145

$\omega = \frac{\theta}{t}$

$\omega \approx \frac{2\pi}{1.65}$

$\omega \approx 3.8$

Its angular velocity is between 3.8 radians/h and 4.1 radians/h.

43a. B clockwise; C counterclockwise

43b. $v_A = r_A\left(\frac{\theta}{t}\right)_A$

$v_A = 3.0\left(\frac{120}{1}\right)$

$v_A = 360$

The linear velocity of each of the three rollers is the same.

$v_B = r_B\left(\frac{\theta}{t}\right)_B$ $\qquad v_C = r_C\left(\frac{\theta}{t}\right)_C$

$360 = 2.0 \cdot \frac{\theta_B}{1}$ $\qquad 360 = 4.8 \cdot \frac{\theta_C}{1}$

$180 = \theta_B$ $\qquad 75 = \theta_C$

180 rpm $\qquad$ 75 rpm

44. $105° = 105° \times \frac{\pi}{180°}$

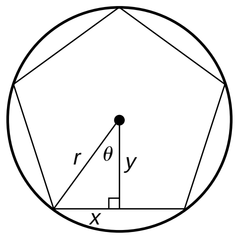
$= \frac{7\pi}{12}$

$A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(7.2^2)\left(\frac{7\pi}{12}\right)$

$A \approx 47.5 \text{ cm}^2$

45.



$r = \frac{1}{2}d$

$r = \frac{1}{2}(7.3)$

$r = 3.65$

$\theta = 360° \div 10$ or $36°$

$\sin \theta = \frac{x}{r}$ $\qquad \cos \theta = \frac{y}{r}$

$\sin 36° = \frac{x}{3.65}$ $\qquad \cos 36° = \frac{y}{3.65}$

$x \approx 2.145416171$ $\qquad y \approx 2.952912029$

$A = \frac{1}{2}bh$

$A \approx \frac{1}{2}(2.15)(2.95)$

$A \approx 3.16761261$

Area of pentagon $\approx 10(3.17)$ or about 31.68 cm^2

46. $35°20'55'' = 35° + 20'\left(\frac{1°}{60'}\right) + 55''\left(\frac{1°}{3600''}\right)$

$\approx 35.349°$

47. $10 + \sqrt{k-5} = 8$ $\qquad$ Check: $10 + \sqrt{k-5} = 8$

$\sqrt{k-5} = -2$ $\qquad 10 + \sqrt{9-5} \stackrel{?}{=} 8$

$\sqrt{k-5} = 4$ $\qquad 10 + \sqrt{4} \stackrel{?}{=} 8$

$k = 9$ $\qquad 10 + 2 \stackrel{?}{=} 8$

$12 \neq 8$

no real solution

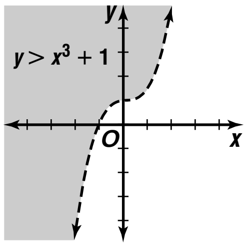
48. $(x - (-4))(x - 3\boldsymbol{i})(x - (-3\boldsymbol{i})) = 0$

$(x + 4)(x - 3\boldsymbol{i})(x + 3\boldsymbol{i}) = 0$

$(x + 4)(x^2 + 9) = 0$

$x^3 + 4x^2 + 9x + 36 = 0$

49.



50. $m = \frac{0-5}{-6-8}$

$m = \frac{-5}{-14}$ or $\frac{5}{14}$

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{5}{14}(x - (-6))$

$y = \frac{5}{14}x + \frac{15}{7}$

51. $P = 2a + 2b$

$P = 2\left(\frac{3}{4}b\right) + 2b$

$P = \frac{3}{2}b + 2b$

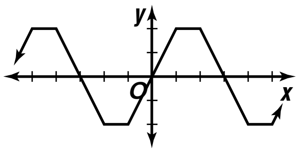
$P = \frac{7}{2}b$

$\frac{2P}{7} = b$ The correct choice is D.

6-3 Graphing Sine and Cosine Functions

Page 363 Check for Understanding

1. Sample answer:

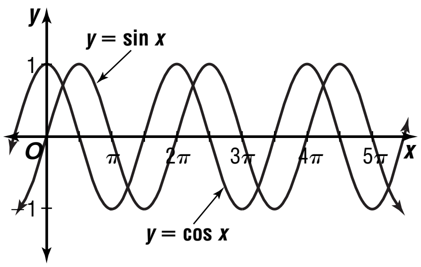


period: 6

2. Sample answers: $-\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}$

3. $\cos x = \cos(x + 2\pi)$

4.

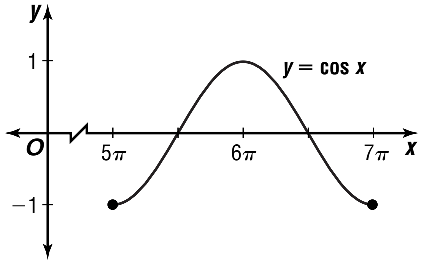


Both functions are periodic functions with the period of 2π. The domain of both functions is the set of real numbers, and the range of both functions is the set of real numbers between -1 and 1, inclusive. The x-intercepts of the sine function are located at πn, but the x-intercepts of the cosine function are located at $\frac{\pi}{2} + \pi n$, where n is an integer. The y-intercept of the sine function is 0, but the y-intercept of the cosine function is 1. The maximum value of the sine function occurs when $x = \frac{\pi}{2} + 2\pi n$ and its minimum value occurs when $x = \frac{3\pi}{2} + 2\pi n$, where n is an integer. The maximum value of the cosine function occurs when $x = \pi n$, where n is an even integer, and its minimum value occurs when $x = \pi n$, where n is an odd integer.

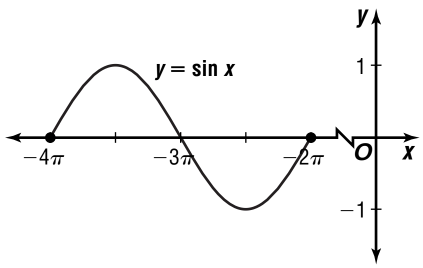
5. yes; 4 6. 0 7. 1

8. $\frac{3\pi}{2} + 2\pi n$, where n is an integer

9.



10.



11. Neither; the period is not 2π.

12. April (month 4):

$y = 49 + 28 \sin\left[\frac{\pi}{6}(t - 4)\right]$

$y = 49 + 28 \sin\left[\frac{\pi}{6}(4 - 4)\right]$

$y = 49$

October (month 10):

$y = 49 + 28 \sin\left[\frac{\pi}{6}(t - 4)\right]$

$y = 49 + 28 \sin\left[\frac{\pi}{6}(10 - 4)\right]$

$y = 49$

The average temperatures are the same.

Pages 363–366 Exercises

13. yes; 6	14. no	15. yes; 20	16. no
17. no	18. no	19. 1	20. 0
21. 0	22. 1	23. −1	24. −1

25. $\sin \pi + \cos \pi = 0 + (-1)$
$= -1$

26. $\sin 2\pi - \cos 2\pi = 0 - 1$
$= -1$

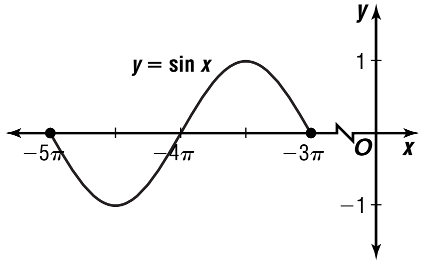
27. $\pi + 2\pi n$, where n is an integer

28. $\frac{\pi}{2} + 2\pi n$, where n is an integer

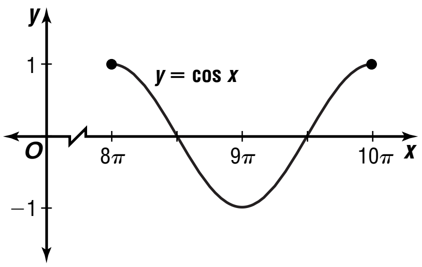
29. $\frac{\pi}{2} + \pi n$, where n is an integer

30. $\theta + 2\pi n$, where n is an integer

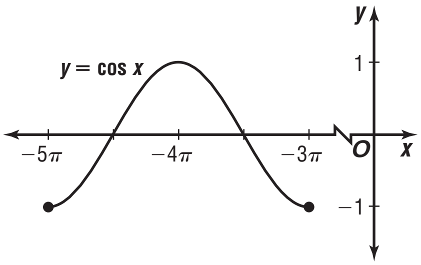
31.



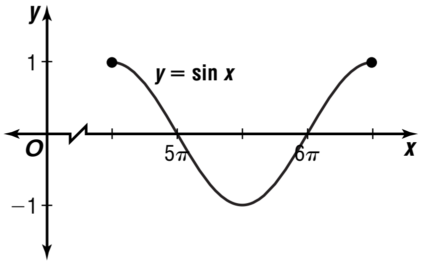
32.



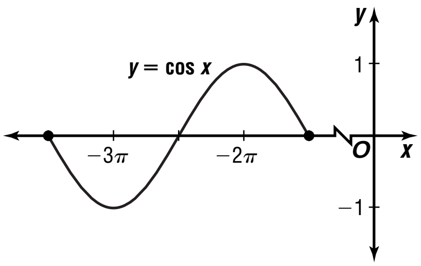
33.



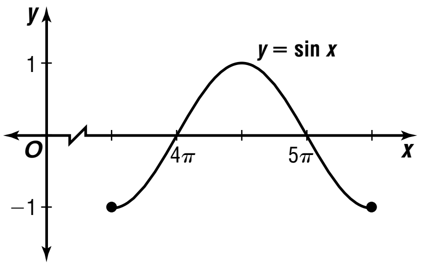
34.



35.



36.



37. $y = \cos x$; the maximum value of 1 occurs when $x = 4\pi$, the minimum value of -1 occurs when $x = 5\pi$, and the x-intercepts are $\frac{7\pi}{2}$, $\frac{9\pi}{2}$, and $\frac{11\pi}{2}$.

38. Neither; the graph does not cross the x-axis.

39. $y = \sin x$; the maximum value of 1 occurs when $x = -\frac{11\pi}{2}$, the minimum value of -1 occurs when $x = -\frac{13\pi}{2}$, and the x-intercepts are -7π, -6π, and -5π.

40. Sample answer: a shift of $\frac{\pi}{2}$ to the left

41. $x = \frac{\pi}{2} + \pi n$, where n is an integer

42. $x = \pi n$, where n is an integer

43a. $\csc\theta = \frac{1}{\sin\theta}$
$1 = \frac{1}{\sin\theta}$
$\sin\theta = 1$
$\frac{\pi}{2} + 2\pi n$, where n is an integer

43b. $\csc\theta = \frac{1}{\sin\theta}$
$-1 = \frac{1}{\sin\theta}$
$\sin\theta = -1$
$\frac{3\pi}{2} + 2\pi n$, where n is an integer

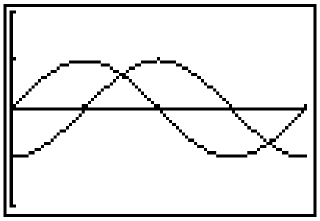
43c. $\csc\theta$ is undefined when $\sin\theta = 0$.
πn, where n is an integer

44a. $\sec\theta = \frac{1}{\cos\theta}$
$1 = \frac{1}{\cos\theta}$
$\cos\theta = 1$
$2\pi n$, where n is an integer

44b. $\sec\theta = \frac{1}{\cos\theta}$
$-1 = \frac{1}{\cos\theta}$
$\cos\theta = -1$
$\pi + 2\pi n$, where n is an integer

44c. $\sec\theta$ is undefined when $\cos\theta = 0$.
$\frac{\pi}{2} + \pi n$, where n is an integer

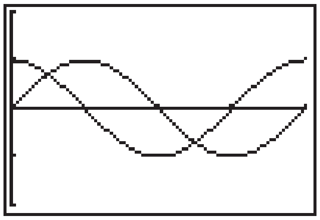
45.



$[0, 2\pi]$ sc1:$\frac{\pi}{2}$ by $[-2, 2]$ sc1:1

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

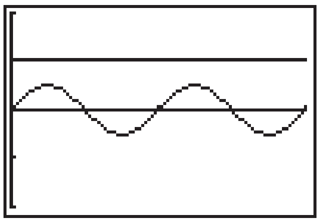
46.



$[0, 2\pi]$ sc1:$\frac{\pi}{2}$ by $[-2, 2]$ sc1:1

$0 \le x \le \frac{\pi}{4}, \frac{5\pi}{4} \le x \le 2\pi$

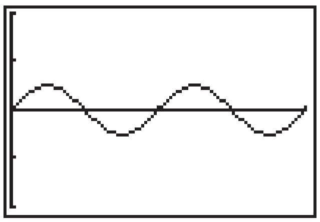
47.



$[0, 2\pi]$ sc1:$\frac{\pi}{2}$ by $[-2, 2]$ sc1:1

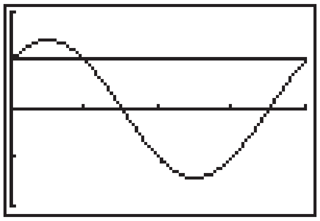
none

48.



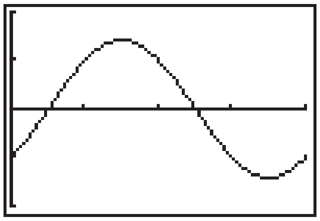
$[0, 2\pi]$ sc1:$\frac{\pi}{2}$ by $[-2, 2]$ sc1:1

$x = 0, \frac{\pi}{2} \le x \le \pi, \frac{3\pi}{2} \le x \le 2\pi$

49. 

$[0, 2\pi]$ scl: $\frac{\pi}{2}$ by $[-2, 2]$ scl: 1

$x = 0, \frac{\pi}{2}, 2\pi$

50. 

$[0, 2\pi]$ scl: $\frac{\pi}{2}$ by $[-2, 2]$ scl: 1

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

51a. July (month 7):

$y = 43 + 31 \sin\left[\frac{\pi}{6}(t - 4)\right]$

$y = 43 + 31 \sin\left[\frac{\pi}{6}(7 - 4)\right]$

$y = 74$

January (month 1):

$y = 43 + 31 \sin\left[\frac{\pi}{6}(t - 4)\right]$

$y = 43 + 31 \sin\left[\frac{\pi}{6}(1 - 4)\right]$

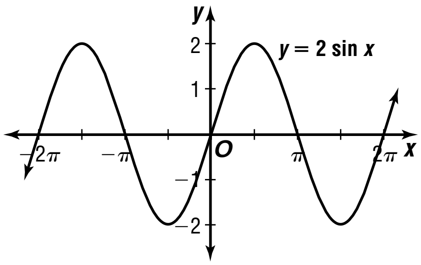
$y = 12$

$74 - 12 = 62$; it is twice the coefficient.

51b. Using answers from 51a., $74 + 12 = 86$; it is twice the constant term.

52a. πn, where n is an integer

52b. 2 **52c.** -2 **52d.** 2π

52e. 

52f. It expands the graph vertically.

53a. $P = 100 + 20 \sin 2\pi t$

$P = 100 + 20 \sin 2\pi(0)$ or 100

$P = 100 + 20 \sin 2\pi(0.25)$ or 120

$P = 100 + 20 \sin 2\pi(0.5)$ or 100

$P = 100 + 20 \sin 2\pi(0.75)$ or 80

$P = 100 + 20 \sin 2\pi(1)$ or 100

53b. 0.25 s **53c.** 0.75 s

54a. $v = 3.5 \cos\left(t\sqrt{\frac{k}{m}}\right)$

$v = 3.5 \cos\left(0.9\sqrt{\frac{19.6}{1.99}}\right)$

$v \approx -3.3$ cm

$v = 3.5 \cos\left(t\sqrt{\frac{k}{m}}\right)$

$v = 3.5 \cos\left(1.7\sqrt{\frac{19.6}{1.99}}\right)$

$v \approx 2.0$ cm

54b. $v = 3.5 \cos\left(t\sqrt{\frac{k}{m}}\right)$

$0 = 3.5 \cos\left(t\sqrt{\frac{19.6}{1.99}}\right)$

$0 = \cos\left(t\sqrt{\frac{19.6}{1.99}}\right)$

$\cos^{-1} 0 = t\sqrt{\frac{19.6}{1.99}}$

$1.570796327 \approx t\sqrt{\frac{19.6}{1.99}}$

$0.5005164776 \approx t$; about 0.5 s

54c. $v = 3.5 \cos\left(t\sqrt{\frac{k}{m}}\right)$

$3.5 = 3.5 \cos\left(t\sqrt{\frac{19.6}{1.99}}\right)$

$1 = \cos\left(t\sqrt{\frac{19.6}{1.99}}\right)$

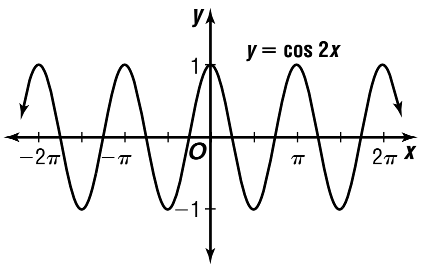
$\cos^{-1} 1 = t\sqrt{\frac{19.6}{1.99}}$

$2\pi = t\sqrt{\frac{19.6}{1.99}}$

$2.00206591 \approx t$; about 2.0 s

55a. $\frac{\pi}{4} + \frac{\pi n}{2}$, where n is an integer

55b. 1 **55c.** -1 **55d.** π

55e. 

56a. $P = 500 + 200 \sin [0.4(t - 2)]$

$P = 500 + 200 \sin [0.4(0 - 2)]$ or about 357 pumas

$D = 1500 + 400 \sin (0.4t)$

$D = 1500 + 400 \sin (0.4(0))$ or 1500 deer

56b. $P = 500 + 200 \sin [0.4(t - 2)]$

$P = 500 + 200 \sin [0.4(10 - 2)]$ or about 488 pumas

$D = 1500 + 400 \sin (0.4t)$

$D = 1500 + 400 \sin (0.4(10))$ or about 1197 deer

56c. $P = 500 + 200 \sin [0.4(t - 2)]$

$P = 500 + 200 \sin [0.4(25 - 2)]$ or about 545 pumas

$D = 1500 + 400 \sin (0.4t)$

$D = 1500 + 400 \sin (0.4(25))$ or about 1282 deer

57. $\frac{500 \text{ revolutions}}{1 \text{ minute}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} \times \frac{2\pi \text{ radians}}{1 \text{ revolution}} \approx 52.4$ radians per second

58. $-1.5 = -1.5 \times \frac{180°}{\pi}$

$\approx -85.9°$

59. 45°, 135°

60. $\frac{2}{x+2} = \frac{x}{2-x} + \frac{x^2+4}{x^2-4}$

$-1(x+2)(x-2)\left(\frac{2}{x+2}\right) = -1(x+2)(x-2)\left(\frac{x}{2-x}\right)$
$+ (-1)(x+2)(x-2)\left(\frac{x^2+4}{x^2-4}\right)$

$-1(x-2)(2) = (x+2)(x) + (-1)(x^2+4)$

$-2x + 4 = x^2 + 2x - x^2 - 4$

$x = 2$

But, $x \neq 2$, so there is no solution.

61. 1 positive real zero

$f(-x) = -2x^3 + 3x^2 + 11x - 6$

2 or 0 negative real zeros

$$\begin{array}{r|rrrr} 2 & 2 & 3 & -11 & -6 \\ & & 4 & 14 & 6 \\ \hline & 2 & 7 & 3 & 0 \end{array}$$

$2x^2 + 7x + 3 = 0$

$(2x+1)(x+3) = 0$

$2x + 1 = 0$ or $x + 3 = 0$

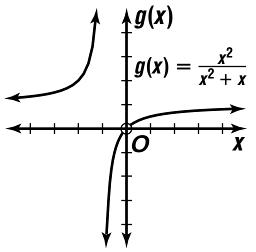
$x = -\frac{1}{2}$ $\quad$ $x = -3$

$-3, -\frac{1}{2}, 2$

62.
$$\begin{array}{r|rrrr} 1 & 1 & 2 & -9 & 18 \\ & & 1 & 3 & -6 \\ \hline & 1 & 3 & -6 & 12 \end{array}$$

12; no

63.



vertical:

$x^2 + x = 0$

$x(x+1) = 0$

$x = 0$ or $x + 1 = 0$

$x = -1$

horizontal:

$y = 1$

64. reflected over the x-axis, expanded vertically by a factor of 3

65. $\begin{vmatrix} -2 & 4 & -1 \\ 1 & -1 & 0 \\ -3 & 4 & 5 \end{vmatrix}$

$= -2\begin{vmatrix} -1 & 0 \\ 4 & 5 \end{vmatrix} - 4\begin{vmatrix} 1 & 0 \\ -3 & 5 \end{vmatrix} + (-1)\begin{vmatrix} 1 & -1 \\ -3 & 4 \end{vmatrix}$

$= -2(-5) - 4(5) - 1(1)$

$= -11$

66. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 & 1 \\ 2 & -4 & 6 \end{bmatrix}$

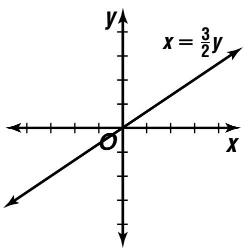
$= \begin{bmatrix} -1(3) + 0(2) & -1(2) + 0(-4) & -1(1) + 0(6) \\ 0(3) + 1(2) & 0(2) + 1(-4) & 0(1) + 1(6) \end{bmatrix}$

$= \begin{bmatrix} -3 & -2 & -1 \\ 2 & -4 & 6 \end{bmatrix}$

$A'(-3, 2), B'(-2, -4), C'(-1, 6)$

67. $x = \frac{3}{2}y$

$y = \frac{2}{3}x$



68. Perimeter of square $RSVW$

$= RS + SV + VW + WR$

$= 5 + 5 + 5 + 5$ or 20

Perimeter of rectangle $RTUW$

$= RT + TU + UW + WR$

$= (5 + 2) + 5 + (5 + 2) + 5$

$= 24$

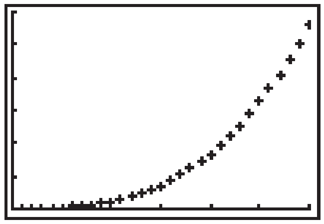
$24 - 20 = 4$

The correct choice is B.

Page 367 History of Mathematics

1.

n	$n^2 + n^3$
1	2
2	12
3	36
4	80
5	150
6	252
7	392
8	576
9	810
10	1100
11	1452
12	1872
13	2366
14	2940
15	3600
16	4352
17	5202
18	6156
19	7220
20	8400
21	9702
22	11,132
23	12,696
24	14,400
25	16,250
26	18,252
27	20,412
28	22,736
29	25,230
30	27,900



[0, 30] sc1:5 by [0, 30,000] sc1:5000

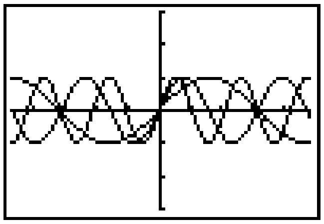
The graph is not a straight line. It curves upward, increasing more rapidly as the value of n increases.

2. See students' work.

6-4 Amplitude and Period of Sine and Cosine Functions

Page 369 Graphing Calculator Exploration

1.



2. The graph is shrunk horizontally.

3. The graph of $f(x) = \sin kx$ for $k < 0$ is the graph of $f(x) = \sin |k| x$ reflected over the y-axis.

Pages 372–373 Check for Understanding

1. Sample answer: $y = 5 \sin 2\theta$

2. The graphs are a reflection of each other over the θ-axis.

3. A: period $= \frac{2\pi}{2}$ or π

B: period $= \frac{2\pi}{5}$

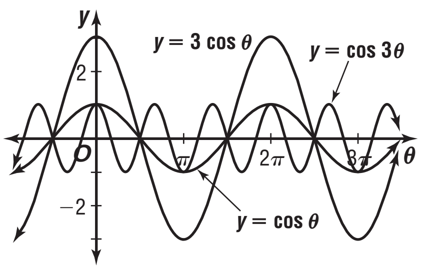
C: period $= \frac{2\pi}{\frac{1}{2}}$ or 4π

D: period $= 2\pi$

C has the greatest period.

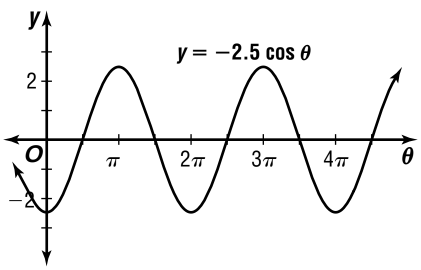
4. Period and frequency are reciprocals of each other.

5.

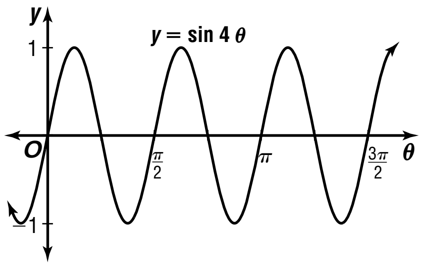


All three graphs are periodic and curve above and below the x-axis. The amplitude of $y = 3 \cos \theta$ is 3, while the amplitude of $y = \cos \theta$ and $y = \cos 3\theta$ is 1. The period of $y = \cos 3\theta$ is $\frac{2\pi}{3}$, while the period of $y = \cos \theta$ and $y = 3 \cos \theta$ is 2π.

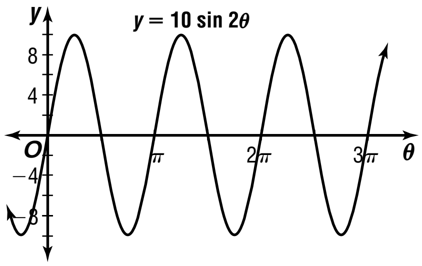
6. $|-2.5| = 2.5$



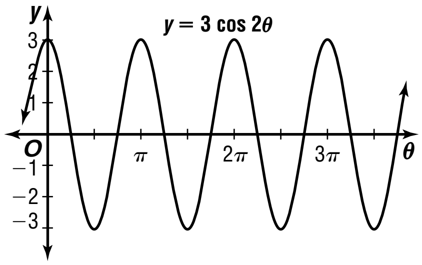
7. $\frac{2\pi}{4} = \frac{\pi}{2}$



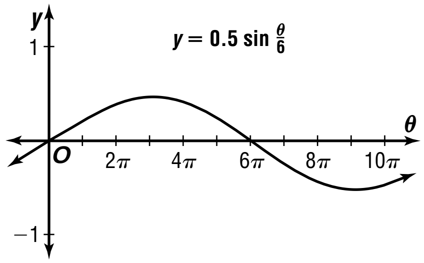
8. $|10| = 10$; $\frac{2\pi}{2} = \pi$



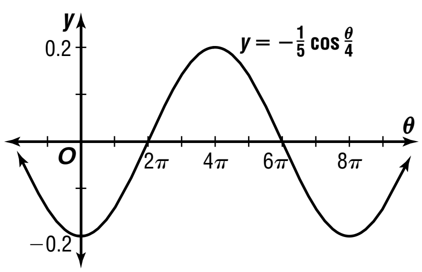
9. $|3| = 3$; $\frac{2\pi}{2} = \pi$



10. $|0.5| = 0.5$; $\frac{2\pi}{\frac{1}{6}} = 12\pi$



11. $\left|-\frac{1}{5}\right| = \frac{1}{5}$; $\frac{2\pi}{\frac{1}{4}} = 8\pi$



12. $|A| = 0.8$ $\qquad \frac{2\pi}{k} = \pi$

$A = \pm 0.8$ $\qquad k = \frac{2\pi}{\pi}$ or 2

$y = \pm 0.8 \sin 2\theta$

13. $|A| = 7$ $\qquad \frac{2\pi}{k} = \frac{\pi}{3}$

$A = \pm 7$ $\qquad k = \frac{2\pi}{\frac{\pi}{3}}$ or 6

$y = \pm 7 \sin 6\theta$

14. $|A| = 1.5$ $\qquad \frac{2\pi}{k} = 5\pi$

$A = \pm 1.5$ $\qquad k = \frac{2\pi}{5\pi}$ or $\frac{2}{5}$

$y = \pm 1.5 \cos \frac{2}{5}\theta$

15. $|A| = \frac{3}{4}$ $\qquad \frac{2\pi}{k} = 6$

$A = \pm\frac{3}{4}$ $\qquad k = \frac{2\pi}{6}$ or $\frac{\pi}{3}$

$y = \pm\frac{3}{4} \cos \frac{\pi}{3}\theta$

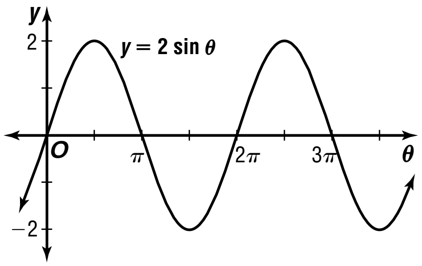
16. $|A| = 0.25$ $\qquad \frac{2\pi}{k} = \frac{1}{294}$

$A = \pm 0.25$ $\qquad k = 588\pi$

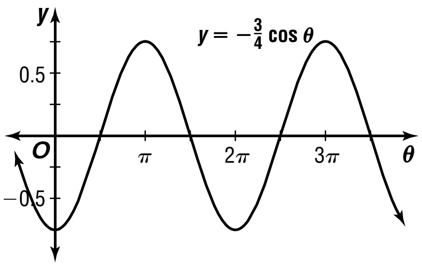
$y = \pm 0.25 \sin (588\pi \times t)$

Pages 373–377 Exercises

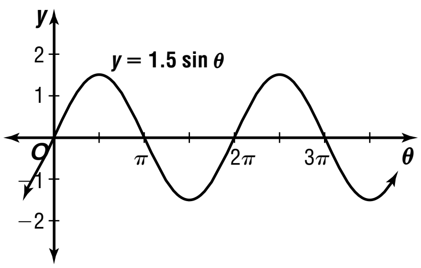
17. $|2| = 2$



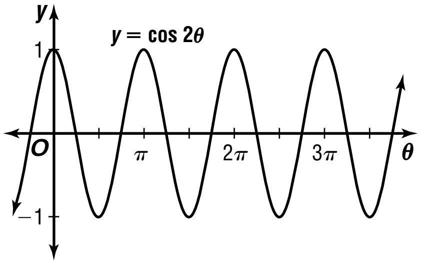
18. $\left|-\frac{3}{4}\right| = \frac{3}{4}$



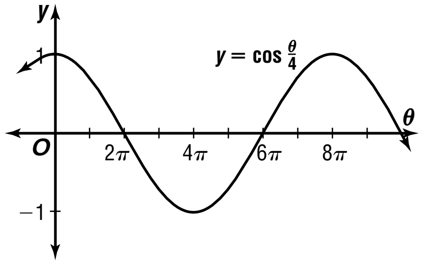
19. $|1.5| = 1.5$



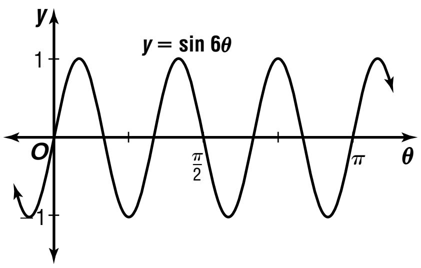
20. $\frac{2\pi}{2} = \pi$



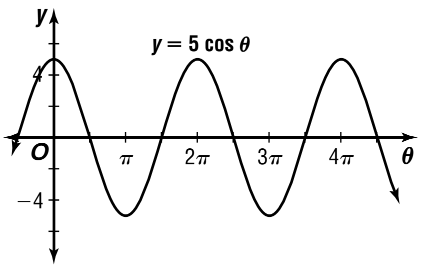
21. $\frac{2\pi}{\frac{1}{4}} = 8\pi$



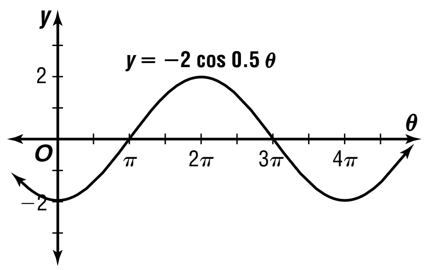
22. $\frac{2\pi}{6} = \frac{\pi}{3}$



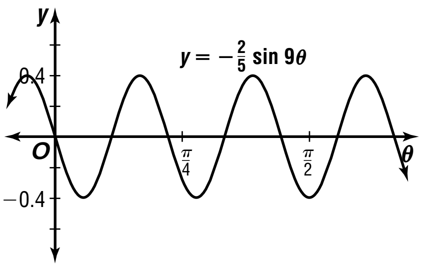
23. $|5| = 5$; $\frac{2\pi}{1} = 2\pi$



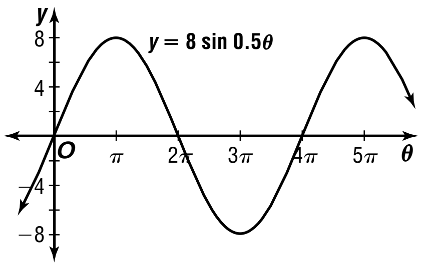
24. $|-2| = 2$; $\frac{2\pi}{0.5} = 4\pi$



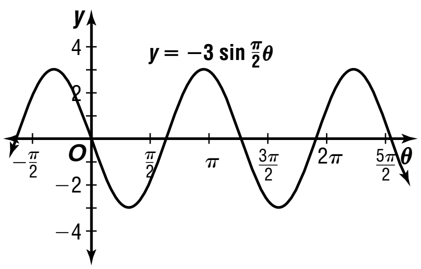
25. $\left|-\frac{2}{5}\right| = \frac{2}{5}; \frac{2\pi}{9}$



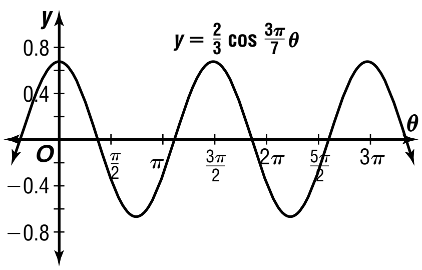
26. $|8| = 8; \frac{2\pi}{0.5} = 4\pi$



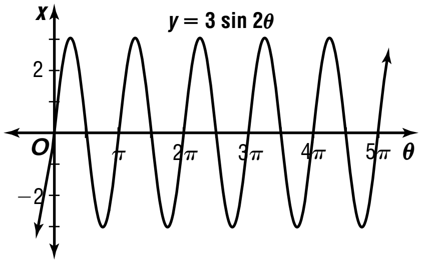
27. $|-3| = 3; \frac{2\pi}{\frac{\pi}{2}} = 4$



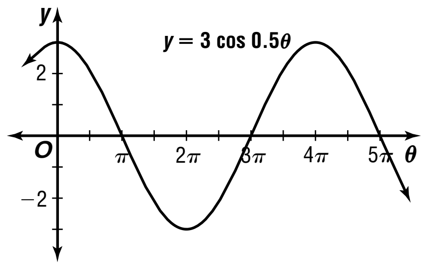
28. $\left|\frac{2}{3}\right| = \frac{2}{3}; \frac{2\pi}{\frac{3\pi}{7}} = \frac{14}{3}$



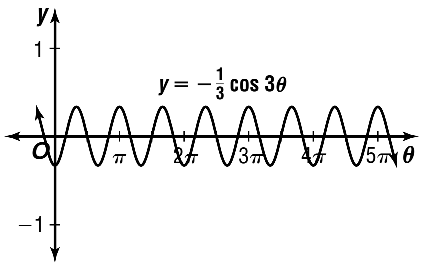
29. $|3| = 3; \frac{2\pi}{2} = \pi$



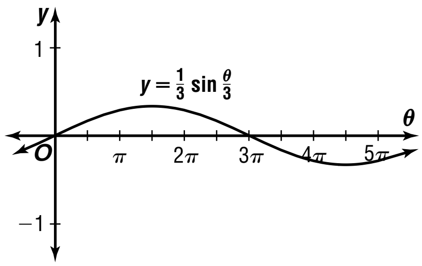
30. $|3| = 3; \frac{2\pi}{0.5} = 4\pi$



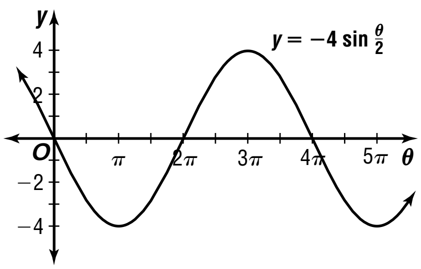
31. $\left|-\frac{1}{3}\right| = \frac{1}{3}; \frac{2\pi}{3}$



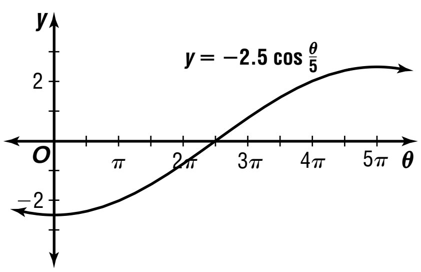
32. $\left|\frac{1}{3}\right| = \frac{1}{3}; \frac{2\pi}{\frac{1}{3}} = 6\pi$



33. $|-4| = 4; \frac{2\pi}{\frac{1}{2}} = 4\pi$



34. $|-2.5| = 2.5; \frac{2\pi}{\frac{1}{5}} = 10\pi$



35. $|0.5| = 0.5; \frac{2\pi}{698\pi} = \frac{1}{349}$

36. $|A| = 0.4$ $\quad \frac{2\pi}{k} = 10\pi$

$A = \pm 0.4$ $\quad k = \frac{1}{5}$

$y = \pm 0.4 \sin \frac{\theta}{5}$

37. $|A| = 35.7$ $\quad \frac{2\pi}{k} = \frac{\pi}{4}$

$A = \pm 35.7$ $\quad k = 8$

$y = \pm 35.7 \sin 8\theta$

38. $|A| = \frac{1}{4}$ $\quad \frac{2\pi}{k} = \frac{\pi}{3}$

$A = \pm \frac{1}{4}$ $\quad k = 6$

$y = \pm \frac{1}{4} \sin 6\theta$

39. $|A| = 0.34$ $\quad \frac{2\pi}{k} = 0.75\pi$

$A = \pm 0.34$ $\quad k = \frac{8}{3}$

$y = \pm 0.34 \sin \frac{8}{3}\theta$

40. $|A| = 4.5$ $\quad \frac{2\pi}{k} = \frac{5\pi}{4}$

$A = \pm 4.5$ $\quad k = \frac{8}{5}$

$y = \pm 4.5 \sin \frac{8}{5}\theta$

41. $|A| = 16$ $\quad \frac{2\pi}{k} = 30$

$A = \pm 16$ $\quad k = \frac{\pi}{15}$

$y = \pm 16 \sin \frac{\pi}{15}\theta$

42. $|A| = 5$ $\quad \frac{2\pi}{k} = 2\pi$

$A = \pm 5$ $\quad k = 1$

$y = \pm 5 \cos \theta$

43. $|A| = \frac{5}{8}$ $\quad \frac{2\pi}{k} = \frac{\pi}{7}$

$A = \pm \frac{5}{8}$ $\quad k = 14$

$y = \pm \frac{5}{8} \cos 14\theta$

44. $|A| = 7.5$ $\quad \frac{2\pi}{k} = 6\pi$

$A = \pm 7.5$ $\quad k = \frac{1}{3}$

$y = \pm 7.5 \cos \frac{\theta}{3}$

45. $|A| = 0.5$ $\quad \frac{2\pi}{k} = 0.3\pi$

$A = \pm 0.5$ $\quad k = \frac{20}{3}$

$y = \pm 0.5 \cos \frac{20}{3}\theta$

46. $|A| = \frac{2}{5}$ $\quad \frac{2\pi}{k} = \frac{3}{5}\pi$

$A = \pm \frac{2}{5}$ $\quad k = \frac{10}{3}$

$y = \pm \frac{2}{5} \cos \frac{10}{3}\theta$

47. $|A| = 17.9$ $\quad \frac{2\pi}{k} = 16$

$A = \pm 17.9$ $\quad k = \frac{\pi}{8}$

$y = \pm 17.9 \cos \frac{\pi}{8}\theta$

48. $|A| = 1.5$ $\quad \frac{2\pi}{k} = \frac{\pi}{2}$

$A = \pm 1.5$ $\quad k = 4$

$y = \pm 1.5 \sin 4\theta$, $y = \pm 1.5 \cos 4\theta$

49. cosine curve $\quad A = 2$ $\quad \frac{2\pi}{k} = 4\pi$

$k = \frac{1}{2}$

$y = 2 \cos \frac{\theta}{2}$

50. sine curve $\quad A = 0.5$ $\quad \frac{2\pi}{k} = \pi$

$k = 2$

$y = 0.5 \sin 2\theta$

51. cosine curve $\quad A = -3$ $\quad \frac{2\pi}{k} = 2\pi$

$k = 1$

$y = -3 \cos \theta$

52. sine curve $\quad A = -1.5$ $\quad \frac{2\pi}{k} = 4\pi$

$k = \frac{1}{2}$

$y = -1.5 \sin \frac{\theta}{2}$

53. $|A| = 3.8$ $\quad \frac{2\pi}{k} = \frac{1}{120}$

$A = \pm 3.8$ $\quad k = 240\pi$

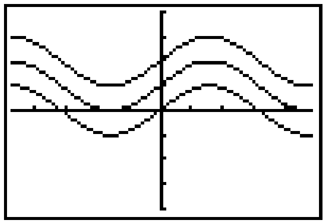
$y = \pm 3.8 \sin(240\pi \times t)$

54. $|A| = 15$ $\quad \frac{2\pi}{k} = \frac{1}{36}$

$A = \pm 15$ $\quad k = 72\pi$

$y = \pm 15 \cos(72\pi \times t)$

55.



All the graphs have the same shape, but have been translated vertically.

56a. $|A| = \left|\frac{A - (-A)}{2}\right|$ $\quad \frac{2\pi}{k} = 8$

$k = \frac{\pi}{4}$

$|A| = \left|\frac{3}{2}\right|$

$A = \pm 1.5$; down first, so $A = -1.5$

$y = -1.5 \sin \frac{\pi}{4}t$

56b. $y = -1.5 \sin \frac{\pi}{4}t$

$y = -1.5 \sin \frac{\pi}{4}(3)$

$y \approx -1.1$ ft

56c. $y = -1.5 \sin \frac{\pi}{4}t$

$y = -1.5 \sin \frac{\pi}{4}(12)$

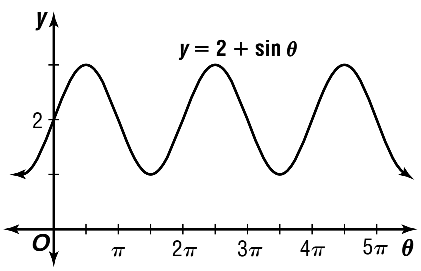
$y = 0$ ft

57a. Maximum value of $\sin \theta = 1$.
Maximum value of $2 + \sin \theta = 2 + 1$ or 3

57b. Minimum value of $\sin \theta = -1$
Minimum value of $2 + \sin \theta = 2 + (-1)$ or 1

57c. $\frac{2\pi}{1} = 2\pi$

37d.



58a. $|A| = 0.2$ $\quad \frac{2\pi}{k} = \frac{1}{262}$

$A = \pm 0.2$ $\quad k = 524\pi$

$y = \pm 0.2 \sin(524\pi \times t)$

58b. $|A| = \frac{1}{2}(0.2)$ $\quad \frac{2\pi}{k} = \frac{1}{524}$

$A = \pm 0.1 \quad k = 1048\pi$

$y = \pm 0.1 \sin(1048\pi \times t)$

58c. $|A| = 2(0.2)$ $\quad \frac{2\pi}{k} = \frac{1}{131}$

$A = \pm 0.4 \quad k = 262\pi$

$y = \pm 0.4 \sin(262\pi \times t)$

59a. $y = A \cos\left(t\sqrt{\frac{g}{\ell}}\right)$

$y = 1.5 \cos\left(t\sqrt{\frac{9.8}{6}}\right)$

59b. $y = 1.5 \cos\left(t\sqrt{\frac{9.8}{6}}\right)$

$y = 1.5 \cos\left(4\sqrt{\frac{9.8}{6}}\right)$

$y \approx 0.6$

about 0.6 m to the right

59c. $y = 1.5 \cos\left(t\sqrt{\frac{9.8}{6}}\right)$

$y = 1.5 \cos\left(7.9\sqrt{\frac{9.8}{6}}\right)$

$y \approx -1.2$

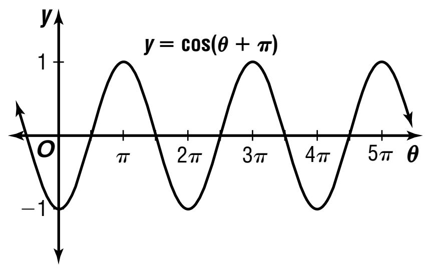
about 1.2 m to the left

60a. $\frac{\pi}{2} + \pi n$, where n is an integer

60b. -1

60c. $\frac{2\pi}{1} = 2\pi$

60d.



61a. $y = 1.5 \cos\left(t\sqrt{\frac{k}{m}}\right)$ $\quad \frac{2\pi}{k} \approx 6.8$

$y = 1.5 \cos\left(t\sqrt{\frac{18.5}{0.4}}\right)$ $\quad k \approx 0.9$ s/cycle

$y \approx 1.5 \cos 6.8t$ $\quad$ frequency: $\frac{1}{0.9} \approx 1.1$ hertz

61b. $y = 1.5 \cos\left(t\sqrt{\frac{k}{m}}\right)$ $\quad \frac{2\pi}{k} \approx 5.6$

$y = 1.5 \cos\left(t\sqrt{\frac{18.5}{0.6}}\right)$ $\quad k \approx 1.1$ s/cycle

$y \approx 1.5 \cos 5.6t$ $\quad$ frequency: $\frac{1}{1.1} \approx 0.9$ hertz

61c. $y = 1.5 \cos\left(t\sqrt{\frac{k}{m}}\right)$ $\quad \frac{2\pi}{k} \approx 4.8$

$y = 1.5 \cos\left(t\sqrt{\frac{18.5}{0.8}}\right)$ $\quad k \approx 1.3$ s/cycle

$y \approx 1.5 \cos 4.8t$ $\quad$ frequency: $\frac{1}{1.3} \approx 0.8$ hertz

61d. It increases. **61e.** It decreases.

62. 0

63. $84 \times 2\pi = 168\pi$ radians

$\omega = \frac{\theta}{t}$

$\omega = \frac{168\pi}{6}$

$\omega \approx 88.0$ radians/s

64. $73° = 73° \times \frac{\pi}{180°}$ $\quad s = r\theta$

$= \frac{73\pi}{180}$ $\quad s = 9\left(\frac{73\pi}{180}\right)$

$s \approx 11.5$ in.

65. $a^2 + b^2 = c^2$ $\quad \tan A = \frac{a}{b}$

$15.1^2 + 19.5^2 = c^2$ $\quad \tan A = \frac{15.1}{19.5}$

$24.66292764 \approx c$ $\quad A = \tan^{-1}\frac{15.1}{19.5}$

$A \approx 37.75273111$

$B = 180° - (90° + 37.8°)$ or $52.2°$

$c = 24.7, A = 37.8°, B = 52.2°$

66. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$4.1 = 2\pi\sqrt{\frac{\ell}{9.8}}$

$0.6525352667 \approx \sqrt{\frac{\ell}{9.8}}$

$0.458022743 \approx \frac{\ell}{9.8}$

$4.17 \approx \ell$; about 4.17 m

67. $b^2 - 4ac = 5^2 - 4(3)(10)$

$= -95$

2 imaginary roots

68a. Let x = the number of Model 28 cards and let y = the number of Model 74 cards.

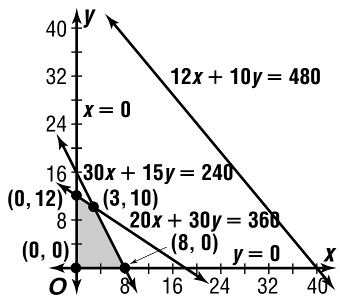
$30x + 15y \le 240$

$20x + 30y \le 360$

$12x + 10y \le 480$

$x \ge 0$

$y \ge 0$



$P(x, y) = 100x + 60y$

$P(0, 0) = 100(0) + 60(0)$ or 0

$P(0, 12) = 100(0) + 60(12)$ or 720

$P(3, 10) = 100(3) + 60(10)$ or 900

$P(0, 8) = 100(0) + 60(8)$ or 480

3 of Model 28, 10 of Model 74

68b. $900

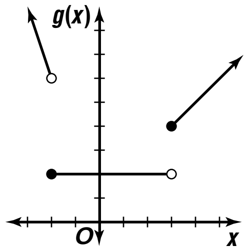
69. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 & 3 & -3 \\ -1 & -1 & -4 & -2 \end{bmatrix}$

$= \begin{bmatrix} 1(-2) + 0(-1) & 1(1) + 0(-1) & 1(3) + 0(-4) & 1(-3) + 0(-2) \\ 0(-2) + (-1)(-1) & 0(1) + (-1)(-1) & 0(3) + (-1)(-4) & 0(-3) + (-1)(-2) \end{bmatrix}$

$= \begin{bmatrix} -2 & 1 & 3 & -3 \\ 1 & 1 & 4 & 2 \end{bmatrix}$

$(-2, 1), (1, 1), (3, 4), (-3, 2)$

70.



71. $y = 14.7x + 140.1$
$y = 14.7(20) + 140.1$
$y = \$434.10$

72.

x	x^2	y
-4	$(-4)^2$	16
-3	$(-3)^2$	9
-2	$(-2)^2$	4

$\{(-4, 16), (-3, 9), (-2, 4)\}$; yes

73. $A = s^2$
$100 = s^2$
$10 = s$

radius $= \frac{1}{2}(10)$ or 5
$A = \pi r^2$
$A = \pi(5)^2$ or 25π
$4(25\pi) = 100\pi$

The correct choice is C.

Page 377 Mid-Chapter Quiz

1. $\frac{5\pi}{6} = \frac{5\pi}{6} \times \frac{180°}{\pi}$
$= 150°$

2. $r = \frac{1}{2}d$ $\qquad s = r\theta$
$r = \frac{1}{2}(0.5)$ or 0.25 $\qquad s = 0.25\left(\frac{5\pi}{3}\right)$
$s \approx 1.3$ m

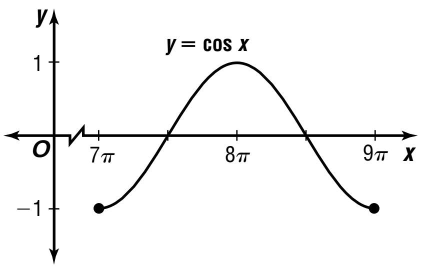
3. $A = \frac{1}{2}r^2\theta$
$A = \frac{1}{2}(8^2)\left(\frac{2\pi}{5}\right)$
$A \approx 40.2$ ft^2

4. $7.8 \times 2\pi = 15.6\pi$ or about 49.0 radians

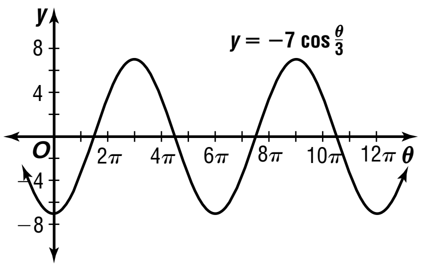
5. $8.6 \times 2\pi = 17.2\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{17.2\pi}{7}$
$\omega \approx 7.7$ radians/s

6. $v = r\omega$
$v = 3(8\pi)$
$v \approx 75.4$ meters/s

7. 1

8. 

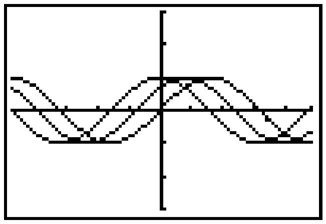
9. $|-7| = 7$; $\frac{2\pi}{\frac{1}{3}} = 6\pi$



10. $|A| = 5$ $\qquad \frac{2\pi}{k} = \frac{\pi}{3}$
$A = \pm 5$ $\qquad k = 6$
$y = \pm 5 \sin 6\theta$

6-5 Translations of Sine and Cosine Functions

Page 378 Graphing Calculator Exploration

1. 

2. The graph shifts farther to the left.
3. The graph shifts farther to the right.

Page 383 Check for Understanding

1. Both graphs are the sine curve. The graph of $y = \sin x + 1$ has a vertical shift of 1 unit upward, while the graph of $y = \sin(x + 1)$ has a horizontal shift of 1 unit to the left.
2. sine function

3a. increase $|A|$

3b. decrease h

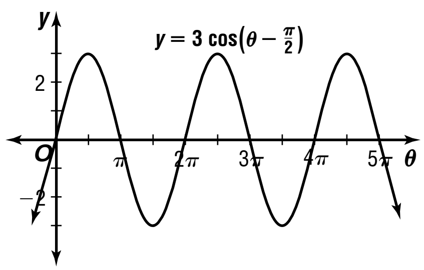
3c. increase $|k|$

3d. increase c

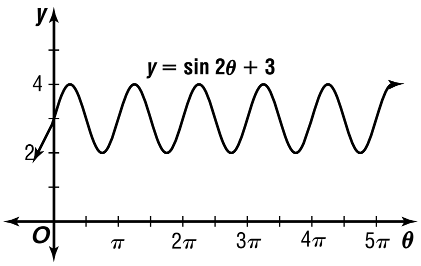
4. Graph $y = \sin x$ and $y = \cos x$, and find the sum of their ordinates.

5. Jamal; $-\frac{c}{k} = -\frac{-\frac{\pi}{2}}{\frac{\pi}{6}}$ or 3

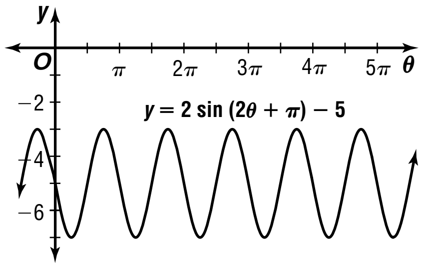
6. $\frac{\pi}{2}$



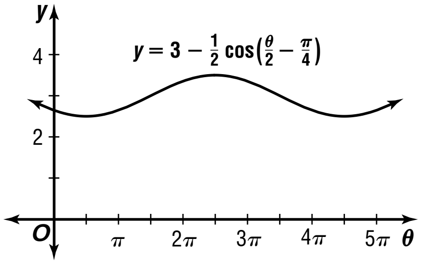
7. 3; $y = 3$
$A = 1$;
$\frac{2\pi}{2} = \pi$



8. $|2| = 2; \frac{2\pi}{2} = \pi; -\frac{\pi}{2}; -5$



9. $\left|-\frac{1}{2}\right| = \frac{1}{2}; \frac{2\pi}{\frac{1}{2}} = 4\pi; -\frac{-\frac{\pi}{4}}{\frac{1}{2}} = \frac{\pi}{2}; 3$



10. $|A| = 20 \qquad \frac{2\pi}{k} = 1 \qquad c = 0$

$A = \pm 20 \qquad k = 2\pi \qquad h = 100$

$y = \pm 20 \sin 2\pi\theta + 100$

11. $|A| = 0.6 \qquad \frac{2\pi}{k} = 12.4$

$A = \pm 0.6 \qquad k = \frac{\pi}{6.2}$

$-\frac{c}{k} = -2.13 \qquad h = 7$

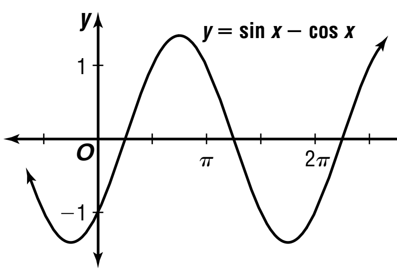
$-\frac{c}{\frac{\pi}{6.2}} = -2.13$

$c = \frac{2.13\pi}{6.2}$

$y = \pm 0.6 \cos\left(\frac{\pi}{6.2}\theta + \frac{2.13\pi}{6.2}\right) + 7$

12.

x	$\sin x - \cos x$	y
0	$\sin 0 - \cos 0$	-1
$\frac{\pi}{2}$	$\sin \frac{\pi}{2} - \cos \frac{\pi}{2}$	1
π	$\sin \pi - \cos \pi$	1
$\frac{3\pi}{2}$	$\sin \frac{3\pi}{2} - \cos \frac{3\pi}{2}$	-1
2π	$\sin 2\pi - \cos 2\pi$	-1



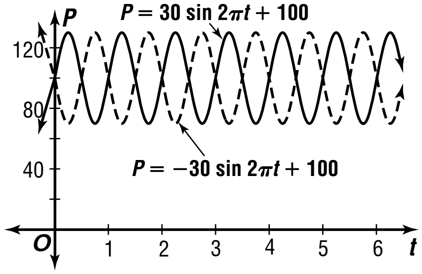
13a. $\frac{130 + 70}{2} = 100; P = 100$

13b. $|A| = \frac{130 - 70}{2} \qquad \frac{2\pi}{k} = 1$

$|A| = 30 \qquad k = 2\pi$

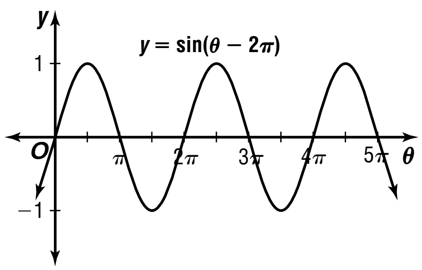
$A = \pm 30 \qquad P = \pm 30 \sin 2\pi t + 100$

13c.

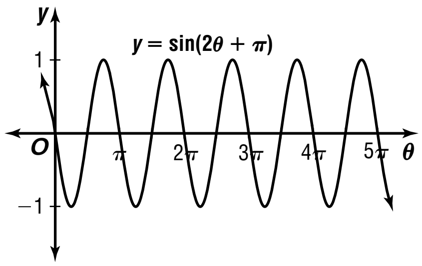


Pages 383–386 Exercises

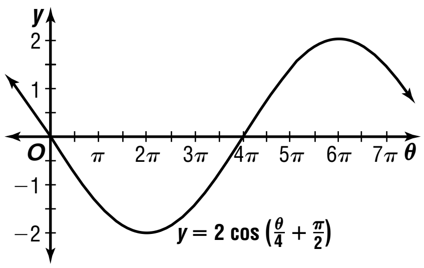
14. $-\frac{c}{k} = -\frac{-2\pi}{1}$ or $2\pi; A = 1; \frac{2\pi}{1} = 2\pi$



15. $-\frac{c}{k} = -\frac{\pi}{2}; A = 1; \frac{2\pi}{2} = \pi$

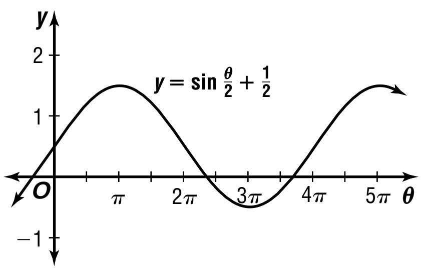


16. $-\frac{c}{k} = -\frac{\frac{\pi}{2}}{\frac{1}{4}}$ or $-2\pi; A = 2; \frac{2\pi}{\frac{1}{4}} = 8\pi$



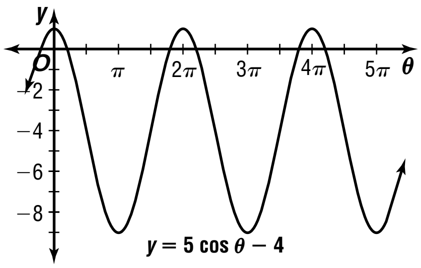
17. $\frac{1}{2}; y = \frac{1}{2}$

$A = 1; \frac{2\pi}{\frac{1}{2}} = 4\pi$



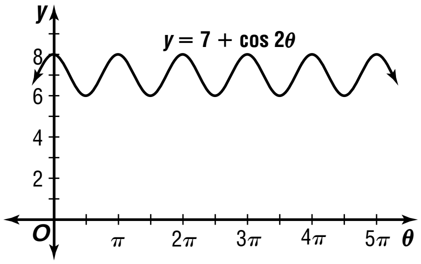
18. $-4; y = -4$

$A = 5; \frac{2\pi}{1} = 2\pi$



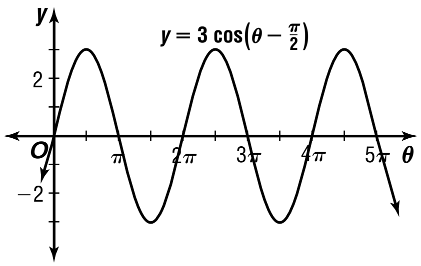
19. $7; y = 7$

$A = 1; \frac{2\pi}{2} = \pi$

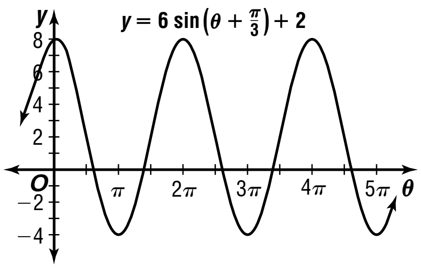


20. $-\frac{c}{k} = -\frac{-4\pi}{2}$ or $2\pi; -3$

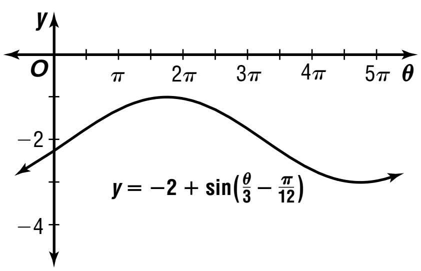
21. $|3| = 3; \frac{2\pi}{1} = 2\pi; \frac{-\frac{\pi}{2}}{1} = \frac{\pi}{2}; 0$



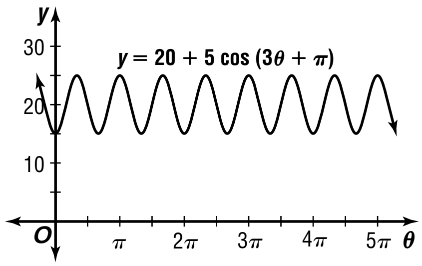
22. $|6| = 6; \frac{2\pi}{1} = 2\pi; -\frac{\frac{\pi}{3}}{1} = -\frac{\pi}{3}; 2$



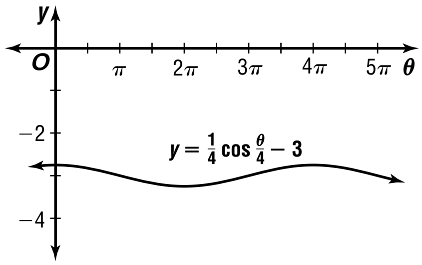
23. $|1| = 1; \frac{2\pi}{\frac{1}{3}} = 6\pi; -\frac{-\frac{\pi}{12}}{\frac{1}{3}} = \frac{\pi}{4}; -2$



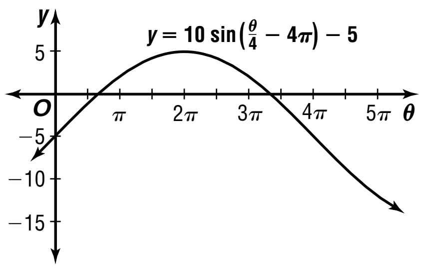
24. $|5| = 5; \frac{2\pi}{3}; -\frac{\pi}{3}; 20$



25. $\left|\frac{1}{4}\right| = \frac{1}{4}; \frac{2\pi}{\frac{1}{2}} = 4\pi; -\frac{0}{\frac{1}{2}} = 0; -3$



26. $|10| = 10; \frac{2\pi}{\frac{1}{4}} = 8\pi; -\frac{-4\pi}{\frac{1}{4}} = 16\pi; -5$



27. $|A| = \frac{2-(-6)}{2}$

$|A| = 4$

$A = \pm 4; 4$

$4\pi; \frac{\pi}{2}$ to the left or $-\frac{\pi}{2}$; down 2, or -2

28. $|A| = 7 \quad \frac{2\pi}{k} = 3\pi \quad -\frac{c}{\frac{2}{3}} = \pi \quad h = -7$

$A = \pm 7 \quad k = \frac{2}{3} \quad c = -\frac{2\pi}{3}$

$y = \pm 7\sin\left(\frac{2}{3}\theta - \frac{2\pi}{3}\right) - 7$

29. $|A| = 50 \quad \frac{2\pi}{k} = \frac{3\pi}{4} \quad -\frac{c}{\frac{8}{3}} = \frac{\pi}{2} \quad h = -25$

$A = \pm 50 \quad k = \frac{8}{3} \quad c = -\frac{4\pi}{3}$

$y = \pm 50\sin\left(\frac{8}{3}\theta - \frac{4\pi}{3}\right) - 25$

30. $|A| = \frac{3}{4} \quad \frac{2\pi}{k} = \frac{\pi}{5} \quad -\frac{c}{10} = \pi \quad h = \frac{1}{4}$

$A = \pm\frac{3}{4} \quad k = 10 \quad c = -10\pi$

$y = \pm\frac{3}{4}\sin(10\theta - 10\pi) + \frac{1}{4}$

31. $|A| = 3.5 \quad \frac{2\pi}{k} = \frac{\pi}{2} \quad -\frac{c}{4} = \frac{\pi}{4} \quad h = 7$

$A = \pm 3.5 \quad k = 4 \quad c = -\pi$

$y = \pm 3.5\cos(4\theta - \pi) + 7$

32. $|A| = \frac{4}{5} \quad \frac{2\pi}{k} = \frac{\pi}{6} \quad -\frac{c}{12} = \frac{\pi}{3} \quad h = \frac{7}{5}$

$A = \pm\frac{4}{5} \quad k = 12 \quad c = -4\pi$

$y = \pm\frac{4}{5}\cos(12\theta - 4\pi) + \frac{7}{5}$

33. $|A| = 100 \qquad \frac{2\pi}{k} = 45 \quad -\frac{c}{\frac{2\pi}{45}} = 0 \qquad h = -110$

$A = \pm 100 \qquad k = \frac{2\pi}{45} \qquad c = 0$

$y = \pm 100 \cos\left(\frac{2\pi}{45}\theta\right) - 110$

34. $|A| = \frac{1-(-3)}{2} \qquad \frac{2\pi}{k} = 4\pi \qquad h = -1$

$|A| = 2 \qquad k = \frac{1}{2}$

$A = \pm 2; -2$

$y = -2\cos\left(\frac{\theta}{2}\right) - 1$

35. $|A| = \frac{3.5-(2.5)}{2} \qquad \frac{2\pi}{k} = \pi \qquad -\frac{c}{2} = 0 \qquad h = 3$

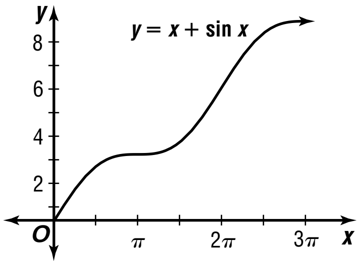
$|A| = 0.5 \qquad k = 2 \qquad c = 0$

$A = \pm 0.5; 0.5$

$y = 0.5 \sin 2\theta + 3$

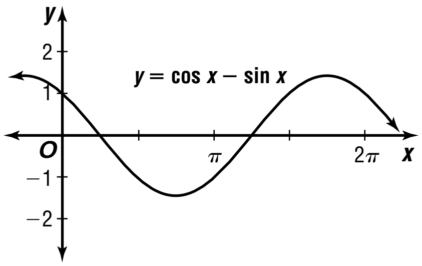
36.

x	$\sin x$	$\sin x + x$
0	0	0
$\frac{\pi}{2}$	1	$\frac{\pi}{2} + 1 \approx 2.57$
π	0	$\pi \approx 3.14$
$\frac{3\pi}{2}$	-1	$\frac{3\pi}{2} - 1 \approx 3.71$
2π	0	$2\pi \approx 6.28$



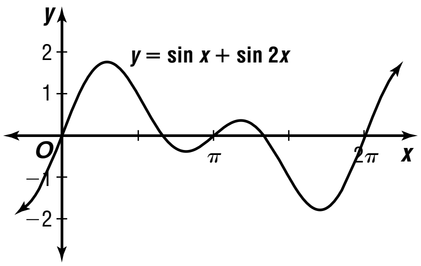
37.

x	$\cos x$	$\sin x$	$\cos x - \sin x$
0	1	0	1
$\frac{\pi}{2}$	0	1	-1
π	-1	0	-1
$\frac{3\pi}{2}$	0	-1	1
2π	1	0	1

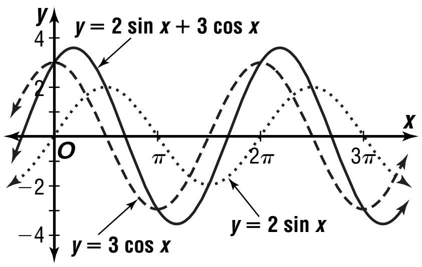


38.

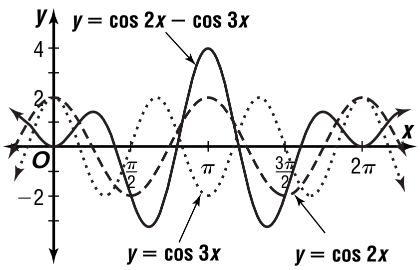
x	$\sin x$	$2x$	$\sin 2x$	$\sin x + \sin 2x$
0	0	0	0	0
$\frac{\pi}{4}$	0.71	$\frac{\pi}{2}$	1	1.71
$\frac{\pi}{2}$	1	π	0	1
π	0	2π	0	0
$\frac{3\pi}{2}$	-1	3π	0	-1
2π	0	4π	0	0



39.



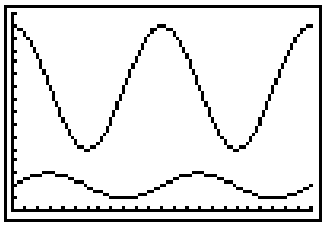
40.



41a. $2000 + 1000 = 3000$
$2000 - 1000 = 1000$

41b. $10{,}000 + 5000 = 15{,}000$
$10{,}000 - 5000 = 5000$

41c.



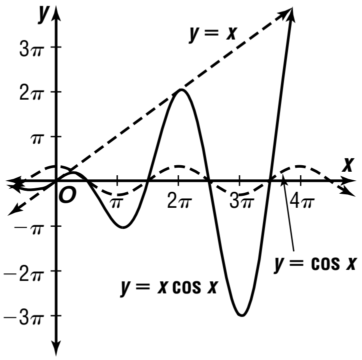
[0, 24] sc1:1 by [0, 16,000] sc1:1000

41d. months number 3 and 15

41e. months number 0, 12, 24

41f. When the sheep population is at a maximum, the wolf population is on the increase because of the maximum availability of food. The upswing in wolf population leads to a maximum later.

42.



43a. $46 - 42 = 4$ ft

43b. $r = \frac{1}{2}d$ $\qquad t = 21 + 4$

$r = \frac{1}{2}(42)$ $\qquad t = 25$

$r = 21$

43c. $\frac{3 \text{ revolutions}}{60 \text{ seconds}} = \frac{1 \text{ revolution}}{x \text{ seconds}}$

$x = 20$ s

43d. $|A| = 21$ $\qquad \frac{2\pi}{k} = 20$ $\qquad h = 25$

$A = \pm 21; 21$ $\qquad k = \frac{\pi}{10}$

$h = 25 + 21 \sin \frac{\pi t}{10}$

43e. $h = 25 + 21 \sin \left(\frac{\pi t}{10}\right)$

$46 = 25 + 21 \sin \left(\frac{\pi t}{10}\right)$

$1 = \sin \left(\frac{\pi t}{10}\right)$

$\sin^{-1} = \sin \left(\frac{\pi t}{10}\right)$

$5 = t$; 5 s

43f. $h = 25 + 21 \sin \left(\frac{\pi t}{10}\right)$

$h = 25 + 21 \sin \left(\frac{\pi \cdot 10}{10}\right)$

$h = 25$ ft

44. $-\frac{c}{k} = -\frac{0}{2}$ or 0

$-\frac{c}{k} = -\frac{-\frac{\pi}{2}}{2}$ or $\frac{\pi}{4}$

There is a $\frac{\pi}{4}$ phase difference.

45a. $y = \sqrt{\sin x}$ $\qquad$ **45b.** $y = \frac{\cos x}{x}$

45c. $y = \cos x^2$ $\qquad$ **45d.** $y = \sin \sqrt{x}$

46. $\frac{2\pi}{k} = \frac{1}{294}$

$k = 588\pi$ $\qquad y = 0.25 \sin 588\pi t$

47. $v = r\omega$

$v = 7(19.2)$

$v = 134.4$ cm/s

48. asymptote: $x = 2$

$y = \frac{x - 3}{x - 2}$

$y(x - 2) = x - 3$

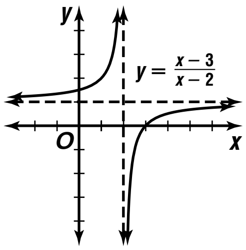
$yx - 2y = x - 3$

$-2y - 3 = x - yx$

$-2y - 3 = x(1 - y)$

$\frac{-2y - 3}{1 - y} = x$

asymptote: $y = 1$



49. $f(x) = \frac{3}{x - 1}$

$y = \frac{3}{x - 1}$

$x = \frac{3}{y - 1}$

$x(y - 1) = 3$

$y - 1 = \frac{3}{x}$

$y = \frac{3}{x} + 1$

50. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ -3 & -5 \end{bmatrix} = X$

$\begin{bmatrix} 1(3) + 1(-3) & 1(5) + 1(-5) \\ 1(3) + 1(-3) & 1(5) + 1(-5) \end{bmatrix} = X$

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = X$

51. $7(3x + 5y) = 7(4)$ $\qquad 21x + 35y = 28$

$14x - 35y = 21$ $\quad \rightarrow \quad$ $\underline{14x - 35y = 21}$

$35x \qquad = 49$

$x = 1.4$

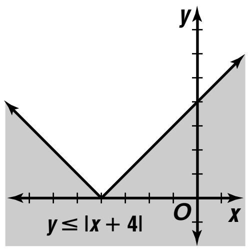
$3x + 5y = 4$

$3(1.4) + 5y = 4$

$y = -0.04$ $\qquad (1.4, -0.04)$

52.

x	$\lvert x + 4 \rvert$	y
-6	$\lvert -6 + 4 \rvert$	2
-4	$\lvert -4 + 4 \rvert$	0
-2	$\lvert -2 + 4 \rvert$	2
0	$\lvert 0 + 4 \rvert$	4



53. $3x - y + 7 = 0$ $\qquad y - y_1 = m(x - x_1)$

$y = 3x + 7$ $\qquad y - (-2) = 3(x - 3)$

slope: 3 $\qquad y + 2 = 3x - 9$

$3x - y - 11 = 0$

54. 4 inches $= \frac{1}{3}$ foot

$75 \times 42 \times \frac{1}{3} = 1050$ cubic feet

$1050 \times 7.48 = 7854$ gal

The correct answer is 7854.

6-6 Modeling Real-World Data with Sinusoidal Functions

Pages 390–391 Check for Understanding

1. any function that can be written as a sine function or a cosine function
2. Both data that can be modeled with a polynomial function and data that can be modeled with a sinusoidal function have fluctuations. However, data that can be modeled with a sinusoidal function repeat themselves periodically, and data that can be modeled with a polynomial function do not.
3. Sample answers: the amount of daylight, the average monthly temperatures, the height of a seat on a Ferris wheel

4a. $y = -5\cos\left(\frac{\pi}{6}t\right)$

$y = -5\cos\left(\frac{\pi}{6}\cdot 0\right)$

$y = -5$

5 units below equilibrium

4b. 5 units above equilibrium

4c. $y = -5\cos\left(\frac{\pi}{6}t\right)$

$y = -5\cos\left(\frac{\pi}{6}\cdot 7\right)$

$y = -4.33$

about 4.33 units above equilibrium

5. $A = \frac{140 - 80}{2}$ $h = \frac{140 + 80}{2}$

$A = 30$ $h = 110$

$\frac{2\pi}{k} = 1$

$k = 2\pi$ $P = 30\sin 2\pi t + 110$

6a. $A = \frac{66° - 41°}{2}$

$A = 12.5°$

6b. $h = \frac{66° + 41°}{2}$

$h = 53.5°$

6c. 12 months

6d. $A = \pm 12.5$ $\frac{2\pi}{k} = 12$ $h = 53.5$

$k = \frac{\pi}{6}$

$$y = -12.5\cos\left(\frac{\pi}{6}t + c\right) + 53.5$$
$$41 = -12.5\cos\left(\frac{\pi}{6}\cdot 1 + c\right) + 53.5$$
$$-12.5 = -12.5\cos\left(\frac{\pi}{6} + c\right)$$
$$1 = \cos\left(\frac{\pi}{6} + c\right)$$
$$\cos^{-1} 1 = \frac{\pi}{6} + c$$
$$\cos^{-1} 1 - \frac{\pi}{6} = c$$
$$-0.5 \approx c$$

Sample answer: $y = -12.5\cos\left(\frac{\pi}{6}t - 0.5\right) + 53.5$

6e. $y = -12.5\cos\left(\frac{\pi}{6}t - 0.5\right) + 53.5$

$y = -12.5\cos\left(\frac{\pi}{6}(2) - 0.5\right) + 53.5$

$y \approx 42.82517529$

Sample answer: About 42.8°; it is somewhat close to the actual average.

6f. $y = -12.5\cos\left(\frac{\pi}{6}t - 0.5\right) + 53.5$

$y = -12.5\cos\left(\frac{\pi}{6}(10) - 0.5\right) + 53.5$

$y \approx 53.20504268$

Sample answer: About 53.2°; it is close to the actual average.

Pages 391–394 Exercises

7a. 0.5

7b. $\frac{2\pi}{k} = 660\pi$

$k = \frac{1}{330}$

7c. $\frac{1}{\frac{1}{330}} = 330$ hertz

8a. $3.5 + |-3| = 6.5$ units

8b. $3.5 - 3 = 0.5$ units

8c. $\frac{2\pi}{k} = \frac{2\pi}{\frac{5\pi}{3}}$ or $\frac{6}{5}$

8d. $h = -3\cos\left(\frac{5\pi}{3}t\right) + 3.5$

$h = -3\cos\left(\frac{5\pi}{3}(25)\right) + 3.5$

$h = 2$ units

9a. $R = 1200 + 300\sin\left(\frac{\pi}{2}t\right)$

$R = 1200 + 300\sin\left(\frac{\pi}{2}\cdot 0\right)$

$R = 1200$

9b. $H = 250 + 25\sin\left(\frac{\pi}{2}t - \frac{\pi}{4}\right)$

$H = 250 + 25\sin\left(\frac{\pi}{2}\cdot 0 - \frac{\pi}{4}\right)$

$H \approx 232$

9c. R: $1200 + 300 = 1500$

H: $250 + 25 = 275$ no

9d.
$$R = 1200 + 300\sin\left(\frac{\pi}{2}t\right)$$
$$1500 = 1200 + 300\sin\left(\frac{\pi}{2}t\right)$$
$$300 = 300\sin\frac{\pi}{2}t$$
$$1 = \sin\frac{\pi}{2}t$$
$$\sin^{-1} 1 = \frac{\pi}{2}t$$
$$\sin^{-1} 1\left(\frac{2}{\pi}\right) = t$$
$$1 = t$$

January 1, 1971

9e. $250 - 25 = 225$

$$H = 250 + 25\sin\left(\frac{\pi}{2}t - \frac{\pi}{4}\right)$$
$$225 = 250 + 25\sin\left(\frac{\pi}{2}t - \frac{\pi}{4}\right)$$
$$-25 = 25\sin\left(\frac{\pi}{2}t - \frac{\pi}{4}\right)$$
$$-1 = \sin\left(\frac{\pi}{2}t - \frac{\pi}{4}\right)$$
$$\sin^{-1} - 1 = \frac{\pi}{2}t - \frac{\pi}{4}$$
$$\frac{2}{\pi}\left(\sin^{-1} - 1 + \frac{\pi}{4}\right) = t$$
$$-0.5 = t$$

July 1, 1969; $\frac{2\pi}{k} = \frac{2\pi}{\frac{\pi}{2}}$

$k = 4$

next minimum: July 1, 1973

9f. See students' work.

10. $A = \frac{4}{2}$ $\frac{2\pi}{k} = 10$

$A = 2$ $k = \frac{\pi}{5}$

$y = 2\cos\left(\frac{\pi}{5}t\right)$

11. $h = 4.25$; $A = 3.55$; $\frac{2\pi}{k} = 12.40$; $-\frac{c}{\frac{\pi}{6.2}} = -4.68$

$k = \frac{\pi}{6.2}$ $c = \frac{2.34\pi}{3.1}$

$y = 3.55\sin\left(\frac{\pi}{6.2}t + \frac{2.34\pi}{3.1}\right) + 4.24$

12a. $h = 47.5$; $A = 23.5$; $\frac{2\pi}{k} = 12$; $-\frac{c}{\frac{\pi}{6}} = 4$

$k = \frac{\pi}{6}$ $c = -\frac{2\pi}{3}$

$y = 23.5\sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right) + 47.5$

12b. $y = 23.5 \sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right) + 47.5$

$y = 23.5 \sin\left(\frac{\pi}{6} \cdot 3 - \frac{2\pi}{3}\right) + 47.5$

$y = 35.75$

about 35.8°

12c. $y = 23.5 \sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right) + 47.5$

$y = 23.5 \sin\left(\frac{\pi}{6} \cdot 8 - \frac{2\pi}{3}\right) + 47.5$

$y \approx 67.9°$

13a. $A = \frac{81° - 73°}{2}$

$A = 4°$

13b. $h = \frac{81° + 73°}{2}$

$h = 77°$

13c. 12 months

13d. $A = \pm 4$ $\quad \frac{2\pi}{k} = 12$ $\quad h = 77$

$k = \frac{\pi}{6}$

$y = -4 \cos\left(\frac{\pi}{6}t + c\right) + 77$

$73 = -4 \cos\left(\frac{\pi}{6} \cdot 1 + c\right) + 77$

$-4 = -4 \cos\left(\frac{\pi}{6} + c\right)$

$1 = \cos\left(\frac{\pi}{6} + c\right)$

$\cos^{-1} 1 = \frac{\pi}{6} + c$

$\cos^{-1} 1 - \frac{\pi}{6} = c$

$-0.5235987756 \approx c$

Sample answer: $y = -4 \cos\left(\frac{\pi}{6}t - 0.5\right) + 77$

13e. $y = -4 \cos\left(\frac{\pi}{6}t - 0.5\right) + 77$

$y = -4 \cos\left(\frac{\pi}{6} \cdot 8 - 0.5\right) + 77$

$y \approx 80.41594391$

Sample answer: About 80.4°; it is very close to the actual average.

13f. $y = -4 \cos\left(\frac{\pi}{6}t - 0.5\right) + 77$

$y = -4 \cos\left(\frac{\pi}{6} \cdot 5 - 0.5\right) + 77$

$y \approx 79.08118409$

Sample answer: About 79.1°; it is close to the actual average.

14. $-\frac{c}{k} = -\frac{-\pi}{1}$ or π

increase shift by $\frac{\pi}{2}$; $\pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$-\frac{c}{k} = \frac{3\pi}{2}$

$-\frac{c}{1} = \frac{3\pi}{2}$

$c = -\frac{3\pi}{2}$

Sample answer: $y = 3 \cos\left(x - \frac{3\pi}{2}\right) + 5$

15a. $A = \frac{13.25 - 1.88}{2}$

$A = 5.685$ ft

15b. $h = \frac{13.25 + 1.88}{2}$

$h = 7.565$ ft

15c. 4:53 P.M. − 4:30 A.M. = 12:23 or about 12.4 h

15d. $A = \pm 5.685$ $\quad \frac{2\pi}{k} = 12.4$ $\quad h = 7.565$

$k = \frac{\pi}{6.2}$

4:30 A.M. = 4.5 hrs

$h = 5.685 \sin\left(\frac{\pi}{6.2}t + c\right) + 7.565$

$13.25 = 5.685 \sin\left(\frac{\pi}{6.2} \cdot 4.5 + c\right)$

$5.685 = 5.685 \sin\left(\frac{4.5\pi}{6.2} + c\right)$

$1 = \sin\left(\frac{4.5\pi}{6.2} + c\right)$

$\sin^{-1} 1 = \frac{4.5\pi}{6.2} + c$

$\sin^{-1} 1 - \frac{4.5\pi}{6.2} = c$

$-0.7093918895 \approx c$

Sample answer: $h = 5.685 \sin\left(\frac{\pi}{6.2}t - 0.71\right) + 7.565$

15e. 7:30 P.M. = 19.5 hrs

$h = 5.685 \sin\left(\frac{\pi}{6.2}t - 0.71\right) + 7.565$

$h = 5.685 \sin\left(\frac{\pi}{6.2} \cdot 19.5 - 0.71\right) + 7.565$

$h \approx 8.993306129$

Sample answer: about 8.99 ft

16a. Table at bottom of page.

Month	Sunrise A.M.	A.M. Time in Decimals	Sunset P.M.	P.M. Time in Decimals	Daylight Hours (P.M.-A.M.)
January	7:19	7.317	4:47	16.783	9.47 h
February	6:56	6.933	5:24	17.4	10.47 h
March	6:16	6.267	5:57	17.95	11.68 h
April	5:25	5.416	6:29	18.483	13.07 h
May	4:44	4.733	7:01	19.017	14.28 h
June	4:24	4.4	7:26	19.433	15.03 h
July	4:33	4.55	7:28	19.467	14.92 h
August	5:01	5.017	7:01	19.017	14 h
September	5:31	5.517	6:14	18.233	12.72 h
October	6:01	6.017	5:24	17.4	11.38 h
November	6:36	6.6	4:43	16.717	10.12 h
December	7:08	7.133	4:28	16.467	9.33 h

16b. $A = \frac{15.03 - 9.33}{2}$

$A = 2.85$ h

16c. $h = \frac{15.03 + 9.33}{2}$

$h = 12.18$ h

16d. 12 months

16e. $A = \pm 2.85 \qquad \frac{2\pi}{k} = 12 \qquad h = 12.18$

$k = \frac{\pi}{6}$

$$y = -2.85 \cos\left(\frac{\pi}{6}t + c\right) + 12.18$$

$$9.47 = -2.85 \cos\left(\frac{\pi}{6} \cdot 1 + c\right) + 12.18$$

$$-2.71 = -2.85 \cos\left(\frac{\pi}{6} + c\right)$$

$$0.950877193 \approx \cos\left(\frac{\pi}{6} + c\right)$$

$$\cos^{-1} 0.950877193 \approx \frac{\pi}{6} + c$$

$$\cos^{-1} 0.950877193 - \frac{\pi}{6} \approx c$$

$$-0.2088597251 \approx c$$

Sample answer: $y = -2.85 \cos\left(\frac{\pi}{6}t - 0.21\right) + 12.18$

17. $70.5 - 19.5 = 51$

$$y = 70.5 + 19.5 \sin\left(\frac{\pi}{6}t + c\right)$$

$$51 = 70.5 + 19.5 \sin\left(\frac{\pi}{6} \cdot 1 + c\right)$$

$$-19.5 = 19.5 \sin\left(\frac{\pi}{6} + c\right)$$

$$-1 = \sin\left(\frac{\pi}{6} + c\right)$$

$$\sin^{-1} - 1 = \frac{\pi}{6} + c$$

$$\sin^{-1} - 1 - \frac{\pi}{6} = c$$

$$-2.094395102 \approx c$$

Sample answer: about -2.09

18a. $\frac{14 \text{ revolutions}}{1 \text{ minute}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} \times \frac{2\pi \text{ radians}}{1 \text{ revolution}} = \frac{7\pi}{15}$ rad/s

$y = -3.5 \cos\left(\frac{7\pi}{15}t\right)$

18b. $y = -3.5 \cos\left(\frac{7\pi}{15}t\right)$

$y = -3.5 \cos\left(\frac{7\pi}{15} \cdot 4\right)$

$y \approx -3.197409102$

about $(4, -3.20)$

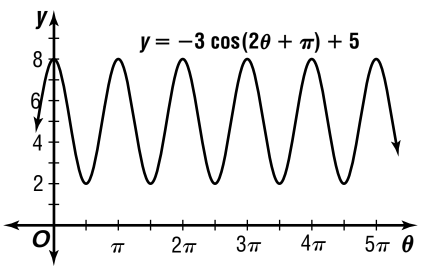
19. $A = \frac{120 - (-120)}{2} \qquad \frac{2\pi}{k} = 60$

$A = 120 \qquad k = \frac{\pi}{30}$

$V_R = 120 \sin\left(\frac{\pi}{30}t\right)$

20. See students' work.

21. $|-3| = 3$; $\frac{2\pi}{2} = \pi$; $-\frac{\pi}{2}$; 5



22. $2\pi n$ where n is an integer

23. $800° \times \frac{\pi}{180°} = \frac{40\pi}{9}$

24. $40^2 = 32^2 + 20^2 - 2(32)(20) \cos \theta$

$\cos \theta = \frac{40^2 - 32^2 - 20^2}{-2(32)(20)}$

$\theta = \cos^{-1}\left(\frac{40^2 - 32^2 - 20^2}{-2(32)(20)}\right)$

$\theta \approx 97.9°$

$180° - 97.9° = 82.1°$

about 97.9°, 82.1°, 97.9°, 82.1°

25. $\frac{2m + 16}{m^2 - 16} = \frac{2m + 16}{(m - 4)(m + 4)}$

$\frac{2m + 16}{(m - 4)(m + 4)} = \frac{A}{m - 4} + \frac{B}{m + 4}$

$2m + 16 = A(m + 4) + B(m - 4)$

Let $m = -4$.

$2(-4) + 16 = A(-4 + 4) + B(-4 - 4)$

$8 = -8B$

$-1 = B$

Let $m = 4$.

$2(4) + 16 = A(4 + 4) + B(4 - 4)$

$24 = 8A$

$3 = A$

$\frac{A}{m - 4} + \frac{B}{m + 4} = \frac{3}{m - 4} + \frac{-1}{m + 4}$

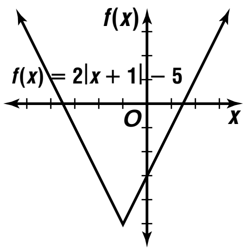
26.

$$\begin{array}{r|rrrr} -2 & 2 & k & -1 & -6 \\ & & -4 & -2k + 8 & 4k - 14 \\ \hline & 2 & k - 4 & -2k + 7 & 4k - 20 \end{array}$$

$4k - 20 = 0$

$k = 5$

27.



increasing: $x > -1$; decreasing: $x < -1$

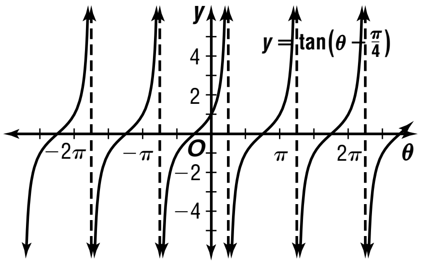
28. The correct choice is E.

6-7 Graphing Other Trigonometric Functions

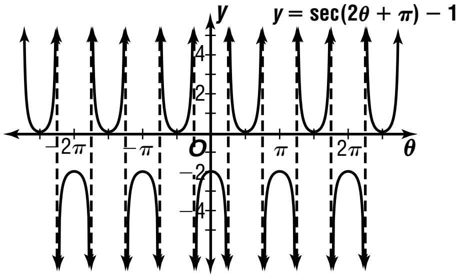
Page 400 Check for Understanding

1. Sample answers: $-\pi$, π, 2π
2. The asymptotes of $y = \tan \theta$ and $y = \sec \theta$ are the same. The period of $y = \tan \theta$ is π and the period of $y = \sec \theta$ is 2π.
3. Sample answers: $\frac{\pi}{2}$, $-\frac{3\pi}{2}$
4. 0
5. 1
6. πn, where n is an odd integer
7. $\frac{\pi}{4} + \pi n$, where n is an integer

8. $\frac{\pi}{1} = \pi$; $\frac{-\frac{\pi}{4}}{1} = -\frac{\pi}{4}$



9. $\frac{2\pi}{2} = \pi$; $-\frac{\pi}{2}$; $h = -1$



10. k: $\frac{2\pi}{k} = 3\pi$ $\quad$ c: $-\frac{c}{\frac{2}{3}} = \frac{\pi}{3}$ $\quad$ h: $h = -4$

$k = \frac{2}{3}$

$c = -\frac{2\pi}{9}$

$y = \csc\left(\frac{2}{3}\theta - \frac{2\pi}{9}\right) - 4$

11. k: $\frac{\pi}{k} = 2\pi$ $\quad$ c: $-\frac{c}{\frac{1}{2}} = -\frac{\pi}{4}$ $\quad$ h: $h = 0$

$k = \frac{1}{2}$

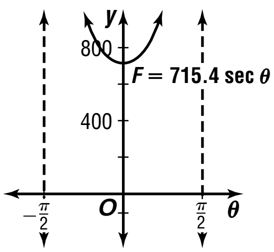
$c = \frac{\pi}{8}$

$y = \cot\left(\frac{1}{2}\theta + \frac{\pi}{8}\right)$

12a. $f = ma$
$f = 73(9.8)$
$f = 715.4$ N

12b. $F = f \sec\theta$
$F = 715.4 \sec\theta$

12c. $\frac{2\pi}{1} = 2\pi$; no phase shift, no vertical shift



12d. 715.4 N

12e. The tension becomes greater.

Pages 400–403 Exercises

13. 0

14. 0

15. undefined

16. −1

17. −1

18. undefined

19. undefined

20. 0

21. πn, where n is an integer

22. πn, where n is an even integer

23. $\frac{3\pi}{2} + 2\pi n$, where n is an integer

24. $\frac{\pi}{4} + \pi n$, where n is an integer

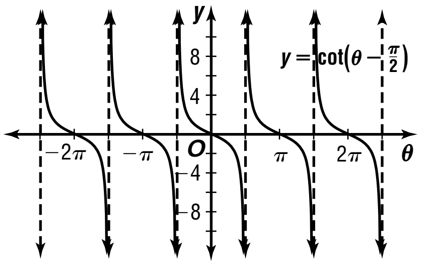
25. $-\frac{\pi}{4} + \pi n$, where n is an integer

26. $\frac{3\pi}{4} + \pi n$, where n is an integer

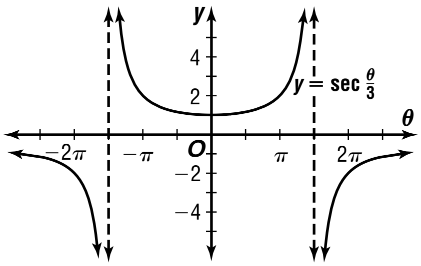
27. $\frac{\pi}{2}n$, where n is an odd integer

28. πn, where n is an integer

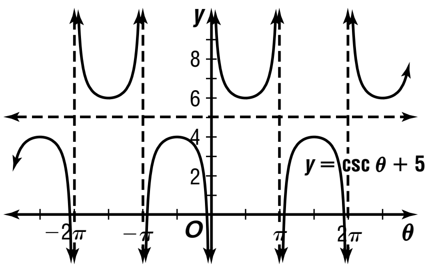
29. $\frac{\pi}{1} = \pi$; $-\frac{-\frac{\pi}{2}}{1} = \frac{\pi}{2}$



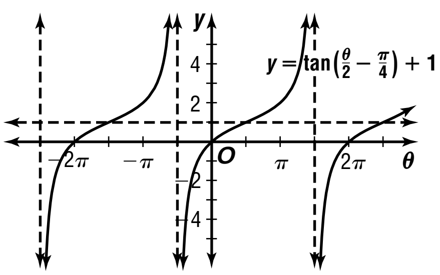
30. $\frac{2\pi}{\frac{1}{3}} = 6\pi$; no phase shift; no vertical shift



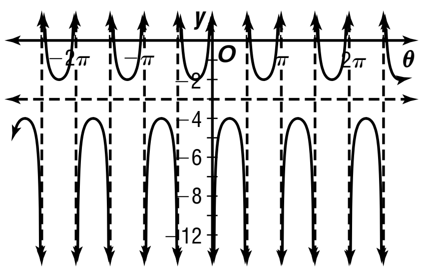
31. $\frac{2\pi}{1} = 2\pi$; no phase shift; $h = 5$



32. $\frac{\pi}{\frac{1}{2}} = 2\pi$; $-\frac{-\frac{\pi}{4}}{\frac{1}{2}} = \frac{\pi}{2}$; $h = 1$

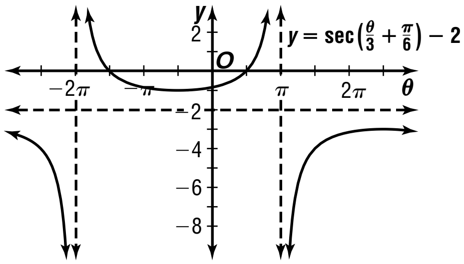


33. $\frac{2\pi}{2} = \pi$; $-\frac{\pi}{2}$; $h = -3$

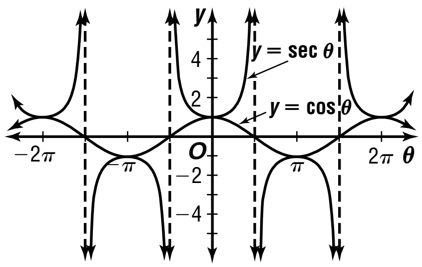


$y = \csc(2\theta + \pi) - 3$

34. $\frac{2\pi}{\frac{1}{3}} = 6\pi$; $-\frac{\frac{\pi}{6}}{\frac{1}{3}} = -\frac{\pi}{2}$; $h = -2$



35.



$-2\pi, -\pi, 0, \pi, 2\pi$

36. k: $\frac{\pi}{k} = 2\pi$ $\quad$ c: $-\frac{c}{\frac{1}{2}} = 0$ $\quad$ h: $h = -6$

$k = \frac{1}{2}$ $\quad$ $c = 0$

$y = \tan\frac{\theta}{2} - 6$

37. k: $\frac{\pi}{k} = \frac{\pi}{2}$ $\quad$ c: $-\frac{c}{2} = \frac{\pi}{8}$ $\quad$ h: $h = 7$

$k = 2$ $\quad$ $c = -\frac{\pi}{4}$

$y = \cot\left(2\theta - \frac{\pi}{4}\right) + 7$

38. k: $\frac{2\pi}{k} = \pi$ $\quad$ c: $-\frac{c}{2} = -\frac{\pi}{4}$ $\quad$ h: $h = -10$

$k = 2$ $\quad$ $c = \frac{\pi}{2}$

$y = \sec\left(2\theta + \frac{\pi}{2}\right) - 10$

39. k: $\frac{2\pi}{k} = 3\pi$ $\quad$ c: $-\frac{c}{\frac{2}{3}} = \pi$ $\quad$ h: $h = -1$

$k = \frac{2}{3}$ $\quad$ $c = -\frac{2\pi}{3}$

$y = \csc\left(\frac{2}{3}\theta - \frac{2\pi}{3}\right) - 1$

40. k: $\frac{\pi}{k} = 5\pi$ $\quad$ c: $-\frac{c}{\frac{1}{5}} = -\pi$ $\quad$ h: $h = 12$

$k = \frac{1}{5}$ $\quad$ $c = \frac{\pi}{5}$

$y = \cot\left(\frac{\theta}{5} + \frac{\pi}{5}\right) + 12$

41. k: $\frac{2\pi}{k} = \frac{\pi}{3}$ $\quad$ c: $-\frac{c}{6} = -\frac{\pi}{2}$ $\quad$ h: $h = -5$

$k = 6$ $\quad$ $c = 3\pi$

$y = \csc\,(6\theta + 3\pi) - 5$

42. k: $\frac{2\pi}{k} = 3\pi$ $\quad$ c: $-\frac{c}{\frac{2}{3}} = -\pi$ $\quad$ h: $h = -8$

$k = \frac{2}{3}$ $\quad$ $c = \frac{2\pi}{3}$

$y = \sec\left(\frac{2}{3}\theta + \frac{2\pi}{3}\right) - 8$

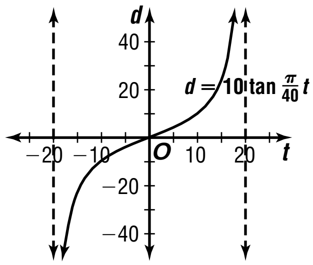
43. k: $\frac{\pi}{k} = \frac{\pi}{2}$ $\quad$ c: $-\frac{c}{2} = \frac{\pi}{4}$ $\quad$ h: $h = 7$

$k = 2$ $\quad$ $c = -\frac{\pi}{2}$

$y = \tan\left(2\theta - \frac{\pi}{2}\right) + 7$

44a. $\frac{\pi}{\frac{\pi}{40}} = 40$

no phase shift
no vertical shift



44b. $d = 10\tan\frac{\pi}{40}t$

$d = 10\tan\frac{\pi}{40}(3)$

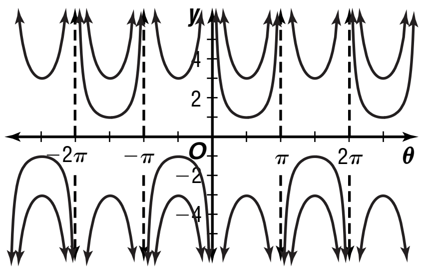
$d \approx 2.4$ ft from the center

44c. $d = 10\tan\frac{\pi}{40}t$

$d = 10\tan\frac{\pi}{40}(15)$

$d \approx 24.1$ ft from the center

45.

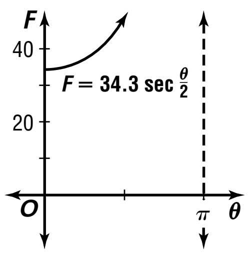


The graph of $y = \csc\theta$ has no range values between -1 and 1, while the graphs of $y = 3\csc\theta$ and $y = -3\csc\theta$ have no range values between -3 and 3. The graphs of $y = 3\csc\theta$ and $y = -3\csc\theta$ are reflections of each other.

46a. $f = m \cdot 9.8$
$f = 7 \cdot 9.8$
$f = 68.6$ N

46b. $F = \frac{1}{2}f\sec\frac{\theta}{2}$
$F = \frac{1}{2}(68.6)\sec\frac{\theta}{2}$
$F = 34.3\sec\frac{\theta}{2}$

46c. $\frac{2\pi}{\frac{1}{2}} = 4\pi$



46d. 34.3 N

46e. The tension becomes greater.

47a. 220 A

47b. $\frac{2\pi}{60\pi} = \frac{1}{30}$ s

47c. $-\frac{-\frac{\pi}{6}}{60\pi} = \frac{1}{360}$

47d. $I = 220 \sin\left(60\pi t - \frac{\pi}{6}\right)$

$I = 220 \sin\left(60\pi \cdot 60 - \frac{\pi}{6}\right)$

$I \approx -110$ A

48. $y = -1 \tan\left(\theta + \frac{\pi}{2}\right)$

49a. $A = \frac{3.99 - 0.55}{2}$

$A = 1.72$ ft

49b. $h = \frac{3.99 + 0.55}{2}$

$h = 2.27$ ft

49c. 12:19 P.M. − 12:03 A.M. = 12:16 or about 12.3 hr

49d. $A = \pm 1.72$ $\quad \frac{2\pi}{k} = 12.3$ $\quad h = 2.27$

$k = \frac{2\pi}{12.3}$

12:03 = 0.05 hr since midnight

$$h = 1.72 \sin\left(\frac{2\pi}{12.3} t + c\right) + 2.27$$
$$3.99 = 1.72 \sin\left(\frac{2\pi}{12.3} \cdot 0.05 + c\right) + 2.27$$
$$1.72 = 1.72 \sin\left(\frac{0.1\pi}{12.3} + c\right)$$
$$1 = \sin\left(\frac{0.1\pi}{12.3} + c\right)$$
$$\sin^{-1} 1 = \frac{0.1\pi}{12.3} + c$$
$$\sin^{-1} 1 - \frac{0.1\pi}{12.3} = c$$
$$1.545254923 \approx c$$

Sample answer: $h = 1.72 \sin\left(\frac{2\pi}{12.3} t + 1.55\right) + 2.27$

49e. noon = 12 hrs since midnight

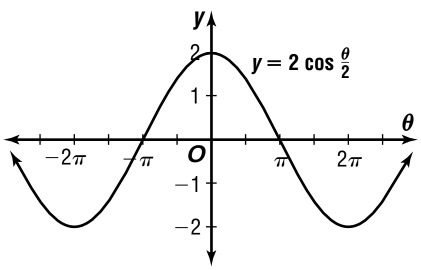
$h = 1.72 \sin\left(\frac{2\pi}{12.3} t + 1.55\right) + 2.27$

$h = 1.72 \sin\left(\frac{2\pi}{12.3} \cdot 12 + 1.55\right) + 2.27$

$h \approx 3.964014939$

Sample answer: 3.96 ft

50. $|2| = 2;\ \frac{2\pi}{\frac{1}{2}} = 4\pi$



51. $s = r\theta$

$s = 18\left(\frac{\pi}{3}\right)$

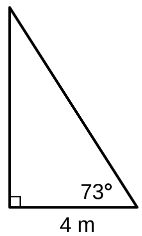
$s = 6\pi$ cm

52. $C = 180° - (62°31' + 75°18')$ or $42°11'$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{57.3}{\sin 62°31'} = \frac{b}{\sin 75°18'}$$
$$b = \frac{57.3 \sin 75°18'}{\sin 62°31'}$$
$$b \approx 62.47505783$$
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{57.3}{\sin 62°31'} = \frac{c}{\sin 42°11'}$$
$$c = \frac{57.3 \sin 42°11'}{\sin 62°31'}$$
$$c \approx 43.37198044$$

$C = 42°11'$, $b = 62.5$, $c = 43.4$

53a.



53b. $\tan 73° = \frac{x}{4}$

$x = 4 \tan 73°$

$x \approx 13.1$ m

53c. $\cos 73° = \frac{4}{y}$

$y = \frac{4}{\cos 73°}$

$y \approx 13.7$ m

54. $a^2 + b^2 = c^2$

$7^2 + 4^2 = c^2$

$\sqrt{65} = c$

$\cos A = \frac{b}{c}$

$\cos A = \frac{4}{\sqrt{65}}$

$\cos A = \frac{4\sqrt{65}}{65}$

$\sin A = \frac{a}{c}$

$\sin A = \frac{7}{\sqrt{65}}$

$\sin A = \frac{7\sqrt{65}}{65}$

$\tan A = \frac{a}{b}$

$\tan A = \frac{7}{4}$

55. $\frac{x^2 - 4}{x^2 - 3x - 10} \le 0$

$\frac{(x-2)(x+2)}{(x-5)(x+2)} \le 0$

zeros: 2, −2

excluded values: 5, −2

Test −3: $\frac{(-3)^2 - 4}{(-3)^2 - 3(-3) - 10} \le 0$

$\frac{5}{5} \le 0$ false

Test 0: $\frac{0^2 - 4}{0^2 - 3(0) - 10} \le 0$

$\frac{-4}{-10} \le 0$ false

Test 3: $\frac{3^2 - 4}{3^2 - 3(3) - 10} \le 0$

$\frac{5}{-10} \le 0$ true

Test 6: $\frac{6^2 - 4}{6^2 - 3(6) - 10} \le 0$

$\frac{32}{8} \le 0$ false

$-2 < x < 5$

56. $k = \frac{6}{0.5}$

$k = 12$

$t = kr$

$10 = 12r$

$r \approx 0.83$

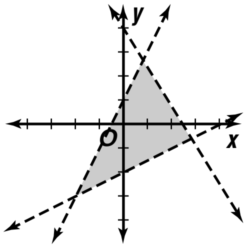
57. $3x + 2y < 8$

$y < -\frac{3}{2}x + 4$

$y < 2x + 1$

$-2y < -x + 4$

$y > \frac{1}{2}x - 2$



58. $y = 17.98x + 35.47$; 0.88

59. A: impossible to tell

B: $6(150) \stackrel{?}{=} 10(90)$

$900 = 900$; true

C: impossible to tell

D: $150 \stackrel{?}{=} 30 + 2(90)$

$150 = 210$; false

E: $3(90) \stackrel{?}{=} 30 + 2(150)$

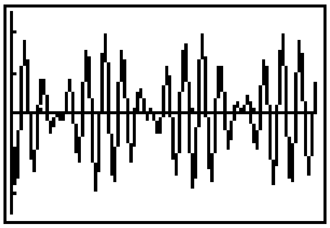
$270 = 330$; false

The correct choice is B.

6-7B Sound Beats

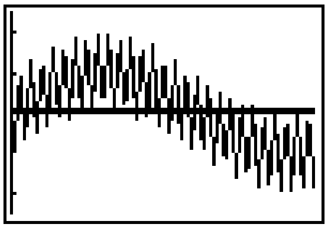
Page 404

1.



the third graph

2.



Sample answer: The graph seems to stay above the x-axis for an interval of x values, and then stay below the x-axis for another interval of x values.

3. 0.38623583

4. no

5. -1.78043; yes; the value for $f(x)$ is negative and corresponds to a point not graphed by the calculator.

6. Sample answer: As you move 1 pixel to the left or right of any pixel on the screen, the x-value for the adjacent pixel decreases or increases by almost 7. Thus, the "find" behavior of the function cannot be observed from the graph unless you change the interval of numbers for the x-axis.

7. See students' work.

8. Yes; no, they only provide plausible visual evidence.

6-8 Trigonometric Inverses and Their Graphs

Page 410 Check for Understanding

1. $y = \sin^{-1} x$ is the inverse relation of $y = \sin x$, $y = (\sin x)^{-1}$ is the function $y = \frac{1}{\sin x}$, and $y = \sin(x^{-1})$ is the function $y = \sin \frac{1}{x}$.

2. For every y value there are more than one x value. The graph of $y = \cos^{-1} x$ fails the vertical line test.

3. The domain of $y = \operatorname{Sin} x$ is the set of real numbers between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, inclusive, while the domain of $y = \sin x$ is the set of all real numbers. The range of both functions is the set of all real numbers between -1 and 1, inclusive.

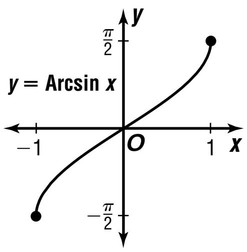
4. Restricted domains are denoted with a capital letter.

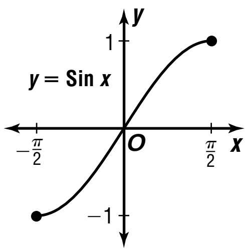
5. Akikta; there are 2 range values for each domain value between 0 and 2π. The principal values are between 0 and π, inclusive.

6. $y = \operatorname{Arcsin} x$

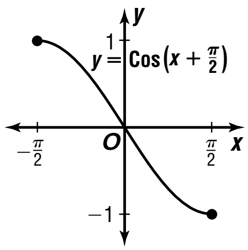
$x = \operatorname{Arcsin} y$

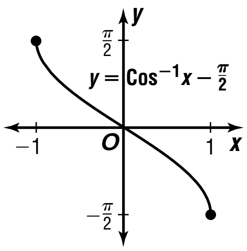
$\operatorname{Sin} x = y$ or $y = \operatorname{Sin} x$





7.
$$y = \operatorname{Cos}\left(x + \frac{\pi}{2}\right)$$
$$x = \operatorname{Cos}\left(y + \frac{\pi}{2}\right)$$
$$\operatorname{Cos}^{-1} x = y + \frac{\pi}{2}$$
$$y = \operatorname{Cos}^{-1} x - \frac{\pi}{2}$$





8. Let $\theta = \operatorname{Arctan} 1$.

$\operatorname{Tan} \theta = 1$

$\theta = \frac{\pi}{4}$

9. Let $\theta = \operatorname{Tan}^{-1} 1$.

$\operatorname{Tan} \theta = 1$

$\theta = \frac{\pi}{4}$

$$\cos(\operatorname{Tan}^{-1} 1) = \cos \theta = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

10. Let $\theta = \text{Cos}^{-1}\left(\frac{\sqrt{2}}{2}\right)$.

$$\text{Cos}\,\theta = \frac{\sqrt{2}}{2}$$
$$\theta = \frac{\pi}{4}$$
$$\cos\left[\text{Cos}^{-1}\left(\frac{\sqrt{2}}{2}\right) - \frac{\pi}{2}\right] = \cos\left(\theta - \frac{\pi}{2}\right)$$
$$= \cos\left(\frac{\pi}{4} - \frac{\pi}{2}\right)$$
$$= \cos -\frac{\pi}{4}$$
$$= \frac{\sqrt{2}}{2}$$

11. true

12. false; sample answer: $x = 1$; when $x = 1$, $\text{Cos}^{-1}(-1) = \pi$, $-\text{Cos}^{-1}(1) = 0$

13a. $C = 2\pi r$
$C = 2\pi(6400)$
$C \approx 40{,}212$ km

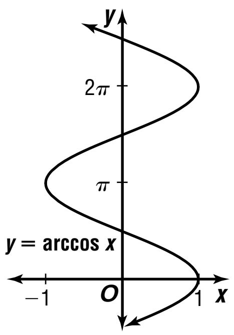
13b. $C = 40{,}212 \cos\theta$

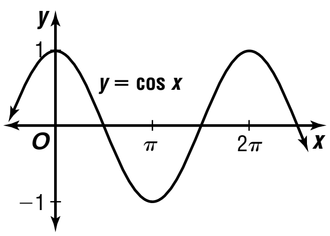
13c.
$$C = 40{,}212 \cos\theta$$
$$3593 = 40{,}212 \cos\theta$$
$$\frac{3593}{40{,}212} = \cos\theta$$
$$\cos^{-1}\left(\frac{3593}{40{,}212}\right) = \theta$$
$1.48 \approx \theta$; about 1.48 radians

13d. $C \approx 40{,}212 \cos\theta$
$C \approx 40{,}212 \cos 0$
$C \approx 40{,}212$ km

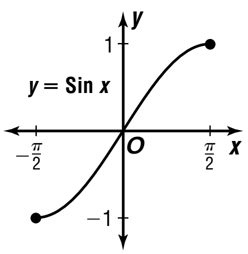
Pages 410–412 Exercises

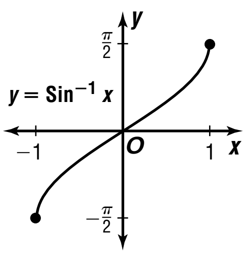
14.
$$y = \arccos x$$
$$x = \arccos y$$
$\cos x = y$ or $y = \cos x$



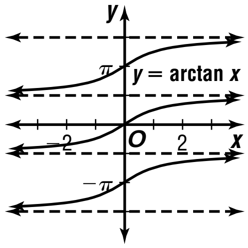


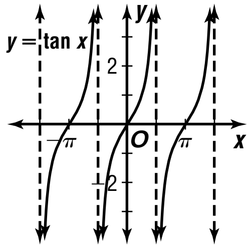
15.
$$y = \text{Sin}\,x$$
$$x = \text{Sin}\,y$$
$\text{Sin}^{-1} x = y$ or $y = \text{Sin}^{-1} x$



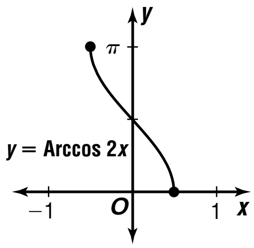


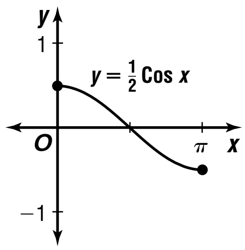
16.
$$y = \arctan x$$
$$x = \arctan y$$
$\tan x = y$ or $y = \tan x$



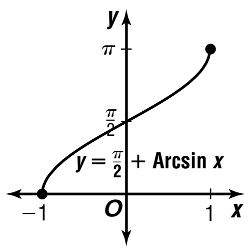


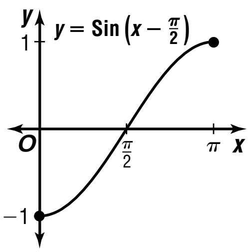
17.
$$y = \text{Arccos}\,2x$$
$$x = \text{Arccos}\,2y$$
$$\text{Cos}\,x = 2y$$
$\frac{1}{2}\text{Cos}\,x = y$ or $y = \frac{1}{2}\text{Cos}\,x$



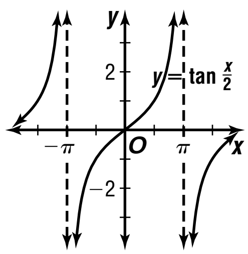


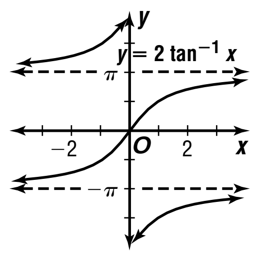
18.
$$y = \frac{\pi}{2} + \text{Arcsin}\,x$$
$$x = \frac{\pi}{2} + \text{Arcsin}\,y$$
$$x - \frac{\pi}{2} = \text{Arcsin}\,y$$
$\text{Sin}\left(x - \frac{\pi}{2}\right) = y$ or $y = \text{Sin}\left(x - \frac{\pi}{2}\right)$





19.
$$y = \tan\frac{x}{2}$$
$$x = \tan\frac{y}{2}$$
$$\tan^{-1} x = \frac{y}{2}$$
$2\tan^{-1} x = y$; $y = 2\tan^{-1} x$





20. $y = \operatorname{Tan}\left(x - \frac{\pi}{2}\right)$

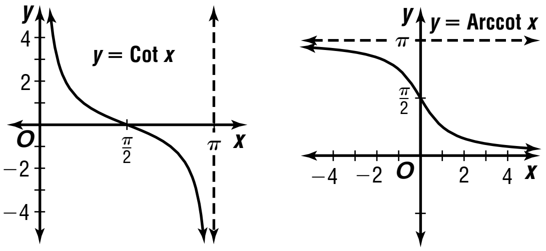
$x = \operatorname{Tan}\left(y - \frac{\pi}{2}\right)$

$\operatorname{Tan}^{-1} x = y - \frac{\pi}{2}$

$\operatorname{Tan}^{-1} x + \frac{\pi}{2} = y$

No; the inverse is $y = \operatorname{Tan}^{-1} x + \frac{\pi}{2}$.

21.



22. Let $\theta = \operatorname{Sin}^{-1} 0$.
$\operatorname{Sin} \theta = 0$
$\theta = 0$

23. Let $\theta = \operatorname{Arccos} 0$.
$\operatorname{Cos} \theta = 0$
$\theta = \frac{\pi}{2}$

24. Let $\theta = \operatorname{Tan}^{-1} \frac{\sqrt{3}}{3}$.
$\operatorname{Tan} \theta = \frac{\sqrt{3}}{3}$
$\theta = \frac{\pi}{6}$

25. If $y = \tan \frac{\pi}{4}$, then $y = 1$.

$\operatorname{Sin}^{-1}\left(\frac{\tan \pi}{4}\right) = \operatorname{Sin}^{-1} y$
$= \operatorname{Sin}^{-1} 1$
$= \frac{\pi}{2}$

26. If $y = \operatorname{Cos}^{-1} \frac{\sqrt{2}}{2}$, then $y = \frac{\pi}{4}$.

$\sin\left(2 \operatorname{Cos}^{-1} \frac{\sqrt{2}}{2}\right) = \sin (2y)$
$= \sin\left(2 \cdot \frac{\pi}{4}\right)$
$= \sin \frac{\pi}{2}$
$= 1$

27. If $y = \operatorname{Tan}^{-1} \sqrt{3}$, then $y = \frac{\pi}{3}$.

$\cos (\operatorname{Tan}^{-1} \sqrt{3}) = \cos y$
$= \cos \frac{\pi}{3}$
$= \frac{1}{2}$

28. Let $\alpha = \operatorname{Tan}^{-1} 1$ and $\beta = \operatorname{Sin}^{-1} 1$.

$\operatorname{Tan} \alpha = 1$ $\quad$ $\operatorname{Sin} \beta = 1$

$\alpha = \frac{\pi}{4}$ $\quad$ $\beta = \frac{\pi}{2}$

$\cos (\operatorname{Tan}^{-1} 1 - \operatorname{Sin}^{-1} 1) = \cos (\alpha - \beta)$
$= \cos\left(\frac{\pi}{4} - \frac{\pi}{2}\right)$
$= \cos -\frac{\pi}{4}$
$= \frac{\sqrt{2}}{2}$

29. Let $\alpha = \operatorname{Cos}^{-1} 0$ and $\beta = \operatorname{Sin}^{-1} \frac{1}{2}$.

$\operatorname{Cos} \alpha = 0$ $\quad$ $\operatorname{Sin} \beta = \frac{1}{2}$

$\alpha = \frac{\pi}{2}$ $\quad$ $\beta = \frac{\pi}{6}$

$\cos\left(\operatorname{Cos}^{-1} 0 + \operatorname{Sin}^{-1}\frac{1}{2}\right) = \cos (\alpha + \beta)$
$= \cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right)$
$= \cos \frac{2\pi}{3}$
$= -\frac{1}{2}$

30. Let $\alpha = \operatorname{Sin}^{-1} 1$ and $\beta = \operatorname{Cos}^{-1} \frac{1}{2}$.

$\operatorname{Sin} \alpha = 1$ $\quad$ $\operatorname{Cos} \beta = \frac{1}{2}$

$\alpha = \frac{\pi}{2}$ $\quad$ $\beta = \frac{\pi}{3}$

$\sin\left(\operatorname{Sin}^{-1} 1 - \operatorname{Cos}^{-1} \frac{1}{2}\right) = \sin (\alpha - \beta)$
$= \sin\left(\frac{\pi}{2} - \frac{\pi}{3}\right)$
$= \sin \frac{\pi}{6}$
$= \frac{1}{2}$

31. No; there is no angle with the sine of 2.

32. false; sample answer: $x = 2\pi$; when $x = 2\pi$, $\operatorname{Cos}^{-1}(\cos 2\pi) = \operatorname{Cos}^{-1} 1$, or 0, not 2π.

33. true

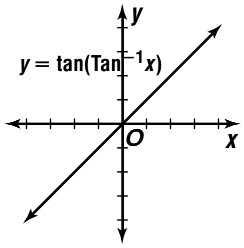
34. false; sample answer: $x = -1$; when $x = -1$, $\operatorname{Arccos} (-1) = \pi$ and $\operatorname{Arccos} (-(-1)) = 0$.

35. true

36. true

37. false; sample answer: $x = \frac{\pi}{2}$; when $x = \frac{\pi}{2}$, $\cos^{-1} \frac{\pi}{2}$ is undefined.

38.



39. $y = 54.5 + 23.5 \sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right)$

$54.5 = 54.5 + 23.5 \sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right)$

$0 = 23.5 \sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right)$

$0 = \sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right)$

$\sin^{-1} 0 = \frac{\pi}{6}t - \frac{2\pi}{3}$

$0 = \frac{\pi}{6}t - \frac{2\pi}{3}$ $\quad$ or $\quad$ $\pi = \frac{\pi}{6}t - \frac{2\pi}{3}$

$\frac{2\pi}{3} = \frac{\pi}{6}t$ $\quad$ $\frac{5\pi}{3} = \frac{\pi}{6}t$

$4 = t$ $\quad$ $10 = t$

April and October

40. $P = VI \operatorname{Cos} \theta$

$7.3 = 122(0.62) \operatorname{Cos} \theta$

$0.0965097832 \approx \operatorname{Cos} \theta$

$\operatorname{Cos}^{-1} 0.0965097832 \approx \theta$

$1.47 \approx \theta$; about 1.47 radians

41. $\frac{\pi}{4} + \pi n$, where n is an integer

42. $I = I_0 \cos^2 \theta$

$1 = 8 \cos^2 \theta$

$\frac{1}{8} = \cos^2 \theta$

$\sqrt{\frac{1}{8}} = \cos \theta$

$\cos^{-1} \sqrt{\frac{1}{8}} = \theta$

$1.21 \approx \theta$; about 1.21 radians

43a. 6:18 + 12:24 = 18:42 or 6:42 P.M.

43b. 12.4 h

43c. $A = \frac{7.05 - (-0.30)}{2}$

$A = 3.675$ ft

43d. $A = \pm 3.675 \qquad \frac{2\pi}{k} = 12.4 \qquad h = \frac{7.05 + (-0.30)}{2}$

$k = \frac{\pi}{6.2} \qquad h = 3.375$

6:18 = 6.3 h

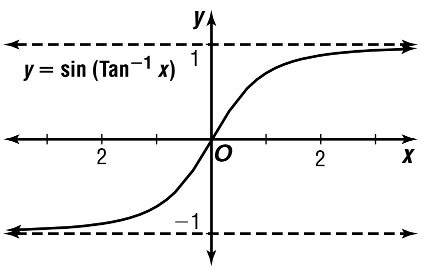
$$y = 3.675 \sin\left(\frac{\pi}{6.2}t + c\right) + 3.375$$
$$7.05 = 3.675 \sin\left(\frac{\pi}{6.2} \cdot 6.3 + c\right) + 3.375$$
$$3.675 = 3.675 \sin\left(\frac{6.3\pi}{6.2} + c\right)$$
$$1 = \sin\left(\frac{6.3\pi}{6.2} + c\right)$$
$$\sin^{-1} 1 = \frac{6.3\pi}{6.2} + c$$
$$\sin^{-1} 1 - \frac{6.3\pi}{6.2} = c$$
$$-1.621467176 \approx c$$

Sample answer:
$y = 3.375 + 3.675 \sin\left(\frac{\pi}{6.2}t - 1.62\right)$

43e.
$$y = 3.375 + 3.675 \sin\left(\frac{\pi}{6.2}t - 1.62\right)$$
$$6 = 3.375 + 3.675 \sin\left(\frac{\pi}{6.2}t - 1.62\right)$$
$$2.625 = 3.675 \sin\left(\frac{\pi}{6.2}t - 1.62\right)$$
$$\frac{2.625}{3.675} = \sin\left(\frac{\pi}{6.2}t - 1.62\right)$$
$$\sin^{-1}\left(\frac{2.625}{3.675}\right) = \frac{\pi}{6.2}t - 1.62$$
$$\sin^{-1}\left(\frac{2.625}{3.675}\right) + 1.62 = \frac{\pi}{6.2}t$$
$$\frac{6.2}{\pi}\left(\sin^{-1}\left(\frac{2.625}{3.675}\right) + 1.62\right) = t$$
$$4.767243867 \approx t$$

$0.767243867 \times 60 \approx 46.03463204$;
Sample answer: about 4:46 A.M.

44.



45a. $\theta = \cos^{-1} \frac{D - d}{2c}$

$\theta = \cos^{-1} \frac{6 - 4}{2(10)}$

$\theta \approx 1.47$ radians

45b. $L = \pi D + (d - D)\theta + 2C \sin \theta$
$L \approx \pi(6) + (4 - 6)1.47 + 2(10) \sin 1.47$
$L \approx 35.81$ in.

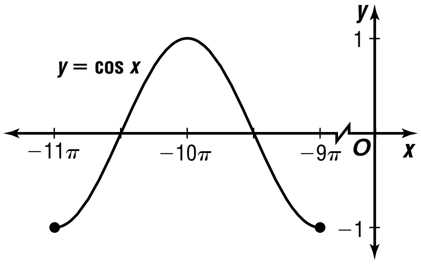
46. πn, where n is an integer

47. $|A| = 5 \qquad \frac{2\pi}{k} = 3\pi \qquad -\frac{c}{\frac{2}{3}} = -\pi \qquad h = -8$

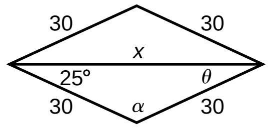
$A = \pm 5 \qquad k = \frac{2}{3} \qquad c = \frac{2\pi}{3}$

$y = \pm 5 \sin\left(\frac{2}{3}\theta + \frac{2\pi}{3}\right) - 8$

48.

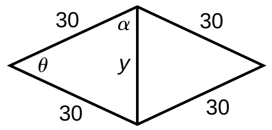


49.



$\theta = 25°$
$\alpha = 180° - (25° + 25°)$ or $130°$

$$\frac{30}{\sin 25°} = \frac{x}{\sin 130°}$$
$$x = \frac{30 \sin 130°}{\sin 25°}$$
$$x \approx 54.4 \text{ units}$$



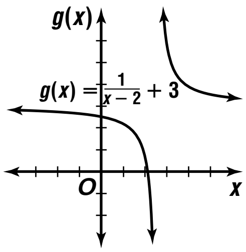
$\theta = 2(25°)$ or $50°$
$\alpha = \frac{1}{2}(180° - 50°)$ or $65°$

$$\frac{30}{\sin 65°} = \frac{y}{\sin 50°}$$
$$y = \frac{30 \sin 50°}{\sin 65°}$$
$$y \approx 25.4 \text{ units}$$

50. $210° - 180° = 30°$

51. p: $\pm 1, \pm 2, \pm 3, \pm 6$
q: $\pm 1, \pm 2$
$\frac{p}{q}$: $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$

52.



decreasing for $x < 2$ and $x > 2$

53. $[f \circ g](x) = f(g(x))$
$= f(3x)$
$= (3x)^3 - 1$
$= 27x^3 - 1$

$[g \circ f](x) = g(f(x))$
$= g(x^3 - 1)$
$= 3(x^3 - 1)$
$= 3x^3 - 3$

54. $D = 4, F = 6, G = 7, H = 8$
value: $(4 + 6 + 7 + 8)4 = (25)4$ or 100
The correct choice is D.

Chapter 6 Study Guide and Assessment

Page 413 Understanding and Using the Vocabulary

1. radian
2. angular
3. the same
4. amplitude
5. angle
6. phase
7. radian
8. frequency
9. sunusoidal
10. domain

Pages 414–416 Skills and Concepts

11. $60° = 60° \times \frac{\pi}{180°}$
$= \frac{\pi}{3}$

12. $-75° = -75° \times \frac{\pi}{180°}$
$= -\frac{5\pi}{12}$

13. $240° = 240° \times \frac{\pi}{180°}$
$= \frac{4\pi}{3}$

14. $\frac{5\pi}{6} = \frac{5\pi}{6} \times \frac{180°}{\pi}$
$= 150°$

15. $-\frac{7\pi}{4} = -\frac{7\pi}{4} \times \frac{180°}{\pi}$
$= -315°$

16. $2.4 = 2.4 \times \frac{180°}{\pi}$
$= 137.5°$

17. $s = r\theta$
$s = 15\left(\frac{3\pi}{4}\right)$
$s \approx 35.3$ cm

18. $75° = 75° \times \frac{\pi}{180°}$
$= \frac{5\pi}{12}$
$s = r\theta$
$s = 15\left(\frac{5\pi}{12}\right)$
$s \approx 19.6$ cm

19. $150° = 150° \times \frac{\pi}{180°}$
$= \frac{5\pi}{6}$
$s = r\theta$
$s = 15\left(\frac{5\pi}{6}\right)$
$s \approx 39.3$ cm

20. $s = r\theta$
$s = 15\left(\frac{\pi}{5}\right)$
$s \approx 9.4$ cm

21. $5 \times 2\pi = 10\pi$ or about 31.4 radians

22. $3.8 \times 2\pi = 7.6\pi$ or about 23.9 radians

23. $50.4 \times 2\pi = 100.8\pi$ or about 316.7 radians

24. $350 \times 2\pi = 700\pi$ or about 2199.1 radians

25. $1.8 \times 2\pi = 3.6\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{3.6\pi}{5}$
$\omega \approx 2.3$ radians/s

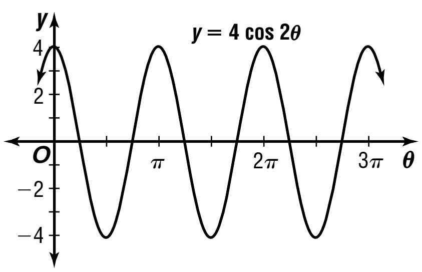
26. $3.6 \times 2\pi = 7.2\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{7.2\pi}{2}$
$\omega \approx 11.3$ radians/min

27. $15.4 \times 2\pi = 30.8\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{30.8\pi}{15}$
$\omega \approx 6.5$ radians/s

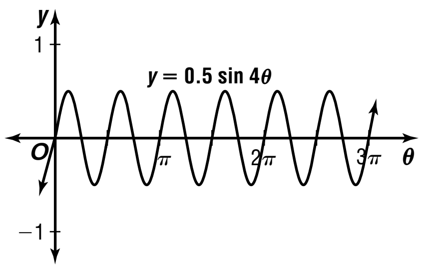
28. $50 \times 2\pi = 100\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{100\pi}{12}$
$\omega \approx 26.2$ radians/min

29. -1 **30.** 0 **31.** 1 **32.** 0

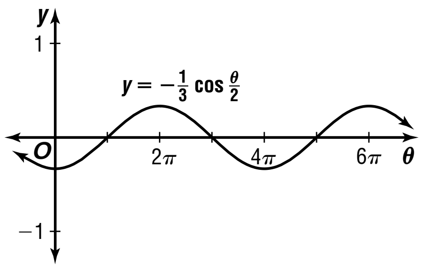
33. $|4| = 4$; $\frac{2\pi}{2} = \pi$



34. $|0.5| = 0.5$; $\frac{2\pi}{4} = \frac{\pi}{2}$



35. $\left|-\frac{1}{3}\right| = \frac{1}{3}$; $\frac{2\pi}{\frac{1}{2}} = 4\pi$



36. $|A| = 4$ $\quad \frac{2\pi}{k} = \frac{\pi}{2}$ $\quad -\frac{c}{4} = -2\pi$ $\quad h = -1$
$A = \pm 4$ $\quad k = 4$ $\quad c = 8\pi$
$y = \pm 4 \sin (4\theta + 8\pi) - 1$

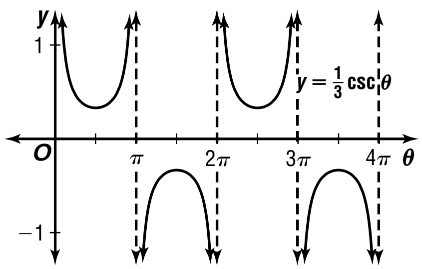
37. $|A| = 0.5$ $\quad \frac{2\pi}{k} = \pi$ $\quad -\frac{c}{2} = \frac{\pi}{3}$ $\quad h = 3$
$A = \pm 0.5$ $\quad k = 2$ $\quad c = -\frac{2\pi}{3}$
$y = \pm 0.5 \sin \left(2\theta - \frac{2\pi}{3}\right) + 3$

38. $|A| = \frac{3}{4}$ $\quad \frac{2\pi}{k} = \frac{\pi}{4}$ $\quad -\frac{c}{8} = 0$ $\quad h = 5$
$A = \pm\frac{3}{4}$ $\quad k = 8$ $\quad c = 0$
$y = \pm\frac{3}{4} \cos 8\theta + 5$

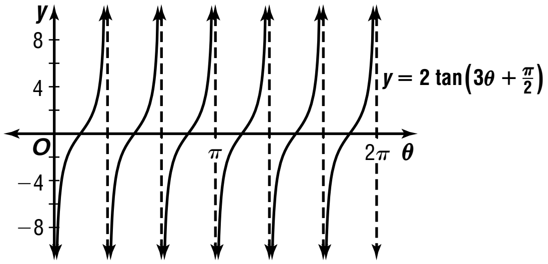
39. $A = \frac{120 - 80}{2}$ $\quad \frac{2\pi}{k} = 1$ $\quad h = \frac{120 + 80}{2}$
$A = 20$ $\quad k = 2\pi$ $\quad h = 100$
$y = 20 \sin 2\pi t + 100$

40. $A = \frac{130 - 100}{2}$ $\quad \frac{2\pi}{k} = 1$ $\quad h = \frac{130 + 100}{2}$
$A = 15$ $\quad k = 2\pi$ $\quad h = 115$
$y = 15 \sin 2\pi t + 115$

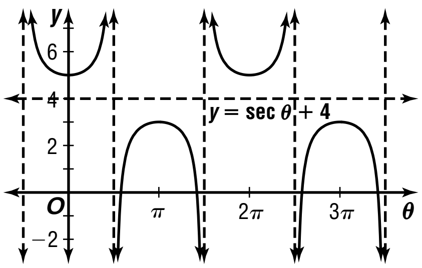
41. period: $\frac{2\pi}{1}$ or 2π, no phase shift, no vertical shift



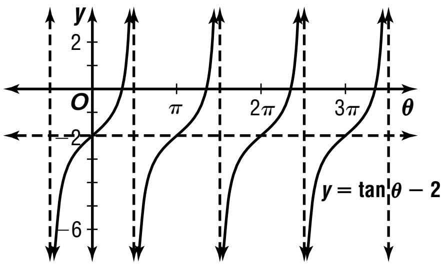
42. $\frac{\pi}{3}$; $-\frac{\frac{\pi}{2}}{3} = -\frac{\pi}{6}$; no vertical shift



43. vertical shift: 4



44. vertical shift: −2



45. Let $\theta = \text{Arctan } -1$.
$\text{Tan } \theta = -1$
$\theta = -\frac{\pi}{4}$

46. Let $\theta = \text{Sin}^{-1} 1$.
$\text{Sin } \theta = 1$
$\theta = \frac{\pi}{2}$

47. If $y = \tan \frac{\pi}{4}$, then $y = 1$.

$$\text{Cos}^{-1}\left(\tan \frac{\pi}{4}\right) = \text{Cos}^{-1} y$$
$$= \text{Cos}^{-1} 1$$

Let $\theta = \text{Cos}^{-1} 1$.
$\text{Cos } \theta = 1$
$\theta = 0$

48. If $y = \text{Sin}^{-1} \frac{\sqrt{3}}{2}$, then $y = \frac{\pi}{3}$.

$$\sin\left(\text{Sin}^{-1} \frac{\sqrt{3}}{2}\right) = \sin y$$
$$= \sin \frac{\pi}{3}$$
$$= \frac{\sqrt{3}}{2}$$

49. Let $\alpha = \text{Arctan } \sqrt{3}$ and $\beta = \text{Arcsin } \frac{1}{2}$.

$\text{Tan } \alpha = \sqrt{3}$ $\quad$ $\text{Sin } \beta = \frac{1}{2}$

$\alpha = \frac{\pi}{3}$ $\quad$ $\beta = \frac{\pi}{6}$

$$\cos(\text{Arctan } \sqrt{3} + \text{Arcsin } \tfrac{1}{2}) = \cos(\alpha + \beta)$$
$$= \cos\left(\frac{\pi}{3} + \frac{\pi}{6}\right)$$
$$= \cos \frac{\pi}{2}$$
$$= 0$$

Page 417 Applications and Problem Solving

50a. $A = 11.5$ $\quad$ $2\frac{\pi}{k} = 12$ $\quad$ $-\frac{c}{\frac{\pi}{6}} = 3$ $\quad$ $h = 64$

$k = \frac{\pi}{6}$ $\quad$ $c = -\frac{\pi}{2}$

$$y = 11.5 \sin\left(\frac{\pi}{6}t - \frac{\pi}{2}\right) + 64$$

50b. April: month 4

$$y = 11.5 \sin\left(\frac{\pi}{6}t - \frac{\pi}{2}\right) + 64$$
$$y = 11.5 \sin\left(\frac{\pi}{6} \cdot 4 - \frac{\pi}{2}\right) + 64$$

$y = 69.75$; about 69.8°

50c. July: month 7

$$y = 11.5 \sin\left(\frac{\pi}{6}t - \frac{\pi}{2}\right) + 64$$
$$y = 11.5 \sin\left(\frac{\pi}{6} \cdot 7 - \frac{\pi}{2}\right) + 64$$

$y \approx 74.0°$

51.
$$B = \frac{F}{IL \sin \theta}$$
$$0.04 = \frac{0.2}{5.0(1) \sin \theta}$$
$$0.04(5.0(1) \sin \theta) = 0.2$$
$$\sin \theta = \frac{0.2}{0.04(5.0)(1)}$$
$$\sin \theta = 1$$
$$\theta = \frac{\pi}{2}$$

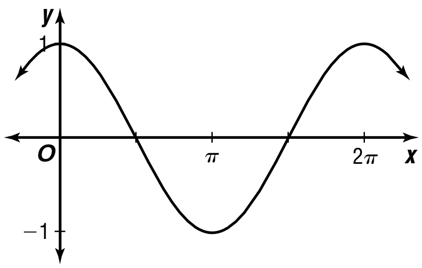
Page 417 Open-Ended Assessment

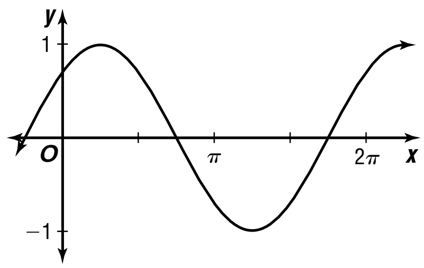
1. $A = \frac{1}{2}r^2\theta$

$26.2 = \frac{1}{2}r^2\theta$

Sample answer: $r = 5$ in., $\theta = \frac{2\pi}{3}$

2a. Sample answer: If the graph does not cross the y-axis at 1, the graph has been translated. The first graph has not been translated and the second graph has been translated.



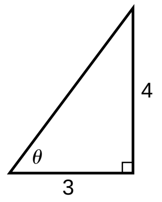


2b. Sample answer: If the equation does not have the form $y = A \cos k\theta$, the graph has been translated. The graph of $y = 2 \cos 2\theta$ has not been translated. The graph of $y = 2 \cos(2\theta + \pi) - 3$ has been translated vertically and horizontally.

Chapter 6 SAT & ACT Preparation

Page 419 SAT and ACT Practice

1. Since there is no diagram, draw one. Sketch a right triangle and mark the information given.



Notice that this is one of the "special" right triangles. Its sides are 3-4-5. So the hypotenuse is 5. The sine is opposite over hypotenuse (SOH).

$\sin \theta = \frac{4}{5}$

The correct choice is B.

2. Let x be the smaller integer. The numbers are two consecutive odd integers. So, the larger integer is 2 more than the first integer. Represent the larger integer by $x + 2$. Write an equation that says that the sum of these two integers is 56. Then solve for x.

$$x + (x + 2) = 56$$
$$2x + 2 = 56$$
$$2x = 54$$
$$x = 27$$

Be sure to read the question carefully. It asks for the value of the larger integer. The smaller integer is 27 and the larger integer is 29.

The correct choice is C.

3. Factor the numerator.

$a^2 - b^2 = (a + b)(a - b)$

$$\frac{(\sin \theta + \cos \theta)(\sin \theta - \cos \theta)}{\sin \theta - \cos \theta} = \sin \theta + \cos \theta$$

The correct choice is B.

4. First find the coordinates of point B. Notice that there are two right triangles. One has a hypotenuse of length 15 and a side of length 12. This is a 3-4-5 right triangle. The coordinates of point B are (9, 12).

Since point A has coordinates (0, 0), each point on side AB must have coordinates in the ratio of 9 to 12 or 3 to 4.

The only point among the answer choices that has this ratio of coordinates is (6, 8).

A slightly different way of solving this problem is to write the equation of the line containing points A and B.

$y = \frac{12}{9} x$

Then test each point to see whether it makes the equation a true statement.

You could also plot each point on the figure and see which point seems to lie on the line segment.

The correct choice is E.

5. Factor the polynomial on the left side of the equation.

$$x^2 - 2x - 8 = 0$$
$$(x - 4)(x + 2) = 0$$

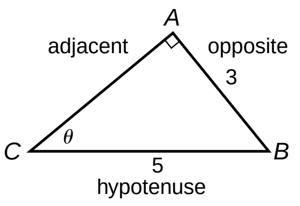
If either of the two factors equals 0, then the statement is true. Set each factor equal to 0 and solve for x.

$x - 4 = 0$ or $x + 2 = 0$

$x = 4$ $\quad$ $x = -2$

The solutions of the equation are 4 and -2. To find the sum of the solutions, add $4 + -2 = 2$. The correct choice is D.

6. You may want to label the triangle with *opposite, adjacent,* and *hypotenuse.*



To find $\cos \theta$, you need to know the length of the adjacent side. Notice that the hypotenuse is 5 and one side is 3, so this is a 3-4-5 right triangle. The adjacent side is 4 units.

Use the ratio for $\cos \theta$.

$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5}$

The correct choice is C.

7. Look at the powers of the variables in the equation. There is an x^2 term, an x term, and a y term, but *no* y^2 term. It cannot represent a line, because of the x^2 term. It cannot represent a circle or an ellipse or a hyperbola because there is no y^2 term. So, it must represent a parabola.

The general form of the equation of a parabola is $y = a(x - h)^2 + k$. The correct choice is A.

8. Factor each of the numerators and determine if the resulting expression could be an integer, that is, the numerator is a multiple of the denominator.

I $\quad \frac{16n + 16}{n + 1} = \frac{16(n + 1)}{n + 1} = 16$; an integer

II $\quad \frac{16n + 16}{16n} = \frac{16(n + 1)}{16n} = \frac{n + 1}{n}$; not an integer

III $\quad \frac{16n^2 + n}{16n} = \frac{n(16n + 1)}{16n} = \frac{16n + 1}{16}$; not an integer

Only expression I is an integer.

The correct choice is A.

9. Since $x > 1$, $1 - x < 0$. So $x^{1 - x} = \frac{1}{x^{x - 1}}$.

Since $x > 1$, $x^{x - 1} > 1$. So $\frac{1}{x^{x - 1}} < 1$.

The correct choice is D.

10. Notice that the triangles are not necessarily isosceles. In $\triangle ADC$, the sum of the angles is 180°, so $m\angle CAD + m\angle ACD = 80$. Since segment AD bisects $\angle BAC$, $m\angle BAD + m\angle CAD$. Similarly, $m\angle BAC = m\angle ACD$. So, $m\angle BAD + m\angle BCD = 80$. Add the two equations. $m\angle CAD + m\angle BAD + m\angle ACD + m\angle BCD = 160$, so two of the angles in $\triangle ABC$ have the combined measure of 160°. Therefore, the third angle in this triangle, $\angle B$, must measure 20°. The correct answer is 20.

Chapter 7 Trigonometric Identities and Equations

7-1 Basic Trigonometric Identities

Page 427 Check for Understanding

1. Sample answer: $x = 45°$

2. Pythagorean identities are derived by applying the Pythagorean Theorem to a right triangle. The opposite angle identities are so named because $-A$ is the opposite of A.

3. $\tan\theta = \frac{1}{\cot\theta}$, $\cot\theta = \frac{1}{\tan\theta}$, $\frac{\cos\theta}{\sin\theta} = \cot\theta$, $1 + \cot^2\theta = \csc^2\theta$

4. $$\begin{aligned}\tan(-A) &= \frac{\sin(-A)}{\cos(-A)}\\ &= \frac{-\sin A}{\cos A}\\ &= -\frac{\sin A}{\cos A}\\ &= -\tan A\end{aligned}$$

5. Rosalinda is correct; there may be other values for which the equation is not true.

6. Sample answer: $\theta = 0°$
$$\begin{aligned}\sin\theta + \cos\theta &\stackrel{?}{=} \tan\theta\\ \sin 0° + \cos 0° &\stackrel{?}{=} \tan 0°\\ 0 + 1 &\stackrel{?}{=} 0\\ 1 &\neq 0\end{aligned}$$

7. Sample answer: $x = 45°$
$$\begin{aligned}\sec^2 x + \csc^2 x &\stackrel{?}{=} 1\\ \sec^2 45° + \csc^2 45° &\stackrel{?}{=} 1\\ (\sqrt{2})^2 + (\sqrt{2})^2 &\stackrel{?}{=} 1\\ 2 + 2 &\stackrel{?}{=} 1\\ 4 &\neq 1\end{aligned}$$

8. $$\begin{aligned}\sec\theta &= \frac{1}{\cos\theta}\\ \sec\theta &= \frac{1}{\frac{2}{3}}\\ \sec\theta &= \frac{3}{2}\end{aligned}$$

9. $$\begin{aligned}\tan\theta &= \frac{1}{\cot\theta}\\ \tan\theta &= \frac{1}{-\frac{\sqrt{5}}{2}}\\ \tan\theta &= -\frac{2}{\sqrt{5}}\\ \tan\theta &= -\frac{2\sqrt{5}}{5}\end{aligned}$$

10. $$\begin{aligned}\sin^2\theta + \cos^2\theta &= 1\\ \left(-\frac{1}{5}\right)^2 + \cos^2\theta &= 1\\ \frac{1}{25} + \cos^2\theta &= 1\\ \cos^2\theta &= \frac{24}{25}\\ \cos\theta &= \pm\frac{2\sqrt{6}}{5}\end{aligned}$$
Quadrant III, so $-\frac{2\sqrt{6}}{5}$

11. $$\begin{aligned}\tan^2\theta + 1 &= \sec^2\theta\\ \left(-\frac{4}{7}\right)^2 + 1 &= \sec^2\theta\\ \frac{16}{49} + 1 &= \sec^2\theta\\ \frac{65}{49} &= \sec^2\theta\\ \pm\frac{\sqrt{65}}{7} &= \sec\theta\end{aligned}$$
Quadrant IV, so $\frac{\sqrt{65}}{7}$

12. $$\begin{aligned}\frac{7\pi}{3} &= 2\pi + \frac{\pi}{3}\\ \cos\frac{7\pi}{3} &= \cos\left(2\pi + \frac{\pi}{3}\right)\\ &= \cos\frac{\pi}{3}\end{aligned}$$

13. $$\begin{aligned}-330° &= -360° + 30°\\ \csc(-330°) &= \frac{1}{\sin(-330°)}\\ &= \frac{1}{\sin(-360° + 30°)}\\ &= \frac{1}{\sin 30°}\\ &= \csc 30°\end{aligned}$$

14. $$\begin{aligned}\frac{\csc\theta}{\cot\theta} &= \frac{\frac{1}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}}\\ &= \frac{1}{\sin\theta}\cdot\frac{\sin\theta}{\cos\theta}\\ &= \frac{1}{\cos\theta}\\ &= \sec\theta\end{aligned}$$

15. $$\begin{aligned}\cos x\csc x\tan x &= \cos x\left(\frac{1}{\sin x}\right)\left(\frac{\sin x}{\cos x}\right)\\ &= 1\end{aligned}$$

16. $$\begin{aligned}\cos x\cot x + \sin x &= \cos x\left(\frac{\cos x}{\sin x}\right) + \sin x\\ &= \frac{\cos^2 x}{\sin x} + \sin x\\ &= \frac{\cos^2 x + \sin^2 x}{\sin x}\\ &= \frac{1}{\sin x}\\ &= \csc x\end{aligned}$$

17. $$\begin{aligned}B &= \frac{F\csc\theta}{I\ell}\\ BI\ell &= F\csc\theta\\ F &= \frac{BI\ell}{\csc\theta}\\ F &= BI\ell\left(\frac{1}{\csc\theta}\right)\\ F &= BI\ell\sin\theta\end{aligned}$$

Pages 427–430 Exercises

18. Sample answer: 45°
$$\begin{aligned}\sin\theta\cos\theta &\stackrel{?}{=} \cot\theta\\ \sin 45°\cos 45° &\stackrel{?}{=} \cot 45°\\ \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) &\stackrel{?}{=} 1\\ \frac{1}{2} &\neq 1\end{aligned}$$

19. Sample answer: 45°
$$\begin{aligned}\frac{\sec\theta}{\tan\theta} &\stackrel{?}{=} \sin\theta\\ \frac{\sec 45°}{\tan 45°} &\stackrel{?}{=} \sin 45°\\ \frac{\sqrt{2}}{1} &\stackrel{?}{=} \frac{\sqrt{2}}{2}\\ \sqrt{2} &\neq \frac{\sqrt{2}}{2}\end{aligned}$$

20. Sample answer: 30°

$$\sec^2 x - 1 \stackrel{?}{=} \frac{\cos x}{\csc x}$$
$$\sec^2 30° - 1 \stackrel{?}{=} \frac{\cos 30°}{\csc 30°}$$
$$\left(\frac{2\sqrt{3}}{3}\right)^2 - 1 \stackrel{?}{=} \frac{\frac{\sqrt{3}}{2}}{2}$$
$$\frac{12}{9} - 1 \stackrel{?}{=} \frac{\sqrt{3}}{4}$$
$$\frac{1}{3} \neq \frac{\sqrt{3}}{4}$$

21. Sample answer: 30°

$$\sin x + \cos x \stackrel{?}{=} 1$$
$$\sin 30° + \cos 30° \stackrel{?}{=} 1$$
$$\frac{1}{2} + \frac{\sqrt{3}}{2} \stackrel{?}{=} 1$$
$$\frac{1+\sqrt{3}}{2} \neq 1$$

22. Sample answer: 0°

$$\sin y \tan y \stackrel{?}{=} \cos y$$
$$\sin 0° \tan 0° \stackrel{?}{=} \cos 0°$$
$$0 \cdot 0 \stackrel{?}{=} 1$$
$$0 \neq 1$$

23. Sample answer: 45°

$$\tan^2 A + \cot^2 A \stackrel{?}{=} 1$$
$$\tan^2 45° + \cot^2 45° \stackrel{?}{=} 1$$
$$1 + 1 \stackrel{?}{=} 1$$
$$2 \neq 1$$

24. Sample answer: 0

$$\cos\left(\theta + \frac{\pi}{2}\right) \neq \cos\theta + \cos\frac{\pi}{2}$$
$$\cos\left(0 + \frac{\pi}{2}\right) \neq \cos 0 + \cos\frac{\pi}{2}$$
$$\cos\frac{\pi}{2} \neq \cos 0 + \cos\frac{\pi}{2}$$
$$0 \neq 1 + 0$$
$$0 \neq 1$$

25. $\csc\theta = \frac{1}{\sin\theta}$

$$\csc\theta = \frac{1}{\frac{2}{5}}$$
$$\csc\theta = \frac{5}{2}$$

26. $\cot\theta = \frac{1}{\tan\theta}$

$$\cot\theta = \frac{1}{\frac{\sqrt{3}}{4}}$$
$$\cot\theta = \frac{4}{\sqrt{3}}$$
$$\cot\theta = \frac{4\sqrt{3}}{3}$$

27. $\sin^2\theta + \cos^2\theta = 1$

$$\left(\frac{1}{4}\right)^2 + \cos^2\theta = 1$$
$$\frac{1}{16} + \cos^2\theta = 1$$
$$\cos^2\theta = \frac{15}{16}$$
$$\cos\theta = \pm\frac{\sqrt{15}}{4}$$

Quadrant I, so $\frac{\sqrt{15}}{4}$

28. $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta + \left(-\frac{2}{3}\right)^2 = 1$$
$$\sin^2\theta + \frac{4}{9} = 1$$
$$\sin^2\theta = \frac{5}{9}$$
$$\sin\theta = \pm\frac{\sqrt{5}}{3}$$

Quadrant II, so $\frac{\sqrt{5}}{3}$

29. $1 + \cot^2\theta = \csc^2\theta$

$$1 + \cot^2\theta = \left(\frac{\sqrt{11}}{3}\right)^2$$
$$1 + \cot^2\theta = \frac{11}{9}$$
$$\cot^2\theta = \frac{2}{9}$$
$$\cot\theta = \pm\frac{\sqrt{2}}{3}$$

Quadrant II, so $-\frac{\sqrt{2}}{3}$

30. $\tan^2\theta + 1 = \sec^2\theta$

$$\tan^2\theta + 1 = \left(-\frac{5}{4}\right)^2$$
$$\tan^2\theta + 1 = \frac{25}{16}$$
$$\tan^2\theta = \frac{9}{16}$$
$$\tan\theta = \pm\frac{3}{4}$$

Quadrant II, so $-\frac{3}{4}$

31. $\sin^2\theta + \cos^2\theta = 1$

$$\left(-\frac{1}{3}\right)^2 + \cos^2\theta = 1$$
$$\frac{1}{9} + \cos^2\theta = 1$$
$$\cos^2\theta = \frac{8}{9}$$
$$\cos\theta = \pm\frac{2\sqrt{2}}{3}$$

Quadrant III, so $\cos\theta = -\frac{2\sqrt{2}}{3}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\tan\theta = \frac{-\frac{1}{3}}{-\frac{2\sqrt{2}}{3}}$$
$$\tan\theta = \frac{1}{2\sqrt{2}} \quad \text{or} \quad \frac{\sqrt{2}}{4}$$

32. $\tan^2\theta + 1 = \sec^2\theta$

$$\left(\frac{2}{3}\right)^2 + 1 = \sec^2\theta$$
$$\frac{4}{9} + 1 = \sec^2\theta$$
$$\frac{13}{9} = \sec^2\theta$$
$$\pm\frac{\sqrt{13}}{3} = \sec\theta$$

Quadrant III, so $\sec\theta = -\frac{\sqrt{13}}{3}$

$$\cos\theta = \frac{1}{\sec\theta}$$
$$\cos\theta = \frac{1}{\frac{-\sqrt{13}}{3}}$$
$$\cos\theta = -\frac{3}{\sqrt{13}} \text{ or } -\frac{3\sqrt{13}}{13}$$

33. $\cos\theta = \frac{1}{\sec\theta}$

$$\cos\theta = \frac{1}{-\frac{7}{5}}$$
$$\cos\theta = -\frac{5}{7}$$

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta + \left(-\frac{5}{7}\right)^2 = 1$$
$$\sin^2\theta + \frac{25}{49} = 1$$
$$\sin^2\theta = \frac{24}{49}$$
$$\sin\theta = \pm\frac{2\sqrt{6}}{7}$$

Quadrant III, so $-\frac{2\sqrt{6}}{7}$

34. $\sec\theta = \frac{1}{\cos\theta}$

$\sec\theta = \frac{1}{\frac{1}{8}}$

$\sec\theta = 8$

$\tan^2\theta + 1 = \sec^2\theta$
$\tan^2\theta + 1 = 8^2$
$\tan^2\theta + 1 = 64$
$\tan^2\theta = 63$
$\tan\theta = \pm 3\sqrt{7}$

Quadrant IV, so $-3\sqrt{7}$

35. $1 + \cot^2\theta = \csc^2\theta$

$1 + \left(-\frac{4}{3}\right)^2 = \csc^2\theta$

$1 + \frac{16}{9} = \csc^2\theta$

$\frac{25}{9} = \csc^2\theta$

$\pm\frac{5}{3} = \csc\theta$

$\sin\theta = \frac{1}{\csc\theta}$

$\sin\theta = \frac{1}{-\frac{5}{3}}$

$\sin\theta = -\frac{3}{5}$

Quadrant IV, so $-\frac{5}{3}$

36. $1 + \cot^2\theta = \csc^2\theta$
$1 + (-8)^2 = \csc^2\theta$
$1 + 64 = \csc^2\theta$
$65 = \csc^2\theta$
$\pm\sqrt{65} = \csc\theta$
Quadrant IV, so $-\sqrt{65}$

37. $\sec\theta = \frac{1}{\cos\theta}$

$\sec\theta = \frac{1}{-\frac{\sqrt{3}}{4}}$

$\sec\theta = -\frac{4}{\sqrt{3}}$ or $-\frac{4\sqrt{3}}{3}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(-\frac{\sqrt{3}}{4}\right)^2 = 1$

$\sin^2\theta + \frac{3}{16} = 1$

$\sin^2\theta = \frac{13}{16}$

$\sin\theta = \pm\frac{\sqrt{13}}{4}$

Quadrant II, so $\frac{\sqrt{13}}{4}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = \frac{\frac{\sqrt{13}}{4}}{-\frac{\sqrt{3}}{4}}$

$\tan\theta = -\frac{\sqrt{13}}{\sqrt{3}}$ or $-\frac{\sqrt{39}}{3}$

$$\frac{\sec^2 A - \tan^2 A}{2\sin^2 A + 2\cos^2 A} = \frac{\left(-\frac{4\sqrt{3}}{3}\right)^2 - \left(-\frac{\sqrt{39}}{3}\right)^2}{2\left(\frac{\sqrt{13}}{4}\right)^2 + 2\left(-\frac{\sqrt{3}}{4}\right)^2}$$

$$= \frac{\frac{48}{9} - \frac{39}{9}}{2\left(\frac{13}{16}\right) + 2\left(\frac{3}{16}\right)}$$

$$= \frac{\frac{9}{9}}{\frac{32}{16}}$$

$$= \frac{1}{2}$$

38. $390° = 360° + 30°$
$\sin 390° = \sin(360° + 30°)$
$= \sin 30°$

39. $\frac{27\pi}{8} = 3\pi + \frac{3\pi}{8}$

$\cos\frac{27\pi}{8} = \cos\left(3\pi + \frac{3\pi}{8}\right)$

$= -\cos\frac{3\pi}{8}$

40. $\frac{19\pi}{5} = 2(2\pi) - \frac{\pi}{5}$

$$\tan\frac{19\pi}{5} = \frac{\sin\frac{19\pi}{5}}{\cos\frac{19\pi}{5}}$$

$$= \frac{\sin\left(2(2\pi) - \frac{\pi}{5}\right)}{\cos\left(2(2\pi) - \frac{\pi}{5}\right)}$$

$$= \frac{-\sin\frac{\pi}{5}}{\cos\frac{\pi}{5}}$$

$$= -\tan\frac{\pi}{5}$$

41. $\frac{10\pi}{3} = 3\pi + \frac{\pi}{3}$

$$\csc\frac{10\pi}{3} = \frac{1}{\sin\frac{10\pi}{3}}$$

$$= \frac{1}{\sin\left(3\pi + \frac{\pi}{3}\right)}$$

$$= \frac{1}{-\sin\frac{\pi}{3}}$$

$$= -\csc\frac{\pi}{3}$$

42. $-1290° = -7(180°) - 30°$

$$\sec(-1290°) = \frac{1}{\cos(-1290°)}$$

$$= \frac{1}{\cos(-7(180°) - 30°)}$$

$$= \frac{1}{-\cos 30°}$$

$$= -\sec 30°$$

43. $-660° = -2(360°) + 60°$

$$\cot(-660°) = \frac{\cos(-660°)}{\sin(-660°)}$$

$$= \frac{\cos(-2(360°) + 60°)}{\sin(-2(360°) + 60°)}$$

$$= \frac{\cos 60°}{\sin 60°}$$

$$= \cot 60°$$

44. $$\frac{\sec x}{\tan x} = \frac{\frac{1}{\cos x}}{\frac{\sin x}{\cos x}}$$

$$= \frac{1}{\sin x}$$

$$= \csc x$$

45. $$\frac{\cot\theta}{\cos\theta} = \frac{\frac{\cos\theta}{\sin\theta}}{\cos\theta}$$

$$= \frac{1}{\sin\theta}$$

$$= \csc\theta$$

46. $$\frac{\sin(\theta + \pi)}{\cos(\theta - \pi)} = \frac{-\sin\theta}{-\cos\theta}$$

$$= \tan\theta$$

47. $(\sin x + \cos x)^2 + (\sin x - \cos x)^2$
$= \sin^2 x + 2\sin x\cos x + \cos^2 x + \sin^2 x$
$- 2\sin x\cos x + \cos^2 x$
$= 2\sin^2 x + 2\cos^2 x$
$= 2(\sin^2 x + \cos^2 x)$
$= 2$

48. $\sin x \cos x \sec x \cot x = \sin x \cos x\left(\frac{1}{\cos x}\right)\left(\frac{\cos x}{\sin x}\right)$
$= \cos x$

49. $\cos x \tan x + \sin x \cot x = \cos x\left(\frac{\sin x}{\cos x}\right) + \sin x\left(\frac{\cos x}{\sin x}\right)$
$= \sin x + \cos x$

50. $(1 + \cos\theta)(\csc\theta - \cot\theta) = (1 + \cos\theta)\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)$
$= (1 + \cos\theta)\left(\frac{1 - \cos\theta}{\sin\theta}\right)$
$= \frac{1 - \cos^2\theta}{\sin\theta}$
$= \frac{\sin^2\theta}{\sin\theta}$
$= \sin\theta$

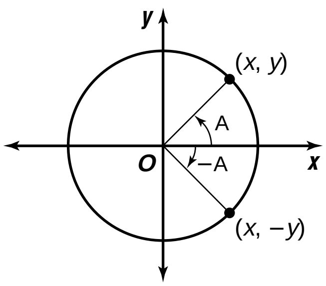
51. $1 + \cot^2\theta - \cos^2\theta - \cos^2\theta\cot^2\theta$
$= 1 + \cot^2\theta - \cos^2\theta(1 + \cot^2\theta)$
$= \csc^2\theta - \cos^2\theta(\csc^2\theta)$
$= \csc^2\theta(1 - \cos^2\theta)$
$= \csc^2\theta(\sin^2\theta)$
$= \frac{1}{\sin^2\theta}(\sin^2\theta)$
$= 1$

52. $\frac{\sin x}{1 + \cos x} + \frac{\sin x}{1 - \cos x}$
$= \frac{\sin x - \sin x\cos x}{1 - \cos^2 x} + \frac{\sin x + \sin x \cos x}{1 - \cos^2 x}$
$= \frac{2\sin x}{1 - \cos^2 x}$
$= \frac{2\sin x}{\sin^2 x}$
$= \frac{2}{\sin x}$
$= 2\csc x$

53. $\cos^4\alpha + 2\cos^2\alpha\sin^2\alpha + \sin^4\alpha = (\cos^2\alpha + \sin^2\alpha)^2$
$= 1^2 \text{ or } 1$

54. $I = I_0\cos^2\theta$
$0 = I_0\cos^2\theta$
$0 = \cos^2\theta$
$0 = \cos\theta$
$\cos^{-1} 0 = \theta$
$90° = \theta$

55. Let (x, y) be the point where the terminal side of A intersects the unit circle when A is in standard position. When A is reflected about the x-axis to obtain $-A$, the y-coordinate is multiplied by -1, but the x-coordinate is unchanged. So,
$\sin(-A) = -y = -\sin A$ and
$\cos(-A) = x = \cos A$.

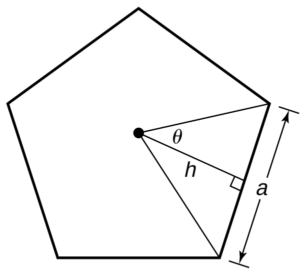


56a. $e = \frac{W\sec\theta}{As}$
$eAs = W\sec\theta$
$\frac{eAs}{\sec\theta} = W$
$W = eAs\cos\theta$

56b. $W = eAs\cos\theta$
$W = 0.80(0.75)(1000)\cos 40°$
$W \approx 459.6266659$
459.63 W

57. $F_N - mg\cos\theta = 0$
$F_N = mg\cos\theta$
$mg\sin\theta - \mu_k F_N = 0$
$mg\sin\theta - \mu_k(mg\cos\theta) = 0$
$\mu_k(mg\cos\theta) = mg\sin\theta$
$\mu_k = \frac{mg\sin\theta}{mg\cos\theta}$
$\mu_k = \frac{\sin\theta}{\cos\theta}$
$\mu_k = \tan\theta$

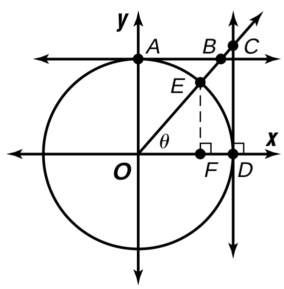
58.



$\theta = \frac{360°}{2n} = \frac{180°}{n}$, $\tan\theta = \frac{\frac{a}{2}}{h}$, so $h = \frac{a}{2\tan\theta} = \frac{a}{2}\cot\theta$.

The area of the isosceles triangle is $\frac{1}{2}(a)\left(\frac{a}{2}\cot\frac{180°}{n}\right)$ $= \frac{a^2}{4}\cot\left(\frac{180°}{n}\right)$. There are n such triangles, so $A = \frac{1}{4}na^2\cot\left(\frac{180°}{n}\right)$.

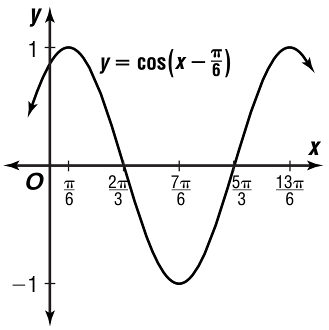
59.



$\sin\theta = EF$ and $\cos\theta = OF$ since the circle is a unit circle. $\tan\theta = \frac{CD}{OD} = \frac{CD}{1} = CD$.
$\sec\theta = \frac{CO}{OD} = \frac{CO}{1} = CO$. $\triangle EOF \sim \triangle OBA$, so
$\frac{OF}{EF} = \frac{BA}{OA} = \frac{BA}{1} = BA$. Then $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{OF}{EF} = BA$.
Also by similar triangles, $\frac{EO}{EF} = \frac{OB}{OA}$, or $\frac{1}{EF} = \frac{OB}{1}$.
Then $\csc\theta = \frac{1}{\sin\theta} = \frac{1}{EF} = \frac{OB}{1} = OB$.

60. $\mathrm{Cos}^{-1}\left(-\frac{\sqrt{2}}{2}\right) = 135°$

61.



62. $2(3° 30') = 7°$ $\qquad 7° = 7° \times \frac{\pi}{180°}$

$= \frac{7\pi}{180}$

$s = r\theta$

$s = 20\left(\frac{7\pi}{180}\right)$

$s \approx 2.44$ cm

63. $B = 180° - (90° + 20°)$ or $70°$

$\sin A = \frac{a}{c}$ $\qquad \cos A = \frac{b}{c}$

$\sin 20° = \frac{a}{35}$ $\qquad \cos 20° = \frac{b}{35}$

$35 \sin 20° = a$ $\qquad 35 \cos 20° = b$

$11.97070502 \approx a$ $\qquad 32.88924173 \approx b$

$a = 12.0, B = 70°, b = 32.9$

64.
$$\begin{array}{r|rrrr} 2 & 2 & 1 & -8 & -4 \\ & & 4 & 10 & 4 \\ \hline & 2 & 5 & 2 & 0 \end{array}$$

$2x^2 + 5x + 2 = 0$

$(2x + 1)(x + 2) = 0$

$2x + 1 = 0$ or $x + 2 = 0$

$x = -\frac{1}{2}$ $\qquad x = -2$

$-2, -\frac{1}{2}, 2$

65. $2x^2 + 7x - 4 = 0$

$x^2 + \frac{7}{2}x - 2 = 0$

$x^2 + \frac{7}{2}x = 2$

$x^2 + \frac{7}{2}x + \frac{49}{16} = 2 + \frac{49}{16}$

$\left(x + \frac{7}{4}\right)^2 = \frac{81}{16}$

$x + \frac{7}{4} = \pm\frac{9}{4}$

$x = -\frac{7}{4} \pm \frac{9}{4}$

$x = 0.5$ or -4

66. continuous

67. $4(x + y - 2z) = 4(3)$ $\qquad$ $4x + 4y - 8z = 12$

$-4x - y - z = 0$ $\quad \rightarrow \quad$ $\underline{-4x - y - z = 0}$

$3y - 9z = 12$

$x + y - 2z = 3$

$\underline{-x - 5y + 4z = 11}$

$-4y + 2z = 14$

$4(3y - 9z) = 4(12)$ $\quad \rightarrow \quad$ $12y - 36z = 48$

$3(-4y + 2z) = 3(14)$ $\qquad$ $\underline{-12y + 6z = 42}$

$-30z = 90$

$z = -3$

$3y - 9z = 12$ $\qquad x + y - 2z = 3$

$3y - 9(-3) = 12$ $\qquad x + (-5) - 2(-3) = 3$

$y = -5$ $\qquad x = 2$

$(2, -5, -3)$

68. $m = \frac{4 - 2}{-4 - 5}$

$= \frac{2}{-9}$ or $-\frac{2}{9}$

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{2}{9}(x - (-4))$

$y = -\frac{2}{9}x + \frac{28}{9}$

69. $m\angle BCD = 40°$

$40 = \frac{1}{2}m(\widehat{BC})$

$80 = m(\widehat{BC})$

$m\angle BAC = \frac{1}{2}m\widehat{BC}$

$m\angle BAC = \frac{1}{2}(80)$

$m\angle BAC = 40°$

The correct choice is C.

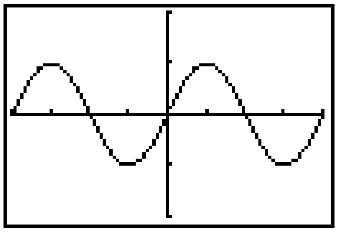
7-2 Verifying Trigonometric Identities

Page 433 Graphing Calculator Exploration

1. yes **2.** no **3.** no

4. No; it is impossible to look at every window since there are an infinite number. The only way an identity can be proven is by showing algebraically that the general case is true.

5.



$[-2\pi, 2\pi]$ sc1:$\frac{\pi}{2}$ by $[-2, 2]$ sc1:1

$\sin x$

Pages 433–434 Check for Understanding

1. Answers will vary.

2. Sample answer: Squaring each side can turn two unequal quantities into equal quantities. For example, $-1 \neq 1$, but $(-1)^2 = 1^2$.

3. Sample answer: They are the trigonometric functions with which most people are most familiar.

4. Answers will vary.

5. $\cos x \stackrel{?}{=} \frac{\cot x}{\csc x}$

$\cos x \stackrel{?}{=} \frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}}$

$\cos x \stackrel{?}{=} \frac{\cos x}{1}$

$\cos x = \cos x$

6.
$$\frac{1}{\tan x + \sec x} \stackrel{?}{=} \frac{\cos x}{\sin x + 1}$$
$$\frac{1}{\frac{\sin x}{\cos x} + \frac{1}{\cos x}} \stackrel{?}{=} \frac{\cos x}{\sin x + 1}$$
$$\frac{1}{\frac{\sin x + 1}{\cos x}} \stackrel{?}{=} \frac{\cos x}{\sin x + 1}$$
$$\frac{\cos x}{\sin x + 1} = \frac{\cos x}{\sin x + 1}$$

7.
$$\csc \theta - \cot \theta \stackrel{?}{=} \frac{1}{\csc \theta + \cot \theta}$$
$$\csc \theta - \cot \theta \stackrel{?}{=} \frac{1}{\csc \theta + \cot \theta} \cdot \frac{\csc \theta - \cot \theta}{\csc \theta - \cot \theta}$$
$$\csc \theta - \cot \theta \stackrel{?}{=} \frac{\csc \theta - \cot \theta}{\csc^2 \theta - \cot^2 \theta}$$
$$\csc \theta - \cot \theta \stackrel{?}{=} \frac{\csc \theta - \cot \theta}{(1 + \cot^2 \theta) - \cot^2 \theta}$$
$$\csc \theta - \cot \theta \stackrel{?}{=} \frac{\csc \theta - \cot \theta}{1}$$
$$\csc \theta - \cot \theta = \csc \theta - \cot \theta$$

8.
$$\sin \theta \tan \theta \stackrel{?}{=} \sec \theta - \cos \theta$$
$$\sin \theta \tan \theta \stackrel{?}{=} \frac{1}{\cos \theta} - \cos \theta$$
$$\sin \theta \tan \theta \stackrel{?}{=} \frac{1}{\cos \theta} - \frac{\cos^2 \theta}{\cos \theta}$$
$$\sin \theta \tan \theta \stackrel{?}{=} \frac{1 - \cos^2 \theta}{\cos \theta}$$
$$\sin \theta \tan \theta \stackrel{?}{=} \frac{\sin^2 \theta}{\cos \theta}$$
$$\sin \theta \tan \theta \stackrel{?}{=} \sin \theta \frac{\sin \theta}{\cos \theta}$$
$$\sin \theta \tan \theta = \sin \theta \tan \theta$$

9.
$$(\sin A - \cos A)^2 \stackrel{?}{=} 1 - 2\sin^2 A \cot A$$
$$\sin^2 A - 2 \sin A \cos A + \cos^2 A \stackrel{?}{=} 1 - 2\sin^2 A \cot A$$
$$1 - 2 \sin A \cos A \stackrel{?}{=} 1 - 2\sin^2 A \cot A$$
$$1 - 2 \sin A \cos A \frac{\sin A}{\sin A} \stackrel{?}{=} 1 - 2\sin^2 A \cot A$$
$$1 - 2 \sin^2 A \frac{\cos A}{\sin A} \stackrel{?}{=} 1 - 2\sin^2 A \cot A$$
$$1 - 2\sin^2 A \cot A = 1 - 2\sin^2 A \cot A$$

10. Sample answer: $\sin x = \frac{1}{4}$
$$\tan x = \frac{1}{4} \sec x$$
$$\frac{\tan x}{\sec x} = \frac{1}{4}$$
$$\frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \frac{1}{4}$$
$$\sin x = \frac{1}{4}$$

11. Sample answer: $\cos x = -1$
$$\cot x + \sin x = -\cos x \cot x$$
$$\frac{\cos x}{\sin x} + \sin x = -\cos x \frac{\cos x}{\sin x}$$
$$\cos x + \sin^2 x = -\cos^2 x$$
$$\cos^2 x + \sin^2 x = -\cos x$$
$$1 = -\cos x$$
$$\cos x = -1$$

12.
$$\frac{I \cos \theta}{R^2} \stackrel{?}{=} \frac{I \cot \theta}{R^2 \csc \theta}$$
$$\frac{I \cos \theta}{R^2} \stackrel{?}{=} \frac{I \frac{\cos \theta}{\sin \theta}}{R^2 \frac{1}{\sin \theta}}$$
$$\frac{I \cos \theta}{R^2} \stackrel{?}{=} \frac{I \frac{\cos \theta}{\sin \theta}}{R^2 \frac{1}{\sin \theta}} \cdot \frac{\sin \theta}{\sin \theta}$$
$$\frac{I \cos \theta}{R^2} = \frac{I \cos \theta}{R^2}$$

Pages 434–436 Exercises

13.
$$\tan A \stackrel{?}{=} \frac{\sec A}{\csc A}$$
$$\tan A \stackrel{?}{=} \frac{\frac{1}{\cos A}}{\frac{1}{\sin A}}$$
$$\tan A \stackrel{?}{=} \frac{\sin A}{\cos A}$$
$$\tan A = \tan A$$

14.
$$\cos \theta \stackrel{?}{=} \sin \theta \cot \theta$$
$$\cos \theta \stackrel{?}{=} \sin \theta \frac{\cos \theta}{\sin \theta}$$
$$\cos \theta = \cos \theta$$

15.
$$\sec x - \tan x = \frac{1 - \sin x}{\cos x}$$
$$\sec x - \tan x \stackrel{?}{=} \frac{1}{\cos x} - \frac{\sin x}{\cos x}$$
$$\sec x - \tan x = \sec x - \tan x$$

16.
$$\frac{1 + \tan x}{\sin x + \cos x} \stackrel{?}{=} \sec x$$
$$\frac{1 + \frac{\sin x}{\cos x}}{\sin x + \cos x} \stackrel{?}{=} \sec x$$
$$\frac{\cos x}{\cos x} \cdot \frac{1 + \frac{\sin x}{\cos x}}{\sin x + \cos x} \stackrel{?}{=} \sec x$$
$$\frac{\cos x + \sin x}{\cos x(\sin x + \cos x)} \stackrel{?}{=} \sec x$$
$$\frac{1}{\cos x} \stackrel{?}{=} \sec x$$
$$\sec x = \sec x$$

17.
$$\sec x \csc x \stackrel{?}{=} \tan x + \cot x$$
$$\sec x \csc x \stackrel{?}{=} \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$
$$\sec x \csc x \stackrel{?}{=} \frac{\sin x}{\cos x} \cdot \frac{\sin x}{\sin x} + \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x}$$
$$\sec x \csc x \stackrel{?}{=} \frac{\sin^2 x}{\cos x \sin x} + \frac{\cos^2 x}{\sin x \cos x}$$
$$\sec x \csc x \stackrel{?}{=} \frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$$
$$\sec x \csc x \stackrel{?}{=} \frac{1}{\cos x \sin x}$$
$$\sec x \csc x \stackrel{?}{=} \frac{1}{\cos x} \cdot \frac{1}{\sin x}$$
$$\sec x \csc x = \sec x \csc x$$

18.
$$\sin \theta + \cos \theta \stackrel{?}{=} \frac{2\sin^2 \theta - 1}{\sin \theta - \cos \theta}$$
$$\sin \theta + \cos \theta \stackrel{?}{=} \frac{2\sin^2 \theta - (\sin^2 \theta + \cos^2 \theta)}{\sin \theta - \cos \theta}$$
$$\sin \theta + \cos \theta \stackrel{?}{=} \frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta - \cos \theta}$$
$$\sin \theta + \cos \theta \stackrel{?}{=} \frac{(\sin \theta - \cos \theta)(\sin \theta + \cos \theta)}{\sin \theta - \cos \theta}$$
$$\sin \theta + \cos \theta = \sin \theta + \cos \theta$$

19.
$$(\sin A + \cos A)^2 \stackrel{?}{=} \frac{2 + \sec A \csc A}{\sec A \csc A}$$
$$(\sin A + \cos A)^2 \stackrel{?}{=} \frac{2}{\sec A \csc A} + \frac{\sec A \csc A}{\sec A \csc A}$$
$$(\sin A + \cos A)^2 \stackrel{?}{=} 2\frac{1}{\sec A} \cdot \frac{1}{\csc A} + 1$$
$$(\sin A + \cos A)^2 \stackrel{?}{=} 2 \cos A \sin A + 1$$
$$(\sin A + \cos A)^2 \stackrel{?}{=} 2 \cos A \sin A + \sin^2 A + \cos^2 A$$
$$(\sin A + \cos A)^2 = (\sin A + \cos A)^2$$

20.
$$\begin{aligned}
(\sin\theta - 1)(\tan\theta + \sec\theta) &\stackrel{?}{=} -\cos\theta \\
\sin\theta\tan\theta - \tan\theta + \sin\theta\sec\theta - \sec\theta &\stackrel{?}{=} -\cos\theta \\
\sin\theta\frac{\sin\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta} + \sin\theta\frac{1}{\cos\theta} - \frac{1}{\cos\theta} &\stackrel{?}{=} -\cos\theta \\
\frac{\sin^2\theta - \sin\theta + \sin\theta - 1}{\cos\theta} &\stackrel{?}{=} -\cos\theta \\
\frac{\sin^2\theta - 1}{\cos\theta} &\stackrel{?}{=} -\cos\theta \\
\frac{-\cos^2\theta}{\cos\theta} &\stackrel{?}{=} -\cos\theta \\
-\cos\theta &= -\cos\theta
\end{aligned}$$

21.
$$\begin{aligned}
\frac{\cos y}{1-\sin y} &\stackrel{?}{=} \frac{1+\sin y}{\cos y} \\
\frac{\cos y}{1-\sin y}\cdot\frac{1+\sin y}{1+\sin y} &\stackrel{?}{=} \frac{1+\sin y}{\cos y} \\
\frac{\cos y(1-\sin y)}{1-\sin^2 y} &\stackrel{?}{=} \frac{1+\sin y}{\cos y} \\
\frac{\cos y(1+\sin y)}{\cos^2 y} &\stackrel{?}{=} \frac{1+\sin y}{\cos y} \\
\frac{1+\sin y}{\cos y} &= \frac{1+\sin y}{\cos y}
\end{aligned}$$

22.
$$\begin{aligned}
\cos\theta\cos(-\theta) - \sin\theta\sin(-\theta) &\stackrel{?}{=} 1 \\
\cos\theta\cos\theta - \sin\theta(-\sin\theta) &\stackrel{?}{=} 1 \\
\cos^2\theta + \sin^2\theta &\stackrel{?}{=} 1 \\
1 &= 1
\end{aligned}$$

23.
$$\begin{aligned}
\csc x - 1 &\stackrel{?}{=} \frac{\cot^2 x}{\csc x + 1} \\
\csc x - 1 &\stackrel{?}{=} \frac{\csc^2 x - 1}{\csc x + 1} \\
\csc x - 1 &\stackrel{?}{=} \frac{(\csc x + 1)(\csc x - 1)}{\csc x + 1} \\
\csc x - 1 &= \csc x - 1
\end{aligned}$$

24.
$$\begin{aligned}
\cos B\cot B &\stackrel{?}{=} \csc B - \sin B \\
\cos B\cot B &\stackrel{?}{=} \frac{1}{\sin B} - \sin B \\
\cos B\cot B &\stackrel{?}{=} \frac{1}{\sin B} - \frac{\sin^2 B}{\sin B} \\
\cos B\cot B &\stackrel{?}{=} \frac{1-\sin^2 B}{\sin B} \\
\cos B\cot B &\stackrel{?}{=} \frac{\cos^2 B}{\sin B} \\
\cos B\cot B &\stackrel{?}{=} \cos B\frac{\cos B}{\sin B} \\
\cos B\cot B &= \cos B\cot B
\end{aligned}$$

25.
$$\begin{aligned}
\sin\theta\cos\theta\tan\theta + \cos^2\theta &\stackrel{?}{=} 1 \\
\sin\theta\cos\theta\frac{\sin\theta}{\cos\theta} + \cos^2\theta &\stackrel{?}{=} 1 \\
\sin^2\theta + \cos^2\theta &\stackrel{?}{=} 1 \\
1 &= 1
\end{aligned}$$

26.
$$\begin{aligned}
(\csc x - \cot x)^2 &\stackrel{?}{=} \frac{1-\cos x}{1+\cos x} \\
\csc^2 x - 2\csc x\cot x + \cot^2 x &\stackrel{?}{=} \frac{1-\cos x}{1+\cos x} \\
\frac{1}{\sin^2 x} - 2\frac{1}{\sin x}\cdot\frac{\cos x}{\sin x} + \frac{\cos^2 x}{\sin^2 x} &\stackrel{?}{=} \frac{1-\cos x}{1+\cos x} \\
\frac{1 - 2\cos x + \cos^2 x}{\sin^2 x} &\stackrel{?}{=} \frac{1-\cos x}{1+\cos x} \\
\frac{(1-\cos x)^2}{1-\cos^2 x} &\stackrel{?}{=} \frac{1-\cos x}{1+\cos x} \\
\frac{(1-\cos x)^2}{(1-\cos x)(1+\cos x)} &\stackrel{?}{=} \frac{1-\cos x}{1+\cos x} \\
\frac{1-\cos x}{1+\cos x} &= \frac{1-\cos x}{1+\cos x}
\end{aligned}$$

27.
$$\begin{aligned}
\sin x + \cos x &\stackrel{?}{=} \frac{\cos x}{1-\tan x} + \frac{\sin x}{1-\cot x} \\
\sin x + \cos x &\stackrel{?}{=} \frac{\cos x}{1-\frac{\sin x}{\cos x}} + \frac{\sin x}{1-\frac{\cos x}{\sin x}} \\
\sin x + \cos x &\stackrel{?}{=} \frac{\cos x}{1-\frac{\sin x}{\cos x}}\cdot\frac{\cos x}{\cos x} + \frac{\sin x}{1-\frac{\cos x}{\sin x}}\cdot\frac{\sin x}{\sin x} \\
\sin x + \cos x &\stackrel{?}{=} \frac{\cos^2 x}{\cos x - \sin x} + \frac{\sin^2 x}{\sin x - \cos x} \\
\sin x + \cos x &\stackrel{?}{=} -\frac{\cos^2 x}{\sin x - \cos x} + \frac{\sin^2 x}{\sin x - \cos x} \\
\sin x + \cos x &\stackrel{?}{=} \frac{\sin^2 x - \cos^2 x}{\sin x - \cos x} \\
\sin x + \cos x &\stackrel{?}{=} \frac{(\sin x + \cos x)(\sin x - \cos x)}{\sin x - \cos x} \\
\sin x + \cos x &= \sin x + \cos x
\end{aligned}$$

28.
$$\begin{aligned}
\sin\theta + \cos\theta + \tan\theta\sin\theta &\stackrel{?}{=} \sec\theta + \cos\theta\tan\theta \\
\sin\theta + \cos\theta + \frac{\sin\theta}{\cos\theta}\sin\theta &\stackrel{?}{=} \sec\theta + \cos\theta\tan\theta \\
\sin\theta + \cos\theta + \frac{\sin^2\theta}{\cos\theta} &\stackrel{?}{=} \sec\theta + \cos\theta\tan\theta \\
\sin\theta + \frac{\cos^2\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos\theta} &\stackrel{?}{=} \sec\theta + \cos\theta\tan\theta \\
\sin\theta + \frac{\cos^2\theta + \sin^2\theta}{\cos\theta} &\stackrel{?}{=} \sec\theta + \cos\theta\tan\theta \\
\sin\theta + \frac{1}{\cos\theta} &\stackrel{?}{=} \sec\theta + \cos\theta\tan\theta \\
\sin\theta + \sec\theta &\stackrel{?}{=} \sec\theta + \cos\theta\tan\theta \\
\sin\theta\frac{\cos\theta}{\cos\theta} + \sec\theta &\stackrel{?}{=} \sec\theta + \cos\theta\tan\theta \\
\cos\theta\frac{\sin\theta}{\cos\theta} + \sec\theta &\stackrel{?}{=} \sec\theta + \cos\theta\tan\theta \\
\cos\theta\tan\theta + \sec\theta &\stackrel{?}{=} \sec\theta + \cos\theta\tan\theta \\
\sec\theta + \cos\theta\tan\theta &= \sec\theta + \cos\theta\tan\theta
\end{aligned}$$

29. Sample answer: $\sec x = \sqrt{2}$
$$\begin{aligned}
\frac{\csc x}{\cot x} &= \sqrt{2} \\
\frac{\frac{1}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}} &= \sqrt{2} \\
\frac{1}{\cos x} &= \sqrt{2} \\
\sec x &= \sqrt{2}
\end{aligned}$$

30. Sample answer: $\tan x = 2$
$$\begin{aligned}
\frac{1+\tan x}{1+\cot x} &= 2 \\
\frac{1+\frac{\sin x}{\cos x}}{1+\frac{\cos x}{\sin x}} &= 2 \\
\frac{\frac{\cos x + \sin x}{\cos x}}{\frac{\sin x + \cos x}{\sin x}} &= 2 \\
\frac{\sin x}{\cos x} &= 2 \\
\tan x &= 2
\end{aligned}$$

31. Sample answer: $\cos x = 0$
$$\begin{aligned}
\frac{1}{\cot x} - \frac{\sec x}{\csc x} &= \cos x \\
\tan x - \frac{\frac{1}{\cos x}}{\frac{1}{\sin x}} &= \cos x \\
\tan x - \frac{\sin x}{\cos x} &= \cos x \\
\tan x - \tan x &= \cos x \\
0 &= \cos x
\end{aligned}$$

32. Sample answer: $\sin x = \frac{1}{2}$

$$\frac{1 + \cos x}{\sin x} + \frac{\sin x}{1 + \cos x} = 4$$
$$\frac{1 + 2\cos x + \cos^2 x}{\sin x(1 + \cos x)} + \frac{\sin^2 x}{\sin x(1 + \cos x)} = 4$$
$$\frac{1 + 2\cos x + \cos^2 x + \sin^2 x}{\sin x(1 + \cos x)} = 4$$
$$\frac{2 + 2\cos x}{\sin x(1 + \cos x)} = 4$$
$$\frac{2(1 + \cos x)}{\sin x(1 + \cos x)} = 4$$
$$\frac{2}{\sin x} = 4$$
$$2 = 4\sin x$$
$$\frac{1}{2} = \sin x$$

33. Sample answer: $\sin x = 1$

$$\cos^2 x + 2\sin x - 2 = 0$$
$$1 - \sin^2 x + 2\sin x - 2 = 0$$
$$0 = \sin^2 x - 2\sin x + 1$$
$$0 = (\sin x - 1)^2$$
$$0 = \sin x - 1$$
$$\sin x = 1$$

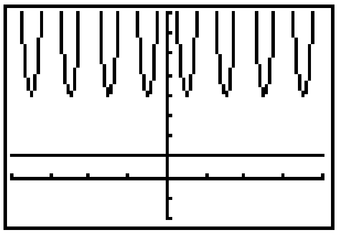
34. Sample answer: $\cot x = 1$

$$\csc x = \sin x \tan x + \cos x$$
$$\csc x = \sin x \frac{\sin x}{\cos x} + \cos x$$
$$\csc x = \frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x}$$
$$\csc x = \frac{1}{\cos x}$$
$$\frac{1}{\sin x} = \frac{1}{\cos x}$$
$$\frac{\cos x}{\sin x} = 1$$
$$\cot x = 1$$

35.
$$\frac{\tan^3\theta - 1}{\tan\theta - 1} - \sec^2\theta - 1 = 0$$
$$\frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{\tan\theta - 1} - (\tan^2\theta + 1) - 1 = 0$$
$$\tan^2\theta + \tan\theta + 1 - \tan^2\theta - 1 - 1 = 0$$
$$\tan\theta - 1 = 0$$
$$\tan\theta = 1$$

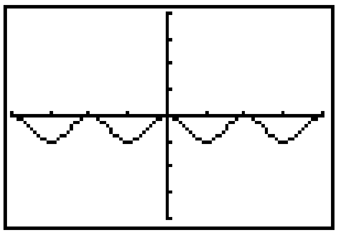
$$\cot\theta = \frac{1}{\tan\theta}$$
$$\cot\theta = \frac{1}{1}$$
$$\cot\theta = 1$$

36. no



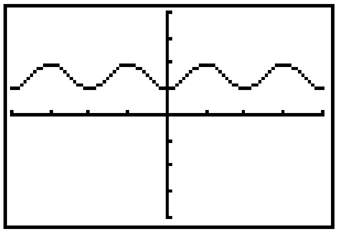
$[-2\pi, 2\pi]$ sc1:$\frac{\pi}{2}$ by $[-2, 8]$ sc1:1

37. yes



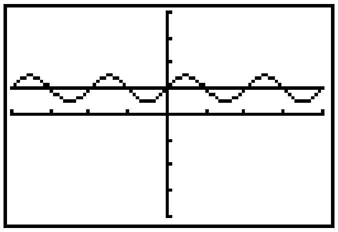
$[-2\pi, 2\pi]$ sc1:$\frac{\pi}{2}$ by $[-4, 4]$ sc1:1

38. yes



$[-2\pi, 2\pi]$ sc1:$\frac{\pi}{2}$ by $[-4, 4]$ sc1:1

39. no



$[-2\pi, 2\pi]$ sc1:$\frac{\pi}{2}$ by $[-4, 4]$ sc1:1

40a. $P = I_0{}^2 R \sin^2 2\pi ft$

$P = I_0{}^2 R(1 - \cos^2 2\pi ft)$

40b. $P = I_0{}^2 R \sin^2 2\pi ft$

$P = \dfrac{I_0{}^2 R}{\csc^2 2\pi ft}$

41. $f(x) = \dfrac{x}{\sqrt{1 + 4x^2}}$

$$f(x) = \frac{\frac{1}{2}\tan\theta}{\sqrt{1 + 4\left(\frac{1}{2}\tan\theta\right)^2}}$$
$$f(x) = \frac{\frac{1}{2}\tan\theta}{\sqrt{1 + \tan^2\theta}}$$
$$f(x) = \frac{\frac{1}{2}\tan\theta}{\sqrt{\sec^2\theta}}$$
$$f(x) = \frac{\frac{1}{2}\tan\theta}{\sec\theta}$$
$$f(x) = \frac{\frac{1}{2}\cdot\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$$
$$f(x) = \frac{1}{2}\sin\theta$$

42. $\sin a = \sin \alpha \sin c \Rightarrow \sin \alpha = \dfrac{\sin a}{\sin c}$

$\cos b = \dfrac{\cos\beta}{\sin\alpha} \Rightarrow \cos\beta = \sin\alpha\cos b$

$\cos c = \cos a \cos b \Rightarrow \cos b = \dfrac{\cos c}{\cos a}$

Then $\cos\beta = \sin\alpha\cos b$

$$= \frac{\sin a}{\sin c}\cdot\frac{\cos c}{\cos a}$$
$$= \frac{\sin a}{\cos a}\cdot\frac{\cos c}{\sin c}$$
$$= \tan a \cot c$$

43. $y = \dfrac{-gv^2}{2v_0{}^2\cos^2\theta} + \dfrac{x\sin\theta}{\cos\theta}$

$y = \dfrac{-gv^2}{2v_0{}^2}\sec^2\theta + x\tan\theta$

$y = -\dfrac{gx^2}{2v_0{}^2}(1 + \tan^2\theta) + x\tan\theta$

44. We find the area of $ABTP$ by subtracting the area of $\triangle OAP$ from the area of $\triangle OBT$.

$$\begin{aligned}\tfrac{1}{2}OB \cdot BT - \tfrac{1}{2}OA \cdot AP &= \tfrac{1}{2} \cdot 1 \cdot \tan\theta - \tfrac{1}{2}\cos\theta\sin\theta \\ &= \tfrac{1}{2}\left(\frac{\sin\theta}{\cos\theta} - \cos\theta\sin\theta\right) \\ &= \tfrac{1}{2}\sin\theta\left(\frac{1}{\cos\theta} - \cos\theta\right) \\ &= \tfrac{1}{2}\sin\theta\left(\frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta}\right) \\ &= \tfrac{1}{2}\sin\theta\left(\frac{1-\cos^2\theta}{\cos\theta}\right) \\ &= \tfrac{1}{2}\sin\theta\left(\frac{\sin^2\theta}{\cos\theta}\right) \\ &= \tfrac{1}{2}\cdot\frac{\sin\theta}{\cos\theta}\sin^2\theta \\ &= \tfrac{1}{2}\tan\theta\sin^2\theta\end{aligned}$$

45. By the Law of Sines, $\frac{b}{\sin\beta} = \frac{a}{\sin\alpha}$, so $b = \frac{a\sin\beta}{\sin\alpha}$. Then

$$\begin{aligned}A &= \tfrac{1}{2}ab\sin\gamma \\ A &= \tfrac{1}{2}a\left(\frac{a\sin\beta}{\sin\alpha}\right)\sin\gamma \\ A &= \frac{a^2\sin\beta\sin\gamma}{2\sin\alpha} \\ A &= \frac{a^2\sin\beta\sin\gamma}{2\sin(180° - (\beta+\gamma))} \\ A &= \frac{a^2\sin\beta\sin\gamma}{2\sin(\beta+\gamma)}\end{aligned}$$

46.
$$\begin{aligned}\frac{\tan x + \cos x + \sin x\tan x}{\sec x + \tan x} &= \frac{\frac{\sin x}{\cos x} + \cos x + \sin x\frac{\sin x}{\cos x}}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}} \\ &= \frac{\frac{\sin x + \cos^2 x + \sin^2 x}{\cos x}}{\frac{1+\sin x}{\cos x}} \\ &= \frac{\sin x + 1}{\cos x}\cdot\frac{\cos x}{1+\sin x} \\ &= 1\end{aligned}$$

47. $|A| = 2$ $\quad \frac{360°}{k} = 180° \quad -\frac{c}{2} = 45°$

$A = \pm 2 \quad k = 2 \quad c = -90°$

$y = \pm 2\sin(2x - 90°)$

48.
$$\begin{aligned}\frac{15\pi}{16} &= \frac{15\pi}{16}\times\frac{180°}{\pi} \\ &= 168.75°\end{aligned}$$
$$\begin{aligned}168.75° &= 168° + \left(0.75° \times \frac{60'}{1°}\right) \\ &= 168° + 45'\end{aligned}$$
$168°\ 45'$

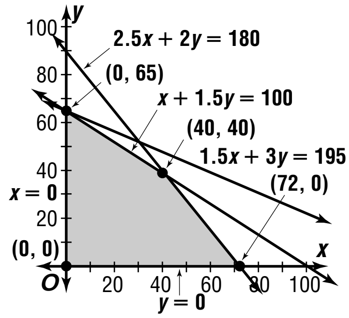
49.
$$\begin{aligned}\sqrt[3]{3y-1} - 2 &= 0 \\ \sqrt[3]{3y-1} &= 2 \\ 3y - 1 &= 8 \\ y &= 3\end{aligned}$$

Check:
$$\begin{aligned}\sqrt[3]{3y-1} - 2 &= 0 \\ \sqrt[3]{3(3)-1} - 2 &\stackrel{?}{=} 0 \\ \sqrt[3]{8} - 2 &\stackrel{?}{=} 0 \\ 2 - 2 &= 0\ \checkmark\end{aligned}$$

50.
$$\begin{aligned}x + 1 &= 0 \\ x &= -1\end{aligned}$$
$$\begin{aligned}f(x) &= \frac{3x}{x+1} \\ y &= \frac{3x}{x+1} \\ y(x+1) &= 3x \\ yx + y &= 3x \\ y &= 3x - yx \\ y &= x(3-y) \\ \frac{y}{3-y} &= x\end{aligned}$$
$$\begin{aligned}3 - y &= 0 \\ y &= 3\end{aligned}$$

51. Let x = the number of shirts and y = the number of pants.

$x + 1.5y \le 100$
$2.5x + 2y \le 180$
$1.5x + 3y \le 195$
$x \ge 0$
$y \ge 0$



$$\begin{aligned}P(x, y) &= 5x + 4.5y \\ P(0, 0) &= 5(0) + 4.5(0) \text{ or } 0 \\ P(0, 65) &= 5(0) + 4.5(65) \text{ or } 292.50 \\ P(40, 40) &= 5(40) + 4.5(40) \text{ or } 380 \\ P(72, 0) &= 5(72) + 4.5(0) \text{ or } 360\end{aligned}$$

40 shirts, 40 pants

52. {16}, {−4, 4}; no, 16 is paired with two elements of the range

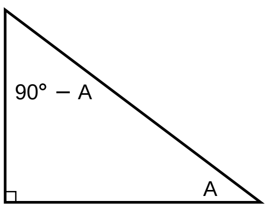
53.
$$\begin{aligned}\frac{a-b}{a+b} \div \frac{b-a}{b+a} &= \frac{a-b}{a+b}\cdot\frac{b+a}{b-a} \\ &= \frac{a-b}{a+b}\cdot\frac{a+b}{-1(a-b)} \\ &= -1\end{aligned}$$

The correct choice is D.

7-3 Sum and Difference Identities

Pages 441–442 Check for Understanding

1. Find a counterexample, such as $x = 30°$ and $y = 60°$.
2. Find the cosine, sine, or tangent, respectively, of the sum or difference, then take the reciprocal.
3. The opposite side for $90° - A$ is the adjacent side for A, so the right-triangle ratio for $\sin(90° - A)$ is the same as that for $\cos A$.



4.
$$\begin{aligned}\cot(\alpha+\beta) &= \frac{1}{\tan(\alpha+\beta)} \\ &= \frac{1}{\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}} \\ &= \frac{1 - \tan\alpha\tan\beta}{\tan\alpha + \tan\beta} \\ &= \frac{1 - \frac{1}{\cot\alpha}\cdot\frac{1}{\cot\beta}}{\frac{1}{\cot\alpha} + \frac{1}{\cot\beta}}\cdot\frac{\cot\alpha\cot\beta}{\cot\alpha\cot\beta} \\ &= \frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta}\end{aligned}$$

5. $\cos 165° = \cos(45° + 120°)$
$= \cos 45° \cos 120° - \sin 45° \sin 120°$
$= \frac{\sqrt{2}}{2} \cdot \left(-\frac{1}{2}\right) - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$
$= -\frac{\sqrt{2} + \sqrt{6}}{4}$

6. $\tan \frac{\pi}{12} = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$
$= \dfrac{\tan \frac{\pi}{3} - \tan \frac{\pi}{4}}{1 + \tan \frac{\pi}{3} \tan \frac{\pi}{4}}$
$= \frac{\sqrt{3} - 1}{1 + \sqrt{3} \cdot 1}$
$= \frac{-4 + 2\sqrt{3}}{-2}$
$= 2 - \sqrt{3}$

7. $795° = 2(360°) + 75°$
$\sec 795° = \sec 75°$
$\cos 75° = \cos(30° + 45°)$
$= \cos 30° \cos 45° - \sin 30° \sin 45°$
$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$
$= \frac{\sqrt{6} - \sqrt{2}}{4}$
$\sec 795° = \frac{4}{\sqrt{6} - \sqrt{2}}$
$= \sqrt{6} + \sqrt{2}$

8. $\cos x = \sqrt{1 - \sin^2 x}$
$= \sqrt{1 - \left(\frac{4}{9}\right)^2}$
$= \sqrt{\frac{65}{81}}$ or $\frac{\sqrt{65}}{9}$

$\cos y = \sqrt{1 - \sin^2 y}$
$= \sqrt{1 - \left(\frac{1}{4}\right)^2}$
$= \sqrt{\frac{15}{16}}$ or $\frac{\sqrt{15}}{4}$

$\sin(x - y) = \sin x \cos y - \cos x \sin y$
$= \left(\frac{4}{9}\right)\left(\frac{\sqrt{15}}{4}\right) - \left(\frac{\sqrt{65}}{9}\right)\left(\frac{1}{4}\right)$
$= \frac{4\sqrt{15} - \sqrt{65}}{36}$

9. $\csc x = \frac{1}{\sin x}$
$\frac{5}{3} = \frac{1}{\sin x}$
$\sin x = \frac{3}{5}$
$\tan x = \frac{\sin x}{\cos x}$
$= \dfrac{\frac{3}{5}}{\frac{4}{5}}$ or $\frac{3}{4}$

$\cos x = \sqrt{1 - \sin^2 x}$
$= \sqrt{1 - \left(\frac{3}{5}\right)^2}$
$= \sqrt{\frac{16}{25}}$ or $\frac{4}{5}$

$\sin y = \sqrt{1 - \cos^2 y}$
$= \sqrt{1 - \left(\frac{5}{13}\right)^2}$
$= \sqrt{\frac{144}{169}}$ or $\frac{12}{13}$

$\tan y = \frac{\sin y}{\cos y}$
$= \dfrac{\frac{12}{13}}{\frac{5}{13}}$ or $\frac{12}{5}$

$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
$= \dfrac{\frac{3}{4} + \frac{12}{5}}{1 - \left(\frac{3}{4}\right)\left(\frac{12}{5}\right)}$
$= \dfrac{\frac{63}{20}}{-\frac{4}{5}}$
$= -\frac{63}{16}$

10. $\sin(90° + A) \stackrel{?}{=} \cos A$
$\sin 90° \cos A + \cos 90° \sin A \stackrel{?}{=} \cos A$
$1 \cdot \cos A + 0 \cdot \sin A \stackrel{?}{=} \cos A$
$\cos A = \cos A$

11. $\tan\left(\theta + \frac{\pi}{2}\right) \stackrel{?}{=} -\cot \theta$
$\dfrac{\sin\left(\theta + \frac{\pi}{2}\right)}{\cos\left(\theta + \frac{\pi}{2}\right)} \stackrel{?}{=} -\cot \theta$
$\dfrac{\sin \theta \cos \frac{\pi}{2} + \cos \theta \sin \frac{\pi}{2}}{\cos \theta \cos \frac{\pi}{2} - \sin \theta \sin \frac{\pi}{2}} \stackrel{?}{=} -\cot \theta$
$\frac{(\sin \theta) \cdot 0 + (\cos \theta) \cdot 1}{(\cos \theta) \cdot 0 - (\sin \theta) \cdot 1} \stackrel{?}{=} -\cot \theta$
$-\frac{\cos \theta}{\sin \theta} \stackrel{?}{=} -\cot \theta$
$-\cot \theta = -\cot \theta$

12. $\sin(x - y) \stackrel{?}{=} \frac{1 - \cot x \tan y}{\csc x \sec y}$
$\sin(x - y) \stackrel{?}{=} \dfrac{1 - \frac{\cos x}{\sin x} \cdot \frac{\sin y}{\cos y}}{\frac{1}{\sin x} \cdot \frac{1}{\cos y}}$
$\sin(x - y) \stackrel{?}{=} \dfrac{1 - \frac{\cos x}{\sin x} \cdot \frac{\sin y}{\cos y}}{\frac{1}{\sin x} \cdot \frac{1}{\cos y}} \cdot \frac{\sin x \cos y}{\sin x \cos y}$
$\sin(x - y) \stackrel{?}{=} \frac{\sin x \cos y - \cos x \sin y}{1}$
$\sin(x - y) = \sin(x - y)$

13. $\sin(n\omega_0 t - 90°) = \sin n\omega_0 t \cos 90° - \cos n\omega_0 t \sin 90°$
$= \sin n\omega_0 t \cdot 0 - \cos n\omega_0 t \cdot 1$
$= -\cos n\omega_0 t$

Pages 442–445 Exercises

14. $\cos 105° = \cos(45° + 60°)$
$= \cos 45° \cos 60° - \sin 45° \sin 60°$
$= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$
$= \frac{\sqrt{2} - \sqrt{6}}{4}$

15. $\sin 165° = \sin(120° + 45°)$
$= \sin 120° \cos 45° + \cos 120° \sin 45°$
$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$
$= \frac{\sqrt{6} - \sqrt{2}}{4}$

16. $\cos \frac{7\pi}{12} = \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$
$= \cos \frac{\pi}{4} \cos \frac{\pi}{3} - \sin \frac{\pi}{4} \sin \frac{\pi}{3}$
$= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$
$= \frac{\sqrt{2} - \sqrt{6}}{4}$

17. $\sin \frac{\pi}{12} = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$
$= \sin \frac{\pi}{3} \cos \frac{\pi}{4} - \cos \frac{\pi}{3} \sin \frac{\pi}{4}$
$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$
$= \frac{\sqrt{6} - \sqrt{2}}{4}$

18. $\tan 195° = \tan(45° + 150°)$
$$= \frac{\tan 45° + \tan 150°}{1 - \tan 45° \tan 150°}$$
$$= \frac{1 + \left(-\frac{\sqrt{3}}{3}\right)}{1 - 1\left(-\frac{\sqrt{3}}{3}\right)}$$
$$= \frac{\frac{3-\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}}$$
$$= \frac{12 - 6\sqrt{3}}{6} \text{ or } 2 - \sqrt{3}$$

19. $\cos\left(-\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$
$$= \cos\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{4}\sin\frac{\pi}{3}$$
$$= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$$
$$= \frac{\sqrt{2} + \sqrt{6}}{4}$$

20. $\tan 165° = \tan(45° + 120°)$
$$= \frac{\tan 45° + \tan 120°}{1 - \tan 45° \tan 120°}$$
$$= \frac{1 + (-\sqrt{3})}{1 - 1(-\sqrt{3})}$$
$$= \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$$
$$= \frac{4 - 2\sqrt{3}}{-2} \text{ or } -2 + \sqrt{3}$$

21. $\tan\frac{23\pi}{12} = \tan\left(\frac{\pi}{4} + \frac{5\pi}{3}\right)$
$$= \frac{\tan\frac{\pi}{4} + \tan\frac{5\pi}{3}}{1 - \tan\frac{\pi}{4}\tan\frac{5\pi}{3}}$$
$$= \frac{1 + (-\sqrt{3})}{1 - 1(-\sqrt{3})}$$
$$= \frac{4 - 2\sqrt{3}}{-2} \text{ or } -2 + \sqrt{3}$$

22. $735° = 2(360°) + 15°$
$\sin 735° = \sin 15°$
$\sin 15° = \sin(45° - 30°)$
$$= \sin 45° \cos 30° - \cos 45° \sin 30°$$
$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$
$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

23. $1275° = 3(360°) + 195°$
$\sec 1275° = \sec 195°$
$\cos 195° = \cos(150° + 45°)$
$$= \cos 150° \cos 45° - \sin 150° \sin 45°$$
$$= -\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$
$$= -\frac{\sqrt{6} - \sqrt{2}}{4}$$
$\sec 1275° = \frac{4}{-\sqrt{6} - \sqrt{2}}$
$$= \sqrt{2} - \sqrt{6}$$

24. $\sin\frac{5\pi}{12} = \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$
$$= \sin\frac{\pi}{6}\cos\frac{\pi}{4} + \cos\frac{\pi}{6}\sin\frac{\pi}{4}$$
$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{2} + \sqrt{6}}{4}$$
$\csc\frac{5\pi}{2} = \frac{4}{\sqrt{2} + \sqrt{6}}$
$$= \sqrt{6} - \sqrt{2}$$

25. $\frac{113\pi}{12} = 4(2\pi) + \frac{17\pi}{12}$
$\cot\frac{113\pi}{12} = \cot\frac{17\pi}{12}$
$\tan\frac{17\pi}{12} = \tan\left(\frac{7\pi}{6} + \frac{\pi}{4}\right)$
$$= \frac{\tan\frac{\pi}{6} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{6}\tan\frac{\pi}{4}}$$
$$= \frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3} \cdot 1}$$
$$= \frac{\frac{\sqrt{3}+3}{3}}{\frac{3-\sqrt{3}}{3}}$$
$$= \sqrt{3} + 2$$
$\cot\frac{113\pi}{12} = \frac{1}{\sqrt{3} + 2}$
$$= 2 - \sqrt{3}$$

26. $\sin x = \sqrt{1 - \cos^2 x}$
$$= \sqrt{1 - \left(\frac{8}{17}\right)^2}$$
$$= \sqrt{\frac{225}{289}} \text{ or } \frac{15}{17}$$
$\cos y = \sqrt{1 - \sin^2 y}$
$$= \sqrt{1 - \left(\frac{12}{37}\right)^2}$$
$$= \sqrt{\frac{1225}{1369}} \text{ or } \frac{35}{37}$$
$\sin(x + y) = \sin x \cos y + \cos x \sin y$
$$= \left(\frac{15}{17}\right)\left(\frac{35}{37}\right) + \left(\frac{8}{17}\right)\left(\frac{12}{37}\right)$$
$$= \frac{621}{629}$$

27. $\sin x = \sqrt{1 - \cos^2 x}$
$$= \sqrt{1 - \left(\frac{3}{5}\right)^2}$$
$$= \sqrt{\frac{16}{25}} \text{ or } \frac{4}{5}$$
$\sin y = \sqrt{1 - \cos^2 y}$
$$= \sqrt{1 - \left(\frac{4}{5}\right)^2}$$
$$= \sqrt{\frac{9}{25}} \text{ or } \frac{3}{5}$$
$\cos(x - y) = \cos x \cos y + \sin x \sin y$
$$= \left(\frac{3}{5}\right)\left(\frac{4}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$$
$$= \frac{24}{25}$$

28. $\cos x = \sqrt{1 - \sin^2 x}$
$$= \sqrt{1 - \left(\frac{8}{17}\right)^2}$$
$$= \sqrt{\frac{225}{289}} \text{ or } \frac{15}{17}$$
$\sin y = \sqrt{1 - \cos^2 y}$
$$= \sqrt{1 - \left(\frac{3}{5}\right)^2}$$
$$= \sqrt{\frac{16}{25}} \text{ or } \frac{4}{5}$$
$\tan x = \frac{\sin x}{\cos x}$
$$= \frac{\frac{8}{17}}{\frac{15}{17}}$$
$$= \frac{8}{15}$$
$\tan y = \frac{\sin y}{\cos y}$
$$= \frac{\frac{4}{5}}{\frac{3}{5}}$$
$$= \frac{4}{3}$$
$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$
$$= \frac{\frac{8}{15} - \frac{4}{3}}{1 + \frac{8}{15} \cdot \frac{4}{3}}$$
$$= \frac{-\frac{12}{15}}{\frac{77}{45}}$$
$$= -\frac{36}{77}$$

29. $\sec x = \sqrt{\tan^2 x + 1}$
$= \sqrt{\left(\frac{5}{3}\right)^2 + 1}$
$= \sqrt{\frac{34}{9}}$ or $\frac{\sqrt{34}}{3}$
$\cos x = \frac{3}{\sqrt{34}}$ or $\frac{3\sqrt{34}}{34}$
$\tan x = \frac{\sin x}{\cos x}$
$\frac{5}{3} = \frac{\sin x}{\frac{3\sqrt{34}}{34}}$
$\sin x = \frac{5\sqrt{34}}{34}$

$\cos y = \sqrt{1 - \sin^2 y}$
$= \sqrt{1 - \left(\frac{1}{3}\right)^2}$
$= \sqrt{\frac{8}{9}}$ or $\frac{2\sqrt{2}}{3}$

$\cos (x + y) = \cos x \cos y - \sin x \sin y$
$= \left(\frac{3\sqrt{34}}{34}\right)\left(\frac{2\sqrt{2}}{3}\right) - \left(\frac{5\sqrt{34}}{34}\right)\left(\frac{1}{3}\right)$
$= \frac{6\sqrt{68}}{102} - \frac{5\sqrt{34}}{102}$
$= \frac{12\sqrt{17} - 5\sqrt{34}}{102}$

30. $\tan x = \frac{1}{\cot x}$
$= \frac{1}{\frac{6}{5}}$
$= \frac{5}{6}$

$\cos y = \frac{1}{\sec y}$
$= \frac{1}{\frac{3}{2}}$
$= \frac{2}{3}$
$\sin y = \sqrt{1 - \cos^2 y}$
$= \sqrt{1 - \left(\frac{2}{3}\right)^2}$
$= \sqrt{\frac{5}{9}}$ or $\frac{\sqrt{5}}{3}$
$\tan y = \frac{\sin y}{\cos y}$
$= \frac{\frac{\sqrt{5}}{3}}{\frac{2}{3}}$ or $\frac{\sqrt{5}}{2}$

$\tan (x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
$= \frac{\frac{5}{6} + \frac{\sqrt{5}}{2}}{1 - \left(\frac{5}{6}\right)\left(\frac{\sqrt{5}}{2}\right)}$
$= \frac{\frac{10 + 6\sqrt{5}}{12}}{\frac{12 - 5\sqrt{5}}{12}}$
$= \frac{10 + 6\sqrt{5}}{12 - 5\sqrt{5}}$
$= \frac{270 + 122\sqrt{5}}{19}$

31. $\sin x = \frac{1}{\csc x}$
$= \frac{1}{\frac{5}{3}}$
$= \frac{3}{5}$
$\cos x = \sqrt{1 - \sin^2 x}$
$= \sqrt{1 - \left(\frac{3}{5}\right)^2}$
$= \sqrt{\frac{16}{25}}$ or $\frac{4}{5}$

$\sec y = \sqrt{\tan^2 y + 1}$
$= \sqrt{\left(\frac{12}{5}\right)^2 + 1}$
$= \sqrt{\frac{169}{25}}$ or $\frac{13}{5}$
$\cos y = \frac{1}{\sec y}$
$= \frac{1}{\frac{13}{5}}$
$= \frac{5}{13}$
$\sin y = \sqrt{1 - \cos^2 y}$
$= \sqrt{1 - \left(\frac{5}{13}\right)^2}$
$= \sqrt{\frac{144}{169}}$ or $\frac{12}{13}$

$\cos(x - y) = \cos x \cos y + \sin x \sin y$
$= \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{12}{13}\right)$
$= \frac{56}{65}$
$\sec (x - y) = \frac{1}{\cos (x - y)}$
$= \frac{1}{\frac{56}{55}}$
$= \frac{65}{56}$

32. $\cos \alpha = \sqrt{1 - \sin^2\alpha}$
$= \sqrt{1 - \left(\frac{1}{5}\right)^2}$
$= \sqrt{\frac{24}{25}}$ or $\frac{2\sqrt{6}}{5}$

$\sin \beta = \sqrt{1 - \cos^2\beta}$
$= \sqrt{1 - \left(\frac{2}{7}\right)^2}$
$= \sqrt{\frac{45}{49}}$ or $\frac{3\sqrt{5}}{7}$

$\sin (\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$
$= \left(\frac{1}{5}\right)\left(\frac{2}{7}\right) - \left(\frac{2\sqrt{6}}{5}\right)\left(\frac{3\sqrt{5}}{7}\right)$
$= \frac{2 - 6\sqrt{30}}{35}$

33. $\sin x = \sqrt{1 - \cos^2 x}$
$= \sqrt{1 - \left(\frac{1}{3}\right)^2}$
$= \sqrt{\frac{8}{9}}$ or $\frac{2\sqrt{2}}{3}$

$\sin y = \sqrt{1 - \cos^2 y}$
$= \sqrt{1 - \left(\frac{3}{4}\right)^2}$
$= \sqrt{\frac{7}{16}}$ or $\frac{\sqrt{7}}{4}$

$\cos (x + y) = \cos x \cos y - \sin x \sin y$
$= \left(\frac{1}{3}\right)\left(\frac{3}{4}\right) - \left(\frac{2\sqrt{2}}{3}\right)\left(\frac{\sqrt{7}}{4}\right)$
$= \frac{3 - 2\sqrt{14}}{12}$

34. $\cos\left(\frac{\pi}{2} + x\right) \stackrel{?}{=} -\sin x$
$\cos \frac{\pi}{2} \cos x - \sin \frac{\pi}{2} \sin x \stackrel{?}{=} -\sin x$
$0 \cdot \cos x - 1 \cdot \sin x \stackrel{?}{=} -\sin x$
$-\sin x = -\sin x$

35. $\cos (60° + A) \stackrel{?}{=} \sin (30° - A)$
$\cos 60° \cos A - \sin 60° \sin A \stackrel{?}{=} \sin 30° \cos A - \cos 30° \sin A$
$\frac{1}{2} \cos A - \frac{\sqrt{3}}{2} \sin A = \frac{1}{2} \cos A - \frac{\sqrt{3}}{2} \sin A$

36. $\sin (A + \pi) \stackrel{?}{=} -\sin A$
$\sin A \cos \pi + \cos A \sin \pi \stackrel{?}{=} -\sin A$
$(\sin A)(-1) + (\cos A)(0) \stackrel{?}{=} -\sin A$
$-\sin A = -\sin A$

37.
$$\cos(180° + x) \overset{?}{=} -\cos x$$
$$\cos 180° \cos x - \sin 180° \sin x \overset{?}{=} -\cos x$$
$$-1 \cdot \cos x - 0 \cdot \sin x \overset{?}{=} -\cos x$$
$$-\cos x = -\cos x$$

38.
$$\tan(x + 45°) \overset{?}{=} \frac{1 + \tan x}{1 - \tan x}$$
$$\frac{\tan x + \tan 45°}{1 - \tan x \tan 45°} \overset{?}{=} \frac{1 + \tan x}{1 - \tan x}$$
$$\frac{\tan x + 1}{1 - (\tan x)(1)} \overset{?}{=} \frac{1 + \tan x}{1 - \tan x}$$
$$\frac{1 + \tan x}{1 - \tan x} = \frac{1 + \tan x}{1 - \tan x}$$

39.
$$\sin(A + B) \overset{?}{=} \frac{\tan A + \tan B}{\sec A \sec B}$$
$$\sin(A + B) \overset{?}{=} \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{1}{\cos A} \cdot \frac{1}{\cos B}}$$
$$\sin(A + B) \overset{?}{=} \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{1}{\cos A} \cdot \frac{1}{\cos B}} \cdot \frac{\cos A \cos B}{\cos A \cos B}$$
$$\sin(A + B) \overset{?}{=} \frac{\sin A \cos B + \cos A \sin B}{1}$$
$$\sin(A + B) = \sin(A + B)$$

40.
$$\cos(A + B) \overset{?}{=} \frac{1 - \tan A \tan B}{\sec A \sec B}$$
$$\cos(A + B) \overset{?}{=} \frac{1 - \frac{\sin A}{\cos A} \cdot \frac{\sin B}{\cos B}}{\frac{1}{\cos A} \cdot \frac{1}{\cos B}}$$
$$\cos(A + B) \overset{?}{=} \frac{1 - \frac{\sin A}{\cos A} \cdot \frac{\sin B}{\cos B}}{\frac{1}{\cos A} \cdot \frac{1}{\cos B}} \cdot \frac{\cos A \cos B}{\cos A \cos B}$$
$$\cos(A + B) \overset{?}{=} \frac{\cos A \cos B - \sin A \sin B}{1}$$
$$\cos(A + B) = \cos(A + B)$$

41.
$$\sec(A - B) \overset{?}{=} \frac{\sec A \sec B}{1 + \tan A \tan B}$$
$$\sec(A - B) \overset{?}{=} \frac{\frac{1}{\cos A} \cdot \frac{1}{\cos B}}{1 + \frac{\sin A}{\cos A} \cdot \frac{\sin B}{\cos B}}$$
$$\sec(A - B) \overset{?}{=} \frac{\frac{1}{\cos A} \cdot \frac{1}{\cos B}}{1 + \frac{\sin A}{\cos A} \cdot \frac{\sin B}{\cos B}} \cdot \frac{\cos A \cos B}{\cos A \cos B}$$
$$\sec(A - B) \overset{?}{=} \frac{1}{\cos A \cos B + \sin A \sin B}$$
$$\sec(A - B) \overset{?}{=} \frac{1}{\cos(A - B)}$$
$$\sec(A - B) = \sec(A - B)$$

42.
$$\sin(x + y)\sin(x - y) \overset{?}{=} \sin^2 x - \sin^2 y$$
$$(\sin x \cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y) \overset{?}{=} \sin^2 x - \sin^2 y$$
$$(\sin x \cos y)^2 - (\cos x \sin y)^2 \overset{?}{=} \sin^2 x - \sin^2 y$$
$$\sin^2 x \cos^2 y - \cos^2 x \sin^2 y \overset{?}{=} \sin^2 x - \sin^2 y$$
$$\sin^2 x \cos^2 y + \sin^2 x \sin^2 y - \sin^2 x \sin^2 y - \cos^2 x \sin^2 y \overset{?}{=} \sin^2 x - \sin^2 y$$
$$\sin^2 x(\cos^2 y + \sin^2 y) - \sin^2 y(\sin^2 x + \cos^2 x) \overset{?}{=} \sin^2 x - \sin^2 y$$
$$(\sin^2 x)(1) - (\sin^2 y)(1) \overset{?}{=} \sin^2 x - \sin^2 y$$
$$\sin^2 x - \sin^2 y = \sin^2 x - \sin^2 y$$

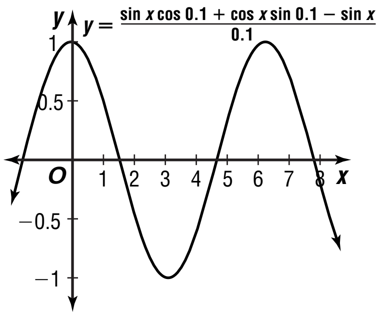
43.
$$V_L = I_0\omega L \cos\left(\omega t + \frac{\pi}{2}\right)$$
$$V_L = I_0\omega L\left(\cos \omega t \cos \frac{\pi}{2} - \sin \omega t \sin \frac{\pi}{2}\right)$$
$$V_L = I_0\omega L(\cos \omega t \cdot 0 - \sin \omega t \cdot 1)$$
$$V_L = I_0\omega L(-\sin \omega t)$$
$$V_L = -I_0\omega L \sin \omega t$$

44.
$$n = \frac{\sin\left[\frac{1}{2}(\alpha + \beta)\right]}{\sin \frac{\beta}{2}}$$
$$n = \frac{\sin\left[\frac{1}{2}(\alpha + 60°)\right]}{\sin \frac{60°}{2}}$$
$$n = \frac{\sin\left(\frac{\alpha}{2} + 30°\right)}{\sin 30°}$$
$$n = \frac{\sin \frac{\alpha}{2} \cos 30° + \cos \frac{\alpha}{2} \sin 30°}{\frac{1}{2}}$$
$$n = 2\left[\left(\sin \frac{\alpha}{2}\right) \cdot \frac{\sqrt{3}}{2} + \left(\cos \frac{\alpha}{2}\right) \cdot \frac{1}{2}\right]$$
$$n = \sqrt{3} \sin \frac{\alpha}{2} + \cos \frac{\alpha}{2}$$

45. The given expression is the expanded form of the sine of the difference of $\frac{\pi}{3} - A$ and $\frac{\pi}{3} + A$. We have
$$\sin\left[\left(\frac{\pi}{3} - A\right) - \left(\frac{\pi}{3} + A\right)\right] = \sin(-2A)$$
$$= -\sin 2A$$

46a.
$$\frac{f(x + h) - f(x)}{h} = \frac{\sin(x + h) - \sin x}{h}$$
$$= \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

46b.



46c. $\cos x$

47.
$$\tan(\alpha + \beta) = \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)}$$
$$\tan(\alpha + \beta) = \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta - \sin \alpha \sin \beta}$$
$$\tan(\alpha + \beta) = \frac{\frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}}$$
$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

Replace β with $-\beta$ to find $\tan(\alpha - \beta)$.
$$\tan(\alpha + (-\beta)) = \frac{\tan \alpha + \tan(-\beta)}{1 - \tan \alpha \tan(-\beta)}$$
$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

48a. Answers will vary.

48b. $\tan A + \tan B + \tan C \stackrel{?}{=} \tan A \tan B \tan C$

$\tan A + \tan B + \tan(180° - (A + B))$

$\stackrel{?}{=} \tan A \tan B \tan(180° - (A + B))$

$\tan A + \tan B + \frac{\tan 180° - \tan (A + B)}{1 + \tan 180° \tan (A + B)}$

$\stackrel{?}{=} \tan A \tan B \frac{\tan 180° - \tan (A + B)}{1 + \tan 180° \tan (A + B)}$

$\tan A + \tan B + \frac{0 - \tan (A + B)}{1 + 0 \cdot \tan (A + B)}$

$\stackrel{?}{=} \tan A \tan B \frac{0 - \tan (A + B)}{1 + 0 \cdot \tan (A + B)}$

$\tan A + \tan B - \tan (A + B)$

$\stackrel{?}{=} -\tan A \tan B \tan (A + B)$

$(\tan A + \tan B) \cdot \frac{1 - \tan A \tan B}{1 - \tan A \tan B} - \tan (A + B)$

$\stackrel{?}{=} -\tan A \tan B (A + B)$

$\tan (A + B)(1 - \tan A \tan B) - \tan (A + B)$

$\stackrel{?}{=} -\tan A \tan B (A + B)$

$(1 - \tan A \tan B - 1) \tan (A + B)$

$\stackrel{?}{=} -\tan A \tan B (A + B)$

$-\tan A \tan B \tan (A + B) = -\tan A \tan B (A + B)$

49. $\sec^2 x \stackrel{?}{=} \frac{1 - \cos^2 x}{1 - \sin^2 x} + \csc^2 x - \cot^2 x$

$\sec^2 x \stackrel{?}{=} \frac{1 - \cos^2 x}{\cos^2 x} + 1 + \cot^2 x - \cot^2 x$

$\sec^2 x \stackrel{?}{=} \frac{1}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x} + 1$

$\sec^2 x \stackrel{?}{=} \sec^2 x - 1 + 1$

$\sec^2 x = \sec^2 x$

50. $\sin^2 \theta + \cos^2 \theta = 1$

$\left(-\frac{1}{8}\right)^2 + \cos^2 \theta = 1$

$\cos^2 \theta = \frac{63}{64}$

$\cos \theta = \pm\frac{3\sqrt{7}}{8}$

Quadrant III, so $-\frac{3\sqrt{7}}{8}$

$\tan \theta = \frac{\sin \theta}{\cos \theta}$

$= \frac{-\frac{1}{8}}{-\frac{3\sqrt{7}}{8}}$

$= \frac{1}{3\sqrt{7}}$

$= \frac{\sqrt{7}}{21}$

51. $\text{Arctan}\sqrt{3} = \frac{\pi}{3}$

$\sin (\text{Arctan}\sqrt{3}) = \sin \frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}$

52. πk, where k is an integer

53. $A = \frac{86 - 50}{2}$ $\quad \frac{2\pi}{4} = \frac{\pi}{2}$ $\quad h = \frac{86 + 50}{2}$

$A = 18 = 68$ $\quad = 68$

$y = 18 \sin\left(\frac{\pi}{2}t + c\right) + 68$

$50 = 18 \sin\left(\frac{\pi}{2} \cdot 1 + c\right) + 68$

$-18 = 18 \sin\left(\frac{\pi}{2} + c\right)$

$-1 = \sin\left(\frac{\pi}{2} + c\right)$

$\sin^{-1}(-1) = \frac{\pi}{2} + c$

$\frac{3\pi}{2} = \frac{\pi}{2} + c$

$\pi = c$

$y = 18\sin\left(\frac{\pi}{2}t - \pi\right) + 68$

54. $|8| = 8$; $\frac{360}{1} = 360$; $\frac{30°}{1} = 30°$

55. $\sin(-540°) = \sin(-360° - 180°)$

$= 0$

56. $s = r\theta$ $\quad A = \frac{1}{2}r^2\theta$

$18 = r(2.9)$ $\quad A \approx \frac{1}{2}(6.2)^2(2.9)$

$6.2 \approx r$; 6.2 ft $\quad A \approx 55.7 \text{ ft}^2$

57. $c^2 = 70^2 + 130^2 - 2(70)(130)\cos 130°$

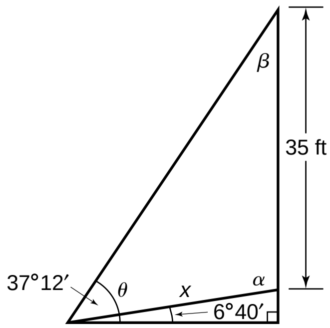
$c^2 \approx 33498.7345$

$c \approx 183$ miles

58. $120° \geq 90°$, consider Case 2.

$4 \leq 12$, 0 solutions

59.



$\theta = 37°\,12' - 6°\,40'$ or $30°\,32'$

$\alpha = 90° + 6°\,40'$ or $96°\,40'$

$\beta = 180° - (30°\,32' + 96°\,40')$ or $52°\,48'$

$\frac{35}{\sin 30°\,32'} = \frac{x}{\sin 52°\,48'}$

$x = \frac{35 \sin 52°\,48'}{\sin 30°\,32'}$

$x \approx 54.87$ ft

60. $4x^3 + 3x^2 - x = 0$

$x(4x^2 + 3x - 1) = 0$

$x(4x - 1)(x + 1) = 0$

$x = 0$ or $4x - 1 = 0$ or $x + 1 = 0$

$x = \frac{1}{4}$ $\quad x = -1$

61. Case 1

$|x + 1| > 4$

$-(x + 1) > 4$

$-x - 1 > 4$

$-x > 5$

$x < -5$

Case 2

$|x + 1| > 4$

$x + 1 > 4$

$x > 3$

$\{x \mid x < -5 \text{ or } x > 3\}$

62. $\begin{vmatrix} -1 & -2 \\ 3 & -6 \end{vmatrix} = -1(-6) - 3(-2)$

$= 6 + 6$ or 12

63. $f \circ g(4) = f(g(4))$

$= f(5(4) + 1)$

$= f(21)$

$= 3(21)^2 - 4$

$= 1319$

$g \circ f(4) = g(f(4))$

$= g(3(4)^2 - 4)$

$= g(44)$

$= 5(44) + 1$

$= 221$

64. $(-8)^{62} \div 8^{62} = \frac{(-8)^{62}}{8^{62}} = \left(\frac{-8}{8}\right)^{62} = (-1)^{62} = 1$

The correct choice is A.

Page 445 Mid-Chapter Quiz

1. $\csc\theta = \frac{1}{\sin\theta}$

$= \frac{1}{\frac{2}{7}}$

$= \frac{7}{2}$

$1 + \cot^2\theta = \csc^2\theta$

$1 + \cot^2\theta = \left(\frac{7}{2}\right)^2$

$1 + \cot^2\theta = \frac{49}{4}$

$\cot^2\theta = \frac{45}{4}$

$\cot\theta = \pm\frac{3\sqrt{5}}{2}$

Quadrant 1, so $\frac{3\sqrt{5}}{2}$

2. $\tan^2\theta + 1 = \sec^2\theta$

$\left(-\frac{4}{3}\right)^2 + 1 = \sec^2\theta$

$\frac{16}{9} + 1 = \sec^2\theta$

$\frac{25}{9} = \sec^2\theta$

$\pm\frac{5}{3} = \sec\theta$

$\cos\theta = \frac{1}{\sec\theta}$

$= \frac{1}{-\frac{5}{3}}$

$= -\frac{3}{5}$

Quadrant II, so $-\frac{5}{3}$

3. $\frac{19\pi}{4} = 5\pi - \frac{\pi}{4}$

$\cos\frac{19\pi}{4} = \cos\left(5\pi - \frac{\pi}{4}\right)$

$= -\cos\frac{\pi}{4}$

4. $\frac{1}{1+\tan^2 x} + \frac{1}{1+\cot^2 x} \stackrel{?}{=} 1$

$\frac{1}{\sec^2 x} + \frac{1}{\csc^2 x} \stackrel{?}{=} 1$

$\cos^2 x + \sin^2 x \stackrel{?}{=} 1$

$1 = 1$

5. $\frac{\csc^2\theta + \sec^2\theta}{\sec^2\theta} \stackrel{?}{=} \csc^2\theta$

$\frac{\csc^2\theta}{\sec^2\theta} + \frac{\sec^2\theta}{\sec^2\theta} \stackrel{?}{=} \csc^2\theta$

$\frac{\frac{1}{\sin^2\theta}}{\frac{1}{\cos^2\theta}} + 1 \stackrel{?}{=} \csc^2\theta$

$\frac{\cos^2\theta}{\sin^2\theta} + 1 \stackrel{?}{=} \csc^2\theta$

$\cot^2\theta + 1 \stackrel{?}{=} \csc^2\theta$

$\csc^2\theta = \csc^2\theta$

6. $\cot x \sec x \sin x \stackrel{?}{=} 2 - \tan x \cos x \csc x$

$\frac{\cos x}{\sin x} \cdot \frac{1}{\cos x} \cdot \sin x \stackrel{?}{=} 2 - \frac{\sin x}{\cos x} \cdot \cos x \cdot \frac{1}{\sin x}$

$1 \stackrel{?}{=} 2 - 1$

$1 = 1$

7. $\tan(\alpha - \beta) \stackrel{?}{=} \frac{1 - \cot\alpha\tan\beta}{\cot\alpha + \tan\beta}$

$\tan(\alpha - \beta) \stackrel{?}{=} \frac{1 - \frac{1}{\tan\alpha}\cdot\tan\beta}{\frac{1}{\tan\alpha} + \tan\beta}$

$\tan(\alpha - \beta) \stackrel{?}{=} \frac{1 - \frac{1}{\tan\alpha}\cdot\tan\beta}{\frac{1}{\tan\alpha} + \tan\beta} \cdot \frac{\tan\alpha}{\tan\alpha}$

$\tan(\alpha - \beta) \stackrel{?}{=} \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\tan(\alpha - \beta) = \tan(\alpha - \beta)$

8. $\cos 75° = \cos(30° + 45°)$

$= \cos 30° \cos 45° - \sin 30° \sin 45°$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$

9. $\cos x = \sqrt{1 - \sin^2 x}$

$= \sqrt{1 - \left(\frac{2}{3}\right)^2}$

$= \sqrt{\frac{5}{9}}$ or $\frac{\sqrt{5}}{3}$

$\cos y = \sqrt{1 - \sin^2 y}$

$= \sqrt{1 - \left(\frac{3}{4}\right)^2}$

$= \sqrt{\frac{7}{16}}$ or $\frac{\sqrt{7}}{4}$

$\cos(x + y) = \cos x \cos y - \sin x \sin y$

$= \left(\frac{\sqrt{5}}{3}\right)\left(\frac{\sqrt{7}}{4}\right) - \left(\frac{2}{3}\right)\left(\frac{3}{4}\right)$

$= \frac{\sqrt{35} - 6}{12}$

10. $\tan x = \frac{5}{4}$

$\tan y = \sqrt{\sec^2 y - 1}$

$= \sqrt{2^2 - 1}$

$= \sqrt{3}$

$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

$= \frac{\frac{5}{4} - \sqrt{3}}{1 + \left(\frac{5}{4}\right)\left(\sqrt{3}\right)}$

$= \frac{\frac{5 - 4\sqrt{3}}{4}}{\frac{4 + 5\sqrt{3}}{4}}$

$= \frac{5 - 4\sqrt{3}}{4 + 5\sqrt{3}}$

$= \frac{80 - 41\sqrt{3}}{-59}$ or $\frac{-80 + 41\sqrt{3}}{59}$

7-3B Reduction Identities

Page 447

1. −sin, −cos, sin

2. −cot, tan, −cot

3. −tan, cot, −tan

4. −csc, −sec, csc

5. sec, −csc, −sec

6a. (1) −cos, −sin, cos
(2) sin, −cos, −sin
(3) −cot, tan, −cot
(4) −tan, cot, −tan
(5) csc, −sec, −csc
(6) −sec, −csc, sec

6b. Sample answer: If a row for sin α were placed above Exercises 1-5, the entries for Exercise 6a could be obtained by interchanging the first and third columns and leaving the middle column alone.

7a. (1) cos, sin, −cos
(2) sin, −cos, −sin
(3) cot, −tan, cot
(4) tan, −cot, tan
(5) csc, −sec, −csc
(6) sec, csc, −sec

7b. Sample answer: The entries in the rows for cos α and sec α are unchanged. All other entries are multiplied by −1.

8a. Sample answer: They can be used to reduce trigonometric functions of large positive or negative angles to those of angles in the first quadrant.

8b. Sample answer: sum or difference identities

7-4 Double-Angle and Half-Angle Identities

Page 453 Check for Understanding

1. If you are only given the value of $\cos\theta$, then $\cos 2\theta = 2\cos^2\theta - 1$ is the best identity to use. If you are only given the value of $\sin\theta$, then $\cos 2\theta = 1 - 2\sin^2\theta$ is the best identity to use. If you are given the values of both $\cos\theta$ and $\sin\theta$, then $\cos 2\theta = \cos^2\theta - \sin^2\theta$ is just as good as the other two.

2. $$\begin{aligned}\cos 2\pi &= 1 - 2\sin^2\theta\\ \cos 2\theta - 1 &= -2\sin^2\theta\\ \frac{\cos 2\theta - 1}{-2} &= \sin^2\theta\\ \frac{1-\cos 2\theta}{2} &= \sin^2\theta\\ \pm\sqrt{\frac{1-\cos 2\theta}{2}} &= \sin\theta\end{aligned}$$
Letting $\theta = \frac{\alpha}{2}$ yields $\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos 2\left(\frac{\alpha}{2}\right)}{2}}$, or $\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$.

3a. III or IV **3b.** I or II **3c.** I, II, III or IV

4. $$\begin{aligned}\sin 2\theta &\stackrel{?}{=} 2\sin\theta\\ \sin 2\left(\frac{\pi}{2}\right) &\stackrel{?}{=} 2\sin\frac{\pi}{2}\\ \sin\pi &\stackrel{?}{=} 2\sin\frac{\pi}{2}\\ 0 &\stackrel{?}{=} 2(1)\\ 0 &\neq 2\end{aligned}$$
Sample answer: $\theta = \frac{\pi}{2}$

5. Both answers are correct. She obtained two different representations of the same number. One way to verify this is to evaluate each expression with a calculator. To verify it algebraically, square each answer and then simplify. The same result is obtained in each case. Since each of the original answers is positive, and they have the same square, the original answers are the same number.

6. $$\begin{aligned}\sin\frac{\pi}{8} &= \sin\frac{\frac{\pi}{4}}{2}\\ &= \sqrt{\frac{1-\cos\frac{\pi}{4}}{2}} \quad \text{(Quadrant I)}\\ &= \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}\\ &= \frac{\sqrt{2-\sqrt{2}}}{2}\end{aligned}$$

7. $$\begin{aligned}\tan 165° &= \tan\frac{330°}{2}\\ &= \sqrt{\frac{1-\cos 330°}{1+\cos 330°}} \quad \text{(Quadrant II)}\\ &= -\sqrt{\frac{1-\frac{\sqrt{3}}{3}}{1+\frac{\sqrt{3}}{3}}}\\ &= -(2-\sqrt{3})\\ &= \sqrt{3}-2\end{aligned}$$

8. $$\begin{aligned}\sin^2\theta + \cos^2\theta &= 1\\ \left(\frac{2}{5}\right)^2 + \cos^2\theta &= 1\\ \cos^2\theta &= \frac{21}{25}\\ \cos\theta &= \frac{\sqrt{21}}{5}\end{aligned}$$
$$\begin{aligned}\tan\theta &= \frac{\sin\theta}{\cos\theta}\\ &= \frac{\frac{2}{5}}{\frac{\sqrt{21}}{5}}\\ &= \frac{2}{\sqrt{21}} \text{ or } \frac{2\sqrt{21}}{21}\end{aligned}$$
(Quadrant I)
$$\begin{aligned}\sin 2\theta &= 2\sin\theta\cos\theta\\ &= 2\left(\frac{2}{5}\right)\left(\frac{\sqrt{21}}{5}\right)\\ &= \frac{4\sqrt{21}}{25}\end{aligned}$$
$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta\\ &= \left(\frac{\sqrt{21}}{5}\right)^2 - \left(\frac{2}{5}\right)^2\\ &= \frac{17}{25}\end{aligned}$$
$$\begin{aligned}\tan 2\theta &= \frac{2\tan\theta}{1-\tan^2\theta}\\ &= \frac{2\left(\frac{2\sqrt{21}}{21}\right)}{1-\left(\frac{2\sqrt{21}}{21}\right)^2}\\ &= \frac{\frac{4\sqrt{21}}{21}}{\frac{17}{21}} \text{ or } \frac{4\sqrt{21}}{17}\end{aligned}$$

9. $$\begin{aligned}\tan^2\theta + 1 &= \sec^2\theta\\ \left(\frac{4}{3}\right)^2 + 1 &= \sec^2\theta\\ \frac{25}{9} &= \sec^2\theta\\ -\frac{5}{3} &= \sec\theta \quad \text{(Quadrant III)}\end{aligned}$$
$$\begin{aligned}\sin^2\theta + \cos^2\theta &= 1\\ \sin^2\theta + \left(-\frac{3}{5}\right)^2 &= 1\\ \sin^2\theta &= \frac{16}{25}\\ \sin\theta &= -\frac{4}{5} \quad \text{(Quadrant III)}\end{aligned}$$
$$\begin{aligned}\cos\theta &= \frac{1}{\sec\theta}\\ &= \frac{1}{\frac{-5}{3}} \text{ or } -\frac{3}{5}\end{aligned}$$
$$\begin{aligned}\sin 2\theta &= 2\sin\theta\cos\theta\\ &= 2\left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right)\\ &= \frac{24}{25}\end{aligned}$$
$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta\\ &= \left(-\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2\\ &= -\frac{7}{25}\end{aligned}$$
$$\begin{aligned}\tan 2\theta &= \frac{2\tan\theta}{1-\tan^2\theta}\\ &= \frac{2\left(\frac{4}{3}\right)}{1-\left(\frac{4}{3}\right)^2}\\ &= \frac{\frac{8}{3}}{-\frac{7}{9}} \text{ or } -\frac{24}{7}\end{aligned}$$

10. $$\begin{aligned}\tan 2\theta &\stackrel{?}{=} \frac{2}{\cot\theta - \tan\theta}\\ \tan 2\theta &\stackrel{?}{=} \frac{2}{\cot\theta - \tan\theta}\cdot\frac{\tan\theta}{\tan\theta}\\ \tan 2\theta &\stackrel{?}{=} \frac{2\tan\theta}{\cot\theta\tan\theta - \tan^2\theta}\\ \tan 2\theta &\stackrel{?}{=} \frac{2\tan\theta}{1-\tan^2\theta}\\ \tan 2\theta &= \tan 2\theta\end{aligned}$$

11. $1 + \frac{1}{2}\sin 2A \stackrel{?}{=} \frac{\sec A + \sin A}{\sec A}$

$$1 + \frac{1}{2}\sin 2A \stackrel{?}{=} \frac{\frac{1}{\cos A} + \sin A}{\frac{1}{\cos A}}$$

$$1 + \frac{1}{2}\sin 2A \stackrel{?}{=} \frac{\frac{1}{\cos A} + \sin A}{\frac{1}{\cos A}} \cdot \frac{\cos A}{\cos A}$$

$$1 + \frac{1}{2}\sin 2A \stackrel{?}{=} 1 + \sin A \cos A$$

$$1 + \frac{1}{2}\sin 2A \stackrel{?}{=} 1 + \frac{1}{2} \cdot 2\sin A \cos A$$

$$1 + \frac{1}{2}\sin 2A = 1 + \frac{1}{2}\sin 2A$$

12. $\sin\frac{x}{2}\cos\frac{x}{2} \stackrel{?}{=} \frac{\sin x}{2}$

$$\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2} \stackrel{?}{=} \frac{\sin x}{2}$$

$$\frac{\sin 2\left(\frac{x}{2}\right)}{2} \stackrel{?}{=} \frac{\sin x}{2}$$

$$\frac{\sin x}{2} = \frac{\sin x}{2}$$

13. $\cos 2\theta = 2\cos^2\theta - 1$

$$\cos 2\theta + 1 = 2\cos^2\theta$$

$$\frac{1}{2}\cos 2\theta + \frac{1}{2} = \cos^2\theta$$

$$P = I_0^2 R \sin^2 \omega t$$

$$P = I_0^2 R (1 - \cos^2 \omega t)$$

$$P = I_0^2 R\left(1 - \left(\frac{1}{2}\cos 2\omega t + \frac{1}{2}\right)\right)$$

$$P = I_0^2 R\left(\frac{1}{2} - \frac{1}{2}\cos 2\omega t\right)$$

$$P = \frac{1}{2} I_0^2 R - \frac{1}{2} I_0^2 R \cos 2\omega t$$

Pages 454–455 Exercises

14. $\cos 15° = \cos\frac{30°}{2}$

$$= \sqrt{\frac{1 + \cos 30°}{2}} \quad \text{(Quadrant I)}$$

$$= \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{2 + \sqrt{3}}{2}}$$

15. $\sin 75° = \sin\frac{150°}{2}$

$$= \sqrt{\frac{1 - \cos 150°}{2}} \quad \text{(Quadrant I)}$$

$$= \sqrt{\frac{1 - \left(-\frac{\sqrt{3}}{2}\right)}{2}}$$

$$= \sqrt{\frac{2 + \sqrt{3}}{2}}$$

16. $\tan\frac{5\pi}{12} = \tan\frac{\frac{5\pi}{6}}{2}$

$$= \sqrt{\frac{1 - \cos\frac{5\pi}{6}}{1 + \cos\frac{5\pi}{6}}} \quad \text{(Quadrant I)}$$

$$= \sqrt{\frac{1 - \left(-\frac{\sqrt{3}}{2}\right)}{1 + \left(-\frac{\sqrt{3}}{2}\right)}}$$

$$= \sqrt{\frac{\frac{2 + \sqrt{3}}{2}}{\frac{2 - \sqrt{3}}{2}}}$$

$$= \sqrt{\frac{(2 + \sqrt{3})(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})}}$$

$$= \sqrt{\frac{(2 + \sqrt{3})^2}{4 - 3}}$$

$$= 2 + \sqrt{3}$$

17. $\sin\frac{3\pi}{8} = \sin\frac{\frac{3\pi}{4}}{2}$

$$= \sqrt{\frac{1 - \cos\frac{3\pi}{4}}{2}} \quad \text{(Quadrant I)}$$

$$= \sqrt{\frac{1 - \left(-\frac{\sqrt{2}}{2}\right)}{2}}$$

$$= \sqrt{\frac{\sqrt{2} + \sqrt{2}}{2}}$$

18. $\cos\frac{7\pi}{12} = \cos\frac{\frac{7\pi}{6}}{2}$

$$= -\sqrt{\frac{1 + \cos\frac{7\pi}{6}}{2}} \quad \text{(Quadrant II)}$$

$$= -\sqrt{\frac{1 + \left(-\frac{\sqrt{3}}{2}\right)}{2}}$$

$$= -\frac{\sqrt{2 - \sqrt{3}}}{2}$$

19. $\tan 22.5° = \tan\frac{45°}{2}$

$$= \sqrt{\frac{1 - \cos 45°}{1 + \cos 45°}} \quad \text{(Quadrant I)}$$

$$= \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{1 + \frac{\sqrt{2}}{2}}}$$

$$= \sqrt{\frac{\frac{2 - \sqrt{2}}{2}}{\frac{2 + \sqrt{2}}{2}}}$$

$$= \sqrt{\frac{(2 - \sqrt{2})(2 - \sqrt{2})}{(2 + \sqrt{2})(2 - \sqrt{2})}}$$

$$= \sqrt{\frac{(2 - \sqrt{2})^2}{4 - 2}}$$

$$= \sqrt{\frac{2 - \sqrt{2}}{\sqrt{2}}}$$

$$= \frac{2\sqrt{2} - 2}{2}$$

$$= \sqrt{2} - 1$$

20. $\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$= \sqrt{\frac{1-\frac{1}{4}}{1+\frac{1}{4}}}$

$= \sqrt{\frac{\frac{3}{4}}{\frac{5}{4}}}$

$= \sqrt{\frac{3}{5}}$ or $\frac{\sqrt{15}}{5}$

21. $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(\frac{4}{5}\right)^2 = 1$

$\sin^2\theta = \frac{9}{25}$

$\sin\theta = \frac{3}{5}$

(Quadrant I)

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{\frac{3}{5}}{\frac{4}{5}}$

$= \frac{3}{4}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$= \frac{24}{25}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{7}{25}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2}$

$= \frac{\frac{3}{2}}{\frac{7}{16}}$ or $\frac{24}{7}$

22. $\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{1}{3}\right)^2 + \cos^2\theta = 1$

$\cos^2\theta = \frac{8}{9}$

$\cos\theta = \frac{2\sqrt{2}}{3}$

(Quadrant I)

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}}$

$= \frac{1}{2\sqrt{2}}$ or $\frac{\sqrt{2}}{4}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{1}{3}\right)\left(\frac{2\sqrt{2}}{3}\right)$

$= \frac{4\sqrt{2}}{9}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{2\sqrt{2}}{3}\right)^2 - \left(\frac{1}{3}\right)^2$

$= \frac{7}{9}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\left(\frac{\sqrt{2}}{4}\right)}{1-\left(\frac{\sqrt{2}}{4}\right)^2}$

$= \frac{\frac{\sqrt{2}}{2}}{\frac{14}{16}}$ or $\frac{4\sqrt{2}}{7}$

23. $\tan^2\theta + 1 = \sec^2\theta$

$(-2)^2 + 1 = \sec^2\theta$

$5 = \sec^2\theta$

$-\sqrt{5} = \sec\theta$ (Quadrant II)

$\cos\theta = \sec\theta$

$= \frac{1}{-\sqrt{5}}$ or $-\frac{\sqrt{5}}{5}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(-\frac{\sqrt{5}}{5}\right)^2 = 1$

$\sin^2\theta = \frac{20}{25}$

$\sin\theta = \frac{2\sqrt{5}}{5}$ (Quadrant II)

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{2\sqrt{5}}{5}\right)\left(-\frac{\sqrt{5}}{5}\right)$

$= -\frac{4}{5}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(-\frac{\sqrt{5}}{5}\right)^2 - \left(\frac{2\sqrt{5}}{5}\right)^2$

$= -\frac{3}{5}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2(-2)}{1-(-2)^2}$

$= \frac{-4}{-3}$ or $\frac{4}{3}$

24. $\cos\theta = \frac{1}{\sec\theta}$

$= \frac{1}{-\frac{4}{3}}$

$= -\frac{3}{4}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(-\frac{3}{4}\right)^2 = 1$

$\sin^2\theta = \frac{7}{16}$

$\sin\theta = \frac{\sqrt{7}}{4}$ (Quadrant II)

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{\frac{\sqrt{7}}{4}}{-\frac{3}{4}}$

$= -\frac{\sqrt{7}}{3}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{\sqrt{7}}{4}\right)\left(-\frac{3}{4}\right)$

$= -\frac{3\sqrt{7}}{8}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(-\frac{3}{4}\right)^2 - \left(\frac{\sqrt{7}}{4}\right)^2$

$= \frac{2}{16}$ or $\frac{1}{8}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\left(-\frac{\sqrt{7}}{3}\right)}{1-\left(-\frac{\sqrt{7}}{3}\right)^2}$

$= -\frac{\frac{2\sqrt{7}}{3}}{\frac{2}{9}}$ or $-3\sqrt{7}$

25. $1 + \cot^2\theta = \csc^2\theta$

$1 + \left(\frac{3}{2}\right)^2 = \csc^2\theta$

$\frac{13}{4} = \csc^2\theta$

$-\frac{\sqrt{13}}{2} = \csc\theta$ (Quadrant III)

$\tan\theta = \frac{1}{\cot\theta} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$

$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{-\frac{\sqrt{13}}{2}} = -\frac{2}{\sqrt{13}}$ or $-\frac{2\sqrt{13}}{13}$

$\sin^2\theta + \cos^2\theta = 1$

$\left(-\frac{2\sqrt{13}}{13}\right)^2 + \cos^2\theta = 1$

$\cos^2\theta = \frac{117}{169}$

$\cos\theta = -\frac{3\sqrt{13}}{13}$

(Quadrant III)

$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(-\frac{2\sqrt{13}}{13}\right)\left(-\frac{3\sqrt{13}}{13}\right) = \frac{12}{13}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(-\frac{3\sqrt{13}}{13}\right)^2 - \left(-\frac{2\sqrt{13}}{13}\right)^2 = \frac{5}{13}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{2}{3}\right)}{1-\left(\frac{2}{3}\right)^2} = \frac{\frac{4}{3}}{\frac{5}{9}}$ or $\frac{12}{5}$

26. $\sin\theta = \frac{1}{\csc\theta} = \frac{1}{-\frac{5}{2}} = -\frac{2}{5}$

$\sin^2\theta + \cos^2\theta = 1$

$\left(-\frac{2}{5}\right)^2 + \cos^2\theta = 1$

$\cos^2\theta = \frac{21}{25}$

$\cos\theta = \frac{\sqrt{21}}{5}$

(Quadrant IV)

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{2}{5}}{\frac{\sqrt{21}}{5}} = -\frac{2}{\sqrt{21}}$ or $-\frac{2\sqrt{21}}{21}$

$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(-\frac{2}{5}\right)\left(\frac{\sqrt{21}}{5}\right) = -\frac{4\sqrt{21}}{25}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{\sqrt{21}}{5}\right)^2 - \left(-\frac{2}{5}\right)^2 = \frac{17}{25}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(-\frac{2\sqrt{21}}{21}\right)}{1-\left(-\frac{2\sqrt{21}}{21}\right)^2} = \frac{-\frac{4\sqrt{21}}{21}}{\frac{17}{21}}$ or $-\frac{4\sqrt{21}}{17}$

27. $\sin^2\alpha + \cos^2\alpha = 1$

$\sin^2\alpha + \left(-\frac{\sqrt{2}}{3}\right)^2 = 1$

$\sin^2\alpha = \frac{7}{9}$

$\sin\alpha = \frac{\sqrt{7}}{3}$ (Quadrant II)

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{\sqrt{7}}{3}}{-\frac{\sqrt{2}}{3}} = -\frac{\sqrt{7}}{\sqrt{2}}$ or $-\frac{\sqrt{14}}{2}$

$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2\left(-\frac{\sqrt{14}}{2}\right)}{1-\left(-\frac{\sqrt{14}}{2}\right)^2} = \frac{-\sqrt{14}}{-\frac{5}{2}}$ or $\frac{2\sqrt{14}}{5}$

28. $\csc 2\theta \stackrel{?}{=} \frac{1}{2}\sec\theta\csc\theta$

$\frac{1}{\sin 2\theta} \stackrel{?}{=} \frac{1}{2}\sec\theta\csc\theta$

$\frac{1}{2\sin\theta\cos\theta} \stackrel{?}{=} \frac{1}{2}\sec\theta\csc\theta$

$\frac{1}{2}\cdot\frac{1}{\sin\theta}\cdot\frac{1}{\cos\theta} \stackrel{?}{=} \frac{1}{2}\sec\theta\csc\theta$

$\frac{1}{2}\csc\theta\sec\theta \stackrel{?}{=} \frac{1}{2}\sec\theta\csc\theta$

$\frac{1}{2}\sec\theta\csc\theta = \frac{1}{2}\sec\theta\csc\theta$

29. $\cos A - \sin A \stackrel{?}{=} \frac{\cos 2A}{\cos A + \sin A}$

$\cos A - \sin A \stackrel{?}{=} \frac{\cos^2 A - \sin^2 A}{\cos A + \sin A}$

$\cos A - \sin A \stackrel{?}{=} \frac{(\cos A - \sin A)(\cos A + \sin A)}{\cos A + \sin A}$

$\cos A - \sin A = \cos A - \sin A$

30. $(\sin\theta + \cos\theta)^2 - 1 \stackrel{?}{=} \sin 2\theta$

$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta - 1 \stackrel{?}{=} \sin 2\theta$

$2\sin\theta\cos\theta + 1 - 1 \stackrel{?}{=} \sin 2\theta$

$2\sin\theta\cos\theta \stackrel{?}{=} \sin 2\theta$

$\sin 2\theta = \sin 2\theta$

31. $\cos x - 1 \stackrel{?}{=} \frac{\cos 2x - 1}{2(\cos x + 1)}$

$\cos x - 1 \stackrel{?}{=} \frac{2\cos^2 x - 1 - 1}{2(\cos x + 1)}$

$\cos x - 1 \stackrel{?}{=} \frac{2\cos^2 x - 2}{2(\cos x + 1)}$

$\cos x - 1 \stackrel{?}{=} \frac{2(\cos^2 x - 1)}{2(\cos x + 1)}$

$\cos x - 1 \stackrel{?}{=} \frac{2(\cos x - 1)(\cos x + 1)}{2(\cos x + 1)}$

$\cos x - 1 = \cos x - 1$

32. $\sec 2\theta \stackrel{?}{=} \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta - \sin^2\theta}$

$\sec 2\theta \stackrel{?}{=} \frac{1}{\cos 2\theta}$

$\sec 2\theta = \sec 2\theta$

33. $\tan\frac{A}{2} \stackrel{?}{=} \frac{\sin A}{1 + \cos A}$

$$\tan\frac{A}{2} \stackrel{?}{=} \frac{\sin 2\left(\frac{A}{2}\right)}{1+\cos 2\left(\frac{A}{2}\right)}$$
$$\tan\frac{A}{2} \stackrel{?}{=} \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{1+2\cos^2\frac{A}{2}-1}$$
$$\tan\frac{A}{2} \stackrel{?}{=} \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{2\cos^2\frac{A}{2}}$$
$$\tan\frac{A}{2} \stackrel{?}{=} \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}}$$
$$\tan\frac{A}{2} = \tan\frac{A}{2}$$

34.
$$\begin{aligned}
\sin 3x &\stackrel{?}{=} 3\sin x - 4\sin^3 x\\
\sin(2x+x) &\stackrel{?}{=} 3\sin x - 4\sin^3 x\\
\sin 2x\cos x + \cos 2x \sin x &\stackrel{?}{=} 3\sin x - 4\sin^3 x\\
2\sin x\cos^2 x + (1-2\sin^2 x)\sin x &\stackrel{?}{=} 3\sin x - 4\sin^3 x\\
2\sin x(1-\sin^2 x) + (1-2\sin^2 x)\sin x &\stackrel{?}{=} 3\sin x - 4\sin^3 x\\
2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x &\stackrel{?}{=} 3\sin x - 4\sin^3 x\\
3\sin x - 4\sin^3 x &= 3\sin x - 4\sin^3 x
\end{aligned}$$

35.
$$\begin{aligned}
\cos 3x &\stackrel{?}{=} 4\cos^3 x - 3\cos x\\
\cos(2x+x) &\stackrel{?}{=} 4\cos^3 x - 3\cos x\\
(2\cos^2 x - 1)\cos x - 2\sin^2 x\cos x &\stackrel{?}{=} 4\cos^3 x - 3\cos x\\
(2\cos^2 x - 1)\cos x - 2(1-\cos^2 x)\cos x &\stackrel{?}{=} 4\cos^3 x - 3\cos x\\
2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x &\stackrel{?}{=} 4\cos^3 x - 3\cos x\\
4\cos^3 x - 3\cos x &= 4\cos^3 x - 3\cos x
\end{aligned}$$

36.
$$\begin{aligned}
\frac{\frac{v^2}{2g}\sin^2 2\theta}{\frac{v^2}{2g}\sin^2\theta} &= \frac{\sin^2 2\theta}{\sin^2\theta}\\
&= \frac{(2\sin\theta\cos\theta)^2}{\sin^2\theta}\\
&= \frac{4\sin^2\theta\cos^2\theta}{\sin^2\theta}\\
&= 4\cos^2\theta
\end{aligned}$$

37. $\angle PBD$ is an inscribed angle that subtends the same arc as the central angle $\angle POD$, so $m\angle PBD = \frac{1}{2}\theta$. By right triangle trigonometry, $\tan\frac{1}{2}\theta = \frac{PA}{BA} = \frac{PA}{1+OA} = \frac{\sin\theta}{1+\cos\theta}$.

38.
$$\begin{aligned}
R &= \frac{2v^2\cos\theta\sin(\theta-\alpha)}{g\cos^2\alpha}\\
R &= \frac{2v^2\cos\theta\sin(\theta-45°)}{g\cos^2 45°}\\
R &= \frac{2v^2\cos\theta(\sin\theta\cos 45° - \cos\theta\sin 45°)}{g\cos^2 45°}\\
R &= \frac{2v^2\cos\theta\left((\sin\theta)\left(\frac{\sqrt{2}}{2}\right) - (\cos\theta)\left(\frac{\sqrt{2}}{2}\right)\right)}{g\left(\frac{\sqrt{2}}{2}\right)2}\\
R &= \frac{2\frac{\sqrt{2}}{2}v^2\cos\theta(\sin\theta-\cos\theta)}{g\cdot\frac{1}{2}}\\
R &= \frac{\sqrt{2}v^2(2\cos\theta\sin\theta - 2\cos^2\theta)}{g}\\
R &= \frac{v^2\sqrt{2}}{g}(2\cos\theta\sin\theta - (2\cos^2\theta - 1) - 1)\\
R &= \frac{v^2\sqrt{2}}{g}(\sin 2\theta - \cos 2\theta - 1)
\end{aligned}$$

39a.
$$\begin{aligned}
\tan\left(45° + \frac{1}{2}\right) &= \frac{\tan 45° + \tan\frac{L}{2}}{1 - \tan 45°\tan\frac{L}{2}}\\
&= \frac{1+\tan\frac{L}{2}}{1 - 1\cdot\tan\frac{L}{2}}\\
&= \frac{1\pm\sqrt{\frac{1-\cos L}{1+\cos L}}}{1\mp\sqrt{\frac{1-\cos L}{1+\cos L}}}
\end{aligned}$$

39b.
$$\begin{aligned}
\frac{1\pm\sqrt{\frac{1-\cos L}{1+\cos L}}}{1\mp\sqrt{\frac{1-\cos L}{1+\cos L}}} &= \frac{1+\sqrt{\frac{1-\cos 60°}{1+\cos 60°}}}{1-\sqrt{\frac{1-\cos 60°}{1+\cos 60°}}}\\
&= \frac{1+\sqrt{\frac{1-\frac{1}{2}}{1+\frac{1}{2}}}}{1-\sqrt{\frac{1-\frac{1}{2}}{1+\frac{1}{2}}}}\\
&= \frac{1+\sqrt{\frac{1}{3}}}{1-\sqrt{\frac{1}{3}}}\\
&= \frac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}}\\
&= \frac{12+6\sqrt{3}}{6} \text{ or } 2+\sqrt{3}
\end{aligned}$$

40.
$$\begin{aligned}
\tan(\alpha+30°) &= \frac{21}{7}\\
\tan(\alpha+30°) &= 3\\
\frac{\tan\alpha + \tan 30°}{1-\tan\alpha\tan 30°} &= 3\\
\tan\alpha + \frac{\sqrt{3}}{3} &= 3 - \sqrt{3}\tan\alpha\\
\tan\alpha + \sqrt{3}\tan\alpha &= 3 - \frac{\sqrt{3}}{3}\\
(1+\sqrt{3})\tan\alpha &= 3 - \frac{\sqrt{3}}{3}\\
\tan\alpha &= \frac{3-\frac{\sqrt{3}}{3}}{1+\sqrt{3}}\\
\tan\alpha &= \frac{9-\sqrt{3}}{3+3\sqrt{3}}\\
\tan\alpha &= \frac{-6+5\sqrt{3}}{3}
\end{aligned}$$

41.
$$\begin{aligned}
\cos\frac{\pi}{12} &= \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)\\
&= \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}\\
&= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}\\
&= \frac{\sqrt{2}+\sqrt{6}}{4}\\
\sec\frac{\pi}{12} &= \frac{1}{\cos\frac{\pi}{12}}\\
&= \frac{1}{\frac{\sqrt{2}+\sqrt{6}}{4}}\\
&= \frac{4\sqrt{2}-4\sqrt{6}}{-4} \text{ or } \sqrt{6}-\sqrt{2}
\end{aligned}$$

42. Sample answer:
$$\begin{aligned}
\sin(\sqrt{\pi})^2 + \cos(\sqrt{\pi})^2 &= \sin\pi + \cos\pi\\
&= 0 + (-1)\\
&= -1\\
&\neq 1
\end{aligned}$$

43. $s = r\theta$

$17 = 10 \cdot \theta$

$\frac{17}{10} = \theta$

$\frac{17}{10} = \frac{17}{10} \cdot \frac{180°}{\pi}$

$\approx 97.4°$

44. Let x = the distance from A to the point beneath the mountain peak.

$\tan 21°10' = \frac{h}{570 + x}$

$h = (570 + x) \tan 21°10'$

$\tan 36°40' = \frac{h}{x}$

$h = x \tan 36°40'$

$(570 + x) \tan 21°10' = x \tan 36°40'$

$570 \tan 21°10' = x \tan 36°40' - x \tan 21°10'$

$570 \tan 21°10' = x(\tan 36°40' - \tan 21°10')$

$\frac{570 \tan 21°10'}{\tan 36°40' - \tan 21°10'} = x$

$617.7646751 \approx x$

$\tan 36°40' = \frac{h}{x}$

$\tan 36°40' \approx \frac{h}{617.8}$

$h \approx 460$ ft

45. $(x - (-3))(x - 0.5)(x - 6)(x - 2) = 0$

$(x + 3)(x - 0.5)(x - 6)(x - 2) = 0$

$(x^2 + 2.5x - 1.5)(x^2 - 8x + 12) = 0$

$x^4 - 5.5x^3 - 9.5x^2 + 42x - 18 = 0$

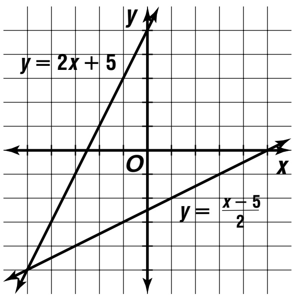
$2x^4 - 11x^3 - 19x^2 + 84x - 36 = 0$

46. $y = 2x + 5$

$x = 2y + 5$

$x - 5 = 2y$

$\frac{x - 5}{2} = y$



47. $x + 2y = 11$

$x = 11 - 2y$

$3x - 5y = 11$

$3(11 - 2y) - 5y = 11$

$33 - 6y - 5y = 11$

$-11y = -22$

$y = 2$

$x + 2y = 11$

$x + 2(2) = 11$

$x = 7 \qquad (7, 2)$

48. $ab = 3$

$b = \frac{3}{a}$

$(a - b)^2 = 64$

$a^2 - 2ab + b^2 = 64$

$a^2 - 2a\left(\frac{3}{a}\right) + \left(\frac{3}{a}\right)^2 = 64$

$a^2 - 6 + \left(\frac{3}{a}\right)^2 = 64$

$a^2 + \left(\frac{3}{a}\right)^2 = 70$

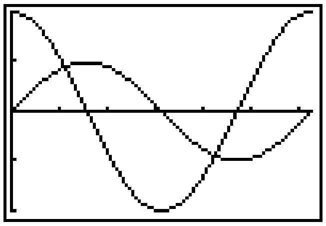
$a^2 + b^2 = 70$

The correct answer is 70.

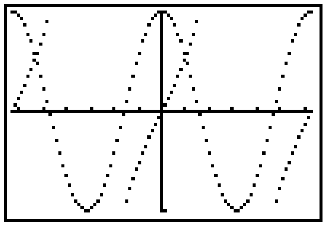
7-5 Solving Trigonometric Equations

Page 458 Graphing Calculator Exploration

1.



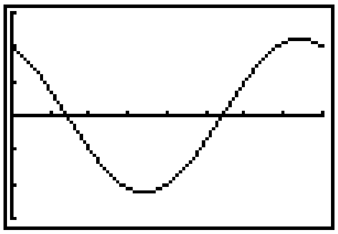
2.



3. Exercise 1: (1.1071, 0.8944), (4.2487, −0.8944)
Exercise 2: (−5.2872, 0.5437), (0.9960, 0.5437)

4. The x-coordinates are the solutions of the equations. Substitute the x-coordinates and see that the two sides of the equation are equal.

5.



$[0, 2\pi]$ sc1: $\frac{\pi}{4}$ by $[-3, 3]$ sc1:1

5a. The x-intercepts of the graph are the solutions of the equation $\sin x = 2 \cos x$. They are the same.

5b. $y = \tan 0.5x - \cos x$ or $y = \cos x - \tan 0.5x$

Page 459 Check for Understanding

1. A trigonometric identity is an equation that is true for all values of the variable for which each side of the equation is defined. A trigonometric equation that is not an identity is only true for certain values of the variable.

2. All trigonometric functions are periodic. Adding the least common multiple of the periods of the functions that appear to any solution to the equation will always produce another solution.

3. $45° + 360x°$ and $135° + 360x°$, where x is any integer

4. Each type of equation may require adding, subtracting, multiplying, or dividing each side by the same number. Quadratic and trigonometric equations can often be solved by factoring. Linear and quadratic equations do not require identities. All linear and quadratic equations can be solved algebraically, whereas some trigonometric equations require a graphing calculator. A linear equation has at most one solution. A quadratic equation has at most two solutions. A trigonometric equation usually has infinitely many solutions unless the values of the variable are restricted.

5. $2\sin x + 1 = 0$

$2\sin x = -1$

$\sin x = -\frac{1}{2}$

$x = -30°$

6. $2\cos x - \sqrt{3} = 0$

$2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

$x = 30°$

7. $\sin x \cot x = \frac{\sqrt{3}}{2}$

$\sin x\left(\frac{\cos x}{\sin x}\right) = \frac{\sqrt{3}}{2}$

$\cos x = \frac{\sqrt{3}}{2}$

$x = 30°$ or $x = 330°$

8. $\cos 2x = \sin^2 x - 2$

$2\cos^2 x - 1 = (1 - \cos^2 x) - 2$

$2\cos^2 x - 1 = -\cos^2 x - 1$

$3\cos^2 x = 0$

$\cos^2 x = 0$

$\cos x = 0$

$x = 90°$ or $x = 270°$

9. $3\tan^2 x - 1 = 0$

$3\tan^2 x = 1$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm\frac{\sqrt{3}}{3}$

$x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}$ or $x = \frac{7\pi}{6}$ or $x = \frac{11\pi}{6}$

10. $2\sin^2 x = 5\sin x + 3$

$2\sin^2 x - 5\sin x - 3 = 0$

$(2\sin x + 1)(\sin x - 3) = 0$

$2\sin x + 1 = 0$ or $\sin x - 3 = 0$

$\sin x = -\frac{1}{2}$ $\quad$ $\sin x = 3$

$x = \frac{7\pi}{6}$ or $x = \frac{11\pi}{6}$ $\quad$ no solutions

11. $\sin^2 2x + \cos^2 x = 0$

$1 - \cos^2 2x + \cos^2 x = 0$

$1 - (2\cos^2 x - 1)^2 + \cos^2 x = 0$

$1 - (4\cos^4 x - 4\cos^2 x + 1) + \cos^2 x = 0$

$-4\cos^4 x + 5\cos^2 x = 0$

$\cos^2 x(-4\cos^2 x + 5) = 0$

$\cos^2 x = 0$ or $-4\cos^2 x + 5 = 0$

$\cos x = 0$ $\quad$ $\cos^2 x = \frac{5}{4}$

$x = \frac{\pi}{2} + \pi k$ $\quad$ $\cos x = \frac{\sqrt{5}}{2}$

no solutions

12. $\tan^2 x + 2\tan x + 1 = 0$

$(\tan x + 1)(\tan x + 1) = 0$

$\tan x + 1 = 0$

$\tan x = -1$

$x = \frac{3\pi}{4} + \pi k$

13. $\cos^2 x + 3\cos x = -2$

$\cos^2 x + 3\cos x + 2 = 0$

$(\cos x + 1)(\cos x + 2) = 0$

$\cos x + 1 = 0$ or $\cos x + 2 = 0$

$\cos x = -1$ $\quad$ $\cos x = -2$

$x = (2k + 1)\pi$ $\quad$ no solutions

14. $\sin 2x - \cos x = 0$

$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ or $2\sin x - 1 = 0$

$x = \frac{\pi}{2} + k\pi$ $\quad$ $\sin x = \frac{1}{2}$

$x = \frac{\pi}{6} + 2\pi k$

or $x = \frac{5\pi}{6} + 2\pi k$

15. $2\cos\theta + 1 < 0$

$2\cos\theta < -1$

$\cos\theta < -\frac{1}{2}$

$\cos\theta = -\frac{1}{2}$ at $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$

$\frac{2\pi}{3} < \theta < \frac{4\pi}{3}$

16. $W = Fd\cos\theta$

$1500 = 100 \cdot 20\cos\theta$

$0.75 = \cos\theta$

$\theta \approx 41.41°$

Pages 459–461 Exercises

17. $\sqrt{2}\sin x - 1 = 0$

$\sqrt{2}\sin x = 1$

$\sin x = \frac{1}{\sqrt{2}}$

$\sin x = \frac{\sqrt{2}}{2}$

$x = 45°$

18. $2\cos x + 1 = 0$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$

$x = 120°$

19. $\sin 2x - 1 = 0$

$2\sin x\cos x - 1 = 0$

$\sin^2 x\cos^2 x = \frac{1}{4}$

$\sin^2 x(1 - \sin^2 x) = \frac{1}{4}$

$\sin^2 x - \sin^4 x - \frac{1}{4} = 0$

$\sin^4 x - \sin^2 x + \frac{1}{4} = 0$

$\left(\sin^2 x - \frac{1}{2}\right)\left(\sin^2 x - \frac{1}{2}\right) = 0$

$\sin^2 x - \frac{1}{2} = 0$

$\sin^2 x = \frac{1}{2}$

$\sin x = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

$x = 45°$

20. $\tan 2x - \sqrt{3} = 0$

$\tan 2x = \sqrt{3}$

$\frac{2\tan x}{1 - \tan^2 x} = \sqrt{3}$

$2\tan x = \sqrt{3}\,(1 - \tan^2 x)$

$2\tan x = \sqrt{3} - \sqrt{3}\tan^2 x$

$\sqrt{3}\tan^2 x + 2\tan x - \sqrt{3} = 0$

$(\sqrt{3}\tan x - 1)(\tan x + \sqrt{3}) = 0$

$\sqrt{3}\tan x - 1 = 0$ $\quad$ $\tan x + \sqrt{3} = 0$

$\tan x = \frac{1}{\sqrt{3}}$ $\quad$ $\tan x = -\sqrt{3}$

$\tan x = \frac{\sqrt{3}}{3}$ $\quad$ $x = -60°$

$x = 30°$

21. $\cos^2 x = \cos x$

$\cos^2 x - \cos x = 0$

$\cos x(\cos x - 1) = 0$

$\cos x = 0$ or $\cos x - 1 = 0$

$x = 90°$ $\quad$ $\cos x = 1$

$x = 0°$

22. $\sin x = 1 + \cos^2 x$

$\sin x = 1 + 1 - \sin^2 x$

$\sin^2 x + \sin x - 2 = 0$

$(\sin x - 1)(\sin x + 2) = 0$

$\sin x - 1 = 0$ or $\sin x + 2 = 0$

$\sin x = 1$ $\quad$ $\sin x = -2$

$x = 90°$ $\quad$ no solution

23. $\sqrt{2}\cos x + 1 = 0$

$\sqrt{2}\cos x = -1$

$\cos x = -\frac{\sqrt{2}}{2}$

$x = 135°$ or $x = 225°$

24. $\cos x \tan x = \frac{1}{2}$

$\cos x \frac{\sin x}{\cos x} = \frac{1}{2}$

$\sin x = \frac{1}{2}$

$x = 30°$ or $x = 150°$

25. $\sin x \tan x - \sin x = 0$

$\sin x\,(\tan x - 1) = 0$

$\sin x = 0$ or $\tan x - 1 = 0$

$x = 0°$ or $x = 180°$ $\quad$ $\tan x = 1$

$x = 45°$ or $x = 225°$

26. $2\cos^2 x + 3\cos x - 2 = 0$

$(2\cos x - 1)(\cos x + 2) = 0$

$2\cos x - 1 = 0$ or $\cos x + 2 = 0$

$2\cos x = 1$ $\quad$ $\cos x = -2$

$\cos x = \frac{1}{2}$ $\quad$ no solution

$x = 60°$ or $x = 300°$

27. $\sin 2x = -\sin x$

$2\sin x\cos x = -\sin x$

$2\sin x\cos x + \sin x = 0$

$\sin x\,(2\cos x + 1) = 0$

$\sin x = 0$ or $2\cos x + 1 = 0$

$x = 0°$ or $x = 180°$ $\quad$ $2\cos x = -1$

$\cos x = -\frac{1}{2}$

$x = 120°$

or $x = 240°$

28. $\cos(x + 45°) + \cos(x - 45°) = \sqrt{2}$

$\cos x\cos 45° - \sin x\sin 45°$

$+ \cos x\cos 45° + \sin x\sin 45° = \sqrt{2}$

$\cos x \cdot \frac{\sqrt{2}}{2} - \sin x \cdot \frac{\sqrt{2}}{2}$

$+ \cos x \cdot \frac{\sqrt{2}}{2} + \sin x \cdot \frac{\sqrt{2}}{2} = \sqrt{2}$

$\sqrt{2}\cos x = \sqrt{2}$

$\cos x = 1$

$x = 0°$

29. $2\sin\theta\cos\theta + \sqrt{3}\sin\theta = 0$

$\sin\theta\,(2\cos\theta + \sqrt{3}) = 0$

$\sin\theta = 0$ or $2\cos\theta + \sqrt{3} = 0$

$\theta = 0°$ or $\theta = 180°$ $\quad$ $2\cos\theta = -\sqrt{3}$

$\cos\theta = -\frac{\sqrt{3}}{2}$

$\theta = 150°$

or $\theta = 210°$

30. $(2\sin x - 1)(2\cos^2 x - 1) = 0$

$2\sin x - 1 = 0$ or $2\cos^2 x - 1 = 0$

$2\sin x = 1$ $\quad$ $2\cos^2 x = 1$

$\sin x = \frac{1}{2}$ $\quad$ $\cos^2 x = \frac{1}{2}$

$x = \frac{\pi}{6}$ $\quad$ $\cos x = \pm\frac{\sqrt{2}}{2}$

or $x = \frac{5\pi}{6}$ $\quad$ $x = \frac{\pi}{4}$ or $x = \frac{3\pi}{4}$

or $x = \frac{5\pi}{4}$ or $x = \frac{7\pi}{4}$

31. $4\sin^2 x + 1 = -4\sin x$

$4\sin^2 x + 4\sin x + 1 = 0$

$(2\sin x + 1)(2\sin x + 1) = 0$

$2\sin x + 1 = 0$

$2\sin x = -1$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}$ or $x = \frac{11\pi}{6}$

32. $\sqrt{2}\tan x = 2\sin x$

$\sqrt{2}\frac{\sin x}{\cos x} = 2\sin x$

$\sqrt{2} = 2\cos x$

$\frac{\sqrt{2}}{2} = \cos x$

$x = \frac{\pi}{4}$ or $x = \frac{7\pi}{4}$

$\sqrt{2}\tan x = 2\sin x$ would also be true if both $\tan x$ and $\sin x$ equal 0. Since $\tan x = \frac{\sin x}{\cos x}$, $\tan x$ equals 0 when $\sin x = 0$. Therefore x can also equal 0 and π.

$0, \frac{\pi}{4}, \pi, \frac{7\pi}{4}$

33. $\sin x = \cos 2x - 1$

$\sin x = 1 - 2\sin^2 x - 1$

$2\sin^2 x + \sin x = 0$

$\sin x(2\sin x + 1) = 0$

$\sin x = 0$ or $2\sin x + 1 = 0$

$x = 0$ or $x = \pi$ $\quad$ $2\sin x = -1$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}$ or

$x = \frac{11\pi}{6}$

34.
$$\begin{aligned}
\cot^2 x - \csc x &= 1\\
\csc^2 x - 1 - \csc x &= 1\\
\csc^2 x - \csc x - 2 &= 0\\
(\csc x - 2)(\csc x + 1) &= 0\\
\csc x - 2 = 0 \quad &\text{or} \quad \csc x + 1 = 0\\
\csc x = 2 \quad &\text{or} \quad \csc x = -1\\
\sin x = \tfrac{1}{2} \quad & \quad \sin x = -1\\
x = \tfrac{\pi}{6} \text{ or } x = \tfrac{5\pi}{6} \quad & \quad x = \tfrac{3\pi}{2}
\end{aligned}$$

35.
$$\begin{aligned}
\sin x + \cos x &= 0\\
\sin x &= -\cos x\\
\sin^2 x &= \cos^2 x\\
\sin^2 x - \cos^2 x &= 0\\
\sin^2 x - 1 + \sin^2 x &= 0\\
2\sin^2 x - 1 &= 0\\
\sin^2 x &= \tfrac{1}{2}\\
\sin x &= \pm\tfrac{1}{\sqrt{2}} \text{ or } \pm\tfrac{\sqrt{2}}{2}
\end{aligned}$$

$\sin x$ and $\cos x$ must be opposites, so $x = \frac{3\pi}{4}$ or $x = \frac{7\pi}{4}$.

36.
$$\begin{aligned}
-1 - 3\sin\theta &= \cos 2\theta\\
-1 - 3\sin\theta &= 1 - 2\sin^2\theta\\
2\sin^2\theta - 3\sin\theta - 2 &= 0\\
(2\sin\theta + 1)(\sin\theta - 2) &= 0\\
2\sin\theta + 1 = 0 \quad &\text{or} \quad \sin\theta - 2 = 0\\
2\sin\theta = -1 \quad & \quad \sin\theta = 2\\
\sin\theta = -\tfrac{1}{2} \quad & \quad \text{no solution}\\
\theta = \tfrac{7\pi}{6} \text{ or } \theta = \tfrac{11\pi}{6} &
\end{aligned}$$

37. $\sin x = -\frac{1}{2}$
$$x = \tfrac{7\pi}{6} + 2\,\pi k \quad \text{or} \quad x = \tfrac{11\pi}{6} + 2\pi k$$

38.
$$\begin{aligned}
\cos x \tan x - 2\cos^2 x &= -1\\
\cos x \tfrac{\sin x}{\cos x} - 2\cos^2 x &= -1\\
\sin x - 2(1 - \sin^2 x) &= -1\\
2\sin^2 x + \sin x - 1 &= 0\\
(2\sin x - 1)(\sin x + 1) &= 0\\
2\sin x - 1 = 0 \quad &\text{or} \quad \sin x + 1 = 0\\
2\sin x = 1 \quad & \quad \sin x = -1\\
\sin x = \tfrac{1}{2} \quad & \quad x = \tfrac{3\pi}{2} + 2\pi k\\
x = \tfrac{\pi}{6} + 2\pi k \text{ or } x = \tfrac{5\pi}{6} + 2\pi k &
\end{aligned}$$

39.
$$\begin{aligned}
3\tan^2 x &= \sqrt{3}\tan x\\
3\tan^2 x - \sqrt{3}\tan x &= 0\\
\tan x(3\tan x - \sqrt{3}) &= 0\\
\tan x = 0 \quad &\text{or} \quad 3\tan x - \sqrt{3} = 0\\
x = \pi k \quad & \quad 3\tan x = \sqrt{3}\\
& \quad \tan x = \tfrac{\sqrt{3}}{3}\\
& \quad x = \tfrac{\pi}{6} + \pi k
\end{aligned}$$

40.
$$\begin{aligned}
2(1 - \sin^2 x) &= 3\sin x\\
2 - 2\sin^2 x &= 3\sin x\\
2\sin^2 x + 3\sin x - 2 &= 0\\
(2\sin x - 1)(\sin x + 2) &= 0\\
2\sin x - 1 = 0 \quad &\text{or} \quad \sin x + 2 = 0\\
2\sin x = 1 \quad & \quad \sin x = -2\\
\sin x = \tfrac{1}{2} \quad & \quad \text{no solution}\\
x = \tfrac{\pi}{6} + 2\pi k \text{ or} &\\
x = \tfrac{5\pi}{6} + 2\pi k &
\end{aligned}$$

41.
$$\begin{aligned}
\tfrac{1}{\cos x - \sin x} &= \cos x + \sin x\\
(\cos x - \sin x)(\cos x + \sin x) &= 1\\
\cos^2 x - \sin^2 x &= 1\\
\cos^2 x - (1 - \cos^2 x) &= 1\\
2\cos^2 x - 1 &= 1\\
2\cos^2 x &= 2\\
\cos^2 x &= 1\\
\cos x &= \pm 1\\
x &= \pi k
\end{aligned}$$

42.
$$\begin{aligned}
2\tan^2 x - 3\sec x &= 0\\
2(\sec^2 x - 1) - 3\sec x &= 0\\
(2\sec x + 1)(\sec x - 2) &= 0\\
2\sec^2 x - 3\sec x - 2 &= 0\\
2\sec x + 1 = 0 \quad &\text{or} \quad \sec x - 2 = 0\\
2\sec x = -1 \quad & \quad \sec x = 2\\
\sec x = -\tfrac{1}{2} \quad & \quad \cos x = \tfrac{1}{2}\\
\cos x = -2 \quad & \quad x = \tfrac{\pi}{3} + 2\pi k \text{ or}\\
\text{no solution} \quad & \quad x = \tfrac{5\pi}{3} + 2\pi k
\end{aligned}$$

43.
$$\begin{aligned}
\sin x \cos x &= \tfrac{1}{2}\\
\sin^2 x \cos^2 x &= \tfrac{1}{4}\\
\sin^2 x(1 - \sin^2 x) &= \tfrac{1}{4}\\
\sin^2 x - \sin^4 x) &= \tfrac{1}{4}\\
\sin^4 x - \sin^2 x + \tfrac{1}{4} &= 0\\
\left(\sin^2 x - \tfrac{1}{2}\right)\left(\sin^2 x - \tfrac{1}{2}\right) &= 0\\
\sin^2 x - \tfrac{1}{2} &= 0\\
\sin^2 x &= \tfrac{1}{2}\\
\sin x &= \pm\tfrac{\sqrt{2}}{2}\\
x &= \tfrac{\pi}{4} + \pi k
\end{aligned}$$

44.
$$\begin{aligned}
\cos^2 x - \sin^2 x &= \tfrac{\sqrt{3}}{2}\\
\cos^2 x - (1 - \cos^2 x) &= \tfrac{\sqrt{3}}{2}\\
2\cos^2 x - 1 &= \tfrac{\sqrt{3}}{2}\\
2\cos^2 x &= \tfrac{2 + \sqrt{3}}{2}\\
\cos^2 x &= \tfrac{2 + \sqrt{3}}{4}\\
\cos x &= \pm\tfrac{\sqrt{2 + \sqrt{3}}}{2}\\
x &= \tfrac{\pi}{12} + \pi k \text{ or } x = \tfrac{11\pi}{12} + \pi k
\end{aligned}$$

45.
$$\begin{aligned}
\sin^4 x - 1 &= 0\\
(\sin^2 x - 1)(\sin^2 x + 1) &= 0\\
\sin^2 x - 1 = 0 \quad &\text{or} \quad \sin^2 x + 1 = 0\\
\sin^2 x = 1 \quad & \quad \sin^2 x = -1\\
\sin x = \pm 1 \quad & \quad \text{no solutions}\\
x = \tfrac{\pi}{2} + \pi k &
\end{aligned}$$

46.
$$\begin{aligned}
\sec^2 x + 2\sec x &= 0\\
\sec x(\sec x + 2) &= 0\\
\sec x = 0 \quad &\text{or} \quad \sec x + 2 = 0\\
\cos x = \tfrac{1}{0} \quad & \quad \sec x = -2\\
\text{no solution} \quad & \quad \cos x = -\tfrac{1}{2}\\
& \quad x = \tfrac{2\pi}{3} + 2\pi k \text{ or}\\
& \quad x = \tfrac{4\pi}{3} + 2\pi k
\end{aligned}$$

47.
$$\begin{aligned} \sin x + \cos x &= 1 \\ \sin^2 x + 2\sin x\cos x + \cos^2 x &= 1 \\ \sin^2 x + 2\sin x\cos x + 1 - \sin^2 x &= 1 \\ 2\sin x\cos x &= 0 \\ \sin x\cos x &= 0 \\ \sin^2 x\cos^2 x &= 0 \\ \sin^2 x(1-\sin^2 x) &= 0 \end{aligned}$$
$\sin^2 x = 0$ or $1 - \sin^2 x = 0$
$\sin x = 0$ $\quad$ $\sin^2 x = 1$
$x = 2\pi k$ $\quad$ $\sin x = \pm 1$
$x = \frac{\pi}{2} + 2\pi k$

48.
$$\begin{aligned} 2\sin x + \csc x &= 3 \\ 2\sin^2 x + 1 &= 3\sin x \\ 2\sin^2 x - 3\sin x + 1 &= 0 \\ (2\sin x - 1)(\sin x - 1) &= 0 \end{aligned}$$
$2\sin x - 1 = 0$ or $\sin x - 1 = 0$
$2\sin x = 1$ $\quad$ $\sin x = 1$
$\sin x = \frac{1}{2}$ $\quad$ $x = \frac{\pi}{2} + 2\pi k$
$x = \frac{\pi}{6} + 2\pi k$ or
$x = \frac{5\pi}{6} + 2\pi k$

49. $\cos\theta \le -\frac{\sqrt{3}}{2}$
$\cos\theta = -\frac{\sqrt{3}}{2}$ at $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$
$\frac{5\pi}{6} \le \theta \le \frac{7\pi}{6}$

50. $\cos\theta - \frac{1}{2} > 0$
$\cos\theta > \frac{1}{2}$
$\cos\theta = \frac{1}{2}$ at $\frac{\pi}{3}$ and $\frac{5\pi}{3}$
$0 \le \theta < \frac{\pi}{3}$ or $\frac{5\pi}{3} < \theta < 2\pi$

51. $\sqrt{2}\sin\theta - 1 < 0$
$\sqrt{2}\sin\theta < 1$
$\sin\theta < \frac{\sqrt{2}}{2}$
$\sin\theta = \frac{\sqrt{2}}{2}$ at $\frac{\pi}{4}$ and $\frac{3\pi}{4}$
$0 \le \theta < \frac{\pi}{4}$ or $\frac{3\pi}{4} < \theta < 2\pi$

52. 0.4636, 3.6052 **53.** 0, 1.8955

54. 0.3218, 3.4633

55. $\sin\theta = \frac{\lambda}{D}$
$\sin\theta = \frac{5.5 \times 10^{-7}}{0.003}$
$\sin\theta \approx 0.0001833333333$
$\theta \approx 0.01°$

56.
$$\begin{aligned} \sin 2x &< \sin x \\ 2\sin x\cos x &< \sin x \\ 2\sin x\cos x - \sin x &< 0 \\ \sin x(2\cos x - 1) &< 0 \end{aligned}$$
The product on the left side of the inequality is equal to 0 when x is 0, $\frac{\pi}{3}$, π, or $\frac{5\pi}{3}$. For the product to be negative, one factor must be positive and the other negative. This occurs if $\frac{\pi}{3} < x < \pi$ or $\frac{5\pi}{3} < x < 2\pi$.

57.
$$\begin{aligned} R &= \frac{v^2}{g}\sin 2\theta \\ 20 &= \frac{15^2}{9.8}\sin 2\theta \end{aligned}$$
$0.8711111111 \approx \sin 2\theta$
$2\theta \approx 60.5880156$ or $2\theta \approx 119.4119844$
$\theta \approx 30.29°$ $\quad$ $\theta \approx 59.71°$

58a.
$$\begin{aligned} n_1 \sin i &= n_2 \sin r \\ 1.00\sin 35° &= 2.42\sin r \\ \sin r &= \frac{1.00\sin 35°}{2.42} \\ \sin r &\approx 0.2370150563 \\ r &\approx 13.71° \end{aligned}$$

58b. Measure the angles of incidence and refraction to determine the index of refraction. If the index is 2.42, the diamond is genuine.

59.
$$\begin{aligned} D &= 0.5\sin(6.5\,x)\sin(2500t) \\ 0.01 &= 0.5\sin(6.5(0.5))\sin(2500t) \\ 0.02 &= \sin 3.25\sin 2500t \\ -0.1848511958 &\approx \sin 2500t \\ -0.1859549654 &\approx 2500t \end{aligned}$$
The first positive angle with sine equivalent to $\sin(-0.1859549654)$ is $\pi + 0.1859549654$ or 3.326477773.
$t \approx \frac{3.326477773}{2500}$
$t \approx 0.0013$ s

60. $a\sin(bx + c) + d = d + \frac{a}{2}$
$a\sin(bx + c) = \frac{a}{2}$
$\sin(bx + c) = \frac{1}{2}$
The period of the function $\sin(bx + c)$ is $\frac{360°}{b}$, so the given interval consists of $\frac{360°}{\frac{360°}{b}} = b$ periods.
The equation $\sin(bx + c) = \frac{1}{2}$ has two solutions per period, so the total number of solutions is $2b$.

61. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$
$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} \sqrt{17} \\ 2\sqrt{2} \end{bmatrix}$$
$$\begin{bmatrix} 3\cos\theta - 4\sin\theta \\ 3\sin\theta + 4\cos\theta \end{bmatrix} = \begin{bmatrix} \sqrt{17} \\ 2\sqrt{2} \end{bmatrix}$$
$3\cos\theta - 4\sin\theta = \sqrt{17}$
$3\sin\theta + 4\cos\theta = 2\sqrt{2}$
$\downarrow$
$$\begin{aligned} 9\cos\theta - 12\sin\theta &= 3\sqrt{17} \\ 16\cos\theta + 12\sin\theta &= 8\sqrt{2} \\ \hline 25\cos\theta \qquad &= 8\sqrt{2} + 3\sqrt{17} \\ \cos\theta &= \frac{8\sqrt{2} + 3\sqrt{17}}{25} \\ \theta &\approx 18.68020037 \end{aligned}$$
$360 - \theta \approx 341.32°$

62. $\cot 67.5° = \cot \frac{135°}{2}$ $\qquad \cot \theta = \frac{1}{\tan \theta}$

$\tan \frac{135°}{2} = \sqrt{\frac{1-\cos 135°}{1+\cos 135°}}$ (Quadrant 1)

$$= \sqrt{\frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{1+\left(-\frac{\sqrt{2}}{2}\right)}}$$

$$= \sqrt{\frac{\frac{2+\sqrt{2}}{2}}{\frac{2-\sqrt{2}}{2}}}$$

$$= \sqrt{\frac{(2+\sqrt{2})(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})}}$$

$$= \sqrt{\frac{(2+\sqrt{2})^2}{4-2}}$$

$$= \frac{2+\sqrt{2}}{\sqrt{2}}$$

$$\cot 67.5 = \frac{1}{\frac{2+\sqrt{2}}{\sqrt{2}}}$$

$$= \frac{\sqrt{2}}{2+\sqrt{2}}$$

$$= \frac{\sqrt{2}(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})}$$

$$= \frac{2\sqrt{2}-2}{4-2}$$

$$= \sqrt{2}-1$$

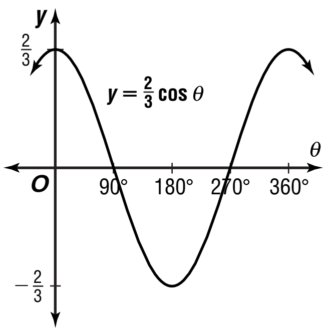
63. $\frac{\tan x}{\sec x} = \frac{\sqrt{2}}{5}$

$$\frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \frac{\sqrt{2}}{5}$$

$$\sin x = \frac{\sqrt{2}}{5}$$

Sample answer: $\sin x = \frac{\sqrt{2}}{5}$

64. $A = \frac{2}{3}$, 2π



65. $\frac{45 \text{ miles}}{\text{hour}} \cdot \frac{5280 \text{ ft}}{\text{mile}} \cdot \frac{12 \text{ inches}}{\text{ft}} \cdot \frac{1 \text{ hour}}{3600 \text{ sec}} = 792$ in/sec

$$v = r\frac{\theta}{t}$$

$$792 = 7\frac{\theta}{1}$$

$$\frac{792}{7} = \pi$$

$\frac{792}{7}$ radians $\div 2\pi \approx 18$ rps

66. undefined

67.

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -3 & -2 \\ & & 2 & 4 & 2 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$

$x^2 + 2x + 1 = 0$

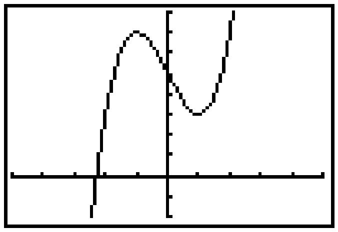
$(x+1)(x+1) = 0$

$x + 1 = 0 \qquad x + 1 = 0$

$x = -1 \qquad x = -1$

$(x-2)(x+1(x+1)$

68.



$[-5, 5]$ sc1:1 by $[-2, 8]$ sc1:1

max: $(-1, 7)$, min: $(1, 3)$

69. $3x + 4 = 16 \qquad 6 = 2y$

$x = 4 \qquad y = 3 \qquad (4, 3)$

70.

$$\begin{array}{rl} x - y + z = 1 & \qquad x - y + z = 1 \\ 2x + y + 3z = 5 & \qquad x + y - z = 11 \\ \hline 3x \quad + 4z = 6 & \qquad 2x \quad = 12 \end{array}$$

$x = 6$

$3x + 4z = 6 \qquad x + y - z = 11$

$3(6) + 4z = 6 \qquad 6 + y - (-3) = 11$

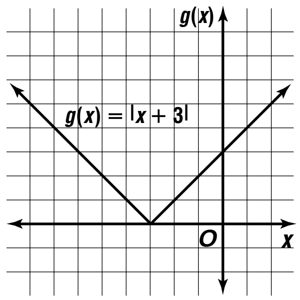
$4z = -12 \qquad y = 2$

$z = -3$

$(6, 2, -3)$

71.

x	$g(x)$
-7	4
-5	2
-3	0
-1	2
1	4



72. $A = \frac{1}{2}bh$

$A = \frac{1}{2}(6)(1)$

$A = 3$

The correct choice is C.

Page 462 History of Mathematics

1. $x^2 = 5^2 + 5^2 - 2(5)(5) \cos 10°$
   $x \approx 0.87$
   $x^2 = 5^2 + 5^2 - 2(5)(5) \cos 20°$
   $x \approx 1.74$
   $x^2 = 5^2 + 5^2 - 2(5)(5) \cos 30°$
   $x \approx 2.59$
   $x^2 = 5^2 + 5^2 - 2(5)(5) \cos 40°$
   $x \approx 3.42$
   $x^2 = 5^2 + 5^2 - 2(5)(5) \cos 50°$
   $x \approx 4.23$
   $x^2 = 5^2 + 5^2 - 2(5)(5) \cos 60°$
   $x = 5$
   $x^2 = 5^2 + 5^2 - 2(5)(5) \cos 70°$
   $x \approx 5.74$
   $x^2 = 5^2 + 5^2 - 2(5)(5) \cos 80°$
   $x \approx 6.43$
   $x^2 = 5^2 + 5^2 - 2(5)(5) \cos 90°$
   $x \approx 7.07$

Angle Measure	Length of Chord (cm)
10°	0.87
20°	1.74
30°	2.59
40°	3.42
50°	4.23
60°	5.00
70°	5.74
80°	6.43
90°	7.07

7-6 Normal Form of a Linear Equation

Page 467 Check for Understanding

1. *Normal* means perpendicular
2. Compute cos 30° and sin 30°. Use these as the coefficients of x and y, respectively, in the normal form. The normal form is $\frac{\sqrt{3}}{2}x + \frac{1}{2}y - 10 = 0$.
3. The statement is true. The given line is tangent to the circle centered at the origin with radius p.
4.

Slope-Intercept Form: $y = mx + b$, displays slope and y-intercept
Point-Slope Form: $y - y_1 = m(x - x_1)$, displays slope and a point on the line
Standard Form: $Ax + by + C = 0$, displays no information
Normal Form: $x \cos \phi + y \sin \phi - p = 0$, displays length of the normal and the angle the normal makes with the x-axis

See students' work for sample problems.

5. $$x \cos \phi + y \sin \phi - p = 0$$
$$x \cos 30° + y \sin 30° - 10 = 0$$
$$\frac{\sqrt{3}}{2}x + \frac{1}{2}y - 10 = 0$$
$$\sqrt{3}x + y - 20 = 0$$

6. $$x \cos \phi + y \sin \phi - p = 0$$
$$x \cos 150° + y \sin 150° - \sqrt{3} = 0$$
$$-\frac{\sqrt{3}}{2}x + \frac{1}{2}y - \sqrt{3} = 0$$
$$\sqrt{3}x - y + 2\sqrt{3} = 0$$

7. $$x \cos \phi + y \sin \phi - p = 0$$
$$x \cos \frac{7\pi}{4} + y \sin \frac{7\pi}{4} - 5\sqrt{2} = 0$$
$$\frac{\sqrt{2}}{2}x + \left(-\frac{\sqrt{2}}{2}\right)y - 5\sqrt{2} = 0$$
$$\sqrt{2}x - \sqrt{2}y - 10\sqrt{2} = 0$$
$$x - y - 10 = 0$$

8. $4x + 3y = -10$ $\quad -\sqrt{A^2 + B^2} = -\sqrt{4^2 + 3^2}$ or -5
$4x + 3y + 10 = 0$ $\quad \frac{4}{-5}x + \frac{3}{-5}y + \frac{10}{-5} = 0$
$$-\frac{4}{5}x - \frac{3}{5}y - 2 = 0$$
$\sin \phi = -\frac{3}{5}$, $\cos \phi = -\frac{4}{5}$, $p = 2$; Quadrant III
$$\tan \phi = \frac{-\frac{3}{5}}{-\frac{4}{5}} \text{ or } \frac{3}{4}$$
ϕ

9. $$y = -3x + 2$$
$$3x + y - 2 = 0$$
$$\sqrt{A^2 + B^2} = \sqrt{3^2 + 1^2} \text{ or } \sqrt{10}$$
$$\frac{3}{\sqrt{10}}x + \frac{1}{\sqrt{10}}y - \frac{2}{\sqrt{10}} = 0$$
$$\frac{3\sqrt{10}}{10}x + \frac{\sqrt{10}}{10}y - \frac{\sqrt{10}}{5} = 0$$
$\sin \phi = \frac{\sqrt{10}}{10}$, $\cos \phi = \frac{3\sqrt{10}}{10}$, $p = \frac{\sqrt{10}}{5}$; Quadrant I
$$\tan \phi = \frac{\frac{\sqrt{10}}{10}}{\frac{3\sqrt{10}}{10}} \text{ or } \frac{1}{3}$$
$$\phi \approx 18°$$

10. $\sqrt{2}x - \sqrt{2}y = 6$

$\sqrt{2}x - \sqrt{2}y - 6 = 0$

$\sqrt{A^2 + B^2} = \sqrt{\sqrt{2}^2 + (-\sqrt{2})^2}$ or 2

$\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y - \frac{6}{2} = 0$

$\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y - 3 = 0$

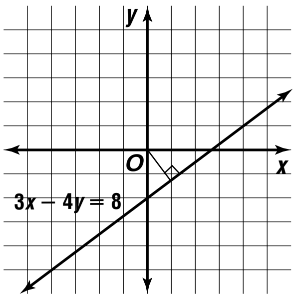
$\sin\phi = -\frac{\sqrt{2}}{2}$, $\cos\phi = \frac{\sqrt{2}}{2}$, $p = 3$; Quadrant IV

$\tan\phi = \dfrac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$ or -1

$\phi \approx 315°$

11a. $3x - 4y = 8$

$y = \frac{3}{4}x - 2$



11b. $3x - 4y = 8$

$3x - 4y - 8 = 0$

$\sqrt{A^2 + B^2} = \sqrt{3^2 + (-4)^2}$ or 5

$\frac{3}{5}x - \frac{4}{5}y - \frac{8}{5} = 0$

$p = \frac{8}{5}$ or 1.6 miles

Pages 467–469 Exercises

12. $x\cos\phi + y\sin\phi - p = 0$

$x\cos 60° + y\sin 60° - 15 = 0$

$\frac{1}{2}x + \frac{\sqrt{3}}{2}y - 15 = 0$

$x + \sqrt{3}y - 30 = 0$

13. $x\cos\phi + y\sin\theta - p = 0$

$x\cos\frac{\pi}{4} + y\sin\frac{\pi}{4} - 12 = 0$

$\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y - 12 = 0$

$\sqrt{2}x + \sqrt{2}y - 24 = 0$

14. $x\cos\phi + y\sin\phi - p = 0$

$x\cos 135° + y\sin 135° - 3\sqrt{2} = 0$

$-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y - 3\sqrt{2} = 0$

$-\sqrt{2}x + \sqrt{2}y - 6\sqrt{2} = 0$

$x - y + 6 = 0$

15. $x\cos\phi + y\sin\phi - p = 0$

$x\cos\frac{5\pi}{6} + y\sin\frac{5\pi}{6} - 2\sqrt{3} = 0$

$-\frac{\sqrt{3}}{2}x + \frac{1}{2}y - 2\sqrt{3} = 0$

$\sqrt{3}x - y + 4\sqrt{3} = 0$

16. $x\cos\phi + y\sin\phi - p = 0$

$x\cos\frac{\pi}{2} + y\sin\frac{\pi}{2} - 2 = 0$

$0x + 1y - 2 = 0$

$y - 2 = 0$

17. $x\cos\phi + y\sin\phi - p = 0$

$x\cos 210° + y\sin 210° - 5 = 0$

$-\frac{\sqrt{3}}{2}x - \frac{1}{2}y - 5 = 0$

$\sqrt{3}x + y + 10 = 0$

18. $x\cos\phi + y\sin\phi - p = 0$

$x\cos\frac{4\pi}{3} + y\sin\frac{4\pi}{3} - 5 = 0$

$-\frac{1}{2}x - \frac{\sqrt{3}}{2}y - 5 = 0$

$x + \sqrt{3}y + 10 = 0$

19. $x\cos\phi + y\sin\phi - p = 0$

$x\cos 300° + y\sin 300° - \frac{3}{2} = 0$

$\frac{1}{2}x - \frac{\sqrt{3}}{2}y - \frac{3}{2} = 0$

$x - \sqrt{3}y - 3 = 0$

20. $x\cos\phi + y\sin\phi - p = 0$

$x\cos\frac{11\pi}{6} + y\sin\frac{11\pi}{6} - 4\sqrt{3} = 0$

$\frac{\sqrt{3}}{2}x - \frac{1}{2}y - 4\sqrt{3} = 0$

$\sqrt{3}x - y - 8\sqrt{3} = 0$

21. $-\sqrt{A^2 + B^2} = -\sqrt{5^2 + 12^2}$ or -13

$\frac{5}{-13}x + \frac{12}{-13}y + \frac{65}{-13} = 0$

$-\frac{5}{13}x - \frac{12}{13}y - 5 = 0$

$\sin\phi = -\frac{12}{13}$, $\cos\phi = -\frac{5}{13}$, $p = 5$; Quadrant III

$\tan\phi = \dfrac{-\frac{12}{13}}{-\frac{5}{13}}$ or $\frac{12}{5}$

$\phi \approx 247°$

22. $x + y = 1$

$x + y - 1 = 0$

$\sqrt{A^2 + B^2} = \sqrt{1^2 + 1^2}$ or $\sqrt{2}$

$\frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{1}{\sqrt{2}} = 0$

$\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y - \frac{\sqrt{2}}{2} = 0$

$\sin\phi = \frac{\sqrt{2}}{2}$, $\cos\phi = \frac{\sqrt{2}}{2}$, $p = \frac{\sqrt{2}}{2}$; Quadrant I

$\tan\phi = \dfrac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$ or 1

$\phi = 45°$

23. $3x - 4y = 15$

$3x - 4y - 15 = 0$

$\sqrt{A^2 + B^2} = \sqrt{3^2 + (-4)^2}$ or 5

$\frac{3}{5}x - \frac{4}{5}y - \frac{15}{5} = 0$

$\frac{3}{5}x - \frac{4}{5}y - 3 = 0$

$\sin\phi = -\frac{4}{5}$, $\cos\phi = \frac{3}{5}$, $p = 3$; Quadrant IV

$\tan\phi = \dfrac{-\frac{4}{5}}{\frac{3}{5}}$ or $-\frac{4}{3}$

$\phi \approx 307°$

24. $y = 2x - 4$

$-2x + y + 4 = 0$

$-\sqrt{A^2 + B^2} = -\sqrt{(-2)^2 + 1^2}$ or $-\sqrt{5}$

$-\frac{2}{-\sqrt{5}}x + \frac{1}{-\sqrt{5}}y + \frac{4}{-\sqrt{5}} = 0$

$\frac{2\sqrt{5}}{5}x - \frac{\sqrt{5}}{5}y - \frac{4\sqrt{5}}{5} = 0$

$\sin\phi = -\frac{\sqrt{5}}{5}$, $\cos\phi = \frac{2\sqrt{5}}{5}$, $p = \frac{4\sqrt{5}}{5}$; Quadrant IV

$\tan\phi = \dfrac{-\frac{\sqrt{5}}{5}}{\frac{2\sqrt{5}}{5}}$ or $-\frac{1}{2}$

$\phi \approx 333°$

25. $x = 3$

$x - 3 = 0$

$\sqrt{A^2 + B^2} = \sqrt{1^2 + 0^2}$ or 1

$\frac{1}{1}x - \frac{3}{1} = 0$

$x - 3 = 0$

$\sin\phi = 0$, $\sin\phi = 1$, $p = 3$

$\tan\phi = \frac{0}{1}$ or 0

$\phi = 0°$

26. $-\sqrt{3}x - y = 2$

$-\sqrt{3}x - y - 2 = 0$

$\sqrt{A^2 + B^2} = \sqrt{(-\sqrt{3})^2 + (-1)^2}$ or 2

$-\frac{\sqrt{3}}{2}x - \frac{1}{2}y - \frac{2}{2} = 0$

$-\frac{\sqrt{3}}{2}x - \frac{1}{2}y - 1 = 0$

$\sin\phi = -\frac{1}{2}$, $\cos\phi = -\frac{\sqrt{3}}{2}$, $p = 1$; Quadrant III

$\tan\phi = \dfrac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$ or $\frac{\sqrt{3}}{3}$

$\phi \approx 210°$

27. $y - 2 = \frac{1}{4}(x + 20)$

$y - 2 = \frac{1}{4}x + 5$

$-x + 4y - 28 = 0$

$\sqrt{A^2 + B^2} = \sqrt{(-1)^2 + 4^2}$ or $\sqrt{17}$

$-\frac{1}{\sqrt{17}}x + \frac{4}{\sqrt{17}}y - \frac{28}{\sqrt{17}} = 0$

$-\frac{\sqrt{17}}{17}x + \frac{4\sqrt{17}}{17}y - \frac{28\sqrt{17}}{17} = 0$

$\sin\phi = \frac{4\sqrt{17}}{17}$, $\cos\phi = -\frac{\sqrt{17}}{17}$, $p = \frac{28\sqrt{17}}{17}$; Quadrant II

$\tan\phi = \dfrac{\frac{4\sqrt{17}}{17}}{-\frac{\sqrt{17}}{17}}$ or -4

$\phi \approx 104°$

28. $\frac{x}{3} = y - 4$

$\frac{x}{3} - y + 4 = 0$

$x - 3y + 12 = 0$

$-\sqrt{A^2 + B^2} = -\sqrt{1^2 + (-3)^2}$ or $-\sqrt{10}$

$\frac{1}{-\sqrt{10}}x - \frac{3}{-\sqrt{10}}y + \frac{12}{-\sqrt{10}} = 0$

$-\frac{\sqrt{10}}{10}x + \frac{3\sqrt{10}}{10}y - \frac{6\sqrt{10}}{5} = 0$

$\sin\phi = \frac{3\sqrt{10}}{10}$, $\cos\phi = -\frac{\sqrt{10}}{10}$, $p = \frac{6\sqrt{10}}{5}$; Quadrant II

$\tan\phi = \dfrac{\frac{3\sqrt{10}}{10}}{-\frac{\sqrt{10}}{10}}$ or -3

$\phi \approx 108°$

29. $\frac{x}{20} + \frac{y}{24} = 1$

$\frac{x}{20} + \frac{y}{24} - 1 = 0$

$6x + 5y - 124 = 0$

$\sqrt{A^2 + B^2} = \sqrt{6^2 + 5^2}$ or $\sqrt{61}$

$\frac{6}{\sqrt{61}}x + \frac{5}{\sqrt{61}}y - \frac{124}{\sqrt{61}} = 0$

$\frac{6\sqrt{61}}{61}x + \frac{5\sqrt{61}}{61}y - \frac{120\sqrt{61}}{61} = 0$

$\sin\phi = \frac{5\sqrt{61}}{61}$, $\cos\phi = \frac{6\sqrt{61}}{61}$, $p = \frac{120\sqrt{61}}{61}$; Quadrant I

$\tan\phi = \dfrac{\frac{5\sqrt{61}}{61}}{\frac{6\sqrt{61}}{61}}$ or $\frac{5}{6}$

$\phi \approx 40°$

30. $\sqrt{A^2 + B^2} = \sqrt{6^2 + 8^2}$ or 10; $p = 10$

$\cos\phi = \frac{6}{10}$ or $\frac{3}{5}$, $\sin\phi = \frac{8}{10}$ or $\frac{4}{5}$

$x\cos\phi + y\sin\phi - p = 0$

$\frac{3}{5}x + \frac{4}{5}y - 10 = 0$

$3x + 4y - 50 = 0$

31. $\sqrt{A^2 + B^2} = \sqrt{(-4)^2 + 4^2}$ or $4\sqrt{2}$; $p = 4\sqrt{2}$

$\cos\phi = \frac{-4}{4\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$, $\sin\phi = \frac{4}{4\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

$x\cos\phi + y\sin\phi - p = 0$

$-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y - 4\sqrt{2} = 0$

$x - y + 8 = 0$

32. $2\sqrt{2}x = y + 18$

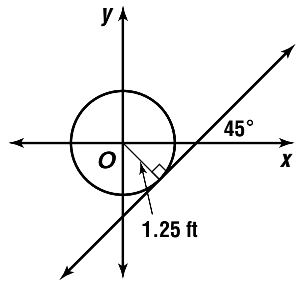
$2\sqrt{2}x - y - 18 = 0$

$\sqrt{A^2 + B^2} = \sqrt{2(2)^2 + (-1)^2} = \sqrt{9} = 3$

$\frac{2\sqrt{2}}{3}x - \frac{1}{3}y - \frac{18}{3} = 0$

$p = \frac{18}{3} = 6$ units

33a.



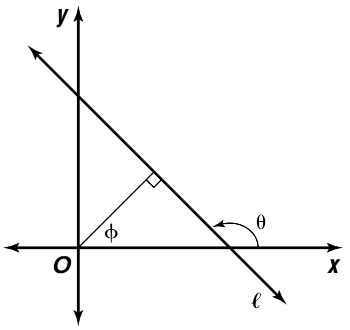
33b. $p = 1.25$, $\phi = 45°$

$x \cos(-45°) + y \sin(-45°) - 1.25 = 0$

$$\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y - 1.25 = 0$$

$$\sqrt{2}x - \sqrt{2}y - 2.5 = 0$$

34a.



ϕ and the supplement of θ are complementary angles of a right triangle, so $\phi + 180° - \theta = 90°$. Simplifying this equation gives $\theta = \phi + 90°$.

34b. $\tan\theta$. The slope of a line is the tangent of the angle the line makes with the positive x-axis

34c. Since the normal line is perpendicular to ℓ, the slope of the normal line is the negative reciprocal of the slope of ℓ. That is, $-\frac{1}{\tan\theta} = -\cot\theta$.

34d. The slope of ℓ is the negative reciprocal of the slope of the normal, or $-\frac{1}{\tan\phi} = -\cot\phi$.

35a. $\sqrt{A^2 + B^2} = \sqrt{5^2 + 12^2}$ or 13

$\frac{5}{13}x + \frac{12}{13}y - \frac{39}{13} = 0$

$\frac{5}{13}x + \frac{12}{13}y - 3 = 0$

35b. $\sin\phi = \frac{12}{13}$, $\cos\phi = \frac{5}{13}$; Quadrant I

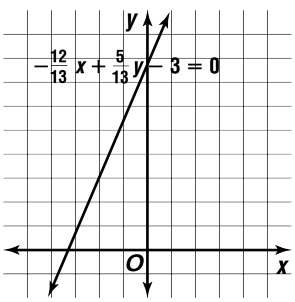
$\tan\phi = \dfrac{\frac{12}{13}}{\frac{5}{13}}$ or $\frac{12}{5}$

$\phi \approx 67°$

$\phi + 90° = 67° + 90°$ or $157°$

$x \cos 157° + y \sin 157° - 3 = 0$

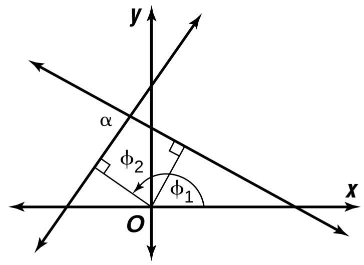
$-\frac{12}{13}x + \frac{5}{13}y - 3 = 0$



35c. See students' work.

35d. The line with normal form $x\cos\phi + y\sin\phi - p = 0$ makes an angle of ϕ with the positive x-axis and has a normal of length p. The graph of Armando's equation is a line whose normal makes an angle of $\phi + \delta$ with the x-axis and also has length p. Therefore, the graph of Armando's equation is the graph of the original line rotated $\delta°$ counterclockwise about the origin. Armando is correct. See students' graphs.

36a.



The angles of the quadrilateral are $180° - \alpha$, $90°$, $\phi_2 - \phi_1$, and $90°$. Then $180° - \alpha + 90° + \phi_2 - \phi_1 + 90° = 360°$, which simplifies to $\phi_2 = \phi_1 + \alpha$. If the lines intersect so that α is an interior angle of the quadrilateral, the equation works out to be $\phi_2 = 180° + \phi_1 - \alpha$.

36b. $\tan\phi_2 = \tan(\phi_1 + \alpha)$

$= \frac{\tan\phi_1 + \tan\alpha}{1 - \tan\phi_1\tan\alpha}$

If the lines intersect so that α is an interior angle of the quadrilateral, the equation works out to be $\tan\phi_2 = \frac{\tan\phi_1 - \tan\alpha}{1 + \tan\phi_1\tan\alpha}$.

37. $5x - y = 15$

$5x - y - 15 = 0$

$\sqrt{A^2 + B^2} = \sqrt{5^2 + (-1)^2}$ or $\sqrt{26}$

$\frac{5}{\sqrt{26}}x - \frac{1}{\sqrt{26}}y - \frac{15}{\sqrt{26}} = 0$

$\frac{5\sqrt{26}}{26}x - \frac{\sqrt{26}}{26}y - \frac{15\sqrt{26}}{26} = 0$, $p = \frac{15\sqrt{26}}{26}$

$3x + 4y = 36$

$3x + 4y - 36 = 0$

$\sqrt{A^2 + B^2} = \sqrt{3^2 + 4^2}$ or 5

$\frac{3}{5}x + \frac{4}{5}y - \frac{36}{5} = 0$, $p = \frac{36}{5}$

$5x - 2y = -20$

$5x - 2y + 20 = 0$

$\sqrt{5^2 + (-2)^2} = \sqrt{25 + 4}$ or $\sqrt{29}$

$\frac{5}{\sqrt{29}}x - \frac{2}{\sqrt{29}}y + \frac{20}{\sqrt{29}} = 0$

$\frac{5\sqrt{29}}{29}x - \frac{2\sqrt{29}}{29}y + \frac{20\sqrt{29}}{29} = 0$, $p = \frac{20\sqrt{29}}{29}$

$\frac{15\sqrt{26}}{26} + \frac{36}{5} + \frac{20\sqrt{29}}{29} \approx 13.85564879$

$13.85564879 \times 500 \approx 6927.824395$; \$6927.82

38. $2\cos^2 x + 7\cos x - 4 = 0$

$(2\cos x - 1)(\cos x + 4) = 0$

$2\cos x - 1 = 0$ or $\cos x + 4 = 0$

$2\cos x = 1$ $\qquad$ $\cos x = -4$

$\cos x = \frac{1}{2}$ $\qquad$ no solution

$x = \frac{\pi}{3}$ or $x = \frac{5\pi}{3}$

39. $\sin x = \sqrt{1 - \cos^2 x}$ $\qquad$ $\sin y = \sqrt{1 - \cos^2 y}$

$= \sqrt{1 - \left(\frac{1}{6}\right)^2}$ $\qquad$ $= \sqrt{1 - \left(\frac{2}{3}\right)^2}$

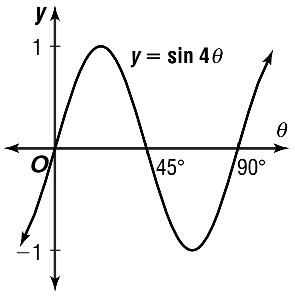
$= \sqrt{\frac{36}{36}}$ or $\frac{\sqrt{35}}{6}$ $\qquad$ $= \sqrt{\frac{5}{9}}$ or $\frac{\sqrt{5}}{3}$

$\sin(x + y) = \sin x \cos y + \cos x \sin y$

$= \left(\frac{\sqrt{35}}{6}\right)\left(\frac{2}{3}\right) + \left(\frac{1}{6}\right)\left(\frac{\sqrt{5}}{3}\right)$

$= \frac{2\sqrt{35} + \sqrt{5}}{18}$

40. $A = 1, \frac{2\pi}{4} = \frac{\pi}{2}$ or $90°$



41. $r = \frac{d}{2}$

$r = \frac{13.4}{2}$ or 6.7

$x^2 = 6.7^2 + 6.7^2 - 2(6.7)(6.7)\cos 26°20'$

$x^2 \approx 9.316604344$

$x \approx 3.05$ cm

42.
$$\frac{x}{x-5} + \frac{17}{25-x^2} = \frac{1}{x+5}$$
$$\frac{x}{x-5} + \frac{-17}{x^2-25} = \frac{1}{x+5}$$
$$(x-5)(x+5)\left(\frac{x}{x-5}\right) +$$
$$(x-5)(x+5)\left(\frac{-17}{(x-5)(x+5)}\right) = (x-5)(x+5)\left(\frac{1}{x+5}\right)$$
$$x(x+5) - 17 = x - 5$$
$$x^2 + 5x - 17 = x - 5$$
$$x^2 + 4x - 12 = 0$$
$$(x+6)(x-2) = 0$$
$$x + 6 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -6 \qquad\qquad x = 2$$

43. original box: $V = \ell wh$

$= 4 \cdot 6 \cdot 2$

$= 48$

new box: $V = \ell wh$

$1.5(48) = (4 + x)(6 + x)(2 + x)$

$72 = x^3 + 12x^2 + 44x + 48$

$0 = x^3 + 12x^2 + 44x - 24$

x	$V(x)$
0.4	−4.416
0.5	1.125

$V(0.5)$ is closer to zero, so $x = 0.5$.

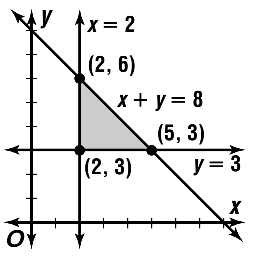
$4 + x = 4 + 0.5$ or 4.5

$6 + x = 6 + 0.5$ or 6.5

$2 + x = 2 + 0.5$ or 2.5

4.5 in. by 6.5 in. by 2.5 in.

44.



$f(x, y) = 3x - y + 4$

$f(2, 3) = 3(2) - 3 + 4$ or 7

$f(2, 6) = 3(2) - 6 + 4$ or 4

$f(5, 3) = 3(5) - 3 + 4$ or 16

16, 4

45.
$$\frac{1}{\begin{vmatrix} -1 & 2 \\ 4 & 3 \end{vmatrix}} \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$$
$$\begin{bmatrix} -1 & 2 \\ -4 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 15 \end{bmatrix}$$
$$\frac{1}{5}\begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 \\ -4 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 15 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -30 \\ -15 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -3 \end{bmatrix}$$

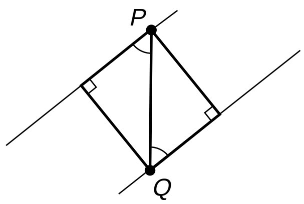
$(-6, -3)$

46. The value of $2a + b$ cannot be determined from the given information. The correct choice is E.

7-7 Distance From a Point to a Line

Page 474 Check for Understanding

1. The distance from a point to a line is the distance from that point to the closest point on the line.
2. The sign should be chosen opposite the sign of C where $Ax + By + C = 0$ is the standard form of the equation of the line.
3. In the figure, P and Q are any points on the lines. The right triangles are congruent by AAS. The corresponding congruent sides of the triangles show that the same distance is always obtained between the two lines.



4. The formula is valid in either case. Examples will vary. For a vertical line, $x = a$, the formula subtracts a from the x-coordinate of the point. For a horizontal line, $y = b$, the formula subtracts b from the y-coordinate of the point.
5. $2x - 3y = -2 \rightarrow 2x - 3y + 2 = 0$
$$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$$
$$d = \frac{2(1) + (-3)(2) + 2}{-\sqrt{2^2 + (-3)^2}}$$
$$d = \frac{-2}{-\sqrt{13}} \text{ or } \frac{2\sqrt{13}}{13}$$
6. $6x - y = -3 \rightarrow 6x - y + 3 = 0$
$$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$$
$$d = \frac{6(-2) + (-1)(3) + 3}{-\sqrt{6^2 + (-1)^2}}$$
$$d = \frac{-12}{-\sqrt{37}} \text{ or } \frac{12\sqrt{37}}{37}$$

7. $3x - 5y = 1$ When $x = 2$, $y = 1$. Use (2, 1).

$3x - 5y = -3 \to 3x - 5y + 3 = 0$

$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$d = \frac{3(2) + (-5)(1) + 3}{-\sqrt{3^2 + (-5)^2}}$

$d = \frac{4}{-\sqrt{34}}$ or $-\frac{2\sqrt{34}}{17}$

$\frac{2\sqrt{34}}{17}$

8. $y = -\frac{1}{3}x + 3$ Use (0, 3).

$y = -\frac{1}{3}x - 7 \to x + 3y + 21 = 0$

$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$d = \frac{1(0) + 3(3) + 21}{-\sqrt{1^2 + 3^2}}$

$d = \frac{30}{-\sqrt{10}}$ or $-3\sqrt{10}$

$3\sqrt{10}$

9. $d_1 = \frac{6x_1 + 8y_1 + 5}{\sqrt{6^2 + 8^2}}$ $d_2 = \frac{2x_1 - 3y_1 - 4}{\sqrt{2^2 + (-3)^2}}$

$\frac{6x_1 + 8y_1 + 5}{10} = \frac{2x_1 - 3y_1 - 4}{\sqrt{13}}$

$6\sqrt{13}x + 8\sqrt{13}y + 5\sqrt{13} = 20x - 30y - 40$

$(20 - 6\sqrt{13})x -$
$(30 + 8\sqrt{13})y - 40 - 5\sqrt{13} = 0;$

$\frac{6x_1 + 8y_1 + 5}{10} = -\frac{2x_1 - 3y_1 = 4}{\sqrt{13}}$

$6\sqrt{13}x + 8\sqrt{13}y + 5\sqrt{13} = -20x + 30y + 40$

$(20 + 6\sqrt{13})x + (8\sqrt{13} - 30)y - 40 + 5\sqrt{13} = 0$

10. (2000, 0)

$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$d = \frac{5(2000) + (-3)(0) + 0}{\sqrt{5^2 + (-3)^2}}$

$d = \frac{10,000}{\sqrt{34}}$ or about 1715 ft

Pages 475–476 Exercises

11. $d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$d = \frac{3(2) + (-4)(0) + 15}{-\sqrt{3^2 + (-4)^2}}$

$d = \frac{21}{-5}$

$\frac{21}{5}$

12. $d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$d = \frac{5(3) + (-3)(5) + 10}{-\sqrt{5^2 + (-3)^2}}$

$d = \frac{10}{-\sqrt{34}}$ or $-\frac{5\sqrt{34}}{17}$

$\frac{5\sqrt{34}}{17}$

13. $-2x - y = -3 \to -2x - y + 3 = 0$

$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$d = \frac{-2(0) + (-1)(0) + 3}{\sqrt{(-2)^2 + (-1)^2}}$

$d = \frac{3}{\sqrt{5}}$ or $\frac{3\sqrt{5}}{5}$

14. $y = 4 - \frac{2}{3}x \to 2x + 3y - 12 = 0$

$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$d = \frac{2(-2) + 3(-3) + (-12)}{\sqrt{2^2 + 3^2}}$

$d = \frac{-25}{\sqrt{13}}$ or $-\frac{25\sqrt{13}}{13}$

$\frac{25\sqrt{13}}{13}$

15. $y = 2x - 5 \to 2x - y - 5 = 0$

$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$d = \frac{2(3) + 3(-1)(1) + (-5)}{\sqrt{2^2 + (-1)^2}}$

$d = \frac{0}{\sqrt{5}}$ or 0

16. $y = -\frac{4}{3}x + 6 \to 4x + 3y - 18 = 0$

$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$d = \frac{4(-1) + 3(2) + (-18)}{\sqrt{4^2 + 3^2}}$

$d = \frac{-16}{5}$ or $-\frac{16}{5}$

$\frac{16}{5}$

17. $d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$d = \frac{3(0) + (-1)(0) + 1}{-\sqrt{3^2 + (-1)^2}}$

$d = \frac{1}{-\sqrt{10}}$ or $-\frac{\sqrt{10}}{10}$

$\frac{\sqrt{10}}{10}$

18. $6x - 8y = 3$ When $x = 0$, $y = -\frac{3}{8}$. Use $\left(0, -\frac{3}{8}\right)$.

$6x - 8y = -5 \to 6x - 8y + 5 = 0$

$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$d = \frac{6(0) + (-8)\left(-\frac{3}{8}\right) + 1}{-\sqrt{6^2 + (-8)^2}}$

$d = \frac{8}{-10}$ or $-\frac{4}{5}$

$\frac{4}{5}$

19. $4x - 5y = 12$ When $x = 3$, $y = 0$. Use (3, 0).

$4x - 5y = 6 \to 4x - 5y - 6 = 0$

$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$d = \frac{4(3) + (-5)(0) + (-6)}{\sqrt{4^2 + (-5)^2}}$

$d = \frac{6}{\sqrt{41}}$ or $\frac{6\sqrt{41}}{41}$

20. $y = 2x + 1$ Use (0, 1).

$2x - y = 2 \to 2x - y - 2 = 0$

$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$d = \frac{2(0) + (-1)(1) + (-2)}{\sqrt{2^2 + (-1)^2}}$

$d = \frac{-3}{\sqrt{5}}$ or $-\frac{3\sqrt{5}}{5}$

$\frac{3\sqrt{5}}{5}$

21. $y = -3x + 6$ Use (0, 6).

$3x + y = 4 \rightarrow 3x + y - 4 = 0$

$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$d = \frac{3(0) + 1(6)(1) + (-4)}{\sqrt{3^2 + 1^2}}$

$d = \frac{2}{\sqrt{10}}$ or $\frac{\sqrt{10}}{5}$

22. $y = \frac{8}{5}x - 1$ Use (0, −1).

$8x + 15 = 5y \rightarrow 8x - 5y + 15 = 0$

$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$d = \frac{8(0) + (-5)(-1) + 15}{-\sqrt{8^2 + (-5)^2}}$

$d = \frac{20}{-\sqrt{89}}$ or $-\frac{20\sqrt{89}}{89}$

$\frac{20\sqrt{89}}{89}$

23. $y = -\frac{3}{2}x$ Use (0, 0).

$y = -\frac{3}{2}x - 4 \rightarrow 3x + 2y + 8 = 0$

$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$d = \frac{3(0) + 2(0) + 8}{-\sqrt{3^2 + 2^2}}$

$d = \frac{8}{-\sqrt{13}}$ or $-\frac{8\sqrt{13}}{13}$

$\frac{8\sqrt{13}}{13}$

24. $y = -x + 6$ Use (0, 6).

$x + y - 1 = 0$

$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$d = \frac{1(0) + 1(6) + (-1)}{\sqrt{1^2 + 1^2}}$

$d = \frac{5}{\sqrt{2}}$ or $\frac{5\sqrt{2}}{2}$

25. $d_1 = \frac{3x_1 + 4y_1 - 10}{\sqrt{3^2 + 4^2}}$ $d_2 = \frac{5x_1 - 12y_1 - 26}{\sqrt{5^2 + (-12)^2}}$

$\frac{3x_1 + 4y_1 - 10}{5} = \frac{5x_1 - 12y_1 - 26}{13}$

$39x + 52y - 130 = 25x - 60y - 130$

$14x + 112y = 0$

$x + 8y = 0$

$\frac{3x_1 + 4y_1 - 10}{5} = -\frac{5x_1 - 12y_1 - 26}{13}$

$39x + 52y - 130 = -25x + 60y + 130$

$64x - 8y - 260 = 0$

$16x - 2y - 65 = 0$

26. $d_1 = \frac{4x_1 + y_1 - 6}{\sqrt{4^2 + 1^2}}$ $d_2 = \frac{-15x_1 + 8y_1 - 68}{\sqrt{(-15)^2 + 8^2}}$

$\frac{4x_1 + y_1 - 6}{\sqrt{17}} = -\frac{-15x_1 + 8y_1 - 68}{17}$

$68x + 17y - 102 = -15\sqrt{12}x + 8\sqrt{17}y - 68\sqrt{17}$

$(68 + 15\sqrt{17})x + (17 - 8\sqrt{17})y - 102 + 68\sqrt{17} = 0$

$\frac{4x_1 + y_1 - 6}{\sqrt{17}} = -\frac{-15x_1 + 8y_1 - 68}{17}$

$68x + 17y - 102 = 15\sqrt{12}x - 8\sqrt{17}y + 68\sqrt{17}$

$(68 - 15\sqrt{17})x + (17 + 8\sqrt{17})y - 102 - 68\sqrt{17} = 0$

27. $y = \frac{2}{3}x + 1 \rightarrow 2x - 3y + 3 = 0$

$y = -3x - 2 \rightarrow 3x + y + 2 = 0$

$d_1 = \frac{2x_1 - 3y_1 + 3}{-\sqrt{2^2 + (-3)^2}}$ $d_2 = \frac{3x_1 - y_1 + 2}{-\sqrt{3^2 + 1^2}}$

$\frac{2x_1 - 3y_1 + 3}{-\sqrt{13}} = -\frac{3x_1 + y_1 + 2}{-\sqrt{10}}$

$2\sqrt{10}x - 3\sqrt{10}y + 3\sqrt{10} = -3\sqrt{13}x - \sqrt{13}y - 2\sqrt{13}$

$(2\sqrt{10} + 3\sqrt{13})x + (\sqrt{13} - 3\sqrt{10})y + 3\sqrt{10} + 2\sqrt{13} = 0$

$\frac{2x_1 - 3y_1 + 3}{-\sqrt{13}} = \frac{-3x_1 + y_1 + 2}{-\sqrt{10}}$

$-2\sqrt{10}x + 3\sqrt{10}y - 3\sqrt{10} = -3\sqrt{13}x - \sqrt{13}y - 2\sqrt{13}$

$(-2\sqrt{10} + 3\sqrt{13})x + (\sqrt{13} + 3\sqrt{10})y - 3\sqrt{10} + 2\sqrt{13} = 0$

28a. Linda: (19, 112)

$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$d = \frac{4(19) + (-3)(112) + 228}{-\sqrt{4^2 + (-3)^2}}$

$d = \frac{-32}{-5}$ or 6.4

Father: (45, 120)

$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$d = \frac{4(45) + (-3)(120) + 228}{-\sqrt{4^2 + (-3)^2}}$

$d = \frac{48}{-5}$ or −9.6

Linda

28b. $4x - 3y + 228 = 0$

$4x - 3(140) + 228 = 0$

$4x = 192$

$x = 48$

29. Let $x = 1$.

$\tan\theta = \frac{y}{x}$

$\tan 40° = \frac{y}{1}$

$y \approx 0.8390996312$

$m = \frac{0.839 - 0}{1 - 0}$

$m \approx 0.839$

$y - y_1 = m(x - x_1)$

$y - 0.839 \approx 0.839(x - 1)$

$y \approx 0.839x$

$-0.839x + y \approx 0$

$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$d \approx \frac{-0.839(16) + 1(12) + 0}{\sqrt{0.839^2 + 1^2}}$

$d \approx -1.092068438$

1.09 m

30. The radius of the circle is $\sqrt{[(-5) - (-2)]^2 + (6 - 2)^2}$ or 5. Now find the distance from the center of the circle to the line.

$d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$

$d = \frac{5(-5) + (-12)(6) + 32}{-\sqrt{5^2 + (-12)^2}}$

$d = \frac{-65}{-13}$

$d = 5$

Since the distance from the center of the circle to the line is the same as the radius of the circle, the line can only intersect the circle in one point. That is, the line is tangent to the circle.

31. $m_1 = \frac{4-7}{-3-1}$ or $\frac{3}{4}$

$y - 7 = \frac{3}{4}(x - 1)$

$3x - 4y + 25 = 0$

$a_1 = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$a_1 = \frac{3(-1) + (-4)(-3) + 25}{-\sqrt{3^2 + (-4)^2}}$

$a_1 = -\frac{34}{5}$

$m_2 = \frac{-3-4}{-1-(-3)}$ or $-\frac{7}{2}$

$y - 4 = -\frac{7}{2}(x - (-3))$

$7x + 2y + 13 = 0$

$a_2 = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$a_2 = \frac{7(1) + 2(7) + 13}{-\sqrt{7^2 + 2^2}}$

$a_2 = \frac{34}{-\sqrt{53}}$ or $-\frac{34\sqrt{53}}{53}$

$m_3 = \frac{7-(-3)}{1-(-1)}$ or 5

$y - 7 = 5(x - 1)$

$5x - y + 2 = 0$

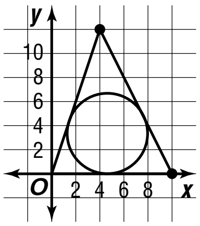
$a_3 = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$a_3 = \frac{5(-3) + (-1)(4) + 2}{-\sqrt{5^2 + (-1)^2}}$

$a_3 = \frac{-17}{-\sqrt{26}}$ or $\frac{17\sqrt{26}}{26}$

$\frac{34}{5}, \frac{34\sqrt{53}}{53}, \frac{17\sqrt{26}}{26}$

32.



The standard form of the equation of the line through (0, 0) and (4, 12) is $3x - y = 0$. The standard form of the equation of the line through (4, 12) and (10, 0) is $2x + y - 20 = 0$. The standard form for the x-axis is $y = 0$. To find the bisector of the angle at the origin, set $\frac{3x - y}{\sqrt{10}} = y$ and solve to obtain $y = \frac{3}{1 + \sqrt{10}}x$. To find the bisector of the angle of the triangle at (10, 0), set $\frac{2x + y - 20}{\sqrt{5}} = -y$ and solve to obtain $2x + (1 + \sqrt{5})y - 20 = 0$. The intersection of these two bisectors is the center of the inscribed circle. To solve the system of equations, substitute $y = \frac{3}{1 + \sqrt{10}}x$ into the equation of the other bisector and solve for x to get $x = \frac{20(1 + \sqrt{10})}{5 + 3\sqrt{5} + 2\sqrt{10}}$. Then $y = \frac{20(1 + \sqrt{10})}{5 + 3\sqrt{5} + 2\sqrt{10}} \cdot \frac{3}{1 + \sqrt{10}} = \frac{60}{5 + 3\sqrt{5} + 2\sqrt{10}}$. This y-coordinate is the inradius of the triangle. The approximate value is 3.33.

33. $-2x + 7y = 5$

$2x - 7y + 5 = 0$

$-\sqrt{A^2 + B^2} = -\sqrt{2^2 + (-7)^2}$ or $-\sqrt{53}$

$\frac{2}{-\sqrt{53}}x - \frac{7}{-\sqrt{53}}y + \frac{5}{-\sqrt{53}} = 0$

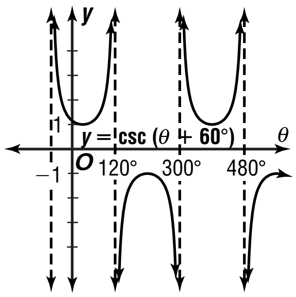
$-\frac{2\sqrt{53}}{53}x + \frac{7\sqrt{53}}{53}y - \frac{5\sqrt{53}}{53} = 0$

34. $\cos 2A = 1 - 2\sin^2 A$

$= 1 - 2\left(\frac{\sqrt{3}}{6}\right)^2$

$= \frac{5}{6}$

35. $\frac{2\pi}{1} = 2\pi, \frac{60°}{1} = 60°$



36. $110 - 3 = 330$ $180° - (60° + 40°) = 80°$

$x^2 = 330^2 + 330^2 - 2(330)(330)\cos 80°$

$x^2 \approx 179979.4269$

$x \approx 424.24$ miles

37. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$T = 2\pi\sqrt{\frac{2}{9.8}}$

$T \approx 2.8$ s

38.

$$\begin{array}{r|rrr} 2 & 1 & 8 & k \\ & & 2 & 20 \\ \hline & 1 & 10 & 20 + k \end{array}$$

$20 + k = 0$

$k = -20$

39. $2x + y - z = -9$
$2(-x + 3y - 2z) = 2(10)$ →

$$\begin{array}{r} 2x + y - z = -9 \\ -2x + 6y - 4z = 20 \\ \hline 7y - 5z = 11 \end{array}$$

$$\begin{array}{r} x - 2y + z = -7 \\ -x + 3y - 2z = 10 \\ \hline y - z = 3 \end{array}$$

$-5(y - z) = -5(3)$
$7y - 5z = 11$ →

$$\begin{array}{r} -5y + 5z = -15 \\ 7y - 5z = 11 \\ \hline 2y = -4 \\ y = -2 \end{array}$$

$y - z = 3$
$-2 - z = 3$
$-5 = z$

$x - 2y + z = -7$
$x - 2(-2) + (-5) = -7$
$x = -6$

$(-6, -2, -5)$

40. square: $A = s^2$
$16 = s^2$
$4 = s$

triangle: $A = \frac{1}{2}bh$
$6 = \frac{1}{2}(4)h$
$3 = h$

$AE = s + h$
$AE = 4 + 3$ or 7
$EF = AE$
$EF = 7$

The correct choice is C.

Chapter 7 Study Guide and Assessment

Page 477 Understanding and Using the Vocabulary

1. b **2.** g **3.** d **4.** a
5. i **6.** j **7.** h **8.** f
9. e **10.** c

Pages 478–480 Skills and Concepts

11. $\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{1}{2}} = 2$

12.
$$\begin{aligned} \tan^2\theta + 1 &= \sec^2\theta \\ 4^2 + 1 &= \sec^2\theta \\ 17 &= \sec^2\theta \\ \sqrt{17} &= \sec\theta \end{aligned}$$

13. $\sin\theta = \frac{1}{\csc\theta} = \frac{1}{\frac{5}{3}} = \frac{3}{5}$

$$\begin{aligned} \sin^2\theta + \cos^2\theta &= 1 \\ \left(\tfrac{3}{5}\right)^2 + \cos^2\theta &= 1 \\ \cos^2\theta &= \tfrac{16}{25} \\ \cos\theta &= \tfrac{4}{5} \end{aligned}$$

14. $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$

$$\begin{aligned} \tan^2\theta + 1 &= \sec^2\theta \\ \tan^2\theta + 1 &= \left(\tfrac{5}{4}\right)^2 \\ \tan^2\theta &= \tfrac{9}{16} \\ \tan\theta &= \tfrac{3}{4} \end{aligned}$$

15.
$$\begin{aligned} \csc x - \cos^2 x \csc x &= \frac{1}{\sin x} - (1 - \sin^2 x)\left(\frac{1}{\sin x}\right) \\ &= \frac{1}{\sin x} - \frac{1}{\sin x} + \sin x \\ &= \sin x \end{aligned}$$

16.
$$\begin{aligned} \cos^2 x + \tan^2 x \cos^2 x &\stackrel{?}{=} 1 \\ \cos^2 x + \frac{\sin^2 x}{\cos^2 x}\cos^2 x &\stackrel{?}{=} 1 \\ \cos^2 x + \sin^2 x &= 1 \\ 1 &= 1 \end{aligned}$$

17.
$$\begin{aligned} \frac{1-\cos\theta}{1+\cos\theta} &\stackrel{?}{=} (\csc\theta - \cot\theta)^2 \\ \frac{1-\cos\theta}{1+\cos\theta} &\stackrel{?}{=} \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 \\ \frac{1-\cos\theta}{1+\cos\theta} &\stackrel{?}{=} \frac{(1-\cos\theta)^2}{\sin^2\theta} \\ \frac{1-\cos\theta}{1+\cos\theta} &\stackrel{?}{=} \frac{(1-\cos\theta)^2}{1-\cos^2\theta} \\ \frac{1-\cos\theta}{1+\cos\theta} &\stackrel{?}{=} \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} \\ \frac{1-\cos\theta}{1+\cos\theta} &\stackrel{?}{=} \frac{1-\cos\theta}{1+\cos\theta} \end{aligned}$$

18.
$$\begin{aligned} \frac{\sec\theta + 1}{\tan\theta} &\stackrel{?}{=} \frac{\tan\theta}{\sec\theta - 1} \\ \frac{\sec\theta + 1}{\tan\theta} &\stackrel{?}{=} \frac{\tan\theta(\sec\theta + 1)}{\sec^2\theta - 1} \\ \frac{\sec\theta + 1}{\tan\theta} &\stackrel{?}{=} \frac{\tan\theta(\sec\theta + 1)}{\tan^2\theta} \\ \frac{\sec\theta + 1}{\tan\theta} &= \frac{\sec\theta + 1}{\tan\theta} \end{aligned}$$

19.
$$\begin{aligned} \frac{\sin^4 x - \cos^4 x}{\sin^2 x} &\stackrel{?}{=} 1 - \cot^2 x \\ \frac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)}{\sin^2 x} &\stackrel{?}{=} 1 - \cot^2 x \\ \frac{\sin^2 x - \cos^2 x}{\sin^2 x} &\stackrel{?}{=} 1 - \cot^2 x \\ 1 - \frac{\cos^2 x}{\sin^2 x} &\stackrel{?}{=} 1 - \cot^2 x \\ 1 - \cot^2 x &= 1 - \cot^2 x \end{aligned}$$

20.
$$\begin{aligned} \cos 195° &= \cos(150° + 45°) \\ &= \cos 150° \cos 45° - \sin 150° \sin 45° \\ &= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \frac{1}{2}\cdot\frac{\sqrt{2}}{2} \\ &= \frac{-\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

21.
$$\begin{aligned} \cos 15° &= \cos(45° - 30°) \\ &= \cos 45° \cos 30° + \sin 45° \sin 30° \\ &= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} \\ &= \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

22.
$$\begin{aligned} \sin\left(-\frac{17\pi}{12}\right) &= -\sin\frac{17\pi}{12} \\ &= -\sin\left(\frac{\pi}{4} + \frac{7\pi}{6}\right) \\ &= -\left(\sin\frac{\pi}{4}\cos\frac{7\pi}{6} + \cos\frac{\pi}{4}\sin\frac{7\pi}{6}\right) \\ &= -\left(\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right)\right) \\ &= -\left(\frac{-\sqrt{6} - \sqrt{2}}{4}\right) \\ &= \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

23.
$$\begin{aligned} \tan\frac{11\pi}{12} &= \tan\left(\frac{2\pi}{3} + \frac{\pi}{4}\right) \\ &= \frac{\tan\frac{2\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{2\pi}{3}\tan\frac{\pi}{4}} \\ &= \frac{-\sqrt{3} + 1}{1 - (-\sqrt{3})(1)} \\ &= \frac{1 - \sqrt{3}}{1 + \sqrt{3}} \\ &= \frac{4 - 2\sqrt{3}}{-2} \text{ or } -2 + \sqrt{3} \end{aligned}$$

24.
$$\begin{aligned} \cos x &= \sqrt{1 - \sin^2 x} \\ &= \sqrt{1 - \left(\frac{7}{25}\right)^2} \\ &= \sqrt{\frac{576}{625}} \text{ or } \frac{24}{25} \end{aligned} \qquad \begin{aligned} \sin y &= \sqrt{1 - \cos^2 x} \\ &= \sqrt{1 - \left(\frac{2}{3}\right)^2} \\ &= \sqrt{\frac{5}{9}} \text{ or } \frac{\sqrt{5}}{3} \end{aligned}$$

$$\begin{aligned} \cos(x - y) &= \cos x \cos y + \sin x \sin y \\ &= \left(\frac{24}{25}\right)\left(\frac{2}{3}\right) + \left(\frac{7}{25}\right)\left(\frac{\sqrt{5}}{3}\right) \\ &= \frac{48 + 7\sqrt{5}}{75} \end{aligned}$$

25. $\cos y = \frac{1}{\sec y}$

$= \frac{1}{\frac{3}{2}}$

$= \frac{2}{3}$

$\sin y = \sqrt{1 - \cos^2 y}$

$= \sqrt{1 - \left(\frac{2}{3}\right)^2}$

$= \sqrt{\frac{5}{9}}$ or $\frac{\sqrt{5}}{3}$

$\tan y = \frac{\sin y}{\cos y}$

$= \frac{\frac{\sqrt{5}}{3}}{\frac{2}{3}}$ or $\frac{\sqrt{5}}{2}$

$$\tan (x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$
$$= \frac{\frac{5}{4} + \frac{\sqrt{5}}{2}}{1 - \left(\frac{5}{4}\right)\left(\frac{\sqrt{5}}{2}\right)}$$
$$= \frac{\frac{5 + 2\sqrt{5}}{4}}{\frac{8 - 5\sqrt{5}}{8}}$$
$$= \frac{10 + 4\sqrt{5}}{8 - 5\sqrt{5}}$$
$$= \frac{180 + 82\sqrt{5}}{-61} \text{ or } -\frac{180 + 25\sqrt{5}}{61}$$

26. $\cos 75° = \cos \frac{150°}{2}$

$= \sqrt{\frac{1 + \cos 150°}{2}}$ (Quadrant I)

$= \sqrt{\frac{1 + \left(-\frac{\sqrt{3}}{2}\right)}{2}}$

$= \frac{\sqrt{2 - \sqrt{3}}}{2}$

27. $\sin \frac{7\pi}{8} = \sin \frac{\frac{7\pi}{4}}{2}$

$= \sqrt{\frac{1 - \cos \frac{7\pi}{4}}{2}}$ (Quadrant II)

$= \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}}$

$= \frac{\sqrt{2 - \sqrt{2}}}{2}$

28. $\sin 22.5° = \sin \frac{45°}{2}$

$= \sqrt{\frac{1 - \cos 45°}{2}}$ (Quadrant I)

$= \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}}$

$= \frac{\sqrt{2 - \sqrt{2}}}{2}$

29. $\tan \frac{\pi}{12} = \tan \frac{\frac{\pi}{6}}{2}$

$= \sqrt{\frac{1 - \cos \frac{\pi}{6}}{1 + \cos \frac{\pi}{6}}}$ (Quadrant I)

$= \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{1 + \frac{\sqrt{3}}{2}}}$

$= \sqrt{\frac{2 - \sqrt{3}}{2 + \sqrt{3}}}$

$= \sqrt{\frac{(2 - \sqrt{3})(2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})}}$

$= \sqrt{\frac{(2 - \sqrt{3})^2}{4 - 3}}$

$= 2 - \sqrt{3}$

30. $\sin^2 \theta + \cos^2 \theta = 1$

$\sin^2 \theta + \left(\frac{3}{5}\right)^2 = 1$

$\sin^2 \theta = \frac{16}{25}$

$\sin \theta = \frac{4}{5}$

$\sin 2\theta = 2 \sin \theta \cos \theta$

$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$

$= \frac{24}{25}$

31. $\cos 2\theta = 2 \cos^2 \theta - 1$

$= 2\left(\frac{3}{5}\right)^2 - 1$

$= -\frac{7}{25}$

32. $\tan \theta = \frac{\sin \theta}{\cos \theta}$

$= \frac{\frac{4}{5}}{\frac{3}{5}}$ or $\frac{4}{3}$

$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

$= \frac{2\left(\frac{4}{3}\right)}{1 - \left(\frac{4}{3}\right)^2}$

$= -\frac{24}{7}$

33. $\sin 4\theta = \sin 2(2\theta)$

$= 2 \sin 2\theta \cos 2\theta$

$= 2\left(\frac{24}{25}\right)\left(-\frac{7}{25}\right)$

$= -\frac{336}{625}$

34.
$$\tan x + 1 = \sec x$$
$$(\tan x + 1)^2 = \sec^2 x$$
$$\tan^2 x + 2 \tan x + 1 = \tan^2 x + 1$$
$$2 \tan x = 0$$
$$\tan x = 0$$
$$x = 0°$$

35.
$$\sin^2 x + \cos 2x - \cos x = 0$$
$$1 - \cos^2 x + 2 \cos^2 x - 1 - \cos x = 0$$
$$\cos^2 x - \cos x = 0$$
$$\cos x (\cos x - 1) = 0$$

$\cos x = 0$ or $\cos x - 1 = 0$

$x = 90°$ or $x = 270°$

$\cos x = 1$

$x = 0°$

36. $\cos 2x + \sin x = 1$

$1 - 2\sin^2 x + \sin x = 1$

$2\sin^2 x - \sin x = 0$

$\sin x(2\sin x - 1) = 0$

$\sin x = 0$ or $2\sin x - 1 = 0$

$x = 0°$ or $x = 180°$; $\sin x = \frac{1}{2}$

$x = 30°$ or $x = 150°$

37. $\sin x \tan x - \frac{\sqrt{2}}{2}\tan x = 0$

$\tan x\left(\sin x - \frac{\sqrt{2}}{2}\right) = 0$

$\tan x = 0$ or $\sin x - \frac{\sqrt{2}}{2} = 0$

$x = \pi k$; $\sin x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4} + 2\pi k$ or $\frac{3\pi}{4} + 2\pi k$

38. $\sin 2x + \sin x = 0$

$2\sin x\cos x + \sin x = 0$

$\sin x(2\cos x + 1) = 0$

$\sin x = 0$ or $2\cos x + 1 = 0$

$x = \pi k$; $\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3} + 2\pi k$

or $x = \frac{4\pi}{3} + 2\pi k$

39. $\cos^2 x = 2 - \cos x$

$\cos^2 x + \cos x - 2 = 0$

$(\cos x - 1)(\cos x + 2) = 0$

$\cos x - 1 = 0$ or $\cos x + 2 = 0$

$\cos x = 1$; $\cos x = -2$

$x = 2\pi k$; no solution

40. $x\cos\phi + y\sin\phi - p = 0$

$x\cos\frac{\pi}{3} + y\sin\frac{\pi}{3} - 2\sqrt{3} = 0$

$\frac{1}{2}x + \frac{\sqrt{3}}{2}y - 2\sqrt{3} = 0$

$x + \sqrt{3}y - 4\sqrt{3} = 0$

41. $x\cos\phi + y\sin\phi - p = 0$

$x\cos 90° + y\sin 90° - 5 = 0$

$0x + 1y - 5 = 0$

$y - 5 = 0$

42. $x\cos\phi + y\sin\phi - p = 0$

$x\cos\frac{2\pi}{3} + y\sin\frac{2\pi}{3} - 3 = 0$

$-\frac{1}{2}x + \frac{\sqrt{3}}{2}y - 3 = 0$

$-x + \sqrt{3}y - 6 = 0$

43. $x\cos\phi + y\sin\phi - p = 0$

$x\cos 225° + y\sin 225° - 4\sqrt{2} = 0$

$-\frac{\sqrt{2}}{2}x + \left(-\frac{\sqrt{2}}{2}\right)y - 4\sqrt{2} = 0$

$x + y + 8 = 0$

44. $\sqrt{A^2 + B^2} = \sqrt{7^2 + 3^2}$ or $\sqrt{58}$

$\frac{7}{\sqrt{58}}x + \frac{3}{\sqrt{58}}y - \frac{8}{\sqrt{58}} = 0$

$\frac{7\sqrt{58}}{58}x + \frac{3\sqrt{58}}{58}y - \frac{4\sqrt{58}}{29} = 0$

$\sin\phi = \frac{3\sqrt{58}}{58}$, $\cos\phi = \frac{7\sqrt{58}}{58}$, $p = \frac{4\sqrt{58}}{29}$; Quadrant I

$\tan\phi = \dfrac{\frac{3\sqrt{58}}{58}}{\frac{7\sqrt{58}}{58}}$ or $\frac{3}{7}$

$\phi \approx 23°$

45. $6x = 4y - 5$

$6x - 4y + 5 = 0$

$-\sqrt{A^2 + B^2} = -\sqrt{6^2 + (-4)^2}$ or $-2\sqrt{13}$

$\frac{6}{-2\sqrt{13}}x - \frac{4}{-2\sqrt{13}}y + \frac{5}{-2\sqrt{13}} = 0$

$-\frac{3\sqrt{13}}{13}x + \frac{2\sqrt{13}}{13}y - \frac{5\sqrt{13}}{26} = 0$

$\sin\phi = \frac{2\sqrt{13}}{13}$, $\cos\phi = -\frac{3\sqrt{13}}{13}$, $p = \frac{5\sqrt{13}}{26}$; Quadrant II

$\tan\phi = \dfrac{\frac{2\sqrt{13}}{13}}{-\frac{3\sqrt{13}}{13}}$ or $-\frac{2}{3}$

$\phi \approx 146°$

46. $9x = -5y + 3$

$9x + 5y - 3 = 0$

$\sqrt{A^2 + B^2} = \sqrt{9^2 + 5^2}$ or $\sqrt{106}$

$\frac{9}{\sqrt{106}}x + \frac{5}{\sqrt{106}}y - \frac{3}{\sqrt{106}} = 0$

$\frac{9\sqrt{106}}{106}x + \frac{5\sqrt{106}}{106}y - \frac{3\sqrt{106}}{106} = 0$

$\sin\phi = \frac{5\sqrt{106}}{106}$, $\cos\phi = \frac{9\sqrt{106}}{106}$, $p = \frac{3\sqrt{106}}{106}$; Quadrant I

$\tan\phi = \dfrac{106}{\frac{9\sqrt{106}}{106}}$ or $\frac{5}{9}$

$\phi \approx 29°$

47. $x - 7y = -5$

$x - 7y + 5 = 0$

$-\sqrt{A^2 + B^2} = -\sqrt{1^2 + (-7)^2}$ or $-5\sqrt{2}$

$\frac{1}{-5\sqrt{2}}x - \frac{7}{-5\sqrt{2}}y + \frac{5}{-5\sqrt{2}} = 0$

$-\frac{\sqrt{2}}{10}x + \frac{7\sqrt{2}}{10}y - \frac{\sqrt{2}}{2} = 0$

$\sin\phi = \frac{7\sqrt{2}}{10}$, $\cos\phi = -\frac{\sqrt{2}}{10}$, $p = \frac{\sqrt{2}}{2}$; Quadrant II

$\tan\phi = \dfrac{\frac{7\sqrt{2}}{10}}{-\frac{\sqrt{2}}{10}}$ or -7

$\phi \approx 98°$

48. $d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$d = \frac{2(5) + (-3)(6) + 2}{-\sqrt{2^2 + (-3)^2}}$

$d = \frac{-6}{-\sqrt{13}}$ or $\frac{6\sqrt{13}}{13}$

49. $2y = -3x + 6 \rightarrow 3x + 2y - 6 = 0$

$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$d = \frac{3(-3) + 2(-4) + (-6)}{\sqrt{3^2 + 2^2}}$

$d = \frac{-23}{\sqrt{13}}$ or $-\frac{23\sqrt{13}}{13}$

$\frac{23\sqrt{13}}{13}$

50. $4y = 3x - 1 \rightarrow 3x - 4y - 1 = 0$

$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$d = \frac{3(-2) + (-4)(4) + (-1)}{\sqrt{3^2 + (-4)^2}}$

$d = \frac{-23}{5}$ or $-\frac{23}{5}$

$\frac{23}{5}$

51. $y = \frac{1}{3}x + 6 \rightarrow x - 3y + 18 = 0$

$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$d = \frac{1(21) + (-3)(20) + 18}{-\sqrt{1^2 + (-3)^2}}$

$d = \frac{-21}{-\sqrt{10}}$ or $\frac{21\sqrt{10}}{10}$

52. $y = \frac{x}{3} - 6$ Use $(0, -6)$.

$y = \frac{x}{3} + 2 \rightarrow x - 3y + 6 = 0$

$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$d = \frac{1(0) + (-3)(-6) + 6}{-\sqrt{1^2 + (-3)^2}}$

$d = \frac{24}{-\sqrt{10}}$ or $-\frac{12\sqrt{10}}{5}$

$d = \frac{12\sqrt{10}}{5}$

53. $y = \frac{3}{4}x + 3$ Use $(0, 3)$.

$y = \frac{3}{4}x - \frac{1}{2} \rightarrow 3x - 4y - 2 = 0$

$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$d = \frac{3(0) + (-4)(3) + (-2)}{\sqrt{3^2 + (-4)^2}}$

$d = \frac{-14}{5}$ or $-\frac{14}{5}$

$d = \frac{14}{5}$

54. $x + y = 1$ Use $(0, 1)$.

$x + y = 5 \rightarrow x + y - 5 = 0$

$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$d = \frac{1(0) + 1(1) + (-5)}{\sqrt{1^2 + 1^2}}$

$d = \frac{-4}{\sqrt{2}}$ or $-2\sqrt{2}$

$d = 2\sqrt{2}$

55. $y = \frac{2}{3}x - 2$ Use $(0, -2)$.

$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$d = \frac{2(0) + (-3)(-2) + 3}{\sqrt{2^2 + (-3)^2}}$

$d = \frac{9}{\sqrt{13}}$ or $\frac{9\sqrt{13}}{13}$

56. $y = -3x - 2 \rightarrow 3x + y + 2 = 0$

$y = -\frac{x}{2} + \frac{3}{2} \rightarrow x + 2y - 3 = 0$

$d_1 = \frac{3x_1 + y_1 + 2}{-\sqrt{3^2 + 1^2}}$ $\quad d_2 = \frac{x_1 + 2y_1 - 3}{\sqrt{1^2 + 2^2}}$

$\frac{3x_1 + y_1 + 2}{-\sqrt{10}} = \frac{x_1 + 2y_1 - 3}{\sqrt{5}}$

$3\sqrt{5}x + \sqrt{5}y + 2\sqrt{5} = -\sqrt{10}x - 2\sqrt{10}y + 3\sqrt{10}$

$(3\sqrt{5} + \sqrt{10})x + (\sqrt{5} + 2\sqrt{10})y + 2\sqrt{5} - 3\sqrt{10} = 0$

$\frac{3x_1 + y_1 + 2}{-\sqrt{10}} = -\frac{x_1 + 2y_1 - 3}{\sqrt{5}}$

$3\sqrt{5}x + \sqrt{5}y + 2\sqrt{5} = \sqrt{10}x + 2\sqrt{10}y - 3\sqrt{10}$

$(3\sqrt{5} - \sqrt{10})x + (\sqrt{5} - 2\sqrt{10})y + 2\sqrt{5} + 3\sqrt{10} = 0$

57. $-x + 3y - 2 = 0$

$y = \frac{3}{5}x + 3 \rightarrow 3x - 5y + 15 = 0$

$d_1 = \frac{-x_1 + 3y_1 - 2}{\sqrt{(-1)^2 + 3^2}}$ $\quad d_2 = \frac{3x_1 - 5y_1 + 15}{-\sqrt{3^2 + (-5)^2}}$

$\frac{-x_1 + 3y_1 - 2}{\sqrt{10}} = -\frac{3x_1 - 5y_1 + 15}{-\sqrt{34}}$

$-\sqrt{34}x + 3\sqrt{34}y - 2\sqrt{34} = 3\sqrt{10}x - 5\sqrt{10}y + 15\sqrt{10}$

$(-\sqrt{34} - 3\sqrt{10})x + (3\sqrt{34} + 5\sqrt{10})y - 2\sqrt{34} - 15\sqrt{10} = 0$

$\frac{-x_1 + 3y_1 - 2}{\sqrt{10}} = \frac{3x_1 - 5y_1 + 15}{-\sqrt{34}}$

$3\sqrt{10}x - 5\sqrt{10}y + 15\sqrt{10} = \sqrt{34}x - 3\sqrt{34}y + 2\sqrt{34}$

$(-\sqrt{34} + 3\sqrt{10})x + (3\sqrt{34} - 5\sqrt{10})y - 2\sqrt{34} + 15\sqrt{10} = 0$

Page 481 Applications and Problem Solving

58. The formulas are equivalent.

$$\frac{v_0^2 \tan^2\theta}{2g\sec^2\theta} = \frac{v_0^2 \cdot \frac{\sin^2\theta}{\cos^2\theta}}{2g \cdot \frac{1}{\cos^2\theta}} = \frac{v_0^2 \cdot \frac{\sin^2\theta}{\cos^2\theta}}{2g \cdot \frac{1}{\cos^2\theta}} \cdot \frac{\cos^2\theta}{\cos^2\theta} = \frac{v_0^2\sin^2\theta}{2g}$$

59. $d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$d = \frac{4(1600) + (-2)(0) + 0}{\sqrt{4^2 + (-2)^2}}$

$d = \frac{6400}{\sqrt{20}}$

$d = 1431$ ft

60. $\sin 30° = \frac{x}{100}$ $\qquad 30° + 45° + \theta = 90°$

$100 \sin 30° = x$ $\qquad \theta = 15°$

$50 = x$

$\cos\theta = \frac{x}{y}$

$\cos 15° = \frac{50}{y}$

$y = \frac{50}{\cos 15°}$

$y \approx 51.76$ yd

Page 481 Open-Ended Assessment

1. Sample answer: 15°; $15° = \frac{30°}{2}$

$\sin\frac{30°}{2} = \sqrt{\frac{1 - \cos 30°}{2}} = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2 - \sqrt{3}}}{2}$

$\cos\frac{30°}{2} = \sqrt{\frac{1 + \cos 30°}{2}} = \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2 + \sqrt{3}}}{2}$

$$\tan \frac{30°}{2} = \sqrt{\frac{1 - \cos 30°}{1 + \cos 30°}}$$
$$= \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{1 + \frac{\sqrt{3}}{2}}}$$
$$= \sqrt{\frac{\frac{2 - \sqrt{3}}{2}}{\frac{2 + \sqrt{3}}{2}}}$$
$$= \sqrt{\frac{(2 - \sqrt{3})(2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})}}$$
$$= \sqrt{\frac{(2 - 3)^2}{4 - 3}}$$
$$= 2 - \sqrt{3}$$

2. Sample answer: $\sin x \tan x = \frac{1 - \cos^2 x}{\cos x}$

$$\sin x \tan x = \frac{1 - \cos^2 x}{\cos x}$$
$$\sin x \frac{\sin x}{\cos x} = \frac{\sin^2 x}{\cos x}$$
$$\frac{\sin^2 x}{\cos x} = \frac{\sin^2 x}{\cos x}$$

SAT & ACT Preparation

Page 483 SAT and ACT Practice

1. The problem states that the measure of $\angle A$ is 80°. Since the measure of $\angle B$ is half the measure of $\angle A$, the measure of $\angle B$ must be 40°. Because $\angle A$, $\angle B$, and $\angle C$ are interior angles of a triangle, the sum of their measures must equal 180°.

$$m\angle A + m\angle B + m\angle C = 180$$
$$80 + 40 + m\angle C = 180$$
$$120 + m\angle C = 180$$
$$m\angle C = 60$$

The correct choice is B.

2. To find the point of intersection, you need to solve a system of two linear equations. Substitution or elimination by addition or subtraction can be used to solve a system of equations. To solve this system of equations, use substitution. Substitute $2x - 2$ for y in the second equation.

$$7x - 3y = 11$$
$$7x - 3(2x - 2) = 11$$
$$7x - 6x + 6 = 11$$
$$x = 5$$

Then use this value for x to calculate the value for y.

$y = 2x - 2$
$y = 2(5) - 2$ or 8

The point of intersection is (5, 8). The correct choice is A.

3. One way to solve this problem is to label the three interior angles of the triangle, a, b, and c. Then write equations using these angles and the exterior angles.

$a + b + c = 180$
$x + a = 180$
$y + b = 180$
$z + c = 180$

Add the last three equations.

$x + a + y + b + z + c = 180 + 180 + 180$
$x + y + z + a + b + c = 180 + 180 + 180$

Replace $a + b + c$ with 180.

$x + y + z + 180 = 180 + 180 + 180$
$x + y + z = 180 + 180$ or 360

The correct choice is D.

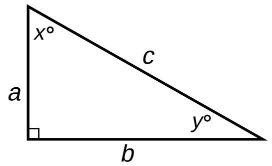
4. Since $x + y = 90°$, $x = 90° - y$.
Then $\sin x = \sin (90° - y)$.
$\sin (90° - y) = \cos y$

$$\frac{\sin x}{\cos y} = \frac{\sin(90° - y)}{\cos y} = \frac{\cos y}{\cos y} = 1$$

The correct choice is D.

Another solution is to draw a diagram and notice that $\sin x = \frac{b}{c}$ and $\cos y = \frac{b}{c}$.

$$\frac{\sin x}{\cos y} = \frac{\frac{b}{c}}{\frac{b}{c}} = 1$$



5. In order to represent the slopes, you need the coordinates of point A. Since A lies on the y-axis, let its coordinates be (0, y). Then calculate the two slopes. The slope of $\overline{AB}$ is $\frac{y - 0}{0 - (-3)} = \frac{y}{3}$. The slope of $\overline{AD}$ is $\frac{y - 0}{0 - 3} = -\frac{y}{3}$. The sum of the slopes is $\frac{y}{3} + \frac{y}{3} = 0$.

The correct choice is B.

6. Since $PQRS$ is a rectangle, its angles measure 90°. The triangles that include the marked angles are right triangles. Write an equation for the measure of $\angle PSR$, using expressions for the unmarked angles on either side of the angle of x°.

$$90 = (90 - a) + x + (90 - b)$$
$$0 = 90 - a - b + x$$
$$a + b = 90 + x$$

The correct choice is A.

7. Simplify the fraction. One method is to multiply both numerator and denominator by $\frac{y^2}{y^2}$.

$$\frac{y - \frac{1}{y}}{1 - \frac{2}{y} + \frac{1}{y^2}} = \frac{y^2}{y^2}\left(\frac{y - \frac{1}{y}}{1 - \frac{2}{y} + \frac{1}{y^2}}\right)$$
$$= \frac{y^3 - y}{y^2 - 2y + 1}$$
$$= \frac{y(y^2 - 1)}{(y - 1)(y - 1)}$$
$$= \frac{y(y - 1)(y + 1)}{(y - 1)(y - 1)}$$
$$= \frac{y^2 + y}{y - 1}$$

Another method is to write both the numerator and denominator as fractions, and then simplify.

$$\frac{y - \frac{1}{y}}{1 - \frac{2}{y} + \frac{1}{y^2}} = \frac{\frac{y^2 - 1}{y}}{\frac{y^2 - 2y + 1}{y^2}}$$
$$= \frac{y^2 - 1}{y}\left(\frac{y^2}{y^2 - 2y + 1}\right)$$
$$= \frac{y(y - 1)(y + 1)}{(y - 1)(y - 1)}$$
$$= \frac{y^2 + y}{y - 1}$$

The correct choice is A.

8. Since the triangles are similar, use a proportion with corresponding sides of the two triangles.

$$\frac{BC}{AC} = \frac{BD}{AE}$$
$$\frac{2}{2 + 3} = \frac{4}{AE}$$
$$2AE = 4(2 + 3)$$
$$AE = 10$$

The correct choice is E.

9. Since the volume V varies directly with the temperature T, the volume and temperature satisfy the equation $V = kT$, where k is a constant. When $V = 12$, $T = 60$. So $12 = 60k$, or $k = \frac{1}{5}$. The relationship is $V = \frac{1}{5}T$.

To find the volume when the temperature is 70°, substitute 70 for T in the equation $V = \frac{1}{5}T$. $V = \frac{1}{5}(70)$ or 14. The volume of the balloon is 14 in^3.

The correct choice is C.

10. Two sides have the same length. The lengths of all sides are integers. The third side is 13. From Triangle Inequality, the sum of the lengths of any two sides must be greater than the length of the third side. Let s be the length of the other two sides. Write and solve an inequality.

$$2s > 13$$
$$s > 6.5$$

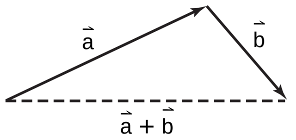
The length of the sides must be greater than 6.5. But the length of the sides must be an integer. The smallest integer greater than 6.5 is 7. The answer is 7. If you answered 6.5, you did not find an integer. If you answered 6, you found a number that is less than 6.5.

Chapter 8 Vectors and Parametric Equations

8-1 Geometric Vectors

Page 490 Check for Understanding

1. Sample answer:



Draw $\vec{a}$. Then draw $\vec{b}$ so that its initial point (tip) is on the terminal point (tail) of $\vec{a}$. Draw a dashed line from the initial point of $\vec{a}$ to the terminal point of $\vec{b}$. The dashed line is the resultant.

2. Sample answer: A vector has magnitude and direction. A line segment has only length. A vector can be represented by a directed line segment.

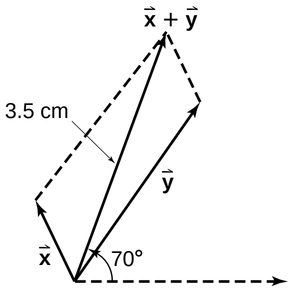
3. Sample answer: the velocities of an airplane and a wind current

4. No, they are opposites.

5-11. Answers may vary slightly.

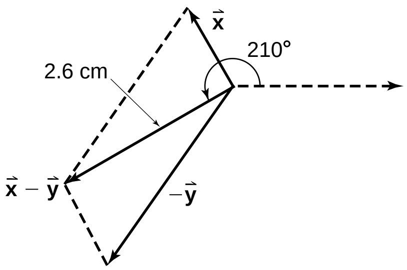
5. 1.2 cm, 120° **6.** 2.9 cm, 55° **7.** 1.4 cm, 20°

8.



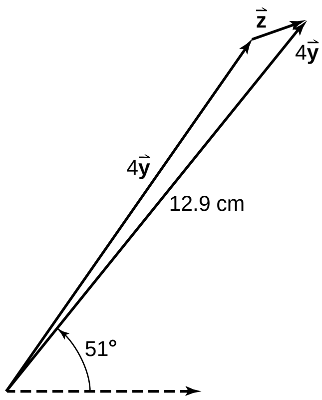
3.5 cm, 70°

9.



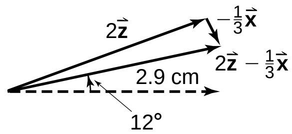
2.6 cm, 210°

10.



12.9 cm, 51°

11.

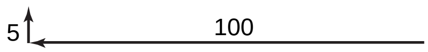


2.9 cm, 12°

12. $h = 2.9 \cos 55°$ $\quad v = 2.9 \sin 55°$

$h \approx 1.66$ cm $\quad v \approx 2.38$ cm

13a.



13b. Use the Pythagorean Theorem.

$c^2 = a^2 + b^2$

$c^2 = (100)^2 + (5)^2$

$c^2 = 10{,}025$

$c = \sqrt{10{,}025}$ or about 100.12 m/s

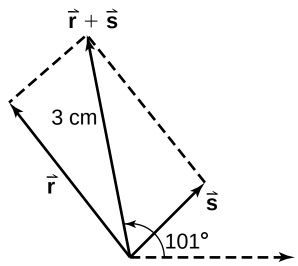
Pages 491–492 Exercises

14. 2.6 cm, 128° **15.** 1.4 cm, 45°

16. 2.1 cm, 14° **17.** 3.0 cm, 340°

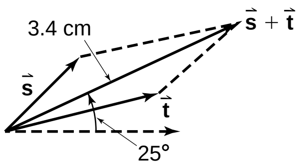
18-30. Answers may vary slightly.

18.



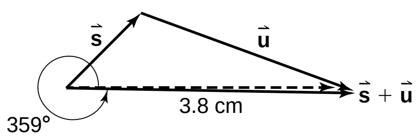
3 cm, 101°

19.



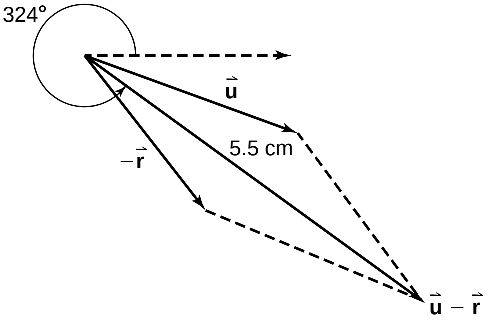
3.4 cm, 25°

20.



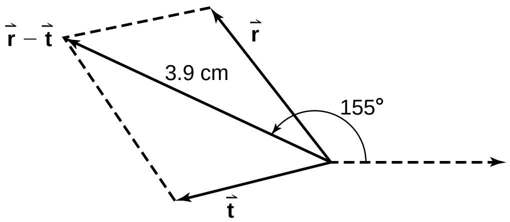
3.8 cm, 359°

21.



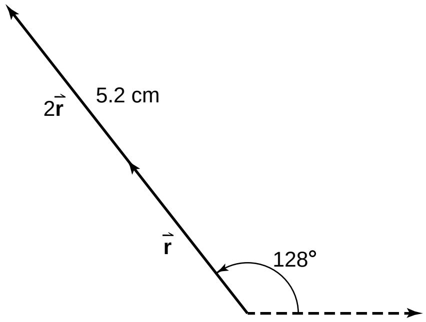
5.5 cm, 324°

22.



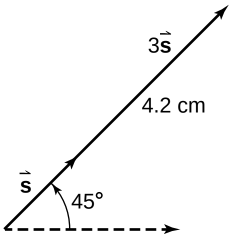
3.9 cm, 155°

23.



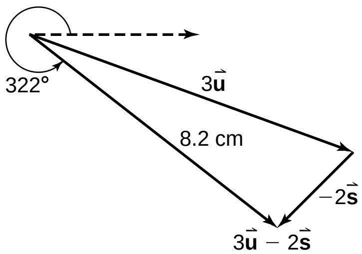
5.2 cm, 128°

24.



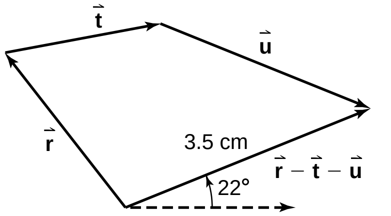
4.2 cm, 45°

25.



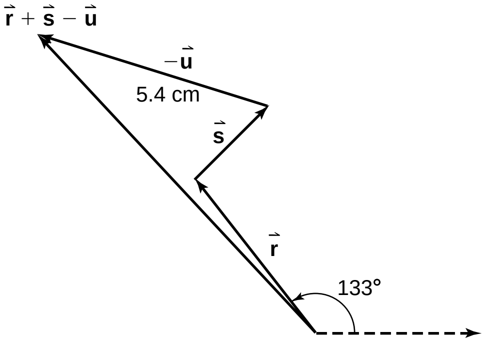
8.2 cm, 322°

26.



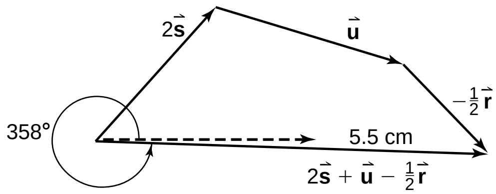
3.5 cm, 22°

27.



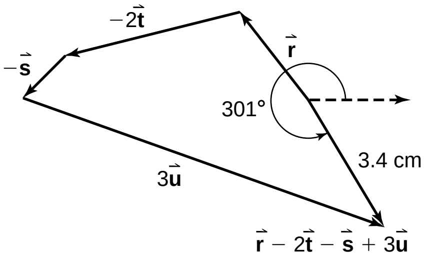
5.4 cm, 133°

28.



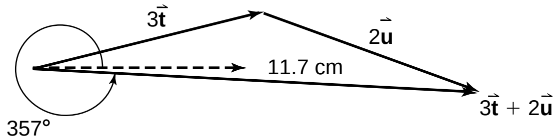
5.5 cm, 358°

29. Draw to scale:



3.4 cm, 301°

30. Draw to scale:



11.7 cm, 357°

31. $h = |2.6 \cos 128°|$ $\quad v = |2.6 \sin 128°|$
$h = 1.60$ cm $\quad v = 2.05$ cm

32. $h = |1.4 \cos 45°|$ $\quad v = |1.4 \sin 45°|$
$h = 0.99$ cm $\quad v = 0.99$ cm

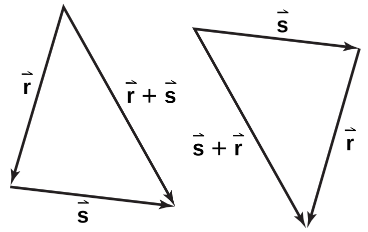
33. $h = |2.1 \cos 14°|$ $\quad v = |2.1 \sin 14°|$
$h = 2.04$ cm $\quad v = 0.51$ cm

34. $h = |3.0 \cos 340°|$ $\quad v = |3.0 \sin 340°|$
$h = 2.82$ cm $\quad v = 1.03$ cm

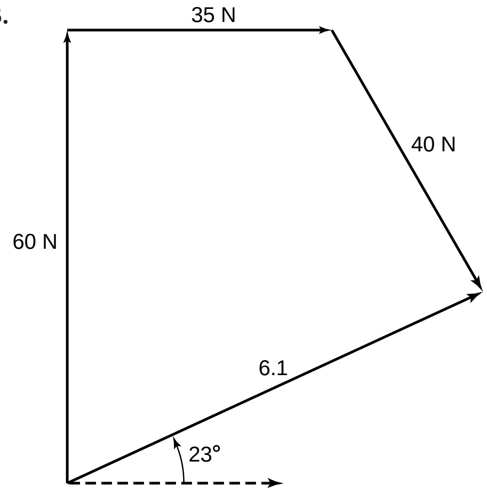
35. $c^2 = a^2 + b^2$
$c^2 = (29.2)^2 + (35.2)^2$
$c^2 = 2091.68$
$c = \sqrt{2091.68}$ or about 45.73 m

36. The difference of the vectors; sample answer: The other diagonal would be the sum of one of the vectors and the opposite of the other vector, so it would be the difference.

37. Yes; sample answer:

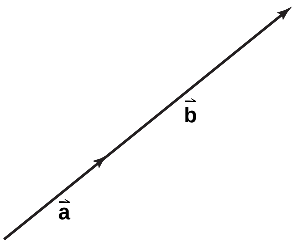


38.

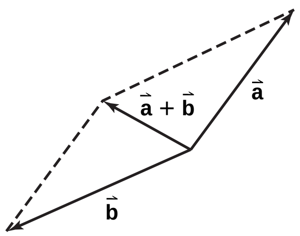


61 N, 23° north of east

39. Sometimes;



$|\vec{a} + \vec{b}| = 5$
$|\vec{a}| + |\vec{b}| = 2.3 + 2.7$
$= 5$



$|\vec{a} + \vec{b}| = 1.5$
$|\vec{a}| + |\vec{b}| = 2.3 + 2.7$
$= 5$

40a. $v = 1.5 \sin 52°$ $\quad h = 1.5 \cos 52°$
$v \approx 1.18$ N $\quad h \approx 0.92$ N

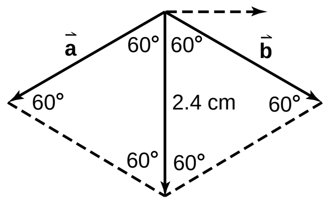
40b. $h = 1.5 \cos 78°$ $\quad v = 1.5 \sin 78°$
$h \approx 0.31$ N $\quad v \approx 1.47$ N

41. $h = 47 \cos 40°$ $\quad v = 47 \sin 40°$
$h \approx 36$ mph $\quad v \approx 30$ mph

42. It is true when $k = 1$ or when $\vec{a}$ is the zero vector.

43. $c^2 = a^2 + b^2$
$c^2 = (50)^2 + (50)^2$
$c = \sqrt{5000}$ or about 71 lb

44.



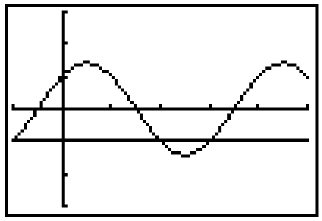
$\vec{a} + \vec{b} = 24$
equilateral triangle $\vec{a} = \vec{b} = 24$ lb

45. The origin is not in the interior of the acute angle.
$d_1 = -d_2$
$d_1 = \frac{x - y + 2}{\sqrt{-1^2 + (-1)^2}}$ or $\frac{x - y + 2}{-\sqrt{2}}$
$d_2 = \frac{y - 5}{\sqrt{0^2 + 1^2}}$ or $y - 5$
$\frac{x - y + 2}{-\sqrt{2}} = -(y - 5)$
$x - y + 2 = \sqrt{2}(y - 5)$
$x - y + 2 = \sqrt{2}y - 5\sqrt{2}$
$x - y + 2 - \sqrt{2}y + 5\sqrt{2} = 0$
$x - (1 + \sqrt{2})y + 2 + 5\sqrt{2} = 0$

46. $\csc \theta \cos \theta \tan \theta = \frac{1}{\sin \theta} \cdot \cos \theta \cdot \frac{\sin \theta}{\cos \theta}$
$= \frac{\sin \theta}{\sin \theta} \cdot \frac{\cos \theta}{\cos \theta}$
$= 1$

47. $\frac{\pi}{4} + \pi n$ where n is an integer

48.



$\left[-\frac{\pi}{2}, \frac{5\pi}{2}\right]$ scl: $\frac{\pi}{2}$ by $[-3, 3]$ scl: 1
$x = \pi, \frac{3\pi}{2}$ for $0 \leq x \leq 2\pi$

49. $\tan 18°29' = \frac{5}{0.5b}$
$b = \frac{5}{0.5 \tan 18°29'}$
$b = 29.9$ cm
$\sin 18°29' = \frac{5}{h}$
$h = \frac{5}{\sin 18°29'}$
$h = 15.8$ cm

50. v_o = volume of original box
v_n = volume of new box
$v_o = \ell_o \times w_o \times h_o$
$= (w + 1) \times w \times 2w$
$= (w + 1)2w^2$
$= 2w^3 + 2w^2$
$v_n = \ell_n \times w_n \times h_n$
$= (w + 2) \times (w + 1) \times (2w + 2)$
$= (w^2 + 3w + 2)(2w + 2)$
$= 2w^3 + 8w^2 + 10w + 4$
$2w^3 + 8w^2 + 10w + 4 = 160$

w	2	8	10	-156
-1	2	6	4	-160
1	2	10	20	-136
2	2	12	34	-88
3	2	14	52	0

$w_o = 3$
$\ell_o = 2w$
$= 2 \cdot 3$ or 6
$h_o = w + 1$
$= 3 + 1$ or 4
So, the dimensions of the original box are 3 ft $\times$ 4 ft $\times$ 6 ft

51. $g(x) = \frac{x + 2}{(x - 1)(x + 3)}$
vertical: As x approaches 1 and -3, the expression approaches $+\infty$ or $-\infty$. So, $x = 1$ and $x = -3$ are vertical asymptotes.
horizontal: $y = \frac{x + 2}{x^2 + 2x - 3}$

$$y = \frac{\frac{x}{x^2} + \frac{2}{x^2}}{\frac{x^2}{x^2} + \frac{2x}{x^2} - \frac{3}{x^2}}$$

$$y = \frac{\frac{1}{x} + \frac{2}{x^2}}{1 + \frac{2}{x} - \frac{3}{x^2}}$$

As x increases positively or negatively, the expression approaches 0. So, $y = 0$ is a horizontal asymptote.

52. Let x, $x + 2$, and $x + 4$ be 3 consecutive odd intergers.
$3x = 2(x + 4) + 3 \qquad x + 4 = 15$
$3x = 2x + 8 + 3$
$3x - 2x = 11$
$x = 11$
The correct answer 15.

8-2 Algebraic Vectors

Pages 496–497 Check for Understanding

1. Sample answer: $\vec{\mathbf{a}} = \langle 8, 6\rangle$, $\vec{\mathbf{b}} = \langle 6, 8\rangle$; equal vectors have the same magnitude and direction.

2. Use $|\overrightarrow{XY}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ and replace the values for x and y.
$x(-5, -6)$, $y(3, -4)$
$|\overrightarrow{XY}| = \sqrt{[3 - (-5)]^2 + [-4 - (-6)]^2}$
$= \sqrt{(8)^2 + (-2)^2}$
$= \sqrt{64 + 4}$ or $\sqrt{68}$

3. Jacqui is correct. The representation is incorrect. $\langle 2, 0\rangle + \langle 0, -5\rangle$ is not equal to $5\langle 1, 0\rangle + (-2)\langle 0, 1\rangle$. The correct expression is $2\vec{\mathbf{i}} - 5\vec{\mathbf{j}}$.

4. $\overrightarrow{MP} = \langle -3 - 2, 4 - (-1)\rangle$ or $\langle -5, 5\rangle$
$|\overrightarrow{MP}| = \sqrt{(-5)^2 + (5)^2}$
$= \sqrt{50}$ or $5\sqrt{2}$ units

5. $\overrightarrow{MP} = \langle 0 - 5, 5 - 6\rangle$ or $\langle -5, -1\rangle$
$|\overrightarrow{MP}| = \sqrt{(-5)^2 + (-1)^2}$
$= \sqrt{26}$ units

6. $\overrightarrow{MP} = \langle 4 - (-19), 0 - 4\rangle$ or $\langle 23, -4\rangle$
$|\overrightarrow{MP}| = \sqrt{(23)^2 + (-4)^2}$
$= \sqrt{545}$ units

7. $\vec{\mathbf{t}} = \vec{\mathbf{u}} + \vec{\mathbf{v}}$
$= \langle -1, 4\rangle + \langle 3, -2\rangle$
$= \langle -1 + 3, 4 + (-2)\rangle$ or $\langle 2, 2\rangle$

8. $\vec{\mathbf{t}} = \frac{1}{2}\vec{\mathbf{u}} - \vec{\mathbf{v}}$
$= \frac{1}{2}\langle -1, 4\rangle - \langle 3, -2\rangle$
$= \langle -\frac{1}{2}, 2\rangle - \langle 3, -2\rangle$
$= \langle -\frac{1}{2} - 3, 2 - (-2)\rangle$ or $\left\langle -3\frac{1}{2}, 4\right\rangle$

9. $\vec{\mathbf{t}} = 4\vec{\mathbf{u}} + 6\vec{\mathbf{v}}$
$= 4\langle -1, 4\rangle + 6\langle 3, -2\rangle$
$= \langle -4, 16\rangle + \langle 18, -12\rangle$
$= \langle -4 + 18, 16 + (-12)\rangle$ or $\langle 14, 4\rangle$

10. $\vec{\mathbf{t}} = -8\vec{\mathbf{u}}$
$= -8\langle -1, 4\rangle$
$= \langle -8(-1), -8(4)\rangle$ or $\langle 8, -32\rangle$

11. $|\langle 8, -6\rangle| = \sqrt{8^2 + (-6)^2}$
$= \sqrt{100}$ or 10
$8\vec{\mathbf{i}} - 6\vec{\mathbf{j}}$

12. $|\langle -7, -5\rangle| = \sqrt{(-7)^2 + (-5)^2}$
$= \sqrt{74}$
$-7\vec{\mathbf{i}} - 5\vec{\mathbf{j}}$

13. Let $\vec{\mathbf{T}}$ represent the force Terrell exerts.
Let $\vec{\mathbf{W}}$ represent the force Mr. Walker exerts.
$|\vec{\mathbf{T}}_x| = 400 \cos 65° \approx 169.05 \qquad |\vec{\mathbf{W}}_x| = 600 \cos 110° \approx -205.21$
$|\vec{\mathbf{T}}_y| = 400 \sin 65° \approx 362.52 \qquad |\vec{\mathbf{W}}_y| = 600 \sin 110° \approx 563.82$
$\vec{\mathbf{T}} = \langle 169.05, 362.52\rangle$, $\vec{\mathbf{W}} = \langle -205.21, 563.82\rangle$
$\vec{\mathbf{T}} + \vec{\mathbf{W}} = \langle -36.16, 926.34\rangle$
$|\vec{\mathbf{T}} + \vec{\mathbf{W}}| = \sqrt{(-36.16)^2 + (926.34)^2}$
≈ 927 N

Pages 497–499 Exercises

14. $\overrightarrow{YZ} = \langle 2 - 4, 8 - 2\rangle$ or $\langle -2, 6\rangle$
$|\overrightarrow{YZ}| = \sqrt{(-2)^2 + 6^2}$
$= \sqrt{40}$ or $2\sqrt{10}$

15. $\overrightarrow{YZ} = \langle -1 - (-5), 2 - 7\rangle$ or $\langle 4, -5\rangle$
$|\overrightarrow{YZ}| = \sqrt{4^2 + (-5)^2}$
$= \sqrt{41}$

16. $\overrightarrow{YZ} = \langle 1 - (-2), 3 - 5\rangle$ or $\langle 3, -2\rangle$
$|\overrightarrow{YZ}| = \sqrt{3^2 + (-2)^2}$
$= \sqrt{13}$

17. $\overrightarrow{YZ} = \langle 0 - 5, -3 - 4\rangle$ or $\langle -5, -7\rangle$
$|\overrightarrow{YZ}| = \sqrt{(-5)^2 + (-7)^2}$
$= \sqrt{74}$

18. $\overrightarrow{YZ} = \langle 0 - 3, 4 - 1\rangle$ or $\langle -3, 3\rangle$
$|\overrightarrow{YZ}| = \sqrt{(-3)^2 + 3^2}$
$= \sqrt{18}$ or $3\sqrt{2}$

19. $\overrightarrow{YZ} = \langle 1 - (-4), 19 - 12\rangle$ or $\langle 5, 7\rangle$
$|\overrightarrow{YZ}| = \sqrt{5^2 + 7^2}$
$= \sqrt{74}$

20. $\overrightarrow{YZ} = \langle 7 - 5, 6 - 0\rangle$ or $\langle 2, 6\rangle$
$|\overrightarrow{YZ}| = \sqrt{2^2 + 6^2}$
$= \sqrt{40}$ or $2\sqrt{10}$

21. $\overrightarrow{YZ} = \langle 23 - 14, -14 - (-23)\rangle$ or $\langle 9, 9\rangle$
$|\overrightarrow{YZ}| = \sqrt{9^2 + 9^2}$
$= \sqrt{162}$ or $9\sqrt{2}$

22. $\overrightarrow{AB} = \langle 36 - 31, -45 - (-33)\rangle$ or $\langle 5, -12\rangle$
$|\overrightarrow{AB}| = \sqrt{5^2 + (-12)^2}$
$= \sqrt{169}$ or 13

23. $\vec{\mathbf{a}} = \vec{\mathbf{b}} + \vec{\mathbf{c}}$
$= \langle 6, 3\rangle + \langle -4, 8\rangle$
$= \langle 6 + (-4), 3 + 8\rangle$ or $\langle 2, 11\rangle$

24. $\vec{\mathbf{a}} = 2\vec{\mathbf{b}} + \vec{\mathbf{c}}$
$= 2\langle 6, 3\rangle + \langle -4, 8\rangle$
$= \langle 12, 6\rangle + \langle -4, 8\rangle$
$= \langle 12 + (-4), 6 + 8\rangle$ or $\langle 8, 14\rangle$

25. $\vec{\mathbf{a}} = \vec{\mathbf{b}} + 2\vec{\mathbf{c}}$
$= \langle 6, 3\rangle + 2\langle -4, 8\rangle$
$= \langle 6, 3\rangle + \langle -8, 16\rangle$
$= \langle 6 + (-8), 3 + 16\rangle$ or $\langle -2, 19\rangle$

26. $\vec{\mathbf{a}} = 2\vec{\mathbf{b}} + 3\vec{\mathbf{c}}$
$= 2\langle 6, 3\rangle + 3\langle -4, 8\rangle$
$= \langle 12, 6\rangle + \langle -12, 24\rangle$
$= \langle 12 + (-12), 6 + 24\rangle$ or $\langle 0, 30\rangle$

27. $\vec{\mathbf{a}} = -\vec{\mathbf{b}} + 4\vec{\mathbf{c}}$
$= -\langle 6, 3\rangle + 4\langle -4, 8\rangle$
$= \langle -6, -3\rangle + \langle -16, 32\rangle$
$= \langle -6 + (-16), -3 + 32\rangle$ or $\langle -22, 29\rangle$

28. $\vec{\mathbf{a}} = \vec{\mathbf{b}} - 2\vec{\mathbf{c}}$
$= \langle 6, 3\rangle - 2\langle -4, 8\rangle$
$= \langle 6, 3\rangle - \langle -8, 16\rangle$
$= \langle 6 - (-8), 3 - (16)\rangle$ or $\langle 14, -13\rangle$

29. $\vec{\mathbf{a}} = 3\vec{\mathbf{b}}$
$= 3\langle 6, 3\rangle$
$= \langle 3 \cdot 6, 3 \cdot 3\rangle$ or $\langle 18, 9\rangle$

30. $\vec{\mathbf{a}} = -\frac{1}{2}\vec{\mathbf{c}}$
$= -\frac{1}{2}\langle -4, 8\rangle$
$= \left\langle -\frac{1}{2} \cdot (-4), -\frac{1}{2} \cdot 8\right\rangle$ or $\langle 2, -4\rangle$

31. $\vec{\mathbf{a}} = 6\vec{\mathbf{b}} + 4\vec{\mathbf{c}}$
$= 6\langle 6, 3\rangle + 4\langle -4, 8\rangle$
$= \langle 36, 18\rangle + \langle -16, 32\rangle$
$= \langle 36 + (-16), 18 + 32\rangle$ or $\langle 20, 50\rangle$

32. $\vec{\mathbf{a}} = 0.4\vec{\mathbf{b}} - 1.2\vec{\mathbf{c}}$
$= 0.4\langle 6, 3\rangle - 1.2\langle -4, 8\rangle$
$= \langle 2.4, 1.2\rangle - \langle -4.8, 9.6\rangle$
$= \langle 2.4 - (-4.8), 1.2 - 9.6\rangle$ or $\langle 7.2, -8.4\rangle$

33. $\vec{\mathbf{a}} = \frac{1}{3}(2\vec{\mathbf{b}} - 5\vec{\mathbf{c}})$
$= \frac{1}{3}(2\langle 6, 3\rangle - 5\langle -4, 8\rangle)$
$= \frac{1}{3}(\langle 12, 6\rangle - \langle -20, 40\rangle)$
$= \frac{1}{3}(\langle 12 - (-20), 6 - 40\rangle)$
$= \frac{1}{3}\langle 32, -34\rangle$ or $\left\langle \frac{32}{3}, -\frac{34}{3}\right\rangle$

34. $\vec{\mathbf{a}} = (3\vec{\mathbf{b}} + \vec{\mathbf{c}}) + 5\vec{\mathbf{b}}$
$= 3\langle 6, 3\rangle + \langle -4, 8\rangle + 5\langle 6, 3\rangle$
$= \langle 18, 9\rangle + \langle -4, 8\rangle + \langle 30, 15\rangle$
$= \langle 18 + (-4) + 30, 9 + 8 + 15\rangle$ or $\langle 44, 32\rangle$

35. $3\vec{\mathbf{m}} - 2.5\vec{\mathbf{n}} = 3\langle -5, -6\rangle - 2.5\langle 6 - 9\rangle$
$= [-15, -18\rangle - \langle 15, -22.5\rangle$
$= [-15 - (15), -18 - (-22.5\rangle$
$= \langle -30, 4.5\rangle$

36. $|\langle 3, 4\rangle| = \sqrt{3^2 + 4^2}$
$= \sqrt{25}$ or 5
$3\vec{\mathbf{i}} + 4\vec{\mathbf{j}}$

37. $|\langle 2, -3\rangle| = \sqrt{2^2 + (-3)^2}$
$= \sqrt{13}$
$2\vec{\mathbf{i}} - 3\vec{\mathbf{j}}$

38. $|\langle -6, -11\rangle| = \sqrt{(-6)^2 + (-11)^2}$
$= \sqrt{157}$
$-6\vec{\mathbf{i}} - 11\vec{\mathbf{j}}$

39. $|\langle 3.5, 12\rangle| = \sqrt{(3.5)^2 + 12^2}$
$= \sqrt{156.25}$ or 12.5
$3.5\vec{\mathbf{i}} + 12\vec{\mathbf{j}}$

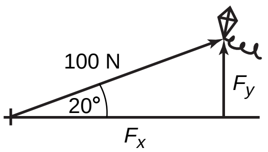
40. $|\langle -4, 1\rangle| = \sqrt{(-4)^2 + 1^2}$
$= \sqrt{17}$
$-4\vec{\mathbf{i}} + \vec{\mathbf{j}}$

41. $|\langle -16, -34\rangle| = \sqrt{(-16)^2 + (-34)^2}$
$= \sqrt{1412}$ or $2\sqrt{353}$
$-16\vec{\mathbf{i}} - 34\vec{\mathbf{j}}$

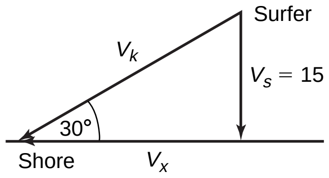
42. $\overrightarrow{ST} = \langle -4 - (-9), -3 - 2\rangle$ or $\langle 5, -5\rangle$
$5\vec{\mathbf{i}} - 5\vec{\mathbf{j}}$

43. Student needs to show that
$(\vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_2) + \vec{\mathbf{v}}_3 = \vec{\mathbf{v}}_1 + (\vec{\mathbf{v}}_2 + \vec{\mathbf{v}}_3)$
$(\vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_2) + \vec{\mathbf{v}}_3 = [\langle a, b\rangle + \langle c, d\rangle] + \langle e, f\rangle$
$= \langle a + c, b + d\rangle + \langle e, f\rangle$
$= \langle (a + c) + e, (b + d) + f\rangle$
$= \langle a + (c + e), b + (d + f)\rangle$
$= \langle a, b\rangle + \langle c + e, d + f\rangle$
$= \langle a, b\rangle + [\langle c, d\rangle + \langle e, f\rangle]$
$= \vec{\mathbf{v}}_1 + (\vec{\mathbf{v}}_2 + \vec{\mathbf{v}}_3)$

44a.



44b. $\sin 20° = \frac{|\vec{\mathbf{F}}_y|}{100}$
$|\vec{\mathbf{F}}_y| = 100 \sin 20°$
≈ 34 N

45a. 

45b. $\sin 30° = \frac{15}{|\vec{\mathbf{v}}_k|}$

$|\vec{\mathbf{V}}_k| = \frac{15}{\sin 30°}$

≈ 30 mph

46a. Since $\overrightarrow{QR} + \overrightarrow{ST} = 0$, $\overrightarrow{QR} = -\overrightarrow{ST}$.
So, they are opposites.

46b. $\overrightarrow{QR}$ and $\overrightarrow{ST}$ have the same magnitude, but opposite direction. So, they are parallel. Quadrilateral $QRST$ is a parallelogram.

47a. $t = \frac{d}{r}$

$= \frac{150 \text{ m}}{5 \text{ m/s}}$ or 30 s

47b. $d = rt$

$= (1.0 \text{ m/s})(30 \text{ s})$ or 30 m

47c. $|\vec{V}_B + \vec{V}_C| = |\langle 0.5 \rangle + \langle 1.0 \rangle|$

$= \sqrt{1^2 + 5^2}$

$= \sqrt{26}$ or about 5.1 m/s

48. $\cos\theta = \frac{(x_2 - x_1)}{|\vec{\mathbf{v}}|} \rightarrow (x_2 - x_1) = \vec{\mathbf{v}} \cos\theta$

$\sin\theta = \frac{(y_2 - y_1)}{|\vec{\mathbf{v}}|} \rightarrow (y_2 - y_1) = \vec{\mathbf{v}} \sin\theta$

49. $\overrightarrow{PQ} = \langle -2 - 8, 5 - (-7) \rangle$

$= \langle -10, 12 \rangle$

$|\overrightarrow{PQ}| = \sqrt{(-10)^2 + 12^2}$

$= \sqrt{244}$

$\overrightarrow{RS} = \langle 7 - 8, 0 - (-7) \rangle$

$= \langle -1, 7 \rangle$

$|\overrightarrow{RS}| = \sqrt{(-1)^2 + 7^2}$

$= \sqrt{50}$

none

50. $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

$d = \frac{|3(-1) - 7(4) - 1|}{\sqrt{3^2 + (-7)^2}}$

$d = \frac{32}{\sqrt{58}}$ or about 4.2

51. $\sin 255° = \sin(225° + 30°)$

$= \sin 225° \cos 30° + \cos 225° \sin 30°$

$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \left(-\frac{\sqrt{2}}{2}\right)\frac{1}{2}$

$= -\frac{\sqrt{6} - \sqrt{2}}{4}$

52. $y = A \sin(kx + c)$

A: $|A| = 17$

$A = 17$ or -17

k: $\frac{2\pi}{k} = \frac{\pi}{4}$

$k = 8$

c: $-\frac{c}{k} = -60°$

$-\frac{c}{8} = -60°$

$c = 480°$

$y = \pm 17 \sin(8x + 480°)$

53. Let $a = 400$, $b = 600$, $C = 46.\overline{3}°$

$c^2 = 400^2 + 600^2 - 2(400)(600)\cos 46.\overline{3}°$

$c^2 \approx 18.8578.39$

$c \approx 434$

$P = a + b + c$

$\approx 400 + 600 + 434$

≈ 1434 ft

$s = \frac{1}{2}(a + b + c)$

$s \approx \frac{1}{2}(1434)$ or 717

$k \approx \sqrt{s(s - a)(s - b)(s - c)}$

$k \approx \sqrt{717(717 - 400)(717 - 600)(717 - 434)}$

$k \approx \sqrt{7{,}525{,}766{,}079}$

$k \approx 86{,}751$ sq ft

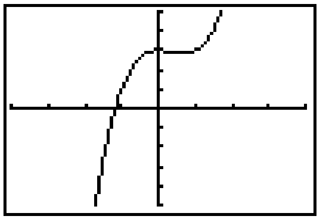
54. Sample answer:

$f(x) = 3x^2 - 2x + 1$			
r	3	-2	1
1	3	1	2
2	3	4	9

An upper bound is 2.

$f(-x) = 3x^2 - 2x + 1$			
r	3	2	1
1	3	5	6
2	3	8	17

A lower bound is -1.

55. 

$[-4, 4]$ scl: 1 by $[-4, 4]$ scl: 1
max: (0, 3), min: (0.67, 2.85)

56.

$f(x) = x^2 + 3x + 1$	
x	$f(x)$
$-10{,}000$	99,970,001
-1000	997,001
-100	9701
-10	71
0	1
10	131
100	10,301
1000	1,003,001
10,000	100,030,001

$y \rightarrow \infty$ as $x \rightarrow \infty$, $y \rightarrow \infty$ as $x \rightarrow -\infty$

57. $7x + 1 > 7x - 1$

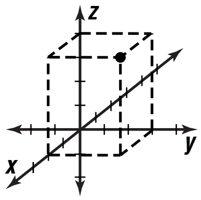
$1 > -1$

This statement is true regardless of the value of x, so it is true for all real values of x.
The correct choice is A.

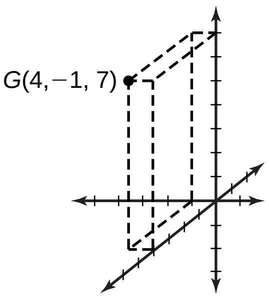
8-3 Vectors in Three-Dimensional Space

Pages 502–503 Check for Understanding

1. Sample answer: sketch a coordinate system with the xy-axes on the horizontal, and the z-axis pointing up. Then, vector $2\vec{\mathbf{i}}$ is two units along the x-axis, vector $3\vec{\mathbf{j}}$ is three units along the y-axis, and vector $4\vec{\mathbf{k}}$ is four units along the z-axis. Draw broken lines to represent three planes.



2. Sample answer: To find the components of the vector, you will need the direction (angle) with the horizontal axis. Using trigonometry, you can obtain the components of the vector.
3. Sample answer: Neither is correct. The sign for the $\vec{\mathbf{j}}$-term must be the same (−), and the coefficient for the $\vec{\mathbf{k}}$-term is 0, so the correct way to express the vector as a sum of unit vectors is $\vec{\mathbf{i}} - 4\vec{\mathbf{j}}$.
4.



$$\left|\overrightarrow{OG}\right| = \sqrt{4^2 + (-1)^2 + 7^2} = \sqrt{66}$$

5. $\overrightarrow{RS} = \langle 3 - (-2), 9 - 5, -3 - 8\rangle$ or $\langle 5, 4, -11\rangle$

$\left|\overrightarrow{RS}\right| = \sqrt{5^2 + 4^2 + (-11)^2} = \sqrt{162}$ or $9\sqrt{2}$

6. $\overrightarrow{RS} = \langle 10 - 3, -4 - 7, 0 - (-1)\rangle$ or $\langle 7, -11, 1\rangle$

$\left|\overrightarrow{RS}\right| = \sqrt{7^2 + (-11)^2 + 1^2} = \sqrt{171}$ or $3\sqrt{19}$

7. $\vec{\mathbf{a}} = 3\vec{\mathbf{f}} + \vec{\mathbf{g}}$
$= 3\langle 1, -3, -8\rangle + \langle 3, 9, -1\rangle$
$= \langle 3, -9, -24\rangle + \langle 3, 9, -1\rangle$
$= \langle 3 + 3, -9 + 9, -24 + (-1)\rangle$ or $\langle 6, 0, -25\rangle$

8. $\vec{\mathbf{a}} = 2\vec{\mathbf{g}} - 5\vec{\mathbf{f}}$
$= 2\langle 3, 9, -1\rangle - 5\langle 1, -3, -8\rangle$
$= \langle 6, 18, -2\rangle - \langle 5, -15, -40\rangle$
$= \langle 6 - 5, 18 - (-15), -2 - (-40)\rangle$ or $\langle 1, 33, 38\rangle$

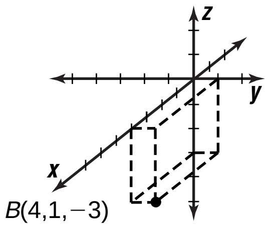
9. $\overrightarrow{EF} = \langle 6 - (-5), -6 - (-2), 6 - 4\rangle$
$= \langle 11, -4, 2\rangle$
$11\vec{\mathbf{i}} - 4\vec{\mathbf{j}} + 2\vec{\mathbf{k}}$

10. $\overrightarrow{EF} = \langle -12 - (-12), 17 - 15, -22 - (-9)\rangle$
$= \langle 0, 2, -13\rangle$
$2\vec{\mathbf{j}} - 13\vec{\mathbf{k}}$

11. $\left|\langle 132, 3454, 0\rangle\right| = \sqrt{132^2 + 3454^2 + 0^2}$
$= \sqrt{11{,}947{,}540}$
≈ 3457 N

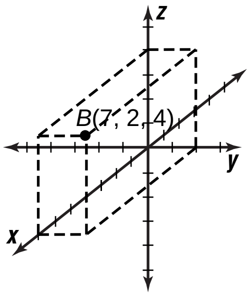
Pages 503–504 Exercises

12.



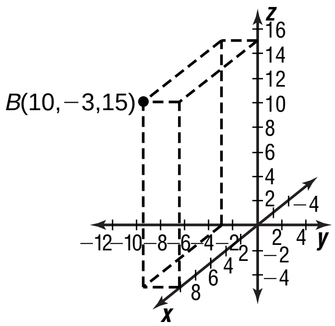
$$\left|\overrightarrow{OB}\right| = \sqrt{4^2 + 1^2 + (-3)^2} = \sqrt{26}$$

13.



$$\left|\overrightarrow{OB}\right| = \sqrt{7^2 + 2^2 + 4^2} = \sqrt{69}$$

14.



$$\left|\overrightarrow{OB}\right| = \sqrt{10^2 + (-3)^2 + 15^2} = \sqrt{334}$$

15. $\overrightarrow{TM} = \langle 3 - 2, 1 - 5, -4 - 4\rangle$ or $\langle 1, -4, -8\rangle$

$\left|\overrightarrow{TM}\right| = \sqrt{1^2 + (-4)^2 + (-8)^2} = \sqrt{81}$ or 9

16. $\overrightarrow{TM} = \langle -3 - (-2), 5 - 4, 2 - 7\rangle$ or $\langle -1, 1, -5\rangle$

$\left|\overrightarrow{TM}\right| = \sqrt{(-1)^2 + 1^2 + (-5)^2} = \sqrt{27}$ or $3\sqrt{3}$

17. $\overrightarrow{TM} = \langle 3 - 2, 1 - 5, 0 - 4\rangle$ or $\langle 1, -4, -4\rangle$

$\left|\overrightarrow{TM}\right| = \sqrt{1^2 + (-4)^2 + (-4)^2} = \sqrt{33}$

18. $\overrightarrow{TM} = \langle -1 - 3, 1 - (-5), 2 - 6\rangle$ or $\langle -4, 6, -4\rangle$

$\left|\overrightarrow{TM}\right| = \sqrt{(-4)^2 + 6^2 + (-4)^2} = \sqrt{68}$ or $2\sqrt{17}$

19. $\overrightarrow{TM} = \langle -2 - (-5), -1 - 8, -6 - 3\rangle$ or $\langle 3, -9, -9\rangle$

$\left|\overrightarrow{TM}\right| = \sqrt{3^2 + (-9)^2 + (-9)^2} = \sqrt{171}$ or $3\sqrt{19}$

20. $\overrightarrow{TM} = \langle 1-0, 4-6, -3-3\rangle$ or $\langle 1, -2, -6\rangle$
$|\overrightarrow{TM}| = \sqrt{1^2+(-2)^2+(-6)^2}$
$= \sqrt{41}$

21. $|\overrightarrow{CJ}| = \langle 3-(-1), -5-3, -4-10\rangle$
or $\langle 4, -8, -14\rangle$
$|\overrightarrow{CJ}| = \sqrt{4^2+(-8)^2+(-14)^2}$
$= \sqrt{276}$ or $2\sqrt{69}$

22. $\vec{\mathbf{u}} = 6\vec{\mathbf{w}} + 2\vec{\mathbf{z}}$
$= 6\langle 2, 6, -1\rangle + 2\langle 3, 0, 4\rangle$
$= \langle 12, 36, -6\rangle + \langle 6, 0, 8\rangle$
$= \langle 18, 36, 2\rangle$

23. $\vec{\mathbf{u}} = \frac{1}{2}\vec{\mathbf{v}} - \vec{\mathbf{w}} + 2\vec{\mathbf{z}}$
$\vec{\mathbf{u}} = \frac{1}{2}\langle 4, -3, 5\rangle - \langle 2, 6, -1\rangle + 2\langle 3, 0, 4\rangle$
$= \left\langle 2, -\frac{3}{2}, \frac{5}{2}\right\rangle - \langle 2, 6, -1\rangle + \langle 6, 0, 8\rangle$
$= \left\langle 6, -7\frac{1}{2}, 11\frac{1}{2}\right\rangle$

24. $\vec{\mathbf{u}} = \frac{3}{4}\vec{\mathbf{v}} - \vec{\mathbf{w}}$
$= \frac{3}{4}\langle 4, -3, 5\rangle - \langle 2, 6, -1\rangle$
$= \left\langle 3, -\frac{9}{4}, \frac{15}{4}\right\rangle - \langle 2, 6, -1\rangle$
$= \left\langle 1, -8\frac{1}{4}, 4\frac{3}{4}\right\rangle$

25. $\vec{\mathbf{u}} = 3\vec{\mathbf{v}} - \frac{2}{3}\vec{\mathbf{w}} + 2\vec{\mathbf{z}}$
$= 3\langle 4, -3, 5\rangle - \frac{2}{3}\langle 2, 6, -1\rangle + 2\langle 3, 0, 4\rangle$
$= \langle 12, -9, 15\rangle - \left\langle \frac{4}{3}, 4, -\frac{2}{3}\right\rangle + \langle 6, 0, 8\rangle$
$= \left\langle 16\frac{2}{3}, -13, 23\frac{2}{3}\right\rangle$

26. $\vec{\mathbf{u}} = 0.75\vec{\mathbf{v}} + 0.25\vec{\mathbf{w}}$
$= 0.75\langle 4, -3, 5\rangle + 0.25\langle 2, 6, -1\rangle$
$= \langle 3, -2.25, 3.75\rangle + \langle 0.5, 1.5, -0.25\rangle$
$= \langle 3.5, -0.75, 3.5\rangle$

27. $\vec{\mathbf{u}} = -4\vec{\mathbf{w}} + \vec{\mathbf{z}}$
$= -4\langle 2, 6, -1\rangle + \langle 3, 0, 4\rangle$
$= \langle -8, -24, 4\rangle + \langle 3, 0, 4\rangle$
$= \langle -5, -24, 8\rangle$

28. $\frac{2}{3}\vec{\mathbf{f}} + 3\vec{\mathbf{g}} - \frac{2}{5}\vec{\mathbf{h}}$
$= \frac{2}{3}\langle -3, 4.5, -1\rangle + 3\langle -2, 1, 6\rangle - \frac{2}{5}\langle 6, -3, -3\rangle$
$= \left\langle -2, 3, -\frac{2}{3}\right\rangle + \langle -6, 3, 18\rangle - \left\langle \frac{12}{5}, -\frac{6}{5}, -\frac{6}{5}\right\rangle$
$= \left\langle -\frac{52}{5}, \frac{36}{5}, \frac{278}{15}\right\rangle$

29. $\overrightarrow{LB} = \langle 5-2, -6-2, 2-7\rangle$ or $\langle 3, -8, -5\rangle$
$3\vec{\mathbf{i}} - 8\vec{\mathbf{j}} - 5\vec{\mathbf{k}}$

30. $\overrightarrow{LB} = \langle -4-(-6), 5-1, -1-0\rangle$ or $\langle 2, 4, -1\rangle$
$2\vec{\mathbf{i}} + 4\vec{\mathbf{j}} - \vec{\mathbf{k}}$

31. $\overrightarrow{LB} = \langle 7-9, 3-7, -2-(-11)\rangle$ or $\langle -2, -4, 9\rangle$
$-2\vec{\mathbf{i}} - 4\vec{\mathbf{j}} + 9\vec{\mathbf{k}}$

32. $\overrightarrow{LB} = \langle -8-12, 7-2, -5-6\rangle$ or $\langle -20, 5, -11\rangle$
$-20\vec{\mathbf{i}} + 5\vec{\mathbf{j}} - 11\vec{\mathbf{k}}$

33. $\overrightarrow{LB} = \langle -8-(-1), 5-2, -10-(-4)\rangle$
or $\langle -7, 3, -6\rangle$
$-7\vec{\mathbf{i}} + 3\vec{\mathbf{j}} - 6\vec{\mathbf{k}}$

34. $\overrightarrow{LB} = \langle 6-(-9), 5-12, -5-(-5)\rangle$
or $\langle 15, -7, 0\rangle$
$15\vec{\mathbf{i}} - 7\vec{\mathbf{j}}$

35. $|\overrightarrow{G_1G_2}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
$= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2} = |\overrightarrow{G_1G_2}|$
because $(x-y)^2 = (y-x)^2$ for all real numbers x and y.

36. If $\vec{\mathbf{m}} = \langle m_1, m_2, m_3\rangle$, then
$|\vec{\mathbf{m}}| = \sqrt{(m_1)^2+(m_2)^2+(m_3)^2}$. If $-\vec{\mathbf{m}} = \langle -m_1, -m_2, -m_3\rangle$, then $|-\vec{\mathbf{m}}| = \sqrt{(-m_1)^2+(-m_2)^2+(-m_3)^2}$.
Since $m_1{}^2 = (-m_1)^2$, $m_2{}^2 = (-m_2)^2$, and $m_3{}^2 = (-m_3)^2$, $|-\vec{\mathbf{m}}| = |\vec{\mathbf{m}}|$.

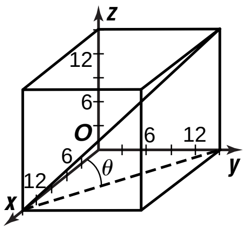
37. $\langle 3, -2, 4\rangle + \langle 6, 2, 5\rangle + \vec{\mathbf{F}} = \vec{\mathbf{O}}$
$\langle 9, 0, 9\rangle + \vec{\mathbf{F}} = \vec{\mathbf{O}}$
$\vec{\mathbf{F}} = -\langle 9, 0, 9\rangle$ or $\langle -9, 0, -9\rangle$

38. $m = \frac{1}{2}(x_1+x_2, y_1+y_2, z_1+z_2)$
$= \frac{1}{2}(2+4, 3+5, 6+2)$
$= \frac{1}{2}(6, 8, 8)$
$= (3, 4, 4)$

39a. $\overrightarrow{OK} = \langle 1-0, 4-0, 0-0\rangle$ or $\langle 1, 4, 0\rangle$
$\vec{\mathbf{i}} + 4\vec{\mathbf{j}}$

39b. $\overrightarrow{TK} = \langle 1-2, 4-4, 0-0\rangle$ or $\langle -1, 0, 0\rangle$
$-\vec{\mathbf{i}}$

40. $\vec{\mathbf{c}} = \vec{\mathbf{b}} - \vec{\mathbf{a}}$
$\vec{\mathbf{c}} = \langle 3, 1, 5\rangle - \langle 1, 3, 1\rangle$
$\vec{\mathbf{c}} = \langle 2, -2, 4\rangle$

41a.


41b. Find distance between (0, 0, 0) and (15, 15, 15).
$d = \sqrt{(15-0)^2+(15-0)^2+(15-0)^2}$
$= \sqrt{675}$ or about 26 feet

41c. $\sin\theta = \frac{15}{26}$
$\theta = \sin^{-1}\left(\frac{15}{\sqrt{675}}\right)$
$\theta = 35.25°$

42. $|\overrightarrow{AB}| = \sqrt{(1-2)^2+(\sqrt{3}-0)^2+(0-0)^2}$
$= \sqrt{4}$ or 2
$|\overrightarrow{BC}| = \sqrt{(1-1)^2+\left(\frac{1}{3}-\sqrt{3}\right)^2+\left(\frac{2\sqrt{2}}{3}-0\right)^2}$
$= \frac{\sqrt{36-6\sqrt{3}}}{3}$ or ≈ 1.69
$|\overrightarrow{AC}| = \sqrt{(1-2)^2+\left(\frac{1}{3}-0\right)^2+\left(\frac{2\sqrt{2}}{3}-0\right)^2}$
$= \sqrt{2}$ or ≈ 1.41
No, the distances between the points are not equal. A and B are 2 units apart, B and C are 1.69 units apart, and A and C are 1.41 units apart.

43. $\langle 3, 5\rangle + \langle -1, 2\rangle = \langle 3+(-1), 5+2\rangle$
$= \langle 2, 7\rangle$

44. $\overrightarrow{AB} = \langle -3 - 5, 3 - 2\rangle$ or $\langle -8, 1\rangle$

$\overrightarrow{CD} = \langle d_1 - 0, d_2 - 0\rangle$ or $\langle d_1, d_2\rangle$

$\overrightarrow{AB} = \overrightarrow{CD}$

$\langle -8, 1\rangle = \langle d_1, d_2\rangle$

$D = (-8, 1)$

45.
$$\frac{\sin 2X}{1 - \cos 2X} = \cot X$$
$$\frac{2 \sin X \cos X}{1 - \cos^2 X + \sin^2 X} = \cot X$$
$$\frac{2 \sin X \cos X}{2 \sin^2 X} = \cot X$$
$$\frac{\cos X}{\sin X} = \cot X$$
$$\cot X = \cot X$$

46. $\cos \theta = \frac{2}{3}$

$\sin^2 \theta = 1 - \cos^2 \theta$

$\sin^2 \theta = 1 - \left(\frac{4}{9}\right)$

$\sin^2 \theta = \frac{5}{9}$

$\sin \theta = \frac{\sqrt{5}}{3}$

47. $y = 6 \sin \frac{\theta}{2}$

amplitude $= |6|$ or 6

period $= \frac{2\pi}{k}$

$= \frac{2\pi}{\frac{1}{2}}$ or 4π

48. $16 \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{8\pi}{15}$ radians per second

49. Yes, because substituting 7 for x and -2 for y results in the inequality $-2 < 180$ which is true.

$y < 4x^2 - 3x + 5$

$-2 < 4(7)^2 - 3(7) + 5$

$-2 < 180$

50. $\frac{3}{2} \colon \frac{3 + 1}{2 + 1} = \frac{4}{3}$ $\qquad \frac{3}{2} > \frac{4}{3}$

So, A, C, and D are not correct.

$\frac{2}{3} \colon \frac{2 + 1}{3 + 1} = \frac{3}{4}$ $\qquad \frac{2}{3} < \frac{3}{4}$

So, B is not correct.

The correct choice is E.

8-4 Perpendicular Vectors

Pages 508–509 Check for Understanding

1. Sample answer: Vector $\vec{\mathbf{v}} \times \vec{\mathbf{w}}$ is the negative of vector $\vec{\mathbf{w}} \times \vec{\mathbf{v}}$.

$$\vec{\mathbf{v}} \times \vec{\mathbf{w}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ -1 & 0 & 3 \\ 1 & 2 & 4 \end{vmatrix}$$
$$= \begin{vmatrix} 0 & 3 \\ 2 & 4 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} -1 & 3 \\ 1 & 4 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} -1 & 0 \\ 0 & 2 \end{vmatrix}\vec{\mathbf{k}}$$
$$= -6\vec{\mathbf{i}} + 7\vec{\mathbf{j}} - 3\vec{\mathbf{k}}$$
$$\vec{\mathbf{v}} \times \vec{\mathbf{w}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 1 & 2 & 4 \\ -1 & 0 & 3 \end{vmatrix}$$
$$= \begin{vmatrix} 2 & 4 \\ 0 & 3 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 1 & 4 \\ -1 & 3 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 1 & 2 \\ -1 & 0 \end{vmatrix}\vec{\mathbf{k}}$$
$$= 6\vec{\mathbf{i}} - 7\vec{\mathbf{j}} + 3\vec{\mathbf{k}}$$

2.
$$\vec{\mathbf{a}} \times \vec{\mathbf{a}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ a_x & a_y & a_z \\ a_x & a_y & a_z \end{vmatrix}$$
$$= \begin{vmatrix} a_y & a_z \\ a_y & a_z \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} a_x & a_z \\ a_x & a_z \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} a_x & a_y \\ a_x & a_y \end{vmatrix}\vec{\mathbf{k}}$$
$$= (a_y a_z - a_y a_z)\vec{\mathbf{i}} - (a_x a_z - a_x a_z)\vec{\mathbf{j}} + (a_x a_y - a_x a_y)\vec{\mathbf{k}}$$
$$= 0\vec{\mathbf{i}} - 0\vec{\mathbf{j}} + 0\vec{\mathbf{k}}$$
$$= \langle 0, 0, 0\rangle$$
$$= \vec{\mathbf{0}}$$

3. Sample answer: No, because a vector cannot be perpendicular to itself.

4. $\langle 5, 2\rangle \cdot \langle -3, 7\rangle = 5(-3) + 2(7)$

$= -15 + 14$

$= -1$, no

5. $\langle -8, 2\rangle \cdot \langle 4.5, 18\rangle = -8(4.5) + 2(18)$

$= -36 + 36$

$= 0$, yes

6. $\langle -4, 9, 8\rangle \cdot \langle 3, 2, -2\rangle = -4(3) + 9(2) + 8(-2)$

$= -12 + 18 - 16$

$= -10$, no

7.
$$\langle 1, -3, 2\rangle \times \langle -2, 1, -5\rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 1 & -3 & 2 \\ -2 & 1 & -5 \end{vmatrix}$$
$$= \begin{vmatrix} -3 & 2 \\ 1 & -5 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 1 & 2 \\ -2 & -5 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 1 & -3 \\ -2 & 1 \end{vmatrix}\vec{\mathbf{k}}$$

$= 13\vec{\mathbf{i}} - \vec{\mathbf{j}} - 5\vec{\mathbf{k}}$ or $\langle 13, 1, -5\rangle$, yes

$\langle 13, 1, -5\rangle \cdot \langle 1, -3, 2\rangle$

$13(1) + 1(-3) + (-5)(2)$

$13 - 3 - 10 = 0$

$\langle 13, 1, -5\rangle \cdot \langle -2, 1, -5\rangle$

$13(-2) + 1(1) + (-5)(-5)$

$-26 + 1 + 25 = 0$

8.
$$\langle 6, 2, 10\rangle \times \langle 4, 1, 9\rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 6 & 2 & 10 \\ 4 & 1 & 9 \end{vmatrix}$$
$$= \begin{vmatrix} 2 & 10 \\ 1 & 9 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 6 & 10 \\ 4 & 9 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 6 & 2 \\ 4 & 1 \end{vmatrix}\vec{\mathbf{k}}$$

$= 8\vec{\mathbf{i}} - 14\vec{\mathbf{j}} - 2\vec{\mathbf{k}}$ or $\langle 8, -14, -2\rangle$, yes

$\langle 8, -14, -2\rangle \cdot \langle 6, 2, 10\rangle$

$8(6) + (-14)(2) + (-2)(10)$

$48 - 28 - 20 = 0$

$\langle 8, -14, -2\rangle \cdot \langle 4, 1, 9\rangle$

$8(4) + (-14)(1) + (-2)(9)$

$32 - 14 - 18 = 0$

9. Sample answer: Let $T(0, 1, 2)$, $U(-2, 2, 4)$, and $V(-1, -1, -1)$

$\overrightarrow{TU} = \langle -2, 1, 2\rangle$

$\overrightarrow{UV} = \langle 1, -3, -5\rangle$

$\overrightarrow{TU} \times \overrightarrow{UV} =$
$$= \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ -2 & 1 & 2 \\ 1 & -3 & -5 \end{vmatrix}$$
$$= \begin{vmatrix} 1 & 2 \\ -3 & -5 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} -2 & 2 \\ 1 & -5 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} -2 & 1 \\ 1 & -3 \end{vmatrix}\vec{\mathbf{k}}$$

$= \vec{\mathbf{i}} - 8\vec{\mathbf{j}} + 5\vec{\mathbf{k}}$ or $\langle 1, -8, 5\rangle$

10. $\overrightarrow{AB} = (0.65, 0, 0.3) - (0, 0, 0)$
$= \langle 0.65, 0, 0.3\rangle$
$\overrightarrow{\mathbf{F}} = \langle 0, 0, -32\rangle$
$\overrightarrow{\mathbf{T}} = \overrightarrow{AB} \times \overrightarrow{\mathbf{F}} =$

$$= \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 0.65 & 0 & 0.3 \\ 0 & 0 & -32 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0.3 \\ 0 & -32 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 0.65 & 0.3 \\ 0 & -32 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 0.65 & 0.3 \\ 0 & 0 \end{vmatrix}\vec{\mathbf{k}}$$

$= 0\vec{\mathbf{i}} - 20.8\vec{\mathbf{j}} + 0\vec{\mathbf{k}}$
$|\vec{\mathrm{T}}| = \sqrt{0^2 + (-20.8)^2 + 0^2}$
$= 20.8$ foot-pounds

Pages 509–511 Exercises

11. $\langle 4.8\rangle \cdot \langle 6, -3\rangle = 4(6) + 8(-3)$
$= 24 - 24$
$= 0$, yes

12. $\langle 3, 5\rangle \cdot \langle 4, -2\rangle = 3(4) + 5(-2)$
$= 12 - 10$
$= 2$, no

13. $\langle 5, -1\rangle \cdot \langle -3, 6\rangle = 5(-3) + (-1)(6)$
$= -15 - 6$
$= -21$, no

14. $\langle 7, 2\rangle \cdot \langle 0, -2\rangle = 7(0) + 2(-2)$
$= 0 - 4$
$= -4$, no

15. $\langle 8, 4\rangle \cdot \langle 2, 4\rangle = 8(2) + 4(4)$
$= 16 + 16$
$= 32$, no

16. $\langle 4, 9, -3\rangle \cdot \langle -6, 7, 5\rangle = 4(-6) + 9(7) + (-3)(5)$
$= -24 + 63 - 15$
$= 24$, no

17. $\langle 3, 1, 4\rangle \cdot \langle 2, 8, -2\rangle = 3(2) + 1(8) + 4(-2)$
$= 6 + 8 - 8$
$= 6$, no

18. $\langle -2, 4, 8\rangle \cdot \langle 16, 4, 2\rangle = -2(16) + 4(4) + 8(2)$
$= -32 + 16 + 16$
$= 0$, yes

19. $\langle 7, -2, 4\rangle \cdot \langle 3, 8, 1\rangle = 7(3) + (-2)(8) + 4(1)$
$= 21 - 16 + 4$
$= 9$, no

20. $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \langle 3, 12\rangle \cdot \langle 8, -2\rangle$
$= 24 - 24$
$= 0$, yes

$\vec{\mathbf{b}} \cdot \vec{\mathbf{c}} = \langle 8, -2\rangle \cdot \langle 3, -2\rangle$
$= 24 + 4$
$= 28$, no

$\vec{\mathbf{a}} \cdot \vec{\mathbf{c}} = \langle 3, 12\rangle \cdot \langle 3, -2\rangle$
$= 9 - 24$
$= -15$, no

21. $$\langle 0, 1, 2\rangle \times \langle 1, 1, 4\rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 0 & 1 & 2 \\ 1 & 1 & 4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 0 & 2 \\ 1 & 4 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}\vec{\mathbf{k}}$$

$= 2\vec{\mathbf{i}} + 2\vec{\mathbf{j}} - \vec{\mathbf{k}}$ or $\langle 2, 2, -1\rangle$, yes

$\langle 2, 2, -1\rangle \cdot \langle 0, 1, 2\rangle$
$2(0) + 2(1) + (-1)(2)$
$2 + 2 - 2 = 0$
$\langle 2, 2, -1\rangle \cdot \langle 1, 1, 4\rangle$
$2(1) + 2(1) + (-1)(4)$
$2 + 2 - 4 = 0$

22. $$\langle 5, 2, 3\rangle \times \langle -2, 5, 0\rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 5 & 2 & 3 \\ -2 & 5 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 3 \\ 5 & 0 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 5 & 3 \\ -2 & 0 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 5 & 2 \\ -2 & 5 \end{vmatrix}\vec{\mathbf{k}}$$

$= 15\vec{\mathbf{i}} - 6\vec{\mathbf{j}} + 29\vec{\mathbf{k}}$ or $\langle -15, -6, 29\rangle$, yes

$\langle -15, -6, 29\rangle \cdot \langle 5, 2, 3\rangle$
$(-15)(5) + (-6)(2) + 29(3)$
$-75 - 12 + 87 = 0$
$\langle -15, -6, 29\rangle \cdot \langle -2, 5, 0\rangle$
$(-15)(-2) + (-6)(5) + 29(0)$
$30 - 30 + 0 = 0$

23. $$\langle 3, 2, 0\rangle \times \langle 1, 4, 0\rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 3 & 2 & 0 \\ 1 & 4 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 0 \\ 4 & 0 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix}\vec{\mathbf{k}}$$

$= 0\vec{\mathbf{i}} - 0\vec{\mathbf{j}} + 10\vec{\mathbf{k}}$ or $\langle 0, 0, 10\rangle$, yes

$\langle 0, 0, 10\rangle \cdot \langle 3, 2, 0\rangle$
$0(3) + 0(2) + 10(0)$
$0 + 0 + 0 = 0$
$\langle 0, 0, 10\rangle \cdot \langle 1, 4, 0\rangle$
$0(1) + 0(4) + 10(0)$
$0 + 0 + 0 = 0$

24. $$\langle 1, -3, 2\rangle \times \langle 5, 1, -2\rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 1 & -3 & 2 \\ 5 & 1 & -2 \end{vmatrix}$$

$$= \begin{vmatrix} -3 & 2 \\ 1 & -2 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 1 & 2 \\ 5 & -2 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 1 & -3 \\ 5 & 1 \end{vmatrix}\vec{\mathbf{k}}$$

$= 4\vec{\mathbf{i}} + 12\vec{\mathbf{j}} + 16\vec{\mathbf{k}}$ or $\langle 4, 12, 16\rangle$, yes

$\langle 4, 12, 16\rangle \cdot \langle 1, -3, 2\rangle$
$4(1) + 12(-3) + 16(2)$
$4 - 36 + 32 = 0$
$\langle 4, 12, 16\rangle \cdot \langle 5, 1, -2\rangle$
$4(5) + 12(1) + 16(-2)$
$20 + 12 - 32 = 0$

25. $$\langle -3, -1, 2\rangle \times \langle 4, -4, 0\rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ -3 & -1 & 2 \\ 4 & -4 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & 2 \\ -4 & 0 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} -3 & 2 \\ 4 & 0 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} -3 & -1 \\ 4 & -4 \end{vmatrix}\vec{\mathbf{k}}$$

$= 8\vec{\mathbf{i}} + 8\vec{\mathbf{j}} + 16\vec{\mathbf{k}}$ or $\langle 8, 8, 16\rangle$, yes

$\langle 8, 8, 16\rangle \cdot \langle -3, -1, 2\rangle$
$8(-3) + 8(-1) + 16(2)$
$-24 - 8 + 32 = 0$
$\langle 8, 8, 16\rangle \cdot \langle 4, -4, 0\rangle$
$8(4) + 8(-4) + 16(0)$
$32 - 32 + 0 = 0$

26. $\langle 4, 0, -2\rangle \times \langle -7, 1, 0\rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 4 & 0 & -2 \\ -7 & 1 & 0 \end{vmatrix}$

$= \begin{vmatrix} 0 & -2 \\ 1 & 0 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 4 & -2 \\ -7 & 0 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 4 & 0 \\ -7 & 1 \end{vmatrix}\vec{\mathbf{k}}$

$= 2\vec{\mathbf{i}} + 14\vec{\mathbf{j}} + 4\vec{\mathbf{k}}$ or $\langle 2, 14, 4\rangle$, yes

$\langle 2, 14, 4\rangle \cdot \langle 4, 0, -2\rangle$
$2(4) + 14(0) + 4(-2)$
$8 + 0 - 8 = 0$
$\langle 2, 14, 4\rangle \cdot \langle -7, 1, 0\rangle$
$2(-7) + 14(1) + 4(0)$
$-14 + 14 + 0 = 0$

27. Sample answer:
Let $\vec{\mathbf{v}} = \langle v_1, v_2, v_3\rangle$ and $-\vec{\mathbf{v}} = \langle -v_1, -v_2, -v_3\rangle$

$\vec{\mathbf{v}} \times (-\vec{\mathbf{v}}) = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ v_1 & v_2 & v_3 \\ -v_1 & -v_2 & -v_3 \end{vmatrix}$

$= \begin{vmatrix} v_2 & v_3 \\ -v_2 & -v_3 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} v_1 & v_3 \\ -v_1 & -v_3 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} v_1 & v_2 \\ -v_1 & -v_2 \end{vmatrix}\vec{\mathbf{k}}$

$= 0\vec{\mathbf{i}} - 0\vec{\mathbf{j}} + 0\vec{\mathbf{k}} = \mathbf{0}$

28. $\vec{\mathbf{a}} \times (\mathbf{b} + \vec{\mathbf{c}}) = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ a_1 & a_2 & a_3 \\ (b_1 + c_1) & (b_2 + c_2) & (b_3 + c_3) \end{vmatrix}$

$= \begin{vmatrix} a_2 & a_3 \\ (b_2 + c_2) & (b_3 + c_3) \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} a_1 & a_3 \\ (b_1 + c_1) & (b_3 + c_3) \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} a_1 & a_2 \\ (b_1 + c_1) & (b_2 + c_2) \end{vmatrix}\vec{\mathbf{k}}$

$= [a_2 (b_3 + c_3) - a_3 (b_2 + c_2)]\vec{\mathbf{i}} -$
$[a_1 (b_3 + c_3) - a_3 (b_1 + c_1)]\vec{\mathbf{j}} +$
$[a_1 (b_2 + c_2) - a_2 (b_1 + c_1)]\vec{\mathbf{k}}$

$= [(a_2b_3 + a_2c_3) - (a_3b_2 + a_3c_2)]\vec{\mathbf{i}} -$
$[(a_1b_3 + a_1c_3) - (a_3b_1 + a_3c_1)]\vec{\mathbf{j}} +$
$[(a_1b_2 + a_1c_2) - (a_2b_1 + a_2c_1)]\vec{\mathbf{k}}$

$= [(a_2b_3 - a_3b_2) + (a_2c_3 - a_3c_2)]\vec{\mathbf{i}} -$
$[(a_1b_3 - a_3b_1) + (a_1c_3 - a_3c_1)]\vec{\mathbf{j}} +$
$[(a_1b_2 - a_2b_1) + (a_1c_2 - a_2c_1)]\vec{\mathbf{k}}$

$= (a_2b_3 - a_3b_2)\vec{\mathbf{i}} + (a_2c_3 - a_3c_2)\vec{\mathbf{i}} -$
$(a_1b_3 - a_3b_1)\vec{\mathbf{j}} - (a_1c_3 - a_3c_1)\vec{\mathbf{j}} +$
$(a_1b_2 - a_2b_1)\vec{\mathbf{k}} + (a_1c_2 - a_2c_1)\vec{\mathbf{k}}$

$= [(a_2b_3 - a_3b_2)\vec{\mathbf{i}} - (a_1b_3 - a_3b_1)\vec{\mathbf{j}} +$
$(a_1b_2 - a_2b_1)\vec{\mathbf{k}}] + [(a_2c_3 - a_3c_2)\vec{\mathbf{i}} -$
$(a_1c_3 - a_3c_1)\vec{\mathbf{j}} + (a_1c_2 - a_2c_1)\vec{\mathbf{k}}]$

$= \left[\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\right]\vec{\mathbf{k}} +$

$= \left[\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\right]\vec{\mathbf{k}} +$

$\left[\begin{vmatrix} a_2 & a_3 \\ c_2 & c_3 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} a_1 & a_3 \\ c_1 & c_3 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} a_1 & a_2 \\ c_1 & c_2 \end{vmatrix}\vec{\mathbf{k}}\right]$

$= (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) + (\vec{\mathbf{a}} \times \vec{\mathbf{c}})$

29. Sample answer:
Let $T(0, -2, 2)$, $U(1, 2, -3)$, and $V(4, 0, -1)$
$\overrightarrow{TU} = \langle 1, 4, -5\rangle$
$\overrightarrow{UV} = \langle 3, -2, 2\rangle$

$\overrightarrow{TU} \times \overrightarrow{UV} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 1 & 4 & -5 \\ 3 & -2 & 2 \end{vmatrix}$

$= \begin{vmatrix} 4 & -5 \\ -2 & 2 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 1 & -5 \\ 3 & 2 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 1 & 4 \\ 3 & -2 \end{vmatrix}\vec{\mathbf{k}}$

$= -2\vec{\mathbf{i}} - 17\vec{\mathbf{j}} - 14\vec{\mathbf{k}}$ or $\langle -2, -17, -14\rangle$

30. Sample answer:
Let $T(-2, 1, 0)$, $U(-3, 0, 0)$, and $V(5, 2, 0)$.
$\overrightarrow{TU} = \langle -1, -1, 0\rangle$
$\overrightarrow{UV} = \langle 8, 2, 0\rangle$

$\overrightarrow{TU} \times \overrightarrow{UV} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ -1 & -1 & 0 \\ 8 & 2 & 0 \end{vmatrix}$

$= \begin{vmatrix} -1 & 0 \\ 2 & 0 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} -1 & 0 \\ 8 & 0 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} -1 & -1 \\ 8 & 2 \end{vmatrix}\vec{\mathbf{k}}$

$= 0\vec{\mathbf{i}} + 0\vec{\mathbf{j}} + 6\vec{\mathbf{k}}$ or $\langle 0, 0, 6\rangle$

31. Sample answer:
Let $T(0, 0, 1)$, $U(1, 0, 1)$, and $V(-1, -1, -1)$.
$\overrightarrow{TU} = \langle 1, 0, 0\rangle$
$\overrightarrow{UV} = \langle -2, -1, -2\rangle$

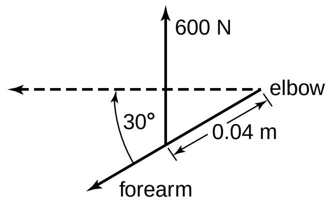
$\overrightarrow{TU} \times \overrightarrow{UV} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 1 & 0 & 0 \\ -2 & -1 & -2 \end{vmatrix}$

$= \begin{vmatrix} 0 & 0 \\ -1 & -2 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 1 & 0 \\ -2 & -2 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 1 & 0 \\ -2 & -1 \end{vmatrix}\vec{\mathbf{k}}$

$= 0\vec{\mathbf{i}} + 2\vec{\mathbf{j}} - \vec{\mathbf{k}}$ or $\langle 0, 2, -1\rangle$

32. The expression is false. $\vec{\mathbf{m}} \times \vec{\mathbf{n}}$ and $\vec{\mathbf{n}} \times \vec{\mathbf{m}}$ have the same magnitude but are opposite in direction.

33a.



33b. $\vec{\mathbf{T}} = \overrightarrow{AB} \times \vec{\mathbf{F}}$
$\overrightarrow{AB} = \langle 0.04 \cos(-30°), 0, 0.04 \sin(-30°)\rangle$
$= \langle 0.02(\sqrt{3}), 0, -0.02\rangle$
$\vec{\mathbf{F}} = \langle 0, 0, 600\rangle$

$\overrightarrow{AB} \times \vec{\mathbf{F}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 0.02\sqrt{3} & 0 & -0.02 \\ 0 & 0 & 600 \end{vmatrix}$

$= 0\vec{\mathbf{i}} - 12\sqrt{3}\,\vec{\mathbf{j}} + 0\vec{\mathbf{k}}$

$\vec{\mathbf{T}} = |\overrightarrow{AB} \times \vec{\mathbf{F}}| = 12\sqrt{3}$ or about 21 N-m

34. $\vec{\mathbf{x}} \times \vec{\mathbf{y}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 2 & 3 & 0 \\ -1 & 1 & 4 \end{vmatrix}$

$= \begin{vmatrix} 3 & 0 \\ 1 & 4 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 2 & 0 \\ -1 & 4 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix}\vec{\mathbf{k}}$

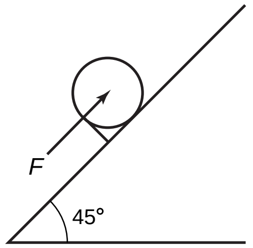
$= 12\vec{\mathbf{i}} - 8\vec{\mathbf{j}} + 5\vec{\mathbf{k}}$

$A = \frac{1}{2}|\vec{\mathbf{x}} \times \vec{\mathbf{y}}|$
$= \frac{1}{2}\sqrt{12^2 + (-8)^2 + (5)^2}$
$= \frac{1}{2}\sqrt{233}$

35a. $\vec{\mathbf{o}} = \langle 120, 310, 60\rangle$
$\vec{\mathbf{c}} = \langle 29, 18, 21\rangle$

35b. $\vec{\mathbf{o}} \cdot \vec{\mathbf{c}} = 120(29) + 310(18) + 60(21)$
$= \$10{,}320$

36a.



36b. $W = |\vec{\mathbf{F}}|\,|\vec{\mathbf{d}}| \cos\theta$
$W = 120 \cdot 4 \cdot \cos 45°$
$W \approx 339$ ft-lb

37a. $\vec{\mathbf{X}} = \langle 2-1, 5-0, 0-3\rangle$ or $\langle 1, 5, -3\rangle$
$\vec{\mathbf{Y}} = \langle 3-2, 1-5, 4-0\rangle$ or $\langle 1, -4, 4\rangle$

$$\vec{\mathbf{X}} \times \vec{\mathbf{Y}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 1 & 5 & -3 \\ 1 & -4 & 4 \end{vmatrix}$$
$$= \begin{vmatrix} 5 & -3 \\ -4 & 4 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 1 & -3 \\ 1 & 4 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 1 & 5 \\ 1 & -4 \end{vmatrix}\vec{\mathbf{k}}$$
$$= 8\vec{\mathbf{i}} - 7\vec{\mathbf{j}} - 9\vec{\mathbf{k}} \text{ or } \langle 8, -7, -9\rangle$$

37b. The cross product of two vectors is always a vector perpendicular to the two vectors and the plane in which they lie.

38a. $v = \vec{\mathbf{p}} \cdot (\vec{\mathbf{q}} \times \vec{\mathbf{r}})$

$$\vec{\mathbf{q}} \times \vec{\mathbf{r}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 2 & 1 & -4 \\ -3 & 1 & -5 \end{vmatrix}$$
$$= \begin{vmatrix} 1 & -4 \\ 1 & -5 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 2 & -4 \\ -3 & -5 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 2 & 1 \\ -3 & 1 \end{vmatrix}\vec{\mathbf{k}}$$
$$= -\vec{\mathbf{i}} - 22\vec{\mathbf{j}} + 5\vec{\mathbf{k}} \text{ or } \langle -1, -22, 5\rangle$$
$$\vec{\mathbf{p}} \cdot (\vec{\mathbf{q}} \times \vec{\mathbf{r}}) = \langle 0, 0, -1\rangle \cdot \langle -1, -22, 5\rangle$$
$$= 0(-1) + 0(-22) + (-1)(5)$$
$$= -5 \text{ or } 5 \text{ units}^3$$

38b. $\begin{vmatrix} 0 & 0 & -1 \\ 2 & 1 & -4 \\ -3 & 1 & -5 \end{vmatrix}$

$$= \begin{vmatrix} 1 & -4 \\ 1 & -5 \end{vmatrix} 0 - \begin{vmatrix} 2 & -4 \\ -3 & -5 \end{vmatrix} 0 + \begin{vmatrix} 2 & 1 \\ -3 & 1 \end{vmatrix}(-1)$$
$= -5$ or 5 units^3
They are the same.

39. Need $(k\vec{\mathbf{v}} + \vec{\mathbf{w}}) \cdot \vec{\mathbf{u}} = 0$.
$[k\langle 1, 2\rangle + \langle -1, 2\rangle] \cdot \langle 5, 12\rangle = 0$
$[\langle k, 2k\rangle + \langle -1, 2\rangle] \cdot \langle 5, 12\rangle = 0$
$\langle k-1, 2k+2\rangle \cdot \langle 5, 12\rangle = 0$
$(k-1)5 + (2k+2)12 = 0$
$5k - 5 + 24k + 24 = 0$
$29k + 19 = 0$
$k = -\frac{19}{29}$

40. $|\overrightarrow{BA}|^2 = |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 - 2|\vec{\mathbf{a}}|\,|\vec{\mathbf{b}}| \cos\theta$
$\left(\sqrt{(a_1-b_1)^2 + (a_2-b_2)^2}\right)^2$
$= \left(\sqrt{a_1^2 + a_2^2}\right)^2 + \left(\sqrt{b_1^2 + b_2^2}\right)^2$
$- 2\left(\sqrt{a_1^2 + a_2^2}\right)\left(\sqrt{b_1^2 + b_2^2}\right)\cos\theta$
$(a_1-b_1)^2 + (a_2-b_2)^2$
$= a_1^2 + a_2^2 + b_1^2 + b_2^2 - 2\left(\sqrt{a_1^2 + a_2^2}\right)$
$\left(\sqrt{b_1^2 + b_2^2}\right)\cos\theta\ a_1^2 - 2a_1b_1 + b_1^2$
$+ a_2^2 - 2a_2b_2 + b_2^2$
$= a_1^2 + a_2^2 + b_1^2 + b_2^2$
$- 2\left(\sqrt{a_1^2 + a_2^2}\right)\left(\sqrt{b_1^2 + b_2^2}\right)\cos\theta -$
$2a_1b_1 - 2a_2b_2$
$= -2\left(\sqrt{a_1^2 + a_2^2}\right)\left(\sqrt{b_1^2 + b_2^2}\right)\cos\theta$
$a_1b_1 + a_2b_2$
$= \sqrt{a_1^2 + a_2^2}\,\sqrt{b_1^2 + b_2^2}\cos\theta$
$a_1b_1 + a_2b_2$
$= |\vec{\mathbf{a}}|\,|\vec{\mathbf{b}}|\cos\theta\ \vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$
$= |\vec{\mathbf{a}}|\,|\vec{\mathbf{b}}|\cos\theta$

41. $\overrightarrow{AB} = \langle 5-3, 3-3, 2-(-1)\rangle$ or $\langle 2, 0, 3\rangle$

42. $D(8, 3)$
$E(0, -2)$
$\overrightarrow{DE} = \langle 0-8, -2-3\rangle$ or $\langle -8, -5\rangle$
$|\overrightarrow{DE}| = \sqrt{(-8)^2 + (-5)^2}$
$= \sqrt{89}$

43. $4x + y - 6 = 0$
$\sqrt{A^2 + B^2} = \sqrt{4^2 + 1^2}$ or $\sqrt{17}$
$\frac{4\sqrt{17}}{17}x + \frac{\sqrt{17}}{17}y - \frac{6\sqrt{17}}{17} = 0$
$p = \frac{6\sqrt{17}}{17} \approx 1.46$ units
$\sin\phi = \frac{\sqrt{17}}{17}$ $\quad\quad$ $\cos\phi = \frac{4\sqrt{17}}{17}$
$\tan\phi = \frac{1}{4}$
$\phi = 14°$

44. $A = 36°$, $b = 13$, and $c = 6$
$a^2 = b^2 + c^2 - 2\,bc\cos A$
$a^2 = 13^2 + 6^2 - 2(13)(6)\cos 36°$
$a \approx 8.9$
$\frac{\sin 36°}{8.9} \approx \frac{\sin B}{13}$
$B \approx \sin^{-1}\left(\frac{13\sin 36°}{8.9}\right)$
$B \approx 59.41°$ or $59°25'$
$C \approx 180° - 36° - 59°25'$
$C \approx 84.59°$ or $84°35'$

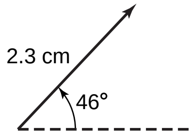
45. $\tan 73° = \frac{h}{4}$ $\quad\quad$ $\cos 73° = \frac{4}{\ell}$
$4\tan 73° = h$ $\quad\quad$ $\ell = \frac{4}{\cos 73°}$
$13.1 = h$; 13.1 m $\quad\quad$ $\ell = 13.7$ m

46. $3 + \sqrt{3x-4} \geq 10$
$\sqrt{3x-4} \geq 7$
$3x - 4 \geq 49$
$x \geq 17.67$

47. $81 = 3^4$
$64 = 2^6 = (2^2)^3$ or $(2^3)^2$
$4 = 2^2$
$2 = 2^1$
$9 = 3^2$
So $64 = 4^3 = 8^2$
The correct choice is B.

Page 511 Mid-Chapter Quiz

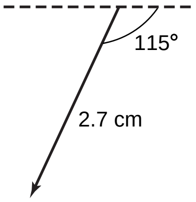
1.



$F_x = 2.3 \cos 46°$
$= 1.6$ cm

$F_y = 2.3 \sin 46°$
$= 1.7$ cm

2.



$F_x = 27 \cos 245°$
$= 11.4$ mm

$F_y = 27 \sin 245°$
$= 24.5$ mm

3. $\overrightarrow{CD} = \langle -4 - (-9), -3 - 2\rangle$ or $\langle 5, -5\rangle$
$|\overrightarrow{CD}| = \sqrt{5^2 + (-5)^2}$
$= 50$ or $5\sqrt{2}$

4. $\overrightarrow{CD} = \langle 5 - 3, 7 - 7, 2 - (-1)\rangle$ or $\langle 2, 0, 3\rangle$
$|\overrightarrow{CD}| = \sqrt{2^2 + 0^2 + 3^2}$
$= \sqrt{13}$

5. $\vec{\mathbf{r}} = \vec{\mathbf{t}} - 2\vec{\mathbf{s}}$
$= \langle -6, 2\rangle - 2\langle 4, -3\rangle$
$= \langle -6, 2\rangle - \langle 8, -6\rangle$
$= \langle -6 - 8, 2 + 6\rangle$ or $\langle -14, 8\rangle$

6. $\vec{\mathbf{r}} = 3\vec{\mathbf{u}} + \vec{\mathbf{v}}$
$= 3\langle 1, -3, -8\rangle + \langle 3, 9, -1\rangle$
$= \langle 3, -9, -24\rangle + \langle 3, 9, -1\rangle$
$= \langle 3 + 3, -9 + 9, -24 + (-1)\rangle$ or $\langle 6, 0, -25\rangle$

7. $\langle 3, 6\rangle \cdot \langle -4, 2\rangle = 3(-4) + 6(2)$
$= -12 + 12$
$= 0$; yes

8. $\langle 3, -2, 4\rangle \cdot \langle 1, -4, 0\rangle = 3(1) + (-2)(-4) + 4(0)$
$= 3 + 8$
$= 11$; no

9. $\langle 1, 3, 2\rangle \times \langle 2, -1, -1\rangle = \begin{vmatrix} \vec{\mathbf{u}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 1 & 3 & 2 \\ 2 & -1 & -1 \end{vmatrix}$

$= \begin{vmatrix} 3 & 2 \\ -1 & -1 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix}\vec{\mathbf{k}}$

$= -\vec{\mathbf{i}} + 5\vec{\mathbf{j}} - 7\vec{\mathbf{k}}$ or $\langle -1, 5, -7\rangle$, yes
$\langle -1, 5, -7\rangle \cdot \langle 1, 3, 2\rangle$
$(-1)(1) + 5(3) + (-7)(2)$
$-1 + 15 - 14 = 0$
$\langle -1, 5, -7\rangle \cdot \langle 2, -1, -1\rangle$
$(-1)(2) + 5(-1) + (-7)(-1)$
$-2 - 5 + 7 = 0$

10. Let $X(2, 0, 4)$ and $Y(7, 4, 6)$.
$|XY| = \sqrt{(7 - 2)^2 + (4 - 0)^2 + (6 - 4)^2}$
$= \sqrt{45}$ or about 6.7 m

8-4B Graphing Calculator Exploration: Finding Cross Products

Page 512

1. $\langle -49, 32\ -55\rangle$
2. $\langle 168, -96, 76\rangle$
3. $\langle 0, 0, 0\rangle$
4. $\langle 11, 15, -3\rangle$
5. $\langle 0, 0, -7\rangle$
6. $\langle 0, 40, 0\rangle$

7. $\vec{\mathbf{u}} \times \vec{\mathbf{x}} = \langle 6, 6, -12\rangle$
$|\vec{\mathbf{u}} \times \vec{\mathbf{v}}| = \sqrt{6^2 + 6^2 + (-12)^2}$
$= \sqrt{216}$

8. $\vec{\mathbf{u}} \times \vec{\mathbf{v}} = \langle 1, -13, -20\rangle$
$|\vec{\mathbf{u}} \times \vec{\mathbf{v}}| = \sqrt{1^2 + (-13)^2 + (-20)^2}$
$= \sqrt{570}$

9. Sample answer: Insert the following lines after the last line of the given program.
:Disp "LENGTH IS"
:Disp $\sqrt{\ }$ ((BZ − CY)^2 + (CX − AZ)^2 + (AY − BX)^2)

8-5 Applications with Vectors

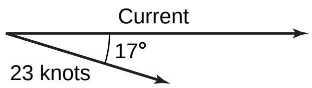
Pages 516–517 Check for Understanding

1. Sample answer: Pushing an object up the slope requires less force because the component of the weight of the object in the direction of motion is $mg \sin \theta$. This is less than the weight mg of the object, which is the force that must be exerted to lift the object straight up.

2. The tension increases.

3. Sample answer: Forces are in equilibrium if the resultant force is $\vec{\mathbf{O}}$.

4.



5. $\vec{\mathbf{F}}_1 = 300\vec{\mathbf{i}}$
$\vec{\mathbf{F}}_2 = (170 \cos 55°)\vec{\mathbf{i}} + (170 \sin 55°)\vec{\mathbf{j}}$
$|\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2| = \sqrt{(300 + 170 \cos 55°)^2 + (170 \sin 55°)^2}$
≈ 421.19 N
$\tan \theta = \frac{170 \sin 55°}{300 + 170 \cos 55°}$
$\theta = \tan^{-1}\left(\frac{170 \sin 55°}{300 + 170 \cos 55°}\right)$
$\approx$

6. $\vec{\mathbf{F}}_1 = 50\vec{\mathbf{i}}$
$\vec{\mathbf{F}}_2 = 100\vec{\mathbf{j}}$
$|\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2| = \sqrt{50^2 + 100^2}$
≈ 111.8 N
$\tan \theta = \frac{100}{50}$ or 2
$\theta = \tan^{-1} 2$
$\approx 63.43°$

7. horizontal $= 18 \cos 40°$
≈ 13.79 N
vertical $= 18 \sin 40°$
≈ 11.57 N

8. $\vec{\mathbf{F}}_1 = (33 \cos 90°)\vec{\mathbf{i}} + (33 \sin 90°)\vec{\mathbf{j}}$ or $33\vec{\mathbf{j}}$

$\vec{\mathbf{F}}_2 = (44 \cos 60°)\vec{\mathbf{i}} + (44 \sin 60°)\vec{\mathbf{j}}$

or $22\vec{\mathbf{i}} + 22\sqrt{3}\,\vec{\mathbf{j}}$

$|\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2| = \sqrt{22^2 + (33 + 22\sqrt{3})^2}$

≈ 74 N

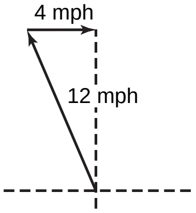
$\tan \theta = \frac{33 + 22\sqrt{3}}{22}$ or $\frac{3 + 2\sqrt{3}}{2}$

$\theta = \tan^{-1}\left(\frac{3 + 2\sqrt{3}}{2}\right)$

$\approx 73°$

A force with magnitude 74 N and direction 73° + 180° or 253° will produce equilibrium.

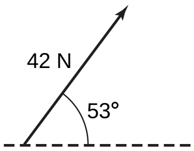
9a.



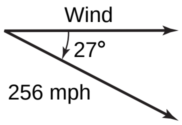
9b. If θ is the angle between the resultant path of the ferry and the line between the landings, then $\sin \theta = \frac{4}{12}$ or $\frac{1}{3}$. So $\theta = \sin^{-1}\frac{1}{3}$, or about 19.5°.

Pages 517–519 Exercises

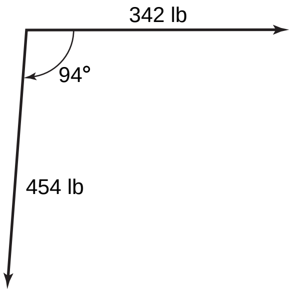
10.



11.



12.



13. $\vec{\mathbf{F}}_1 = 425\vec{\mathbf{i}}$

$\vec{\mathbf{F}}_2 = 390\vec{\mathbf{j}}$

$|\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2| = \sqrt{425^2 + 390^2}$

≈ 576.82 N

$\tan \theta = \frac{390}{425}$ or $\frac{78}{85}$

$\theta = \tan^{-1}\frac{78}{85}$

$\approx 42.5°$

14. $\vec{\mathbf{v}}_1 = 65\vec{\mathbf{i}}$

$\vec{\mathbf{v}}_2 = (50 \cos 300°)\vec{\mathbf{i}} + (50 \sin 300°)\vec{\mathbf{j}}$ or $25\vec{\mathbf{i}} - 25\sqrt{3}\,\vec{\mathbf{j}}$

$|\vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_2| = \sqrt{90^2 + (-25\sqrt{3})^2}$

≈ 99.87 mph

$\tan \theta = -\frac{25\sqrt{3}}{90}$ or $-\frac{5\sqrt{3}}{18}$

$\theta = \tan^{-1}\left(-\frac{5\sqrt{3}}{18}\right)$

A positive value for θ is about 334.3°.

15. $\vec{\mathbf{v}}_1 = (115 \cos 60°)\vec{\mathbf{i}} + (115 \sin 60°)\vec{\mathbf{j}}$

or $57.5\vec{\mathbf{i}} + 57.5\sqrt{3}\,\vec{\mathbf{j}}$

$\vec{\mathbf{v}}_2 = (115 \cos 120°)\vec{\mathbf{i}} + (115 \sin 120°)\vec{\mathbf{j}}$

or $-57.5\vec{\mathbf{i}} + 57.5\sqrt{3}\,\vec{\mathbf{j}}$

$|\vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_2| = \sqrt{0^2 + (115\sqrt{3})^2}$

$= 115\sqrt{3}$

≈ 199.19 km/h

Since $\tan \theta$ is undefined and the vertical component is positive, $\theta = 90°$.

16. The force must be at least as great as the component of the weight of the object in the direction of the ramp. This is 100 sin 10°, or about 17.36 lb.

17. $\vec{\mathbf{F}}_1 = 105\vec{\mathbf{i}}$

$\vec{\mathbf{F}}_2 = (110 \cos 50°)\vec{\mathbf{i}} + (110 \sin 50°)\vec{\mathbf{j}}$

$|\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2| = \sqrt{(105 + 110 \cos 50°)^2 + (110 \sin 50°)^2}$

≈ 194.87 N

$\tan \theta = \frac{110 \sin 50°}{105 + 110 \cos 50°}$

$\theta = \tan^{-1}\left(\frac{110 \sin 50°}{105 + 110 \cos 50°}\right)$

$\approx 25.62°$

18.

$F = w \sin \theta$

$52.1 = 75 \sin \theta$

$\frac{52.1}{75} = \sin \theta$

$\sin^{-1}\left(\frac{52.1}{75}\right) = \theta$

$44° \approx \theta$

19. $\vec{\mathbf{F}}_1 = (250 \cos 25°)\vec{\mathbf{i}} + (250 \sin 25°)\vec{\mathbf{j}}$

$\vec{\mathbf{F}}_2 = (45 \cos 250°)\vec{\mathbf{i}} + (45 \sin 250°)\vec{\mathbf{j}}$

$|\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2| =$

$\sqrt{(250 \cos 25° + 45 \cos 250°)^2 + (250 \sin 25° + 45 \sin 250°)^2}$

≈ 220.5 lb

$\tan \theta = \frac{250 \sin 25° + 45 \sin 250°}{250 \cos 25° + 45 \cos 250°}$

$\theta = \tan^{-1}\left(\frac{250 \sin 25° + 45 \sin 250°}{250 \cos 25° + 45 \cos 250°}\right)$

$\approx 16.7°$

20. $\vec{\mathbf{F}}_1 = (70 \cos 330°)\vec{\mathbf{i}} + (70 \sin 330°)\vec{\mathbf{j}}$ or $35\sqrt{3}\,\vec{\mathbf{i}} - 35\vec{\mathbf{j}}$

$\vec{\mathbf{F}}_2 = (40 \cos 45°)\vec{\mathbf{i}} + (40 \sin 45°)\vec{\mathbf{j}}$ or $20\sqrt{2}\,\vec{\mathbf{i}} + 20\sqrt{2}\,\vec{\mathbf{j}}$

$\vec{\mathbf{F}}_3 = (60 \cos 135°)\vec{\mathbf{i}} + (60 \sin 135°)\vec{\mathbf{j}}$ or $-30\sqrt{2} + 30\sqrt{2}\,\vec{\mathbf{j}}$

$\tan \theta = \frac{-35 + 50\sqrt{2}}{35\sqrt{3} - 10\sqrt{2}}$

$\theta = \tan^{-1}\left(\frac{-35 + 50\sqrt{2}}{35\sqrt{3} - 10\sqrt{2}}\right)$

$\approx 37.5°$

$|\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3| = \sqrt{(35\sqrt{3} - 10\sqrt{2})^2 + (-35 + 50\sqrt{2})^2}$

≈ 58.6 lb

21. $\vec{\mathbf{F}}_1 = (23 \cos 60°)\vec{\mathbf{i}} + (23 \sin 60°)\vec{\mathbf{j}}$

or $11.5\vec{\mathbf{i}} + 11.5\sqrt{3}\vec{\mathbf{j}}$

$\vec{\mathbf{F}}_2 = (23 \cos 120°)\vec{\mathbf{i}} + (23 \sin 120°)\vec{\mathbf{j}}$

or $-11.5\vec{\mathbf{i}} + 11.5\sqrt{3}\vec{\mathbf{j}}$

$|\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2| = \sqrt{0^2 + (23\sqrt{3})^2}$

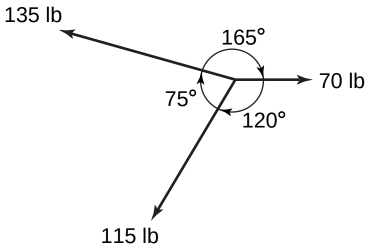
$= 23\sqrt{3}$

≈ 39.8 N

Since $\tan \theta$ is undefined and the vertical component is positive , $\theta = 90°$. A force with magnitude 39.8 N and direction 90° + 180° or 270° will produce equilibrium.

22. $a = g \sin 40°$
$= 32 \sin 40°$
$\approx 20.6 \text{ ft/s}^2$

23. $\vec{\mathbf{F_1}} = (36 \cos 20°)\vec{\mathbf{i}} + (36 \sin 20°)\vec{\mathbf{j}}$
$\vec{\mathbf{F_2}} = (48 \cos 222°)\vec{\mathbf{i}} + (48 \sin 222°)\vec{\mathbf{j}}$
$|\vec{\mathbf{F_1}} + \vec{\mathbf{F_2}}| =$
$\sqrt{(36 \cos 20° + 48 \cos 222°)^2 + (36 \sin 20° + 48 \sin 222°)^2}$
≈ 19.9 N
$\tan \theta = \frac{36 \sin 20° + 48 \sin 222°}{36 \cos 20° + 48 \cos 222°}$
$\theta = \tan^{-1}\left(\frac{36 \sin 20° + 48 \sin 222°}{36 \cos 20° + 48 \cos 222°}\right)$
$\approx 264.7°$ or $5.3°$ west of south

24a.


24b. $\vec{\mathbf{F_1}} = 70\vec{\mathbf{i}}$
$\vec{\mathbf{F_2}} = (135 \cos 165°)\vec{\mathbf{i}} + (135 \sin 165°)\vec{\mathbf{j}}$
$|\vec{\mathbf{F_1}} + \vec{\mathbf{F_2}} + \vec{\mathbf{F_3}}| =$
$\sqrt{(70 + 135 \cos 165° + 115 \cos 240°)^2 + (135 \sin 165° + 115 \sin 240°)^2}$
≈ 134.5 lb
$\tan \theta = \frac{135 \sin 165° + 115 \sin 240°}{70 + 135 \cos 165° + 115 \cos 240°}$
$\theta = \tan^{-1}\left(\frac{135 \sin 165° + 115 \sin 240°}{70 + 135 \cos 165° + 115 \cos 240°}\right)$
$\approx 208.7°$ or $28.7°$ south of west
Since $\vec{\mathbf{F_1}} + \vec{\mathbf{F_2}} + \vec{\mathbf{F_3}} \neq 0$, the vectors are not in equilibrium.

25. $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}}$
$= [(1600 \cos 50°)\vec{\mathbf{i}} + (1600 \sin 50°)\vec{\mathbf{j}}] \cdot 1500\vec{\mathbf{i}}$
$= (1600 \cos 50°)(1500) + (1600 \sin 50°)(0)$
$\approx 1{,}542{,}690$ N-m

26a. Sample answer: The horizontal forward force is $\vec{F} \cos \theta$. You can increase the horizontal forward force by decreasing the angle θ between the handle and the lawn.

26b. Sample answer: Pushing the lawnmower at a lower angle may cause back pain.

27a. $\tan \theta = \frac{3}{18}$ or $\frac{1}{6}$
$\theta = \tan^{-1} \frac{1}{6}$
$\approx 9.5°$ south of east

27b. $s = \sqrt{18^2 + 3^2}$
≈ 18.2 mph

28. $F \cos \theta = 100 \cos 25°$
≈ 90.63 N

29. $F_1 \cos 174.5° + F_2 \cos 6.2° = 0$
$F_1 \sin 174.5° + F_2 \sin 6.2° - 155 = 0$
The first equation gives $F_2 = -\frac{\cos 174.5°}{\cos 6.2°} F_1$.
Substitute into the second equation.
$F_1 \sin 174.5° - \frac{\cos 174.5° \sin 6.2°}{\cos 6.2°} F_1 - 155 = 0$
$F_1 (\sin 174.5° - \cos 174.5° \tan 6.2°) = 155$
$F_1 = \frac{155}{\sin 174.5° - \cos 174.5° \tan 6.2°}$
≈ 760 lb
$F_2 = -\frac{\cos 174.5°}{\cos 6.2°} F_1$
≈ 761 lb

30. Sample answer: Method b is better. Let F be the force exerted by the tractor, T be the tension in the two halves of the rope, and θ be the angle between the original line of the rope and half of the rope after it is pulled. At equilibrium, $2T \sin \theta - F = 0$, or $T = \frac{F}{2 \sin \theta}$. So, if $0° < \theta < 30°$, the force applied to the stump using method b is greater than the force exerted by the tractor.

31. Let T be the tension in each towline and suppose the axis of the ship is the vertical direction.
$2T \sin 70° - 6000 = 0$
$T = \frac{6000}{2 \sin 70°}$
≈ 3192.5 tons

32. Let T be the tension in each wire. The halves of the wire make angles of 30° and 150° with the horizontal.
$T \sin 30° + T \sin 150° - 25 = 0$
$\frac{1}{2}T + \frac{1}{2}T - 25 = 0$
$T = 25$ lb

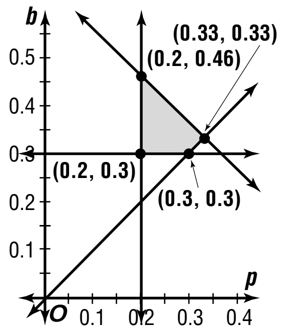
33. $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = 9(-3) + 5(2) + 3(5)$
$= -2$
The vectors are not perpendicular since $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} \neq 0$.

34. $\overrightarrow{AB} = \langle 0 - 12, -11 - (-5), 21 - 18 \rangle$
$= \langle -12, -6, 3 \rangle$

35. $d = \frac{2{v_0}^2}{g} \sin \theta \cos \theta$
$= \frac{2 \cdot 100^2}{32} \sin 65° \cos 65°$
≈ 239.4 ft

36. Sample answer: A plot of the data suggests a quadratic function. Performing a quadratic regression and rounding the coefficients gives $y = 1.4x^2 - 2x + 3.9$.

37. $b \geq 0.3$
$p \geq 0.2$
$b + p \leq 0.66$
$b \geq p$



The vertices are at (0.2, 0.3), (0.3, 0.3), (0.33, 0.33) and (0.2, 0.46).
cost function $C(p, b) = 90p + 140b + 32(1 - p - b)$
$= 32 + 58p + 108b$
$C(0.2, 0.3) = 32 + 58(0.2) + 108(0.3)$ or 76
$C(0.3, 0.3) = 32 + 58(0.3) + 108(0.3)$ or 81.8
$C(0.33, 0.33) = 32 + 58(0.33) + 108(0.33)$ or 86.78
$C(0.2, 0.46) = 32 + 58(0.2) + 108(0.46)$ or 93.28
The minimum cost is $76, using 30% beef and 20% pork.

38. $*4 - *(-3) = (4^3 - 4) - [(-3)^3 - (-3)]$
$= 60 - (-24)$
$= 84$
The correct choice is A.

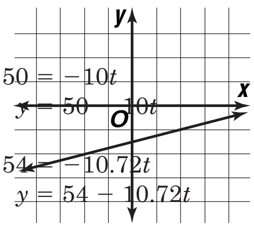
8-6 Vectors and Parametric Equations

Pages 523–524 Check for Understanding

1. When $t = 0$, $x = 3$ and $y = -1$. When $t = 1$, $x = 7$ and $y = 1$. The graph is a line through $(3, -1)$ and $(7, 1)$.
2. Sample answer: For every single unit increment of t, x increases 1 unit and y increases 2 units. Then, the parametric equations of the line are $x = 3 + t$, $y = 6 + 2t$.
3. When $t = 0$, $x = 1$ and $y = 0$, so the line passes through $(1, 0)$. When $t = -1$, $x = 0$ and $y = 1$, so the line passes through $(0, 1)$, its y-intercept. The slope of the line is $\frac{1-0}{0-1}$ or -1.
4. $\langle x - (-4), y - 11\rangle = t\langle -3, 8\rangle$
$\langle x + 4, y - 11\rangle = t\langle -3, 8\rangle$
$x + 4 = -3t$ $\quad$ $y - 11 = 8t$
$x = -3t - 4$ $\quad$ $y = 8t + 11$
5. $\langle x - 1, y - 5\rangle = t\langle -7, 2\rangle$
$x - 1 = -7t$ $\quad$ $y - 5 = 2t$
$x = 1 - 7t$ $\quad$ $y = 5 + 2t$
6. $3x + 2y = 5$
$2y = -3x + 5$
$y = -\frac{3}{2}x + \frac{5}{2}$
$x = t$
$y = -\frac{3}{2}t + \frac{5}{2}$
7. $4x - 6y = -12$
$-6y = -4x - 12$
$y = \frac{2}{3}x + 2$
$x = t$
$y = \frac{2}{3}t + 2$
8. $x = -4t + 3$
$x - 3 = -4t$
$-\frac{1}{4}x + \frac{3}{4} = t$
$y = 5t - 3$
$y = 5\left(-\frac{1}{4}x + \frac{3}{4}\right) - 3$
$y = -\frac{5}{4}x + \frac{3}{4}$
9. $x = 9t$
$\frac{x}{9} = t$
$y = 4t + 2$
$y = 4\left(\frac{x}{9}\right) + 2$
$y = \frac{4}{9}x + 2$
10.

t	x	y
-1	-2	-2
0	2	-1
1	6	0
2	10	1

11a. receiver:
$x - 5 = 0t$ $\quad$ $y - 50 = -10t$
$x = 5$ $\quad$ $y = 50 - 10t$
defensive player:
$x - 10 = -0.9t$ $\quad$ $y - 54 = -10.72t$
$x = 10 - 0.9t$ $\quad$ $y = 54 - 10.72t$



11b. $50 - 10t = 0$
$50 = 10t$
$5 = t$
When $t = 5$, the coordinates of the defensive player are $(10 - 0.9(5), 54 - 10.72(5))$ or $(5.5, 0.4)$, so the defensive player has not yet caught the receiver.

524–525 Exercises

12. $\langle x - 5, y - 7\rangle = t\langle 2, 0\rangle$
$x - 5 = 2t$ $\quad$ $y - 7 = 0t$
$x = 5 + 2t$ $\quad$ $y = 7$
13. $\langle x - (-1), y - 4\rangle = t\langle 6, -10\rangle$
$\langle x + 1, y - 4\rangle = t\langle 6, -10\rangle$
$x + 1 = 6t$ $\quad$ $y - 4 = -10t$
$x = -1 + 6t$ $\quad$ $y = 4 - 10t$
14. $\langle x - (-6), y - 10\rangle = t\langle 3, 2\rangle$
$\langle x + 6, y - 10\rangle = t\langle 3, 2\rangle$
$x + 6 = 3t$ $\quad$ $y - 10 = 2t$
$x = -6 + 3t$ $\quad$ $y = 10 + 2t$
15. $\langle x - 1, y - 5\rangle = t\langle -7, 2\rangle$
$x - 1 = -7t$ $\quad$ $y - 5 = 2t$
$x = 1 - 7t$ $\quad$ $y = 5 + 2t$
16. $\langle x - 1, y - 0\rangle = t\langle -2, -4\rangle$
$\langle x - 1, y\rangle = t\langle -2, -4\rangle$
$x - 1 = -2t$ $\quad$ $y = -4t$
$x = 1 - 2t$
17. $\langle x - 3, y - (-5)\rangle = t\langle -2, 5\rangle$
$\langle x - 3, y + 5\rangle = t\langle -2, 5\rangle$
$x - 3 = -2t$ $\quad$ $y + 5 = 5t$
$x = 3 - 2t$ $\quad$ $y = -5 + 5t$
18. $x = t$
$y = 4t - 5$
19. $-3x + 4y = 7$
$4y = 3x + 7$
$y = \frac{3}{4}x + \frac{7}{4}$
$x = t$
$y = \frac{3}{4}t + \frac{7}{4}$
20. $2x - y = 3$
$-y = -2x + 3$
$y = 2x - 3$
$x = t$
$y = 2t - 3$
21. $9x + y = -1$
$y = -9x - 1$
$x = t$
$y = -9t - 1$
22. $2x + 3y = 11$
$3y = -2x + 11$
$y = -\frac{2}{3}x + \frac{11}{3}$
$x = t$
$y = -\frac{2}{3}t + \frac{11}{3}$
23. $-4x + y = -2$
$y = 4x - 2$
$x = t$
$y = 4t - 2$
24. $3x - 6y = -8$
$-6y = -3x - 8$
$y = \frac{1}{2}x + \frac{4}{3}$
The slope is $\frac{1}{2}$.
$y - 5 = \frac{1}{2}(x + 2)$
$y = \frac{1}{2}x + 6$
$x = t$
$y = \frac{1}{2}t + 6$

25. $x = 2t$
$\frac{x}{2} = t$
$y = 1 - t$
$y = 1 - \frac{x}{2}$
$y = -\frac{1}{2}x + 1$

26. $x = -7 + \frac{1}{2}t$
$x + 7 = \frac{1}{2}t$
$2x + 14 = t$
$y = 3t$
$y = 3(2x + 14)$
$y = 6x + 42$

27. $x = 4t - 11$
$x + 11 = 4t$
$\frac{1}{4}x + \frac{11}{4} = t$
$y = t + 3$
$y = \frac{1}{4}x + \frac{11}{4} + 3$
$y = \frac{1}{4}x + \frac{23}{4}$

28. $x = 4t - 8$
$x + 8 = 4t$
$\frac{1}{4}x + 2 = t$
$y = 3 + t$
$y = 3 + \frac{1}{4}x + 2$
$y = \frac{1}{4}x + 5$

29. $x = 3 + 2t$
$x - 3 = 2t$
$\frac{1}{2}x - \frac{3}{2} = t$
$y = -1 + 5t$
$y = -1 + 5\left(\frac{1}{2}x - \frac{3}{2}\right)$
$y = \frac{5}{2}x - \frac{17}{2}$

30. Regardless of the value of t, x is always 8, so the parametric equations represent the vertical line with equation $x = 8$.

31a. $\langle x - 11, y - (-4)\rangle = t\langle 3, 7\rangle$
$\langle x - 11, y + 4\rangle = t\langle 3, 7\rangle$

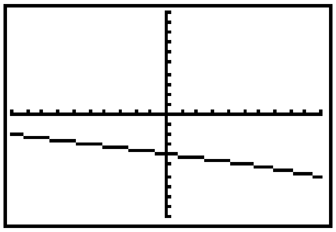
31b. $x - 11 = 3t$ $\quad$ $y + 4 = 7t$
$x = 3t + 11$ $\quad$ $y = 7t - 4$

31c. $x = 3t + 11$
$x - 11 = 3t$
$\frac{1}{3}x - \frac{11}{3} = t$
$y = 7t - 4$
$y = 7\left(\frac{1}{3}x - \frac{11}{3}\right) - 4$
$y = \frac{7}{3}x - \frac{89}{3}$

32.

T	X1T	Y1T
1.0000	5.0000	-5.000
2.0000	10.000	-6.000
3.0000	15.000	-7.000
4.0000	20.000	-8.000
5.0000	25.000	-9.000
6.0000	30.000	-10.00
14.000	70.000	-18.00

T=1

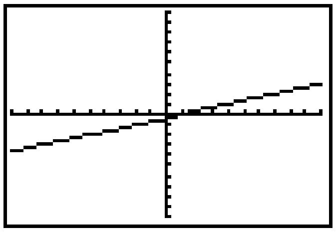


[−5, 5] Tstep: 1
[−10, 10] Xscl: 1
[−10, 10] Yscl: 1

33.

T	X1T	Y1T
1.0000	8.0000	2.0000
2.0000	11.000	3.0000
3.0000	14.000	4.0000
4.0000	17.000	5.0000
5.0000	20.000	6.0000
6.0000	23.000	7.0000
14.000	47.000	15.000

Y1T=2

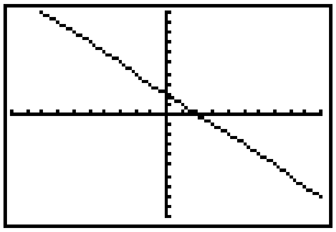


[−10, 10] Tstep: 1
[−20, 20] Xscl: 2
[−20, 20] Yscl: 2

34.

T	X1T	Y1T
1.0000	2.0000	0.0000
2.0000	3.0000	-1.000
3.0000	4.0000	-2.000
4.0000	5.0000	-3.000
5.0000	6.0000	-4.000
6.0000	7.0000	-5.000
14.000	15.000	-13.00

T=1

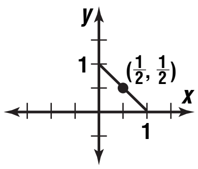


[−10, 10] Tstep: 1
[−10, 10] Xscl: 1
[−10, 10] Yscl: 1

35a. $x = 2 + 3t$ and $y = 4 + 7t$
If $t \geq 0$, then $x \geq 2$ and $y \geq 4$, so the part of the line to the right of point (2, 4) is obtained.

35b. $x < 0$
$2 + 3t < 0$
$3t < -2$
$t < -\frac{2}{3}$

36. $x + y = \cos^2 t + \sin^2 t$
$= 1$
$0 \leq \cos^2 t \leq 1$ and $0 \leq \sin^2 t \leq 1$, so the graph is the segment of the line with equation $x + y = 1$ from (1, 0) to (0, 1).



37a. target drone:

$x = 3 + (-1)t \qquad y = 4 + 0t$

$x = 3 - t \qquad y = 4$

missile:

$x = 2 + t \qquad y = 2 + 2t$

37b. $3 - t = 2 + t$

$1 = 2t$

$\frac{1}{2} = t$

When $t = \frac{1}{2}$, the missile has a y-coordinate of 3, not 4, so it does not intercept the drone.

38a. Ceres: $x = -1 + t, y = 4 - t, z = -1 + 2t$
Pallas: $x = -7 + 2t, y = -6 + 2t, z = -1 + t$

38b. Adding the equations for x and y for Ceres gives $x + y = 3$. Subtracting the equations for x and y for Pallas results in $x - y = -1$. The solution of this system is $x = 1$ and $y = 2$. Eliminating t from the equations for y and z results in the system $2y + z = 7$, $y - 2z = -4$ which has solution $y = 2$ and $z = 3$. Hence, the paths cross at (1, 2, 3).

38c. $-1 + t = 1 \Rightarrow t = 2$

$-7 + 2t = 1 \Rightarrow t = 4$

Ceres is at (1, 2, 3) when $t = 2$ but Pallas is at (1, 2, 3) when $t = 4$. The asteroids will not collide.

39. The line is parallel to the vector $\left\langle 0 - \left(-\frac{1}{3}\right), 5 - 1, -8 - 1\right\rangle$ or $\left\langle \frac{1}{3}, 4, -9\right\rangle$. The vector equation of the line is $\left\langle x - \left(-\frac{1}{3}\right), y = 1, z - 1\right\rangle = t\left\langle \frac{1}{3}, 4, -9\right\rangle$ or $\left\langle x + \frac{1}{3}, y - 1, z - 1\right\rangle = t\left\langle \frac{1}{3}, 4, -9\right\rangle$.

$x + \frac{1}{3} = \frac{1}{3}t \qquad y - 1 = 4t$

$x = -\frac{1}{3} + \frac{1}{3}t \qquad y = 1 + 4t$

$z - 1 = -9t$

$z = 1 - 9t$

40. $\vec{\mathbf{v}}_1 = (150 \cos 330°)\vec{\mathbf{i}} + (150 \sin 330°)\vec{\mathbf{j}}$

$\vec{\mathbf{v}}_2 = (50 \cos 245°)\vec{\mathbf{i}} + (50 \sin 245°)\vec{\mathbf{j}}$

$|\vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_2| =$

$\sqrt{(150 \cos 330° + 50 \cos 245°)^2 + (150 \sin 330° + 50 \sin 245°)^2}$

≈ 162.2 km/h

$\tan \theta = \frac{150 \sin 330° + 50 \sin 245°}{150 \cos 330° + 50 \cos 245°}$

$\theta = \tan^{-1}\left(\frac{150 \sin 330° + 50 \sin 245°}{150 \cos 330° + 50 \cos 245°}\right)$

$\approx -47°53'4''$ or $47°53'4''$ south of east

41. $\langle 1, 3\rangle \cdot \langle 3, -2\rangle = 1(3) + 3(-2)$

$= -3$

Since the inner product is not 0, the vectors are not perpendicular.

42. Since $A < 90°$, $a < b$, and $a < b \sin A$, no solution exists.

43. A graphing calculator indicates that there is one real zero and that it is close to 1. $f(1) = 0$, so the zero is exactly 1.

44. $x = \frac{3}{2}y - 2$

$x + 2 = \frac{3}{2}y$

$\frac{2}{3}x + \frac{4}{3} = y$

$y = \frac{2}{3}x + \frac{4}{3}$

45. The slope is 1.

$y - 1 = 1[x - (-3)]$

$y - 1 = x + 3$

$x - y + 4 = 0$

46. The linear velocity of the belt around the larger pulley is $(120 \text{ rpm})\left(2\pi \cdot \frac{9}{2} \text{ in./rev}\right) = 1080\pi$ in./min. The linear velocity around the smaller pulley must be the same, so its angular velocity is $(1080\pi \text{ in./min})\left(\frac{1 \text{ rev}}{2\pi \cdot 3 \text{ in.}}\right) = 180$ rpm. The correct choice is D.

8-6B Graphing Calculator Exploration: Modeling with Parametric Equations

Page 526

1. $408.7t = 418.3(t - 0.0083)$

$408.7t = 418.3t - 3.47189$

$-9.6t = -3.47189$

$t = \frac{3.47189}{9.6}$

≈ 0.362 hr or 21.7 min

2. $d = rt$

$= 408.7\left(\frac{3.47189}{9.6}\right)$

≈ 147.8 mi

3. The time for plane 1 to fly 500 miles is $\frac{500}{408.7}$. The time for plane 2 is $\frac{500}{418.3} + 0.0083$. Suppose the speed of plane 1 is increased by a mph.

$\frac{500}{408.7 + a} = \frac{500}{418.3} + 0.0083$

$\frac{408.7 + a}{500} = \frac{1}{\frac{500}{418.3} + 0.0083}$

$a = \frac{500}{\frac{500}{418.3} + 0.0083} - 408.7$

≈ 6.7 mph

8-7 Modeling Motion Using Parametric Equations

Page 531 Check for Understanding

1. Sample answer: a rocket launched at 90° to the horizontal; tip-off in basketball

2. Equal magnitude with opposite direction.

3. The greater the angle of the head of the golf club, the greater the angle of initial velocity of the ball.

4. $|\vec{\mathbf{v}}_y| = |\vec{\mathbf{v}}| \sin \theta$

$= 50 \sin 40°$

≈ 32.14 ft/s

5. $|\vec{\mathbf{v}}_x| = |\vec{\mathbf{v}}| \cos \theta$

$= 20 \cos 50°$

≈ 12.86 m/s

6. $|\vec{\mathbf{v}}_x| = |\vec{\mathbf{v}}| \cos \theta$

$= 45 \cos 32°$

≈ 38.16 ft/s

$|\vec{\mathbf{v}}_y| = |\vec{\mathbf{v}}| \sin \theta$

$= 45 \sin 32°$

≈ 23.85 ft/s

7. $|\vec{\mathbf{v}}_x| = |\vec{\mathbf{v}}| \cos \theta$

$= 7.5 \cos 20°$

≈ 7.05 m/s

$|\vec{\mathbf{v}}_y| = |\vec{\mathbf{v}}| \sin \theta$

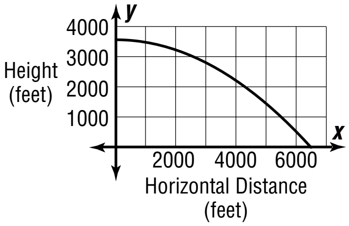
$= 7.5 \sin 20°$

≈ 2.57 m/s

8a. $300 \text{ mph} \left(\frac{5280 \text{ ft}}{\text{mile}}\right)\left(\frac{\text{h}}{3600 \text{ s}}\right) = 440 \text{ ft/s}$

$$\begin{aligned} x &= t|\vec{\mathbf{v}}| \cos \theta \\ x &= t(440) \cos 0^\circ \\ x &= 440t \\ y &= t|\vec{\mathbf{v}}| \sin \theta - \tfrac{1}{2}gt^2 + h \\ y &= t(440) \sin 0^\circ - \tfrac{1}{2}(32)t^2 + 3500 \\ y &= -16t^2 + 3500 \end{aligned}$$

8b. Sample graph:



8c.
$$\begin{aligned} -16t^2 + 3500 &= 0 \\ -16t^2 &= -3500 \\ t^2 &= \tfrac{-3500}{-16} \\ t &= \sqrt{\tfrac{3500}{16}} \\ t &\approx 14.8 \text{ s} \end{aligned}$$

8d.
$$\begin{aligned} x &= 440t \\ &= 440(14.8) \\ &= 6512 \text{ ft} \end{aligned}$$

Pages 531–533 Exercises

9. $|\vec{\mathbf{v}}_x| = |\vec{\mathbf{v}}| \cos \theta = 65 \cos 60^\circ = 32.5$ ft/s; $|\vec{\mathbf{v}}_y| = |\vec{\mathbf{v}}| \sin \theta = 65 \sin 60^\circ = 56.29$ ft/s

10. $|\vec{\mathbf{v}}_x| = |\vec{\mathbf{v}}| \cos \theta = 47 \cos 10.7^\circ \approx 46.18$ m/s; $|\vec{\mathbf{v}}_y| = |\vec{\mathbf{v}}| \sin \theta = 47 \sin 10.7^\circ \approx 8.73$ m/s

11. $|\vec{\mathbf{v}}_x| = |\vec{\mathbf{v}}| \cos \theta = 1200 \cos 42^\circ \approx 891.77$ ft/s; $|\vec{\mathbf{v}}_y| = |\vec{\mathbf{v}}| \sin \theta = 1200 \sin 42^\circ \approx 802.96$ ft/s

12. $|\vec{\mathbf{v}}_x| = |\vec{\mathbf{v}}| \cos \theta = 17 \cos 28^\circ \approx 15.01$ ft/s; $|\vec{\mathbf{v}}_y| = |\vec{\mathbf{v}}| \sin \theta = 17 \sin 28^\circ \approx 7.98$ ft/s

13. $|\vec{\mathbf{v}}_x| = |\vec{\mathbf{v}}| \cos \theta = 69 \cos 37^\circ \approx 55.11$ yd/s; $|\vec{\mathbf{v}}_y| = |\vec{\mathbf{v}}| \sin \theta = 69 \sin 37^\circ \approx 41.53$ yd/s

14. $|\vec{\mathbf{v}}_x| = |\vec{\mathbf{v}}| \cos \theta = 46 \cos 19^\circ \approx 43.49$ km/h; $|\vec{\mathbf{v}}_y| = |\vec{\mathbf{v}}| \sin \theta = 46 \sin 19^\circ \approx 14.98$ km/h

15a. $x = t|\vec{\mathbf{v}}| \cos \theta$, $x = 175t \cos 35^\circ$; $y = t|\vec{\mathbf{v}}| \sin \theta - \frac{1}{2}gt^2$, $y = 175t \sin 35^\circ - 16t^2$

15b.
$$\begin{aligned} y &= 0 \\ 175t \sin 35^\circ - 16t^2 &= 0 \\ t(175 \sin 35^\circ - 16t) &= 0 \\ 175 \sin 35^\circ - 16t &= 0 \\ 175 \sin 35^\circ &= 16t \\ \tfrac{175 \sin 35^\circ}{16} &= t \end{aligned}$$
$$\begin{aligned} x &= 175t \cos 35^\circ \\ &= 175\left(\tfrac{175 \sin 35^\circ}{16}\right) \cos 35^\circ \\ &\approx 899.32 \text{ ft or } 299.77 \text{ yd} \end{aligned}$$

16. To find the time the projectile stays in the air, set $y = 0$ and solve for t.
$$\begin{aligned} t|\vec{\mathbf{v}}| \sin \theta - \tfrac{1}{2}gt^2 &= 0 \\ t(|\vec{\mathbf{v}}| \sin \theta - \tfrac{1}{2}gt) &= 0 \\ |\vec{\mathbf{v}}| \sin \theta - \tfrac{1}{2}gt &= 0 \\ |\vec{\mathbf{v}}| \sin \theta &= \tfrac{1}{2}gt \\ \tfrac{2|\vec{\mathbf{v}}| \sin \theta}{g} &= t \end{aligned}$$
The greater the angle, the greater the time the projectile stays in the air. To find the horizontal distance covered, substitute the expression for t in the equation for x.
$$\begin{aligned} x &= t|\vec{\mathbf{v}}| \cos \theta \\ &= \tfrac{2|\vec{\mathbf{v}}| \sin \theta}{g} |\vec{\mathbf{v}}| \cos \theta \\ &= \tfrac{|\vec{\mathbf{v}}|^{2^g} \sin 2\theta}{g} \end{aligned}$$
As the angle increases from 0° to 45°, the horizontal distance increases. As the angle increases from 45° to 90°, the horizontal distance decreases.

17a. $y = 300$ when $t = 7$
$$\begin{aligned} 7|\vec{\mathbf{v}}| \sin 78^\circ - \tfrac{1}{2}(32)7^2 &= 300 \\ 7|\vec{\mathbf{v}}| \sin 78^\circ - 784 &= 300 \\ 7|\vec{\mathbf{v}}| \sin 78^\circ &= 1084 \\ |\vec{\mathbf{v}}| &= \tfrac{1084}{7 \sin 78^\circ} \\ |\vec{\mathbf{v}}| &\approx 158.32 \text{ ft/s} \end{aligned}$$

17b.
$$\begin{aligned} x &= \tfrac{1}{3}t|\vec{\mathbf{v}}| \cos \theta + 50 \text{ yd} \\ &= \tfrac{1}{3}(7)(158.32) \cos 78^\circ + 50 \\ &\approx 127 \text{ yd} \end{aligned}$$

18.
$$\begin{aligned} x &= t|\vec{\mathbf{v}}| \cos \theta \\ \tfrac{x}{|\vec{\mathbf{v}}| \cos \theta} &= t \\ y &= t|\vec{\mathbf{v}}| \sin \theta - \tfrac{1}{2}gt^2 \\ y &= \tfrac{x}{|\vec{\mathbf{v}}| \cos \theta} |\vec{\mathbf{v}}| \sin \theta - \tfrac{1}{2}g\left(\tfrac{x}{|\vec{\mathbf{v}}| \cos \theta}\right)^2 \\ y &= x \tan \theta - \tfrac{g}{2|\vec{\mathbf{v}}|^2 \cos^2 \theta} \end{aligned}$$
The presence of the x^2-term (due to the force of gravity) means that y is a quadratic function of x. Therefore, the path of a projectile is a parabolic arc.

19. To find the time the projectile stays in the air if the initial velocity is $\vec{\mathbf{v}}$, set $y = 0$ and solve for t.
$$\begin{aligned} t|\vec{\mathbf{v}}| \sin \theta - \tfrac{1}{2}gt^2 &= 0 \\ t\left(|\vec{\mathbf{v}}| \sin \theta - \tfrac{1}{2}gt\right) &= 0 \\ |\vec{\mathbf{v}}| \sin \theta - \tfrac{1}{2}gt &= 0 \\ |\vec{\mathbf{v}}| \sin \theta &= \tfrac{1}{2}gt \\ \tfrac{2|\vec{\mathbf{v}}| \sin \theta}{g = t} &= t \end{aligned}$$
To find the range, substitute this expression for t in the equation for x.
$$\begin{aligned} x &= t|\vec{\mathbf{v}}| \cos \theta \\ &= \tfrac{2|\vec{\mathbf{v}}| \sin \theta}{g} |\vec{\mathbf{v}}| \cos \theta \\ &= \tfrac{|\vec{\mathbf{v}}|^2 \sin 2\theta}{g} \end{aligned}$$

If the magnitude of the initial velocity is doubled to $2|\vec{\mathbf{v}}|$, the range becomes $\frac{(2|\vec{\mathbf{v}}|)^2 \sin 2\theta}{g}$ or $4\frac{|\vec{\mathbf{v}}|^2 \sin 2\theta}{g}$. The projectile will travel four times as far.

20a. $800 \text{ km/h}\left(\frac{1000 \text{ m}}{\text{km}}\right)\left(\frac{\text{h}}{3600 \text{ s}}\right) \approx 222.2 \text{ m/s}$

$x = t|\vec{\mathbf{v}}| \cos\theta$

$x = -222.2\, t \cos 45°$

$y = t|\vec{\mathbf{v}}| \sin\theta - \frac{1}{2}gt^2$

$y = -222.2t \sin 45° - \frac{1}{2}(9.8)t^2$

$y = -222.2t \sin 45° - 4.9t^2$

The negative coefficient in the t-term in the equation for y indicates that the aircraft is descending. The negative coefficient in the equation for x is arbitrary.

20b. $y = -222.2t \sin 45° - 4.9t^2$

$= -222.2(2.5) \sin 45° - 4.9\,(2.5)^2$

≈ -423.4

The aircraft has descended about 423.4 m.

20c. $\frac{423.4 \text{ m}}{2.5 \text{ s}} \approx 169 \text{ m/s}$

or

$169 \text{ m/s}\left(\frac{\text{km}}{1000 \text{ m}}\right)\left(\frac{3600 \text{ s}}{\text{h}}\right) = 608.4 \text{ km/h}$

21a. $70 \text{ mph}\left(\frac{5280 \text{ ft}}{\text{mi}}\right)\left(\frac{\text{h}}{3600 \text{ s}}\right) = \frac{308}{3} \text{ ft/s}$

$y = 0$

$t\left(\frac{308}{3}\right) \sin 35° - 16t^2 + 10 = 0$

$t = \frac{\frac{-308}{3}\sin 35° - \sqrt{\left(\frac{308}{3}\sin 35°\right)^2 - 4(-16)10}}{}$

$t \approx$

3.84 s

$x = t|\vec{\mathbf{v}}| \cos\theta$

$\approx 323.2 \text{ ft}$

21b. $y = 8$

$t\left(\frac{308}{3}\right) \sin 35° - 16t^2 + 10 = 8$

$-16t^2 + t\left(\frac{308}{3}\right) \sin 35° + 2 = 0$

$t = \frac{-\frac{308}{3}\sin 35° - \sqrt{\left(\frac{308}{3}\sin 35°\right)^2 - 4(-16)2}}{}$

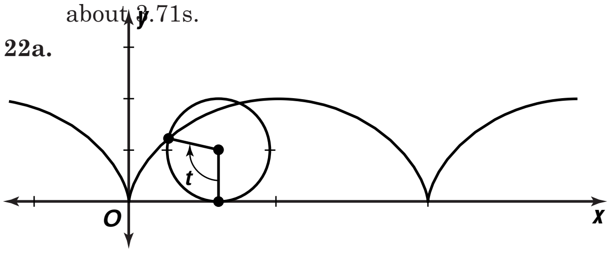
$t \approx 3.71 \text{ s}$

$x = t|\vec{\mathbf{v}}| \cos\theta$

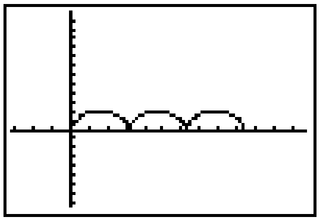
$\approx 312.4 \text{ ft}$

21c. From the calculations in part b, the time is about 3.71s.

22a.



22b.



23a. $y = 300$ when $t = 4.8$

$4.8|\vec{\mathbf{v}}| \sin 82° - \frac{1}{2}(32)(4.8)^2 = 300$

$4.8|\vec{\mathbf{v}}| \sin 82° - 368.64 = 668.64$

$|\vec{\mathbf{v}}| = \frac{668.64}{4.8 \sin 82°}$

$|\vec{\mathbf{v}}| \approx 140.7 \text{ ft/s}$

23b. $x = \frac{1}{3}t|\vec{\mathbf{v}}| \cos\theta + 100$

$\approx 131.3 \text{ yd}$

24a. $x = t|\vec{\mathbf{v}}| \cos\theta \qquad y = t|\vec{\mathbf{v}}| \sin\theta - \frac{1}{2}gt^2 + h$

$x = 155t \cos 22° \qquad y = 155t \sin 22° - 16t^2 + 3$

24b. $x = 420$

$155t \cos 22° = 420$

$t = \frac{420}{155 \cos 22°}$

$y = 155t \sin 22° - 16t^2 + 3$

$= 155\left(\frac{420}{155 \cos 22°}\right) \sin 22° - 16\left(\frac{420}{155 \cos 22°}\right)^2 + 3$

$\approx 36.04 \text{ ft}$

Since $36.04 > 15$, the ball will clear the fence.

24c. $y = 0$

$155t \sin 22° - 16t^2 + 3 = 0$

$t = \frac{-155 \sin 22° - \sqrt{(155 \sin 22°)^2 - 4(-16)3}}{}$

$t \approx$

3.68 s

$x = t|\vec{\mathbf{v}}| \cos\theta$

$\approx 528.86 \text{ ft}$

25. $x = 11 - t$

$x - 11 = -t$

$-x + 11 = t$

$y = 8 - 6t$

$y = 8 - 6(-x + 11)$

$y = 6x - 58$

26a. $mg \sin\theta = 300(9.8) \sin 22°$

$\approx 1101.3 \text{ N}$

26b. $mg \cos\theta = 300(9.8) \cos 22°$

$\approx 2725.9 \text{ N}$

27. $\cos A = \frac{17.4}{21.9}$

$A = \cos^{-1}\frac{17.4}{21.9}$

$A \approx 37°$

28. $2(2x - y + z) = 2(2)$

$$\begin{array}{l} x + 3y - 2z = -3.25 \\ \hline 5x + y \qquad = 0.75 \end{array}$$

$$-1(2x - y + z) = -1(2)$$

$$\begin{array}{l} -4x - 5y + z = 2.5 \\ \hline -6x - 4y \qquad = 0.5 \end{array}$$

$$4(5x + y) = 4(0.75)$$

$$\begin{array}{l} -6x - 4y = 0.5 \\ \hline 14x = 3.5 \end{array}$$

$$x = \frac{3.5}{14}$$

$$x = 0.25$$

$$\begin{aligned} 5x + y &= 0.75 & 2x - y + z &= 2 \\ 5(0.25) + y &= 0.75 & 2(0.25) + 0.5 + z &= 2 \\ y &= -0.5 & z &= 1 \end{aligned}$$

29. $\pi \cdot 5^2 - \pi \cdot 3^2 = 25\pi - 9\pi$

$$= 16\pi$$

The correct choice is B.

Page 534 History of Mathematics

1. $\frac{7°12'}{360°} = \frac{7.2°}{360°}$

$$= \frac{1}{50}$$

$$\frac{1}{50} = \frac{5000 \text{ stadia}}{x}$$

$$x = 50(5000)$$

$$x = 250{,}000 \text{ stadia}$$

$250{,}000(500) = 125{,}000{,}000$ ft

$125{,}000{,}000 \div 5280 \approx 23{,}674$ mi

The actual circumference of Earth is about 24,901.55 miles.

2. See students' work. No solution exists.
3. See students' work.

8-8 Transformation Matrices in Three-Dimensional Space

Pages 539–540 Check for Understanding

1. Matrix T multiplies x-coordinates by -2 and y- and z-coordinates by 2, so it produces a reflection over the yz-plane and increases the dimensions two-fold.
2. $\overrightarrow{CC'} = \langle 8 - 6, 8 - 7, 2 - 3\rangle$ or $\langle 2, 1, -1\rangle$

The matrix is $\begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}$.

3. $VU = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = T$, so the transformations are the same.

4a-c.

Transformation	Orientation	Site	Shape
Reflection	yes	no	no
Translation	no	no	no
Dilation	no	yes	no

5a. $\overrightarrow{BE} = \langle 0 - 5, 2 - 5, 4 - 0\rangle$ or $\langle -5, -3, 4\rangle$

$A(5, 5 + (-3), 0) = A(5, 2, 0)$

$C(5 + (-5), 5, 0) = C(0, 5, 0)$

$D(5 + (-5), 5 + (-3), 0) = D(0, 2, 0)$

$F(5, 5 + (-3), 0 + 4) = F(5, 2, 4)$

$G(5, 5, 0 + 4) = G(5, 5, 4)$

$H(5 + (-5), 5, 0 + 4) = H(0, 5, 4)$

The matrix is $\begin{bmatrix} 5 & 5 & 0 & 0 & 0 & 5 & 5 & 0 \\ 2 & 5 & 5 & 2 & 2 & 2 & 5 & 5 \\ 0 & 0 & 0 & 0 & 4 & 4 & 4 & 4 \end{bmatrix}$.

5b.

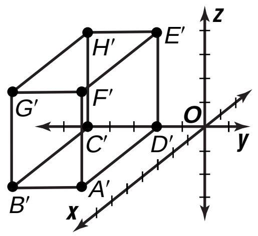
$$\begin{bmatrix} 5+4 & 5+4 & 0+4 & 0+4 & 0+4 & 5+4 & 5+4 & 0+4 \\ 2+(-1) & 5+(-1) & 5+(-1) & 2+(-1) & 2+(-1) & 2+(-1) & 5+(-1) & 5+(-1) \\ 0+2 & 0+2 & 0+2 & 0+2 & 4+2 & 4+2 & 4+2 & 4+2 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 9 & 4 & 4 & 4 & 9 & 9 & 4 \\ 1 & 4 & 4 & 1 & 1 & 1 & 4 & 4 \\ 2 & 2 & 2 & 2 & 6 & 6 & 6 & 6 \end{bmatrix}$$

5c.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 5 & 5 & 0 & 0 & 0 & 5 & 5 & 0 \\ 2 & 5 & 5 & 2 & 2 & 2 & 5 & 5 \\ 0 & 0 & 0 & 0 & 4 & 4 & 4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 5 & 0 & 0 & 0 & 5 & 5 & 0 \\ -2 & -5 & -5 & -2 & -2 & -2 & -5 & -5 \\ 0 & 0 & 0 & 0 & 4 & 4 & 4 & 4 \end{bmatrix}$$



The image is the reflection over the xz-plane.

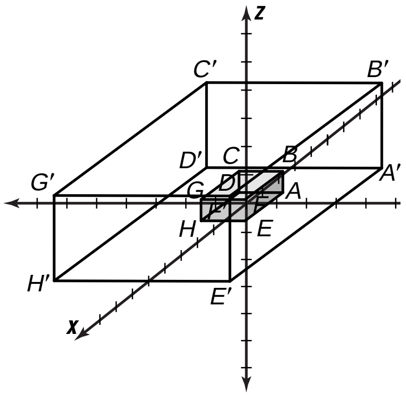
5d. The dimensions of the resulting figure are half the original.

6a. The scale factor of the dilation is 4. The translation increases x-coordinates by 2. The matrices are

$D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ and

$T = \begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

6b. Sample answer: If the original prism has vertices $A(-3, 3, 0)$ $B(-3, 3, 3)$, $C(-3, -3, 3)$, $D(-3, -3, 0)$, $E(5, 3, 0)$, $F(5, 3, 3)$, $G(5, -3, 3)$, and $H(5, -3, 0)$, then the image has vertices $A'(-10, 12, 0)$, $B'(-10, 12, 12)$, $C'(-10, -12, 12)$, $D'(-10, -12, 0)$, $E'(22, 12, 0)$, $F'(22, 12, 12)$, $G'(22, -12, 12)$, and $H'(22, -12, 0)$.



Pages 540–542 Exercises

7. $\overrightarrow{FB} = \langle 3-3, 1-7, 4-4\rangle$ or $\langle 0, -6, 0\rangle$
$A(2, 3, + (-6), 2) = A(2, -3, 2)$
$C(4, 7 + (-6), -1) = C(4, 1, -1)$
The matrix is $\begin{bmatrix} 2 & 3 & 4 & 4 & 2 & 3 \\ -3 & 1 & 1 & 7 & 3 & 7 \\ 2 & 4 & -1 & -1 & 2 & 4 \end{bmatrix}$.

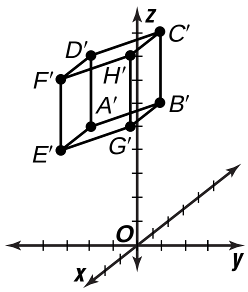
8. $\overrightarrow{AH} = \langle 4-(-3), 1-(-2), -2-2\rangle$ or $\langle 7, 3, -4\rangle$
$B(-3, -2+3, 2) = B(-3, 1, 2)$
$C(-3, -2+3, 2+(-4)) = C(-3, 1, -2)$
$D(-3, -2, 2+(-4)) = D(-3, -2, -2)$
$E(-3+7, -2, 2+(-4)) = E(4, -2, -2)$
$F(-3+7, -2, 2) = F(4, -2, 2)$
$G(-3+7, -2+3, 2) = G(4, 1, 2)$
The matrix is $\begin{bmatrix} -3 & -3 & -3 & -3 & 4 & 4 & 4 & 4 \\ -2 & 1 & 1 & -2 & -2 & -2 & 1 & 1 \\ 2 & 2 & -2 & -2 & -2 & 2 & 2 & -2 \end{bmatrix}$

9. $\overrightarrow{CF} = \langle 6-4, 0-(-1), 0-2\rangle$ or $\langle 2, 1, -2\rangle$
$D(2+2, -2+1, 3+(-2)) = D(4, -1, 1)$
$E(1+2, 0+1, 4+(-2)) = E(3, 1, 2)$
The matrix is $\begin{bmatrix} 2 & 1 & 4 & 4 & 3 & 6 \\ -2 & 0 & -1 & -1 & 1 & 0 \\ 3 & 4 & 2 & 1 & 2 & 0 \end{bmatrix}$.

10. $\left[\begin{matrix} 0+0 & 0+0 & 0+0 & 0+0 \\ 0+(-2) & 3+(-2) & 3+(-2) & 0+(-2) \\ 1+4 & 2+4 & 5+4 & 4+4 \end{matrix}\right.$

$\left.\begin{matrix} 2+0 & 2+0 & 2+0 & 2+0 \\ 0+(-2) & 0+(-2) & 3+(-2) & 3+(-2) \\ 1+4 & 4+4 & 5+4 & 2+4 \end{matrix}\right]$

$= \begin{bmatrix} 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 \\ -2 & 1 & 1 & -2 & -2 & -2 & 1 & 1 \\ 5 & 6 & 9 & 8 & 5 & 8 & 9 & 6 \end{bmatrix}$

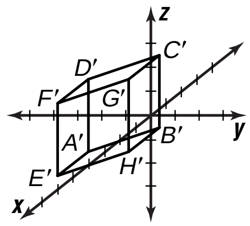


The result is a translation of -2 units along the y-axis and 4 units along the z-axis.

11. $\left[\begin{matrix} 0+1 & 0+1 & 0+1 & 0+1 \\ 0+(-2) & 3+(-2) & 3+(-2) & 0+(-2) \\ 1+(-2) & 2+(-2) & 5+(-2) & 4+(-2) \end{matrix}\right.$

$\left.\begin{matrix} 2+1 & 2+1 & 2+1 & 2+1 \\ 0+(-2) & 0+(-2) & 3+(-2) & 3+(-2) \\ 1+(-2) & 4+(-2) & 5+(-2) & 2+(-2) \end{matrix}\right]$

$= \begin{bmatrix} 1 & 1 & 1 & 1 & 3 & 3 & 3 & 3 \\ -2 & 1 & 1 & -2 & -2 & -2 & 1 & 1 \\ -1 & 0 & 3 & 2 & -1 & 2 & 3 & 0 \end{bmatrix}$

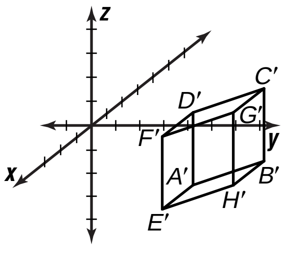


The result is a translation of 1 unit along the x-axis, -2 units along the y-axis, and -2 units along the z-axis.

12. $\left[\begin{matrix} 0+1 & 0+1 & 0+1 & 0+1 \\ 0+5 & 3+5 & 3+5 & 0+5 \\ 1+(-3) & 2+(-3) & 5+(-3) & 4+(-3) \end{matrix}\right.$

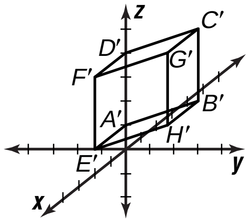
$\left.\begin{matrix} 2+1 & 2+1 & 2+1 & 2+1 \\ 0+5 & 0+5 & 3+5 & 3+5 \\ 1+(-3) & 4+(-3) & 5+(-3) & 2+(-3) \end{matrix}\right]$

$= \begin{bmatrix} 1 & 1 & 1 & 1 & 3 & 3 & 3 & 3 \\ 5 & 8 & 8 & 5 & 5 & 5 & 8 & 8 \\ -2 & -1 & 2 & 1 & -2 & 1 & 2 & -1 \end{bmatrix}$



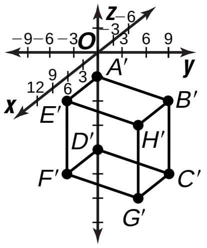
The results is a translation of 1 unit along the x-axis, 5 units along the y-axis, and -3 units along the z-axis.

13. $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}0&0&0&0&2&2&2&2\\0&3&3&0&0&0&3&3\\1&2&5&4&1&4&5&2\end{bmatrix}$
$=\begin{bmatrix}0&0&0&0&2&2&2&2\\0&3&3&0&0&0&3&3\\1&2&5&4&1&4&5&2\end{bmatrix}$



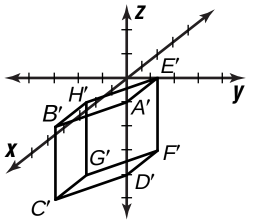
The transformation does not change the figure.

14. $\begin{bmatrix}1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}\begin{bmatrix}0&0&0&0&2&2&2&2\\0&3&3&0&0&0&3&3\\1&2&5&4&1&4&5&2\end{bmatrix}$
$=\begin{bmatrix}0&0&0&0&2&2&2&2\\0&-3&-3&0&0&0&-3&-3\\-1&-2&-5&-4&-1&-4&-5&-2\end{bmatrix}$



The transformation results in reflections over the $xy-$ and xz-planes.

15. $\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}\begin{bmatrix}0&0&0&0&2&2&2&2\\0&3&3&0&0&0&3&3\\1&2&5&4&1&4&5&2\end{bmatrix}$
$=\begin{bmatrix}0&0&0&0&-2&-2&-2&-2\\0&-3&-3&0&0&0&-3&-3\\-1&-2&-5&-4&-1&-4&-5&-2\end{bmatrix}$



The transformation results in reflections over all three coordinate planes.

16. The matrix results in a dilation of scale factor 2, so the figure is twice the original size.

17. $\begin{bmatrix}3&0&0\\0&3&0\\0&0&-3\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&-1\end{bmatrix}\begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}$, so the figure is three times the original size and reflected over the xy-plane.

18. $\begin{bmatrix}-0.75&0&0\\0&-0.75&0\\0&0&-0.75\end{bmatrix}=\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}$
$\begin{bmatrix}0.75&0&0\\0&0.75&0\\0&0&0.75\end{bmatrix}$, so the figure is three-fourths the original site and reflected over all three coordinate planes.

19a. $\begin{bmatrix}2x\\2y\\5z\end{bmatrix}=\begin{bmatrix}2&0&0\\0&2&0\\0&0&5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}$, so the transformation can be represented by the matrix $\begin{bmatrix}2&0&0\\0&2&0\\0&0&5\end{bmatrix}$.

19b. The transformation will magnify the x- and y-dimensions two-fold, and the z-dimension 5-fold.

20a. $\begin{bmatrix}23.6&23.6&23.6&23.6&23.6\\72&72&72&72&72\\0&0&0&0&0\end{bmatrix}$

20b. $\begin{bmatrix}20+23.6&136+23.6&247+23.6&302+23.6&351+23.6\\-58+72&-71+72&-74+72&-83+72&-62+72\\27+0&53+0&59+0&37+0&52+0\end{bmatrix}$
$=\begin{bmatrix}43.6&159.6&270.6&325.6&374.6\\14&1&-2&-11&10\\27&53&59&37&52\end{bmatrix}$

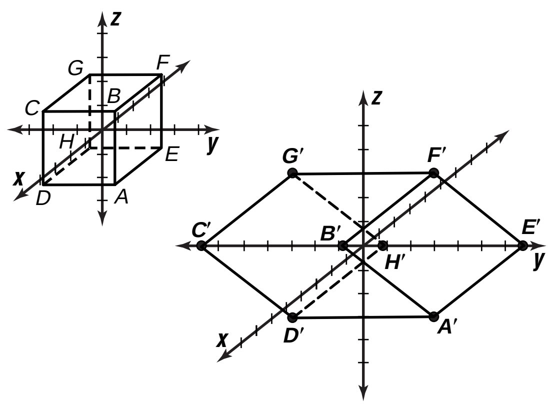
20c. The result is a translation 23.6 units along the x-axis and 72 units along the y-axis.

21. The matrix $\begin{bmatrix}-1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ would reflect the prism over the yz-plane. The matrix $\begin{bmatrix}0.5&0&0\\0&0.5&0\\0&0&0.5\end{bmatrix}$ would reduce its dimensions by half.
$\begin{bmatrix}0.5&0&0\\0&0.5&0\\0&0&0.5\end{bmatrix}\begin{bmatrix}-1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}-0.5&0&0\\0&0.5&0\\0&0&0.5\end{bmatrix}$

22a. Placing a non zero element in the first row and third column will skew the cube so that the top is no longer directly above the bottom.

Sample answer: $\begin{bmatrix}1&0&1\\0&1&0\\0&0&1\end{bmatrix}$

22b. Sample graphs:



23. The first transformation reflects the figure over all three coordinate planes. The second transformation stretches the dimensions along the y- and z-axes and skews it along the xy-plane. (The first row of T changes the x-coordinate of (x, y, z) to $x + 2z$.)

24. To multiply the x-coordinate by 3, the first row of the matrix must be 3 0 0. Since the y-coordinate is multiplied by 2, the second row is 0 2 0. To convert a z-coordinate to $x - 4z$, use a third row of 1 0 −4.

The matrix is $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & -4 \end{bmatrix}$.

25a. The x-coordinates are unchanged, the y-coordinates increase, and the z-coordinates decrease, so the movement is dip-slip.

25b. $$\begin{bmatrix} 123.9 & -41.3 & 201.7 & 73.8 & -129.4 & 36.4 \\ 88.0 & 145.8 & -28.3 & -82.6 & 97.1 & -123.9 \\ 205.3 & 246.6 & 261.5 & 212.0 & -166.4 & -85.3 \end{bmatrix}$$
$$- \begin{bmatrix} 123.9 & -41.3 & 201.7 & 73.8 & -129.4 & 36.4 \\ 86.4 & 144.2 & -29.9 & -84.2 & 95.5 & -125.5 \\ 206.5 & 247.8 & 262.7 & 213.2 & -165.2 & -84.1 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1.6 & 1.6 & 1.6 & 1.6 & 1.6 & 1.6 \\ -1.2 & -1.2 & -1.2 & -1.2 & -1.2 & -1.2 \end{bmatrix}$$

26a. La Shawna | Jaimie

$x = 0$ | $x = 35t$

$y = -16t^2 + 150$ | $y = -16t^2 + 150$

$$\begin{aligned} -16t^2 + 150 &= 0 \\ 150 &= 16t^2 \\ \frac{150}{16} &= t^2 \\ \sqrt{\frac{150}{16}} &= t \\ 3.06 &\approx t \end{aligned}$$

$$\begin{aligned} x &= 35t \\ &= 35\sqrt{\frac{150}{16}} \\ &\approx 107 \text{ ft} \end{aligned}$$

26b. Since the stones have the same parametric equations for y, they land at the same time. In part a, it was calculated that the elapsed time is about 3.06 seconds.

27. $$\begin{aligned} x &= -5t - 1 \\ x + 1 &= -5t \\ \frac{x+1}{-5} &= t \\ y &= 2t + 10 \\ y &= 2\left(\frac{x+1}{-5}\right) + 10 \\ y &= -\frac{2}{5}x + \frac{48}{5} \end{aligned}$$

28. $$\sec\left(\cos^{-1}\frac{2}{5}\right) = \frac{1}{\cos\left(\cos^{-1}\frac{2}{5}\right)} = \frac{1}{\frac{2}{5}} = \frac{5}{2}$$

29. $$\begin{aligned} 80x^3 + 80x^2 + 80x &= 24.2 \\ 80x^3 + 80x^2 + 80x - 24.2 &= 0 \end{aligned}$$

A graphing calculator indicates that there is a solution between 0 and 1. By Descartes' Rule of Signs, it is the only solution. When $x = 0.2$, $80x^3 + 80x^2 + 80x - 24.2 = -4.36$ and when $x = 0.3$, $80x^3 + 80x^2 + 80x - 24.2 = 9.16$. So the solution to the nearest tenth is 0.2.

30. Divide each side of the equations by 2, 3, 4, and 6, respectively, so that the left side is $x + 2y$.

I. $x + 2y = 4$ II. $y = 4$

III. $x + 2y = 2$ IV. $x + 2y = \frac{8}{3}$

Only I and II are equivalent, so the correct choice is A.

Chapter 8 Study Guide and Assessment

Page 543 Understanding and Using the Vocabulary

1. resultant **2.** unit
3. magnitude **4.** cross
5. inner **6.** vector
7. parallel **8.** standard
9. direction **10.** components

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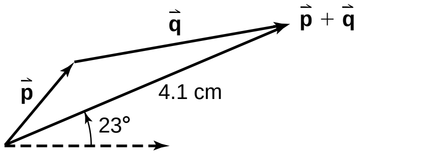
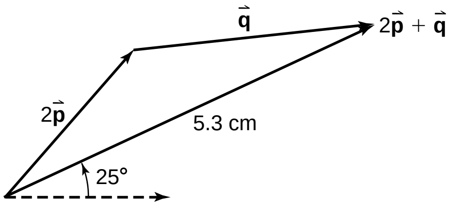
11. 1.3 cm, 50° **12.** 2.9 cm, 10°

13.



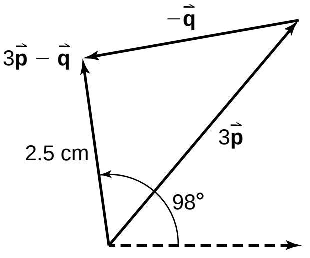
4.1 cm, 23°

14.



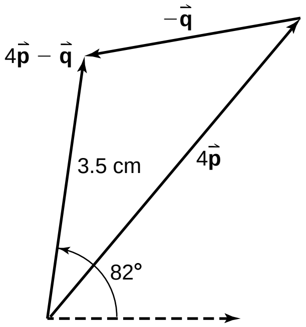
5.3 cm, 25°

15.



2.5 cm, 98°

16.



3.5 cm, 82°

17. $h = 1.3 \cos 50°$ $\quad v = 1.3 \sin 50°$
$h = 0.8$ cm $\quad v = 1$ cm

18. $h = 2.9 \cos 10°$ $\quad v = 2.9 \sin 10°$
$h = 2.9$ cm $\quad v = 0.5$ cm

19. $\overrightarrow{CD} = \langle 7 - 2, 15 - 3 \rangle$ or $\langle 5, 12 \rangle$
$|\overrightarrow{CD}| = \sqrt{5^2 + 12^2}$
$= \sqrt{169}$ or 13

20. $\overrightarrow{CD} = \langle 4 - (-2), 12 - 8 \rangle$ or $\langle 6, 4 \rangle$
$|\overrightarrow{CD}| = \sqrt{6^2 + 4^2}$
$= \sqrt{52}$ or $2\sqrt{13}$

21. $\overrightarrow{CD} = \langle 0 - 2, 9 - (-3) \rangle$ or $\langle -2, 12 \rangle$
$|\overrightarrow{CD}| = \sqrt{(-2)^2 + 12^2}$
$= \sqrt{148}$ or $2\sqrt{37}$

22. $\overrightarrow{CD} = \langle -5 - (-6), -4 - 4 \rangle$ or $\langle 1, -8 \rangle$
$|\overrightarrow{CD}| = \sqrt{1^2 + (-8)^2}$
$= \sqrt{65}$

23. $\vec{\mathbf{u}} = \vec{\mathbf{v}} + \vec{\mathbf{w}}$
$\vec{\mathbf{u}} = \langle 2, -5 \rangle + \langle 3, -1 \rangle$
$\vec{\mathbf{u}} = \langle 2 + 3, -5 + (-1) \rangle$ or $\langle 5, -6 \rangle$

24. $\vec{\mathbf{u}} = \vec{\mathbf{v}} - \vec{\mathbf{w}}$
$\vec{\mathbf{u}} = \langle 2, -5 \rangle - \langle 3, -1 \rangle$
$\vec{\mathbf{u}} = \langle 2 - 3, -5 - (-1) \rangle$ or $\langle -1, -4 \rangle$

25. $\vec{\mathbf{u}} = 3\vec{\mathbf{v}} + 2\vec{\mathbf{w}}$
$\vec{\mathbf{u}} = 3\langle 2, -5 \rangle + 2\langle 3, -1 \rangle$
$\vec{\mathbf{u}} = \langle 6, -15 \rangle + \langle 6, -2 \rangle$
$\vec{\mathbf{u}} = \langle 6 + 6, -15 + (-2) \rangle$ or $\langle 12, -17 \rangle$

26. $\vec{\mathbf{u}} = 3\vec{\mathbf{v}} - 2\vec{\mathbf{w}}$
$\vec{\mathbf{u}} = 3\langle 2, -5 \rangle - 2\langle 3, -1 \rangle$
$\vec{\mathbf{u}} = \langle 6, -15 \rangle - \langle 6, -2 \rangle$
$\vec{\mathbf{u}} = \langle 6 - 6, -15 - (-2) \rangle$ or $\langle 0, -13 \rangle$

27. $\overrightarrow{EF} = \langle 6 - 2, -2 - (-1), 1 - 4 \rangle$ or $\langle 4, -1, -3 \rangle$
$|\overrightarrow{EF}| = \sqrt{4^2 + (-1)^2 + (-3)^2}$
$= \sqrt{26}$

28. $\overrightarrow{EF} = \langle -1 - 9, 5 - 8, 11 - 5 \rangle$ or $\langle -10, -3, 6 \rangle$
$|\overrightarrow{EF}| = \sqrt{(-10)^2 + (-3)^2 + 6^2}$
$= \sqrt{145}$

29. $\overrightarrow{EF} = \langle 2 - (-4), -1 - (-3), 7 - 0 \rangle$ or $\langle 6, 2, 7 \rangle$
$|\overrightarrow{EF}| = \sqrt{6^2 + 2^2 + 7^2}$
$= \sqrt{89}$

30. $\overrightarrow{EF} = \langle -4 - 3, 0 - 7, 5 - (-8) \rangle$ or $\langle -7, -7, 13 \rangle$
$|\overrightarrow{EF}| = \sqrt{(-7)^2 + (-7)^2 + 13^2}$
$= \sqrt{267}$

31. $\vec{\mathbf{u}} = 2\vec{\mathbf{w}} - 5\vec{\mathbf{v}}$
$\vec{\mathbf{u}} = 2\langle 4, -1, 5 \rangle - 5\langle -1, 7, -4 \rangle$
$\vec{\mathbf{u}} = \langle 8, -2, 10 \rangle - \langle -5, 35, -20 \rangle$
$\vec{\mathbf{u}} = \langle 8 - (-5), -2 - 35, 10 - (-20) \rangle$
$\vec{\mathbf{u}} = \langle 13, -37, 30 \rangle$

32. $\vec{\mathbf{u}} = 0.25\vec{\mathbf{v}} + 0.4\vec{\mathbf{w}}$
$\vec{\mathbf{u}} = 0.25\langle -1, 7, -4 \rangle + 0.4\langle 4, -1, 5 \rangle$
$\vec{\mathbf{u}} = \langle -0.25, 1.75, -1 \rangle + \langle 1.6, -0.4, 2 \rangle$
$\vec{\mathbf{u}} = \langle -0.25 + 1.6, 1.75 + (-0.4), -1 + 2 \rangle$
$\vec{\mathbf{u}} = \langle 1.35, 1.35, 1 \rangle$

33. $\langle 5, -1 \rangle \cdot \langle -2, 6 \rangle = 5(-2) + (-1)6$
$= -10 - 6$
$= -16$; no

34. $\langle 2, 6 \rangle \cdot \langle 3, -4 \rangle = 2(3) + 6(-4)$
$= 6 - 24$
$= -18$; no

35. $\langle 4, 1, -2 \rangle \cdot \langle 3, -4, 4 \rangle = 4(3) + 1(-4) + (-2)4$
$= 12 - 4 - 8$
$= 0$; yes

36. $\langle 2, -1, 4 \rangle \cdot \langle 6, -2, 1 \rangle = 2(6) + (-1)(-2) + 4(1)$
$= 12 + 2 + 4$
$= 18$; no

37. $\langle 5, 2, -10 \rangle \cdot \langle 2, -4, -4 \rangle$
$= 5(2) + 2(-4) + (-10)(-4)$
$= 10 - 8 + 40$
$= 42$; no

38. $\langle 5, -2, 5 \rangle \times \langle -1, 0, -3 \rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 5 & -2 & 5 \\ -1 & 0 & -3 \end{vmatrix}$
$= \begin{vmatrix} -2 & 5 \\ 0 & -3 \end{vmatrix}\vec{\mathbf{u}} - \begin{vmatrix} 5 & 5 \\ -1 & -3 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 5 & -2 \\ -1 & 0 \end{vmatrix}\vec{\mathbf{k}}$
$= 6\vec{\mathbf{i}} + 10\vec{\mathbf{j}} - 2\vec{\mathbf{k}}$ or $\langle 6, 10, -2 \rangle$
$\langle 6, 10, -2 \rangle \cdot \langle 5, -2, 5 \rangle$
$6(5) + 10(-2) + (-2)(5)$
$30 - 20 - 10 = 0$
$\langle 6, 10, -2 \rangle \cdot \langle -1, 0, -3 \rangle$
$6(-1) + 10(0) + (-2)(-3)$
$-6 + 0 + 6 = 0$

39. $\langle -2, -3, 1 \rangle \times \langle 2, 3, -4 \rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ -2 & -3 & 1 \\ 2 & 3 & -4 \end{vmatrix}$
$= \begin{vmatrix} -3 & 1 \\ 3 & -4 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} -2 & 1 \\ 2 & -4 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} -2 & -3 \\ 2 & 3 \end{vmatrix}\vec{\mathbf{k}}$
$= 9\vec{\mathbf{i}} - 6\vec{\mathbf{j}} + 0\vec{\mathbf{k}}$ or $\langle 9, -6, 0 \rangle$
$\langle 9, -6, 0 \rangle \cdot \langle -2, -3, 1 \rangle$
$9(-2) + (-6)(-3) + 0(1)$
$-18 + 18 + 0 = 0$
$\langle 9, -6, 0 \rangle \cdot \langle 2, 3, -4 \rangle$
$9(2) + (-6)(3) + 0(-4)$
$18 - 18 + 0 = 0$

40. $\langle -1, 0, 4\rangle \times \langle 5, 2, -1\rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ -1 & 0 & 4 \\ 5 & 2 & -1 \end{vmatrix}$

$= \begin{vmatrix} 0 & 4 \\ 2 & -1 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} -1 & 4 \\ 5 & -1 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} -1 & 0 \\ 5 & 2 \end{vmatrix}\vec{\mathbf{k}}$

$= -8\vec{\mathbf{i}} + 19\vec{\mathbf{j}} - 2\vec{\mathbf{k}}$ or $\langle -8, 19, -2\rangle$

$\langle -8, 19, -2\rangle \cdot \langle -1, 0, 4\rangle$

$(-8)(-1) + 19(0)(-2)(4)$

$8 + 0 - 8 = 0$

$\langle -8, 19, -2\rangle \cdot \langle 5, 2, -1\rangle$

$(-8)(5) + 19(2) + (-2)(-1)$

$-40 + 38 + 2 = 0$

41. $\langle 7, 2, 1\rangle \times \langle 2, 5, 3\rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 7 & 2 & 1 \\ 2 & 5 & 3 \end{vmatrix}$

$= \begin{vmatrix} 2 & 1 \\ 5 & 3 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 7 & 1 \\ 2 & 3 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 7 & 2 \\ 2 & 5 \end{vmatrix}\vec{\mathbf{k}}$

$= \vec{\mathbf{i}} - 19\vec{\mathbf{j}} + 31\vec{\mathbf{k}}$ or $\langle 1, -19, 31\rangle$

$\langle 1, -19, 31\rangle \cdot \langle 7, 2, 1\rangle$

$1(7) + (-19)2 + 31(1)$

$7 + (-38) + 31 = 0$

$\langle 1, -19, 31\rangle \cdot \langle 2, 5, 3\rangle$

$1(2) + (-19)5 + 31(3)$

$2 + (-95) + 93 = 0$

42. Sample answer:

Let $x(1, 2, 3)$, $y(-4, 2, -1)$ and $z(5, -3, 0)$

$\overrightarrow{xy} = \langle -4 - 1, 2 - 2, -1 - 3\rangle$ or $\langle -5, 0, -4\rangle$

$\overrightarrow{yz} = \langle 5 - (-4), -3 - 2, 0 - (-1)\rangle$ or $\langle 9, -5, 1\rangle$

$\langle -5, 0, -4\rangle \times \langle 9, -5, 1\rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ -5 & 0 & -4 \\ 9 & -5 & 1 \end{vmatrix}$

$= \begin{vmatrix} 0 & -4 \\ -5 & 1 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} -5 & -4 \\ 9 & 1 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} -5 & 0 \\ 9 & -5 \end{vmatrix}\vec{\mathbf{k}}$

$= -20\vec{\mathbf{i}} - 31\vec{\mathbf{j}} + 25\vec{\mathbf{k}}$ or $\langle -20, -31, 25\rangle$

43. $\vec{\mathbf{F}}_1 = 320\vec{\mathbf{i}}$

$\vec{\mathbf{F}}_2 = 260\vec{\mathbf{j}}$

$|\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2| = \sqrt{320^2 + 260^2}$

≈ 412.31 N

$\tan\theta = \frac{260}{320}$ or $\frac{13}{16}$

$\theta = \tan^{-1}\frac{13}{16}$

$\approx 39.09°$

44. $\vec{\mathbf{v}}_1 = 12\vec{\mathbf{j}}$

$\vec{\mathbf{v}}_2 = (30\cos 116°)\vec{\mathbf{i}} + (30\sin 116°)\vec{\mathbf{j}}$

$|\vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_2| = \sqrt{(30\cos 116°)^2 + (12 + 30\sin 116°)^2}$

≈ 41 m/s

$\tan\theta = \frac{12 + 30\sin 116°}{30\cos 116°}$

$\theta = \tan^{-1}\left(\frac{12 + 30\sin 116°}{30\cos 116°}\right)$

$\approx 108.65°$

45. $\langle x - 3, y - (-5)\rangle = t\langle 4, 2\rangle$

$\langle x - 3, y + 5\rangle = t\langle 4, 2\rangle$

$x - 3 = 4t$ $\qquad$ $y + 5 = 2t$

$x = 3 + 4t$ $\qquad$ $y = -5 + 2t$

46. $\langle x - (-1), y - 9\rangle = t\langle -7, -5\rangle$

$\langle x + 1, y - 9\rangle = t\langle -7, -5\rangle$

$x + 1 = -7t$ $\qquad$ $y - 9 = -5t$

$x = -1 - 7t$ $\qquad$ $y = 9 - 5t$

47. $\langle x - 4, y - 0\rangle = t\langle 3, -6\rangle$

$\langle x - 4, y\rangle = t\langle 3, -6\rangle$

$x - 4 = 3t$ $\qquad$ $y = -6t$

$x = 4 + 3t$

48. $x = t$

$y = -8t - 7$

$= -7 - 8t$

49. $x = t$

$y = -\frac{1}{2}t + \frac{5}{2}$

50. $|\vec{\mathbf{v}}_x| = |\vec{\mathbf{v}}|\cos\theta$ $\qquad$ $|\vec{\mathbf{v}}_y| = |\vec{\mathbf{v}}|\sin\theta$

$= 15\cos 55°$ $\qquad$ $= 15\sin 55°$

≈ 8.60 ft/s $\qquad$ ≈ 12.29 ft/s

51. $|\vec{\mathbf{v}}_x| = |\vec{\mathbf{v}}|\cos\theta$ $\qquad$ $|\vec{\mathbf{v}}_y| = |\vec{\mathbf{v}}|\sin\theta$

$= 13.2\cos 66°$ $\qquad$ $= 13.2\sin 66°$

≈ 5.37 ft/s $\qquad$ ≈ 12.06 ft/s

52. $|\vec{\mathbf{v}}_x| = |\vec{\mathbf{v}}|\cos\theta$ $\qquad$ $|\vec{\mathbf{v}}_y| = |\vec{\mathbf{v}}|\sin\theta$

$= 18\cos 28°$ $\qquad$ $= 18\sin 28°$

≈ 15.89 m/s $\qquad$ ≈ 8.45 m/s

53. $\overrightarrow{CH} = \langle -4 - 3, -2 - 4, 2 - (-1)\rangle$ or $\langle -7, -6, 3\rangle$

$A(3, 4 + (-6), -1 + 3) = A(3, -2, 2)$

$B(3, 4 + (-6), -1) = B(3, -2, -1)$

$D(3, 4, -1 + 3) = D(3, 4, 2)$

$E(3 + (-7), 4, -1 + 3) = E(-4, 4, 2)$

$F(3 + (-7), 4, -1) = F(-4, 4, -1)$

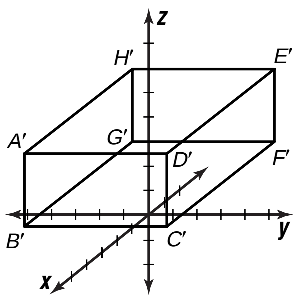
$G(3 + (-7), 4 + (-6), -1) = G(-4, -2, -1)$

The matrix for the figure is

$$\begin{bmatrix} 3 & 3 & 3 & 3 & -4 & -4 & -4 & -4 \\ -2 & -2 & 4 & 4 & 4 & 4 & -2 & -2 \\ 2 & -1 & -1 & 2 & 2 & -1 & -1 & 2 \end{bmatrix}.$$

The matrix for the translated figure is

$$\begin{bmatrix} 5 & 5 & 5 & 5 & -2 & -2 & -2 & -2 \\ -2 & -2 & 4 & 4 & 4 & 4 & -2 & -2 \\ 5 & 2 & 2 & 5 & 5 & 2 & 2 & 5 \end{bmatrix}.$$

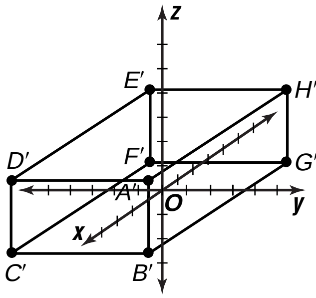


The figure moves 2 units along the x-axis and 3 units along the z-axis.

54. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 3 & 3 & 3 & 3 & -4 & -4 & -4 & -4 \\ -2 & -2 & 4 & 4 & 4 & 4 & -2 & -2 \\ 2 & -1 & -1 & 2 & 2 & -1 & -1 & 2 \end{bmatrix}$

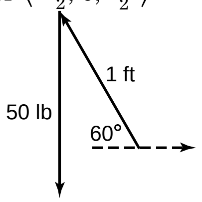
$= \begin{bmatrix} 3 & 3 & 3 & 3 & -4 & -4 & -4 & -4 \\ 2 & 2 & -4 & -4 & -4 & -4 & 2 & 2 \\ 2 & -1 & -1 & 2 & 2 & -1 & -1 & 2 \end{bmatrix}$



The figure is reflected over the xz-plane.

Page 547 Applications and Problem Solving

55. $\overrightarrow{AB} = \langle 1 \cos 120°, 0, 1 \sin 120° \rangle$ or $\left\langle -\frac{1}{2}, 0, \frac{\sqrt{3}}{2} \right\rangle$



$\vec{\mathbf{F}} = \langle 0, 0, -50 \rangle$

$\vec{\mathbf{T}} = \overrightarrow{AB} \times \vec{\mathbf{F}}$

$= \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ -\frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 0 & -50 \end{vmatrix}$

$= \begin{vmatrix} 0 & \frac{\sqrt{3}}{2} \\ 0 & -50 \end{vmatrix} \vec{\mathbf{i}} - \begin{vmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -50 \end{vmatrix} \vec{\mathbf{j}} + \begin{vmatrix} -\frac{1}{2} & 0 \\ 0 & 0 \end{vmatrix} \vec{\mathbf{k}}$

$= 0\vec{\mathbf{i}} - 25\vec{\mathbf{j}} + 0\vec{\mathbf{k}}$ or $\langle 0, -25, 0 \rangle$

$|\vec{\mathbf{T}}| = \sqrt{0^2 + (-25)^2 + 0^2}$

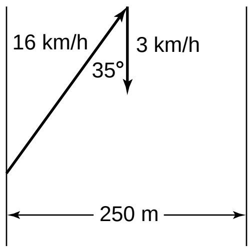
$= 25$ lb-ft

56. $y = t|\vec{\mathbf{v}}| \sin \theta - \frac{1}{2}gt^2 + h$

$= 0.5(38) \sin 40° - \frac{1}{2}(32)(0.5)^2 + 2$

≈ 10.2 ft

57a.



$\vec{\mathbf{b}} = (16 \cos 55°)\vec{\mathbf{c}} + (16 \sin 55°)\vec{\mathbf{j}}$

$\vec{\mathbf{c}} = -3\vec{\mathbf{j}}$

$|\vec{\mathbf{b}} + \vec{\mathbf{c}}| = \sqrt{(16 \cos 55°)^2 + (16 \sin 55° - 3)^2}$

≈ 13.7 km/h

57b. $\frac{u}{250} = \frac{16 \sin 55° - 3}{16 \cos 55°}$

$u = 250\left(\frac{16 \sin 55° - 3}{16 \cos 55°}\right)$

$u \approx 275.3$ m

58. $\vec{\mathbf{F}}_1 = 90\mathbf{i}$

$\vec{\mathbf{F}}_2 = (70 \cos 30°)\vec{\mathbf{i}} + (70 \sin 30°)\vec{\mathbf{j}}$ or $35\sqrt{3}\,\vec{\mathbf{i}} + 35\vec{\mathbf{j}}$

$|\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2| = \sqrt{(90 + 35\sqrt{3})^2 + 35^2}$

≈ 154.6 N

$\tan \theta = \frac{35}{90 + 35\sqrt{3}}$ or $\frac{7}{18 + 7\sqrt{3}}$

$\theta = \tan^{-1}\left(\frac{7}{18 + 7\sqrt{3}}\right)$

$\approx 13.1°$

Page 547 Open-Ended Assessment

1a. Sample answer: $X(4, -1)$, $Y(1, 1)$

$\overrightarrow{XY} = \langle 1 - 4, 1 - (-1) \rangle$ or $\langle -3, 2 \rangle$

1b. $|\overrightarrow{XY}| = \sqrt{(-3)^2 + 2^2}$ or $\sqrt{13}$

The magnitude of $\overrightarrow{XY}$ only depends on the differences of the coordinates of X and Y, not the actual coordinates.

2a. Sample answer: $P(1, 1)$, $Q(3, 3)$, $R(3, 1)$, $S(5, 3)$

$\overrightarrow{PQ} = \langle 3 - 1, 3 - 1 \rangle$ or $\langle 2, 2 \rangle$

$\overrightarrow{RS} = \langle 5 - 3, 3 - 1 \rangle$ or $\langle 2, 2 \rangle$

$\overrightarrow{PQ}$ and $\overrightarrow{RS}$ are parallel because they have the same direction. In fact, they are the same vector.

2b. Sample answer: $\vec{\mathbf{a}} = \langle 8, -4 \rangle$, $\vec{\mathbf{b}} = \langle 3, 6 \rangle$

$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 8(3) + (-4)6$ or 0

$\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are perpendicular because their inner product is 0.

Chapter 8 SAT & ACT Preparation

Page 549 SAT and ACT Practice

1. Recall that the formula for the area of a parallelogram is base times height. You know the base is 5, but you don't know the height. Don't be fooled by the segment BD; it is not the height of the parallelogram. Try another method to find the area. The parallelogram is made up of two triangles. Find the area of each triangle. Since $ABCD$ is a parallelogram, $AB = DC$ and $AD = BC$. The two triangles are both right triangles, and they share a common side, BD. By SAS, the two triangles are congruent. So you can find the area of one triangle and multiply by 2. The hypotenuse of the triangle is 5 and one side is 3. Use the Pythagorean Theorem to find the other side.

$5^2 = 3^2 + b^2$

$25 = 9 + b^2$

$16 = b^2$

$4 = b$

The height is 4.

Use the formula for the area of a triangle.

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(4)(3)$ or 6

Since the parallelogram consists of two triangles, the area of the parallelogram is 2×6 or 12. The correct choice is A.

2. In order to write the equation of a circle, you need to know the coordinates of the center and the length of the radius. The general equation for a circle is $(x - h)^2 + (y - k)^2 = r^2$, where the center is (h, k) and the radius is r. From the coordinates of points A and B, you know the length of the side is 4. So the center Q, has coordinates (0, 4).
To calculate the length of the radius, draw the radius OB. This creates a 45°-45°-90° right triangle. The two legs each have length 2. The hypotenuse has length $2\sqrt{2}$.

$$(x - 4)^2 + (y - 0)^2 = (2\sqrt{2})^2$$
$$(x - 4)^2 + y^2 = 4(2)$$
$$(x - 4)^2 + y^2 = 8$$

The correct choice is B.

3. Write the equation for the perimeter of a rectangle. then replace x with its value in terms of y. Solve the equation for y.

$$p = 2x + 2y$$
$$p = 2\left(\frac{2}{3}y\right) + 2y$$
$$p = \frac{4}{3}y + 2y$$
$$p = \frac{10}{3}y$$
$$\frac{3p}{10} = y$$

The correct choice is B.

4. Recall the triangle Inequality Theorem: the sum of the lengths of any two sides of a triangle is greater than the length of the third side. Let x represent the length of the third side.

$$40 + 80 > x$$
$$120 > x$$
$$40 + x > 80$$
$$x > 40$$

Since x must be greater than 40, x cannot be equal to 40. The correct choice is A. To check your answer, notice that the other answer choices are greater than 40 and less than 120, so they are all possible values for x.

5. Since the answer choices have fractional exponents of x, start by rewriting the expression with fractional exponents. Simplify the fractions and use the rules for exponents to combine terms.

$$\sqrt[3]{x^2} \cdot \sqrt[9]{x^3} = x^{\frac{2}{3}} \cdot x^{\frac{3}{9}}$$
$$= x^{\frac{2}{3}} \cdot x^{\frac{1}{3}}$$
$$= x^{\left(\frac{2}{3} + \frac{1}{3}\right)}$$
$$= x^1 \text{ or } x$$

The correct choice is E.

6. This figure looks more complex than it is. A *semi-circle* is just one half of a circle. Notice that the answer choices include π, so don't convert to decimals. Find the radius of each semi-circle. Calculate the area of each semi-circle.
The area of the shaded region is the area of the large semi-circle minus the area of the medium semi-circle plus the area of the small semi-circle.

Large semi-circle area $= \frac{1}{2}\pi\, 3^2 = \frac{9\pi}{2}$

Medium semi-circle area $= \frac{1}{2}\pi\, 2^2 = \frac{4\pi}{2}$

Small semi-circle area $= \frac{1}{2}\pi\, 1^2 = \frac{1\pi}{2}$

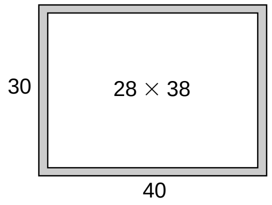
Shaded area $= \frac{9\pi}{2} - \frac{4\pi}{2} + \frac{1\pi}{2} = \frac{6\pi}{2} = 3\pi$

The correct choice is A.

7. The only values for which a rational function is undefined are values which make the denominator 0. Since $f(x) = \frac{x^2 - 3x + 2}{x - 1}$,
the denominator is only 0 when $x - 1 = 0$ or $x = 1$.

The correct choice is D.

8. Start by sketching a diagram of the counter



Use your calculator to find the area of the whole counter and then subtract the area of the white tiles in the center. The white tiles cover an area of $(30 - 2)(40 - 2)$ or $(28)(38)$.

$$(30)(40) = 1200$$
$$(28)(38) = 1064$$
$$\text{Red tiles} = 1200 - 1064 = 136$$

The correct choice is B.

9. First, find the slope of the line containing the points (–2, 6) and (4, –3).

$$m = \frac{-3 - 6}{4 - -2}$$
$$m = \frac{-9}{6} \text{ or } -\frac{3}{2}$$

The point-slope form of the line is

$$y - 6 = -\frac{3}{2}(x - -2).$$
$$y - 6 = -\frac{3}{2}x - 3$$
$$y = -\frac{3}{2}x + 3$$

So the y-intercept of the line is 3.

The correct choice is B.

10. Write an expression for the sum of the areas of the two triangles. Recall the area of a triangle is one half the base times the height.

$\frac{1}{2}(AC)(AB) + \frac{1}{2}(CE)(ED)$

From the figure, you know that $\triangle ABC$ and $\triangle CDE$ are both isosceles, because of the angles marked $x°$ and because $\overline{BCD}$ is a line segment. These two triangles have equal corresponding angles.

Since they are isosceles triangles, $AC = AB$ and $CE = ED$. Use these equivalent lengths in the expressions for the area sum.

$$\frac{1}{2}(AC)(AB) + \frac{1}{2}(CE)(ED) = \frac{1}{2}(AC)^2 + \frac{1}{2}(CE)^2$$
$$= \frac{1}{2}[(AC)^2 + (CE)^2]$$

Using the Pythagorean Theorem for $\triangle ACE$, you know that $(AC)^2 + (CE)^2 = (AE)^2$ or 1.

So the sum of the two areas is $\frac{1}{2}(1) = \frac{1}{2}$. You can grid the answer either as .5 or as 1/2.

Chapter 9 Polar Coordinates and Complex Numbers

9-1 Polar Coordinates

Pages 557–558 Check for Understanding

1. There are infinitely many ways to represent the angle θ. Also, r can be positive or negative.

2. Draw the angle θ in standard position. Extend the terminal side of the angle in the opposite direction. Locate the point that is $|r|$ units from the pole along this extension.

3. Sample answer: $-60°$ and $300°$
Plot (4, 120) such that θ is in standard position and $|r|$ is 4 units from the pole. Extend the terminal side of the angle in the opposite direction. Locate the point that is 4 units from the pole along this extension.

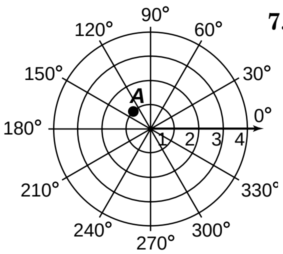
$r = -4$

$\theta = 120 - 180$ or $\theta = 120 + 180$
$\quad = -60 \qquad\qquad = 300$

4. The points 3 units from the origin in the opposite direction are on the circle where $r = 3$.

5. All ordered pairs of the form (r, θ) where $r = 0$.

6.



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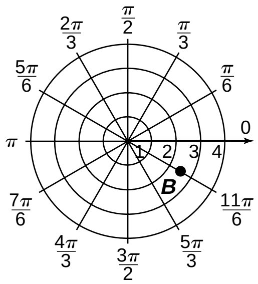

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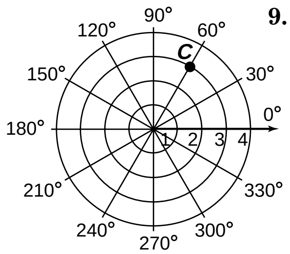

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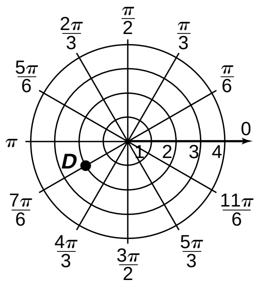


10. Sample answer: $\left(-2, \frac{13\pi}{7}\right), \left(-2, \frac{25\pi}{6}\right), \left(2, \frac{7\pi}{6}\right), \left(2, \frac{19\pi}{6}\right)$

$(r, \theta + 2k\pi)$

$\rightarrow \left(-2, \frac{\pi}{6} + 2(1)\pi\right) \rightarrow \left(-2, \frac{13\pi}{6}\right)$

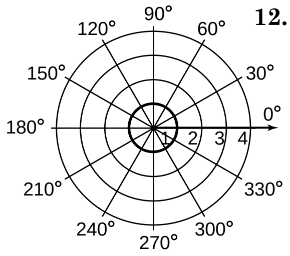
$\rightarrow \left(-2, \frac{\pi}{6} + 2(2)\pi\right) \rightarrow \left(-2, \frac{25\pi}{6}\right)$

$(-r, \theta + (2k + 1)\pi)$

$\rightarrow \left(2, \frac{\pi}{6} + (1)\pi\right) \rightarrow \left(2, \frac{7\pi}{6}\right)$

$\rightarrow \left(2, \frac{\pi}{6} + (3)\pi\right) \rightarrow \left(2, \frac{19\pi}{6}\right)$

11.



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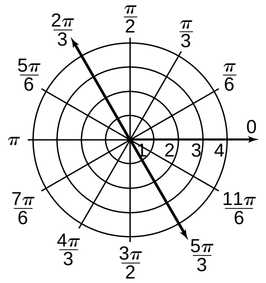

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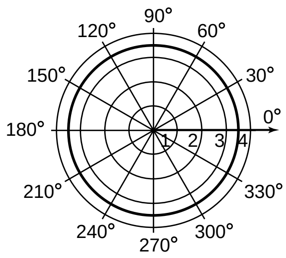


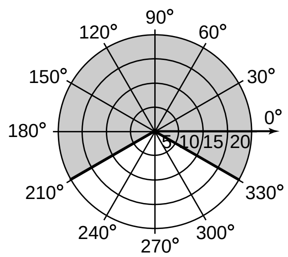
14. $P_1P_2 = \sqrt{2.5^2 + (-3)^2 - 2(2.5)(-3)\cos\left(-\frac{\pi}{4} - \frac{\pi}{6}\right)}$

$= \sqrt{6.25 + 9 + 15\cos\left(-\frac{5\pi}{12}\right)}$

$= \sqrt{5.25 + 15\cos\left(-\frac{5\pi}{12}\right)}$

≈ 4.37

15a.



15b. $210 - (-30) = 240$

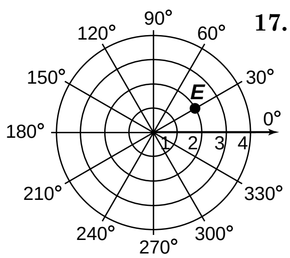
$A = \frac{N}{360}(\pi r^2)$

$= \frac{240}{360}(\pi 20^2)$

$\approx 838 \text{ ft}^2$

Pages 558–560 Exercises

16.



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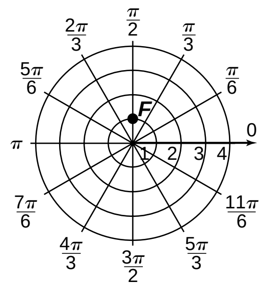
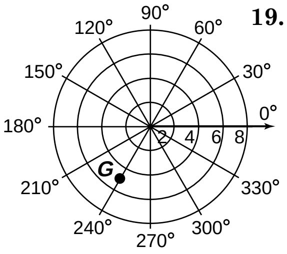
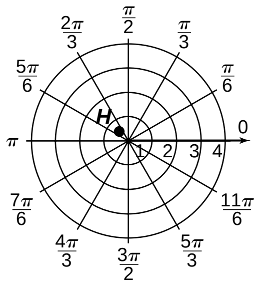
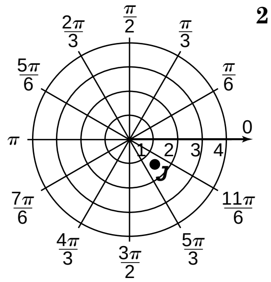
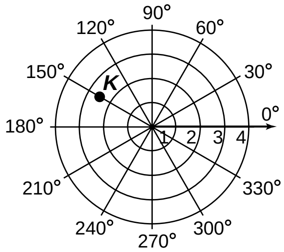
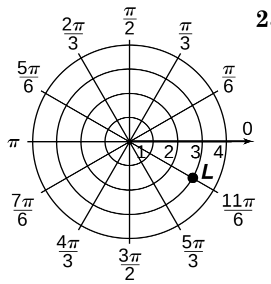


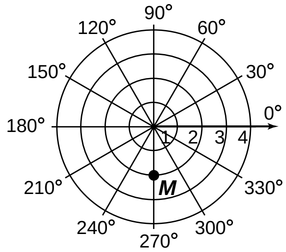
18. 

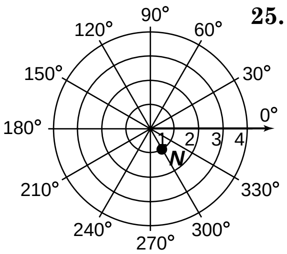
19. 

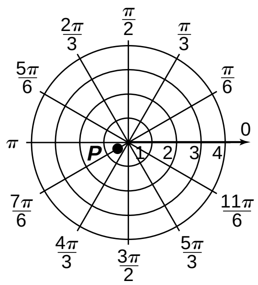
20. 

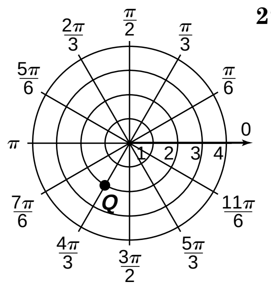
21. 

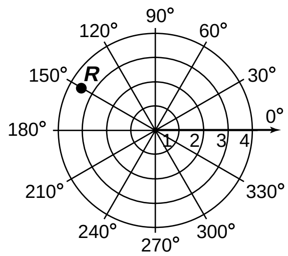
22. 

23. 

24. 

25. 

26. 

27. 

28. Sample answer: $\left(2, \frac{\pi}{3}\right), \left(2, \frac{7\pi}{3}\right)$, $(-2, 240°)$, $(-2, 600°)$

$(r, \theta + 2k\pi)$

$\rightarrow \left(2, \frac{\pi}{3} + 2(0)\pi\right) \rightarrow \left(2, \frac{\pi}{3}\right)$

$\rightarrow \left(2, \frac{\pi}{3} + 2(1)\pi\right) \rightarrow \left(2, \frac{7\pi}{3}\right)$

$(-r, \theta + (2k + 1)180°)$

$\rightarrow (-2, 60° + (1)180°) \rightarrow (-2, 240°)$

$\rightarrow (-2, 60° + (3)180°) \rightarrow (-2, 600°)$

29. Sample answer: $(1.5, 540°)$, $(1.5, 900°)$, $(-1.5, 0°)$, $(-1.5, 360°)$

$(r, \theta + 360k°)$

$\rightarrow (1.5, 180° + 360(1)°) \rightarrow (1.5, 540°)$

$\rightarrow (1.5, 180° + 360(2)°) \rightarrow (1.5, 900°)$

$(-r, \theta + (2k + 1)180°)$

$\rightarrow (-1.5, 180° + (-1)180°) \rightarrow (-1.5, 0°)$

$\rightarrow (-1.5, 180° + (1)180°) \rightarrow (-1.5, 360°)$

30. Sample answer: $\left(-1, \frac{7\pi}{3}\right), \left(-1, \frac{13\pi}{3}\right), \left(1, \frac{4\pi}{3}\right), \left(1, \frac{10\pi}{3}\right)$

$(r, \theta + 2k\pi)$

$\rightarrow \left(-1, \frac{\pi}{3} + 2(1)\pi\right) \rightarrow \left(-1, \frac{7\pi}{3}\right)$

$\rightarrow \left(-1, \frac{\pi}{3} + 2(2)\pi\right) \rightarrow \left(-1, \frac{13\pi}{3}\right)$

$(-r, \theta + (2k + 1)\pi)$

$\rightarrow \left(1, \frac{\pi}{3} + (1)\pi\right) \rightarrow \left(1, \frac{4\pi}{3}\right)$

$\rightarrow \left(1, \frac{\pi}{3} + (3)\pi\right) \rightarrow \left(1, \frac{10\pi}{3}\right)$

31. Sample answer: $(4, 675°)$, $(4, 1035°)$, $(-4, 135°)$, $(-4, 495°)$

$(r, \theta + 360k°)$

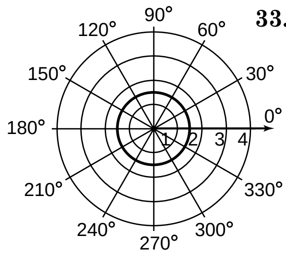
$\rightarrow (4, 315 + 360(1)°) \rightarrow (4, 675°)$

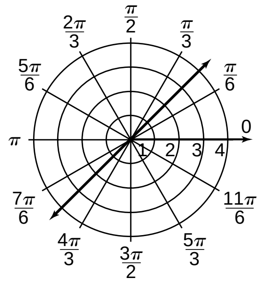
$\rightarrow (4, 315 + 360(2)°) \rightarrow (4, 1035°)$

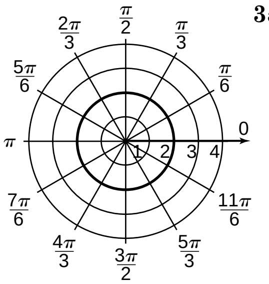
$(-r, \theta + (2k + 1)180°)$

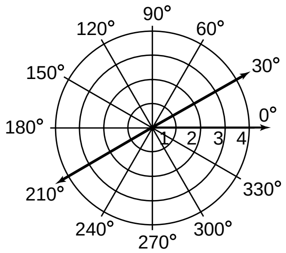
$\rightarrow (-4, 315 + (-1)180°) \rightarrow (-4, 135°)$

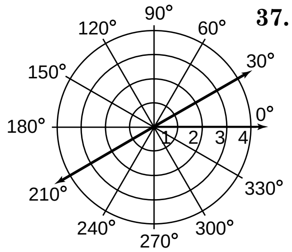
$\rightarrow (-4, 315 + (1)180°) \rightarrow (-4, 495°)$

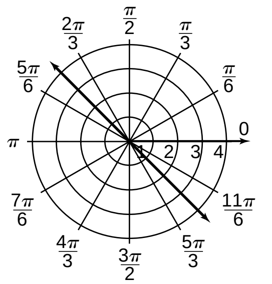
32. 

33. 

34. 

35. 

36. 

37. 

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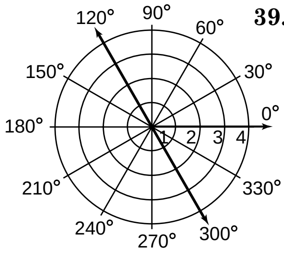

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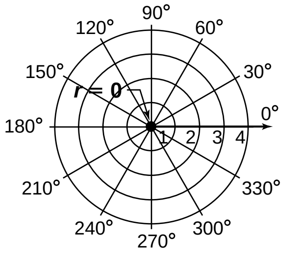

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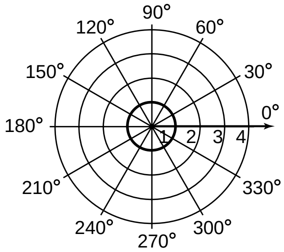


41. $r = \sqrt{2}$ or $r = -\sqrt{2}$ for any θ.

42. $$\begin{aligned} P_1P_2 &= \sqrt{4^2 + 6^2 - 2(4)(6)\cos(105° - 170°)} \\ &= \sqrt{16 + 36 - 48\cos(-65°)} \\ &= \sqrt{52 - 48\cos(-65)} \\ &\approx 5.63 \end{aligned}$$

43. $$\begin{aligned} P_1P_2 &= \sqrt{1^2 + 5^2 - 2(1)(5)\cos\left(\frac{3\pi}{4} - \frac{\pi}{6}\right)} \\ &= \sqrt{1 + 25 - 10\cos\left(\frac{7\pi}{12}\right)} \\ &= \sqrt{26 - 10\cos\left(\frac{7\pi}{12}\right)} \\ &\approx 5.35 \end{aligned}$$

44. $$\begin{aligned} P_1P_2 &= \\ &\sqrt{(-2.5)^2 + (-1.75)^2 - 2(-2.5)(-1.75)\cos\left(-\frac{2\pi}{5} - \frac{\pi}{8}\right)} \\ &= \sqrt{6.25 + 30.0625 - 8.75\cos\left(-\frac{21\pi}{40}\right)} \\ &= \sqrt{9.3125 - 8.75\cos\left(-\frac{21\pi}{40}\right)} \\ &\approx 3.16 \end{aligned}$$

45. $$\begin{aligned} P_1P_2 &= \sqrt{1.3^2 + (-3.6)^2 - 2(1.3)(-3.6)\cos(-62° - (-47°))} \\ &= \sqrt{1.69 + 12.96 + 9.36\cos(-62° + 47°)} \\ &= \sqrt{14.65 + 9.36\cos(-15°)} \\ &\approx 4.87 \end{aligned}$$

46. $r = \sqrt{(-3)^2 + 4^2} = 5$

$\sin\theta = \frac{4}{5}$, $\theta \approx 53°$

$180° - 53° = 127°$

Sample answer: $(5, 127°)$

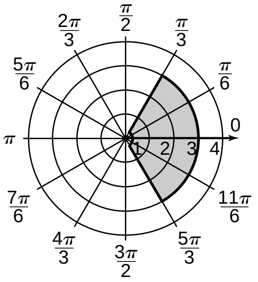
47. There are 360° in a circle. If the circle is cut into 6 equal pieces, each slice measures $\frac{360}{6}$ or 60°. Beginning at the origin, the equation of the first line is $\theta = 0°$. The equation of the next line, rotating counterclockwise, is $\theta = 0 + 60$ or 60°. The equation of the last line is $\theta = 60 + 60$ or 120°. Note that lines extend through the origin, so 3 lines create 6 pieces.

48. $$\begin{aligned} P_1P_2 &= \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta - \theta)} \\ &= \sqrt{r_1^2 - r_2^2 - 2r_1r_2\cos 0} \\ &= \sqrt{r_1^2 + r_2^2 - 2r_1r_2} \\ &= \sqrt{(r_1 - r_2)^2} \\ &= |r_1 - r_2| \end{aligned}$$

49a. When $\theta = 120°$, $r = 17$. The maximum speed at 120° is 17 knots.

49b. When $\theta = 150°$, $r = 13$. The maximum speed at 150° is 13 knots.

50a.



50b. $\frac{\pi}{3} - \left(-\frac{\pi}{3}\right) = \frac{2\pi}{3}$ or 120°

Let $R = 3 \cdot 100$ or 300 and let $r = 0.25 \cdot 100$ or 25.

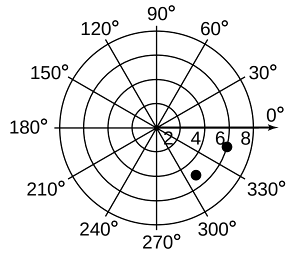
$$\begin{aligned} A &= \frac{N}{360}(\pi R^2) - \frac{N}{360}(\pi r^2) \\ &= \frac{120}{360}(\pi(300^2) - \frac{120}{360}(\pi(25)^2) \\ &= \frac{120}{360}\pi(90{,}000 - 625) \\ &\approx 93{,}593 \text{ ft}^2 \end{aligned}$$

If each person's seat requires 6 ft^2 of space, there are $\frac{93{,}593}{6}$ or 15,599 seats.

51. The distance formula is symmetric with respect to (r_1, θ_1) and (r_2, θ_2). That is,

$$\begin{aligned} &\sqrt{r_2^2 + r_1^2 - 2r_2r_1\cos(\theta_1 - \theta_2)} \\ &= \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos[-(\theta_2 - \theta_1)]} \\ &= \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)} \end{aligned}$$

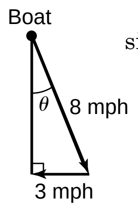
52a.



52b. $$\begin{aligned} P_1P_2 &= \sqrt{5^2 + 6^2 - 2(5)(6)\cos(345° - 310°)} \\ &= \sqrt{25 + 36 - 60\cos(35°)} \\ &= \sqrt{61 - 60\cos(35°)} \\ &\approx 3.44 \end{aligned}$$

No; the planes are 3.44 miles apart.

53. Draw a picture.



$$\begin{aligned} \sin\theta &= \frac{3}{8} \\ \sin^{-1}(\sin\theta) &= \sin^{-1}\left(\frac{3}{8}\right) \\ \theta &\approx 22.0° \end{aligned}$$

54. $$\begin{aligned} \langle 3, -2, 4\rangle \cdot \langle 1, -4, 0\rangle &= (3)(1) + (-2)(-4) + (4)(0) \\ &= 3 + 8 + 0 \\ &= 11 \end{aligned}$$

No, the vectors are not perpendicular because their inner product is not 0.

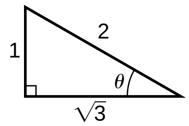
55. Rewrite $y = 9x - 3$ as $9x - y - 3 = 0$.

$$\begin{aligned} d &= \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}} \\ &= \frac{9(-3) + (-1)(2) + (-3)}{\pm\sqrt{9^2 + (-1)^2}} \\ &= \frac{-32}{\pm\sqrt{82}} \\ &= \frac{-32}{\pm\sqrt{82}} \cdot \frac{\sqrt{82}}{\sqrt{82}} \\ &= \pm\frac{-32\sqrt{82}}{82} \\ &= \frac{16\sqrt{82}}{41} \end{aligned}$$
Distance is always positive.

56. $\frac{1 - \sin^2\alpha}{\sin^2\alpha} = \frac{1}{\sin^2\alpha} - 1$
$= \csc^2\alpha - 1$
$= \cot^2\alpha$

57. Arc cos $\frac{\sqrt{3}}{2} = 30°$

In a 30°-60°-90° right triangle, the angle opposite the smallest leg is 30°.



58. $y = 5 \cos 4\theta$

Amplitude $= 5$; Period $= \frac{2\pi}{4}$ or $\frac{\pi}{2}$

59. $b \sin A = 18.6 \sin 30°$
$= 9.3$

Since $a = b \sin A$, there is one solution.

Find B.

$\frac{18.6}{\sin B} = \frac{9.3}{\sin 30°}$

$18.6 \sin 30° = 9.3 \sin B$

$\frac{18.6 \sin 30°}{9.3} = \sin B$

$90° \approx B$

Find C.

$C = 180° - 90° - 30°$
$= 60°$

Find c.

$\frac{c}{\sin 60°} = \frac{9.3}{\sin 30°}$

$c \sin 30° = 9.3 \sin 60°$

$c = \frac{9.3 \sin 60°}{\sin 30°}$

$c = 16.1$

60. 3 or 1 positive

$f(-x) = -x^3 - 4x^2 - 4x - 1$

0 negative

$\frac{P}{Q}: \pm 1$

Since there are only positive real zeros, the only rational real zero is 1.

61.
$$\begin{array}{r} x - 3 \\ x + 5\overline{)x^2 + 2x - 3} \\ \underline{x^2 + 5x} \\ -3x - 3 \\ \underline{-3x - 15} \\ 10 \end{array}$$

As $x \to \pm\infty$, $\frac{10}{x + 5} \to 0$. Therefore, the slant asymptote is $y = x - 3$.

62. y-axis:

For x: $f(x) = x^4 - 3x^2 + 2$

For $-x$: $f(-x) = (-x)^4 - 3(-x)^2 + 2$
$= x^4 - 3x^2 + 2$

So, in general, point $(-x, y)$ is on the graph if and only if (x, y) is on the graph.

63. $\begin{vmatrix} -2 & 4 & -1 \\ 1 & -1 & 0 \\ -3 & 4 & 5 \end{vmatrix} = -2\begin{vmatrix} -1 & 0 \\ 4 & 5 \end{vmatrix} - 4\begin{vmatrix} 4 & 0 \\ -3 & 5 \end{vmatrix} + (-1)\begin{vmatrix} 1 & -1 \\ -3 & 4 \end{vmatrix}$

$= -2(-5) - 4(5) - 1(1)$
$= -11$

64. $11 - (-3) = 14$
$11 - (-2) = 13$
$11 - (-1) = 12$
$11 - 0 = 11$
$\{(-3, 14), (-2, 13), (-1, 12), (0, 11)\}$
For each x-value, there ia a unique v-value.
Yes, the relation is a function.

65. Since the two triangles formed are right triangles, the side opposite the right angles, $\overline{AB}$, intercept an arc measuring 180°, or half the circle. $\overline{AB}$ is a diameter.

$C = \pi d$
$50\pi = \pi d$
$50 = d$

The correct choice is E.

9-2 Graphs of Polar Equations

Page 565 Check for Understanding

1. Sample answer: $r = \sin 2\theta$

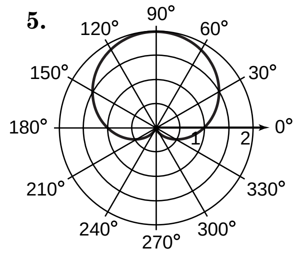
The graph of a polar equation whose form is $r = a \cos n\theta$ or $a \sin n\theta$, where n is a positive integer, is a rose.

2. $-1 \le \sin \theta \le 1$ for any value of θ. Therefore, the maximum value of $r = 3 + 5 \sin \theta$ is $r = 3 + 5(1)$ or 8. Likewise, the minimum value of $r = 3 + 5 \sin \theta$ is $r = 3 + 5(-1)$ or -2.

3. The polar coordinates of a point are not unique. A point of intersection may have one representation that satisfies one equation in a system, another representation that satisfies the other equation, but no representation that satisfies both simultaneously.

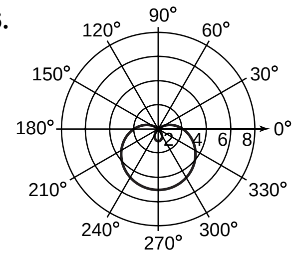
4. Barbara is correct. The interval $0 \le \theta \le \pi$ is not always sufficient. For example, the interval $0 \le \theta \le \pi$ only generates two of the four petals for the rose $r = \sin 2\theta$. $r = \sin \frac{\theta}{2}$ is an example where values of θ from 0 to 4π would have to be considered.

5.



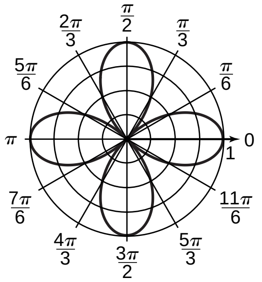
cardioid

6.



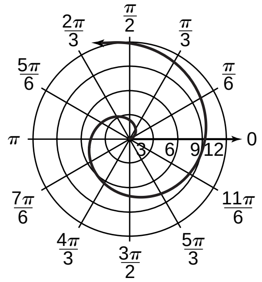
limaçon

7.



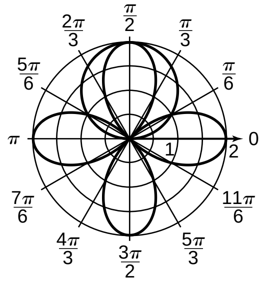
rose

8.



spiral of Archimedes

9.



$$
\begin{aligned}
2 \sin \theta &= 2 \cos 2\theta \\
\sin \theta &= \cos 2\theta \\
\sin \theta &= 1 - 2 \sin^2 \theta \\
2 \sin^2 \theta + \sin \theta - 1 &= 0 \\
(2 \sin \theta - 1)(\sin \theta + 1) &= 0
\end{aligned}
$$

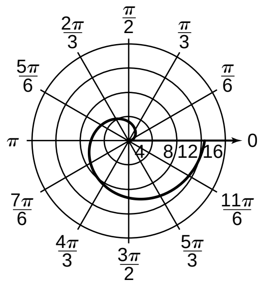
$2 \sin \theta - 1 = 0$ or $\sin \theta + 1 = 0$

$\sin \theta = \frac{1}{2}$ $\qquad$ $\sin \theta = -1$

$\theta = \frac{\pi}{6}$ or $\theta = \frac{5\pi}{6}$ or $\theta = \frac{3\pi}{2}$

If $\theta = \frac{\pi}{6}$ or $\theta = \frac{5\pi}{6}$ is substituted in either original equation, $r = 1$. If $\theta = \frac{3\pi}{2}$ is substituted in either original equation, $r = -2$. The solutions are $\left(1, \frac{\pi}{6}\right)$, $\left(1, \frac{5\pi}{6}\right)$, and $\left(-2, \frac{3\pi}{2}\right)$.

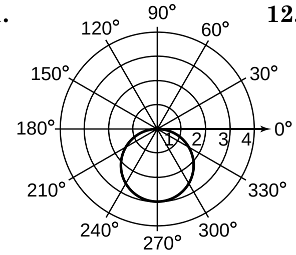
10a.



10b. Sample answer: $0 \leq \theta \leq \frac{14\pi}{3}$
Begin at the origin and "spiral" twice around it, or through 4π radians. Move straight up through $4\pi + \frac{\pi}{2}$ or $\frac{9\pi}{2}$ radians. Now move to the left slightly, through approximately $\frac{9\pi}{2} + \frac{\pi}{6}$ or $\frac{14\pi}{3}$ radians.

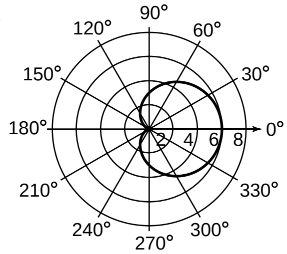
Pages 565–567 Exercises

11.



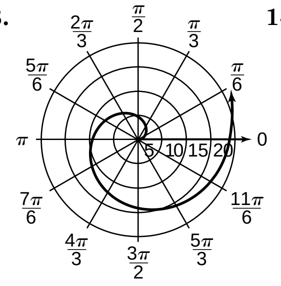
circle

12.



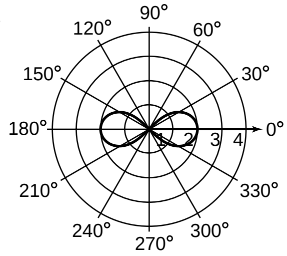
cardioid

13.



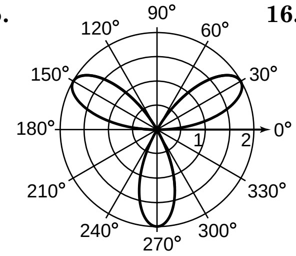
spiral of Archimedes

14.



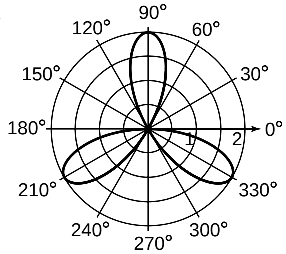
lemniscate

15.



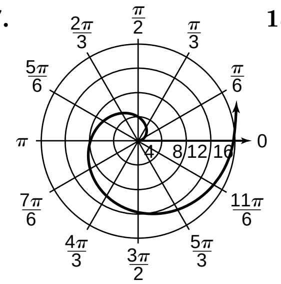
rose

16.



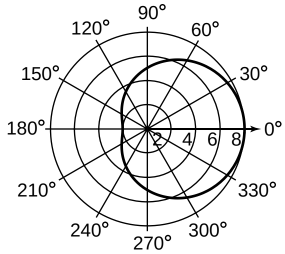
rose

17.



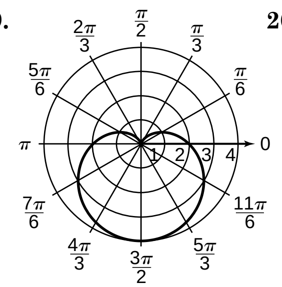
Spiral of Archimedes

18.



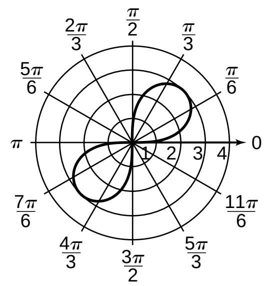
limaçon

19.

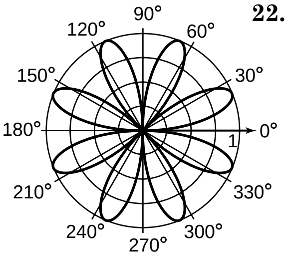


cardioid

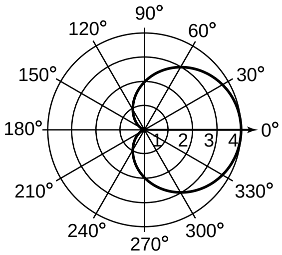
20.



lemniscate

21. 

rose

22. 

cardioid

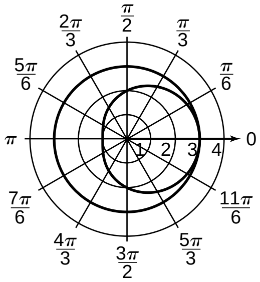
23. Sample answer: $r = \sin 3\theta$
The graph of a polar equation of the form $r = a \cos 3\theta$ or $r = a \sin 3\theta$ is a rose with 3 petals.

24. Sample answer: $r = \frac{\theta}{2}$

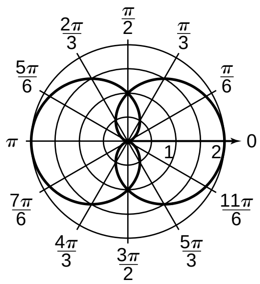
$\frac{\pi}{4} = a\left(\frac{\pi}{2}\right)$ $\quad$ $r = \frac{1}{2}\theta$

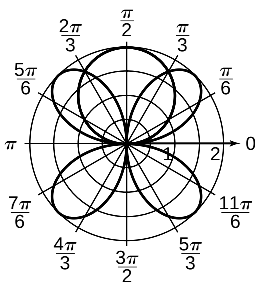
$\frac{1}{2} = a$ $\quad$ $r = \frac{\theta}{2}$

25. $3 = 2 + \cos \theta$
$1 = \cos \theta$
$\theta = 0$
The solution is (3, 0)



26. $1 + \cos \theta = 1 - \cos \theta$
$2 \cos \theta = 0$
$\cos \theta = 0$
$\theta = \frac{\pi}{2}$ or $\theta = \frac{3\pi}{2}$
Substituting each angle into either of the original equations gives $r = 1$, so the solutions of the system are $\left(1, \frac{\pi}{2}\right)$ and $\left(1, \frac{3\pi}{2}\right)$.

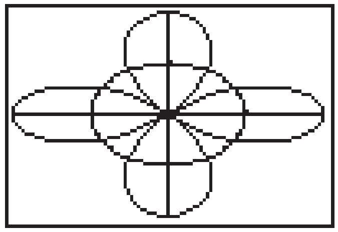


27. 

$2 \sin \theta = 2 \sin 2\theta$
$\sin \theta = \sin 2\theta$
$\sin \theta = 2 \cos \theta \sin \theta$
$0 = 2 \cos \theta \sin \theta - \sin \theta$
$0 = \sin \theta\,(2 \cos \theta - 1)$
$\sin \theta = 0$ or $2 \cos \theta - 1 = 0$
$\cos \theta = \frac{1}{2}$
$\theta = 0$ or π or $\theta = \frac{\pi}{3}$ or $\theta = \frac{5\pi}{3}$

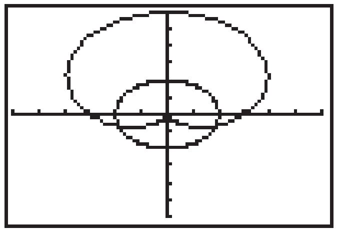
If $\theta = 0$ or $\theta = \pi$ is substituted in either original equation, $r = 0$. If $\theta = \frac{\pi}{3}$ or $\theta = \frac{5\pi}{3}$ is substituted in either original equation, $r = \sqrt{3}$ or $r = -\sqrt{3}$, respectively. The solutions are (0, 0), (0, π), $\left(\sqrt{3}, \frac{\pi}{3}\right)$, and $\left(-\sqrt{3}, \frac{5\pi}{3}\right)$.

28. (1, 0.5), (1, 1.0), (1, 2.1), (1, 2.6), (1, 3.7), (1, 4.2), (1, 5.2), (1, 5.8)



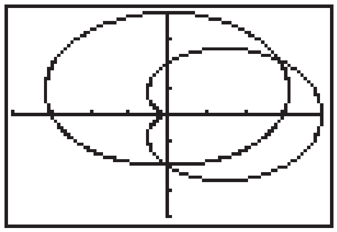
[−2, 2] scl: 1 by [−2, 2] scl: 1

29. (2, 3.5), (2, 5.9)



[−6, 6] scl:1 by [−6, 6] scl: 1

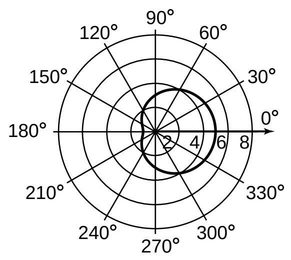
30. (3.6, 0.6), (2.0, 4.7)



[−4, 4] scl: 1 by [−4, 4] scl: 1

31a. If the lemniscate is 6 units from end to end, then $a = \frac{1}{2}(6)$ or 3.
$r^2 = 9 \cos 2\theta$ or $r^2 = 9 \sin 2\theta$

31b. If the lemniscate is 8 units from end to end, then $a = \frac{1}{2}(8)$ or 4.
$r^2 = 16 \cos 2\theta$ or $r^2 = 16 \sin 2\theta$

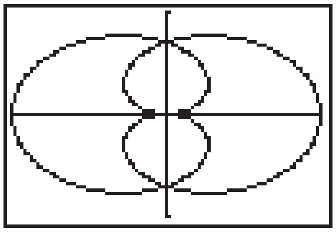
32. 

This microphone will pick up more sounds from behind than the cardioid microphone.

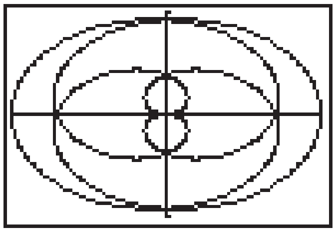
33. $0 \le \theta \le 4\pi$: Begin at the origin and curl around once, or through 2π radians. Curl around a second time and go through $2\pi + 2\pi$ or 4π radians.

34. All screens are [−1, 1] scl: 1 by [−1, 1] scl: 1

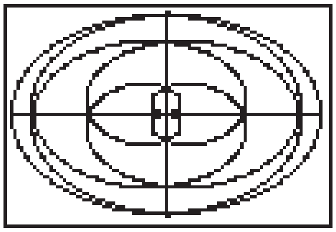
34a. $r = \cos\frac{\theta}{2}$



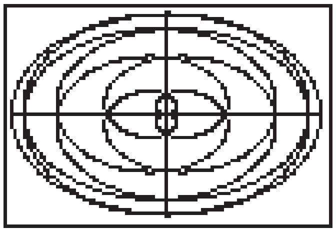
$r = \cos\frac{\theta}{4}$



$r = \cos\frac{\theta}{6}$

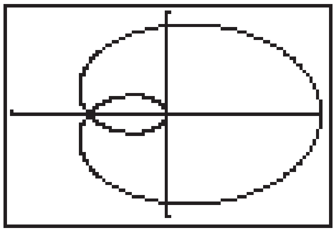


$r = \cos\frac{\theta}{8}$

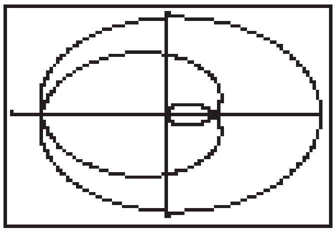


When $n = 10$, two more outer rings will appear.

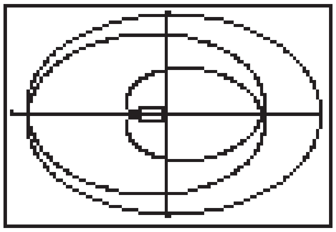
34b. $r = \cos\frac{\theta}{3}$



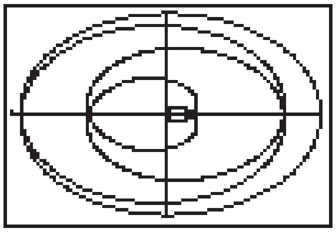
$r = \cos\frac{\theta}{5}$



$r = \cos\frac{\theta}{7}$



$r = \cos\frac{\theta}{9}$



When $n = 11$, the innermost loop will be on the left and there will be an additional outer ring.

35. Sample answer: $r = -1 - \sin\theta$
A heart resembles the shape of a cardioid. The sine function orients the heart so that the axis of symmetry is along the y-axis. If $a = -1$, the heart points in the right direction.

36a. For a limaçon to go back on itself and have an inner loop, r must change sign. This will happen if $|b| > |a|$.

36b. For the other two cases, $|a| \geq |b|$. Experimentation shows that the dimple disappears when $|a| = |2b|$, so there is a dimple if $|b| \leq |a| < |2b|$.

36c. For this remaining case, there is neither an inner loop nor a dimple if $|a| \geq |2b|$.

37a. Subtracting α from θ rotates the graph counterclockwise by an angle of α.

37b. Multiplying θ by -1 reflects the graph about the polar axis or x-axis.

37c. Multiplying the function by -1 changes r to its opposite, so the graph is reflected about the origin.

37d. Multiplying the function by c results in a dilation by a factor of c. (Points on the graph move closer to the origin if $c < 1$, or farther away from the origin if $c > 1$.)

38. Sample answer: (4, 405°), (4, 765°), (−4, −135°), (−4, 225°)
$(r, \theta + 360k°)$
$\rightarrow (4, 45° + 360(1)°) \rightarrow (4, 405°)$
$\rightarrow (4, 45° + 360(2)°) \rightarrow (4, 765°)$
$(r, \theta + (2k + 1)180°)$
$\rightarrow (-4, 45° + (-1)180°) \rightarrow (-4, -135°)$
$\rightarrow (-4, 45° + (1)180°) \rightarrow (-4, 225°)$

39. $\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 0 \\ -1 & 2 & 4 \end{vmatrix}$

$= \begin{vmatrix} 3 & 0 \\ 2 & 4 \end{vmatrix} \vec{i} - \begin{vmatrix} 2 & 0 \\ -1 & 4 \end{vmatrix} \vec{j} + \begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix} \vec{k}$

$= 12\vec{i} - 8\vec{j} + 7\vec{k}$

$= \langle 12, -8, 7 \rangle$

$\langle 2, 3, 0 \rangle \cdot \langle 12, -8, 7 \rangle = 24 + (-24) + 0$ or 0

$\langle -1, 2, 4 \rangle \cdot \langle 12, -8, 7 \rangle = -12 + (-16) + 28$ or 0

40. 3.5 cm, 87°

41. $\frac{\sin^2 x}{\cos^4 x + \cos^2 x \sin^2 x} \stackrel{?}{=} \tan^2 x$

$\frac{\sin^2 x}{\cos^2 x (\cos^2 x + \sin^2 x)} \stackrel{?}{=} \tan^2 x$

$\frac{\sin^2 x}{(\cos^2 x)(1)} \stackrel{?}{=} \tan^2 x$

$\frac{\sin^2 x}{\cos^2 x} \stackrel{?}{=} \tan^2 x$

$\tan^2 x = \tan^2 x$

42. Find C.

$C = 180° - 21°15' - 49°40'$
$= 109°5'$

Find b.

$\frac{b}{\sin 49°40'} = \frac{28.9}{\sin 109°5'}$

$b \sin 109°5' = 28.9 \sin 49°40'$

$b = \frac{28.9 \sin 49°40'}{\sin 109°5'}$

$b \approx 23.3$

Find a.

$\frac{a}{\sin 21°15'} = \frac{28.9}{\sin 109°5'}$

$a \sin 109°5' = 28.9 \sin 21°15'$

$a = \frac{28.9 \sin 21°15'}{\sin 109°5'}$

$a \approx 11.1$

43.

$$\begin{array}{l} \\ \text{Bus} \\ \text{Train} \end{array} \begin{array}{c} \begin{array}{ccc} \text{NY} & \text{LA} & \text{Miami} \end{array} \\ \begin{bmatrix} \$240 & \$199 & \$260 \\ \$254 & \$322 & \$426 \end{bmatrix} \end{array}$$

44. $\frac{1}{8} + \frac{6}{4} = \frac{1}{8} + \frac{12}{8} = \frac{13}{8}$

So $\dfrac{\frac{1}{8} + \frac{6}{4}}{\frac{3}{16}} = \dfrac{\frac{13}{8}}{\frac{3}{16}} = \frac{13}{8} \cdot \frac{16}{3}$

$= \frac{26}{3}$

The correct choice is A.

9-3 Polar and Rectangular Coordinates

Page 571 Check for Understanding

1. Sample answer: $(2\sqrt{2}, 45°)$

$r = \sqrt{2^2 + 2^2}$ $\quad\theta = \text{Arctan}\left(\frac{2}{2}\right)$

$= \sqrt{8}$ $\quad = 45°$

$= 2\sqrt{2}$

2. The quadrant that the point lies in determines whether θ is given by Arctan $\frac{y}{x}$ or Arctan $\frac{y}{x} + \pi$.

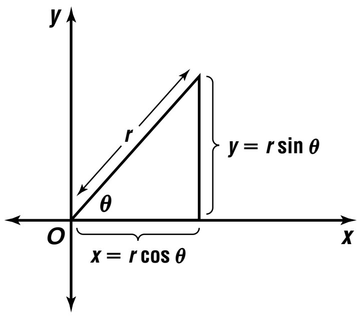
3. $x = 2$

$r \cos \theta = 2$

$r = \frac{2}{\cos \theta}$

$r = 2 \sec \theta$

4. To convert from polar coordinates to rectangular coordinates, substitute r and θ into the equations $x = r \cos \theta$ and $y = r \sin \theta$. To convert from rectangular coordinates to polar coordinates, use the equation $r = \sqrt{x^2 + y^2}$ to find r. If $x > 0$, $\theta =$ Arctan $\frac{y}{x}$. If $x < 0$, $\theta =$ Arctan $\frac{y}{x} + \pi$. If $x = 0$, you can use $\frac{\pi}{2}$ or any coterminal angle for θ.



5. $r = \sqrt{(-\sqrt{2})^2 + (\sqrt{2})^2}$ $\quad\theta = \text{Arctan}\left(\frac{\sqrt{2}}{-\sqrt{2}}\right)$

$= \sqrt{4}$ or 2 $\quad = \frac{3\pi}{4}$

$\left(2, \frac{3\pi}{4}\right)$

6. $r = \sqrt{(-2)^2 + (-5)^2}$ $\quad\theta = \text{Arctan}\left(\frac{-5}{-2}\right) + \pi$

$= \sqrt{29} \approx 5.39$ $\quad \approx 4.33$

$(5.39, 4.33)$

7. $x = -2 \cos\left(\frac{4\pi}{3}\right)$ $\quad y = -2 \sin\left(\frac{4\pi}{3}\right)$

$= 1$ $\quad = \sqrt{3}$

$(1, \sqrt{3})$

8. $x = 2.5 \cos 250°$ $\quad y = 2.5 \sin 250°$

≈ -0.86 $\quad = -2.35$

$(-0.86, -2.35)$

9. $y = 2$

$r \sin \theta = 2$

$r = \frac{2}{\sin \theta}$

$r = 2 \csc \theta$

10. $x^2 + y^2 = 16$

$(r \cos \theta)^2 + (r \sin \theta)^2 = 16$

$r^2(\cos^2 \theta + \sin^2 \theta) = 16$

$r^2 = 16$

$r = 4$ or $r = -4$

11. $r = 6$

$\sqrt{x^2 + y^2} = 6$

$x^2 + y^2 = 36$

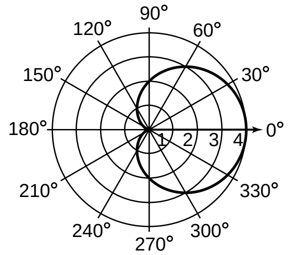
12. $r = -\sec \theta$

$\frac{r}{r} = \frac{-1}{r \cos \theta}$

$1 = \frac{-1}{x}$

$x = -1$

13a.



13b. No. The given point is on the negative x-axis, directly behind the microphone. The polar pattern indicates that the microphone does not pick up any sound from this direction.

Pages 572–573 Exercises

14. $r = \sqrt{2^2 + (-2)^2}$ $\quad\theta = \text{Arctan}\left(\frac{-2}{2}\right)$

$= \sqrt{8}$ or $2\sqrt{2}$ $\quad = -\frac{\pi}{4}$

Add 2π to obtain $\theta = \frac{7\pi}{4}$.

$\left(2\sqrt{2}, \frac{7\pi}{4}\right)$

15. $r = \sqrt{0^2 + 1^2}$

$= \sqrt{1}$ or 1

Since $x = 0$ when $y = 1$, $\theta = \frac{\pi}{2}$.

$\left(1, \frac{\pi}{2}\right)$

16. $r = \sqrt{1^2 + (\sqrt{3})^2}$ $\quad\theta = \text{Arctan}\ \frac{\sqrt{3}}{1}$

$= \sqrt{4}$ or 2 $\quad = \frac{\pi}{3}$

$\left(2, \frac{\pi}{3}\right)$

17. $r = \sqrt{\left(-\frac{1}{4}\right)^2 + \left(-\frac{\sqrt{3}}{4}\right)^2}$ $\quad\theta = \text{Arctan}\left(\frac{-\frac{\sqrt{3}}{4}}{-\frac{1}{4}}\right)$

$= \sqrt{\frac{4}{16}}$ $\quad = \text{Arctan}\left(\frac{\sqrt{3}}{1}\right)$ or $\frac{4\pi}{3}$

$= \frac{2}{4}$ or $\frac{1}{2}$

$\left(\frac{1}{2}, \frac{4\pi}{3}\right)$

18. $r = \sqrt{3^2 + 8^2}$ $\quad\theta = \text{Arctan}\left(\frac{8}{3}\right)$

$= \sqrt{73} \approx 8.54$ $\quad \approx 1.21$

(8.54, 1.21)

19. $r = \sqrt{4^2 + (-7)^2}$ $\quad\theta = \text{Arctan}\left(\frac{-7}{4}\right)$

$= \sqrt{65} \approx 8.06$ $\quad \approx -1.05$

Add 2π to obtain $\theta = 5.23$.

(8.06, 5.23)

20. $x = 3\cos\left(\frac{\pi}{2}\right)$ $\quad y = 3\sin\left(\frac{\pi}{2}\right)$

$= 0$ $\quad = 3$

(0, 3)

21. $x = \frac{1}{2}\cos\left(\frac{3\pi}{4}\right)$ $\quad y = \frac{1}{2}\sin\left(\frac{3\pi}{4}\right)$

$= \frac{1}{2}\left(-\frac{\sqrt{2}}{2}\right)$ $\quad = \frac{1}{2}\left(-\frac{\sqrt{2}}{2}\right)$

$= -\frac{\sqrt{2}}{4}$ $\quad = \frac{\sqrt{2}}{4}$

$\left(-\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4}\right)$

22. $x = -1\cos\left(-\frac{\pi}{6}\right)$ $\quad y = -1\sin\left(-\frac{\pi}{6}\right)$

$= -1\left(\frac{\sqrt{3}}{2}\right)$ $\quad = -1\left(-\frac{1}{2}\right)$

$= -\frac{\sqrt{3}}{2}$ $\quad = \frac{1}{2}$

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

23. $x = -2\cos 270°$ $\quad y = -2\sin 270°$

$= 0$ $\quad = 2$

(0, 2)

24. $x = 4\cos 210°$ $\quad y = 4\sin 210°$

$= 4\left(-\frac{\sqrt{3}}{2}\right)$ $\quad = 4\left(-\frac{1}{2}\right)$

$= -2\sqrt{3}$ $\quad = -2$

$(-2\sqrt{3}, -2)$

25. $x = 14\cos 130°$ $\quad y = 14\sin 130°$

≈ -9.00 $\quad \approx 10.72$

(−9.00, 10.72)

26.
$$\begin{aligned} x &= -7 \\ r\cos\theta &= -7 \\ r &= \frac{-7}{\cos\theta} \\ r &= -7\sec\theta \end{aligned}$$

27.
$$\begin{aligned} y &= 5 \\ r\sin\theta &= 5 \\ r &= \frac{5}{\sin\theta} \\ r &= 5\csc\theta \end{aligned}$$

28.
$$\begin{aligned} x^2 + y^2 &= 25 \\ (r\cos\theta)^2 + (r\sin\theta)^2 &= 25 \\ r^2(\cos^2\theta + \sin^2\theta) &= 25 \\ r^2 &= 25 \end{aligned}$$
$r = 5$ or $r = -5$

29.
$$\begin{aligned} x^2 + y^2 &= 2y \\ (r\cos\theta)^2 + (r\sin\theta)^2 &= 2r\sin\theta \\ r^2(\cos^2\theta + \sin^2\theta) &= 2r\sin\theta \\ r^2 &= 2r\sin\theta \\ r &= 2\sin\theta \end{aligned}$$

30.
$$\begin{aligned} x^2 - y^2 &= 1 \\ (r\cos\theta)^2 - (r\sin\theta)^2 &= 1 \\ r^2(\cos^2\theta - \sin^2\theta) &= 1 \\ r^2(\cos 2\theta) &= 1 \\ r^2 &= \frac{1}{\cos 2\theta} \\ r^2 &= \sec 2\theta \end{aligned}$$

31.
$$\begin{aligned} x^2 + (y-2)^2 &= 4 \\ x^2 + y^2 - 4y + 4 &= 4 \\ (r\cos\theta)^2 + (r\sin\theta)^2 - 4r\sin\theta &= 0 \\ r^2(\cos^2\theta + \sin^2\theta) - 4r\sin\theta &= 0 \\ r^2 - 4r\sin\theta &= 0 \\ r^2 &= 4r\sin\theta \\ r &= 4\sin\theta \end{aligned}$$

32.
$$\begin{aligned} r &= 2 \\ \sqrt{x^2 + y^2} &= 2 \\ x^2 + y^2 &= 4 \end{aligned}$$

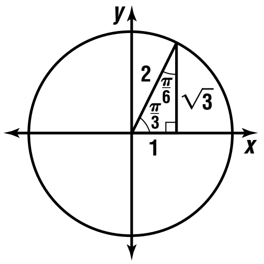
33.
$$\begin{aligned} r &= -3 \\ \sqrt{x^2 + y^2} &= -3 \\ x^2 + y^2 &= 9 \end{aligned}$$

34. $\theta = \frac{\pi}{3}$

$\text{Arctan}\,\frac{y}{x} = \frac{\pi}{3}$

$\frac{y}{x} = \frac{\sqrt{3}}{1}$

$y = \sqrt{3}x$



35. $r = 2\csc\theta$

$\frac{r}{r} = \frac{2}{r\sin\theta}$

$1 = \frac{2}{y}$

$y = 2$

36. $r = 3\cos\theta$

$r^2 = 3r\cos\theta$

$x^2 + y^2 = 3x$

37. $r^2\sin 2\theta = 8$

$r^2 2\sin\theta\cos\theta = 8$

$2r\sin\theta\, r\cos\theta = 8$

$2yx = 8$

$xy = 4$

38. $y = x$

$\frac{y}{x} = 1$

$\text{Arctan}\,\frac{y}{x} = \text{Arctan}\,1$

$\theta = \frac{\pi}{4}$

39. $r = \sin\theta$

$r^2 = r\sin\theta$

$x^2 + y^2 = y$

40. $x = 325\cos 70°$ $\qquad$ $y = 325\sin 70°$

≈ 111.16 $\qquad$ ≈ 305.40

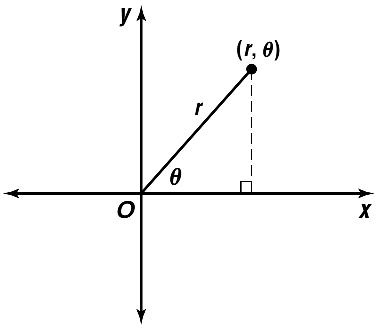
$(111.16, 305.40)$

41. $\dfrac{\frac{5\pi}{4}}{6} - \dfrac{\frac{\pi}{4}}{6} = \frac{5\pi}{24} - \frac{\pi}{24}$

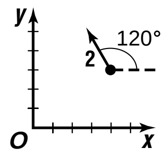
$= \frac{4\pi}{24}$

≈ 0.52 unit

42. Drop a perpendicular from the point with polar coordinates (r, θ) to the x-axis. r is the length of the hypotenuse in the resulting right triangle. x is the length of the side adjacent to angle θ, so $\cos\theta = \frac{x}{r}$. Solving for x gives $x = r\cos\theta$. y is the length of the side opposite angle θ, so $\sin\theta = \frac{y}{r}$. Solving for y gives $y = r\sin\theta$. (The figure is drawn for a point in the first quadrant, but the signs work out correctly regardless of where in the plane the point is located.)



43. horizontal distance:
$25(4 + 2\cos 120°) = 75$ m east
vertical distance:
$25(3 + 2\sin 120°) = 118.30$ m north



44a. $x = 4\cos 20°$ $\qquad$ $y = 4\sin 20°$

≈ 3.76 $\qquad$ $= 1.37$

$\langle 3.76, 1.37\rangle$

$x = 5\cos 70°$ $\qquad$ $y = 5\sin 70°$

≈ 1.71 $\qquad$ ≈ 4.70

$\langle 1.71, 4.70\rangle$

44b. $\langle 3.76, 1.37\rangle + \langle 1.71, 4.70\rangle$

$= \langle 3.76 + 1.71, 1.37 + 4.70\rangle$

$= \langle 5.47, 6.07\rangle$

44c. $5.47 = r\cos\theta$; $6.07 = r\sin\theta$

$\frac{6.07}{5.47} = \frac{r\sin\theta}{r\cos\theta}$

$\frac{6.07}{5.47} = \tan\theta$

$47.98 \approx \theta$; $47.98°$

$5.47 = r\cos 47.98°$

$\frac{5.47}{\cos 47.98°} = r$

$8.17 = r$

$8.17\angle 47.98°$

44d. $8.17\sin(3.14t + 47.98°)$

45. $r = 2a\sin\theta + 2a\cos\theta$

$r^2 = 2ar\sin\theta + ar\cos\theta$

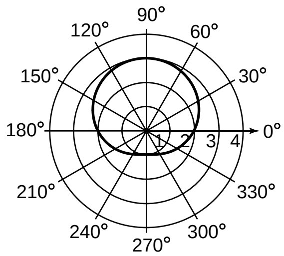
$x^2 + y^2 = 2ay + 2ax$

$x^2 - 2ax + y^2 - 2ay = 0$

$(x - a)^2 + (y - a)^2 = 2a^2$

The graph of the equation is the circle centered at (a, a) with radius $\sqrt{2}|a|$.

46.



47. Sample answer: $(-2, 405°)$, $(-2, 765°)$, $(2, 225°)$, $(2, 585°)$

$(r, \theta + 360k°)$

$\rightarrow (-2, 45° + 360(1)°) \rightarrow (-2, 405°)$

$\rightarrow (-2, 45° + 360(2)°) \rightarrow (-2, 765°)$

$(-r, \theta + (2k + 1)180°)$

$\rightarrow (2, 45° + (1)180°) \rightarrow (2, 225°)$

$\rightarrow (2, 45° + (3)180°) \rightarrow (2, 585°)$

48. $|\vec{r}|^2 = 50^2 + 425^2 = 2 \cdot 50 \cdot 425\cos 30°$

$|\vec{r}| \approx 382.52$ mph

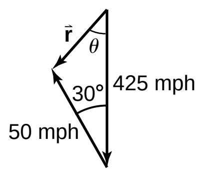
$\frac{50}{\sin\theta} = \frac{382.52}{\sin 30°}$

$50\sin 30° = 382.52\sin\theta$

$\frac{50\sin 30°}{382.52} = \sin\theta$

$3°45' \approx \theta$

The direction is $3°45'$ west of south.



49. $\sin^2 A = \cos A - 1$

$1 - \cos^2 A = \cos A - 1$

$0 = \cos^2 A + \cos A - 2$

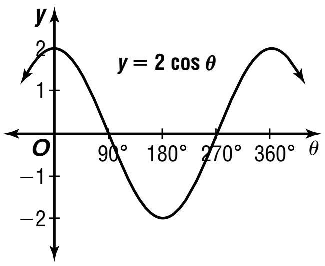
$0 = (\cos A + 2)(\cos A - 1)$

$\cos A + 2 = 0$ or $\cos A - 1 = 0$

$\cos A = -2$ $\quad$ $\cos A = 1$

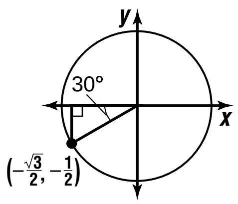
$A = 0°$

50.



51. The terminal side is in the third quadrant and the reference angle is $210 - 180$ or $30°$.

$\cos 210° = -\frac{\sqrt{3}}{2}$



52. Enter the x-values in L1 and the $f(x)$-values in L2 of your graphing calculator. Make a scatter plot. The data points are in the shape of parabola. Perform a quadratic regression.
$a \approx -0.07$, $b \approx 0.73$, $c \approx -1.36$
Sample answer:
$y = -0.07x^2 + 0.73x - 1.36$

53.
$$\begin{array}{r|rrrrrr} 2 & 1 & 0 & 0 & -3 & 0 & -20 \\ & & 2 & 4 & 8 & 10 & 20 \\ \hline & 1 & 2 & 4 & 5 & 10 & 0 \end{array}$$

$x^4 + 2x^3 + 4x^2 + 5x + 10$

54. $m = \frac{625 - 145}{25 - 17}$ $\quad$ $(y - 145) = 60(x - 17)$

$= 60$ $\quad$ $y = 60x - 875$

55. $x > y$ and $y > z$, so $x > z$.

If $x > z$, then $0 < \frac{z}{x} < 1$.

The correct choice is C.

9-4 Polar Form of a Linear Equation

Pages 577–578 Check for Understanding

1. The polar equation of a line is $p = r \cos (\theta - \phi)$. r and θ are the variables. p is the length of the normal segment from the line to the origin and ϕ is the angle the normal makes with the positive x-axis.
2. For r to be equal to p, we must have $\cos (\theta - \phi) = 1$. The first positive value of θ for which this is true is $\theta = \phi$.
3. The graph of the equation $x = k$ is a vertical line. Since the line is vertical, the x-axis is the normal line through the origin. Therefore, $\phi = 0°$ or $\phi = 180°$, depending on whether k is positive or negative, respectively. The origin is $|k|$ units from the given vertical line, so $p = |k|$. The polar form of the given line is $k = r \cos (\theta - 0°)$ if k is positive or $-k = r \cos (\theta - 180°)$ if k is negative. Both equations simplify to $k = r \cos \theta$.
4. You can use the extra ordered pairs as a check on your work. If all the ordered pairs you plot are not collinear, then you have made a mistake.
5. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{3^2 + (-4)^2}$

   $= \pm 5$

   Since C is negative, use $+5$.

   $\frac{3}{5}x - \frac{4}{5}y - 2 = 0$

   $\cos \phi = \frac{3}{5}$, $\sin \phi = -\frac{4}{5}$, $p = 2$

   $\phi = \text{Arctan } -\frac{4}{3}$

   $\approx -53°$ or $307°$

   $p = r \cos (\theta - \phi)$

   $2 = r \cos (\theta - 307°)$
6. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{(-2)^2 + 4^2}$

   $= \pm 2\sqrt{5}$

   Since C is negative, use $+2\sqrt{5}$.

   $-\frac{2}{2\sqrt{5}} x + \frac{4}{2\sqrt{5}} y - \frac{9}{2\sqrt{5}} = 0$

   $\cos \phi = -\frac{\sqrt{5}}{5}$, $\sin \phi = \frac{2\sqrt{5}}{5}$, $p = \frac{9\sqrt{5}}{10}$

   $\phi = \text{Arctan}(-2)$

   $\approx -63°$

   Since $\cos \phi < 0$, but $\sin \phi > 0$, the normal lies in the second quadrant.

   $\phi = 180° - 63°$ or $117°$

   $p = r \cos (\theta - \phi)$

   $\frac{9\sqrt{5}}{10} = r \cos (\theta - 117°)$
7. $3 = r \cos (\theta - 60°)$

   $0 = r \cos (\theta - 60°) - 3$

   $0 = r (\cos \theta \cos 60° + \sin \theta \sin 60°) - 3$

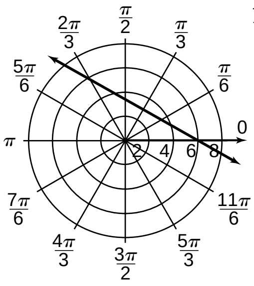
   $0 = \frac{1}{2}r \cos \theta + \frac{\sqrt{3}}{2} r \sin \theta - 3$

   $0 = \frac{1}{2}x + \frac{\sqrt{3}}{2}y - 3$

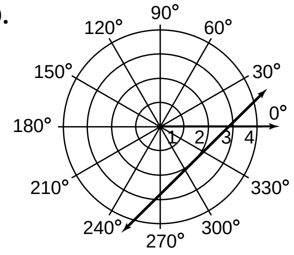
   $0 = x + \sqrt{3}y - 6$ or

   $x + \sqrt{3}y - 6 = 0$
8. $$r = 2 \sec \left(\theta + \frac{\pi}{4}\right)$$
   $$r \cos \left(\theta + \frac{\pi}{4}\right) = 2$$
   $$r \left(\cos \theta \cos \frac{\pi}{4} - \sin \theta \sin \frac{\pi}{4}\right) - 2 = 0$$
   $$\frac{\sqrt{2}}{2} r \cos \theta - \frac{\sqrt{2}}{2} r \sin \theta - 2 = 0$$
   $$\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y - 2 = 0$$
   $$\sqrt{2}x - \sqrt{2}y - 4 = 0$$

9.



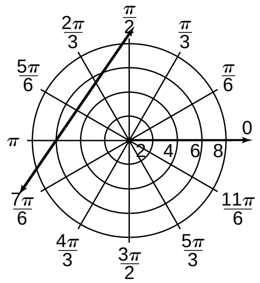
10.



11a. $p = r \cos (\theta - \phi) \rightarrow 5 = r \cos \left(\theta - \frac{5\pi}{6}\right)$

Since the shortest distance is along the normal, the answer is (p, ϕ) or $\left(5, \frac{5\pi}{6}\right)$.

11b.



Pages 578–579 Exercises

12. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{7^2 + (-24)^2}$
$= \pm 25$

Since C is positive, use -25.

$-\frac{7}{25}x + \frac{24}{25}y - 4 = 0$

$\cos \phi = -\frac{7}{25}, \sin \phi = \frac{24}{25}, p = 4$

$\phi = \text{Arctan } -\frac{24}{7}$

$\approx -74°$

Since $\cos \phi < 0$, but $\sin \phi > 0$, the normal lies in the second quadrant.

$\phi = 180° - 74°$ or $106°$

$p = r \cos (\theta - \phi)$

$4 = r \cos (\theta - 106°)$

13. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{21^2 + 20^2}$
$= \pm 29$

Since C is negative, use $+29$.

$\frac{21}{29}x + \frac{20}{29}y - \frac{87}{29} = 0$

$\cos \phi = \frac{21}{29}, \sin \phi = \frac{20}{29}, p = 3$

$\phi = \text{Arctan } \frac{20}{21}$

$\approx 44°$

$p = r \cos (\theta - \phi)$

$3 = r \cos (\theta - 44°)$

14. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{6^2 + (-8)^2}$
$= \pm 10$

Since C is negative, use $+10$.

$\frac{6}{10}x - \frac{8}{10}y - \frac{21}{10} = 0$

$\cos \phi = \frac{3}{5}, \sin \phi = -\frac{4}{5}, p = 2.1$

$\phi = \text{Arctan } -\frac{4}{3}$

$\approx -53°$

Since $\cos \phi > 0$, but $\sin \phi < 0$, the normal lies in the fourth quadrant.

$\phi = 360° - 53°$ or $307°$

$p = r \cos (\theta - \phi)$

$2.1 = r \cos (\theta - 307°)$

15. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{3^2 + 2^2}$
$= \pm\sqrt{13}$

Sinc C is negative, use $\pm\sqrt{13}$.

$\frac{3}{\sqrt{13}}x + \frac{2}{\sqrt{13}}y - \frac{5}{\sqrt{13}} = 0$

$\cos \phi = \frac{3\sqrt{13}}{13}, \sin \phi = \frac{2\sqrt{13}}{13}, p = \frac{5\sqrt{13}}{13}$

$\phi = \text{Arctan } \left(\frac{2}{3}\right)$

$\approx 34°$

$p = r \cos (\theta - \phi)$

$\frac{5\sqrt{13}}{13} = r \cos (\theta - 34°)$

16. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{4^2 + (-5)^2}$
$= \pm\sqrt{14}$

Since C is negative, use $\pm\sqrt{41}$.

$\frac{4}{\sqrt{41}}x - \frac{5}{\sqrt{41}}y - \frac{10}{\sqrt{41}} = 0$

$\cos \phi = \frac{4\sqrt{41}}{41}, \sin \phi = \frac{-5\sqrt{41}}{41}, p = \frac{10\sqrt{41}}{41}$

$\phi = \text{Arctan } \left(-\frac{5}{4}\right)$

$\approx -51°$

Since $\cos \phi > 0$, but $\sin \phi < 0$, the normal lies in the fourth quadrant.

$\phi = 360° - 51°$ or $309°$

$p = r \cos (\theta - \phi)$

$\frac{10\sqrt{41}}{41} = r \cos (\theta - 309°)$

17. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{(-1)^2 + 3^2}$
$= \pm\sqrt{10}$

Since C is negative, use $+\sqrt{10}$.

$-\frac{1}{\sqrt{10}}x + \frac{3}{\sqrt{10}}y - \frac{7}{\sqrt{10}} = 0$

$\cos \phi = -\frac{\sqrt{10}}{10} \sin \phi = \frac{3\sqrt{10}}{10}, p = \frac{7\sqrt{10}}{10}$

$\phi = \text{Arctan } (-3)$

$\approx -72°$

Since $\cos \phi < 0$, but $\sin \phi > 0$, the normal lies in the second quadrant.

$\phi = 180° - 72°$ or $108°$

$p = r \cos (\theta - \phi)$

$\frac{7\sqrt{10}}{10} = r \cos (\theta - 108°)$

18. $6 = r \cos (\theta - 120°)$

$0 = r (\cos \theta \cos 120° + \sin \theta \sin 120°) - 6$

$0 = -\frac{1}{2} r \cos \theta + \frac{\sqrt{3}}{2} r \sin \theta - 6$

$0 = -\frac{1}{2}x + \frac{\sqrt{3}}{2}y - 6$

$0 = -x + \sqrt{3}y - 12$ or

$-x + \sqrt{3}y - 12 = 0$

19. $4 = r\cos\left(\theta + \frac{\pi}{4}\right)$

$0 = r\left(\cos\theta\cos\frac{\pi}{4} - \sin\theta\sin\frac{\pi}{4}\right) - 4$

$0 = \frac{\sqrt{2}}{2}r\cos\theta - \frac{\sqrt{2}}{2}r\sin\theta - 4$

$0 = \frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y - 4$

$0 = \sqrt{2}x - \sqrt{2}y - 8$ or

$\sqrt{2}x - \sqrt{2}y - 8 = 0$

20. $2 = r\cos(\theta + \pi)$

$0 = r(\cos\theta\cos\pi - \sin\theta\sin\pi) - 2$

$0 = -r\cos\theta - 2$

$0 = -x - 2$

$x = -2$

21. $1 = r\cos(\theta - 330°)$

$0 = r(\cos\theta\cos 330° + \sin\theta\sin 330°) - 1$

$0 = \frac{\sqrt{3}}{2}r\cos\theta + \frac{1}{2}r\sin\theta - 1$

$0 = \frac{\sqrt{3}}{2}x - \frac{1}{2}y - 1$

$0 = \sqrt{3}x - y - 2$ or

$\sqrt{3}x - y - 2 = 0$

22. $r = 11\sec\left(\theta + \frac{7\pi}{6}\right)$

$r\cos\left(\theta + \frac{7\pi}{6}\right) = 11$

$r\left(\cos\theta\cos\frac{7\pi}{6} - \sin\theta\sin\frac{7\pi}{6}\right) - 11 = 0$

$-\frac{\sqrt{3}}{2}r\cos\theta + \frac{1}{2}r\sin\theta - 11 = 0$

$-\frac{\sqrt{3}}{2}x + \frac{1}{2}y - 11 = 0$

$-\sqrt{3}x + y - 22 = 0$

23. $r = 5\sec(\theta - 60°)$

$r\cos(\theta - 60°) = 5$

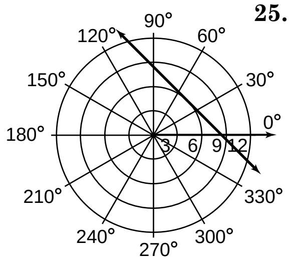
$r(\cos\theta\cos 60° + \sin\theta\sin 60°) - 5 = 0$

$\frac{1}{2}r\cos\theta + \frac{\sqrt{3}}{2}r\sin\theta - 5 = 0$

$\frac{1}{2}x + \frac{\sqrt{3}}{2}y - 5 = 0$

$x + \sqrt{3}y - 10 = 0$

24.



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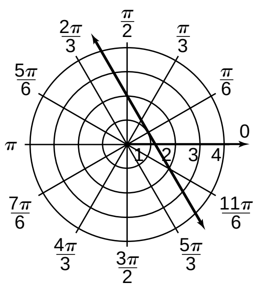

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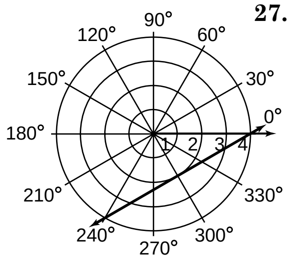

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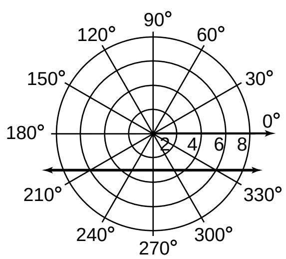

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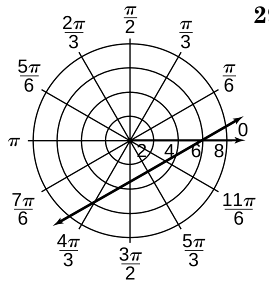
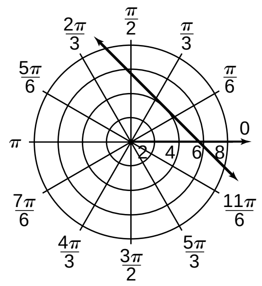


29.



30. $m = \frac{4}{-6}$ or $-\frac{2}{3}$

$(y + 1) = -\frac{2}{3}(x - 4) \rightarrow 2x + 3y - 5 = 0$

$\pm\sqrt{A^2 + B^2} = \pm\sqrt{2^2 + 3^2}$

$= \pm\sqrt{13}$

Since C is negative, use $+\sqrt{13}$.

$\frac{2}{\sqrt{13}} + \frac{3}{\sqrt{13}} - \frac{5}{\sqrt{13}} = 0$

$\cos\phi = \frac{2\sqrt{13}}{13}$, $\sin\phi = \frac{3\sqrt{13}}{13}$, $p = \frac{5\sqrt{13}}{13}$

$\phi = \text{Arctan}\,\frac{3}{2}$

$\approx 56°$

$p = r\cos(\theta - \phi)$

$\frac{5\sqrt{13}}{13} = r\cos(\theta - 56°)$

31. $p = r\cos(\theta - \phi)$

$\rightarrow p = 3\cos\left(\frac{\pi}{4} - \phi\right)$

$\rightarrow p = 2\cos\left(\frac{7\pi}{6} - \phi\right)$

Use a graphing calculator and the INTERJECT feature to find solutions to the system at (2.25, 0.31) and (5.39, −0.31). Since p, the length of the normal, must be positive, use $\phi = 2.25$ and $p = 0.31$.

$0.31 = r\cos(\theta - 2.25)$

32a. $p = r\cos(\theta - \phi) \rightarrow 6 = r\cos(\theta - 16°)$

Since the shortest distance is along the normal, the closest the fly came was p or 6 cm.

32b. (p, ϕ) or $(6, 15°)$

33. Since both normal segments have length 2, p must be 2 in both equations. Since the two lines intersect at right angles, their normals also intersect at right angles. This can be achieved by having the two ϕ-values differ by 90°. To make sure neither line is vertical, neither ϕ-value should be a multiple of 90°. Therefore, a sample answer is $2 = r\cos(\theta - 45°)$ and $2 = r\cos(\theta - 135°)$.

34. $m = 0$

$(y - 4) = 0(x - 5) \rightarrow y - 4 = 0$

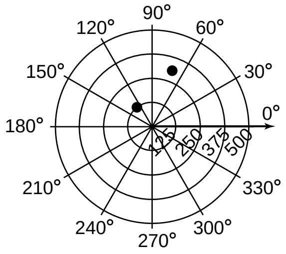
$\cos\phi = 0$, $\sin\phi = 1$, $p = 4$

Since $\cos\phi = 0$ when $\sin\phi = 1$, $\phi = 90°$.

$p = r\cos(\theta - \phi)$

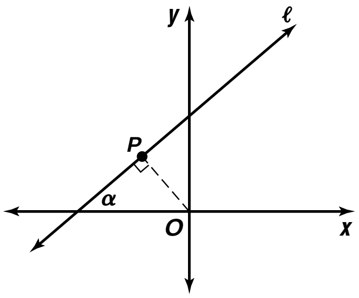
$4 = r\cos(\theta - 90°)$

35a.



35b. $p = r\cos(\theta - \phi)$
$\rightarrow p = 125\cos(130 - \phi)$
$\rightarrow p = 300\cos(70 - \phi)$
Use a graphing calculator and the INTERSECT feature to find the solutions to the system at $(-45, -124.43)$ and $(135, 124.43)$. Sinc p, the length of the normal, must be positive, use $\phi = 135°$ and $p = 124.43$.
$124.43 = r\cos(\theta - 135°)$

36. $k = r\sin(\theta - \alpha)$
$k = r[\sin\theta\cos\alpha - \cos\theta\sin\alpha]$
$k = r\sin\theta\cos\alpha - r\cos\theta\sin\alpha$
$k = y\cos\alpha - x\sin\alpha$
This is the equation of a line in rectangular coordinates. Solving the last equation for y yields $y = (\tan\alpha)x + \frac{k}{\cos\alpha}$. The slope of the line shows that α is the angle the line makes with the x-axis. To find the length of the normal segment in the figure, observe that the complementary angle to α in the right triangle is $90° - \alpha$, so the θ-coordinate of P in polar coordinates is $180° - (90° - \alpha) = \alpha + 90°$. Substitute into the original polar equation to find the r-coordinate of P:
$k = r\sin(\alpha + 90° - \alpha)$
$k = r\sin 90°$
$k = r$
Therefore, k is the length of the normal segment.



37. $p = r\cos(\theta - \phi)$
$\rightarrow p = 40\cos(0° - \phi)$
$\rightarrow p = 40\cos(72° - \phi)$
Use a graphing calculator and the INTERSECT feature to find the solutions of the system at $(-144, -32.36)$ and $(36, 32.36)$. Since p, the length of the normal, must be positive, use $\phi = 36°$ and $p = 32.36$.
$32.36 = r\cos(\theta - 36°)$

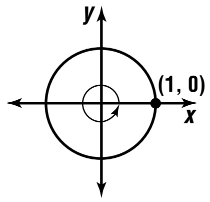
38.
$$r = 6$$
$$\sqrt{x^2 + y^2} = 6$$
$$x^2 + y^2 = 36$$

39. The graph of a polar equation of the form $r = a\sin n\theta$ is a rose.

40. $x - 3y = 6$
$$y = \frac{-x + 6}{-3}$$
$$y = \frac{1}{3}x - 2$$
$$x = t,\ y = \frac{1}{3}t - 2$$

41. $A = \frac{N}{360}\left(\pi r^2\right)$
$$= \frac{65}{360}\left(\pi 6^2\right)$$
$$\approx 20.42 \text{ ft}^2$$

42. Since 360° lies on the x-axis of the unit circle at (1, 0), $\sin 360° = y$ or 0.

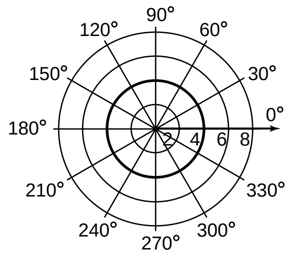


43. $2x^3 + 5x^2 - 12x = 0$
$$x(2x^2 + 5x - 12) = 0$$
$$x(2x - 3)(x + 4) = 0$$
$$x = 0 \text{ or } x = \frac{3}{2} \text{ or } x = -4$$

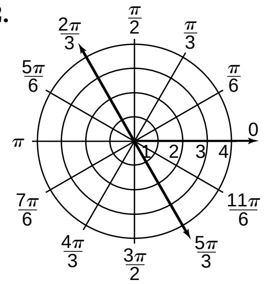
44.
$$c^2 - d^2 = 48$$
$$(c + d)(c - d) = 48$$
$$12(c - d) = 48$$
$$c - d = 4$$

Page 579 Mid-Chapter Quiz

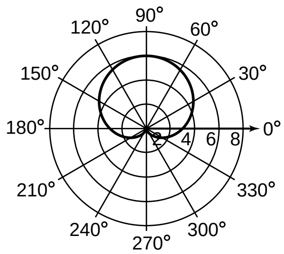
1.



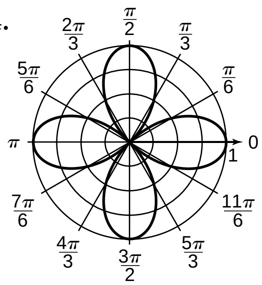
2.



3.



4.



5. $r = \sqrt{(-\sqrt{2})^2 + (-\sqrt{2})^2}$ $\quad\theta = \text{Arctan}\left(\frac{-\sqrt{2}}{-\sqrt{2}}\right)$
$= \sqrt{4}$ or 2 $\quad = \frac{\pi}{4}$
Since $(-\sqrt{2}, -\sqrt{2})$ is in the third quadrant, $\theta = \pi + \frac{\pi}{4}$ or $\frac{5\pi}{4}$.
$\left(2, \frac{5\pi}{4}\right)$

6. $r = \sqrt{0^2 + (-4)^2}$
$= \sqrt{16}$ or 4
Since $x = 0$ when $y = -4$, $\theta = \frac{3\pi}{2}$.
$\left(4, \frac{3\pi}{2}\right)$

7. $x^2 + y^2 = 36$
$\sqrt{x^2 + y^2} = \pm\sqrt{36}$
$r = 6$ or $r = -6$

8. $r = 2 \csc \theta$
$r \sin \theta = 2$
$y = 2$

9. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{5^2 + (-12)^2}$
$= \pm 13$
Since C is positive, use -13.
$-\frac{5}{13}x + \frac{12}{13}y - \frac{3}{13} = 0$
$\cos \phi = -\frac{5}{13}$, $\sin \phi = \frac{12}{13}$, $p = \frac{3}{13}$
$\phi = \text{Arctan}\left(-\frac{12}{5}\right)$
$\approx -67°$
Since $\cos \phi < 0$, but $\sin \phi > 0$, the normal lies in the second quadrant.
$\phi = 180° - 67°$ or $113°$
$p = r \cos(-\phi)$
$\frac{3}{13} = r \cos(\theta - 113°)$

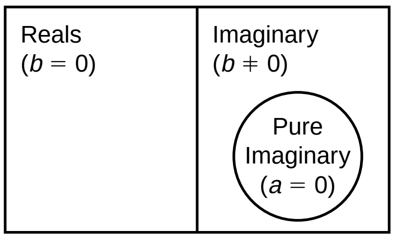
10. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{(-2)^2 + (-6)^2}$
$= \pm 2\sqrt{10}$
Since C is negative, use $+2\sqrt{10}$.
$-\frac{2}{2\sqrt{10}x} - \frac{6}{2\sqrt{10}}y - \frac{2}{2\sqrt{10}} = 0$
$\cos \phi = -\frac{\sqrt{10}}{10}$, $\sin \phi = -\frac{3\sqrt{10}}{10}$, $p = \frac{\sqrt{10}}{10}$
$\phi = \text{Arctan}\left(\frac{-3}{-1}\right)$
$\approx 72°$
Since $\cos \phi < 0$ and $\sin \phi < 0$, the normal lies in the third quadrant.
$\phi = 180° + 72°$ or $252°$
$p = r \cos(\theta - \phi)$
$\frac{\sqrt{10}}{10} = r \cos(\theta - 252°)$

9-5 Simplifying Complex Numbers

Page 583 Check for Understanding

1. Find the (positive) remainder when the exponent is divided by 4. If the remainder is 0, the answer is 1; if the remainder is 1, the answer is $\boldsymbol{i}$; if the remainder is 2, the answer is -1; and if the remainder is 3, the answer is $-\boldsymbol{i}$.

2. **Complex Numbers ($a + bi$)**



3. When you multiply the denominators, you will be multiplying a complex number and its conjugate. This makes the denominator of the product a real number, so you can then write the answer in the form $a + b\boldsymbol{i}$.

4. Sample answer: $x^2 + 1 = 0$
$(x - \boldsymbol{i})(x + \boldsymbol{i}) = 0$, where the solutions are $x = \pm\boldsymbol{i}$.
$x^2 + x\boldsymbol{i} - x\boldsymbol{i} - \boldsymbol{i}^2 = 0$
$x^2 - (-1) = 0$
$x^2 + 1 = 0$

5. $\boldsymbol{i}^{-6} = (\boldsymbol{i}^4)^{-2} \cdot \boldsymbol{i}^2$
$= 1^{-2} \cdot (-1)$
$= -1$

6. $\boldsymbol{i}^{10} + \boldsymbol{i}^2 = (\boldsymbol{i}^4)^2 \cdot \boldsymbol{i}^2 + \boldsymbol{i}^2$
$= (1)^2\boldsymbol{i}^2 + \boldsymbol{i}^2$
$= -1 + (-1)$ or -2

7. $(2 + 3\boldsymbol{i}) + (-6 + \boldsymbol{i}) = (2 + (-6)) + (3\boldsymbol{i} + \boldsymbol{i})$
$= -4 + 4\boldsymbol{i}$

8. $(2.3 + 4.1\boldsymbol{i}) - (-1.2 - 6.3\boldsymbol{i})$
$= (2.3 - (-1.2)) + (4.1\boldsymbol{i} - (-6.3\boldsymbol{i}))$
$= 3.5 + 10.4\boldsymbol{i}$

9. $(2 + 4\boldsymbol{i}) + (-1 + 5\boldsymbol{i}) = (2 + (-1)) + (4\boldsymbol{i} + 5\boldsymbol{i})$
$= 1 + 9\boldsymbol{i}$

10. $(-2 - \boldsymbol{i})^2 = (-2 - \boldsymbol{i})(-2 - \boldsymbol{i})$
$= 4 + 4\boldsymbol{i} + \boldsymbol{i}^2$
$= 3 + 4\boldsymbol{i}$

11. $\frac{\boldsymbol{i}}{1 + 2\boldsymbol{i}} = \frac{\boldsymbol{i}}{1 + 2\boldsymbol{i}} \cdot \frac{1 - 2\boldsymbol{i}}{1 - 2\boldsymbol{i}}$
$= \frac{\boldsymbol{i} - 2\boldsymbol{i}^2}{1 - 4\boldsymbol{i}^2}$
$= \frac{\boldsymbol{i} + 2}{5}$
$= \frac{2}{5} + \frac{1}{5}\boldsymbol{i}$

12. $(2.5 + 3.1\boldsymbol{i}) + (-6.2 + 4.3\boldsymbol{i})$
$= (2.5 + (-6.2)) + (3.1\boldsymbol{i} + 4.3\boldsymbol{i})$
$= -3.7 + 7.4\boldsymbol{i}$ N

Pages 583–585 Exercises

13. $\boldsymbol{i}^6 = \boldsymbol{i}^4 \cdot \boldsymbol{i}^2$
$= 1 \cdot -1$
$= -1$

14. $\boldsymbol{i}^{19} = (\boldsymbol{i}^4)^4 \cdot \boldsymbol{i}^3$
$= 1^4 \cdot -\boldsymbol{i}$
$= -\boldsymbol{i}$

15. $\boldsymbol{i}^{1776} = (\boldsymbol{i}^4)^{444}$
$= 1^{444}$
$= 1$

16. $\boldsymbol{i}^9 + \boldsymbol{i}^{-5} = (\boldsymbol{i}^4)^2 \cdot \boldsymbol{i} + (\boldsymbol{i}^4)^{-2} \cdot \boldsymbol{i}^3$
$= 1^2 \cdot \boldsymbol{i} + 1^{-2} \cdot -\boldsymbol{i}$
$= \boldsymbol{i} + (-\boldsymbol{i})$ or 0

17. $(3 + 2\boldsymbol{i}) + (-4 + 6\boldsymbol{i}) = (3 + (-4)) + (2\boldsymbol{i} + 6\boldsymbol{i})$
$= -1 + 8\boldsymbol{i}$

18. $(7 - 4\boldsymbol{i}) + (2 - 3\boldsymbol{i}) = (7 + 2) + (-4\boldsymbol{i} - 3\boldsymbol{i})$
$= 9 - 7\boldsymbol{i}$

19. $\left(\frac{1}{2} + \boldsymbol{i}\right) - (2 - \boldsymbol{i}) = \left(\frac{1}{2} + (-2)\right) + (\boldsymbol{i} - (-\boldsymbol{i}))$
$= -\frac{3}{2} + 2\boldsymbol{i}$

20. $(-3 - \boldsymbol{i}) - (4 - 5\boldsymbol{i}) = (-3 + (-4)) + (-\boldsymbol{i} - (-5\boldsymbol{i}))$
$= -7 + 4\boldsymbol{i}$

21. $(2 + \boldsymbol{i})(4 + 3\boldsymbol{i}) = 8 + 10\boldsymbol{i} + 3\boldsymbol{i}^2$
$= 5 + 10\boldsymbol{i}$

22. $(1 + 4\boldsymbol{i})^2 = (1 + 4\boldsymbol{i})(1 + 4\boldsymbol{i})$
$= 1 + 8\boldsymbol{i} + 16\boldsymbol{i}^2$
$= -15 + 8\boldsymbol{i}$

23. $(1 + \sqrt{7}i)(-2 - \sqrt{5}i)$
$= -2 - \sqrt{5}i - 2\sqrt{7}i - \sqrt{35}i^2$
$= (-2 + \sqrt{35}) + (-2\sqrt{7} - \sqrt{5})i$

24. $(2 + \sqrt{-3})(-1 + \sqrt{-12}) = (2 + \sqrt{3}i)(-1 + \sqrt{12}i)$
$= -2 + 2\sqrt{12}i - \sqrt{3}i + \sqrt{36}i^2$
$= -2 + 4\sqrt{3}i - \sqrt{3}i - 6$
$= -8 + 3\sqrt{3}i$

25. $\frac{2 + i}{1 + 2i} = \frac{2 + i}{1 + 2i} \cdot \frac{1 - 2i}{1 - 2i}$
$= \frac{2 - 3i - 2i^2}{1 - 4i^2}$
$= \frac{4 - 3i}{5}$
$= \frac{4}{5} - \frac{3}{5}i$

26. $\frac{3 - 2i}{-4 - i} = \frac{3 - 2i}{-4 - i} \cdot \frac{-4 + i}{-4 + i}$
$= \frac{-12 + 11i - 2i^2}{16 - i^2}$
$= \frac{-10 + 11i}{17}$
$= -\frac{10}{17} + \frac{11}{17}i$

27. $\frac{5 - i}{5 + i} = \frac{5 - i}{5 + i} \cdot \frac{5 - i}{5 - i}$
$= \frac{25 - 10i + i^2}{25 - i^2}$
$= \frac{24 - 10i}{26}$
$= \frac{12}{13} - \frac{5}{13}i$

28. $(x - i)(x + i) = 0$
$x^2 - i^2 = 0$
$x^2 + 1 = 0$

29. $(x - (2 + i))(x - (2 - i)) = 0$
$(x - 2 - i)(x - 2 + i) = 0$
$x^2 - 2x + xi - 2x + 4 - 2i - xi + 2i - i^2 = 0$
$x^2 - 4x + 4 + 1 = 0$
$x^2 - 4x + 5 = 0$

30. $(2 - i)(3 + 2i)(1 - 4i) = (6 + i - 2i^2)(1 - 4i)$
$= (8 + i)(1 - 4i)$
$= 8 - 31i - 4i^2$
$= 12 - 31i$

31. $(-1 - 3i)(2 + 2i)(1 - 2i) = (-2 - 8i - 6i^2)(1 - 2i)$
$= (4 - 8i)(1 - 2i)$
$= 4 - 16i + 16i^2$
$= -12 - 16i$

32. $\dfrac{\frac{1}{2} + \sqrt{3}i}{1 - \sqrt{2}i} = \dfrac{\frac{1}{2} + \sqrt{3}i}{1 - \sqrt{2}i} \cdot \dfrac{1 + \sqrt{2}i}{1 + \sqrt{2}i}$
$= \dfrac{\frac{1}{2} + \frac{\sqrt{2}}{2}i + 3i + 6i^2}{1 - 2i^2}$
$= \dfrac{\left(\frac{1}{2} - \sqrt{6}\right) + \left(\frac{\sqrt{2}}{2} + \sqrt{3}\right)i}{3}$
$= \left(\frac{1}{6} - \frac{\sqrt{6}}{3}\right) + \left(\frac{\sqrt{3}}{3} + \frac{\sqrt{2}}{6}\right)i$

33. $\frac{2 - \sqrt{2}i}{3 + \sqrt{6}i} = \frac{2 - \sqrt{2}i}{3 + \sqrt{6}i} \cdot \frac{3 - \sqrt{6}i}{3 - \sqrt{6}i}$
$= \frac{6 - 2\sqrt{6}i - 3\sqrt{2}i + \sqrt{12}i^2}{9 - 6i^2}$
$= \frac{(6 - 2\sqrt{3}) + (-2\sqrt{6} - 3\sqrt{2})i}{15}$
$= \left(\frac{2}{5} - \frac{2\sqrt{3}}{15}\right) + \left(-\frac{\sqrt{2}}{5} - \frac{2\sqrt{6}}{15}\right)i$

34. $\frac{3 + i}{(2 + i)^2} = \frac{3 + i}{(2 + i)(2 + i)}$
$= \frac{3 + i}{4 + 4i + i^2}$
$= \frac{3 + i}{3 + 4i}$
$= \frac{3 + i}{3 + 4i} \cdot \frac{3 - 4i}{3 - 4i}$
$= \frac{9 - 9i - 4i^2}{9 - 16i^2}$
$= \frac{13 - 9i}{25}$
$= \frac{13}{25} - \frac{9}{25}i$

35. $\frac{(1 + i)^2}{(-3 + 2i)^2} = \frac{(1 + i)(1 + i)}{(-3 + 2i)(-3 + 2i)}$
$= \frac{1 + 2i + i^2}{9 - 12i + 4i^2}$
$= \frac{2i}{5 - 12i}$
$= \frac{2i}{5 - 12i} \cdot \frac{5 + 12i}{5 + 12i}$
$= \frac{10i + 24i^2}{25 - 144i^2}$
$= \frac{-24 + 10i}{169}$
$= -\frac{24}{169} + \frac{10}{169}i$

36a. $Z = R + (X_L - X_C)j$
$\rightarrow Z = 10 + (1 - 2)j \rightarrow Z = 10 - j$ ohms
$\rightarrow Z = 3 + (1 - 1)j \rightarrow Z = 3 + 0j$ ohms

36b. $(10 - j) + (3 + 0j) = (10 + 3) + (-1j + 0j)$
$= 13 - j$ ohms

36c. $S = \frac{1}{Z} \rightarrow S = \frac{1}{6 + 3j}$
$= \frac{1}{6 + 3j} \cdot \frac{6 - 3j}{6 - 3j}$
$= \frac{6 - 3j}{36 - 9j^2}$
$= \frac{6 - 3j}{45}$
$\approx 0.13 - 0.07j$ siemens

37a. $x = \frac{-8i \pm \sqrt{(8i)^2 - 4(1)(-25)}}{2(1)}$
$= \frac{-8i \pm \sqrt{36}}{2}$
$= \pm 3 - 4i$

37b. No

37c. The solutions need not be complex conjugates because the coefficients in the equation are not all real.

37d. $(3 - 4i)^2 + 8i(3 - 4i) - 25 \stackrel{?}{=} 0$
$-7 - 24i + 24i + 32 - 25 \stackrel{?}{=} 0$
$0 = 0$
$(-3 - 4i)^2 + 8i(-3 - 4i) - 25 \stackrel{?}{=} 0$
$-7 + 24i - 24i + 32 - 25 \stackrel{?}{=} 0$
$0 = 0$

38. $f(x + yi) = (x + yi)^2$
$= x^2 + 2xyi - y^2$
$= (x^2 - y^2) + 2xyi$

39a. $z_0 = 2 - i$
$z_1 = i(2 - i) + i^2$ or $1 + 2i$
$z_2 = i(2i + 1) = 2i^2 + i$ or $-2 + i$
$z_3 = i(-2 + i) = -2i + i^2$ or $-1 - 2i$
$z_4 = i(-1 - 2i) = -i - 2i^2$ or $2 - i$
$z_5 = i(2 - i) = 2i - i^2$ or $1 + 2i$

39b. $z_0 = 1 + 0\boldsymbol{i}$

$z_1 = (0.5 - 0.866\boldsymbol{i})(1 + 0\boldsymbol{i}) = 0.5 - 0.866\boldsymbol{i}$

$z_2 = (0.5 - 0.866\boldsymbol{i})(0.5 - 0.866\boldsymbol{i})$
$= 0.25 - 0.866\boldsymbol{i} - 0.75$
$= -0.500 - 0.866\boldsymbol{i}$

$z_3 = (0.5 - 0.866\boldsymbol{i})(-0.500 - 0.866\boldsymbol{i})$
$= -0.250 - 0.750$
$= -1.000 - 0.000\boldsymbol{i}$

$z_4 = (0.5 - 0.866\boldsymbol{i})(-1.000) = -0.500 + 0.866\boldsymbol{i}$

$z_5 = (0.5 - 0.866\boldsymbol{i})(-0.500 + 0.866\boldsymbol{i})$
$= -0.250 + 0.866\boldsymbol{i} + 0.75$
$= 0.500 + 0.866\boldsymbol{i}$

40. $(1 + 2\boldsymbol{i})^{-3} = \frac{1}{(1 + 2\boldsymbol{i})^3}$
$= \frac{1}{(-3 + 4\boldsymbol{i})(1 + 2\boldsymbol{i})}$
$= \frac{1}{-11 - 2\boldsymbol{i}}$
$= \frac{1}{-11 - 2\boldsymbol{i}} \cdot \frac{-11 + 2\boldsymbol{i}}{-11 + 2\boldsymbol{i}}$
$= \frac{-11 + 2\boldsymbol{i}}{125}$
$= -\frac{11}{125} + \frac{2}{125}\boldsymbol{i}$

41. $c_1(\cos 2t + \boldsymbol{i} \sin 2t) + c_2(\cos 2t - \boldsymbol{i} \sin 2t)$
$= c_1 \cos 2t + c_1\boldsymbol{i} \sin 2t + c_2 \cos 2t - c_2\boldsymbol{i} \sin 2t$
$= (c_1 + c_2)(\cos 2t) + (c_1 - c_2)(\boldsymbol{i} \sin 2t)$
$= (c_1 + c_2)(\cos 2t)$ only if $c_1 = c_2$

42. $\pm\sqrt{A^2 + b^2} = \pm\sqrt{6^2 + (-2)^2}$
$= \pm 2\sqrt{10}$

Since C is positive, use $-2\sqrt{10}$.

$-\frac{6}{2\sqrt{10}}x + \frac{2}{2\sqrt{10}}y - \frac{3}{2\sqrt{10}} = 0$

$\cos \phi = -\frac{3\sqrt{10}}{10}$, $\sin \phi = \frac{\sqrt{10}}{10}$, $p = \frac{3\sqrt{10}}{20}$

$\phi = \text{Arctan}\left(-\frac{1}{3}\right)$
$\approx -18°$

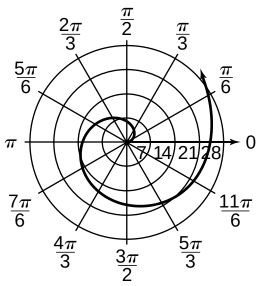
Since $\cos \phi < 0$, but $\sin \phi > 0$, the normal lies in the second quadrant.

$\phi = 180° - 18°$ or $162°$

$p = r \cos (\theta - \phi)$

$\frac{3\sqrt{10}}{20} = r \cos (\theta - 162°)$

43.



44. $\langle x - (-3), y - 6\rangle = t\langle 1, -4\rangle$
$\langle x + 3, y - 6\rangle = t\langle 1, -4\rangle$

45. $\vec{\boldsymbol{u}} = \frac{1}{4}\langle -8, 6, 4\rangle - 2\langle 2, -6, 3\rangle$
$= \left\langle -2, \frac{3}{2}, 1\right\rangle - \langle 4, -12, 6\rangle$
$= \left\langle -6, \frac{27}{2}, -5\right\rangle$

46. $\tan \alpha = \frac{4}{3}$
$\tan^2 \alpha + 1 = \sec^2 \alpha$
$\left(\frac{4}{3}\right)^2 + 1 = \sec^2 \alpha$
$\frac{25}{9} = \sec^2 \alpha$
$\frac{9}{25} = \cos^2 \alpha$
$\frac{3}{5} = \cos \alpha$
$\sin^2 \alpha + \cos^2 \alpha = 1$
$\sin^2 \alpha + \left(\frac{3}{5}\right)^2 = 1$
$\sin^2 \alpha = \frac{16}{25}$
$\sin \alpha = \frac{4}{5}$

$\cot B = \frac{5}{12}$
$1 + \cot^2 B = \csc^2 B$
$1 + \left(\frac{5}{12}\right)^2 = \csc^2 B$
$\frac{169}{144} = \csc^2 B$
$\frac{144}{169} = \sin^2 B$
$\frac{12}{13} = \sin B$
$\sin^2 B + \cos^2 B = 1$
$\left(\frac{12}{13}\right)^2 + \cos^2 B = 1$
$\cos^2 B = \frac{25}{169}$
$\cos B = \frac{5}{13}$

$\cos (\alpha + B) = \cos \alpha \cos B - \sin \alpha \sin B$
$= \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$
$= -\frac{33}{65}$

47. amplitude $= \frac{1}{2}(7)$ or 3.5

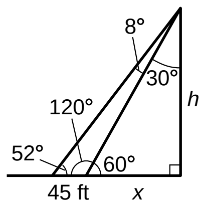
period $= \frac{2\pi}{12}$ or $\frac{\pi}{6}$

$y = 3.5 \cos \left(\frac{\pi}{6}t\right)$

48. $h = x\sqrt{3}$

$\tan 52° = \frac{x\sqrt{3}}{x + 45}$
$x \tan 52° + 45 \tan 52° = x\sqrt{3}$
$x \tan 52° - x\sqrt{3} = -45 \tan 52°$
$x = \frac{-45 \tan 52°}{\tan 52° - \sqrt{3}}$
$x \approx 127.40$

$h = x\sqrt{3} = 127.40(\sqrt{3}) \approx 221$ ft



49. Enter the x-values in L1 and the $f(x)$-values in L2 of your graphing calculator. Make a scatter plot. The data points are in the shape of a parabola, so a quadratic function would best model the set of data.

50. Let d = depth of the original pool.
The second pool's width $= 5d + 4$, the length $= 10d + 6$, and the depth $= d + 2$.

$(5d + 4)(10d + 6)(d + 2) = 3420$
$(50d^2 + 70d + 24)(d + 2) = 3420$
$50d^3 + 100d^2 + 70d^2 + 140d + 24d + 48 = 3420$
$50d^3 + 170d^2 + 164d - 3372 = 0$
$25d^3 + 85d^2 + 82d - 1686 = 0$

Use a graphing calculator to find the solution $d = 3$.
The dimensions of the original pool are 15 ft by 30 ft by 3 ft.

51. $80 = k(5)(8)$
$2 = k$
$y = 2(16)(2)$
$= 64$

52. $y = 7 - x^2$

$x = 7 - y^2$

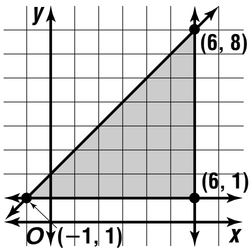
$x - 7 = -y^2$

$-x + 7 = y^2$

$\pm\sqrt{-x + 7} = y$

$f^{-1}(x) = \pm\sqrt{7 - x}$

53.



$f(x, y) = -2x + y$

$f(-1, 1) = -2(-1) + 1$ or 3

$f(6, 1) = -2(6) + 1$ or -11

$f(6, 8) = -2(6) + 8$ or -4

The maximum value is 3 and the minimum value is -11.

54.
$$\begin{aligned} x + 2y - 7z &= 14 \\ -x - 3y + 5z &= -21 \\ \hline -y - 2z &= -7 \end{aligned}$$

$$-x - 3y + 5z = -21 \rightarrow -5x - 15y + 25z = -105$$
$$5x - y + 2z = -7 \qquad \begin{aligned} 5x - y + 2z &= -7 \\ \hline -16y + 27z &= -112 \end{aligned}$$

$$-y - 2z = -7 \rightarrow 16y + 32z = 112$$
$$-16y + 27z = -112 \qquad \begin{aligned} -16y + 27z &= -112 \\ \hline 59z &= 0 \\ z &= 0 \end{aligned}$$

$-y - 2(0) = -7 \rightarrow y = 7$

$x + 2(7) - 7(0) = 14 \rightarrow x = 0$

$(0, 7, 0)$

55. Since $BC = BD$, $m\angle BDC = m\angle DCB = x$

$m\angle DBC = 180 - 120$ or 60.

$x + x + 60 = 180$

$2x = 120$

$x = 60$

$x + 40 = 60 + 40$ or 100

The correct choice is A.

9-6 The Complex and Polar Form of Complex Numbers

Pages 589–590 Check for Understanding

1. To find the absolute value of $a + b\boldsymbol{i}$, square a and b, add the squares, then take the square root of the sum.

2. $\boldsymbol{i} = 0 + \boldsymbol{i}$; $\cos\frac{\pi}{2} = 0$ and $\sin\frac{\pi}{2} = 1$

$\boldsymbol{i} = \cos\frac{\pi}{2} + \boldsymbol{i}\sin\frac{\pi}{2}$

3. Sample answer: $z_1 = \boldsymbol{i}$, $z_2 = -\boldsymbol{i}$

$|z_1 + z_2| = |z_1| + |z_2|$

$|\boldsymbol{i} + (-\boldsymbol{i})| \stackrel{?}{=} |\boldsymbol{i}| + |-\boldsymbol{i}|$

$|0| \stackrel{?}{=} \boldsymbol{i} + \boldsymbol{i}$

$0 \neq 2\boldsymbol{i}$

4. The conjugate of $a + b\boldsymbol{i}$ is $a - b\boldsymbol{i}$.
$\sqrt{(a + b\boldsymbol{i})(a - b\boldsymbol{i})} = \sqrt{a^2 + b^2}$, so the friend's method gives the same answer.
Sample answer: The absolute value of $2 + 3\boldsymbol{i}$ is $\sqrt{2^2 + 3^2} = \sqrt{13}$. Using the friend's method, the absolute value is $\sqrt{(2 + 3\boldsymbol{i})(2 - 3\boldsymbol{i})} = \sqrt{4 + 9} = \sqrt{13}$.

5. $2x + y + (x + y)\boldsymbol{i} = 5 + 4\boldsymbol{i}$

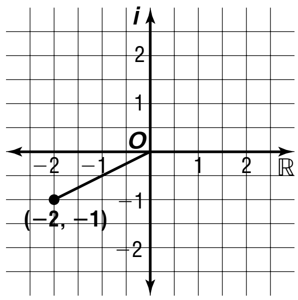
$2x + y = 5 \qquad x + y = 4$

$2x + (-x + 4) = 5 \qquad y = -x + 4$

$x = 1$

$y = -(1) + 4$ or 3

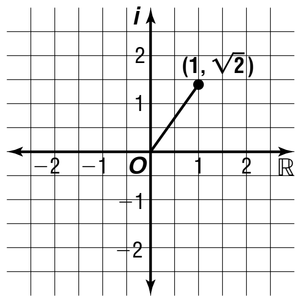
6.



$|z| = \sqrt{(-2^2 + (-1)^2}$

$= \sqrt{5}$

7.



$|z| = \sqrt{1^2 + (\sqrt{2})^2}$

$= \sqrt{3}$

8. $r = \sqrt{2^2 + (-2)^2}$ $\qquad \theta = \text{Arctan}\left(\frac{-2}{2}\right) + 2\pi$

$= \sqrt{8}$ or $2\sqrt{2}$ $\qquad = \frac{7\pi}{4}$

θ is in the fourth quadrant.

$2 - 2\boldsymbol{i} = 2\sqrt{2}\left(\cos\frac{7\pi}{4} + \boldsymbol{i}\sin\frac{7\pi}{4}\right)$

9. $r = \sqrt{4^2 + 5^2}$ $\qquad \theta = \text{Arctan}\left(\frac{5}{4}\right)$

$= \sqrt{41}$ $\qquad = 0.90$

$4 + 5\boldsymbol{i} = \sqrt{41}(\cos 0.90 + \boldsymbol{i}\sin 0.90)$

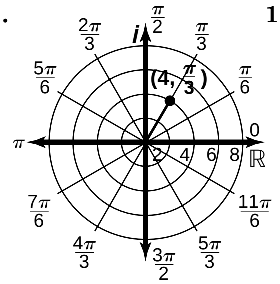
10. $r = \sqrt{(-2)^2 + 0^2}$ $\qquad \theta = \text{Arctan}\frac{0}{-2} + \pi$

$= \sqrt{4}$ or 2 $\qquad = \pi$

θ is on the x-axis at -2.

$-2 = 2(\cos\pi + \boldsymbol{i}\sin\pi)$

11.

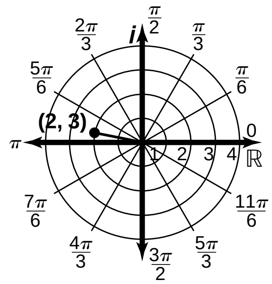


$4\left(\cos\frac{\pi}{3} + \boldsymbol{i}\sin\frac{\pi}{3}\right)$

$= 4\left(\frac{1}{2} + \boldsymbol{i}\left(\frac{\sqrt{3}}{2}\right)\right)$

$= 2 + 2\sqrt{3}\boldsymbol{i}$

12.

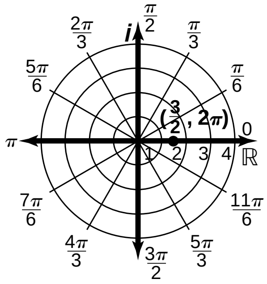


$2(\cos 3 + \boldsymbol{i}\sin 3)$

$\approx 2(-0.99 + \boldsymbol{i}(0.14))$

$= -1.98 + 0.28\boldsymbol{i}$

13.

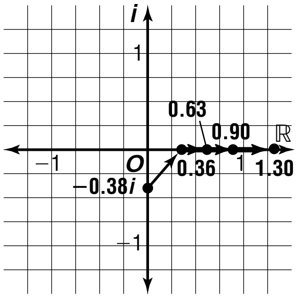


$\frac{3}{2}(\cos 2\pi + \boldsymbol{i} \sin 2\pi)$

$= \frac{3}{2}(1 + \boldsymbol{i}(0))$

$= \frac{3}{2}$

14.



15a. magnitude $= \sqrt{10^2 + 15^2}$

$= \sqrt{325}$

≈ 18.03 N

15b. $\theta = \text{Arctan } \frac{15}{10}$

$\approx 56.31°$

Pages 590–591 Exercises

16. $2x - 5y\boldsymbol{i} = 12 + 15\boldsymbol{i}$

$2x = 12 \qquad -5y = 15$

$x = 6 \qquad y = -3$

17. $1 + (x + y)\boldsymbol{i} = y + 3x\boldsymbol{i}$

$1 = y \qquad x + y = 3x$

$x + (1) = 3x$

$1 = 2x$

$\frac{1}{2} = x$

18. $4x + (y - 5)\boldsymbol{i} = 2x - y + (x + 7)\boldsymbol{i}$

$y - 5 = x + 7 \qquad 4x = 2x - y$

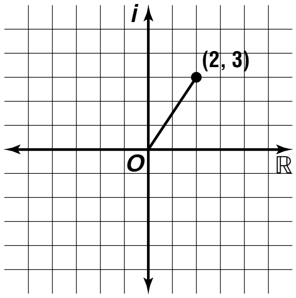
$y = x + 12 \qquad 4x = 2x - (x + 12)$

$3x = -12$

$x = -4$

$y = (-4) + 12$ or 8

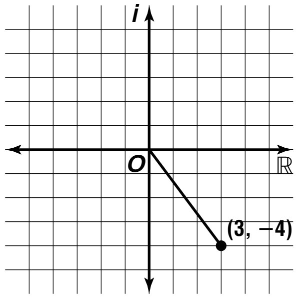
19.



$|z| = \sqrt{2^2 + 3^2}$

$= \sqrt{13}$

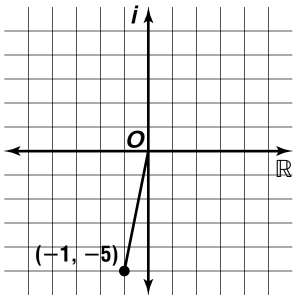
20.



$|z| = \sqrt{3^2 + (-4)^2}$

$= \sqrt{25}$ or 5

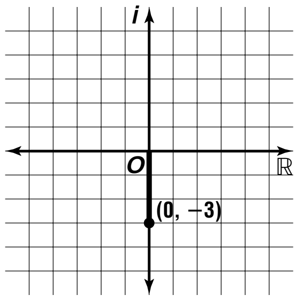
21.



$|z| = \sqrt{(-1)^2 + (-5)^2}$

$= \sqrt{26}$

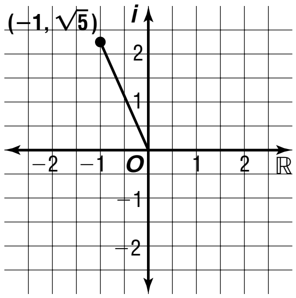
22.



$|z| = \sqrt{0^2 + (-3)^2}$

$= \sqrt{9}$ or 3

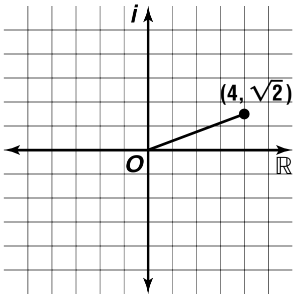
23.



$|z| = \sqrt{(-1)^2 + (\sqrt{5})^2}$

$= \sqrt{6}$

24.



$|z| = \sqrt{4^2 + (\sqrt{2})^2}$

$= \sqrt{18}$ or $3\sqrt{2}$

25. $r = \sqrt{(-4)^2 + 6^2}$

$= \sqrt{52}$ or $2\sqrt{13}$

26. $r = \sqrt{3^2 + 3^2} \qquad \theta = \text{Arctan}\left(\frac{3}{3}\right)$

$= \sqrt{18}$ or $3\sqrt{2} \qquad = \frac{\pi}{4}$

$3 + 3\boldsymbol{i} = 3\sqrt{2}\left(\cos \frac{\pi}{4} + \boldsymbol{i} \sin \frac{\pi}{4}\right)$

27. $r = \sqrt{(-1)^2 + (-\sqrt{3})^2} \qquad \theta = \text{Arctan}\left(\frac{-\sqrt{3}}{-1}\right) + \pi$

$= \sqrt{4}$ or 2 $\qquad = \frac{4\pi}{3}$

θ is in the third quadrant.

$-1 - \sqrt{3}\boldsymbol{i} = 2\left(\cos \frac{4\pi}{3} + \boldsymbol{i} \sin \frac{4\pi}{3}\right)$

28. $r = \sqrt{6^2 + (-8)^2} \qquad \theta = \text{Arctan}\left(\frac{-8}{6}\right) + 2\pi$

$= \sqrt{100}$ or 10 $\qquad \approx 5.36$

θ is in the fourth quadrant.

$6 - 8\boldsymbol{i} = 10(\cos 5.36 + \boldsymbol{i} \sin 5.36)$

29. $r = \sqrt{(-4)^2 + 1^2}$ $\qquad \theta = \text{Arctan}\left(\frac{1}{-4}\right) + \pi$

$= \sqrt{17}$ $\qquad \approx 2.90$

θ is in the second quadrant.

$-4 + \boldsymbol{i} = \sqrt{17}(\cos 2.90 + \boldsymbol{i} \sin 2.90)$

30. $r = \sqrt{20^2 + (-21)^2}$ $\qquad \theta = \text{Arctan}\left(\frac{-21}{20}\right) + 2\pi$

$= \sqrt{841}$ or 29 $\qquad \approx 5.47$

θ is in the fourth quadrant.

$20 - 21\boldsymbol{i} = 29(\cos 5.47 + \boldsymbol{i} \sin 5.47)$

31. $r = \sqrt{(-2)^2 + 4^2}$ $\qquad \theta = \text{Arctan}\left(\frac{4}{-2}\right) + \pi$

$= \sqrt{20}$ or $2\sqrt{5}$ $\qquad \approx 2.03$

θ is in the second quadrant.

$-2 + 4\boldsymbol{i} = 2\sqrt{5}(\cos 2.03 + \boldsymbol{i} \sin 2.03)$

32. $r = \sqrt{3^2 + 0^2}$ $\qquad \theta = \text{Arctan}\left(\frac{0}{3}\right)$

$= \sqrt{9}$ or 3 $\qquad = 0$

θ is on the x-axis at 3.

$3 = 3(\cos 0 + \boldsymbol{i} \sin 0)$

33. $r = \sqrt{(-4\sqrt{2})^2 + 0^2}$ $\qquad \theta = \text{Arctan}\, \frac{0}{-4\sqrt{2}} + \pi$

$= \sqrt{32}$ or $4\sqrt{2}$ $\qquad = \pi$

θ is on the x-axis at $-4\sqrt{2}$.

$-4\sqrt{2} = 4\sqrt{2}\,(\cos \pi + \boldsymbol{i} \sin \pi)$

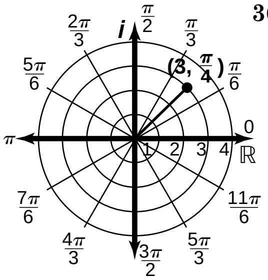
34. $r = \sqrt{0^2 + (-2)^2}$

$= \sqrt{4}$ or 2

Since $x = 0$ when $y = -2$, $\theta = \frac{3\pi}{2}$.

$-2\boldsymbol{i} = 2\left(\cos \frac{3\pi}{2} + \boldsymbol{i} \sin \frac{3\pi}{2}\right)$

35.

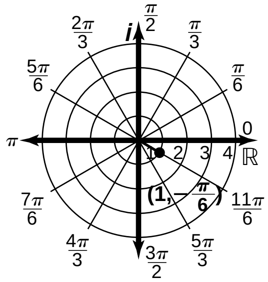


$3\left(\cos \frac{\pi}{4} + \boldsymbol{i} \sin \frac{\pi}{4}\right)$

$= 3\left(\frac{\sqrt{2}}{2} + \boldsymbol{i}\left(\frac{\sqrt{2}}{2}\right)\right)$

$= \frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}\boldsymbol{i}$

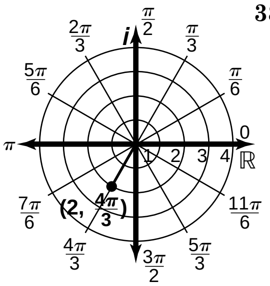
36.



$\cos\left(-\frac{\pi}{6}\right) + \boldsymbol{i} \sin\left(-\frac{\pi}{6}\right)$

$= \frac{\sqrt{3}}{2} - \frac{1}{2}\boldsymbol{i}$

37.

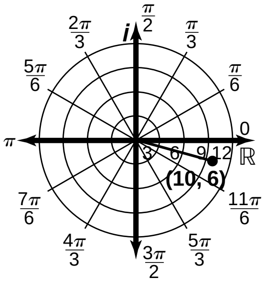


$2\left(\cos \frac{4\pi}{3} + \boldsymbol{i} \sin \frac{4\pi}{3}\right)$

$= 2\left(-\frac{1}{2} - \boldsymbol{i}\left(\frac{\sqrt{3}}{2}\right)\right)$

$= -1 - \sqrt{3}\boldsymbol{i}$

38.

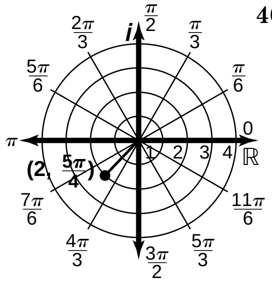


$10(\cos 6 + \boldsymbol{i} \sin 6)$

$\approx 10(0.960 + \boldsymbol{i}(-0.279))$

$= 9.60 - 2.79\boldsymbol{i}$

39.

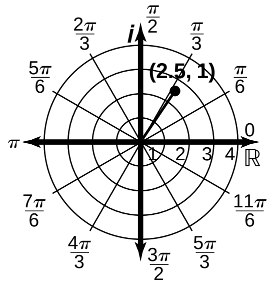


$2\left(\cos \frac{5\pi}{4} + \boldsymbol{i} \sin \frac{5\pi}{4}\right)$

$= 2\left(-\frac{\sqrt{2}}{2} + \boldsymbol{i}\left(-\frac{\sqrt{2}}{2}\right)\right)$

$= -\sqrt{2} - \sqrt{2}\boldsymbol{i}$

40.

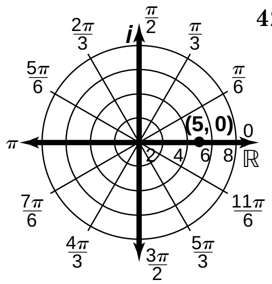


$2.5(\cos 1 + \boldsymbol{i} \sin 1)$

$\approx 2.5(0.54 + \boldsymbol{i}(0.84))$

$= 1.35 + 2.10\boldsymbol{i}$

41.

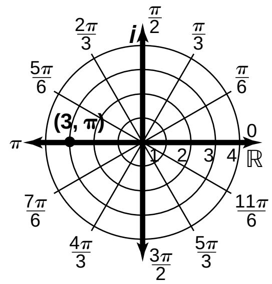


$5(\cos 0 + \boldsymbol{i} \sin 0)$

$= 5(1 + 0)$

$= 5$

42.

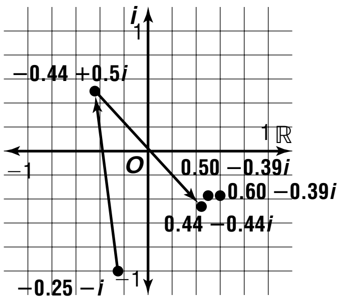


$3(\cos \pi + \boldsymbol{i} \sin \pi)$

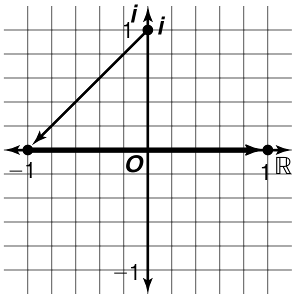
$= 3(-1 + 0)$

$= -3$

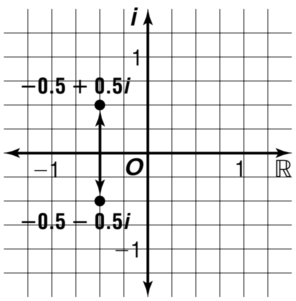
43.



44.



45.



46a. $40\angle 30° = 40(\cos 30° + \boldsymbol{j}\sin 30°)$
$= 40\left(\frac{\sqrt{3}}{2} + \boldsymbol{j}\left(\frac{1}{2}\right)\right)$
$= 34.64 + 20\boldsymbol{j}$
$60\angle 60° = 60(\cos 60° + \boldsymbol{j}\sin 60°)$
$= 60\left(\frac{1}{2} + \boldsymbol{j}\left(\frac{\sqrt{3}}{2}\right)\right)$
$= 30 + 51.96\boldsymbol{j}$

46b. $(34.64 + 20\boldsymbol{j}) + (30 + 51.96\boldsymbol{j})$
$= (34.64 + 30) + (20\boldsymbol{j} + 51.96\boldsymbol{j})$
$= 64.64 + 71.96\boldsymbol{j}$

46c. $v(t) = r\sin(250t + \theta°)$
$r = \sqrt{64.64^2 + 71.96^2}$ $\quad \theta = \text{Arctan}\,\frac{71.96}{64.64}$
≈ 96.73 $\quad \approx 48°$
$v(t) = 96.73\sin(250t + 48°)$

47. The graph of the conjugate of a complex number is obtained by reflecting the original number about the real axis. This reflection does not change the modulus. Since the amplitude is reflected, we can write the amplitude of the conjugate as the opposite of the original amplitude. In other words, the conjugate of $r(\cos\theta + \boldsymbol{i}\sin\theta)$ can be written as $r(\cos(-\theta) + \boldsymbol{i}\sin(-\theta))$, or $r(\cos\theta - \boldsymbol{i}\sin\theta)$.

48a. $10(\cos 0.7 + \boldsymbol{j}\sin 0.7) \approx 7.65 + 6.44\boldsymbol{j}$
$16(\cos 0.5 + \boldsymbol{j}\sin 0.5) \approx 14.04 + 7.67\boldsymbol{j}$

48b. $(7.65 + 6.44\boldsymbol{j}) + (14.04 + 7.67\boldsymbol{j})$
$= (7.65 + 14.04) + (6.44\boldsymbol{j} + 7.67\boldsymbol{j})$
$= 21.69 + 14.11\boldsymbol{j}$ ohms

48c. $r = \sqrt{21.69^2 + 14.11^2}$ $\quad \theta = \text{Arctan}\,\frac{14.11}{21.69}$
≈ 25.88 $\quad \approx 0.58$
$21.69 + 14.11\boldsymbol{j} = 25.88(\cos 0.58 + \boldsymbol{j}\sin 0.58)$ ohms

49a. Translate 2 units to the right and down 3 units.

49b. Rotate 90° counterclockwise about the origin.

49c. Dilate by a factor of 3.

49d. Reflect about the real axis.

50a. Sample answer: let $z_1 = 1 + \boldsymbol{i}$ and $z_2 = 3 + 4\boldsymbol{i}$.
$z_1z_2 = (1 + \boldsymbol{i})(3 + 4\boldsymbol{i})$
$= -1 + 7\boldsymbol{i}$

50b. $z_1 = \sqrt{2}\left(\cos\frac{\pi}{4} + \boldsymbol{i}\sin\frac{\pi}{4}\right) \approx 1.41(\cos 0.79 + \boldsymbol{i}\sin 0.79)$
$z_2 = 5(\cos 0.93 + \boldsymbol{i}\sin 0.93)$
$z_1z_2 = 5\sqrt{2}(\cos 1.71 + \boldsymbol{i}\sin 1.71)$
$= 7.07(\cos 1.71 + \boldsymbol{i}\sin 1.71)$

50c. Sample answer: Let $z_1 = 2 - 4\boldsymbol{i}$ and $z_2 = -1 + 3\boldsymbol{i}$. Then
$z_1 = 2\sqrt{5}(\cos 5.18 + \boldsymbol{i}\sin 5.18)$
$\approx 4.47(\cos 5.18 + \boldsymbol{i}\sin 5.18)$,
$z_2 = \sqrt{10}(\cos 1.89 + \boldsymbol{i}\sin 1.89)$
$\approx 3.16(\cos 1.89 + \boldsymbol{i}\sin 1.89)$, and
$z_1z_2 = (2 - 4\boldsymbol{i})(-1 + 3\boldsymbol{i})$
$= 10 + 10\boldsymbol{i}$
$= 10\sqrt{2}\left(\cos\frac{\pi}{4} + \boldsymbol{i}\sin\frac{\pi}{4}\right)$
$= 14.14(\cos 0.79 + \boldsymbol{i}\sin 0.79)$.

50d. To multiply two complex numbers in polar form, multiply the moduli and add the amplitudes. (In the sample answer for 50c, note that $5.18 + 1.89 = 7.07$, which is coterminal with 0.79.)

51. $(6 - 2\boldsymbol{i})(-2 + 3\boldsymbol{i}) = -12 + 22\boldsymbol{i} - 6\boldsymbol{i}^2$
$= -6 + 22\boldsymbol{i}$

52. $x = -3\cos -135°$ $\quad y = -3\sin -135°$
$= -3\left(-\frac{\sqrt{2}}{2}\right)$ $\quad = -3\left(-\frac{\sqrt{2}}{2}\right)$
$= \frac{3\sqrt{2}}{2}$ $\quad = \frac{3\sqrt{2}}{2}$
$\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$

53. magnitude $= \sqrt{(-3)^2 + 7^2}$
$= \sqrt{58}$
$\langle -3, 7\rangle = -3\vec{i} + 7\vec{j}$

54. $\tan 105° = \tan(60° + 45°)$
$= \frac{\tan 60° + \tan 45°}{1 - \tan 60° \tan 45°}$
$= \frac{\sqrt{3} + 1}{1 - (\sqrt{3})(1)}$
$= \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}}$
$= \frac{\sqrt{3} + 3 + 1 + \sqrt{3}}{}$
$= \frac{4 + 2\sqrt{3}}{-2}$
$= -2 - \sqrt{3}$

55. $w = \frac{\theta}{t}$
$= \frac{12(2\pi)}{1}$ or 24π
$v = r\omega$
$= 18(24\pi)$ or 432π cm/s
432π cm/s $= 4.32\pi$ m/s
≈ 13.57 m/s

56. $\sin A = \frac{12}{18}$
$A = \sin^{-1}\left(\frac{2}{3}\right)$
$A \approx 41.8°$

47. $\sqrt{2a - 1} = \sqrt{3a - 5}$
$2a - 1 = 3a - 5$
$4 = a$

58. as $x \to \infty$, $y \to \infty$; as $x \to -\infty$, $y \to \infty$

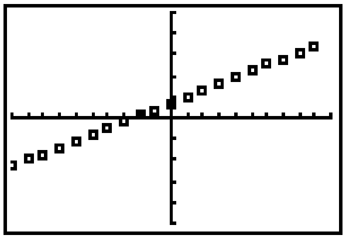
$y = 2x^2 + 2$	
x	y
-1000	2×10^6
-10	202
0	2
10	202
1000	2×10^6

59. In the fourth month, the person will have received 3 pay raises.
$\$500(1.10)^3 = \$665.50$
The correct choice is D.

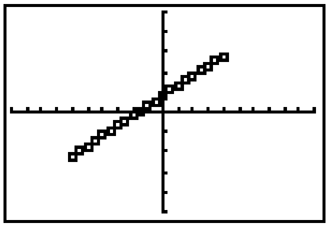
9-6B Graphing Calculator Exploration: Geometry in the Complex Plane

Page 592

1. They are collinear.



2. Yes. M is the point obtained when $T = 0$, and N is the point obtained when $T = 1$.
3. The points are again collinear, but closer together.



4. The points are on the line through M and N.
5. If one of a, b, or c equals 0, then $aK + bM + cN$ is on $\triangle KMN$. If none of a, b, or c equals 0, then $aK + bM + cN$ lies on or inside $\triangle KMN$.
6. M is the point obtained when $T = 0$ and N is the point obtained when $T = 1$. Thus, a point between M and N is obtained when $0 < T < 1$.
7. The distance between z and $1 - \boldsymbol{i}$ is 5. This defines a circle of radius 5 centered at $1 - \boldsymbol{i}$.
8. The distance between a point z and a point at $2 + 3\boldsymbol{i}$ is 2.
$|z - (2 + 3\boldsymbol{i})| = 2$

9-7 Products and Quotients of Complex Numbers in Polar Form

Pages 596 Check for Understanding

1. The modulus of the quotient is the quotient of the moduli of the two complex numbers. The amplitude of the quotient is the difference of the amplitudes of the two complex numbers.
2. Square the modulus of the given complex number and double its amplitude.
3. Addition and subtraction are easier in rectangular form. Multiplication and division are easier in polar form. See students' work for examples.
4. $r = 2 \cdot 2$ or 4 $\qquad \theta = \frac{\pi}{2} + \frac{3\pi}{2} = \frac{4\pi}{2}$ or 2π

$4(\cos 2\pi + \boldsymbol{i} \sin 2\pi) = 4(1 + \boldsymbol{i}(0)) = 4$

5. $r = \frac{3}{4}$ $\qquad \theta = \frac{\pi}{6} - \frac{2\pi}{3} = -\frac{3\pi}{6}$ or $-\frac{\pi}{2}$

$\frac{3}{4}\left(\cos\left(-\frac{\pi}{2}\right) + \boldsymbol{i} \sin\left(-\frac{\pi}{2}\right)\right) = \frac{3}{4}(0 + (-1)\boldsymbol{i}) = -\frac{3}{4}\boldsymbol{i}$

6. $r = \frac{4}{2}$ or 2 $\qquad \theta = \frac{9\pi}{4} - \left(-\frac{\pi}{2}\right) = \frac{9\pi}{4} + \frac{2\pi}{4}$ or $\frac{11\pi}{4}$

$2\left(\cos \frac{11\pi}{4} + \boldsymbol{i} \sin \frac{11\pi}{4}\right) = 2\left(-\frac{\sqrt{2}}{2} + \boldsymbol{i}\left(\frac{\sqrt{2}}{2}\right)\right) = -\sqrt{2} + \sqrt{2}\boldsymbol{i}$

7. $r = \frac{1}{2}(6)$ or 3 $\qquad \theta = \frac{\pi}{3} + \frac{5\pi}{6} = \frac{2\pi}{6} + \frac{5\pi}{6}$ or $\frac{7\pi}{6}$

$3\left(\cos \frac{7\pi}{6} + \boldsymbol{i} \sin \frac{7\pi}{6}\right) = 3\left(-\frac{\sqrt{3}}{2} + \boldsymbol{i}\left(-\frac{1}{2}\right)\right) = -\frac{3\sqrt{3}}{2} - \frac{3}{2}\boldsymbol{i}$

8. $r_1 = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{16}$ or 4 $\qquad r_2 = \sqrt{(-3)^2 + (\sqrt{3})^2} = \sqrt{12}$ or $2\sqrt{3}$

$r = 4(2\sqrt{3})$ or $8\sqrt{3}$

$\theta_1 = \text{Arctan}\left(\frac{2\sqrt{3}}{2}\right) = \frac{\pi}{3}$ $\qquad \theta_2 = \text{Arctan}\left(\frac{\sqrt{3}}{-3}\right) + \pi = \frac{5\pi}{6}$

$\theta = \frac{\pi}{3} + \frac{5\pi}{6} = \frac{2\pi}{6} + \frac{5\pi}{6}$ or $\frac{7\pi}{6}$

$8\sqrt{3}\left(\cos \frac{7\pi}{6} + \boldsymbol{i} \sin \frac{7\pi}{6}\right) = 8\sqrt{3}\left(-\frac{\sqrt{3}}{2} + \boldsymbol{i}\left(\frac{1}{2}\right)\right) = -12 - 4\sqrt{3}\boldsymbol{i}$

9. $E = IZ$
$= 2\left(\cos \frac{11\pi}{6} + \boldsymbol{j} \sin \frac{11\pi}{6}\right) \cdot 3\left(\cos \frac{\pi}{3} + \boldsymbol{j} \sin \frac{\pi}{3}\right)$

$r = 2(3)$ or 6 $\qquad \theta = \frac{11\pi}{6} + \frac{\pi}{3} = \frac{11\pi}{6} + \frac{2\pi}{6} = \frac{13\pi}{6}$ or $\frac{\pi}{6}$

$V = 6\left(\cos \frac{\pi}{6} + \boldsymbol{j} \sin \frac{\pi}{6}\right)$ volts

Pages 596–598 Exercises

10. $r = 4(7)$ or 28 $\qquad \theta = \frac{\pi}{3} + \frac{2\pi}{3} = \frac{3\pi}{3}$ or π

$28(\cos \pi + \boldsymbol{i} \sin \pi) = 28(-1 + \boldsymbol{i}(0)) = -28$

11. $r = \frac{6}{2}$ or 3 $\qquad \theta = \frac{3\pi}{4} - \frac{\pi}{4} = \frac{2\pi}{4}$ or $\frac{\pi}{2}$

$3\left(\cos \frac{\pi}{2} + \boldsymbol{i} \sin \frac{\pi}{2}\right) = 3(0 + \boldsymbol{i}(1)) = 3\boldsymbol{i}$

12. $r = \frac{\frac{1}{2}}{3}$ or $\frac{1}{6}$ $\qquad \theta = \frac{\pi}{3} - \frac{\pi}{6} = \frac{2\pi}{6} - \frac{\pi}{6}$ or $\frac{\pi}{6}$

$\frac{1}{6}\left(\cos \frac{\pi}{6} + \boldsymbol{i} \sin \frac{\pi}{6}\right) = \frac{1}{6}\left(\frac{\sqrt{3}}{2} + \boldsymbol{i}\left(\frac{1}{2}\right)\right) = \frac{\sqrt{3}}{12} + \frac{1}{12}\boldsymbol{i}$

13. $r = 5(2)$ or 10 $\quad \theta = \pi + \frac{3\pi}{4}$

$= \frac{4\pi}{4} + \frac{3\pi}{4}$ or $\frac{7\pi}{4}$

$10\left(\cos \frac{7\pi}{4} + \boldsymbol{i} \sin \frac{7\pi}{4}\right) = 10\left(\frac{\sqrt{2}}{2} + \boldsymbol{i}\left(-\frac{\sqrt{2}}{2}\right)\right)$

$= 5\sqrt{2} - 5\sqrt{2}\boldsymbol{i}$

14. $r = 6(3)$ or 18 $\quad \theta = -\frac{\pi}{3} + \frac{5\pi}{6}$

$= -\frac{2\pi}{6} + \frac{5\pi}{6}$

$= \frac{3\pi}{6}$ or $\frac{\pi}{2}$

$18\left(\cos \frac{\pi}{2} + \boldsymbol{i} \sin \frac{\pi}{2}\right) = 18(0 + \boldsymbol{i}(1))$

$= 18\boldsymbol{i}$

15. $r = \frac{3}{1}$ or 3 $\quad \theta = \frac{7\pi}{3} - \frac{\pi}{2}$

$= \frac{14\pi}{6} - \frac{3\pi}{6}$ or $\frac{11\pi}{6}$

$3\left(\cos \frac{11\pi}{6} + \boldsymbol{i} \sin \frac{11\pi}{6}\right) = 3\left(\frac{\sqrt{3}}{2} + \boldsymbol{i}\left(-\frac{1}{2}\right)\right)$

$= \frac{3\sqrt{3}}{2} - \frac{3}{2}\boldsymbol{i}$

16. $r = 2(3)$ or 6 $\quad \theta = 240° + 60°$

$= 300°$

$6(\cos 300° + \boldsymbol{i} \sin 300°) = 6\left(\frac{1}{2} + \boldsymbol{i}\left(-\frac{\sqrt{3}}{2}\right)\right)$

$= 3 - 3\sqrt{3}\boldsymbol{i}$

17. $r = \dfrac{\sqrt{2}}{\frac{\sqrt{2}}{2}}$ $\quad \theta = \frac{7\pi}{4} - \frac{3\pi}{4}$

$= \frac{4\pi}{4}$ or π

$= \frac{2\sqrt{2}}{\sqrt{2}}$ or 2

$2(\cos \pi + \boldsymbol{i} \sin \pi) = 2(-1 + \boldsymbol{i}(0))$

$= -2$

18. $r = 3(0.5)$ or 1.5 $\quad \theta = 4 + 2.5$ or 6.5

$1.5(\cos 6.5 + \boldsymbol{i} \sin 6.5) \approx 1.46 + 0.32\boldsymbol{i}$

19. $r = \frac{4}{1}$ or 4 $\quad \theta = -2 - 3.6$ or -5.6

$4[\cos(-5.6) + \boldsymbol{i} \sin(-5.6)] \approx 3.10 + 2.53\boldsymbol{i}$

20. $r = \frac{20}{15}$ or $\frac{4}{3}$ $\quad \theta = \frac{7\pi}{6} - \frac{11\pi}{3}$

$= \frac{7\pi}{6} - \frac{22\pi}{6}$

$= -\frac{15\pi}{6}$ or $-\frac{\pi}{2}$

$\frac{4}{3}\left(\cos -\frac{\pi}{2} + \boldsymbol{i} \sin -\frac{\pi}{2}\right) = \frac{4}{3}(0 + \boldsymbol{i}(-1))$

$= -\frac{4}{3}\boldsymbol{i}$

21. $r = 2(\sqrt{2})$ or $2\sqrt{2}$ $\quad \theta = \frac{3\pi}{4} + \frac{\pi}{2}$

$= \frac{3\pi}{4} + \frac{2\pi}{4}$ or $\frac{5\pi}{4}$

$2\sqrt{2}\left(\cos \frac{5\pi}{4} + \boldsymbol{i} \sin \frac{5\pi}{4}\right) = 2\sqrt{2}\left(-\frac{\sqrt{2}}{2} + \boldsymbol{i}\left(-\frac{\sqrt{2}}{2}\right)\right)$

$= -2 - 2\boldsymbol{i}$

22. $r = 2(6)$ or 12 $\quad \theta = \frac{\pi}{3} + \left(-\frac{\pi}{6}\right)$

$= \frac{2\pi}{6} - \frac{\pi}{6}$ or $\frac{\pi}{6}$

$12\left(\cos \frac{\pi}{6} + \boldsymbol{i} \sin \frac{\pi}{6}\right) = 12\left(\frac{\sqrt{3}}{2} + \boldsymbol{i}\left(\frac{1}{2}\right)\right)$

$= 6\sqrt{3} + 6\boldsymbol{i}$

23. $r = \dfrac{4}{\frac{1}{2}}$ or 8 $\quad \theta = \frac{5\pi}{3} - \frac{\pi}{3}$

$= \frac{4\pi}{3}$

$8\left(\cos \frac{4\pi}{3} + \boldsymbol{i} \sin \frac{4\pi}{3}\right) = 8\left(-\frac{1}{2} + \boldsymbol{i}\left(-\frac{\sqrt{3}}{2}\right)\right)$

$= -4 - 4\sqrt{3}\boldsymbol{i}$

24. $r_1 = \sqrt{2^2 + (-2)^2}$ $\quad r_2 = \sqrt{(-3)^2 + 3^2}$

$= \sqrt{8}$ or $2\sqrt{2}$ $\quad = \sqrt{18}$ or $3\sqrt{2}$

$r = 2\sqrt{2}(3\sqrt{2})$ or 12

$\theta_1 = \text{Arctan}\left(-\frac{2}{2}\right) + 2\pi$ $\quad \theta_2 = \text{Arctan}\left(\frac{3}{-3}\right) + \pi$

$= -\frac{7\pi}{4}$ $\quad = \frac{3\pi}{4}$

$\theta = \frac{7\pi}{4} + \frac{3\pi}{4}$

$= \frac{10\pi}{4}$ or $-\frac{\pi}{2}$

$12\left(\cos \frac{\pi}{2} + \boldsymbol{i} \sin \frac{\pi}{2}\right) = 12(0 + \boldsymbol{i}(1))$

$= 12\boldsymbol{i}$

25. $r_1 = \sqrt{(\sqrt{2})^2 + (-\sqrt{2})^2}$ $\quad r_2 = \sqrt{(-3\sqrt{2})^2 + (-3\sqrt{2})^2}$

$= \sqrt{4}$ or 2 $\quad = \sqrt{36}$ or 6

$r = 2 \cdot 6$ or 12

$\theta_1 = \text{Arctan}\left(\frac{-\sqrt{2}}{\sqrt{2}}\right) + 2\pi$ $\quad \theta_2 = \text{Arctan}\left(\frac{-3\sqrt{2}}{-3\sqrt{2}}\right)$

$= \frac{7\pi}{4}$ $\quad = \frac{5\pi}{4}$

$\theta = \frac{7\pi}{4} + \frac{5\pi}{4}$

$= \frac{12\pi}{4}$ or π

$12(\cos \pi + \boldsymbol{i} \sin \pi) = 12(-1 + \boldsymbol{i}(0))$

$= -12$

26. $r_1 = \sqrt{(\sqrt{3})^2 + (-1)^2}$ $\quad r_2 = \sqrt{2^2 + (-2\sqrt{3})^2}$

$= \sqrt{4}$ or 2 $\quad = \sqrt{16}$ or 4

$r = \frac{2}{4}$ or $\frac{1}{2}$

$\theta_1 = \text{Arctan}\left(\frac{-1}{\sqrt{3}}\right) + 2\pi$ $\quad \theta_2 = \text{Arctan}\left(\frac{-2\sqrt{3}}{2}\right) + 2\pi$

$= \frac{11\pi}{6}$ $\quad = \frac{5\pi}{3}$

$\theta = \frac{11\pi}{6} - \frac{5\pi}{3}$

$= \frac{11\pi}{6} - \frac{10\pi}{6}$ or $\frac{\pi}{6}$

$\frac{1}{2}\left(\cos \frac{\pi}{6} + \boldsymbol{i} \sin \frac{\pi}{6}\right) = \frac{1}{2}\left(\frac{\sqrt{3}}{2} + \boldsymbol{i}\left(\frac{1}{2}\right)\right)$

$= \frac{\sqrt{3}}{4} + \frac{1}{4}\boldsymbol{i}$

27. $r_1 = \sqrt{(-4\sqrt{2})^2 + (4\sqrt{2})^2}$ $\quad r_2 = \sqrt{6^2 + 6^2}$

$= \sqrt{64}$ or 8 $\quad = \sqrt{72}$ or $6\sqrt{2}$

$r = \frac{8}{6\sqrt{2}}$

$= \frac{4}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{4\sqrt{2}}{6}$ or $\frac{2\sqrt{2}}{3}$

$\theta_1 = \text{Arctan}\left(\frac{4\sqrt{2}}{-4\sqrt{2}}\right) + \pi$ $\quad \theta_2 = \text{Arctan}\left(\frac{6}{6}\right)$

$= \frac{3\pi}{4}$ $\quad = \frac{\pi}{4}$

$\theta = \frac{3\pi}{4} - \frac{\pi}{4}$

$= \frac{2\pi}{4}$ or $\frac{\pi}{2}$

$\frac{2\sqrt{2}}{3}\left(\cos \frac{\pi}{2} + \boldsymbol{i} \sin \frac{\pi}{2}\right) = \frac{2\sqrt{2}}{3}(0 + \boldsymbol{i}(1))$

$= \frac{2\sqrt{2}}{3}\boldsymbol{i}$

28. $I = \frac{E}{Z}$

$= \frac{13}{3 - 2\boldsymbol{j}}$

$r_1 = 13 \qquad r_2 = \sqrt{3^2 + (-2)^2}$
$\qquad\qquad\quad = \sqrt{13}$

$r = \frac{13}{\sqrt{13}}$ or $\sqrt{13}$

$\theta_1 = 0 \qquad \theta_2 = \text{Arctan}\left(-\frac{2}{3}\right)$
$\qquad\qquad\quad \approx -0.59$

$\theta = 0 - (-0.59)$ or 0.59

$I = \sqrt{13}(\cos 0.59 + \boldsymbol{j} \sin 0.59) \approx 3 + 2\boldsymbol{j}$ amps

29. $Z = \frac{E}{I}$

$= \frac{100}{4 - 3\boldsymbol{j}}$

$r_1 = 100 \qquad r_2 = \sqrt{4^2 + (-3)^2}$
$\qquad\qquad\quad = \sqrt{25}$ or 5

$r = \frac{100}{5}$ or 20

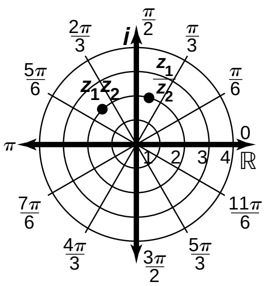
$\theta_1 = 0 \qquad \theta_2 = \text{Arctan}\left(\frac{-3}{4}\right)$
$\qquad\qquad\quad \approx -0.64$

$\theta = 0 - (-0.64)$ or 0.64

$z = 20(\cos 0.64 + \boldsymbol{j} \sin 0.64)$

$= 16 + 12\boldsymbol{j}$ ohms

30. Start at z_1 in the complex plane. Since the modulus of z_2 is 1, z_1z_2 and $\frac{z_1}{z_2}$ will both have the same modulus as z_1. Then z_1z_2 and $\frac{z_1}{z_2}$ can be located by rotating z_1 by $\frac{\pi}{6}$ counterclockwise and clockwise, respectively.



31a. The point is rotated counterclockwise about the origin by an angle of θ.

31b. The point is rotated 60° counterclockwise about the origin.

32. Since $a = 1$, the equation will be the form $z^2 + bz + c = 0$. The coefficient c is the product of the solutions, which is $6\left(\cos \frac{7\pi}{6} + \boldsymbol{i} \sin \frac{7\pi}{6}\right)$, or $-3\sqrt{3} - 3\boldsymbol{i}$ in rectangular form. The coefficient b is the opposite of the sum of the solutions, so convert the solutions to rectangular form to do the addition.

$b = -\left[3\left(\cos \frac{\pi}{3} + \boldsymbol{i} \sin \frac{\pi}{3}\right) + 2\left(\cos \frac{5\pi}{6} + \boldsymbol{i} \sin \frac{5\pi}{6}\right)\right]$

$= -\left[\left(\frac{3}{2} + \frac{3\sqrt{3}}{2}\boldsymbol{i}\right) + (-\sqrt{3} + \boldsymbol{i})\right]$

$= \left(-\frac{3}{2} + \sqrt{3}\right) + \left(\frac{-3\sqrt{3} - 2}{2}\right)\boldsymbol{i}$

Therefore, the equation is $z^2 + \left[\left(-\frac{3}{2} + \sqrt{3}\right) + \left(\frac{-3\sqrt{3} - 2}{2}\right)\boldsymbol{i}\right]z + (-3\sqrt{3} - 3\boldsymbol{i}) = 0.$

33. $r = \sqrt{5^2 + (-12)^2} \qquad \theta = \text{Arctan}\left(\frac{-12}{5}\right) + 2\pi$

$= \sqrt{169}$ or $13 \qquad \approx 5.11$

$5 - 12\boldsymbol{i} = 13(\cos 5.11 + \boldsymbol{i} \sin 5.11)$

34.

$r = 5 \sec\left(\theta - \frac{5\pi}{6}\right)$

$r \cos\left(\theta - \frac{5\pi}{6}\right) = 5$

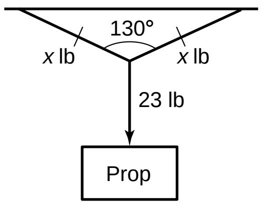
$r\left(\cos\theta \cos \frac{5\pi}{6} + \sin\theta \sin \frac{5\pi}{6}\right) - 5 = 0$

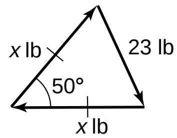
$-\frac{\sqrt{3}}{2}r \cos\theta + \frac{1}{2}r \sin\theta - 5 = 0$

$-\frac{\sqrt{3}}{2}x + \frac{1}{2}y - 5 = 0$

$-\sqrt{3}x + y - 10 = 0$

35.





Since the triangle is isosceles, the base angles are congruent. Each measures $\frac{180 - 50}{2}$ or 65°.

$\frac{23}{\sin 50°} = \frac{x}{\sin 65°}$

$23 \sin 65° = x \sin 50°$

$\frac{23 \sin 65°}{\sin 50°} = x$

$27.21 \approx x$; 27.21 lb

36.

$\cos 2x + \sin x = 1$

$1 - 2\sin^2 x + \sin x = 1$

$2\sin^2 x - \sin x = 0$

$\sin x\,(2\sin x - 1) = 0$

$\sin x = 0$ or $2\sin x - 1 = 0$

$x = 0° \qquad \sin x = \frac{1}{2}$

$x = 30°$

37.

$y = \cos x$

$x = \cos y$

$\text{arccos}\ x = y$

38. $BC = ED = BE = AF = CD = 3$

$AB = FE = 2$

$AC = AB + BC$

$= 2 + 3$ or 5

$FD = FE + ED$

$= 2 + 3$ or 5

perimeter of rectangle $ACDF = 3 + 5 + 3 + 5$ or 16

perimeter of square $BCDE = 4(3)$ or 12

$16 - 12 = 4$

The correct choice is C.

9-8 Powers and Roots of Complex Numbers

Page 602 Graphing Calculator Exploration

1. Rewrite 1 in polar form as $1(\cos 0 + \boldsymbol{i} \sin 0)$. Follow the keystrokes to find the roots at 1, $-0.5 + 0.87\boldsymbol{i}$, and $-0.5 - 0.87\boldsymbol{i}$.
2. Rewrite $\boldsymbol{i}$ in polar form as $1\left(\cos \frac{\pi}{2} + \boldsymbol{i} \sin \frac{\pi}{2}\right)$.

   Follow the keystrokes to find the roots at $0.92 + 0.38\boldsymbol{i}$, $-0.38 + 0.92\boldsymbol{i}$, $-0.92 - 0.38\boldsymbol{i}$, and $0.38 - 0.92\boldsymbol{i}$.

3. Rewrite $1 + \boldsymbol{i}$ in polar form as $\sqrt{2}\left(\cos \frac{\pi}{4} + \boldsymbol{i} \sin \frac{\pi}{4}\right)$.

Follow the keystrokes to find the roots at $1.06 + 0.17\boldsymbol{i}$, $0.17 + 1.06\boldsymbol{i}$, $-0.95 + 0.49\boldsymbol{i}$, $-0.76 - -0.76\boldsymbol{i}$, and $0.49 - 0.95\boldsymbol{i}$.

4. equilateral triangle
5. regular pentagon
6. If $a > 0$ and $b = 0$, then $a + b\boldsymbol{i} = a$. The principal roots of a positive real number is a positive real number which would lie on the real axis in a complex plane.

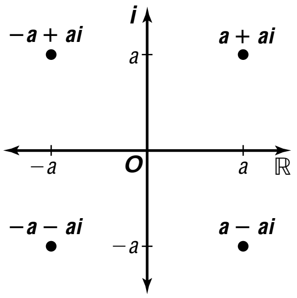
Pages 604-605 Check for Understanding

1. Same results, $-4 - 4\boldsymbol{i}$; answers may vary.

$(1 + \boldsymbol{i})(1 + \boldsymbol{i})(1 + \boldsymbol{i})(1 + \boldsymbol{i})(1 + \boldsymbol{i})$
$= (1 + 2\boldsymbol{i} + \boldsymbol{i}^2)(1 + 2\boldsymbol{i} + \boldsymbol{i}^2)(1 + \boldsymbol{i})$
$= (2\boldsymbol{i})(2\boldsymbol{i})(1 + \boldsymbol{i})$
$= -4(1 + \boldsymbol{i})$
$= -4 - 4\boldsymbol{i}$
$(1 + \boldsymbol{i})^5$
$\rightarrow r = \sqrt{2}, \theta = \frac{\pi}{4}$
$(\sqrt{2})^5 \left(\cos (5)\left(\frac{\pi}{4}\right) + \boldsymbol{i} \sin (5)\left(\frac{\pi}{4}\right)\right)$
$= 4\sqrt{2}\left(\cos \frac{5\pi}{4} + \boldsymbol{i} \sin \frac{5\pi}{4}\right)$
$= 4\sqrt{2}\left(-\frac{\sqrt{2}}{2} + \boldsymbol{i}\left(-\frac{\sqrt{2}}{2}\right)\right)$
$= -4 - 4\boldsymbol{i}$

2. Finding a reciprocal is the same as raising a number to the -1 power, so take the reciprocal of the modulus and multiply the amplitude by -1.

3.



4. Shembala is correct. The polar form of $a + a\boldsymbol{i}$ is $a\sqrt{2}\left(\cos \frac{\pi}{4} + \boldsymbol{i} \sin \frac{\pi}{4}\right)$. By De Moivre's Theorem, the polar form of $(a + a\boldsymbol{i})^2$ is $2a^2\left(\cos \frac{\pi}{2} + \boldsymbol{i} \sin \frac{\pi}{2}\right)$. Since $\cos \frac{\pi}{2} = 0$, this is a pure imaginary number.

5. $r = \sqrt{(\sqrt{3})^2 + (-1)^2}$ or 2 $\quad \theta = \text{Arctan}\left(\frac{-1}{\sqrt{3}}\right)$ or $-\frac{\pi}{6}$

$2^3\left(\cos (3)\left(-\frac{\pi}{6}\right) + \boldsymbol{i} \sin (3)\left(-\frac{\pi}{6}\right)\right)$
$= 8\left[\cos\left(-\frac{\pi}{2}\right) + \boldsymbol{i} \sin\left(-\frac{\pi}{2}\right)\right]$
$= = 8(0 + \boldsymbol{i}(-1))$
$= -8\boldsymbol{i}$

6. $r = \sqrt{3^2 + (-5)^2}$ or $\sqrt{34}$ $\quad \theta = \text{Arctan}\left(\frac{-5}{3}\right)$
≈ -1.030376827

$\sqrt{34}^4 (\cos (4)(\theta) + \boldsymbol{i} \sin (4)(\theta))$
$= -644 + 960\boldsymbol{i}$

7. $r = \sqrt{0^2 + 1^2}$ or 1 $\quad \theta = \frac{\pi}{2}$

$1^{\frac{1}{6}}\left(\cos\left(\frac{1}{6}\right)\left(\frac{\pi}{2}\right) + \boldsymbol{i} \sin\left(\frac{1}{6}\right)\left(\frac{\pi}{2}\right)\right)$
$= 1\left(\cos \frac{\pi}{12} + \boldsymbol{i} \sin \frac{\pi}{12}\right)$
$\approx 0.97 + 0.26\boldsymbol{i}$

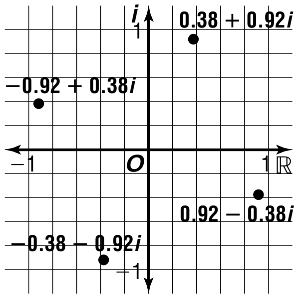
8. $r = \sqrt{(-2)^2 + (-1)^2}$ or $\sqrt{5}$ $\quad \theta = \text{Arctan}\left(\frac{-1}{-2}\right) - \pi$
≈ -2.677945045

$\sqrt{5}^{\frac{1}{3}}\left(\cos\left(\frac{1}{3}\right)(\theta) + \boldsymbol{i} \sin (3)(\theta)\right)$
$= 0.82 - 1.02\boldsymbol{i}$

9. $x^4 + \boldsymbol{i} = 0 \rightarrow x^4 = -\boldsymbol{i}$

Find the fourth roots of $-\boldsymbol{i}$.

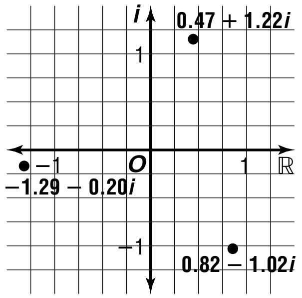
$r = \sqrt{0^2 + (-1)^2} = 1 \quad \theta = \frac{3\pi}{2}$
$(-\boldsymbol{i})^{\frac{1}{4}} = \left[1\left(\cos\left(\frac{3\pi}{2} + 2n\pi\right) + \boldsymbol{i} \sin\left(\frac{3\pi}{2} + 2n\pi\right)\right)\right]^{\frac{1}{4}}$
$= \cos \frac{3\pi + 4n\pi}{8} + \boldsymbol{i} \sin \frac{3\pi + 4n\pi}{8}$
$x_1 = \cos \frac{3\pi}{8} + \boldsymbol{i} \sin \frac{3\pi}{8} \approx 0.38 + 0.92\boldsymbol{i}$
$x_2 = \cos \frac{7\pi}{8} + \boldsymbol{i} \sin \frac{7\pi}{8} \approx -0.92 + 0.38\boldsymbol{i}$
$x_3 = \cos \frac{11\pi}{8} + \boldsymbol{i} \sin \frac{11\pi}{8} \approx -0.38 - 0.92\boldsymbol{i}$
$x_4 = \cos \frac{15\pi}{8} + \boldsymbol{i} \sin \frac{15\pi}{8} \approx 0.92 - 0.38\boldsymbol{i}$



10. $2x^3 + 4 + 2\boldsymbol{i} = 0 \rightarrow x^3 = -2 - \boldsymbol{i}$

Find the third roots of $-2 - \boldsymbol{i}$.

$r = \sqrt{(-2)^2 + (-1)^2} = \sqrt{5} \quad \theta = \text{Arctan}\left(\frac{-1}{-2}\right) + \pi$
≈ 3.605240263
$(-2 - \boldsymbol{i})^{\frac{1}{3}} = [\sqrt{5}(\cos (\theta + 2\boldsymbol{i}\pi) + \boldsymbol{i} \sin (\theta + 2n\pi))]^{\frac{1}{3}}$
$= (\sqrt{5})^{\frac{1}{3}}\left(\cos \frac{\theta + 2n\pi}{3} + \boldsymbol{i} \sin \frac{\theta + 2n\pi}{3}\right)$
$x_1 = (\sqrt{5})^{\frac{1}{3}}\left(\cos \frac{\theta}{3} + \boldsymbol{i} \sin \frac{\theta}{3}\right) = 0.47 + 1.22\boldsymbol{i}$
$x_2 = (\sqrt{5})^{\frac{1}{3}}\left(\cos \frac{\theta + 2\pi}{3} + \boldsymbol{i} \sin \frac{\theta + 2\pi}{3}\right) \approx -1.29 - 0.20\boldsymbol{i}$
$x_3 = (\sqrt{5})^{\frac{1}{3}}\left(\cos \frac{\theta + 4\pi}{3} + \boldsymbol{i} \sin \frac{\theta + 4\pi}{3}\right) \approx 0.81 - 1.02\boldsymbol{i}$



11. For w_1, the modulus $= (\sqrt{0.8^2 + (-0.7)^2})^2$ or 1.13.
For w_2, the modulus $= 1.13^2$ or 1.28.
For w_3, the modulus $= 1.28^2$ or 1.64.
This moduli will approach infinity as the number of iterations increases. Thus, it is an escape set.

Pages 605–606 Exercises

12. $3^3\left(\cos (3)\left(\frac{\pi}{6}\right) + \boldsymbol{i} \sin (3)\left(\frac{\pi}{6}\right)\right)$
$= 27\left(\cos \frac{\pi}{2} + \boldsymbol{i} \sin \frac{\pi}{2}\right)$
$= 27(0 + \boldsymbol{i}(1))$
$= 27\boldsymbol{i}$

13. $2^5\left(\cos (5)\left(\frac{\pi}{4}\right) + \boldsymbol{i} \sin (5)\left(\frac{\pi}{4}\right)\right)$
$= 32\left(\cos \frac{5\pi}{4} + \boldsymbol{i} \sin \frac{5\pi}{4}\right)$
$= 32\left(-\frac{\sqrt{2}}{2} + \boldsymbol{i}\left(-\frac{\sqrt{2}}{2}\right)\right)$
$= -16\sqrt{2} - 16\sqrt{2}\boldsymbol{i}$

14. $r = \sqrt{(-2)^2 + 2^2} = 2\sqrt{2}$ $\quad \theta = \text{Arctan}\left(\frac{2}{-2}\right) = \frac{7\pi}{4}$
$(2\sqrt{2})^3\left(\cos (3)\left(\frac{7\pi}{9}\right) + \boldsymbol{i} \sin (3)\left(\frac{7\pi}{4}\right)\right)$
$= 16\sqrt{2}\left(\cos \frac{21\pi}{4} + \boldsymbol{i} \sin \frac{21\pi}{4}\right)$
$= 16\sqrt{2}\left(\frac{\sqrt{2}}{2} + \boldsymbol{i}\left(\frac{\sqrt{2}}{2}\right)\right)$
$= 16 + 16\boldsymbol{i}$

15. $r = \sqrt{1^2 + (\sqrt{3})^2} = 2$ $\quad \theta = \text{Arctan}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$
$2^4\left(\cos (4)\left(\frac{\pi}{3}\right) + \boldsymbol{i} \sin (4)\left(\frac{\pi}{3}\right)\right)$
$= 16\left(\cos \frac{4\pi}{3} + \boldsymbol{i} \sin \frac{4\pi}{3}\right)$
$= 16\left(-\frac{1}{2} + \boldsymbol{i}\left(-\frac{\sqrt{3}}{2}\right)\right)$
$= -8 - 8\sqrt{3}\boldsymbol{i}$

16. $r = \sqrt{3^2 + (-6)^2} = 3\sqrt{5}$ $\quad \theta = \text{Arctan}\left(\frac{-6}{3}\right)$
≈ -1.107148718
$(3\sqrt{5})^4 (\cos (4)(\theta) + \boldsymbol{i} \sin (4)(\theta))$
$= -567 + 1944\boldsymbol{i}$

17. $r = \sqrt{2^2 + 3^2} = \sqrt{13}$ $\quad \theta = \text{Arctan}\left(\frac{3}{2}\right)$
≈ 0.9827937232
$(\sqrt{13})^{-2} (\cos (-2)(\theta) + \boldsymbol{i} \sin (-2)(\theta))$
$= -0.03 - 0.07\boldsymbol{i}$

18. $r = \sqrt{2^2 + 4^2} = 2\sqrt{5}$ $\quad \theta = \text{Arctan}\left(\frac{4}{2}\right)$
$= 1.107148718$
$(2\sqrt{5})^4 (\cos (4)(\theta) + \boldsymbol{i} \sin (4)(\theta))$
$= -112 - 384\boldsymbol{i}$

19. $32^{\frac{1}{5}}\left(\cos \left(\frac{1}{5}\right)\left(\frac{2\pi}{3}\right) + \boldsymbol{i} \sin \left(\frac{1}{5}\right)\left(\frac{2\pi}{3}\right)\right)$
$= 2\left(\cos \frac{2\pi}{15} + \boldsymbol{i} \sin \frac{2\pi}{15}\right)$
$\approx 1.83 + 0.81\boldsymbol{i}$

20. $r = \sqrt{(-1)^2 + 0^2} = 1$ $\quad \theta = \pi$
$1^{\frac{1}{4}}\left(\cos \left(\frac{1}{4}\right)(\pi) + \boldsymbol{i} \sin \left(\frac{1}{4}\right)(\pi)\right)$
$= \cos \frac{\pi}{4} + \boldsymbol{i} \sin \frac{\pi}{4}$
$\approx 0.71 + 0.71\boldsymbol{i}$

21. $r = \sqrt{(-2)^2 + 1^2} = \sqrt{5}$ $\quad \theta = \text{Arctan}\left(\frac{1}{-2}\right) + \pi$
≈ 2.677945045
$(\sqrt{5})^{\frac{1}{4}}\left(\cos \left(\frac{1}{4}\right)(\theta) + \boldsymbol{i} \sin \left(\frac{1}{4}\right)(\theta)\right)$
$= 0.96 + 0.76\boldsymbol{i}$

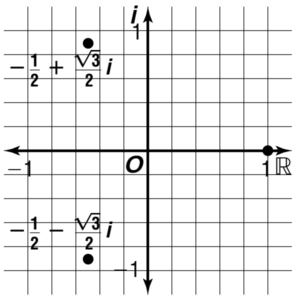
22. $r = \sqrt{4^2 + (-1)^2} = \sqrt{17}$ $\quad \theta = \text{Arctan}\left(\frac{-1}{4}\right)$
≈ -0.2449786631
$(\sqrt{17})^{\frac{1}{3}}\left(\cos \left(\frac{1}{3}\right)(\theta) + \boldsymbol{i} \sin \left(\frac{1}{3}\right)(\theta)\right)$
$= 1.60 - 0.13\boldsymbol{i}$

23. $r = \sqrt{2^2 + 2^2} = 2\sqrt{2}$ $\quad \theta = \text{Arctan}\left(\frac{2}{2}\right) = \frac{\pi}{4}$
$(2\sqrt{2})^{\frac{1}{3}}\left(\cos \left(\frac{1}{3}\right)\left(\frac{\pi}{4}\right) + \boldsymbol{i} \sin \left(\frac{1}{3}\right)\left(\frac{\pi}{4}\right)\right)$
$= 1.37 + 0.37\boldsymbol{i}$

24. $r = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$ $\quad \theta = \text{Arctan}\left(\frac{-1}{-1}\right) - \pi$
$= -\frac{3\pi}{4}$
$(\sqrt{2})^{\frac{1}{4}}\left(\cos \left(\frac{1}{4}\right)\left(-\frac{3\pi}{4}\right) + \boldsymbol{i} \sin \left(\frac{1}{4}\right)\left(-\frac{3\pi}{4}\right)\right)$
$= 0.91 - 0.61\boldsymbol{i}$

25. $r = \sqrt{0^2 + 1^2} = 1$ $\quad \theta = \frac{\pi}{2}$
$1^{\frac{1}{2}}\left(\cos \left(\frac{1}{2}\right)\left(\frac{\pi}{2}\right) + \boldsymbol{i} \sin \left(\frac{1}{2}\right)\left(\frac{\pi}{2}\right)\right)$
$= 0.71 + 0.71\boldsymbol{i}$

26. $x^3 - 1 = 0 \rightarrow x^3 = 1$
Find the third roots of 1.
$r = \sqrt{1^2 + 0^2} = 1$ $\quad \theta = 0$
$1^{\frac{1}{3}} = [1 (\cos (0 + 2n\pi) + \boldsymbol{i} \sin (0 + 2n\pi))]^{\frac{1}{3}}$
$= \cos \frac{2n\pi}{3} + \boldsymbol{i} \sin \frac{2n\pi}{3}$
$x_1 = \cos 0 + \boldsymbol{i} \sin 0 = 1$
$x_2 = \cos \frac{2\pi}{3} + \boldsymbol{i} \sin \frac{2\pi}{3} = -\frac{1}{2} + \frac{\sqrt{3}}{2}\boldsymbol{i}$
$x_3 = \cos \frac{4\pi}{3} + \boldsymbol{i} \sin \frac{4\pi}{3} = -\frac{1}{2} - \frac{\sqrt{3}}{2}\boldsymbol{i}$



27. $x^5 + 1 \rightarrow x^5 = -1$

Find the fifth roots of -1.

$r = \sqrt{(-1)^2 + 0^2} = 1 \qquad \theta = \pi$

$(-1)^{\frac{1}{5}} = [1\,(\cos\,(\pi + 2n\pi) + \boldsymbol{i}\sin\,(\pi + 2n\pi))]^{\frac{1}{5}}$

$= \cos\frac{\pi + 2n\pi}{5} + \boldsymbol{i}\sin\frac{\pi + 2n\pi}{5}$

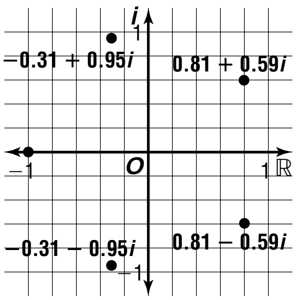
$x_1 = \cos\frac{\pi}{5} + \boldsymbol{i}\sin\frac{\pi}{5} = 0.81 + 0.59\boldsymbol{i}$

$x_2 = \cos\frac{3\pi}{5} + \boldsymbol{i}\sin\frac{3\pi}{5} = -0.31 + 0.95\boldsymbol{i}$

$x_3 = \cos\frac{5\pi}{5} + \boldsymbol{i}\sin\frac{5\pi}{5} = -1$

$x_4 = \cos\frac{7\pi}{5} + \boldsymbol{i}\sin\frac{7\pi}{5} = -0.31 - 0.95\boldsymbol{i}$

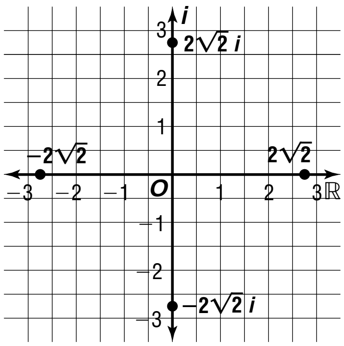
$x_5 = \cos\frac{9\pi}{5} + \boldsymbol{i}\sin\frac{9\pi}{5} = 0.81 - 0.59\boldsymbol{i}$



28. $2x^4 - 128 = 0 \rightarrow x^4 = 64$

Find the fourth roots of 64.

$r = \sqrt{64^2 + 0^2} = 64 \qquad \theta = 0$



$64^{\frac{1}{4}} = [64\,(\cos\,(0 + 2n\pi) + \boldsymbol{i}\sin\,(0 + 2n\pi))]^{\frac{1}{4}}$

$= 2\sqrt{2}\left(\cos\frac{n\pi}{2} + \boldsymbol{i}\sin\frac{n\pi}{2}\right)$

$x_1 = 2\sqrt{2}\,(\cos 0 + \boldsymbol{i}\sin 0) = 2\sqrt{2}$

$x_2 = 2\sqrt{2}\left(\cos\frac{\pi}{2} + \boldsymbol{i}\sin\frac{\pi}{2}\right) = 2\sqrt{2}\boldsymbol{i}$

$x_3 = 2\sqrt{2}\,(\cos\pi + \boldsymbol{i}\sin\pi) = -2\sqrt{2}$

$x_4 = 2\sqrt{2}\left(\cos\frac{3\pi}{2} + \boldsymbol{i}\sin\frac{3\pi}{2}\right) = -2\sqrt{2}\boldsymbol{i}$

29. $3x^4 + 48 = 0 \rightarrow x^4 = -16$

Find the fourth roots of -16.

$r = \sqrt{(-16)^2 + 0^2} = 16 \qquad \theta = \pi$

$(-16)^{\frac{1}{4}} = [16\,(\cos\,(\pi + 2n\pi) + \boldsymbol{i}\sin\,(\pi + 2n\pi))]^{\frac{1}{4}}$

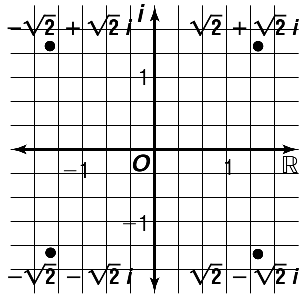
$= 2\left(\cos\frac{\pi + 2n\pi}{4} + \boldsymbol{i}\sin\frac{\pi + 2n\pi}{4}\right)$

$x_1 = 2\left(\cos\frac{\pi}{4} + \boldsymbol{i}\sin\frac{\pi}{4}\right) = \sqrt{2} + \sqrt{2}\boldsymbol{i}$

$x_2 = 2\left(\cos\frac{3\pi}{4} + \boldsymbol{i}\sin\frac{3\pi}{4}\right) = -\sqrt{2} + \sqrt{2}\boldsymbol{i}$

$x_3 = 2\left(\cos\frac{5\pi}{4} + \boldsymbol{i}\sin\frac{5\pi}{4}\right) = -\sqrt{2} - \sqrt{2}\boldsymbol{i}$

$x_4 = 2\left(\cos\frac{7\pi}{4} + \boldsymbol{i}\sin\frac{7\pi}{4}\right) = \sqrt{2} - \sqrt{2}\boldsymbol{i}$



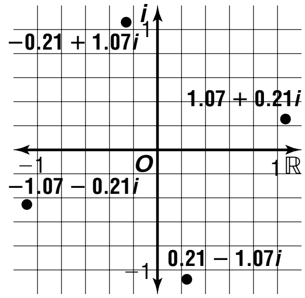
30. $x^4 - (1 + \boldsymbol{i}) = 0 \rightarrow x^4 = 1 + \boldsymbol{i}$

Find the fourth roots of $1 + \boldsymbol{i}$.

$r = \sqrt{1^2 + 1^2} = \sqrt{2} \qquad \theta = \text{Arctan}\left(\frac{1}{1}\right) = \frac{\pi}{4}$

$(1 + \boldsymbol{i})^{\frac{1}{4}} = \left[\sqrt{2}\left(\cos\left(\frac{\pi}{4} + 2n\pi\right) + \boldsymbol{i}\sin\left(\frac{\pi}{4} + 2n\pi\right)\right)\right]^{\frac{1}{4}}$

$= (\sqrt{2})^{\frac{1}{4}}\left(\cos\frac{\pi + 8n\pi}{16} + \boldsymbol{i}\sin\frac{\pi + 8n\pi}{16}\right)$



$x_1 = (\sqrt{2})^{\frac{1}{4}}\left(\cos\frac{\pi}{16} + \boldsymbol{i}\sin\frac{\pi}{16}\right) = 1.07 + 0.21\boldsymbol{i}$

$x_2 = (\sqrt{2})^{\frac{1}{4}}\left(\cos\frac{9\pi}{16} + \boldsymbol{i}\sin\frac{9\pi}{16}\right) = -0.21 + 1.07\boldsymbol{i}$

$x_3 = (\sqrt{2})^{\frac{1}{4}}\left(\cos\frac{17\pi}{16} + \boldsymbol{i}\sin\frac{17\pi}{16}\right) = -1.07 - 0.21\boldsymbol{i}$

$x_4 = (\sqrt{2})^{\frac{1}{4}}\left(\cos\frac{25\pi}{16} + \boldsymbol{i}\sin\frac{25\pi}{16}\right) = 0.21 - 1.07\boldsymbol{i}$

31. $2x^4 + 2 + 2\sqrt{3}\boldsymbol{i} = 0 \rightarrow x^4 = -1 - \sqrt{3}\boldsymbol{i}$.

Find the fourth roots of $-1 - \sqrt{3}\boldsymbol{i}$.

$r = \sqrt{(-1)^2 + (-\sqrt{3})^2} = 2$

$\theta = \text{Arctan}\left(\frac{-\sqrt{3}}{-1}\right) = \frac{\pi}{3} + \pi \text{ or } \frac{4\pi}{3}$

$(-1 - \sqrt{3}\boldsymbol{i})^{\frac{1}{4}}$

$= \left[2\left(\cos\left(\frac{4\pi}{3} + 2n\pi\right) + \boldsymbol{i}\sin\left(\frac{4\pi}{3} + 2n\pi\right)\right)\right]^{\frac{1}{4}}$

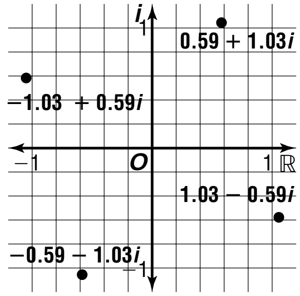
$= 2^{\frac{1}{4}}\left(\cos\frac{4\pi + 6n\pi}{12} + \boldsymbol{i}\sin\frac{4\pi + 6n\pi}{12}\right)$

$x_1 = 2^{\frac{1}{4}}\left(\cos\frac{4\pi}{12} + \boldsymbol{i}\sin\frac{4\pi}{12}\right) = 0.59 + 1.03\boldsymbol{i}$

$x_2 = 2^{\frac{1}{4}}\left(\cos\frac{10\pi}{12} + \boldsymbol{i}\sin\frac{10\pi}{12}\right) = -1.03 + 0.59\boldsymbol{i}$

$x_3 = 2^{\frac{1}{4}}\left(\cos\frac{22\pi}{12} + \boldsymbol{i}\sin\frac{22\pi}{12}\right) = -0.59 - 1.03\boldsymbol{i}$

$x_4 = 2^{\frac{1}{4}}\left(\cos\frac{28\pi}{12} + \boldsymbol{i}\sin\frac{28\pi}{12}\right) = 1.03 - 0.59\boldsymbol{i}$



32. Rewrite $10 - 9\boldsymbol{i}$ in polar form as
$\sqrt{181}\left[\cos\left(\tan^{-1}\left(\frac{-9}{10}\right)\right) + \boldsymbol{i}\sin\left(\tan^{-1}\left(\frac{-9}{10}\right)\right)\right]$.
Use a graphing calculator to find the fifth roots at $0.75 + 1.51\boldsymbol{i}$, $-1.20 + 1.18\boldsymbol{i}$, $-1.49 - 0.78\boldsymbol{i}$, $0.28 - 1.66\boldsymbol{i}$, and $1.66 - 0.25\boldsymbol{i}$.

33. Rewrite $2 + 4\boldsymbol{i}$ in polar form as
$2\sqrt{5}[\cos(\tan^{-1}(2)) + \boldsymbol{i}\sin(\tan^{-1}(2))]$.
Use a graphing calculator to find the sixth roots at $1.26 + 0.24\boldsymbol{i}$, $0.43 + 1.21\boldsymbol{i}$, $-0.83 + 0.97\boldsymbol{i}$, $-1.26 - 0.24\boldsymbol{i}$, $-0.43 - 1.21\boldsymbol{i}$, and $0.83 - 0.97\boldsymbol{i}$.

34. Rewrite $36 + 20\boldsymbol{i}$ in polar form as
$4\sqrt{106}\left[\cos\left(\tan^{-1}\left(\frac{5}{9}\right)\right) + \boldsymbol{i}\sin\left(\tan^{-1}\left(\frac{5}{9}\right)\right)\right]$.
Use a graphing calculator to find the eighth roots at $1.59 + 0.10\boldsymbol{i}$, $1.05 + 1.19\boldsymbol{i}$, $-0.10 + 1.59\boldsymbol{i}$, $-1.19 + 1.05\boldsymbol{i}$, $-1.59 - 0.10\boldsymbol{i}$, $-1.05 - 1.19\boldsymbol{i}$, $0.10 - 1.59\boldsymbol{i}$, and $1.19 - 1.05\boldsymbol{i}$.

35. For w_1, the modulus $= \left(\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{3}{4}\right)^2}\right)^2$ or 0.81.
For w_2, the modulus $= (0.81)^2$ or 0.66.
For w_3, the modulus $= (0.66)^2$ or 0.44.
This moduli will approach 0 as the number of iterations increase. Thus, it is a prisoner set.

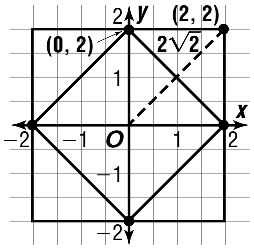
36a. In polar form the 31st roots of 1 are given by $\cos\frac{2n\pi}{31} + \boldsymbol{i}\sin\frac{2n\pi}{31}$, $n = 0, 1, \ldots, 30$. Then $a = \cos\frac{2n\pi}{31}$. The maximum value of a cosine expression is 1, and it is achieved in this situation when $n = 0$.

36b. From the polar form in the solution to part a, we get $b = \sin\frac{2n\pi}{31}$. b will be maximized when $\frac{2n\pi}{31}$ is as close to $\frac{\pi}{2}$ as possible. This occurs when $n = 8$, so the maximum value of b is $\sin\frac{16\pi}{31}$, or about 0.9987.

37. $x^6 - 1 = 0 \rightarrow x^6 = 1$
Find the sixth roots of 1.
$r = \sqrt{1^2 + 0^2} = 1 \qquad \theta = 0$

$$1^{\frac{1}{6}} = [1(\cos(0 + 2n\pi) + \boldsymbol{i}\sin(0 + 2n\pi))]^{\frac{1}{6}}$$
$$= \cos\frac{n\pi}{3} + \boldsymbol{i}\sin\frac{n\pi}{3}$$
$$x_1 = \cos 0 + \boldsymbol{i}\sin 0 = 1$$
$$x_2 = \cos\frac{\pi}{3} + \boldsymbol{i}\sin\frac{\pi}{3} = \frac{1}{2} + \frac{\sqrt{3}}{2}\boldsymbol{i}$$
$$x_3 = \cos\frac{2\pi}{3} + \boldsymbol{i}\sin\frac{2\pi}{3} = -\frac{1}{2} + \frac{\sqrt{3}}{2}\boldsymbol{i}$$
$$x_4 = \cos\frac{3\pi}{3} + \boldsymbol{i}\sin\frac{3\pi}{3} = -1$$
$$x_5 = \cos\frac{4\pi}{3} + \boldsymbol{i}\sin\frac{4\pi}{3} = -\frac{1}{2} - \frac{\sqrt{3}}{2}\boldsymbol{i}$$
$$x_6 = \cos\frac{5\pi}{3} + \boldsymbol{i}\sin\frac{5\pi}{3} = \frac{1}{2} - \frac{\sqrt{3}}{2}\boldsymbol{i}$$

38a. The point at (2, 2) becomes the point at (0, 2). From the origin, the point at (2, 2) had a length of $2\sqrt{2}$ and the new point at (0, 2) has a length of 2. The dilation factor is $\frac{\sqrt{2}}{2}$.



$$\frac{\sqrt{2}}{2}(\cos 45° + \boldsymbol{i}\sin 45°)$$
$$= \frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2} + \boldsymbol{i}\sin\left(\frac{\sqrt{2}}{2}\right)\right)$$
$$= 0.5 + 0.5\boldsymbol{i}$$

38b. $\left[\frac{\sqrt{2}}{2}(\cos 45° + \boldsymbol{i}\sin 45°)\right]^2 = \frac{1}{2}(\cos 90° + \boldsymbol{i}\sin 90°)$
The square is rotated 90° counterclockwise and dilated by a factor of 0.5.

39. The roots are the vertices of a regular polygon. Since one of the roots must be a positive real number, a vertex of the polygon lies on the positive real axis and the polygon is symmetric about the real axis. This means that the non-real complex roots occur in conjugate pairs. Since the imaginary part of the sum of two complex conjugates is 0, the imaginary part of the sum of all the roots must be 0.

40. $r = 2(3)$ or 6 $\qquad \theta = \frac{\pi}{6} + \frac{5\pi}{3}$
$\qquad = \frac{\pi}{6} + \frac{10\pi}{6}$ or $\frac{11\pi}{6}$

$$6\left(\cos\frac{11\pi}{6} + \boldsymbol{i}\sin\frac{11\pi}{6}\right) = 6\left(\frac{\sqrt{3}}{2} + \boldsymbol{i}\left(-\frac{1}{2}\right)\right)$$
$$= 3\sqrt{3} - 3\boldsymbol{i}$$

41. $(2 - 5\boldsymbol{i}) + (-3 + 6\boldsymbol{i}) - (-6 + 2\boldsymbol{i})$
$= (2 + (-3) - (-6)) + (-5\boldsymbol{i} + 6\boldsymbol{i} - 2\boldsymbol{i})$
$= 5 - \boldsymbol{i}$

42. $x = t$, $y = -2t + 7$

43. $\cos 22.5° = \cos\left(\frac{45°}{2}\right)$

$$= \pm\sqrt{\frac{1 + \cos 45°}{2}}$$
$$= \pm\sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}}$$
$$= \pm\sqrt{\frac{2 + \sqrt{2}}{4}}$$
$$= \pm\frac{\sqrt{2 + \sqrt{2}}}{2}$$

44. Find B.
$B = 180° - 90° - 81°15'$
$= 8°45'$
Find a.

$$\tan 81°15' = \frac{a}{28}$$
$$28\tan 81°15' = a$$
$$181.9 = a$$

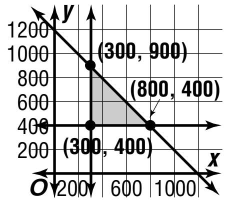
Find c.

$\cos 81°15' = \frac{28}{c}$

$c = \frac{28}{\cos 81°15'}$

$c = 184.1$

45. Let x = the number of large bears produced.
Let y = the number of small bears produced.



$x \geq 300$

$y \geq 400$

$x + y \leq 1200$

$f(x, y) = 9x + 5y$

$f(300, 400) = 9(300) + 5(400)$
$= 4700$

$f(300, 900) = 9(300) + 5(900)$
$= 7200$

$f(800, 400) = 9(800) + 5(400)$
$= 9200$

Producing 800 large bears and 400 small bears yields the maximum profit.

46. $0.20(6) = 1.2$ quarts of alcohol

$0.60(4) = 2.4$ quarts of alcohol

$\frac{1.2 + 2.4}{6 + 4} = \frac{3.6}{10}$ or 36% alcohol

The correct choice is A.

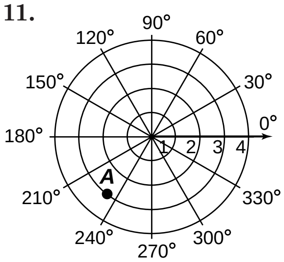
Chapter 9 Study Guide and Assessment

Pages 607 Check for Understanding

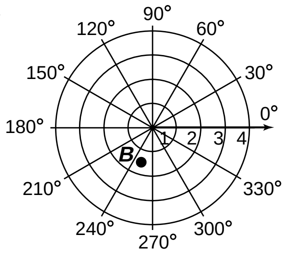
1. absolute value
2. Polar
3. prisoner
4. iteration
5. pure imaginary
6. cardioid
7. rectangular
8. spiral of Archimedes
9. Argand
10. modulus

Pages 608–610 Skills and Concepts

11.



12.



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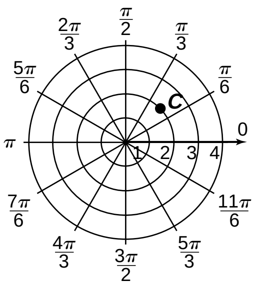

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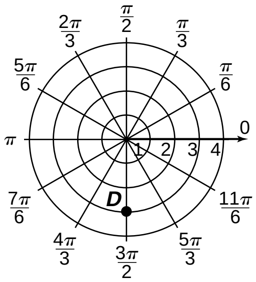


15. Sample answer: (4, 585°), (4, 945°), (−4, 45°), (−4, 405°)

$(r, \theta + 360k°)$

$\rightarrow (4, 225° + 360(1)°) \rightarrow (4, 585°)$

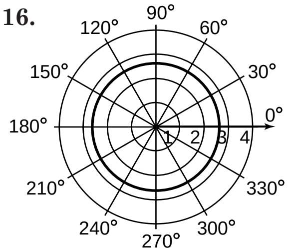
$\rightarrow (4, 225° + 360(2)°) \rightarrow (4, 945°)$

$(-r, \theta + (2k + 1)180°)$

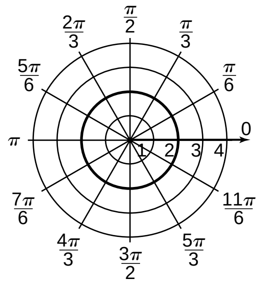
$\rightarrow (-4, 225° + (-1)180°) \rightarrow (-4, 45°)$

$\rightarrow (-4, 225° + (1)180°) \rightarrow (-4, 405°)$

16.



17.



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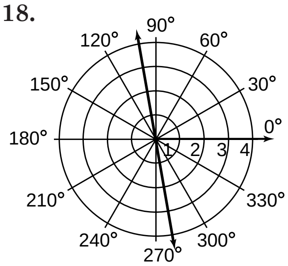
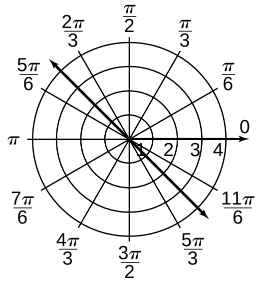
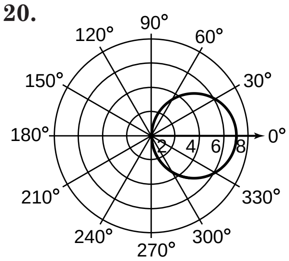


19.

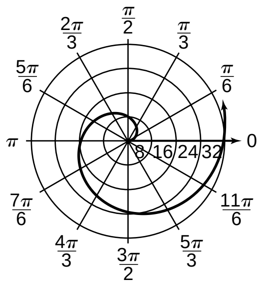


20.



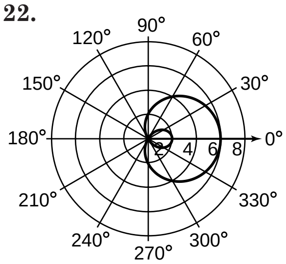
circle

21.



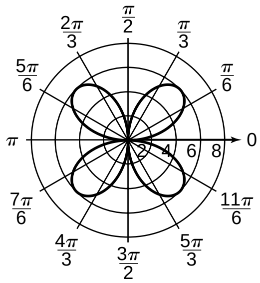
Spiral of Archimedes

22.



limaçon

23.



rose

24. $x = 6 \cos 45°$
$= 6\left(\frac{\sqrt{2}}{2}\right)$
$= 3\sqrt{2}$

$y = 6 \sin 45°$
$= 6\left(\frac{\sqrt{2}}{2}\right)$
$= 3\sqrt{2}$

$(3\sqrt{2}, 3\sqrt{2})$

25. $x = 2 \cos 330°$
$= 2\left(\frac{\sqrt{3}}{2}\right)$
$= \sqrt{3}$

$y = 2 \sin 330°$
$= 2\left(-\frac{1}{2}\right)$
$= -1$

$(\sqrt{3}, -1)$

26. $x = -2\cos\left(\frac{3\pi}{4}\right)$ $\quad$ $y = -2\sin\left(\frac{3\pi}{4}\right)$

$= -2\left(-\frac{\sqrt{2}}{2}\right)$ $\quad$ $= -2\left(\frac{\sqrt{2}}{2}\right)$

$= \sqrt{2}$ $\quad$ $= -\sqrt{2}$

$(\sqrt{2}, -\sqrt{2})$

27. $x = 1\cos\left(\frac{\pi}{2}\right)$ $\quad$ $y = 1\sin\left(\frac{\pi}{2}\right)$

$= 0$ $\quad$ $= 1$

$(0, 1)$

28. $r = \sqrt{(-\sqrt{3})^2 + (-3)^2}$ $\quad$ $\theta = \text{Arctan}\left(\frac{-3}{-\sqrt{3}}\right) + \pi$

$= \sqrt{12}$ or $2\sqrt{3}$ $\quad$ $= \frac{4\pi}{3}$

$\left(2\sqrt{3}, \frac{4\pi}{3}\right)$

29. $r = \sqrt{5^2 + 5^2}$ $\quad$ $\theta = \text{Arctan}\left(\frac{5}{5}\right)$

$= \sqrt{50}$ or $5\sqrt{2}$ $\quad$ $= \frac{\pi}{4}$

$\left(5\sqrt{2}, \frac{\pi}{4}\right)$

30. $r = \sqrt{(-3)^2 + 1^2}$ $\quad$ $\theta = \text{Arctan}\left(\frac{1}{-3}\right) + \pi$

$= \sqrt{10} \approx 3.16$ $\quad$ ≈ 2.82

$(3.16, 2.82)$

31. $r = \sqrt{4^2 + 2^2}$ $\quad$ $\theta = \text{Arctan}\left(\frac{2}{4}\right)$

$= \sqrt{20} \approx 4.47$ $\quad$ ≈ 0.46

$(4.47, 0.46)$

32. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{2^2 + 1^2}$

$= \pm\sqrt{5}$

Since C is positive, use $-\sqrt{5}$.

$-\frac{2}{\sqrt{5}}x - \frac{1}{\sqrt{5}}y - \frac{3}{\sqrt{5}} = 0$

$\cos\phi = -\frac{2\sqrt{5}}{5}$, $\sin\phi = -\frac{\sqrt{5}}{5}$, $p = \frac{3\sqrt{5}}{5}$

$\phi = \text{Arctan}\left(\frac{1}{2}\right)$

$\approx 27°$

Since $\cos\phi < 0$ and $\sin\phi < 0$, the normal lies in the third quadrant.

$\phi = 180° + 27°$ or $207°$

$p = r\cos(\theta - \phi)$

$\frac{3\sqrt{5}}{5} = r\cos(\theta - 207°)$

33. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{3^2 + 1^2}$

$= \pm\sqrt{10}$

Since C is positive, use $-\sqrt{10}$.

$\frac{3}{\sqrt{10}}x - \frac{1}{\sqrt{10}}y - \frac{4}{\sqrt{10}} = 0$

$\cos\phi = -\frac{3\sqrt{10}}{10}$, $\sin\phi = -\frac{\sqrt{10}}{10}$, $p = \frac{2\sqrt{10}}{5}$

$\phi = \text{Arctan}\left(\frac{1}{3}\right)$

$\approx 18°$

Since $\cos\phi < 0$ and $\sin\phi < 0$, the normal lies in the third quadrant.

$\phi = 180° + 18°$ or $198°$

$p = r\cos(\theta - \phi)$

$\frac{2\sqrt{10}}{5} = r\cos(\theta - 198°)$

34. $3 = r\cos\left(\theta - \frac{\pi}{3}\right)$

$0 = r\cos\theta\cos\frac{\pi}{3} + r\sin\theta\sin\frac{\pi}{3} - 3$

$0 = \frac{1}{2}r\cos\theta + \frac{\sqrt{3}}{2}r\sin\theta - 3$

$0 = \frac{1}{2}x + \frac{\sqrt{3}}{2}y - 3$

$0 = x + \sqrt{3}y - 6$ or

$x + \sqrt{3}y - 6 = 0$

35. $4 = r\cos\left(\theta + \frac{\pi}{2}\right)$

$0 = r\cos\theta\cos\frac{\pi}{2} - r\sin\theta\sin\frac{\pi}{2} - 4$

$0 = 0 - r\sin\theta - 4$

$0 = -y - 4$

$0 = y + 4$ or

$y + 4 = 0$

36. $\boldsymbol{i}^{10} + \boldsymbol{i}^{25} = (\boldsymbol{i}^4)^2 \cdot \boldsymbol{i}^2 + (\boldsymbol{i}^4)^6 \cdot \boldsymbol{i}$

$= (1)^2 \cdot (-1) + (1)^6 \cdot \boldsymbol{i}$

$= -1 + \boldsymbol{i}$

37. $(2 + 3\boldsymbol{i}) - (4 - 4\boldsymbol{i}) = (2 + (-4)) + (3\boldsymbol{i} - (-4\boldsymbol{i}))$

$= -2 + 7\boldsymbol{i}$

38. $(2 + 7\boldsymbol{i}) + (-3 - \boldsymbol{i}) = (2 + (-3)) + (7\boldsymbol{i} + (-\boldsymbol{i}))$

$= -1 + 6\boldsymbol{i}$

39. $\boldsymbol{i}^3(4 - 3\boldsymbol{i}) = 4\boldsymbol{i}^3 - 3\boldsymbol{i}^4$

$= 4(-\boldsymbol{i}) - 3(1)$

$= -3 - 4\boldsymbol{i}$

40. $(\boldsymbol{i} - 7)(-\boldsymbol{i} + 7) = -\boldsymbol{i}^2 + 14\boldsymbol{i} - 49$

$= 1 + 14\boldsymbol{i} - 49$

$= -48 + 14\boldsymbol{i}$

41. $\frac{4 + 2\boldsymbol{i}}{5 - 2\boldsymbol{i}} = \frac{4 + 2\boldsymbol{i}}{5 - 2\boldsymbol{i}} \cdot \frac{5 + 2\boldsymbol{i}}{5 + 2\boldsymbol{i}}$

$= \frac{20 + 18\boldsymbol{i} + 4\boldsymbol{i}^2}{25 - 4\boldsymbol{i}^2}$

$= \frac{16 + 18\boldsymbol{i}}{29}$

$= \frac{16}{29} + \frac{18}{29}\boldsymbol{i}$

42. $\frac{5 + \boldsymbol{i}}{1 - \sqrt{2}\boldsymbol{i}} = \frac{5 + \boldsymbol{i}}{1 - \sqrt{2}\boldsymbol{i}} \cdot \frac{1 + \sqrt{2}\boldsymbol{i}}{1 + \sqrt{2}\boldsymbol{i}}$

$= \frac{5 + 5\sqrt{2}\boldsymbol{i} + \boldsymbol{i} + \sqrt{2}\boldsymbol{i}^2}{1 - 2\boldsymbol{i}^2}$

$= \frac{(5 - \sqrt{2}) + (5\sqrt{2} + 1)\boldsymbol{i}}{3}$

$= \frac{5 - \sqrt{2}}{3} + \frac{1 + 5\sqrt{2}}{3}\boldsymbol{i}$

43. $r = \sqrt{2^2 + 2^2}$ $\quad$ $\theta = \text{Arctan}\left(\frac{2}{2}\right)$

$= \sqrt{8}$ or $2\sqrt{2}$ $\quad$ $= \frac{\pi}{4}$

$2\sqrt{2}\left(\cos\frac{\pi}{4} + \boldsymbol{i}\sin\frac{\pi}{4}\right)$

44. $r = \sqrt{1^2 + (-3)^2}$ $\quad$ $\theta = \text{Arctan}\left(\frac{-3}{1}\right) + 2\pi$

$= \sqrt{10}$ $\quad$ ≈ 5.03

$\sqrt{10}(\cos 5.03 + \boldsymbol{i}\sin 5.03)$

45. $r = \sqrt{(-1)^2 + (\sqrt{3})^2}$ $\quad$ $\theta = \text{Arctan}\left(\frac{\sqrt{3}}{-1}\right) + \pi$

$= \sqrt{4}$ or 2 $\quad$ $= \frac{2\pi}{3}$

$2\left(\cos\frac{2\pi}{3} + \boldsymbol{i}\sin\frac{2\pi}{3}\right)$

46. $r = \sqrt{(-6)^2 + (-4)^2}$ $\quad$ $\theta = \text{Arctan}\left(\frac{-4}{-6}\right) + \pi$

$= \sqrt{52}$ or $2\sqrt{13}$ $\quad$ ≈ 3.73

$2\sqrt{13}(\cos 3.73 + \boldsymbol{i}\sin 3.73)$

47. $r = \sqrt{(-4)^2 + (-1)^2}$ $\quad$ $\theta = \text{Arctan}\left(\frac{-1}{-4}\right) + \pi$

$= \sqrt{17}$ $\quad$ ≈ 3.39

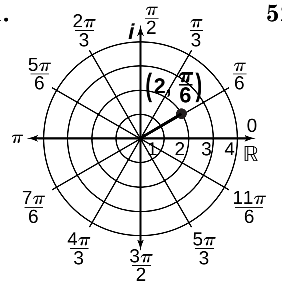
$\sqrt{17}(\cos 3.39 + \boldsymbol{i}\sin 3.39)$

48. $r = \sqrt{4^2 + 0^2}$ $\qquad \theta = 0$
$= \sqrt{16}$ or 4
$4(\cos 0 + \boldsymbol{i} \sin 0)$

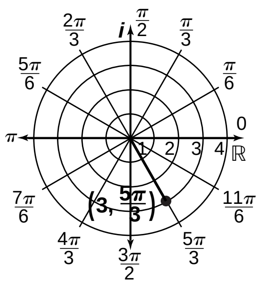
49. $r = \sqrt{(-2\sqrt{2})^2 + 0^2}$ $\qquad \theta = \pi$
$= \sqrt{8}$ or $2\sqrt{2}$
$2\sqrt{2}(\cos \pi + \boldsymbol{i} \sin \pi)$

50. $r = \sqrt{0^2 + 3^2}$ $\qquad \theta = \frac{\pi}{2}$
$= \sqrt{9}$ or 3
$3\left(\cos \frac{\pi}{2} + \boldsymbol{i} \sin \frac{\pi}{2}\right)$

51.



52.



51. $2\left(\cos \frac{\pi}{6} + \boldsymbol{i} \sin \frac{\pi}{6}\right)$
$= 2\left(\frac{\sqrt{3}}{2} + \boldsymbol{i}\left(\frac{1}{2}\right)\right)$
$= \sqrt{3} + \boldsymbol{i}$

52. $3\left(\cos \frac{5\pi}{3} + \boldsymbol{i} \sin \frac{5\pi}{3}\right)$
$= 3\left(\frac{1}{2} + \boldsymbol{i}\left(-\frac{\sqrt{3}}{2}\right)\right)$
$= \frac{3}{2} - \frac{3\sqrt{3}}{2}\boldsymbol{i}$

53. $r = 4(3)$ or 12 $\qquad \theta = \frac{\pi}{3} + \frac{\pi}{3}$ or $\frac{2\pi}{3}$
$12\left(\cos \frac{2\pi}{3} + \boldsymbol{i} \sin \frac{2\pi}{3}\right)$
$= 12\left(-\frac{1}{2} + \boldsymbol{i}\left(\frac{\sqrt{3}}{2}\right)\right)$
$= -6 + 6\sqrt{3}\boldsymbol{i}$

54. $r = 8(4)$ or 32 $\qquad \theta = \frac{\pi}{4} + \frac{\pi}{2}$
$32\left(\cos \frac{3\pi}{4} + \boldsymbol{i} \sin \frac{3\pi}{4}\right)$ $\qquad = \frac{\pi}{4} + \frac{2\pi}{4}$ or $\frac{3\pi}{4}$
$= 32\left(-\frac{\sqrt{2}}{2} + \boldsymbol{i}\left(\frac{\sqrt{2}}{2}\right)\right)$
$= -16\sqrt{2} + 16\sqrt{2}\boldsymbol{i}$

55. $r = 2(5)$ or 10 $\qquad \theta = 2 + 0.5$ or 2.5
$10(\cos 2.5 + \boldsymbol{i} \sin 2.5)$
$\approx -8.01 + 5.98\boldsymbol{i}$

56. $r = \frac{8}{2}$ or 4 $\qquad \theta = \frac{7\pi}{6} - \frac{5\pi}{3}$
$4\left[\cos\left(-\frac{\pi}{2}\right) + \boldsymbol{i} \sin\left(-\frac{\pi}{2}\right)\right]$ $\qquad = \frac{7\pi}{6} - \frac{10\pi}{6}$ or $-\frac{\pi}{2}$
$= 4(0 + \boldsymbol{i}(-1))$
$= -4\boldsymbol{i}$

57. $r = \frac{6}{4}$ or $\frac{3}{2}$ $\qquad \theta = \frac{\pi}{2} - \frac{\pi}{6}$
$\frac{3}{2}\left(\cos \frac{\pi}{3} + \boldsymbol{i} \sin \frac{\pi}{3}\right)$ $\qquad = \frac{3\pi}{6} - \frac{\pi}{6}$ or $\frac{\pi}{3}$
$= \frac{3}{2}\left(\frac{1}{2} + \boldsymbol{i}\left(\frac{\sqrt{3}}{2}\right)\right)$
$= \frac{3}{4} + \frac{3\sqrt{2}}{4}\boldsymbol{i}$

58. $r = \frac{2.2}{4.4}$ or 0.5 $\qquad \theta = 1.5 - 0.6$ or 0.9
$0.5(\cos 0.9 + \boldsymbol{i} \sin 0.9)$
$\approx 0.31 + 0.39\boldsymbol{i}$

59. $r = \sqrt{2^2 + 2^2} = 2\sqrt{2}$ $\qquad \theta = \text{Arctan}\left(\frac{2}{2}\right) = \frac{\pi}{4}$
$(\sqrt{2})^8\left(\cos (8)\left(\frac{\pi}{4}\right) + \boldsymbol{i} \sin (8)\left(\frac{\pi}{4}\right)\right)$
$= 4096(\cos 2\pi + \boldsymbol{i} \sin 2\pi)$
$= 4096$

60. $r = (\sqrt{\sqrt{3})^2 + (-1)^2} = 2$
$\theta = \text{Arctan}\ \frac{-1}{\sqrt{3}} = -\frac{\pi}{6}$
$2^7\left(\cos (7)\left(-\frac{\pi}{6}\right) + \boldsymbol{i} \sin (7)\left(-\frac{\pi}{6}\right)\right)$
$= 128\left(\cos -\frac{7\pi}{6} + \boldsymbol{i} \sin -\frac{7\pi}{6}\right)$
$= 128\left(-\frac{\sqrt{3}}{2} + \boldsymbol{i}\left(\frac{1}{2}\right)\right)$
$= -64\sqrt{3} + 64\boldsymbol{i}$

61. $r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$ $\qquad \theta = \text{Arctan}\left(\frac{1}{-1}\right) + \pi$
$\qquad = \frac{3\pi}{4}$
$(\sqrt{2})^4\left(\cos (4)\left(\frac{3\pi}{4}\right) + \boldsymbol{i} \sin (4)\left(\frac{3\pi}{4}\right)\right)$
$= 4(\cos 3\pi + \boldsymbol{i} \sin 3\pi)$
$= -4$

62. $r = \sqrt{(-2)^2 + (-2)^2} = 2\sqrt{2}$ $\qquad \theta = \text{Arctan}\left(\frac{-2}{-2}\right) + \pi$
$\qquad = \frac{5\pi}{4}$
$(2\sqrt{2})^3\left(\cos (3)\left(\frac{5\pi}{4}\right) + \boldsymbol{i} \sin (3)\left(\frac{5\pi}{4}\right)\right)$
$= 16\sqrt{2}\left(\cos \frac{15\pi}{4} + \boldsymbol{i} \sin \frac{15\pi}{4}\right)$
$= 16\sqrt{2}\left(\frac{\sqrt{2}}{2} + \boldsymbol{i}\left(-\frac{\sqrt{2}}{2}\right)\right)$
$= 16 - 16\boldsymbol{i}$

63. $r = \sqrt{0^2 + 1^2} = 1$ $\qquad \theta = \frac{\pi}{2}$
$1^{\frac{1}{4}}\left(\cos\left(\frac{1}{4}\right)\left(\frac{\pi}{2}\right) + \boldsymbol{i} \sin\left(\frac{1}{4}\right)\left(\frac{\pi}{2}\right)\right)$
$= \cos \frac{\pi}{8} + \boldsymbol{i} \sin \frac{\pi}{8}$
$\approx 0.92 + 0.38\boldsymbol{i}$

64. $r = \sqrt{(\sqrt{3})^2 + 1^2} = 2$ $\qquad \theta = \text{Arctan}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$
$2^{\frac{1}{3}}\left(\cos\left(\frac{1}{3}\right)\left(\frac{\pi}{6}\right) + \boldsymbol{i} \sin\left(\frac{1}{3}\right)\left(\frac{\pi}{6}\right)\right)$
$= 2^{\frac{1}{3}}\left(\cos\left(\frac{\pi}{18}\right) + \boldsymbol{i} \sin\left(\frac{\pi}{18}\right)\right)$
$\approx 1.24 + 0.22\boldsymbol{i}$

Page 611 Applications and Problem Solving

65. lemniscate

66. $r = \sqrt{75^2 + 125^2}$ $\qquad \theta = \text{Arctan}\left(\frac{125}{75}\right)$
$= \sqrt{21{,}250} \approx 145.77$ $\qquad \approx 59.04°$
$(145.77, 59.04°)$

67.
$$r \cos\left(\theta - \frac{\pi}{2}\right) + 5 = 0$$
$$r \cos \theta \cos \frac{\pi}{2} + r \sin \theta \sin \frac{\pi}{2} + 5 = 0$$
$$r \sin \theta + 5 = 0$$
$$y + 5 = 0$$
$$y = -5$$

68. $I = \frac{E}{Z}$
$= \frac{50 + 180\boldsymbol{j}}{4 + 5\boldsymbol{j}}$
$= \frac{50 + 180\boldsymbol{j}}{4 + 5\boldsymbol{j}} \cdot \frac{4 - 5\boldsymbol{j}}{4 - 5\boldsymbol{j}}$
$= \frac{200 + 470\boldsymbol{j} - 900\boldsymbol{j}^2}{16 - 25\boldsymbol{j}^2}$
$= \frac{1100 + 470\boldsymbol{j}^2}{41}$
$\approx 26.83 + 11.46\boldsymbol{j}$ amps

Page 611 Open-Ended Assessment

1a. Sample answer: $4 - 6\boldsymbol{i}$ and $3 + 2\boldsymbol{i}$

$$(4 - 6\boldsymbol{i}) + (3 + 2\boldsymbol{i}) = (4 + 3) + (-6\boldsymbol{i} + 2\boldsymbol{i})$$
$$= 7 - 4\boldsymbol{i}$$

1b. No. Sample explanation: $2 - 3\boldsymbol{i}$ and $5 - \boldsymbol{i}$ also have this sum.

$$(2 - 3\boldsymbol{i}) + (5 - \boldsymbol{i}) = (2 + 5) + (-3\boldsymbol{i} + (-\boldsymbol{i}))$$
$$= 7 - 4\boldsymbol{i}$$

2a. Sample answer: $4 + \boldsymbol{i}$

$$|z| = \sqrt{4^2 + 1^2}$$
$$= \sqrt{17}$$

2b. No. Sample explanation: $1 + 4\boldsymbol{i}$ also has this absolute value.

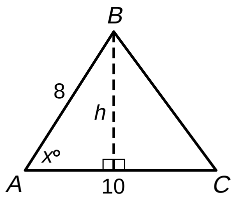
$$|z| = \sqrt{1^2 + 4^2}$$
$$= \sqrt{17}$$

Chapter 9 SAT & ACT Preparation

Page 613 SAT and ACT Practice

1. $\angle a$ and $\angle b$ form a linear pair, so $\angle b$ is supplementary to $\angle a$. Since $\angle b$ and $\angle d$ are vertical angles, they are equal in measure. So $\angle d$ is also supplementary to $\angle a$. Since $\angle d$ and $\angle f$ are alternate interior angles, they are equal. So $\angle f$ is supplementary to $\angle a$. And since $\angle f$ and $\angle h$ are vertical angles, $\angle h$ is supplementary to $\angle a$. The angles supplementary to $\angle a$ are angles b, d, f, and h. The correct choice is A.

2. Draw the given triangle and draw the height h from point B.

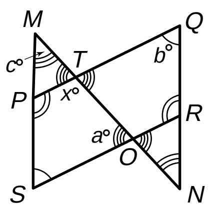


The answer choices include sin x. Write an expression for the height, using the sine of x.

$$\sin x = \frac{h}{8} \qquad A = \frac{1}{2}bh$$
$$8 \sin x = h \qquad = \frac{1}{2}(10)(8 \sin x)$$
$$= 40 \sin x$$

The correct choice is B.

3. Since $PQRS$ is a parallelogram, sides PQ and SR are parallel and $m\angle Q = m\angle S = b$.



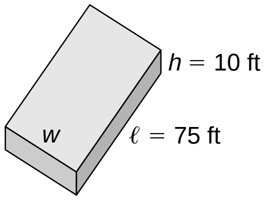
In $\triangle SMO$, $c + b + a = 180$ or $a = 180 - (c + b)$. Also, $x + a = 180$ or $a = 180 - x$ since consecutive interior angles are supplementary.

$$180 - (c + b) = 180 - x$$
$$x = c + b$$

The correct choice is E.

4. Volume $= \ell wh$

$$16{,}500 = 75 \cdot w \cdot 10$$
$$16{,}500 = 750w$$
$$22 = w$$



The correct choice is A.

5. $\frac{1}{100^{100}} - \frac{1}{10^{99}} = \frac{1}{100^{100}} - \frac{10}{10^{100}}$

$$= \frac{-9}{10^{100}}$$

The correct choice is A.

6. Consider the three unmarked angles at the intersection point. One of these angles, say the top one, is the supplement of the other two unmarked angles, because of vertical angles. So the sum of the measures of the unmarked angles is 180°.

The sum of the measures of the marked angles and the three unmarked angles is 3(180), since these angles are the interior angles of three triangles.

m(sum of marked angles) +
m(sum of unmarked angles) = 3(180)
m(sum of marked angles) + 180 = 3(180)
m(sum of marked angles) = 360

The correct choice is C.

7. Subtract the second equation from the first.

$$\begin{array}{r} 5x^2 + 6x = 70 \\ -5x^2 + 6y = 10 \\ \hline 6x + 6y = 60 \end{array}$$

$x + y = 10$, so $10x + 10y = 100$.
The correct choice is E.

8. Since $\angle B$ is a right angle, $\angle C$ is a right angle also, because they are alternate interior angles.
In the triangle containing $\angle C$, $90 + x + y = 180$ or $x + y = 90$.
The straight angle at D is made up of 3 angles.

$$120 + x + x = 180$$
$$2x = 60 \text{ or } x = 30$$
$$x + y = 90$$
$$(30) + y = 90$$
$$y = 60$$

The correct choice is B.

9. In the slope-intercept form of a line, $y = mx + b$, m represents the slope of the line, and b represents the y-intercept. Since the slope is given as $\frac{3}{2}$, the slope-intercept form of the line is $y = \frac{3}{2}x + b$.
Since (–3, 0) is on the line, it satisfies the equation. $0 = \frac{3}{2}(-3) + b$. So $b = \frac{9}{2}$.
The correct choice is D.

10. Note that consecutive interior angles are supplementary.

$$110 + 2x = 180 \qquad y + x = 180$$
$$2x = 70 \qquad y + (35) = 180$$
$$x = 35 \qquad y = 145$$

The answer is 145.

Chapter 10 Conics

10-1 Introduction to Analytic Geometry

Pages 619–620 Check for Understanding

1. negative distances have no meaning

2. Use the distance formula to show that the measure of the distance from the midpoint to either endpoint is the same.

3a. Yes; the distance from B to A is $\frac{a\sqrt{5}}{2}$ and the distance from B to C is also $\frac{a\sqrt{5}}{2}$.

3b. Yes; the distance from B to A is $\sqrt{a^2 + b^2}$ and the distance from C to A is also $\sqrt{a^2 + b^2}$.

3c. No; the distance from A to B is $\sqrt{a^2 + b^2}$, the distance from A to C is $a + b$, and the distance from B to C is $b\sqrt{2}$.

4. (1) Show that two pairs of opposite sides are parallel by showing that slopes of the lines through each pair of opposite sides are equal.
(2) Show that two pairs of opposite sides are congruent by showing that the distance between the vertices forming each pair of opposite sides are equal.
(3) Show that one pair of opposite sides is parallel and congruent by showing that the slopes of the lines through that pair of sides are equal and that the distances between the endpoints of each pair of segments are equal.
(4) Show that the diagonals bisect each other by showing that the midpoints of the diagonals coincide.

5. $d = \sqrt{(x_2 - x_1)^2 + (y^2 - y_1)^2}$
$d = \sqrt{(5 - 5)^2 + (11 - 1)^2}$
$d = \sqrt{0^2 + 10^2}$
$d = \sqrt{100}$ or 10
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{5 + 5}{2}, \frac{1 + 11}{2}\right)$
$= (5, 6)$

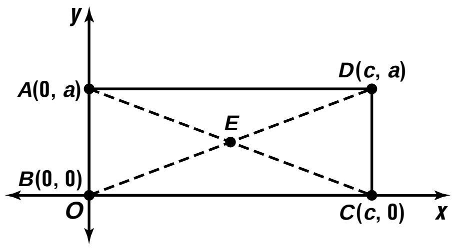
6. $d = \sqrt{(x_2 - x_2)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(-4 - 0)^2 + (-3 - 0)^2}$
$d = \sqrt{(-4)^2 + (-3)^2}$
$d = \sqrt{25}$ or 5
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0 + (-4)}{2}, \frac{0 + (-3)}{2}\right)$
$= (-2, -1.5)$

7. $d = \sqrt{[0 - (-2)]^2 + (4 - 2)^2}$
$d = \sqrt{2^2 + 2^2}$
$d = \sqrt{8}$ or $2\sqrt{2}$
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 0}{2}, \frac{2 + 4}{2}\right)$
$= (-1, 3)$

8. $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(6 - 3)^2 + (2 - 4)^2}$
$= \sqrt{13}$
slope of $\overline{AB}$
$m = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{2 - 4}{6 - 3}$ or $-\frac{2}{3}$
$DC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(8 - 5)^2 + (7 - 9)^2}$
$= \sqrt{13}$
slope of $\overline{DC}$
$m = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{7 - 9}{8 - 5}$ or $-\frac{2}{3}$
Yes; $\overline{AB} \cong \overline{DC}$ since $AB = \sqrt{13}$ and $DC = \sqrt{13}$, and $\overline{AB} \parallel \overline{DC}$ since the slope of $\overline{AB}$ is $-\frac{2}{3}$ and the slope of $\overline{DC}$ is also $-\frac{2}{3}$.

9. $XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[-1 - (-3)]^2 + (-6 - 2)^2}$
$= \sqrt{68}$ or $2\sqrt{17}$
$XZ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[5 - (-3)]^2 + (0 - 2)^2}$
$= \sqrt{68}$ or $2\sqrt{17}$
Yes; $\overline{XY} \cong \overline{XZ}$, since $XY = 2\sqrt{17}$ and $XZ = 2\sqrt{17}$, therefore $\triangle XYZ$ is isosceles.

10a.



10b. $BD = \sqrt{(c - 0)^2 + (a - 0)^2}$
$= \sqrt{c^2 + a^2}$
$AC = \sqrt{(c - 0)^2 + (0 - a)^2}$
$= \sqrt{c^2 + a^2}$
Thus, $\overline{AC} \cong \overline{BD}$.

10c. The midpoint of $\overline{AC}$ is $\left(\frac{c}{2}, \frac{a}{2}\right)$. The midpoint of $\overline{BD}$ is $\left(\frac{c}{2}, \frac{a}{2}\right)$. Therefore, the diagonals intersect at their common midpoint, $E\left(\frac{c}{2}, \frac{a}{2}\right)$. Thus, $\overline{AE} \cong \overline{EC}$ and $\overline{BE} \cong \overline{ED}$.

10d. The diagonals of a rectangle are congruent and bisect each other.

11a. Both players are located along a diagonal of the field with endpoints (0, 0) and (80, 120). The kicker's teammate is located at the midpoint of this diagonal.
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0 + 80}{2}, \frac{0 + 120}{2}\right)$
$= (40, 60)$

11b. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(40 - 0)^2 + (60 - 0)^2}$
$d = \sqrt{40^2 + 60^2}$
$d = \sqrt{5200}$
$d = 20\sqrt{13}$ or about 72 yards

Pages 620–622 Exercises

12. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{[4 - (-1)]^2 + (13 - 1)^2}$
$d = \sqrt{5^2 + 12^2}$
$d = \sqrt{169}$ or 13
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-1 + 4}{2}, \frac{1 + 13}{2}\right)$
$= (1.5, 7)$

13. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(-1 - 1)^2 + (-3 - 3)^2}$
$d = \sqrt{(-2)^2 + (-6)^2}$
$d = \sqrt{40}$ or $2\sqrt{10}$
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1 + (-1)}{2}, \frac{3 + (-3)}{2}\right)$
$= (0, 0)$

14. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(0 - 8)^2 + (8 - 0)^2}$
$d = \sqrt{(-8)^2 + 8^2}$
$d = \sqrt{128}$ or $8\sqrt{2}$
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{8 + 0}{2}, \frac{8 + 0}{2}\right)$
$= (4, 4)$

15. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{[5 - (-1)]^2 + [-3 - (-6)]^2}$
$d = \sqrt{6^2 + 3^2}$
$d = \sqrt{45}$ or $3\sqrt{5}$
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-1 + 5}{2}, \frac{-6 + (-3)}{2}\right)$
$= (2, -4.5)$

16. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(7\sqrt{2} - 3\sqrt{2})^2 + [-1 - (-5)]^2}$
$d = \sqrt{(4\sqrt{2})^2 + 4^2}$
$d = \sqrt{48}$ or $4\sqrt{3}$
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3\sqrt{2} + 7\sqrt{2}}{2}, \frac{-5 + (-1)}{2}\right)$
$= (5\sqrt{2}, -3)$

17. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(a - a)^2 + (-9 - 7)^2}$
$d = \sqrt{0^2 + (-16)^2}$
$d = \sqrt{256}$ or 16
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{a + a}{2}, \frac{7 + (-9)}{2}\right)$
$= (a, -1)$

18. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{[r - 2 - (6 + r)]^2 + (s - s)^2}$
$d = \sqrt{(-8)^2 + 0^2}$
$d = \sqrt{64}$ or 8
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{6 + r + r - 2}{2}, \frac{s + s}{2}\right)$
$= \left(\frac{2r + 4}{2}, \frac{2s}{2}\right)$
$= (r + 2, s)$

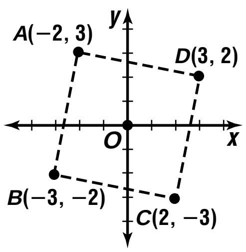
19. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(c + 2 - c)^2 + (d - 1 - d)^2}$
$d = \sqrt{2^2 + (-1)^2}$
$d = \sqrt{5}$
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{c + c + 2}{2}, \frac{d + d - 1}{2}\right)$
$= \left(\frac{2c + 2}{2}, \frac{2d - 1}{2}\right)$
$= \left(c + 1, d - \frac{1}{2}\right)$

20. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{[w - (w - 2)]^2 + (4w - w)^2}$
$d = \sqrt{2^2 + (3w)^2}$
$d = \sqrt{4 + 9w^2}$ or $\sqrt{9w^2 + 4}$
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{w - 2 + w}{2}, \frac{w + 4w}{2}\right)$
$= \left(w - 1, \frac{5}{2}w\right)$

21. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$20 = \sqrt{(-2a - a)^2 + [7 - (-9)]^2}$
$20 = \sqrt{(-3a)^2 + 16^2}$
$20 = \sqrt{9a^2 + 256}$
$400 = 9a^2 + 256$
$144 = 9a^2$
$a^2 = 16$
$a = \pm\sqrt{16}$ or ± 4

22. Let D have coordinates (x_2, y_2).
$\left(\frac{4 + x_2}{2}, \frac{-1 + y_2}{2}\right) = \left(-3, \frac{5}{2}\right)$
$\frac{4 + x_2}{2} = -3$ $\qquad$ $\frac{-1 + y_2}{2} = \frac{5}{2}$
$4 + x_2 = -6$ $\qquad$ $-1 + y_2 = 5$
$x_2 = -10$ $\qquad$ $y_2 = 6$
Then D has coordinates $(-10, 6)$.

23. Let the vertices of the quadrilateral be $A(-2, 3)$, $B(-2, -3)$, $C(2, -3)$, and $D(3, 2)$.



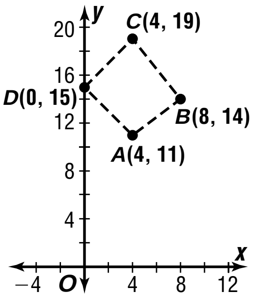
A quadrilateral is a parallelogram if one pair of opposite sides are parallel and congruent.
$\overline{AD}$ and $\overline{BC}$ are one pair of opposite sides.

slope of $\overline{AD}$
$m = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{2 - 3}{3 - (-2)}$
$= -\frac{1}{5}$

slope of $\overline{BC}$
$m = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{-3 - (-2)}{2 - (-3)}$
$= -\frac{1}{5}$

Their slopes are equal, therefore $\overline{AD} \parallel \overline{BC}$.
$AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[3 - (-2)]^2 + (2 - 3)^2}$
$= \sqrt{5^2 + (-1)^2}$
$= \sqrt{26}$
$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[2 - (-3)]^2 + [-3 - (-2)]^2}$
$= \sqrt{5^2 + (-1)^2}$
$= \sqrt{26}$
The measures of $\overline{AD}$ and $\overline{BC}$ are equal.
Therefore $\overline{AD} \cong \overline{BC}$. Since $\overline{AD} \parallel \overline{BC}$ and $\overline{AD} \cong \overline{BC}$, quadrilateral $ABCD$ is a parallelogram; yes.

24. Let the vertices of the quadrilateral be $A(4, 11)$, $B(8, 14)$, $C(4, 19)$, and $D(0, 15)$.



A quadrilateral is a parallelogram if both pairs of opposite sides are parallel.
$\overline{AB}$ and $\overline{DC}$ are one pair of opposite sides.

slope of $\overline{AB}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14 - 11}{8 - 4} = \frac{3}{2}$$

slope of $\overline{DC}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{19 - 15}{4 - 0} = \frac{4}{4} \text{ or } 1$$

Since $\overline{AB} \nparallel \overline{DC}$, quadrilateral $ABCD$ is not a parallelogram; no.

25. The slope of the line through $(15, 1)$ and $(-3, -8)$ should be equal to the slope of the line through $(-3, -8)$ and $(3, k)$ since all three points lie on the same line.

slope through $(15, 1)$ and $(-3, -8)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-8 - 1}{-3 - 15} = \frac{-9}{-18} \text{ or } \frac{1}{2}$$

slope through $(-3, -8)$ and $(3, k)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{k - (-8)}{3 - (-3)} = \frac{k + 8}{6}$$

$$\frac{k + 8}{6} = \frac{1}{2} \Rightarrow k = -5$$

26. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$AB = \sqrt{[-1 - (-3)]^2 + (2\sqrt{3} - 0)^2} = \sqrt{16} \text{ or } 4$$

$$BC = \sqrt{[1 - (-1)]^2 + (0 - 2\sqrt{3})^2} = \sqrt{16} \text{ or } 4$$

$$CA = \sqrt{(-3 - 1)^2 + (0 - 0)^2} = \sqrt{16} \text{ or } 4$$

Yes, $AB = 4$, $BC = 4$, and $CA = 4$. Thus, $\overline{AB} \cong \overline{BC} \cong CA$. Therefore the points A, B, and C form an equilateral triangle.

27. $EF = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(4 - 2)^2 + (4 - 5)^2} = \sqrt{5}$$

$$HG = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2 - 0)^2 + (0 - 1)^2} = \sqrt{5}$$

slope of $\overline{EF}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 5}{4 - 2} \text{ or } -\frac{1}{2}$$

slope of $\overline{FG}$

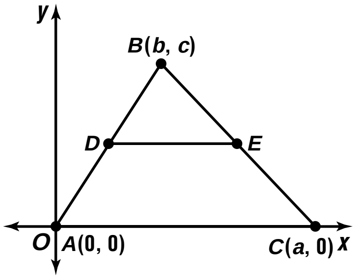
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 4}{2 - 4} \text{ or } \frac{2}{1}$$

slope of $\overline{HG}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 1}{2 - 0} \text{ or } -\frac{1}{2}$$

$\overline{EF} \cong \overline{HG}$ since $EF = \sqrt{5}$ and $HG = \sqrt{5}$.
$\overline{EF} \parallel \overline{HG}$ since the slope of $\overline{EF}$ is $-\frac{1}{2}$ and the slope of $\overline{HG}$ is $-\frac{1}{2}$. Thus the points form a parallelogram. $\overline{EF} \perp \overline{FG}$ since the product of the slopes of $\overline{EF}$ and $\overline{FG}$, $-\frac{1}{2} \cdot \frac{2}{1}$, is -1. Therefore, the points form a rectangle.

28. Let $A(0, 0)$, $B(b, c)$, and $C(a, 0)$ be the vertices of a triangle. Let D be the midpoint of $\overline{AB}$ and E be the midpoint of $\overline{BC}$.



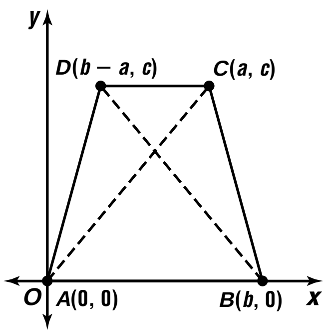
The coordinates of D are $\left(\frac{0 + b}{2}, \frac{0 + c}{2}\right)$ or $\left(\frac{b}{2}, \frac{c}{2}\right)$.
The coordinates of E are $\left(\frac{b + a}{2}, \frac{c + 0}{2}\right)$ or $\left(\frac{b + a}{2}, \frac{c}{2}\right)$.

$$AC = \sqrt{(a - 0)^2 + (0 - 0)^2} = \sqrt{a^2} \text{ or } a$$

$$DE = \sqrt{\left(\frac{b + a}{2} - \frac{b}{2}\right)^2 + \left(\frac{c}{2} - \frac{c}{2}\right)^2} = \sqrt{\frac{a^2}{4}} \text{ or } \frac{a}{2}$$

Since $DE = \frac{1}{2}AC$, the line segment joining the midpoints of two sides of a triangle is equal in length to one-half the third side.

29. In trapezoid $ABCD$, let A and B have coordinates $(0, 0)$ and $(b, 0)$, respectively. To make the trapezoid isosceles, let C have coordinates $(b - a, c)$ and let D have coordinates (a, c).

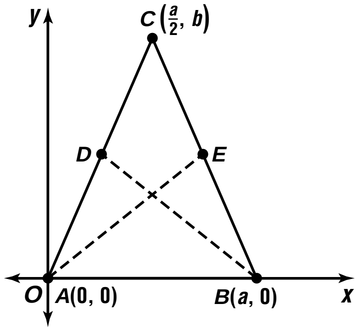


$$AC = \sqrt{(a - 0)^2 + (c - 0)^2} = \sqrt{a^2 + c^2}$$

$$BD = \sqrt{(b - a - b)^2 + (c - 0)^2} = \sqrt{a^2 + c^2}$$

$AC = \sqrt{a^2 + c^2} = \sqrt{a^2 + c^2} = BD$, so the diagonals of an isosceles trapezoid are congruent.

30. In $\triangle ABC$, let the vertices be $A(0, 0)$ and $B(a, 0)$. Since $\overline{AC}$ and $\overline{BC}$ are congruent sides, let the third vertex be $C\left(\frac{a}{2}, b\right)$. Let D be the midpoint of $\overline{AC}$ and let E be the midpoint of $\overline{BC}$.



The coordinates of D are: $\left(\frac{\frac{a}{2}+0}{2}, \frac{b+0}{2}\right)$ or $\left(\frac{a}{4}, \frac{b}{2}\right)$.

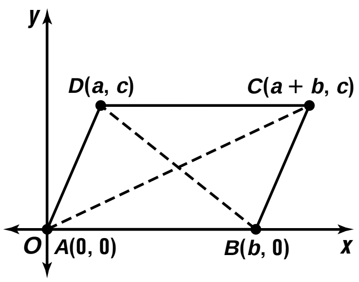
The coordinates of E are: $\left(\frac{\frac{a}{2}+a}{2}, \frac{b+0}{2}\right)$ or $\left(\frac{3a}{4}, \frac{b}{2}\right)$.

$$AE = \sqrt{\left(\frac{3a}{4} - 0\right)^2 + \left(\frac{b}{2} - 0\right)^2} = \frac{1}{2}\sqrt{\frac{9a^2}{4} + b^2}$$

$$BD = \sqrt{\left(a - \frac{a}{4}\right)^2 + \left(0 - \frac{b}{2}\right)^2} = \frac{1}{2}\sqrt{\frac{9a^2}{4} + b^2}$$

Since $AE = BD$, the medians to the congruent sides of an isosceles triangle are congruent.

31. Let A and B have coordinates $(0, 0)$ and $(b, 0)$ respectively. To make a parallelogram, let C have coordinates $(b + a, c)$ and let D have coordinates (a, c).

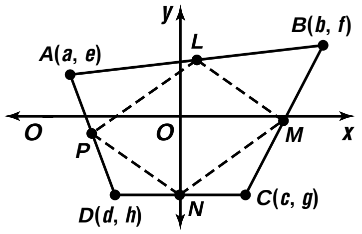


The midpoint of $\overline{BD}$ is $\left(\frac{a+b}{2}, \frac{0+c}{2}\right)$ or $\left(\frac{a+b}{2}, \frac{c}{2}\right)$.

The midpoint of $\overline{AC}$ is $\left(\frac{a+b+0}{2}, \frac{c+0}{2}\right)$ or $\left(\frac{a+b}{2}, \frac{c}{2}\right)$.

Since the diagonals have the same midpoint, the diagonals bisect each other.

32. Let the vertices of quadrilateral $ABCD$ be $A(a, e)$, $B(b, f)$, $C(c, g)$, and $D(d, h)$. The midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, respectively, are $L\left(\frac{a+b}{2}, \frac{e+f}{2}\right)$, $M\left(\frac{b+c}{2}, \frac{f+g}{2}\right)$, $N\left(\frac{c+d}{2}, \frac{g+h}{2}\right)$, and $P\left(\frac{a+d}{2}, \frac{e+h}{2}\right)$.



The slope of $\overline{LM}$ is $\dfrac{\frac{f+g}{2} - \frac{e+f}{2}}{\frac{b+c}{2} - \frac{a+b}{2}}$ or $\frac{g-e}{c-a}$.

The slope of $\overline{NP}$ is $\dfrac{\frac{e+h}{2} - \frac{g+h}{2}}{\frac{a+d}{2} - \frac{c+d}{2}}$ or $\frac{e-g}{a-c}$.

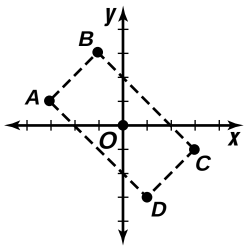
These slopes are equal, so $\overline{LM} \parallel \overline{NP}$.

The slope of $\overline{MN}$ is $\dfrac{\frac{f+g}{2} - \frac{g+h}{2}}{\frac{b+c}{2} - \frac{c+d}{2}}$ or $\frac{f-h}{b-d}$.

The slope of $\overline{PL}$ is $\dfrac{\frac{e+h}{2} - \frac{e+f}{2}}{\frac{a+d}{2} - \frac{a+b}{2}}$ or $\frac{h-f}{d-b}$.

These slopes are equal, so $\overline{MN} \parallel \overline{PL}$. Since $\overline{LM} \parallel \overline{NP}$, and $\overline{MN} \parallel \overline{PL}$, $PLMN$ is a parallelogram.

33. Let the vertices of the rectangle be $A(-3, 1)$, $B(1, 3)$, $C(3, -1)$, and $D(1, -3)$. Since the area of a rectangle is the length times the width, find the measure of two consecutive sides, $\overline{AD}$ and $\overline{DC}$.



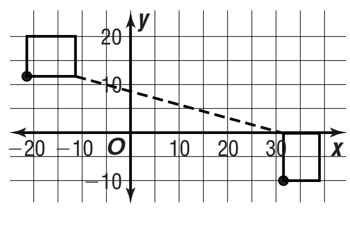
$$\begin{aligned}
AD &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{[1 - (-3)]^2 + (-3 - 1)^2} \\
&= \sqrt{4^2 + (-4)^2} \\
&= \sqrt{32} \text{ or } 4\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
DC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{(3 - 1)^2 + [-1 - (-3)]^2} \\
&= \sqrt{2^2 + 2^2} \\
&= \sqrt{8} \text{ or } 2\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
\text{Area} &= \ell w \\
&= (4\sqrt{2})(2\sqrt{2}) \\
&= 16
\end{aligned}$$

The area of the rectangle is 16 units^2.

34a.



34b. The two regions are closest between $(-12, 12)$ and $(31,0)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

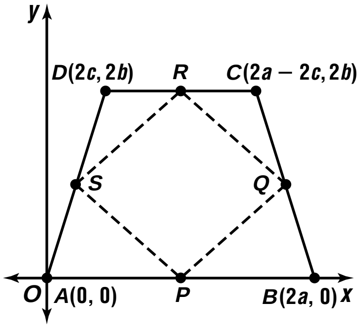
$= \sqrt{[31 - (-12)]^2 + (0 - 12)^2}$

$= \sqrt{43^2 + (-12)^2}$

$= \sqrt{1993}$ or about 44,64

The distance between these two points is about 44.64 pixels, which is greater than 40 pixels. therefore, the regions meet the criteria.

35. Let the vertices of the isosceles trapezoid have the coordinates $A(0, 0)$, $B(2a, 0)$, $C(2a - 2c, 2b)$, $D(2c, 2b)$. The coordinates of the midpoints are: $P(a, 0)$, $Q(2a - c, b)$, $R(a, 2b)$, $S(c, b)$.



$PQ = \sqrt{(2a - c - a)^2 + (b - 0)^2}$

$= \sqrt{(a - c)^2 + b^2}$

$QR = \sqrt{(2a - c - a)^2 + (b - 2b)^2}$

$= \sqrt{(a - c)^3 + b^2}$

$RS = \sqrt{(a - c)^2 + (2b - b)^2}$

$= \sqrt{(a - c)^2 + b^2}$

$PS = \sqrt{(a - c)^2 + (0 - b)^2}$

$= \sqrt{(a - c)^2 + b^2}$

So, all of the sides are congruent and quadrilateral $PQRS$ is a rhombus.

36a. distance from fountain to rosebushes:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{[1 - (-3)]^2 + (-3 - 2)^2}$

$d = \sqrt{41}$ or $2\sqrt{41}$ meters

distance from rosebushes to bench:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(3 - 1)^2 + [3 - (-3)]^2}$

$d = 2\sqrt{10}$ or $4\sqrt{10}$ meters

distance from bench to fountain:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(-3 - 3)^2 + (2 - 3)^2}$

$d = \sqrt{37}$ or $2\sqrt{37}$ meters

Yes; the distance from the fountain to the rosebushes is $2\sqrt{41}$ or about 12.81 meters. The distance from the rosebushes to the bench is $4\sqrt{10}$ or about 12.65 meters. The distance from the bench to the fountain is $2\sqrt{37}$ or about 12.17 meters.

36b. The fountain is located at $(-3, 2)$ and the rosebushes are located at $(1, -3)$.

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-3 + 1}{2}, \frac{2 + (-3)}{2}\right)$

$= \left(-1, -\frac{1}{2}\right)$

37a. Find a representation for MA and for MB.

$MA = \sqrt{t^2 + (3t - 15)^2}$

$= \sqrt{t^2 + 9t^2 - 90t + 225}$

$= \sqrt{10t^2 - 90t + 225}$

$MB = \sqrt{(t - 9)^2 + (3t - 12)^2}$

$= \sqrt{t^2 - 18t + 81 + 9t^2 - 72t + 144}$

$= \sqrt{10t^2 - 90t + 225}$

By setting these representations equal to each other, you find a value for t that would make the two distances equal.

$MA = MB$

$\sqrt{10t^2 - 90t + 225} = \sqrt{10t^2 - 90t + 225}$

Since the above equation is a true statement, t can take on any real values.

37b. A line; this line is the perpendicular bisector of $\overline{AB}$.

38. $r = \sqrt{a^2 + b^2}$ $\qquad \theta = \text{Arctan } \frac{b}{a}$

$= \sqrt{(-5)^2 + 12^2}$ $\qquad = \text{Arctan } \frac{12}{-5}$

$= \sqrt{169}$ or 13 $\qquad \approx -1.176005207$

$(-5 + 12\boldsymbol{i})^2 = 13^2 [\cos 2\theta + \boldsymbol{i} \sin 2\theta]$

$= -119 - 120\boldsymbol{i}$

39. If $\vec{\mathbf{v}} = (115, 2018, 0)$, then

$|\vec{\mathbf{v}}| = \sqrt{115^2 + 2018^2 + 0^2}$

$= \sqrt{4085549}$ or about 2021

The magnitude of the force is about 2021 N.

40. $2 \sec^2 x \stackrel{?}{=} \frac{1}{1 + \sin x} + \frac{1}{1 - \sin x}$

$2 \sec^2 x \stackrel{?}{=} \frac{(1 - \sin x) + (1 + \sin x)}{(1 + \sin x)(1 - \sin x)}$

$2 \sec^2 x \stackrel{?}{=} \frac{2}{1 - \sin^2 x}$

$2 \sec^2 x \stackrel{?}{=} = \frac{2}{\cos^2 x}$

$2 \sec^2 x = 2 \sec^2 x$

41. $s = r\theta$

$11.5 = 12\theta$

$\theta = \frac{11.5}{12}$ radians

$\frac{11.5}{12} \cdot \frac{180°}{\pi} \approx 54.9°$

42. $\sin 390° = \sin (390° - 360°)$

$= \sin 30°$ or $\frac{1}{2}$

43. $z^2 - 8z = -14$

$z^2 - 8z + 16 = -14 + 16$

$(z - 4)^2 = 2$

$z - 4 = \pm\sqrt{2}$

$z = 4 \pm \sqrt{2}$

44. $x^2 = 16$

$x = \pm\sqrt{16}$ or ± 4

$y^2 = 4$

$y = \pm\sqrt{4}$ or ± 2

Evaluating $(x - y)^2$ when $x = 4$ and $y = -2$ results in the greatest possible value, $[4 - (-2)]^2$ or 36.

10-2 Circles

Page 627 Check for Understanding

1. complete the square on each variable

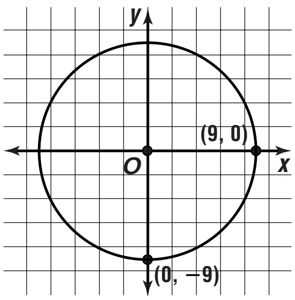
2. Sample answer: $(x + 4)^2 + (y - 9)^2 = 1$, $(x + 4)^2 + (y - 9)^2 = 2$, $(x + 4)^2 + (y - 9)^2 = 3$, $(x + 4)^2 + (y - 9)^2 = 4$, $(x + 4)^2 + (y - 9)^2 = 5$

3. Find the center of the circle, (h, k), by finding the midpoint of the diameter. Next find the radius of the circle, r, by finding the distance from the center to one endpoint. Then write the equation of the circle in standard form as $(x - h)^2 + (y - k)^2 = r^2$.

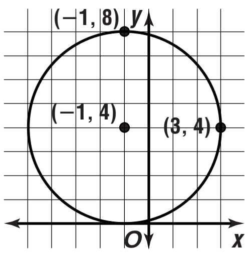
4. The equation $x^2 + y^2 + 8x + 8y + 36 = 0$ written in standard form is $(x + 4)^2 + (y + 4)^2 = -4$. Since a circle cannot have a negative radius, the graph of the equation is the empty set.

5. Ramon; the square root of a sum does not equal the sum of the square roots.

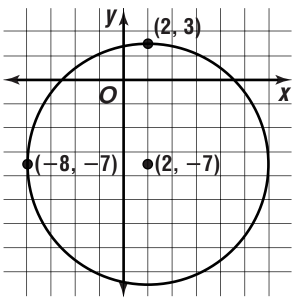
6.
$$\begin{aligned}(x - h)^2 + (y - k)^2 &= r^2\\ (x - 0)^2 + (y - 0)^2 &= 9^2\\ x^2 + y^2 &= 81\end{aligned}$$



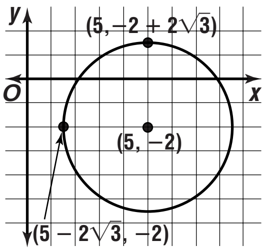
7.
$$\begin{aligned}(x - h)^2 + (y - k)^2 &= r^2\\ [x - (-1)]^2 + (y - 4)^2 &= [3 - (-1)]^2\\ (x + 1)^2 + (y - 4)^2 &= 16\end{aligned}$$



8.
$$\begin{aligned}x^2 + y^2 - 4x + 14y - 47 &= 0\\ x^2 - 4x + 4 + y^2 + 14y + 49 &= 47 + 4 + 49\\ (x - 2)^2 + (y + 7)^2 &= 100\end{aligned}$$



9.
$$\begin{aligned}2x^2 + 2y^2 - 20x + 8y + 34 &= 0\\ 2x^2 - 20x + 2y^2 + 8y &= -34\\ 2(x^2 - 10x + 25) + 2(y^3 + 4y + 4) &= -34 + 2(25)\\ &\quad + 2(4)\\ 2(x - 5)^2 + 2(y + 2)^2 &= 24\\ (x - 5)^2 + (y + 2)^2 &= 12\end{aligned}$$



10.
$$\begin{aligned}x^2 + y^2 + Dx + Ey + F &= 0\\ 0^2 + 0^2 + D(0) + E(0) + F &= 0\\ 4^2 + 0^2 + D(4) + E(0) + F &= 0\\ 0^2 + 4^2 + D(0) + E(4) + F &= 0\end{aligned}$$

$F = 0 \qquad\qquad F = 0$
$4D + F = -16 \Rightarrow D = -4$
$4E + F = -16 \qquad E = -4$

$$\begin{aligned}x^2 + y^2 - 4x - 4y &= 0\\ x^2 - 4x + 4 + y^2 - 4y + 4 &= 0 + 4 + 4\\ (x - 2)^2 + (y - 2)^2 &= 8\end{aligned}$$

center: $(h, k) = (2, 2)$
radius: $r^2 = 8$
$r = \sqrt{8}$ or $2\sqrt{2}$

11.
$$\begin{aligned}x^2 + y^2 + Dx + Ey + F &= 0\\ 1^2 + 3^2 + D(1) + E(3) + F = 0 &\Rightarrow D + 3E + F = -10\\ 5^2 + 5^2 + D(5) + E(5) + F = 0 &\Rightarrow 5D + 5E + F = -50\\ 5^2 + 3^2 + D(5) + E(3) + F = 0 &\Rightarrow 5D + 3E + F = -34\end{aligned}$$

$$\begin{aligned}D + 3E + F &= -10\\ \underline{(-1)(5D + 5E + F)} &\underline{= (-1)(-50)}\\ -4D - 2E &= 40\end{aligned}$$

$$\begin{aligned}5D + 5E + F &= -50\\ \underline{(-1)(5D + 3E + F)} &\underline{= (-1)(-34)}\\ 2E &= -16\\ E &= -8\end{aligned}$$

$$\begin{aligned}-4D - 2(-8) &= 40\\ -4D &= 24 \qquad (-6) + 3(-8) + F = -10\\ D &= -6 \qquad\qquad\qquad\qquad F = 20\end{aligned}$$

$$\begin{aligned}x^2 + y^2 - 6x - 8y + 20 &= 0\\ x^2 - 6x + 9 + y^2 - 8y + 16 &= -20 + 9 + 16\\ (x - 3)^2 + (y - 4)^2 &= 5\end{aligned}$$

center: $(h, k) = (3, 4)$
radius: $r^2 = 5$
$r = \sqrt{5}$

12.
$$\begin{aligned}(x - h)^2 + (y - k)^2 &= r^2\\ [x - (-2)]^2 + (y - 1)^2 &= r^2\\ (x + 2)^2 + (y - 1)^2 &= r^2\\ (1 + 2)^2 + (5 - 1)^2 &= r^2\\ 25 &= r^2\\ (x + 2)^2 + (y - 1)^2 &= 25\end{aligned}$$

13. midpoint of diameter:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 10}{2}, \frac{6 + (-10)}{2}\right)$$
$$= (4, -2)$$

$$\text{radius: } r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{[4 - (-2)]^2 + [6 - (-2)]^2}$$
$$= \sqrt{6^2 + 8^2}$$
$$= \sqrt{100} \text{ or } 10$$

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 4)^2 + [y - (-2)]^2 = 10^2$$
$$(x - 4)^2 + (y + 2)^2 = 100$$

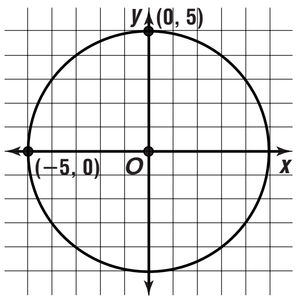
14. $(x - h)^2 + (y - k)^2 = r^2$

$$(x - 0)^2 + (y - 0)^2 = (1740 + 185)^2$$
$$x^2 + y^2 = 1925^2$$

Pages 627–630 Exercises

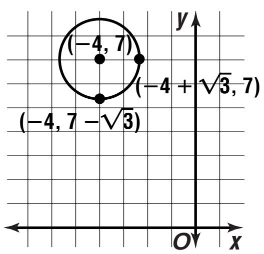
15. $(x - h)^2 + (y - k)^2 = r^2$

$$(x - 0)^2 + (y - 0)^2 = 5^2$$
$$x^2 + y^2 = 25$$



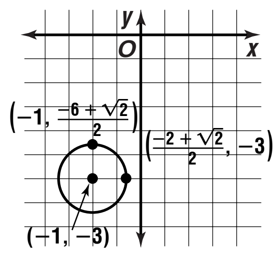
16. $(x - h)^2 + (y - k)^2 = r^2$

$$[x - (-4)]^2 + (y - 7)^2 = (\sqrt{3})^2$$
$$(x + 4)^2 + (y - 7)^2 = 3$$



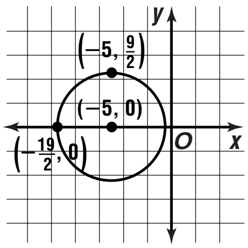
17. $(x - h)^2 + (y - k)^2 = r^2$

$$[x - (-1)]^2 + [y - (-3)]^2 = \left(\frac{\sqrt{2}}{2}\right)^2$$
$$(x + 1)^2 + (y + 3)^2 = \frac{1}{2}$$



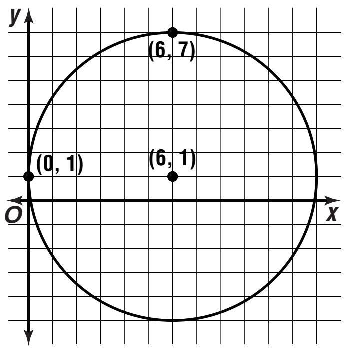
18. $(x - h)^2 + (y - k)^2 = r^2$

$$[x - (-5)]^2 + (y - 0)^2 = \left(\frac{9}{2}\right)^2$$
$$(x + 5)^2 + y^2 = \frac{81}{4}$$



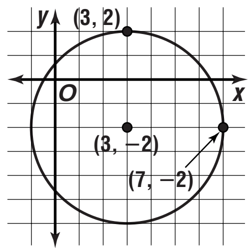
19. $(x - h)^2 + (y - k)^2 = r^2$

$$(x - 6)^2 + (y - 1)^2 = 6^2$$
$$(x - 6)^2 + (y - 1)^2 = 36$$



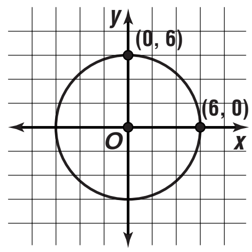
20. $(x - h)^2 + (y - k)^2 = r^2$

$$(x - 3)^2 + [y - (-2)]^2 = [2 - (-2)]^2$$
$$(x - 3)^2 + (y + 2)^2 = 16$$



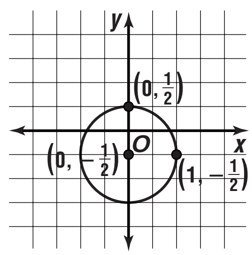
21. $36 - x^2 = y^2$

$$x^2 + y^2 = 36$$



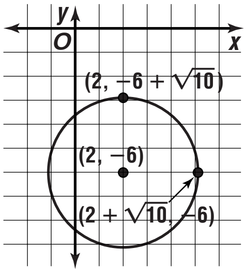
22. $x^2 + y^2 + y = \frac{3}{4}$

$$x^2 + y^2 + y + \frac{1}{4} = \frac{3}{4} + \frac{1}{4}$$
$$x^2 + \left(y + \frac{1}{2}\right)^2 = 1$$



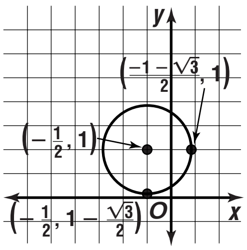
23.

$$\begin{aligned} x^2 + y^2 - 4x + 12y + 30 &= 0 \\ x^2 - 4x + 4 + y^2 + 12y + 36 &= -30 + 4 + 36 \\ (x - 2)^2 + (y + 6)^2 &= 10 \end{aligned}$$



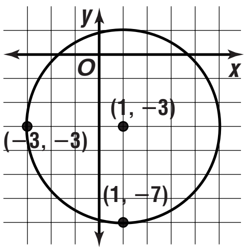
24.

$$\begin{aligned} 2x^2 + 2y^2 + 2x - 4y &= -1 \\ 2x^2 + 2x + 2y^2 - 4y &= -1 \\ 2\left(x^2 + 1x + \tfrac{1}{4}\right) + 2(y^2 - 2y + 1) &= -1 + 2\left(\tfrac{1}{4}\right) + 2(1) \\ 2\left(x + \tfrac{1}{2}\right)^2 + 2(y - 1)^2 &= \tfrac{3}{2} \\ \left(x + \tfrac{1}{2}\right)^2 + (y - 1)^2 &= \tfrac{3}{4} \end{aligned}$$



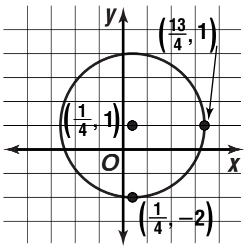
25.

$$\begin{aligned} 6x^2 - 12x + 6y^2 + 36y &= 36 \\ 6(x^2 - 2x + 1) + 6(y^2 + 6y + 9) &= 36 + 6(1) + 6(9) \\ 6(x - 1)^2 + 6(y + 3)^2 &= 96 \\ (x - 1)^2 + (y + 3)^2 &= 16 \end{aligned}$$



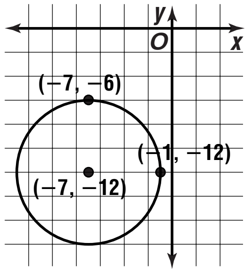
26.

$$\begin{aligned} 16x^2 + 16y^2 - 8x - 32y &= 127 \\ 16x^2 - 8x + 16y^2 - 32y &= 127 \\ 16\left(x^2 - \tfrac{1}{2}x + \tfrac{1}{16}\right) + 16(y^2 - 2y + 1)^2 &= 127 + 16\left(\tfrac{1}{16}\right) \\ &\quad + 16(1) \\ 16\left(x - \tfrac{1}{4}\right)^2 + 16(y - 1)^2 &= 144 \\ \left(x - \tfrac{1}{4}\right)^2 + (y - 1)^2 &= 9 \end{aligned}$$



27.

$$\begin{aligned} x^2 + y^2 + 14x + 24y + 157 &= 0 \\ x^2 + 14x + 49 + y^2 + 24y + 144 &= -157 + 49 + 144 \\ (x + 7)^2 + (y + 12)^2 &= 36 \end{aligned}$$



28.

$$\begin{aligned} x^2 + y^2 + Dx + Ey + F &= 0 \\ 0^2 + (-1)^2 + D(0) + E(-1) + F = 0 &\Rightarrow \\ -E + F &= -1 \\ (-3)^2 + (-2)^2 + D(-3) + E(-2) + F = 0 &\Rightarrow \\ -3D - 2E + F &= -13 \\ (-6)^2 + (-1)^2 + D(-6) + E(-1) + F = 0 &\Rightarrow \\ -6D - E + F &= -37 \end{aligned}$$

$$\begin{aligned} -E + F &= -1 \\ \underline{(-1)(-3D - 2E + F)} &= \underline{(-1)(-13)} \\ 3D + E &= 12 \end{aligned}$$

$$\begin{aligned} -3D - 2E + F &= -13 \\ \underline{(-1)(-6D - E + F)} &= \underline{(-1)(-37)} \\ 3D - E &= 24 \end{aligned}$$

$$\begin{aligned} 3D + E &= 12 \\ \underline{3D - E} &= \underline{24} \\ 6D &= 36 \\ D &= 6 \end{aligned} \qquad \begin{aligned} 3(6) + E &= 12 \\ E &= -6 \\ -(-6) + F &= -1 \\ F &= -7 \end{aligned}$$

$$\begin{aligned} x^2 + y^2 + 6x - 6y - 7 &= 0 \\ x^2 + 6x + 9 + y^2 - 6y + 9 &= 7 + 9 + 9 \\ (x + 3)^2 + (y - 3)^2 &= 25 \end{aligned}$$

center: $(h, k) = (-3, 3)$

radius: $r^2 = 25$

$r = \sqrt{25}$ or 5

29.

$$\begin{aligned} x^2 + y^2 + Dx + Ey + F &= 0 \\ 7^2 + (-1)^2 + D(7) + E(-1) + F = 0 &\Rightarrow \\ 7D - E + F &= -50 \\ 11^2 + (-5)^2 + D(11) + E(-5) + F = 0 &\Rightarrow \\ 11D - 5E + F &= -146 \\ 3^2 + (-5)^2 + D(3) + E(-5) + F = 0 &\Rightarrow \\ 3D - 5E + F &= -34 \end{aligned}$$

$$\begin{aligned} 7D - E + F &= -50 \\ \underline{(-1)(11D - 5E + F)} &= \underline{-1(-146)} \\ -4D + 4E &= 96 \end{aligned}$$

$$\begin{aligned} 11D - 5E + F &= -146 \\ \underline{(-1)(3D - 5E + F)} &= \underline{-1(-34)} \\ 8D &= -12 \\ D &= -14 \end{aligned}$$

$$\begin{aligned} -4(-14) + 4E &= 96 \\ 4E &= 40 \\ E &= 10 \end{aligned} \qquad \begin{aligned} 7(-14) - (10) + F &= -50 \\ F &= 58 \end{aligned}$$

$$\begin{aligned} x^2 + y^2 - 14x + 10y + 58 &= 0 \\ x^2 - 14x + 49 + y^2 + 10y + 25 &= -58 + 49 + 25 \\ (x - 7)^2 + (y + 5)^2 &= 16 \end{aligned}$$

center: $(h, k) = (7, -5)$

radius: $r^2 = 16$

$r = \sqrt{16}$ or 4

30.

$$\begin{aligned}
x^2 + y^2 + Dx + Ey + F &= 0\\
(-2)^2 + 7^2 + D(-2) + E(7) + F = 0 &\Rightarrow\\
-2D + 7E + F &= -53\\
(-9)^2 + 0^2 + D(-9) + E(0) + F = 0 &\Rightarrow\\
-9D + F &= -81\\
(-10)^2 + (-5)^2 + D(-10) + E(-5) + F = 10 &\Rightarrow\\
-10D - 5E + F &= -125
\end{aligned}$$

$$\begin{aligned}
-2D + 7E + F &= -53\\
(-1)(-9D \qquad + F) &= (-1)(-81)\\
\hline
7D + 7E \qquad &= 28\\
D + E &= 4
\end{aligned}$$

$$\begin{aligned}
-10D - 5E + F &= -125\\
(-1)(-9D \qquad + F) &= (-1)(-81)\\
\hline
-D - 5E \qquad &= -44
\end{aligned}$$

$$\begin{aligned}
-D - 5E &= -44\\
D + E &= 4\\
\hline
-4E &= -40\\
E &= 10
\end{aligned}$$

$$\begin{aligned}
D + (10) &= 4\\
D &= -6\\
-9(-6) + F &= -81\\
F &= -135
\end{aligned}$$

$$\begin{aligned}
x^2 + y^2 - 6x + 10y - 135 &= 10\\
x^2 - 6x + 9 + y^2 + 10y + 25 &= 135 + 9 + 25\\
(x - 3)^2 + (y + 5)^2 &= 169
\end{aligned}$$

center: $(h, k) = (3, -5)$

radius: $r^2 = 169$

$r = \sqrt{169}$ or 13

31.

$$\begin{aligned}
x^2 + y^2 + Dx + Ey + F &= 0\\
(-2)^2 + 3^2 + D(-2) + E(3) + F = 0 &\Rightarrow\\
-2D + 3E + F &= -13\\
6^2 + (-5)^2 + D(6) + E(-5) + F = 0 &\Rightarrow\\
6D - 5E + F &= -61\\
0^2 + 7^2 + D(0) + E(7) + F = 0 &\Rightarrow\\
7E + F &= -49
\end{aligned}$$

$$\begin{aligned}
-2D + 3E + F &= -13\\
(-1)(6D - 5E + F) &= (-1)(-61)\\
\hline
-8D + 8E \qquad &= 48\\
-D + E &= 6
\end{aligned}$$

$$\begin{aligned}
6D - 5E + F &= -61\\
(-1)(7E + F) &= (-1)(-49)\\
\hline
6D - 12E \qquad &= -12\\
D - 2E &= -2
\end{aligned}$$

$$\begin{aligned}
-D + E &= 6\\
D - 2E &= -2\\
\hline
-E &= 4\\
E &= -4
\end{aligned}$$

$$\begin{aligned}
D - 2(-4) &= -2\\
D &= -10\\
7(-4) + F &= -49\\
F &= -21
\end{aligned}$$

$$\begin{aligned}
x^2 + y^2 - 10x - 4y - 21 &= 0\\
x^2 - 10x + 25 + y^2 - 4y + 4 &= 21 + 25 + 4\\
(x - 5)^2 + (y - 2)^2 &= 50
\end{aligned}$$

center: $(h, k) = (5, 2)$

radius: $r^2 = 50$

$r = \sqrt{50}$ or $5\sqrt{2}$

32.

$$\begin{aligned}
x^2 + y^2 + Dx + Ey + F &= 0\\
4^2 + 5^2 + D(4) + E(5) + F = 0 &\Rightarrow\\
4D + 5E + F &= -41\\
(-2)^2 + 3^2 + D(-2) + E(3) + F = 0 &\Rightarrow\\
-2D + 3E + F &= -13\\
(-4)^2 + (-3)^2 + D(-4) + E(-3) + F = 0 &\Rightarrow\\
-4D - 3E + F &= -25
\end{aligned}$$

$$\begin{aligned}
4D + 5E + F &= -41\\
(-1)(-2D + 3E + F) &= (-1)(-13)\\
\hline
6D + 2E \qquad &= -28\\
3D + E &= -14
\end{aligned}$$

$$\begin{aligned}
4D + 5E + F &= -41\\
(-1)(-4D - 3E + F) &= (-1)(-25)\\
\hline
8D + 8E \qquad &= -16\\
D + E &= -2
\end{aligned}$$

$$\begin{aligned}
3D + E &= -14\\
(-1)(D + E) &= (-1)(-2)\\
\hline
2D \qquad &= -12\\
D &= -6
\end{aligned}$$

$$\begin{aligned}
-6 + E &= -2\\
E &= 4\\
-2(-6) + 3(4) + F &= -13\\
F &= -37
\end{aligned}$$

$$\begin{aligned}
x^2 + y^2 - 6x + 4y - 37 &= 0\\
x^2 + 6x + 9 + y^2 + 4y + 4 &= 37 + 9 + 4\\
(x - 3)^2 + (y + 2)^2 &= 50
\end{aligned}$$

center: $(h, k) = (3, -2)$

radius: $r^2 = 50$

$r = \sqrt{50}$ or $5\sqrt{2}$

33.

$$\begin{aligned}
x^2 + y^2 + Dx + Ey + F &= 0\\
1^2 + 4^2 + D(1) + E(4) + F = 0 &\Rightarrow\\
D + 4E + F &= -17\\
2^2 + (-1)^2 + D(2) + E(-1) + F = 0 &\Rightarrow\\
2D - E + F &= -5\\
(-3)^2 + 0^2 + D(-3) + E(0) + F = 0 &\Rightarrow\\
-3D + F &= -9
\end{aligned}$$

$$\begin{aligned}
D + 4E + F &= -17\\
(-1)(2D - E + F) &= (-1)(-5)\\
\hline
-D + 5E \qquad &= -12
\end{aligned}$$

$$\begin{aligned}
D + 4E + F &= -17\\
(-1)(-3D \qquad + F) &= (-1)(-9)\\
\hline
4D + 4E \qquad &= -8\\
D + E &= -2
\end{aligned}$$

$$\begin{aligned}
-D + 5E &= -12\\
D + E &= -2\\
\hline
6E &= -14\\
E &= -\tfrac{7}{3}
\end{aligned}$$

$$\begin{aligned}
D + \left(-\tfrac{7}{3}\right) &= -2\\
D &= \tfrac{1}{3}\\
-3\left(\tfrac{1}{3}\right) + F &= -9\\
F &= -8
\end{aligned}$$

$$\begin{aligned}
x^2 + y^2 + Dx + Ey + F &= 0\\
x^2 + y^2 + \tfrac{1}{3}x - \tfrac{7}{3}y - 8 &= 0\\
x^2 + \tfrac{1}{3}x + \tfrac{1}{36} + y^2 - \tfrac{7}{3}y + \tfrac{49}{36} &= 8 + \tfrac{1}{36} + \tfrac{49}{36}\\
\left(x + \tfrac{1}{6}\right)^2 + \left(y - \tfrac{7}{6}\right)^2 &= \tfrac{169}{18}
\end{aligned}$$

center: $(h, k) = \left(-\frac{1}{6}, \frac{7}{6}\right)$

radius: $r^2 = \frac{169}{18}$

$r = \sqrt{\frac{169}{18}}$

$= \frac{13}{3\sqrt{2}}$ or $\frac{13\sqrt{2}}{6}$

34. $x^2 + y^2 + Dx + Ey + F = 0$

$0^2 + 0^2 + D(0) + E(0) + F = 0 \Rightarrow$

$F = 0$

$(2.8)^2 + 0^2 + D(2.8) + E(0) + F = 0 \Rightarrow$

$2.8D + F = -7.84$

$(5)^2 + 2^2 + D(5) + E(2) + F = 0 \Rightarrow$

$5D + 2E + F = -29$

$2.8D + 0 = -7.84 \qquad 5(-2.8) + 2E + (0) = -29$

$2.8D = -7.84 \qquad 2E = -15$

$D = -2.8 \qquad E = -7.5$

$x^2 + y^2 - 2.8x - 7.5y + 0 = 0$

$x^2 - 2.8x + 1.96 + y^2 - 7.5y + 14.0625 = 1.96 + 14.0625$

$(x - 1.4)^2 + (y - 3.75)^2 = 16.0225$

or about $(x - 1.4)^2 + (y - 3.75)^2 = 16.02$

35. $(x - h)^2 + (y - k)^2 = r^2$

$[x - (-4)]^2 + (y - 3)^2 = r^2$

$(x + 4)^2 + (y - 3)^2 = r^2$

$(0 + 4)^2 + (0 - 3)^2 = r^2$

$25 = r^2$

$(x + 4)^2 + (y - 3)^2 = 25$

36. $(x - h)^2 + (y - k)^2 = r^2$

$(x - 2)^2 + (y - 3)^2 = r^2$

$(5 - 2)^2 + (6 - 3)^2 = r^2$

$18 = r^2$

$(x - 2)^2 + (y - 3)^2 = 18$

37. midpoint of diameter:

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2 + (-6)}{2}, \frac{3 + (-5)}{2}\right)$

$= (-2, -1)$

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-2 - 2)^2 + (-1 - 3)^2}$

$= \sqrt{(-4)^2 + (-4)^2}$

$= \sqrt{32}$

$(x - h)^2 + y - k)^2 = r^2$

$[x - (-2)]^2 + [y - (-1)]^2 = \left(\sqrt{32}\right)^2$

$(x + 2)^2 + (y + 1)^2 = 32$

38. midpoint of diameter:

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-3 + 2}{2}, \frac{4 + 1}{2}\right)$

$= \left(-\frac{1}{2}, \frac{5}{2}\right)$

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\left(-\frac{1}{2} - 2\right)^2 + \left(\frac{5}{2} - 1\right)^2}$

$= \sqrt{\left(-\frac{5}{2}\right)^2 + \left(\frac{3}{2}\right)^2}$

$= \sqrt{\frac{17}{2}}$

$(x - h)^2 + (y - k)^2 = r^2$

$\left[x - \left(-\frac{1}{2}\right)\right]^2 + \left(y - \frac{5}{2}\right)^2 = \left(\sqrt{\frac{17}{2}}\right)^2$

$\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{17}{2}$

39. $(x - h)^2 + (y - k)^2 = r^2$

$(x - 5)^2 + (y - 1)^2 = r^2$

$x + 3y = -2$

$x + 3y + 2 = 0 \Rightarrow A = 1, B = 3$, and $C = 2$

$r = \left|\frac{Ax_1 + By_1 + \mathrm{C}}{\pm\sqrt{A^2 + B^2}}\right|$

$= \left|\frac{(1)(5) + (3)(1) + 2}{-\sqrt{1^2 + 3^2}}\right|$

$= \left|\frac{10}{-\sqrt{10}}\right|$ or $\sqrt{10}$

$(x - 5)^2 + (y - 1)^2 = \left(\sqrt{10}\right)^2$

$(x - 5)^2 + (y - 1)^2 = 10$

40. center: $(h, 0)$, radius: $r = 1$

$(x - h)^2 + (y - k)^2 = r^2$

$\left(\frac{\sqrt{2}}{2} - h\right)^2 + \left(\frac{\sqrt{2}}{2} - 0\right)^2 = 1^2$

$-\frac{1}{2} - \sqrt{2}h + h^2 + \frac{1}{2} = 1$

$h^2 - \sqrt{2}h = 1 - 1$

$h\left(h - \sqrt{2}\right) = 0$

$h = 0$ or $h = \sqrt{2}$

$(x - 0)^2 + (y - 0)^2 = 1 \qquad \left(x - \sqrt{2}\right)^2 + (y - 0)^2 = 1$

$x^2 + y^2 = 1$ or $\left(x - \sqrt{2}\right)^2 + y^2 = 1$

41a. $(x - h)^2 + (y - k)^2 = r^2$

$(x - 0)^2 + (y - 0)^2 = \left(\frac{12}{2}\right)^2$

$x^2 + y^2 = 36$

41b. $x^2 + y^2 = 36$

$y^2 = 36 - x^2$

$y = \pm\sqrt{36 - x^2}$

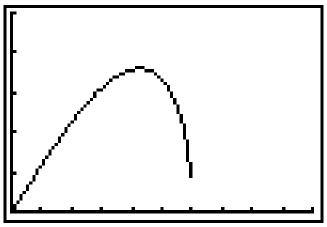
dimensions of rectangle:

$2x$ by $2y \Rightarrow 2x$ by $2\sqrt{36 - x^2}$

41c. $A(x) = 2x\left(2\sqrt{36 - x^2}\right)$

$= 4x\sqrt{36 - x^2}$

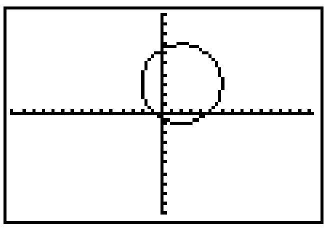
41d.



[0, 10] scl:1 by [0, 100] scl:20

41e. Use 4: maximum on the CALC menu of the calculator. The x-coordinate of this point is about 4.2. The maximum area of the rectangle is the corresponding y-value of 72, for an area of 72 units^2.

42a.

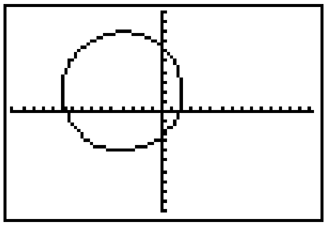


[−15.16, 15.16] scl:1 by [−5, 5] scl:1

42b. a circle centered at (2, 3) with radius 4

42c. $(x - 2)^2 + (y - 3)^2 = 16$

42d. center: $(h, k) = (-4, 2)$
radius: $r^2 = 36$
$r = \sqrt{36}$ or 6
$\boxed{\text{2nd}}$ [DRAW] 9:Circle($\boxed{-}$ 4 $\boxed{,}$ 2 $\boxed{,}$ 6 $\boxed{)}$



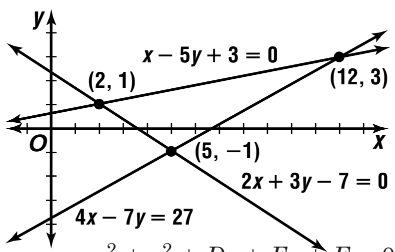
[−15.16, 15.16] scl:1 by [5, 5] scl:1

43a. $(x - h)^2 + (y - k)^2 = r^2$
$(x - 0)^2 + (y - 0)^2 = \left(\frac{24}{2}\right)^2$
$x^2 + y^2 = 144$

43b. $x^2 + y^2 = 6.25 \Rightarrow r_1{}^2 = 6.25$
$r_1 = \sqrt{6.25}$ or 2.5
If the circles are equally spaced apart then radius r_2 of the middle circle is found by adding the radius of the smallest circle to the radius of the largest circle and dividing by two.
$r_2 = \frac{12 + 2.5}{2}$ or 7.25

$$\begin{array}{c} \text{area of} \\ \text{region } B \end{array} = \begin{array}{c} \text{area of} \\ \text{middle circle} \end{array} - \begin{array}{c} \text{area of} \\ \text{smallest circle} \end{array}$$
$$\begin{aligned} &= \pi r_2{}^2 - \pi r_1{}^2 \\ &= \pi\,(r_2{}^2 - r_1{}^2) \\ &= \pi\,(7.25^2 - 2.5^2) \\ &= \pi\,(46.3125) \text{ or about } 145.50 \end{aligned}$$
The area of region B is about 145.50 in^2.

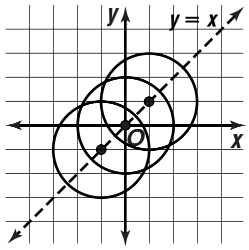
44.



$$x^2 + y^2 + Dx + Ey + F = 0$$
$$2^2 + 1^2 + D(2) + E(1) + F = 0 \Rightarrow$$
$$2D + E + F = -5$$
$$5^2 + (-1)^2 + D(5) + E(-1) + F = 0 \Rightarrow$$
$$5D - E + F = -26$$
$$12^2 + 3^2 + D(12) + E(3) + F = 0 \Rightarrow$$
$$12D + 3E + F = -153$$
$$\begin{array}{r} 2D + E + F = -5 \\ \underline{(-1)(5D - E + F) = (-1)(-26)} \\ -3D + 2E \quad = 21 \end{array}$$
$$\begin{array}{r} 2D + E + F = -5 \\ \underline{(-1)(12D + 3E + F) = (-1)(-153)} \\ -10D - 2E \quad = 148 \end{array}$$
$$\begin{array}{r} -3D + 2E = 21 \\ \underline{-10D - 2E = 148} \\ -13D \quad = 169 \\ D = -13 \end{array}$$
$$\begin{aligned} -3(-13) + 2E &= 21 \\ 2E &= -18 \\ E &= -9 \end{aligned}$$
$$\begin{aligned} 2(-13) + (-9) + F &= -5 \\ F &= 30 \end{aligned}$$
$$x^2 + y^2 - 13x - 9y + 30 = 0$$
$$x^2 - 13x + 42.25 + y^2 - 9y + 20.25 = -30 + 42.25 + 20.25$$
$$(x - 6.5)^2 + (y - 4.5)^2 = 32.5$$

45a. $(x - h)^2 + (y - k)^2 = r^2$
$(x - k)^2 + (y - k)^2 = 2^2$
$(x - k)^2 + (y - k)^2 = 4$

45b. $(x - 1)^2 + (y - 1)^2 = 4$
$(x - 0)^2 + (y - 0)^2 = 4$
$(x + 1)^2 + (y + 1)^2 = 4$



45c. All of the circles in this family have a radius of 2 and centers located on the line $y = x$.

46a. $(x - h)^2 + (y - k)^2 = r^2$
$(x - 0)^2 + (y - 0)^2 = 14^2$
$x^2 + y^2 = 196$
$y^2 = 196 - x^2$
$y = \pm\sqrt{196 - x^2}$
$y = \sqrt{196 - x^2}$

46b. No, if $x = 7$, then $y = \sqrt{147} \approx 12.1$ ft, so the truck cannot pass.

47. $x^2 + y^2 - 8x + 6y + 25 = 0$
$x^2 - 8x + 16 + y^2 + 6y + 9 = -25 + 16 + 9$
$(x - 4)^2 + (y + 3)^2 = 0$
radius: $r^2 = 0$
$r = \sqrt{0}$ or 0
center: $(h, k) = (4, -3)$
Graph is a point located at $(4, -3)$.

48a. $(x - h)^2 + (y - k)^2 = r^2$
$(x - 0)^2 + (y - 0)^2 = r^2$
$x^2 + y^2 = r^2$
$(475)^2 + (1140)^2 = r^2$
$1{,}525{,}225 = r^2$
$x^2 + y^2 = 1{,}525{,}225$

48b. $r^2 = 1{,}525{,}225$
$r = \sqrt{1{,}525{,}225}$ or 1235
$A = \pi r^2$
$= \pi\,(1235)^2$ or approximately 4,792,000 ft^2

48c. $\frac{2500^2 - 4{,}792{,}000}{2500^2} = 0.23328$
about 23%

49a. $\overline{PA}$ has a slope of $\frac{y - 4}{x - 3}$ and $\overline{PB}$ has slope of $\frac{y + 4}{x + 3}$. If $\overline{PA} \perp \overline{PB}$ then $\frac{y - 4}{x - 3} \cdot \frac{y + 4}{x + 3} = -1$.
$\frac{y - 4}{x - 3} \cdot \frac{y + 4}{x + 3} = -1$
$\frac{y^2 - 16}{x^2 - 9} = -1$
$y^2 - 16 = -x^2 + 9$
$x^2 + y^2 = 25$

49b. If $\overline{PA} \perp \overline{PB}$, then A, P, and B are on the circle $x^2 + y^2 = 25$.

50. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(-2 - 4)^2 + [6 - (-3)]^2}$
$d = \sqrt{(-6)^2 + 9^2}$
$d = \sqrt{117}$

51. $(2 + \boldsymbol{i})(3 - 4\boldsymbol{i})(1 + 2\boldsymbol{i}) = (6 - 8\boldsymbol{i} + 3\boldsymbol{i} - 4\boldsymbol{i}^2)(1 + 2\boldsymbol{i})$
$= [6 - 8\boldsymbol{i} + 3\boldsymbol{i} - 4(-1)](1 + 2\boldsymbol{i})$
$= (10 - 5\boldsymbol{i})(1 + 2\boldsymbol{i})$
$= 10 + 20\boldsymbol{i} - 5\boldsymbol{i} - 10\boldsymbol{i}^2$
$= 10 + 20\boldsymbol{i} - 5\boldsymbol{i} - 10(-1)$
$= 20 + 15\boldsymbol{i}$

52. $x = t|\vec{\mathbf{v}}| \cos\theta \qquad y = t|\vec{\mathbf{v}}| \sin\theta - \frac{1}{2}gt^2$
$x = t(60)\cos 60° \qquad y = t(60)\sin 60° - \frac{1}{2}(32)t^2$
$x = 60t\cos 60° \qquad y = 60t\sin 60° - 16t^2$
$x = 60(0.5)\cos 60° \qquad y = 60(0.5)\sin 60° - 16(0.5)^2$
$x = 15 \qquad y \approx 21.98076211$
15 ft horizontally, about 22 ft vertically

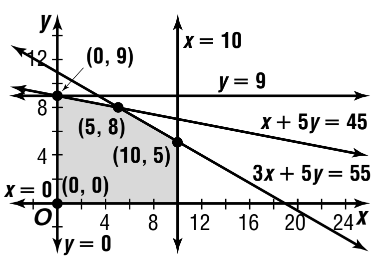
53. $A = \frac{5}{2}$ or 2.5
$20 = \frac{2\pi}{k}$ and $k = \frac{\pi}{10}$
$y = A\cos(kt)$
$y = 2.5\cos\left(\frac{\pi}{10}t\right)$

54. $s = \frac{1}{2}(a + b + c)$
$= \frac{1}{2}(15 + 25 + 35)$
$= 37.5$
$k = \sqrt{s(s - a)(s - b)(s - c)}$
$= \sqrt{37.5(37.5 - 15)(37.5 - 25)(37.5 - 3.5)}$
$= \sqrt{26{,}367.1875}$
≈ 162 units^2

55. $v = \sqrt{v_0^2 + 64h}$
$95 = \sqrt{15^2 + 64h}$
$95^2 = 15^2 + 64h$
$h = \frac{95^2 - 15^2}{64}$
$h = 137.5$ ft

56. $y = 6x^4 - 3x^2 + 1$
$b = 6a^4 - 3a^2 + 1 \qquad (x, y) = (a, b)$
x-axis: $(x, y) = (a, -b)$
$-b = 6a^4 - 3a^2 + 1$; no
y-axis: $(x, y) = (-a, b)$
$b = 6(-a)^4 - 3(-a)^2 + 1$
$b = 6a^4 - 3a^2 + 1$; yes
$y = x$: $(x, y) = (b, a)$
$a = 6b^4 - 3b^2 + 1$; no
origin: $f(-x) = -f(x)$
$f(-x) = 6(-x)^4 - 3(-x)^2 + 1 \qquad -f(x) = -(6x^4 - 3x^2 + 1)$
$f(-x) = 6x^4 - 3x^2 + 1 \qquad -f(x) = -6x^4 + 3x^2 - 1$
no
The graph is symmetric with respect to the y-axis.

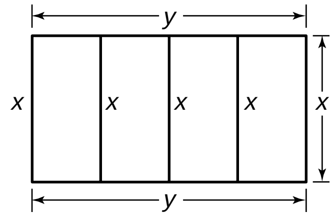
57a. Let x = number of cases of drug A.
Let y = number of cases of drug B.
$x = \leq 10$
$y \leq 9$
$3x + 5y \leq 55$
$x + 5y \leq 45$



(0, 0), (0, 9), (5, 8), (10, 5)
$f(x, y) = 320x + 500y$
$f(0, 0) = 320(0) + 500(0) = 0$
$f(0, 0) = 320(0) + 500(9) = 4500$
$f(5, 8) = 320(5) + 500(8) = 5600$
$f(10, 5) = 320(10) + 500(5) = 5700$
The maximum profit occurs when 10 cases of drug A and 5 cases of drug B are produced.

57b. When 10 cases of drug A and 5 cases of drug B are produced, the profit is $5700.

58.



$x + x + x + x + x + y + y = 5x + 2x$
The correct choice is A

10-3 Ellipses

Pages 637–638 Check For Understanding

1. $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$
$\frac{y^2}{8^2} + \frac{x^2}{5^2} = 1$
$\frac{y^2}{64} + \frac{x^2}{25} = 1$

2. Since the foci lie on the major axis, determine whether the major axis is horizontal or vertical. If the a^2 is the denominator of the x terms, the major axis is horizontal. If the a^2 is the denominator of the y terms, the major axis is vertical.

3. When the foci and center of an ellipse coincide, $c = 0$.
$c^2 = a^2 - b^2 \qquad e = \frac{c}{a}$
$0 = a^2 - b^2 \qquad e = \frac{0}{a}$
$b^2 = a^2$
$b = a \qquad e = 0$
The figure is a circle.

4. In an ellipse, $b^2 = a^2 - c^2$ and $\frac{c}{a} = e$.
$\frac{c}{a} = e \qquad b^2 = a^2 - c^2$
$c = ae \qquad b^2 = a^2 - a^2e^2$
$c^2 = a^2e^2 \qquad b^2 = a^2(1 - e^2)$

5. Shanice; an equation with only one squared term cannot be the equation of an ellipse.

6. center: $(h, k) = (-7, 0)$
$a = |0 - 6|$ or 6
$b = |-7 - (-4)|$ or 3
$\frac{(y - k)^2}{a^2} + \frac{(x - h)^2}{b^2} = 1$
$\frac{(y - 0)^2}{6^2} + \frac{[x - (-7)]^2}{3^2} = 1$
$\frac{y^2}{36} + \frac{(x + 7)^2}{9} = 1$
$c = \sqrt{a^2 - b^2} \qquad$ foci: $(h, k \pm c) = (-7, 0 \pm 3\sqrt{3})$
$c = \sqrt{6^2 - 3^2} \qquad = (-7, \pm 3\sqrt{3})$
$c = 3\sqrt{3}$

7. center: $(h, k) = (0, 0)$

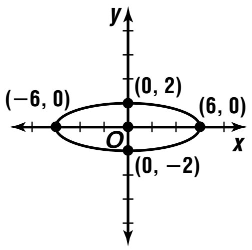
$a^2 = 36 \qquad b^2 = 4 \qquad c = \sqrt{a^2 - b^2}$

$a = \sqrt{36}$ or $6 \quad b = \sqrt{4}$ or $2 \quad c = \sqrt{36 - 4}$ or $4\sqrt{2}$

foci: $(h \pm c, k) = (0 \pm 4\sqrt{2}, 0)$ or $(\pm 4\sqrt{2}, 0)$

major axis vertices: $(h \pm a, k) = (0 \pm 6, 0)$ or $(\pm 6, 0)$

minor axis vertices: $(h, k \pm b) = (0, 0 \pm 2)$ or $(0, \pm 2)$



8. center: $(h, k) = (0, 4)$

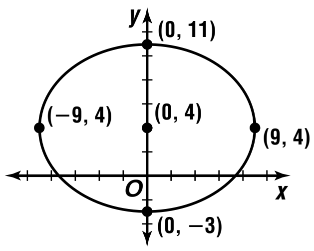
$a^2 = 81 \qquad b^2 = 49 \qquad c = \sqrt{a^2 - b^2}$

$a = \sqrt{81}$ or $9 \quad b = \sqrt{49}$ or $7 \quad c = \sqrt{81 - 49}$ or $4\sqrt{2}$

foci: $(h \pm c, k) = (0 \pm 4\sqrt{2}, 4)$ or $(\pm 4\sqrt{2}, 4)$

major axis vertices: $(h \pm a, k) = (0 \pm 9, 4)$ or $(\pm 9, 4)$

minor axis vertices: $(h, k \pm b) = (0, 4 \pm 7)$ or $(0, 11), (0, -3)$



9.

$$\begin{aligned} 25x^2 + 9y^2 + 100x - 18y &= 116 \\ 25(x^2 + 4x + ?) + 9(y^2 - 2y + ?) &= 116 + ? + ? \\ 25(x^2 + 4x + 4) + 9(y^2 - 2y + 1) &= 116 + 25(4) + 9(1) \\ 25(x + 2)^2 + 9(y - 1)^2 &= 225 \\ \frac{(x + 2)^2}{9} + \frac{(y - 1)^2}{25} &= 1 \end{aligned}$$

center: $(h, k) = (-2, 1)$

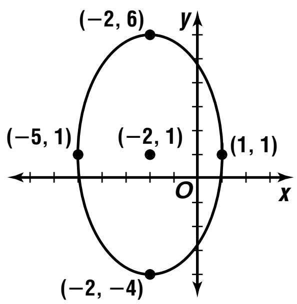
$a^2 = 25 \qquad b^2 = 9 \qquad c = \sqrt{a^2 - b^2}$

$a = \sqrt{25}$ or $5 \quad b = \sqrt{9}$ or $3 \quad c = \sqrt{25 - 9}$ or 4

foci: $(h, k \pm c) = (-2, 1 \pm 4)$ or $(-2, 5), (-2, -3)$

major axis vertices: $(h, k \pm a) = (-2, 1 \pm 5)$ or $(-2, 6), (-2, -4)$

minor axis vertices: $(h \pm b, k) = (-2 \pm 3, 1)$ or $(1, 1), (-5, 1)$



10.

$$\begin{aligned} 9x^2 + 4y^2 - 18x + 16y &= 11 \\ 9(x^2 - 2x + ?) + 4(y^2 + 4y + ?) &= 11 + ? + ? \\ 9(x^2 - 2x + 1) + 4(y^2 + 4y + 4) &= 11 + 9(1) + (4) \\ 9(x - 1)^2 + 4(y + 2)^2 &= 36 \\ \frac{(x - 1)^2}{4} + \frac{(y + 2)^2}{9} &= 36 \end{aligned}$$

center: $(h, k) = (1, -2)$

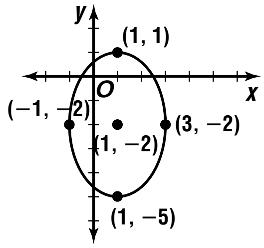
$a^2 = 9 \qquad b^2 = 4 \qquad c = \sqrt{a^2 - b^2}$

$a = \sqrt{9}$ or $3 \quad b = \sqrt{4}$ or $2 \quad c = \sqrt{9 - 4}$ or $\sqrt{5}$

foci: $(h, k \pm c) = \left(1, -2 \pm \sqrt{5}\right)$

major axis vertices: $(h, k \pm a) = (1, -2 \pm 3)$ or $(1, 1), (1, -5)$

minor axis vertices: $(h + b, k) = (1 \pm 2, -2)$ or $(3, -2), (-1, -2)$



11. center: $(h, k) = (-2, -3)$

$a = \frac{8}{2}$ or 4

$b = \frac{2}{2}$ or 1

$$\begin{aligned} \frac{(y - k)^2}{a^2} + \frac{(x - h)^2}{b^2} &= 1 \\ \frac{[y - (-3)]^2}{4^2} + \frac{[x - (-2)]^2}{1^2} &= 1 \\ \frac{(y + 3)^2}{16} + \frac{(x + 2)^2}{1} &= 1 \end{aligned}$$

12. The major axis contains the foci and it is located on the x-axis.

center: $(h, k) = \left(\frac{-1 + 1}{2}, \frac{0 + 0}{2}\right)$ or $(0, 0)$

$c = 1, a = 4$

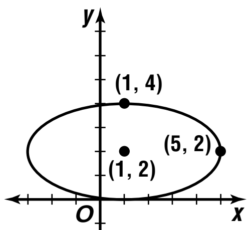
$c^2 = a^2 - b^2$

$1^2 = 4^2 - b^2$

$b^2 = 15$

$$\begin{aligned} \frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} &= 1 \\ \frac{(x - 0)^2}{4^2} + \frac{(y - 0)^2}{15} &= 1 \\ \frac{x^2}{16} + \frac{y^2}{15} &= 1 \end{aligned}$$

13. center: $(h, k) = (1, 2)$



The points at $(1, 4)$ and $(5, 2)$ are vertices of the ellipse.

$a = 4, b = 2$

$$\begin{aligned} \frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} &= 1 \\ \frac{(x - 1)^2}{4^2} + \frac{(y - 2)^2}{2^2} &= 1 \\ \frac{(x - 1)^2}{16} + \frac{(y - 2)^2}{4} &= 1 \end{aligned}$$

14. center: $(h, k) = (3, 1)$

$a = 6$

$e = \frac{c}{a}$

$\frac{1}{3} = \frac{c}{6}$

$2 = c$

$c^2 = a^2 - b^2$

$2^2 = 6^2 - b^2$

$4 = 36 - b^2$

$b^2 = 32$

$\frac{(y-k)^2}{a^2} + \frac{(h-h)^2}{b^2} = 1$

$\frac{(y-1)^2}{6^2} + \frac{(x-3)^2}{32} = 1$

$\frac{(y-1)^2}{36} + \frac{(x-3)^2}{32} = 1$

15. The major axis contains the foci and is located on the x-axis.

center: $(h, k) = (0, 0)$

$c = 0.141732$

$a = \frac{1}{2}(3.048)$ or 1.524

$c^2 = a^2 - b^2$

$(0.141732)^2 = (1.524)^2 - b^2$

$0.020 = 2.323 - b^2$

$b^2 = 2.302$

$b \approx 1.517$

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$\frac{(x-0)^2}{1.524^2} + \frac{(y-0)^2}{1.517^2} = 1$

$\frac{x^2}{1.524^2} + \frac{y^2}{1.517^2} = 1$

Pages 638–641 Exercises

16. center: $(h, k) = (0, -5)$

$a = |0 - (-7)|$ or 7

$b = |-5 - 0|$ or 5

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$\frac{(x-0)^2}{7^2} + \frac{[y-(-5)]^2}{5^2} = 1$

$\frac{x^2}{49} + \frac{(y+5)^2}{25} = 1$

$c = \sqrt{a^2 - b^2}$

$c = \sqrt{7^2 - 5^2}$ or $2\sqrt{6}$

foci: $(h \pm c, k) = (0 \pm 2\sqrt{6}, -5)$

$= (\pm 2\sqrt{6}, -5)$

17. center: $(h, k) = (-2, 0)$

$a = |-2 - 2|$ or 4

$b = |0 - 2|$ or 2

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$\frac{[x-(-2)]^2}{4^2} + \frac{(y-0)^2}{2^2} = 1$

$\frac{(x+2)^2}{16} + \frac{y^2}{4} = 1$

$c = \sqrt{a^2 - b^2}$

$c = \sqrt{4^2 - 2^2}$ or $2\sqrt{3}$

foci: $(h \pm c, k) = (-2 \pm 2\sqrt{3}, 0)$

18. centers: $(h, k) = (-3, 4)$

$a = |4 - 12|$ or 8

$b = |-3 - 2|$ or 5

$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$

$\frac{(y-4)^2}{8^2} + \frac{[x-(-3)]^2}{5^2} = 1$

$\frac{(y-4)^2}{64} + \frac{(x+3)^2}{25} = 1$

$c = \sqrt{a^2 - b^2}$

$c = \sqrt{8^2 - 5^2}$ or $\sqrt{39}$

foci: $(h, k \pm c) = (3, 4 \pm \sqrt{39})$

19. center: $(h, k) = (-2, 1)$

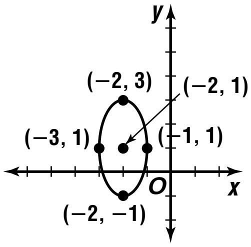
$a^2 = 4$ $\quad b^2 = 1$ $\quad c = \sqrt{a^2 - b^2}$

$a = \sqrt{4}$ or 2 $\quad b = \sqrt{1}$ or 1 $\quad c = \sqrt{4 - 1}$ or $\sqrt{3}$

foci: $(h, k \pm c) = (-2, 1 \pm \sqrt{3})$

major axis vertices: $(h, k \pm a) = (-2, 1 \pm 2)$ or $(-2, 3), (-2, -1)$

minor axis vertices: $(h \pm b, k) = (-2 \pm 1, 1)$ or



20. center: $(h, k) = (6, 7)$

$a^2 = 121$ $\quad b^2 = 100$

$a = \sqrt{121}$ or 11 $\quad b = \sqrt{100}$ or 10

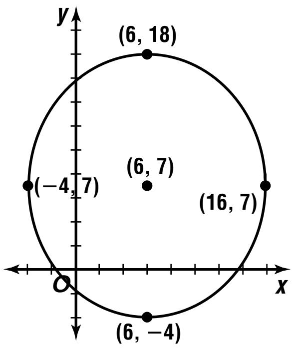
$c = \sqrt{a^2 - b^2}$

$c = \sqrt{121 - 100}$ or $\sqrt{21}$

foci: $(h, k \pm c) = (6, 7 \pm \sqrt{21})$

major axis vertices: $(h, k \pm a) = (6, 7 \pm 11)$ or $(6, 18), (6, -4)$

minor axis vertices: $(h \pm b, k) = (6 \pm 10, 7)$ or



21. center: $(h, k) = (4, -6)$

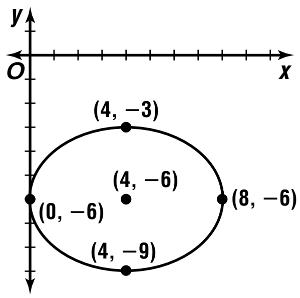
$a^2 = 16$ $\quad b^2 = 9$ $\quad c = \sqrt{a^2 - b^2}$

$a = \sqrt{16}$ or 4 $\quad b = \sqrt{9}$ or 3 $\quad c = \sqrt{16 - 9}$ or $\sqrt{7}$

foci: $(h \pm c, k) = (4 \pm \sqrt{7}, -6)$

major axis vertices: $(h \pm a, k) = (4 \pm 4, -6)$ or $(8, -6), (0, -6)$

minor axis vertices: $(h, k \pm b) = (4, -6 \pm 3)$ or



22. $(h, k) = (0, 0)$

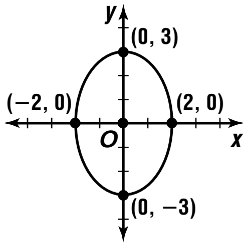
$a^2 = 9 \qquad b^2 = 4 \qquad c = \sqrt{a^2 - b^2}$

$a = \sqrt{9}$ or $3 \qquad b = \sqrt{4}$ or $2 \qquad c = \sqrt{9 - 4}$ or $\sqrt{5}$

foci: $(h, k \pm c) = \left(0, 0 \pm \sqrt{5}\right)$ or $\left(0, \pm \sqrt{5}\right)$

major axis vertices: $(h, k \pm a) = (0, 0 \pm 3)$ or $(0, \pm 3)$

minor axis vertices: $(h \pm b, k) = (0 \pm 2, 0)$ or $(\pm 2, 0)$



23.

$$4x^2 + y^2 - 8x + 6y + 9 = 0$$
$$4(x^2 - 2x + ?) + (y^2 + 6y + ?) = -9 + ? + ?$$
$$4(x^2 - 2x + 1) + (y^2 + 6y + 9) = -9 + 4(1) + 9$$
$$4(x - 1)^2 - (y + 3)^2 = 4$$
$$\frac{(x - 1)^2}{1} + \frac{(y + 3)^2}{4} = 1$$

center: $(h, k) = (1, -3)$

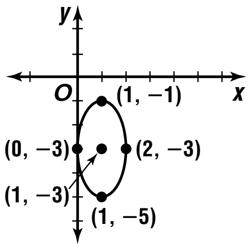
$a^2 = 4 \qquad b^2 = 1 \qquad c = \sqrt{a^2 - b^2}$

$a = \sqrt{4}$ or $2 \qquad b = \sqrt{1}$ or $1 \qquad c = \sqrt{4 - 1}$ or $\sqrt{3}$

foci: $(h, k \pm c) = \left(1, -3 \pm \sqrt{3}\right)$

major axis vertices: $(h, k \pm a) = (1, -3 \pm 2)$ or $(1, -1), (1, -5)$

minor axis vertices: $(h \pm b, k) = (1 \pm 1, -3)$ or $(2, -3), (0, -3)$



24.

$$16x^2 + 25y^2 - 96x - 200y = -144$$
$$16(x^2 - 6x + ?) + 25(y^2 - 8y + ?) = -144 + ? + ?$$
$$16(x^2 - 6x + 9) + 25(y^2 - 8y + 16) = -144 + 16(9) + 25(16)$$
$$16(x - 3)^2 + 25(y - 4)^2 = 400$$
$$\frac{(x - 3)^2}{25} + \frac{(y - 4)^2}{16} = 1$$

center: $(h, k) = (3, 4)$

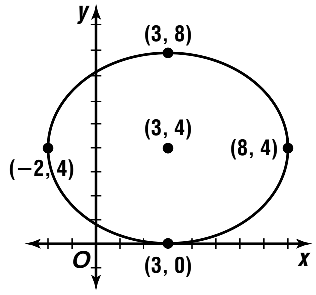
$a^2 = 25 \qquad b^2 = 16 \qquad c = \sqrt{a^2 - b^2}$

$a = \sqrt{25}$ or $5 \qquad b = \sqrt{16}$ or $4 \qquad c = \sqrt{25 - 16}$ or 3

foci: $(h \pm c, k) = (3 \pm 3, 4)$ or $(6, 4), (0, 4)$

major axis vertices: $(h \pm a, k) = (3 \pm 5, 4)$ or $(8, 4), (-2, 4)$

major axis vertices: $(h, k \pm b) = (3, 4 \pm 4)$ or $(3, 8), (3, 0)$



25.

$$3x^2 + y^2 + 18x - 2y + 4 = 0$$
$$3(x^2 + 6x + ?) + (y^2 - 2 + ?) = -4$$
$$3(x^2 + 6x + 9) + (y^2 - 2y + 1) = -4 + 3(9) + 1$$
$$3(x + 3)^2 + (y - 1)^2 = 24$$
$$\frac{(x + 3)^2}{8} + \frac{(y - 1)^2}{24} = 1$$

center: $(h, k) = (-3, 1)$

$a^2 = 24 \qquad b^2 = 8$

$a = \sqrt{24}$ or $2\sqrt{6} \qquad b = \sqrt{8}$ or $2\sqrt{2}$

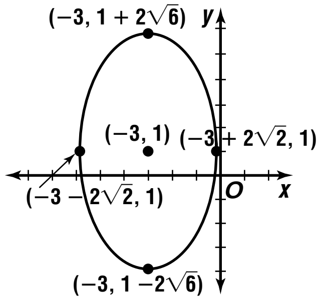
$c = \sqrt{a^2 - b^2}$

$c = \sqrt{24 - 8}$ or 4

foci: $(h, k \pm c) = (-3, 1 \pm 4)$ or $(-3, 5), (-3, -3)$

major axis vertices: $(h, k \pm a) = (-3, 1 \pm 2\sqrt{6})$

minor axis vertices: $(h \pm b, k) = (-3 \pm 2\sqrt{2}, 1)$



26.

$$6x^2 - 12x + 6y + 36y = 36$$
$$6(x^2 - 2x + ?) + 6(y^2 + 6y + ?) = 36$$
$$6(x^2 - 2x + 1) + 6(y^2 + 6y + 9) = 36 + 6(1) + 6(9)$$
$$6(x - 1)^2 + 6(y + 3)^2 = 96$$
$$\frac{(x - 1)^2}{16} + \frac{(y + 3)^2}{16} = 1$$

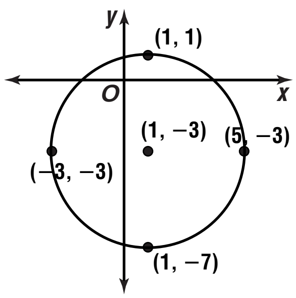
center: $(h, k) = (1, -3)$

$a^2 = 16 \qquad b^2 = 16 \qquad c = \sqrt{a^2 - b^2}$

$a = \sqrt{16}$ or $4 \qquad b = \sqrt{16}$ or $4 \qquad c = \sqrt{16 - 16}$ or 0

foci: $(h \pm c, k)$ or $(h, k \pm c) = (1, -3)$

Since $a = b = 4$, the vertices are $(h \pm 4, k)$ and $(h, k \pm 4)$ or $(5, -3), (-3, -3), (1, 1), (1, -7)$



27. $18y^2 + 12x^2 - 144y - 48x = -120$

$18(y^2 - 8y + ?) + 12(x^2 - 4x + ?) = -120 + ? + ?$

$18(y^2 - 8y + 16) + 12(x^2 - 4x + 4) =$
$-120 + 18(16) + 12(4)$

$18(y - 4)^2 + 12(x - 2)^2 = 216$

$\frac{(y-4)^2}{12} + \frac{(x-2)^2}{18} = 1$

center: $(h, k) = (2, 4)$

$a^2 = 18$ $\quad b^2 = 12$

$a = \sqrt{18}$ or $3\sqrt{2}$ $\quad b = \sqrt{12}$ or $2\sqrt{3}$

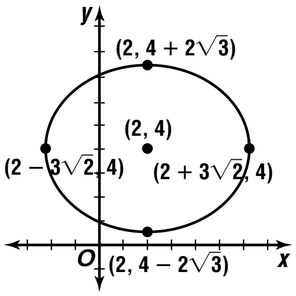
$c = \sqrt{a^2 - b^2}$

$c = \sqrt{18 - 12}$ or $\sqrt{6}$

foci: $(h \pm c, k) = (2 \pm \sqrt{6}, 4)$

major axis vertices: $(h \pm g, k) = (2 \pm 3\sqrt{2}, 4)$

minor axis vertices: $(h, k \pm b) = (2, 4 \pm 2\sqrt{3})$



28. $4x^2 - 8y + 9x^2 - 54x + 49 = 0$

$4(y^2 - 2y + ?) + 9(x^2 - 6x + ?) = -49 + ? + ?$

$4(y^2 - 2y + 1) + 9(x^2 - 6x + 9) = -49 + 4(1) + 9(9)$

$4(y - 1)^2 + 9(x - 3)^2 = 36$

$\frac{(y-1)^2}{9} + \frac{(x-3)^2}{4} = 1$

center: $(h, k) = (3, 1)$

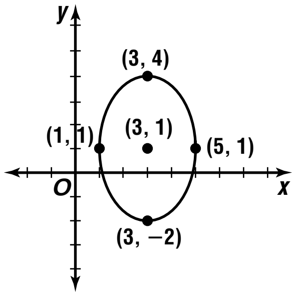
$a^2 = 9$ $\quad b^2 = 4$ $\quad c = \sqrt{a^2 - b^2}$

$a = \sqrt{9}$ or 3 $\quad b = \sqrt{4}$ or 2 $\quad c = \sqrt{9 - 4}$ or $\sqrt{5}$

foci: $(j, k \pm c) = (3, 1 \pm 5)$

major axis vertices: $(h, k \pm a) = (3, 1 \pm 3)$ or $(3, 4)$, $(3, 2)$

minor axis vertices: $(h \pm b, k) = (3 \pm 2, 1)$ or $(5, 1)$, $(1, 1)$



29. $49x^2 + 16y^2 + 160y - 384 = 0$

$49x^2 + 16(y^2 + 10y + ?) = 384 + ?$

$49x^2 + 16(y^2 + 10y + 25) = 384 + 16(25)$

$49x^2 + 16(y - 5)^2 = 784$

$\frac{x^2}{16} + \frac{(y+5)^2}{49} = 1$

center: $(h, k) = (0, -5)$

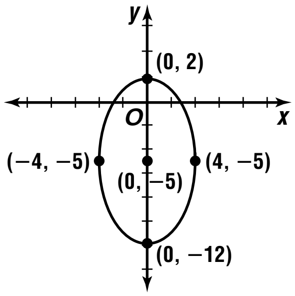
$a^2 = 49$ $\quad b^2 = 16$ $\quad c = \sqrt{a^2 - b^2}$

$a = \sqrt{49}$ or 7 $\quad b = \sqrt{16}$ or 4 $\quad c = \sqrt{49 - 16}$ or $\sqrt{33}$

foci: $(h, k \pm c) = (0, -5 \pm \sqrt{33})$

major axis vertices: $(h, k \pm a) = (0, -5 \pm 7)$ or $(0, 2)$, $(0, -12)$

minor axis vertices: $(h \pm b, k) = (0 \pm 4, -5)$ or $(\pm 4, -5)$



30. $9y^2 + 108y + 4x^2 - 56x = -484$

$9(y^2 + 12y + ?) + 4(x^2 - 14x + ?) = -484 + ? + ?$

$9(y^2 + 12y + 36) + 4(x^2 - 14x + 49) =$
$-484 + 9(36) + 4(49)$

$9(y + 6)^2 + 4(x - 7)^2 = 36$

$\frac{(y+6)^2}{4} + \frac{(x-7)^2}{9} = 1$

center: $(h, k) = (7, -6)$

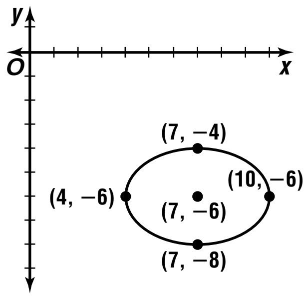
$a^2 = 9$ $\quad b^2 = 4$ $\quad c = \sqrt{a^2 - b^2}$

$a = \sqrt{9}$ or 3 $\quad b = \sqrt{4}$ or 2 $\quad c = \sqrt{9 - 4}$ or $\sqrt{5}$

foci: $(h \pm c, k) = (7 \pm \sqrt{5}, -6)$

major axis vertices: $(h \pm a, k) = (7 \pm 3, -6)$ or $(10, -6)(4, -6)$

minor axis vertices: $(h, k \pm b) = (7, -6 \pm 2)$ or $(7, -4)$, $(7, -8)$



31. $a = 7, b = 5$

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$\frac{[x-(-3)]^2}{7^2} + \frac{[y-(-1)]^2}{5^2} = 1$

$\frac{(x+3)^2}{49} + \frac{(y+1)^2}{25} = 1$

32. The major axis contains the foci and it is located on the x-axis.

center: $(h, k) = \left(\frac{-2+2}{0}, \frac{0+0}{2}\right)$ or $(0, 0)$

$$c = 2, a = 7$$
$$c^2 = a^2 - b^2$$
$$2^2 = 7^2 - b^2$$
$$b^2 = 45$$
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-0)^2}{7^2} + \frac{(y-0)^2}{45} = 1$$
$$\frac{x^2}{49} + \frac{y^2}{45} = 1$$

33. $b = \frac{3}{4}a$

$$6 = \frac{3}{4}a$$
$$8 = a$$
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-0)^2}{8^2} + \frac{(y-0)^2}{6^2} = 1$$
$$\frac{x^2}{64} + \frac{y^2}{36} = 1$$

34. The major axis contains the foci and it is the vertical axis of the ellipse.

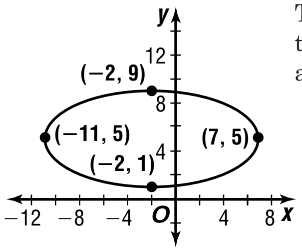
center: $(h, k) = \left(\frac{-1+(-1)}{2}, \frac{1+(-5)}{2}\right)$ or $(-1, -2)$

$c = |1 - k|$ $\qquad a = 2\sqrt{13}$

$c = |1 - (-2)|$ or 3

$$c^2 = a^2 - b^2$$
$$3^2 = \left(2\sqrt{13}\right)^2 - b^2$$
$$b^2 = 52 - 9$$
$$b^2 = 43$$
$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$
$$\frac{[y-(-2)]^2}{(2\sqrt{13})^2} + \frac{[x-(-1)]^2}{43} = 1$$
$$\frac{(y+2)^2}{52} + \frac{(x+1)^2}{43} = 1$$

35.



The horizontal axis of the ellipse is the major axis.

enter: $(h, k) = \left(\frac{-11+7}{2}, \frac{5+5}{2}\right)$ or $(-2, 5)$

$h + a = 7$ $\qquad k + b = 9$

$-2 + a = 7$ $\qquad 5 + b = 9$

$a = 9$ $\qquad b = 4$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$
$$\frac{[x-(-2)]^2}{9^2} + \frac{(y-5)^2}{4^2} = 1$$
$$\frac{(x+2)^2}{81} + \frac{(y-5)^2}{16} = 1$$

36. The major axis contains the foci and it is the vertical axis of the ellipse.

$c = \frac{5-(-1)}{2}$ or 3

center: $(h, k) = \left(\frac{1+1}{2}, \frac{-1+5}{2}\right)$ or $(1, 2)$

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$
$$\frac{(2-2)^2}{a^2} + \frac{(y-1)^2}{b^2} = 1$$
$$\frac{0^2}{a^2} + \frac{3^2}{b^2} = 1$$
$$\frac{9}{b^2} = 1$$
$$9 = b^2$$

$$c^2 = a^2 - b^2$$
$$3^2 = a^2 - 9$$
$$18 = a^2$$

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$
$$\frac{(y-2)^2}{18} + \frac{(x-1)^2}{9} = 1$$

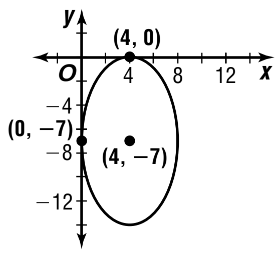
37. $\frac{1}{2} = \frac{c}{a}$

$$\frac{a}{2} = c$$
$$\frac{10}{2} = c$$
$$5 = c$$

$$b^2 = a^2 - c^2$$
$$b^2 = 10^2 - 5^2$$
$$b^2 = 75$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-0)^2}{10^2} + \frac{(y-0)^2}{75} = 1$$
$$\frac{x^2}{100} + \frac{y^2}{75} = 1$$

38.



tangent vertices: $(4, 0)$, $(0, -7)$

$a = |-7 - 0|$ or 7

$b = |4 - 0|$ or 4

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$
$$\frac{[y-(-7)]}{7^2} + \frac{(x-4)^2}{4^2} = 1$$
$$\frac{(y+7)^2}{49} + \frac{(x-4)^2}{16} = 1$$

39. $b^2 = a^2(1 - e^2)$

$$b^2 = 2^2\left[1 - \left(\frac{3}{4}\right)^2\right]$$

$b^2 = \frac{28}{16}$ or 1.75

Case 1: Horizontal axis is major axis.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)}{b^2} = 1$$
$$\frac{(x-0)^2}{2^2} + \frac{(y-0)^2}{1.75} = 1$$
$$\frac{x^2}{4} + \frac{y^2}{1.75} = 1$$

Case 2: Vertical axis is major axis.

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$
$$\frac{(y-0)^2}{2^2} + \frac{(x-0)^2}{1.75} = 1$$
$$\frac{y^2}{4} + \frac{x^2}{1.75} = 1$$

40. The major axis contains the foci and it is the horizontal axis of the ellipse.

center: $(h, k) = \left(\frac{3+1}{2}, \frac{5+5}{2}\right)$ or $(2, 5)$

foci: $(3, 5) = (h + c, k)$

$3 = h + c$

$3 = 2 + c$

$1 = c$

$e = \frac{c}{a}$ $\qquad$ $b^2 = a^2(1 - e^2)$

$0.25 = \frac{1}{a}$ $\qquad$ $b^2 = 4^2(1 - 0.25^2)$

$a = 4$ $\qquad$ $b^2 = 15$

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$\frac{(x-2)^2}{4^2} + \frac{(y-5)^2}{15} = 1$

$\frac{(x-2)^2}{16} + \frac{(y-15)^2}{15} = 1$

41. $a = \frac{20}{2}$ or 10

$b^2 = a^2(1 - e^2)$

$b^2 = 10^2\left[1 - \left(\frac{7}{10}\right)^2\right]$ or 51

$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$

$\frac{(y-0)^2}{10^2} + \frac{(x-3)^2}{51} = 1$

$\frac{y^2}{100} + \frac{(x-3)^2}{51} = 1$

42. focus: $(1, -1 + \sqrt{5}) = (h, k + c)$

$-1 + \sqrt{5} = k + c$

$-1 + \sqrt{5} = -1 + c$

$\sqrt{5} = c$

$e = \frac{c}{a}$ $\qquad$ $b^2 = a^2(1 - e^2)$

$\frac{\sqrt{5}}{3} = \frac{\sqrt{5}}{a}$ $\qquad$ $b^2 = 3^2\left[1 - \left(\frac{\sqrt{5}}{3}\right)^2\right]$

$a = 3$ $\qquad$ $b^2 = 4$

major axis: vertical axis

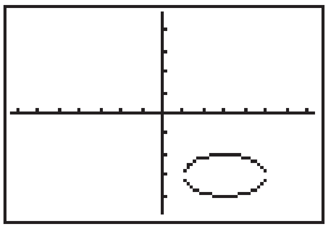
$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$

$\frac{[y-(-1)]^2}{3^2} + \frac{(x-1)^2}{4} = 1$

$\frac{(y+1)^2}{9} + \frac{(x-1)^2}{4} = 1$

43.

$$x^2 + 4y^2 - 6x + 24y = -41$$
$$(x^2 - 6x + ?) + 4(y^2 + 6y + ?) = -41 + ? + ?$$
$$(x^2 - 6x + 9) + 4(y^2 + 6y + 9) = -41 + 9 + 4(9)$$
$$(x-3)^2 + 4(y+3)^2 = 4$$
$$4(y+3)^2 = 4 - (x-3)^2$$
$$(y+3)^2 = \frac{4 - (x-3)^2}{4}$$
$$y + 3 = \pm\sqrt{\frac{4 - (x-3)^2}{4}}$$
$$y = \pm\sqrt{\frac{4 - (x-3)^2}{4}} - 3$$

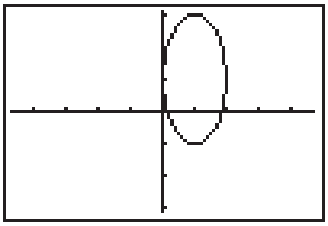


vertices: $(5, -3)$, $(1, -3)$, $(3, -2)$, $(3, -4)$

$[-7.28, 7.28]$ scl:1 by $[-4.8, 4.8]$ scl:1

44.

$$4x^2 + y^2 - 8x - 2y = -1$$
$$4(x^2 - 2x + ?) + (y^2 - 2y + ?) = -1 + ? + ?$$
$$4(x^2 - 2x + 1) + (y^2 - 2y + 1) = -1 + 4(1) + 1$$
$$4(x-1)^2 + (y-1)^2 = 4$$
$$(y-1)^2 = 4 - 4(x-1)^2$$
$$y - 1 = \pm\sqrt{-4 - 4(x-1)^2}$$
$$y = \pm\sqrt{4 - 4(x-1)^2} + 1$$

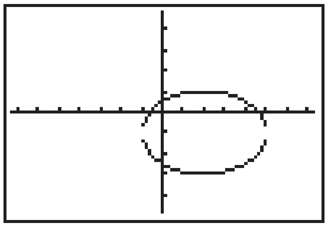


Vertices: $(0, 1)$, $(2, 1)$, $(1, -1)$, $(1, 3)$

$[-4.7, 4.7]$ scl:1 by $[-3.1, 3.1]$ scl:1

45.

$$4x^2 + 9y^2 - 16x + 18y = 11$$
$$4(x^2 - 4x + ?) + 9(y^2 + 2y + ?) = 11 + ? + ?$$
$$4(x^2 - 4x + 4) + 9(y^2 + 2y + 1) = 11 + 4(4) + 9(1)$$
$$4(x-2)^2 + 9(y+1)^2 = 36$$
$$9(y+1)^2 = 36 - 4(x-2)^2$$
$$y + 1 = \pm\sqrt{\frac{36 - 4(x-2)^2}{9}}$$
$$y = \pm\sqrt{\frac{36 - 4(x-2)^2}{9}} - 1$$

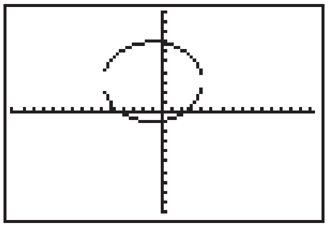


Vertices: $(-1, -1)$, $(5, -1)$, $(2, -3)$, $(2, 1)$

$[-7.28, 7.28]$ scl:1 by $[-4.8, 4.8]$ scl:1

46.

$$25y^2 + 16x^2 - 150y + 32x = 159$$
$$25(y^2 - 6y + ?) + 16(x^2 + 2x - ?) = 159 + ? + ?$$
$$25(y^2 - 6y + 9) + 16(x^2 - 2x + 1) = 159 + 25(9) + 16(1)$$
$$25(y-3)^2 + 16(x+1)^2 = 400$$
$$25(y-3)^2 = 400 - 16(x+1)^2$$
$$(y-3) = \pm\sqrt{\frac{400 - 16(x+1)^2}{25}}$$
$$y = \pm\sqrt{\frac{400 - 16(x+1)^2}{5}} + 3$$



Vertices: $(4, 3)$, $(-6, 3)$, $(-1, 7)$, $(-1, -1)$

$[-15.16, 15.16]$ scl:1 by $[-10, 10]$ scl:1

47. The target ball should be placed opposite the pocket, $\sqrt{5}$ feet from the center along the major axis of the ellipse. The cue ball can be placed anywhere on the side opposite the pocket. The ellipse has a semi-major axis of length 3 feet and a semi-minor axis of length 2 feet. Using the equation $c^2 = a^2 - b^2$, the focus of the ellipse is found to be $\sqrt{5}$ feet from the center of the ellipse. Thus the hole is located at one focus of the ellipse. The reflective properties of an ellipse should insure that a ball placed $\sqrt{5}$ feet from the center of the ellipse and hit so that it rebounds once off the wall should fall into the pocket at the other focus of the ellipse.

48. A horizontal line; see students' work.

49a. $a = \frac{96}{2}$ or 48

$b = \frac{46}{2}$ or 23

$(h, k) = (0, 0)$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-0)^2}{48^2} + \frac{(y-0)^2}{23^2} = 1$$
$$\frac{x^2}{2304} + \frac{y^2}{529} = 1$$

49b. $c = \sqrt{a^2 - b^2}$

$c = \sqrt{2304 - 529}$

$c \approx 42.13$

He could have stood at a focal point, about 42 feet on either side of the center along the major axis.

49c. The distance between the focal points is $2c$.

$2c = 2(42)$

$= 84$

about 84 ft

50a. $x^2 + y^2 = r^2$

$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$ $\qquad A = \pi r^2$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\qquad A = \pi \cdot r \cdot r$

$A = \pi \cdot a \cdot b$

$A = \pi ab$

50b. $\frac{x^2}{9} + \frac{y^2}{4} = 1$

$a^2 = 9$ $\qquad b^2 = 4$

$a = 3$ $\qquad b = 2$

$A = \pi ab$

$A = \pi(3)(2)$

$A = 6\pi$ units^2

51. If (x, y) is a point on the ellipse, then show that $(-x, -y)$ is also on the ellipse.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
$$\frac{(-x)^2}{a^2} + \frac{(-y)^2}{b^2} = 1$$
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Thus, $(-x, -y)$ is also a point on the ellipse and the ellipse is symmetric with respect to the origin.

52a. $a = \frac{8}{2}$ or 4

$b = 3$

$c = \sqrt{a^2 - b^2}$

$c = \sqrt{4^2 - 3^2}$

$c = \sqrt{7}$

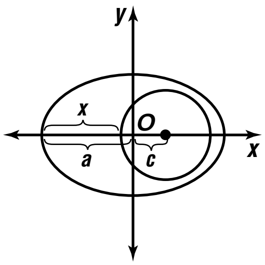
foci: $(h \pm c, 0) = (0 \pm \sqrt{7}, 0)$ or $(\pm\sqrt{7}, 0)$

The thumbtacks should be placed $(\pm\sqrt{7}, 0)$ from the center of the arch.

52b. With the string anchored by thumbtacks at the foci of the arch and held taunt by a pencil, the sum of the distances from each thumbtack to the pencil will remain constant.

53a. GOES 4; its eccentricity is closest to 0.

53b.



$\frac{c}{a} = e$

$\frac{c}{6955} = 0.052$

$c = 361.66$

[figure not drawn to scale]

$x = a + c -$ Earth's radius

$x = 6955 + 361.66 - 6357$

$x = 959.66$

$x \approx 960$ km

54.

$$\begin{aligned}
x^2 + y^2 + Dx + Ey + F &= 0\\
0^2 + (-9)^2 + D(0) + E(-9) + F = 0 &\Rightarrow\\
-9E + F &= -81\\
7^2 + (-2)^2 + D(7) + E(-2) + F = 0 &\Rightarrow\\
7D - 2E + F &= -53\\
(-5)^2 + (-10)^2 + D(-5) + E(-10) + F = 0 &\Rightarrow\\
-5D - 10E + F &= -125
\end{aligned}$$

$$\begin{aligned}
-9E + F &= -81\\
(-1)(7D - 2E + F) &= (-1)(-53)\\
\hline
-7D - 7E &= -28\\
D + E &= 4
\end{aligned}$$

$$\begin{aligned}
7D - 2E + F &= -53\\
(-1)(-5D - 10E + F) &= (-1)(-125)\\
\hline
12D + 8E &= 72
\end{aligned}$$

$$\begin{aligned}
(-8)(D + E) &= (8)(4)\\
12D + 8E &= 72\\
\hline
4D &= 40\\
D &= 10
\end{aligned}$$

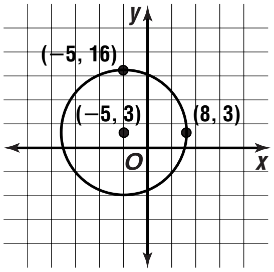
$D + E = 4$ $\qquad -9E + F = -81$

$10 + E = 4$ $\qquad -9(-6) + F = -81$

$E = -6$ $\qquad F = -135$

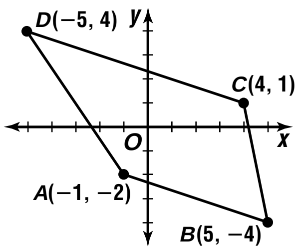
$$x^2 + y^2 + Dx + Ey + F = 0$$
$$x^2 + y^2 + 10x - 6y - 135 = 0$$
$$(x^2 + 10x + 25) + (y^2 - 6y + 9) = 135 + 25 + 9$$
$$(x + 5)^2 + (y - 3)^2 = 169$$

center: $(h, k) = (-5, 3)$
radius: $r^2 = 169$
$r = 13$



55. Graph the quadrilateral with vertices $A(-1, -2)$, $B(5, -4)$, $C(4, 1)$, and $D(-5, 4)$.

A quadrilateral is a parallelogram if one pair of opposite sides are parallel and congruent.



slope of $\overline{DA}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 4}{-1 - (-5)} = \frac{-6}{4} \text{ or } \frac{3}{2}$$

slope of $\overline{CB}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 1}{5 - 4} = -\frac{5}{1}$$

The slopes are not equal, so $\overline{DA} \nparallel \overline{CB}$. The quadrilateral is not a parallelogram; no.

56. $\cos 2\theta = 1 - 2\sin^2 \theta$
$\cos 2\theta = 1 - 2\left(\frac{7}{8}\right)^2$
$\cos 2\theta = \frac{-34}{64}$ or $-\frac{17}{32}$

57. $|A| = 4$
$A = \pm 4$
$-\frac{c}{k} = 20°$
$-\frac{c}{2} = 20°$
$c = -40°$

$\frac{2\pi}{k} = 180$
$k = 2$
$h = 0$

$y = A\cos(kx + c) + h$
$y = \pm 4\cos[2x + (-40°)] + 0$
$y = \pm 4\cos(2x - 40°)$

58. $A = 180° - (121° 32' + 42° 5')$ or $16° 23'$

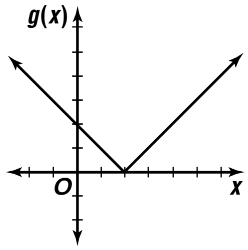
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{4.1}{\sin 16° 23'} = \frac{b}{\sin 42° 5'}$$
$$\frac{4.1 \sin 42° 5'}{\sin 16° 23'} = b$$
$$9.7 \approx b$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{4.1}{\sin 16° 23'} = \frac{c}{\sin 121° 32'}$$
$$\frac{4.1 \sin 121° 32'}{\sin 16° 23'} = c$$
$$12.4 \approx c$$

59. $P(x) = x^4 - 4x^3 - 2x^2 - 1$
$P(5) = 5^4 - 4(5)^3 - 2(5)^2 - 1$
$P(5) = 74$
$P(5) \neq 0$; no, the binomial is not a factor of the polynomial.

60. Let $h = 0.1$.
$x - h = x - 0.1$
$= -1 - 0.1$ or -1.1
$f(x - 0.1) = f(-1.1)$
$= (-1.1)^2 + 4(-1.1) - 12$
$= -15.19$
$x + h = x + 0.1$
$= -1 + 0.1$ or -0.9
$f(x + 0.1) = f(-0.9)$
$= (-0.9)^2 + 4(-0.9) - 12$
$= -14.79$
$f(x) = -16$
$f(x) < f(x - 0.1)$ and $f(x) < f(x - 0.1)$, so the point is a location of a minimum.

61. The graph of the parent function $g(x) = |x|$ is translated 2 units right.



62. Initial location: (2, 0)

$$\text{Rot}_{90} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \text{ or } (0, 2)$$

$$\text{Rot}_{80} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix} \text{ or } (-2, 0)$$

$$\text{Rot}_{270} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix} \text{ or } (0, -2)$$

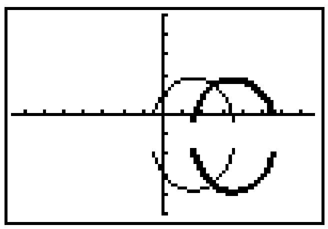
63. $m\angle QTS + m\angle TSR = 180$
$a + b + c + d = 180$
$b + b + c + c = 180$
$2b + 2c = 180$
$b + c = 90$

$p + b + c = 180$
$p + 90 = 180$
$p = 90$

The correct choice is C.

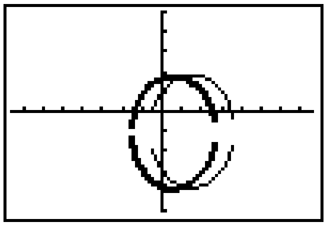
Page 641 Graphing Calculator Exploration

1. Sample answer: The graph will shift 4 units to the right.



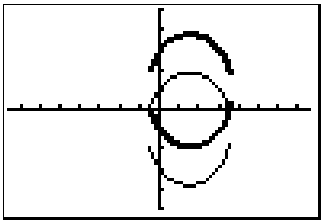
$[-15.16, 15.16]$ scl:2 by $[-10, 10]$ scl:2

2. Sample answer: The graph will shift 4 units to the left.



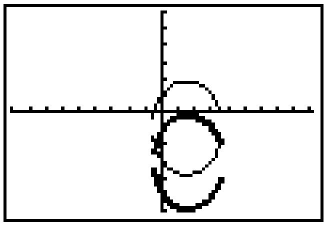
$[-15.16, 15.16]$ scl:2 by $[-10, 10]$ scl:2

3. Sample answer: The graph will shift 4 units up.



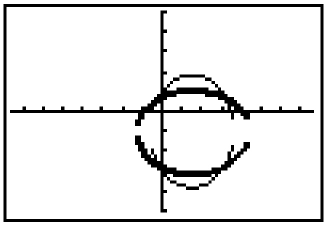
$[-15.16, 15.16]$ scl:2 by $[-10, 10]$ scl:2

4. Sample answer: The graph will shift 4 units down.



$[-18.19, 18.19]$ scl:2 by $[-12, 12]$ scl:2

5. Sample answer: The graph will rotate 90°.



$[-15.16, 15.16]$ scl:2 by $[-10, 10]$ scl:2

6. For $(x + c)$, the graph will shift c units to the left. For $(x - c)$, the graph will shift c units to the right.
7. For $(y + c)$, the graph will shift c units down. For $(y - c)$, the graph will shift c units up.
8. The graph will rotate 90°.

10-4 Hyperbolas

Pages 649–650 Check For Understanding

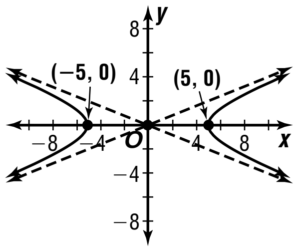
1. The equations of both hyperbolas and ellipses have x^2 terms and y^2 terms. In an ellipse, the terms are added and in a hyperbola these terms are subtracted.

2. transverse axis: vertical
$2a = 4$
$a = 2$
An equation in standard form of the hyperbola must have $\frac{y^2}{2^2}$ or $\frac{y^2}{4}$ as the first term; b.

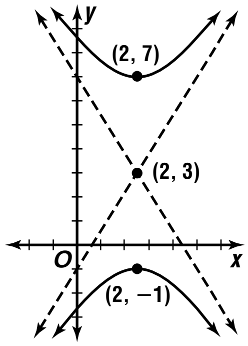
3. $e = \frac{c}{a}$, so $ae = c$ and $a^2e^2 = c^2$.
Since $c^2 = a^2 + b^2$ we have
$a^2e^2 = a^2 + b^2$
$a^2e^2 - a^2 = b^2$
$a^2(e^2 - 1) = b^2$

4. With the equation in standard form, if the first expression contains "x", the transverse axis is horizontal. If the first expression contains "y", the transverse axis is vertical.

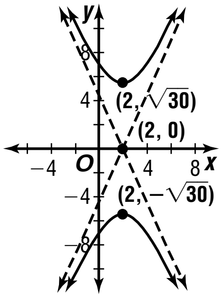
5. center: $(h, k) = (0, 0)$
$a^2 = 25$ $\quad b^2 = 4$ $\quad c = \sqrt{a^2 + b^2}$
$a = 5$ $\quad b = 2$ $\quad c = \sqrt{25 + 4}$ or $\sqrt{29}$
transverse axis: horizontal
foci: $(h + c, k) = (0 \pm \sqrt{29}, 0)$ or $(\pm\sqrt{29}, 0)$
vertices: $(h \pm a, k) = (0 \pm 5, 0)$ or $(\pm 5, 0)$
asymptotes: $y - k = \pm\frac{b}{a}(x - h)$
$y - 0 = \pm\frac{2}{5}(x - 0)$
$y = \pm\frac{2}{5}x$



6. center: $(h, k) = (2, 3)$
$a^2 = 16$ $\quad b^2 = 4$ $\quad c = \sqrt{a^2 + b^2}$
$a = \sqrt{16}$ or 4 $\quad b = \sqrt{4}$ or 2 $\quad c = \sqrt{16 + 4}$ or $2\sqrt{5}$
transverse axis: vertical
foci: $(h, k \pm c) = \left(2, 3 \pm 2\sqrt{5}\right)$
vertices: $(h, k \pm a) = (2, 3 \pm 4)$ or $(2, 7), (2, -1)$
asymptotes: $y - k = \pm\frac{a}{b}(x - h)$
$y - 3 = \pm\frac{4}{2}(x - 2)$
$y - 3 = \pm 2(x - 2)$

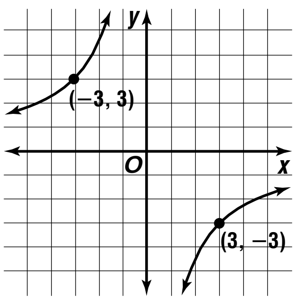


7. $$y^2 - 5x^2 + 20x = 50$$
$$y^2 - 5(x^2 - 4x + ?) = 50 + ?$$
$$y^2 - 5(x^2 - 4x + 4) = 50 + (-5)(4)$$
$$y^2 - 5(x - 2)^2 = 30$$
$$\frac{y^2}{30} - \frac{(x-2)^2}{6} = 1$$
$x - h = x - 2$ $\quad$ $y - k = y$
$h = 2$ $\quad$ $k = 0$
center: $(h, k) = (2, 0)$
$a^2 = 30$ $\quad$ $b^2 = 6$ $\quad$ $c = \sqrt{a^2 - b^2}$
$a = \sqrt{30}$ $\quad$ $b = \sqrt{6}$ $\quad$ $c = \sqrt{30 + 6}$ or 6
transverse axis: vertical
foci: $(h, k \pm c) = (2, 0 \pm 6)$ or $(2, \pm 6)$
vertices: $(h, k \pm a) = \left(2, 0 \pm \sqrt{30}\right)$ or $\left(2, \pm\sqrt{30}\right)$
asymptotes: $y - k = \pm \frac{a}{b}(x - h)$
$$y - 0 = \pm\frac{\sqrt{30}}{\sqrt{6}}(x - 2)$$
$$y = \pm\sqrt{5}(x - 2)$$



8. center: $(h, k) = (0, 5)$
transverse axis: horizontal
$a = 5, b = 3$
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-0)^2}{5^2} - \frac{(y-5)^2}{3^2} = 1$$
$$\frac{x^2}{25} - \frac{(y-5)^2}{9} = 1$$

9. $c = -9$
quadrants: II and IV
transverse axis: $y = -x$
vertices: $xy = -9$ $\quad$ $xy = -9$
$3(-3) = -9$ $\quad$ $-3(3) = -9$
$(3, -3)$ $\quad$ $(-3, 3)$



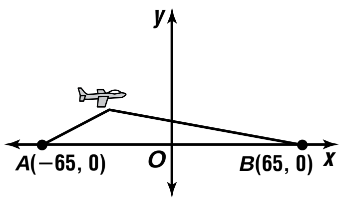
10. center: $(h, k) = (1, -4)$
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-1)^2}{5^2} - \frac{[y-(-4)]^2}{2^2} = 1$$
$$\frac{(x-1)^2}{25} - \frac{(y+4)^2}{4} = 1$$

11. $2b = 6$
$b = 3$
center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3+3}{2}, \frac{4-0}{2}\right)$
$= (3, 2)$
transverse axis: vertical
$a = |4 - 2|$ or 2
$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$
$$\frac{(y-2)^2}{2^2} - \frac{(x-3)^2}{3^2} = 1$$
$$\frac{(y-2)^2}{4} - \frac{(x-3)^2}{9} = 1$$

12. center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0+0}{2}, \frac{6+(-6)}{2}\right)$
$= (0, 0)$
transverse axis: vertical
c = distance from center to a focus
$= |0 - 6|$ or 6
$b^2 = c^2 - a^2$ $\quad$ $b^2 = a^2$
$a^2 = c^2 - a^2$ $\quad$ $b^2 = 18$
$2a^2 = c^2$
$a^2 = \frac{c^2}{2}$
$a^2 = \frac{6^2}{2}$ or 18
$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$
$$\frac{(y-0)^2}{18} - \frac{(x-0)^2}{18} = 1$$
$$\frac{y^2}{18} - \frac{x^2}{18} = 1$$

13. center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{10+(-10)}{2}, \frac{0+0}{2}\right)$
$= (0, 0)$
transverse axis: horizontal
c = distance from center to a focus
$= |10 - 0|$ or 10
$e = \frac{c}{a}$ $\quad$ $b^2 = c^2 - a^2$
$\frac{5}{3} = \frac{10}{a}$ $\quad$ $b^2 = 10^2 - 6^2$
$b^2 = 64$
$a = 6$
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-0)^2}{6^2} - \frac{(y-0)^2}{64} = 1$$
$$\frac{x^2}{36} - \frac{y^2}{64} = 1$$

14a. The origin is located midway between stations A and B; $(h, k) = (0, 0)$. The stations are located at the foci, so $2c = 130$ or $c = 65$.



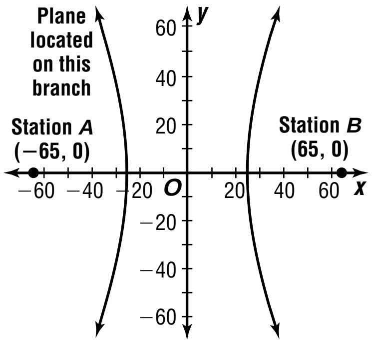
The difference of the distances from the plane to each station is 50 miles.
$50 = 2a$ $\quad$ (Definition of hyperbola)
$25 = a$
$b^2 = c^2 - a^2$
$b^2 = 65^2 - 25^2$
$b^2 = 3600$

transverse axis: horizontal

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-0)^2}{25^2} - \frac{(y-0)^2}{3600} = 1$$
$$\frac{x^2}{625} - \frac{y^2}{3600} = 1$$

14b. Vertices: $(h \pm a, k) = (0 \pm 25, 0)$ or $(\pm 25, 0)$

asymptotes: $y - k = \pm\frac{b}{a}(x - h)$
$$y - 0 = \pm\frac{\sqrt{3600}}{25}(x - 0)$$
$$y = \pm\frac{12}{5}x$$



14c. Let $y = 6$.
$$\frac{x^2}{625} - \frac{y^2}{3600} = 1$$
$$\frac{x^2}{625} - \frac{6^2}{3600} = 1$$
$$\frac{x^2}{625} - \frac{36}{3600} = 1$$
$$\frac{x^2}{625} = 1 + \frac{36}{3600}$$
$$\frac{x^2}{625} = 1.01$$
$$x^2 = 625(1.01)$$
$$x^2 = 631.25$$
$$x = \pm\sqrt{631.25}$$
$$x \approx \pm 25.1$$

Since the phase is closer to station A than station B, use the negative value of x to locate the ship at $(-25.1, 6)$.

Pages 650–652 Exercises

15. center: $(h, k) = (0, 0)$

$a^2 = 100$ $b^2 = 16$

$a = \sqrt{100}$ or 10 $b = \sqrt{16}$ or 4

$c = \sqrt{a^2 + b^2}$

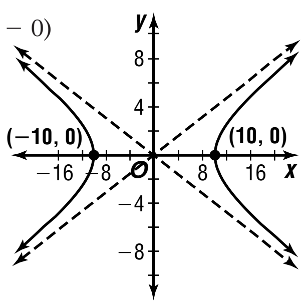
$c = \sqrt{100 + 16}$ or $2\sqrt{29}$

transverse axis: horizontal

foci: $(h \pm c, k) = (0 \pm 2\sqrt{29}, 0)$ or $(\pm 2\sqrt{29}, 0)$

vertices: $(h \pm a, k) = (0 \pm 10, 0)$ or $(\pm 10, 0)$

asymptotes: $y - k = \pm\frac{b}{a}(x - h)$
$$y - 0 = \pm\frac{4}{10}(x - 0)$$
$$y = \pm\frac{2}{5}x$$



16. center: $(h, k) = (0, 5)$

$a^2 = 9$ $b^2 = 81$

$a = \sqrt{9}$ or 3 $b = \sqrt{81}$ or 9

$c = \sqrt{a^2 + b^2}$

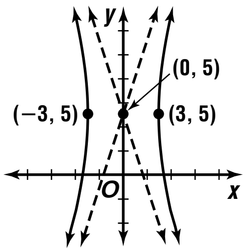
$c = \sqrt{9 + 81}$ or $3\sqrt{10}$

transverse axis: horizontal

foci: $(h \pm c, k) = (0 \pm 3\sqrt{10}, 5)$ or $(\pm 3\sqrt{10}, 5)$

vertices: $(h \pm a, k) = (0 \pm 3, 5)$ or $(\pm 3, 5)$

asymptotes: $y - k = \pm\frac{b}{a}(x - h)$
$$y - 5 = \pm\frac{9}{3}(x - 0)$$
$$y - 5 = \pm 3x$$



17. center: $(h, k) = (0, 0)$

$a^2 = 4$ $b^2 = 49$

$a = \sqrt{4}$ or 2 $b = \sqrt{49}$ or 7

$c = \sqrt{a^2 + b^2}$

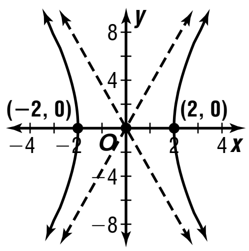
$c = \sqrt{4 + 49}$ or $\sqrt{53}$

transverse axis: horizontal

foci: $(h \pm c, k) = (0 \pm \sqrt{53}, 0)$ or $(\pm\sqrt{53}, 0)$

vertices: $(h \pm a, k) = (0 \pm 2, 0)$ or $(\pm 2, 0)$

asymptotes: $y - k = \pm\frac{b}{a}(x - h)$
$$y - 0 = \pm\frac{7}{2}(x - 0)$$
$$y = \pm\frac{7}{2}x$$



18. center: $(h, k) = (-1, 7)$

$a^2 = 64$ $b^2 = 4$

$a = \sqrt{64}$ or 8 $b = \sqrt{4}$ or 2

$c = \sqrt{a^2 + b^2}$

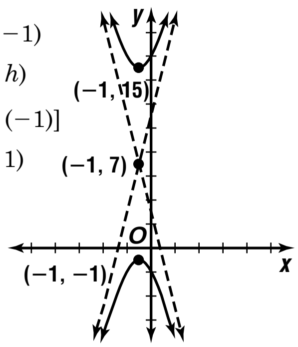
$c = \sqrt{64 + 4}$ or $2\sqrt{17}$

transverse axis: vertical

foci: $(h, k \pm c) = (-1, 7 \pm 2\sqrt{17})$

vertices: $(h, k \pm a) =$
$(-1, 7 \pm 8)$ or $(-1, 15), (-1, -1)$

asymptotes: $y - k = \pm\frac{a}{b}(x - h)$
$$y - 7 = \pm\frac{8}{2}[x - (-1)]$$
$$y - 7 = \pm 4(x + 1)$$



19. $x^2 - 4y^2 + 6x - 8y = 11$

$(x^2 + 6x + ?) - 4(y^2 + 2y + ?) = 11 + ? + ?$

$(x^2 + 6x + 9) - 4(y^2 + 2y + 1) = 11 + 9 + (-4)(1)$

$(x + 3)^2 - 4(y + 1)^2 = 16$

$\frac{(x+3)^2}{16} - \frac{(y+1)^2}{4} = 1$

center: $(h, k) = (-3, -1)$

$a^2 = 16$ $\quad b^2 = 4$ $\quad c = \sqrt{a^2 + b^2}$

$a = \sqrt{16}$ or 4 $\quad b^2 = \sqrt{4}$ or 2 $\quad c = \sqrt{16 + 4}$ or $2\sqrt{5}$

transverse axis: horizontal

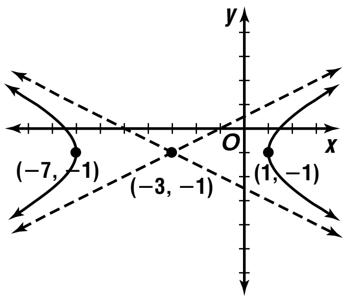
foci: $(h \pm c, k) = (-3 \pm 2\sqrt{5}, -1)$

vertices: $(h \pm a, k) = (-3 \pm 4, -1)$ or $(1, -1)$, $(-7, -1)$

asymptotes: $y - k = \pm \frac{b}{a}(x - h)$

$y - (-1) = \pm \frac{2}{4}[x - (-3)]$

$y + 1 = \pm\frac{1}{2}(x + 3)$



20. $-4x + 9y^2 - 24x - 90y + 153 = 0$

$9(y^2 - 10y + ?) - 4(x^2 + 6x + ?) = -153$

$9(y^2 - 10y + 25) - 4(x^2 + 6x + 9) = -153 + 9(25) - 4(9)$

$9(y - 5)^2 - 4(x + 3)^2 = 36$

$\frac{(y-5)^2}{4} - \frac{(x+3)^2}{9} = 1$

center: $(h, k) = (-3, 5)$

$a^2 = 4$ $\quad b^2 = 9$ $\quad c = \sqrt{a^2 + b^2}$

$a = \sqrt{4}$ or 2 $\quad b = \sqrt{9}$ or 3 $\quad c = \sqrt{4 + 9}$ or $\sqrt{13}$

transverse axis: vertical

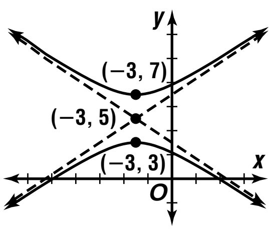
foci: $(h, k \pm c) = (-3, 5 \pm \sqrt{13})$

vertices: $(h, k \pm a) = (-3, 5 \pm 2)$ or $(-3, 7)$, $(-3, 3)$

asymptotes: $y - k = \pm \frac{a}{b}(x - h)$

$y - 5 = \pm\frac{2}{3}[x - (-3)]$

$y - 5 = \pm\frac{2}{3}(x + 3)$



21. $16y^2 - 25x^2 - 96y + 100x - 356 = 0$

$16(y^2 - 6y + ?) - 25(x^2 - 4x + ?) = 356$

$16(y^2 - 6y + 9) - 25(x^2 - 4x + 4) = 356 + 16(9) - 25(4)$

$16(y - 3)^2 - 25(x - 2)^2 = 400$

$\frac{(y-3)^2}{25} - \frac{(x-2)^2}{16} = 1$

center: $(h, k) = (2, 3)$

$a^2 = 25$ $\quad b^2 = 16$ $\quad c = \sqrt{a^2 + b^2}$

$a = \sqrt{25}$ or 5 $\quad b = \sqrt{16}$ or 4 $\quad c = \sqrt{25 + 16}$ or $\sqrt{41}$

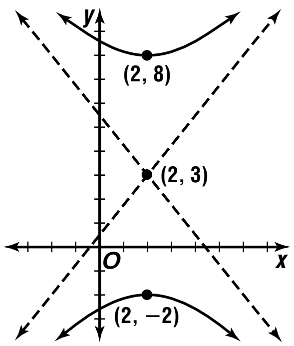
transverse axis: vertical

foci: $(h, k \pm c) = (2, 3 \pm \sqrt{41})$

vertices: $(h, k \pm a) = (2, 3 \pm 5)$ or $(2, 8)$, $(2, -2)$

asymptotes: $y - k = \pm\frac{a}{b}(x - h)$

$y - 3 = \pm\frac{5}{4}(x - 2)$



22. $36x^2 - 49y^2 - 72x - 294y = 2169$

$36(x^2 - 2x + ?) - 49(y^2 + 6y + ?) = 2169 + ? + ?$

$36(x - 2x + 1) - 49(y^2 + 6y + 9) = 2169 + 36(1) - 49(9)$

$36(x - 1)^2 - 49(y + 3)^2 = 1764$

$\frac{(x-1)^2}{49} - \frac{(y+3)^2}{36} = 1$

center: $(h, k) = (1, -3)$

$a^2 = 49$ $\quad b^2 = 36$ $\quad c = \sqrt{a^2 + b^2}$

$a = \sqrt{49}$ or 7 $\quad b = \sqrt{36}$ or 6 $\quad c = \sqrt{49 + 36}$ or $\sqrt{85}$

transverse axis: horizontal

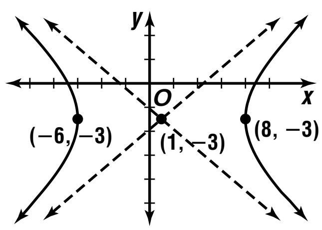
foci: $(h \pm c, k) = (1 \pm \sqrt{85}, -3)$

vertices: $(h \pm a, k) = (1 \pm 7, -3)$ or $(8, -3)$, $(-6, -3)$

asymptotes: $y - k = \pm\frac{b}{a}(x - h)$

$y - (-3) = \pm\frac{6}{7}(x - 1)$

$y + 3 = \pm\frac{6}{7}(x - 1)$



23. $25y^2 - 9x^2 - 100y - 72x - 269 = 0$

$$25(y^2 - 4y + ?) - 9(x^2 + 8x + ?) = 269 + ? + ?$$
$$25(y^2 - 4y + 4) - 9(x^2 + 8x + 16) = 269 + 25(4) - 9(16)$$
$$25(y - 2)^2 - 9(x + 4)^2 = 225$$
$$\frac{(y-2)^2}{9} - \frac{(x+4)^2}{25} = 1$$

center: $(h, k) = (-4, 2)$

$a^2 = 9$ $\quad b^2 = 25$ $\quad c = \sqrt{a^2 + b^2}$

$a = \sqrt{9}$ $\quad b = \sqrt{25}$ $\quad c = \sqrt{9 + 25}$

or 3 $\quad$ or 5 $\quad$ or $\sqrt{34}$

transverse axis: vertical

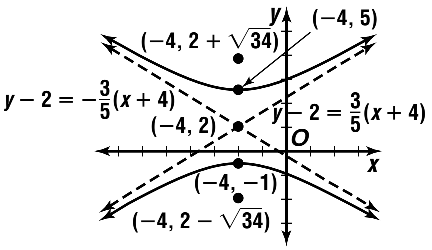
foci: $(h, k \pm c) = \left(-4, 2 \pm \sqrt{34}\right)$

vertices: $(h, k \pm a) =$

$(-4, 2 \pm 3)$ or $(-4, 5)$, $(-4, -1)$

asymptotes: $y - k = \pm\frac{a}{b}(x - h)$

$$y - 2 = \pm\frac{3}{5}[x - (-4)]$$
$$y - 2 = \pm\frac{3}{5}(x + 4)$$



24. center: $(h, k) = (4, 3)$

transverse axis: vertical

$a = 4$, $b = 3$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$
$$\frac{(y-3)^2}{4^2} - \frac{(x-4)^2}{3^2} = 1$$
$$\frac{(y-3)^2}{16} - \frac{(x-4)^2}{9} = 1$$

25. center: $(h, k) = (0, 0)$

transverse axis: horizontal

$a = 3$, $b = 3$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-0)^2}{3^2} - \frac{(y-0)^2}{3^2} = 1$$
$$\frac{x^2}{9} - \frac{y^2}{9} = 1$$

26. center: $(h, k) = (-4, 0)$

transverse axis: vertical

$a = 2$, $b = 1$

$$\frac{(y-k)^2}{a^2} - \frac{(x-b)^2}{b^2} = 1$$
$$\frac{(y-0)^2}{2^2} - \frac{[x-(-4)]^2}{1^2} = 1$$
$$\frac{y^2}{4} - \frac{(x+4)^2}{1} = 1$$

27. $c = 49$

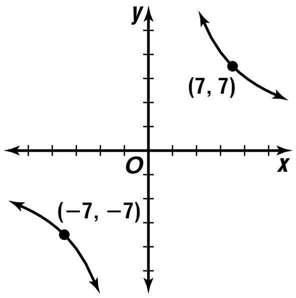
quadrants: I and III

transverse axis: $y = x$

vertices: $xy = 49$ $\quad xy = 49$

$7(7) = 49$ $\quad -7(-7) = 49$

$(7, 7)$ $\quad (-7, -7)$



28. $c = -36$

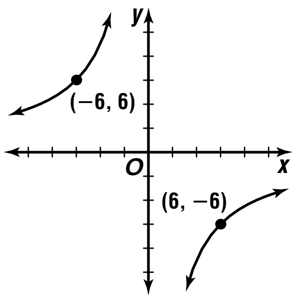
quadrants: II and IV

transverse axis: $y = -x$

vertices: $xy = -36$ $\quad xy = -36$

$6(-6) = 36$ $\quad -6(6) = -36$

$(6, -6)$ $\quad (-6, 6)$



29. $4xy = -25$

$$xy = -\frac{25}{4}$$
$$c = -\frac{25}{4}$$

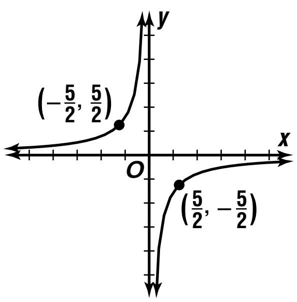
quadrants: II and IV

transverse axis: $y = -x$

vertices: $xy = -\frac{25}{4}$ $\quad xy = -\frac{25}{4}$

$\frac{5}{2}\left(-\frac{5}{2}\right) = -\frac{25}{4}$ $\quad -\frac{5}{2}\left(\frac{5}{2}\right) = -\frac{25}{4}$

$\left(\frac{5}{2}, -\frac{5}{2}\right)$ $\quad \left(-\frac{5}{2}, \frac{5}{2}\right)$



30. $9xy = 16$

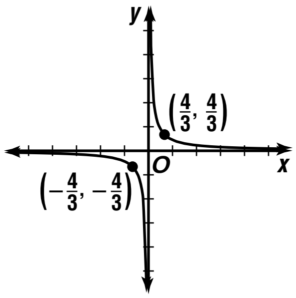
$$xy = \frac{16}{9}$$
$$c = \frac{16}{9}$$

quadrants: I and III

transverse axis: $y = x$

vertices: $xy = \frac{16}{9}$ $\qquad$ $xy = \frac{16}{9}$

$$\frac{4}{3}\left(\frac{4}{3}\right) = \frac{16}{9} \qquad -\frac{4}{3}\left(-\frac{4}{3}\right) = \frac{16}{9}$$
$$\left(\frac{4}{3}, \frac{4}{3}\right) \qquad \left(-\frac{4}{3}, -\frac{4}{3}\right)$$



31. center: $(h, k) = (4, -2)$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$
$$\frac{[y-(-2)]^2}{2^2} - \frac{(x-4)^2}{3^2} = 1$$
$$\frac{(y+2)^2}{4} - \frac{(x-4)^2}{9} = 1$$

32. center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0+0}{2}, \frac{3+(-3)}{2}\right)$

$= (0, 0)$

transverse axis: vertical

a = distance from center to a vertex

$= |0 - 3]$ or 3

c = distance from center to a focus

$= |0 - (-9)|$ or 9

$b^2 = c^2 - a^2$

$b^2 = 9^2 - 3^2$ or 72

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$
$$\frac{(y-0)^2}{3^2} - \frac{(x-0)^2}{72} = 1$$
$$\frac{y^2}{9} - \frac{x^2}{72} = 1$$

33. $2a = 6$

$a = 3$

center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{5+(-5)}{2}, \frac{2+2}{2}\right)$

$= (0, 2)$

transverse axis: horizontal

c = distance from center to a focus

$= |0 - 5|$ or 5

$b^2 = c^2 - a^2$

$b^2 = 5^2 - 3^2$ or 16

$$\frac{(x-b)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-0)^2}{3^2} - \frac{(y-2)^2}{16} = 1$$
$$\frac{x^2}{9} - \frac{(y-2)^2}{16} = 1$$

34. $2b = 8$

$b = 4$

center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-3+(-3)}{2}, \frac{9+(-5)}{2}\right)$

$= (-3, 2)$

transverse axis: vertical

a = distance from center to a vertex

$= |2 - 9|$ or 7

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$
$$\frac{(y-2)^2}{7^2} - \frac{[x-(-3)]^2}{4^2} = 1$$
$$\frac{(y-2)^2}{49} - \frac{(x+3)^2}{16} = 1$$

35. center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{8+(-8)}{2}, \frac{0+0}{2}\right)$

$= (0, 0)$

transverse axis: horizontal

c = distance from center to a focus

$= |0 - 8|$ or 8

$b^2 = c^2 - a^2$ $\qquad$ $b^2 = a^2$

$a^2 = c^2 - a^2$ $\qquad$ $b^2 = 32$

$2a^2 = c^2$

$$a^2 = \frac{c^2}{2}$$
$$a^2 = \frac{8^2}{2} \text{ or } 32$$
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-0)^2}{32} - \frac{(y-0)^2}{32} = 1$$
$$\frac{x^2}{32} - \frac{y^2}{32} = 1$$

36. centers: $(h, k) = (-3, 1)$

c = distance from center to a focus

$= |-3 - 2|$ or 5

$e = \frac{c}{a}$ $\qquad$ $b^2 = c^2 - a^2$

$\frac{5}{4} = \frac{5}{a}$ $\qquad$ $b^2 = 5^2 - 4^2$

$\qquad$ $b^2 = 9$

$a = 4$

transverse axis: horizontal

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$
$$\frac{[x-(-3)]^2}{4^2} - \frac{(y-1)^2}{9} = 1$$
$$\frac{(x+3)^2}{16} - \frac{(y-1)^2}{9} = 1$$

37. center: $(h, k) = (4, 2)$

a = distance from center to a vertex

$= |2 - 5|$ or 3

transverse axis: vertical

$$4y + 4 = 3x$$
$$4y + 4 - 12 = 3x - 12$$
$$4y - 8 = 3x - 12$$
$$4(y - 2) = 3(x - 4)$$
$$y - 2 = \frac{3}{4}(x - 4)$$
$$\frac{a}{b} = \frac{3}{4}$$
$$\frac{3}{b} = \frac{3}{4}$$
$$b = 4$$
$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$
$$\frac{(y-2)^2}{3^2} - \frac{(x-4)^2}{4^2} = 1$$
$$\frac{(y-2)^2}{9} - \frac{(x-4)^2}{16} = 1$$

38. center: $(h, k) = (3, -1)$
a = distance from center to a vertex
$= |3 - 5|$ or 2
transverse axis: horizontal
$$3x - 11 = 2y$$
$$3x - 11 - 4 = 2y - 4$$
$$3x - 15 = 2y - 4$$
$$3(x - 5) = 2(y - 2)$$
$$\frac{3}{2}(x - 5) = y - 2$$
$$y - 2 = \frac{3}{2}(x - 5)$$
$$\frac{b}{a} = \frac{3}{2}$$
$$\frac{b}{2} = \frac{3}{2}$$
$$b = 3$$
$$\frac{(x - b)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$
$$\frac{(x - 3)^2}{2^2} - \frac{[y - (-1)]^2}{3^2} = 1$$
$$\frac{(x - 3)^2}{4} - \frac{(y + 1)^2}{9} = 1$$

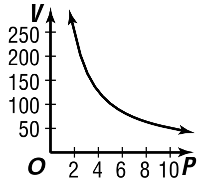
39. center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0 + 0}{2}, \frac{8 + (-8)}{2}\right)$
$= (0, 0)$
c = distance from center to a focus
$= |0 - 8|$ or 8
$$e = \frac{c}{a} \qquad b^2 = c^2 - a^2$$
$$\frac{4}{3} = \frac{8}{a} \qquad b^2 = 8^2 - 6^2$$
$$b^2 = 28$$
$$a = 6$$
transverse axis: vertical
$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$$
$$\frac{(y - 0)^2}{6^2} - \frac{(x - 0)^2}{28} = 1$$
$$\frac{y^2}{36} - \frac{x^2}{28} = 1$$

40. centers: $\left(\frac{x_1 - x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{10 + (-2)}{2}, \frac{-3 + (-3)}{2}\right)$
$= (4, -3)$
c = distance from center to a focus
$= |4 - 10|$ or 6
$$e = \frac{c}{a} \qquad b^2 = c^2 - a^2$$
$$\frac{6}{5} = \frac{6}{a} \qquad b^2 = 6^2 - 5^2$$
$$b^2 = 11$$
$$a = 5$$
transverse axis: horizontal
$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$
$$\frac{(x - 4)^2}{5^2} - \frac{[y - (-3)]}{11} = 1$$
$$\frac{(x - 4)^2}{25} - \frac{(y + 3)^2}{11} = 1$$

41. center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{9 + (-9)}{0}, \frac{0 + 0}{2}\right)$
$= (0, 0)$
c = distance from center to a focus
$= |0 - 9|$ or 9
$$b^2 = c^2 - a^2 \qquad b^2 = a^2$$
$$a^2 = 9^2 - a^2 \qquad b^2 = \frac{81}{2}$$
$$2a^2 = 81$$
$$a^2 = \frac{81}{2}$$
transverse axis: horizontal
$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$
$$\frac{(x - 0)^2}{\frac{81}{2}} - \frac{(y - 0)^2}{\frac{81}{2}} = 1$$
$$\frac{2x^2}{81} - \frac{2y^2}{81} = 1$$

42. center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1 + 1}{2}, \frac{5 + (-3)}{2}\right)$
$= (1, 1)$
c = distance from center to a focus
$= |1 - 5|$ or 4
transverse axis: vertical
$$\frac{a}{b} = \pm 2 \qquad a^2 = (2b)^2$$
$$a = 2b \qquad a^2 = 4\left(\frac{16}{5}\right)$$
$$c^2 = a^2 + b^2 \qquad a^2 = \frac{64}{5}$$
$$4^2 = (2b)^2 + b^2$$
$$16 = 5b^2$$
$$\frac{16}{5} = b^2$$
$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$$
$$\frac{(y - 1)^2}{\frac{64}{5}} - \frac{(x - 1)^2}{\frac{16}{5}} = 1$$
$$\frac{5(y - 1)^2}{64} - \frac{5(x - 1)^2}{16} = 1$$

43a. quadrants: I and II
transverse axis: $y = x$



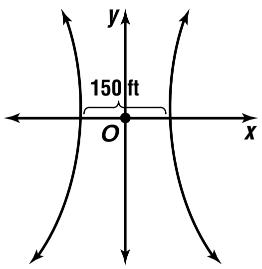
43b.
$$PV = 505$$
$$(101)V = 505$$
$$V = 5.0 \text{ dm}^3$$

43c.
$$PV = 505$$
$$(50.5)V = 505$$
$$V = 10.0 \text{ dm}^3$$

43d. If the pressure is halved, then the volume is doubled, or $V = 2$(original V).

44. In an equilateral hyperbola, $a = b$ and $c^2 = a^2 + b^2$.
$$c^2 = a^2 + a^2 \qquad a = b$$
$$c^2 = 2a^2$$
$$c = a\sqrt{2}$$
Since $e = \frac{c}{a}$, we have
$$e = \frac{c}{a}$$
$$e = \frac{a\sqrt{2}}{a}$$
$$e = \sqrt{2}$$
Thus, the eccentricity of any equilateral hyperbola is $\sqrt{2}$.

45a.



$2a = 150$
$a = 75$

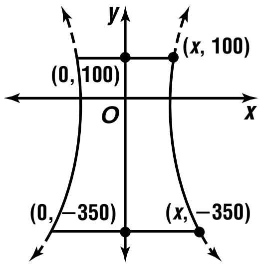
$e = \frac{c}{a}$
$\frac{5}{3} = \frac{c}{75}$
$125 = c$

$b^2 = c^2 - a^2$
$b^2 = 125^2 - 75^2$
$b^2 = 10{,}000$
$b = 100$

transverse axis: horizontal
center: $(h, k) = (0, 0)$

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
$\frac{(x-0)^2}{75^2} - \frac{(y-0)^2}{100^2} = 1$
$\frac{x^2}{75^2} - \frac{y^2}{100^2} = 1$

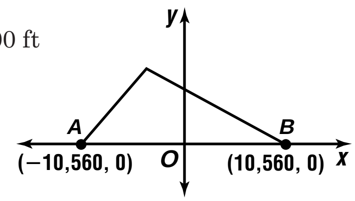
45b.



top: $\frac{x^2}{75^2} - \frac{y^2}{100^2} = 1$
$\frac{x^2}{75^2} - \frac{100^2}{100^2} = 1 \qquad (x, y) = (x, 100)$
$\frac{x^2}{75^2} - 1 = 1$
$\frac{x^2}{75^2} = 2$
$x^2 = 11{,}250$
$x \approx 106.07$ ft

base: $\frac{x^2}{75^2} - \frac{y^2}{100^2} = 1$
$\frac{x^2}{75^2} - \frac{(-350)^2}{100^2} = 1 \qquad (x, y) = (x, -350)$
$\frac{x^2}{75^2} - 12.25 = 1$
$\frac{x^2}{75^2} = 13.25$
$x^2 = 74{,}531.25$
$x \approx 273.00$ ft

46. The origin is located midway between stations A and B. The stations are located at the foci, so $2c = 4$ or $c = 2$ miles.
$c = 2$ mi
$c = 2 \text{ mi} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}}$
$c = 10{,}560$ ft
$d = rt$
$d = 1100(2)$ or 2200 ft

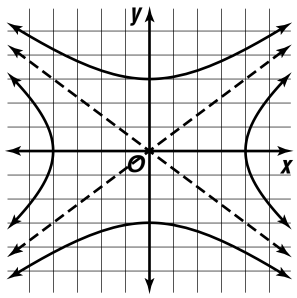


The lightning is 2200 feet farther from station B than from station A. The difference of distances equals $2a$.
$2200 = 2a$ (Definition of hyperbola)
$1100 = a$
$b^2 = c^2 - a^2$
$b = \sqrt{c^2 - a^2}$
$b = \sqrt{10{,}560^2 - 1100^2}$
$b \approx 10{,}503$
center: $(h, k) = (0, 0)$
transverse axis: horizontal
$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
$\frac{(x-0)^2}{1100^2} - \frac{(y-0)^2}{10{,}503^2} = 1$
$\frac{x^2}{1100^2} - \frac{y^2}{10{,}503^2} = 1$

47. center: $(h, k) = (0, 0)$
$c = |0 - 6|$ or 6
$|PF_1 - PF_2| = 10$
$2a = 10 \qquad |PF_1 - PF_2| = 2a$
$a = 5$
$b^2 = c^2 - a^2$
$b^2 = 6^2 - 5^2$
$b^2 = 11$
transverse axis: horizontal
$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
$\frac{(x-0)^2}{5^2} - \frac{(y-0)^2}{11} = 1$
$\frac{x^2}{25} - \frac{y^2}{11} = 1$

48a. $\frac{x^2}{16} - \frac{y^2}{9} = 1$
center: $(h, k) = (0, 0)$
$a^2 = 16 \qquad b^2 = 9$
$a = \sqrt{16}$ or 4 $\qquad b = \sqrt{9}$ or 3
transverse axis: horizontal
vertices: $(h \pm a, k) = (0 \pm 4, 0)$ or $(\pm 4, 0)$
asymptotes: $y - k = \pm\frac{b}{a}(x - h)$
$y - 0 = \pm\frac{3}{4}(x - h)$
$y = \pm\frac{3}{4}x$

$\frac{y^2}{9} - \frac{x^2}{16} = 1$
center: $(h, k) = (0, 0)$
$a^2 = 9 \qquad b^2 = 16$
$a = \sqrt{9}$ or 3 $\qquad b = \sqrt{16}$ or 4
transverse axis: vertical
vertices: $(h, k \pm a) = (0, 0 \pm 3)$ or $(0, \pm 3)$
asymptotes: $y - k = \pm\frac{a}{b}(x - h)$
$y - 0 = \pm\frac{3}{4}(x - 0)$
$y = \pm\frac{3}{4}x$



48b. They are the same lines.

48c. $\frac{(y-2)^2}{25} - \frac{(x-3)^2}{16} = 1$

48d. $\frac{(x-3)^2}{16} - \frac{(y-2)^2}{25} = 1$

center: $(h, k) = (3, 2)$

$a^2 = 16$ $\qquad$ $b^2 = 25$

$a = \sqrt{16}$ or 4 $\qquad$ $b = \sqrt{25}$ or 5

transverse axis: horizontal

vertices: $(h \pm a, k) = (3 \pm 4, 2)$ or $(7, 2), (-1, 2)$

asymptotes: $y - k = \pm\frac{b}{a}(x - h)$

$$y - 2 = \pm\frac{5}{4}(x - 3)$$

$\frac{(y-2)^2}{25} - \frac{(x-3)^2}{16} = 1$

center: $(h, k) = (3, 2)$

$a^2 = 25$ $\qquad$ $b^2 = 16$

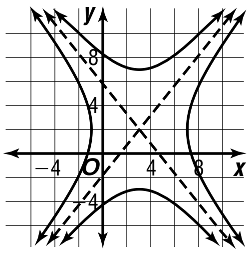
$a = \sqrt{25}$ or 5 $\qquad$ $b = \sqrt{16}$ or 4

transverse axis: vertical

vertices: $(h, k \pm a) = (3, 2 \pm 5)$ or $(3, 7), (3, -3)$

asymptotes: $y - k = \pm\frac{a}{b}(x - h)$

$$y - 2 = \pm\frac{5}{4}(x - 3)$$



49. center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2+2}{2}, \frac{3+(-3)}{2}\right)$

$= (2, 0)$

$a = 4$

$c =$ distance from center to a focus

$= |0 - 3|$ or 3

$b^2 = a^2 - c^2$

$b^2 = 4^2 - 3^2$ or 7

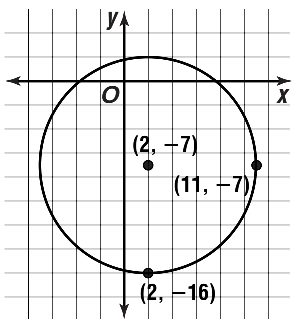
major axis: vertical

$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$

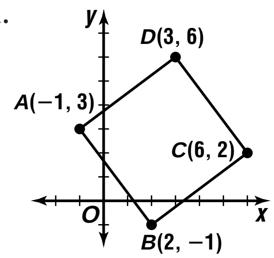
$\frac{(y-0)^2}{4^2} + \frac{(x-2)^2}{7} = 1$

$\frac{y^2}{16} + \frac{(x-2)^2}{7} = 1$

50.
$$x^2 + y^2 - 4x + 14y - 28 = 0$$
$$(x^2 - 4x + ?) + (y^2 + 14y + ?) = 28 + ? + ?$$
$$(x^2 - 4x + 4) + (y^2 + 14y + 49) = 28 + 4 + 49$$
$$(x - 2)^2 + (y + 7)^2 = 81$$



51.



$AB = \sqrt{(2+1)^2 + (-1-3)^2} = 5$

$BC = \sqrt{(2-6)^2 + (-1-2)^2} = 5$

$CD = \sqrt{(6-3)^2 + (2-6)^2} = 5$

$AD = \sqrt{(3+1)^2 + (6-3)^2} = 5$

Thus, $ABCD$ is a rhombus. The slope of $\overline{AD} = \frac{6-3}{3+1}$ or $\frac{3}{4}$ and the slope of $\overline{AB} = \frac{3+1}{-1-2}$ or $-\frac{4}{3}$. Thus, $\overline{AD}$ is perpendicular to $\overline{AB}$ and $ABCD$ is a square.

52. $(r, \theta) = (90, 208°)$

$(r, \theta + 360\,k°) = (90, 208° + 360(-1)°)$

$= (90, -152°)$

$\big(-r, \theta + (2k+1)(180°)\big)$

$= \big(-90, 208 + (2(-1) + 1)(180°)\big)$

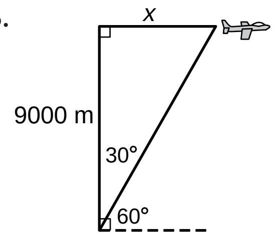
$= (-90, 28°)$

53. $4(-5) + -1(2) + 8(2) = -6$

No, the inner product of the two vectors is not zero.

54.
$$x \cos \phi + y \sin \phi - p = 0$$
$$x \cos 60 + y \sin 60 - 3 = 0$$
$$\frac{1}{2}x + \frac{\sqrt{3}}{2}y - 3 = 0$$
$$x + \sqrt{3}y - 6 = 0$$

55.



$\tan 30° = \frac{x}{9000}$

$9000 \tan 30° = x$

$5196 \approx x$

$d = rt$

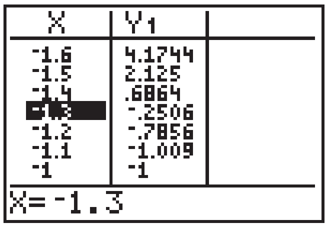
$\frac{d}{t} = r$

$\frac{5196}{15} = r$

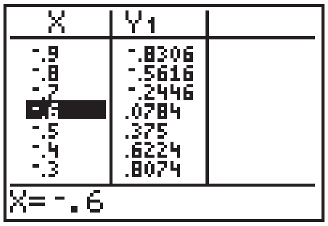
$346.4 \approx r$

about 346 m/s

56.



Since -0.2506 is closer to zero than 0.6864, the zero is about -1.3.



Since 0.0784 is closer to zero than -0.2446, the zero is about -0.6.

57. Case 1: r is positive and s is negative.
Case 2: r is negative and s is positive.
I. $r^3 > s^3$ is false if r is negative.
II. $r^3 = s^2$ is false for each case.
III. $r^4 = s^4$ is true for each case.
The correct choice is C.

10-5 Parabolas

Pages 658–659 Check for Understanding

1. The equation of a parabola will have only one squared term, while the equation of a hyperbola will have two squared terms.

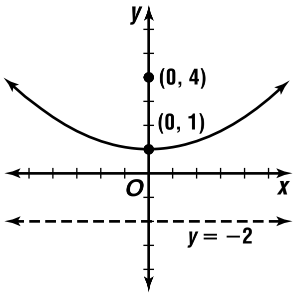
2. vertex: $(h, k) = (2, 1)$
$p = -4$
$(x - h)^2 = 4p(y - k)$
$(x - 2)^2 = 4(-4)(y - 1)$
$(x - 2)^2 = -16(y - 1)$

3. The vertex and focus both lie on the axis of symmetry. The directrix and axis of symmetry are perpendicular to each other. The focus and the point on the directrix collinear with the focus are equidistant from the vertex.

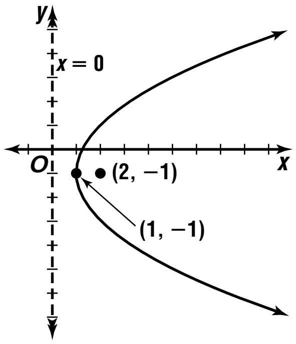
4. $(h, k) = (-4, 5)$
$p = -5$
$(y - k)^2 = 4p(x - h)$
$(y - 5)^2 = 4(-5)[x - (-4)]$
$(y - 5)^2 = -20(x + 4)$

5a. ellipse **5b.** parabola
5c. hyperbola **5d.** circle

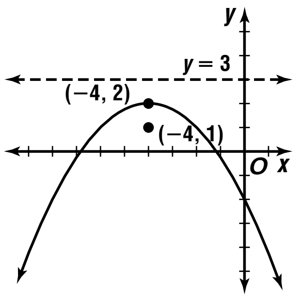
6. vertex: $(h, k) = (0, 1)$
$4p = 12$
$p = 3$
focus: $(h, k + p) = (0, 1 + 3)$ or $(0, 4)$
directrix: $y = k - p$
$y = 1 - 3$
$y = -2$
axis of symmetry: $x = h$
$x = 0$



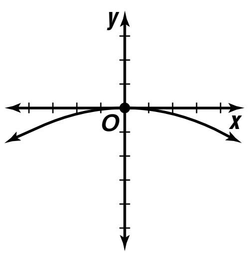
7. $y^2 - 4x + 2y + 5 = 0$
$y^2 + 2y = 4x - 5$
$y^2 + 2y + ? = 4x - 5 + ?$
$y^2 + 2y + 1 = 4x - 5 + 1$
$(y + 1) = 4(x - 1)$
vertex: $(h, k) = (1, -1)$
$4p = 4$
$p = 1$
focus: $(h + p, k) = (1 + 1, -1)$ or $(2, -1)$
directrix: $x = h - p$
$x = 1 - 1$
$x = 0$
axis of symmetry: $y = k$
$y = -1$



8. $x^2 + 8x + 4y + 8 = 0$
$x^2 + 8x = -4y - 8$
$x^2 + 8x + ? = -4y - 8 + ?$
$x^2 + 8x + 16 = -4y - 8 + 16$
$(x + 4)^2 = -4(y - 2)$
vertex: $(h, k) = (-4, 2)$
$4p = -4$
$p = -1$
focus: $(h, k + p) = (-4, 2 + (-1))$ or $(-4, 1)$
directrix: $y = k - p$
$y = 2 - (-1)$
$y = 3$
axis of symmetry: $x = h$
$x = -4$



9. vertex: $(h, k) = (0, 0)$
opening: downward
p $= -4$
$(x - h)^2 = 4p(y - k)$
$(x - 0)^2 = 4(-4)(y - 0)$
$x^2 = -16y$



10. $(y - k)^2 = 4p(x - h)$

$(-1 - 5)^2 = 4p[2 - (-7)]$ *(h, k) = (−7, 5); (x, y) = (2, −1)*

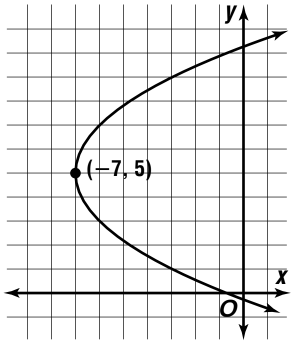
$(-6)^2 = 36p$

$1 = p$

$(y - k)^2 = 4p(x - h)$

$(y - 5)^2 = 4(1)[x - (-7)]$

$(y - 5)^2 = 4(x + 7)$



11. vertex: $(h, k) = (4, -3)$

opening: upward

$(x - h)^2 = 4p(y - k)$

$(5 - 4)^2 = 4p[2 - (-3)]$ *(h, k) = (4, −3); (x, y) = (5, 2)*

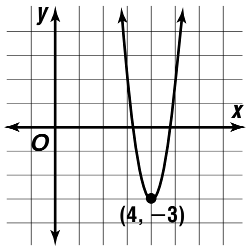
$1^2 = 20p$

$\frac{1}{20} = p$

$(x - h)^2 = 4p(y - k)$

$(x - 4)^2 = 4\left(\frac{1}{20}\right)(y + 3)$

$(x - 4)^2 = \frac{1}{5}(y + 3)$

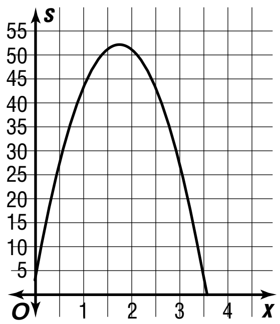


12a.

$$s = v_0t - 16t^2 + 3$$
$$s = 56t - 16t^2 + 3$$
$$-16t^2 + 56t = s - 3$$
$$-16\left(t^2 - \frac{7}{5}t + ?\right) = s - 3 + ?$$
$$-16\left(t^2 - \frac{7}{2}t + \frac{49}{16}\right) = s - 3 + (-16)\left(\frac{49}{16}\right)$$
$$-16\left(t - \frac{7}{4}\right)^2 = s - 52$$
$$\left(t - \frac{7}{4}\right)^2 = -\frac{1}{16}(s - 52)$$
$$\left(t - \frac{7}{4}\right)^2 = 4\left(-\frac{1}{64}\right)(s - 52)$$

vertex: $(h, k) = \left(\frac{7}{4}, 52\right)$

opening: downward



12b. The maximum height is $s = 52$ ft.

12c. Let $s = 0$.

$s = 56t - 16t^2 + 3$

$0 = -16t^2 + 56t + 3$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{-56 \pm \sqrt{56^2 - 4(-16)(3)}}{2(-16)}$

$t \approx 3.6$ or -0.05

3.6 s

Pages 659–661 Exercises

13. vertex: $(h, k) = (0, 0)$

$4p = 8$

$p = 2$

focus: $(h + p, k) = (0 + 2, 0)$ or $(2, 0)$

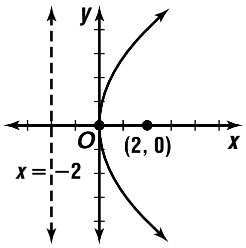
directrix: $x = h - p$

$x = 0 - 2$

$x = -2$

axis of symmetry: $y = k$

$y = 0$



14. vertex: $(h, k) = (0, 3)$

$4p = -4$

$p = 1$

focus: $(h, k + p) = (0, 3 + (-1))$ or $(0, 2)$

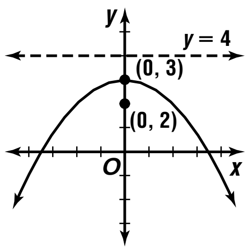
directrix: $y = k - p$

$y = 3 - (-1)$

$y = 4$

axis of symmetry: $x = h$

$x = 0$



15. vertex: $(h, k) = (0, 6)$

$4p = 4$

$p = 1$

focus: $(h + p, k) = (0 + 1, 6)$ or $(1, 6)$

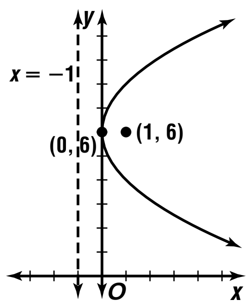
directrix: $x = h - p$

$x = 0 - 1$

$x = -1$

axis of symmetry: $y = k$

$y = 6$



16.

$$
\begin{aligned}
y^2 + 12x &= 2y - 13\\
y^2 - 2y &= -12x - 13\\
y^2 - 2y + ? &= -12x - 13 + ?\\
y^2 - 2y + 1 &= -12x - 13 + 1\\
(y - 1)^2 &= -12(x + 1)
\end{aligned}
$$

vertex: $(h, k) = (-1, 1)$

$4p = -12$

$p = -3$

focus: $(h + p, k) = (-1 + (-3), 1)$ or $(-4, 1)$

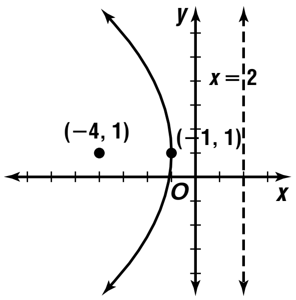
directrix: $x = h - p$

$x = -1 - (-3)$

$x = 2$

axis of symmetry: $y = k$

$y = 1$



17.

$$
\begin{aligned}
y - 2 &= x^2 - 4x\\
x^2 - 4x &= y - 2\\
x^2 - 4x + ? &= y - 2 + ?\\
x^2 - 4x + 4 &= y - 2 + 4\\
(x - 2)^2 &= y + 2
\end{aligned}
$$

vertex: $(h, k) = (2, -2)$

$4p = 1$

$p = \frac{1}{4}$

focus: $(h, k + p) = \left(2, -2 + \frac{1}{4}\right)$ or $\left(2, -\frac{7}{4}\right)$

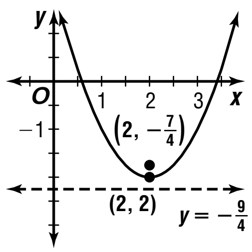
directrix: $y = k - p$

$y = -2 - \frac{1}{4}$

$y = -\frac{9}{4}$

axis of symmetry: $x = h$

$x = 2$



18.

$$
\begin{aligned}
x^2 + 10x + 25 &= -8y + 24\\
(x + 5)^2 &= -8(y - 3)
\end{aligned}
$$

vertex: $(h, k) = (-5, 3)$

$4p = -8$

$p = -2$

focus: $(h, k + p) = (-5, 3 + (-2))$ or $(-5, 1)$

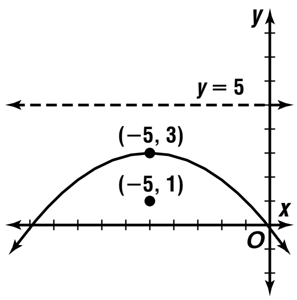
directrix: $y = k - p$

$y = 3 - (-2)$

$y = 5$

axis of symmetry: $x = h$

$x = -5$



19.

$$
\begin{aligned}
y^2 - 2x + 14y &= -41\\
y^2 + 14y &= 2x - 41\\
y^2 + 14y + ? &= 2x - 41 + ?\\
y^2 + 14y + 49 &= 2x - 41 + 49\\
(y + 7)^2 &= (2x + 4)
\end{aligned}
$$

vertex: $(h, k) = (-4, -7)$

$4p = 2$

$p = \frac{2}{4}$ or $\frac{1}{2}$

focus: $(h + p, k) = \left(-4 + \frac{1}{2}, -7\right)$ or $\left(-\frac{7}{2}, -7\right)$

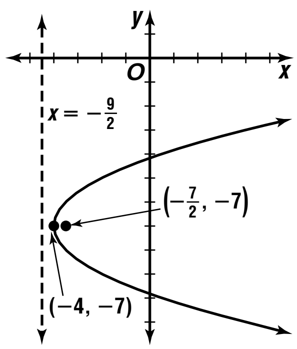
directrix: $x = h - p$

$x = -4 - \frac{1}{2}$

$x = -\frac{9}{2}$

axis of symmetry: $y = k$

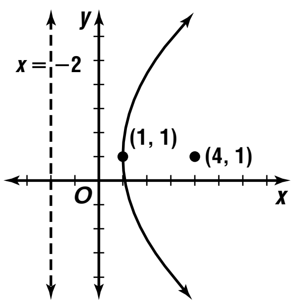
$y = -7$



20. $y^2 - 2y - 12x + 13 = 0$

$$y^2 - 2y = 12x - 13$$
$$y^2 - 2y + ? = 12x - 13 + ?$$
$$y^2 - 2y + 1 = 12x - 13 + 1$$
$$(y - 1)^2 = 12(x - 1)$$

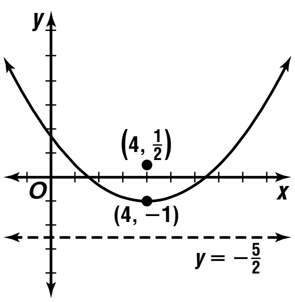
vertex: $(h, k) = (1, 1)$
$4p = 12$
$p = 3$
focus: $(h + p, k) = (1 + 3, 1)$ or $(4, 1)$
directrix: $x = h - p$
$x = 1 - 3$
$x = -2$
axis of symmetry: $y = k$
$y = 1$



21. $2x^2 - 12y - 16x + 20 = 0$

$$2x^2 - 16x = 12y - 20$$
$$2(x^2 - 8x + ?) = 12y - 20 + ?$$
$$2(x^2 - 8x + 16) = 12y - 20 + 2(16)$$
$$2(x - 4)^2 = 12y + 12$$
$$(x - 4)^2 = 6(y + 1)$$

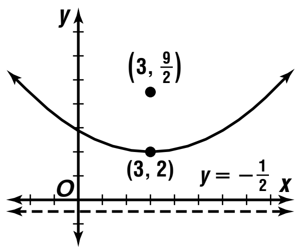
vertex: $(h, k) = (4, -1)$
$4p = 6$
$p = \frac{6}{4}$ or $\frac{3}{2}$
focus: $(h, k + p) = \left(4, -1 + \frac{3}{2}\right)$ or $\left(4, \frac{1}{2}\right)$
directrix: $y = k - p$
$y = -1 - \frac{3}{2}$
$y = -\frac{5}{2}$
axis of symmetry: $x = h$
$x = 4$



22. $3x^2 - 30x - 18x + 87 = 0$

$$3x^2 - 18x = 30y - 87$$
$$3(x^2 - 6x + ?) = 30y - 87 + ?$$
$$3(x^2 - 6x + 9) = 30y - 87 + 3(9)$$
$$3(x - 3)^2 = 30y - 60$$
$$(x - 3)^2 = 10y - 20$$
$$(x - 3)^2 = 10(y - 2)$$

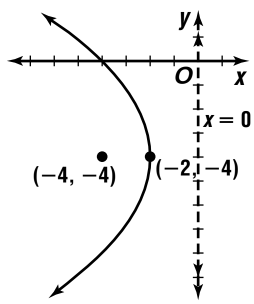
vertex: $(h, k) = (3, 2)$
$4p = 10$
$p = \frac{10}{4}$ or $\frac{5}{2}$
focus: $(h, k + p) = \left(3, 2 + \frac{5}{2}\right)$ or $\left(3, \frac{9}{2}\right)$
directrix: $y = k - p$
$y = 2 - \frac{5}{2}$
$y = -\frac{1}{2}$
axis of symmetry: $x = h$
$x = 3$



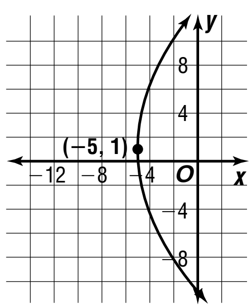
23. $2y^2 + 16y + 16x + 64 = 0$

$$2y^2 + 16y = -16x - 64$$
$$2(y^2 + 8y + ?) = -16x - 64 + ?$$
$$2(y^2 + 8y + 16) = -16x - 64 + 2(16)$$
$$2(y + 4)^2 = -16x - 32$$
$$(y + 4)^2 = -8x - 16$$
$$(y + 4)^2 = -8(x + 2)$$

vertex: $(h, k) = (-2, -4)$
$4p = -8$
$p = -2$
focus: $(h + p, k) = (-2 + (-2), -4)$ or $(-4, -4)$
directrix: $x = h - p$
$x = -2 - (-2)$
$x = 0$
axis of symmetry: $y = k$
$y = -4$



24. vertex: $(h, k) = (-5, 1)$
opening: right
$h + p = 2$
$-5 + p = 2$
$p = 7$
$(y - k)^2 = 4p(x - h)$
$(y - 1)^2 = 4(7)[x - (-5)]$
$(y - 1)^2 = 28(x + 5)$



25. opening: left

focus: $(h + p, k) = (0, 6)$

$h + p = 0$ $\quad\quad$ $k = 6$

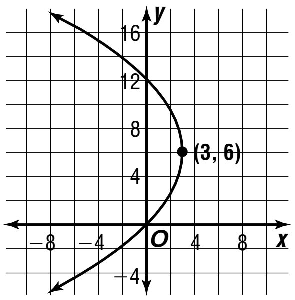
$h + (-3) = 0$

$h = 3$

$(y - k)^2 = 4p(x - h)$

$(y - 6)^2 = 4(-3)(x - 3)$

$(y - 6)^2 = -12(x - 3)$



26. opening: upward

vertex: $(h, k) = \left(4, \frac{5-1}{2}\right)$ or $(4, -3)$

focus: $(h, k + p) = (4, -1)$

$k + p = -1$

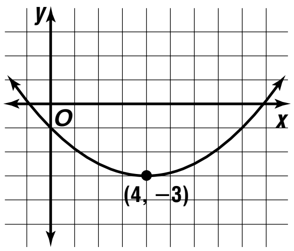
$-3 + p = -1$

$p = 2$

$(x - b)^2 = 4p(y - k)$

$(x - 4)^2 = 4(2)[y - (-3)]$

$(x - 4)^2 = 8(y + 3)$



27. opening: downward

vertex: $(h, k) = (4, 3)$

$(x - h)^2 = 4p(y - k)$

$(5 - 4)^2 = 4p(2 - 3)$ $\quad\quad$ *(h, k) = (4, 3);*

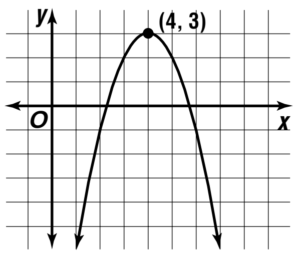
$1^2 = -4p$ $\quad\quad$ *(x, y) = (5, 2)*

$-\frac{1}{4} = p$

$(x - h)^2 = 4p(y - k)$

$(x - 4)^2 = 4\left(-\frac{1}{4}\right)(y - 3)$

$(x - 4)^2 = -(y - 3)$



28. vertex: $(h, k) = (-2, -3)$

$(y - k)^2 = 4p(x - h)$

$[1 - (-3)]^2 = 4p[-3 - (-2)]$ $\quad\quad$ *(h, k) = (−2, −3);*

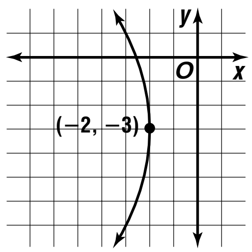
$4^2 = 4p$ $\quad\quad$ *(x, y) = (−3, 1)*

$-4 = p$

$(y - k)^2 = 4p(x - h)$

$(y + 3)^2 = 4(-4)(x + 2)$

$(y + 3)^2 = -16(x + 2)$



29. opening: upward

$p = 2$

focus: $(h, k + p) = (-1, 7)$

$h = -1$ $\quad\quad$ $k + p = 7$

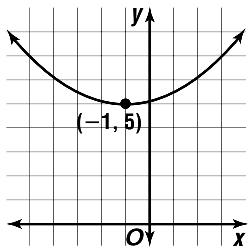
$k + 2 = 7$

$k = 5$

$(x - h)^2 = 4p(y - k)$

$[x - (-1)]^2 = 4(2)(y - 5)$

$(x + 1)^2 = 8(y - 5)$



30. opening: downward

vertex: $(h, k) = (5, -3)$ $\quad\quad$ (maximum)

$(x - h)^2 = 4p(y - k)$

$(1 - 5)^2 = 4p[-7 - (-3)]$ $\quad\quad$ *(h, k) = (5, −3);*

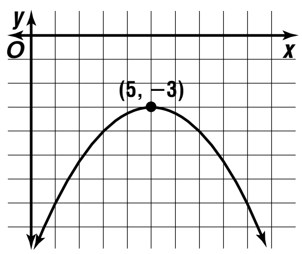
$(-4)^2 = -16p$ $\quad\quad$ *(x, y) = (1, −7)*

$-1 = p$

$(x - h)^2 = 4p(y - k)$

$(x - 5)^2 = 4(-1)(y + 3)$

$(x - 5)^2 = -4(y + 3)$



31. opening: right

vertex: $(h, k) = (-1, 2)$

$(y - k)^2 = 4p(x - h)$

$(0 - 2)^2 = 4p[0 - (-1)]$ $\quad\quad$ *(h, k) = (−1, 2);*

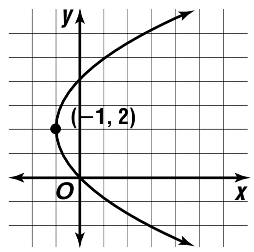
$(-2)^2 = 4p$ $\quad\quad$ *(x, y) = (0, 0)*

$1 = p$

$(y - k)^2 = 4p(x - h)$

$(y - 2)^2 = 4(1)[x - (-1)]$

$(y - 2)^2 = 4(x - 1)$



32. opening: upward

$h = \frac{x_1 + x_2}{2}$

$= \frac{1+2}{2}$ or $\frac{3}{2}$

vertex: $(h, k) = \left(\frac{3}{2}, 0\right)$

$(x - h)^2 = 4p(y - k)$

$\left(1 - \frac{3}{2}\right)^2 = 4p(1 - 0)$ $\quad$ *$(h, k) = \left(\frac{3}{2}, 0\right)$;*

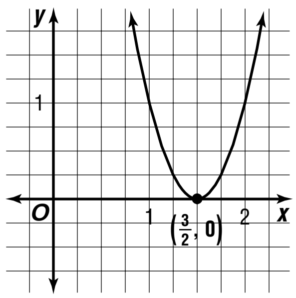
$\frac{1}{4} = 4p$ $\quad$ *$(x, y) = (1, 1)$*

$\frac{1}{16} = p$

$(x - h)^2 = 4p(y - k)$

$\left(x - \frac{3}{2}\right)^2 = 4\left(\frac{1}{16}\right)(y - 0)$

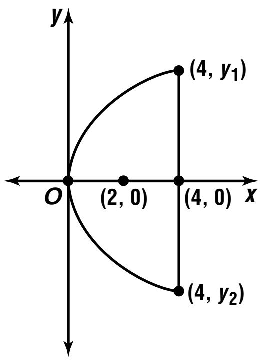
$\left(y - \frac{3}{2}\right) = \frac{1}{4}y$



33a. vertex: $(h, k) = (0, 0)$

depth: $x = 4$

$p = 2$



$(y - k)^2 = 4p(x - h)$

$(y - 0)^2 = 4(2)(x - 0)$ $\quad$ *$(h, k) = (0, 0)$;*

$y^2 = 8x$ $\quad$ *$p = 2$*

$y = \pm\sqrt{8x}$

$y = \pm\sqrt{8(4)}$

$y = \pm 4\sqrt{2}$

$y_1 = 4\sqrt{2}, y_2 = -4\sqrt{2}$

diameter $= |y_1 - y_2|$

$= |4\sqrt{2} - (-4\sqrt{2})|$

$= 8\sqrt{2}$ in.

33b. depth: $x = 1.25(4)$

$x = 5$

$(y - k)^2 = 4p(x - h)$

$(y - 0)^2 = 4(2)(x - 0)$ $\quad$ *$(h, k) = (0, 0)$;*

$y^2 = 8x$ $\quad$ *$p = 2$*

$y = \pm\sqrt{8x}$

$y = \pm\sqrt{8(5)}$

$y = \pm 2\sqrt{10}$

$y_1 = 2\sqrt{10}, y_2 = -2\sqrt{10}$

diameter $= |y_1 - y_2|$

$= |2\sqrt{10} - (-2\sqrt{10})|$

$= 4\sqrt{10}$ in.

34a. Let y = income per flight.

Let x = the number of \$10 price decreases.

Income = number of passengers · cost of a ticket

$y = (110 + 20x) \cdot (140 - 10x)$

$y =$

$15{,}400 - 1100x + 2800x - 200x^2$

$y = -200\left(x^2 - \frac{17}{2}x\right) + 15{,}400$

$y - 15{,}400 = -200\left(x^2 - \frac{17}{2}x\right)$

$y - 15{,}400 - 200\left(\frac{17}{4}\right)^2 = -200\left[x^2 - \frac{17}{2}x + \left(\frac{17}{9}\right)^2\right]$

$-\frac{1}{200}(y - 19{,}012.5) = \left(x - \frac{17}{4}\right)^2$

The vertex of the parabola is at $\left(\frac{17}{4}, 19{,}012.5\right)$, and because p is negative it opens downward. So the vertex is a maximum and the number of \$10 price decreases is $\frac{17}{4}$ or 4.25.

number of passengers $= 110 + 20x$

$= 110 + 20(4.25)$

$= 195$

However, the flight can transport only 180 people.

number of passengers $= 110 + 20x$

$180 = 110 + 20x$

$3.5 = x$

Therefore, there should be 3.5 \$10 price decreases.

cost of ticket $= 140 - 10x$

$= 140 - 10(3.5)$

$= \$105$

34b. Let y = income per flight.

Let x = the number of \$10 price decreases.

Income = number of passengers · cost of a ticket

$y = (110 + 10x) \cdot (140 - 10x)$

$y = 15{,}400 + 300x - 100x^2$

$y = -100(x^2 - 3x) + 15{,}400$

$y - 15{,}400 = -100(x^2 - 3x)$

$y - 15{,}400 - 100\left(\frac{3}{2}\right)^2 = -100\left[x^2 - 3x + \left(\frac{3}{2}\right)^2\right]$

$y - 15{,}625 = -100\left(x - \frac{3}{2}\right)^2$

$-\frac{1}{100}(y - 15{,}625) = \left(x - \frac{3}{2}\right)^2$

The vertex of the parabola is at $\left(\frac{3}{2}, 15{,}625\right)$, and because p is negative, it opens downward. So the vertex is a maximum and the number of \$10 price decreases is $\frac{3}{2}$ or 1.5.

number of passengers $= 110 + 10x$

$= 110 + 10(1.5)$

$= 125$

This is less than 180, so the new ticket price can be found using 1.5 \$10 price decreases.

cost of a ticket $= 140 - 10x$

$= 140 - 10(1.5)$

$= \$125$

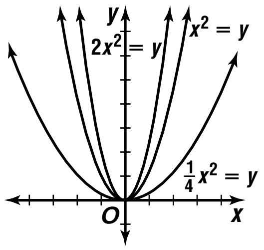
35a. Let $(h, k) = (0, 0)$.

$x^2 = 4py$
$x^2 = 4\left(\frac{1}{8}\right)y \qquad p = \frac{1}{8}$
$2x^2 = y$

$x^2 = 4py$
$x^2 = 4\left(\frac{1}{4}\right)y \qquad p = \frac{1}{4}$
$x^2 = y$

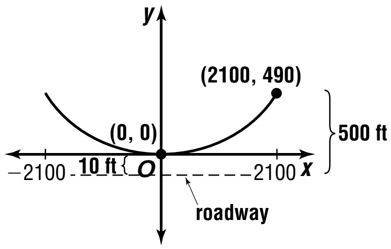
$x^2 = 4py$
$x^2 = -1(1)y \qquad p = 1$
$\frac{1}{4}x = y$

The opening becomes narrower.



35b. The opening becomes wider.

36a. Sample answer:
opening: upward
vertex: $(h, k) = (0, 0)$



$(x - h)^2 = 4p(y - k)$
$(2100 - 0)^2 = 4p(490 - 0) \qquad (h, k) = (0, 0)$
$2250 = p \qquad (x, y) = (2100, 490)$
$(x - h)^2 = 4p(y - k)$
$(x - 0)^2 = 4(2250)(y - 0)$
$x^2 = 9000y$

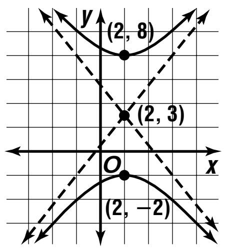
36b. $x^2 = 9000y$
$(720)^2 = 9000y$
$57.6 = y$
$57.6 + 10 = 67.6$ ft

37. $(y - k)^2 = 4p(x - h)$
$y^2 - 2ky + k^2 = 4px - 4ph$
$y^2 - 4px - 2ky + k^2 + 4ph = 0$
$y^2 + Dx + Ey + F = 0$
$(x - h)^2 = 4p(y - k)$
$x^2 - 2hx + h^2 = 4py - 4pk$
$x^2 - 4py - 2hx + h^2 + 4pk = 0$
$x^2 + Dx + Ey + F = 0$

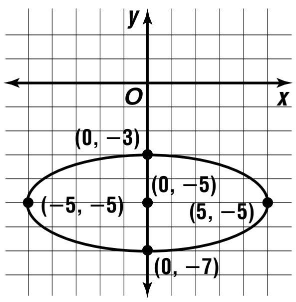
38a. $4p = 8$
$p = 2$ or $p = -2$
opening: right
$(y - k)^2 = 4p(x - h)$
$(y - 2)^2 = 4(2)[x - (-3)]$
$(y - 2)^2 = 8(x + 3)$
opening: left
$(y - k)^2 = 4p(x - h)$
$(y - 2)^2 = 4(-2)[x - (-3)]$
$(y - 2)^2 = -8(x + 3)$

38b. $4p = -16$
$p = -4$
focus of parabola = center of circle
vertex: $(h, k) = (1, 4)$
focus: $(h, k + p) = (1, 4 + (-4))$ or $(1, 0)$
diameter = latus rectum
= 16
radius $= \frac{1}{2}(16)$
$= 8$
$(x - h)^2 + (y - k)^2 = r^2$
$(x - 1)^2 + (y - 0)^2 = 8^2$
$(x - 1)^2 + y^2 = 64$

39. center: $(h, k) = (2, 3)$
$a^2 = 25 \qquad b^2 = 16$
$a = \sqrt{25}$ or 5 $\qquad b = \sqrt{16}$ or 4
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{25 + 16}$ or $\sqrt{41}$
transverse axis: vertical
foci: $(h, k \pm c) = (2, 3 \pm \sqrt{41})$
vertices: $(h, k \pm a) = (2, 3 \pm 5)$ or $(2, 8), (2, -2)$
asymptotes: $y - k = \pm\frac{a}{b}(x - h)$
$y - 3 = \pm\frac{5}{4}(x - 2)$

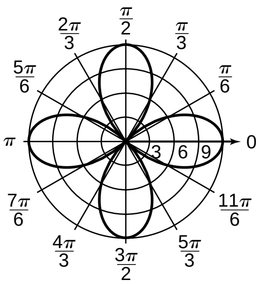


40. $4x^2 + 25y^2 + 250y + 525 = 0$
$4x^2 + 25(y^2 + 10y + ?) = -525 + ?$
$4x^2 + 25(y^2 + 10y + 25) = -525 + 25(25)$
$4x^2 + 25(y + 5)^2 = 100$
$\frac{x^2}{25} + \frac{(y + 5)^2}{4} = 1$
center: $(h, k) = (0, -5)$
$a^2 = 25 \qquad b^2 = 4$
$a = 25$ or 5 $\qquad b = \sqrt{4}$ or 2
$c = \sqrt{a^2 - b^2}$
$c = \sqrt{25 - 4}$ or $\sqrt{21}$
foci: $(h \pm c, k) = (0 \pm \sqrt{21}, -5)$ or $(\pm\sqrt{21}, -5)$
major axis vertices: $(h \pm a, k) = (0 \pm 5, -5)$ or $(\pm 5, -5)$
minor axis vertices: $(h, k \pm b) = (0, -5 \pm 2)$ or $(0, -3), (0, -7)$



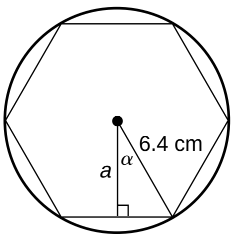
41.

θ	$12 \cos 2\theta$	(r, θ)
0	12	(12, 0)
$\frac{\pi}{6}$	6	$\left(6, \frac{\pi}{6}\right)$
$\frac{\pi}{3}$	-6	$\left(-6, \frac{\pi}{3}\right)$
$\frac{\pi}{2}$	-12	$\left(-12, \frac{\pi}{2}\right)$
$\frac{2\pi}{3}$	-6	$\left(-6, \frac{2\pi}{3}\right)$
$\frac{5\pi}{6}$	6	$\left(6, \frac{5\pi}{6}\right)$
π	12	$(12, \pi)$
$\frac{7\pi}{6}$	6	$\left(6, \frac{7\pi}{6}\right)$
$\frac{4\pi}{3}$	-6	$\left(-6, \frac{4\pi}{3}\right)$
$\frac{3\pi}{2}$	-12	$\left(-12, \frac{3\pi}{2}\right)$
$\frac{5\pi}{3}$	-6	$\left(-6, \frac{5\pi}{3}\right)$
$\frac{11\pi}{6}$	6	$\left(6, \frac{11\pi}{6}\right)$



42. $2\pi n$, where n is any integer

43.



The measure of α is $360° \div 12$ or $30°$.

$$\cos 30° = \frac{a}{6.4}$$
$$6.4 \cos 30° = a$$
$$5.5 \approx a;\ 5.5 \text{ cm}$$

44.

$g(x) = \frac{4}{x^2 + 1}$	
x	$y = g(x)$
$-10{,}000$	4×10^{-8}
-1000	4×10^{-6}
-100	4×10^{-4}
-10	0.04
0	4
10	0.04
100	4×10^{-4}
1000	4×10^{-6}
10,000	4×10^{-8}

$y \to 0$ as $x \to \infty$, $y \to 0$ as $x \to -\infty$

45. $19 < t < 14 + 19$
$19 < t < 33$
$t = 3^3$ or 27
$$\begin{aligned} \text{perimeter} &= 14 + 19 + t \\ &= 14 + 19 + 27 \\ &= 60 \end{aligned}$$
The correct choice is C.

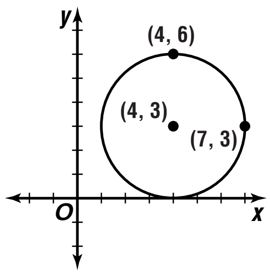
Page 661 Mid-Chapter Quiz

1a.
$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(6 - 3)^2 + (9 - 3)^2} \\ &= \sqrt{3^2 + 6^2} \\ &= \sqrt{45} \end{aligned}$$
$$\begin{aligned} BC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(9 - 6)^2 + (3 - 9)^2} \\ &= \sqrt{3^2 + (-6)^2} \\ &= \sqrt{45} \end{aligned}$$
Since $AB = BC$, triangle ABC is isosceles.

1b.
$$\begin{aligned} AC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(9 - 3)^2 + (3 - 3)^2} \\ &= \sqrt{6^2 + 0^2} \\ &= 6 \end{aligned}$$
$$\begin{aligned} \text{perimeter} &= AB + BC + AC \\ &= \sqrt{45} + \sqrt{45} + 6 \\ &\approx 19.42 \text{ units} \end{aligned}$$

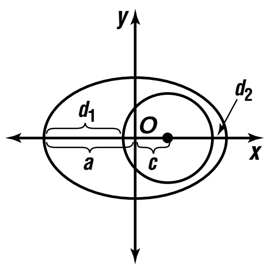
2. Diagonals of a rectangle intersect at their midpoint.
$$\begin{aligned} \text{midpoint of } \overline{AC} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\ &= \left(\frac{-4 + 5}{2}, \frac{9 + 5}{2}\right) \\ &= (0.5, 7) \end{aligned}$$

3.
$$\begin{aligned} x^2 + y^2 - 6y - 8x &= -16 \\ (x^2 - 8x + ?) + (y^2 - 6y + ?) &= -16 + ? + ? \\ (x^2 - 8x + 16) + (y^2 - 6y + 9) &= -16 + 16 + 9 \\ (x - 4)^2 + (y - 3)^2 &= 9 \end{aligned}$$
center: (4, 3): radius: $\sqrt{9}$ or 3



4.
$$\begin{aligned} (x - h)^2 + (y - k)^2 &= r^2 \\ [x - (-5)]^2 + (y - 2)^2 &= \left(\sqrt{7}\right)^2 \\ (x + 5)^2 + (y - 2)^2 &= 7 \end{aligned}$$

5a.

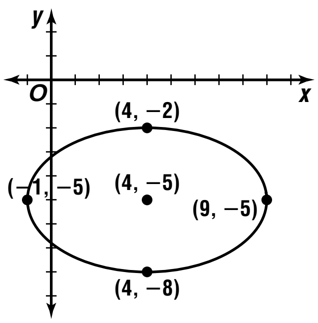


Let d_1 be the greatest distance from the satellite to Earth. Let d_2 be the least distance from the satellite to Earth.

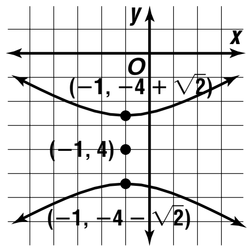
$a = \frac{1}{2}(10{,}440)$
$a = 5220$
$\frac{c}{a} = e$
$\frac{c}{5220} = 0.16$
$c = 835.20$
radius of Earth $= \frac{1}{2}(7920)$
$= 3960$
$d_1 = a + c -$ Earth radius
$d_1 = 5220 + 835.20 - 3960$
$d_1 = 2095.2$ miles
$d_2 =$ major axis $- d_1 -$ Earth diameter
$d_2 = 10{,}440 - 2095.2 - 7920$
$d_2 = 424.8$ miles

5b. $(h, k) = (0,0)$
$a = 5220$
$b^2 = a^2(1 - e^2)$
$b^2 = (5220)^2\,(1 - 0.16^2)$
$b^2 = 26{,}550{,}840.96$
$$\frac{(x-b)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-0)^2}{5220} + \frac{(y-0)^2}{26{,}550{,}840.96} = 1$$
$$\frac{x^2}{27{,}248{,}400} + \frac{y^2}{26{,}550{,}840.96} = 1$$

6. $9x^2 + 25y^2 - 72x + 250y + 554 = 0$
$9(x^2 - 8x + ?) + 25(y^2 + 10y + ?) = -544 + ? + ?$
$9(x^2 - 8x + 16) + 25(y^2 + 10y + 25) = -544 + 9(16) + 25(25)$
$9(x - 4)^2 + 25(y + 5)^2 = 225$
$$\frac{(x-4)^2}{25} + \frac{(y+5)^2}{9} = 1$$
center: $(h, k) = (4, -5)$
$a^2 = 25$ $\quad b^2 = 9$ $\quad c = \sqrt{a^2 - b^2}$
$a = 5$ $\quad b = 3$ $\quad c = \sqrt{25 - 9}$ or 4
major axis vertices: $(h \pm a, k) = (4 \pm 5, -5)$ or $(9, -5), (-1, -5)$
minor axis vertices: $(h, k \pm b) = (4, -5 \pm 3)$ or $(4, -2), (4, -8)$
foci: $(h \pm c, k) = (4 \pm 4, -5)$ or $(8, -5), (0, -5)$

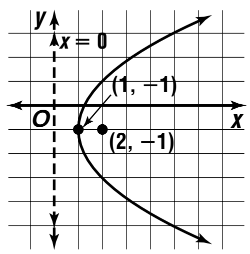


7. $3y^2 + 24y - x^2 - 2x + 41 = 0$
$3(y^2 + 8y + ?) - (x^2 + 2x + ?) = -41 + ? + ?$
$3(y^2 + 8y + 16) - (x^2 + 2x - 1) = -41 + 3(16) - 1$
$3(y + 4)^2 - (x + 1)^2 = 6$
$$\frac{(y+4)^2}{2} - \frac{(x+1)^2}{6} = 1$$
center: $(h, k) = (-1, -4)$
$a^2 = 2$ $\quad b^2 = 6$ $\quad c = \sqrt{a^2 - b^2}$
$a = \sqrt{2}$ $\quad b = \sqrt{6}$ $\quad c = \sqrt{2 + 6}$ or $2\sqrt{2}$
transverse axis: vertical
vertices: $(h, k \pm a) = \left(-1, -4 \pm \sqrt{2}\right)$
foci: $(h, k \pm c) = \left(-1, -4 \pm 2\sqrt{2}\right)$
asymptotes: $y - k = \pm\frac{a}{b}(x - h)$
$$y - (-4) = \pm\frac{\sqrt{2}}{\sqrt{6}}\,[x - (-1)]$$
$$y + 4 = \pm\frac{\sqrt{3}}{3}\,(x + 1)$$



8. To find the center, find the intersection of the asymptotes.
$y = -2x + 4$
$2x = -2x + 4$
$4x = 4$
$x = 1$
$y = 2 \cdot 1$
$y = 2$
The center is at (1, 2).
Notice that (4, 2) must be a vertex and a equals $4 - 1$ or 3.
Point A has an x-coordinate of 4.
Since $y = 2x$, the y-coordinate is $2 \cdot 4$ or 8.
The value of b is $8 - 2$ or 6.
The equation is $\frac{(x-1)^2}{9} - \frac{(y-2)^2}{36} = 1$.

9. $y^2 - 4x + 2y + 5 = 0$
$y^2 + 2y = 4x - 5$
$y^2 + 2y + ? = 4x - 5 + ?$
$y^2 + 2y + 1 = 4x - 5 + 1$
$(y + 1)^2 = 4(x - 1)$
vertex: $(h, k) = (1, -1)$
$4p = 4$
$p = 1$
focus: $(h + p, k) = (1 + 1, -1)$ or $(2, -1)$
axis of symmetry: $y = k$
$y = -1$
directrix: $x = h - p$
$x = 1 - 1$
$x = 0$



10. vertex: $(h, k) = (5, -1)$

$(x - h)^2 = 4p(y - k)$ $\quad$ $(h, k) = (5, -1)$

$(9 - 5)^2 = 4p[-2 - (-1)]$ $\quad$ $(x, y) = (9, -2)$

$4^2 = -4p$

$-4 = p$

$(x - h)^2 = 4p(y - k)$

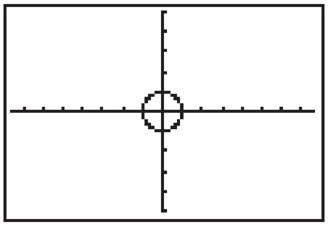
$(x - 5)^2 = 4(-4)[y - (-1)]$

$(x - 5)^2 = -16(y + 1)$

10-6 Rectangular and Parametric Forms of Conic Sections

Page 665 Graphing Calculator Exploration

1.

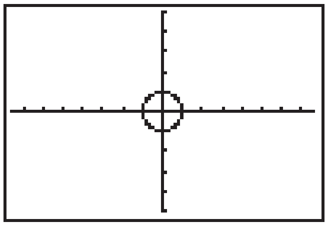


Tmin: [0, 6.28] step: 0.1

$[-7.58, 7.58]$ scl: 1 by $[-5, 5]$ scl: 1

1a. $(-1, 0)$ **1b.** clockwise

2.

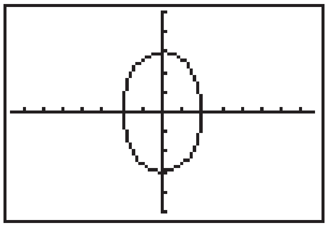


Tmin: [0, 6.28] step: 0.1

$[-7.58, 7.58]$ scl: 1 by $[-5, 5]$ scl: 1

2a. $(0, 1)$ **2b.** clockwise

3.



Tmin: [0, 6.28] step: 0.1

$[-7.58, 7.58]$ scl: 1 by $[-5, 5]$ scl: 1

an ellipse

4. The value of a determines the length of the radius of the circle.

5. Each graph is traced out twice.

Page 667 Check for Understanding

1. For the general equation of a conic, A and C have the same sign and $A \neq C$ for an ellipse. A and C have opposite signs for a hyperbola. $A = C$ for a circle. Either $A = 0$ or $C = 0$ for a parabola.

2. $-\infty < t < \infty$

3. Sample answer:

rectangular equation: $y^2 = -x$

parametric equations: $y = t$ $\quad$ $y^2 = -x$

$t^2 = -x$

$-t^2 = x$

$y = t,\ x = -t^2,\ -\infty < t < \infty$

4. $A = 1$, $c = 9$; since A and C have the same sign and are not equal, the conic is an ellipse.

$$x^2 + 9y^2 + 2x - 18y + 1 = 0$$
$$(x^2 + 2x + ?) + 9(y^2 - 2y + ?) = -1 + ? + ?$$
$$(x^2 + 2x + 1) + 9(y^2 - 2y + 1) = -1 + 1 + 9(1)$$
$$(x + 1)^2 + 9(y - 1)^2 = 9$$
$$\frac{(x + 1)^2}{9} + \frac{(y - 1)^2}{1} = 1$$

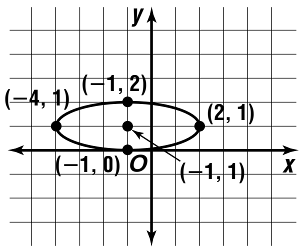
center: $(h, k) = (-1, 1)$

$a^2 = 9$ $\quad$ $b^2 = 1$

$a = 3$ $\quad$ $b = 1$

vertices: $(h \pm a, k) = (-1 \pm 3, 1)$ or $(2, 1), (-4, 1)$

$(h, k \pm b) = (-1, 1 \pm 1)$ or $(-1, 2), (-1, 0)$



5. $A = 1$, $C = 0$; since $C = 0$, the conic is a parabola.

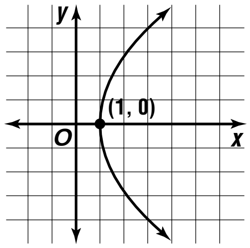
$y^2 - 8x = -8$

$y^2 = 8x - 8$

$y^2 = 8(x - 1)$

vertex: $(h, k) = (1, 0)$

opening: right



6. $A = 1$, $C = -1$; since A and C have different signs, the conic is a hyperbola.

$$x^2 - 4x - y^2 - 5 - 4y = 0$$
$$(x^2 - 4x + ?) - (y^2 + 4y + ?) = 5 + ? + ?$$
$$(x^2 - 4x + 4) - (y^2 + 4y + 4) = 5 + 4 - 4$$
$$(x - 2)^2 - (y + 2)^2 = 5$$
$$\frac{(x - 2)^2}{5} - \frac{(y + 2)^2}{5} = 1$$

center: $(h, k) = (2, -2)$

$a^2 = 5$

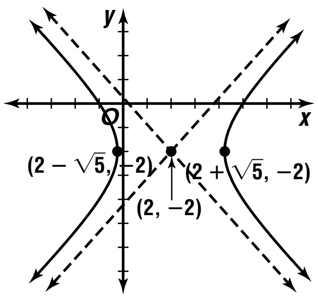
$a = \sqrt{5}$

vertices: $(h \pm a, k) = (2 \pm \sqrt{5}, -2)$

asymptotes: $y - k = \pm \frac{b}{a}(x - h)$

$y - (-2) = \pm \frac{\sqrt{5}}{\sqrt{5}}(x - 2)$

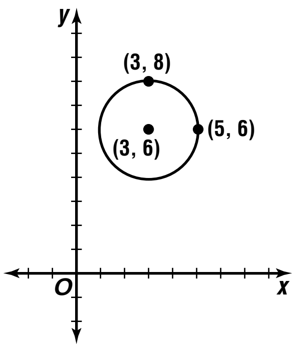
$y + 2 = \pm(x - 2)$



7. $A = 1$, $C = 1$; since $A = C$, the conic is a circle.

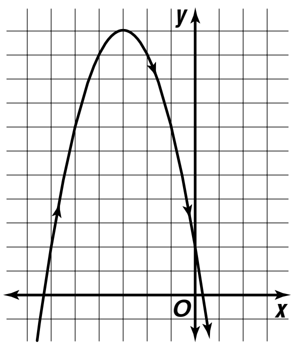
$$x^2 - 6x + y^2 - 12y + 41 = 0$$
$$(x^2 - 6x + ?) + (y^2 - 12y + ?) = -41 + ? + ?$$
$$(x^2 - 6x + 9) + (y^2 - 12y + 36) = -41 + 9 + 36$$
$$(x - 3)^2 + (y - 6)^2 = 4$$

center: $(h, k) = (3, 6)$
radius: $r^2 = 4$
$r = 2$

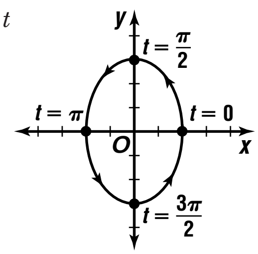


8. $y = -t^2 - 6t + 2$
$y = -x^2 - 6x + 2$

t	x	y	(x, y)
−6	−6	2	(−6, 2)
−5	−5	7	(−5, 7)
−4	−4	10	(−4, 10)
−3	−3	11	(−3, 11)
−2	−2	10	(−2, 10)
−1	−1	7	(−1, 7)
0	0	2	(0, 2)



9. $x = 2 \cos t$ $\quad$ $y = 3 \sin t$
$\frac{x}{2} = \cos t$ $\quad$ $\frac{y}{3} = \sin t$
$\cos^2 t + \sin^2 t = 1$
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$$
$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$



t	x	y	(x, y)
0	2	0	(2, 0)
$\frac{\pi}{2}$	0	3	(0, 3)
π	−2	0	(−2, 0)
$\frac{3\pi}{2}$	0	−3	(0, −3)

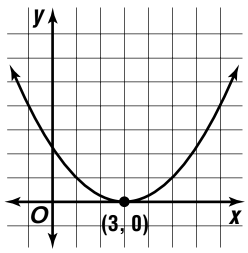
10. Sample answer:
Let $x = t$.
$y = 2x^2 - 5x$
$y = 2t^2 - 5t$
$x = t$, $y = 2t^2 - 5t$, $-\infty \le t \le \infty$

11. Sample answer:
$$x^2 + y^2 = 36$$
$$\frac{x^2}{36} + \frac{y^2}{36} = 1$$
$$\left(\frac{x}{6}\right)^2 + \left(\frac{y}{6}\right)^2 = 1$$
$\cos^2 t + \sin^2 t = 1$
$\left(\frac{x}{6}\right)^2 = \cos^2 t$ $\quad$ $\left(\frac{y}{6}\right)^2 = \sin^2 t$
$\frac{x}{6} = \cos t$ $\quad$ $\frac{y}{6} = \sin t$
$x = 6 \cos t$ $\quad$ $y = 6 \sin t$
$x = 6 \cos t$, $y = 6 \sin t$, $0 \le t \le 2\pi$

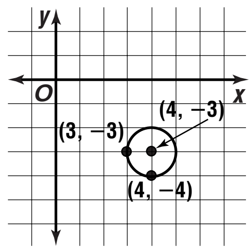
12. $x = \frac{t^2}{80}$
$x = \frac{y^2}{80}$
$80x = y^2$
$y^2 = 80x$

Pages 667–669 Exercises

13. $A = 1$, $C = 0$; since $C = 0$, the conic is a parabola.
$$x^2 - 4y - 6x + 9 = 0$$
$$x^2 - 6x + ? = 4y - 9 + ?$$
$$x^2 - 6x + 9 = 4y - 9 + 9$$
$$(x - 3)^2 = 4y$$
vertex: $(h, k) = (3, 0)$
opening: upward



14. $A = 1$, $C = 1$; since $A = C$, the conic is a circle.
$$x^2 - 8x + y^2 + 6y + 24 = 0$$
$$(x^2 - 8x + ?) + (y^2 + 6y + ?) = -24 + ? + ?$$
$$(x^2 - 8x + 16) + (y^2 + 6y + 9) = -24 + 16 + 9$$
$$(x - 4)^2 + (y + 3)^2 = 1$$
center: $(h, k) = (4, -3)$
radians: $r^2 = 1$
$r = 1$



15. $A = -1$, $C = 3$; since A and C have different signs, the conic is a hyperbola.

$$x^2 - 3y^2 + 2x - 24y - 41 = 0$$
$$-x^2 + 3y^2 - 2x + 24y + 41 = 0$$
$$3(y^2 + 8y + ?) - (x^2 + 2x + ?) = -41 + ? + ?$$
$$3(y^2 + 8y + 16) - (x^2 + 2x + 1) = -41 + 3(16) - 1$$
$$3(y + 4)^2 - (x + 1)^2 = 6$$
$$\frac{(y+4)^2}{2} - \frac{(x+1)^2}{6} = 1$$

center: $(h, k) = (-1, -4)$

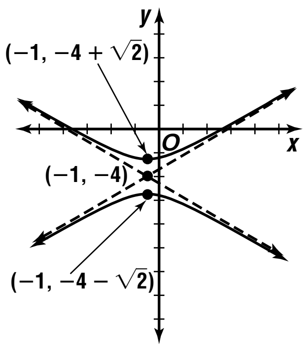
$a^2 = 2$

$a = \sqrt{2}$

vertices: $(h, k + a) = (-1, -4 \pm \sqrt{2})$

asymptotes: $y - k = \pm \frac{a}{b}(x - h)$

$$y - (-4) = \pm \frac{\sqrt{2}}{\sqrt{6}}[x - (-1)]$$
$$y + 4 = \pm \frac{\sqrt{3}}{3}(x + 1)$$



16. $A = 9$, $C = 25$; since A and C have the same sign and are not equal, the conic is an ellipse.

$$9x^2 + 25y^2 - 54x - 50y - 119 = 0$$
$$9(x^2 - 6x + ?) + 25(y^2 - 2y + ?) = 119 + ? + ?$$
$$9(x^2 - 6x + 9) + 25(y^2 - 2y + 1) = 119 + 9(9) + 25(1)$$
$$9(x - 3)^2 + 25(y - 1)^2 = 225$$
$$\frac{(x-3)^2}{25} + \frac{(y-1)^2}{9} = 1$$

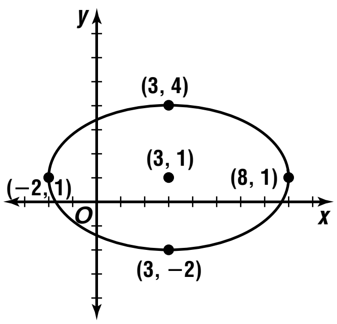
center: $(h, k) = (3, 1)$

$a^2 = 25$ $\qquad b^2 = 9$

$a = 5$ $\qquad b = 3$

vertices: $(h \pm a, k) = (3 \pm 5, 1)$ or $(8,1)$, $(-2, 1)$

$(h, k \pm b) = (3, 1 \pm 3)$ or $(3, 4)$, $(3, -2)$

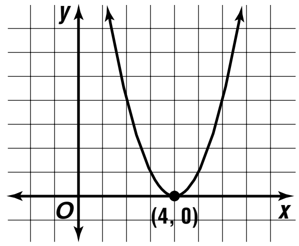


17. $A = 1$, $C = 0$; since $C = 0$, the conic is a parabola.

$$x^2 = y + 8x - 16$$
$$x^2 - 8x + ? = y - 16 + ?$$
$$x^2 - 8x + 16 = y - 16 + 16$$
$$(x - 4)^2 = y$$

vertex: $(h, k) = (4, 0)$

opening: upward



18. $A = C = D = E = 0$; the conic is a hyperbola.

$2xy = 3$

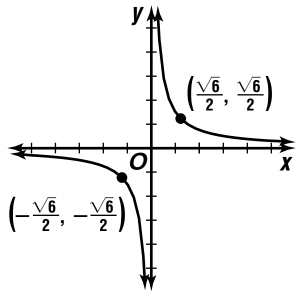
$xy = \frac{3}{2}$

quadrants: I and III

transverse axis: $y = x$

vertices: $\left(\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}\right)$ or $\left(\sqrt{\frac{6}{2}}, \sqrt{\frac{6}{2}}\right)$,

$\left(-\sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}}\right)$ or $\left(-\frac{\sqrt{6}}{2}, -\frac{\sqrt{6}}{2}\right)$



19. $A = 5$, $C = 2$; since A and C have the same sign and are not equal, the conic is an ellipse.

$$5x^2 + 2y^2 - 40x - 20y + 110 = 0$$
$$5(x^2 - 8x + ?) + 2(y^2 - 10y + ?) = -110 + ? + ?$$
$$5(x^2 - 8x + 16) + 2(y^2 - 10y + 25) = -110 + 5(16) + 2(25)$$
$$5(x - 4)^2 + 2(y - 5)^2 = 20$$
$$\frac{(x-4)^2}{4} + \frac{(y-5)^2}{10} = 1$$
$$\frac{(y-5)^2}{10} + \frac{(x-4)^2}{4} = 1$$

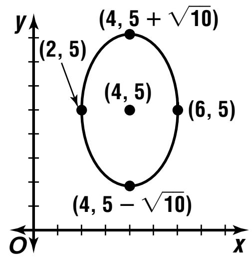
center: $(h, k) = (4, 5)$

$a^2 = 10$ $\qquad b^2 = 4$

$a = \sqrt{10}$ $\qquad b = 2$

vertices: $(h, k \pm a) = (4, 5 \pm \sqrt{10})$

$(h \pm b, k) = (4 \pm 2, 5)$ or $(6, 5)$, $(2, 5)$



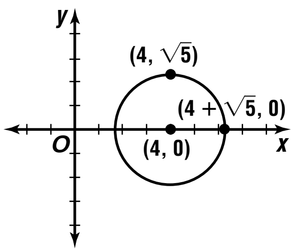
20. $A = 1$, $C = 1$; since $A = C$, the conic is a circle.

$$x^2 - 8x + 11 = -y^2$$
$$(x^2 - 8x + ?) + y^2 = -11 + ?$$
$$(x^2 - 8x + 16) + y^2 = -11 + 16$$
$$(x - 4)^2 + y^2 = 5$$

center: $(h, k) = (4, 0)$

radius: $r^2 = 5$

$r = \sqrt{5}$



21. $A = -9$, $C = 8$; since A and C have different signs, the conic is a hyperbola.

$$8y^2 - 9x^2 - 16y + 36x - 100 = 0$$
$$8(y^2 - 2y + ?) - 9(x^2 - 4x + ?) = 100 + ? + ?$$
$$8(y^2 - 2y + 1) - 9(x^2 - 4x + 4) = 100 + 8(1) - 9(4)$$
$$8(y - 1)^2 - 9(x - 2)^2 = 72$$
$$\frac{(y-1)^2}{9} - \frac{(x-2)^2}{8} = 1$$

center: $(h, k) = (2, 1)$

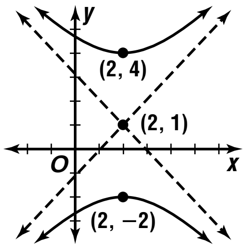
$a^2 = 9$

$a = 3$

vertices: $(h, k \pm a) = (2, 1 \pm 3)$ or $(2, 4)$, $(2, -2)$

asymptotes: $y - k = \pm \frac{a}{b}(x - h)$

$$y - 1 = \pm \frac{3}{2\sqrt{2}}(x - 2)$$
$$y - 1 = \pm \frac{3\sqrt{2}}{4}(x - 2)$$

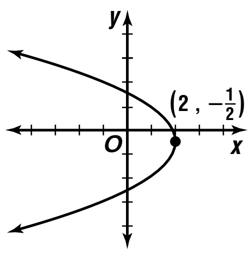


22. $A = 0$, $C = 4$; since $A = 0$, the conic is a parabola.

$$4y^2 + 4y + 8x = 15$$
$$4(y^2 + y + ?) = -8x + 15 + ?$$
$$4\left(y^2 + y + \frac{1}{4}\right) = -8x + 15 + 4\left(\frac{1}{4}\right)$$
$$4\left(y + \frac{1}{2}\right)^2 = -8x + 16$$
$$\left(y + \frac{1}{2}\right)^2 = -2x + 4$$
$$\left(y + \frac{1}{2}\right)^2 = -2(x - 2)$$

vertex: $(h, k) = \left(2, -\frac{1}{2}\right)$

opening: left



23.

$$-4y^2 + 10x = 16y - x^2 - 5$$
$$x^2 - 4y^2 + 10x - 16y + 5 = 0$$

$A = 1$, $C = -4$; since A and C have different signs, the conic is a hyperbola.

$$x^2 - 4y^2 + 10x - 16y + 5 = 0$$
$$(x^2 + 10x + ?) - 4(y^2 + 4y + ?) = -5 + ? + ?$$
$$(x^2 + 10x + 25) - 4(y^2 + 4y + 4) = -5 + 25 - 4(4)$$
$$(x + 5)^2 - 4(y + 2)^2 = 4$$
$$\frac{(x+5)^2}{4} - \frac{(y+2)^2}{1} = 1$$

center: $(h, k) = (-5, -2)$

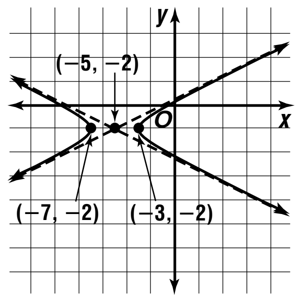
$a^2 = 4$

$a = 2$

vertices: $(h \pm a, k) = (-5 \pm 2, -2)$ or $(-3, -2)$, $(-7, -2)$

asymptotes: $y - k = \pm \frac{b}{a}(x - h)$

$$y - (-2) = \pm \frac{1}{2}[x - (-5)]$$
$$y + 2 = \pm \frac{1}{2}(x + 5)$$



24.

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$
$$2x^2 + 0 + 2y^2 + (-8)x + 12y + 6 = 0$$
$$2x^2 + 2y^2 - 8x + 12y = -6$$

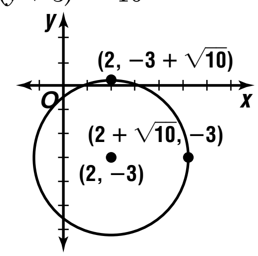
A = C; circle

$$2(x^2 - 4x + ?) + 2(y^2 + 6y + ?) = 6 + ? + ?$$
$$2(x^2 - 4x + 4) + 2(y^2 + 6y + 6y + 9) =$$
$$-6 + 2(4) + 2(9)$$
$$2(x - 2)^2 + 2(y + 3)^2 = 20$$
$$(x - 2)^2 + (y + 3)^2 = 10$$

center: $(h, k) = (2, -3)$

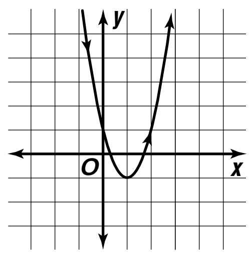
radius: $r^2 = 10$

$r = \sqrt{10}$



25. $y = 2t^2 - 4t + 1$

$y = 2x^2 - 4x + 1$



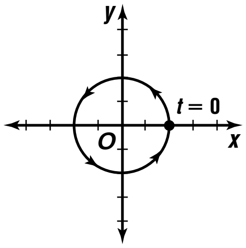
t	x	y	(x, y)
−1	−1	7	(−1, 7)
0	0	1	(0, 1)
1	1	−1	(1, −1)
2	2	7	(2, 7)

26. $x = \cos 2t$ $\qquad$ $y = \sin 2t$

$\cos^2 2t + \sin^2 2t = 1$

$x^2 + y^2 = 1$

t	x	y	(x, y)
0	1	0	(1, 0)
$\frac{\pi}{2}$	0	1	(0, 1)
π	-1	0	$(-1, 0)$
$\frac{3\pi}{2}$	0	-1	$(0, -1)$



27. $x = -\cos t$ $\qquad$ $y = \sin t$

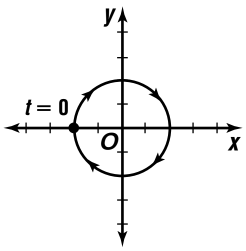
$-x = \cos t$

$\cos^2 t + \sin^2 t = 1$

$(-x)^2 + y^2 = 1$

$x^2 + y^2 = 1$

t	x	y	(x, y)
0	-1	0	$(-1, 0)$
$\frac{\pi}{2}$	0	1	(0, 1)
π	1	0	(1, 0)
$\frac{3\pi}{2}$	0	-1	$(0, -1)$



28. $x = 3 \sin t$ $\qquad$ $y = 2 \cos t$

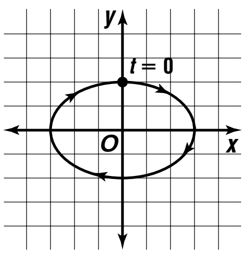
$\frac{x}{3} = \sin t$ $\qquad$ $\frac{y}{2} = \cos t$

$\cos^2 t + \sin^2 t = 1$

$\left(\frac{y}{2}\right)^2 + \left(\frac{x}{3}\right)^2 = 1$

$\frac{y^2}{4} + \frac{x^2}{9} = 1$

$\frac{x^2}{9} + \frac{y^2}{4} = 1$



t	x	y	(x, y)
0	2	0	(0, 2)
$\frac{\pi}{2}$	3	0	(3, 0)
π	0	-2	$(0, -2)$
$\frac{3\pi}{2}$	-3	0	$(-3, 0)$

29. $x = -\sin 2t$ $\qquad$ $y = 2 \cos 2t$

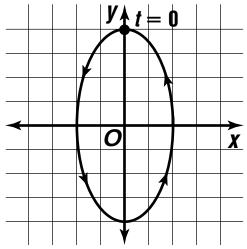
$-x = \sin 2t$ $\qquad$ $\frac{y}{2} = \cos 2t$

$\cos^2 2t + \sin^2 2t = 1$

$\left(\frac{y}{2}\right)^2 + (-x)^2 = 1$

$\frac{y^2}{4} + x^2 = 1$

$x^2 + \frac{y^2}{4} = 1$



t	x	y	(x, y)
0	0	2	(0, 2)
$\frac{\pi}{4}$	-1	0	$(-1, 0)$
$\frac{\pi}{2}$	0	-2	$(0, -2)$
$\frac{3\pi}{4}$	1	0	(1, 0)

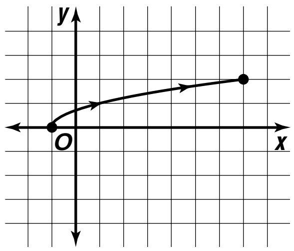
30. $x = 2t - 1$

$x + 1 = 2t$

$\frac{x+1}{2} = t$

$y = \sqrt{t}$

$y = \sqrt{\frac{x+1}{2}}$



t	x	y	(x, y)
0	-1	0	$(-1, 0)$
1	1	1	(1, 1)
2	3	$\sqrt{2}$	$(3, \sqrt{2})$
3	5	$\sqrt{3}$	$(5, \sqrt{3})$
4	7	2	(7, 2)

31. $x = -3 \cos 2t$ $\qquad$ $y = 3 \sin 2t$

$-\frac{x}{3} = \cos 2t$ $\qquad$ $\frac{y}{3} = \sin 2t$

$\cos^2 2t + \sin^2 2t = 1$

$\left(-\frac{x}{3}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$

$\frac{x^2}{9} + \frac{y^2}{9} = 1$

$x^2 + y^2 = 9$

32. Sample answer:

$x^2 + y^2 = 25$

$\frac{x^2}{25} + \frac{y^2}{25} = 1$

$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$

$\cos^2 t + \sin^2 t = 1$

$\left(\frac{x}{5}\right)^2 = \cos^2 t$ $\qquad$ $\left(\frac{y}{5}\right)^2 = \sin^2 t$

$\frac{x}{5} = \cos t$ $\qquad$ $\frac{y}{5} = \sin t$

$x = 5 \cos t$ $\qquad$ $y = 5 \sin t$

$x = 5 \cos t$, $y = 5 \sin t$, $0 \leq t \leq 2\pi$

33. Sample answer:

$$x^2 + y^2 - 16 = 0$$
$$x^2 + y^2 = 16$$
$$\frac{x^2}{16} + \frac{y^2}{16} = 1$$
$$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$$
$$\cos^2 t + \sin^2 t = 1$$
$$\left(\frac{x}{4}\right)^2 = \cos^2 t \qquad \left(\frac{y}{4}\right)^2 = \sin^2 t$$
$$\frac{x}{4} = \cos t \qquad \frac{y}{4} = \sin t$$
$$x = 4\cos t \qquad y = 4\sin t$$
$$x = 4\cos t,\ y = 4\sin t,\ 0 \le t \le 2\pi$$

34. Sample answer:

$$\frac{x^2}{4} + \frac{y^2}{25} = 1$$
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$$
$$\cos^2 t + \sin^2 t = 1$$
$$\left(\frac{x}{2}\right)^2 = \cos^2 t \qquad \left(\frac{y}{5}\right)^2 = \sin^2 t$$
$$\frac{x}{2} = \cos t \qquad \frac{y}{5} = \sin t$$
$$x = 2\cos t \qquad y = 5\sin t$$
$$x = 2\cos t,\ y = 5\sin t,\ 0 \le t \le 2\pi$$

35. Sample answer:

$$\frac{y^2}{16} + x^2 = 1$$
$$x^2 + \left(\frac{y}{4}\right)^2 = 1$$
$$\cos^2 t + \sin^2 t = 1$$
$$x^2 = \cos^2 t \qquad \left(\frac{y}{4}\right)^2 = \sin^2 t$$
$$x = \cos t \qquad \frac{y}{4} = \sin t$$
$$y = 4\sin t$$
$$x = \cos t,\ y = 4\sin t,\ 0 \le t \le 2\pi$$

36. Sample answer:

Let $x = t$.

$y = x^2 - 4x + 7$

$y = t^2 - 4t + 7$

$x = t,\ y = t^2 - 4t + 7,\ -\infty < t < \infty$

37. Sample answer:

Let $y = t$.

$x = y^2 + 2y - 1$

$x = t^2 + 2t - 1$

$x = t^2 + 2t - 1,\ y = t,\ -\infty < t < \infty$

38. Sample answer:

Let $y = t$.

$$(y + 3)^2 = 4(x - 2)$$
$$(t + 3)^2 = 4(x - 2)$$
$$0.25(t + 3)^2 = x - 2$$
$$0.25(t + 3)^2 + 2 = x$$
$$x = 0.25(t + 3)^2 + 2,\ y = t,\ -\infty < t < \infty$$

39a. Answers will vary. Sample answers:

Let $x = t$.

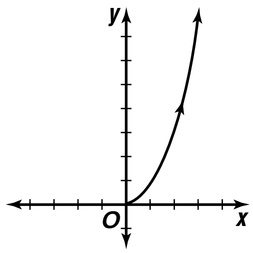
$$x = \sqrt{y}$$
$$t = \sqrt{y}$$
$$t^2 = y$$
$$x = t,\ y = t^2,\ t \ge 0$$

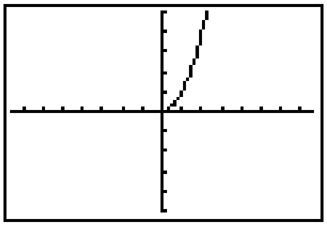
Let $y = t$.

$$x = \sqrt{y}$$
$$x = \sqrt{t}$$
$$x = \sqrt{t},\ y = t,\ t \ge 0$$

39b.

t	x	y	(x, y)
0	0	0	(0, 0)
1	1	1	(1, 1)
2	2	4	(2, 4)
3	3	9	(3, 9)





Tmin: [0, 5] step: 0.1

[−7.58, 7.58] scl: 1 by [−5, 5] scl: 1

39c. yes

39d. There is usually more than one parametric representation for the graph of a rectangular equation.

40a. a circle with center (0, 0) and radius 6 feet

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 0)^2 + (y - 0)^2 = 6^2$$
$$x^2 + y^2 = 36$$

40b. Sample answer:

$$x^2 + y^2 = 36$$
$$\frac{x^2}{36} + \frac{y^2}{36} = 1$$
$$\left(\frac{x}{6}\right)^2 + \left(\frac{y}{6}\right)^2 = 1$$
$$\sin^2(\omega t) + \cos^2(\omega t) = 1$$

Since the paddlewheel completes a revolution in 2 seconds, the period is $\frac{2\pi}{\omega} = 2$, so $\omega = \pi$.

$$\sin^2(\pi t) + \cos^2(\pi t) = 1$$
$$\left(\frac{x}{6}\right)^2 = \sin^2(\pi t) \qquad \left(\frac{y}{6}\right)^2 = \cos^2(\pi t)$$
$$\frac{x}{6} = \sin(\pi t) \qquad \frac{y}{6} = \cos(\pi t)$$
$$x = 6\sin(\pi t) \qquad y = 6\cos(\pi t)$$
$$x = 6\sin(\pi t),\ y = 6\cos(\pi t),\ 0 \le t \le 2$$

40c. $C = 2\pi r$

$C = 2\pi 6$

$C \approx 37.7$ ft

The paddlewheel makes 1 revolution, or moves 37.7 ft in 2 seconds.

$\frac{37.7 \text{ ft}}{2 \text{ s}} \cdot 60 \text{ s} = 1131$ ft

The paddlewheel moves about 1131 ft in 1 minute.

41a. $A = 2$, $C = 5$; since A and C have the same sign and $A \neq C$, the graph is an ellipse.

$2x^2 + 5y^2 = 0$

$5y^2 = -2x$

$y^2 = -\frac{2}{5}x$

$y = \sqrt{-\frac{2}{5}x}$

This equation is true for $(x, y) = (0, 0)$.
The graph is a point at $(0, 0)$; the equation is that of a degenerate ellipse.

41b. $A = 1$, $C = 1$; since $A = C$, the graph is a circle.

$x^2 + y^2 - 4x - 6y + 13 = 0$

$(x^2 - 4x + ?) + (y^2 - 6y + ?) = -13$

$(x^2 - 4x + 4) + (y^2 - 6y + 9) = -13 + 4 + 9$

$(x - 2)^2 + (y - 3)^2 = 0$

center: $(h, k) = (2, 3)$
radius: 0
The graph is a point at $(2, 3)$; the equation is that of a degenerate circle.

41c. $A = -9$, $C = 1$; since A and C have different signs, the graph is a hyperbola.

$y^2 - 9x^2 = 0$

$y^2 = 9x^2$

$y = \pm 3x$

The graph is two intersecting lines $y = \pm 3x$; the equation is that of a degenerate hyperbola.

42. The substitution for x must be a function that allows x to take on all of the values stipulated by the domain of the rectangular equation. The domain of $y = x^2 - 5$ is all real numbers, but using a substitution of $x = t^2$ would only allow for values of x such that $x \geq 0$.

43a. center: $(h, k) = (0, 0)$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 0)^2 + (y - 0)^2 = 6$

$x^2 + y^2 = 36$

43b. $x^2 + y^2 = 36$

$\frac{x^2}{36} + \frac{y^2}{36} = 1$

$\left(\frac{x}{6}\right)^2 + \left(\frac{y}{6}\right)^2 = 1$

$\sin^2 t + \cos^2 t = 1$

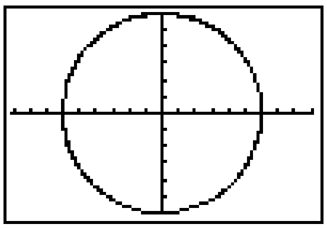
$\left(\frac{x}{6}\right)^2 = \sin^2 t$ $\qquad$ $\left(\frac{y}{6}\right)^2 = \cos^2 t$

$\frac{x}{6} = \sin t$ $\qquad$ $\frac{y}{6} = \cos t$

$x = 6 \sin t$ $\qquad$ $y = 6 \cos t$

$x = 6 \sin t$, $y = 6 \cos t$
Since the second hand makes 2 revolutions, $0 \leq t \leq 4\pi$.

43c.



Tmin: $[0, 4\pi]$ step: 0.1
$[-9.10, 9.10]$ scl:1 by $[-6, 6]$ scl:1

44. After drawing a vertical line through (x, y) and a horizontal line through the endpoint opposite (x, y), two right triangles are formed. Both triangles contain an angle t, since corresponding angles are congruent when two parallel lines are cut by a transversal. Using the larger triangle, $\cos t = \frac{x}{a}$ or $x = a \cos t$. Using the smaller triangle, $\sin t = \frac{y}{b}$ or $y = b \sin t$.

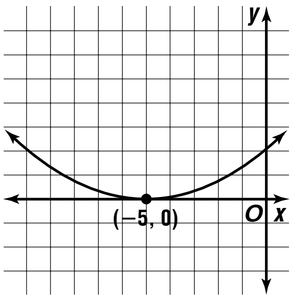
45. $x^2 - 12y + 10x = -25$

$x^2 + 10x + ? = 12y - 25 + ?$

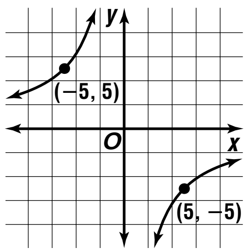
$x^2 + 10x + 25 = 12y - 25 + 25$

$(x + 5)^2 = 12y$

vertex: $(h, k) = (-5, 0)$
$4p = 12$
$p = 3$
focus: $(h, k + p) = (-5, 0 + 3)$ or $(-5, 3)$
axis of symmetry: $x = h$
$x = -5$
directrix: $y = k - p$
$y = 0 - 3$
$y = -3$



46. $c = -25$
quadrants: II and IV
transverse axis: $y = -x$
vertices: $xy = -25$ $\qquad$ $xy = -25$
$5(-5) = -25$ $\qquad$ $-5(5) = -25$
$(5, -5)$ $\qquad$ $(-5, 5)$



47. $3x^2 + 3y^2 - 18x + 12y = 9$

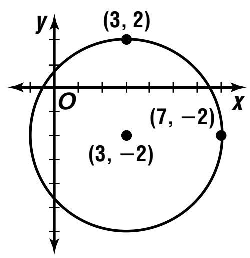
$3(x^2 - 6x + ?) + 3(y^2 + 4y + ?) = 9 + ? + ?$

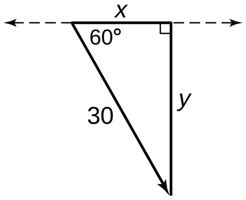
$3(x^2 - 6x + 9) + 3(y^2 + 4y + 4) = 9 + 3(9) - 3(4)$

$3(x - 3)^2 + 3(y + 2)^2 = 48$

$(x - 3)^2 + (y + 2)^2 = 16$

center: $(h, k) = (3, -2)$
radians: $r^2 = 16$
$r = 4$



48. 

$\cos 60° = \frac{x}{30}$ $\qquad\qquad$ $\sin 60° = \frac{y}{30}$

$x = 30 \cos 60°$ $\qquad\qquad$ $y = 30 \sin 60°$

$x = 15$ lb $\qquad\qquad$ $y = 15\sqrt{3}$ lb

49. $y = -0.13x + 37.8$

$0.13x + y - 37.8 = 0$

$A = 0.13, B = 1, C = -37.8$

Car 1: $(x_1, y_1) = (135, 19)$

$$d_1 = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$$

$$d_1 = \frac{0.13(135) + 1(19) + (-37.8)}{\pm\sqrt{(0.13)^2 + 1^2}}$$

$d_1 \approx -1.24$

The point (135, 19) is about 1 unit from the line $y = -0.13x + 37.8$.

Car 2: $(x_2, y_2) = (245, 16)$

$$d_2 = \frac{Ax_2 + By_2 + C}{\pm\sqrt{A^2 + B^2}}$$

$$d_2 = \frac{0.13(245) + 1(16) + (-37.8)}{\pm\sqrt{(0.13)^2 + 12}}$$

$d_2 \approx 9.97$

The point (245, 16) is about 10 units from the line $y = -0.13x + 37.8$.

Car 1: the point (135, 19) is about 9 units closer to the line $y = -0.13x + 37.8$ than the point (245, 16).

50. Let $\theta = \text{Sin}^{-1} \frac{1}{2}$.

$\text{Sin}\, \theta = \frac{1}{2}$

$\theta = 30$

$$\begin{aligned}\sin\left(2 \,\text{Sin}^{-1} \tfrac{1}{2}\right) &= \sin(2\theta)\\ &= \sin(2 \cdot 30)\\ &= \sin 60\\ &= \tfrac{\sqrt{3}}{2}\end{aligned}$$

51. $s = \frac{1}{2}(a + b + c)$

$s = \frac{1}{2}(48 + 32 + 44)$

$s = 62$

$K = \sqrt{s(s - a)(s - b)(s - c)}$

$K = \sqrt{62(62 - 48)(62 - 32)(62 - 44)}$

$K = \sqrt{468720}$

$K \approx 685$ units^2

52.
$$\begin{aligned}\sqrt{2y - 3} - \sqrt{2y + 3} &= -1\\ \sqrt{2y - 3} &= \sqrt{2y + 3} - 1\\ 2y - 3 &= 2y + 3 - 2\sqrt{2y + 3} + 1\\ -7 &= -2\sqrt{2y + 3}\\ \tfrac{7}{2} &= \sqrt{2y + 3}\\ \tfrac{49}{4} &= 2y + 3\\ \tfrac{37}{4} &= 2y\\ \tfrac{37}{8} &= y\end{aligned}$$

53. $y = kxz$ $\qquad\qquad$ $y = kxz$

$16 = k(5)(2)$ $\qquad\qquad$ $y = 1.6(8)(3)$

$1.6 = k$ $\qquad\qquad$ $y = 38.4$

54.
$$\begin{vmatrix} 5 & 9 \\ 7 & -3 \end{vmatrix} = 5(-3) - 7(9) = -78$$

Yes, an inverse exists since the determinant of the matrix $\neq 0$.

55.
$$\begin{aligned}m &= \frac{y_2 - y_1}{x_2 - x_1}\\ &= \frac{7 - 4}{3 - (-6)}\\ &= \frac{1}{3}\end{aligned}$$

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{1}{3}(x + 6)$ or $y - 7 = \frac{1}{3}(x - 3)$

$y = mx + b$ $\qquad\qquad$ $y = mx + b$

$4 = \frac{1}{3}(-6) + b$ $\qquad\qquad$ $y = \frac{1}{3}x + 6$

$6 = b$

56. $(1 \# 4)\ @\ (2 \# 3) = 1\ @\ 2$

$= 2$

The correct choice is *B*.

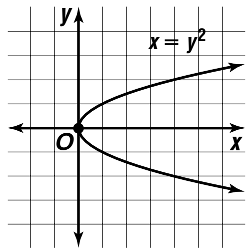
10-7 Transformation of Conics

Pages 674–675 Check for Understanding

1. Sample answers:

$(h, k) = (0, 0)$

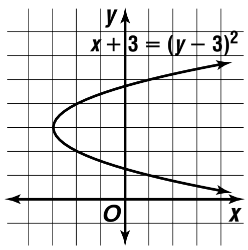
$x = y^2$



$(h, k) = (-3, 3)$

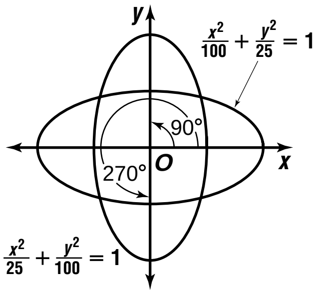
$(x - h) = (y - k)^2$

$(x + 3) = (y - 3)^2$



2. Replace x with $x' \cos 30° + y' \sin 30°$ or $\frac{\sqrt{3}}{2}x' + \frac{1}{2}y'$.

Replace y with $-x' \sin 30° + y' \cos 30°$ or $-\frac{1}{2}x', + \frac{\sqrt{2}}{2}y'$.

3. 

90° or 270°

4. Ebony; $B^2 - 4AC = \left(6\sqrt{3}\right)^2 - 4(7)(13) < 0$
and $A \neq C$

5. $B^2 - 4AC = 0 - 4(1)(1)$
$= -4$
$A = C = 1$; circle

$$(x-h)^2 + (y-k)^2 = 7$$
$$(x-3)^2 + (y-2)^2 = 7 \qquad (h, k) = (3, 2)$$
$$x^2 - 6x + 9 + y^2 - 4y + 4 = 7$$
$$x^2 + y^2 - 6x - 4y + 6 = 0$$

6. $B^2 - 4AC = 0 - 4(2)(0)$
$= 0$
parabola

$$y = 2x^2 - 7x + 5$$
$$y - 5 = 2x^2 - 7x$$
$$y - 5 = 2\left(x^2 - \tfrac{7}{2}x\right)$$
$$y - 5 + 2\left(\tfrac{7}{4}\right)^2 = 2\left[x^2 - \tfrac{7}{2}x + \left(\tfrac{7}{4}\right)^2\right]$$
$$y - 5 + \tfrac{49}{8} = 2\left(x - \tfrac{7}{4}\right)^2$$
$$y + \tfrac{9}{8} = 2\left(x - \tfrac{7}{4}\right)^2$$
$$y + \tfrac{9}{8} - k = 2\left(x - \tfrac{7}{4} - h\right)^2$$
$$y + \tfrac{9}{8} - 5 = 2\left(x - \tfrac{7}{4} + 4\right)^2 \qquad (h, k) = (-4, 5)$$
$$y - \tfrac{31}{8} = 2\left(x + \tfrac{9}{4}\right)^2$$
$$y = \tfrac{31}{8} = 2\left(x^2 + \tfrac{18}{4}x + \tfrac{81}{16}\right)$$
$$y - \tfrac{31}{8} = 2x^2 + 9x + \tfrac{81}{8}$$
$$0 = 2x^2 + 9x - y + 14$$
$$2x^2 + 9x - y + 14 = 0$$

7. $B^2 - 4AC = 0 - 4(1)(-1)$
$= 4$
hyperbola

$$x^2 - y^2 = 9$$
$$(x' \cos 60° + y' \text{ sub } 60°)^2 - (-x' \sin 60° + y' \cos 60°)^2 = 9$$
$$\left(\tfrac{1}{2}x' + \tfrac{\sqrt{3}}{2}y'\right)^2 - \left(-\tfrac{\sqrt{3}}{2}x' + \tfrac{1}{2}y'\right)^2 = 9$$
$$\tfrac{1}{4}(x')^2 + \tfrac{\sqrt{3}}{2}x'y' + \tfrac{3}{4}(y')^2 - \left[\tfrac{3}{4}(x')^2 - \tfrac{\sqrt{3}}{2}x'y' + \tfrac{1}{4}(y')^2\right] = 9$$
$$-\tfrac{1}{2}(x')^2 + \sqrt{3}x'y' + \tfrac{1}{2}(y')^2 = 9$$
$$(x')^2 - 2\sqrt{3}x'y' - (y')^2 = -18$$
$$(x')^2 - 2\sqrt{3}x'y' - (y')^2 + 18 = 0$$

8. $B^2 - 4AC = 0 - 4(1)(1)$
$= -4$
$A = C = 1$; circle

$$x^2 - 5x + y^2 = 3$$
$$\left(x' \cos \tfrac{\pi}{4} + y' \sin \tfrac{\pi}{4}\right)^2 - 5\left(x' \cos \tfrac{\pi}{4} + y' \sin \tfrac{\pi}{4}\right) + \left(-x' \sin \tfrac{\pi}{4} + y' \cos \tfrac{\pi}{4}\right)^2 = 3$$
$$\left(\tfrac{\sqrt{2}}{2}x' + \tfrac{\sqrt{2}}{2}y'\right)^2 - 5\left(\tfrac{\sqrt{2}}{2}x' + \tfrac{\sqrt{2}}{2}y'\right) + \left(-\tfrac{\sqrt{2}}{2}x' + \tfrac{\sqrt{2}}{2}y'\right)^2 = 3$$
$$\tfrac{1}{2}(x')^2 + x'y' + \tfrac{1}{2}(y')^2 - \tfrac{5\sqrt{2}}{2}x' - \tfrac{5\sqrt{2}}{2}y' + \tfrac{1}{2}(x')^2 - x'y' + \tfrac{1}{2}(y')^2 = 3$$
$$(x')^2 + (y')^2 - \tfrac{5\sqrt{2}}{2}x' - \tfrac{5\sqrt{2}}{2}y' = 3$$
$$2(x')^2 + 2(y')^2 - 5\sqrt{2}x' - 5\sqrt{2}y' = 6$$
$$2(x')^2 + 2(y')^2 - 5\sqrt{2}x' - 5\sqrt{2}y' - 6 = 0$$

9. $B^2 - 4AC = 4^2 - 4(9)(4)$
$= -128$
A ≠ C; ellipse

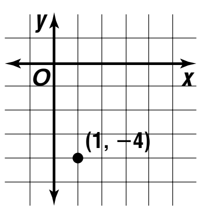
$$\tan 2\theta = \frac{B}{A - C}$$
$$\tan 2\theta = \frac{4}{9 - 4}$$
$$\tan 2\theta = 0.8$$
$$2\theta \approx 38.65980825°$$
$$\theta \approx 19°$$

10. $B^2 - 4AC = 5^2 - 4(8)(-4)$
$= 153$
hyperbola

$$\tan 2\theta = \frac{B}{A - C}$$
$$\tan 2\theta = \frac{5}{8 - (-4)}$$
$$\tan 2\theta \approx 0.416666667$$
$$2\theta \approx 22.61986495°$$
$$\theta \approx 11°$$

11.
$$3(x-1)^2 + 4(y+4)^2 = 0$$
$$3(x^2 - 2x + 1) + 4(y^2 + 8y + 16) = 0$$
$$3x^2 - 6x + 3 + 4y^2 + 32y + 64 = 0$$
$$\underset{\substack{\uparrow \\ a}}{4}y^2 + \underset{\substack{\uparrow \\ b}}{32}y + \underbrace{(3x^2 - 6x + 67)}_{\substack{\uparrow \\ c}} = 0$$
$$y = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$y = \frac{-32 \pm \sqrt{32^2 - 4(4)(3x^2 - 6x + 67)}}{2(4)}$$
$$y = \frac{-32 \pm \sqrt{-48x^2 + 96x - 48}}{8}$$
$$y = \frac{-32 \pm \sqrt{-48(x-1)^2}}{8}$$

$x = 1$, $y = -4$; point

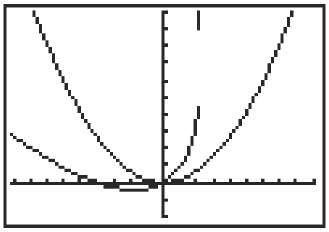


12a.

$$y = \frac{1}{6}x^2$$
$$-x' \sin 30° + y' \cos 30° = \frac{1}{6}(x' \cos 30° + y' \sin 30°)^2$$
$$-\frac{1}{2}x' + \frac{\sqrt{3}}{2}y' = \frac{1}{6}\left(\frac{\sqrt{3}}{2}x' + \frac{1}{2}y'\right)^2$$
$$-\frac{1}{2}x' + \frac{\sqrt{3}}{2}y' = \frac{1}{6}\left[\frac{3}{4}(x')^2 + \frac{2\sqrt{3}}{4}x'y' + \frac{1}{4}(y')^2\right]$$
$$-\frac{1}{2}x' + \frac{\sqrt{3}}{2}y' = \frac{1}{8}(x')^2 + \frac{\sqrt{3}}{12}x'y' + \frac{1}{24}(y')^2$$
$$-12x' + 12\sqrt{3}y' = 3(x')^2 + 2\sqrt{3}x'y' + (y')^2$$
$$0 = 3(x')^2 + 2\sqrt{3}x'y' + (y')^2 + 12x' - 12\sqrt{3}y'$$
$$3(x') + 2\sqrt{3}x'y' + (y')^2 + 12x' - 12\sqrt{3}y' = 0$$

12b.

$$3x^2 + 2\sqrt{3}xy + y^2 + 12x - 12\sqrt{3}y = 0$$
$$\underset{\substack{\uparrow \\ a}}{1}y^2 + \underbrace{(2\sqrt{3}x - 12\sqrt{3})}_{\substack{\uparrow \\ b}}y + \underbrace{(3x^2 + 12x)}_{\substack{\uparrow \\ c}} = 0$$
$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$y = \frac{-(2\sqrt{3}x - 12\sqrt{3}) \pm \sqrt{(2\sqrt{3}x - 12\sqrt{3})^2 - 4(1)(3x^2 + 12x)}}{2(1)}$$
$$y = -\sqrt{3}x + 6\sqrt{3} \pm \frac{\sqrt{12x^2 - 144x + 432 - 12x^2 - 48x}}{2}$$
$$y = -\sqrt{3}x + 6\sqrt{3} \pm \frac{\sqrt{-192x + 432}}{2} \text{ and } y = \frac{1}{6}x^2$$



Pages 675–677 Exercises

13. $B^2 - 4AC = 0 - 4(3)(0)$
$= 0$

parabola

$$y = 3x^2 - 2x + 5$$
$$y - 5 = 3x^2 - 2x$$
$$y - 5 = 3\left(x^2 - \frac{2}{3}x\right)$$
$$y - 5 + 3\left(\frac{1}{3}\right)^2 = 3\left[x^2 - \frac{2}{3}x + \left(\frac{1}{3}\right)^2\right]$$
$$y - \frac{14}{3} = 3\left(x - \frac{1}{3}\right)^2$$
$$y - \frac{14}{3} - k = 3\left(x - \frac{1}{3} - h\right)^2$$
$$y - \frac{14}{3} + 3 = 3\left(x - \frac{1}{3} - 2\right)^2 \qquad (h, k) = (2, -3)$$
$$y - \frac{5}{3} = 3\left(x - \frac{7}{3}\right)^2$$
$$y - \frac{5}{3} = 3\left(x^2 - \frac{14}{3}x + \frac{49}{9}\right)$$
$$y - \frac{5}{3} = 3x^2 - 14x + \frac{49}{3}$$
$$0 = 3x^2 - 14x - y + 18$$
$$3x^2 - 14x - y + 18 = 0$$

14. $B^2 - 4AC = 0 - 4(4)(5)$
$= -80$

$A \neq C$; ellipse

$$4x^2 + 5y^2 = 20$$
$$4(x - h)^2 + 5(y - k)^2 = 20$$
$$4(x - 5)^2 + 5(y + 6)^2 = 20 \qquad (h, k) = (5, -6)$$
$$4(x^2 - 10x + 25) + 5(y^2 + 12y + 36) = 20$$
$$4x^2 - 40x + 100 + 5y^2 + 60y + 180 = 20$$
$$4x^2 + 5y^2 - 40x + 60y + 260 = 0$$

15. $B^2 - 4AC = 0 - 4(3)(1)$
$= -12$

$A \neq C$; ellipse

$$3x^2 + y^2 = 9$$
$$3(x - h)^2 + (y - k)^2 = 9$$
$$3(x + 1)^2 (y - 3)^2 = 9 \qquad (h, k) = (-1, 3)$$
$$3(x^2 + 2x + 1) + y^2 - 6y + 9 = 9$$
$$3x^2 + 6x + 3 + y^2 - 6y + 9 = 9$$
$$3x^2 + y^2 + 6x - 6y + 3 = 0$$

16. $B^2 - 4AC = 0 - 4(12)(4)$
$= -192$

$A \neq C$; ellipse

$$4y^2 + 12x^2 = 24$$
$$4(y - k)^2 + 12(x - h)^2 = 24$$
$$4(y - 4)^2 + 12(x + 1)^2 = 24 \qquad (h, k) = (-1, 4)$$
$$4(y^2 - 8y + 16) + 12(x^2 + 2x + 1) = 24$$
$$4y^2 - 32y + 64 + 12x^2 + 24x + 12 = 24$$
$$y^2 - 8y + 16 + 3x^2 + 6x + 3 = 6$$
$$3x^2 + y^2 + 6x - 8y + 13 = 0$$

17. $B^2 - 4AC = 0 - 4(9)(-25)$
$= 900$

hyperbola

$$9x^2 - 25y^2 = 225$$
$$9(x - h)^2 - 25(y - k)^2 = 225$$
$$9(x - 0)^2 - 25(y - 5)^2 = 225 \qquad (h, k) = (0, 5)$$
$$9x^2 - 25(y^2 - 10y + 25) = 225$$
$$9x^2 - 25y^2 + 250y - 850 = 0$$

18. $(x - 3)^2 = 4y$

$x^2 + 6x + 9 - 4y = 0$

$B^2 - 4AC = 0 - 4(1)(0)$
$= 0$

parabola

$$(x + 3)^2 = 4y$$
$$(x + 3 - h)^2 = 4(y - k)$$
$$(x + 3 + 7)^2 = 4(y - 2) \qquad (h, k) = (-7, 2)$$
$$(x + 10)^2 = 4y - 8$$
$$x^2 + 20x + 100 = 4y - 8$$
$$x^2 + 20x - 4y + 108 = 0$$

19. $B^2 - 4AC = 0 - 4(1)(0)$
$= 0$

parabola

$$x^2 - 8y = 0$$
$$(x' \cos 90° + y' \sin 90°)^2 - 8(-x' \sin 90° + y' \cos 90°) = 0$$
$$(y')^2 - 8(-x') = 0$$
$$(y')^2 + 8x = 0$$

20. $B^2 - 4AC = 0 - 4(2)(2)$
$= -16$
$A = C$; circle

$$\begin{aligned}
2x^2 + 2y^2 &= 8\\
2(x' \cos 30° + y' \sin 30°)^2 \\
+ 2(-x' \sin 30° + y' \cos 30°)^2 &= 8\\
2\left(\tfrac{\sqrt{3}}{2}x' + \tfrac{1}{2}y'\right)^2 + 2\left(-\tfrac{1}{2}x' + \tfrac{\sqrt{3}}{2}y'\right)^2 &= 8\\
2\left[\tfrac{3}{4}(x')^2 + \tfrac{\sqrt{3}}{2}x'y' + \tfrac{1}{4}(y')^2\right] \\
+ 2\left[\tfrac{1}{4}(x')^2 - \tfrac{\sqrt{3}}{2}x'y' + \tfrac{3}{4}(y')^2\right] &= 8\\
\tfrac{3}{2}(x')^2 + \sqrt{3}x'y' + \tfrac{1}{2}(y')^2 + \tfrac{1}{2}(x')^2 \\
- \sqrt{3}x'y' + \tfrac{3}{2}(y')^2 &= 8\\
2(x')^2 + 2(y')^2 &= 8\\
(x')^2 + (y')^2 - 4 &= 0
\end{aligned}$$

21. $B^2 - 4AC = 0 - 4(1)(0)$
$= 0$
$A \neq C$; parabola

$$\begin{aligned}
y^2 + 8x &= 0\\
\left(-x' \sin \tfrac{\pi}{6} + y' \cos \tfrac{\pi}{6}\right)^2 + 8\left(x' \cos \tfrac{\pi}{6} + y' \sin \tfrac{\pi}{6}\right) &= 0\\
\left(-\tfrac{1}{2}x' + \tfrac{\sqrt{3}}{2}y'\right)^2 + 8\left(\tfrac{\sqrt{3}}{2}x' + \tfrac{1}{2}y'\right) &= 0\\
\tfrac{1}{4}(x')^2 - \tfrac{\sqrt{3}}{2}x'y' + \tfrac{3}{4}(y')^2 + 4\sqrt{3}x' + 4y' &= 0\\
(x')^2 - 2\sqrt{3}x'y' + 3(y')^2 + 16\sqrt{3}x' + 16y' &= 0
\end{aligned}$$

22. $B^2 - 4AC = 1^2 - 4(0)(0)$
$= 1$
hyperbola

$$\begin{aligned}
xy &= -8\\
\left(x' \cos \tfrac{\pi}{4} + y' \sin \tfrac{\pi}{4}\right)\left(-x' \sin \tfrac{\pi}{4} + y' \cos \tfrac{\pi}{4}\right) &= -8\\
\left(\tfrac{\sqrt{2}}{2}x' + \tfrac{\sqrt{2}}{2}y'\right)\left(-\tfrac{\sqrt{2}}{2}x' + \tfrac{\sqrt{2}}{2}y'\right) &= -8\\
-\tfrac{1}{2}(x')^2 + \tfrac{1}{2}x'y' - \tfrac{1}{2}x'y' + \tfrac{1}{2}(y')^2 &= -8\\
(x')^2 - (y')^2 &= 16\\
(x')^2 - (y')^2 - 16 &= 0
\end{aligned}$$

23. $B^2 - 4AC = 0 - 4(1)(1)$
$= -4$
$A = C$; circle

$$\begin{aligned}
x^2 - 5x + y^2 &= 3\\
\left(x' \cos \tfrac{\pi}{3} + y' \sin \tfrac{\pi}{3}\right)^2 - 5\left(x' \cos \tfrac{\pi}{3} + y' \sin \tfrac{\pi}{3}\right) \\
+ \left(-x' \sin \tfrac{\pi}{3} + y' \cos \tfrac{\pi}{3}\right)^2 &= 3\\
\left(\tfrac{1}{2}x' + \tfrac{\sqrt{3}}{2}y'\right)^2 - 5\left(\tfrac{1}{2}x' + \tfrac{\sqrt{3}}{2}y'\right) \\
+ \left(-\tfrac{\sqrt{3}}{2}x' + \tfrac{1}{2}y'\right)^2 &= 3\\
\tfrac{1}{4}(x')^2 + \tfrac{\sqrt{3}}{2}x'y' + \tfrac{3}{4}(y')^2 - \tfrac{5}{2}x' - \tfrac{5\sqrt{3}}{2}y' \\
+ \tfrac{3}{4}(x')^2 - \tfrac{\sqrt{3}}{2}x'y' + \tfrac{1}{4}(y')^2 &= 3\\
(x')^2 + (y')^2 - \tfrac{5}{2}x' - \tfrac{5\sqrt{3}}{2}y &= 3\\
2(x')^2 + 2(y')^2 - 5x' - 5\sqrt{3}y' - 6 &= 0
\end{aligned}$$

24. $B^2 - 4AC = 0 - 4(16)(-4)$
$= 256$
hyperbola

$$\begin{aligned}
16x^2 - 4y^2 &= 64\\
16(x' \cos 60° + y' \sin 60°)^2 \\
- 4(-x' \sin 60° + y' \cos 60°) &= 64\\
16\left(\tfrac{1}{2}x' + \tfrac{\sqrt{3}}{2}y'\right)^2 - 4\left(-\tfrac{\sqrt{3}}{2}x' + \tfrac{1}{2}y'\right)^2 &= 64\\
16\left[\tfrac{1}{4}(x')^2 + \tfrac{\sqrt{3}}{2}x'y' + \tfrac{3}{4}(y')^2\right] \\
- 4\left[\tfrac{3}{4}(x')^2 - \tfrac{\sqrt{3}}{2}x'y' + \tfrac{1}{4}(y')^2\right] &= 64\\
4(x')^2 + 8\sqrt{3}x'y' + 12(y')^2 \\
- 3(x')^2 + 2\sqrt{3}x'y' - (y')^2 &= 64\\
(x')^2 + 10\sqrt{3}x'y' + 11y')^2 - 64 &= 0
\end{aligned}$$

25.

$$\begin{aligned}
6x^2 + 5y^2 &= 30\\
6(x' \cos 30° + y' \sin 30°)^2 \\
+ 5(-x' \sin 30° + y' \cos 30°)^2 &= 30\\
6\left(\tfrac{\sqrt{3}}{2}x' + \tfrac{1}{2}y'\right)^2 + 5\left(-\tfrac{1}{2}x' + \tfrac{\sqrt{3}}{2}y'\right)^2 &= 30\\
6\left[\tfrac{3}{4}(x')^2 + \tfrac{\sqrt{3}}{2}x'y' + \tfrac{1}{4}(y')^2\right] \\
+ 5\left[\tfrac{1}{4}(x')^2 - \tfrac{\sqrt{3}}{2}x'y' + \tfrac{3}{4}(y')^2\right] &= 30\\
\tfrac{18}{4}(x')^2 + \tfrac{6\sqrt{3}}{2}x'y' + \tfrac{6}{4}(y')^2 + \tfrac{5}{4}(x')^2 - \tfrac{5\sqrt{3}}{2}x'y' \\
+ \tfrac{15}{4}(y')^2 &= 30\\
\tfrac{23}{4}(x')^2 + \tfrac{\sqrt{3}}{2}x'y' + \tfrac{21}{4}(y')^2 - 30 &= 0\\
23(x')^2 + 2\sqrt{3}x'y' + 21(y')^2 - 120 &= 0
\end{aligned}$$

26. $3^2 - 4AC = 4^2 - 4(9)(5)$
$= -164$
$A \neq C$; ellipse

$$\begin{aligned}
\tan 2\theta &= \tfrac{B}{A - C}\\
\tan 2\theta &= \tfrac{4}{9 - 5}\\
\tan 2\theta &= 1\\
2\theta &= 45°\\
\theta &\approx 23°
\end{aligned}$$

27. $B^2 - 4AC = (-1)^2 - 4(1)(-4)$
$= 17$
hyperbola

$$\begin{aligned}
\tan 2\theta &= \tfrac{B}{A - C}\\
\tan 2\theta &= \tfrac{-1}{1 - (-4)}\\
\tan 2\theta &= -\tfrac{1}{5}\\
2\theta &\approx -11.30993247°\\
\theta &\approx -6°
\end{aligned}$$

28. $B^2 - 4AC = 8^2 - 4(8)(2)$
$= 0$
parabola

$$\begin{aligned}
\tan 2\theta &= \tfrac{B}{A - C}\\
\tan 2\theta &= \tfrac{8}{8 - 2}\\
\tan 2\theta &= \tfrac{4}{3}\\
2\theta &\approx 53.13010235°\\
\theta &\approx 27°
\end{aligned}$$

29. $B^2 - 4AC = 9^2 - 4(2)(14)$
$= -31$
$A \neq C$; ellipse

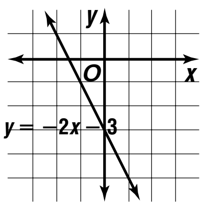
$$\begin{aligned}
\tan 2\theta &= \tfrac{B}{A - C}\\
\tan 2\theta &= \tfrac{9}{2 - 14}\\
\tan 2\theta &= -\tfrac{3}{4}\\
2\theta &\approx -36.86989765°\\
\theta &\approx -18°
\end{aligned}$$

30. $B^2 - 4AC = 4^2 - 4(2)(5)$
$= -24$
$A \neq C$; ellipse
$\tan 2\theta = \frac{B}{A - C}$
$\tan 2\theta = \frac{4}{2 - 5}$
$\tan 2\theta = -\frac{4}{3}$
$2\theta \approx -53.13010235°$
$\theta \approx -27°$

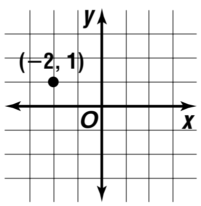
31. $B^2 - 4AC = \left(4\sqrt{3}\right)^2 - 4(2)(6)$
$= 0$
parabola
$\tan 2\theta = \frac{B}{A - C}$
$\tan 2\theta = \frac{4\sqrt{3}}{2 - 6}$
$\tan 2\theta = -\sqrt{3}$
$2\theta = -60°$
$\theta = -30°$

32. $B^2 - 4AC = 4^2 - 4(2)(2)$
$= 0$
parabola
$A = C$; $\theta = \frac{\pi}{4}$ or 45°

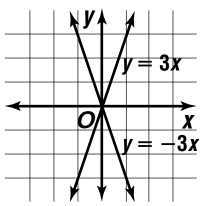
33. $(x - 2)^2 - (x + 3)^2 = 5(y + 2)$
$x^2 - 4x + 4 - x^2 - 6x - 9 = 5(y + 2)$
$-10x - 5 = 5(y + 2)$
$-2x - 1 = y + 2$
$-2x - 3 = y$
$y = -2x - 3$ line



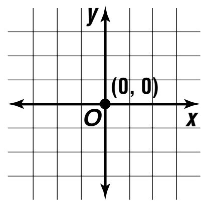
34. $2x^2 + 6y^2 + 8x - 12y + 14 = 0$
$x^2 + 3y^2 + 4x - 6y + 7 = 0$
$\underbrace{3}_{a}y^2 + \underbrace{(-6)}_{b}y + \underbrace{(x^2 + 4x + 7)}_{c} = 0$
$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$y = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(x^2 + 4x + 7)}}{2(3)}$
$y = \frac{6 \pm \sqrt{-12x^2 - 48x - 48}}{6}$
$y = \frac{6 \pm \sqrt{-12(x + 2)^2}}{6}$
$x = -2$, $y = 1$; point



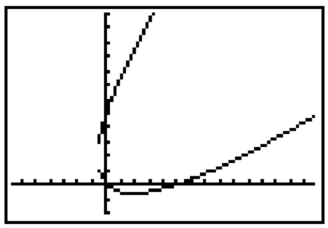
35. $y^2 - 9x^2 = 0$
$y^2 = 9x^2$
$y = \sqrt{9x^2}$
$y = \pm 3x$
intersecting lines



36. $(x - 2)^2 + (y - 2)^2 + 4(x + y) = 8$
$x^2 - 4x + 4 + y^2 - 4y + 4 + 4x + 4y = 8$
$\underbrace{(1)}_{a}y^2 + \underbrace{0}_{b}y + \underbrace{x^2}_{c} = 0$
$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$y = \frac{0 \pm \sqrt{0 - 4(1)(x^2)}}{2(1)}$
$y = \pm\frac{\sqrt{-4x^2}}{2}$
$x = 0$, $y = 0$; point

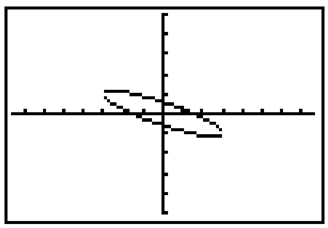


37. $x^2 - 2xy + y^2 - 5x - 5y = 0$
$\underbrace{(1)}_{a}y^2 + \underbrace{(-2x - 5)}_{b}y + \underbrace{(x^2 - 5x)}_{c} = 0$
$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$y = \frac{-(-2x - 5) \pm \sqrt{(-2x - 5)^2 - 4(1)(x^2 - 5x)}}{2(1)}$
$y = \frac{2x + 5 \pm \sqrt{4x^2 + 20x + 25 - 4x^2 + 20x}}{2}$
$y = \frac{2x + 5 \pm \sqrt{40x + 25}}{2}$



[−6.61, 14.6] scl:1 by [−2, 12] scl:1

38. $2x^2 + 9xy + 14y^2 = 5$
$\underbrace{14}_{a}y^2 + \underbrace{(9x)}_{b}y + \underbrace{(2x^2 - 5)}_{c} = 0$
$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$y = \frac{-9x \pm \sqrt{(9x)^2 - 4(14)(2x^2 - 5)}}{2(14)}$
$y = \frac{-9x \pm \sqrt{-21x^2 + 280}}{28}$



[−7.58, 7.58] scl:1 by [−5, 5] scl:1

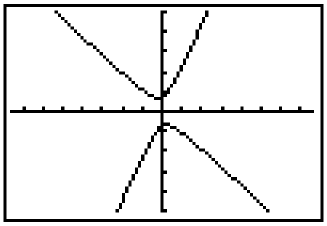
39. $8x^2 + 5xy - 4y^2 = -2$

$\underset{a}{\uparrow}(-4)y^2 + \underset{b}{\uparrow}(5x)y + \underbrace{(8x^2 + 2)}_{c} = 0$

$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \frac{-5x \pm \sqrt{(5x)^2 - 4(-4)(8x^2 + 2)}}{2(-4)}$

$y = \frac{-5x \pm \sqrt{153x^2 + 32}}{-8}$



$[-7.58, 7.58]$ scl:1 by $[-5, 5]$ scl:1

40. $2x^2 + 4\sqrt{3}xy + 6y^2 + 3x = y$

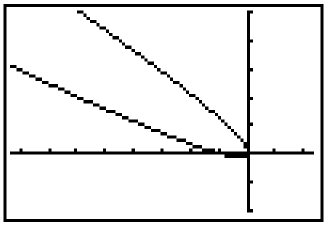
$6y^2 + \underbrace{(4\sqrt{3}x - 1)}_{b}y + \underbrace{(2x^2 + 3x)}_{c} = 0$, $a = 6$

$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \frac{-(4\sqrt{3}x - 1) \pm \sqrt{(4\sqrt{3}x - 1)^2 - 4(6)(2x^2 + 3x)}}{2(6)}$

$y = \frac{-4\sqrt{3}x + 1 \pm \sqrt{48x^2 - 8\sqrt{3}x + 1 - 48x^2 - 72x}}{12}$

$y = \frac{-4\sqrt{3}x + 1 \pm \sqrt{-8\sqrt{3}x - 72x + 1}}{12}$



$[-8.31, 2.31]$ scl:1 by $[-2, 5]$ scl:1

41. $2x^2 + 4xy + 2y^2 + 2\sqrt{2}x - 2\sqrt{2}y = -12$

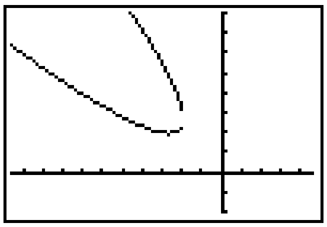
$2y^2 + \underbrace{(4x - 2\sqrt{2})}_{b}y + \underbrace{(2x^2 + 2\sqrt{2}x + 12)}_{c} = 0$, $a = 2$

$y = \frac{-b \pm b^2 - 4ac}{2a}$

$y = \frac{-(4x - 2\sqrt{2}) \pm \sqrt{(4x - 2\sqrt{2})^2 - 4(2)(2x^2 + 2\sqrt{2}x + 12)}}{2(2)}$

$y = \frac{-4x + 2\sqrt{2} \pm \sqrt{16x^2 - 16\sqrt{2}x + 8 - 16x^2 - 16\sqrt{2}x - 96}}{4}$

$y = \frac{-4x + 2\sqrt{2} \pm \sqrt{-32\sqrt{2}x - 88}}{4}$



$[-10.58, 4.58]$ scl:1 by $[-2, 8]$ scl:1

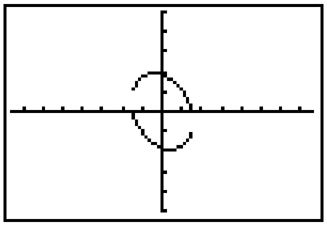
42. $9x^2 + 4xy + 6y^2 = 20$

$6y^2 + (4x)y + \underbrace{(9x^2 - 20)}_{c} = 0$, $a = 6$, $b = 4x$

$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

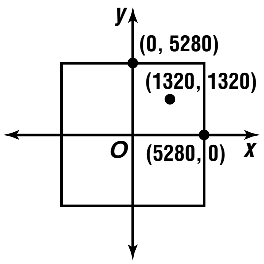
$y = \frac{-4x \pm \sqrt{(4x)^2 - 4(6)(9x^2 - 20)}}{2(6)}$

$y = \frac{-4x \pm \sqrt{-212x^2 + 480}}{12}$



$[-7.85, 7.85]$ scl:1 by $[-5, 5]$ scl:1

43a.



$T_{(1320,\ 1320)}$

43b. circle
center: $(h, k) = (1320, 1320)$
radius: $r = 1320$

$(x - h)^2 + (y - k)^2 = r^2$
$(x - 1320)^2 + (y - 1320)^2 = 1320^2$
$(x - 1320)^2 + (y - 1320)^2 = 1{,}742{,}400$

44a. $B^2 - 4AC = 0 - 4(1)(0)$
$= 0$
parabola; 360°

44b. $B^2 - 4AC = 0 - 4(8)(6)$
$= -192$
$A \neq C$; ellipse; 180°

44c. $B^2 - 4AC = 4^2 - 4(0)(0)$
$= 16$
hyperbola; 180°

44d. $3^2 - 4AC = 0 - 4(15)(15)$
$= -900$
$A = C$; circle; There is no minimum angle of rotation, since any degree of rotation will result in a graph that coincides with the original.

45. Let $x = x' \cos\theta + y' \sin\theta$ and $y = -x' \sin\theta + y' \cos\theta$.

$$x^2 + y^2 = r^2$$
$$(x' \cos\theta + y' \sin\theta)^2 + (-x' \sin\theta + y' \cos\theta)^2 = r^2$$
$$(x')^2 \cos^2\theta + x'y' \cos\theta \sin\theta + (y')^2 \sin^2\theta + (x')^2 \sin^2\theta - x'y' \cos\theta \sin\theta + (y')^2 \cos\theta = r^2$$
$$[(x')^2 + (y')^2] \cos^2\theta + [(x')^2 + (y')^2] \sin^2\theta = r^2$$
$$[(x')^2 + (y')^2](\cos^2\theta + \sin^2\theta) = r^2$$
$$[(x')^2 + (y')^2](1) = r^2$$
$$(x')^2 + (y')^2 = r^2$$

46a. $B^2 - 4AC = \left(-10\sqrt{3}\right)^2 - 4(31)(21)$
$= -2304$
$A \neq C$; elliptical

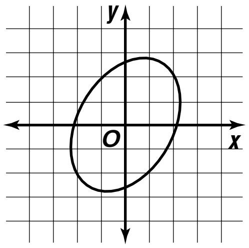
46b. $$31x^2 - 10\sqrt{3}xy + 21y^2 = 144$$

$$\underset{\substack{\uparrow\\ a}}{21}y^2 + \underset{\substack{\uparrow\\ b}}{(-10\sqrt{3}x)}y + \underset{\substack{\uparrow\\ c}}{\underbrace{(31x^2 - 144)}} = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-10\sqrt{3}x) \pm \sqrt{(-10\sqrt{3}x)^2 - 4(21)(31x^2 - 144)}}{2(21)}$$

$$y = \frac{10\sqrt{3}x \pm \sqrt{-2304x^2 + 12{,}096}}{42}$$



46c. $\tan 2\theta = \frac{B}{A - C}$

$$\tan 2\theta = \frac{-10\sqrt{3}}{31 - 21}$$
$$\tan 2\theta = -\sqrt{3}$$
$$2\theta = -60°$$
$$\theta = -30°$$

47a. $\tan 2\theta = \frac{B}{A - C}$

$$\tan 2\theta = \frac{-2\sqrt{3}}{9 - 11}$$
$$\tan 2\theta = \sqrt{3}$$
$$2\theta = 60°$$
$$\theta = 30°$$

The graph of this equation has been rotated 30°. To transform the graph so the axes are on the x- and y-axes, rotate the graph $-30°$.

47b. $B^2 - 4AC = \left(-2\sqrt{3}\right)^2 - 4(9)(11)$
$= -384$

$A \neq C$; the graph is an ellipse.

$$9x^2 - 2\sqrt{3}xy + 11y^2 - 24 = 0$$

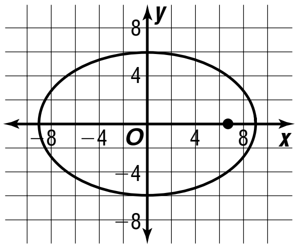
$$9[x' \cos(-30°) + y' \sin(-30°)]^2 - 2\sqrt{3}$$
$$[(x' \cos(-30°) + y' \sin(-30°)]$$
$$[-x' \sin(-30°) + y' \cos(-30°)] + 11$$
$$[-x' \sin(-30°) + y' \cos(-30°)^2] - 24 = 0$$

$$9\left(\frac{\sqrt{3}}{2}x' - \frac{1}{2}y'\right)^2 - 2\sqrt{3}\left(\frac{\sqrt{3}}{2}x' - \frac{1}{2}y'\right)$$
$$\left(\frac{1}{2}x' + \frac{\sqrt{3}}{2}y'\right) + 11\left(\frac{1}{2}x' + \frac{\sqrt{3}}{2}y'\right)^2 - 24 = 0$$

$$9\left[\frac{3}{4}(x')^2 - \frac{\sqrt{3}}{2}x'y' + \frac{1}{4}(y')^2\right] -$$
$$2\sqrt{3}\left[\frac{\sqrt{3}}{4}(x')^2 + \frac{3}{4}x'y' - \frac{1}{4}x'y' - \frac{\sqrt{3}}{4}(y')^2\right]$$
$$+ 11\left[\frac{1}{4}(x')^2 + \frac{\sqrt{3}}{2}x'y' + \frac{3}{4}(y')^2\right] - 24 = 0$$

$$\frac{27}{4}(x')^2 - \frac{9\sqrt{3}}{2}x'y' + \frac{9}{4}(y')^2 - \frac{6}{4}(x')^2$$
$$- \frac{6\sqrt{3}}{4}x'y' + \frac{2\sqrt{3}}{4}x'y' + \frac{6}{4}(y')^2$$
$$+ \frac{11}{4}(x')^2 + \frac{11\sqrt{3}}{2}x'y' + \frac{33}{4}(y')^2 - 24 = 0$$
$$8(x')^2 + 12(y')^2 - 24 = 0$$
$$\frac{(x')^2}{3} + \frac{(y')^2}{2} = 1$$

48a. center: $(h, k) = (0, 0)$
major axis: horizontal
$a = \sqrt{81}$ or 9
$b = \sqrt{36}$ or 6
$c = \sqrt{a^2 - b^2}$
$c = \sqrt{81 - 36}$
$c = \sqrt{45}$ or $3\sqrt{5}$



48b. $T_{(3\sqrt{5},\, 0)}$

$$\frac{x^2}{81} + \frac{y^2}{36} = 1$$
$$\frac{(x - h)^2}{81} + \frac{(y - k)^2}{36} = 1$$

48c. $\frac{(x' - 3\sqrt{5})^2}{36} + \frac{(y')^2}{81} = 1$

49. $A = -3$, $C = 5$; since A and C have different signs, the conic is a hyperbola.

50. $(h, k) = (2, -3)$

$$e = \frac{c}{a}$$
$$\frac{2\sqrt{6}}{5} = \frac{c}{1}$$
$$\frac{2\sqrt{6}}{5} = c$$
$$b^2 = a^2 - c^2$$
$$b^2 = 1^2 - \left(\frac{2\sqrt{6}}{5}\right)^2$$
$$b^2 = \frac{1}{25}$$

major axis: horizontal

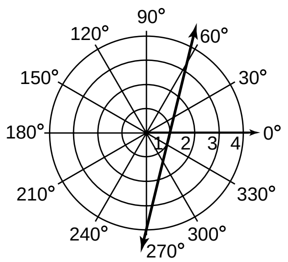
$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$
$$\frac{(x - 2)^2}{1} + \frac{[y - (-3)]^2}{\frac{1}{25}} = 1$$
$$(x - 2)^2 + 25(y + 3)^2 = 1$$

major axis : vertical

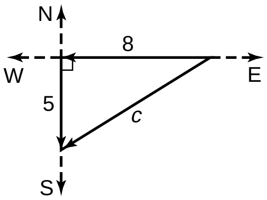
$$\frac{(y - k)^2}{a^2} + \frac{(x - h)^2}{b^2} = 1$$
$$\frac{[y - (-3)]^2}{1^2} + \frac{(x - 2)^2}{\frac{1}{25}} = 1$$
$$(y + 3)^2 + 25(x - 2)^2 = 1$$

51.

θ	0	30°	60°	90°	120°	150°	180°
r	1	1.4	3.9	−3.9	−1.4	−1	−1



52.



$c^2 = a^2 + b^2$
$c^2 = 8^2 + 5^2$
$c = \sqrt{89}$
$c \approx 9.4$
about 9.4 m/s

53. $\cos 70° \approx 0.34$
$\cos 170° \approx -0.98$
$\cos 70°$

54. $\frac{5\pi}{16} = \frac{5\pi}{16} \times \frac{180°}{\pi}$
$= 56.25$
$= 56\frac{15}{60}$
$= 56°\ 15'$

55. $\frac{2y+5}{y^2+3y+2} = \frac{2y+5}{(y+2)(y+1)}$
$\frac{2y+5}{y^2+3y+2} = \frac{A}{y+2} + \frac{B}{y+1}$
$2y + 5 = A(y+1) + B(y+2)$
$2(-2) + 5 = A(-2+1) + B(-2+2)$
$1 = -A$
$-1 = A$
$2y + 5 = A(y+1) + B(y+2)$
$2(-1) + 5 = A(-1+1) + B(-1+2)$
$3 = B$
$\frac{A}{y+2} + \frac{B}{y+1} = \frac{-1}{y+2} + \frac{3}{y+1}$

56. $\frac{x_1}{y_2} = \frac{x_2}{y_1}$
$\frac{12}{y_2} = \frac{5}{4}$
$y_2 = 9.6$

57. $8m - 3n - 4p = 6$
$8m = 6 + 3n + 4p$
$m = \frac{3}{4} + \frac{3}{8}n + \frac{1}{2}p$
$4m + 9n - 2p = -4$
$4\left(\frac{3}{4} + \frac{3}{8}n + \frac{1}{2}p\right) + 9n - 2p = -4$
$3 + \frac{3}{2}n + 2p + 9n - 2p = -4$
$\frac{21}{2}n = -7$
$n = -\frac{2}{3}$

$8m - 3n - 4p = 6$
$8m - 3\left(-\frac{2}{3}\right) - 4p = 6$
$8m + 2 - 4p = 6$
$8m - 4p = 4$
$2m - p = 1$

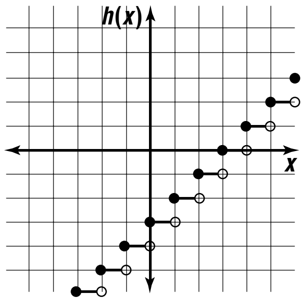
$6m + 12n + 5p = -1$
$6m + 12\left(-\frac{2}{3}\right) + 5p = -1$
$6m - 8 + 5p = -1$
$6m + 5p = 7$

$2m - p = 1 \rightarrow \quad 10m - 5p = 5$
$6m + 5p = 7 \rightarrow \quad (+)\ \ 6m + 5p = 7$
$16m = 12$
$m = \frac{3}{4}$

$8m - 3n - 4p = 6$
$8\left(\frac{3}{4}\right) - 3\left(-\frac{2}{3}\right) - 4p = 6$
$6 + 2 - 4p = 6$
$p = \frac{1}{2}$
$\left(\frac{3}{4}, -\frac{2}{3}, \frac{1}{2}\right)$

58.

x	$h(x) = [[x]] - 3$
$-3 \le x < -2$	-6
$-2 \le x < -1$	-5
$-1 \le x < 0$	-4
$0 \le x < 1$	-3
$1 \le x < 2$	-2
$2 \le x < 3$	-1
$3 \le x < 4$	0
$4 \le x < 5$	1
$5 \le x < 6$	2
$6 \le x < 7$	3

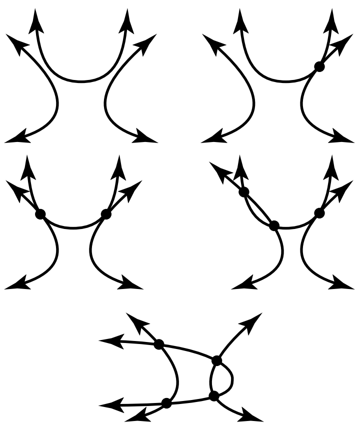


59. The expression $\frac{5a^8b^5}{180a^6b^2}$ simplifies to $\frac{a^2b^3}{36}$. Since $1 < b$ and $2 < a$, the expression is always larger than $\frac{2^2 1^3}{36} = \frac{1}{9}$. Since $b < 2$ and $a < 3$, the expression is always less than $\frac{3^2 2^3}{36} = \frac{72}{36}$ or 2. The correct choice is B.

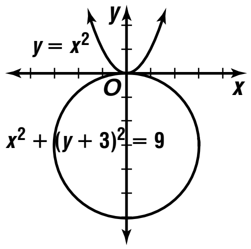
10-8 Systems of Second-Degree Equations and Inequalities

Page 682 Check for Understanding

1. Possible number of solutions: 0, 1, 2, 3, or 4



2. Sample answer: $y = x^2$, $x^2 + (y + 3)^2 = 9$



3. The system contains equation(s) that are equivalent. The graphs coincide.

4. Graph each second-degree inequality. The region in which the graphs overlap represents the solution to the system.

5. $x - y = 0$

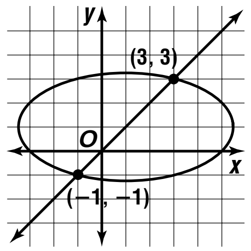
$x = y$

$$\frac{(x-1)^2}{20} + \frac{(y-1)^2}{5} = 1$$
$$\frac{(x-1)^2}{20} + \frac{(x-1)^2}{5} = 1$$
$$(x-1)^2 + 4(x-1)^2 = 20$$
$$x^2 - 2x + 1 + 4(x^2 - 2x + 1) = 20$$
$$5x^2 - 10x + 5 - 20 = 0$$
$$5(x^2 - 2x - 3) = 0$$
$$5(x - 3)(x + 1) = 0$$

$x - 3 = 0$ $x + 1 = 0$

$x = 3$ $x = -1$

(3, 3), (−1, −1)



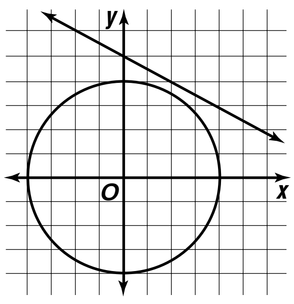
6. $x + 2y = 10$

$x = 10 - 2y$

$$x^2 + y^2 = 16$$
$$(10 - 2y)^2 + y^2 = 16$$
$$100 - 40y + 5y^2 = 16$$
$$5y^2 - 40y + 84 = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$y = \frac{-(-40) \pm \sqrt{(-40)^2 - 4(5)(84)}}{2(5)}$$
$$y = \frac{40 \pm \sqrt{-80}}{10}$$

no solution

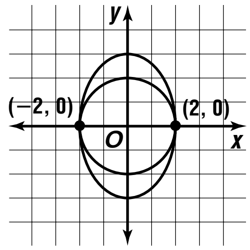


7. $x^2 + y^2 = 4$

$$y^2 = 4 - x^2$$
$$9x^2 - 4y^2 = 36$$
$$9x^2 - 4(4 - x^2) = 36$$
$$13x^2 - 16 = 36$$
$$x^2 = 4$$
$$x = \pm 2$$

$$x^2 + y^2 = 4$$
$$2^2 + y^2 = 4$$
$$y^2 = 0$$
$$y = 0$$

(±2, 0)

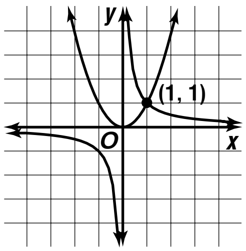


8. $xy = 1$

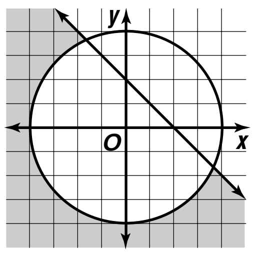
$$x(x^2) = 1$$
$$x^3 = 1$$
$$x = 1$$

$$x^2 = y$$
$$1^2 = y$$
$$1 = y$$

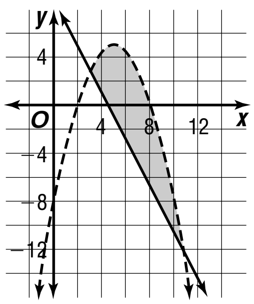
(1, 1)



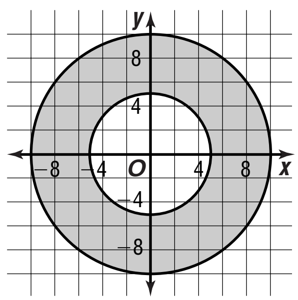
9.



10.

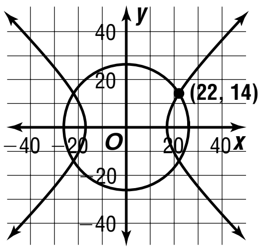


11.



12a. Let x = side length of flowerbed 1.
Let y = side length of flowerbed 2.
$A_1 = x \cdot x$ or x^2
$A_2 = y \cdot y$ or y^2
Total Area $= x^2 + y^2$
$680 = x^2 + y^2$
$x^2 + y^2 = 680$
Difference of Areas $= x^2 - y^2$
$288 = x^2 - y^2$
$x^2 - y^2 = 288$

12b.



Since side length cannot be negative, an estimated solution is (22, 14).

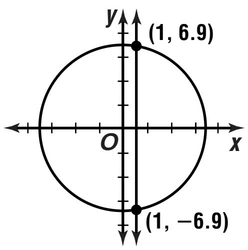
12c. $x^2 + y^2 = 680$
$y^2 = 680 - x^2$

$x^2 - y^2 = 288$	$x^2 + y^2 = 680$
$x^2 - (680 - x^2) = 288$	$22^2 + y^2 = 680$
$2x^2 = 968$	$y^2 = 196$
$x^2 = 484$	$y = 14$
$x = 22$	

22 ft and 14 ft

Pages 682–684 Exercises

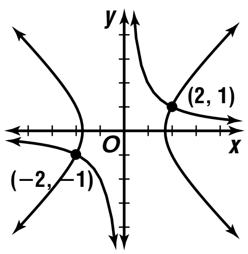
13. $x - 1 = 0$
$x = 1$
$y^2 = 49 - x^2$
$y^2 = 49 - (-1)^2$
$y^2 = 48$
$y \approx \pm 6.9$
$(1, \pm 6.9)$



14. $xy = 2$
$y = \frac{2}{x}$
$x^2 = 3 + y^2$
$x^2 = 3 + \left(\frac{2}{x}\right)^2$
$x^2 = 3 + \frac{4}{x^2}$
$x^4 = 3x^2 + 4$
$x^4 - 3x^2 - 4 = 0$
$(x^2 - 4)(x^2 + 1) = 0$

$x^2 - 4 = 0$	$x^2 + 1 = 0$
$x^2 = 4$	$x^2 = -1$
$x = \pm 2$	
$xy = 2$	$xy = 2$
$2(y) = 2$	$(-2)y = 2$
$y = 1$	$y = -1$

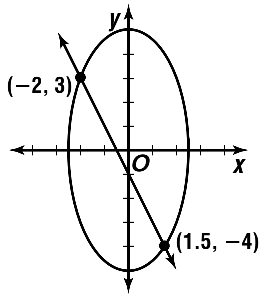
$(2, 1), (-2, -1)$



15. $-1 = 2x + y$
$-1 - 2x = y$
$4x^2 + y^2 = 25$
$4x^2 + (-1 - 2x)^2 = 25$
$4x^2 + 1 + 4x + 4x^2 = 25$
$8x^2 + 4x - 24 = 0$
$4(2x^2 + x - 6) = 0$
$4(2x - 3)(x + 2) = 0$

$2x - 3 = 0$	$x + 2 = 0$
$x = 1.5$	$x = -2$
$-1 = 2x + y$	$-1 = 2x + y$
$-1 = 2(1.5) + y$	$-1 = 2(-2) + y$
$-4 = y$	$3 = y$

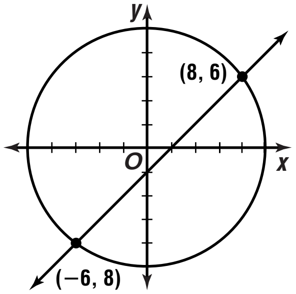
$(1.5, -4), (-2, 3)$



16. $x - y = 2$
$x = 2 + y$
$x^2 = 100 - y^2$
$(2 + y)^2 = 100 - y^2$
$4 + 4y + y^2 = 100 - y^2$
$2y^2 + 4y - 96 = 0$
$2(y^2 + 2y - 48) = 0$
$2(y - 6)(y + 8) = 0$

$y - 6 = 0$	$y + 8 = 0$
$y = 6$	$y = -8$
$x - y = 2$	$x - y = 2$
$x - 6 = 2$	$x - (-8) = 2$
$x = 8$	$x = -6$

$(8, 6), (-6, -8)$



17. $x - y = 0$

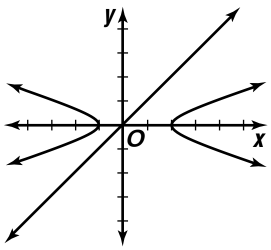
$$
\begin{aligned}
x &= y\\
\frac{(x-1)^2}{9} - y^2 &= 1\\
\frac{(y-1)^2}{9} - y^2 &= 1\\
(y-1)^2 - 9y^2 &= 9\\
y^2 - 2y + 1 - 9y^2 &= 9\\
-8y^2 - 2y - 8 &= 0\\
4y^2 + y + 4 &= 0
\end{aligned}
$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-1 \pm \sqrt{1^2 - 4(4)(4)}}{2(1)}$$

$$y = \frac{-1 \pm \sqrt{-63}}{2}$$

no solution

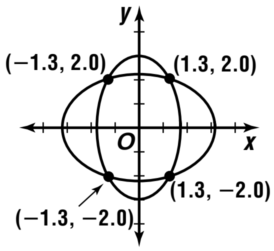


18. $x^2 + 2y^2 = 10$

$$
\begin{aligned}
x^2 &= 10 - 2y^2\\
3x^2 &= 9 - y^2\\
3(10 - 2y^2) &= 9 - y^2\\
30 - 6y^2 &= 9 - y^2\\
-5y^2 &= -21\\
y^2 &= 4.2\\
y &\approx \pm 2.0
\end{aligned}
$$

$$
\begin{aligned}
3x^2 &= 9 - y^2 & 3x^2 &= 9 - y^2\\
3x^2 &\approx 9 - (2.0)^2 & 3x^2 &\approx 9 - (-2.0)^2\\
x^2 &\approx 1.6 & x^2 &\approx 1.6\\
x &\approx \pm 1.3 & x &= \pm 1.3
\end{aligned}
$$

$(\pm 1.3, 2.0), (\pm 1.3, -2.0)$

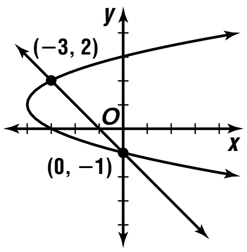


19. $x + y = -1$

$$
\begin{aligned}
y &= -1 - x\\
(y-1)^2 &= 4 + x\\
(-1 - x - 1)^2 &= 4 + x\\
(-2 - x)^2 &= 4 + x\\
4 + 4x + x^2 &= 4 + x\\
x^2 + 3x &= 0\\
x(x + 3) &= 0
\end{aligned}
$$

$$
\begin{aligned}
x &= 0 & x + 3 &= 0\\
& & x &= -3\\
x + y &= -1 & x + y &= -1\\
0 + y &= -1 & -3 + y &= -1\\
y &= -1 & y &= 2
\end{aligned}
$$

$(0, -1), (-3, 2)$

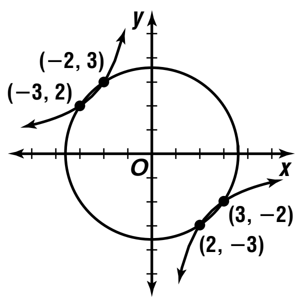


20. $xy + 6 = 0$

$$
\begin{aligned}
y &= -\frac{6}{x}\\
x^2 + y^2 &= 13\\
x^2 + \left(-\frac{6}{x}\right)^2 &= 13\\
x^2 + \frac{36}{x^2} &= 13\\
x^4 + 36 &= 13x^2\\
x^4 - 13x^2 + 36 &= 0\\
(x^2 - 9)(x^2 - 4) &= 0
\end{aligned}
$$

$$
\begin{aligned}
x^2 - 9 &= 0 & x^2 - 4 &= 0\\
x^2 &= 9 & x^2 &= 4\\
x &= \pm 3 & x &= \pm 2\\
xy + 6 &= 0 & xy + 6 &= 0\\
3y + 6 &= 0 & 2y + 6 &= 0\\
y &= -2 & y &= -3\\
xy + 6 &= 0 & xy + 6 &= 0\\
-3y + 6 &= 0 & -2y + 6 &= 0\\
y &= 2 & y &= 3
\end{aligned}
$$

$(3, -2), (-3, 2)$ $\quad$ $(2, -3), (-2, 3)$



21. $x^2 + y - 3 = 0$

$x^2 = -y + 3$

$x^2 + 4y^2 = 36$

$-y + 3 + 4y^2 = 36$

$4y^2 - y - 33 = 0$

$(y - 3)(4y + 11) = 0$

$y - 3 = 0$ $\qquad$ $4y + 11 = 0$

$y = 3$ $\qquad$ $y \approx -2.8$

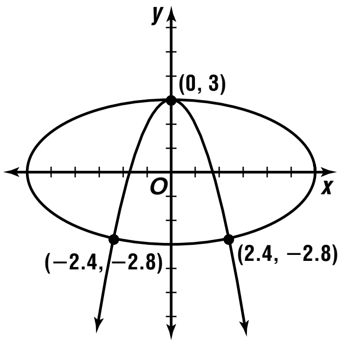
$x^2 + y - 3 = 0$ $\qquad$ $x^2 + y - 3 = 0$

$x^2 + 3 - 3 = 0$ $\qquad$ $x^2 + (-2.8) - 3 \approx 0$

$x^2 = 0$ $\qquad$ $x^2 \approx 5.8$

$x = 0$ $\qquad$ $x \approx \pm 2.4$

$(0, 3), (\pm 2.4, -2.8)$



22. $2y - x + 3 = 0$

$2y + 3 = x$

$x^2 = 16 - y^2$

$(2y + 3)^2 = 16 - y^2$

$4y^2 + 12y + 9 = 16 - y^2$

$5y^2 + 12y - 7 = 0$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-12 \pm \sqrt{12^2 - 4(5)(-7)}}{2(5)}$$

$$y = \frac{-12 \pm \sqrt{284}}{10}$$

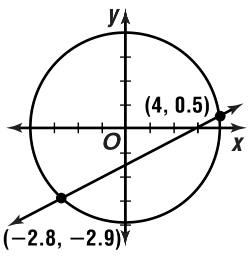
$y \approx 0.5$ $\qquad$ or $\qquad$ $y \approx -2.9$

$2y - x + 3 = 0$ $\qquad$ $2y - x + 3 = 0$

$2(0.5) - x + 3 \approx 0$ $\qquad$ $2(-2.9) - x + 3 \approx 0$

$x \approx 4.0$ $\qquad$ $x \approx -2.8$

$(4.0, 0.5), (-2.8, -2.9)$



23. $xy = -4$

$y = -\frac{4}{x}$

$x^2 = 25 - 9y^2$

$x^2 = 25 - 9\left(-\frac{4}{x}\right)^2$

$x^2 = 25 - \frac{144}{x^2}$

$x^4 = 25x^2 - 144$

$x^4 - 25x^2 + 144 = 0$

$(x^2 - 9)(x^2 - 16) = 0$

$x^2 - 9 = 0$ $\qquad$ $x^2 - 16 = 0$

$x^2 = 9$ $\qquad$ $x^2 = 16$

$x = \pm 3$ $\qquad$ $x = \pm 4$

$xy = -4$ $\qquad$ $xy = -4$

$3y = -4$ $\qquad$ $4y = -4$

$y \approx -1.3$ $\qquad$ $y = -1$

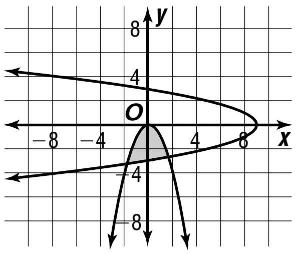
$xy = -4$ $\qquad$ $xy = -4$

$-3y = -4$ $\qquad$ $-4y = -4$

$y \approx 1.3$ $\qquad$ $y = 1$

$(3, -1.3), (-3, 1.3)$ $\qquad$ $(4, -1), (-4, 1)$

24.



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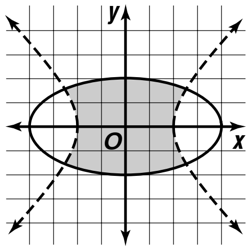

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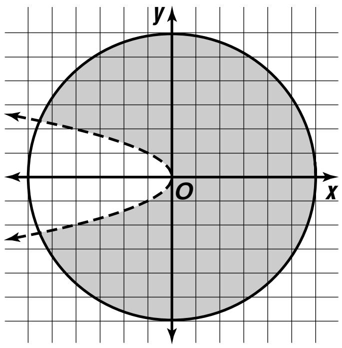
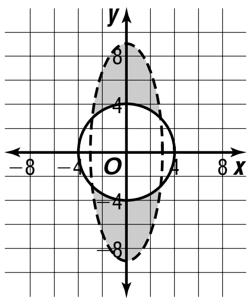


27.



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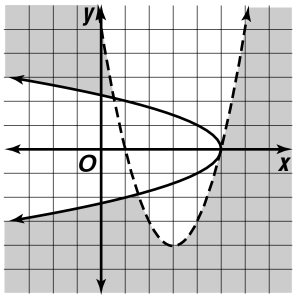

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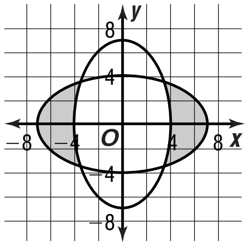

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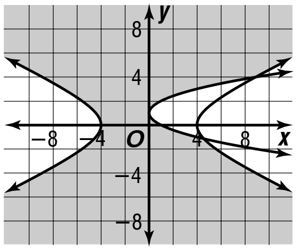

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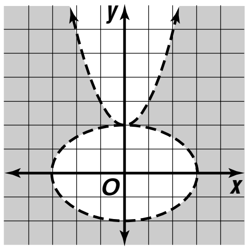
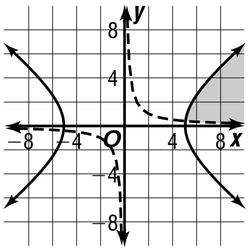
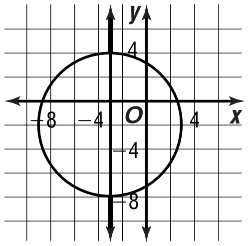


32.



33.



34. parabola:

vertex: $(1, -3)$

$(y - k)^2 = 4p(x - h)$

$(-5 + 3)^2 = 4p(-1 - 1)$

$-\frac{1}{2} = p$

$(y - 3)^2 = 4\left(-\frac{1}{2}\right)(x - 1)$

$(y + 3)^2 = -2(x - 1)$

line:

$m = -2,\ b = -7$

$y = mx + b$

$y = -2x - 7$

35. circle:

center: $(0, 0)$, radius: $2\sqrt{2}$

$x^2 + y^2 = r^2$

$x^2 + y^2 = 8$

hyperbola:

$(-2)(-2) = 4$

$xy = 4$

36. large ellipse:

$a = 5,\ b = 4$, center $= (0, 0)$

$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$

$\frac{y^2}{25} + \frac{x^2}{16} \leq 1$ (interior is shaded)

small ellipse:

$a = 3,\ b = 2$, center $= (0, -1)$

$\frac{x^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$

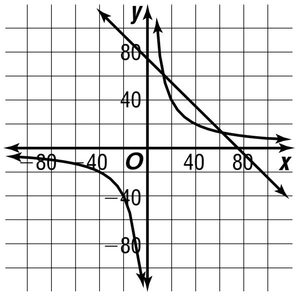
$\frac{x^2}{9} + \frac{(y + 1)^2}{4} > 1$ (exterior is shaded)

37a. $2x + 2y = P$ $\quad xy = A$

$2x + 2y = 150$ $\quad xy = 800$

37b. A system of a line and a hyperbola may have 0, 1, or 2 solutions.

37c.



37d. $xy = 800$

$y = \frac{800}{x}$

$2x + 2y = 150$

$2x + 2\left(\frac{800}{x}\right) = 150$

$2x + \frac{1600}{x} = 150$

$2x^2 + 1600 = 150x$

$x^2 - 75x + 800 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-75) \pm \sqrt{(-75)^2 - 4(1)(800)}}{2(1)}$

$x = \frac{75 \pm \sqrt{2425}}{2}$

$x \approx 12.88$ or $x \approx 62.12$

$xy = 800$ $\quad xy = 800$

$12.88y \approx 800$ $\quad 62.12y \approx 800$

$y \approx 62.11$ $\quad y \approx 12.88$

12.9 m by 62.1 m or 62.1 m by 12.9 m

38a. $(h, k) = (0, 4)$

$(x, y) = (6, 0)$

$(x - h)^2 = 4p(y - k)$

$(6 - 0)^2 = 4p\ (0, 4)$

$36 = -16p$

$-2.25 = p$

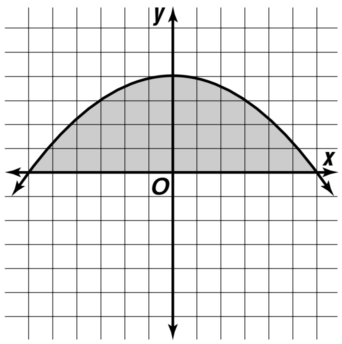
$(x - h)^2 = 4p(y - k)$

$(x - 0)^2 = 4(-2.25)(y - 4)$

$x^2 = -9(y - 4)$

$x^2 \leq -9(y - 4),\ y \geq 0$

38b.



38c. $(h, k) = (0, 3)$
$(x, y) = (6, 0)$
$(x - h)^2 = 4p(y - k)$
$(6 - 0)^2 = 4p(0 - 3)$
$36 = -12p$
$-3 = p$
$(x - h)^2 = 4p(y - k)$
$(x - 0)^2 = 4(-3)(y - 3)$
$x^2 = -12(y - 3)$
$x^2 \leq -12(y - 3), y \geq 0$

39. $xy = -12$
$y = -\frac{12}{x}$

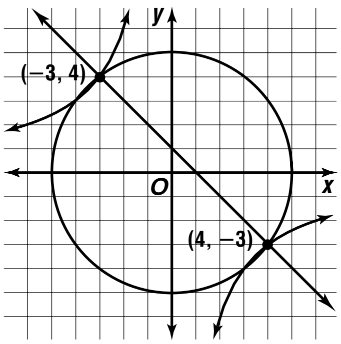
$x = -y + 1$
$x = -\left(-\frac{12}{x}\right) + 1$
$x^2 = 12 + x$
$x^2 - x - 12 = 0$
$(x - 4)(x + 3) = 0$

$x - 4 = 0$	$x + 3 = 0$
$x = 4$	$x = -3$
$xy = -12$	$xy = -12$
$4y = -12$	$-3y = -12$
$y = -3$	$y = 4$
$(4, -3)$	$(-3, 4)$

Check that $(4, -3)$ and $(-3, 4)$ are also solutions of $y^2 = 25 - x^2$.

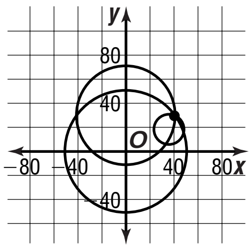
$y^2 = 25 - x^2$	$y^2 = 25 - x^2$
$(-3)^2 \stackrel{?}{=} 25 - (4)^2$	$(4)^2 \stackrel{?}{=} 25 - (-3)^2$
$9 = 9$ ✓	$16 = 16$ ✓

$(4, -3), (-3, 4)$



40a. first station: $(h, k = (0, 0)$
$x^2 + y^2 = r^2$
$x^2 + y^2 = 50^2$
$x^2 + y^2 = 2500$
second station: $(h, k) = (0, 30)$
$(x - h)^2 + (y - k)^2 = r^2$
$x^2 + (y - 30)^2 = 40^2$
$x^2 + (y - 30)^2 = 1600$
third station: $(h, k) = (35, 18)$
$(x - h)^2 + (y - k)^2 = r^2$
$(x - 35)^2 + (y - 18)^2 = 13^2$
$(x - 35)^2 + (y - 18)^2 = 169$

40b.



estimate: (40, 30)

40c. $x^2 + y^2 = 2500$
$x^2 = 2500 - y^2$
$x^2 + (y - 30)^2 = 1600$
$2500 - y^2 + y^2 - 60y + 900 = 1600$
$-60y + 1800 = 0$
$y = 30$
$(x - 35)^2 + (y - 18)^2 = 169$
$x^2 - 70x + 1225 + (30 - 18)^2 = 169$
$x^2 - 70x + 1200 = 0$
$(x - 30)(x - 40) = 0$

$x - 30 = 0$	or	$x - 40 = 0$
$x = 30$		$x = 40$

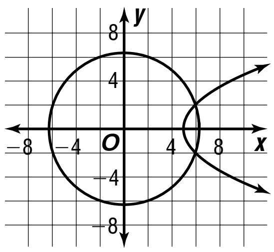
Check (30, 30) and (40, 30):

$x^2 + y^2 = 2500$	$x^2 + y^2 = 2500$
$30^2 + 30^2 \stackrel{?}{=} 2500$	$40^2 + 30^2 \stackrel{?}{=} 2500$
$1800 \neq 2500$	$2500 = 2500$ ✓

(40, 30)

41. $x + 3y = k$
$2y^2 + 3y = k$
$2y^2 + 3y - k = 0$
$y^2 + \frac{3}{2}y - \frac{1}{2}k = 0$
$y^2 + \frac{3}{2}y + \left(\frac{3}{4}\right)^2 = 0$ *Complete the square.*
$\left(y + \frac{3}{4}\right)^2 = 0$
$-\frac{1}{2}k = \left(\frac{3}{4}\right)^2$
$-\frac{1}{2}k = \frac{9}{16}$
$k = -\frac{9}{8}$

42a.



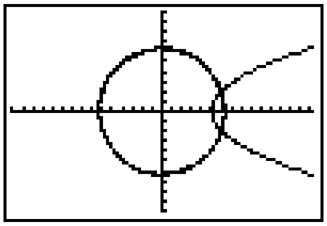
42b. yes; (6, 2) or (6, −2)

42c. Earth's surface:
$x_1{}^2 + y_1{}^2 = 40$
$\frac{x_1{}^2}{40} + \frac{y_1{}^2}{40} = 1$
$\left(\frac{x_1}{\sqrt{40}}\right)^2 + \left(\frac{y_1}{\sqrt{40}}\right)^2 = 1$
$\cos^2 t + \sin^2 t = 1$

$\left(\frac{x_1}{\sqrt{40}}\right)^2 = \cos^2 t$	$\left(\frac{y_1}{\sqrt{40}}\right)^2 = \sin^2 t$
$\frac{x_1}{2\sqrt{10}} = \cos t$	$\frac{y_1}{2\sqrt{10}} = \sin t$
$x_1 = 2\sqrt{10}\cos t$	$y_1 = 2\sqrt{10}\sin t$

asteroid:
Let $y_2 = -t$.
$x_2 = 0.25y^2 + 5$
$x_2 = 0.25\,t^2 + 5$

42d.



Tmin = −8, Tmax = 8, Tstep = 0.13
$[-15.16, 15.16]$ scl: 1 by $[-10, 10]$ scl = 1

43.

$$\frac{x^2}{9} + y^2 = 1$$

$$\frac{(x'\cos 30° + y'\sin 30°)^2}{9} + (-x'\sin 30° + y'\cos 30°)^2 = 1$$

$$\frac{\left(\frac{\sqrt{3}}{2}x' + \frac{1}{2}y'\right)^2}{9} + \left(-\frac{1}{2}x' + \frac{\sqrt{3}}{2}y'\right)^2 = 1$$

$$\frac{\frac{3}{4}(x')^2 + \frac{\sqrt{3}}{2}x'y' + \frac{1}{4}(y')^2}{9} + \frac{1}{4}(x')^2 - \frac{\sqrt{3}}{2}x'y' + \frac{3}{4}(y')^2 = 1$$

$$\frac{3}{4}(x')^2 + \frac{\sqrt{3}}{2}x'y' + \frac{1}{4}(y')^2 + \frac{9}{4}(x')^2 - \frac{9\sqrt{3}}{2}x'y' + \frac{27}{4}(y')^2 = 9$$

$$3(x')^2 - 4\sqrt{3}x'y' + 7(y')^2 - 9 = 0$$

44. $x = 4t + 1$ $\qquad$ $y = 5t - 7$

$\frac{x-1}{4} = t$ $\qquad$ $\frac{y+7}{5} = t$

$$\frac{x-1}{4} = \frac{y+7}{5}$$

$(x - 1)(5) = (4)(y + 7)$
$5x - 5 = 4y + 28$
$5x - 4y - 33 = 0$

45. $4\csc\theta\cos\theta\tan\theta = 4\left(\frac{1}{\sin\theta}\right)(\cos\theta)\left(\frac{\sin\theta}{\cos\theta}\right)$
$= 4$

46. $r = 10$ cm or 0.10 m
$v = r\omega$
$v = (0.10)\left(\frac{5 \cdot 2\pi}{1}\right)$
$v \approx 3.14$ m/s

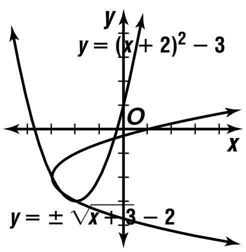
47.

r	1	0	0	−4
0	1	0	0	−4
1	1	1	1	−3
2	1	2	4	4

between 1 and 2

48.

$$y = (x + 2)^2 - 3$$
$$x = (y + 2)^2 - 3$$
$$x + 3 = (y + 2)^2$$
$$\pm\sqrt{x + 3} = y + 2$$
$$\pm\sqrt{x + 3} - 2 = y$$



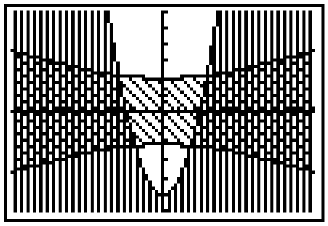
49. No; the domain value 4 is mapped to two elements in the range, 0 and −3.

50. area of rectangle $= \ell\omega$
$= 8(4)$ or 32
area of circles $= 2\,(\pi r^2)$
$= 2(4\pi)$ or 8π
area of shaded region $= 32 - 8\pi$
The correct choice is E.

10-8B Graphing Calculator Exploration: Shading Areas on a Graph

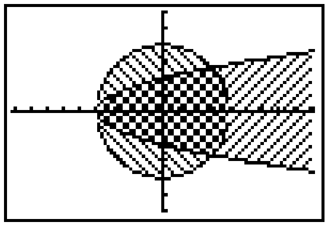
Page 686

1.



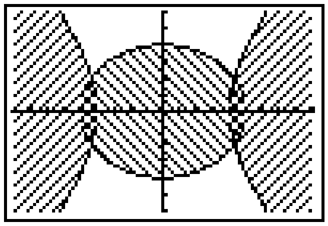
$[-9.1, 9.1]$ scl: 1 by $[-6, 6]$ scl: 1

2.



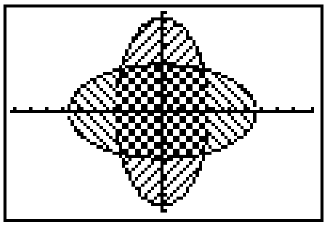
$[-9.1, 9.1]$ scl: 1 by $[-6, 6]$ scl: 1

3.



$[-9.1, 9.1]$ scl: 1 by $[-6, 6]$ scl: 1

4.

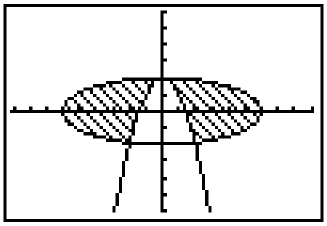


$[-9.1, 9.1]$ scl: 1 by $[-6, 6]$ scl: 1

5a. 3

5b. Find the points of intersection for the boundary equation by using the TRACE function.

5c. SHADE(−√((36−X²)/9),√((36−X²)/9),−6,−2,3,4);
SHADE(−X²+2,√((36−X²)/9),−2,2,3,4,);
SHADE(−√((36−X²)/9), √((36−X²)/9),2,6,3,4)



[−9.1, 9.1] scl: 1 by [−6, 6] scl: 1

6. See students' work.

Chapter 10 Study Guide and Assessment

Page 687 Understanding and Using the Vocabulary

1. true
2. false; center
3. false; transverse
4. true
5. false; hyperbola
6. false; axis or axis of symmetry
7. true
8. false; parabola
9. true
10. false; ellipse

Pages 688–690 Skills and Concepts

11. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(-3 - 1)^2 + [-4 - (-6)]^2}$

$d = \sqrt{20}$ or $2\sqrt{5}$

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1 + (-3)}{2}, \frac{-6 + (-4)}{2}\right)$

$= (-1, -5)$

12. $d = \sqrt{x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(a + 3 - a)^2 + (b + 4 - b)^2}$

$d = \sqrt{25}$ or 5

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{a + a + 3}{2}, \frac{b + b + 4}{2}\right)$

$= (a + 1.5, b + 2)$

13. $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[3 - (-5)]^2 + [4 - (-2)]^2}$

$= \sqrt{100}$ or 10

$DC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(10 - 2)^2 + [3 - (-3)]^2}$

$= \sqrt{100}$ or 10

$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(10 - 3)^2 + (3 - 4)^2}$

$= \sqrt{50}$ or $5\sqrt{2}$

$AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[2 - (-5)]^2 + [-3 - (-2)]^2}$

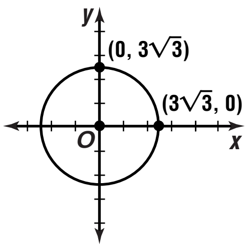
$= \sqrt{50}$ or $5\sqrt{2}$

Yes; $AB = DC = 10$ and $BC = AD = 5\sqrt{2}$. Since opposite sides of quadrilateral $ABCD$ are congruent, $ABCD$ is a parallelogram.

14. $(x - h)^2 + (y - k)^2 = r^2$

$(x - 0)^2 + (y - 0)^2 = \left(3\sqrt{3}\right)^2$

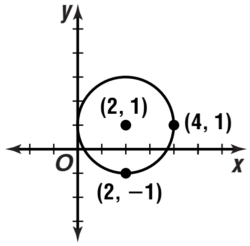
$x^2 + y^2 = 27$



15. $(x - h)^2 + (y - k)^2 = r^2$

$(x - 2)^2 + (y - 1)^2 = 2^2$

$(x - 2)^2 + (y - 1)^2 = 4$

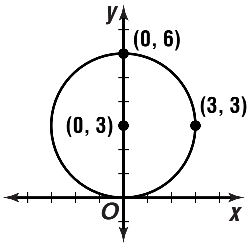


16. $x^2 + y^2 = 6y$

$x^2 + (y^2 - 6y + ?) = 0 + ?$

$x^2 + (y^2 - 6y + 9) = 0 + 9$

$x^2 + (y - 3)^2 = 9$

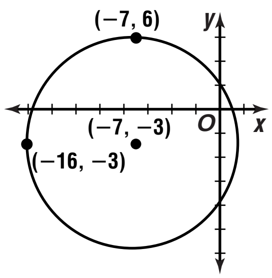


17. $x^2 + 14x + y^2 + 6y = 23$

$(x^2 + 14x + ?) + (y^2 + 6y + ?) = 23 + ? + ?$

$(x^2 + 14x + 49) + (y^2 + 6y + 9) = 23 + 49 + 9$

$(x + 7)^2 + (y + 3)^2 = 81$



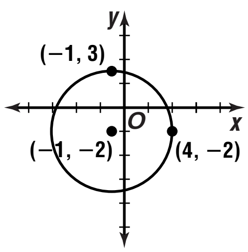
18. $3x^2 + 3y^2 + 6x + 12y - 60 = 0$

$x^2 + y^2 + 2x + 4y - 20 = 0$

$(x^2 + 2x + ?) + (y^2 + 4y + ?) = 20 + ? + ?$

$(x^2 + 2x + 1) + (y^2 + 4y + 4) = 20 + 1 + 4$

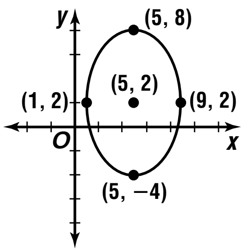
$(x + 1)^2 + (y + 2)^2 = 25$



19. $$x^2 + y^2 + Dx + Ey + F = 0$$
$$1^2 + 1^2 + D(1) + E(1) + F = 0 \Rightarrow$$
$$D + E + F = -2$$
$$(-2)^2 + 2^2 + D(-2) + E(2) + F = 0 \Rightarrow$$
$$-2D + 2E + F = -8$$
$$(-5)^2 + 1^2 + D(-5) + E(1) + F = 0 \Rightarrow$$
$$-5D + E + F = -26$$
$$\begin{aligned} D + E + F &= -2 \\ \underline{(-1)(-5D + E + F)} &\underline{= (-1)(-26)} \\ 6D \qquad\quad &= 24 \\ D &= 4 \end{aligned}$$
$$\begin{aligned} -2D + 2E + F &= -8 \\ \underline{(-1)(D + \quad E + F)} &\underline{= (-1)(-2)} \\ -3D + \quad E \qquad &= -6 \\ -3(4) + E &= -6 \\ E &= 6 \end{aligned}$$
$$\begin{aligned} D + E + F &= -2 \\ 4 + (6) + F &= -2 \\ F &= -12 \end{aligned}$$
$$x^2 + y^2 + Dx + Ey - F = 0$$
$$x^2 + y^2 + 4x + 6y - 12 = 0$$
$$(x^2 + 4x + ?) + (y^2 + 6y + ?) = 12 + ? + ?$$
$$(x^2 + 4x + 4) + (y^2 + 6y + 9) = 12 + 4 + 9$$
$$(x + 2)^2 + (y + 3)^2 = 25$$

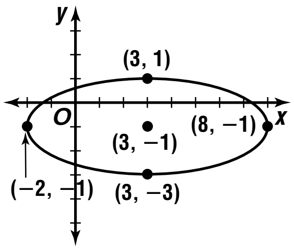
center: $(h, k) = (-2 - 3)$
$r = \sqrt{25}$ or 5

20. center: $(h, k) = (5, 2)$
$a^2 = 36$ $\qquad b^2 = 16$
$a = \sqrt{36}$ or 6 $\qquad b = \sqrt{16}$ or 4
$c = \sqrt{a^2 - b^2}$
$c = \sqrt{6 - 4}$ or $\sqrt{2}$
foci: $(h, k \pm c) = (5, 2 \pm \sqrt{2})$
major axis vertices: $(h, k \pm a) = (5, 2 \pm 6)$ or
$(5, 8), (5, -4)$
minor axis vertices: $(h \pm b, k) = (5 \pm 4, 2)$ or
$(9, 2), (1, 2)$



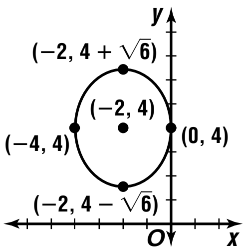
21. $$4x^2 + 25y^2 - 24x + 50y = 39$$
$$4(x^2 - 6x + ?) + 25(y^2 + 2y + ?) = 39 + ? + ?$$
$$4(x^2 - 6x + 9) + 25(y^2 + 2y + 1) = 39 + 4(9) + 25(1)$$
$$4(x - 3)^2 + 25(y + 1)^2 = 100$$
$$\frac{(x - 3)^2}{25} + \frac{(y + 1)^2}{4} = 1$$

center: $(h, k) = (3, -1)$
$a^2 = 25$ $\qquad b^2 = 4$
$a = \sqrt{25}$ or 5 $\qquad b = \sqrt{4}$ or 2
$c = \sqrt{a^2 - b^2}$
$c = \sqrt{25 - 4}$ or $\sqrt{21}$
foci: $(h \pm c, k) = (3 \pm \sqrt{21}, -1)$
major axis vertices: $(h \pm a, k) = (3 \pm 5, -1)$ or
$(8, -1), (-2, -1)$
minor axis vertices: $(h, k \pm b) = (3, -1 \pm 2)$ or
$(3, 1), (3, -3)$



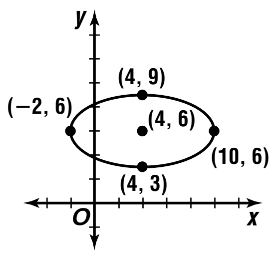
22. $$6x^2 + 4y^2 + 24x - 32y + 64 = 0$$
$$6(x^2 + 4x + ?) + 4(y^2 - 8y + ?) = -64 + ? + ?$$
$$6(x^2 + 4x + 4) + 4(y^2 - 8y + 16) = -64 + 6(4) + 4(16)$$
$$6(x + 2)^2 + 4(y - 4)^2 = 24$$
$$\frac{(x + 2)^2}{4} + \frac{(y - 4)^2}{6} = 24$$

center: $(h, k) = (-2, 4)$
$a^2 = 6$ $\qquad b^2 = 4$
$a = \sqrt{6}$ $\qquad b = \sqrt{4}$ or 2
$c = \sqrt{a^2 - b^2}$
$c = \sqrt{6 - 4}$ or $\sqrt{2}$
foci: $(h, k \pm c) = (-2, 4 \pm \sqrt{2})$
major axis vertices: $(h, k \pm a) = \left(-2, 4 \pm \sqrt{6}\right)$
minor axis vertices: $(h \pm b, k) = (-2 \pm 2, 4)$ or
$(0, 4), (-4, 4)$



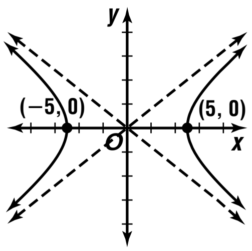
23. $$x^2 + 4y^2 + 124 = 8x + 48y$$
$$(x^2 - 8x + ?) + 4(y^2 - 12y + ?) = -124 + ? + ?$$
$$(x^2 - 8x + 16) + 4(y^2 - 12y + 36) = -124 + 16 + 4(36)$$
$$(x - 4)^2 + 4(y - 6)^2 = 36$$
$$\frac{(x - 4)^2}{36} + \frac{(y - 6)^2}{9} = 36$$

center: $(h, k) = (4, 6)$
$a^2 = 36$ $\qquad b^2 = 9$
$a = \sqrt{36}$ or 6 $\qquad b = \sqrt{9}$ or 3
$c = \sqrt{a^2 - b^2}$
$c = \sqrt{36 - 9}$ or $3\sqrt{3}$
foci: $(h \pm c, k) = (4 \pm 3\sqrt{3}, 6)$
major axis vertices: $(h \pm a, k) = (4 \pm 6, 6)$ or
$(10, 6), (-2, 6)$
minor axis vertices: $(h, k \pm b) = (4, 6 \pm 3)$ or
$(4, 9), (4, 3)$

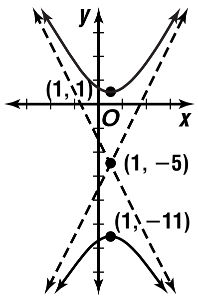


24. $(h, k) = (-4, 1)$
$a = 9$
$b = 6$
$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$
$$\frac{(y-1)^2}{9^2} + \frac{[x-(-4)]^2}{6^2} = 1$$
$$\frac{(y-1)^2}{81} + \frac{(x+4)^2}{36} = 1$$

25. center: $(h, k) = (0, 0)$
$a^2 = 25$ $\quad$ $b^2 = 16$
$a = \sqrt{25}$ or 5 $\quad$ $b = \sqrt{16}$ or 4
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{25 + 16}$ or $\sqrt{41}$
transverse axis: horizontal
foci: $(h \pm c, k) = (0 \pm \sqrt{41}, 0)$ or $(\pm\sqrt{41}, 0)$
vertices: $(h \pm a, k) = (0 \pm 5, 0)$ or $(-5, 0), (5, 0)$
asymptotes: $y - k = \pm \frac{b}{a}(x - h)$
$$y - 0 = \pm \frac{4}{5}(x - 0)$$
$$y = \pm \frac{4}{5}x$$

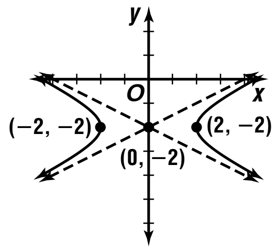


26. center: $(h, k) = (1, -5)$
$a^2 = 36$ $\quad$ $b^2 = 9$
$a = \sqrt{36}$ or 6 $\quad$ $b = \sqrt{9}$ or 3
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{36 + 9}$ or $3\sqrt{5}$
transverse axis: vertical
foci: $(h, k \pm c) = \left(1, -5 \pm 3\sqrt{5}\right)$
vertices: $(h, k \pm a) = (1, -5 \pm 6)$ or $(1, 1), (1, -11)$
asymptotes: $y - k = \pm \frac{a}{b}(x - h)$
$$y - (-5) = \pm \frac{6}{3}(x - 1)$$
$$y + 5 = \pm 2(x - 1)$$



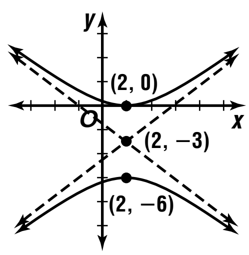
27.
$$x^2 - 4y^2 - 16y = 20$$
$$x^2 - 4(y^2 + 4y + ?) = 20 + ?$$
$$x^2 - 4(y^2 + 4y + 4) = 20 - 4(4)$$
$$x^2 - 4(y + 2)^2 = 4$$
$$\frac{x^2}{4} - \frac{(y+2)^2}{1} = 1$$

center: $(h, k) = (0, -2)$
$a^2 = 4$ $\quad$ $b^2 = 1$
$a = \sqrt{4}$ or 2 $\quad$ $b = \sqrt{1}$ or 1
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{4 + 1}$ or $\sqrt{5}$
transverse axis: horizontal
foci: $(h \pm c, k) = (0 \pm \sqrt{5}, -2)$ or $(\pm\sqrt{5}, -2)$
vertices: $(h \pm a, k) = (0 \pm 2, -2)$ or $(-2, -2)$, $(2, -2)$
asymptotes: $y - k = \pm \frac{b}{a}(x - h)$
$$y - (-2) = \pm \frac{1}{2}(x - 0)$$
$$y + 2 = \pm \frac{1}{2}x$$



28.
$$9x^2 - 16y^2 - 36x - 96y + 36 = 0$$
$$9(x^2 - 4x + ?) - 16(y^2 + 6y + ?) = -36 + ? + ?$$
$$9(x^2 - 4x + 4) - 16(y^2 + 6y + 9) = -36 + 9(4) - 16(9)$$
$$9(x-2)^2 - 16(y+3)^2 = -144$$
$$\frac{(y+3)^2}{9} - \frac{(x-2)^2}{16} = 1$$

center: $(h, k) = (2, -3)$
$a^2 = 9$ $\quad$ $b^2 = 16$
$a = \sqrt{9}$ or 3 $\quad$ $b = \sqrt{16}$ or 4
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{9 + 16}$ or 5
transverse axis: vertical
foci: $(h, k \pm c) = (2, -3 \pm 5)$ or $(2, 2), (2, -8)$
vertices: $(h, k \pm a) = (2, -3 \pm 3)$ or $(2, 0), (2, -6)$
asymptotes: $y - k = \pm \frac{a}{b}(x - h)$
$$y - (-3) = \pm \frac{3}{4}(x - 2)$$
$$y + 3 = \pm \frac{3}{4}(x - 2)$$



29. $c = 9$

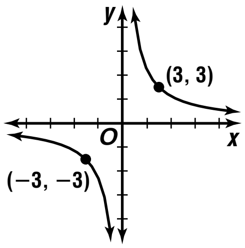
quadrants: I and III

transverse axis: $y = x$

vertices: $xy = 9$ $\qquad$ $xy = 9$

$3(3) = 9$ $\qquad$ $(-3)(-3) = 9$

$(3, 3)$ $\qquad$ $(-3, -3)$



30. $2b = 10$

$b = 5$

center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1 + 1}{2}, \frac{-1 + 5}{2}\right)$

$= (1, 2)$

transverse axis: vertical

a = distance from center to a vertex

$= |2 - (-1)|$ or 3

$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$$

$$\frac{(y - 2)^2}{3^2} - \frac{(x - 1)^2}{5^2} = 1$$

$$\frac{(y - 2)^2}{9} - \frac{(x - 1)^2}{25} = 1$$

31. center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 6}{2}, \frac{-3 + (-3)}{2}\right)$

$= (2, -3)$

a = distance from center to a vertex

$= |2 - (-2)|$ or 4

c = distance from center to a focus

$= |2 - (-4)|$ or 6

$b^2 = c^2 - a^2$

$b^2 = 6^2 - 4^2$

$b^2 = 20$

transverse axis: horizontal

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

$$\frac{(x - 2)^2}{4^2} - \frac{[y - (-3)]^2}{20} = 1$$

$$\frac{(x - 2)^2}{16} - \frac{(y + 3)^2}{20} = 1$$

32. vertex: $(h, k) = (5, 3)$

$4p = 8$

$p = 2$

focus $(h, k + p) = (5, 3 + 2)$ or $(5, 5)$

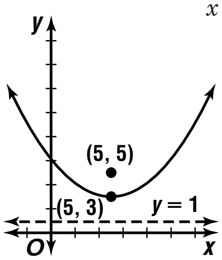
directrix: $y = k - p$

$y = 3 - 2$

$y = 1$

axis of symmetry: $x = h$

$x = 5$



33. vertex: $(h, k) = (1, -2)$

$4p = -16$

$p = -4$

focus: $(h + p, k) = (1 + (-4), -2)$ or $(-3, -2)$

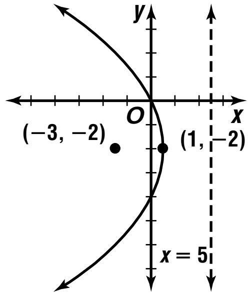
directrix: $x = h - p$

$x = 1 - (-4)$

$x = 5$

axis of symmetry: $y = k$

$y = -2$



34. $y^2 + 6y - 4x = -25$

$y^2 + 6y + ? = 4x - 25 + ?$

$y^2 + 6y + 9 = 4x - 25 + 9$

$(y + 3)^2 = 4(x - 4)$

vertex: $(h, k) = (4, -3)$

$4p = 4$

$p = 1$

focus: $(h + p, k) = (4 + 1, -3)$ or $(5, -3)$

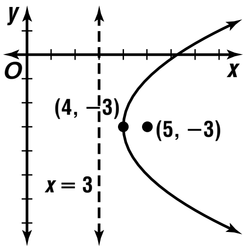
directrix: $x = h - p$

$x = 4 - 1$

$x = 3$

axis of symmetry: $y = k$

$y = -3$



35. $x^2 + 4x = y - 8$

$x^2 + 4x + 4 = y - 8 + 4$

$(x + 2)^2 = y - 4$

vertex: $(h, k) = (-2, 4)$

$4p = 1$

$p = \frac{1}{4}$

focus: $(h, k + p) = \left(-2, 4 + \frac{1}{4}\right)$ or $(-2, 4.25)$

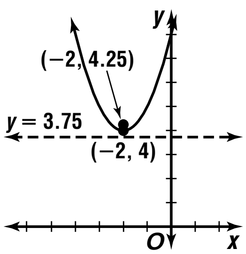
directrix: $y = k - p$

$y = 4 - \frac{1}{4}$

$y = 3.75$

axis of symmetry: $x = h$

$x = -2$



36. vertex: $(h, k) = (-1, 3)$
$(y - k)^2 = 4p(x - h)$
$(7 - 3)^2 = 4p[-3 - (-1)]$
$16 = 8p$
$2 = p$
Since parabola opens left, $p = -2$.
$(y - k)^2 = 4p(x - h)$
$(y - 3)^2 = 4(-2)[x - (-1)]$
$(y - 3)^2 = -8(x + 1)$

37. vertex: $(h, k) = \left(5, \frac{2-4}{2}\right)$ or $(5, -1)$
focus: $(h, k + p) = (5, 2)$
$k + p = 2$
$-1 + p = 2$
$p = 3$
$(x - h)^2 = 4p(y - k)$
$(x - 5)^2 = 4(3)[y - (-1)]$
$(x - 5)^2 = 12(y + 1)$

38. $A = 5$, $c = 2$; ellipse

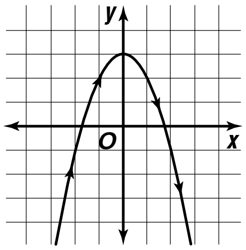
39. $A = C = 0$; equilateral hyperbola

40. $A = C = 5$; circle

41. $C = 0$; parabola

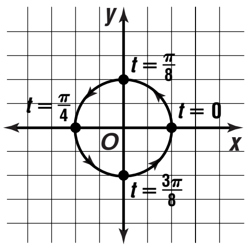
42. $y = -t^2 + 3$
$y = -x^2 + 3$

t	x	y	(x, y)
-2	-2	-1	$(-2, -1)$
-1	-1	2	$(-1, 2)$
0	0	3	$(0, 3)$
1	1	2	$(1, 2)$
2	2	-1	$(2, -1)$



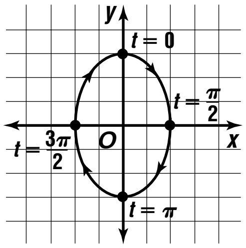
43. $x = \cos 4t$ $\qquad$ $y = \sin 4t$
$\cos^2 4t + \sin^2 4t = 1$
$x^2 + y^2 = 1$

t	x	y	(x, y)
0	1	0	$(1, 0)$
$\frac{\pi}{8}$	0	1	$(0, 1)$
$\frac{\pi}{4}$	-1	0	$(-1, 0)$
$\frac{3\pi}{8}$	0	-1	$(0, -1)$



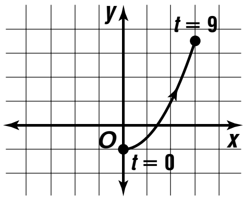
44. $x = 2 \sin t$ $\qquad$ $y = 3 \cos t$
$\frac{x}{2} = \sin t$ $\qquad$ $\frac{y}{3} = \cos t$
$\sin^2 t + \cos^2 t = 1$
$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$
$\frac{x^2}{4} + \frac{y^2}{9} = 1$

t	x	y	(x, y)
0	0	3	$(0, 3)$
$\frac{\pi}{2}$	2	0	$(2, 0)$
π	0	-3	$(0, -3)$
$\frac{3\pi}{2}$	-2	0	$(-2, 0)$



45. $x = \sqrt{t}$
$x^2 = t$
$y = \frac{t}{2} - 1$
$y = \frac{x^2}{2} - 1$

t	x	y	(x, y)
0	0	-1	$(0, -1)$
4	2	1	$(2, 1)$
9	3	3.5	$(3, 3.5)$



46. Sample answer:
Let $x = t$.
$y = 2x^2 + 4$
$y = 2t^2 + 4, -\infty < t < \infty$

47. Sample answer:
$x^2 + y^2 = 49$
$\frac{x^2}{49} + \frac{y^2}{49} = 1$
$\left(\frac{x}{7}\right)^2 + \left(\frac{y}{7}\right)^2 = 1$
$\sin^2 t + \cos^2 t = 1$
$\left(\frac{x}{7}\right)^2 = \sin^2 t$ $\qquad$ $\left(\frac{y}{7}\right)^2 = \cos^2 t$
$\frac{x}{7} = \sin t$ $\qquad$ $\frac{y}{7} = \cos t$
$x = 7 \sin t$ $\qquad$ $y = 7 \cos t, 0 \le t \le 2\pi$

48. Sample answer:

$$\frac{x^2}{36} + \frac{y^2}{81} = 1$$
$$\left(\frac{x}{6}\right)^2 + \left(\frac{y}{9}\right)^2 = 1$$
$$\cos^2 t + \sin^2 t = 1$$
$$\left(\frac{x}{6}\right)^2 = \cos^2 t \qquad \left(\frac{y}{9}\right)^2 = \sin^2 t$$
$$\frac{x}{6} = \cos t \qquad \frac{y}{9} = \sin t$$
$$x = 6 \cos t \qquad y = 9 \sin t,\ 0 \le t \le 2\pi$$

49. Sample answer:

Let $y = t$.

$x = -y^2$

$x = -t^2,\ -\infty < t < \infty$

50. $B^2 - 4AC = 0 - 4(4)(9)$

$= -144$

$A \neq C$; ellipse

$$\begin{aligned}
4x^2 + 9y^2 &= 36\\
4\left(x' \cos \tfrac{\pi}{6} + y' \sin \tfrac{\pi}{6}\right)^2 + 9\left(-x' \sin \tfrac{\pi}{6} + y' \cos \tfrac{\pi}{6}\right)^2 &= 36\\
4\left(\tfrac{\sqrt{3}}{2}x' + \tfrac{1}{2}y'\right)^2 + 9\left(-\tfrac{1}{2}x' + \tfrac{\sqrt{3}}{2}y'\right)^2 &= 36\\
4\left[\tfrac{3}{4}(x')^2 + \tfrac{\sqrt{3}}{2}x'y' + \tfrac{1}{4}(y')^2\right] &\\
+ 9\left[\tfrac{1}{4}(x')^3 - \tfrac{\sqrt{3}}{2}x'y' + \tfrac{3}{4}(y')^2\right] &= 36\\
3(x')^2 + 2\sqrt{3}x'y' + (y')^2 &\\
+ \tfrac{9}{4}(x')^2 - \tfrac{9\sqrt{3}}{2}x'y' + \tfrac{27}{4}(y')^2 &= 36\\
\tfrac{21}{4}(x')^2 - \tfrac{5\sqrt{3}}{2}x'y' + \tfrac{31}{4}(y')^2 &= 36\\
21(x')^2 - 10\sqrt{3}x'y' + 31(y')^2 - 144 &= 0
\end{aligned}$$

51. $B^2 - 4AC = 0 - 4(0)(1)$

$= 0$

parabola

$$\begin{aligned}
y^2 - 4x &= 0\\
(-x' \sin 45° + y' \cos 45°)^2 - 4(x' \cos 45° + y' \sin 45°) &= 0\\
\left(-\tfrac{\sqrt{2}}{2}x' + \tfrac{\sqrt{2}}{2}y'\right)^2 - 4\left(\tfrac{\sqrt{2}}{2}x' - \tfrac{\sqrt{2}}{2}y'\right) &= 0\\
\tfrac{1}{2}(x')^2 - x'y' + \tfrac{1}{2}(y')^2 - 2\sqrt{2}x' + 2\sqrt{2}y' &= 0\\
(x')^2 - 2x'y' + (y')^2 - 4\sqrt{2}x' + 4\sqrt{2}y' &= 0
\end{aligned}$$

52. $B^2 - 4AC = 0 - 4(4)(-16)$

$= 256$

hyperbola

$$\begin{aligned}
4x^2 - 16(y-1)^2 &= 64\\
4(x-h)^2 - 16(y-k-1)^2 &= 64\\
4(x-1)^2 - 16(y+2-1)^2 &= 64\\
4(x-1)^2 - 16(y+1)^2 &= 64\\
4(x^2 - 2x + 1) - 16(y^2 + 2y + 1) &= 64\\
4x^2 - 8x + 4 - 16y^2 - 32y - 16 - 64 &= 0\\
x^2 - 4y^2 - 2x - 8y - 19 &= 0
\end{aligned}$$

53. $B^2 - 4AC = \left(2\sqrt{3}\right)^2 - 4(6)(8)$

$= -180$

$A \neq C$; ellipse

$\tan 2\theta = \frac{B}{A - C}$

$\tan 2\theta = \frac{2\sqrt{3}}{6 - 8}$

$\tan 2\theta = -\sqrt{3}$

$2\theta = -60°$

$\theta = -30°$

54. $B^2 - 4AC = (-6)^2 - 4(1)(9)$

$= 0$

parabola

$\tan 2\theta = \frac{B}{A - C}$

$\tan 2\theta = \frac{-6}{1 - 9}$

$\tan 2\theta = \frac{3}{4}$

$2\theta \approx 36.86989765°$

$\theta \approx 18°$

55. $(x-1)^2 + 4(y-1)^2 = 20$

$(y-1)^2 + 4(y-1)^2 = 20$

$5(y-1)^2 = 20$

$(y-1)^2 = 4$

$y - 1 = \pm 2$

$y = 3$ or -1

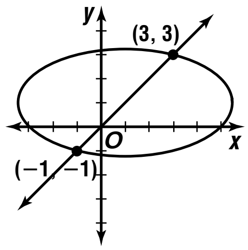
$y = 3 \qquad x = y$

$\qquad\qquad x = 3$

$y = -1 \qquad x = y$

$\qquad\qquad x = -1$

$(3, 3), (-1, -1)$



56. $2x - y = 0$

$2x = y$

$y^2 = 49 + x^2$

$(2x)^2 = 49 + x^2$

$4x^2 = 49 + x^2$

$3x^2 = 49$

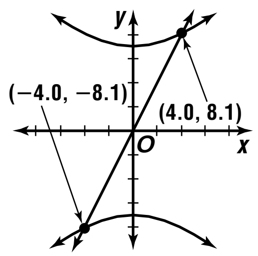
$x \approx \pm 4.04$

$2x - y = 0 \qquad 2x - y = 0$

$2(4.04) - y \approx 0 \qquad 2(-4.04) - y \approx 0$

$y \approx 8.08 \qquad y \approx -8.08$

$(4.0, 8.1), (-4.0, -8.1)$



57. $x^2 - 4x - 4y = 4$

$x^2 - 4x - 4 = 4y$

$(x - 2)^2 + 4y = 0$

$(x - 2)^2 + x^2 - 4x - 4 = 0$

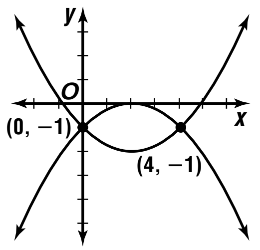
$x^2 - 4x + 4 + x^2 - 4x - 4 = 0$

$2x^2 - 8x = 0$

$2x(x - 4) = 0$

$2x = 0$	$x - 4 = 0$
$x = 0$	$x = 4$
$(x - 2)^2 + 4y = 0$	$(x - 2)^2 + 4y = 0$
$(0 - 2)^2 + 4y = 0$	$(4 - 2)^2 + 4y = 0$
$y = -1$	$y = -1$

$(0, -1), (4, -1)$



58. $xy = -4$

$y = -\frac{4}{x}$

$x^2 + y^2 = 12$

$x^2 + \left(-\frac{4}{x}\right)^2 = 12$

$x^2 + \frac{16}{x^2} = 12$

$x^4 + 16 - 12x^2 = 0$

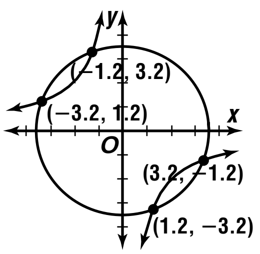
$(x^2)^2 - 12(x^2) + 16 = 0$

$x^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

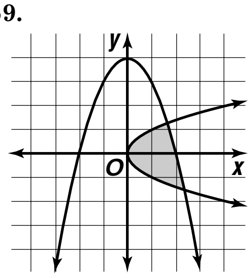
$x^2 = \frac{12 \pm \sqrt{12^2 - 4(1)(16)}}{2(1)}$

$x^2 \approx 10.472$	or	$x^2 \approx 1.528$
$x \approx \pm 3.236$		$x \approx 1.236$
$xy = -4$		$xy = -4$
$3.236y \approx -4$		$1.236y \approx -4$
$y \approx -1.236$		$y \approx -3.236$
$xy = -4$		$xy = -4$
$-3.236y \approx -4$		$-1.236y \approx -4$
$y \approx 1.236$		$y \approx 3.236$

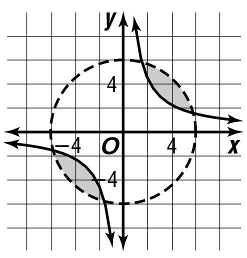
$(3.2, -1.2), (-3.2, 1.2), (1.2, -3.2), (-1.2, 3.2)$



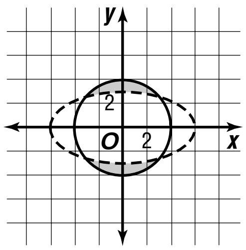
59.



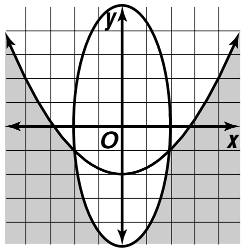
60.



61.



62.



Page 691 Applications and Problem Solving

63a. $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$r = \sqrt{(12 - 0)^2 + (16 - 0)^2}$

$r = 20$

$x^2 + y^2 = r^2$

$x^2 + y^2 = 20^2$

$x^2 + y^2 = 400$

63b. area of watered portion $= \pi r^2$

$= \pi 20^2$

$\approx 1256.6 \text{ ft}^2$

area of backyard $= \ell\omega$

$= 50(40)$

$= 2000 \text{ ft}^2$

area of nonwatered portion $\approx 2000 - 1256.6$

$\approx 743.4 \text{ ft}^2$

percent not watered $\approx \frac{743.4}{2000}$

≈ 0.37

about 37%

64. $2a = 2{,}000$ $\quad$ $e = \frac{c}{a}$

$a = 6000$ $\quad$ $0.2 = \frac{c}{6000}$

$1200 = c$

$b^2 = a^2 - c^2$

$b^2 = 6000^2 - 1200^2$

$b^2 = 34{,}560{,}000$

$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$

$\frac{(x - 0)^2}{6000^2} + \frac{(y - 0)^2}{34{,}560{,}000} = 1$

$\frac{x^2}{36{,}000{,}000} + \frac{y^2}{34{,}560{,}000} = 1$

65. $a = 3.5$

$b = 3$

$c = \sqrt{a^2 - b^2}$

$c = \sqrt{3.5^2 - 3^2}$

$c \approx 1.8$

about 1.8 feet from the center

Page 691 Open-Ended Assessment

1. Sample answer:

$e = \frac{c}{a}$

$\frac{1}{9} = \frac{c}{a}$

Let $a = 9$.

$\frac{1}{9} = \frac{c}{9}$

$c = 1$

$b^2 = a^2 - c^2$

$b^2 = 9^2 - 1^2$

$b^2 = 80$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{9^2} + \frac{y^2}{80} = 1$

$\frac{x^2}{81} + \frac{y^2}{80} = 1$

2. Sample answer:

axis of symmetry: $x = h$

$x = 2$, so $h = 2$

focus: $(h, k + p) = (2, 5)$

$k + p = 5$

Let $k = 2$, $p = 3$.

$(x - h)^2 = 4p(y - k)$

$(x - 2)^2 = 4(3)(y - 2)$

$(x - 2)^2 = 12(y - 2)$

SAT & ACT Preparation

Page 693 SAT & ACT Practice

1. Add the two numbers of parts to get the whole, 8. The fraction of red jelly beans to the whole is $\frac{3}{8}$. The total number of jelly beans is 160. The number of red jelly beans is $\frac{3}{8}(160)$ or 60. The correct choice is C.

Or you can use a ratio box. Multiply by 20.

Green	Red	Whole
5	3	8
	60	160

2. Notice the capitalized word *EXCEPT*. You might want to try the plug-in method on this problem. Choose a value for b that is an odd integer, say 1. Then substitute that value for b in the equation.

$a^2b = 12^2$

$a^2(1) = 12^2$

$a^2 = 12^2$

$a = 12$

Check the answer choices for divisors of this value of a. 12 is divisible by 3, 4, 6, and 12, but not by 9. The correct choice is D.

3. The information in the question confirms the information given in the figure. Recall the formula for the area of a triangle — one half the base times the height. The triangle DCB is obtuse, so the height will lie outside of the triangle.

Let $\overline{DC}$ be the base. The length of the base is 6. The height will be *equal* to 7, since it is a line segment parallel to $\overline{AD}$ through point B.

$A = \frac{1}{2}bh$

$= \frac{1}{2}(6)(7)$ or 21

The correct choice is A.

4. The problem asks how many more girls there are than boys. First find how many girls and how many boys there are in the class.

One method is to find the fraction of girls in the whole class and the fraction of boys in the whole class. Since the ratio of girls to boys is 4 to 3, the fraction of girls in the whole class is $\frac{4}{7}$. Find the number of girls in the class by multiplying this fraction by 35.

$\frac{4}{7}(35) = 20$ There are 20 girls in the class.

Using the same process, the fraction of boys is $\frac{3}{7}$.

$\frac{3}{7}(35) = 15$ There are 15 boys in the class.

So there are 5 more girls than boys. The correct choice is D.

Another method is to use a "Ratio Box." First enter the given information, shown in the darker cells below. Then enter the number for the total of the first row, 7. To go from the total of 7 to the total of 35, you must multiply by 5. Write a 5 in each cell in the second row.

Girls	Boys	Total
4	3	7
×5	×5	×5
20	15	35

Then multiply the two numbers in the first column to get 20 girls, shown with a dark border. Multiply the second column to get 15 boys. Subtract to find there are 5 more girls than boys.

5. Set A is the set of all positive integers less than 30. Set B is the set of all positive multiples of 5. The intersection of Sets A and B is the set of all elements that are in both Set A and Set B. The intersection consists of all positive multiples of 5 which are also less than 30. The intersection of the two sets is {5, 10, 15, 20, 25}.

The correct choice is A.

6. For a quadratic equation in the form $y = a(x - h)^2 + k$, the coordinates of the vertex of the graph of the function are given by the ordered pair (h, k). So the vertex of the graph of $y = \frac{1}{2}(x - 3)^2 + 4$ has coordinates (3, 4). The correct choice is C.

7. On the SAT, if you forget the relationships for 45° right triangles, look at the Reference Information in the gray box at the beginning of each mathematics section of the exam. The measure of each leg of a 45–45–90 triangle is equal to the length of the hypotenuse divided by $\sqrt{2}$. Multiply both numerator and denominator by $\sqrt{2}$ and simplify.

$$\begin{aligned} BC &= \frac{8}{\sqrt{2}} \\ &= \frac{8}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{8\sqrt{2}}{2} \text{ or } 4\sqrt{2} \end{aligned}$$

The correct choice is D. You could also use the Pythagorean Theorem and the fact that the two legs must be equal in length, but that method might take more time.

8. Form a ratio using the given fractions as numerator and denominator. Write a proportion, using x as the unknown. Multiply the cross-products. Solve for x.

$$\frac{\frac{1}{7}}{\frac{1}{5}} = \frac{100}{x}$$

$$\frac{1}{7}x = \frac{1}{5}(100)$$

$$\frac{1}{7}x = 20$$

$$x = 140$$

The correct choice is E.

9. Let d represent the number of dimes in the jar. Since there are 4 more nickels than dimes, there are $d + 4$ nickels in the jar. So, the ratio of dimes to nickles in the jar is $\frac{d}{d + 4}$. This ratio is less than 1. The only answer choice that is less than 1 is choice A, $\frac{8}{10}$. If $\frac{d}{d + 4} = \frac{8}{10}$, then $d = 16$. So, there are 16 dimes and 16 + 4 or 20 nickels in the jar, and $\frac{16}{20} = \frac{8}{10}$.

The correct choice is A.

10. Set up a proportion.

$$\frac{\text{total liters}}{\text{total bottles}} = \frac{x \text{ liters}}{1 \text{ bottle}}$$

$$\frac{8}{20} = \frac{x}{1}$$

$$20x = 8$$

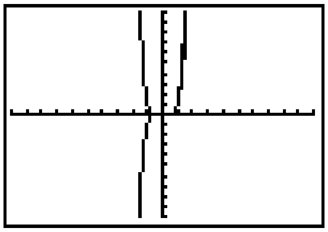
$$x = 0.4 \text{ or } \frac{2}{5}$$

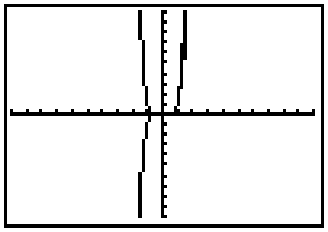
The correct answer is .4 or 2/5.

Chapter 11 Exponential and Logarithmic Function

11-1 Real Exponents

Page 695 Graphing Calculator Exploration

1. 

2. 

3. $a^m \cdot a^n = a^{m+n}$

4. $(a^m)^n = a^{mn}$

5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, when $b \neq 0$

Page 700 Check for Understanding

1. The quantities are not the same. When the negative is enclosed inside of the parentheses and the base is raised to an even power, the answer is positive. When the negative is not enclosed inside of the parentheses and the base is raised to an even power, the answer is negative.

2. If the base were negative and the denominator were even, then we would be taking an even root of a negative number, which is undefined as a real number.

3. Laura is correct. The negative exponent of 10 represents a fraction with a numerator of 1 and a denominator of a positive power of 10. The product of this fraction and a number between 1 and 10 is between 0 and 1.

4. $$\begin{aligned} 5^{-4} &= \frac{1}{5^4} \\ &= \frac{1}{625} \end{aligned}$$

5. $$\begin{aligned} \left(\frac{9}{16}\right)^{-2} &= \frac{1}{\left(\frac{9}{16}\right)^2} \\ &= \left(\frac{16}{9}\right)^2 \\ &= \frac{16^2}{9^2} \\ &= \frac{256}{81} \end{aligned}$$

6. $$\begin{aligned} 216^{\frac{1}{3}} &= (6^3)^{\frac{1}{3}} \\ &= 6^{\frac{3}{3}} \\ &= 6 \end{aligned}$$

7. $$\begin{aligned} \sqrt{27} \cdot \sqrt{3} &= 27^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} \\ &= (3^3)^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} \\ &= 3^{\frac{3}{2}} \cdot 3^{\frac{1}{2}} \\ &= 3^{\frac{4}{2}} \\ &= 3^2 \\ &= 9 \end{aligned}$$

8. $$\begin{aligned} 32^{\frac{3}{5}} &= (2^5)^{\frac{3}{5}} \\ &= 2^{\frac{15}{5}} \\ &= 2^3 \\ &= 8 \end{aligned}$$

9. $$\begin{aligned} (3a^{-2})^3 \cdot 3a^5 &= 3^3 \cdot a^{-6} \cdot 3a^5 \\ &= 3^4 a^{-1} \\ &= 81a^{-1} \text{ or } \frac{81}{a} \end{aligned}$$

10. $$\begin{aligned} \sqrt{m^3n^2} \cdot \sqrt{m^4n^5} &= (m^3n^2)^{\frac{1}{2}} \cdot (m^4n^5)^{\frac{1}{2}} \\ &= m^{\frac{3}{2}}n^{\frac{2}{2}} \cdot m^{\frac{4}{2}}n^{\frac{5}{2}} \\ &= m^{\frac{7}{2}}n^{\frac{7}{2}} \text{ or } m^3n^3\sqrt{mn} \end{aligned}$$

11. $$\begin{aligned} \sqrt{\frac{8^n \cdot 2^7}{4^{-n}}} &= \left(\frac{8^n \cdot 2^7}{4^{-n}}\right)^{\frac{1}{2}} \\ &= \frac{8^{\frac{n}{2}} \cdot 2^{\frac{7}{2}}}{4^{-\frac{n}{2}}} \\ &= \frac{(2^3)^{\frac{n}{2}} \cdot 2^{\frac{7}{2}}}{(2^2)^{-\frac{n}{2}}} \\ &= \frac{2^{\frac{3n}{2}} \cdot 2^{\frac{7}{2}}}{2^{-\frac{2n}{2}}} \\ &= 2^{\frac{3n}{2}} \cdot 2^{\frac{7}{2}} \cdot 2^{\frac{2n}{2}} \\ &= 2^{\frac{5n+7}{2}} \\ &= 2^{2n+3} \cdot 2^{\frac{n+1}{2}} \\ &= 2^{2n+3}\sqrt{2^{n+1}} \end{aligned}$$

12. $$\begin{aligned} (2x^4y^8)^{\frac{1}{2}} &= 2^{\frac{1}{2}} \cdot (x^4)^{\frac{1}{2}} \cdot (y^8)^{\frac{1}{2}} \\ &= 2^{\frac{1}{2}} \cdot x^{\frac{4}{2}} \cdot y^{\frac{8}{2}} \\ &= 2^{\frac{1}{2}} \cdot x^2y^4 \quad \text{or} \quad x^2 2^4\sqrt{2} \end{aligned}$$

13. $$\begin{aligned} \sqrt{169x^5} &= (169x^5)^{\frac{1}{2}} \\ &= 169^{\frac{1}{2}} \cdot (x^5)^{\frac{1}{2}} \\ &= 13x^{\frac{5}{2}} \end{aligned}$$

14. $$\begin{aligned} \sqrt[4]{a^2b^3c^4d^5} &= (a^2b^3c^4d^5)^{\frac{1}{4}} \\ &= (a^2)^{\frac{1}{4}}(b^3)^{\frac{1}{4}}(c^4)^{\frac{1}{4}}(d^5)^{\frac{1}{4}} \\ &= |a|^{\frac{1}{2}}\,|b|^{\frac{3}{4}}\,|c|\,\,|d|^{\frac{5}{4}} \end{aligned}$$

15. $$\begin{aligned} 6^{\frac{1}{4}}b^{\frac{3}{4}}c^{\frac{1}{4}} &= (6b^3c)^{\frac{1}{4}} \\ &= \sqrt[4]{6b^3c} \end{aligned}$$

16. $$\begin{aligned} 15x^{\frac{1}{3}}y^{\frac{1}{5}} &= 15x^{\frac{5}{15}}y^{\frac{3}{15}} \\ &= 15\,(x^5y^3)^{\frac{1}{15}} \\ &= 15\sqrt[15]{x^5y^3} \end{aligned}$$

17. $$\begin{aligned} \sqrt[3]{p^4q^6r^5} &= (p^4q^6r^5)^{\frac{1}{3}} \\ &= (p^4)^{\frac{1}{3}}\,(q^6)^{\frac{1}{3}}\,(r^5)^{\frac{1}{3}} \\ &= p^{\frac{4}{3}}q^2r^{\frac{5}{3}} \\ &= pq^2r\sqrt[3]{pr^2} \end{aligned}$$

18. $$\begin{aligned} y^{\frac{4}{5}} &= 34 \\ \left(y^{\frac{4}{5}}\right)^{\frac{5}{4}} &= 34^{\frac{5}{4}} \\ y &= (34^5)^{\frac{1}{4}} \\ y &\approx 82.1 \end{aligned}$$

19. $A = \pi r^2$ $\quad r = 3.875 \times 10^{-7}$ m

$$\begin{aligned} A &= \pi(3.875 \times 10^{-7}\text{ m})^2 \\ &= \pi(3.875)^2 \times (10^{-7})^2\text{ m}^2 \\ &= \pi(15.015625 \times 10^{-14})\text{ m}^2 \\ &\approx 4.717 \times 10^{-13}\text{ m}^2 \end{aligned}$$

Pages 700–703 Exercises

20. $(-6)^{-4} = \frac{1}{(-6)^4} = \frac{1}{1296}$

21. $-6^{-4} = -\left(\frac{1}{6^4}\right) = -\frac{1}{1296}$

22. $(5 \cdot 3)^2 = 15^2 = 225$

23. $\frac{2^4}{2^{-1}} = 2^{4-(-1)} = 2^5 = 32$

24. $\left(\frac{7}{8}\right)^{-3} = \frac{1}{\left(\frac{7}{8}\right)^3} = \left(\frac{8}{7}\right)^3 = \frac{8^3}{7^3} = \frac{512}{343}$

25. $(3^{-1} + 3^{-3})^{-1} = \frac{1}{3^{-1} + 3^{-1}} = \frac{1}{\frac{1}{3} + \frac{1}{9}} = \frac{1}{\frac{4}{9}} = \frac{9}{4}$

26. $81^{\frac{1}{2}} = (9^2)^{\frac{1}{2}} = 9$

27. $729^{\frac{1}{3}} = (9^3)^{\frac{1}{3}} = 9$

28. $\frac{27}{27^{\frac{2}{3}}} = \frac{3^3}{(3^3)^{\frac{2}{3}}} = \frac{3^3}{3^2} = 3$

29. $2^{\frac{1}{2}} \cdot 12^{\frac{1}{2}} = 2^{\frac{1}{2}} \cdot (2 \cdot 6)^{\frac{1}{2}} = 2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} \cdot 6^{\frac{1}{2}} = 2 \cdot 6^{\frac{1}{2}} = 2\sqrt{6}$

30. $64^{\frac{1}{2}} = (2^6)^{\frac{1}{12}} = 2^{\frac{1}{2}}$ or $\sqrt{2}$

31. $16^{-\frac{1}{4}} = \frac{1}{16^{\frac{1}{4}}} = \frac{1}{(2^4)^{\frac{1}{4}}} = \frac{1}{2}$

32. $\frac{(3^7)(9^4)}{\sqrt{27^6}} = \frac{3^7(3^2)^4}{(3^3)^{\frac{6}{2}}} = \frac{3^{15}}{3^9} = 3^6 = 729$

33. $\left(\sqrt[3]{216}\right)^2 = (216^{\frac{1}{3}})^2 = 216^{\frac{2}{3}} = (6^3)^{\frac{2}{3}} = 6^2 = 36$

34. $81^{\frac{1}{2}} - 81^{-\frac{1}{2}} = (9^2)^{\frac{1}{2}} - \frac{1}{(9^2)^{\frac{1}{2}}} = 9 - \frac{1}{9} = 8\frac{8}{9}$

35. $\frac{1}{\sqrt[7]{(-128)^4}} = \frac{1}{(-128)^{\frac{4}{7}}} = \frac{1}{[(-2)^7]^{\frac{4}{7}}} = \frac{1}{(-2)^4} = \frac{1}{16}$

36. $(3n^2)^3 = 3^3(n^2)^3 = 27n^6$

37. $(y^2)^{-4} \cdot y^8 = \frac{1}{(y^2)^4} \cdot y^8 = \frac{1}{y^8} \cdot y^8 = 1$

38. $(4y^4)^{\frac{3}{2}} = 4^{\frac{3}{2}}(y^4)^{\frac{3}{2}} = (2^2)^{\frac{3}{2}}(y^4)^{\frac{3}{2}} = 2^{\frac{6}{2}}y^{\frac{12}{2}} = 8y^6$

39. $(27p^3q^6r^{-1})^{\frac{1}{3}} = 27^{\frac{1}{3}}(p^3)^{\frac{1}{3}}(q^6)^{\frac{1}{3}}\left(\frac{1}{r}\right)^{\frac{1}{3}} = (3^3)^{\frac{1}{3}}(p^3)^{\frac{1}{3}}(q^6)^{\frac{1}{3}}\left(\frac{1}{r}\right)^{\frac{1}{3}} = 3^{\frac{3}{3}}p^{\frac{3}{3}}q^{\frac{6}{3}}\ \frac{1}{r^{\frac{1}{3}}} = 3pq^2r^{-\frac{1}{3}}$

40. $[(2x^4]^{-2} = \frac{1}{[(2x)^4]^2} = \frac{1}{2^8x^8} = 2^{-8}x^{-8}$ or $\frac{1}{256x^8}$

41. $(36x^6)^{\frac{1}{2}} = 36^{\frac{1}{2}}(x^6)^{\frac{1}{2}} = 6|x|^3$

42. $\left(\frac{b^{2n}}{b^{-2n}}\right)^{\frac{1}{2}} = (b^{2n} \cdot b^{2n})^{\frac{1}{2}} = (b^{4n})^{\frac{1}{2}} = b^{2n}$

43. $\frac{2n}{4n^{\frac{1}{2}}} = \frac{2n \cdot n^{-\frac{1}{2}}}{4} = \frac{2n^{\frac{1}{2}}}{4} = \frac{\sqrt{\text{n}}}{2}$

44. $\left(3m^{\frac{1}{2}} \cdot 27n^{\frac{1}{4}}\right)^4 = 3^4\left(m^{\frac{1}{2}}\right)^4 27^4\left(n^{\frac{1}{4}}\right)^4$
$= 3^4 m^2 (3^3)^4 n$
$= 3^{16} m^2 n$

45. $\left(\frac{f^{-16}}{256g^4h^{-4}}\right)^{-\frac{1}{4}} = (f^{-16} \cdot 256^{-1} \cdot g^{-4} \cdot h^4)^{-\frac{1}{4}}$
$= (f^{-16})^{-\frac{1}{4}}(256^{-1})^{-\frac{1}{4}}(g^{-4})^{-\frac{1}{4}}(h^4)^{-\frac{1}{4}}$
$= f^4\, 256^{\frac{1}{4}} \cdot |g| \cdot |h|^{-1}$
$= 4\, f^4 |g|\, |h|^{-1}$ or $\frac{4 f^4 |g|}{|h|}$

46. $\sqrt[6]{x^2\left(x^{\frac{3}{4}} + x^{-\frac{3}{4}}\right)} = \left[x^2\left(x^{\frac{3}{4}} + x^{-\frac{3}{4}}\right)\right]^{\frac{1}{6}}$
$= \left[x^2 \cdot x^{-\frac{3}{4}}\left(x^{\frac{6}{4}} + 1\right)\right]^{\frac{1}{6}}$
$= \left[x^{\frac{5}{4}}\left(x^{\frac{3}{2}} + 1\right)\right]^{\frac{1}{6}}$

47. $\left(2x^{\frac{1}{4}}y^{\frac{1}{3}}\right)\left(3x^{\frac{1}{4}}y^{\frac{2}{3}}\right) = 6x^{\frac{2}{4}}y^{\frac{3}{3}}$
$= 6x^{\frac{1}{2}}y$

48. $\sqrt[m]{\sqrt[n]{a}} = \left(a^{\frac{1}{n}}\right)^{\frac{1}{m}}$
$= a^{\frac{1}{n}\cdot\frac{1}{m}}$
$= a^{\frac{1}{nm}}$
$= a^{\frac{1}{mn}}$
$= \sqrt[mn]{a}$

49. $\sqrt{m^6 n} = (m^6)^{\frac{1}{2}}\left(n^{\frac{1}{2}}\right)$
$= |m|^3 n^{\frac{1}{2}}$

50. $\sqrt{xy^3} = (x)^{\frac{1}{2}}(y^3)^{\frac{1}{2}}$
$= |x|^{\frac{1}{2}}|y|^{\frac{3}{2}}$

51. $\sqrt[3]{8x^3y^6} = 8^{\frac{1}{3}}(x^3)^{\frac{1}{3}}(y^6)^{\frac{1}{3}}$
$= 2xy^2$

52. $17\sqrt[7]{x^{14}y^7z^{12}} = 17(x^{14})^{\frac{1}{2}}(y^7)^{\frac{1}{7}}(z^{12})^{\frac{1}{7}}$
$= 17x^2yz^{\frac{12}{7}}$

53. $\sqrt[5]{a^{10}b^2} \cdot \sqrt[4]{c^2} = (a^{10})^{\frac{1}{5}}(b^2)^{\frac{1}{5}}(c^2)^{\frac{1}{4}}$
$= a^2b^{\frac{2}{5}}|c|^{\frac{1}{2}}$

54. $60\sqrt[8]{r^{80}s^{56}t^{27}} = 60(r^{80})^{\frac{1}{8}}(s^{56})^{\frac{1}{8}}(t^{27})^{\frac{1}{8}}$
$= 60r^{10}|s|\,t^{\frac{27}{8}}$

55. $16^{\frac{1}{5}} = \sqrt[5]{16}$

56. $(7a)^{\frac{5}{8}}b^{\frac{3}{8}} = \sqrt[8]{7^5a^5b^3}$

57. $p^{\frac{2}{3}}q^{\frac{1}{2}}r^{\frac{1}{3}} = p^{\frac{4}{6}}q^{\frac{3}{6}}r^{\frac{2}{6}}$
$= \sqrt[6]{p^4q^3r^2}$

58. $\frac{2^{\frac{2}{3}}}{2^{\frac{1}{3}}} = 2^{\frac{1}{3}}$
$= \sqrt[3]{2}$

59. $13a^{\frac{1}{7}}b^{\frac{1}{3}} = 13a^{\frac{3}{21}}b^{\frac{7}{21}}$
$= 13\,\sqrt[21]{a^3b^7}$

60. $(n^3m^9)^{\frac{1}{2}} = (n^3)^{\frac{1}{2}}(m^9)^{\frac{1}{2}}$
$= n^{\frac{3}{2}}m^{\frac{9}{2}}$
$= |n|\,m^4\sqrt{mn}$

61. $x = \sqrt[3]{(-245)^{-\frac{1}{5}}}$
$\approx \sqrt[3]{-0.33}$
≈ -0.69

62. $d^3e^2f^2 = (d^3)^{\frac{1}{2}}(e^2)^{\frac{1}{2}}(f^2)^{\frac{1}{2}}$
$= d\,|e|\,|f|\sqrt{d}$

63. $\sqrt[3]{a^5b^7c} = (a^5)^{\frac{1}{3}}(b^7)^{\frac{1}{3}}(c)^{\frac{1}{3}}$
$= ab^2\sqrt[3]{a^2bc}$

64. $\sqrt{20x^3y^6} = (20)^{\frac{1}{2}}(x^3)^{\frac{1}{2}}(y^6)^{\frac{1}{2}}$
$= 2x|y|^3\sqrt{5x}$

65. $14.2 = x^{-\frac{3}{2}}$
$(14.2)^{-\frac{2}{3}} = (x^{-\frac{3}{2}})^{-\frac{2}{3}}$
$0.17 \approx x$

66. $724 = 15a^{\frac{5}{2}} + 12$
$712 = 15a^{\frac{5}{2}}$
$\frac{712}{15} = a^{\frac{5}{2}}$
$\left(\frac{712}{15}\right)^{\frac{2}{5}} = (a^{\frac{5}{2}})^{\frac{2}{5}}$
$4.68 \approx a$

67. $\frac{1}{8}\sqrt{x^5} = 3.5$
$x^{\frac{5}{2}} = 28$
$(x^{\frac{5}{2}})^{\frac{2}{5}} = (28)^{\frac{2}{5}}$
$x \approx 3.79$

68. $d = 6.794 \times 10^3$ km so $r = 3.397 \times 10^3$ km
$V = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi(3.397 \times 10^3 \text{ km})^3$
$\approx 1.64 \times 10^{11} \text{ km}^3$

69. $y = 3^x$; $x = -8, -6, -5, \frac{10}{33}, \frac{1}{2}, \frac{2}{3}, \frac{10}{9}, \frac{5}{3}, \frac{7}{2}$

x	3^x	y
-8	3^{-8}	$\frac{9}{6561}$
-6	3^{-6}	$\frac{1}{729}$
-5	3^{-5}	$\frac{1}{243}$
$\frac{10}{33}$	$3^{\frac{10}{33}}$	1.395
$\frac{1}{2}$	$3^{\frac{1}{2}}$	1.732
$\frac{2}{3}$	$3^{\frac{2}{3}}$	2.080
$\frac{10}{9}$	$3^{\frac{10}{9}}$	3.389
$\frac{5}{3}$	$3^{\frac{5}{3}}$	14.620
$\frac{7}{2}$	$3^{\frac{7}{2}}$	46.765

69a. If $x < 0$ then $y > 0$. If $x = 0$ then $y = 1$. Since $x < 0$, $y > 0$ and $y < 1$. So, $0 < y < 1$.

69b. If $x > 0$ then $y > 1$. If $x < 1$ then $y < 3$. So, $1 < y < 3$.

69c. If $x > 1$ then $y > 3$. So, $y > 3$.

69d. If the exponent is less than 0, the power is greater than 0 and less than 1. If the exponent is greater than 0 and less than 1, the power is greater than 1 and less than the base. If the exponent is greater than 1, the power is greater than the base.

Any number to the zero power is 1. Thus, if the exponent is less than zero, the power is less than 1. A power of a positive number is never negative, so the power is greater than 0.

Any number to the zero power is 1 and to the first power is itself. Thus, if the exponent is greater than zero and less than 1, the power is between 1 and the base.

Any number to the first power is itself. Thus, if the exponent is greater than 1, the power is greater than the base.

70. $r = (1.2 \times 10^{-15})A^{\frac{1}{3}}$

If $r = 2.75 \times 10^{-15}$ then

$$2.75 \times 10^{-5} = (1.2 \times 10^{-15})A^{\frac{1}{3}}$$
$$\frac{2.75 \times 10^{-15}}{1.2 \times 10^{-15}} = A^{\frac{1}{3}}$$
$$2.29 \approx A^{\frac{1}{3}}$$
$$12.04 \approx A$$

Since $12.04 \approx 12$, which is the mass number of carbon, the atom is carbon.

71.
$$32^{(x^2+4x)} = 16^{(x^2+4x+3)}$$
$$(2^5)^{(x^2+4x)} = (2^4)^{(x^2+4x+3)}$$
$$2^{(5x^2+20x)} = 2^{(4x^2+16x+12)}$$
$$5x^2 + 20x = 4x^2 + 16x + 12$$
$$x^2 + 4x - 12 = 0$$
$$(x + 6)(x - 2) = 0$$
$$x + 6 = 0 \qquad x - 2 = 0$$
$$x = -6 \qquad x = 2$$

72a.

Wind Speed	Wind Chill
5	0.8
10	−12.7
15	−22.6
20	−29.9
25	−35.3
30	−39.2

72b. A 5-mile per hour increase in the wind speed when the wind is light has more of an effect on perceived temperature than a 5-mile per hour increase in the wind speed when the wind is heavy.

73a. $r^3 = \frac{GM_e t^2}{4\pi^2}$

$G = 6.67 \times 10^{-11}$
$M_t = 5.98 \times 10^{24}$
$t = 1$ day $= 86{,}4000$ seconds

$$r^3 = \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(86{,}400)^2}{4\pi^2}$$
$$r \approx 42{,}250{,}474.31 \text{ m}$$

73b. $42{,}250{,}474.31$ m $= 42{,}250.47431$ km

$42.250.47431 - 6380 = 35870.47431$
$\approx 35{,}870$ km

74a. $a^m a^n = \overbrace{a \cdot a \cdot \ldots \cdot a}^{m\text{ factors}} \cdot \overbrace{a \cdot a \cdot \ldots \cdot a}^{n\text{ factors}} = \overbrace{a \cdot a \cdot \ldots \cdot a}^{m+n\text{ factors}} = a^{m+n}$

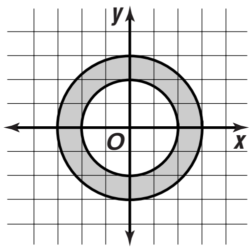
74b. $(a^m)^n = \overbrace{\overbrace{a \cdot a \cdot \ldots \cdot a}^{m\text{ factors}} \cdot \overbrace{a \cdot a \cdot \ldots \cdot a}^{m\text{ factors}} \cdot \ldots \cdot \overbrace{a \cdot a \cdot \ldots \cdot a}^{m\text{ factors}}}^{n\text{ factors}} = \overbrace{a \cdot a \cdot \ldots \cdot a}^{m \cdot n\text{ factors}} = a^{m+n}$

74c. $(ab)^m = \overbrace{ab \cdot ab \cdot \ldots \cdot ab}^{m\text{ factors}} = \overbrace{a \cdot a \cdot \ldots \cdot a}^{m\text{ factors}} \cdot \overbrace{b \cdot b \cdot \ldots \cdot b}^{m\text{ factors}} = a^m b^m$

74d. $\left(\frac{a}{b}\right)^m = \overbrace{\frac{a}{b} \cdot \frac{a}{b} \cdot \ldots \cdot \frac{a}{b}}^{m\text{ factors}} = \frac{a^m}{b^m}$

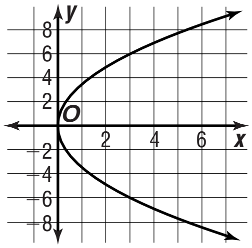
74e. $\frac{a^m}{a^n} = \frac{\overbrace{a \cdot a \cdot \ldots \cdot a}^{m\text{ factors}}}{\overbrace{a \cdot a \cdot \ldots \cdot a}^{n\text{ factors}}} \; a^{m-n}$

75.



76.
$$y^2 = 12x$$
$$(y - 0)^2 = 4(3)(x - 0)$$

Vertex is at (0, 0) and $p = 3$. The parabola opens to the right so the focus is at $(0 + 3, 0)$ or $(3, 0)$. Since the directrix is 3 units to the left of the vertex, the equation of the directrix is $x = -3$.



77. $(2\sqrt{3} + 2i)^{\frac{1}{5}}$

Convert to polar form $r(\cos\theta + \boldsymbol{i}\sin\theta)$.

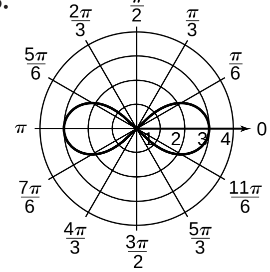
$$r = \sqrt{\left(2\sqrt{3}\right)^2 + 2^2} \qquad \theta = \text{Arctan}\,\frac{2}{2\sqrt{3}}$$
$$= \sqrt{16} \qquad\qquad = \text{Arctan}\,\frac{\sqrt{3}}{3}$$
$$= 4 \qquad\qquad = \frac{\pi}{6}$$

So, $(2\sqrt{3} + 2\boldsymbol{i}) = 4\left(\cos\frac{\pi}{6} + \boldsymbol{i}\sin\frac{\pi}{6}\right)$.

Use De Moivre's Theorem.

$$\left[4\left(\cos\frac{\pi}{6} + \boldsymbol{i}\sin\frac{\pi}{6}\right)\right]^{\frac{1}{5}} = 4^{\frac{1}{5}}\left(\cos\left(\frac{1}{5}\cdot\frac{\pi}{6}\right) + \sin\left(\frac{1}{5}\cdot\frac{\pi}{6}\right)\right)$$
$$= 4^{\frac{1}{5}}\cos\frac{\pi}{30} + 4^{\frac{1}{5}}\boldsymbol{i}\sin\frac{\pi}{30}$$
$$= 1.31 + 0.14\boldsymbol{i}$$

78.



Lemniscate

79. Use the equation $y = t|\vec{v}|\sin\theta - \frac{1}{2}gt^2 + h$.

$|\vec{v}| = 105 \qquad g = 32 \qquad h = 3 \qquad \theta = 42$

$y = t(105)(\sin 42°) - \frac{1}{2}(32)t^2 + 3$

$= 16t^2 - (105 \sin 42°)t - 3$

Find t when $y = 0$ (i.e., the ball is on the ground).

$t = \frac{105 \sin 42° \pm \sqrt{(105 \sin 42°) - 4(16)(-3)}}{2(16)}$

$t = -0.04, 4.43$

So, the ball hits the ground after about 4.43 s.

80. $\overrightarrow{TC} = \langle(2-3), (6-(-4)), (-5-6)\rangle$

$= \langle -1, 10, -11\rangle$

$|\overrightarrow{TC}| = \sqrt{(2-3)^2 + (6-(-4))^2 + (-5-6)^2}$

$= \sqrt{222}$

81. Sample answer:

$\tan S \cos S = \frac{1}{2}$

$\frac{\sin S}{\cos S} \cdot \frac{\cos S}{1} = \frac{1}{2}$

$\sin S = \frac{1}{2}$

82. $\cot\theta = 0$

$\theta = \frac{\pi}{2}$

Period of $\cot\theta$ is π so

$\cot\theta = 0$

$\theta = \frac{\pi}{2} + \pi n$, where n is an integer.

83. $r \cdot 6h = 150\pi$ m

$r = \frac{150\pi \text{ m}}{6 \text{ h}}$

$r = 25\pi$ m/h

84. 90°, 270°

85. $x^3 - 25x = 0$

Degree of 3 so there are 3 complex roots.

$x^3 - 25x = 0$

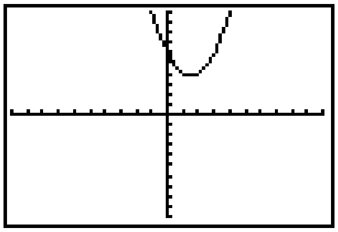
$x(x-5)(x+5) = 0$

$x = 0 \qquad x - 5 = 0 \qquad x + 5 = 0$

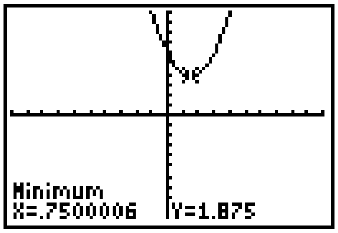
$\qquad\qquad x = 5 \qquad\qquad x = 5$

3; −5, 0, 5

86.



[−5, 5] scl:0.5 by [−5, 5] scl:0.5



abs. min; (0.75, 1.88)

87. The time it takes to paint a building is inversely proportional to the number of painters.

$48 = \frac{k}{8}$

$k = 384$

So $t = \frac{384}{16}$

$t = 24$

The correct choice is E.

11-2 Exponential Functions

Page 705 Graphing Calculator Exploration

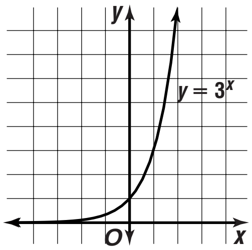
1. positive reals
2. (0, 1)
3. For $a = 0.5$ and 0.75, $y \to \infty$ as $x \to -\infty$, $y \to 0$ as $x \to \infty$. For $a = 2$ and 5, $y \to 0$ as $x \to -\infty$, $y \to \infty$ as $x \to \infty$.
4. horizontal asymptote at $y = 0$, no vertical asymptotes
5. Yes; the range of an exponential function is all positive reals because the value of any positive real number raised to any power is positive.
6. Any nonzero number raised to the zero power is 1.
7. The graph of $y = b^x$ is decreasing when $0 < b < 1$ because multiplying by number between 0 and 1 results in a product less than the original number. The graph of $y = b^x$ is increasing when $b > 1$ because multiplying by a number greater than 1 results in a product greater than the original number.
8. There is a horizontal asymptote at $y = 0$ because a power of a positive real number is never 0 or less. As a number between 0 and 1 is raised to greater and greater powers, its value approaches 0. As a number greater than 1 is raised to powers approaching negative infinity, its value approaches 0.

Page 708 Check for Understanding

1. Power; in a power function the variable is the base, in an exponential function the variable is the exponent.
2. Both graphs are one-to-one, have the domain of all reals, a range of positive reals, a horizontal asymptote of $y = 0$, a y-intercept of (0, 1), and no vertical asymptote. The graph of $y = b^x$ is decreasing when $0 < b < 1$ and increasing when $b > 1$.
3. If the base is greater than 1, the equation represents exponential growth. If base is between 0 and 1, the equation represents exponential decay.
4. The graphs of $y = 4^x$ and $y = 4^x - 3$ are the same except the graph of $y = 4^x - 3$ is shifted down three units from the graph of $y = 4^x$.

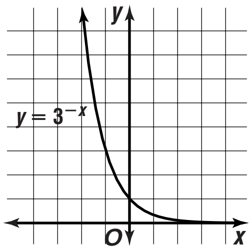
5.

x	y
-1	$\frac{1}{9}$
0	1
1	3
2	9



6.

x	y
-2	9
-1	3
0	1
1	$\frac{1}{9}$



7.

x	y
-2	$-3\frac{3}{4}$
-1	$-3\frac{1}{2}$
0	-4
1	-2
2	0

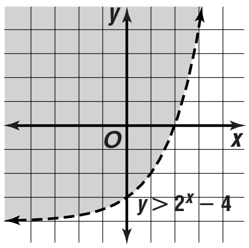
Use (0, 0) as a test point.

$0 \overset{?}{>} 2^0 - 4$

$0 \overset{?}{>} 1 - 4$

$0 > -3$ ✓

The statement is true so shade the region containing (0, 0).



8. Use $N = N_0\,(1 + r)^t$ where $N_0 = 3750$, $r = -0.25$, and $t = 2$.

$N = 3750\,(1 - 0.25)^2$

$= 3750\,(0.75)^2$

≈ 2109.38

9a. $9{,}145{,}219 - 8{,}863{,}052 = 282{,}167$

$\frac{282{,}167}{7} = 40{,}309.57$

$\frac{40{,}309.57}{8{,}863{,}052} \approx 0.0045$

$\approx 0.45\%$

9b. Use $N = N_0(1 + r)^t$.

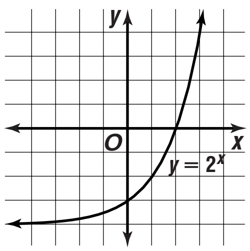
$N \approx 8{,}863{,}052(1 + 0.0045)^{20}$

$N \approx 9{,}695{,}766$

Pages 708–711 Exercises

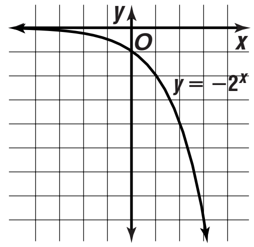
10.

x	y
-1	$\frac{1}{2}$
0	1
1	2
2	4



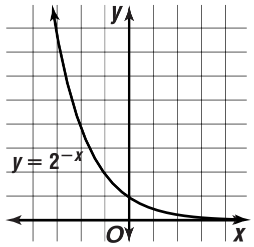
11.

x	y
-1	$-\frac{1}{2}$
0	-1
1	-2
2	-4



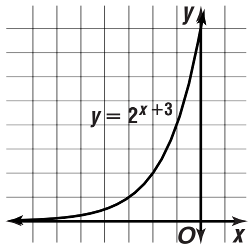
12.

x	y
-2	4
-1	2
0	1
1	$\frac{1}{2}$



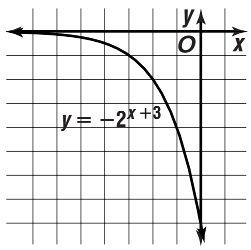
13.

x	y
-3	1
-2	2
-1	4
0	8



14.

x	y
-3	-1
-2	-2
-1	-4
0	-8



15.

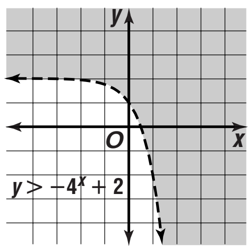
x	y
-1	$1\frac{3}{4}$
0	1
1	-2
2	-14

(0, 0)

$0 \overset{?}{>} -4^2 + 2$

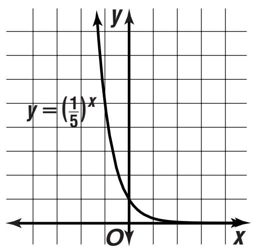
$0 \overset{?}{>} -1 + 2$

$0 \not> 1$; no



16.

x	y
-1	5
0	1
1	$\frac{1}{5}$



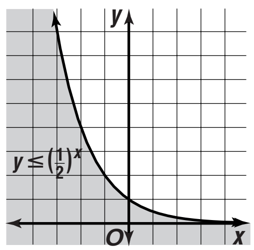
17.

x	y
-2	4
-1	2
0	1
1	$\frac{1}{2}$

(0, 0)

$0 \overset{?}{\le} \left(\frac{1}{2}\right)^0$

$0 \overset{?}{\le} 1$ ✓



18.

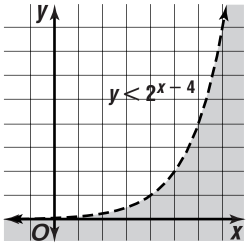
x	y
3	$\frac{1}{2}$
4	1
5	2
6	4

(0, 0)

$0 \overset{?}{<} 2^{0-4}$

$0 \overset{?}{<} 2^{-4}$

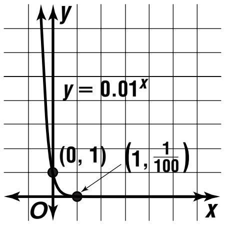
$0 < \frac{1}{16}$ ✓



19.

x	y
−1	100
0	1
1	$\frac{1}{100}$

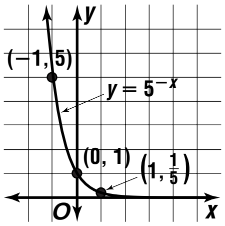
B



20.

x	y
−1	5
0	1
1	$\frac{1}{5}$

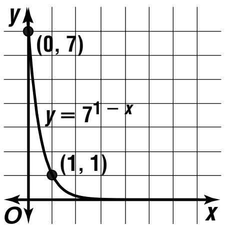
C



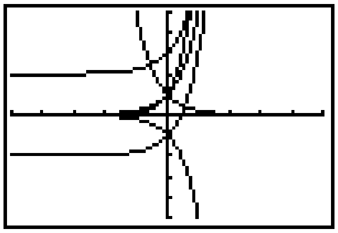
21.

x	y
−1	49
0	7
1	1

A



22.



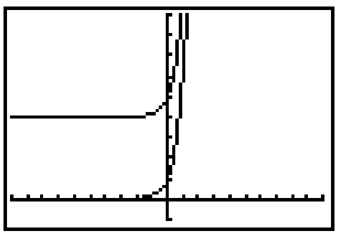
$[-5, 5]$ scl:1 by $[-5, 5]$ scl:1

22a. The graph of $y = -5^x$ is a reflection of $y = 5^x$ across the x-axis. The graph of $y = 5^x$ is a reflection of $y = 5^x$ across the y-axis.

22b. The graph of $y = 5^x + 2$ is shifted up two units, while the graph of $y = 5^x - 2$ is shifted down two units.

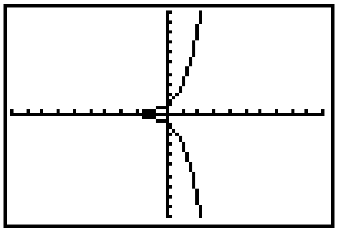
22c. The graph of $y = 10^x$ increases more quickly than the graph of $y = 5^x$. The graphs are not the same because $5^{2x} \neq 10^x$.

23a. The graph of $y = 6^x + 4$ is shifted up four units from the graph of $y = 6^x$.



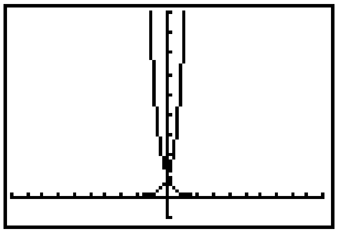
$[-10, 10]$ scl:1 by $[-1, 9]$ scl:1

23b. The graph of $y = -3^x$ is a reflection of the graph of $y = 3^x$ across the x-axis.



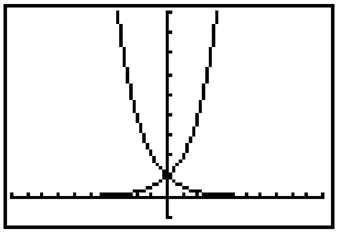
$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

23c. The graph of $y = 7^{-x}$ is a reflection of the graph of $y = 7^x$ across the y-axis.



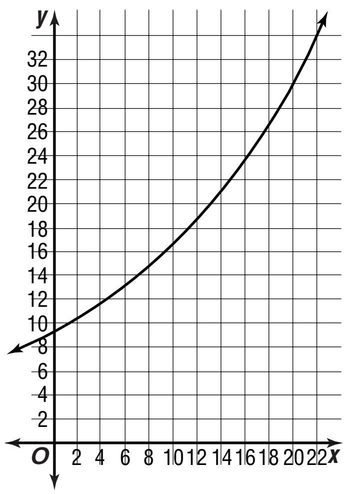
$[-10, 10]$ scl:1 by $[-1, 9]$ scl:1

23d. The graph of $y = \left(\frac{1}{2}\right)^x$ is a reflection of the graph of $y = 2^x$ across the y-axis.



$[-10, 10]$ scl:1 by $[-1, 9]$ scl:1

24a.

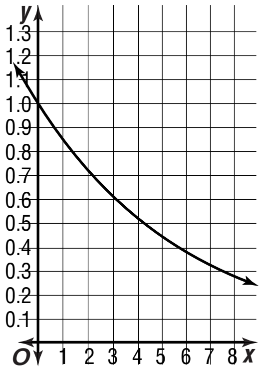


24b. $y = 9.25(1.06)^{50}$

$y \approx 170.386427$ thousand

$y \approx \$170{,}400$

25a. $y = (0.85)^x$

25b. 

25c. $y = (0.85)^{12}$
≈ 0.14 or 14%

25d. No; the graph has an asymptote at $y = 0$, so the percent of impurities, y, will never reach 0.

26a. $N = 876{,}156(1 + 0.0074)^{15}$
$\approx 978{,}612.2261$ or $978{,}612$

26b. $N = 2{,}465{,}326(1 - 0.0053)^{15}$
$\approx 2{,}668{,}760.458$ or $2{,}668{,}760$

26c. $152{,}307 = 139{,}510(1 + r)^{10}$
$\frac{152{,}307}{139{,}510} = (1 + r)^{10}$
$\left(\frac{152{,}307}{139{,}510}\right)^{\frac{1}{10}} = 1 + r$
$N = 139{,}510(1 + r)^{25}$
$N = 173{,}736.7334$ or $173{,}737$

26d. $191{,}701 = 168{,}767(1 + r)^{10}$
$\frac{191{,}701}{168{,}767} = (1 + r)^{10}$
$\left(\frac{191{,}701}{168{,}767}\right)^{\frac{1}{10}} = 1 + r$
$N = 168{,}767(1 + r)^{25}$
$N = 232{,}075.6889$ or $232{,}076$

27a. $O = 100\left(3^{\frac{3 \cdot 5}{5}}\right)$
$= 100(3^3)$
$= 2700$ units

27b. $s = \frac{4.2 \text{ mi}}{1 \text{ hr}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ hr}}{3600 \text{ s}}$
$= 6.16$ ft/s
$O = 100\left(3^{\frac{3 \cdot 6.16}{5}}\right)$
≈ 5800.16
≈ 5800 units

28a. P_n 121,000, $n = 30 \cdot 12$ or 360, $i = 0.075 \div 12$ or 0.00625
$121{,}000 = P\left[\frac{1 - (1 + 0.00625)^{-360}}{0.00625}\right]$
$P \approx$
846.04955; \$846.05

28b. $P_n = 121{,}000$, $n = 20 \cdot 12$ or 240, $i = 0.0725 \div 12$ or 0.00604
$$121{,}000 = P\left[\frac{1 - \left(1 + \frac{0.0725}{12}\right)^{-240}}{\left(\frac{0.0725}{12}\right)}\right]$$
$P \approx 956.35494$; \$956.35

28c. 30 year: $I = 360(846.05) - 121{,}000$
$= \$183.578$
20 year: $I = 240(956.35) - 121{,}000$
$= \$108{,}524$

28d. Sample answer: A borrower might choose the 30-year mortgage in order to have a lower monthly payment. A borrower might choose the 20-year mortgage in order to have a lower interest expense.

29a. $P = 4000$, $n = 43$, $i = 0.0475$
$F_n = 4000\left[\frac{(1 + 0.0475)^{43} - 1}{0.0475}\right]$
$\approx 535{,}215.918$; \$535,215.92

29b. $P = 4000$, $n = 43$, $i = 0.0525$
$F_n = 4000\left[\frac{(1 + 0.0525)^{43} - 1}{0.0525}\right]$
$\approx 611{,}592.1194$ or \$611,592.12
\$611,592.12 − 535,215.92 = \$76,376.20

30. The function $y = a^x$ is undefined when $a < 0$ and the exponent x is a fraction with an even denominator.

31a. Compounded once:
$I = 1000[(1 + 0.05)^1 - 1]$
$= 50$; \$50
Compounded twice:
$I = 1000\left[\left(1 + \frac{0.05}{2}\right)^2 - 1\right]$
≈ 50.625; \$50.63
Compounded four times:
$I = 1000\left[\left(1 + \frac{0.05}{4}\right)^4 - 1\right]$
≈ 50.9453; \$50.94
Compounded twelve times:
$I = 1000\left[\left(1 + \frac{0.05}{12}\right)^{12} - 1\right]$
≈ 51.1619; \$51.16
Compounded 365 times:
$I = 1000\left[\left(1 + \frac{0.05}{365}\right)^{365} - 1\right]$
≈ 51.2675; \$51.26

31b. Let x represent the investment.
Statement savings: $I = x[(1 + 0.051)^1 - 1]$
$= 0.051x$
The return is 5.1%
Money Market Savings: $I = x\left[\left(1 + \frac{0.0505}{12}\right)^{12} - 1\right]$
$= 0.517x$
The return is 5.17%
Super Saver: $I = x\left[\left(1 + \frac{0.05}{365}\right)^{365} - 1\right]$
$= 0.513x$
The return is 5.13%
Money Market Savings

31c. $(1 + 0.05) = \left(1 + \frac{x}{365}\right)^{365}$
$(1.05)^{\frac{1}{365}} = 1 + \frac{x}{365}$
$365[(1.05)^{\frac{1}{365}} - 1] = x$
$0.04879 = x$; 4.88%

32. $4x^2(4x)^{-2} = 4x^2(4)^{-2}(x)^{-2}$
$= \frac{4x^2}{16x^2}$
$= \frac{1}{4}$

33. $y = 15$
Use $y = r \sin \theta$.
$y = 15$
So, $15 = r \sin \theta$.

34. $\langle -3, 9\rangle - \langle 2, 1\rangle = (-3)(2) + (9)(1)$
$= 3$
3; no because the inner product does not equal 0.

35. $\frac{1}{3}\left(\cos \frac{7\pi}{8} + \boldsymbol{i} \sin \frac{7\pi}{8}\right) \cdot 3\sqrt{3}\left[\cos\left(-\frac{\pi}{4}\right) + \boldsymbol{i} \sin\left(-\frac{\pi}{4}\right)\right]$

$= \left(\frac{1}{3} \cdot 3\sqrt{3}\right)\left[\cos\left(\frac{7\pi}{8} - \frac{\pi}{4}\right) + \boldsymbol{i} \sin\left(\frac{7\pi}{8} - \frac{\pi}{4}\right)\right]$

$= \sqrt{3}\left(\cos \frac{5\pi}{8} + \boldsymbol{i} \sin \frac{5\pi}{8}\right)$

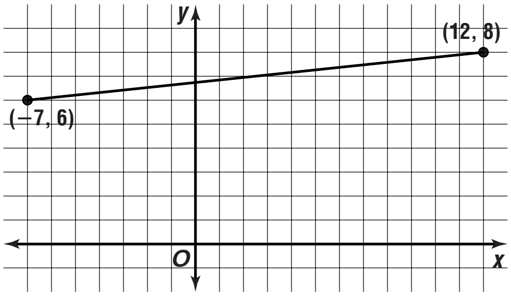
$= -0.66 + 1.60\boldsymbol{i}$

36.

$$\begin{aligned} s &= 72t - 16t^2 + 4 \\ s - 4 &= -16t^2 + 72t \\ s - 4 + (-16)(5.0625) &= -16(t^2 - 4.5 + 5.0625) \\ (s - 85) &= -16(t - 2.25)^2 \end{aligned}$$

Vertex: (2.25, 85)
Maximum height: 85 feet.

37a.



37b. $\left(\frac{(12 + (-7))}{2}, \frac{(8 + 6)}{2}\right) = (2.5, 7)$

38.

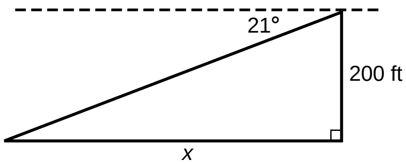
$$\begin{aligned} \sin^4 A + \cos^2 A &= \cos^4 A + \sin^2 A \\ (\sin^2 A)^2 + \cos^2 A &= \cos^4 A + \sin^2 A \\ (1 - \cos^2 A)^2 + \cos^2 A &= \cos^4 A + \sin^2 A \\ (1 - 2\cos^2 A + \cos^4 A) + \cos^2 A &= \cos^4 A + \sin^2 A \\ \cos^4 A + 1 - \cos^2 A &= \cos^4 A + \sin^2 A \\ \cos^4 A + \sin^2 A &= \cos^4 A + \sin^2 A \end{aligned}$$

39. $\frac{2900 \text{ rev}}{1} \cdot \frac{2\pi}{1 \text{ rev}} = 4800\pi$

$V = 9.2\ \frac{4800\pi}{1}$

$\approx 138{,}732.73$ or about 139,000 cm/s

40.



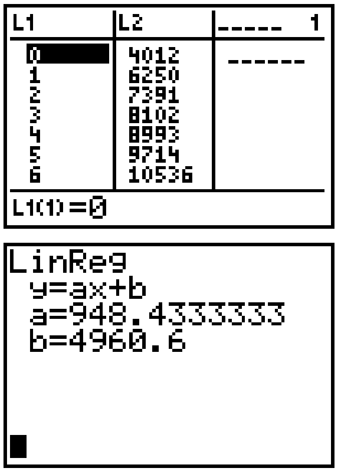
$\tan 69° = \frac{x}{200}$

$200 \tan 69° = x$

$521.02 = x$

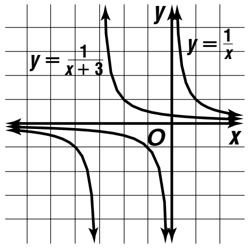
about 521 feet

41.



Sample answer: $y = 948.4x + 4960.6$

42.



The parent graph is translated 3 units left. The vertical asymptote is now $x = -3$. The horizontal asymptote, $y = 0$, is unchanged.

43. $C_{AC} = 32\pi \approx 100.53$ $\qquad C_{AB} = 16\pi \approx 50.27$

$100.53 - 50.27 = 50.26$ or about 50.

The correct choice is E.

11-3 The Number *e*

Page 714 Check for Understanding

1. C

2. If k is positive, the equation models growth. If k is negative, the equation models decay.

3. Amount in an account with a beginning balance of \$3000 and interest compounded continuously at an annual rate of 5.5%.

4. reals, positive reals

5. Sample answer: Continuously compounded interest is a continuous function, but interest compounded monthly is a discrete function.

6a. growth

6b. 33,430

6c. $y = 33{,}430e^{0.0397(60)}$

$\approx 361{,}931.0414$ or 361,931

7. $A = 12{,}000e^{0.064(12)}$

$\approx 25{,}865.412$ or \$25,865.41

Pages 714–717 Exercises

8. $p = (100 - 18)e^{-06(2)} + 18$

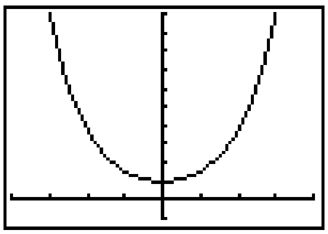
$= 42.6$ or 43%

9a. $y = 84e^{-0.23(15)} + 76$

$= 78.66$ or 78.7°F

9b. too cold; After 5 minutes, his coffee will be about 90°F.

10a.



[−4, 4] scl:1 by [−1, 10] scl:1

10b. symmetric about y-axis

11a. Annually: $I = 100[(1 + 0.08)^1 - 1]$

$= 80$ $80.00; 8%

Semi-annually: $I = 1000\left[\left(1 + \frac{0.08}{2}\right)^2 - 1\right]$

$= 81.6$ $81.60; 8.16%

Quarterly: $I = 1000\left[\left(1 + \frac{0.08}{4}\right)^4 - 1\right]$

≈ 82.4316 $82.43; 8.243%

Monthly: $I = 1000\left[\left(1 + \frac{0.08}{12}\right)^{12} - 1\right]$

≈ 82.9995 $83.00; 8.3%

Daily: $I = 1000\left[\left(1 + \frac{0.08}{365}\right)^{365} - 1\right]$

≈ 83.2776 $83.28; 8.328%

Continuously: $I = 1000(e^{0.08(1)} - 1)$

≈ 83.2871 $83.98; 8.329%

Interest Compounded	Interest	Effective Annual Yield
Annually	$80.00	8%
Semi-annually	$81.60	8.16%
Quarterly	$82.43	8.243%
Monthly	$83.00	8.3%
Daily	$83.28	8.328%
Continuously	$83.29	8.329%

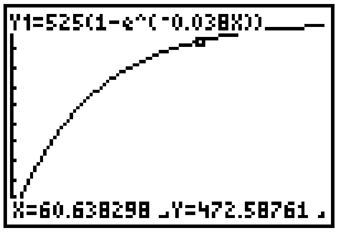
11b. continuously **11c.** $E = \left(1 + \frac{r}{n}\right)^n - 1$

11d. $E = e^r - 1$

12a. $y = 525(1 - e^{-0.038(24)})$

≈ 314.097 314 people

12b.



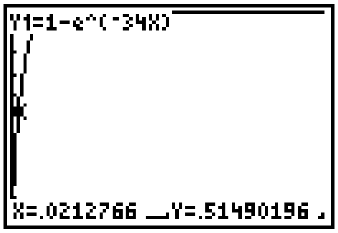
after about 61h

[0, 100] scl:10 by [0, 550] scl:50

13a. $P = 1 - e^{-6(0.5)}$

≈ 0.95021 95%

13b.



$x = 0.02$; about 0.02 h

$\frac{0.02\text{h}}{1} \cdot \frac{60 \text{ min}}{1 \text{ h}} = 1.2$

[0, 1] scl:0.1 by [0, 1] scl:0.1 about 1.2 min

14a. For $x = 10$: $\left(\frac{2(10) + 1}{2(10) - 1}\right)^{10} = \left(\frac{21}{19}\right)^{10}$

≈ 2.720551414

For $x = 100$: $\left(\frac{2(100) + 1}{2(100) - 1}\right)^{100} = \left(\frac{201}{199}\right)^{100}$

≈ 2.718304481

For $x = 1000$: $\left(\frac{2(1000) + 1}{2(1000) - 1}\right)^{1000} = \left(\frac{2001}{1999}\right)^{1000}$

≈ 2.718282055

2.720551414; 2.718304481; 2.718282055

14b. 2 decimal places; 4 decimal places; 6 decimal places

14c. always greater

15a. 5 days: $P = 1 - e^{-0.047(5)}$

≈ 0.20943 20.9%

20 days: $P = 1 - e^{-0.047(26)}$

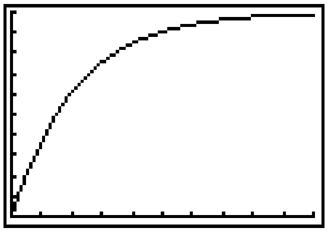
≈ 0.60937 60.9%

90 days: $P = 1 - e^{-0.047(90)}$

≈ 0.98545 98.5%

20.9%; 60.9% 98.5%

15b.



about 29 days

15c. Sample answer: The probability that a person who is going to respond has responded approaches 100% as t approaches infinity. New ads may be introduced after a high percentage of those who will respond have responded. The graph appears to level off after about 50 days. So, new ads can be introduced after an ad has run about 50 days.

16a. all reals

16b. $0 < f(x) < 1$

16c. c shifts the graph to the right or left

17. $120{,}000 = P\left[\frac{(1 + 0.035)^8 - 1}{0.035}\right]$

$120{,}000 \approx P(9.051687)$

$P \approx 13{,}257.19725$

$13,257.20

18. $x^{\frac{8}{5}}y^{\frac{3}{5}}z^{\frac{1}{5}} = x\sqrt[5]{x^3y^3z}$

19. $y = 6x^2$ $\theta = 45°$

$-x \sin 45° + y \cos 45° = 6(x \cos 45° + y \sin 45°)^2$

$-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y = 6\left(\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y\right)^2$

$-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y = 6\left(\frac{1}{2}x^2 + xy + \frac{1}{2}y^2\right)$

$-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y = 3x^2 + 6xy + 3y^2$

$-\sqrt{2}x + \sqrt{2}y = 6x^2 + 12xy + 6y^2$

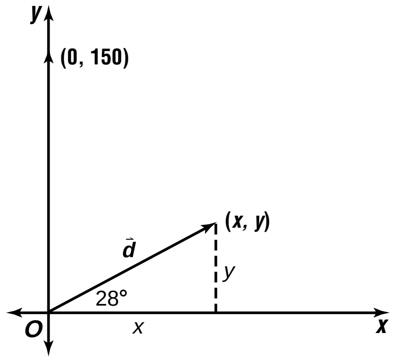
$6x^2 + 12xy + 6y^2 + \sqrt{2}x - \sqrt{2}y = 0$

20. $r = \sqrt{(-5)^2 + (-1)^2}$ $\theta = \text{Arctan } \frac{-1}{-5} + \pi$

$= \sqrt{26}$ ≈ 3.34

$\sqrt{26}(\cos 3.34 + \boldsymbol{i} \sin 3.34)$

21.



$\vec{d} = \langle x, y\rangle \qquad \vec{F} = \langle 0, 150\rangle$

$\cos 28° = \frac{x}{10} \qquad \sin 28° = \frac{x}{10}$

$x = 10 \cos 28° \qquad x = 10 \sin 28°$

$x \approx 8.8295 \qquad x \approx 4.6947$

$\boldsymbol{d} = \langle 8.8295, 4.6947\rangle$

$W = \vec{F} \cdot \vec{d}$

$W = \langle 0, 150\rangle \cdot \langle 8.8295, 4.6947\rangle$

$= 0 + 704.205$

$= 704.2$ ft-lb

22. $y = -1.5(10)^2 + 13.3(14) + 19.4$

$= 2.4$

23. $\sqrt{2x + 3} = 4$

$2x + 3 = 16$

$2x = 13$

$x = \frac{13}{2}$

24. $|3x + 2| \le 6$

$3x + 2 \le 6 \qquad 3x + 2 \ge -6$

$3x \le 4 \qquad 3x \ge -8$

$x \le \frac{4}{3} \qquad x \ge -\frac{8}{3}$

$\left\{x \mid -\frac{8}{3} \le x \le \frac{4}{3}\right\}$

25. $3\begin{bmatrix} -3 & -2 & 2 & 3 \\ -2 & 6 & 5 & -1 \end{bmatrix} = \begin{bmatrix} -9 & -6 & 6 & 9 \\ -6 & 18 & 15 & -3 \end{bmatrix}$

$J'(-9, -6)$, $K'(-6, 18)$, $L'(6, 15)$, $M'(9, -3)$; The dilated image has sides that are 3 times the length of the original figure.

26. $\begin{bmatrix} 4x + y \\ x \end{bmatrix} = \begin{bmatrix} 6 \\ 2y - 12 \end{bmatrix}$

$4x + y = 6 \qquad 4(2y - 12) + y = 6 \qquad x = 2(6) - 12$

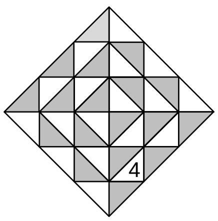
$x = 2y - 12 \qquad 9y = 54 \qquad x = 0$

$y = 6$

(0, 6)

27. $\{-4, 2, 5\}$; $\{5, 7\}$; yes

28.



The correct choice is D.

Page 717 Mid-Chapter Quiz

1. $64^{2} = 8$

2. $(\sqrt[3]{343})^{-2} = (343^{\frac{1}{3}})^{-2}$

$= ((7^3)^{\frac{1}{3}})^{-2}$

$= 7^{-2}$

$= \frac{1}{49}$

3. $\left(\frac{8x^3y^{-6}}{27w^6z^{-9}}\right)^{\frac{1}{3}} = \left(\frac{8x^3z^9}{27w^6y^6}\right)^{\frac{1}{3}}$

$= \frac{(2^3)^{\frac{1}{3}}(x^3)^{\frac{1}{3}}(z^9)^{\frac{1}{3}}}{(3^3)^{\frac{1}{3}}(w^6)^{\frac{1}{3}}(y^6)^{\frac{1}{3}}}$

$= \frac{2xz^3}{3w^2y^2}$ or $\frac{2}{3}w^{-2}xy^{-2}z^3$

4. $\sqrt{a^6b^3} = (a^4b^3)^{\frac{1}{2}}$

$= (a^6)^{\frac{1}{2}}(b^3)^{\frac{1}{2}}$

$= |a|^3 b^{\frac{3}{2}}$

5. $(125a^2b^3)^{\frac{1}{3}} = \sqrt[3]{125a^2b^3}$

$= \sqrt[3]{5^3a^2b^3}$

$= 5b\sqrt[3]{a^2}$

6. $1.75 \times 10^2 = 0.094\sqrt{A^3}$

$1.75 \times 10^2 = 0.94\, A^{\frac{3}{2}}$

$\frac{1.75 \times 10^2}{0.94} = A^{\frac{3}{2}}$

$\left(\frac{1.75 \times 10^2}{0.94}\right)^{\frac{2}{3}} = (A^{\frac{3}{2}})^{\frac{2}{3}}$

$151.34 \approx A$

1.51×10^2 mm^2

7. $1{,}786{,}691 = 1{,}637{,}859\,(1 + r)^8$

$\frac{1{,}786{,}691}{1{,}637{,}859} = (1 + r)^8$

$\left(\frac{1{,}786{,}691}{1{,}637{,}859}\right)^{\frac{1}{8}} = [(1 + r)^8]^{\frac{1}{8}}$

$\left(\frac{1{,}786{,}691}{1{,}637{,}859}\right)^{\frac{1}{8}} - 1 = r$

$r \approx 0.011$ Store the exact value in your calculator's memory.

$N = 1{,}637{,}859\,(1 + 0.011)^{24}$ Use the stored value for r.

$= 2{,}216{,}156.979$

2,126,157

8. $A = 3500\left(1 + \frac{0.052}{4}\right)^{(4)(3.5)}$

$= 3500(1.013)^{14}$

$= 4193.728$

\$4193.73

9a. $y = 6.7e^{\frac{-48.1}{15}}$

≈ 0.271292

271,292 ft^3

9b. $y = 6.7e^{\frac{-48.1}{50}}$

≈ 2.560257

2,560,257 ft^3

10. 2 years: $n = \frac{200}{1 + 20e^{-0.35(2)}}$

≈ 18.3

15 years: $n = \frac{200}{1 + 20e^{-0.35(15)}}$

≈ 181

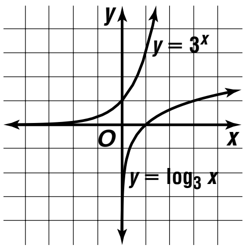
60 years: $n = \frac{200}{1 + 20e^{-0.35(60)}}$

≈ 200

18.3; 181; 200

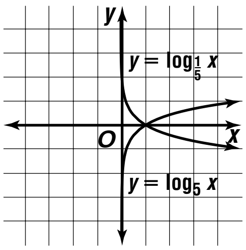
11-4 Logarithmic Functions

Pages 722–723 Check for Understanding

1. 

$y = 3^x$ and $\log_3 x$ are similar in that they are both continuous, one-to-one, increasing and inverses.
$y = 3^x$ and $\log_3 x$ are not similar in that they are inverses. The domain of one is the range of another and the range of one is the domain of the other. $y = 3^x$ has a y-intercept and a horizontal asymptote whereas $y = \log_3 x$ has a x-intercept and a vertical asymptote.

2. Let $b^x = m$, then $\log_b m = x$.
$$(b^x)^p = m^p$$
$$b^{xp} = m^p$$
$$\log_b b^{xp} = \log_b m^p$$
$$xp = \log_b m^p$$
$$p \log_b m = \log_b m^p$$

3. 

$\text{Log}_5\, x$ is an increasing function and $\log_{\frac{1}{5}} x$ is a decreasing function.

4. Sean is correct. The product property states that $\log_b mn = \log_b m + \log_b n$.

5. In half-life applications $r = -\frac{1}{2}$. So, $(1 + r)$ becomes $\left(1 - \frac{1}{2}\right)$ or $\left(\frac{1}{2}\right)$. Thus, the formula $N = N_0(1 + r)^t$ becomes $N = N_0\left(\frac{1}{2}\right)^t$.

6. $9^{\frac{3}{2}} = 27$

7. $\left(\frac{1}{25}\right)^{-\frac{1}{2}} = 5$

8. $\log_7 y = -6$

9. $\log_8 \frac{1}{4} = -\frac{2}{3}$

10. $\log_2 \frac{1}{16} = x$
$$2^x = \frac{1}{16}$$
$$2^x = 2^{-4}$$
$$x = -4$$

11. $\log_{10} 0.01 = x$
$$10^x = 0.01$$
$$10^x = 10^{-2}$$
$$x = -2$$

12. $\log_7 \frac{1}{343} = x$
$$7^x = \frac{1}{343}$$
$$7^x = 7^{-3}$$
$$x = -3$$

13. $\log_2 x = 5$
$$2^5 = x$$
$$32 = x$$

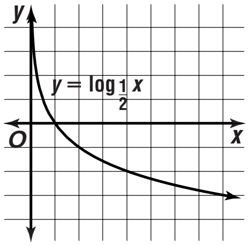
14. $\log_7 n = \frac{2}{3} \log_7 8$
$$\log_7 n = \log_7 8^{\frac{2}{3}}$$
$$n = 8^{\frac{2}{3}}$$
$$n = 4$$

15. $\log_6 (4x + 4) = \log_6 64$
$$4x + 4 = 64$$
$$x = 15$$

16. $2 \log_6 4 - \frac{1}{4} \log_6 16 = \log_6 x$
$$\log_6 4^2 - \log_6 16^{\frac{1}{2}} = \log_6 x$$
$$\log_6 \frac{4^2}{16^{\frac{1}{4}}} = \log_6 x$$
$$\frac{4^2}{16^{\frac{1}{4}}} = x$$
$$x = 8$$

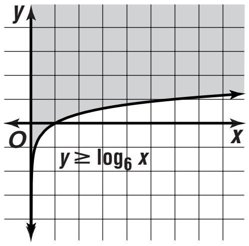
17.

x	y
1	0
2	-1
4	-2



18.

x	y
1	0
6	1



19. $16 = \frac{t}{3.3 \log_4 1024}$
$$t = 16(33 \log_4 1024)$$
$$t = 16(33 \cdot 5)$$
$$t = 264 \text{ h}$$

$$\log_4 1024 = x$$
$$4^x = 1024$$
$$2^{2x} = 2^{10}$$
$$2x = 10$$
$$x = 5$$

Pages 723–725 Exercises

20. $27^{\frac{1}{3}} = 3$

21. $16^{\frac{1}{2}} = 4$

22. $7^{-4} = \frac{1}{2401}$

23. $4^{\frac{5}{2}} = 32$

24. $e^x = 65.98$

25. $\left(\sqrt{6}\right)^4 = 36$

26. $\log_{81} 9 = \frac{1}{2}$

27. $\log_{36} 216 = \frac{3}{2}$

28. $\log_{\frac{1}{8}} 512 = -3$

29. $\log_6 \frac{1}{36} = -2$

30. $\log_{16} 1 = 0$

32. $\log_8 64 = x$

$8^x = 64$
$8^x = 8^2$
$x = 2$

33. $\log_{125} 5 = x$

$2^x = 32$
$2^x = 2^5$
$x = 5$

35. $\log_4 128 = x$

$9^x = 9^6$
$x = 6$

37. $\log_{49} 343 = x$

$8^x = 16$
$2^{3x} = 2^4$
$3x = 4$
$x = \frac{4}{3}$

39. $\log_{\sqrt{8}} 4096 = x$

$10^{\log_{10} 24} = x$
$2^4 = x$
$16 = x$

42. $\log_3 3x = \log_3 36$
$3x = 36$
$x = 12$

43. $\log_6 x + \log_6 9 = \log_6 54$
$\log_6 9x = \log_6 54$
$9x = 54$
$x = 6$

44. $\log_8 48 - \log_8 w = \log_8 6$
$\log_8 \frac{48}{w} = \log_8 6$
$\frac{48}{w} = 6$
$6w = 48$

$w = 8$

45. $\log_6 216 = x$
$6^x = 216$
$6^x = 6^3$

31. $\log_x 14.36 = 1.238$

$125^x = 5$
$(5^3)^x = 5^1$
$3x = 1$
$x = \frac{1}{3}$

34. $\log_2 32 = x$
$4^x = 128$
$2^{2x} = 2^7$
$2x = 7$
$x = \frac{7}{2}$ or 3.5

36. $\log_9 9^6 = x$
$49^x = 343$
$7^{2x} = 7^3$
$2x = 3$
$x = \frac{3}{2}$ or 1.5

38. $\log_8 16 = x$
$\left(\sqrt{8}\right)^x = 4096$
$8^{\frac{x}{2}} = 8^4$
$\frac{x}{2} = 4$
$x = 8$

40. $10^{4 \log_{10} 2} = x$

41. $\log_x 49 = 2$
$x^2 = 49$
$x = 7$

$x = 3$

46. $\log_5 0.04 = x$
$5^x = 0.04$
$5^x = 5^{-2}$

$x = -2$

47. $\log_{10} \sqrt[3]{10} = x$
$10^x = \sqrt[3]{10}$
$10^x = 10^{\frac{1}{3}}$
$x = \frac{1}{3}$

48. $\log_{12} x = \frac{1}{2} \log_{12} 9 + \frac{1}{3} \log_{12} 27$
$\log_{12} x = \log_{12} 9^{\frac{1}{2}} + \log_{12} 27^{\frac{1}{3}}$
$\log_{12} x = \log_{12} 9^{\frac{1}{2}} \cdot 27^{\frac{1}{3}}$
$x = 9^{\frac{1}{2}} \cdot 27^{\frac{1}{3}}$
$x = 3 \cdot 3$
$x = 9$

49. $\log_5 (x + 4) + \log_5 8 = \log_5 64$
$\log_5 (x + 4)(8) = \log_5 64$
$(x + 4)(8) = 64$
$x + 4 = 8$
$x = 4$

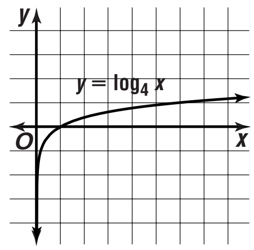
50. $\log_4 (x - 3) + \log_4 (x + 3) = 2$
$\log_4 (x - 3)(x + 3) = 2$
$4^2 = (x - 3)(x + 3)$
$16 = x^2 - 9$
$25 = x^2$
$5 = x$

51. $\frac{1}{2}(\log_7 x + \log_7 8) = \log_7 16$
$\frac{1}{2}(\log_7 8x) = \log_7 16$
$\log_7 (8x)^{\frac{1}{2}} = \log_7 16$
$(8x)^{\frac{1}{2}} = 16$
$8x = 256$
$x = 32$

52. $2 \log_5 (x - 2) = \log_5 36$
$\log_5 (x - 2)^2 = \log_5 36$
$(x - 2)^2 = 36$
$x - 2 = 6$
$x = 8$

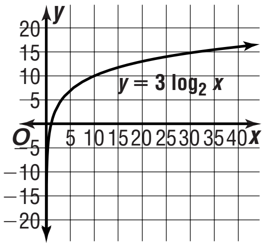
53.

x	y
1	0
2	$\frac{1}{2}$
4	1



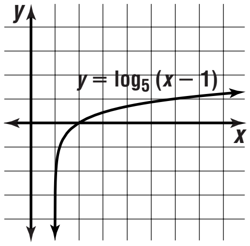
54.

x	y
1	0
2	3
4	6



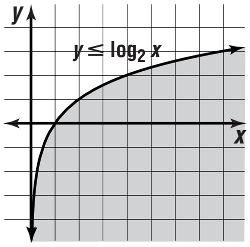
55.

x	y
2	0
6	1



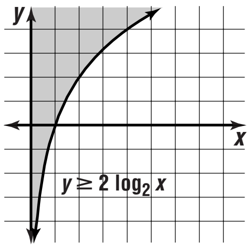
56.

x	y
1	0
2	1
4	2



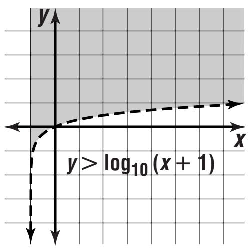
57.

x	y
1	0
2	2
4	4



58.

x	y
0	0
9	1



59. Use $N = N_0(1 + r)^t$; $r = 1$ since the rate of growth is 100% every t time periods.

$64{,}000 = 1000\,(1 + 1)^t$
$64 = 2^t$
$\log_2 2^6 = \log_2 2^t$
$6 = t$
$t \cdot 15 = 90$ min

60. All powers of 1 are 1, so the inverse of $y = 1^x$ is not a function.

61. Let $\log_b m = x$ and $\log_b n = y$.
So, $b^x = m$ and $b^y = n$.

$\frac{m}{n} = \frac{b^x}{b^y} = b^{x-y}$
$\frac{m}{n} = b^{x-y}$
$\log_b \frac{m}{n} = x - y$
$\log_b \frac{m}{n} = \log_b m - \log_b n$

62a. $5000 \ge 2500\left(1 + \frac{r}{4}\right)^{4\cdot 10}$
$2^7 \ge \left(1 + \frac{r}{4}\right)^{40}$

62b. $2 = \left(1 + \frac{r}{4}\right)^{40}$
$2^{\frac{1}{40}} = \left[\left(1 + \frac{r}{4}\right)^{40}\right]^{\frac{1}{40}}$
$1.0175 \approx 1 + \frac{r}{4}$
$0.0699 \approx r$
6.99%

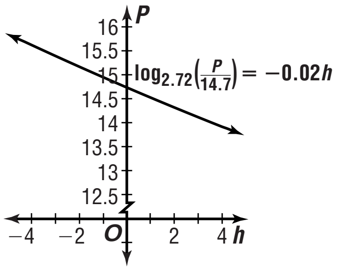
62c. $2 = \left(1 + \frac{r}{4}\right)^{28}$
$2^{\frac{1}{28}} = 1 + \frac{r}{4}$
$1.0251 \approx 1 + \frac{r}{4}$
$0.1004 \approx r$
10.04%

63a. $n = \log_2 \frac{1}{\frac{1}{4}}$
$n = \log_2 4$
$2^n = 4$
$n = 2$

63b. $3 = \log_2 \frac{1}{p}$
$2^3 = \frac{1}{p}$
$8 = p^{-1}$
$\frac{1}{8} = p$
less light; $\frac{1}{8}$

64. Let $y = \log_a x$, so $x = a^y$.
$x = a^y$
$\log_b x = \log_b a^y$
$\log_b x = y \log_b a$
$y = \frac{\log_b x}{\log_b a}$
$\log_a x = \frac{\log_b x}{\log_b a}$

65a.



65b. $\log_{2.72} \frac{P}{14.7} = -0.02(1)$
$\frac{P}{14.7} = 2.72^{-0.02}$
$P = 14.7\,(272^{-0.02})$
≈ 14.4 psi

65c. $\log_{2.72} \frac{P}{14.7} = -0.02(-6.8)$
$P = 14.7(2.72^{0.136})$
≈ 16.84 psi

66. $6.8 = 38\left(\frac{1}{2}\right)^t$
$\frac{6.8}{38} = \left(\frac{1}{2}\right)^t$
$\log \frac{6.8}{38} = \log\left(\frac{1}{2}\right)^t$
$\log \frac{6.8}{38} = t \log \frac{1}{2}$
$t = \frac{\log \frac{6.8}{38}}{\log \frac{1}{2}}$
$t = 2.5$
$2.5 \cdot 3.82 = 9.55$
about 9 days

67. 69.6164

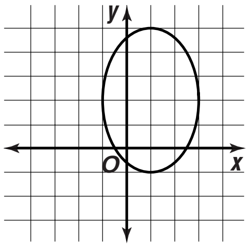
68. $90{,}000 = P\left[\dfrac{1-\left(1+\frac{0.115}{12}\right)^{-12.30}}{\frac{0.115}{12}}\right]$

$P \approx 891.262$

\$891.26

69. ellipse

$$9x^2 - 18x + 4y^2 - 16y - 11 = 0$$
$$9x^2 - 18x + 4y^2 - 16y = 11$$
$$9(x^2 - 2x + 1) + 4(y^2 - 4y + 4) = 11 + 9 + 16$$
$$9(x-1)^2 + 4(y-2)^2 = 36$$
$$\frac{(x-1)^2}{4} + \frac{(y-2)^2}{9} = 1$$



70. $r = 3, \theta = 2$

$(3 \cos 2t, 3 \sin 2t)$

71. $AB = \sqrt{(-1-(-1))^2 + (-3-3)^2}$
$= \sqrt{0^2 + (-6)^2}$
$= 6$

$BC = \sqrt{(3-(-1))^2 + (0-(-3))^2}$
$= \sqrt{4^2 + 3^2}$
$= 5$

$AC = \sqrt{(3-(-1))^2 + (0-3)^2}$
$= \sqrt{4^2 + 3^2}$
$= 5$

72. $5\left(\cos \frac{3\pi}{4} + \boldsymbol{i} \sin \frac{3\pi}{4}\right) \cdot 2\left(\cos \frac{2\pi}{3} + \boldsymbol{i} \sin \frac{2\pi}{3}\right)$

$= 5 \cdot 2\left(\cos\left(\frac{3\pi}{4} + \frac{2\pi}{3}\right) + \boldsymbol{i} \sin\left(\frac{3\pi}{4} + \frac{2\pi}{3}\right)\right)$

$= 10\left(\cos \frac{17\pi}{12} + \boldsymbol{i} \sin \frac{17\pi}{12}\right)$

$= 10 \cos \frac{17\pi}{12} + 10\boldsymbol{i} \sin \frac{17\pi}{12}$

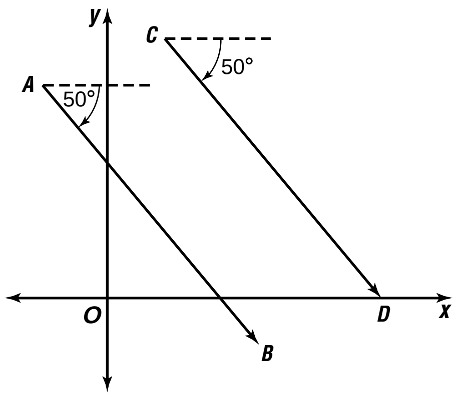
$\approx -2.59 - 9.66\boldsymbol{i}$

$10\left(\cos \frac{17\pi}{12} + \boldsymbol{i} \sin \frac{17\pi}{12}\right)$, $-2.59 - 9.66\boldsymbol{i}$

73. $(3 - 4\boldsymbol{j})(12 + 7\boldsymbol{j}) = 36 + 21\boldsymbol{j} - 48\boldsymbol{j} - 28\boldsymbol{j}^2$
$= 64 - 27\boldsymbol{j}$

$64 - 27\boldsymbol{j}$ volts

74.



Both vectors have the same direction. 50° south of east. Therefore, $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel.

75. $\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cos A = \frac{5}{13}$ $\quad\quad$ $\cos B = \frac{35}{37}$

$x^2 + 5^2 = 13^2$ $\quad\quad$ $x^2 + 35^2 = 37^2$

$x^2 = 144$ $\quad\quad$ $x^2 = 144$

$x = 12$ $\quad\quad$ $x = 12$

So, $\sin A = \frac{12}{13}$ $\quad\quad$ So, $\sin B = \frac{12}{37}$

$\cos(A + B) = \frac{5}{13} \cdot \frac{35}{37} - \frac{12}{13} - \frac{12}{37}$
$= \frac{175}{481} - \frac{144}{481}$
$= \frac{31}{481}$

76. $y = A \sin(kt + c) + h$

$A = \frac{90-64}{2}$ $\quad$ $h = \frac{90+64}{2}$ $\quad$ $\frac{2\pi}{k} = 4$

$= 13$ $\quad$ $= 77$ $\quad$ $k = \frac{\pi}{2}$

$$y = 13 \sin\left(\frac{\pi}{2}t + c\right) + 77$$
$$64 = 13 \sin\left(\frac{\pi}{2}(1) + c\right) + 77$$
$$-13 = 13 \sin\left(\frac{\pi}{2} + c\right)$$
$$-1 = \sin\left(\frac{\pi}{2} + c\right)$$

$\sin^{-1}(-1) - \frac{\pi}{2} = c$

$3.14 \approx c$

So, $y = 13 \sin\left(\frac{\pi}{2}k - 3.14\right) + 77$

77. $c^2 = (6.11)^2 + (5.84)^2 - 2(6.11)(5.84) \cos 105.3$

$c^2 = 37.3321 + 34.1056 - 71.3648 \cos 105.3$

$c^2 \approx 90.2689$

$c \approx 9.5$

$(6.11)^2 \approx (5.84)^2 + (9.5)^2 - 2(5.84)(9.5) \cos A$

$37.3321 \approx 34.1056 + 90.25 - 110.96 \cos A$

$\cos A \approx 0.7843$

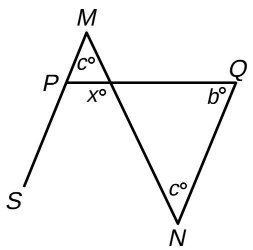
$A \approx 38.34$ or $38° 20'$

$B \approx 180 - (105° 18' + 38° 20')$

$\approx 36° 22'$

$c = 9.5, A = 38° 20', B = 36° 22'$

78.



$m\angle SMN = m\angle QNM$ alternate interior angles

$x° = b° + c°$ Exterior Angle Theorem

The correct choice is E.

11-5 Common Logarithms

Page 730 Check for Understanding

1. $\log 1 = 0$ means $\log_{10} 1 = 0$. So, $10° = 1$.
$\log 10 = 1$ means $\log_{10} 10 = 1$. So, $10^1 = 10$.

2. Write the number in scientific notation. The exponent of the power of 10 is the characteristic.

3. antilog $2.835 = 10^{2.835} = 683.9116$

4. $\log 15 = 1.1761$
$\log 5 = 0.6990$
$\log 3 = 0.4771$
$\log 5 + \log 3 = 0.6990 + 0.4771 = 1.1761$

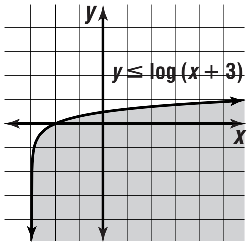
5.
$$\begin{aligned} \log 80{,}000 &= \log(10{,}000 \times 8) \\ &= \log 10^4 + \log 8 \\ &= 4 + 0.9031 \\ &= 4.9031 \end{aligned}$$

6.
$$\begin{aligned} \log 0.003 &= \log(0.001 \times 3) \\ &= \log 10^{-3} + \log 3 \\ &= -3 + 0.4771 \\ &= -2.5229 \end{aligned}$$

7.
$$\begin{aligned} \log 0.0081 &= \log(0.0001 \times 3^4) \\ &= \log 10^{-4} + 4 \log 3 \\ &= -4 + 4(0.4771) \\ &= -2.0915 \end{aligned}$$

8. 2.6274 **9.** 74,816.95

10.



11.
$$\begin{aligned} \log_{12} 18 &= \frac{\log 18}{\log 12} \\ &\approx 1.1632 \end{aligned}$$

12.
$$\begin{aligned} \log_8 15 &= \frac{\log 15}{\log 8} \\ &\approx 1.3023 \end{aligned}$$

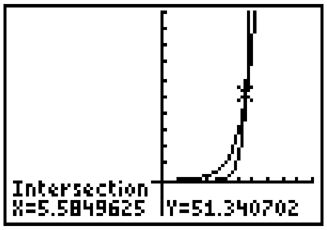
13.
$$\begin{aligned} 2.2^{x-5} &= 9.32 \\ (x-5)\log 2.2 &= \log 9.32 \\ (x-5) &= \frac{\log 9.32}{\log 2.2} \\ x &\approx 7.83 \end{aligned}$$

14.
$$\begin{aligned} 6^{x-2} &= 4^x \\ (x-2)\log 6 &= x \log 4 \\ x\log 6 - 2\log 6 &= x\log 4 \\ -2\log 6 &= x\log 4 - x\log 6 \\ -2\log 6 &= x(\log 4 - \log 6) \\ \frac{-2\log 6}{\log 4 - \log 6} &= x \\ 8.84 &\approx x \end{aligned}$$

15.
$$\begin{aligned} 4.3^x &< 76.2 \\ x\log 4.3 &< \log 76.2 \\ x &< \frac{\log 76.2}{\log 4.3} \\ x &< 2.97 \end{aligned}$$

16.
$$\begin{aligned} 3^{x-3} &\ge 2\sqrt[4]{4^{x-1}} \\ 3^{x-3} &\ge 2\left(4^{\frac{x-1}{4}}\right) \\ (x-3)\log 3 &\ge \log 2 + \frac{x-1}{4}\log 4 \\ (4x-12)\log 3 &\ge 4\log 2 + (x-1)\log 4 \\ 4x\log 3 - 12\log 3 &\ge 4\log 2 + x\log 4 - \log 4 \\ 4x\log 3 - x\log 4 &\ge 4\log 2 - \log 4 + 12\log 3 \\ x(4\log 3 - \log 4) &\ge 4\log 2 - \log 4 + 12\log 3 \\ x &\ge \frac{4\log 2 - \log 4 + 12\log 3}{4\log 3 - \log 4} \\ x &\ge 4.84 \end{aligned}$$

17.



[10, 10] scl:1 by [−20, 100] scl:10
5.5850

18a.
$$\begin{aligned} R &= \log\left(\frac{200}{1.6}\right) + 4.2 \\ &= 6.3 \end{aligned}$$

18b. 10 times; According to the definition of logarithms, R in the equation $R = \log\left(\frac{a}{T}\right) + B$ is an exponent of the base of the logarithm, 10. 10^5 is ten times greater than 10^4.

Pages 730–732 Exercises

19.
$$\begin{aligned} \log 4000{,}000 &= \log(100{,}000 \times 4) \\ &= \log 100{,}000 + \log 4 \\ &= 5 + 0.6021 \\ &= 5.6021 \end{aligned}$$

20.
$$\begin{aligned} \log 0.00009 &= \log(0.00001 \times 9) \\ &= \log 0.00001 + \log 9 \\ &= -5 + 0.9542 \\ &= -4.0458 \end{aligned}$$

21.
$$\begin{aligned} \log 1.2 &= \log(0.1 \times 12) \\ &= \log 0.1 + \log 12 \\ &= -1 + 1.0792 \\ &= 0.0792 \end{aligned}$$

22.
$$\begin{aligned} \log 0.06 &= \log\left(0.01 \times \frac{12}{2}\right) \\ &= \log 0.01 + \log \frac{12}{4^{\frac{1}{2}}} \\ &= \log 0.01 + \log 12 - \frac{1}{2}\log 4 \\ &= -2 + 1.0792 - \frac{1}{2}(0.6021) \\ &= -1.2218 \end{aligned}$$

23.
$$\begin{aligned} \log 36 &= \log(4 \times 9) \\ &= \log 4 + \log 9 \\ &= 0.6021 + 0.9542 \\ &= 1.5563 \end{aligned}$$

24.
$$\begin{aligned} \log 108{,}000 &= \log(1000 \times 12 \times 9) \\ &= \log 1000 + \log 12 + \log 9 \\ &= 3 + 1.0792 + 0.9542 \\ &= 5.0334 \end{aligned}$$

25. $\log 0.0048 = \log(0.0001 \times 12 \times 4)$
$= \log 0.0001 + \log 12 + \log 4$
$= -4 + 1.0792 + 0.6021$
$= -2.3188$

26. $\log 4.096 = \log(0.001 + 4^6)$
$= \log 0.001 + 6 \log 4$
$= -3 + 6(0.6021)$
$= 0.6124$

27. $\log 1800 = \log\left(100 \times 9 \times 4^{\frac{1}{2}}\right)$
$= \log 100 + \log 9 + \frac{1}{2}\log 4$
$= 2 + 0.9542 + \frac{1}{2}(0.6021)$
$= 3.2553$

28. 1.9921

29. 2.9515

30. 0.871

31. 2.001

32. 3.2769

33. 2.1745

34. $\log_2 8 = \frac{\log 8}{\log 2}$
$= 3$

35. $\log_5 625 = \frac{\log 625}{\log 5}$
$= 4$

36. $\log_6 24 = \frac{\log 24}{\log 6}$
≈ 1.7737

37. $\log_7 4 = \frac{\log 4}{\log 7}$
≈ 0.7124

38. $\log_{6.5} 0.0675 = \frac{\log 0.0675}{\log 0.5}$
≈ 3.8890

39. $\log_{\frac{1}{2}} 15 = \frac{\log 15}{\log \frac{1}{2}}$

40. $2^x = 95$
$x \log 2 = \log 95$
$x = \frac{\log 95}{\log 2}$
$x \approx 6.5699$

41. $5x = 4^{x+3}$
$x \log 5 = (x + 3) \log 4$
$x \log 5 = x \log 4 + 3 \log 4$
$x(\log 5 - \log 4) = 3 \log 4$
$x = \frac{3 \log 4}{\log 5 - \log 4}$
$x \approx 18.6377$

42. $\frac{1}{3}\log x = \log 8$
$x^{\frac{1}{3}} = 8$
$x \approx 512$

43. $0.16^{4+3x} = 0.3^{8-x}$
$(4+3x)\log 0.16 = (8 - x)\log 0.3$
$4 \log 0.16 + 3x \log 0.16 = 8 \log 0.3 - x \log 0.3$
$3x \log 0.16 + x \log 0.3 = 8 \log 0.3 - 4 \log 0.16$
$x(3 \log 0.16 + \log 0.3) = 8 \log 0.3 - 4 \log 0.16$
$x = \frac{8 \log 0.3 - 4 \log 0.16}{3 \log 0.16 + \log 0.3}$
$x \approx 0.3434$

44. $4 \log(x + 3) = 9$
$\log(x + 3) = \frac{9}{4}$
$(x + 3) = \text{antilog } \frac{9}{4}$
$x = \text{antilog } \frac{9}{4} - 3$
$x \approx 174.8297$

45. $0.25 = \log 16^x$
$0.25 = x \log 16$
$x = \frac{0.25}{\log 16}$
$x \approx 0.2076$

46. $3^{x-1} \le 2^{x-7}$
$(x - 1)\log 3 \le (x - 7)\log 2$
$x \log 3 - \log 3 \le x \log 2 - 7 \log 2$
$x \log 3 - x \log 2 \le \log 3 - 7 \log 2$
$x(\log 3 - \log 2) \le \log 3 - 7 \log 2$
$x \le \frac{\log 3 - 7 \log 2}{\log 3 - \log 2}$
$x = -9.2571$

47. $\log_x 6 > 1$
$\frac{\log 6}{\log x} > 1$
$\log 6 > \log x$
$6 > x$
When $x = 1$, $\log 1 = 0$, which means $\frac{\log 6}{\log x}$ is undefined. When $x < 1$, $\frac{\log 6}{\log x}$ is negative, which is not greater than 1. So, x must also be greater than 1. Therefore, $1 < x < 6$.

48. $4^{2x-5} \le 3^{x-3}$
$(2x - 5)\log 4 \le (x - 3)\log 3$
$2x \log 4 - 5 \log 4 \le x \log 3 - 3 \log 3$
$2x \log 4 - x \log 3 \le 5 \log 4 - 3 \log 3$
$x(2 \log 4 - \log 3) \le 5 \log 4 - 3 \log 3$
$x \le \frac{5 \log 4 - 3 \log 3}{2 \log 4 - \log 3}$
$x \le 2.1719$

49. $0.5^{2x-4} \le 0.1^{5-x}$
$(2x - 4)\log 0.5 \le (5 - x)\log 0.1$
$2x(\log 0.5) - 4 \log 0.5 \le 5 \log 0.1 - x \log 0.1$
$2x \log 0.5 + x \log 0.1 \le 5 \log 0.1 + 4 \log 0.5$
$x(2 \log 0.5 + \log 0.1) \le 5 \log 0.1 + 4 \log 0.5$
$x \ge \frac{5 \log 0.1 + 4 \log 0.5}{2 \log 0.5 + \log 0.1}$

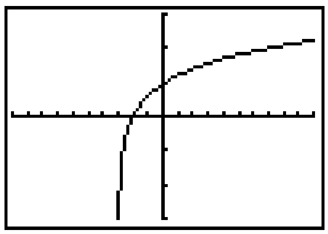
Change inequality sign because (2 log 0.5 + log 0.1) is negative.

$x \ge 3.8725$

50. $\log_2 x = -3$
$x = 2^{-3}$
$x = 0.1250$

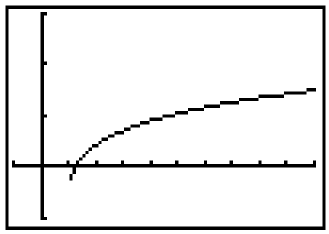
51. $x < \frac{\log 52.7}{\log 3}$
$x < 3.6087$

52.



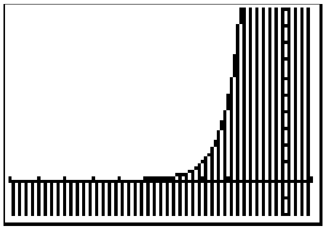
$[-10, 10]$ scl:1 by $[-3, 3]$ scl:1

53.



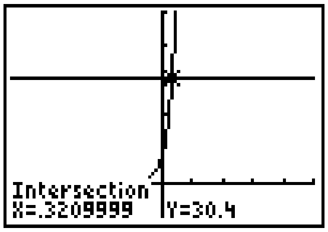
$[-1, 10]$ scl:1 by $[-1, 3]$ scl:1

54.



[−10, 1] scl:1 by [−2, 10] scl:1

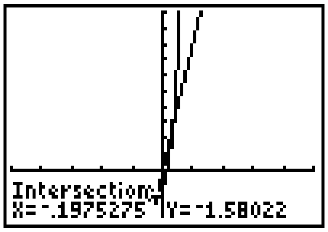
55.



$x \approx 0.3210$

[−5, 5] scl:1 by [−10, 50] scl:10

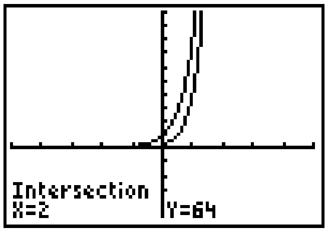
56.



$x \approx -0.1975$

[−5, 5] scl:1 by [−3, 10] scl:1

57.



$x = 2$

[−5, 5] scl:1 by [−5, 10] scl:1

58a. $h = -\frac{100}{9} \log \frac{10.3}{14.7}$
≈ 1.7 mi

58b.
$$\begin{aligned} 4.3 &= -\tfrac{100}{9} \log \tfrac{P}{14.7} \\ -0.3870 &= \log P - \log 14.7 \\ -0.3870 + \log 14.7 &= \log P \\ 0.7803 &\approx \log P \\ 6.03 &\approx P\text{; 6 psi} \end{aligned}$$

59a. $M = 5.3 + 5 + 5 \log 0.018$
≈ 1.58

59b.
$$\begin{aligned} 5.3 &= 8.6 + 5 + 5 \log P \\ -8.3 &= 5 \log P \\ -1.66 &= \log P \\ 0.0219 &\approx P \end{aligned}$$

60a. $q = \left(\frac{1}{2}\right)^{0.8^9}$
$= \left(\frac{1}{2}\right)^{0.1342}$
$= 0.9112$
\$91,116

60b.
$$\begin{aligned} 0.9535 &= \left(\tfrac{1}{2}\right)^{0.8^t} \\ \log 0.9535 &= 0.8^t \log \tfrac{1}{2} \\ \frac{\log 0.9535}{\log \frac{1}{2}} &= 0.8^t \\ \log\left[\frac{\log 0.9535}{\log \frac{1}{2}}\right] &= t \log 0.8 \\ 12.0016 &\approx t \end{aligned}$$
12 years

61. Sample answer: x is between 2 and 3 because 372 is between 100 and 1000, and log 100 = 2 and log 1000 = 3.

62a. $L = 10 \log \frac{1}{1.0 \times 10^{-12}}$
$= 10(\log 1 - \log (1.0 \times 10^{-12}))$
$= 120$ dB

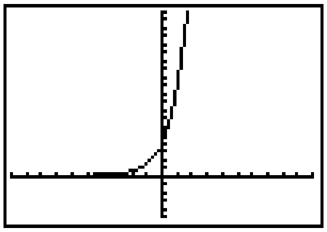
62b.
$$\begin{aligned} 20 &= \log \tfrac{I}{1.0 \times 10^{-12}} \\ 2 &= \log I - \log (1.0 \times 10^{-12}) \\ 2 &= \log I + 12 \\ -10 &= \log I \end{aligned}$$
$1 \times 10^{-10} = \text{I}$; 1×10^{-10} W/m^2

63. Use $N = N_0\left(\frac{1}{2}\right)^t$.
$N = 630$ micrograms $= 63 \times 10^{-4}$ gram
$N_0 = 1$ milligram $= 1.0 \times 10^{-3}$ gram
$6.3 \times 10^{-4} = (1.0 \times 10^{-3})\left(\frac{1}{2}\right)^t$
$\log \frac{6.3 \times 10^{-4}}{1.0 \times 10^{-3}} = t \log \frac{1}{2}$
$0.6666 \approx t$
$0.6666 \times 5730 \approx 3819$ yr

64. $\log_a y = \log_a P - \log_a q + \log_a r$
$\log_a y = \log_a \frac{p}{q} + \log_a r$
$\log_a y = \log_a \frac{pr}{q}$
$y = \frac{pr}{q}$

65. $\log_x 243 = 5$
$x^5 = 243$
$x = 3$

66.



increasing from $-\infty$ to ∞

67. $(a^4b^2)^{\frac{1}{3}}c^{\frac{2}{3}} = (a^4)^{\frac{1}{3}}(b^2)^{\frac{1}{3}}(c^2)^{\frac{1}{3}}$
$= a\sqrt[3]{ab^2c^2}$

68. $(5)^2 + (0)^2 + D(5) + E(0) + F = 0$
$5D + F + 25 = 0$
$(1)^2 + (-2)^2 + D(1) + E(-2) + F = 0$
$D - 2E + F + 5 = 0$
$(4)^2 + (-3)^2 + D(4) + E(-3) + F = 0$
$4D - 3E + F + 25 = 0$

$$\begin{array}{rl} & 5D + 0E + F + 25 = 0 \\ (-) & \underline{D - 2E + F + 5 = 0} \\ & 4D + 2E \qquad + 20 = 0 \end{array}$$

$$\begin{array}{rl} & 4D + 3E + F + 25 = 0 \\ (-) & \underline{D - 2E + F + 5 = 0} \\ & 3D - E \qquad + 20 = 0 \end{array}$$

$$\begin{aligned} 4D + 2E + 20 &= 0 \\ \underline{+2(3D - E + 20} &\underline{= 0)} \\ 10D + 60 &= 0 \\ D &= -6 \end{aligned}$$

$$\begin{aligned} 4(-6) + 2E + 20 &= 0 \\ 2E - 4 &= 0 \\ E &= 2 \end{aligned}$$

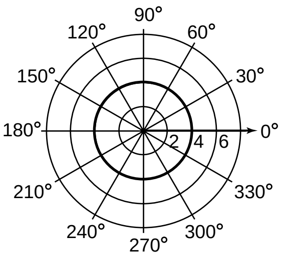
$$\begin{aligned} 5(-6) + 0(2) + F + 25 &= 0 \\ F - 5 &= 0 \\ F &= 5 \end{aligned}$$

$$\begin{aligned} x^2 + y^2 - 6x + 2y + 5 &= 0 \\ (x^2 - 6x + 9) + (y^2 + 2y + 1) &= -5 + 9 + 1 \\ (x - 3)^2 + (y + 1)^2 &= 5 \end{aligned}$$

69. $\left(\frac{6\sqrt{5} - 2\sqrt{5}}{2}, \frac{-18 - 4}{2}\right) = (2\sqrt{5}, -11)$

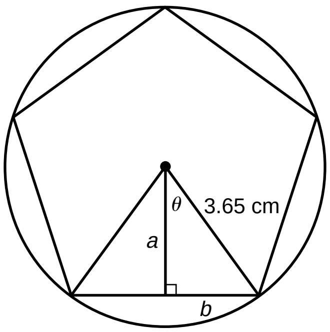
70. $$\begin{aligned} r &= 6 \\ r^2 &= 36 \\ x^2 + y^2 &= 36 \end{aligned}$$

71.



72. $$\begin{aligned} \overrightarrow{AB} &= \langle(6 - 5), (-5 + 6)\rangle \\ &= \langle 1, 1\rangle \\ |\overrightarrow{AB}| &= \sqrt{(6 - 5)^2 + (-5 + 6)^2} \\ &= \sqrt{2} \approx 1.414 \end{aligned}$$

73.



$\theta = 36 \div 10 = 360°$

$\cos 36° = \frac{a}{3.65}$ $\qquad$ $\sin 36° = \frac{b}{3.65}$

$a = 3.65 \cos 36°$ $\qquad$ $b = 3.65 \sin 36°$

≈ 2.9529 $\qquad$ ≈ 2.1454

Use $A = \frac{1}{2}aP$, where $P \approx 10(2.1454) \approx 21.454$.

$$\begin{aligned} A &\approx \tfrac{1}{2}(2.9529)(21.454) \\ &\approx 31.6758 \text{ or } 31.68 \text{ cm}^2 \end{aligned}$$

74. $f(x) = x^3 - 2x^2 - 11x + 12$

$f(1) = 1 - 2(1) - 11(1) + 12$ $\qquad$ Test $f(1)$.

$f(1) = 0$ $\qquad$ $(x - 1)$ is a factor.

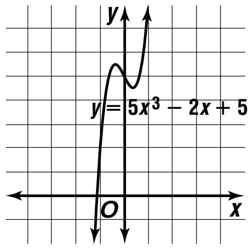
$$\begin{array}{r|rrrr} \underline{1} & 1 & -2 & -11 & 12 \\ & & 1 & -1 & -12 \\ \hline & 1 & -1 & -12 & 0 \end{array}$$

$x^2 - x - 12 = 0$

$(x - 4)(x + 3) = 0$

So, the factors are $(x - 4)(x + 3)(x - 1)$.

75.



Neither; the graph of the function is not symmetric with respect to either the origin or the y-axis.

76. $7 + 5 + 4 + 1 = 17$

17,000,000

The correct choice is D.

11-6 Natural Logarithms

Page 735 Check for Understanding

1. $\ln e = 1$ is the same as $\log_e e = 1$. And $e^1 = e$. So, $\ln e = 1$.

2. The two logarithms have different bases.

$\log 17 \Rightarrow 10^x = 17$ or $x = 1.23$

$\ln 17 \Rightarrow e^x = 17$ or $x = 2.83$

3. $\ln 64 = 4.1589$

$\ln 16 = 2.7726$

$\ln 4 = 1.3863$

$\ln 16 + \ln 4 = 2.7726 + 1.3863 = 4.1589$

4. The two equations represent the same thing, $A = Pe^{rt}$ is a special case of the equation $N = N_0e^{kt}$ and is used primarily for computations involving money.

5. -4.7217 $\qquad$ **6.** 1.1394

7. -15.606 $\qquad$ **8.** 0.4570

9. $$\begin{aligned} \log_5 132 &= \frac{\ln 132}{\ln 5} \\ &\approx 3.0339 \end{aligned}$$

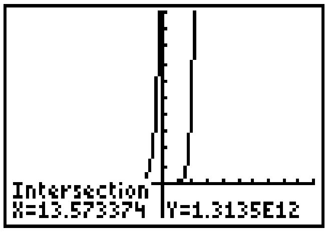
10. $$\begin{aligned} \log_3 64 &= \frac{\ln 64}{\ln 3} \\ &\approx 3.7856 \end{aligned}$$

11. $$\begin{aligned} 18 &= e^{3x} \\ \ln 18 &= 3x \ln e \\ \frac{\ln 18}{3} &= x \\ 0.9635 &= x \end{aligned}$$

12.
$$\begin{aligned} 10 &= 5e^{5k} \\ 2 &= e^{5k} \\ \ln 2 &= 5k \ln e \\ \tfrac{\ln 2}{5} &= k \\ 0.1386 &\approx k \end{aligned}$$

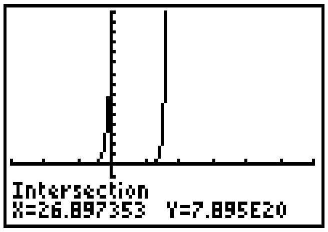
13.
$$\begin{aligned} 25e^x &< 100 \\ e^x &< 4 \\ x \ln e &< \ln 4 \\ x &< 1.3863 \end{aligned}$$

14.
$$\begin{aligned} 4.5 &\geq e^{0.031t} \\ \ln 4.5 &\geq 0.031t \ln e \\ \tfrac{\ln 4.5}{0.031} &\geq t \\ 48.5186 &\geq t \end{aligned}$$

15. 

$x \approx 13.57$

$[-20, 20]$ scl:2 by $[-4, 20]$ scl:2

16. 

$x > 26.90$

$[-15, 30]$ scl:5 by $[-50, 150]$ scl:10

17a.
$$\begin{aligned} p &= 760e^{-0.125(3.3)} \\ &= 760e^{-0.4125} \\ &\approx 503.1 \text{ torrs} \end{aligned}$$

17b.
$$\begin{aligned} 450 &= 760e^{-0.125a} \\ \tfrac{450}{760} &= e^{-0.125a} \\ \ln\left(\tfrac{450}{760}\right) &= -0.125a \ln e \\ \frac{\ln\left(\frac{450}{760}\right)}{-0.125} &= a \\ 4.1926 &\approx \text{a; } 4.2 \text{ km} \end{aligned}$$

Pages 736–737 Exercise

18. 5.4931
19. -0.2705
20. 6.8876
21. 0.9657
22. 0
23. 2.2322
24. 10.4395
25. 1.2134
26. 0.0233
27. 0.9966
28. 146.4963
29. 0.2417

30. $\log_{12} 56 = \frac{\ln 56}{\ln 12} \approx 1.6199$

31. $\log_5 36 = \frac{\ln 36}{\ln 5} \approx 2.2266$

32. $\log_4 83 = \frac{\ln 83}{\ln 4} \approx 3.1875$

33. $\log_8 0.512 = \frac{\ln 0.512}{\ln 8} \approx -0.3219$

34. $\log_6 323 = \frac{\ln 303}{1.6} \approx 3.2246$

35. $\log_5 \sqrt{288} = \frac{\ln \sqrt{288}}{\ln 5} \approx 1.7593$

36.
$$\begin{aligned} 6^x &= 72 \\ x \ln 6 &= \ln 72 \\ x &= \tfrac{\ln 72}{\ln 6} \\ &\approx 2.3869 \end{aligned}$$

37.
$$\begin{aligned} 2^x &= 27 \\ x \ln 2 &= \ln 27 \\ x &= \tfrac{\ln 27}{\ln 2} \\ &\approx 4.7549 \end{aligned}$$

38.
$$\begin{aligned} 9^{x-4} &= 7.13 \\ (x-4) \ln 9 &= \ln 7.13 \\ x \ln 9 - 4 \ln 9 &= \ln 7.13 \\ x \ln 9 &= \ln 7.13 + 4 \ln 9 \\ x &= \tfrac{\ln 7.13 + 4 \ln 9}{\ln 9} \\ x &\approx 4.8940 \end{aligned}$$

39.
$$\begin{aligned} 3x &= 3\sqrt{2} \\ x \ln 3 &= \ln 3 + \ln \sqrt{2} \\ x &= \tfrac{\ln 3 + \ln \sqrt{2}}{\ln 3} \\ x &\approx 1.3155 \end{aligned}$$

40.
$$\begin{aligned} 25e^x &= 1000 \\ e^x &= 40 \\ x \ln e &= \ln 40 \\ x &\approx 3.6889 \end{aligned}$$

41.
$$\begin{aligned} 60.3 &< e^{0.1t} \\ \ln 60.3 &< 0.1t \ln e \\ \tfrac{\ln 60.3}{0.1} &< t \\ 40.9933 &< t \end{aligned}$$

42.
$$\begin{aligned} 6.2e^{0.64t} &= 3e^{t+1} \\ \ln 6.2 + 0.64t \ln e &= \ln 3 + (t+1) \ln e \\ \ln 6.2 - \ln 3 - 1 &= 0.36t \\ \tfrac{\ln 6.2 - \ln 3 - 1}{0.36} &= t \\ 0.7613 &\approx t \end{aligned}$$

43.
$$\begin{aligned} 22 &= 44\,(1 - e^{2x}) \\ \tfrac{1}{2} - 1 &= -e^{2x} \\ \tfrac{1}{2} &= e^{2x} \\ \ln \tfrac{1}{2} &= 2x \ln e \\ \frac{\ln \frac{1}{2}}{2} &= x \\ -0.3466 &\approx x \end{aligned}$$

44.
$$\begin{aligned} 25 &< e^{0.075y} \\ \ln 25 &< 0.075y \ln e \\ \tfrac{\ln 25}{0.075} &< y \\ y &> 42.9183 \end{aligned}$$

45. $5^x \le 7\sqrt{6}$

$x \ln 5 \le \ln 7 + \ln \sqrt{6}$

$x \le \frac{\ln 7 + \ln \sqrt{6}}{\ln 5}$

$x \le 1.7657$

46. $12^{x-4} > 4^x$

$(x - 4) \ln 12 > x \ln 4$

$x \ln 12 - 4 \ln 12 > x \ln 4$

$x \ln 12 - x \ln 4 > 4 \ln 12$

$x(\ln 12 - \ln 4) > 4 \ln 12$

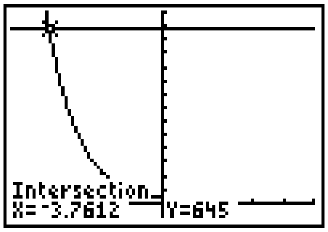
$x > \frac{4 \ln 12}{\ln 12 - \ln 4}$

$x > 9.0474$

47. $x^{\frac{2}{3}} \ge 27.6$

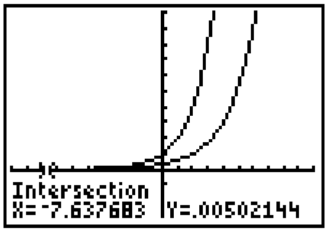
$x \ge (27.6)^{\frac{3}{2}}$

$x \ge 144.9985$

48. 

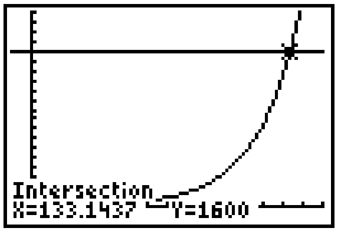
$x \approx -3.76$

$[-5, 5]$ scl:1 by $[-50, 700]$ scl:50

49. 

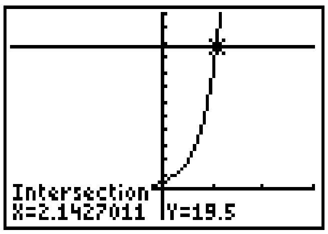
$x \approx -7.64$

$[70, 10]$ scl:1 by $[-3, 10]$ scl:1

50. 

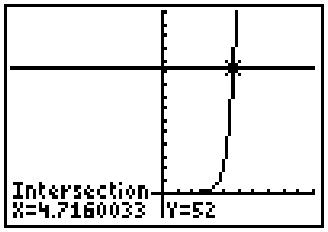
$t < 133.14$

$[-10, 150]$ scl:10 by $[-100, 2000]$ scl:100

51. 

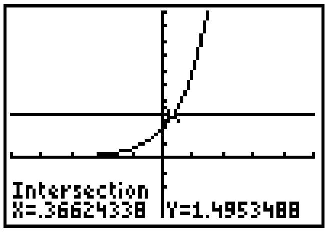
$x \approx 2.14$

$[-6, 6]$ scl:1 by $[-4, 24]$ scl:2

52. 

$x \ge 4.72$

$[-10, 10]$ scl:1 by $[-10, 75]$ scl:5

53. 

$x \ge 0.37$

$[-5, 5]$ scl:1 by $[-2, 5]$ scl: 0.5

54. $0.6 = 1e^{\frac{t}{-20,000(4 \times 10^{-11})}}$

$\ln 0.6 = \frac{t}{-20,000(4 \times 10^{-11})} \ln e$

$-20,000(4 \times 10^{-11}) \ln 0.6 = t$

$4.09 \times 10^{-7} \approx t$

4.09×10^{-7} s

55. $2.8 = 9\left(\frac{1}{2}\right)^t$

$\ln \frac{2.8}{9} = t \ln \frac{1}{2}$

$\frac{\ln \frac{2.8}{9}}{\ln \frac{1}{2}} = t$

$1.6845 \approx t$

$1.6845 \times 8 \times 24 = 323.4236$

324 h

56a. $\ln |180 - 72| = -k(0) + c$

$4.6821 \approx c$

56b. $\ln |150 - 72| = -k(2) + 4.6821$

$\frac{\ln 78 - 4.6821}{-2} = k$

$0.1627 \approx k$

56c. $\ln |100 - 72| = -(0.1622)t + 4.6821$

$\frac{\ln 28 - 4.6821}{-0.1627} = t$

$8.3 \approx t$ $8.3 - 2 = 6.3$

about 6.3 min

57. $e^{-2x} - 4e^{-x} + 3 = 0$

$(e^{-x} - 3)(e^{-x} - 1) = 0$

$e^{-x} - 3 = 0$	$e^{-x} - 1 = 0$
$e^{-x} = 3$	$e^{-x} = 1$
$-x \ln e = \ln 3$	$-x \ln e = \ln 1$
$x \approx -1.0986$	$x = 0$

0 or -1.0986

58a. $2 = e^{0.063t}$

$\ln 2 = 0.063t \ln e$

$\frac{\ln 2}{0.063} = t$

$11.0023 \approx t$

about 11 years

58b. See students' work.

59. $1800 = -5000 \ln r$

$\frac{1800}{-5000} = \ln r$

$\text{antiln}\left(\frac{-1800}{5000}\right) = r$

$0.6977 \approx r$; about 70%

60a. $\frac{1}{2} = 1e^{k(1622)}$

$\ln \frac{1}{2} = 1622\, k \ln e$

$\frac{\ln \frac{1}{2}}{1622} = k$

$-0.000427 \approx k$

60b. $1.7 = 23e^{(-0.000427)(t)}$

$\ln \frac{1.7}{2.3} = -0.000427t \ln e$

$\frac{\ln \frac{1.7}{2.3}}{-0.000427} = t$

$707.9177 \approx t$

about 708 yr

61. y is a logarithmic function of x. The pattern in the table can be determined by $3^y = x$ which can be expressed as $\log_3 x = y$.

62. 1.2844

63. $16^{\frac{3}{4}} = 8$

64. $x^2 = y + 4$

$x^2 + 4y^2 = 8$

$(y + 4) + 4y^2 = 8$

$4y^2 + y - 4 = 0$

$y = \frac{-1 \pm \sqrt{1 - 4(4)(-4)}}{8}$

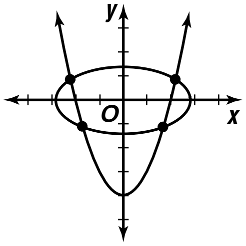
$y = \frac{-1 \pm \sqrt{65}}{8}$

$y \approx 0.9, -1.1$

$x^2 \approx 0.9 + 4$ $x^2 \approx -11 + 4$

$x \approx \pm\sqrt{4.9}$ $x \approx \pm\sqrt{2.9}$

$x \approx -2.2, 2.2$ $x \approx -1.7, 1.7$



65. $\frac{52.4 \text{ N}}{\text{m}^2} \cdot 146 \text{ cm}^3 \cdot \frac{\text{m}^3}{100^3 \text{ cm}^3} = c$

$0.00765 \approx c$; $0.00765 \text{ N} \cdot \text{m}$

66. $x = 0.25 \cos \pi$ $y = 0.25 \sin \pi$

$= -0.25$ $= 0$

$(-0.25, 0)$

67. $\vec{a} = \langle 1, -2\rangle + 3\langle 4, 3\rangle$

$= \langle 1, -2\rangle + \langle 12, 9\rangle$

$= \langle 13, 7\rangle$

68. $2x - 5y + 3 = 0$

$-\sqrt{A^2 + B^2} = -\sqrt{2^2 + (-5)^2} = -\sqrt{29}$

$-\frac{2x}{\sqrt{29}} + \frac{5y}{\sqrt{29}} - \frac{3}{29} = 0$

$-\frac{2\sqrt{29}}{29}x + \frac{5\sqrt{29}}{29}y - \frac{3\sqrt{29}}{29} = 0$

$p = \frac{3\sqrt{29}}{29} \approx 0.56$ units

$\sin \phi = \frac{5}{\sqrt{29}}$ $\cos \phi = -\frac{2}{\sqrt{29}}$

$\tan \phi = \frac{\frac{5\sqrt{29}}{29}}{-\frac{2\sqrt{29}}{29}}$

$\tan \phi = -\frac{5}{2}$

$\phi = 112°$

$-\frac{2\sqrt{29}}{29}x + \frac{5\sqrt{29}}{29}y - \frac{3\sqrt{29}}{29} = 0$; $\frac{3\sqrt{29}}{29} \approx 0.56$; 112°

69. $y = \pm 70 \cos 4\theta$

70. $d = 800 - (10 \cdot 55)$

$= 250$

The correct answer is 250.

11-6B Graphing Calculator Exploration: Natural Logarithms and Area

Pages 738–739

1. 0.69314718

2. 0.6931471806; It is the same value as found in Exercise 1 expressed to 10 decimal places.

3a. The result is the opposite of the result in Exercise 1.

3b. Sample answer: a negative value

4a. 0.69314718

4b. 1.0986123

4c. 1.3862944

4d. 0.6931471806, 1.098612289, 1.386294361

4e. The value for each area is the same as the value of each natural logarithm.

5. −0.5108256238; −0.6931471806; −0.9162907319; These values are equal to the value of ln 0.6, ln 0.5, and ln 0.4.

6. If $k \geq 1$, then the area of the region is equal to $\ln k$. If $0 < k < 1$, then the opposite of the area is equal to $\ln k$.

7. The value of a should be equal to or very close to 1, and the value of b should be very close to e. This prediction is confirmed when you display the actual regression equation.

8. Sample answer: Define $\ln k$ for $k > 0$ to be the area between the graph of $y = \frac{1}{x}$, the x-axis, and the vertical lines $x = 1$ and $x = k$ if $k \geq 1$ and to be the opposite of this area if $0 < k < 1$. Define e to be the value of k for which the area of the region is equal to 1.

11-7 Modeling Real-World Data with Exponential and Logarithmic Functions

Page 744 Check for Understanding

1. Replace N by $4N_0$ in the equation $N = N_0e^{kt}$, where N_0 is the amount invested and k is the interest rate. Then solve for t.

2. The data should be modeled with an exponential function. The points in the scatter plot approach a horizontal asymptote. Exponential functions have horizontal asymptotes, but logarithmic functions do not.

3. $y = 2e^{(\ln 4)x}$ or $y = 2e^{1.3863x}$; $\ln y = \ln 2 + (\ln 4)x$ or $\ln y = 0.6931 + 1.3863x$

4. $t = \frac{\ln 2}{0.0175}$
≈ 39.61 yr

5. $t = \frac{\ln 2}{0.08}$
≈ 8.66 yr

6a. $y = 10.0170(0.9703)^x$

6b. $y = 10.0170(0.9703)^x$
$y = 10.0170(e^{\ln 0.9703})^x$
$y = 10.0170e^{(\ln 0.9703)x}$
$y \approx 10.0170e^{-0.0301x}$

6c. $5 \approx 10.0170e^{-0.0301x}$
$\ln \frac{5}{10.0170} \approx -0.0301x$
$\frac{\ln \frac{5}{10.0170}}{-0.0301} \approx x$
$23.08 \approx x$; 23.08 min

Pages 745–748 Exercises

7. $t = \frac{\ln 2}{0.0225}$
≈ 30.81 yr

8. $t = \frac{\ln 2}{0.05}$
≈ 13.86 yr

9. $t = \frac{\ln 2}{0.07125}$
≈ 9.73

10. exponential; the graph has a horizontal asymptote

11. logarithmic; the graph has a vertical asymptote

12. logarithmic; the graph has a vertical asymptote

13. exponential; the graph has a horizontal asymptote

14a. $y = 4.7818(1.7687)^x$

14b. $y = 4.7818(1.7687)^x$
$y = 4.7818(e^{\ln 1.7687})^x$
$y = 4.7818e^{(\ln 1.7687)x}$
$y = 4.7818e^{0.5702x}$

14c. Use $t = \frac{\ln 2}{k}$; $k = 0.5702$.
$t = \frac{\ln 2}{0.5702}$
≈ 1.215 hr

15a. $y = 1.0091(0.9805)^x$

15b. $y = 1.0091(0.9805)^x$
$y = 1.0091(e^{\ln 0.9805})^x$
$y = 1.0091e^{(\ln 0.9805)x}$
$y = 1.0091e^{-0.0197x}$

15c. $0.415 = 1.0091e^{-0.0197x}$
$\ln \frac{0.415}{1.0091} = -0.0197x$
$\frac{\ln \frac{0.415}{1.0091}}{-0.0197} = x$
$45.10 \approx x$
$45.10 - 10 = 35.10$ min

16a. $y = 2137.5192(1.0534)^x$

16b. $y = 2137.5192(1.0534)^x$
$y = 2137.5192(e^{\ln 1.0534})^x$
$y = 2137.5192e^{(\ln 1.0534)x}$
$y = 2137.5192e^{0.0520x}$

16c. $2631.74 = 2137.52e^{4r}$
$\ln \frac{2631.74}{2137.52} = 4r$
$\ln \frac{\frac{2631.74}{2137.52}}{4} = r$
$0.0520 \approx r$; 5.2%

17. $y = 40 + 14.4270 \ln x$

18a. $y = -826.4217 + 520.4168 \ln x$

18b. The year 1960 would correspond to $x = 0$ and $\ln 0$ is undefined.

19. Take the square root of each side.
$y = cx^2$
$\sqrt{y} = \sqrt{cx^2}$
$\sqrt{y} = \sqrt{c}x$

20a. $1034.34 = 1000(1 + r)^1$
$1.03034 = 1 + r$
$0.03034 \approx r$; 3.034%

20b. $y = 1000.0006(1.0303)^x$

20c. $y = 1000.0006(1.0303)^x$
$y = 1000.0006(e^{\ln 1.0303})^x$
$y = 1000.0006e^{(\ln 1.0303)x}$
$y = 1000.0006e^{0.0299x}$

20d. $1030.34 = 1000e^r$
$\ln \frac{1030.34}{1000} = r$
$0.0299 \approx r$; 2.99%

21a.

x	0	50	100	150	190	200
$\ln y$	1.81	2.07	3.24	3.75	4.25	4.38

21b. $\ln y = 0.0136x + 1.6889$

21c. $\ln y = 0.0136x + 1.6889$
$y = e^{0.0136x+1.6889}$

21d. $y = e^{0.0136(225)+1.6889}$
≈ 115.4572
115.5 persons per square mile

22a. The graph appears to have a horizontal asymptote at $y = 2$, so you must subtract 2 from each y-value before a calculator can perform exponential regression.

22b. $y = 2 + 1.0003(2.5710)^x$

23a. $\ln y$ is a linear function of $\ln x$.
$y = cx^a$
$\ln y = \ln(cx^a)$
$\ln y = \ln c + \ln x^a$
$\ln y = \ln c + a \ln x$

23b. The result of part a indicates that we should take the natural logarithms of both the x- and y-values.

$\ln x$	6.21	6.91	8.52	9.21	9.62
$\ln y$	4.49	4.84	5.65	5.99	6.19

23c. $\ln y = 0.4994 \ln x + 1.3901$

23d.
$$\begin{aligned} \ln y &= 0.4994 \ln x + 1.3901 \\ e^{\ln y} &= e^{0.4994\ln x + 1.3901} \\ y &= e^{0.4994\ln x} \cdot e^{1.3901} \\ y &= (e^{\ln x})^{0.4994} \cdot 4.0153 \\ y &= 4.0153x^{0.4994} \end{aligned}$$

24.
$$\begin{aligned} 2 &= e^{k(85)} \\ \ln 2 &= 85k \\ 0.0082 &\approx k \\ 12 &= e^{0.0082t} \\ \ln 12 &= 0.0082t \\ 303 &\approx t \end{aligned}$$
303.04 min or about 5 h

25. 0.01

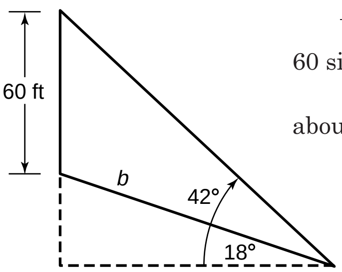
26.
$$\begin{aligned} \log_5 (7x) &= \log_5 (5x + 16) \\ 7x &= 5x + 16 \\ 2x &= 16 \\ x &= 8 \end{aligned}$$

27a.
$$\begin{aligned} y &= x(400 - 20(x - 3)) \\ y &= x(460 - 20x) \\ y &= -20x^2 + 460x \\ y - 2645 &= 20(x^2 - 23x + 132.25) \\ y - 2645 &= -20(x - 11.5) \end{aligned}$$
vertex at (11.5, 2645), maximum at $x = 11.5$
\$11.50

27b. At maximum, $y = 2645$.
\$2645

28.
$$\begin{aligned} 5\left(\cos \tfrac{\pi}{6} + \boldsymbol{i} \sin \tfrac{\pi}{6}\right) &= 5\left(\tfrac{\sqrt{3}}{2} + \tfrac{1}{2}\boldsymbol{i}\right) \\ &= \tfrac{5\sqrt{3}}{2} + \tfrac{5}{2}\boldsymbol{i} \end{aligned}$$

29.

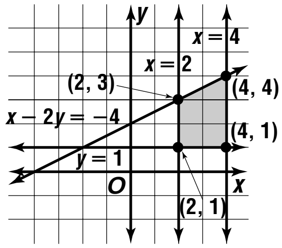


$$\begin{aligned} \frac{60}{\sin 24°} &= \frac{b}{\sin 48°} \\ 60 \sin 48° &= b \sin 24° \\ b &= 109.625 \end{aligned}$$
about 109.6 ft

30. $5x^2 - 8x + 12 = 0$
Discriminant: $(-8)^2 - 4(5)(12) = -176$
The discriminant is negative, so there are 2 imaginary roots.
$$\begin{aligned} x &= \frac{8 \pm \sqrt{-176}}{10} \\ &= \frac{8 \pm 4\boldsymbol{i}\sqrt{11}}{10} \text{ or } \frac{4 \pm 2\boldsymbol{i}\sqrt{11}}{5} \end{aligned}$$

31. 4 units left and 8 units down

32.



$$\begin{aligned} (2, 1)\text{: } f(x) &= 2(2) + 8(1) + 10 \\ &= 22 \\ (4, 1)\text{: } f(x) &= 2(4) + 8(1) + 10 \\ &= 26 \\ (2, 8)\text{: } f(x) &= 2(2) + 8(3) + 10 \\ &= 38 \\ (4, 4)\text{: } f(x) &= 2(4) + 8(4) + 10 \\ &= 50 \end{aligned}$$

50; 22

33. Circle X contains the regions a, b, d, and e. Circle Z contains the regions d, e, f, and g. Six regions are contained in one or both of circles X and Z.
The correct choice is C.

Chapter 11 Study Guide and Assessment

Page 749 Understanding the Vocabulary

1. common logarithm
2. exponential growth
3. logarithmic function
4. scientific notation
5. mantissa
6. natural logarithm
7. linearizing data
8. exponential function
9. nonlinear regression
10. exponential equation

Pages 750–752 Skills and Concepts

11. $\left(\frac{1}{4}\right)^{-2} = \dfrac{1}{\left(\frac{1}{4}\right)^2}$
$= 16$

12. $(64)^{\frac{1}{2}} = 8$

13.
$$\begin{aligned} (27)^{\frac{4}{3}} &= (3^3)^{\frac{4}{3}} \\ &= 3^4 \\ &= 81 \end{aligned}$$

14.
$$\begin{aligned} \left(\sqrt[4]{256}\right)^3 &= (256)^{\frac{3}{4}} \\ &= (4^4)^{\frac{3}{4}} \\ &= 4^3 \\ &= 64 \end{aligned}$$

15.
$$\begin{aligned} 3x^2(3x)^{-2} &= \frac{3x^2}{(3x)^2} \\ &= \frac{3x^2}{9x^2} \\ &= \frac{1}{3} \end{aligned}$$

16.
$$\begin{aligned} \left(6a^{\frac{1}{3}}\right)^3 &= 6^3\left(a^{\frac{1}{3}}\right)^3 \\ &= 216a \end{aligned}$$

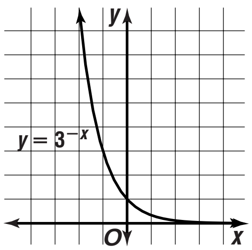
17.
$$\begin{aligned} \left(\tfrac{1}{2}x^4\right)^3 &= \left(\tfrac{1}{2}\right)^3(x^4)^3 \\ &= \tfrac{1}{8}x^{12} \end{aligned}$$

18.
$$\begin{aligned} (w^3)^4 \cdot (4w^2)^2 &= w^{12} \cdot 4^2 \cdot w^4 \\ &= 16w^{16} \end{aligned}$$

19.
$$\begin{aligned} \left((2a)^{\frac{1}{3}}(a^2b)^{\frac{1}{3}}\right)^3 &= \left[(2a)^{\frac{1}{3}}\right]^3 \cdot \left[(a^2b)^{\frac{1}{3}}\right]^2 \\ &= (2a)(a^2b) \\ &= 2a^3b \end{aligned}$$

20. $\left(3x^{\frac{1}{2}}y^{\frac{1}{4}}\right)(4x^2y^2) = 12x^{\frac{5}{2}}y^{\frac{9}{4}}$

21.



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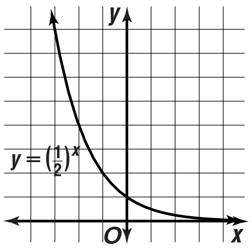

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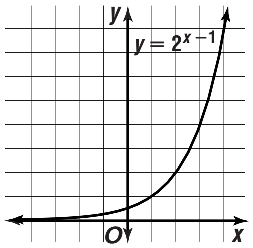

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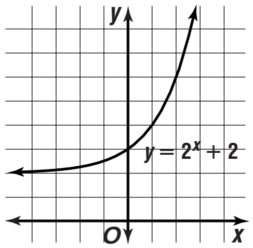

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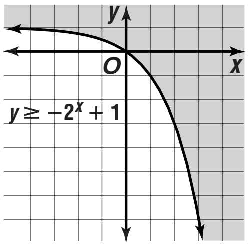

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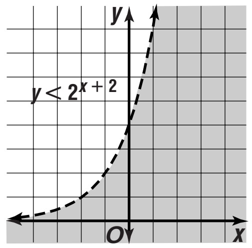


27. $A = 2500e^{0.065(10)}$
≈ 4788.8520; \$4788.85

28. $A = 6000e^{0.0725(10)}$
$\approx 12{,}388.3866$; \$12,388.39

29. $A = 12{,}000e^{0.059(10)}$
$\approx 21{,}647.8610$, \$21,647.86

30. $8^{\frac{2}{3}} = 4$

31. $3^{-4} = \frac{1}{81}$

32. $\log_2 16 = 4$

33. $\log_5 \frac{1}{25} = -2$

34. $2^x = 32$
$2^x = 2^5$
$x = 5$

35. $10^x = 0.001$
$10^x = 10^{-3}$
$x = -3$

36. $4^x = \frac{1}{16}$
$4^x = 4^{-2}$
$x = -2$

37. $2^x = 0.5$
$2^x = 2^{-1}$
$x = -1$

38. $6^x = 216$
$6^x = 6^3$
$x = 3$

39. $9^x = \frac{1}{9}$
$9^x = 9^{-1}$
$x = -1$

40. $4^x = 1024$
$4^x = 4^5$
$x = 5$

41. $8^x = 512$
$8^x = 8^3$
$x = 3$

42. $x^4 = 81$
$x = (81)^{\frac{1}{4}}$
$x = 3$

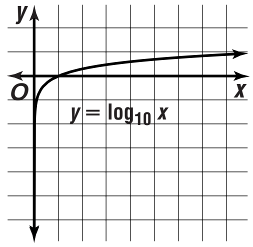
43. $\left(\frac{1}{2}\right)^{-4} = x$
$16 = x$

44. $\log_3 3 + \log_3 x = \log_3 45$
$\log_3 3x = \log_3 45$
$3x = 45$
$x = 15$

45. $2 \log_6 4 - \frac{1}{3} \log_6 8 = \log_6 x$
$\log_6 4^2 - \log_6 8^{\frac{1}{3}} = \log_6 x$
$\log_6 \frac{4^2}{8^{\frac{1}{3}}} = \log_6 x$
$\frac{4^2}{8^{\frac{1}{3}}} = x$
$8 = x$

46. $\log_2 x = \frac{1}{3} \log_6 27$
$\log_2 x = \log_2 27^{\frac{1}{3}}$
$x = 27^{\frac{1}{3}}$
$x = 3$

47.



48. $\log 300{,}000 = \log (100{,}000 \times 3)$
$= \log 100{,}000 + \log 3$
$= 5 + 0.4771$
$= 5.4771$

49. $\log 0.0003 = \log (0.0001 \times 3)$
$= \log 0.0001 + \log 3$
$= -4 + 0.4771$
$= -3.5229$

50. $\log 140 = \log (10 \times 14)$
$= \log 10 + \log 14$
$= 1 + 1.1461$
$= 2.1461$

51. $\log 0.014 = \log (0.001 \times 14)$
$= \log 0.001 + \log 14$
$= -3 + 1.1461$
$= -1.8539$

52. $4x = 6^{x+2}$
$x \log 4 = (x + 2) \log 6$
$x \log^4 = x \log 6 + 2 \log 6$
$x \log 4 - x \log 6 = 2 \log 6$
$x(\log 4 - \log 6) = 2 \log 6$
$x = \frac{2 \log 6}{\log 4 - \log 6}$
$x \approx -8.84$

53. $12^{0.5x} = 8^{0.1x-4}$
$0.5x \log 12 = (0.1x - 4) \log 8$
$0.5x \log 12 = 0.1x \log 8 - 4 \log 8$
$0.5x \log 12 - 0.1x \log 8 = -4 \log 8$
$x(0.5 \log 12 - 0.1 \log 8) = -4 \log 8$
$x = \frac{-4 \log 8}{0.5 \log 12 - 0.1 \log 8}$
$x \approx -8.04$

54. $\left(\frac{1}{4}\right)^{3x} < 6^{x-2}$

$$3x \log \frac{1}{4} < (x-2) \log 6$$
$$3x \log \frac{1}{4} < x \log 6 - 2 \log 6$$
$$3x \log \frac{1}{4} - x \log 6 < -2 \log 6$$
$$x\left(3 \log \frac{1}{4} - \log 6\right) < -2 \log 6$$
$$x > \frac{-2 \log 6}{3 \log \frac{1}{4} - \log 6}$$

Change the inequality because 3 log $\frac{1}{4}$ − log 6 is negative.

$$x > 0.6$$

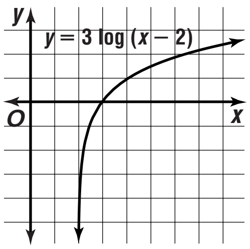
55. $0 - 1^{2x+8} \geq 7^{x+4}$

$$(2x+8) \log 0.1 \geq (x+4) \log 7$$
$$2x \log 0.1 + 8 \log 0.1 \geq x \log 7 + 4 \log 7$$
$$2x \log 0.1 - x \log 7 \geq 4 \log 7 - 8 \log 0.1$$
$$x(2 \log 0.1 - \log 7) \geq 4 \log 7 - 8 \log 0.1$$
$$x \geq \frac{4 \log 7 - 7 \log 0.1}{2 \log 0.1 - \log 7}$$
$$x \geq -4$$

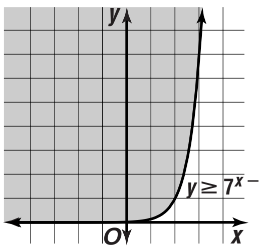
56. $\log(2x+3) = -\log(3-x)$

$$\log(2x+3) = \log(3-x)^{-1}$$
$$(2x+3) = (3-x)^{-1}$$
$$(2x+3)(3-x) = 1$$
$$2x^2 - 3x - 8 = 0$$
$$x = \frac{3 \pm \sqrt{73}}{4}$$
$$\approx -1.39, 2.89$$

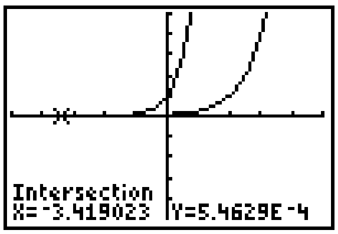
57.



58.



59.



$x = 3.42$

[5, 5] scl:1 by [−5, 5] scl:1

60. $\log_4 15 = \frac{\log 15}{\log 4}$
≈ 1.9534

61. $\log_8 24 = \frac{\log 24}{\log 8}$
≈ 1.5283

62. $\log_4 100 = \frac{\log 100}{\log 9}$
≈ 2.0959

63. $\log_{15} 125 = \frac{\log 125}{\log 15}$
≈ 1.7829

64. $4x = 100$

$$x \ln 4 = \ln 100$$
$$x = \frac{\ln 100}{\ln 4}$$
$$x \approx 3.3219$$

65. $6^{x-2} = 30$

$$(x-2) \ln 6 = \ln 30$$
$$x - 2 = \frac{\ln 30}{\ln 6}$$
$$x = \frac{\ln 30}{\ln 6} + 2$$
$$x \approx 3.8982$$

66. $3^{x+1} = 4^{2x}$

$$(x+1) \ln 3 = 2x \ln 4$$
$$x \ln 3 + \ln 3 = 2x \ln 4$$
$$x \ln 3 - 2x \ln 4 = -\ln 3$$
$$x(\ln 3 - 2 \ln 4) = -\ln 3$$
$$x = \frac{-\ln 3}{\ln 3 - 2 \ln 4}$$
$$x \approx 0.6563$$

67. $9^{4x} = 5^{x-4}$

$$4x \ln 9 = (x-4) \ln 5$$
$$4x \ln 9 = x \ln 5 - 4 \ln 5$$
$$4x \ln 9 - x \ln 5 = -4 \ln 5$$
$$x(4 \ln 9 - \ln 5) = -4 \ln 5$$
$$x = \frac{-4 \ln 5}{4 \ln 9 - \ln 5}$$
$$x \approx -0.8967$$

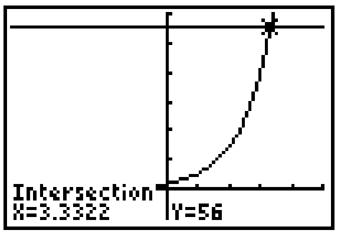
68. $24 < e^{2x}$

$$\ln 24 < 2x$$
$$x > \frac{\ln 24}{2}$$
$$x > 1.5890$$

69. $15e^x \geq 200$

$$e^x \geq \frac{200}{15}$$
$$x \geq \ln \frac{200}{15}$$
$$x \geq 2.5903$$

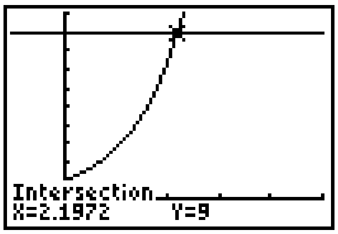
70.



$x \approx 3.333$

[−5, 5] scl:1 by [−10, 60] scl:10

71.



$x < 2.20$

[−1, 5] scl:1 by [−1, 10] scl:1

72. $t = \frac{\ln 2}{0.028}$

≈ 24.76

73. $t = \frac{\ln 2}{0.05125}$

≈ 13.52

74. $18 = \frac{\ln 2}{k}$

$k = \frac{\ln 2}{18}$

≈ 0.0385; 3.85%

Page 753 Applications and Problem solving

75. $0.065 = \left(\frac{1}{2}\right)^t$

$\log 0.65 = t \log \frac{1}{2}$

$\frac{\log 0.65}{\log \frac{1}{2}} = t$

$0.6215 \approx t$

$0.6215 \times 5730 \approx 3561.13$ or 3561 yr.

76a. $\beta = 10 \log \frac{1.15 \times 10^{-10}}{10^{-12}}$

≈ 20.6 20.6 dB

76b. $\beta = 10 \log \frac{9 \times 10^{-9}}{10^{-2}}$

≈ 39.5 39.5 dB

76c. $\beta = 10 \log \frac{8.95 \times 10^{-3}}{10^{-12}}$

≈ 99.5 99.5 dB

77. $200{,}000 = 142{,}000e^{0.014t}$

$\frac{100}{71} = e^{0.014t}$

$\ln \frac{100}{71} = 0.014t$

$\frac{\ln \frac{100}{71}}{0.014} = t$

$24.4 \approx t$

$1990 + 24 = 2014$

78a. $N = 65 - 30e^{-0.20(2)}$

≈ 44.89; 45 words per minute

78b. $N = 65 - 30e^{-0.20(15)}$

$= 63.50$; 64 words per minute

78c. $50 = 65 - 30e^{-0.20t}$

$\frac{1}{2} = e^{-0.20t}$

$\ln \frac{1}{2} = -0.20t$

$\frac{\ln \frac{1}{2}}{-0.20} = t$

$3.47 \approx t$; 3.5 weeks

Page 753 Open-Ended Assessment

1. Sample answer: $(n^4)^{\frac{1}{4}}(4m)^{-1}$
2. Sample answer:

   $\log 2 + \log(x + 2) = \frac{1}{2} \log 36$

Chapter SAT & ACT Preparation

Page 755 SAT and ACT Practice

1. To find the greatest possible value, the other 3 values must be as small as possible. Since they are distinct positive integers, they must be 1, 2, and 3. The sum of all 4 integers is 4(11) or 44. The sum of the 3 smallest is 1 + 2 + 3 or 6, so the fourth integer cannot be more than 44 − 6 or 38. The correct choice is B.

2. Since one root is $\frac{1}{2}$, $x = \frac{1}{2}$, $2x = 1$, and $2x = 0$. Similarly for the root that is $\frac{1}{3}$, $x = \frac{1}{3}$, $3x = 1$, and $3x - 1 = 0$.

   To find the quadratic equation, multiply these two factors and let the product equal zero.

   $(2x - 1)(3x - 1) = 0$

   $6x^2 - 5x + 1 = 0$

   The correct choice is E.

3. The result of dividing T by 6 is 14 less than the correct average.

   $\frac{T}{6} = \text{correct answer} - 14$

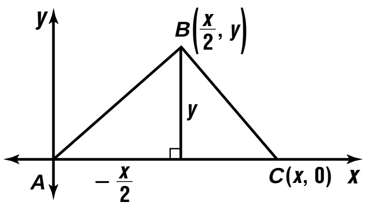
   $\frac{T}{6} + 14 = \text{correct average}$

   The correct average is the total divided by the number of scores, 5.

   $\text{correct average} = \frac{T}{5}$

   $\frac{T}{6} + 14 = \frac{T}{5}$

   The correct choice is E.

4. 

   $\tan A = \frac{y}{\frac{x}{2}} = \frac{2y}{x}$

   Find the area of $\triangle ABC$.

   $A = \frac{1}{2}bh = \frac{1}{2}xy = \frac{xy}{2}$

   Simplify the ratio.

   $\frac{\text{area of } \triangle ABC}{\tan A} = \frac{\frac{xy}{2}}{\frac{2y}{x}} = \frac{x}{2y}\left(\frac{xy}{2}\right) = \frac{x^2}{4}$

   The correct choice is E.

5. $\frac{x}{y} = \frac{10}{2y}$

   $2yx = 10y$

   $2x = 10$

   $x = 5$

   The correct choice is B.

6. $C = \pi D$

$\frac{2\pi}{3} = \pi D$

$D = \frac{2}{3}$ and, therefore, $r = \frac{1}{3}$.

Now use this value for the radius to calculate *half* of the area.

$\frac{1}{2}A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi\left(\frac{1}{3}\right)^2 = \frac{1}{2}\pi\left(\frac{1}{9}\right) = \frac{\pi}{18}$

The correct choice is A.

7. The average of 8 numbers is 20.

$20 = \frac{\text{sum of eight numbers}}{8}$

sum of eight numbers = 160
The average of 5 of the numbers is 14.

$14 = \frac{\text{sum of five numbers}}{5}$

sum of five numbers = 70

The sum of the other three numbers must be $160 - 70$ or 90. Calculate the average of these three numbers.

$\text{average} = \frac{\text{sum of three numbers}}{3} = \frac{90}{3} = 30$

The correct choice is D.

8. The sum of the angles in a triangle is 180°. Since $\angle B$ is a right angle, it is 90°. So the sum of the other two angles is 90°. Write and solve an equation using the expressions for the two angles.

$2x + 3x = 90$
$5x = 90$
$x = 18$

The question asks for the measure of $\angle A$.

$\angle A = 2x = 2(18) = 36$

The correct choice is C.

9. A is the arithmetic mean of three consecutive positive even integers, so $A = \frac{x + (x + 2) + (x + 4)}{3} = \frac{3x + 6}{3} = x + 2$, where x is a positive even integer. Then A is also a positive even integer. Since A is even, when A is divided by 6, the remainder must also be an even integer. The possible even remainders are 0, 2, and 4.

The correct choice is C.

10. First notice that b must be a prime integer. Next notice that $3b$ is greater than 10. So b could be 5, since $3(5) = 15$. (b cannot be 3.) Check to be sure that 5 fits the rest of the inequality.

$3(5) = 15 > \frac{5}{6}(5) = \frac{25}{6} = 4\frac{1}{6}$

So 5 is one possible answer. You can check to see that 7 and 11 are also valid answers.
The correct answer is 5, 7, or 11.

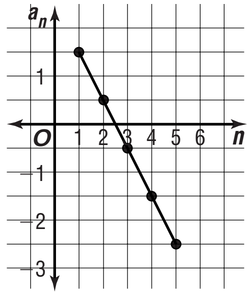
Chapter 12 Sequences and Series

12-1 Arithmetic Sequences and Series

Pages 762–763 Check for Understanding

1. $a_1 = 6 - 4(1)$ or 2
$a_2 = 6 - 4(2)$ or -2
$a_3 = 6 - 4(3)$ or -6
$a_4 = 6 - 4(4)$ or -10
$a_5 = 6 - 4(5)$ or -14
2, -2, -6, -10, -14; yes, there is a common difference of -4.

2a.



2b. linear

2c. The common difference is -1. This is the slope of the line through the points of the sequence.

3a. The number of houses sold cannot be negative.

3b. $S_n = \frac{n}{2}[2a, + (n - 1)d]$
$S_{10} = \frac{10}{2}[2 \cdot 3750 + (10 - 1)500]$
$= \$60{,}000$

4. Negative; let n and n − 1 be two consecutive numbers in the sequence.
$d = (n - 1) - n$ or -1

5. Neither student is correct, since neither sequence has a common difference. The difference fluctuates between -1 and 1. The second sequence has a difference that fluctuates between 2 and -2.

6. $d = 11 - 6$ or 5
$16 + 5 = 21, 21 + 5 = 26, 26 + 5 = 31, 31 + 5 = 36$
21, 26, 31, 36

7. $d = -7 - (-15)$ or 8
$1 + 8 = 9, 9 + 8 = 17, 17 + 8 = 25, 25 + 8 = 33$
9, 17, 25, 33

8. $d = (a - 2) - (a - 6)$
$= a - a - 2 + 6$ or 4
$a + 2 + 4 = a + 6, a + 6 + 4 = a + 10,$
$a + 10 + 4 = a + 14, a + 14 + 4 = 4 = a + 18$
$a + 6, a + 10, a + 14, a + 18$

9. $a_n = a_1 + (n - 1)d$
$a_{17} = 10 + (17 - 1)(-3)$
$= -38$

10. $a_n = a_1 + (n - 1)d$
$37 = -13 + (n - 1)5$
$50 = 5(n - 1)$
$10 = n - 1$
$11 = n$

11. $a_n = a_1 + (n - 1)d$
$3 = a_1 + (7 - 1)(-2)$
$3 = a_1 - 12$
$15 = a_1$

12. $a_n = a_1 + (n - 1)d$
$34 = 100 + (12 - 1)d$
$-66 = 11d$
$-6 = d$

13. $a_n = a_1 + (n - 1)d$
$24 = 9 + (4 - 1)d$
$15 = 3d$
$5 = d$
$9 + 5 = 14, 14 + 5 = 19$
9, 14, 19, 24

14. $S_n = \frac{n}{2}[2a_1 + (n - 1)d]$
$S_{35} = \frac{35}{2}[2.7 + (35 - 1) \cdot 2]$
$= 1435$

15. $S_n = \frac{n}{2}[2a_1 + (n - 1)d]$
$-210 = \frac{n}{2}[2 \cdot 30 + (n - 1)(-4)]$
$-420 = 60n - 4n^2 + 4n$
$4n^2 - 64n - 420 = 0$
$4(n - 21)(n + 5) = 0$
$n = 21$ or $n = -5$
Since n cannot be negative, $n = 21$.

16. $n = 19, a_{19} = 27, d = 1$
$a_n = a_1 + (n - 1)d$
$27 = a_1 + (19 - 1)1$
$9 = a_1$
$S_{19} = \frac{n}{2}(a_1 + a_{19})$
$= \frac{19}{2}(9 + 27)$
$= 342$ seats

Pages 763–765 Exercises

17. $d = -1 - 5$ or -6
$-7 + (-6) = -13, -13 + (-6) = -19,$
$-19 + (-6) = -25, -25 + (-6) = -31$
$-13, -19, -25, -31$

18. $d = -7 - (-18) = 11$
$4 + 11 = 15, 15 + 11 = 26, 26 + 11 = 37, 37 + 11 = 48$
15, 26, 37, 48

19. $d = 4.5 - 3$ or 1.5
$6 + 1.5 = 7.5, 7.5 + 1.5 = 9, 9 + 1.5 = 10.5,$
$10.5 + 1.5 = 12$
7.5, 9, 10.5, 12

20. $d = 3.8 - 5.6$ or -1.8
$2 + (-1.8) = 0.2, 0.2 + (-1.8) = -1.6,$
$-1.6 + (-1.8) = -3.4, -3.4 + (-1.8) = -5.2$
$0.2, -1.6, -3.4, -5.2$

21. $d = b + 4 - b$ or 4
$b + 8 + 4 = b + 12, b + 12 + 4 = b + 16,$
$b + 16 + 4 = b + 20, b + 20 + 4 = b + 24$
$b + 12, b + 16, b + 20, b + 24$

22. $d = 0 - (x)$ or x
$x + x = 2x, 2x + x = 3x, 3x + x = 4x, 4x + x = 5x$
$2x, 3x, 4x, 5x$

23. $d = -n - 5n$ or $-6x$
$-7n + (-6n) = -13n - 13n + (-6n) = 19n,$
$-19n + (-6n) = -25n, -25n + (-6n) = -31n$
$-13n, -19n, -25n, -31n$

24. $d = 5 - (5 + k)$ or $-k$
$5 - k + (-k) = 5 - 2k, 5 - 2k + (-k) = 5 - 3k,$
$5 - 3k + (-k) = 5 - 4k, 5 - 4k + (-k) = 5 - 5k$
$5 - 2k, 5 - 3k, 5 - 4k, 5 - 5k$

25. $d = (2a + 2) - (2a - 5)$ or 7
$2a + 9 + 7 = 2a + 16, 2a + 16 + 7 = 2a + 23,$
$2a + 23 + 7 = 2a + 30, 2a + 30 + 7 = 2a + 37$
$2a + 16, 2a + 23, 2a + 30, 2a + 37$

26. $d = 5 - (3 + \sqrt{7})$ or $2 - \sqrt{7}$
$7 - \sqrt{7} + 2 - \sqrt{7} = 9 - 2\sqrt{7}, 9 - 2\sqrt{7} + 2 - \sqrt{7}$
$= 11 - 3\sqrt{7}, 11 - 3\sqrt{7} + 2 - \sqrt{7} = 13 - 4\sqrt{7}$
$9 - 2\sqrt{7}, 11 - 3\sqrt{7}, 13 - 4\sqrt{7}$

27. $a_{25} = 8 + (25 - 1)3$
$= 80$

28. $a_{18} = 1.4 + (18 - 1)(0.5)$
$= 9.9$

29. $-41 = 19 + (n - 1)(-5)$
$-60 = -5(n - 1)$
$12 = n - 1$
$13 = n$

30. $138 = -2 + (n - 1)7$
$140 = 7(n - 1)$
$20 = n - 1$
$21 = n$

31. $38 = a_1 + (15 - 1)(-3)$
$38 = a_1 - 42$
$80 = a_1$

32. $10\frac{2}{3} = a_1 + (7 - 1)\frac{1}{3}$
$10\frac{2}{3} = a_1 + 2$
$8\frac{2}{3} = a_1$

33. $58 = 6 + (14 - 1)d$
$52 = 13d$
$4 = d$

34. $26 = 8 + (11 - 1)d$
$18 = 10d$
$1\frac{4}{5} = d$

35. $d = (-1 + \sqrt{5}) - (-4 + \sqrt{5})$ or 3
$a_8 = -4 + \sqrt{5} + (8 - 1)3$
$= 17 + \sqrt{5}$

36. $d = 6 - (5 - \boldsymbol{i}) = 1 + \boldsymbol{i}$
$a_{12} = 5 - \boldsymbol{i} + (12 - 1)(1 + \boldsymbol{i})$
$= 5 - \boldsymbol{i} + 11 + 11\boldsymbol{i}$
$= 16 + 10\boldsymbol{i}$

37. $d = 10.5 - 12.2$ or -1.7
$a_{33} = 12.2 + (33 - 1)(-1.7)$
$= -42.2$

38. $d = -4 - (-7)$ or 3
$a_{79} = -7 + (79 - 1)3$
$= 227$

39. $21 = 12 + (3 - 1)d$
$9 = 2d$
$4.5 = d$
$12 + 4.5 = 16.5$
12, 16.5, 21

40. $4 = -5 + (4 - 1)d$
$9 = 3d$
$3 = d$
$-5 + 3 = -2, -2 + 3 = 1$
$-5, -2, 1, 4$

41. $12 = \sqrt{3} + (4 - 1)d$
$12 - \sqrt{3} = 3d$
$\frac{12 - \sqrt{3}}{3} = d$
$\sqrt{3} + \frac{12 - \sqrt{3}}{3} = \frac{12 + 2\sqrt{3}}{3},$
$\frac{12 + 2\sqrt{3}}{3} + \frac{12 - \sqrt{3}}{3} = \frac{24 + \sqrt{3}}{3}$
$\sqrt{3}, \frac{12 + 2\sqrt{3}}{3}, \frac{24 + \sqrt{3}}{3}, 12$

42. $5 = 2 + (5 - 1)d$
$3 = 4d$
$0.75 = d$
$2 + 0.75 = 2.75, 2.75 + 0.75 = 3.5,$
$3.5 + 0.75 = 4.25$
2, 2.75, 3.5, 4.25, 5

43. $d = 1 - \frac{3}{2}$ or $-\frac{1}{2}$
$a_{11} = \frac{3}{2} + (11 - 1)\left(-\frac{1}{2}\right)$
$= -3.5$
$S_{11} = \frac{11}{2}\left(\frac{3}{2} + \left(-3\frac{1}{2}\right)\right)$
$= -11$

44. $d = -4.8 - (-5)$ or 0.2
$a_{100} = -5 + (100 - 1)0.2$
$= 14.8$
$S_{100} = \frac{100}{2}(-5 + 14.8)$
$= 490$

45. $d = -13 - (-19)$ or 6
$a_{26} = -19 + (26 - 1)6$
$= 131$
$S_{26} = \frac{26}{2}(-19 + 131)$
$= 1456$

46. $-14 = \frac{n}{2}[2(-7) + (n - 1)1.5]$
$-28 = -14n + 1.5n^2 - 1.5n$
$0 = 1.5n^2 - 15.5n + 28$
$n = \frac{15.5 \pm \sqrt{(-15.5)^2 - 4(1.5)(28)}}{2(1.5)}$
$n = 8$ or $n = 2\frac{1}{3}$
Since there cannot be a fractional number of terms, $n = 8$.

47. $31.5 = \frac{n}{2}[2(-3) + (n - 1)2.5]$
$63 = -6n + 2.5n^2 - 25n$
$0 = 2.5n^2 - 8.5n - 63$
$n = \frac{8.5 \pm \sqrt{(-8.5)^2 - 4(2.5)(-63)}}{2(2.5)}$
$n = 7$ or $n = 3.6$
Since n cannot be negative, $n = 7$.

48. $d = 7 - 5$ or 2
$a_n = 5 + (n - 1)2$
$= 2n + 3$

49. $d = -2 - 6$ or -8
$a_n = 6 + (n - 1)(-8)$
$= -8n + 14$

50. 9:00, 9:30, 10:00, 10:30, 11:00, 11:30, 12:00
$n = 7, d = 2, a_1 = 3$
$a_7 = 3 + (7 - 1)2$
$= 15$ data items per minute

51. Let d be the common difference. Then, $y = x + d$, $z = x + 2d$, and $w = x + 3d$. Substitute these values into the expression $x + w - y$ and simplify.
$x + (x + 3d) - (x + d) = x + 2d$ or z.

52. $a_1 = 5, d = 4, n = 25$
$a_{25} = 5 + (25 - 1)4$
$= 101$
$S_{25} = \frac{25}{2}(5 + 101)$
$= 1325$ bricks

53. $S_n = \frac{n}{2}(128° + 172°)$; $S_n = (n - 2)180°$
$150n = 180n - 360$
$-30n = -360$
$n = 12$

54a. $S_4 = (4 - 2)180°$ or $360°$
$S_5 = (5 - 2)180°$ or $540°$
$S_6 = (6 - 2)180°$ or $720°$
$S_7 = (7 - 2)180°$ or $900°$
360°, 540°, 720°, 900°

54b. The common difference between each consecutive term in the sequence is 180, therefore the sequence is arithmetic.

54c. $a_{35} = -180 + (35 - 1)180$
$= 5940°$

55a. $a_1 = 1, d = 2$
$S_5 = \frac{5}{2}[2(1) + (5 - 1)2]$
$= 25$

55b. $S_{10} = \frac{10}{2}[2(1) + (10 - 1)2]$
$= 100$

55c. Conjecture: The sum of the first n terms of the sequence of natural numbers is n^2.
Proof:
Let $a_n = 2n - 1$. The first term of the sequence of natural numbers is 1, so $a_1 = 1$.
Then, using the formula for the sum of an arithmetic series,
$S_n = \frac{n}{2}(a_1 + a_n)$
$S_n = \frac{n}{2}[1 + (2n - 1)]$
$= \frac{n}{2}(2n)$ or n^2

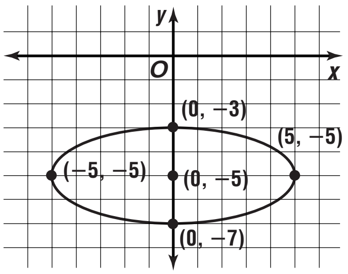
56. $a_1 = 5, d = 7, n = 15$
$S_{15} = \frac{15}{2}[2(5) + (15 - 1)7]$
$= 810$ feet

57. $n = 10, S_{10} = 5510, d = 100$
$5510 = \frac{10}{2}[2a_1 + (10 - 1)100]$
$5510 = 10a_1 + 4500$
$1010 = 10a_1$
$101 = a_1$
$a_{10} = 101 + (10 - 1)100$
$= 1001$
least: \$101, greatest: \$1001

58. $S_n = a_1 + a_2 + (a_{3-1} - a_{3-2}) + (a_{4-1} - a_{4-2}) + (a_{5-1} - a_{5-2}) + \cdots$
$= a_1 + a_2 + a_2 - a_1 + a_3 - a_2 + a_4 - a_3 + \cdots$
$= a_1 + a_2 + a_2 - a_1 + (a_2 - a_1) - a_2 + a_3 - a_2 - a_3 + \cdots$
$= 0$

59. $A = Pe^{rt}$
$= 100e^{0.07(15)}$
$= \$285.77$

60. $4x^2 + 25y^2 + 250y + 525 = 0$
$4x^2 + 25(y^2 + 10y) = -525$
$4x^2 + 25(y + 5)^2 = 100$
$\frac{x^2}{25} + \frac{(y + 5)^2}{4} = 1$
$h = 0, k = -5, a = 5, b = 2, c = \sqrt{21}$
center: $(0, -5)$
foci: $(\pm\sqrt{21}, -5)$
vertices: major $\rightarrow (\pm5, -5)$
minor $\rightarrow (0, -3)$ and $(0, -7)$

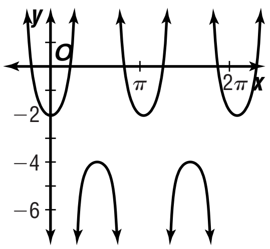


61. $r = \frac{6}{12}$ or 0.5
$\theta = \frac{5\pi}{8} - \frac{\pi}{2}$ or $\frac{\pi}{8}$
$0.5\left(\cos\frac{\pi}{8} + \boldsymbol{i}\sin\frac{\pi}{8}\right) \approx 0.46 + 0.19\boldsymbol{i}$

62. $\langle 2, -1, 3\rangle \cdot \langle 5, 3, 0\rangle = 2(5) + (-1)(3) + (3)(0)$
$= 7$

63. $x\cos 30° + y\sin 30° - 5 = 0$
$\frac{\sqrt{3}}{2}x + \frac{1}{2}y - 5 = 0$
$\sqrt{3}x + y - 10 = 0$

64.



65. Find A.
$A = 90° - 19° 32'$
$= 70° 28'$
Find a.
$\cos 19° 32' = \frac{a}{4.5}$
$4.2 \approx a$
Find b.
$\sin 19° 32' = \frac{b}{4.5}$
$1.5 \approx b$

66. discriminant $= (-3)^2 - 4(4)(2)$
$= -23$
Since the discriminant is negative, this indicates two imaginary roots.

67.

$$\begin{array}{r} x - 1 \\ x - 3\overline{)x^2 - 4x + 2} \\ -\underline{(x^2 - 3x)} \\ -x + 2 \\ -\underline{(-x + 3)} \\ -1 \end{array}$$

$y = x - 1$

68. $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -2 & 0 & 1 \\ 1 & 3 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 3 & -4 \\ 2 & 0 & -1 \end{bmatrix}$

$A'(1, 2)$, $B'(3, 0)$, $C'(-4, -1)$

69. $a - 4b = 15 \rightarrow a = 15 + 4b$

$$\begin{aligned} 4a - b &= 15 \\ 4(15 + 4b) - b &= 15 \\ 60 + 16b - b &= 15 \\ 15b &= -45 \\ b &= -3 \\ 4a - (-3) &= 15 \\ 4a &= 12 \\ a &= 3 \end{aligned}$$

$a - b = 3 - (-3)$ or 6

The correct choice is C.

12-2 Geometric Sequences and Series

Page 771 Check for Understanding

1. Both arithmetic and geometric sequences are recursive. Each term of an arithmetic sequence is the sum of a fixed difference and the previous term. Each term of a geometric sequence is the product of a common ratio and the previous term.

2. $a_n = (-3)^{1+1}$ or 9
$a_2 = (-3)^{2+1}$ or -27
$a_3 = (-3)^{3+1}$ or 81
The expression generates the following sequence: 9, -27, 81, $\cdots$. The common ratio is -3, therefore it is a geometric sequence.

3. If the first term in a geometric sequence were zero, then finding the common ratio would mean dividing by zero. Division by zero is undefined.

4. Sample answer: The first term of the series $5 - 10 + 20 - \cdots$ is 5 and the sum of the first 6 terms of the sequence is -105, but -105 is not greater than 5.

5a. No; the ratio between the first two terms is 2, but the ratio between the next two terms is 3.

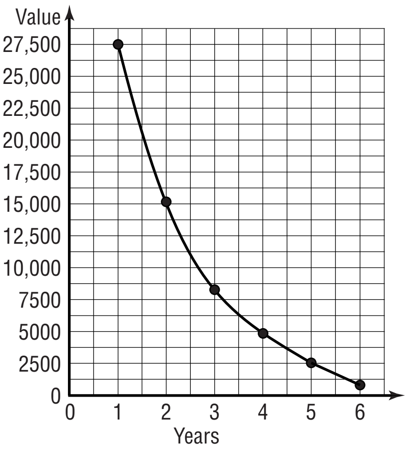
5b. Yes; the common ratio is $\sqrt{3}$.

5c. Yes; the common ratio is x.

6a.

Beginning of Year	Value of Computers
1	27,500.00
2	15,125.00
3	8318.75
4	4575.31
5	2516.42
6	1384.03

64b.



6c. an exponential function

7. $r = \dfrac{4}{\frac{2}{3}}$ or 6

$24(6) = 144$, $144(6) = 864$, $864(6) = 5184$
144, 864, 5184

8. $r = \frac{3}{2}$

$\frac{9}{2}\left(\frac{3}{2}\right) = \frac{27}{4}$, $\frac{27}{4}\left(\frac{3}{2}\right) = \frac{81}{8}$, $\frac{81}{8}\left(\frac{3}{2}\right) = \frac{243}{16}$

$\frac{27}{4}$, $\frac{81}{8}$, $\frac{243}{16}$

9. $r = \frac{-7.2}{1.8}$ or -4

$28.8(-4) = -115.2$, $-115.2(-4) = 460.8$,
$460.8(-4) = -1843.2$
-115.2, 460.8, -1843.2

10. $r = \frac{2.1}{7}$ or 0.3

$$\begin{aligned} a_n &= a_1 r^{n-1} \\ a_7 &= 7(0.3)^{7-1} \\ &= 0.005103 \end{aligned}$$

11.
$$\begin{aligned} a_n &= a_1 r^{n-1} \\ 24 &= a_1(2)^{5-1} \\ 24 &= 16a_1 \\ \tfrac{3}{2} &= a_1 \end{aligned}$$

12. $a_3 = \frac{2.5}{2}$ or 1.25

$a_2 = \frac{1.25}{2}$ or 0.625

$a_1 = \frac{0.825}{2}$ or 0.3125

0.3125, 0.625, 1.25

13.
$$\begin{aligned} a_n &= a_1 r^{n-1} \\ 27 &= 1(r)^{4-1} \\ 27 &= r^3 \\ 3 &= r \end{aligned}$$

$1(3) = 3$, $3(3) = 9$
1, 3, 9, 27

14. $r = \frac{-1}{0.5}$ or -2

$$\begin{aligned} S_n &= \frac{a_1 - a_1 r^n}{1 - r} \\ S_9 &= \frac{0.5 - 0.5(-2)^9}{1 - (-2)} \\ &= 85.5 \end{aligned}$$

15. $r = 1.035$
The value of the car after 10, 20, and 40 years will be the 11th, 21st, and 41st terms of the sequence respectively.
$a_{11} = 20{,}000(1.035)^{11-1}$
$\approx \$28{,}211.98$
$a_{21} = 20{,}000(1.035)^{21-1}$
$\approx \$39{,}795.78$
$a_{42} = 20{,}000(1.035)^{41-1}$
$\approx \$79{,}185.19$

Pages 771–773 Exercises

16. $r = \frac{2}{10}$ or 0.2
$0.4(0.2) = 0.08$, $0.08(0.2) = 0.016$,
$0.016(0.2) = 0.0032$
0.08, 0.016, 0.0032

17. $r = \frac{-20}{8}$ or -2.5
$50(-2.5) = -125$, $-125(-2.5) = 312.5$,
$312.5(-2.5) = -781.25$
-125, 312.5, -781.25

18. $r = \frac{\frac{2}{3}}{\frac{2}{9}}$ or 3
$2(3) = 6$, $6(3) = 18$, $18(3) = 54$
6, 18, 54

19. $r = \frac{\frac{3}{10}}{\frac{3}{4}}$ or $\frac{2}{5}$
$\frac{3}{25}\left(\frac{2}{5}\right) = \frac{6}{125}$, $\frac{6}{125}\left(\frac{2}{5}\right) = \frac{12}{625}$, $\frac{12}{625}\left(\frac{2}{5}\right) = \frac{24}{3125}$
$\frac{6}{125}$, $\frac{12}{625}$, $\frac{24}{3125}$

20. $r = \frac{3.5}{-7}$ or -0.5
$-1.75(-0.5) = 0.875$, $0.875(-0.5) = -0.4375$,
$-0.4375(-0.5) = 0.21875$
0.875, -0.4375, 0.21875

21. $r = \frac{6}{3\sqrt{2}}$ or $\sqrt{2}$
$6\sqrt{2}\left(\sqrt{2}\right) = 12$, $12\left(\sqrt{2}\right) = 12\sqrt{2}$, $12\sqrt{2}\left(\sqrt{2}\right) = 24$
12, $12\sqrt{2}$, 24

22. $r = \frac{3\sqrt{3}}{9}$ or $\frac{\sqrt{3}}{3}$
$3\left(\frac{\sqrt{3}}{3}\right) = \sqrt{3}$, $\sqrt{3}\left(\frac{\sqrt{3}}{3}\right) = 1$, $1\left(\frac{\sqrt{3}}{3}\right) = \frac{\sqrt{3}}{3}$
$\sqrt{3}$, 1, $\frac{\sqrt{3}}{3}$

23. $r = \frac{-1}{\boldsymbol{i}} = \frac{-1}{\sqrt{-1}}$ or $\boldsymbol{i}$
$-\boldsymbol{i}(\boldsymbol{i}) = 1$, $1(\boldsymbol{i}) = \boldsymbol{i}$, $\boldsymbol{i}(\boldsymbol{i}) = -1$
1, $\boldsymbol{i}$, -1

24. $r = \frac{t^5}{t^8}$ or t^{-3}
$t^2(t^{-3}) = t^{-1}$, $t^{-1}(t^{-3}) = t^{-4}$, $t^{-4}(t^{-3}) = t^{-7}$
t^{-1}, t^{-4}, t^{-7}

25. $\frac{a}{b^2}\left(\frac{b}{a^2}\right) = \frac{1}{ab}$, $\frac{1}{ab}\left(\frac{b}{a^2}\right) = \frac{1}{a^3}$, $\frac{1}{a^3}\left(\frac{b}{a^2}\right) = \frac{b}{a^5}$,
$\frac{b}{a^5}\left(\frac{b}{a^2}\right) = \frac{b^2}{a^7}$, $\frac{b^2}{a^7}\left(\frac{b}{a^2}\right) = \frac{b^3}{a^9}$
$\frac{1}{ab}$, $\frac{1}{a^3}$, $\frac{b}{a^5}$, $\frac{b^2}{a^7}$, $\frac{b^3}{a^9}$

26. $a_5 = 8\left(\frac{3}{2}\right)^{5-1}$
$= \frac{81}{2}$

27. $r = \frac{-\frac{3}{8}}{\frac{1}{2}}$ or $-\frac{3}{4}$
$a_6 = \frac{1}{2}\left(-\frac{3}{4}\right)^{6-1}$
$= -\frac{243}{2048}$

28. $r = \frac{0.4}{4.0}$ or 0.01
$a_7 = 40(0.01)^{7-1}$
$= 4 \times 10^{-11}$

29. $r = \frac{\sqrt{10}}{\sqrt{5}}$ or $\sqrt{2}$
$a_9 = \sqrt{5}\left(\sqrt{2}\right)^{9-1}$
$= 16\sqrt{5}$

30. $192 = a_1(4)^{6-1}$
$192 = 1024a_1$
$0.1875 = a_1$

31. $32\sqrt{2} = a_1(-\sqrt{2})^{5-1}$
$32\sqrt{2} = 4a_1$
$8\sqrt{2} = a_1$

32. $-6 = a_1\left(-\frac{1}{3}\right)^{5-1}$
$-6 = \frac{1}{81}a_1$
$-486 = a_1$
$a_2 = -486\left(-\frac{1}{3}\right)$ or 162
$a_3 = 162\left(-\frac{1}{3}\right)$ or -54
-486, 162, -54

33. $0.32 = a_1(0.2)^{5-1}$
$0.32 = 0.0016a_1$
$200 = a_1$
$a_2 = 200(0.2)$ or 40
$a_3 = 40(0.2)$ or 8
200, 40, 8

34. $81 = 256r^{5-1}$
$\frac{81}{256} = r^4$
$\pm\frac{3}{4} = r$
$256\left(\pm\frac{3}{4}\right) = \pm 192$, $192\left(\pm\frac{3}{4}\right) = \pm 144$,
$144\left(\pm\frac{3}{4}\right) = \pm 108$
256, ± 192, ± 144, ± 108, 81

35. $54 = -2r^{4-1}$
$-27 = r^3$
$-3 = r$
$-2(-3) = 6$, $6(-3) = -18$
-2, 6, -18, 54

36. $7 = \frac{4}{7}r^{3-1}$
$\frac{49}{4} = r^2$
$\pm\frac{7}{2} = r$
$\frac{4}{7}\left(\pm\frac{7}{2}\right) = \pm 2$
$\frac{4}{7}$, ± 2, 7

37. $r = \frac{5}{\frac{5}{3}}$ or 3

$S_5 = \frac{\frac{5}{3} - \frac{5}{3}(3)^5}{1-3}$

$= \frac{605}{3}$

38. $r = \frac{13}{65}$ or 0.2

$S_6 = 65 - \frac{65(0.2)^6}{1-0.2}$

$= 81.2448$

39. $r = \frac{-\frac{3}{2}}{1}$ or $-\frac{3}{2}$

$S_{10} = \frac{1 - 1\left(-\frac{3}{2}\right)^{10}}{1 - \left(-\frac{3}{2}\right)}$

$= \frac{-11{,}605}{512}$

40. $r = \frac{2\sqrt{3}}{2}$ or $\sqrt{3}$

$S_8 = \frac{2 - 2(\sqrt{3})^8}{1 - \sqrt{3}}$

$= \frac{-160}{1-\sqrt{3}}$

$= \frac{-160}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}}$

$= \frac{-160(1+\sqrt{3})}{-2}$

$= 80(1 + \sqrt{3})$

41a. The population doubles every half-hour, so $r = 2$. After 1 hour, the number of bacteria is the third term in the sequence and $n - 1 = 2$. After 2 hours, it is the fifth term and $n - 1 = 4$. After 3 hours, it is the seventh term and $n - 1 = 6$. After t hours it is the $2t + 1$ term and $n - 1 = 2t$.
$b_t = b_0 \cdot 2^{2t}$

41b. $b_t = 30 \cdot 2^{2(5)}$
$= 30{,}720$

41c. Sample answer: It is assumed that favorable conditions are maintained for the growth of the bacteria, such as an adequate food and oxygen supply, appropriate surrounding temperature, and adequate room for growth.

42a. $a_7 = a_4 r^3$
$12 = 4r^3$
$3 = r^3$
$\sqrt[3]{3} = r$

42b. $a_4 = a_1 r^{4-1}$
$4 = a_1\left(\sqrt[3]{3}\right)^3$
$4 = 3a_1$
$\frac{4}{3} = a_1$
$a_{28} = \frac{4}{3}\left(\sqrt[3]{3}\right)^{28-1}$
$= 26{,}244$

43a. $r = \frac{5.50}{5}$ or 1.1
$a_{10} = 5(1.1)^{10-1}$
$\approx \$11.79$
$a_{20} = 5(1.1)^{20-1}$
$\approx \$30.58$
$a_{10} = 5(1.1)^{40-1}$
$\approx \$205.72$
\$11.79, \$30.58, \$205.72

43b. $S_{52} = \frac{5 - 5(1.1)^{52}}{1 - 1.1}$
$\approx \$7052.15$

43c. Each payment made is rounded to the nearest penny, so the sum of the payments will actually be more than the sum found in b.

44a. $\frac{\frac{1}{5} + \frac{1}{8}}{2} = \frac{1}{x}$
$\frac{13}{80} = \frac{1}{x}$
$\frac{80}{13} = x$

44b. $\frac{1}{8} = \frac{\frac{1}{20} + \frac{1}{z}}{2}$
$\frac{1}{4} = \frac{1}{20} + \frac{1}{z}$
$\frac{1}{5} = \frac{1}{z}$
$5 = z$

45. $a_2 = -3(a_1)$ *by definition*
$= -3(-2)$
$= 6$
$r = \frac{a_2}{a_1} = -3$
Then $a_n = a_1 r^{n-1}$
So, $a_n = (-2)(-3)^{n-1}$

46. $a_1 = 1$, $r = 2.5$, $n = 15$
$a_{15} = 1(2.5)^{15-1}$
$= 372{,}529$

47a. $25\left(1 + \frac{0.024}{12}\right) = \$25.05$

47b. No; at the end of two years, she will have only \$615.23 in her account.

$S_{24} = \frac{25.05 - 25.05\left(1 + \frac{0.024}{12}\right)^{24}}{1 - \left(1 + \frac{0.024}{12}\right)}$

$\approx \$615.23$

47c. $750 \le \frac{a_1 - a_1\left(1 + \frac{0.024}{12}\right)^{24}}{1 - \left(1 + \frac{0.24}{12}\right)}$

$-1.5 \ge a_1\left(1 - \left(1 + \frac{0.024}{12}\right)^{24}\right)$
$\$30.54 \le a_1$
$a_0\left(1 + \frac{0.024}{12}\right) \ge \$30.54$
$a_0 \ge \$30.48$
The least monthly deposit is \$30.48.

48. $r = \frac{a_2}{a_1}$

$r = \frac{\frac{1}{27}}{\frac{1}{81}} = 3$

$a_n = a_1 r^{n-1}$

$6561 = \frac{1}{81} \cdot 3^{n-1}$

$(6561)(81) = 3^{n-1}$

$(3^8)(3^4) = 3^{n-1}$

$3^{12} = 3^{n-1}$

$12 = n - 1$

$13 = n$

6561 is the 13th term of the sequence.

49. $S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$650 = \frac{n}{2}[2(20) + (n-1)5]$

$1300 = 40n + 5n^2 - 5n$

$0 = 5n^2 + 35n - 1300$

$0 = 5(n-13)(n+20)$

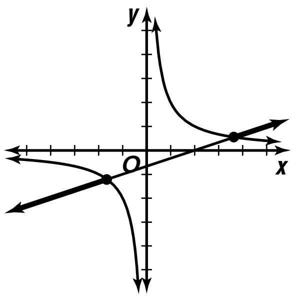
$n = 13$ or $n = -20$

Since n cannot be negative, $n = 13$ weeks.

50. $\log_{11} 265 = \frac{\log 265}{\log_{11}}$

≈ 2.3269

51.



52. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{3^2 + (-5^2)}$

$= \pm\sqrt{34}$

Since C is positive, use $-\sqrt{34}$.

$-\frac{3}{\sqrt{34}}x + \frac{5}{\sqrt{34}}y - \frac{5}{\sqrt{34}} = 0$

$p = \frac{5}{\sqrt{34}}$ or $\frac{5\sqrt{34}}{34}$, $\cos\phi = -\frac{3}{\sqrt{34}}$, $\sin\phi = \frac{5}{\sqrt{34}}$

$\tan\phi = \frac{\frac{5}{\sqrt{34}}}{-\frac{3}{\sqrt{34}}}$

$\tan\phi = -\frac{5}{3}$

$\phi \approx -59°$

Since cosine is negative and sine is positive, $\phi = -59° + 180°$ or $121°$.

$p = r\cos(\theta - \phi)$

$\frac{5\sqrt{34}}{34} = r\cos(\theta - 121°)$

53. $3x + 4y = 5$

$y = -\frac{3}{4}x + \frac{5}{4}$

$x = t,\ y = -\frac{3}{4}t + \frac{5}{4}$

54. $\csc\theta = 3$

$\frac{1}{\sin\theta} = 3$

$\frac{1}{3} = \sin\theta$

55. Find the amplitude.

$A = \frac{86 - 36}{2}$ or 25

Find h.

$h = \frac{86 + 36}{2}$ or 61

Find k.

$\frac{2\pi}{k} = 4$

$k = \frac{\pi}{2}$

$y = 25\sin\left(\frac{\pi}{2}t + c\right) + 61$

$36 = 25\sin\left(\frac{\pi}{2} \cdot 1 + c\right) + 61$

$-1 = \sin\left(\frac{\pi}{2} + c\right)$

$-\frac{\pi}{2} = \frac{\pi}{2} + c$

$-\pi = c$

$y = 25\sin\left(\frac{\pi}{2}t - 3.14\right) + 61$

56. Since $43° < 90°$, consider the following.

$b\sin A = 20\sin 43°$

≈ 13.64

Since $11 < 13.64$, no triangle exists.

57. $-(n^2) \le -\sqrt{49}$

$n^2 \ge 7$

Solution set: $\{n \mid n \le -3 \text{ or } n \ge 3\}$

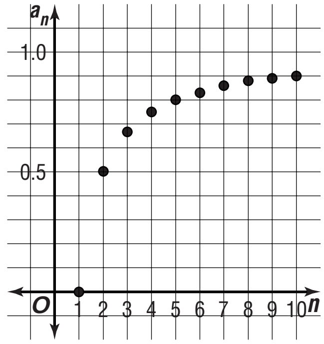
n	$m = n + 1$	nm	
-5	$-5 + 1$ or -4	$(-5)(-4)$ or 20	
-4	$-4 + 1$ or -3	$(-4)(-3)$ or 12	
-3	$-3 + 1$ or -2	$(-3)(-2)$ or 6	← least possible value
3	$3 + 1$ or 4	$3(4)$ or 12	
4	$4 + 1$ or 5	$4(5)$ or 20	
5	$5 + 1$ or 6	$5(6)$ or 30	

The answer is 6.

12-3 Infinite Sequences and Series

Pages 780–781 Check for Understanding

1a.



1b. The value of a_n approaches 1 as the value of n increases.

1c. $\lim_{n\to\infty} \frac{n-1}{n} = 1$

1d. $\lim\limits_{n\to\infty} \frac{n-1}{n} = \lim\limits_{n\to\infty} \frac{n}{n} - \lim\limits_{n\to\infty} \frac{1}{n}$
$= 1 - 0$
$= 1$

The limits are equal.

2a. See students' work. Student's should draw the following conclusions:

$\lim\limits_{n\to\infty}\left(\frac{1}{2}\right)^n = 0$

$\lim\limits_{n\to\infty}\left(\frac{1}{4}\right)^n = 0$

$\lim\limits_{n\to\infty}(1)^n = 1$

$\lim\limits_{n\to\infty}(2)^n =$ no limit

$\lim\limits_{n\to\infty}(5)^n =$ no limit

2b. If $|r| < 1$, then $\lim\limits_{n\to\infty} r^n = 0$. If $|r| = 1$, then $\lim\limits_{n\to\infty} r^n = 1$. If $|r| > 1$ then $\lim\limits_{n\to\infty} r^n$ does not exist.

3. Sample answer: $2 + 4 + 8 + \cdots$

4. Zonta is correct. As n approaches infinity the expression $2n - 3$ will continue to grow larger and larger. Tyree applied the method of dividing by the highest powered term incorrectly. Both the numerator and the denominator of the expression must be divided by the highest-powered term. It is not appropriate to apply this method here since the denominator of the expression $2n - 3$ is 1.

5. 0; as $n \to \infty$, 5^n becomes increasingly large and thus the value $\frac{1}{5^n}$ becomes smaller and smaller, approaching zero. So the sequence has a limit of zero.

6. $\lim\limits_{n\to\infty} \frac{5-n^2}{2n} = \lim\limits_{n\to\infty}\left(\frac{5}{2n} - \frac{1}{2}n\right)$

$\lim\limits_{n\to\infty} \frac{5}{2n} = \lim\limits_{n\to\infty} \frac{5}{2} \cdot \frac{1}{n} = \frac{5}{2} \cdot 0 = 0$

As n approaches infinity, $\frac{1}{2}n$ becomes increasingly large, so the sequence has no limit.

7. $\frac{3}{7}$, $\lim\limits_{n\to\infty} \frac{3n-6}{7n} = \lim\limits_{n\to\infty}\left(\frac{3}{7} - \frac{6}{7} \cdot \frac{1}{n}\right)$
$= \lim\limits_{n\to\infty} \frac{3}{7} - \lim\limits_{n\to\infty} \frac{6}{7} \cdot \lim\limits_{n\to\infty} \frac{1}{n}$
$= \frac{3}{7} - \frac{6}{7} \cdot 0$ or $\frac{3}{7}$

8. $0.\overline{7} = \frac{7}{10} + \frac{7}{100} + \cdots$

$a_1 = \frac{7}{10}$, $r = \frac{1}{10}$

$S_n = \frac{\frac{7}{10}}{1 - \frac{1}{10}}$

$= \frac{7}{9}$

9. $5.\overline{126} = 5 + \frac{126}{1000} + \frac{126}{1{,}000{,}000} + \cdots$

$a_1 = \frac{126}{1000}$, $r = \frac{1}{1000}$

$S_n = 5 + \frac{\frac{126}{1000}}{1 - \frac{1}{1000}}$

$= 5 + \frac{126}{999}$

$= 5\frac{14}{11}$

10. $r = \frac{3}{-6}$ or $-\frac{1}{2}$

$S_n = \frac{-6}{1 - \left(-\frac{1}{2}\right)}$

$= -4$

11. $r = \frac{\frac{1}{4}}{\frac{3}{4}}$ or $\frac{1}{3}$

$S_n = \frac{\frac{3}{4}}{1 - \frac{1}{3}}$

$= 1\frac{1}{8}$

12. $r = \frac{3}{\sqrt{3}}$ or $\sqrt{3}$

The sum does not exist since $|r| = |\sqrt{3}| > 1$.

13. $a_1 = 75$, $r = \frac{2}{5}$

$S_n = \frac{75}{1 - \frac{2}{5}}$

$= 125$ m

Pages 781–783 Exercises

14. $\lim\limits_{n\to\infty} \frac{7-2n}{5n} = \lim\limits_{n\to\infty}\left(\frac{7}{5} \cdot \frac{1}{n} - \frac{2}{5}\right)$
$= \lim\limits_{n\to\infty} \frac{7}{5} \cdot \lim\limits_{n\to\infty} \frac{1}{n} - \lim\limits_{n\to\infty} \frac{2}{5}$
$= \frac{7}{5} \cdot 0 - \frac{2}{5}$ or $-\frac{2}{5}$

15. $\lim\limits_{n\to\infty} \frac{n^3-2}{n^2} = \lim\limits_{n\to\infty}\left(n - \frac{2}{n}\right)$.

$\lim\limits_{n\to\infty} \frac{2}{\text{n}} = \lim\limits_{n\to\infty} 2 \cdot \frac{1}{n} = 2 \cdot 0$ or 0, but as n approaches infinity, n becomes increasingly large, so the sequence has no limit.

16. $\lim\limits_{n\to\infty} \frac{6n^2+5}{3n^2} = \lim\limits_{n\to\infty}\left(\frac{6}{3} + \frac{5}{3n^2}\right)$
$= \lim\limits_{n\to\infty} 2 + \lim\limits_{n\to\infty} \frac{5}{3} \cdot \lim\limits_{n\to\infty} \frac{1}{n^2}$
$= 2 + \frac{5}{3} \cdot 0$ or 2

17. $\lim\limits_{n\to\infty} \frac{9n^3+5n-2}{2n^3} = \lim\limits_{n\to\infty}\left(\frac{9}{2} + \frac{5}{2n^2} - \frac{1}{n^3}\right)$
$= \lim\limits_{n\to\infty} \frac{9}{2} + \lim\limits_{n\to\infty} \frac{5}{2} \cdot \lim\limits_{n\to\infty} \frac{1}{n^2} - \lim\limits_{n\to\infty} \frac{1}{n^3}$
$= \frac{9}{2} + \frac{5}{2} \cdot 0 - 0$ or $\frac{9}{2}$

18. $\lim\limits_{n\to\infty} \frac{(3n+4)(1-n)}{n^2} = \lim\limits_{n\to\infty} \frac{-3n^2-n+4}{n^2}$
$= \lim\limits_{n\to\infty}\left(-3 - \frac{1}{n} + \frac{4}{n^2}\right)$
$= \lim\limits_{n\to\infty}(-3) - \lim\limits_{n\to\infty} \frac{1}{n} + \lim\limits_{n\to\infty} 4 \cdot \lim\limits_{n\to\infty} \frac{1}{n^2}$
$= -3 - 0 + 4 \cdot 0$ or -3

19. Dividing by the highest powered term, n^2, we find $\lim\limits_{n\to\infty} \frac{8 + \frac{5}{n} + \frac{2}{n^2}}{\frac{3}{n^2} + \frac{2}{n}}$ which as n approaches infinity simplifies to $\frac{8+0+0}{0+0} = \frac{8}{0}$. Since this fraction is undefined, the limit does not exist.

20. $\lim_{n\to\infty} \frac{4-3n+n^2}{2n^3-3n^2+5} = \lim_{n\to\infty} \frac{\frac{4}{n^3}-\frac{3n}{n^3}+\frac{n^2}{n^3}}{\frac{2n^3}{n^3}-\frac{3n^2}{n^3}+\frac{5}{n^3}}$

$= \lim_{n\to\infty} \frac{\frac{4}{n^3}-\frac{3}{n^2}+\frac{1}{n}}{2-\frac{3}{n}+\frac{5}{n^3}}$

$= \frac{\lim_{n\to\infty} 4 \cdot \lim_{n\to\infty} \frac{1}{n^3} - \lim_{n\to\infty} 3 \cdot \lim_{n\to\infty} \frac{1}{n^2} + \lim_{n\to\infty} \frac{1}{n}}{\lim_{n\to\infty} 2 - \lim_{n\to\infty} 3 \cdot \lim_{n\to\infty} \frac{1}{n} + \lim_{n\to\infty} 5 \cdot \lim_{n\to\infty} \frac{1}{n^3}}$

$= \frac{4\cdot 0 - 3\cdot 0 + 0}{2 - 3\cdot 0 + 5\cdot 0}$ or 0

21. As $n \to \infty$, 3^n becomes increasingly large and thus the value $\frac{1}{3^n}$ becomes smaller and smaller, approaching zero. So the sequence has a limit of zero.

22. Dividing by the highest powered term, n, we find

$\lim_{n\to\infty} \frac{\frac{(-2)^n}{n}}{\frac{4}{n}+1} = \lim_{n\to\infty}\left(\frac{(-2)^n}{n} \cdot \frac{1}{\frac{4}{n}+1}\right)$

which as n approaches infinity

$\lim_{n\to\infty} \frac{1}{\frac{4}{n}+1} = 1$, but $\lim_{n\to\infty} \frac{(-2)^n}{n}$ has no limit since $|-2| > 1$.

23. $\lim_{n\to\infty} \frac{5n+(-1)^n}{n^2} = \lim_{n\to\infty} \frac{5n}{n^2} + \lim_{n\to\infty} \frac{(-1)^n}{n^2}$

$= \lim_{n\to\infty} \frac{5}{n} + \lim_{n\to\infty} \frac{(-1)^n}{n^2}$

$= \lim_{n\to\infty} \frac{(-1)^n}{n^2}$

As n increases, the value of the numerator alternates between -1 and 1. As n approaches infinity, the value of the denominator becomes increasingly large, causing the value of the fraction to become increasingly small. Thus, the terms of the sequence alternate between smaller and smaller positive and negative values, approaching zero. So the sequence has a limit of zero.

24. $0.\overline{4} = \frac{4}{10} + \frac{4}{100} + \ldots$

$a_1 = \frac{4}{10}, r = \frac{1}{10}$

$S_n = \frac{\frac{4}{10}}{1-\frac{1}{10}}$

$= \frac{4}{9}$

25. $0.\overline{51} = \frac{51}{100} + \frac{51}{10,000} + \ldots$

$a_1 = \frac{51}{100}, r = \frac{1}{100}$

$S_n = \frac{\frac{51}{100}}{1-\frac{1}{100}}$

$= \frac{51}{99}$ or $\frac{17}{33}$

26. $0.\overline{370} = \frac{370}{1000} + \frac{370}{1,000,000} + \ldots$

$a_1 = \frac{370}{1000}, r = \frac{1}{1000}$

$S_n = \frac{\frac{370}{1000}}{1-\frac{1}{1000}}$

$= \frac{370}{999}$ or $\frac{10}{27}$

27. $6.\overline{259} = 6 + \frac{259}{1000} + \frac{259}{1,000,000} + \ldots$

$a_1 = \frac{259}{1000}, r = \frac{1}{1000}$

$S_n = 6 + \frac{\frac{259}{1000}}{1-\frac{1}{1000}}$

$= 6 + \frac{259}{999}$

$= 6\frac{7}{27}$

28. $0.\overline{15} = \frac{15}{100} + \frac{15}{10,000} + \ldots$

$a_1 = \frac{15}{100}, r = \frac{1}{100}$

$S_n = \frac{\frac{15}{100}}{1-\frac{1}{100}}$

$= \frac{15}{99}$ or $\frac{5}{33}$

29. $0.2\overline{63} = \frac{2}{10} + \frac{63}{1000} + \frac{63}{100,000} + \ldots$

$a_1 = \frac{63}{1000}, r = \frac{1}{100}$

$S_n = \frac{1}{5} + \frac{\frac{63}{1000}}{1-\frac{1}{100}}$

$= \frac{1}{5} + \frac{63}{990}$

$= \frac{29}{110}$

30. The series is geometric, having a common ratio of 0.1. Since this ratio is less than 1, the sum of the series exists and is $\frac{2}{9}$.

31. $r = \frac{12}{16}$ or $\frac{3}{4}$

$S_n = \frac{16}{1-\frac{3}{4}}$

$= 64$

32. $r = \frac{7.5}{5}$ or 1.5

This series is geometric with a common ratio of 1.5. Since this ratio is greater than 1, the sum of the series does not exist.

33. $r = \frac{5}{10}$ or $\frac{1}{2}$

$S_n = \frac{10}{1-\frac{1}{2}}$

$= 20$

34. The series is arithmetic, having a general term of $7 - n$. Since $\lim_{n\to\infty} 7 - n$ does not equal zero, this series has no sum.

35. $r = \frac{\frac{1}{4}}{\frac{1}{8}}$ or 2

This series is geometric with a common ratio of 2. Since this ratio is greater than 1, the sum of the series does not exist.

36. $r = \dfrac{\frac{1}{9}}{-\frac{2}{3}}$ or $-\frac{1}{6}$

$S_n = \dfrac{-\frac{2}{3}}{1 - \left(-\frac{1}{6}\right)}$

$= -\frac{4}{7}$

37. $r = \dfrac{\frac{4}{5}}{\frac{6}{5}}$ or $\frac{2}{3}$

$S_n = \dfrac{\frac{6}{5}}{1 - \frac{2}{3}}$

$= 3\frac{3}{5}$

38. $r = \frac{1}{\sqrt{5}}$ or $\frac{\sqrt{5}}{5}$

$S_n = \dfrac{\sqrt{5}}{1 - \frac{\sqrt{5}}{5}}$

$= \dfrac{\sqrt{5}}{1 - \frac{\sqrt{5}}{5}} \cdot \left(\dfrac{1 + \frac{\sqrt{5}}{5}}{1 + \frac{\sqrt{5}}{5}}\right)$

$= \dfrac{\sqrt{5} + 1}{1 - \frac{5}{25}}$

$= \frac{4}{5}(\sqrt{5} + 1)$

39. $r = -\frac{4\sqrt{3}}{8}$ or $-\frac{\sqrt{3}}{2}$

$S_n = \dfrac{8}{1 - \left(-\frac{\sqrt{3}}{2}\right)}$

$= \dfrac{8}{1 + \frac{\sqrt{3}}{2}} \cdot \dfrac{\left(1 - \frac{\sqrt{3}}{2}\right)}{\left(1 - \frac{\sqrt{3}}{2}\right)}$

$= \dfrac{8\left(1 - \frac{\sqrt{3}}{2}\right)}{1 - \frac{3}{4}}$

$= 32\left(1 - \frac{\sqrt{3}}{2}\right)$

$= 32 - 16\sqrt{3}$

40a. $a_1 = 35\ r = \frac{2}{5}$

$a_2 = 35\left(\frac{2}{5}\right)$ or 14

$a_3 = 14$

$a_4 = 14\left(\frac{2}{5}\right)$ or 5.6

$a_5 = 5.6$

35, 14, 14, 5.6, 5.6

40b. $S_n = 35 + \dfrac{14}{1 - \frac{2}{5}} + \dfrac{14}{1 - \frac{2}{5}}$

$= 35 + \frac{20}{3} + \frac{70}{3}$

$= 81\frac{2}{3}$ m or about 82 m

41a. The limit of a difference equals the difference of the limits only if the two limits exist. Since neither $\lim\limits_{n\to\infty} \frac{n^2}{2n + 1}$ nor $\lim\limits_{n\to\infty} \frac{n^2}{2n - 1}$ exists, this property of limits does not apply.

41b. $\lim\limits_{n\to\infty} \left(\dfrac{n^2}{2n + 1} - \dfrac{n^2}{2n - 1}\right) = \lim\limits_{n\to\infty}\left[\dfrac{n^2(2n - 1) - n^2(2n + 1)}{(2n + 1)(2n - 1)}\right]$

$= \lim\limits_{n\to\infty} \dfrac{2n^3 - n^2 - 2n^3 - n^2}{4n^2 - 1}$

$= \lim\limits_{n\to\infty} \dfrac{-2n^2}{4n^2 - 1}$

$= \lim\limits_{n\to\infty} \dfrac{\frac{-2n^2}{n^2}}{\frac{4n^2}{n^2} - \frac{1}{n^2}}$

$= \lim\limits_{n\to\infty} \dfrac{-2}{4 - \frac{1}{n^2}}$

$= -\frac{1}{2}$

42a. $12 \div 4 = 3$

$r = \sqrt[3]{3}$

$a_4 = a_1 r^{4-1}$

$4 = a_1(\sqrt[3]{3})^3$

$4 = a_1(3)$

$\frac{4}{3} = a_1$

42b. $a_{28} = a_1 r^{28-1}$

$= \frac{4}{3}(\sqrt[3]{3})^{27}$

$= \frac{4}{3}(3^9)$

$= 4(3^8)$

$= 26{,}244$

43. No; if n is even, $\lim\limits_{n\to\infty} \cos \frac{n\pi}{2} = \frac{1}{2}$, but if n is odd, $\lim\limits_{n\to\infty} \cos \frac{n\pi}{2} = -\frac{1}{2}$.

44a. After 2 hours, $\frac{1}{2}D$ exists. After 4 hours, $\frac{1}{2} \cdot \frac{1}{2}D$ or $\frac{1}{4}D$ exists. After 6 hours and before the second dose, $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}D$ or $\frac{1}{8}D$ exists.

44b. $a_1 = D$, $r = \frac{1}{8}$

$S_n = D\ \dfrac{\left[1 - \left(\frac{1}{8}\right)^n\right]}{1 - \frac{1}{8}}$

$= \frac{8}{7}D\left[1 - \left(\frac{1}{8}\right)^n\right]$

44c. $\lim\limits_{n\to\infty} S_n = S$

$S = \frac{a_1}{1 - r}$

$= \dfrac{D}{1 - \frac{1}{8}}$

$= \frac{8}{7}D$

44d. $350 \geq \frac{8}{7}D$

$306.25 \geq D$

The largest possible dose is 306.25 mg.

45a. A side of the original square measures $\frac{20}{4}$ or 5 feet. Half of 5 feet is $\frac{5}{2}$ feet.

$\left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^2 = s^2$

$\frac{50}{4} = s^2$

$\frac{5\sqrt{2}}{2} = s$

Perimeter $= 4 \cdot \frac{5\sqrt{2}}{2}$ or $10\sqrt{2}$ feet.

45b. $a_1 = 20$, $r = \frac{10\sqrt{2}}{20}$ or $\frac{\sqrt{2}}{2}$

$$S = \frac{20}{1 - \frac{\sqrt{2}}{2}}$$

$$= \frac{20}{1 - \frac{\sqrt{2}}{2}} \cdot \frac{\left(1 + \frac{\sqrt{2}}{2}\right)}{\left(1 + \frac{\sqrt{2}}{2}\right)}$$

$$= \frac{20 + 10\sqrt{2}}{1 - \frac{2}{4}}$$

$40 + 20\sqrt{2}$ ft or about 68 ft

46a.

n	$17.3032181(0.864605)^n$
1	15.0
2	12.9
3	11.2
4	9.7
5	8.4
6	7.2
7	6.2
8	5.4
9	4.7
10	4.0

15.0, 12.9, 11.2, 9.7, 8.4, 7.2, 6.2, 5.4, 4.7, 4.0

46b. The 2000–2001 school year corresponds to the 9th term of the sequence, 4.7. The model is 0.3 below the actual statistic.

46c. The 2006–2007 school year would correspond to the 15th term of the sequence.
$17.3032181(0.864605)^{15} \approx 2.0$

46d. Yes; as $n \to \infty$, $17.3032181(0.864605)^n \to 0$.

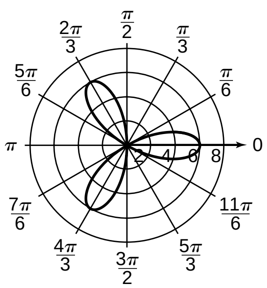
46e. No, the number of students per computer must be greater than zero.

47. $-3\left(\frac{2}{3}\right) = -2$, $-2\left(\frac{2}{3}\right) = -1\frac{1}{3}$, $-1\frac{1}{3}\left(\frac{2}{3}\right) = -\frac{8}{9}$,
$-\frac{8}{9}\left(\frac{2}{3}\right) = -\frac{16}{27}$
$-2, -1\frac{1}{3}, -\frac{8}{9}, -\frac{16}{27}$

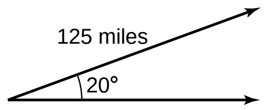
48. $a_{16} = 1.5 + (16 - 1)0.5$
$= 9$

49. $x^2 - 4y^2 - 12x - 16y = -16$
$(x - 6)^2 - 4(y + 2)^2 = 4$
$\frac{(x-6)^2}{4} - \frac{(y+2)^2}{1} = 1$
$h = 6, k = -2, a = 2, b = 1, c = \sqrt{5}$
center: $(6, -2)$
foci: $(6 \pm \sqrt{5}, -2)$
vertices: $(8, -2)$ and $(4, -2)$

50.



51. $v_y = 125 \sin 20°$
$= 42.75$ miles
$v_x = 125 \cos 20°$
≈ 117.46 miles



52. $\cos 112.5° = \cos\left(\frac{225°}{2}\right)$

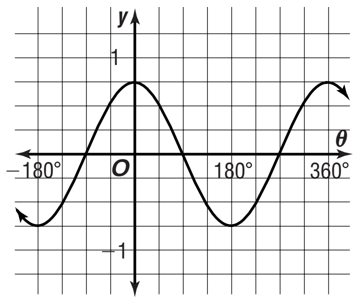
$$= -\sqrt{\frac{1 + \cos 225°}{2}}$$

$$= -\sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}}$$

$$= -\sqrt{\frac{2 - \sqrt{2}}{4}}$$

$$= -\frac{\sqrt{2 - \sqrt{2}}}{2}$$

53.



54. possible values of p: $\pm1, \pm2$
possible values of q: $\pm1, \pm2, \pm4, \pm8$
possible rational zeros, $\frac{p}{q}$: $\pm1, \pm2, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{1}{8}$

55. If $b = 1$, then $4b + 26 = 30$ which is divisible by 2, 5, and 6.
If $b = 11$, then $4b + 26 = 70$, which is divisible by 7.
$4b + 26$ is not divisible by 4 since 4 divides $4b$ evenly, but does not divide 26 evenly. The correct choice is B.

12-3B Graphing Calculator Exploration: Continued Fractions

Page 784

1. 1.618181818

2. $N = 24$

3.
$$x = 1 + \frac{1}{x}$$
$$x^2 = x + 1$$
$$x^2 - x - 1 = 0$$
$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$
$$x = \frac{1 + \sqrt{5}}{2}$$

Since the sum $1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}$ cannot be negative, the value of $1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}$ is $\frac{1 + \sqrt{5}}{2}$.

4. Let $x = 3 + \cfrac{1}{3 + \frac{1}{3}} \cdots$, then $x = 3 + \frac{1}{x}$.

Solve for x.

$$x = 3 + \frac{1}{x}$$
$$x^2 = 3x + 1$$
$$x^2 - 3x - 1 = 0$$
$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)}$$
$$= \frac{3 \pm \sqrt{13}}{2}$$

Since the sum of positive numbers must remain positive, $x = \frac{3 + \sqrt{13}}{2}$.

5. The output and the decimal approximation are equal.

6. Let $x = A + \cfrac{1}{A + \frac{1}{A}} \cdots$, then $x = A + \frac{1}{x}$.

Solve for x.

$$x = A + \frac{1}{x}$$
$$x^2 = Ax + 1$$
$$x^2 - Ax - 1 = 0$$
$$x = \frac{A \pm \sqrt{(-A)^2 - 4(1)(-1)}}{2(1)}$$
$$= \frac{A \pm \sqrt{A^2 + 4}}{2}$$

Since the sum of positive numbers must remain positive, $x = \frac{A + \sqrt{A^2 + 4}}{2}$

7. Sample program:

```
Program: CCFRAC
:Prompt A :Prompt B
:Disp "INPUT TERM"
:Disp "NUMBER N, N ≥ 3"
:Prompt N
:1 → K
:B + 1/(2A) → C
:Lbl 1
:B + 1/(2A + C) → C
:K + 1 → K
:If K < N − 1
:Then: Goto 1
:Else: Disp C + A
```

8. For large values of N, the program output and the decimal approximation of $\sqrt{A^2 + B}$ are equal.

9.
$$x + A = 2A + \cfrac{B}{2A + \frac{B}{2A + \cdots}}$$
$$x + A = 2A + \frac{B}{x + A}$$
$$(x + A)^2 = 2A(x + A) + B$$
$$x^2 + 2Ax + A^2 = 2Ax + 2A^2 + B$$
$$x^2 = A^2 + B$$
$$x = \pm\sqrt{A^2 + B}$$

Since the sum $A + \cfrac{B}{2A + \frac{B}{2A + \cdots}}$ cannot be negative, the value of $A + \cfrac{B}{2A + \frac{B}{2A + \cdots}}$ is $\sqrt{A^2 + B}$.

10. They will be opposites.

11. Answers will vary. Sample answers: $A = 1$, $B = 14$; $A = 4$, $B = -1$.

$A^2 + B$: If $A = 1$ and $B = 14$, $\sqrt{1^2 + 14} = \sqrt{15}$.

If $A = 4$ and $B = -1$, $\sqrt{4^2 + (1)} = 15$.

12-4 Convergent and Divergent Series

Pages 790–791 Check for Understanding

1a. See students' work.

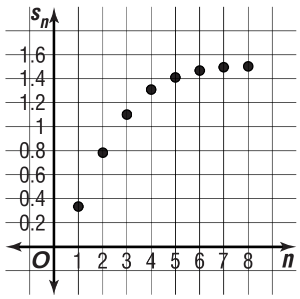
1b. See students' work.

1c. See students' work.

1d. In a given trigonometric series where $|r| > 1$, each succeeding term is larger than the one preceding it. Therefore, the series approaches ∞ and thus does not converge.

2. As $n \to \infty$, $S \to 6$.

3a.



3b. convergent

3c. $\frac{n^2}{3^n}$

3d. We can use the ratio test to determine whether the series is indeed convergent.

$a_n = \frac{n^2}{3^n}$ and $a_{n+1} = \frac{(n+1)^2}{3^{n+1}}$

$$r = \lim_{n\to\infty} \frac{\frac{(n+1)^2}{3^{n+1}}}{\frac{n^2}{3^n}}$$
$$r = \lim_{n\to\infty} \frac{(n+1)^2}{3^{n+1}} \cdot \frac{3^n}{n^2}$$
$$r = \lim_{n\to\infty} \frac{n^2 + 2n + 1}{3n^2}$$
$$r = \lim_{n\to\infty} \frac{1 + \frac{2}{n} + \frac{1}{n}}{3}$$

$r = \frac{1}{3}$ Since $r < 1$, the series is convergent.

4. Consider the infinite series a_n

1. If the $\lim_{n\to\infty} a_n \neq 0$, the sum of the series does not exist, and thus the series is divergent. If the $\lim_{n\to\infty} a_n = 0$, the sum may or may not exit and therefore it cannot be determined from this test if the series is convergent or divergent.
2. If the series is arithmetic then it is divergent.
3. If the series is geometric then the series converges for $|r| < 1$ and diverges for $|r| \geq 1$.
4. Ratio test: the series converges for $r < 1$ and diverges for $r > 1$. If $r = 1$, the test fails. This test can only be used if all the terms of the

series are positive and if the series can be expressed in general form.

5. Comparison test: may only be used if all the terms in the series are positive.

5. $a_n = \frac{n}{2^n}$, $a_{n+1} = \frac{n+1}{2^{n+1}}$

$$r = \lim_{n\to\infty} \frac{a_{n+1}}{a^n}$$
$$= \lim_{n\to\infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}}$$
$$= \lim_{n\to\infty} \frac{(n+1)2^n}{n2^{n+1}}$$
$$= \lim_{n\to\infty} \frac{(n+1)}{2n}$$
$$= \lim_{n\to\infty} \left(\frac{1}{2} + \frac{1}{2n}\right)$$
$$= \frac{1}{2}$$

convergent

6. $a_n = \frac{4n-1}{4n}$, $a_{n+1} = \frac{4(n+1)-1}{4(n+1)}$

$$r = \lim_{n\to\infty} \frac{\frac{4(n+1)-1}{4(n+1)}}{\frac{4n-1}{4n}}$$
$$= \lim_{n\to\infty} \frac{(4n+3)(4n)}{(4n+4)(4n-1)}$$
$$= \lim_{n\to\infty} \frac{16n^2+12n}{16n^2+12n-4}$$
$$= \lim_{n\to\infty} \frac{\frac{16n^2}{n^2} + \frac{12n}{n^2}}{\frac{16n^2}{n^2} + \frac{12n}{n^2} - \frac{4}{n^2}}$$
$$= \frac{16}{16} \text{ or } 1$$

test provides no information

7. The general term is $\frac{(n+1)}{n}$.

$\frac{n+1}{n} > \frac{1}{n}$ for all n, so divergent

8. The series is arithmetic, so it is divergent.

9. The general term is $\frac{1}{2+n^2}$.

$\frac{1}{2+n^2} \le \frac{1}{n}$ for all n, so convergent

10. $a_n = \frac{1}{n \cdot 2^n}$, $a_{n+1} = \frac{1}{(n+1)2^{n+1}}$

$$r = \frac{\frac{1}{(n+1)2^{n+1}}}{\frac{1}{n2^n}}$$
$$= \lim_{n\to\infty} \frac{n2^n}{(n+1)2^{n+1}}$$
$$= \lim_{n\to\infty} \frac{n}{2(n+1)}$$
$$= \lim_{n\to\infty} \frac{\frac{n}{n}}{2\left(\frac{n}{n} + \frac{1}{n}\right)}$$
$$= \frac{1}{2(1+0)}$$
$$= \frac{1}{2}$$

convergent

11. The series is geometric where $r = \frac{3}{4}$.

Since $\frac{3}{4} < 1$, it is convergent.

12a. The series is geometric where $r = \frac{900}{1500}$ or $\frac{3}{5}$.

$$S_{10} = \frac{1500 - 1500\left(\frac{3}{5}\right)^{10}}{1 - \frac{3}{5}}$$
$$\approx 3727 \text{ m}$$

12b. $S = \frac{1500}{1 - \frac{3}{5}}$

$$= 3750 \text{ m}$$

No, the sum of the infinite series modeling this situation is 3750. Thus, the spill will spread no more than 3750 meters.

Pages 791–793 Exercises

13. $a_n = \frac{4}{3^n}$, $a_{n+1} = \frac{4}{3^{n+1}}$

$$r = \lim_{n\to\infty} \frac{\frac{4}{3^{n+1}}}{\frac{4}{3^n}}$$
$$= \lim_{n\to\infty} \frac{4 \cdot 3^n}{4 \cdot 3^n \cdot 3}$$
$$= \frac{1}{3}$$

convergent

14. $a_n = \frac{2^n}{5n}$, $a_{n+1} = \frac{2^{n+1}}{5(n+1)}$

$$r = \lim_{n\to\infty} \frac{\frac{2^{n+1}}{5(n+1)}}{\frac{2^n}{5n}}$$
$$= \lim_{n\to\infty} \frac{2^n \cdot 2 \cdot 5n}{2^n \cdot (5n+5)}$$
$$= \lim_{n\to\infty} \frac{10n}{5n+5}$$
$$= \lim_{n\to\infty} \frac{\frac{10n}{n}}{\frac{5n}{n} + \frac{5}{n}}$$
$$= \frac{10}{5} \text{ or } 2$$

divergent

15. $a_n = \frac{2^n}{n^2}$, $a_{n+1} = \frac{2^{n+1}}{(n+1)^2}$

$$r = \lim_{n\to\infty} \frac{\frac{2^{n+1}}{(n+1)^2}}{\frac{2^n}{n^2}}$$
$$= \lim_{n\to\infty} \frac{2^n \cdot 2 \cdot n^2}{2^n(n^2+2n+1)}$$
$$= \lim_{n\to\infty} \frac{2n^2}{n^2+2n+1}$$
$$= \lim_{n\to\infty} \frac{\frac{2n^2}{n^2}}{\frac{n^2}{n^2} + \frac{2n}{n^2} + \frac{1}{n^2}}$$
$$= 2$$

divergent

16. $a_n = \frac{2}{(n+1)(n+2)}$, $a_{n+1} = \frac{2}{(n+2)(n+3)}$

$$r = \lim_{n\to\infty} \frac{\frac{2}{(n+2)(n+3)}}{\frac{2}{(n+1)(n+2)}}$$
$$= \lim_{n\to\infty} \frac{2(n^2+3n+2)}{2(n^2+5n+6)}$$
$$= \lim_{n\to\infty} \frac{\frac{n^2}{n^2} + \frac{3n}{n^2} + \frac{2}{n^2}}{\frac{n^2}{n^2} + \frac{5n}{n^2} + \frac{6}{n^2}}$$
$$= 1$$

test provides no information

17. $a_n = \frac{2n-1}{1 \cdot 2 \cdot \ldots \cdot (2n-1)}$, $a_{n+1} = \frac{2(n+1)-1}{1 \cdot 2 \cdot \ldots \cdot [2(n+1)-1]}$

$$r = \lim_{n\to\infty} \frac{\frac{2(n+1)-1}{1 \cdot 2 \cdot \ldots \cdot [2(n+1)-1]}}{\frac{2n-1}{1 \cdot 2 \cdot \ldots \cdot (2n-1)}}$$
$$= \lim_{n\to\infty} \frac{(2n+1)(1 \cdot 2 \cdot \ldots \cdot (2n-1))}{(2n-1)(1 \cdot 2 \cdot \ldots \cdot (2n+1))}$$
$$= \lim_{n\to\infty} \frac{1}{(2n-1)(2n)}$$
$$= \lim_{n\to\infty} \frac{1}{4n^2-2n}$$
$$= 0$$

convergent

18. $a_n = \frac{5^n}{1 \cdot 2 \cdot \ldots \cdot n}$, $a_{n+1} = \frac{5^{n+1}}{1 \cdot 2 \cdot \ldots \cdot (n+1)}$

$$r = \lim_{n\to\infty} \frac{\frac{5^{n+1}}{1 \cdot 2 \cdot \ldots \cdot (n+1)}}{\frac{5^n}{1 \cdot 2 \cdot \ldots \cdot n}}$$
$$= \lim_{n\to\infty} \frac{5^n \cdot 5(1 \cdot 2 \cdot \ldots \cdot n)}{5^n(1 \cdot 2 \cdot \ldots \cdot (n+1))}$$
$$= \lim_{n\to\infty} \frac{5}{n+1}$$
$$= 0$$

convergent

19. $a_n = \frac{2n \cdot 2(n+1)}{2n}$, $a_{n+1} = \frac{2(n+1) \cdot 2(n+2)}{2^{n+1}}$

$$r = \lim_{n\to\infty} \frac{\frac{2(n+1) \cdot 2(n+2)}{2^{n+1}}}{\frac{2n \cdot 2(n+1)}{2^n}}$$
$$= \lim_{n\to\infty} \frac{2(n+1) \cdot 2(n+2) \cdot 2^n}{2n \cdot 2(n+1) \cdot 2^n \cdot 2}$$
$$= \lim_{n\to\infty} \frac{n+2}{2n}$$
$$= \lim_{n\to\infty} \frac{n}{2n} + \frac{2}{2n}$$
$$= \frac{1}{2}$$

convergent

20. The general term is $\frac{1}{(2n)^2}$ or $\frac{1}{4n^2}$.

$\frac{1}{4n^2} \le \frac{1}{n}$ for all n, so convergent

21. The general term is $\frac{1}{n^3+1}$.

$\frac{1}{n^3+1} \le \frac{1}{n}$ for all n, so convergent

22. The general term is $\frac{n}{n+1}$.

$\frac{n}{n+1} \ge \frac{1}{n}$ for all n, so divergent

23. The general term is $\frac{5}{n+2}$.

$\frac{5}{n+2} \ge \frac{1}{n}$ for all n, so divergent

24. The general term is $\frac{1}{2n+1}$.

$\frac{1}{2n+1} \le \frac{1}{n}$ for all n, so convergent

25. The series is geometric where $r = -\frac{3}{4}$.

Since $-\frac{3}{4} < 1$, it is convergent.

26. The general term is $\frac{2n+1}{2n-1}$.

$\frac{2n+1}{2n-1} \ge \frac{1}{n}$ for all n, so divergent

27. The general term is $\frac{1}{5+n^2}$.

$\frac{1}{5+n^2} \le \frac{1}{n}$ for all n, so convergent

28. The general term is $\frac{1}{\sqrt{n}}$.

$\frac{1}{\sqrt{n}} \ge \frac{1}{n}$ for all n, so divergent

29. The series is arithmetic, so it is divergent

30. $a_n = \frac{2n-1}{2^{n+1}}$, $a_{n+1} = \frac{2(n+1)-1}{2^{n+2}}$

$$r = \lim_{n\to\infty} \frac{\frac{2(n+1)-1}{2^{n+2}}}{\frac{2n-1}{2^{n+1}}}$$
$$= \lim_{n\to\infty} \frac{(2n+1) \cdot 2^n \cdot 2}{(2n-1) \cdot 2^n \cdot 2 \cdot 2}$$
$$= \lim_{n\to\infty} \frac{2n+1}{4n-2}$$
$$= \lim_{n\to\infty} \frac{\frac{2n}{n} + \frac{1}{n}}{\frac{4n}{n} - \frac{2}{n}}$$
$$= \frac{2}{4} \text{ or } \frac{1}{2}$$

convergent

31a. No, MagicSoft let $a_1 = 1{,}000{,}000$ to arrive at their figure. The first term of this series is $1{,}000{,}000 \cdot 0.70$ or 700,000.

31b. The series is geometric where $a_1 = 700{,}000$ and $r = 0.70$.

$$S = \frac{700{,}000}{1 - 0.70}$$
$$= \$2.3 \text{ million}$$

32. the harmonic series: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$

33a. Culture A: 1400 cells, Culture B; 713 cells
Culture A generates an arithmetic sequence where $a_1 = 1000$, $d = 200$ and $n = 8$

$$a_8 = 1000 + (8-1)200$$
$$= 2400$$

Only considering cell growth, there are $2400 - 1000$ or 1400 new cells.
Culture B generates a geometric sequence where $a_1 = 1000$ and $r = 1.08$

$$a_8 = 1000(1.08)^{8-1}$$
$$=1713 \quad \text{A part of a cell cannot be generated.}$$

Only considering cell growth, there are $1713 - 1000$ or 713 new cells.

33b. Culture A: $a_{31} = 1000 + (31-1)200$
$= 7000$
$7000 - 1000 = 6000$ cells

Culture B: $a_{31} = 1000(1.08)^{31-1}$
$\approx 10{,}062$
$10{,}062 - 1000 = 9062$

Culture B; at the end of one month, culture A will have produced 6000 cells while culture B will have produced 9062 cells.

34a. $\frac{2^n - 1}{2^{n-1}}$

34b. $S = \lim\limits_{n\to\infty} S_n = \lim\limits_{n\to\infty} \frac{2^n - 1}{2^{n-1}}$
$= \lim\limits_{n\to\infty} \frac{2(2^n - 1)}{2^n}$
$= \lim\limits_{n\to\infty} \left(2 - \frac{2}{2^n}\right)$
$= 2$ seconds

35a. When the minute hand is at 4, 20 minutes have passed. This is $20\left(\frac{1}{60}\right)$ or $\frac{1}{3}$ hour.

35b. $\frac{1}{3}$ the distance between 4 and 5 is $\frac{1}{3}(5)$ or $\frac{5}{3}$ minutes. This is $\frac{5}{3}\left(\frac{1}{60}\right)$ or $\frac{1}{36}$ hour.

35c. $\left(\frac{1}{3} + \frac{1}{36}\right)(5) = \frac{65}{36}$ minutes
$\frac{65}{36} - \frac{5}{3} = \frac{5}{36}$ minute
$\frac{5}{36}\left(\frac{1}{60}\right) = \frac{1}{432}$ hour
$\left(\frac{1}{3} + \frac{1}{36} + \frac{1}{432}\right)(5) = \frac{785}{432}$ minutes
$\frac{785}{432} - \frac{65}{36} = \frac{5}{432}$ minute
$\frac{5}{432}\left(\frac{1}{60}\right) = \frac{1}{5184}$ hour
$\frac{1}{432}, \frac{1}{5184}$

35d. The sequence $\frac{1}{3}, \frac{1}{36}, \frac{1}{432}, \frac{1}{5184}$ is geometric where $r = \frac{1}{12}$.

$\lim\limits_{n\to\infty} S_n = S = \frac{a_1}{1 - r}$
$= \dfrac{\frac{1}{3}}{1 - \frac{1}{12}}$
$= \frac{4}{11}$ hour

The hands will coincide at $4 + \frac{4}{11}$ o'clock, approximately 21 min 49 s after 4:00.

36. $\lim\limits_{n\to\infty} \frac{4n^2 + 5}{3n^2 - 2n} = \lim\limits_{n\to\infty} \dfrac{\frac{4n^2}{n^2} + \frac{5}{n^2}}{\frac{3n^2}{n^2} - \frac{2n}{n^2}}$
$= \frac{4}{3}$

37. $r = \frac{2}{\sqrt{2}}$ or $\sqrt{2}$
$a_9 = \sqrt{2}\left(\sqrt{2}\right)^{9-1}$
$= 16\sqrt{2}$

38. $19 = -11 + (7 - 1)d$
$30 = 6d$
$5 = d$
$-11 + 5 = -6, -6 + 5 = -1, -1 + 5 = 4,$
$4 + 5 = 9, 9 + 5 = 14$
$-11, -6, -1, 4, 9, 14, 19$

39. $45.9 = e^{0.075t}$
$\ln 45.9 = 0.075t$
$51.02 \approx t$

40. $6 = 12r \cos(\theta - 30°)$
$\frac{1}{2} = r(\cos\theta \cos 30° + \sin\theta \sin 30°)$
$\frac{1}{2} = \frac{\sqrt{3}}{2} r\cos\theta + \frac{1}{2} r\sin\theta$
$0 = \frac{\sqrt{3}}{2}x + \frac{1}{2}y - \frac{1}{2}$
$0 = \sqrt{3}x + y - 1$

41. $\overrightarrow{AB} = \langle 5 - 8, -1 - (-3)\rangle$
$= \langle -3, 2\rangle$

42. List all cubes from 1 to 200. There are five. The correct choice is E.

n	n^3
1	1
2	8
3	27
4	64
5	125

Page 793 Mid-Chapter Quiz

1. $a_{12} = 11 + (19 - 1)(-2)$
$= -25$

2. $S_{20} = \frac{20}{2}[2(-14) + (20 - 1)6]$
$= 860$

3. $189 = 56r^{4-1}$
$\frac{27}{8} = r^3$
$\frac{3}{2} = r$
$56\left(\frac{3}{2}\right) = 84,\ 84\left(\frac{3}{2}\right) = 126$
56, 84, 126, 189

4. $r = \frac{-6}{3}$ or -2
$S_8 = \frac{3 - 3(-2)^8}{1 - (-2)}$
$= -255$

5. $\lim\limits_{n\to\infty} \frac{n^2 + 2n - 5}{n^2 - 1} = \lim\limits_{n\to\infty} \dfrac{\frac{n^2}{n^2} + \frac{2n}{n^2} - \frac{5}{n^2}}{\frac{n^2}{n^2} - \frac{1}{n^2}}$
$= 1$

6. $a_1 = 250, r = 0.55$
$a_2 = 250(0.55)$ or 137.5
$a_3 = 137.5$
$a_4 = 137.5(0.55)$ or 75.625
$a_5 = 75.625 \cdots$
$S_n = 250 + \frac{137.5}{1 - 0.55} + \frac{137.5}{1 - 0.55}$
≈ 861 ft

7. $a_1 = \frac{1}{25}, r = \frac{1}{10}$
$S = \dfrac{\frac{1}{25}}{1 - \frac{1}{10}}$
$= \frac{2}{45}$

8. $a_n = \frac{1 \cdot 2 \cdots n}{10^n},\ a_{n+1} = \frac{1 \cdot 2 \cdot \ldots \cdot (n + 1)}{10^{n+1}}$
$r = \lim\limits_{n\to\infty} \dfrac{\frac{1 \cdot 2 \cdot \ldots \cdot (n+1)}{10^{n+1}}}{\frac{1 \cdot 2 \cdot \ldots \cdot n}{10^n}}$
$= \lim\limits_{n\to\infty} \frac{10^n[1 \cdot 2 \cdot \ldots \cdot (n + 1)]}{10^n \cdot 10(1 \cdot 2 \cdot \ldots \cdot n)}$
$= \lim\limits_{n\to\infty} \frac{n + 1}{10}$
As $n \to \infty$, $\frac{n+1}{10} \to \infty$. Since $r > 1$, the series is divergent.

9. The series is geometric where $r = \frac{1}{3}$.
Since $r < 1$, it is convergent.

10. $500\left(1 + \frac{0.12}{4}\right) = 515$

$a_1 = 515,\ r = 1.03\ n = 4$

$S_4 = \frac{515 - 515(1.03)^4}{1 - 1.03}$

$= \$2154.57$

12-5 Sigma Notation and the n^{th} term.

Pages 797–798 Check for Understanding

1. The series 4 + 6 + 8 + 10 + 12 can be represented by $\sum_{n=0}^{4} 2n + 4$ or by $\sum_{n=1}^{5} 2n + 2$.

2a. Sample answer: $(-1)^{n-1}$

2b. Sample answer: $(-1)^n$

3a. 9; 2, 3, 4, 5, 6, 7, 8, 9, 10

3b. $t = b - a + 1$

3c. $t = b - a + 1$
$= 3 - (-2) + 1$
$= 6$

3d. $\sum_{k=2}^{3} \frac{1}{k+3} = \frac{1}{-2+3} + \frac{1}{-1+3} + \frac{1}{0+5} + \frac{1}{1+3} + \frac{1}{2+3} + \frac{1}{3+3}$
$= \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$
$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{5}$

There are 6 terms.

4. $\sum_{n=1}^{6} (n-3) = (1-3) + (2-3) + (3-3) + (4-3) + (5-3) + (6-3)$
$= (-2) + (-1) + 0 + 1 + 2 + 3$
$= 3$

5. $\sum_{k=2}^{5} 4k = 4 \cdot 2 + 4 \cdot 3 + 4 \cdot 4 + 4 \cdot 5$
$= 8 + 12 + 16 + 20$
$= 56$

6. $\sum_{a=0}^{4} \frac{1}{2^a} = \frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4}$
$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$
$= 1\frac{15}{16}$

7. $\sum_{p=0}^{\infty} 5\left(\frac{3}{4}\right)^p = 5\left(\frac{3}{4}\right)^0 + 5\left(\frac{3}{4}\right)^1 + 5\left(\frac{3}{4}\right)^2 + 5\left(\frac{3}{4}\right)^3 + \cdots + 5\left(\frac{3}{4}\right)^{10}$
$= 5 + \frac{15}{4} + \frac{45}{16} + \frac{135}{64} + \cdots + 5\left(\frac{3}{4}\right)^\infty$

$S_n = \dfrac{5}{1 - \frac{3}{4}}$
$= 20$

8. $\sum_{n=1}^{5} 5n$

9. $\sum_{k=0}^{3} (3^k + 1)$

10. $\sum_{n=1}^{4} (8 - 6n)$

11. $\sum_{n=2}^{\infty} 3\left(\frac{1}{2}\right)^n$

12. $\sum_{n=1}^{\infty} (-3)^n$

13a. $\sum_{n=1}^{60} 389(0.63)^{n-1}$

$S_{60} = \frac{389 - 389(0.63)^{60}}{1 - 0.63}$
≈ 1051 ft

13b. $S = \frac{389}{1 - 0.63}$
≈ 1051 ft

Pages 798–800 Exercises

14. $\sum_{n=1}^{4} (2n-7) = (2 \cdot 1 - 7) + (2 \cdot 2 - 7) + (2 \cdot 3 - 7) + (2 \cdot 4 - 7)$
$= (-5) + (-3) + (-1) + 1$
$= -8$

15. $\sum_{a=2}^{5} 5a = 5(2) + 5(3) + 5(4) + 5(5)$
$= 10 + 15 + 20 + 25$
$= 70$

16. $\sum_{b=3}^{8} (6 - 4b) = (6 - 4 \cdot 3) + (6 - 4 \cdot 4) + (6 - 4 \cdot 5) + (6 - 4 \cdot 6) + (6 - 4 \cdot 7) + (6 - 4 \cdot 8)$
$= (-6) + (-10) + (-14) + (-18) + (-22) + (-26)$
$= -96$

17. $\sum_{k=2}^{6} (k + k^2) = (2 + 2^2) + (3 + 3^2) + (4 + 4^2) + (5 + 5^2) + (6 + 6^2)$
$= 6 + 12 + 20 + 30 + 42$
$= 110$

18. $\sum_{n=5}^{8} \frac{n}{n-4} = \frac{5}{5-4} + \frac{6}{6-4} + \frac{7}{7-4} + \frac{8}{8-4}$
$= 5 + 3 + \frac{7}{3} + 2$
$= 12\frac{1}{3}$

19. $\sum_{j=4}^{8} 2^j = 2^4 + 2^5 + 2^6 + 2^7 + 2^8$
$= 16 + 32 + 64 + 128 + 256$
$= 496$

20. $\sum_{m=0}^{3} 3^{m-1} = 3^{0-1} + 3^{1-1} + 3^{2-1} + 3^{3-1}$
$= \frac{1}{3} + 1 + 3 + 9$
$= 13\frac{1}{3}$

21. $\sum_{r=1}^{3} \left(\frac{1}{2} + 4^r\right) = \left(\frac{1}{2} + 4^1\right) + \left(\frac{1}{2} + 4^2\right) + \left(\frac{1}{2} + 4^3\right)$
$= 4\frac{1}{2} + 16\frac{1}{2} + 64\frac{1}{2}$
$= 85\frac{1}{2}$

22. $\sum_{i=3}^{5} (0.5)^{-i} = (0.5)^{-3} + (0.5)^{-4} + (0.5)^{-5}$
$= 8 + 16 + 32$
$= 56$

23. $\sum_{k=3}^{7} k! = 3! + 4! + 5! + 6! + 7!$
$= 6 + 24 + 120 + 720 + 5040$
$= 5910$

24. $\sum_{p=0}^{10} 4(0.75)^p = 4(0.75)^0 + 4(0.75)^1 + 4(0.75)^2 + 4(0.75)^3 + \cdots + 4(0.75)^\infty$
$= 4 + 3 + 2.25 + 1.6875 + \cdots + 4(0.75)^\infty$

$S = \frac{4}{1 - 0.75}$
$= 16$

25. $\sum_{n=1}^{\infty} 4\left(\frac{2}{5}\right)^n = 4\left(\frac{2}{5}\right)^1 + 4\left(\frac{2}{5}\right)^2 + 4\left(\frac{2}{5}\right)^3 + \cdots + 4\left(\frac{2}{5}\right)^{\infty}$
$= \frac{8}{5} + \frac{16}{25} + \frac{32}{125} + \cdots + 4\left(\frac{2}{5}\right)^{\infty}$

$$S = \frac{\frac{8}{5}}{1 - \frac{2}{5}}$$
$= \frac{8}{3}$ or $2\frac{2}{3}$

26. $\sum_{n=2}^{5} n + \boldsymbol{i}^n = (2 + \boldsymbol{i}^2) + (3 + \boldsymbol{i}^3) + (4 + \boldsymbol{i}^4) + (5 + \boldsymbol{i}^5)$
$= (1) + (3 - \boldsymbol{i}) + (5) + (5 + \boldsymbol{i})$
$= 14$

27. $\sum_{k=1}^{4} (3k + 3)$ **28.** $\sum_{k=0}^{4} 4k$

29. $\sum_{k=4}^{12} 2k$ **30.** $\sum_{k=0}^{3} (-2)^{3-k}$

31. $\sum_{k=1}^{4} 2 \cdot 5^k$ **32.** $\sum_{k=0}^{3} (13 - 4k)$

33. $\sum_{k=2}^{10} \frac{1}{5k - 1}$ **34.** $\sum_{k=1}^{\infty} \frac{2^k}{2k + 1}$

35. $\sum_{k=2}^{\infty} (-1)^k k^2$ **36.** $\sum_{k=0}^{\infty} \frac{5}{k!}$

37. $\sum_{n=0}^{\infty} \left[(-1)^{n+1} \frac{32}{2^n}\right]$ **38.** $\sum_{k=1}^{\infty} \frac{(k + 1)!}{k}$

39. $\sum_{k=1}^{\infty} \frac{k}{2^k + 3}$ **40.** $\sum_{k=2}^{\infty} \frac{k^2 - 1}{3^k k!}$

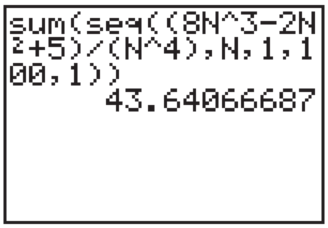
41. $\sum_{k=11}^{\infty} \frac{2\frac{k}{2}}{3k!}$

42. $\frac{(a - 2)!}{a!} = \frac{(a - 2)!}{a(a - 1)(a - 2)!}$
$= \frac{1}{a(a - 1)}$

43. $\frac{(a + 1)!}{(a - 2)!} = \frac{(a + 1)(a)(a - 1)(a - 2)!}{(a - 2)!}$
$= a(a + 1)(a - 1)$

44. $\frac{(a + b)!}{(a + b - 1)!} = \frac{(a + b)(a + b - 1)!}{(a + b - 1)!}$
$= a+$

45. 43.64



46a. $\sum_{n=1}^{\infty} 500{,}000(0.35)^n$

46b. $S = \frac{175{,}000}{1 - 0.35}$
$= 269{,}239$ people

46c. $\frac{269{,}230}{500{,}000} \approx 53.8\%$

46d. The ad agency assumes that the people who buy the tennis shoes will be satisfied with their purchase.

47a. $(x - 3) + (x - 6) + (x - 9) + (x - 12) + (x - 15) + (x - 18) = -3$
$6x - 63 = -3$
$6x = 60$
$x = 60$

47b. $0 + 1(1 - x) + 2(2 - x) + 3(3 - x) + 4(4 - x) + 5(5 - x) = 25$
$1 - x + 4 - 2x + 9 - 3x + 16 - 4x + 25 - 5x = 25$
$55 - 15x = 25$
$-15x = -30$
$x = 2$

48a. false; $\sum_{k=3}^{7} 3^k = 3^3 + 3^4 + \cdots + 3^7$
$\sum_{b=7}^{9} 3^b = 3^7 + 3^8 + 3^9$
Since there are two 3^7 terms, $\sum_{k=3}^{7} 3^k + \sum_{b=7}^{9} 3^b \neq \sum_{a=3}^{9} 3^a$.

48b. true; $\sum_{n=2}^{8} (2n - 3) = 1 + 3 + \cdots + 13 = 49$
$\sum_{m=3}^{9} (2m - 5) = 1 + 3 + \cdots + 13 = 49$
Since 49 = 49, $\sum_{n=2}^{8} (2n - 3) = \sum_{m=3}^{9} (2m - 5)$.

48c. true; $2\sum_{n=3}^{7} n^2 = 18 + 32 + \cdots + 98 = 270$
$\sum_{n=3}^{7} 2n^2 = 18 + 32 + \cdots + 98 = 270$
Since 270 = 270, $2\sum_{n=3}^{7} n^2 = \sum_{n=3}^{7} 2n^2$.

48d. false; $\sum_{k=1}^{10} (5 + n) = 6 + 7 + \cdots + 15 = 105$
$\sum_{p=0}^{9} (4 + p) = 4 + 5 + \cdots + 13 = 85$
Since $85 \neq 105$, $\sum_{k=1}^{10} (5 + n) \neq \sum_{p=0}^{9} (4 + p)$.

49a. 6!

49b. 5! or 120

49c. 4! or 24, "LISTEN"

50a. On an 8 × 8 chessboard, there is 1-8 × 8 square. On an 8 × 8 chessboard, there are 4-7 × 7 squares, one in each of the four corners.

50b. For the 6 × 6 squares, begin in one corner. For different configurations, you can move it over, up to 2 more spaces, or down, up to 2 more spaces. Thus, there are 3 · 3 or 9-6 × 6 squares. Continue this procedure for the other sizes of squares.
9-6 × 6, 16-5 × 5, 25-4 × 4, 36-3 × 3, 49-2 × 2, and 64-1 × 1

50c. $\sum_{n=1}^{8} n^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2$
$= 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64$
$= 204$

51. The general term is $\frac{3}{n + 2}$.
$\frac{3}{n + 2} > \frac{1}{n}$ for all n, so divergent

52. 0.42(21) = 8.82 liters
If with each stroke 20% is removed, then 80% remains.
$a_1 = 21(0.80) = 16.8$
$a_2 = 16.8(0.80) = 13.44$
$a_3 = 13.44(0.80) = 10.752$
$a_4 = 10.752(0.80) = 8.6016$
It will take 4 strokes for 42% of the air to remain.

53. $32\sqrt{2} = a_1\left(-\sqrt{2}\right)^{5-1}$

$32\sqrt{2} = 4a_1$

$8\sqrt{2} = a_1$

$8\sqrt{2}\left(-\sqrt{2}\right) = -16,\ -16\left(-\sqrt{2}\right) = 16\sqrt{2},$

$16\sqrt{2}\left(-\sqrt{2}\right) = -32$

$8\sqrt{2},\ -16,\ 16\sqrt{2},\ -32$

54. $\log_{10} 0.001 = -3$

55. $x^2 + y^2 + Dx + Ey + F = 0$

$(0, 9) \rightarrow 81 + 9E + F = 0$

$(-7, 2) \rightarrow 49 + 4 - 7D + 2E + F = 0$

$(0, -5) \rightarrow 25 - 5E + F = 0$

$$\begin{array}{rlr} 9E + F = -81 & \qquad & 9(-4) + F = -81 \\ \underline{-5E + F = -25} & & F = -45 \\ 14E \quad = -56 & & 53 - 7D + 2(-4) - 45 = 0 \\ E = -4 & & -7D = 0 \\ & & D = 0 \end{array}$$

$x^2 + y^2 - 4y - 45 = 0$

$x^2 + (y - 2)^2 = 49$

56. $(\sqrt{2} + \boldsymbol{i})(4\sqrt{2} + \boldsymbol{i}) = 8 + 5\sqrt{2}\boldsymbol{i} + \boldsymbol{i}^2$

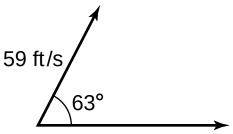
$= 7 + 5\boldsymbol{i}\sqrt{2}$

57. $v_y = 59 \sin 63°$

≈ 52.57 ft/s

$v_x = 59 \cos 63°$

≈ 26.79 ft/s



58. $d_1 = \frac{2x_1 - 3y_1 + 9}{-\sqrt{13}},\ d_2 = \frac{x_1 + 4y_1 + 4}{-\sqrt{17}}$

$\frac{2x_1 - 3y_1 + 9}{-\sqrt{13}} = \frac{x_1 + 4y_1 + 4}{-\sqrt{17}}$

$2\sqrt{17}x_1 - 3\sqrt{17}y_1 + 9\sqrt{17} =$

$-\sqrt{13}x_1 - 4\sqrt{13}y_1 - 4\sqrt{13}$

$\left(2\sqrt{17} + \sqrt{13}\right)x + \left(4\sqrt{13} -\right.$

$\left.3\sqrt{17}\right)y + 9\sqrt{17} + 4\sqrt{13} = 0$

59. $\frac{5 + m}{9 + m} = \frac{2}{3}$

$3(5 + m) = 2(9 + m)$

$15 + 3m = 18 + 2m$

$m = 3$ The correct choice is D.

12-6 The Binomial Theorem

Pages 803–804 Check for Understanding

1a. $n = 0$: 1

$n = 1$: $1 + 1$ or 2

$n = 2$: $1 + 2 + 1$ or 4

$n = 3$: $1 + 3 + 3 + 1$ or 8

$n = 4$: $1 + 4 + 6 + 4 + 1$ or 16

$n = 5$: $1 + 5 + 10 + 10 + 5 + 1$ or 32

1, 2, 4, 8, 16, 32

1b. 2^n

2a. The second term of $(x - y)^3$ is $-3x^2y$.
It is negative.
The second term of $(x - y)^4$ is $-4x^3y$.
It is negative.
The second term of $(x - y)^5$ is $-5x^4y$.
It is negative.

2b. The third term of $(x - y)^3$ is $3xy^2$. It is positive.
The third term of $(x - y)^4$ is $6x^2y^2$. It is positive.
The third term of $(y - y)^5$ is $10x^3y^2$. It is positive.

2c. Even indexed terms are negative and odd indexed terms are positive.

3. The sum of the exponents of each term is n.

4. The exponents must add to 12, so the exponent of y is $12 - 7$ or 5.
To find the coefficient of the term use the formula
$(x + y)^n = \sum_{r=0}^{n} \frac{n!}{r!(n - r)!}x^{n-r}y^r$.
Evaluate the general term for $n = 12$ and $r = 5$.
$\frac{12!}{5!7!}\,x^7y^5 = 792x^7y^5$.

5. $c^5 + 5c^4d + 10c^3d^2 + 10c^2d^3 + 5cd^4 + d^5$

6. $(a + 3)^6 = a^6 + 3 \cdot 6a^5 + \frac{3^2 \cdot 6 \cdot 5}{1 \cdot 2}a^4$

$+ \frac{3^3 \cdot 6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3}\,a^3$

$+ \frac{3^4 \cdot 6 \cdot 5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3 \cdot 4}\,a^2$

$+ \frac{3^5 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}\,a$

$+ \frac{3^6 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6}$

$= a^6 + 18a^5 + 135a^4 + 540a^3 + 1215a^2 +$
$1458a + 729$

7. $(5 - y)^3 = 5^3(-y)^0 + 3 \cdot 5^2(-y) + \frac{3 \cdot 2 \cdot 5 \cdot (-y)^2}{2 \cdot 1}$

$+ \frac{3 \cdot 2 \cdot 1 \cdot 5^0\,(-y)^3}{3 \cdot 2 \cdot 1}$

$= 125 - 75y + 15y^2 - y^3$

8. $(3p - 2q)^4 = (3p)^4\,(-2q)^0 + 4(3p)^3\,(-2q)^1$

$+ \frac{4 \cdot 3(3p)^2\,(-2q)^2}{2 \cdot 1} + \frac{4 \cdot 3 \cdot 2(3p)(-2q)^3}{3 \cdot 2 \cdot 1}$

$+ \frac{4 \cdot 3 \cdot 2 \cdot 1(3p)^0(-2q)^4}{4 \cdot 3 \cdot 2 \cdot 1}$

$= 81p^4 - 216p^3q + 216p^2q^2 - 96pq^3$
$+ 16q^4$

9. $\frac{7!}{r!(7 - r)!}(a)^{7-r}(-b)^r$

$= \frac{7!}{5!(7 - 5)!}(a)^{7-5}(-b)^5$

$= -\frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}\,a^2b^5$

$= -21a^2b^5$

10. $\frac{9!}{r!(9 - r)!}(x)^{9-r}\left(\sqrt{3}\right)^r$

$= \frac{9!}{3!(9 - 3)!}(x)^{9-3}\left(\sqrt{3}\right)^3$

$= \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}\,x^6 \cdot 3\sqrt{3}$

$= 252\sqrt{3}\,x^6$

11. $(H + T)^5 = H^5 + 5H^4T + 10H^3T^2 + 10H^2T^3 + 5HT^2 + T^5$

11a. 1

11b. 10

11c. $1 + 5$ or 6

11d. $10 + 10 + 5 + 1$ or 26

Pages 804–805 Exercises

12. $(a + b)^8 = a^8 + 8a^7b + 28a^6b^2 + 56a^5b^3 + 70a^4b^4$
$+ 56a^3b^5 + 28a^2b^6 + 8ab^7 + b^8$

13. $(n - 4)^6 = n^6 - 24n^5 + 240n^4 - 1280n^3 + 3840n^2$
$- 6144n + 4096$

14. $(3c - d)^4 = 81c^4 - 108c^3d + 54c^2d^2 - 12cd^3 + d^4$

15. $(2 + a)^9 = 512 + 2304a + 4608a^2 + 5376a^3 +$
$4032a^4 + 2016a^5 + 672a^6 + 144a^7 +$
$18a^8 + a^9$

16. $(d+2)^7 = d^7 \cdot 2^0 + 7 \cdot d^6 \cdot 2^1 + \frac{7 \cdot 6 \cdot d^5 \cdot 2^2}{2 \cdot 1}$
$+ \frac{7 \cdot 6 \cdot 5 \cdot d^4 \cdot 2^3}{3 \cdot 2 \cdot 1} + \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot d^3 \cdot 2^4}{4 \cdot 3 \cdot 2 \cdot 1}$
$+ \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot d^2 \cdot 2^5}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$+ \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot d^1 \cdot 2^6}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$+ \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot d^0 \cdot 2^7}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= d^7 + 14d^6 + 84d^5 + 280d^4 + 560d^3 +$
$672d^2 + 448d + 128$

17. $(3-x)^5 = 3^5(-x)^0 + 5 \cdot 3^4(-x^1)^1 + \frac{5 \cdot 4 \cdot 3^3(-x)^2}{2 \cdot 1}$
$+ \frac{5 \cdot 4 \cdot 3 \cdot 3^2(-x)^3}{3 \cdot 2 \cdot 1} + \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 3^1(-x)^4}{4 \cdot 3 \cdot 2 \cdot 1}$
$+ \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3^0(-x)^5}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 243 - 405x + 270x^2 - 90x^3 + 15x^4 - x^5$

18. $(4a+b)^4 = (4a)^4(b)^0 + 4(4a)^3(b)^1 + \frac{4 \cdot 3(4a)^2(b)^2}{2 \cdot 1}$
$+ \frac{4 \cdot 3 \cdot 2(4a)^1(b)^3}{3 \cdot 2 \cdot 1} + \frac{4 \cdot 3 \cdot 2 \cdot 1(4a)^0(b)^4}{4 \cdot 3 \cdot 2 \cdot 1}$
$= 256a^4 + 256a^3b + 96a^2b^2 + 16ab^3 + b^4$

19. $(2x-3y)^3 = (2x)^3(-3y)^0 + 3(2x)^2(-3y)^1$
$+ \frac{3 \cdot 2(2x)(-3y)^2}{2 \cdot 1} + \frac{3 \cdot 2 \cdot 1(2x)^0(-3y)^3}{3 \cdot 2 \cdot 1}$
$= 8x^3 - 36x^2y + 54xy^2 - 27y^3$

20. $\left(3m + \sqrt{2}\right)^4 = (3m)^4\left(\sqrt{2}\right)^0 + 4(3m)^3\left(\sqrt{2}\right)^1 +$
$\frac{4 \cdot 3(3m)^2(\sqrt{2})^2}{2 \cdot 1} + \frac{4 \cdot 3 \cdot 2(3m)^1(\sqrt{2})^3}{3 \cdot 2 \cdot 1}$
$+ \frac{4 \cdot 3 \cdot 2 \cdot 1(3m)^0(\sqrt{2})^4}{4 \cdot 3 \cdot 2 \cdot 1}$
$= 81m^4 + 108\sqrt{2}m^3 + 108m^2 +$
$24\sqrt{2}m + 4$

21. $\left(\sqrt{c} - 1\right)^6 = \left(\sqrt{c}\right)^6(-1)^0 + 6\left(\sqrt{c}\right)^5(-1)^1$
$+ \frac{6 \cdot 5(\sqrt{c})^4(-1)^2}{2 \cdot 1} + \frac{6 \cdot 5 \cdot 4(\sqrt{c})^3(-1)^3}{3 \cdot 2 \cdot 1}$
$+ \frac{6 \cdot 5 \cdot 4 \cdot 3(\sqrt{c})^2(-1)^4}{4 \cdot 3 \cdot 2 \cdot 1}$
$+ \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2(\sqrt{c})^1(-1)^5}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$+ \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1(\sqrt{c})^0(-1)^6}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= c^3 - 6c^2\sqrt{c} + 15c^2 - 20c\sqrt{c} + 15c$
$- 6\sqrt{c} + 1$

22. $\left(\frac{1}{2}n + 2\right) = \left(\frac{1}{2}n\right)^5(2)^0 + 5\left(\frac{1}{2}n\right)^4(2)^1 + \frac{5 \cdot 4\left(\frac{1}{2}n\right)^3(2)^2}{2 \cdot 1}$
$+ \frac{5 \cdot 4 \cdot 3\left(\frac{1}{2}n\right)^2(2)^3}{3 \cdot 2 \cdot 1} + \frac{5 \cdot 4 \cdot 3 \cdot 2\left(\frac{1}{2}n\right)^1(2)^4}{4 \cdot 3 \cdot 2 \cdot 1}$
$+ \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1\left(\frac{1}{2}n\right)^0(2)^5}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= \frac{1}{32}n^5 + \frac{5}{8}n^4 + 5n^3 + 20n^2 + 40n + 32$

23. $\left(3a + \frac{2}{3}b\right)^4 = (3a)^4\left(\frac{2}{3}b\right)^0 + 4(3a)^3\left(\frac{2}{3}b\right)^1$
$+ \frac{4 \cdot 3(3a)^2\left(\frac{2}{3}b\right)^2}{2 \cdot 1} + \frac{4 \cdot 3 \cdot 2(3a)^1\left(\frac{2}{3}b\right)^3}{3 \cdot 2 \cdot 1}$
$+ \frac{4 \cdot 3 \cdot 2 \cdot 1(3a)^0\left(\frac{2}{3}b\right)^4}{4 \cdot 3 \cdot 2 \cdot 1}$
$= 81a^4 + 72a^3b + 24a^2b^2 +$
$\frac{32}{9}ab^3 + \frac{16}{81}b^4$

24. $(p^2+q)^8 = (p^2)^8(q)^0 + 8(p^2)^7(q)^1 + \frac{8 \cdot 7(p^2)^6(q)^2}{2 \cdot 1}$
$+ \frac{8 \cdot 7 \cdot 6(p^2)^5(q)^3}{3 \cdot 2 \cdot 1} + \frac{8 \cdot 7 \cdot 6 \cdot 5(p^2)^4(q)^4}{4 \cdot 3 \cdot 2 \cdot 1}$
$+ \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4(p^2)^3(q)^5}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$+ \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3(p^2)^2(q)^6}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$+ \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2(p^2)^1(q)^7}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$+ \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1(p^2)^0(q)^8}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= p^{16} + 8p^{14}q + 28p^{12}q^2 + 56p^{10}q^3$
$+ 70p^8q^4 + 56p^6q^5 + 28p^4q^6 + 8p^2q^7$
$+ q^8$

25. $(xy - 2z^3)^6 = (xy)^6(-2x^3)^0 + 6(xy)^5(-2z^3)^1$
$+ \frac{6 \cdot 5(xy)^4(-2z^3)^2}{2 \cdot 1} + \frac{6 \cdot 5 \cdot 4(xy)^3(-2z^3)^3}{3 \cdot 2 \cdot 1}$
$+ \frac{6 \cdot 5 \cdot 4 \cdot 3(xy)^2(-2z^3)^4}{4 \cdot 3 \cdot 2 \cdot 1}$
$+ \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2(xy)^1(-2z^3)^5}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$+ \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1(xy)^0(-2z^3)^6}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= x^6y^6 - 12x^5y^5z^3 + 60x^4y^4z^6$
$- 160x^3y^3z^9 + 240x^2y^2z^{12}$
$- 192xyz^{15} + 64z^{18}$

26. $\frac{9!}{4!(9-4)!}(x)^{9-4}(y)^4 = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}\ x^5y^4$
$= 126x^5y^4$

27. $\frac{8!}{3!(8-3)!}(a)^{8-3}\left(-\sqrt{2}\right)^3$
$= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}\ a^5 \cdot -2\sqrt{2}$
$= -112\sqrt{2}a^5$

28. $\frac{7!}{3!(7-3)!}(2a)^{7-3}(-b)^3 = 35 \cdot 16a^4 \cdot (-b^3)$
$= -560a^4b^3$

29. $\frac{9!}{6!(9-6)!}(3c)^{9-6}(2d)^6 = 84 \cdot 27c^3 \cdot 64d^6$
$= 145{,}152c^3d^6$

30. $\frac{10!}{7!(10-7)!}(\frac{1}{2}x)^{10-7}(-y)^7 = 120 \cdot \frac{1}{8}x^3 \cdot (-y^7)$
$= -15x^3y^7$

31. $\frac{11!}{5!(11-5)!}(2p)^{11-5}(-3q)^5 = 462 \cdot 64p^6 \cdot (-243q^5)$
$= -7{,}185{,}024p^6q^5$

32. The middle term is the fifth term.
$\frac{8!}{4!(8-4)!}\left(\sqrt{x}\right)^{8-4}\left(\sqrt{y}\right)^4 = 70x^2y^2$

33. $(M+W)^8 = M^8 + 8M^7W + 28M^6W^2 + 56M^5W^3$
$+ 70M^4W^4 + 56M^3W^5 + 28M^2W^6$
$+ 8MW^7 + W^8$
$70 + 56 + 28 + 8 + 1$ or 163

34. Sample answer: Treat $a + b$ as a single term and expand $[a + b) + c]^{12}$ using the Binomial Theorem. Then evaluate each $(a + b)^n$ term in the expansion using the Binomial Theorem.

35. $(\mathrm{T} + \mathrm{F})^{12} = \mathrm{T}^{12} + 12\mathrm{T}^{11}\mathrm{F} + 66^{10}\mathrm{F}^2 + 220\mathrm{T}^9\mathrm{F}^3$
$+ 495\mathrm{T}^8\mathrm{F}^4 + 792\mathrm{T}^7\mathrm{F}^5 + 924\mathrm{T}^6\mathrm{F}^6$
$+ 792\mathrm{T}^5\mathrm{F}^7 + 495\mathrm{T}^4\mathrm{F}^8 + 220\mathrm{T}^3\mathrm{F}^9$
$+ 66\mathrm{T}^2\mathrm{F}^{10} + 12\mathrm{T}\mathrm{F}^{11} + \mathrm{F}^{12}$

35a. 495

35b. $924 + 792 + 495 + 220 + 66 + 12 + 1$ or 2510

36. Find the term for which both x's have the same exponent. This will occur for the middle term of the expansion, the 4th term when $n = 6$. Use the

Binomial Theorem to find the 4th term for the expansion of $\left(3x - \frac{1}{4x}\right)^6$.

$\frac{6!}{3!3!}(3x)^3\left(-\frac{1}{4x}\right)^3 = -\frac{135}{16}$

37a. Sample answer: 1 + 0.01

37b. Sample answer: 1.04060401

$$\begin{aligned}(1 + 0.01)^4 &= 1^4 + 4 \cdot 1^3 \cdot 0.01^1 + 6 \cdot 1^2 \cdot 0.01^2 \\ &\quad + 4 \cdot 1^1 \cdot 0.01^3 + 0.01^4 \\ &= 1.04060401\end{aligned}$$

37c. 1.04060401; the two values are equal.

38. $$\begin{aligned}\sum_{x=2}^{7} 5 - 2k &= (5 - 2 \cdot 2) + (5 - 2 \cdot 3) + (5 - 2 \cdot 4) \\ &\quad + (5 - 2 \cdot 5) + (5 - 2 \cdot 6) + (5 - 2 \cdot 7) \\ &= 1 + (-1) + (-3) + (-5) + (-7) + (-9) \\ &= -24\end{aligned}$$

39. $a_n = \frac{2^n}{n!}\ a_{n+1} = \frac{2^{n+1}}{(n+1)!}$

$$\begin{aligned}r &= \lim_{n\to\infty} \frac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}} \\ &= \lim_{n\to\infty} \frac{2^n \cdot 2 \cdot n!}{2^n \cdot (n+1) \cdot n!} \\ &= \lim_{n\to\infty} \frac{2}{n+1} \\ &= 0\end{aligned}$$

convergent

40. This is a geometric series where $r = \frac{1}{2}$.

$$\begin{aligned}S &= \frac{\frac{2}{3}}{1 - \frac{1}{2}} \\ &= 1\tfrac{1}{3}\end{aligned}$$

41. $$\begin{aligned}P_n &= P\left[\frac{1 - (1+i)^{-n}}{i}\right] \\ 150{,}000 &= P\left[\frac{1 - \left(1 + \frac{0.08}{12}\right)^{-12(30)}}{\frac{0.08}{12}}\right] \\ 150{,}000 &= P(136.283491) \\ \$1100.65 &= P\end{aligned}$$

42. $$\begin{aligned}\overrightarrow{MK} &= [4 - (-2)]\vec{\mathbf{i}} + (8 - 6)\vec{\mathbf{j}} + (-2 - 3)\vec{\mathbf{k}} \\ &= 6\vec{\mathbf{i}} + 2\vec{\mathbf{j}} - 5\vec{\mathbf{k}}\end{aligned}$$

43. $$\begin{aligned}s &= r\theta \\ 0.25 &= r\left(\frac{\pi}{4}\right) \\ r &\approx 0.3183098862 \text{ mi} \\ r &\approx 0.3183098862(5280) \\ &\approx 1681 \text{ feet}\end{aligned}$$

44. Test all answer choices that are prime integers. You can eliminate answer choice C.

A: $3(2) \overset{?}{>} 10 \overset{?}{>} \frac{5}{6}(2)$

$6 \overset{?}{>} 10 \overset{?}{>} \frac{5}{3}$; false

B: $3(3) \overset{?}{>} 10 \overset{?}{>} \frac{5}{6}(3)$

$9 \overset{?}{>} 10 \overset{?}{>} \frac{5}{2}$; false

D: $3(11) \overset{?}{>} 10 \overset{?}{>} \frac{5}{6}(11)$

$33 \overset{?}{>} 10 \overset{?}{>} \frac{55}{6}$; true   The correct choice is D.

12-7 Special Sequences and Series

Page 809 Graphing Calculator Exploration

1. Sample answer (without zooming): $-2.4 \le x \le 2.4$
2. Sample answer for greatest difference: about 0.08; The least difference is 0.
3. Sample answer: $-3.4 \le x \le 3.4$; sample answer: about 0.15; 0
4. Sample answer: $-3.8 \le x \le 3.8$; sample answer: about 0.05; 0
5. larger
6. $-\frac{x^{11}}{11!}$

Pages 811–812 Check for Understanding

1. The approximation given in Example 1 only used the first five terms of the exponential series. Using more terms of the exponential series would give an approximation closer to that given by the calculator.
2. Sample answer: $-2 \le x \le 1.5$
3. The problem seems to imply that siblings mate. Genetically, this can lead to problems. Another problem is the assumption that each birth produces only two offspring, one male and one female. Rabbits are more likely to give birth to more than two offspring and the ratio of male to female births is not guaranteed to be 1 to 1.
4. $a_{n+1} = a_n + a_{n-1}$ for $n \ge 2$
5. $$\begin{aligned}\ln(-7) &= \ln(-1) + \ln(7) \\ &\approx \boldsymbol{i}\pi + 1.9459\end{aligned}$$
6. $$\begin{aligned}\ln(-0.379) &= \ln(-1) + \ln(0.379) \\ &\approx \boldsymbol{i}\pi - 0.9702\end{aligned}$$
7. $$\begin{aligned}e^{0.8} &= 1 + 0.8 + \frac{(0.8)^2}{2!} + \frac{(0.8)^3}{3!} + \frac{(0.8)^4}{4!} \\ &\approx 1 + 0.8 + 0.32 + 0.08 + 0.017 \\ &\approx 2.22\end{aligned}$$
8. $$\begin{aligned}e^{1.36} &= 1 + 1.36 + \frac{(1.36)^2}{2!} + \frac{(1.36)^3}{3!} + \frac{(1.36)^4}{4!} \\ &\approx 1 + 1.36 + 0.925 + 0.419 + 0.143 \\ &\approx 3.85\end{aligned}$$
9. $\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$
$$\begin{aligned}\sin \pi &\approx \sin 3.1416 \\ &\approx 3.1416 - \frac{(3.1416)^3}{3!} + \frac{(3.1416)^5}{5!} - \frac{(3.1416)^7}{7!} \\ &\quad + \frac{(3.1416)^9}{9!} \\ &\approx 3.1416 - 5.1677 + 2.5502 - 0.5993 + 0.0821 \\ &\approx 0.0069\end{aligned}$$
10. $\sqrt{2}\left(\cos \frac{3\pi}{4} + \boldsymbol{i} \sin \frac{3\pi}{4}\right) = \sqrt{2}e^{\boldsymbol{i}\frac{3\pi}{4}}$
11. $$\begin{aligned}-1 + \sqrt{3}\boldsymbol{i} &= 2\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\boldsymbol{i}\right) \\ &= 2\left(\cos \frac{2\pi}{3} + \boldsymbol{i} \sin \frac{2\pi}{3}\right) \\ &= 2e^{\boldsymbol{i}\frac{2\pi}{3}}\end{aligned}$$

12a. $A = Pe^{rt}$

$2P = Pe^{(0.06)5}$

$P = e^{0.3}$

$= 1 + 0.3 + \frac{(0.3)^2}{2!}$

≈ 1.345

approximately $1.345P$

12b. No, in five years she will have increased her savings by about 34.5%, not 100%.

12c. The approximation is accurate to two decimal places.

Pages 812–814 Exercises

13. $\ln(-4) = \ln(-1) + \ln 4 \approx \boldsymbol{i}\pi + 1.3863$

14. $\ln(-3.1) = \ln(-1) + \ln(3.1) \approx \boldsymbol{i}\pi + 1.1314$

15. $\ln(-0.25) = \ln(-1) + \ln(0.25) \approx \boldsymbol{i}\pi - 1.3863$

16. $\ln(-0.033) = \ln(-1) + \ln(0.033) \approx \boldsymbol{i}\pi - 3.4112$

17. $\ln(-238) = \ln(-1) + \ln(238) \approx \boldsymbol{i}\pi + 5.4723$

18. $\ln(-1207) = \ln(-1) + \ln(1207) \approx \boldsymbol{i}\pi + 7.0959$

19. $e^{1.1} = 1 + 1.1 + \frac{(1.1)^2}{2!} + \frac{(1.1)^3}{3!} + \frac{(1.1)^4}{4!}$

$\approx 1 + 1.1 + 0.605 + 0.222 + 0.06$

≈ 2.99

20. $e^{-0.2} = 1 + (-0.2) + \frac{(-0.2)^2}{2!} + \frac{(-0.2)^3}{3!} + \frac{(-0.2)^4}{4!}$

$\approx 1 - 0.2 + 0.02 - 0.0013 + 0.00007$

≈ 0.82

21. $e^{4.2} = 1 + 4.2 + \frac{(4.2)^2}{2!} + \frac{(4.2)^3}{3!} + \frac{(4.2)^4}{4!}$

$\approx 1 + 4.2 + 8.82 + 12.348 + 12.965$

≈ 39.33

22. $e^{0.55} = 1 + 0.55 + \frac{(0.55)^2}{2!} + \frac{(0.55)^3}{3!} + \frac{(0.55)^4}{4!}$

$\approx 1 + 0.55 + 0.151 + 0.028 + 0.004$

≈ 1.73

23. $e^{3.5} = 1 + 3.5 + \frac{(3.5)^2}{2!} + \frac{(3.5)^3}{3!} + \frac{(3.5)^4}{4!}$

$\approx 1 + 3.5 + 6.125 + 7.146 + 6.253$

≈ 24.02

24. $e^{2.73} = 1 + 2.73 + \frac{(2.73)^2}{2!} + \frac{(2.73)^3}{3!} + \frac{(2.73)^4}{4!}$

$\approx 1 + 2.73 + 3.726 + 3.391 + 2.314$

≈ 13.16

25. $\cos x \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!}$

$\cos \pi \approx \cos 3.1416$

$\approx 1 - \frac{(3.1416)^2}{2!} + \frac{(3.1416)^4}{4!} - \frac{(3.1416)^6}{6!} + \frac{(3.1416)^8}{8!}$

$\approx 1 - 4.9348 + 4.0588 - 1.3353 + 0.2353$

≈ -0.9760

actual value: $\cos \pi = -1$

26. $\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$

$\sin \frac{\pi}{4} \approx \sin 0.7854$

$\approx 0.7854 - \frac{(0.7854)^3}{3!} + \frac{(0.7854)^5}{5!} - \frac{(0.7854)^7}{7!}$

$+ \frac{(0.7854)^9}{9!}$

$\approx 0.7854 - \frac{0.4845}{6} + \frac{0.2989}{120} - \frac{0.1843}{5040} + \frac{0.1137}{362{,}880}$

≈ 0.7071

actual value: $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} \approx 0.7071$

27. $\cos x \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!}$

$\cos \frac{\pi}{6} \approx \cos 0.5236$

$\approx 1 - \frac{(0.5236)^2}{2!} + \frac{(0.5236)^4}{4!} - \frac{(0.5236)^6}{6!} + \frac{(0.5236)^8}{8!}$

$\approx 1 - \frac{0.2742}{2} + \frac{0.7516}{24} - \frac{0.0206}{720} + \frac{0.0056}{40{,}320}$

≈ 0.8660

actual value: $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \approx 0.8660$

28. $\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$

$\sin \frac{\pi}{2} \approx \sin 1.5708$

$\approx 1.5708 - 0.6460 + 0.0797 - 0.0047 + 0.0002$

≈ 1.0000

actual value: $\sin \frac{\pi}{2} = 1$

29. $5\left(\cos \frac{5\pi}{3} + \boldsymbol{i} \sin \frac{5\pi}{3}\right) = 5e^{i\frac{5\pi}{3}}$

30. $\boldsymbol{i} = \cos \frac{\pi}{2} + \boldsymbol{i} \sin \frac{\pi}{2} = e^{\boldsymbol{i}\frac{\pi}{2}}$

31. $r = \sqrt{1^2 + 1^2}$ or $\sqrt{2}$

$Q = \text{Arctan } \frac{1}{1}$ or $\frac{\pi}{4}$

$1 + \boldsymbol{i} = \sqrt{2}\left(\cos \frac{\pi}{4} + \boldsymbol{i} \sin \frac{\pi}{4}\right) = \sqrt{2}e^{\boldsymbol{i}\frac{\pi}{4}}$

32. $r = \sqrt{(\sqrt{3})^2 + 1^2}$ or 2

$\theta = \text{Arctan } \frac{1}{\sqrt{3}}$ or $\frac{\pi}{6}$

$\sqrt{3} + \boldsymbol{i} = 2\left(\cos \frac{\pi}{6} + \boldsymbol{i} \sin \frac{\pi}{6}\right) = 2e^{\boldsymbol{i}\frac{\pi}{6}}$

33. $r = \sqrt{\left(-\sqrt{2}\right)^2 - \left(\sqrt{2}\right)^2}$ or 2

$\theta = \text{Arctan}\left(\frac{\sqrt{2}}{-\sqrt{2}}\right)$ or $3\frac{\pi}{4}$

$-\sqrt{2} + \sqrt{2}\boldsymbol{i} = 2\left(\cos \frac{3\pi}{4} + \boldsymbol{i} \sin \frac{3\pi}{4}\right) = 2e^{\boldsymbol{i}\frac{3\pi}{4}}$

34. $r = \sqrt{\left(-4\sqrt{3}\right)^2 + (-4)^2}$ or 8

$\theta = \text{Arctan}\left(\frac{-4}{-4\sqrt{3}}\right) + \pi$ or $\frac{7\pi}{6}$

$-4\sqrt{3} - 4\boldsymbol{i} = 8\left(\cos \frac{7\pi}{6} + \boldsymbol{i} \sin \frac{7\pi}{6}\right) = 8e^{\boldsymbol{i}\frac{7\pi}{6}}$

35. $r = \sqrt{3^2 + 3^2}$ or $3\sqrt{2}$

$\theta = \text{Arctan}\left(\frac{3}{3}\right)$ or $\frac{\pi}{4}$

$3 + 3\boldsymbol{i} = 3\sqrt{2}\left(\cos \frac{\pi}{4} + \boldsymbol{i} \sin \frac{\pi}{4}\right) = 3\sqrt{2}e^{\boldsymbol{i}\frac{\pi}{4}}$

36. Sample answer: A transcendental number is one that cannot be the root of an algebraic equation with rational coefficients. Examples are π and e.

37. $\frac{e^{ix} - e^{-ix}}{2\boldsymbol{i}} = \frac{\cos x + \boldsymbol{i} \sin x - (\cos x - \boldsymbol{i} \sin x)}{2\boldsymbol{i}}$

$= \frac{2\boldsymbol{i} \sin x}{2\boldsymbol{i}}$

$= \sin x$

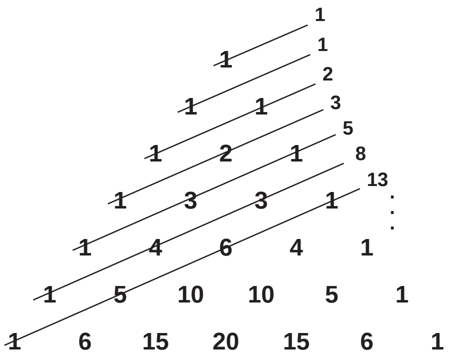
$\frac{e^{ix} + e^{-ix}}{2} = \frac{\cos x + \boldsymbol{i} \sin x + \cos x - \boldsymbol{i} \sin x}{2}$

$= \frac{2 \cos x}{2}$

$= \cos x$

38. See students' work.

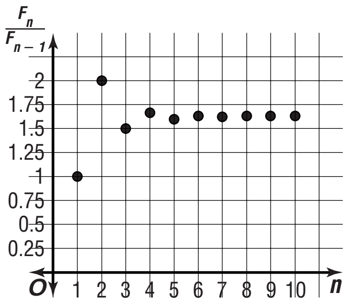
39. If you add the numbers on the diagonal lines as shown, the sums are the terms of the Fibonacci sequence.



40a. $\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \frac{55}{34}, \frac{89}{55}$

40b. neither

40c.



40d. yes; 1.618

40e. The two ratios are equivalent to three decimal places.

40f. See students' work.

41a. $A = Pe^{rt}$

$= 5000e^{0.05(13)}$

$= 5000e^{0.65}$

$\approx 5000\left[1 + 0.65 + \frac{(0.65)^2}{2!} + \frac{(0.65)^3}{3!} + \frac{(0.65)^4}{4!}\right]$

$\approx \$9572.29$

41b. No, the account will be short by more than \$30,000.

41c. about 42 years; 47 years old

41d. $40{,}000 = Pe^{0.65}$

$\$20{,}882 = P$

42a. Every third Fibonacci number is an even number.

42b. Every fourth Fibonacci number is a multiple of 3.

43.
$$(2x + y)^6 = (2x)^6(y)^0 + 6(2x)^5(y)^1 + \frac{6 \cdot 5(2x)^4(y)^2}{2 \cdot 1} + \frac{6 \cdot 5 \cdot 4(2x)^3(y)^3}{3 \cdot 2 \cdot 1} + \frac{6 \cdot 5 \cdot 4 \cdot 3(2x)^2(y)^4}{4 \cdot 3 \cdot 2 \cdot 1} + \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2(2x)^1(y)^5}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} + \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1(2x)^0(y)^6}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$
$$= 64x^6 + 192x^5y + 240x^4y^2 + 160x^3y^3 + 60x^2y^4 + 12xy^5 + y^6$$

44. $\sum_{k=1}^{6} 2^k$

45a. $a_1 = 0.005$, $r = \frac{0.01}{0.005}$ or 2

$a_3 = 0.005(2)^{3-1}$

$= 0.020$ cm

$a_4 = 0.020(2)$

$= 0.040$ cm

45b. $0.005(2)^{n-1}$

45c. $a_{10} = 0.005(2)^{10-1}$

$= 2.56$ cm

$a_{100} = 0.005(2)^{100-1}$

$\approx 3.169 \times 10^{27}$ cm

46. $\frac{8^{\frac{2}{3}}}{8^{\frac{1}{3}}} = 8^{\frac{1}{3}}$ or 2

47. $y^2 + Dx + Ey + F = 0$

$(0, 0)$: $F = 0$

$(2, -1)$: $1 + 2D - E + F = 0$

$(4, -4)$: $16 + 4D - 4E + F = 0$

$$\begin{aligned} 2D - E &= -1 \\ 4D - 4E &= -16 \end{aligned} \quad \rightarrow \quad \begin{aligned} 4D - 2E &= -2 \\ \underline{4D - 4E} &\underline{= -16} \\ 2E &= 14 \\ E &= 7 \end{aligned}$$

$2D - E = -1$

$2D - 7 = -1$

$2D = 6$

$D = 3$

$y^2 + 3x + 7y = 0$

48. $r = \frac{14}{6}$ or 4

$\theta = \frac{\pi}{8} - \frac{\pi}{4}$

$= -\frac{\pi}{8}$ or $\frac{15\pi}{8}$

$4\left(\cos \frac{15\pi}{8} + \boldsymbol{i} \sin \frac{15\pi}{8}\right)$

49. $\vec{\mathbf{r}}^{\,2} = 30^2 + 50^2 - 2(30)(50) \cos 140°$

$\vec{\mathbf{r}} = 75.5$ N

$\frac{75.5}{\sin 140°} = \frac{30}{\sin \theta}$

$\sin \theta = \frac{30 \sin 140°}{75.5}$

$\theta = \text{Arcsin}\left(\frac{30 \sin 140°}{75.5}\right)$

$= 14°48'$

50a. $\omega = \frac{\theta}{t}$

$= \frac{5(2\pi) \text{ radians}}{1 \text{ second}}$

$= 10\pi$ radians per second

50b. $v = r\frac{\theta}{t}$

$= \left(\frac{1}{2} \text{ ft}\right)(10\pi \text{ radians/s})$

≈ 15.7 ft/s

51. Let m = multiple choice. Let e = essay.

$m + e \leq 30$

$1m + 12e \leq 96$

$f(m, e) = 5m + 20e$

$f(24, 6) = 5(24) + 20(6)$

$= 240$

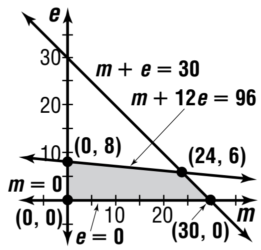
$f(0, 8) = 5(0) + 20(8)$

$= 160$

$f(30, 0) = 5(30) + 20(0)$

$= 150$

$f(0, 0) = 0$



To receive the highest score, answer 24 multiple choice and 6 essay.

52. $(DC)^2 + 2^2 = \left(\sqrt{18}\right)^2$
$(DC)^2 = 14$
$DC = \sqrt{14}$
$\left(\sqrt{14}\right)^2 + 5^2 = (BC)^2$
$39 = (BC)^2$
$\sqrt{39} = BC$ The correct choice is C.

12-8 Sequences and Iteration

Page 819 Check for Understanding

1. Iteration is the repeated composition of a function upon itself.
2. It is the sequence of iterates produced when a complex number is iterated for a function $f(z)$.
3. If the prisoner set is connected, then the Julia set is the boundary between the prisoner set and the escape set. If the prisoner set is disconnected, then the Julia set is the prisoner set.
4. $f(-1) = (-1)^2 = 1$
$f(1) = 1^2 = 1$
$f(1) = 1^2 = 1$
$f(1) = 1^2 = 1$
1, 1, 1, 1
5. $f(2) = 2 \cdot 2 - 5 = -1$
$f(-1) = 2 \cdot (-1) - 5 = -7$
$f(-7) = 2 \cdot (-7) - 5 = -19$
$f(-19) = 2 \cdot (-19) - 5 = -43$
$-1, -7, -19, -43$
6. $z_0 = 6\boldsymbol{i}$
$z_1 = 0.6(6\boldsymbol{i}) + 2\boldsymbol{i} = 5.6\boldsymbol{i}$
$z_2 = 0.6(5.6\boldsymbol{i}) + 2\boldsymbol{i} = 5.36\boldsymbol{i}$
$z_3 = 0.6(5.36\boldsymbol{i}) + 2\boldsymbol{i} = 5.216\boldsymbol{i}$
7. $z_0 = 25 + 40\boldsymbol{i}$
$z_1 = 0.6(25 + 40\boldsymbol{i}) + 2\boldsymbol{i} = 15 + 26\boldsymbol{i}$
$z_2 = 0.6(15 + 26\boldsymbol{i}) + 2\boldsymbol{i} = 9 + 17.6\boldsymbol{i}$
$z_3 = 0.6(9 + 17.6\boldsymbol{i}) + 2\boldsymbol{i} = 5.4 + 12.56\boldsymbol{i}$
8. $z_0 = 0$, $f(z) = z^2 + (1 + 2\boldsymbol{i})$
$z_1 = 0^2 + (1 + 2\boldsymbol{i}) = 1 + 2\boldsymbol{i}$
$z_2 = (1 + 2\boldsymbol{i})^2 + (1 + 2\boldsymbol{i})$
$= 1 + 4\boldsymbol{i} + 4\boldsymbol{i}^2 + 1 + 2\boldsymbol{i} = -2 + 6\boldsymbol{i}$
$z_3 = (-2 + 6\boldsymbol{i})^2 + (1 + 2\boldsymbol{i})$
$= 4 - 24\boldsymbol{i} + 35\boldsymbol{i}^2 + 1 + 2\boldsymbol{i} = -31 - 22\boldsymbol{i}$
9. $z_0 = 1 + 2\boldsymbol{i}$, $f(z) = z^2 + (2 - 3\boldsymbol{i})$
$z_1 = (1 + 2\boldsymbol{i})^2 + (2 - 3\boldsymbol{i})$
$= 1 + 4\boldsymbol{i} + 4\boldsymbol{i}^2 + 2 - 3\boldsymbol{i} = -1 + \boldsymbol{i}$
$z_2 = (-1 + \boldsymbol{i})^2 + (2 - 3\boldsymbol{i})$
$= 1 - 2\boldsymbol{i} + \boldsymbol{i}^2 + 2 - 3\boldsymbol{i} = 2 - 5\boldsymbol{i}$
$z_3 = (2 - 5\boldsymbol{i})^2 + (2 - 3\boldsymbol{i})$
$= 4 - 20\boldsymbol{i} + 25\boldsymbol{i}^2 + 2 - 3\boldsymbol{i} = -19 - 23\boldsymbol{i}$
10. $p_n = \frac{24}{40} = 0.60$
$p_1 = 0.60 + 1.75(0.60)(1 - 0.60)$
$= 1.02$
$(1.02)(40) \approx 41$
$p_2 = 1.02 + 1.75(1.02)(1 - 1.02)$
$= 0.9843$
$(0.9843)(40) \approx 39$
$p_3 = 0.9843 + 1.75(0.9843)(1 - 0.9843)$
$= 1.0113$
$(1.0113)(40) \approx 40$
$p_4 = 1.0113 + 1.75(1.0113)(1 - 1.0113)$
$= 0.9913$
$(0.9913)(40) \approx 40$
$p_5 = 0.9913 + 1.75(0.9913)(1 - 0.9913)$
$= 1.0064$
$(1.0064)(40) \approx 40$
$p_6 = 1.0064 + 1.75(1.0064)(1 - 1.0064)$
≈ 0.9951
$(0.9951)(40) \approx 40$
$p_7 = 0.9951 + 1.75(0.9951)(1 - 0.9951)$
≈ 1.0036
$(1.0036)(40) \approx 40$
$p_8 = 1.0036 + 1.75(1.0036)(1 - 1.0036)$
≈ 0.9973
$(0.9973)(40) \approx 40$
$p_9 = 0.9973 + 1.75(0.9973)(1 - 0.9973)$
≈ 1.002
$(1.002)(40) \approx 40$
$p_{10} = 1.002 + 1.75(1.002)(1 - 1.002)$
≈ 0.9985
$(0.9985)(40) \approx 40$
41, 39, 40, 40, 40, 40, 40, 40, 40, 40

Pages 820–821 Exercises

11. $f(x_0) = f(4)$
$= 3(4) - 7$
$= 5$
$f(x_1) = f(5)$
$= 3(5) - 7$
$= 8$
$f(x_2) = f(8)$
$= 3(8) - 7$
$= 17$
$f(x_3) = f(17)$
$= 3(17) - 7$
$= 44$
12. $f(-2) = (-2)^2 = 4$
$f(4) = 4^2 = 16$
$f(16) = 16^2 = 256$
$f(256) = 256^2 = 65.536$
13. $f(4) = (4 - 5)^2 = 1$
$f(1) = (1 - 5)^2 = 16$
$f(16) = (16 - 5)^2 = 121$
$f(121) = (121 - 5)^2 = 13{,}456$
14. $f(-1) = (-1)^2 - 1 = 0$
$f(0) = 0^2 - 1 = -1$
$f(-1) = (-1)^2 - 1 = 0$
$f(0) = 0^2 - 1 = -1$

15. $f(0.1) = 2(0.1)^2 - 0.1$
$= -0.08$
$f(-0.08) = 2(-0.08)^2 - (-0.08)$
≈ 0.09
$f(0.09) = 2(0.09)^2 - 0.09$
≈ -0.07
$f(-0.07) = 2(-0.07)^2 - (-0.07)$
≈ 0.08

16a. $t_1 = \frac{2}{1} = 2$
$t_2 = \frac{2}{2} = 1$
$t_3 = \frac{2}{1} = 2$
$t_4 = \frac{2}{2} = 1$
$\vdots$
$t_{10} = \frac{2}{2} = 1$

16b. $t_1 = \frac{2}{4} = \frac{1}{2}$
$t_2 = \frac{2}{\frac{1}{2}} = 4$
$t_3 = \frac{2}{4} = \frac{1}{2}$
$t_4 = \frac{2}{\frac{1}{2}} = 4$
$\vdots$
$t_{10} = \frac{2}{\frac{1}{2}} = 4$

16c. $t_1 = \frac{2}{7}$
$t_2 = \frac{2}{\frac{2}{7}} = 7$
$t_3 = \frac{2}{7}$
$t_4 = \frac{2}{\frac{2}{7}} = 7$
$\vdots$
$t_{10} = \frac{2}{\frac{2}{7}} = 7$

16d. The values of the iterates alternate between $\frac{2}{x_0}$ and x_0.

17. $z_0 = 5\boldsymbol{i}$
$z_1 = 2(5\boldsymbol{i}) + (3 - 2\boldsymbol{i})$
$= 3 + 8\boldsymbol{i}$
$z_2 = 2(3 + 8\boldsymbol{i}) + (3 - 2\boldsymbol{i})$
$= 6 + 16\boldsymbol{i} + 3 - 2\boldsymbol{i}$
$= 9 + 14\boldsymbol{i}$
$z_3 = 2(9 + 14\boldsymbol{i}) + (3 - 2\boldsymbol{i})$
$= 18 + 28\boldsymbol{i} + 3 - 2\boldsymbol{i}$
$= 21 + 26\boldsymbol{i}$

18. $z_0 = 4$
$z_1 = 2(4) + (3 - 2\boldsymbol{i})$
$= 11 - 2\boldsymbol{i}$
$z_2 = 2(11 - 2\boldsymbol{i}) + (3 - 2\boldsymbol{i})$
$= 22 - 4\boldsymbol{i} + 3 - 2\boldsymbol{i}$
$= 25 - 6\boldsymbol{i}$
$z_3 = 2(25 - 6\boldsymbol{i}) + (3 - 2\boldsymbol{i})$
$= 50 - 12\boldsymbol{i} + 3 - 2\boldsymbol{i}$
$= 53 - 14\boldsymbol{i}$

19. $z_0 = 1 + 2\boldsymbol{i}$
$z_1 = 2(1 + 2\boldsymbol{i}) + (3 - 2\boldsymbol{i})$
$= 2 + 4\boldsymbol{i} + 3 - 2\boldsymbol{i}$
$= 5 + 2\boldsymbol{i}$
$z_2 = 2(5 + 2\boldsymbol{i}) + (3 - 2\boldsymbol{i})$
$= 10 + 4\boldsymbol{i} + 3 - 2\boldsymbol{i}$
$= 13 + 2\boldsymbol{i}$
$z_3 = 2(13 + 2\boldsymbol{i}) + (3 - 2\boldsymbol{i})$
$= 26 + 4\boldsymbol{i} + 3 - 2\boldsymbol{i}$
$= 29 + 2\boldsymbol{i}$

20. $z_0 = -1 - 2\boldsymbol{i}$
$z_1 = 2(-1 - 2\boldsymbol{i}) + (3 - 2\boldsymbol{i})$
$= -2 - 4\boldsymbol{i} + 3 - 2\boldsymbol{i}$
$= 1 - 6\boldsymbol{i}$
$z_2 = 2(1 - 6\boldsymbol{i}) + (3 - 2\boldsymbol{i})$
$= 2 - 12\boldsymbol{i} + 3 - 2\boldsymbol{i}$
$= 5 - 14\boldsymbol{i}$
$z_3 = 2(5 - 14\boldsymbol{i}) + (3 - 2\boldsymbol{i})$
$= 10 - 28\boldsymbol{i} + 3 - 2\boldsymbol{i}$
$= 13 - 30\boldsymbol{i}$

21. $z_0 = 6 + 2\boldsymbol{i}$
$z_1 = 2(6 + 2\boldsymbol{i}) + (3 - 2\boldsymbol{i})$
$= 12 + 4\boldsymbol{i} + 3 - 2\boldsymbol{i}$
$= 15 + 2\boldsymbol{i}$
$z_2 = 2(15 + 2\boldsymbol{i}) + (3 - 2\boldsymbol{i})$
$= 30 + 4\boldsymbol{i} + 3 - 2\boldsymbol{i}$
$= 33 + 2\boldsymbol{i}$
$z_3 = 2(33 + 2\boldsymbol{i}) + (3 - 2\boldsymbol{i})$
$= 66 + 4\boldsymbol{i} + 3 - 2\boldsymbol{i}$
$= 69 + 2\boldsymbol{i}$

22. $z_0 = 0.3 - \boldsymbol{i}$
$z_1 = 2(0.3 - \boldsymbol{i}) + (3 - 2\boldsymbol{i})$
$= 0.6 - 2\boldsymbol{i} + 3 - 2\boldsymbol{i}$
$= 3.6 - 4\boldsymbol{i}$
$z_2 = 2(3.6 - 4\boldsymbol{i}) + (3 - 2\boldsymbol{i})$
$= 7.2 - 8\boldsymbol{i} + 3 - 2\boldsymbol{i}$
$= 10.2 - 10\boldsymbol{i}$
$z_3 = 2(10.2 - 10\boldsymbol{i}) + (3 - 2\boldsymbol{i})$
$= 20.4 - 20\boldsymbol{i} + 3 - 2\boldsymbol{i}$
$= 23.4 - 22\boldsymbol{i}$

23. $z_0 = \frac{1}{3} + \frac{2}{3}\boldsymbol{i}$
$z_1 = 3\left(\frac{1}{3} + \frac{2}{3}\boldsymbol{i}\right) - 2\boldsymbol{i}$
$= 1 + 2\boldsymbol{i} - 2\boldsymbol{i}$
$= 1$
$z_2 = 3(1) - 2\boldsymbol{i}$
$= 3 - 2\boldsymbol{i}$
$z_3 = 3(3 - 2\boldsymbol{i}) - 2\boldsymbol{i}$
$= 9 - 6\boldsymbol{i} - 2\boldsymbol{i}$
$= 9 - 8\boldsymbol{i}$

24. $z_0 = 0 - \boldsymbol{i}$, $f(z) = z^2 - 1$
$z_1 = (-\boldsymbol{i})^2 - 1$
$= -2$
$z_2 = (-2)^2 - 1$
$= 3$
$z_3 = 3^2 - 1$
$= 8$

25. $z_0 = \boldsymbol{i}, f(z) = z^2 + 1 - 3\boldsymbol{i}$
$z_1 = \boldsymbol{i}^2 + 1 - 3\boldsymbol{i}$
$= -3\boldsymbol{i}$
$z_2 = (-3\boldsymbol{i})^2 + 1 - 3\boldsymbol{i}$
$= -8 - 3\boldsymbol{i}$
$z_3 = (-8 - 3\boldsymbol{i})^2 + 1 - 3\boldsymbol{i}$
$= 64 + 48\boldsymbol{i} - 9 + 1 - 3\boldsymbol{i}$
$= 56 + 45\boldsymbol{i}$

26. $z_0 = 1, f(z) = z^2 + 3 + 2\boldsymbol{i}$
$z_1 = 1^2 + 3 + 2\boldsymbol{i}$
$= 4 + 2\boldsymbol{i}$
$z_2 = (4 + 2\boldsymbol{i})^2 + 3 + 2\boldsymbol{i}$
$= 16 + 16\boldsymbol{i} - 4 + 3 + 2\boldsymbol{i}$
$= 15 + 18\boldsymbol{i}$
$z_3 = (15 + 18\boldsymbol{i})^2 + 3 + 2\boldsymbol{i}$
$= 225 + 540\boldsymbol{i} - 324 + 3 + 2\boldsymbol{i}$
$= -96 + 542\boldsymbol{i}$

27. $z_0 = 1 + \boldsymbol{i}, f(z) = z^2 - 4\boldsymbol{i}$
$z_1 = (1 + \boldsymbol{i})^2 - 4\boldsymbol{i}$
$= 1 + 2\boldsymbol{i} - 1 - 4\boldsymbol{i}$
$= -2\boldsymbol{i}$
$z_2 = (-2\boldsymbol{i})^2 - 4\boldsymbol{i}$
$= -4 - 4\boldsymbol{i}$
$z_3 = (-4 - 4\boldsymbol{i})^2 - 4\boldsymbol{i}$
$= 16 + 32\boldsymbol{i} - 16 - 4\boldsymbol{i}$
$= 28\boldsymbol{i}$

28. $z_0 = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\boldsymbol{i}, f(z) = z^2$
$z_1 = \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\boldsymbol{i}\right)^2 = \frac{1}{2} - \boldsymbol{i} + \frac{1}{2}\boldsymbol{i}^2 = -1$
$z_2 = (-\boldsymbol{i})^2 = -1$
$z_3 = 1^2 = 1$

29. $z_0 = 1 - \boldsymbol{i}, f(z) = z^2 + 2 + 3\boldsymbol{i}$
$z_1 = (1 - \boldsymbol{i})^2 + 2 + 3\boldsymbol{i}$
$= 1 - 2\boldsymbol{i} - 1 + 2 + 3\boldsymbol{i}$
$= 2 + \boldsymbol{i}$
$z_2 = (2 + \boldsymbol{i})^2 + 2 + 3\boldsymbol{i}$
$= 4 + 4\boldsymbol{i} - 1 + 2 + 3\boldsymbol{i}$
$= 5 + 7\boldsymbol{i}$
$z_3 = (5 + 7\boldsymbol{i})^2 + 2 + 3\boldsymbol{i}$
$= 25 + 70\boldsymbol{i} - 49 + 2 + 3\boldsymbol{i}$
$= -22 + 73\boldsymbol{i}$

30. $p_1 = p_0 + rp_0$
$p_1 = 2000 + (0.052)(2000)$
$= \$2104$
$p_2 = 2104 + (0.052)(2104)$
$= \$2213.41$
$p_3 = 2213.41 + (0.052)(2213.14)$
$= \$2328.51$
$p_4 = 2328.51 + (0.052)(2328.51)$
$= \$2449.59$
$p_5 = 2449.59 + (0.052)(2449.59)$
$= \$2576.97$

31.

Iteration	x = percent	$x + 2.5x(1 - x)$
1	0.10	0.325
2	0.325	0.8734
3	0.8734	1.1498
4	1.1498	0.7192
5	0.7192	1.2241
6	1.2241	0.5383
7	0.5383	1.1596
8	1.1596	0.6969
9	0.6969	1.225
10	1.225	0.5359
11	0.5359	1.1577
12	1.1577	0.7013
13	0.7013	1.225
14	1.225	0.5359
15	0.5359	1.1577
16	1.1577	0.7013
17	0.7013	1.225
18	1.225	0.5359

$2002 - 1984 = 18$; After 18 years, about 54% of the maximum sustainable population is present.

32. $f(z) = z^2 + c$
$-1 + 15\boldsymbol{i} = (2 + 3\boldsymbol{i})^2 + c$
$-1 + 15\boldsymbol{i} = 4 + 12\boldsymbol{i} + 9\boldsymbol{i}^2 + c$
$4 + 3\boldsymbol{i} = c$

33. $\pm\sqrt{2}$
$x_1 = \frac{2}{\sqrt{2}} = \sqrt{2}$
$x_1 = \frac{2}{\sqrt{2}} = \sqrt{2}$
$\vdots$

34. See students' work. Sample topics for discussion are judging soil quality and detection of heat stress in cows.

35a. 1.414213562, 1.189207115, 1.090507733, 1.044273782

35b. $f(z) = \sqrt{z}, z_0 = 2$

35c. 1

35d. 1

36. $2\left(\cos\frac{\pi}{3} + \boldsymbol{i}\sin\frac{\pi}{3}\right) = 2e^{\boldsymbol{i}\frac{\pi}{3}}$

37. $\frac{8!}{4!(8-4)!}(2a)^{8-4}(-3b)^4 = 70 \cdot 16a^4 \cdot 81b^4$
$= 90{,}720a^4b^4$

38. Convergent; the series is geometric with $|r| = \frac{1}{4} < 1$.

39. The distance between the vertices is 130 ft.
$2a = 130$, so $a = 75$.
$e = \frac{c}{a} = \frac{7}{5} = \frac{91}{65}$
$b^2 = c^2 - a^2$
$b^2 = 91^2 - 65^2$
$= 4056$
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{4225} - \frac{y^2}{4056} = 1$

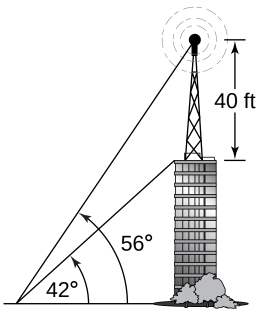
40. $n_1 \sin I = n_2 \sin r$

$1.00 \sin 42° = 2.42 \sin r$

$r = \text{Arcsin}\left[\frac{1.00 \sin 42°}{2.42}\right]$

$r \approx 16°$

41a.



41b. Let h = height of the building.

Let x = distance from the point of elevation to the center of the base of the building.

$\tan 42° = \frac{h}{x}$

$x = \frac{h}{\tan 42°}$

$\tan 56° = \frac{h + 40}{x}$

$\tan 56° = \frac{h + 40}{\frac{h}{\tan 42°}}$

$\tan 56° = \frac{(h + 40) \tan 42°}{h}$

$1.6466 \approx 1 + \frac{40}{h}$

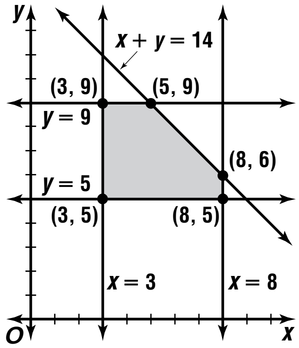
$0.6466 = \frac{40}{h}$

$h \approx 62$ feet

No, the height of the building is about 62 feet for a total of about 102 feet with the tower.

42. $x \neq -2$ for all x, so infinite discontinuity

43.



$f(x, y) = 2x + 8y + 10$

$f(3, 9) = 2(3) + 8(9) + 10$
$= 88$

$f(5, 9) = 2(5) + 8(9) + 10$
$= 92$

$f(8, 6) = 2(8) + 8(6) + 10$
$= 74$

$f(8, 5) = 2(8) + 8(5) + 10$
$= 66$

$f(3, 5) = 2(3) + 8(5) + 10$
$= 56$

max: 92, min: 56

44. $H - L = \frac{H + L}{2}$

$2H - 2L = H + L$

$H = 3L$

$\frac{H}{L} = 3$ The correct choice is D.

12-9 Mathematical Induction

Page 826 Check for Understanding

1. The $n + 1$ case shows that the premise is true for an infinite number of cases.

2. Provide a counterexample.

3a. $n(n + 2)$

3b. Since 3 is the first term in the sequence of partial sums and $1(1 + 2) = 3$, the formula is valid for $n = 1$.

Since 8 is the second term in the sequence of partial sums and $2(2 + 2) = 8$, the formula is valid for $n = 2$.

Since 15 is the third term in the sequence of partial sums and $3(3 + 2) = 15$, the formula is valid for $n = 3$.

3c. $S_k \Rightarrow k(k + 2)$; $S_{k+1} \Rightarrow (k + 1)(k + 3)$

4. $8^n - 1 = 7r$ for some integer r.

5. Sample answer: If we wish to prove that we can climb a ladder with an indefinite number of steps, we must prove the following. First, we must show that we can climb off the ground to rung 1. Next, we must show that if we can climb to rung k, then we can climb to rung $k + 1$.

6. Step 1: Verify that the formula is valid for $n = 1$. Since 3 is the first term in the sequence and $1(1 + 2) = 3$, the formula is valid for $n = 1$.
Step 2: Assume that the formula is valid for $n = k$ and derive a formula for $n = k + 1$.

$S_k \Rightarrow 3 + 5 + 7 + \ldots + (2k + 1) = k(k + 2)$

$S_{k+1} \Rightarrow 3 + 5 + 7 + \ldots$
$+ (2k + 1) + (2k + 3) = k(k + 2) + (2k + 3)$
$= k^2 + 4k + 3$
$= (k + 1)(k + 3)$

Apply the original formula for $n = k + 1$.

$(k + 1)[(k + 1) + 2] = (k + 1)(k + 3)$

The formula gives the same result as adding the $(k + 1)$ term directly. Thus if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Thus, the formula is valid for all positive integral values of n.

7. Step 1: Verify that the formula is valid for $n = 1$. Since 2 is the first term in the sequence and $2(2^1 - 1) = 2$, the formula is valid for $n = 1$.
Step 2: Assume that the formula is valid for $n = k$ and derive a formula for $n = k + 1$.

$S_k \Rightarrow 2 + 2^2 + 2^3 + \ldots + 2^k = 2(2^k - 1)$

$S_{k+1} \Rightarrow 2 + 2^2 + 2^3 + \ldots + 2^k + 2^{k+1}$
$= 2(2^k - 1) + 2^{k+1}$
$= 2 \cdot 2^{k+1} - 2$
$= 2(2^{k+1} - 1)$

When the original formula is applied for $n = k + 1$, the same result is obtained. Thus if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Thus, the formula is valid for all positive integral values of n.

8. Step 1: Verify that the formula is valid for $n = 1$. Since $\frac{1}{2}$ is the first term in the sequence and $1 - \frac{1}{2^1} = \frac{1}{2}$, the formula is valid for $n = 1$.

Step 2: Assume that the formula is valid for $n = k$ and derive a formula for $n = k + 1$.

$S_k \Rightarrow \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots + \frac{1}{2^k} = 1 - \frac{1}{2^k}$

$$\begin{aligned} S_{k+1} \Rightarrow \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots + \frac{1}{2^k} + \frac{1}{2^{k+1}} &= 1 - \frac{1}{2^k} + \frac{1}{2^{k+1}} \\ &= 1 - \frac{2}{2 \cdot 2^k} + \frac{1}{2^{k+1}} \\ &= 1 - \frac{2}{2^{k+1}} + \frac{1}{2^{k+1}} \\ &= 1 - \frac{1}{2^{k+1}} \end{aligned}$$

When the original formula is applied for $n = k + 1$, the same result is obtained. Thus if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Thus, the formula is valid for all positive integral values of n.

9. S_n: $3^n - 1 = 2r$ for some integer r

Step 1: Verify that S_n is valid for $n = 1$.

$S_1 \Rightarrow 3^1 - 1$ or 2. Since $2 = 2 \cdot 1$, S_n is valid for $n = 1$.

Step 2: Assume that S_n is valid for $n = k$ and show that it is also valid for $n = k + 1$.

$S_k \Rightarrow 3^k - 1 = 2r$ for some integer r

$S_{k+1} \Rightarrow 3^{k+1} - 1 = 2t$ for some integer t

$$\begin{aligned} 3^k - 1 &= 2r \\ 3(3^k - 1) &= 3 \cdot 2r \\ 3^{k+1} - 3 &= 6r \\ 3^{k+1} - 1 &= 6r + 2 \\ 3^{k+1} - 1 &= 2(3r + 1) \end{aligned}$$

Thus, $3^{k+1} - 1 = 2t$, where $t = 3r + 1$ is an integer, and we have shown that if S_n is valid, then S_{k+1} is also valid. Since S_n is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Hence, $3^n - 1$ is divisible by 2 for all integral values of n.

10a. $6 + 4 = 10$
$10 + 5 = 15$
$15 + 6 = 21$
$21 + 7 = 28$
$28 + 8 = 36$
10, 15, 21, 28, 36

10b. $a_n = \frac{n(n+1)}{2}$

10c. Step 1: Verify that the formula is valid for $n = 1$. Since 1 is the first term in the sequence and $\frac{1(1+1)(1+2)}{6} = 1$, the formula is valid for $n = 1$.

Step 2: Assume that the formula is valid for $n = k$ and derive a formula for $n = k + 1$.

$S_k \Rightarrow 1 + 3 + 6 + \ldots + \frac{k(k+1)}{2} = \frac{(k+1)(k+2)}{6}$

$$\begin{aligned} S_k \Rightarrow 1 + 3 + 6 + \ldots + \frac{k(k+1)}{2} + \frac{(k+1)(k+2)}{2} &= \frac{k(k+1)(k+2)}{6} + \frac{(k+1)(k+2)}{2} \\ &= \frac{k(k+1)(k+2) + 3(k+1)(k+2)}{6} \\ &= \frac{(k+1)(k+2)(k+3)}{6} \end{aligned}$$

Apply the original formula for $n = k + 1$.

$\frac{(k+1)[(k+1)+1][(k+1)+2]}{6} = \frac{(k+1)(k+2)(k+3)}{6}$

The formula gives the same result as adding the $(k + 1)$ term directly. Thus if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Thus, the formula is valid for all positive integral values of n.

Pages 826–828 Exercises

11. Step 1: Verify that the formula is valid for $n = 1$. Since 1 is the first term in the sequence and $(1)[2(1) - 1] = 1$, the formula is valid for $n = 1$.

Step 2: Assume that the formula is valid for $n = k$ and derive a formula for $n = k + 1$.

$S_k \Rightarrow 1 + 5 + 9 + \ldots + (4k - 3) = k(2k - 1)$

$$\begin{aligned} S_{k+1} \Rightarrow 1 + 5 + 9 + \ldots + (4k - 3) + (4k + 1) \\ &= k(2k - 1) + (4k + 1) \\ &= 2k^2 + 3k + 7 \\ &= (k + 1)(2k + 1) \end{aligned}$$

Apply the original formula for $n = k + 1$.

$(k + 1)[2(k + 1) - 1] = (k + 1)(2k + 1)$

The formula gives the same result as adding the $(k + 1)$ term directly. Thus if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 1$, it is also valid for $n = 2$. Since it is valid for $n = 2$, it is also valid for $n = 3$, and so on indefinitely. Thus, the formula is valid for all positive integral values of n.

12. Step 1: Verify that the formula is valid for $n = 1$. Since 1 is the first term in the sequence and $\frac{(1)[3(1) - 1]}{2} = 1$, the formula is valid for $n = 1$.

Step 2: Assume that the formula is valid for $n = k$ and derive a formula for $n = k + 1$.

$S_k \Rightarrow 1 + 4 + 7 + \ldots + (3k - 2) = \frac{k(3k - 1)}{2}$

$$\begin{aligned} S_{k+1} \Rightarrow 1 + 4 + 7 + \ldots + (3k - 2) + (3k + 1) \\ &= \frac{k(3k - 1)}{2} + (3k + 1) \\ &= \frac{k(3k - 1)}{2} + \frac{2(3k + 1)}{2} \\ &= \frac{3k^2 + 5k + 2}{2} \\ &= \frac{(k + 1)(3k + 2)}{2} \end{aligned}$$

Apply the original formula for $n = k + 1$.

$\frac{(k + 1)[3(k + 1) - 1]}{2} = \frac{(k + 1)(3k + 2)}{2}$

The formula gives the same result as adding the $(k + 1)$ term directly. Thus if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Thus, the formula is valid for all positive integral values of n.

13. Step 1: Verify that the formula is valid for $n = 1$. Since $-\frac{1}{2}$ is the first term in the sequence and $\frac{1}{2^1} - 1 = -\frac{1}{2}$, the formula is valid for $n = 1$.
Step 2: Assume that the formula is valid for $n = k$ and derive a formula for $n = k + 1$.

$$S_k \Rightarrow -\frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \ldots - \frac{1}{2^k} = \frac{1}{2^k} - 1$$

$$\begin{aligned} S_{k+1} &\Rightarrow -\frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \ldots - \frac{1}{2^k} - \frac{1}{2^{k+1}} \\ &= \frac{1}{2^k} - 1 - \frac{1}{2^{k+1}} \\ &= \frac{2}{2 \cdot 2^k} - 1 - \frac{1}{2^{k+1}} \\ &= \frac{2}{2^{k+1}} - 1 - \frac{1}{2^{k+1}} \\ &= \frac{1}{2^{k+1}} - 1 \end{aligned}$$

When the original formula is applied for $n = k + 1$, the same result is obtained. Thus if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Thus, the formula is valid for all positive integral values of n.

14. Step 1: Verify that the formula is valid for $n = 1$. Since 1 is the first term in the sequence and $\frac{1^2(1 + 1)^2}{4} = 1$, the formula is valid for $n = 1$.
Step 2: Assume that the formula is valid for $n = k$ and derive a formula for $n = k + 1$.

$$S_k \Rightarrow 1 + 8 + 27 + \ldots + k^3 = \frac{k^2(k + 1)^2}{4}$$

$$\begin{aligned} S_{k+1} &\Rightarrow 1 + 8 + 27 + \ldots + k^3 + (k + 1)^3 \\ &= \frac{k^2(k + 1)^2}{4} + (k + 1)^3 \\ &= \frac{k^2(k + 1)^2 + 4(k + 1)^3}{4} \\ &= \frac{(k + 1)^2[k^2 + 4(k + 1)]}{4} \\ &= \frac{(k + 1)^2(k^2 + 4k + 4)}{4} \\ &= \frac{(k + 1)^2(k + 2)^2}{4} \end{aligned}$$

Apply the original formula for $n = k + 1$.

$$\frac{(k + 1)^2[(k + 1) + 1]^2}{4} = \frac{(k + 1)^2(k + 2)^2}{4}$$

The formula gives the same result as adding the $(k + 1)$ term directly. Thus if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Thus, the formula is valid for all positive integral values of n.

15. Step 1: Verify that the formula is valid for $n = 1$. Since 1 is the first term in the sequence and $\frac{1[2(1) - 1][2(1) + 1]}{3} = 1$, the formula is valid for $n = 1$.
Step 2: Assume that the formula is valid for $n = k$ and derive a formula for $n = k + 1$.

$$S_k \Rightarrow 1^2 + 3^2 + 5^2 + \ldots + (2k - 1)^2 = \frac{k(2k - 1)(2k + 1)}{3}$$

$$\begin{aligned} S_{k+1} &\Rightarrow 1^2 + 3^2 + 5^2 + \ldots + (2k - 1)^2 + (2k + 1)^2 \\ &= \frac{k(2k - 1)(2k + 1)}{3} + (2k + 1)^2 \\ &= \frac{k(2k - 1)(2k + 1) + 3(2k + 1)^2}{3} \\ &= \frac{[k(2k - 1) + 3(2k + 1)](2k + 1)}{3} \\ &= \frac{(2k^2 + 5k + 3)(2k + 1)}{3} \\ &= \frac{(2k + 3)(k + 1)(2k + 1)}{3} \end{aligned}$$

Apply the original formula for $n = k + 1$.

$$\frac{(k + 1)[2(k + 1) - 1][2(k + 1) + 1]}{3} = \frac{(k + 1)(2k + 1)(2k + 3)}{3}$$

The formula gives the same result as adding the $(k + 1)$ term directly. Thus if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Thus, the formula is valid for all positive integral values of n.

16. Step 1: Verify that the formula is valid for $n = 1$. Since $S_1 \Rightarrow 1$ and $2^1 - 1 = 1$, the formula is valid for $n = 1$.
Step 2: Assume that the formula is valid for $n = k$ and derive a formula for $n = k + 1$.

$$S_k \Rightarrow 1 + 2 + 4 + \ldots + 2^{k-1} = 2^k - 1$$

$$\begin{aligned} S_{k+1} \Rightarrow 1 + 2 + 4 + \ldots + 2^{k-1} + 2^k &= 2^k - 1 + 2^k \\ &= 2(2^k) - 1 \\ &= 2^{k+1} - 1 \end{aligned}$$

When the original formula is applied for $n = k + 1$, the same result is obtained. Thus if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so indefinitely. Thus, the formula is valid for all positive integral values of n.

17. $S_n \Rightarrow 7^n + 5 = 6r$ for some integer r
Step 1: Verify that S_n is valid for $n = 1$.
$S_1 \Rightarrow 7^1 + 5$ or 12. Since $12 = 6 \cdot 2$, S_n is valid for $n = 1$.
Step 2: Assume that S_n is valid for $n = k$ and show that it is also valid for $n = k + 1$.

$S_k \Rightarrow 7^k + 5 = 6r$ for some integer r
$S_{k+1} \Rightarrow 7^{k+1} + 5 = 6t$ for some integer t

$$\begin{aligned} 7^k + 5 &= 6r \\ 7(7^k + 5) &= 7 \cdot 6r \\ 7^{k+1} + 35 &= 42r \\ 7^{k+1} + 5 &= 42r - 30 \\ 7^{k+1} + 5 &= 6(7r - 5) \end{aligned}$$

Thus, $7^{k+1} + 5 = 6t$, where $t = 7r - 5$ is an integer, and we have shown that if S_n is valid, then S_{k+1} is also valid. Since S_n is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Hence, $7^n + 5$ is divisible by 6 for all integral values of n.

18. $S_n \Rightarrow 8^n - 1 = 7r$ for some integer r

Step 1: Verify that S_n is valid for $n = 1$.

$S_1 \Rightarrow 8^1 - 1$ or 7. Since $7 = 7 \cdot 1$, S_n is valid for $n = 1$.

Step 2: Assume that S_n is valid for $n = k$ and show that it is also valid for $n = k + 1$.

$S_k \Rightarrow 8^k - 1 = 7r$ for some integer r

$S_{k+1} \Rightarrow 8^{k+1} - 1 = 7t$ for some integer t

$$\begin{aligned} 8^k - 1 &= 7r \\ 8(8^k - 1) &= 8 \cdot 7r \\ 8^{k+1} - 8 &= 56r \\ 8^{k+1} - 1 &= 56r + 7 \\ 8^{k+1} - 1 &= 7(8r + 1) \end{aligned}$$

Thus, $8^{k+1} - 1 = 7t$, where $t = 8r + 1$ is an integer, and we have shown that if S_n is valid, then S_{k+1} is also valid. Since S_n is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. hence, $8^n - 1$ is divisible by 7 for all integral values of n.

19. $S_n \Rightarrow 5^n - 2^n = 3r$ for some integer r

Step 1: Verify that S_n is valid for $n = 1$.

$S_1 \Rightarrow 5^1 - 2^1$ or 3. Since $3 = 3 \cdot 1$, S_n is valid for $n = 1$.

Step 2: Assume that S_n is valid for $n = k$ and show that it is also valid for $n = k + 1$.

$S_k \Rightarrow 5^k - 2^k = 3r$ for some integer r

$S_{k+1} \Rightarrow 5^{k+1} - 2^{k+1} = 3t$ for some integer t

$$\begin{aligned} 5^k - 2^k &= 3r \\ 5^k &= 2^k + 3r \\ 5^k \cdot 5 &= (2^k + 3r)(2 + 3) \\ 5^{k+1} &= 2^{k+1} + 3(2^k) + 6r + 9r \\ 5^{k+1} - 2^{k+1} &= 2^{k+1} + 3(2^k) + 6r + 9r \\ &\quad - 2^{k+1} \\ &= 3(2^k) + 15r \\ &= 3(2^k + 5r) \end{aligned}$$

Thus, $5^{k+1} - 2^{k+1} = 3t$, where $t = 2^k + 5r$ is an integer, and we have shown that if S_n is valid, then S_{k+1} is also valid. Since S_n is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Hence, $5^n - 2^n$ is divisible by 3 for all integral values of n.

20. Step 1: Verify that the formula is valid for $n = 1$. Since a is the first term in the sequence and $\frac{1}{2}[2a + (1 - 1)d] = a$, the formula is valid for $n = 1$.

Step 2: Assume that the formula is valid for $n = k$ and derive a formula for $n = k + 1$.

$$S_k \Rightarrow a + (a + d) + (a + 2d) + \ldots + [a + (k - 1)d] = \frac{k}{2}[2a + (k - 1)d]$$

$$\begin{aligned} S_{k+1} \Rightarrow a + (a + d) + (a + 2d) + \ldots + [a + (k - 1)d] \\ + (a + kd) &= \frac{k}{2}[2a + (k - 1)d] + (a + kd) \\ &= \frac{k[2a + (k - 1)d] + 2(a + kd)}{2} \\ &= \frac{2ak + k(k - 1)d + 2a + 2kd}{2} \\ &= \frac{(k + 1)2a + [k(k - 1) + 2k]d}{2} \\ &= \frac{(k + 1)2a + (k^2 + k)d}{2} \\ &= \frac{(k + 1)2a + k(k + 1)d}{2} \\ &= \frac{(k + 1)}{2}(2a + kd) \end{aligned}$$

Apply the original formula for $n = k + 1$.

$$\frac{(k + 1)}{2}\{2a + [(k + 1) - 1]d\} = \frac{(k + 1)}{2}(2a + kd)$$

The formula gives the same result as adding the $(k + 1)$ term directly. Thus if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 1$, it is also valid for $n = 2$, $n =3$, and so on indefinitely. Thus , the formula is valid for all positive integral values of n.

21. Step 1: Verify that the formula is valid for $n = 1$. Since $\frac{1}{2}$ is the first term in the sequence and $\frac{1}{1 + 1} = \frac{1}{2}$, the formula is valid for $n = 1$.

Step 2: Assume that the formula is valid for $n = k$ and derive a formula for $n = k + 1$.

$$S_k \Rightarrow + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \ldots + \frac{1}{k(k + 1)} = \frac{k}{k + 1}$$

$$\begin{aligned} S_{k+1} \Rightarrow \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \ldots + \frac{1}{k(k + 1)} + \frac{1}{(k + 1)(k + 2)} \\ &= \frac{k}{k + 1} + \frac{1}{(k + 1)(k + 2)} \\ &= \frac{k(k + 2) + 1}{(k + 1)(k + 2)} \\ &= \frac{k^2 + 2k + 1}{(k + 1)(k + 2)} \\ &= \frac{(k + 1)^2}{(k + 1)(k + 2)} \\ &= \frac{k + 1}{k + 2} \end{aligned}$$

Apply the original formula for $n = k + 1$.

$$\frac{(k + 1)}{(k + 1) + 1} = \frac{k + 1}{k + 2}$$

The formula gives the same result as adding the $(k + 1)$ term directly. Thus if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Thus, the formula is valid for all positive integral values of n.

22. $S_n \Rightarrow 2^{2n+1} + 3^{2n+1} = 5r$ for some integer r

Step 1: Verify that S_n is valid for $n = 1$.

$S_1 \Rightarrow 2^{2(1)+1} + 3^{2(1)+1}$ or 35. Since $35 = 5 \cdot 7$, S_n is valid for $n = 1$.

Step 2: Assume that S_n is valid for $n = k$ and show that it is also valid for $n = k + 1$.

$S_k \Rightarrow 2^{2k+1} + 3^{k+1} = 5r$ for some integer r

$S_{k+1} \Rightarrow 2^{k+3} + 3^{2k+3} = 5t$ for some integer t

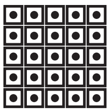
$$\begin{aligned} 2^{2k+1} + 3^{2k+1} &= 5r \\ 2^{2k+1} &= 5r - 3^{2k+1} \\ 2^{2k+1} \cdot 2^2 &= (5r - 3^{2k+1})(3^2 - 5) \\ 2^{2k+3} &= 45r - 25r - 3^{2k+3} + 5(3^{2k+1}) \\ 2^{2k+3} + 3^{2k+3} &= 45r - 25r - 3^{2k+3} + 5(3^{2k+1}) \\ &\quad + 3^{2k+3} \\ &= 20r + 5(3^{2k+1}) \\ &= 5(4 + 3^{2k+1}) \end{aligned}$$

Thus, $2^{2k+3} + 3^{2k+3} = 5t$, where $t = 4 + 3^{2k+1}$ is an integer; and we have shown that if S_n is valid, then S_{k+1} is also valid. Since S_n is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Hence, $2^{2n+1} + 3^{2k+1}$ is divisible by 5 for all integral values of n.

23. Step 1: Verify that the formula is valid for $n = 1$. Since $S_1 \Rightarrow [r(\cos\theta + \boldsymbol{i}\sin\theta]^1$ or $r(\cos\theta + \boldsymbol{i}\sin\theta)$ and $r^1[\cos(1)\theta + \boldsymbol{i}\sin(1)\theta] = r(\cos\theta + \boldsymbol{i}\sin\theta)$, the formula is valid for $n = 1$.
Step 2: Assume that the formula is valid for $n = k$ and derive a formula for $n = k + 1$.
That is, assume that $[r(\cos\theta + \boldsymbol{i}\sin\theta)]^k = r^k(\cos k\theta + \boldsymbol{i}\sin k\theta)$.
Multiply each side of the equation by $r(\cos\theta + \boldsymbol{i}\sin\theta)$.

$$\begin{aligned}
&[r(\cos\theta + \boldsymbol{i}\sin\theta)]^{k+1}\\
&\quad= [r^k(\cos k\theta + \boldsymbol{i}\sin k\theta] \cdot [r(\cos\theta + \boldsymbol{i}\sin\theta)]\\
&\quad= r^{k+1}[\cos k\theta\cos\theta + (\cos k\theta(\boldsymbol{i}\sin\theta)\\
&\qquad+ \boldsymbol{i}\sin k\theta\cos\theta + \boldsymbol{i}^2\sin k\theta\sin\theta]\\
&\quad= r^{k+1}[(\cos k\theta\cos\theta - \sin k\theta\sin\theta)\\
&\qquad+ \boldsymbol{i}(\sin k\theta\cos\theta + \cos k\theta\sin\theta)]\\
&\quad= r^{k+1}[\cos(k+1)\theta + \boldsymbol{i}\sin(k+1)\theta]
\end{aligned}$$

When the original formula is applied for $n = k + 1$, the same result is obtained. Thus if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Thus, the formula is valid for all positive integral values of n.

24a. 

24b. $4 - 1 = 3$
$9 - 4 = 5$
$16 - 9 = 7$
$1, 3, 5, 7, \ldots$

24c. $2n - 1$

24d. n^2

24e. Step 1: Verify that the formula is valid for $n = 1$. Since 1 is the first term in the sequence and $1^2 = 1$, the formula is valid for $n = 1$.
Step 2: Assume that the formula is valid for $n = k$ and derive a formula for $n = k + 1$.

$$\begin{aligned}
S_k &\Rightarrow 1 + 3 + 5 + 7 + \ldots + (2k - 1) = k^2\\
S_{k+1} &\Rightarrow 1 + 3 + 5 + 7 + \ldots + (2k - 1) + (2k + 1)\\
&= k^2 + (2k + 1)\\
&= k^2 + 2k + 1\\
&= (k + 1)^2
\end{aligned}$$

When the original formula is applied for $n = k + 1$, the same result is obtained. Thus if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Thus, the formula is valid for all positive integral values of n.

25. $S_1 \Rightarrow n^2 + 5n = 2r$ for some positive integer r
Step 1: Verify that S_1 is valid for $n = 1$.
$S_1 \Rightarrow 1^2 + 5 \cdot 1$ or 6. Since $6 = 2 \cdot 3$, S_1 is valid for $n = 1$.
Step 2: Assume that S_n is valid for $n = k$ and show that it is valid for $n = k + 1$.
$S_k \Rightarrow k^2 + 5k = 2r$ for some positive integer r
$S_{k+1} \Rightarrow (k + 1)^2 + 5(k + 1) = 2t$ for some positive integer t

$$\begin{aligned}
(k+1)^2 + 5(k+1) &= k^2 + 2k + 1 + 5k + 5\\
&= (k^2 + 5k) + (2k + 6)\\
&= 2r + 2(k + 3)\\
&= 2(r + k + 3)
\end{aligned}$$

Thus, $k^2 + 5k = 2t$, where $t = r + k + 3$ is an integer, and we have shown that if S_n is valid, then S_{k+1} is also valid. Since S_n is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Hence, $n^2 + 5n$ is divisible by 2 for all positive integral values of n.

26a.

Number of people	Number of Interactions
2	$1 \quad 2 = 1$
3	$1 \quad 2 \rightarrow 3 = 3$
4	$1 \quad 2 \quad 3 \rightarrow 4 = 6$
$\vdots$	$\vdots$
n	$\frac{n(n-1)}{2}$

26b. Step 1: Verify that $S_n \Rightarrow 0 + 1 + 2 + 3 + \ldots + (n - 1) = \frac{n(n-1)}{2}$ is valid for $n = 1$.
Since 0 is the first term in the sequence and $\frac{1(1-1)}{2} = 0$, the formula is valid for $n = 1$.
Step 2: Assume that the formula is valid for $n = k$ and derive a formula for $n = k + 1$.

$$\begin{aligned}
S_k &\Rightarrow 0 + 1 + 2 + \ldots + (k - 1) = \frac{k(k-1)}{2}\\
S_{k+1} &\Rightarrow 0 + 1 + 2 + \ldots + (k - 1) + k = \frac{k(k-1)}{2} + k\\
&= \frac{k(k-1) + 2k}{2}\\
&= \frac{k^2 - k + 2k}{2}\\
&= \frac{k(k+1)}{2}
\end{aligned}$$

Apply the original formula for $n = k + 1$.

$$\frac{(k+1)[(k+1) - 1]}{2} = \frac{k(k+1)}{2}$$

The formula gives the same result as adding the $(k + 1)$ term directly. Thus if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Thus, the formula is valid for all positive integral values of n.

26c. Yes; 15 people will require $\frac{15(14)}{2}$ or 105 interactions and last approximately 105(0.5) or 52.5 minutes.

27. Step 1: Verify that $S_n \Rightarrow (x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}y^3 + \ldots + y^n$ is valid for $n = 1$. Since $S_1 \Rightarrow (x+y)^1 = x^1 + y^1$ or $x + y$, S_n is valid for $n = 1$.

Step 2: Assume that the formula is valid for $n = k$ and derive a formula for $n = k + 1$.

$S_k \Rightarrow (x+y)^k = x^k + kx^{k-1}y + \frac{k(k-1)}{2!}x^{k-2}y^2 + \frac{k(k-1)(k-2)}{3!}x^{k-3}y^3 + \ldots + y^k$

$S_{k+1} \Rightarrow (x+y)^k(x+y) = (x+y)(x^k + kx^{k-1}y + \frac{k(k-1)}{2!}x^{k-2}y^2 + \frac{k(k-1)(k-2)}{3!}x^{k-3}y^3 + \ldots + y^k)$

$(x+y)^{k+1} = x(x^k + kx^{k-1}y + \frac{k(k-1)}{2!}x^{k-2}y^2 + \frac{k(k-1)(k-2)}{3!}x^{k-3}y^3 + \ldots + y^k) + y(x^k + kx^{k-1}y + \frac{k(k-1)}{2!}x^{k-2}y^2 + \frac{k(k-1)(k-2)}{3!}x^{k-3}y^3 + \ldots + y^k)$

$= x^{k+1} + kx^ky + \frac{k(k-1)}{2!}x^{k-1}y^2 + \ldots + xy^k + x^ky + kx^{k-1}y^2 + \frac{k(k-1)}{2!}x^{k-2}y^3 + \ldots + y^{k+1}$

$= x^{k+1} + (k+1)x^ky + kx^{k-1}y^2 + \frac{k(k-1)}{2!}x^{k-1}y^2 + \ldots + y^{k+1}$

$= x^{k+1} + (k+1)x^ky + \frac{k(k+1)}{2!}x^{k-1}y^2 + \ldots + y^{k+1}$

When the original formula is applied for $n = k + 1$, the same result is obtained. Thus if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Thus, the formula is valid for all positive integral values of n.

28a. $0.9 + 0.09 + 0.009 + \ldots$

28b. $\frac{9}{10^n}$

28c. $S = \frac{a_1 - a_1 r^n}{1 - r}$

$= \dfrac{\frac{9}{10} - \frac{9}{10}\left(\frac{1}{10}\right)^n}{1 - \frac{1}{10}}$

$= \dfrac{\frac{9}{10}\left(1 - \frac{1}{10}\right)^n}{\frac{9}{10}}$

$= 1 - \frac{1}{10^1}$

$= \frac{10^n - 1}{10^n}$

28d. Step 1: Verify that S_n: $0.9 + 0.9 + 0.009 + \ldots + \frac{9}{10^n} = \frac{10^n - 1}{10^n}$ is valid for $n = 1$.

Since 0.9 is the first term in the sequence and $\frac{10^1 - 1}{10^1} = 0.9$, the formula is valid for $n = 1$.

Step 2: Assume that the formula is valid for $n = k$ and derive a formula for $n = k + 1$.

$S_k \Rightarrow 0.9 + 0.09 + 0.009 + \ldots + \frac{9}{10^k} = \frac{10^k - 1}{10^k}$

$S_{k+1} \Rightarrow 0.9 + 0.09 + 0.009 + \ldots + \frac{9}{10^k} + \frac{9}{10^{k+1}}$

$= \frac{10^k - 1}{10^k} + \frac{9}{10^{k+1}}$

$= \frac{10(10^k - 1) + 9}{10^{k+1}}$

$= \frac{10^{k+1} - 10 + 9}{10^{k+1}}$

$= \frac{10^{k+1} - 1}{10^{k+1}}$

When the original formula is applied for $n = k + 1$, the same result is obtained. Thus if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on in indefinitely. Thus, the formula is valid for all positive integral values of n.

28e. $\lim\limits_{n\to\infty} \frac{10^k - 1}{10^n} = \lim\limits_{n\to\infty} 1 - \frac{1}{10^n}$

$= 1 - 0$ or 1

Thus, $0.999\ldots = 1$.

29. $z_1 = 2(4 - \boldsymbol{i}) + \boldsymbol{i}$

$= 8 - \boldsymbol{i}$

$z_2 = 2(8 - \boldsymbol{i}) + \boldsymbol{i}$

$= 16 - \boldsymbol{i}$

$z_3 = 2(16 - \boldsymbol{i}) + \boldsymbol{i}$

$= 32 - \boldsymbol{i}$

30. $64 = -6 + (29 - 1)d$

$70 = 28d$

$\frac{5}{2} = d$

31. $25x^2 + 4y^2 - 100x - 40y + 100 = 0$

$25(x^2 - 4x) + 4(y^2 - 10y) = -100$

$25(x-2)^2 + 4(y-5)^2 = -100 + 100 + 100$

$25(x-2)^2 + 4(y-5)^2 = 100$

$\frac{(x-2)^2}{4} + \frac{(y-5)^2}{25} = 1$

ellipse

32. $\ell = 250$, $k = 4$

$A = \frac{(250)^2\pi}{4(4)}$

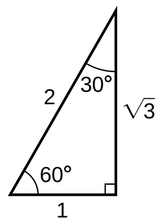
$\approx 12{,}271.85 \text{ m}^2$

33. $\frac{2\pi}{\pi} = 2$; $y = \pm\frac{3}{4}\sin 2x$

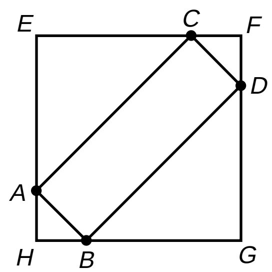
34. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30°$ Since $\frac{\sqrt{3}}{2}$ is negative, x is in the second or third quadrants.

$x = 180° - 30°$ or $150°$

$x = 180° + 30°$ or $210°$



35. Since the area of the square is 25, each side measures 5.



Since $\overline{AE}$ and $\overline{BE}$ measure 1 unit, $\overline{AH}$ and $\overline{BF}$ each measure 4 units. $\overline{AB}$ is the hypotenuse of an isosceles right triangle with legs that measure 1 unit. So $\overline{AB}$ measures $\sqrt{2}$ units. $\overline{CA}$ is the hypotenuse of an isosceles right triangle with legs that measure 4 units. So $\overline{CA}$ measures $4\sqrt{2}$ units. Since $ABCD$ is a rectangle, the perimeter is $2(\sqrt{2} + 4\sqrt{2}) = 2(5\sqrt{2}) = 10\sqrt{2}$ units.

The correct choice is B.

Chapter 12 Study Guide and Assessment

Page 829 Understanding the Vocabulary

1. d **2.** i **3.** m **4.** j
5. k **6.** f **7.** c **8.** e
9. b **10.** h

Pages 830–831 Skills and Concepts

11. $d = 4.3 - 3$ or 1.3
$5.6 + 1.3 = 6.9,\ 6.9 + 1.3 = 8.2,$
$8.2 + 1.3 = 9.5,\ 9.5 + 1.3 = 10.8$
$6.9, 8.2, 9.5, 10.8$

12. $a_{20} = 5 + (20 - 1)(-3)$
$= -52$

13. $-4 = 6 + (5 - 1)d$
$-10 = 4d$
$-2.5 = d$
$6 + (-2.5) = 3.5,\ 3.5 + (-2.5) = 1,$
$1 + (-2.5) = -1.5$
$6, 3.5, 1, -1.5, -4$

14. $d = -23 - (-30)$ or 7
$a_{14} = -30 + (14 - 1)\,7$
$= 61$
$S_{14} = \frac{14}{2}(-30 + 61)$
$= 217$

15. $S_n = \frac{n}{2}[2a_1 + (n - 1)d]$
$250.2 = \frac{n}{2}[2(2) + (n - 1)(1.4)]$
$250.2 = 2n + 0.7n(n - 1)$
$0 = 0.7n^2 + 1.3n - 250.2$
$n = \frac{-1.3 \pm \sqrt{(1.3)^3 - 4(0.7)(-250.2)}}{2(0.7)}$
$= \frac{-1.3 \pm 26.5}{1.4}$
$n = 18$ or $n = -19.86$
Since n is a positive whole number, $n = 18$.

16. $r = \frac{7}{49}$ or $\frac{1}{7}$
$1 \cdot \frac{1}{7} = \frac{1}{7},\ \frac{1}{7} \cdot \frac{1}{7} = \frac{1}{49},\ \frac{1}{49} \cdot \frac{1}{7} = \frac{1}{343}$
$\frac{1}{7}, \frac{1}{49}, \frac{1}{343}$

17. $a_{15} = 2.2(2)^{15-1}$
$= 36{,}044.8$

18. $8 = a_1(0.2)^{7-1}$
$8 = 0.000064a_1$
$125{,}000 = a_1$

19. $125 = 0.2r^{5-1}$
$625 = r^4$
$\pm 5 = r$
$0.2(\pm 5) = \pm 1,\ \pm 1(\pm 5) = \pm 5,\ \pm 5(\pm 5) = \pm 25$
$0.2, \pm 1, \pm 5, \pm 25, 125$

20. $r = \frac{-2.4}{1.2}$ or -2
$S_9 = \frac{1.2 - 1.2(-2)^9}{1 - (-2)}$
$= \frac{1.2 + 614.4}{3}$
$= 205.2$

21. $r = \frac{4\sqrt{2}}{4}$ or $\sqrt{2}$
$S_8 = \frac{4 - 4(\sqrt{2})^8}{1 - \sqrt{2}}$
$= \frac{-60}{1 - \sqrt{2}} \cdot \frac{1 + \sqrt{2}}{1 + \sqrt{2}}$
$= \frac{-60(1 + \sqrt{2})}{1 - 2}$
$= 60\left(1 \pm \sqrt{2}\right)$

22. $\lim\limits_{n\to\infty} \frac{3n}{4n + 1} = \lim\limits_{n\to\infty} \frac{\frac{3n}{n}}{\frac{4n}{n} + \frac{1}{n}}$
$= \frac{3}{4}$

23. $\lim\limits_{n\to\infty} \frac{6n - 3}{n} = \lim\limits_{n\to\infty} \frac{6n}{n} - \lim\limits_{n\to\infty} \frac{3}{n}$
$= 6 - 0$
$= 6$

24. Does not exist; $\lim\limits_{n\to\infty} \frac{2^n n^3}{3n^3} = \lim\limits_{n\to\infty} \frac{2^n}{3}$; since $\lim\limits_{n\to\infty} \frac{2^n}{3}$ becomes increasingly large as n approaches infinity, the sequence has no limit.

25. $\lim\limits_{n\to\infty} \frac{4n^3 - 3n}{n^4 - 4n^3} = \lim\limits_{n\to\infty} \frac{\frac{4n^3}{n^4} - \frac{3n}{n^4}}{\frac{n^4}{n^4} - \frac{4n^3}{n^4}}$
$= \frac{0}{1}$
$= 0$

26. $5.\overline{123} = 5 + \frac{123}{1000} + \frac{123}{1{,}000{,}000} + \ldots$
$a_1 = \frac{123}{1000},\ r = \frac{1}{1000}$
$S_n = 5 + \frac{\frac{123}{1000}}{1 - \frac{1}{1000}}$
$= 5 + \frac{123}{999}$
$= 5\frac{41}{333}$

27. $r = \frac{504}{1260}$ or 0.4
$S_n = \frac{1260}{1 - 0.4}$
$= 2100$

28. $a_n = \frac{n^2}{5^n},\ a_{n+1} = \frac{(n + 1)^2}{5^{n+1}}$
$r = \lim\limits_{n\to\infty} \frac{\frac{(n + 1)^2}{5^{n+1}}}{\frac{n^2}{5n}}$
$= \lim\limits_{n\to\infty} \frac{5^n(n^2 + 2n + 1)}{5^n \cdot 5 \cdot n^2}$
$= \lim\limits_{n\to\infty} \frac{n^2}{5n^2} + \lim\limits_{n\to\infty} \frac{2n}{n^2} + \lim\limits_{n\to\infty} \frac{1}{n^2}$
$= \frac{1}{5} + 0 + 0$
$= \frac{1}{5}$
convergent

29. The general term is $\frac{n + 5}{n}$ or $1 + \frac{5}{n}$.
$1 + \frac{5}{n} > \frac{1}{n}$ for all n, so divergent

30. The general term is $\frac{2}{n}$.
$\frac{2}{n} > \frac{1}{n}$ for all n, so divergent

31. $\sum_{a=5}^{9}(3a-3) = (3 \cdot 5 - 3) + (3 \cdot 6 - 3) + (3 \cdot 7 - 3) + (3.8 - 3) + (3.9 - 3)$

$= 12 + 15 + 18 + 21 + 24$
$= 90$

32. $\sum_{k=1}^{\infty}(0.4)^k = (0.4)^1 + (0.4)^2 + (0.4)^3 + \ldots + (0.4)^{\infty}$

$S = \frac{0.4}{1-0.4}$
$= \frac{2}{3}$

33. $\sum_{a=0}^{\infty}(2n-1)$ **34.** $\sum_{a=1}^{9} = 1(n^2+1)$

35. $(a-4)^6 = a^6 + \frac{6!}{1!(6-1)!} \cdot a^5 \cdot (-4)^1 + \frac{6!}{2!(6-2)!} \cdot a^4 \cdot (-4)^2 + \frac{6!}{3!(6-3)!} \cdot a^3 \cdot (-4)^3 + \frac{6!}{4!(6-4)!} \cdot a^2 \cdot (-4)^4 + \frac{6!}{5!(6-5)!} \cdot a^1 \cdot (-4)^5 + \frac{6!}{6!(6-6)!} \cdot a^0 \cdot (-4)^6$

$= a^6 - 24a^5 + 240a^4 - 1280a^3 + 3840a^2 - 6144a + 4096$

36. $(2r+3s)^4 = (2r)^4 + \frac{4!}{1!(4-1)!}(2r)^3(3s)^1 + \frac{4!}{2!(4-2)!}(2r)^2(3s)^2 + \frac{4!}{3!(4-3)!}(2r)(3s)^3 + \frac{4!}{4!0!}(2r)^0(3s)^4$

$= 16r^4 + 96r^3s + 216r^2s^2 + 216rs^3 + 81s^4$

37. $\frac{10!}{4!(10-4)!} \cdot x^{10-4} \cdot (-2)^4 = 210 \cdot x^6 \cdot 16$
$= 3360x^6$

38. $\frac{8!}{2!(8-2)!} \cdot 4m^{8-2} \cdot 1^2 = 28 \cdot 4096m^6 \cdot 1$
$= 114{,}688m^6$

39. $\frac{10!}{7!(10-7)!} \cdot x^{10-7} \cdot (3y)^7 = 120 \cdot x^3 \cdot 2187y^7$
$= 262{,}440x^3y^7$

40. $\frac{12!}{5!(12-5)!} \cdot (2c)^{12-5} \cdot (-d)^4 = 792 \cdot 128c^7 \cdot (-d)^5$
$= -101{,}376c^7d^5$

41. $2\left(\cos\frac{3\pi}{4} + \boldsymbol{i}\sin\frac{3\pi}{4}\right) = 2e^{\boldsymbol{i}\frac{3\pi}{4}}$

42. $4\boldsymbol{i} = 4\left(\cos\frac{\pi}{2} + \boldsymbol{i}\sin\frac{\pi}{2}\right)$
$= 4e^{\boldsymbol{i}\frac{\pi}{2}}$

43. $r = \sqrt{2^2 + (-2)^2}$ or $2\sqrt{2}$

$\theta = \text{Arctan}\left(\frac{-2}{2}\right)$ or $\frac{7\pi}{4}$

$2 - 2\boldsymbol{i} = 2\sqrt{2}\left(\cos\frac{7\pi}{4} + \boldsymbol{i}\sin\frac{7\pi}{4}\right)$
$= 2\sqrt{2}\,e^{\boldsymbol{i}\frac{7\pi}{4}}$

44. $r = \sqrt{(3\sqrt{3})^2 + 3^2}$ or 6

$\theta = \text{Arctan}\left(\frac{3}{3\sqrt{3}}\right)$ or $\frac{\pi}{6}$

$3\sqrt{3} + 3\boldsymbol{i} = 6\left(\cos\frac{\pi}{6} + \boldsymbol{i}\sin\frac{\pi}{6}\right)$
$= 6e^{\boldsymbol{i}\frac{\pi}{6}}$

45. $f(2) = 6 - 3 \cdot 2 = 0$
$f(0) = 6 - 3 \cdot 0 = 6$
$f(6) = 6 - 3 \cdot 6 = -12$
$f(-12) = 6 - 3 \cdot (-12) = 42$
0, 6, −12, 42

46. $f(-3) = (-3)^2 + 4 = 13$
$f(13) = 13^2 + 4 = 173$
$f(173) = 173^2 + 4 = 29{,}933$
$f(29{,}933) = 29{,}933^2 + 4 = 895{,}984{,}493$
13; 173; 29.933; 895, 984, 493

47. $z_0 = 4\boldsymbol{i}$
$z_1 = 0.5(4\boldsymbol{i}) + (4 - 2\boldsymbol{i}) = 2\boldsymbol{i} + 4 - 2\boldsymbol{i} = 4$
$z_2 = 0.5(4) + (4 - 2\boldsymbol{i}) = 2 + 4 - 2\boldsymbol{i} = 6 - 2\boldsymbol{i}$
$z_3 = 0.5(6 - 2\boldsymbol{i}) + (4 - 2\boldsymbol{i})$
$= 3 - \boldsymbol{i} + 4 - 2\boldsymbol{i} = 7 - 3\boldsymbol{i}$

48. $z_0 = -8$
$z_1 = 0.5(-8) + (4 - 2\boldsymbol{i}) = -4 + 4 - 2\boldsymbol{i} = -2\boldsymbol{i}$
$z_2 = 0.5(-2\boldsymbol{i}) + (4 - 2\boldsymbol{i}) = -\boldsymbol{i} + 4 - 2\boldsymbol{i} = 4 - 3\boldsymbol{i}$
$z_3 = 0.5(4 - 3\boldsymbol{i}) + (4 - 2\boldsymbol{i})$
$= 2 - 1.5\boldsymbol{i} + 4 - 2\boldsymbol{i} = 6 - 3.5\boldsymbol{i}$

49. $z_0 = -4 + 6\boldsymbol{i}$
$z_1 = 0.5(-4 + 6\boldsymbol{i}) + (4 - 2\boldsymbol{i})$
$= -2 + 3\boldsymbol{i} + 4 - 2\boldsymbol{i} = 2 + \boldsymbol{i}$
$z_2 = 0.5(2 + \boldsymbol{i}) + (4 - 2\boldsymbol{i})$
$= 1 + 0.5\boldsymbol{i} + 4 - 2\boldsymbol{i} = 5 - 1.5\boldsymbol{i}$
$z_3 = 0.5(5 - 1.5\boldsymbol{i}) + (4 - 2\boldsymbol{i})$
$= 2.5 - 0.5\boldsymbol{i} + 4 - 2\boldsymbol{i} = 6.5 - 2.75\boldsymbol{i}$

50. $z_0 = 12 - 8\boldsymbol{i}$
$z = 0.5(12 - 8\boldsymbol{i}) + (4 - 2\boldsymbol{i})$
$= 6 - 4\boldsymbol{i} + 4 - 2\boldsymbol{i} = 10 - 6\boldsymbol{i}$
$z_2 = 0.5(10 - 6\boldsymbol{i}) + (4 - 2\boldsymbol{i})$
$= 5 - 3\boldsymbol{i} + 4 - 2\boldsymbol{i} = 9 - 5\boldsymbol{i}$
$z_3 = 0.5(9 - 5\boldsymbol{i}) + (4 - 2\boldsymbol{i})$
$= 4.5 - 2.5\boldsymbol{i} + 4 - 2\boldsymbol{i} = 8.5 - 4.5\boldsymbol{i}$

51. Step 1: Verify that the formula is valid for $n = 1$. Since the first term in the sequence is 1 and $\frac{1(1+1)}{2} = 1$, the formula is valid for $n = 1$.

Step 2: Assume that the formula is valid for $n = k$ and derive a formula for $n = k + 1$.

$S_k \Rightarrow 1 + 2 + 3 + \ldots + k = \frac{k(k+1)}{2}$

$S_{k+1} \Rightarrow 1 + 2 + 3 + \ldots + k + (k+1)$

$= \frac{k(k+1)}{2} + \frac{2(k+1)}{2}$

$= \frac{k^2+k}{2} + \frac{2k+2}{2}$

$= \frac{k^2+k+2k+2}{2}$

$= \frac{k^2+3k+2}{2}$

$= \frac{(k+1)(k+2)}{2}$

Apply the original formula for $n = k + 1$.

$\frac{(k+1)[(k+1)+1]}{2} = \frac{(k+1)(k+2)}{2}$

The formula gives the same result as adding the $(k + 1)$ term directly. Thus, if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Thus, the formula is valid for all positive integral values of n.

52. Step 1: Verify that the formula is valid for $n = 1$. Since the first term in the sequence is 3 and $\frac{1(1+1)(2 \cdot 1 + 7)}{6} = 3$, the formula is valid for $n = 1$.

Step 2: Assume that the formula is valid for $n = k$ and derive formula for $n = k + 1$.

$S_k \Rightarrow 3 + 8 + 15 + \ldots + k(k+2) = \frac{k(k+1)(2k+7)}{6}$

$$\begin{aligned} S_{k+1} &\Rightarrow 3 + 8 + 15 + \ldots + k(k+2) + (k+1)(k+3) \\ &= \frac{k(k+1)(2k+7)}{6} + (k+1)(k+3) \\ &= \frac{k(k+1)(2k+7)}{6} + \frac{6(k+1)(k+3)}{6} \\ &= \frac{k(k+1)(2k+7) + 6(k+1)(k+3)}{6} \\ &= \frac{(k+1)[k(2k+7) + 6(k+3)]}{6} \\ &= \frac{(k+1)(2k^2 + 7k + 6k + 18)}{6} \\ &= \frac{(k+1)(2k^2 + 13k + 18)}{6} \\ &= \frac{(k+1)(k+2)(2k+9)}{6} \end{aligned}$$

Apply the original formula for $n = k + 1$.

$\frac{(k+1)[(k+1)+1][(2(k+1)+7]}{6} = \frac{(k+1)(k+2)(2k+9)}{6}$

The formula gives the same result as adding the $(k + 1)$ term directly. Thus, if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Thus, the formula is valid for all positive integral values of n.

53. $S_n \Rightarrow 9^n - 4^n = 5r$ for some integer r

Step 1: Verify that S_n is valid for $n = 1$.

$S_1 \Rightarrow 9^1 - 4^1$ or 5. Since $5 = 5 \cdot 1$, S_n is valid for $n = 1$.

Step 2: Assume that S_n is valid for $n = k$ and show that it is also valid for $n = k + 1$.

$S_k \Rightarrow 9^k - 4^k = 5r$ for some integer r

$S_{k+1} \Rightarrow 9^{k+1} - 4^{k+1} = 5t$ for some integer t

$$\begin{aligned} 9^k - 4^k &= 5r \\ 9^k &= 4^k + 5r \\ 9(9^k) &= (4^k + 5r)(4 + 5) \\ 9^{k+1} &= 4^{k+1} + 5(4^k) + 20r + 25r \\ 9^{k+1} - 4^{k+1} &= 4^{k+1} + 5(4^k) + 20r + 25r - 4^{k+1} \\ &= 5(4^k) + 45r \\ &= 5(4^k + 9r) \end{aligned}$$

Thus, $9^{k+1} - 4^{k+1} = 5t$, where $t = 4^k + 9r$ is an integer, and we have shown that if S_n is valid, then S_{k+1} is also valid. Since S_n is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Hence, $9^n - 4^n$ is divisible by 5 for all integral values of n.

Page 833 Applications and Problem Solving

54a. $a_1 = 16$, $d = 48 - 16$ or 32

$A_{12} = 16 + (12 - 1)32$
$= 368$ ft

54b. $S_{12} = \frac{12}{2}(16 + 368)$
$= 2304$ ft

55. If the budget is cut 3% each year, 97% remains after each year.

$a_1 = 160{,}000{,}000$, $r = 0.97$

$a_{11} = 160{,}000{,}000(0.97)^{11-1}$
$= \$117{,}987{,}860.30$

56a. One side of the original triangle measures $\frac{6}{3}$ or 2 units. Half of 2 units is 1 unit. Each side of the new triangle measures 1 unit, so its perimeter is 1 + 1 + 1 or 3 units.

56b. $a_1 = 6$, $r = \frac{3}{6}$ or $\frac{1}{2}$

$S = \frac{6}{1 - \frac{1}{2}}$

$=1$

Page 833 Open-Ended Assessment

1a. Arithmetic; arithmetic sequences have common differences, while geometric sequences have common ratios.

1b. Sample answer: 1, 4, 7, 10, …; $a_n = 1 + 3(n - 1)$

2. Sample answer: $\frac{6 + 5n^2}{3n}$; $\lim\limits_{n\to\infty} \frac{6 + 5n^2}{3n} = \lim\limits_{n\to\infty} \left(\frac{2}{n} + \frac{5n}{3}\right)$ $\lim\limits_{n\to\infty} \frac{2}{n} = 0$, but since $\lim\limits_{n\to\infty} \frac{5n}{3}$ becomes increasingly large as n approaches infinity, the sequence has no limit.

Chapter 12 SAT & ACT Preparation

Page 835 SAT and ACT Practice

1. If 40% of the tapes are jazz then 60% of the tapes must be blues. There are 80 tapes. Find 60% of 80.

$0.60(80) = 48$

The correct choice is D.

2. Because of alternate interior angles,

150 = 130 + unmarked angle of right △

20 = unmarked angle of right △

In the right triangle, $x + 20 = 90$ or $x = 70$.

The correct choice is C.

3. fraction pumped $= \frac{d \text{ gallons}}{k \text{ gallons}} = \frac{d}{k}$

Change this fraction into a percent by multiplying by 100.

percent pumped $= \frac{d}{k} \cdot 100$ or $\frac{100d}{k}\%$

The correct choice is A.

4. First calculate the number of caps Andrei has now. He starts with 48 and gives away 13, so he has $48 - 13 = 35$ left. Then he buys 17, so he has $35 + 17 = 52$. Then he trades 6 caps for 8 caps. this leaves him with $52 - 6 + 8$ or 54. His total is now 54.

Percent increase $= \frac{54 - 48}{48} \cdot 100$

$= 12.5\%$

The correct choice is B.

5. Since the figure is not drawn to scale, do not assume that the two lines are parallel, even though they may appear parallel. Since $AB = AC$, $\triangle ABC$ is isosceles. So $m\angle B = m\angle ACB$.

$m\angle B + m\angle ACB + 80 = 180$

$2 \cdot m\angle ACB = 100$

$m\angle ACB = 50$

Since AD is a line segment, $x + 70 + 50 = 180$. So, $x = 60$.

Consider right triangle CDE.

$x + y = 90$

$60 + y = 90$

$y = 30$

$x - y = 60 - 30$ or 30

The correct choice is C.

6. The population of Rockville is now 20,000 and will double every 8 years. So in 8 years the population will be 40,000 and in 16 years will be 80,000. So $f(8) = 40{,}000$ and $f(16) = 80{,}000$. Choice A is incorrect since $f(8) = 20{,}000$. Choice B is incorrect since $f(16) = 2(20{,}000)^2$ or 800,000,000. Choice C is incorrect since $f(8) = \frac{20{,}000}{64}$. Choice E is incorrect since $f(8) = 20{,}000$. For Choice D, $f(8) = 40{,}000$ and $f(16) = 80{,}000$.

The correct choice is D.

7. Notice that the figure is not drawn to scale. $\angle A$ could be a right angle. To be sure it is, find the slope of AB and compare it to the slope of AC.

Slope of $AB = \frac{10 - 4}{6 - 5} = \frac{6}{1}$

Slope of $AC = -\frac{1}{6}$

Since the slopes are negative reciprocals, the line segments are perpendicular, so $m\angle A = 90$. Therefore, $\triangle ABC$ is a 30°-60°-90° right triangle. The hypotenuse, AC, is twice the length of the leg opposite the 30° angle, AB.

$AB = \sqrt{(6 - 5)^2 + (10 - 4)^2} = \sqrt{1 + 36}$ or $\sqrt{37}$

$AC = 2\sqrt{37}$

The correct choice is D.

8. Method 1: Substitute each answer choice for x to test both inequalities.

A: $(-6) + 6 > 0$ and $1 - 2(-6) > -1$.

$0 > 0$ and $13 > -1$; false

Method 2: Solve each inequality for x.

$x + 6 > 0$ and $1 - 2x > -1$

$x > -6$ $\quad -2x > -2$

$x < 1$

The solution is $-6 < x < 1$. All of the answer choices except A are in this range.

The correct choice is A.

9. The increase from 99 to 100 is 1. So the percent increase from 99 to 100 is $\frac{1}{99}$.

$\frac{1}{99}$ is greater than $\frac{1}{100}$, or 1%.

The correct choice is A.

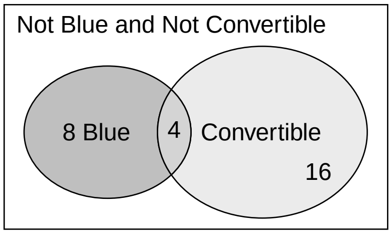
10. Choose a number for the total number of cars in the parking lot. Since the fractions have denominators of 2, 4, and 5, choose a number that is divisible by 2, 4, and 5. Let the number of cars in the parking lot equal 40.

$\frac{1}{5} \cdot 40 = 8$ blue cars

$\frac{1}{2} \cdot 8$ blue cars $= 4$ blue convertibles

$\frac{1}{4} \cdot$ the number of convertibles $= 4$

the number of convertibles $= 16$



The number of cars that are neither blue nor convertible is the total number minus the blue cars minus the convertibles plus the number that are both blue and convertible.

Neither blue nor convertible

$= 40 - 8 - 16 + 4$

$= 20$

percent that are neither blue nor convertible

$= \frac{20}{40} \cdot 100$

$= 50\%$

The answer is 50.

Chapter 13 Combinatorics and Probability

13-1 Permutations and Combinations

Pages 842–843 Check for Understanding

1. Sample answer: Both are used to determine the number of arrangements of a group of objects. However, order of the objects is important in permutations. When order of the objects is not important, combinations are computed.

2. Select 2 jacks out of 4—$C(4, 2)$
Select 3 queens out of 4—$C(4, 3)$
number of hands—$C(4, 2) \cdot C(4, 3)$

3. Sam is correct. The room assignments are an ordered selection of 5 rooms from the 7 rooms. A permutation should be used.

4.

S	blue	S-blue
	green	S-green
	gray	S-gray
M	blue	M-blue
	green	M-green
	gray	M-gray
L	blue	L-blue
	green	L-green
	gray	L-gray
XL	blue	XL-blue
	green	XL-green
	gray	XL-gray

5. Using the Basic Counting Principle, $4 \cdot 3 \cdot 5 \cdot 5 = 300$.

6. independent

7. $P(6, 6) = \frac{6!}{(6-6)!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} = 720$

8. $P(5, 3) = \frac{5!}{(5-3)!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 60$

9. $\frac{P(12, 8)}{P(6, 4)} = \frac{\frac{12!}{(12-8)!}}{\frac{6!}{(6-4)!}} = \frac{12!\,2!}{6!\,4!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 55{,}440$

10. $C(7, 4) = \frac{7!}{(7-4)!\,4!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 35$

11. $C(20, 15) = \frac{20!}{(20-15!)\,15!} = \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 15{,}504$

12. $C(4, 3) \cdot C(5, 2) = \frac{4!}{(4-3)!3!} \cdot \frac{5!}{(5-2)!\,2!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 3 \cdot 2 \cdot 1} \cdot \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 4 \cdot 10$ or 40

13. Using the Basic Counting Principle, $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 3{,}628{,}800$.

14. $C(15, 9) = \frac{15!}{(15-9)!\,9!} = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 5005$

15a. Using the Basic Counting Principle, $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 100{,}000$.

15b. Using the Basic Counting Principle, $1 \cdot 9 \cdot 9 \cdot 9 \cdot 10 = 7290$

15c. Using the Basic Counting Principle, $(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) - (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) = 999{,}900{,}000$

Pages 543–545 Exercises

16. Using the Basic Counting Principle, $2 \cdot 6 \cdot 4 = 48$.

17. $P(7, 7) = \frac{7!}{(7-7)!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} = 5040$

18a. Using the Basic Counting Principle, $9 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 9{,}000{,}000$.

18b. Using the Basic Counting Principle, $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 78{,}125$.

18c. Using the Basic Counting Principle, $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 1 = 1{,}000{,}000$.

18d. Using the Basic Counting Principle, $1 \cdot 1 \cdot 1 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$.

19. dependent **20.** independent

21. dependent

22. $P(8, 8) = \frac{8!}{(8-8)!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} = 40{,}320$

23. $P(6, 4) = \frac{6!}{(6-4)!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 360$

24. $P(5, 3) = \frac{5!}{(5-3)!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 60$

25. $P(7, 4) = \frac{7!}{(7-4)!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 840$

26. $P(9, 5) = \frac{9!}{(9-5)!}$
$= \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1}$
$= 15{,}120$

27. $P(10, 7) = \frac{10!}{(10-7)!}$
$= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1}$
$= 604{,}800$

28. $\frac{P(6, 3)}{P(4, 2)} = \frac{\frac{6!}{(6-3)!}}{\frac{4!}{(4-2)!}}$
$= \frac{6!\,2!}{4!\,3!}$
$= \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}$
$= 10$

29. $\frac{P(6, 4)}{P(5, 3)} = \frac{\frac{6!}{(6-4)!}}{\frac{5!}{(5-3)!}}$
$= \frac{6!\,2!}{5!\,2!}$
$= \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 6$

30. $\frac{P(6, 3) \cdot P(7, 5)}{P(9, 6)} = \frac{\frac{6!}{(6-3)!} \cdot \frac{7!}{(7-5)!}}{\frac{9!}{(9-6)!}}$
$= \frac{6!\,7!\,3!}{9!\,3!\,2!}$
$= \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}$
$= 5$

31. $C(5, 3) = \frac{5!}{(5-3)!\,3!}$
$= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}$
$= 10$

32. $C(10, 5) = \frac{10!}{(10-5)!\,5!}$
$= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 252$

33. $C(4, 2) = \frac{4!}{(4-2)!\,2!}$
$= \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1}$
$= 6$

34. $C(12, 4) = \frac{12!}{(12-4)!\,4!}$
$= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 495$

35. $C(9, 9) = \frac{9!}{(9-9)!\,9!}$
$= 1$

36. $C(14, 7) = \frac{14!}{(14-7)!\,7!}$
$= \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 3432$

37. $C(3, 2) \cdot C(8, 3) = \frac{3!}{(3-2)!\,2!} \cdot \frac{8!}{(8-3)!\,3!}$
$= \frac{3 \cdot 2 \cdot 1}{1 \cdot 2 \cdot 1} \cdot \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}$
$= 3 \cdot 56$ or 168

38. $C(7, 3) \cdot C(8, 5)$
$= \frac{7!}{(7-3)!\,3!} \cdot \frac{8!}{(8-5)!\,5!}$
$= \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} \cdot \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 35 \cdot 56$ or 1960

39. $C(5, 1) \cdot C(4, 2) \cdot C(8, 2)$
$= \frac{5!}{(5-1)!\,1!} \cdot \frac{4!}{(4-2)!\,2!} \cdot \frac{8!}{(8-2)!\,2!}$
$= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 1} \cdot \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1} \cdot \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}$
$= 5 \cdot 6 \cdot 28$ or 840

40. $C(14, 4) = \frac{14!}{(14-4)!\,4!}$
$= \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 1001$

41. $C(14, 5) = \frac{14!}{(14-5)!\,5!}$
$= \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 2002$

42. $C(18, 12)$
$= \frac{18!}{(18-12)!\,12!}$
$= \frac{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 18{,}564$

43. $C(3, 2) \cdot C(5, 1) \cdot C(8, 2)$
$= \frac{3!}{(3-2)!\,2!} \cdot \frac{5!}{(5-1)!\,1!} \cdot \frac{8!}{(8-2)!\,2!}$
$= \frac{3 \cdot 2 \cdot 1}{1 \cdot 2 \cdot 1} \cdot \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 1} \cdot \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}$
$= 3 \cdot 5 \cdot 28$ or 420

44. $P(11, 11) = \frac{11!}{(11-1)!}$
$= \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1}$
$= 39{,}916{,}800$

45a. $C(13, 3) \cdot C(13, 2) = \frac{13!}{(13-3)!\,3!} \cdot \frac{13!}{(13-2)!\,2!}$
$= \frac{13 \cdot 12 \cdot 11}{3 \cdot 2 \cdot 1} \cdot \frac{13 \cdot 12}{2 \cdot 1}$
$= 286 \cdot 78$ or $22{,}308$

45b. $C(4, 1) \cdot C(4, 2) \cdot C(4, 2)$
$= \frac{4!}{(4-1)!\,1!} \cdot \frac{4!}{(4-2)!\,2!} \cdot \frac{4!}{(4-2)!\,2!}$
$= \frac{4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 1} \cdot \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1} \cdot \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1}$
$= 4 \cdot 6 \cdot 6$ or 144

45c. $C(12, 5) = \frac{12!}{(12-5)!\,5!}$
$= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 792$

46a. Using the Basic Counting Principle,
$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 100{,}000$

46b. Using the Basic Counting Principle,
$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 30{,}240$

46c. Using the Basic Counting Principle,
$5 \cdot 5 \cdot 4 \cdot 4 \cdot 4 = 1600$.
$P(5, 2) \cdot P(4, 3) = \frac{5!}{(5-2)!} \cdot \frac{4!}{(4-3)!}$
$= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} \cdot \frac{4 \cdot 3 \cdot 2 \cdot 1}{1}$
$= 20 \cdot 24$ or 480

47. $C(3, 1) \cdot C(4, 1) \cdot C(6, 1) \cdot C(14, 6)$

$= \frac{3!}{(3-1)!\,1!} \cdot \frac{4!}{(4-1)!\,1!} \cdot \frac{6!}{(6-1)!\,1!} \cdot \frac{14!}{(14-6)!\,6!}$

$= \frac{3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 1} \cdot \frac{4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 1} \cdot \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1}$

$\cdot \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$= 3 \cdot 4 \cdot 6 \cdot 3003$ or $216{,}216$

48a. $P(42, 42) = \frac{42!}{(42-42)!}$

$= 42!$

$\approx 1.4 \times 10^{51}$

48b. $C(42, 30) = \frac{42!}{(42-30)!\,30!}$

$= \frac{42!}{12!\,30!}$

$\approx 1.1 \times 10^{10}$

48c. $C(5, 3) \cdot C(12, 6) \cdot C(10, 6) \cdot C(15, 5)$

$= \frac{5!}{(5-3)!\,3!} \cdot \frac{12!}{(12-6)!\,6!} \cdot \frac{10!}{(10-6)!\,6!} \cdot \frac{15!}{(15-5)!\,5!}$

$= 10 \cdot 924 \cdot 210 \cdot 3003$

$\approx 5.8 \times 10^9$

49. $P(n, n-1) \stackrel{?}{=} P(n, n)$

$\frac{n!}{[n-(n-1)]!} \stackrel{?}{=} \frac{n!}{(n-n)!}$

$\frac{n!}{1!} \stackrel{?}{=} \frac{n!}{0!}$ *0! = 1*

$n! = n!$

50a. $P(6, 6) = \frac{6!}{(6-6)!}$

$= \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1}$

$= 720$

50b. There are 3 ways to arrange the 3 couples, and 2 ways to arrange each of the two members within a couple.

$P(3, 3) \cdot P(2, 2) \cdot P(2, 2) \cdot P(2, 2)$

$= \frac{3!}{(3-3)!} \cdot \frac{2!}{(2-2)!} \cdot \frac{2!}{(2-2)!} \cdot \frac{2!}{(2-2)!}$

$= \frac{3 \cdot 2 \cdot 1}{1} \cdot \frac{2 \cdot 1}{1} \cdot \frac{2 \cdot 1}{1} \cdot \frac{2 \cdot 1}{1}$

$= 6 \cdot 2 \cdot 2 \cdot 2$ or 48

51a. $C(11, 4) = \frac{11!}{(11-4)!\,4!}$

$= \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$= 330$

51b. $C(6, 2) \cdot C(5, 2) = \frac{6!}{(6-2)!\,2!} \cdot \frac{5!}{(5-2)!\,2!}$

$= \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} \cdot \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}$

$= 15 \cdot 10$ or 150

52. $C(10, 2) = \frac{10!}{(10-2)!\,2!}$

$= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}$

$= 45$

53a. $\frac{3552}{6}$ or $37(2 + 5 + 9) = 592$

53b. Yes; let h, t, and u be the digits.

$$\begin{array}{l} 100h + 10t + u \\ 100h + 10u + t \\ 100t + 10h + u \\ 100t + 10u + h \\ 100u + 10t + h \\ +\ 100u + 10h + t \\ \hline 200(h + t + u) + 20(h + t + u) + 2(h + t + u) \\ = 222(h + t + u) \end{array}$$

$\frac{222(h + t + u)}{6} = 37(h + t + u)$

54. $2140 \cdot (1.058) = \$2264.12$

$2264.12(1.058) = \$2395.44$

$2395.44(1.058) = \$2534.38$

55. $\sum_{n=1}^{10} n^3 = 1^3 + 2^3 + \ldots + 10^3 = 3025$

56. $7.1^x = 83.1$

$x \ln 7.1 = \ln 83.1$

$x = \frac{\ln 83.1}{\ln 7.1}$

$x \approx 2.26$

57. $x = e^{0.346}$ *Use a calculator.*

≈ 1.4

58.

$y = 4x^2$

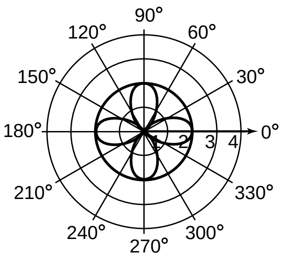
$-x' \sin 45° + y' \cos 45° = 4(x' \cos 45° + y' \sin 45°)^2$

$-\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y' = 4\left(\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'\right)^2$

$-\sqrt{2}x' + \sqrt{2}y' = 8\left(\frac{1}{2}x'^2 + x'y' + \frac{1}{2}y'^2\right)$

$0 = 4(x')^2 + 8x'y' + 4(y')^2 + \sqrt{2}x' - \sqrt{2}y'$

59.



$r = 2$, $r = 2 \cos 2\theta$

$2 = 2 \cos 2\theta$

$1 = \cos 2\theta$

$2\theta = 0°$ or $2\theta = 360°$

$\theta = 0°$ $\theta = 180°$

$(2, 180°)$, $(2, 0°)$

60. $v_x = 28 \cos 45°$, $v_y = 28 \sin 45°$

$= 28\left(\frac{\sqrt{2}}{2}\right)$ $= 28\left(\frac{\sqrt{2}}{2}\right)$

≈ 19.80 ft/s ≈ 19.80 ft/s

61. $\sin 2x + 2 \sin x = 0$

$2 \sin x \cos x + 2 \sin x = 0$

$2 \sin x (\cos x + 1) = 0$

$2 \sin x = 0$ or $\cos x + 1 = 0$

$\sin x = 0$ $\cos x = -1$

$x = 0°$, $x = 180°$, $x = 360°$, or $x = 180°$

So, $x = 0°, 180°, 360°$

62. $y = 8 \cos(\theta - 30°)$

amplitude $= 8$

period $= \frac{360°}{1}$ or $360°$

phase shift $= 30°$

63. Find B.
$B = 180° - 90° - 27°$ or $63°$
Find a.
$$\tan 27° = \frac{a}{15.2}$$
$$15.2 \tan 27° = a$$
$$7.7 \approx a$$
Find c.
$$\cos 27° = \frac{15.2}{c}$$
$$c = \frac{15.2}{\cos 27°}$$
$$c \approx 17.1$$

64. Each hour, an hour hand moves through $\frac{360}{12}$ or $30°$. Since 12 minutes is $\frac{1}{5}$ of an hour, the hour hand moves through an additional $\frac{1}{5}(30)$ or $6°$.
$2(30°) + 6° = 66°$
The correct choice is A.

13-2 Permutations with Repetitions and Circular Permutations

Pages 848–849 Check for Understanding

1. The circular permutation has no beginning or end. Therefore, the number of different arrangements is always $\frac{1}{n}$ of a revolution.

2. Sample answer: house or phone numbers where some of the digits repeat

3. Sample answer: The number of permutations of n charms on a bracelet with a clasp is $\frac{n!}{2}$.

4. $\frac{8!}{2!\,2!} = 10{,}080$

5. $\frac{9!}{2!\,2!\,2!\,2!} = 22{,}680$

6. $\frac{14!}{2!\,4!\,5!\,2!} = 7{,}567{,}560$

7. circular; $(11 - 1)!$ or 3,628,800

8. circular; $(8 - 1)!$ or 5040

9. circular; $(12 - 1)!$ or 39,916,800

10. linear; $\frac{5!}{2}$ or 60

11. $\frac{9!}{5!\,2!\,2!} = 756$

Pages 849–851 Exercises

12. $\frac{8!}{2!\,2!} = 10{,}080$

13. $\frac{10!}{2!\,2!} = 907{,}200$

14. $\frac{8!}{2!} = 20{,}160$

15. $\frac{10!}{3!\,2!} = 302{,}400$

16. $\frac{12!}{2!\,2!\,2!} = 59{,}875{,}200$

17. $\frac{9!}{4!\,2!\,2!} = 3780$

18. $\frac{7!}{2!\,2!\,2!} = 630$

19. $\frac{9!}{5!\,4!} = 126$

20. $\frac{50!}{4!\,3!\,4!\,2!\,8!\,8!\,3!\,2!\,2!\,2!\,4!} \approx 2.35 \times 10^{48}$

21. circular; $(12 - 1)! = 39{,}916{,}800$

22. linear; $\frac{26!}{6!\,3!\,7!\,10!} \approx 5.1 \times 10^{12}$

23. circular; $(9 - 1)!$ 40,320

24. circular; $(5 - 1)! = 24$

25. circular; $(8 - 1)! = 5040$

26. linear; $6! = 720$

27. linear; $10! = 3{,}628{,}800$

28. circular; $(9 - 1)! = 40{,}320$

29. circular; $\frac{(14 - 1)!}{2} = 3{,}113{,}510{,}400$

30. circular; $(20 - 1)! \approx 1.22 \times 10^{17}$

31. circular; $(32 - 1)! \approx 8.22 \times 10^{33}$

32. linear; $25! \approx 1.55 \times 10^{25}$

33. $\frac{8!}{2!2!2!2!} = 2520$

34a. $(7 - 1)! = 720$

34b. $7! = 5040$

35. $\frac{11!}{3!4!3!} = 46{,}200$

36a. $\frac{11!}{2!\,2!\,2!} = 4{,}989{,}600$

36b. integral calculus

37a. $\frac{43!}{20!\,14!\,9!} \approx 7.85 \times 10^{17}$

37b. $(43 - 1)! \approx 1.41 \times 10^{51}$

37c. $43! \approx 6.04 \times 10^{52}$

38. Let n = total number of symbols.
Let $n - 3$ = number of dashes.
$$\frac{n!}{3!\,(n-3)!} = 35$$
$$\frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = 210$$
$$n^3 - 3n^2 + 2n - 210 = 0$$
Use a graphing calculator to find the solution at $n = 7$.

39.
$$C(6, 3) = \frac{6!}{(6-3)!\,3!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = 20$$

40.
$$\begin{aligned}(5x - 1)^3 &= C(3, 0) \cdot (5x)^3 \cdot (-1)^0 \\ &\quad + C(3, 1) \cdot (5x)^2 \cdot (-1)^1 \\ &\quad + C(3, 2) \cdot (5x)^1 \cdot (-1)^2 \\ &\quad + C(3, 3) \cdot (5x)^0 \cdot (-1)^3 \\ &= 125x^3 - 75x^2 + 15x - 1\end{aligned}$$

41. $x < \log_2 413$
$$x < \frac{\log_{10} 413}{\log_{10} 2}$$
$$x < 8.69$$

42. $h = 6,\ k = -1$
$$3 = h + p$$
$$3 = 6 + p$$
$$-3 = p$$
$$(y - k)^2 = 4p(x - h)$$
$$(y + 1)^2 = -12(x - 6)$$

43.
$$\begin{aligned}2(4 - 3\boldsymbol{i})(7 - 2\boldsymbol{i}) &= 2(28 - 29\boldsymbol{i} + 6\boldsymbol{i}^2) \\ &= 56 - 58\boldsymbol{i} + 12(-1) \\ &= 44 - 58\boldsymbol{i}\end{aligned}$$

44. $\vec{\mathbf{v}} \times \vec{\mathbf{w}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 2 & 0 & 3 \\ 2 & 5 & 0 \end{vmatrix}$

$= \begin{vmatrix} 0 & 3 \\ 5 & 0 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 2 & 3 \\ 2 & 0 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 2 & 0 \\ 2 & 5 \end{vmatrix}\vec{\mathbf{k}}$

$= -15\vec{\mathbf{i}} + 6\vec{\mathbf{j}} + 10\vec{\mathbf{k}}$

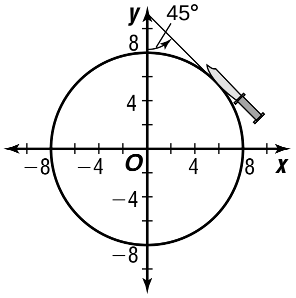
$= \langle -15, 6, 10\rangle$

since $\langle 2, 0, 3\rangle \cdot \langle -15, 6, 10\rangle = -30 + 0 + 30$ or 0 and $\langle 2, 5, 0\rangle \cdot \langle -15, 6, 10\rangle = -30 + 30 + 0$ or 0, then the resulting vector is perpendicular to $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$.

45. $x \cos 45° + y \sin 45° - 8 = 0$

$\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y - 8 = 0$

$\sqrt{2}x + \sqrt{2}y - 16 = 0$



The sparks will be highest at the y-intercept, $8\sqrt{2}$ inches above the center of the wheel. This is $8\sqrt{2} - 8$ or about 3.31 inches above the wheel.

46. If $x^2 = 36$, then $x - 6$ or $x = 6$.

$2^{x-1} = 2^{6-1}$ or 32

$2^{x-1} = 2^{-6-1}$ or $\frac{1}{128}$

The correct choice is E.

13-3 Probability and Odds

855–856 Check for Understanding

1. The probability of the event happening is 50−50.
2. Answers will vary; see students' work.
3. Sample answer: The probability of the successful outcome of an event is the ratio of the number of successful outcomes to the total number of outcomes possible. The odds of the successful outcome of an event is the ratio of the probability of its success to the probability of its failure.
4. Geraldo is correct. $P(\text{win}) = \frac{3}{2+3} = \frac{3}{5}$ or 60%.
5. $P(\text{softball}) = \frac{7}{3+7+11} = \frac{7}{21}$ or $\frac{1}{3}$
6. $P(\text{not a baseball}) = \frac{3+7}{3+7+11}$ or $\frac{10}{21}$
7. $P(\text{golf ball}) = \frac{0}{3+7+11}$ or 0
8. $P(\text{woman}) = \frac{7}{7+4}$ or $\frac{7}{11}$
9. $P(s) = \frac{C(4, 3)}{C(7, 3)}$ $\qquad P(f) = 1 - P(s)$

$= \frac{4}{35}$ $\qquad = 1 - \frac{4}{35}$ or $\frac{31}{35}$

odds $= \dfrac{\frac{4}{35}}{\frac{31}{35}}$ or $\frac{4}{31}$

10. $P(s) = \frac{C(4, 1) \cdot C(3, 2)}{C(7, 3)}$ $\qquad P(f) = 1 - P(s)$

$= \frac{4 \cdot 3}{35}$ $\qquad = 1 - \frac{12}{35}$

$= \frac{12}{35}$ $\qquad = \frac{23}{35}$

odds $= \dfrac{\frac{12}{35}}{\frac{23}{35}}$ or $\frac{12}{23}$

11. $P(s) = \frac{C(3, 1) \cdot C(4, 2)}{C(7, 3)}$ $\qquad P(f) = 1 - P(s)$

$= \frac{3 \cdot 6}{35}$ $\qquad = 1 - \frac{18}{35}$

$= \frac{18}{35}$ $\qquad = \frac{17}{35}$

odds $= \dfrac{\frac{18}{35}}{\frac{17}{35}}$ or $\frac{18}{17}$

12. $P(\text{rain}) = \frac{80}{100}$ or $\frac{4}{5}$

$P(\text{not rain}) = 1 - \frac{4}{5}$

$= \frac{1}{5}$

odds $= \dfrac{\frac{1}{5}}{\frac{4}{5}}$ or $\frac{1}{4}$

Pages 856–858 Exercises

13. $P(\text{face card}) = \frac{4+4+4}{52}$

$= \frac{12}{52}$ or $\frac{3}{13}$

14. $P(\text{a card of 6 or less}) = \frac{6+6+6+6}{52}$

$= \frac{24}{52}$ or $\frac{6}{13}$

15. $P(\text{a black, non-face card}) = \frac{10+10}{52}$

$= \frac{20}{52}$ or $\frac{5}{13}$

16. $P(\text{not a face card}) = \frac{10+10+10+10}{52}$

$= \frac{40}{52}$ or $\frac{10}{13}$

17. $P(\text{red}) = \frac{5}{5+2+3} = \frac{5}{10}$ or $\frac{1}{2}$
18. $P(\text{white}) = \frac{2}{5+2+3} = \frac{2}{10}$ or $\frac{1}{5}$
19. $P(\text{not pink}) = \frac{5+2}{5+2+3}$ or $\frac{7}{10}$
20. $P(\text{red or pink}) = \frac{5+3}{5+2+3} = \frac{8}{10}$ or $\frac{4}{5}$
21. $P(\text{2 pop}) = \frac{C(4, 2)}{C(40, 2)}$

$= \frac{6}{780}$ or $\frac{1}{130}$

22. $P(\text{2 country}) = \frac{C(8, 2)}{C(40, 2)}$

$= \frac{28}{780}$ or $\frac{7}{195}$

23. $P(\text{1 rap and 1 rock}) = \frac{C(10, 1) - C(18, 1)}{C(40, 2)}$

$= \frac{10 \cdot 18}{735}$

$= \frac{180}{780}$ or $\frac{3}{13}$

24. $P(\text{not rock}) = \frac{C(22, 2)}{C(40, 2)}$

$= \frac{231}{780}$ or $\frac{77}{260}$

25. Using the Basic Counting Principle, there are $1 \cdot 1$ or 1 way to roll both fives. Using the Basic Counting Principle, $P(\text{both fives}) = \frac{1}{36}$.

26. $P(s) = \frac{C(3, 2)}{C(6, 2)} = \frac{3}{15} = \frac{1}{5}$

$P(f) = 1 - P(s) = 1 - \frac{1}{5} = \frac{4}{5}$

odds $= \frac{\frac{1}{5}}{\frac{4}{5}}$ or $\frac{1}{4}$

27. $P(s) = \frac{C(4, 2)}{C(6, 2)} = \frac{6}{15} = \frac{2}{5}$

$P(f) = 1 - P(s) = 1 - \frac{2}{5} = \frac{3}{5}$

odds $= \frac{\frac{2}{5}}{\frac{3}{5}}$ or $\frac{2}{3}$

28. $P(s) = \frac{C(1, 1) \cdot C(3, 1)}{C(6, 2)} = \frac{1 \cdot 3}{15} = \frac{3}{15}$ or $\frac{1}{5}$

$P(f) = 1 - P(s) = 1 - \frac{1}{5} = \frac{4}{5}$

odds $= \frac{\frac{1}{5}}{\frac{4}{5}}$ or $\frac{1}{4}$

29. $P(s) = \frac{C(1, 1) \cdot C(2, 1)}{C(6, 2)} + \frac{C(1, 1) \cdot C(3, 1)}{C(6, 2)} + \frac{C(2, 1) \cdot C(3, 1)}{C(6, 2)} = \frac{2}{15} + \frac{3}{15} + \frac{6}{15}$ or $\frac{11}{15}$

$P(f) = 1 - P(s) = 1 - \frac{11}{15}$ or $\frac{4}{15}$

odds $= \frac{\frac{11}{15}}{\frac{4}{15}}$ or $\frac{11}{4}$

30. $P(s) = \frac{C(11, 3)}{C(27, 3)} = \frac{165}{2925} = \frac{11}{195}$

$P(f) = 1 - P(s) = 1 - \frac{11}{195} = \frac{184}{195}$

odds $= \frac{\frac{11}{195}}{\frac{184}{195}}$ or $\frac{11}{184}$

31. $P(s) = \frac{C(13, 2) \cdot C(11, 1)}{C(27, 3)} = \frac{78 \cdot 11}{2925} = \frac{858}{2925}$ or $\frac{22}{75}$

$P(f) = 1 - P(s) = 1 - \frac{22}{75} = \frac{53}{75}$

odds $= \frac{\frac{22}{75}}{\frac{53}{75}}$ or $\frac{22}{53}$

32. $P(s) = \frac{C(14, 3)}{C(27, 3)} = \frac{364}{2925} = \frac{28}{225}$

$P(f) = 1 - P(s) = 1 - \frac{28}{225} = \frac{197}{225}$

odds $= \frac{\frac{28}{225}}{\frac{197}{225}}$ or $\frac{28}{197}$

33. $P(s) = \frac{C(3, 1) \cdot C(24, 2)}{C(27, 3)} = \frac{3 \cdot 276}{2925} = \frac{828}{2925}$ or $\frac{92}{325}$

$P(f) = 1 - P(s) = 1 - \frac{92}{325} = \frac{233}{325}$

odds $= \frac{\frac{92}{325}}{\frac{233}{325}}$ or $\frac{92}{233}$

34. $P(s) = \frac{1}{1 + 249}$ or $\frac{1}{250}$

35. $P(s) = \frac{4}{5}$

$P(f) = 1 - P(s) = 1 - \frac{4}{5}$ or $\frac{1}{5}$

odds $= \frac{\frac{4}{5}}{\frac{1}{5}}$ or $\frac{4}{1}$

36. $P(s) = \frac{C(13, 3) \cdot C(13, 2)}{C(52, 5)} \cdot P(4, 2) = \frac{286 \cdot 78}{2{,}598{,}960} \cdot 12 = \frac{267{,}696}{2{,}598{,}960} = \frac{429}{4165}$

$P(f) = 1 - P(s) = 1 - \frac{429}{4165}$ or $\frac{3736}{4165}$

odds $= \frac{\frac{429}{4165}}{\frac{3736}{4165}}$ or $\frac{429}{3736}$

37. $P(s) = \frac{1}{1 + 4}$ or $\frac{1}{5}$

38. $P(s) = 0.325$

$P(f) = 1 - P(s) = 1 - 0.325$ or 0.675

odds $= \frac{0.325}{0.675}$ or $\frac{13}{27}$

39a. $P(s) = \frac{1}{10} \cdot \frac{1}{9} \cdot \frac{1}{8}$ or $\frac{1}{720}$

39b. $P(f) = \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$ or $\frac{1}{1000}$

$P(s) = 1 - P(f) = 1 - \frac{1}{1000}$ or $\frac{999}{1000}$

odds $= \frac{\frac{999}{1000}}{\frac{1}{1000}}$ or $\frac{999}{1}$

40a. $P(\text{both males}) = \frac{1}{2} \cdot \frac{1}{2}$ or $\frac{1}{4}$

40b. $P(s) = 1 \cdot \frac{1}{2} = \frac{1}{2}$

$P(f) = 1 - P(s) = 1 - \frac{1}{2}$ or $\frac{1}{2}$

odds $= \frac{\frac{1}{2}}{\frac{1}{2}}$ or $\frac{1}{1}$

41a. $P(s) = \frac{C(15, 10) \cdot C(5, 0)}{C(20, 10)} = \frac{3003 \cdot 1}{184{,}756}$ or $\frac{21}{1292}$

41b. $P(s) = \frac{C(15, 8) \cdot C(5, 2)}{C(20, 10)} = \frac{6435 \cdot 10}{184{,}756} = \frac{64{,}350}{184{,}756} = \frac{225}{646}$

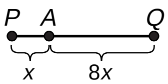
$P(f) = 1 - P(s) = 1 - \frac{225}{646} = \frac{421}{646}$

odds $= \frac{\frac{225}{646}}{\frac{421}{646}}$ or $\frac{225}{421}$

42a. $P = \frac{179{,}820 + 151{,}322}{84{,}475 + 3273 + 179{,}820 + 151{,}322}$
$= \frac{331{,}142}{418{,}890}$
≈ 0.791

42b. $P(f) = \frac{151{,}322}{418{,}890}$ $\qquad P(s) = 1 - P(f)$
$= 1 - \frac{151{,}322}{418{,}890}$
$= \frac{267{,}568}{418{,}890}$

odds $= \frac{\frac{267{,}568}{418{,}890}}{\frac{151{,}322}{418{,}890}}$
$= \frac{267{,}568}{151{,}322}$ or $\frac{133{,}784}{75{,}661}$

43.


Given a pipe $\overrightarrow{PQ}$ and a random cut point, A, $AP{:}AQ = 1{:}8$. If AP is x inches long, then AQ is $8x$ inches long. Now, the cut must be made along $\overrightarrow{AP}$ so that the longer piece will be 8 or more times as long as the shorter piece. Thus, the probability that the cut is on $\overrightarrow{AP}$ is $\frac{x}{x + 8x} = \frac{1}{9}$. Since the cut can be made on either end of the pipe, the actual probability is $\frac{2}{9}$.

44. This is a circular permutation.
$(6 - 1)! = 5!$ or 120

45. $C(10, 4) = \frac{10!}{(10 - 4)!\,4!}$
$= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 210$

46. $S_n = \frac{n}{2}[2a_1 + (n - 1)d]$
$S_{14} = \frac{14}{2}[2(3.2) + (14 - 1)1.5]$
$= 7(25.9)$
$= 181.3$

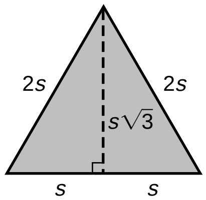
47. Let $y = 7^{\log_7 2x}$
$\log_7 y = \log_7 2x$
$y = 2x$
So, $7^{\log_7 2x} = 2x$.

48. Center: $(7, 2)$
$r^2 = (10 - 7)^2 + (-8 - 2)^2$
$= 3^2 + (-10)^2$ or 109
$(x - 7)^2 + (y - 2)^2 = 109$

49. $r = 3 \cdot 2$ or 6
$\theta = (\pi + \frac{\pi}{4})$ or $\frac{5\pi}{4}$
$6(\cos \frac{5\pi}{4} + \boldsymbol{i} \sin \frac{5\pi}{4}) = 6\left(-\frac{\sqrt{2}}{2} + \left(-\frac{\sqrt{2}}{2}\right)\boldsymbol{i}\right)$
$= -3\sqrt{2} - 3\sqrt{2}\boldsymbol{i}$

50. $\vec{\mathbf{u}} = \langle 3, -5\rangle + \langle -4, 2\rangle$
$= \langle 3 + (-4), -5 + 2\rangle$
$= \langle -1, -3\rangle$

51. Drawing the altitude from one vertex to the opposite side forms a 30°-60°-90° right triangle with hypotenuse $2s$. The short side of this right triangle measures s. So the altitude drawn has length $s\sqrt{3}$. This is the height of the equilateral triangle. The base measures $2s$. So the area of the equilateral triangle is $\frac{1}{2} \cdot 2s \cdot s\sqrt{3} = \sqrt{3s^2}$. The correct choice is B.



Page 858 Mid-Chapter Quiz

1. $P(15, 5) = \frac{15!}{(15 - 5)!}$
$= \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 360{,}360$

2. $C(20, 9) = \frac{20!}{(20 - 9)!\,9!}$
$= 167{,}960$

3. Using the Basic Counting Principle,
$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 175{,}760{,}000$.

4. $P(12, 5) = \frac{12!}{(12 - 5)!}$
$= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 95{,}040$

5. Using the Basic Counting Principle,
$18 \cdot 18 \cdot 3 \cdot 6 = 5832$.

6. $\frac{9!}{2!} = 181{,}440$

7. $\frac{10!}{3!\,4!\,3!} = 4200$

8. This is a circular permutation.
$(8 - 1)! = 5040$

9. $P(\text{both hearts}) = \frac{C(13, 2)}{C(52, 2)}$
$= \frac{78}{1326}$ or $\frac{1}{17}$

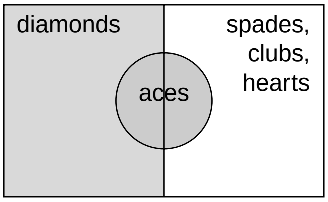
10. $P(s) = \frac{C(3, 1) \cdot C(3, 1)}{C(12, 2)}$ $\qquad P(f) = 1 - P(s)$
$= \frac{3 \cdot 3}{66}$ $\qquad = 1 - \frac{3}{22}$
$= \frac{9}{66}$ or $\frac{3}{22}$ $\qquad = \frac{19}{22}$

odds $= \frac{\frac{3}{22}}{\frac{19}{22}}$ or $\frac{3}{19}$

13-4 Probabilities of Compound Events

Pages 863–864 Check for Understanding

1. The occurrence of one event does not affect another for independent events. The occurrence of the first event affects the occurrence of a second for dependent events.

2a. 

2b. No, one of the aces can be an ace of diamonds.

2c. $P(\text{ace or diamond}) = P(\text{ace}) + P(\text{diamond}) - P(\text{ace and diamond})$

3. Answers will vary; see students' work.

4. independent, $\frac{6}{36} \cdot \frac{3}{36} = \frac{1}{72}$

5. dependent, $\frac{4}{10} \cdot \frac{3}{9} = \frac{2}{15}$

6. dependent, $\frac{4}{6} \cdot \frac{3}{5} = \frac{2}{5}$

7. exclusive, $\frac{4}{13} + \frac{6}{13} = \frac{10}{13}$

8. inclusive, $\frac{15}{27} + \frac{11}{27} - \frac{6}{27} = \frac{20}{27}$

9. exclusive, $\frac{4}{52} + \frac{4}{52} = \frac{8}{52}$ or $\frac{2}{13}$

10. $P(\text{selecting 5 even numbers}) = \frac{37}{75} \cdot \frac{36}{74} \cdot \frac{35}{73} \cdot \frac{34}{72} \cdot \frac{33}{71} \approx 0.025$

11. $P(\text{selecting 5 two digit numbers}) = \frac{66}{75} \cdot \frac{65}{74} \cdot \frac{64}{73} \cdot \frac{63}{72} \cdot \frac{62}{71} \approx 0.518$

12. $P(\text{5 odd numbers or 5 multiples of 4})$
$= P(\text{5 odd numbers}) + P(\text{5 multiples of 4})$
$= \left(\frac{38}{75} \cdot \frac{37}{74} \cdot \frac{36}{73} \cdot \frac{35}{72} \cdot \frac{34}{71}\right) + \left(\frac{17}{75} \cdot \frac{16}{74} \cdot \frac{15}{73} \cdot \frac{14}{72} \cdot \frac{13}{71}\right)$
$\approx 0.029 + 0.0004$
≈ 0.029

13. $P(\text{5 even numbers or 5 numbers less than 30}) = P(\text{5 even numbers}) + P(\text{5 numbers less than 30}) - P(\text{5 even numbers and 5 numbers less than 30})$
$= \left(\frac{37}{75} \cdot \frac{36}{74} \cdot \frac{35}{73} \cdot \frac{34}{72} \cdot \frac{33}{71}\right) + \left(\frac{29}{75} \cdot \frac{28}{74} \cdot \frac{27}{73} \cdot \frac{26}{72} \cdot \frac{25}{71}\right) - \left(\frac{14}{75} \cdot \frac{13}{74} \cdot \frac{12}{73} \cdot \frac{11}{72} \cdot \frac{10}{71}\right)$
$\approx 0.025 + 0.007 - 0.001$
≈ 0.032

14. $P(\text{none if 6 clocks are damaged})$
$= \frac{96}{100} \cdot \frac{95}{99} \cdot \frac{94}{98} \cdot \frac{93}{97} \cdot \frac{92}{96} \cdot \frac{91}{95}$
$= \frac{435{,}643}{560{,}175}$

15. P(at least 1 right handed pitcher = P(1 right-handed pitcher) + P(2 right-handed pitchers)
$= \frac{C(8, 1) \cdot C(5, 1)}{C(13, 2)} + \frac{C(8, 2) \cdot C(5, 0)}{C(13, 2)}$
$= \frac{40}{78} + \frac{28}{78}$
$= \frac{68}{78}$ or $\frac{34}{39}$

Pages 864–867 Exercises

16. dependent, $\frac{5}{9} \cdot \frac{4}{8} = \frac{5}{18}$

17. independent, $\frac{5}{9} \cdot \frac{5}{9} = \frac{25}{81}$

18. independent, $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

19. dependent, $\frac{4}{7} \cdot \frac{3}{6} = \frac{2}{7}$

20. dependent, $\frac{4}{15} \cdot \frac{4}{14} \cdot \frac{7}{13} = \frac{8}{195}$

21. dependent, $\frac{26}{52} \cdot \frac{25}{51} \cdot \frac{24}{50} \cdot \frac{23}{49} \cdot \frac{22}{48} \cdot \frac{21}{47} \cdot \frac{20}{46} \cdot \frac{19}{45} \cdot \frac{18}{44} \cdot \frac{17}{43} \cdot \frac{16}{42} \cdot \frac{15}{41} \cdot \frac{14}{40} = \frac{19}{1{,}160{,}054}$

22. dependent, $\frac{12}{28} \cdot \frac{8}{27} \cdot \frac{8}{26} = \frac{32}{819}$

23. independent, $\frac{5}{16} \cdot \frac{4}{16} \cdot \frac{7}{16} = \frac{35}{1024}$

24. independent
$P(\text{winning}) = \frac{4}{7}$
$P(\text{winning next four games}) = \frac{4}{7} \cdot \frac{4}{7} \cdot \frac{4}{7} \cdot \frac{4}{7}$
$= \frac{256}{2401}$

25. inclusive, $\frac{1}{6} + \frac{1}{6} - \frac{1}{36} = \frac{11}{36}$

26. inclusive, $\frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{7}{13}$

27. inclusive, $\frac{26}{52} + \frac{12}{52} - \frac{6}{52} = \frac{8}{13}$

28. exclusive
$P(\text{at least 3 males})$
$= P(\text{3 males}) + P(\text{4 males}) + P(\text{5 males})$
$= \frac{C(5, 3) \cdot C(4, 2)}{C(9, 5)} + \frac{C(5, 4) \cdot C(4, 1)}{C(9, 5)} + \frac{C(5, 5) \cdot C(5, 0)}{C(9, 5)}$
$= \frac{60}{126} + \frac{20}{126} + \frac{1}{126}$
$= \frac{81}{126}$ or $\frac{9}{14}$

29. exclusive
$P(\text{sum of 6 or sum of 9}) = P(\text{sum of 6}) + P(\text{sum of 9})$
$= \frac{5}{36} + \frac{4}{36}$
$= \frac{9}{36}$ or $\frac{1}{4}$

30. exclusive
$P(\text{at least three women})$
$= P(\text{3 women}) + P(\text{4 women}) + P(\text{5 women}) + P(\text{6 women})$
$= \frac{C(7, 3) \cdot C(7, 3)}{C(14, 6)} + \frac{C(7, 4) \cdot C(7, 2)}{C(14, 6)} + \frac{C(7, 5) \cdot C(7, 1)}{C(14, 6)} + \frac{C(7, 6) \cdot C(7, 0)}{C(14, 6)}$
$= \frac{1225}{3003} + \frac{735}{3003} + \frac{147}{3003} + \frac{7}{3003}$
$= \frac{2114}{3003}$ or $\frac{302}{429}$

31. exclusive
$P(\text{at least 4 tails})$
$= P(\text{4 tails}) + P(\text{5 tails}) + P(\text{6 tails})$
$= C(6, 4)\left(\frac{1}{2}\right)^6 + C(6, 5)\left(\frac{1}{2}\right)^6 + C(6, 6)\left(\frac{1}{2}\right)^6$
$= \frac{15}{64} + \frac{6}{64} + \frac{1}{64}$
$= \frac{22}{64}$ or $\frac{11}{32}$

32. inclusive
$\left(\frac{4}{52} \cdot \frac{3}{51}\right) + \left(\frac{26}{52} \cdot \frac{25}{51}\right) - \left(\frac{2}{52} \cdot \frac{1}{51}\right)$
$= \frac{12}{2652} + \frac{650}{2652} - \frac{2}{2652}$
$= \frac{660}{2652}$ or $\frac{55}{221}$

33. exclusive
$P(\text{at least 2 rock}) = P(\text{2 rock}) + P(\text{3 rock})$
$= \frac{C(6, 2) \cdot C(5, 1)}{C(11, 3)} + \frac{C(6, 3) \cdot C(5, 0)}{C(11, 3)}$
$= \frac{75}{165} + \frac{20}{165}$
$= \frac{95}{165}$ or $\frac{19}{33}$

34. $P(\text{all red cards}) = \frac{26}{52} \cdot \frac{25}{51} \cdot \frac{24}{50} \cdot \frac{23}{49} \cdot \frac{22}{48}$
$= \frac{253}{9996}$

35. $P(\text{both kings or both aces})$
$= P(\text{both kings}) + P(\text{both aces})$
$= \left(\frac{4}{52} \cdot \frac{3}{51}\right) + \left(\frac{4}{52} \cdot \frac{3}{51}\right)$
$= \frac{12}{2652} + \frac{12}{2652}$
$= \frac{24}{2652}$ or $\frac{2}{221}$

36. $P(\text{all diamonds}) = \left(\frac{13}{52}\right)^{10}$

37. $P(\text{both red or both queens})$
$= P(\text{both red}) + P(\text{both queens}) - P(\text{both red queens})$
$= \left(\frac{26}{52} \cdot \frac{25}{51}\right) + \left(\frac{4}{52} \cdot \frac{3}{51}\right) - \left(\frac{2}{52} \cdot \frac{1}{51}\right)$
$= \frac{650}{2652} + \frac{12}{2652} - \frac{2}{2652}$
$= \frac{660}{2652}$ or $\frac{55}{221}$

38. $P(\text{2 pennies}) = \frac{5}{21} \cdot \frac{4}{20}$
$= \frac{1}{21}$

39. $P(\text{2 nickels or 2 silver-colored coins})$
$= \left(\frac{7}{21} \cdot \frac{6}{20}\right) + \left(\frac{16}{21} \cdot \frac{15}{20}\right) - \left(\frac{7}{21} \cdot \frac{6}{20}\right)$
$= \frac{240}{420}$ or $\frac{4}{7}$

40. $P(\text{at least 1 nickel}) = 1 - P(\text{no nickels})$
$= 1 - \left(\frac{14}{21} \cdot \frac{13}{20}\right)$
$= \frac{17}{30}$

41. $P(\text{2 dimes or 1 penny and 1 nickel})$
$= P(\text{2 dimes}) + P(\text{1 penny and 1 nickel})$
$= \frac{C(9, 2)}{C(21, 2)} + \frac{C(5, 1) \cdot C(7, 1)}{C(21, 2)}$
$= \frac{36}{210} + \frac{35}{210}$
$= \frac{71}{210}$

42. $P(\text{all female}) = \frac{5}{10} \cdot \frac{4}{9} \cdot \frac{3}{8} \cdot \frac{2}{7}$
$= \frac{1}{42}$

43. $P(\text{all female or all male})$
$= P(\text{all female}) + P(\text{all male})$
$= \frac{5}{10} \cdot \frac{4}{9} \cdot \frac{3}{8} \cdot \frac{2}{7} + \frac{5}{10} \cdot \frac{4}{9} \cdot \frac{3}{8} \cdot \frac{2}{7}$
$= \frac{1}{42} + \frac{1}{42}$
$= \frac{2}{42}$ or $\frac{1}{21}$

44. $P(\text{at least 3 females})$
$= P(\text{3 females}) + P(\text{4 female})$
$= \frac{C(5, 3) \cdot C(5, 1)}{C(10, 4)} + \frac{C(5, 4) \cdot C(5, 0)}{C(10, 4)}$
$= \frac{50}{210} + \frac{5}{210}$
$= \frac{55}{210}$ or $\frac{11}{42}$

45. $P(\text{at least 2 females and at least 1 male})$
$= P(\text{2 females and 2 males}) +$
$P(\text{3 females and 1 male})$
$= \frac{C(5, 2) \cdot C(5, 2)}{C(10, 4)} + \frac{C(5, 3) \cdot C(5, 1)}{C(10, 4)}$
$= \frac{100}{210} + \frac{50}{210}$
$= \frac{150}{210}$ or $\frac{5}{7}$

46. $P(\text{word processing or playing games})$
$= P(\text{word processing}) + P(\text{playing games}) - P(\text{both})$
$= \frac{2}{5} + \frac{1}{3} - \frac{1}{4}$
$= \frac{29}{60}$

47. $P(\text{rain or lightning})$
$= P(\text{rain}) + P(\text{lightning}) - P(\text{both})$
$= \frac{3}{5} + \frac{2}{5} - \frac{1}{5}$
$= \frac{4}{5}$

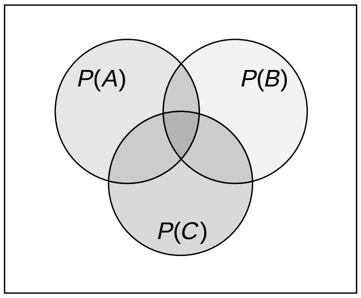
48. $P(\text{even sum})$
$= P(\text{3 even cards}) + P(\text{2 odd cards and 1 even card})$
$= \frac{C(5, 3) \cdot C(4, 0)}{C(9, 3)} + \frac{C(4, 2) \cdot C(5, 1)}{C(9, 3)}$
$= \frac{10}{84} + \frac{30}{84}$
$= \frac{40}{84}$ or $\frac{10}{21}$

49. $P(\text{at least 3 women})$
$= P(\text{3 women}) + P(\text{4 women}) + P(\text{5 women})$
$= \frac{C(6, 3) \cdot C(7, 2)}{C(13, 5)} + \frac{C(6, 4) \cdot C(7, 1)}{C(13, 5)} + \frac{C(6, 5) \cdot C(7, 0)}{C(13, 5)}$
$= \frac{420}{1287} + \frac{105}{1287} + \frac{6}{1287}$
$= \frac{531}{1287}$ or $\frac{59}{143}$

50. $P(\text{at least 1 doctor})$
$= P(\text{1 doctor}) + P(\text{both doctors})$
$= \left(\frac{93}{100} \cdot \frac{3}{100} + \frac{97}{100} + \frac{7}{100}\right) + \left(\frac{93}{100} \cdot \frac{97}{100}\right)$
$= \frac{958}{10{,}000} + \frac{9021}{10{,}000}$
$= \frac{9979}{10{,}000}$

51. $P(\text{supplies or money})$
$= P(\text{supplies}) + P(\text{money}) - P(\text{both})$
$= \frac{812}{2500} + \frac{625}{2500} - \frac{375}{2500}$
$= \frac{1062}{2500}$ or $\frac{531}{1250}$

52a.



52b. $P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) - P(A \text{ and } B) - P(A \text{ and } C) - P(B \text{ and } C) + P(A \text{ and } B \text{ and } C)$. You must add the intersection of all three sets which have not been accounted for.

53. $P(\text{action video or pop/rock CD or romance DVD})$
$\frac{4}{7} + \frac{1}{2} + \frac{5}{11} - \frac{2}{9} - \frac{1}{7} - \frac{1}{4} + \frac{1}{44} \approx 0.93$

54a. First consider the probability that no 2 students have the same birthday. The first person in the class can have any birthday; there are 366 choices out of 366 days. The second person has only 365 choices out of 366 days, and so on.
So, P(2 students with the same birthday)
$= 1 - P$(no 2 students have the same birthday).

$$= 1 - \left[\frac{366}{366} \cdot \frac{365}{366} \cdot \frac{364}{366} \cdot \ldots \cdot \frac{349}{366}\right]$$

$$= 1 - \frac{\frac{366!}{348!}}{366^{18}}$$

$$= 1 - \frac{\frac{366!}{(366-348)!}}{366^{18}}$$

$$= 1 - \frac{P(366, 18)}{366^{18}}$$

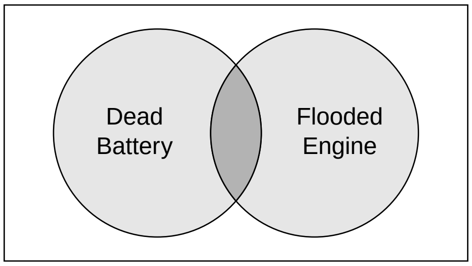
≈ 0.346

54b. $1 - \frac{P(366, n)}{366^n} > \frac{1}{2}$

54c. In part a, there is only a 0.346 probability that 2 students have the same birthday. This is too small. Substitute numbers greater than 18 for n in the inequality of part b. When n is 23, P is about 0.51. So, 23 − 18 or 5 more students are needed in the class.

55a. inclusive

55b.



55c. P(flooded engine or dead battery)
$= P$(flooded engine) $+ P$(dead battery) $- P$(both)

$$= \frac{1}{2} + \frac{2}{5} - \frac{1}{10}$$

$$= \frac{4}{5}$$

56. P(two threes given a sum of six) $= \frac{1}{5}$

57. $(7 - 1)! = 720$

58. Let b = basket. Let m = miss.

Expand $(b + m)^{20} = \sum_{r=0}^{20} \frac{20!}{r!(20-r)!} b^{20-r} m^r$

Find the coefficient of the $b^{15}m^5$ term where $r = 5$.

$\frac{20!}{5!(20-5)!} b^{20-5}m^5 = 15{,}504b^{15}m^5$

15,504

59. No, the spill will spread no more than 2000 meters away.

$a_1 = 1200$; $r = \frac{480}{1200}$ or 0.4

$$s = \frac{a_1}{1 - r}$$

$$= \frac{1200}{1 - 0.04}$$

$$= 2000$$

60.

$$\begin{aligned} 12^{x+2} &= 3^{x-4} \\ (x + 2) \log 12 &= (x - 4) \log 3 \\ x \log 12 + 2 \log 12 &= x \log 3 - 4 \log 3 \\ x \log 12 - x \log 3 &= -4 \log 3 - 2 \log 12 \\ x (\log 12 - \log 3) &= -4 \log 3 - 2 \log 12 \\ x &= \frac{-4 \log 3 - 2 \log 12}{\log 12 - \log 3} \\ x &\approx -6.7549 \end{aligned}$$

61. Let y = income and x = number of \$1.00 increases.

income = (number of customers) · (cost of a ticket)

$$\begin{aligned} y &= (400 - 20x)(3 + x) \\ y &= 1200 + 340x - 20x^2 \\ y - 1200 &= -20(x^2 - 17x) \\ y - 1200 - 1445 &= -20(x^2 - 17x + 72.25) \\ (y - 2645) &= -20(x - 8.5)^2 \end{aligned}$$

The vertex of the parabola is (8.5, 2645). An increase of \$8.50 will give a maximum profit of \$2645. The price of each ticket should be 3 + 8.5 or \$11.50.

62.

$$\begin{aligned} A &= \frac{\ell^2\pi}{4k} \\ 8270 &= \frac{\ell^2\pi}{4(7)} \\ \ell^2 &= \frac{231{,}560}{\pi} \\ \ell &\approx 271.5 \text{ yards} \end{aligned}$$

63.

$x = x_1 + ta_1$ $\quad$ $y = y_1 + ta_2$
$x = 1 + t(-2)$ $\quad$ $y = -5 + t(-4)$
$x - 1 = -2t$ $\quad$ $y + 5 = -4t$
$\langle x - 1, y + 5\rangle = t\langle -2, -4\rangle$

64.

$$\begin{aligned} 2 \tan x - 4 &= 0 \\ \tan x &= 2 \\ \tan^{-1}(\tan x) &= \tan^{-1} 2 \\ x &= 63° 26' \end{aligned}$$

65. Since $\angle ADB \cong \angle CBD$ and they are alternate interior angles, $\overline{AD} \parallel \overline{BC}$. Simply because $a = 45$ does not mean $b = 45$, so you cannot conclude that ℓ_3 bisects $\angle ABC$.
The correct choice is B.

13-5 Conditional Probability

Pages 870–871 Check for Understanding

1. Sample answer: If A and B are independent events, then $P(A|B) = P(A)$. Thus, the formula for conditional probability becomes $P(A) = \frac{P(A \text{ and } B)}{P(B)}$ or $P(A) \cdot P(B) = P(A \text{ and } B)$. This is the formula for the probability of independent events.
2. S = {J spades, Q spades, K spades, J clubs, Q clubs, K clubs}
3. Answers will vary; see students' work.
4. P(cubes match $|$ sum greater than 5) $= \frac{\frac{4}{36}}{\frac{26}{36}}$

$$= \frac{2}{13}$$

5. $P(\text{queen} \mid \text{face card}) = \dfrac{\frac{4}{52}}{\frac{12}{52}}$
$= \frac{1}{3}$

6. $P(\text{all heads} \mid \text{first coin is a head}) = \dfrac{\frac{1}{8}}{\frac{4}{8}}$
$= \frac{1}{4}$

7. $P(\text{all heads} \mid \text{at least 1 head}) = \dfrac{\frac{1}{8}}{\frac{7}{8}}$
$= \frac{1}{7}$

8. $P(\text{all heads} \mid \text{at least 2 heads}) = \dfrac{\frac{1}{8}}{\frac{4}{8}}$
$= \frac{1}{4}$

9. $P(\text{numbers match} \mid \text{sum greater than or equal to 9})$
$= \dfrac{\frac{2}{36}}{\frac{10}{36}}$
$= \frac{1}{5}$

10. $P(\text{sum is even} \mid \text{sum greater than or equal to 9})$
$= \dfrac{\frac{4}{36}}{\frac{10}{36}}$
$= \frac{2}{5}$

11. $P(\text{numbers match or sum is even} \mid \text{sum greater than or equal to 9}) = \dfrac{\frac{4}{36}}{\frac{10}{36}}$
$= \frac{2}{5}$

12a. $P(\text{disease prevented}) = \frac{68 + 62}{100 + 100}$
$= \frac{13}{20}$

12b. $P(\text{disease prevented} \mid \text{vaccine}) = \dfrac{\frac{68}{200}}{\frac{100}{200}}$
$= \frac{17}{25}$

12c. $P(\text{disease prevented} \mid \text{conventional treatment})$
$= \dfrac{\frac{62}{200}}{\frac{100}{200}}$
$= \frac{31}{50}$

13a. $P(\text{legal} \mid \text{accepted}) = \dfrac{\frac{69}{100}}{\frac{70}{100}}$
$= \frac{69}{70}$

13b. $P(\text{rejected} \mid \text{legal}) = \dfrac{\frac{6}{100}}{\frac{75}{100}}$
$= \frac{2}{25}$

13c. $P(\text{not rejected} \mid \text{counterfeit}) = \dfrac{\frac{1}{100}}{\frac{25}{100}}$
$= \frac{1}{25}$

Pages 872–874 Exercises

14. $P(\text{1 head} \mid \text{at least 1 tail}) = \dfrac{\frac{2}{4}}{\frac{3}{4}}$
$= \frac{2}{3}$

15. $P(\text{Democrat} \mid \text{man}) = \dfrac{\frac{4}{12}}{\frac{8}{12}}$
$= \frac{1}{2}$

16. $P(\text{first bag} \mid \text{first chip is blue}) = \dfrac{\frac{4}{16}}{\frac{10}{16}}$
$= \frac{2}{5}$

17. $P(\text{girls are separated} \mid \text{girl at an end}) = \dfrac{\frac{12}{24}}{\frac{20}{24}}$
$= \frac{3}{5}$

18. $P(\text{number end in 52} \mid \text{number is even})$
$= \dfrac{\frac{3 \cdot 2 \cdot 1}{5!}}{\frac{4 \cdot 3 \cdot 2 \cdot 1 + 4 \cdot 3 \cdot 2 \cdot 1}{5!}}$
$= \frac{1}{8}$

19. $P(\text{2 odd numbers} \mid \text{sum is even}) = \dfrac{\frac{20}{72}}{\frac{32}{72}}$
$= \frac{5}{8}$

20. $P(\text{ace} \mid \text{black}) = \dfrac{\frac{2}{52}}{\frac{26}{52}}$
$= \frac{1}{13}$

21. $P(4 \mid \text{black}) = \dfrac{\frac{2}{52}}{\frac{26}{52}}$
$= \frac{1}{13}$

22. $P(\text{face card} \mid \text{black}) = \dfrac{\frac{6}{52}}{\frac{26}{52}}$
$= \frac{3}{13}$

23. $P(\text{queen of hearts} \mid \text{black}) = \dfrac{\frac{0}{52}}{\frac{26}{52}}$
$= 0$

24. $P(\text{6 of clubs} \mid \text{black}) = \dfrac{\frac{1}{52}}{\frac{26}{52}}$
$= \frac{1}{26}$

25. $P(\text{jack or ten} \mid \text{black}) = \dfrac{\frac{4}{52}}{\frac{26}{52}}$

$= \frac{2}{13}$

26. $P(\text{second marble is green} \mid \text{first marble was green}) = \dfrac{\frac{3}{8} \cdot \frac{2}{7}}{\frac{3}{8}}$

$= \frac{2}{7}$

27. $P(\text{second marble is yellow} \mid \text{first marble was green}) = \dfrac{\frac{3}{8} \cdot \frac{5}{7}}{\frac{3}{8}}$

$= \frac{5}{7}$

28. $P(\text{second marble is yellow} \mid \text{first marble is yellow})$

$= \dfrac{\frac{5}{8} \cdot \frac{4}{7}}{\frac{5}{8}}$

$= \frac{4}{7}$

29. $P(\text{salmon} \mid \text{bass}) = \dfrac{\frac{C(1, 1) \cdot C(1, 1) \cdot C(4, 1)}{C(6, 3)}}{\frac{C(1, 1) \cdot C(5, 2)}{C(6, 3)}}$

$= \frac{4}{10}$ or $\frac{2}{5}$

30. $P(\text{not walleye} \mid \text{trout and perch})$

$= \dfrac{\frac{C(1, 1) \cdot C(1, 1) \cdot C(3, 1)}{C(6, 3)}}{\frac{C(1, 1) \cdot C(1, 1) \cdot (4, 1)}{C(6, 3)}}$

$= \frac{3}{4}$

31. $P(\text{bass and perch} \mid \text{not catfish})$

$= \dfrac{\frac{C(1, 1) \cdot C(1, 1) \cdot C(3, 1)}{C(6, 3)}}{\frac{C(5, 3)}{C(6, 3)}}$

$= \frac{3}{10}$

32. $P(\text{perch and trout} \mid \text{neither bass nor walleye})$

$= \dfrac{\frac{C(1, 1) \cdot C(1, 1) \cdot (2, 1)}{C(6, 3)}}{\frac{C(4, 3)}{C(6, 3)}}$

$= \frac{2}{4}$ or $\frac{1}{2}$

33.

Exercises 33–35

$P(\text{brown eyes} \mid \text{brown hair}) = \frac{0.10}{0.60}$

$= \frac{1}{6}$

34. $P(\text{no brown hair} \mid \text{brown eyes}) = \frac{0.20}{0.30}$

$= \frac{2}{3}$

35. $P(\text{no brown eyes} \mid \text{no brown hair}) = \frac{0.20}{0.40}$

$= \frac{1}{2}$

36. A = the sum of the cards is 7 or less
B = at least one card is an ace
B' = both cards not an ace

$P(B') = \frac{C(48, 2)}{C(52, 2)} = \frac{188}{221}$

$P(B) = 1 - P(B') = 1 - \frac{188}{221} = \frac{33}{221}$

$P(A \text{ and } B) = \frac{C(4, 2) + C(4, 1) \cdot C(20, 1)}{C(52, 2)}$

$= \frac{43}{663}$

$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$

$= \dfrac{\frac{43}{663}}{\frac{33}{221}}$

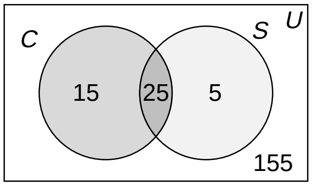
$= \frac{43}{99}$

37. $P(\text{sum greater than 18} \mid \text{queen of hearts})$

$= \dfrac{\frac{1}{52} \cdot \frac{19}{51}}{\frac{1}{52} \cdot \frac{51}{51}}$

$= \frac{19}{51}$

38a.



38b. $P(\text{cancer} \mid \text{smokes}) = \dfrac{\frac{25}{200}}{\frac{30}{200}}$

$= \frac{5}{6}$

39. A = person buys something
B = person asks questions

$P(A \mid B) = \dfrac{\frac{120}{500}}{\frac{150}{500}}$ or $\frac{4}{5}$

Four out of five people who ask questions will make a purchase. Therefore, they are more likely to buy something if they ask questions.

40. Sample answers: The rolls are independent. The number cubes do not have a memory, whether they are fair or biased. Probability does not guarantee an outcome.

41. $P(\text{passes} \mid \text{studied}) = \frac{P(\text{passes and studied})}{P(\text{studied})}$

$\frac{4}{5} = \dfrac{\frac{2}{3}}{P(\text{studied})}$

$P(\text{studied}) = \frac{2}{3} \cdot \frac{5}{4}$ or $\frac{5}{6}$

42a. $P(\text{defective}) = \frac{66}{1000}$ or $\frac{33}{500}$

42b. $P(\text{chip from 3-D Images} \mid \text{defective})$

$= \dfrac{\frac{21}{1000}}{\frac{66}{1000}}$

$= \frac{7}{22}$

42c. $P(\text{functioning}) = \frac{934}{1000}$ or $\frac{467}{500}$

42d. Sample answer: A chip from CyberChip Corp. has the least probability of being defective.

$P(\text{defective from CyberChip}) = \frac{25}{500}$
$= 0.05$

$P(\text{defective from 3-D Images}) = \frac{21}{300}$
$= 0.07$

$P(\text{defective from MegaView Designs}) = \frac{20}{200}$
$= 0.10$

43. $P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$ by definition.
So, if $P(A) = P(A \mid B)$ then by substitution $P(A) = \frac{P(A \text{ and } B)}{P(B)}$ or $P(A \text{ and } B) = P(A) \cdot P(B)$. Therefore, the events are independent.

44. $P(\text{at least 3 women})$

$$= P(3 \text{ women}) + P(4 \text{ women}) + P(5 \text{ women})$$
$$= \frac{C(6, 3) \cdot C(7, 2)}{C(13, 5)} + \frac{C(6, 4) \cdot C(7, 1)}{C(13, 5)} + \frac{C(6, 5) \cdot C(7, 0)}{C(13, 5)}$$
$$= \frac{420}{1287} + \frac{105}{1287} + \frac{6}{1287}$$
$$= \frac{531}{1287} \text{ or } \frac{59}{143}$$

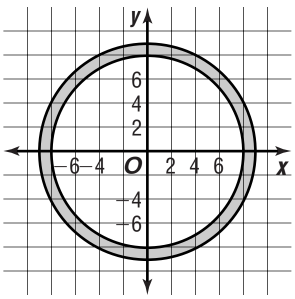
45. $C(9, 4) = 126$

46. $\sum_{b=1}^{\infty} 3(0.5)^b = 3(0.5)^1 + 3(0.5)^2 + 3(0.5)^3 + \ldots$
$= 1.5 + 0.75 + 0.375 + \ldots$

$S = \frac{a_1}{r}$ $\quad a_1 = 1.5, r = 0.5$
$= \frac{1.5}{0.5}$ or 3

47. They are reflections of each other over the x-axis.

48.



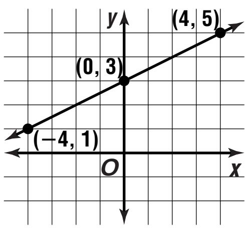
49.
$$r\left(\cos \theta - \frac{\pi}{2}\right) + 5 = 0$$
$$r\left(\cos \theta \cos \frac{\pi}{2} + \sin \theta \sin \frac{\pi}{2}\right) = -5$$
$$0 + r \sin \theta = -5$$
$$y = -5$$

50. $x = 4t$ $\qquad y = 3 + 2t$

$\frac{x}{4} = t$ $\qquad \frac{y-3}{2} = t$

$\frac{x}{4} = \frac{y-3}{2}$

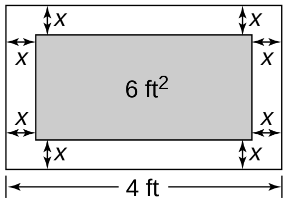
$y = \frac{x}{2} + 3$



51. $A = \frac{1}{2}r^2 \theta$
$= \frac{1}{2}(8)^2\left(\frac{98}{1} \cdot \frac{71}{180}\right)$
$\approx 54.7 \text{ ft}^2$

52. $\tan 27° = \frac{x}{25}$
$12.7 \approx x$; 12.7 m

53.



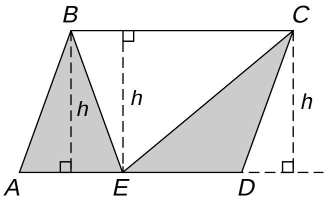
$$(4 - 2x)(3 - 2x) = 6$$
$$12 - 14x + 4x^2 = 6$$
$$4x^2 - 14x + 6 = 0$$
$$2(2x - 1)(x - 3) = 0$$
$$x = \frac{1}{2} \text{ or } x = 3$$

If $x = 3$ ft then the photo would have a negative length and width. So, $x = \frac{1}{2}$ ft or 6 in.

54. $f(x) = \frac{5}{x^2 - 4}$
$= \frac{5}{(x + 2)(x - 2)}$

$f(x)$ is discontinuous when $x = \pm 2$;
$f(x)$ is undefined when $x = \pm 2$.

55. Drop an altitude from B to $\overline{AE}$, from C to $\overleftrightarrow{ED}$, and from E to $\overline{BC}$. Label the diagram as shown.



area of unshaded region $= \frac{1}{2}(BC)h$

area of shaded region $= \frac{1}{2}(AE)h + \frac{1}{2}(ED)h$
$= \frac{1}{2}h(AE + ED)$

$BC = AE + ED$ since opposite sides of a parallelogram are equal. So, the ratio of the areas is $\frac{1}{2}h(BC):\frac{1}{2}h(BC)$ or 1:1.

The correct choice is B.

13-6 The Binomial Theorem and Probability

Page 877 Graphing Calculator Exploration

1. $S = \{0, 1, 2\}$
2. P (Bobby wins) $= \frac{2}{3}$
3. Answers will vary. In 40 repetitions, it may be around 0.22. This means that there were exactly 5 wins for 8 or 9 of the 40 repetitions.
4. $P(\text{winning 5 games}) = C(6, 5) \cdot \left(\frac{2}{3}\right)^5 \cdot \left(\frac{1}{3}\right)^1$
≈ 0.26
5. There is not a large enough sample of trials.
6. Increase the number of repetitions.

Page 878 Check for Understanding

1a. Yes, it meets all three conditions.
1b. No, there are more than 2 possible outcomes.
1c. No, the events are not independent.
2. Sample answer: the probabilities derived from a simulation rather than an actual event

3. First, determine P(right) and P(wrong). Second, set up the binomial expansion $(p_r + p_w)^5$. Third, determine the term of the expansion. Fourth, substitute the probability values for p_r and p_w. Last, compute the probability of getting exactly 2 correct answers.

4. $P(\text{only one } 4) = C(5, 1)\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^4 = \frac{3125}{7776}$

5. $P(\text{no more than two } 4\text{s})$
$= P(\text{no } 4\text{s}) + P(\text{one } 4) + P(\text{two } 4\text{s})$
$= C(5, 0)\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^5 + C(5, 1)\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^4 + C(5, 2)\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3$
$= \frac{3125}{7776} + \frac{3125}{7776} + \frac{1250}{7776}$
$= \frac{625}{648}$

6. $P(\text{at least three } 4\text{s})$
$= P(\text{three } 4\text{s}) + P(\text{four } 4\text{s}) + P(5 \text{ fours})$
$= C(5, 3)\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2 + C(5, 4)\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^1 + C(5, 5)\left(\frac{1}{6}\right)^5\left(\frac{5}{6}\right)^0$
$= \frac{250}{7776} + \frac{25}{7776} + \frac{1}{7776}$
$= \frac{23}{648}$

7. $P(\text{exactly five } 4\text{s}) = C(5, 5)\left(\frac{1}{6}\right)^5\left(\frac{5}{6}\right)^0 = \frac{1}{7776}$

8. $P(\text{not having rain on any day}) = C(5, 5)\left(\frac{7}{10}\right)^5\left(\frac{3}{10}\right)^0 = \frac{16{,}807}{100{,}000}$

9. $P(\text{having rain on exactly one day})$
$= C(5, 1)\cdot\left(\frac{3}{10}\right)^1\left(\frac{7}{10}\right)^4$
$= \frac{36{,}015}{100{,}000}$ or $\frac{7203}{20{,}000}$

10. $P(\text{having rain no more than three days})$
$= 1 - [P(\text{rain on 4 days}) + P(\text{rain on 5 days})]$
$= 1 - \left[C(5, 4)\cdot\left(\frac{3}{10}\right)^4\left(\frac{7}{10}\right)^1 + C(5, 5)\cdot\left(\frac{3}{10}\right)^5\left(\frac{7}{10}\right)^0\right]$
$= 1 - \left[\frac{2835}{100{,}000} + \frac{243}{100{,}000}\right]$
$= \frac{96{,}922}{100{,}000}$ or $\frac{48{,}461}{50{,}000}$

11. $P(\text{4 do not collapse}) = C(6, 4)\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)^2 = \frac{3840}{15{,}625}$ or $\frac{768}{3125}$

12. $P(\text{10 stocks make money}) = C(13, 10)\left(\frac{5}{8}\right)^{10}\left(\frac{3}{8}\right)^3 \approx 0.1372$

Pages 878–880 Exercises

13. $P(\text{never the correct color}) = C(4, 0)\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^4 = 1\cdot 1\cdot\frac{1}{81} = \frac{1}{81}$

14. $P(\text{correct at least 3 times})$
$= C(4, 3)\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^1 + C(4, 4)\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^0$
$= 4\cdot\frac{8}{27}\cdot\frac{1}{3} + 1\cdot\frac{16}{81}\cdot 1$
$= \frac{16}{27}$

15. $P(\text{no more than 3 times correct})$
$= 1 - P(\text{correct 4 times})$
$= 1 - C(4, 4)\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^0$
$= 1 - 1\cdot\frac{16}{81}\cdot 1$
$= 1 - \frac{16}{81}$
$= \frac{65}{81}$

16. $P(\text{correct exactly 2 times}) = C(4, 2)\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^2 = 6\cdot\frac{4}{9}\cdot\frac{1}{9} = \frac{8}{27}$

17. $P(\text{7 correct}) = C(10, 7)\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^3 = 120\cdot\frac{1}{128}\cdot\frac{1}{8} = \frac{15}{128}$

18. $P(\text{at least 6 correct})$
$= C(10, 6)\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^4 + C(10, 7)\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^3 + C(10, 8)\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 + C(10, 9)\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^1 + C(10, 10)\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^0$
$= 210\cdot\frac{1}{64}\cdot\frac{1}{16} + 120\cdot\frac{1}{128}\cdot\frac{1}{8} + 45\cdot\frac{1}{256}\cdot\frac{1}{4} + 10\cdot\frac{1}{512}\cdot\frac{1}{2} + 1\cdot\frac{1}{1024}\cdot 1$
$= \frac{386}{1024}$
$= \frac{193}{512}$

19. $P(\text{all incorrect}) = C(10, 0)\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{10} = 1\cdot 1\cdot\frac{1}{1024} = \frac{1}{1024}$

20. $P(\text{at least half correct})$
$= C(10, 5)\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^5 + \text{answer to 18}$
$= 252\cdot\frac{1}{32}\cdot\frac{1}{32} + \frac{386}{1024}$
$= \frac{252}{1024} + \frac{386}{1024}$
$= \frac{319}{512}$

21. $P(\text{4 heads}) = C(4, 4)\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^0 = 1\cdot\frac{1}{81}\cdot 1 = \frac{1}{81}$

22. $P(\text{3 heads}) = C(4, 3)\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^1 = 4\cdot\frac{1}{27}\cdot\frac{2}{3} = \frac{8}{81}$

23. $P(\text{at least 2 heads})$
$= C(4, 2)\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2 + \text{answer to 22} + \text{answer to 21}$
$= 6\cdot\frac{1}{9}\cdot\frac{4}{9} + \frac{8}{81} + \frac{1}{81}$
$= \frac{24}{81} + \frac{9}{81}$
$= \frac{11}{27}$

24. $P(\text{6 correct answers}) = C(10, 6)\left(\frac{1}{4}\right)^6\left(\frac{3}{4}\right)^4 = 210\cdot\frac{1}{4096}\cdot\frac{81}{256} \approx 0.016$

25. $P(\text{half answers correct}) = C(10, 5)\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^5$
$= 252 \cdot \frac{1}{1024} \cdot \frac{243}{1024}$
≈ 0.058

26. $P(\text{from 3 to 5 correct answers})$
$= C(10, 3)\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^7 + C(10, 4)\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^6$
$+ C(10, 5)\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^5$
$= 120 \cdot \frac{1}{64} \cdot \frac{2187}{16{,}384} + 210 \cdot \frac{1}{256} \cdot \frac{729}{4096}$
$+ 252 \cdot \frac{1}{1024} \cdot \frac{243}{1024}$
$\approx 0.25 + 0.15 + 0.06$
≈ 0.46

27. $P(\text{all point up}) = C(10, 10)\left(\frac{2}{5}\right)^{10}\left(\frac{3}{5}\right)^0$
$= 1 \cdot \frac{1024}{9{,}765{,}625} \cdot 1$
$\approx 1.049 \times 10^{-4}$

28. $P(\text{exactly 3 point up}) = C(10, 3)\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^7$
$= 120 \cdot \frac{8}{125} \cdot \frac{2187}{78{,}125}$
≈ 0.215

29. $P(\text{exactly 5 point up}) = C(10, 5)\left(\frac{2}{5}\right)^5\left(\frac{3}{5}\right)^5$
$= 252 \cdot \frac{32}{3125} \cdot \frac{243}{3125}$
≈ 0.201

30. $P(\text{at least 6 point up})$
$= C(10, 6)\left(\frac{2}{5}\right)^6\left(\frac{3}{5}\right)^4 + C(10, 7)\left(\frac{2}{5}\right)^7\left(\frac{3}{5}\right)^3$
$+ C(10, 8)\left(\frac{2}{5}\right)^8\left(\frac{3}{5}\right)^2 + C(10, 9)\left(\frac{2}{5}\right)^9\left(\frac{3}{5}\right)^1$
$+ C(10, 10)\left(\frac{2}{5}\right)^{10}\left(\frac{3}{5}\right)^0$
$= 210 \cdot \frac{64}{15{,}625} \cdot \frac{81}{625} + 120 \cdot \frac{128}{78{,}125} \cdot \frac{27}{125}$
$+ 45 \cdot \frac{256}{390{,}625} \cdot \frac{9}{25} + 10 \cdot \frac{512}{1{,}953{,}125} \cdot \frac{3}{5}$
$+ 1 \cdot \frac{1024}{9{,}765{,}625} \cdot 1$
≈ 0.166

31. $P(\text{3 heads or 3 tails})$
$= C(3, 3)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0 + C(3, 3)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0$
$= 1 \cdot \frac{1}{8} \cdot 1 + 1 \cdot \frac{1}{8} \cdot 1$
$= \frac{1}{4}$

32. $P(\text{at least 2 heads})$
$= C(3, 2)\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1 + C(3, 3)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0$
$= 3 \cdot \frac{1}{4} \cdot \frac{1}{2} + 1 \cdot \frac{1}{8} \cdot 1$
$= \frac{4}{8}$
$= \frac{1}{2}$

33. $P(\text{exactly 2 tails}) = C(3, 2)\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1$
$= 3 \cdot \frac{1}{4} \cdot \frac{1}{2}$
$= \frac{3}{8}$

34a. The values of the function for $0 \le x \le 6$ are the coefficients of the binomial expansion.

34b. Change 6 nCr X to 8 nCr X on the Y-menu.

35. Enter 16 nCr X on the Y-menu of your calculator. 16 nCr X represents the coefficients of the binomial expansion where X is the number of games won.
$P(\text{winning at least 12 games})$
$= 1820 \cdot \left(\frac{7}{10}\right)^{12}\left(\frac{3}{10}\right)^4 + 560 \cdot \left(\frac{7}{10}\right)^{13}\left(\frac{3}{10}\right)^3$
$+ 120 \cdot \left(\frac{7}{10}\right)^{14}\left(\frac{3}{10}\right)^2$
$+ 16 \cdot \left(\frac{7}{10}\right)^{15}\left(\frac{3}{10}\right)^1 + 1 \cdot \left(\frac{7}{10}\right)^{16}\left(\frac{3}{10}\right)^1$
≈ 0.45 or 45%

36a. A success means that a missile hits its target. There are 6 trials and the probability of success on each trial is 20% or $\frac{1}{5}$.

36b. $P(\text{between 2 and 6 missiles hit the target})$
$= 1 - [P(\text{0 missiles hit the garget}) + P(\text{1 missile hits the target})]$
$= 1 - \left[C(6, 0)\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^6 + C(6, 1)\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^5\right]$
$= 1 - \left(1 \cdot 1 \cdot \frac{4096}{15{,}625} + 6 \cdot \frac{1}{5} \cdot \frac{1024}{3125}\right)$
$= \frac{1077}{3125}$

37. $P(\text{all men or all women})$
$= C(10, 10)\left(\frac{4}{10}\right)^{10}\left(\frac{6}{10}\right)^0 + C(10, 10)\left(\frac{6}{10}\right)^{10}\left(\frac{4}{10}\right)^0$
≈ 0.0062

38a. $P(\text{all carry the disease}) = C(20, 20)\left(\frac{1}{10}\right)^{20}\left(\frac{9}{10}\right)^0$
$= 1 \times 10^{-20}$

38b. $P(\text{exactly half have the disease})$
$= C(20, 10)\left(\frac{1}{10}\right)^{10}\left(\frac{9}{10}\right)^{10}$
$\approx 6.4 \times 10^{-6}$

39. $P(\text{at least 2 people do not show up})$
$= 1 - [P(\text{0 people do not show up})$
$+ P(\text{1 person does not show up})]$
$= 1 - \left[C(75, 0)\left(\frac{4}{100}\right)^0\left(\frac{96}{100}\right)^{75}\right.$
$\left.+ C(75, 1)\left(\frac{4}{100}\right)^1\left(\frac{96}{100}\right)^{74}\right]$
≈ 0.807

40. $P(\text{less than or equal to 3 policies})$
$= 1 - P(\text{4 policies})$
$= 1 - \left[C(4, 4)\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0\right]$
$= 1 - \left(1 \cdot \frac{1}{16} \cdot 1\right)$
$= \frac{15}{16}$ or about 0.94

41a. If Trina walks 100 meters, then she has flipped the coin 10 times. To end up where she began, she walked north and south 5 times each.
$P(\text{back at her starting point}) = C(10, 5)\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^5$
$= 252 \cdot \frac{1}{32} \cdot \frac{1}{32}$
≈ 0.246

41b. The closest Trina can come to her starting point is if she flips 6 heads and 4 tails or 4 heads and 6 tails. However, this places her 20 meters from her starting point. The answer for part b is the same as that for part a, 0.246.

41c. $P(\text{exactly 20 meters from the starting point})$

$$= C(10, 6)\left(\tfrac{1}{2}\right)^6\left(\tfrac{1}{2}\right)^4 + C(10, 4)\left(\tfrac{1}{2}\right)^4\left(\tfrac{1}{2}\right)^6$$
$$= 210 \cdot \tfrac{1}{64} \cdot \tfrac{1}{16} + 210 \cdot \tfrac{1}{16} \cdot \tfrac{1}{64}$$
$$\approx 0.41$$

42. $P(\text{sum less than 9} \mid \text{both cubes are the same})$

$$= \frac{\frac{4}{36}}{\frac{6}{36}}$$
$$= \frac{2}{3}$$

43. $P(\text{letter is contained in house or phone})$

$$= \tfrac{5}{26} + \tfrac{5}{26} - \tfrac{3}{26}$$
$$= \tfrac{7}{26}$$

44a. 80, 75, 70, . . .

44b. $T = 80 - 5n$

44c. $T = -125$

$n = \frac{40{,}000}{1000}$ or 40

$-125 = g - 5(40)$

$75 = g$; 75° F

45.

$$3^{x-1} = 6^{-x}$$
$$(x - 1)\log 3 = -x \log 6$$
$$x \log 3 - \log 3 = -x \log 6$$
$$x \log 3 + x \log 6 = \log 3$$
$$x(\log 3 + \log 6) = \log 3$$
$$x = \frac{\log 3}{\log 3 + \log 6}$$
$$x \approx 0.38$$

46. $h = 0$, $k = -3$, $a = 7$, $b = 5$, $c = 2\sqrt{6}$

Center: $(0, -3)$

Foci: $(\pm 2\sqrt{6}, -3)$

Vertices: major axis $\rightarrow (7, -3)$ and $(-7, -3)$

minor axis $\rightarrow (0, 2)$ and $(0, -8)$

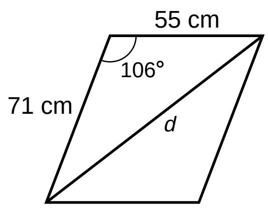
47. $\sqrt{2}\left(\cos -\frac{\pi}{2} + \boldsymbol{i} \sin -\frac{\pi}{2}\right) = \sqrt{2} \cdot 0 + \sqrt{2} \cdot -\boldsymbol{i}$

$= 0 - \sqrt{2}\boldsymbol{i}$

48. $\overrightarrow{WX} = \langle 6 - 8, 5 - (-3)\rangle$ or $\langle -2, 8\rangle$

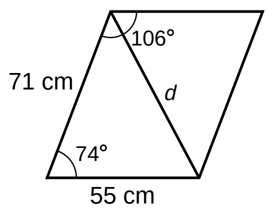
$$|\overrightarrow{WX}| = \sqrt{(6 - 8)^2 + (5 - (-3))^2}$$
$$= \sqrt{(-2)^2 + 8^2}$$
$$= \sqrt{68} \text{ or } 2\sqrt{17}$$

49.



$d^2 = 71^2 + 55^2 - 2(71)(55) \cos 106°$

$d \approx 101.1$ cm



$d^2 = 71^2 + 55^2 - 2(71)(55) \cos 74°$

$d \approx 76.9$ cm

50.

$$\begin{array}{r|rrrrr} -4 & 1 & 12 & 21 & -62 & -72 \\ & & -4 & -32 & 44 & 72 \\ \hline & 1 & 8 & -11 & -18 & 0 \end{array}$$

Since the remainder is 0, $x + 4$ is a factor of the polynomial.

51. $$\frac{\text{Area of page for text}}{\text{Area of entire page}} = \frac{(9 - 1 - 1)(12 - 1.5 - 1.5)}{9 \cdot 12}$$
$$= \frac{63}{108}$$
$$= \frac{7}{12}$$

The answer is 7/12.

Chapter 13 Study Guide and Assessment

Page 881 Understanding the Vocabulary

1. independent
2. failure
3. 1
4. probability
5. permutation
6. permutation with repetitions
7. mutually exclusive
8. sample space
9. conditional
10. combinatorics

Pages 882–884 Skills and Concepts

11. Using the Basic Counting Principle, $3 \cdot 2 \cdot 1$ or 6.

12. Using the Basic Counting Principle, $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ or 120.

13. Using the Basic Counting Principle, $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ or 720.

14. $$P(6, 3) = \frac{6!}{(6 - 3)!}$$
$$= \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1}$$
$$= 120$$

15. $$P(8, 6) = \frac{8!}{(8 - 6)!}$$
$$= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1}$$
$$= 20{,}160$$

16. $$C(5, 3) = \frac{5!}{(5 - 3)!\,3!}$$
$$= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}$$
$$= 10$$

17. $$C(11, 8) = \frac{11!}{(11 - 8)!\,8!}$$
$$= \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$
$$= 165$$

18. $$\frac{P(6, 3)}{P(5, 3)} = \frac{\frac{6!}{(6 - 3)!}}{\frac{5!}{(5 - 3)!}}$$
$$= \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}$$
$$= 2$$

19. $$C(5, 5) \cdot C(3, 2) = \frac{5!}{(5 - 5)!\,5!} \cdot \frac{3!}{(3 - 2)!\,2!}$$
$$= \frac{1}{1} \cdot \frac{3 \cdot 2 \cdot 1}{1 \cdot 2 \cdot 1}$$
$$= 3$$

20. There are $P(5, 5)$ ways to arrange the other books if the dictionary is on the left end. The same is true if the dictionary is on the right end.

$$2 \cdot P(5, 5) = 2 \cdot \frac{5!}{(5-5)!} = 2 \cdot \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} = 240$$

21. $$C(3, 2) \cdot C(7, 2) = \frac{3!}{(3-2)!\,2!} \cdot \frac{7!}{(7-2)!\,2!} = \frac{3 \cdot 2 \cdot 1}{1 \cdot 2 \cdot 1} \cdot \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 3 \cdot 21 \text{ or } 63$$

22. $$\frac{5!}{2!\,2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1} = 30$$

23. $$\frac{10!}{2!\,3!\,3!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = 50{,}400$$

24. $$\frac{8!}{2!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 20{,}160$$

25. $$\frac{6!}{3!\,2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 60$$

26. $$\frac{9!}{3!\,2!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 30{,}240$$

27. $$P(3 \text{ pennies}) = \frac{C(7, 3) \cdot C(4, 0) \cdot C(5, 0)}{C(16, 3)} = \frac{35 \cdot 1 \cdot 1}{560} = \frac{1}{16}$$

28. $$P(2 \text{ pennies and } 1 \text{ nickel}) = \frac{C(7, 2) \cdot C(4, 1) \cdot C(5, 0)}{C(16, 3)} = \frac{21 \cdot 4 \cdot 1}{560} = \frac{3}{20}$$

29. $$P(3 \text{ nickels}) = \frac{C(7, 0) \cdot C(4, 3) \cdot C(5, 0)}{C(16, 3)} = \frac{1 \cdot 4 \cdot 1}{560} = \frac{1}{140}$$

30. $$P(1 \text{ nickel and } 2 \text{ dimes}) = \frac{C(7, 0) \cdot C(4, 1) \cdot C(5, 2)}{C(16, 3)} = \frac{1 \cdot 4 \cdot 10}{560} = \frac{1}{14}$$

31. $P(s) = \frac{1}{16}$; $P(f) = \frac{15}{16}$

$$\text{odds} = \frac{\frac{1}{16}}{\frac{15}{16}} = \frac{1}{15}$$

32. $P(s) = \frac{3}{20}$; $P(f) = \frac{17}{20}$

$$\text{odds} = \frac{\frac{3}{20}}{\frac{17}{20}} = \frac{3}{17}$$

33. $P(s) = \frac{1}{140}$; $P(f) = \frac{139}{140}$

$$\text{odds} = \frac{\frac{1}{140}}{\frac{139}{140}} = \frac{1}{139}$$

34. $P(s) = \frac{1}{14}$; $P(f) = \frac{13}{14}$

$$\text{odds} = \frac{\frac{1}{14}}{\frac{13}{14}} = \frac{1}{13}$$

35. independent, $P(\text{sum of } 2) \cdot P(\text{sum of } 6) = \frac{1}{36} \cdot \frac{5}{36} = \frac{5}{1296}$

36. dependent, $P(\text{two yellow markets}) = \frac{4}{10} \cdot \frac{3}{9} = \frac{2}{15}$

37. $P(\text{selecting a prime number or a multiple of } 4)$
$$= P(\text{prime number}) + P(\text{a multiple of } 4) = \frac{6}{14} + \frac{3}{14} = \frac{9}{14}$$

38. $P(\text{selecting a multiple of } 2 \text{ or a multiple of } 3)$
$$= P(\text{multiple of } 2) + P(\text{multiple of } 3) - P(\text{multiple of } 2 \text{ and } 3) = \frac{7}{14} + \frac{4}{14} - \frac{2}{14} = \frac{9}{14}$$

39. $$P(\text{selecting a } 3 \text{ or a } 4) = P(3) \text{ or } P(4) = \frac{1}{14} + \frac{1}{14} = \frac{1}{7}$$

40. $P(\text{selecting an } 8 \text{ or a number less than } 8)$
$$= P(8) + P(\text{less than } 8) = \frac{1}{14} + \frac{7}{14} = \frac{4}{7}$$

41. $P(\text{sum less than } 5 \mid \text{exactly one cube shows } 1)$
$$= \frac{\frac{4}{36}}{\frac{10}{36}} = \frac{2}{5}$$

42. $$P(\text{different numbers} \mid \text{sum is } 8) = \frac{\frac{4}{36}}{\frac{5}{36}} = \frac{4}{5}$$

43. $P(\text{numbers match} \mid \text{sum is greater than or equal to } 5)$
$$= \frac{\frac{4}{36}}{\frac{30}{36}} = \frac{2}{15}$$

44. $$P(\text{exactly } 1 \text{ head}) = C(4, 1)\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3 = 4 \cdot \frac{1}{2} \cdot \frac{1}{8} = \frac{1}{4}$$

45. $$P(\text{no heads}) = C(4, 0)\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4 = 1 \cdot 1 \cdot \frac{1}{16} = \frac{1}{16}$$

46. $P(\text{2 heads and 2 tails}) = C(4, 2)\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2$

$= 6 \cdot \frac{1}{4} \cdot \frac{1}{4}$

$= \frac{3}{8}$

47. $P(\text{at least 3 tails})$

$= P(\text{3 tails}) + P(\text{4 tails})$

$= C(4, 3)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1 + C(4, 4)\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0$

$= 4 \cdot \frac{1}{8} \cdot \frac{1}{2} + 1 \cdot \frac{1}{16} \cdot 1$

$= \frac{1}{4} + \frac{1}{16}$ or $\frac{5}{16}$

Page 885 Applications and Problem Solving

48. $C(1, 1) \cdot C(6, 4) = 1 \cdot 15$ or 15

49. $\frac{7!}{2} = 2520$

50. $P(\text{at least 1 good chip})$

$= 1 - P(\text{both defective chips})$

$= 1 - \frac{C(3, 2)}{C(15, 2)}$

$= 1 - \frac{1}{35}$

$= \frac{34}{35}$

51a. $P(\text{female name excluding Reba}) = \frac{7}{15}$

51b. $P(\text{Reba's name, then a male name}) = \frac{1}{15} \cdot \frac{7}{14}$

$= \frac{1}{30}$

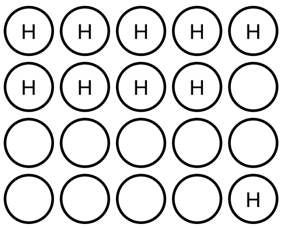
Page 885 Open-Ended Assessment

1. Yes; sample answer: $x \cdot \frac{1}{2} = \frac{1}{12}$, so $x = \frac{1}{6}$; Two marbles are chosen from a box containing 6 red, 4 blue, and 2 green marbles. What is the probability of choosing a red and a green marble?
2. Sample answer: In a permutation, the order of objects is very important. In a combination, the order of objects is not important.

Chapter 13 SAT & ACT Preparation

Page 887 SAT and ACT Practice

1. You might want to draw a diagram of the 20 coins. The first and last coins are heads. The total number of heads is 10. There could be 9 consecutive heads followed by 10 tails and then the final head. The correct choice is D.



2. $4\sqrt{x-3} + 8 = 32$

$4\sqrt{x-3} = 24$

$\sqrt{x-3} = 6$

$(\sqrt{x-3})^2 = 6^2$

$x - 3 = 36$

$x - 3 = 39$

The correct choice is E.

3. Find the probability of selecting a green marble from the jar now.

$\frac{\text{number of green marbles}}{\text{total marbles}} = \frac{3}{15}$ or $\frac{1}{5}$

Let x represent the number of green marbles added so the probability equals $2 \cdot \frac{1}{5}$ or $\frac{2}{5}$.

$\frac{\text{number of green marbles}}{\text{total marbles}} = \frac{3 + x}{15 + x} = \frac{2}{5}$

$5(3 + x) = 2(15 + x)$

$15 + 5x = 30 + 2x$

$3x = 15$

$x = 5$

The correct choice is C.

4. $\text{Average} = \frac{\text{sum of terms}}{\text{number of terms}}$

$20 = \frac{\text{sum of five terms}}{5}$

sum of five terms $= 100$

Since one of the numbers is 18, the sum of the other four is $100 - 18$ or 82. The correct choice is C.

5. Select specific numbers for the problem. Let $x + y = 8$. Let $x + z = 12$. Let $z = 7$.
$x + 7 = 12$, so $x = 5$, an odd number.
$5 + y = 8$, so $y = 3$, an odd number.
Statement I is false, and statement III is true. Since $y + z$ or $3 + 7 = 10$, an even number, statement II is true also. The correct choice is E.

6. Since the probability of selecting a blue marble is $\frac{1}{5}$ and the total number of blue and white marbles is 200, the number of blue marbles must be 40. So the number of white marbles must be 160. After 100 white marbles are added, the total number of white marbles is 260 and the total of all marbles is 300. The probability of selecting a white marble is $\frac{260}{300}$ or $\frac{13}{15}$. The correct choice is E.

7. $\angle A$ and $\angle C$ must be equal because they are corresponding angles. The correct choice is A.

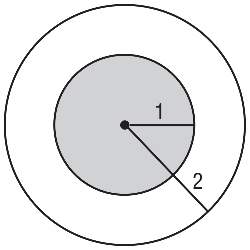
8. The sample space, or total possible outcomes, is the 52 cards in a deck. The outcome "drawing a diamond" consists of the 13 cards that are diamonds.

$P(\text{diamond}) = \frac{13}{52}$ or $\frac{1}{4}$

The correct choice is C.

9. 7 different entrees are offered. 3 are chosen. The number of combinations that can be chosen is $C(7, 3) = \frac{7!}{3!(7-3)!} = \frac{7!}{3!4!} = 35$.
The correct choice is B.

10.



The probability that a dart thrown randomly at the target will land in the shaded region is equal to the ratio of the area of the shaded region to the area of the entire target.

$$P = \frac{\text{area of shaded region}}{\text{area of target}}$$

$$= \frac{\pi(1)^2}{\pi(2)^2}$$

$$= \frac{\pi}{4\pi} \text{ or } \frac{1}{4}$$

The answer is $\frac{1}{4}$.

Chapter 14 Statistics and Data Analysis

14-1 The Frequency Distribution

Pages 892–893 Check for Understanding

1. A line plot, a bar graph, a histogram, and a frequency polygon all show data visually. A line plot shows the frequency of specific quantities by using symbols and a bar graph shows the frequency of specific quantities by using bars. A histogram is a special bar graph in which the width of each bar represents a class interval. A frequency polygon shows the frequency of a class interval using a broken line graph.

2. Choose an appropriate class interval. Use tally marks to determine the number of elements in each class interval.

3a. No; there would be too many classes.

3b. Yes; there would be 9 classes.

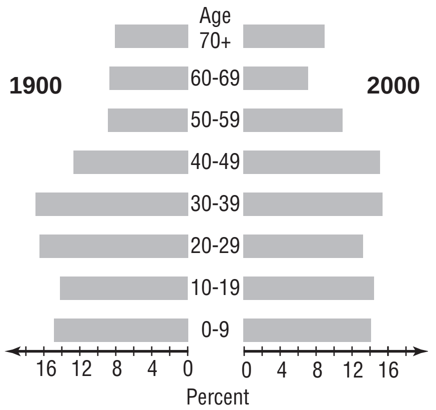
3c. Yes; there would be 5 classes.

3d. No; there would only be 3 classes.

3e. No; there would only be 2 classes.

4. See students' work.

5a.



5b. In 1999, there are larger percents of older citizens than in 1990.

6a. range = 69 − 42 or 27

6b. Sample answer: 5

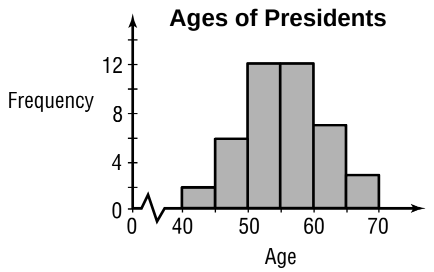
6c. Sample answer: 40, 45, 50, 55, 60, 65, 70

6d. Sample answer: 42.5, 47.5, 52.5, 57.5, 62.5, 67.5

6e. Sample answer:

Ages	Frequency
40–45	2
45–50	6
50–55	12
55–60	12
60–65	7
65–70	3

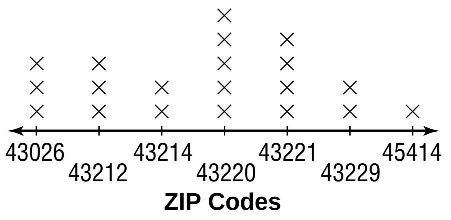
6f. Sample answer:



6g. Sample answer: 50–60

Pages 893–896 Exercises

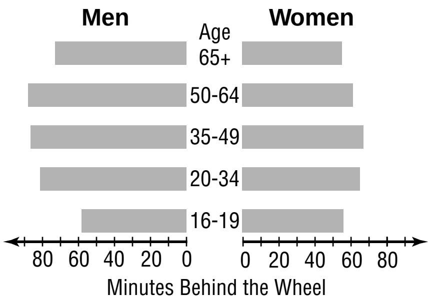
7a.



7b. 43220

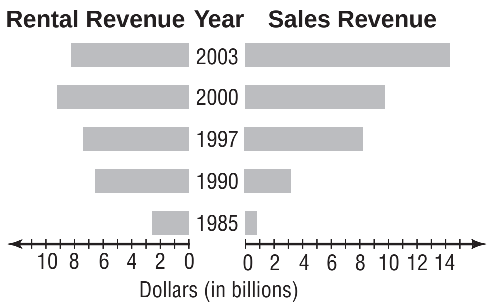
7c. Sample answer: to determine where most of their customers live so they can target their advertising accordingly

8a.



8b. Sample answer: Men spend more hours driving than women.

9a.



9b. Sales; the sales revenue is increasing, and the rental revenue has started to decrease.

10a. range = 53 − 4 or 49

10b. Sample answer: 10

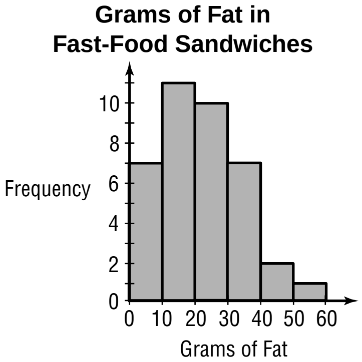
10c. Sample answer: 0, 10, 20, 30, 40, 50, 60

10d. Sample answer: 5, 15, 25, 35, 45, 55

10e. Sample answer:

Grams of Fat	Frequency
0–10	7
10–20	11
20–30	10
30–40	7
40–50	2
50–60	1

10f. Sample answer:



10g. Sample answer: 10–20

11a. range = 72 − 16 or 56

11b. Sample answer: 10

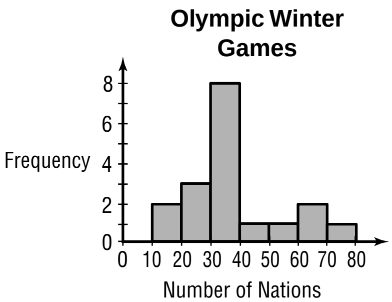
11c. Sample answer: 10, 20, 30, 40, 50, 60, 70, 80

11d. Sample answer: 15, 25, 35, 45, 55, 65, 76

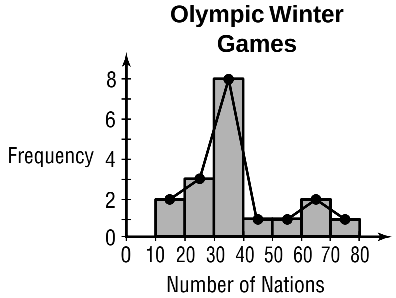
11e. Sample answer:

Number of Nations	Frequency
10–20	2
20–30	3
30–40	8
40–50	1
50–60	1
60–70	2
70–80	1

11f. Sample answer:



11g. Sample answer:



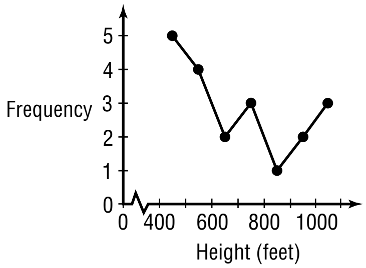
12a. range = 1023 − 404 or 619

12b. Sample answer: 100

12c. Sample answer:

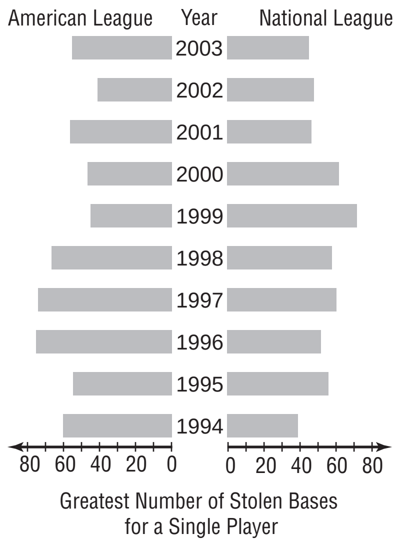
Height (feet)	Frequency
400–500	5
500–600	4
600–700	2
700–800	3
800–900	1
900–1000	2
1000–1100	3

12d. Sample answer:



12e. Sample answer: 400–500

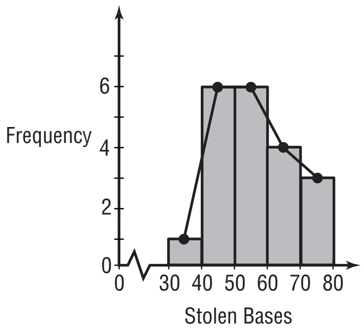
13a.



13b. Sample answer:

Stolen Bases	Frequency
30–40	1
40–50	6
50–60	6
60–70	4
70–80	3

13c. Sample answer:

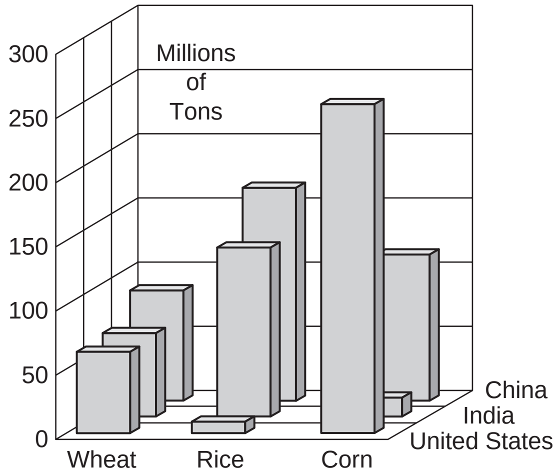


13d. 3 players

13e. 7 players

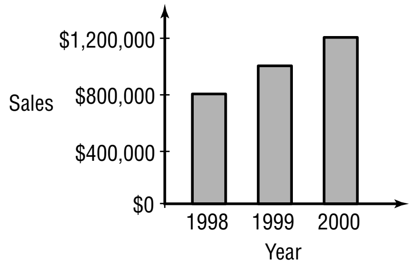
14. Sample answer: 0.1, 0.2, 0.3, 0.4, 0.6, 0.7, 0.8, 0.9, 1.1, 1.2, 1.3, 1.4, 1.6, 1.7, 1.8, 1.9, 2.1, 2.2, 2.3, 2.4

15.



16a. The first interval on the vertical axis represents \$800,000, but the other intervals represent only \$200,000. Therefore, the sales for 1999 appear to be twice the sales of 1998, but in reality they are not.

16b.



16c. See students' work.

17. See students' work.

18. This is a biannual experiment, where 8 mums are involved and there are only two possible outcomes, survival S and not survival N.

$$\begin{aligned} P(\text{exactly 6 surviving}) &= C(8, 6)(S)^6(N)^2 \\ &= 28(0.8)^6(0.2)^2 \\ &\approx 0.29 \text{ or } 29\% \end{aligned}$$

19. $(c - 2d)^7 = \sum_{r=0}^{7} \frac{7!}{r!(7-r)!}(c)^{7-r}(-2d)^r$

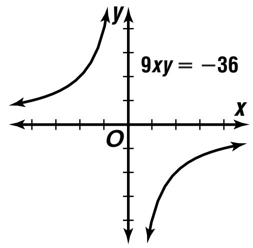
To find the second term, evaluate the general term for $r = 1$.

$$\begin{aligned} \frac{7!}{r!(7-r)!}(c)^{7-r}(-2d)^r &= \frac{7!}{1!(7-1)!}(c)^{7-1}(-2d)^1 \\ &= 7c^6 \cdot -2d \\ &= -14c^6d \end{aligned}$$

20.

$$\begin{aligned} 3.6^x &= 58.9 \\ x \ln 3.6 &= \ln 58.9 \\ x &\approx 3.18 \end{aligned}$$

21.



22.

$$\begin{aligned} (x + y)^2 &= x^2 + 2xy + y^2 \\ &= (x^2 + y^2) + 2(xy) \\ &= 16 + 2(8) \\ &= 32 \end{aligned}$$

14-2 Measures of Central Tendency

Page 903 Check for Understanding

1. Mean, median, mode; to find the mean, add the values in a set of data and divide the sum by the number of values in the set. To find the median, arrange the values in a set of data from least to greatest. If there is an odd number of values in the set, the median is the middle value. If there is an even number of values in the set, the median is the mean of the two middle values. To find the mode, find the item of data that appears more frequently than any other in the set.

2. Sample answer: {1, 2, 2, 3, 4, 4, 5}
The modes are 2 and 4.

3. Write the stems 9, 10, 11, 12, 13, and 14 on the left. Write the tens digits as leaves to the right of the appropriate stems. Be sure to order the leaves.

4. Tia; the median 2.5 and the mode 2 do not represent the greater numbers. The mean 8.5 is more representative of all 8 items in the data.

5. $\overline{X} = \frac{1}{4}(10 + 10 + 45 + 58)$ or 30.75

$M_d = \frac{10 + 45}{2}$ or 27.5

Mode = 10

6. $\overline{X} = \frac{1}{10}(21 + 22 + 23 + 24 + 28 + 29 + 31 + 31 + 34 + 37)$

$= 28$

$M_d = \frac{28 + 29}{2}$ or 28.5

Mode = 31

7. $\overline{X} = \frac{1}{3}(91 + 94 + 95 + 98 + 99 + 105 + 105 + 107 + 107 + 107 + 111 + 111 + 112) \cdot 100$

$\approx 103.23 \cdot 100$

$\approx 10{,}323$

$M_d = 10{,}500$

Mode = 10,700

8a. $2 + 8 + 15 + 6 + 38 + 31 + 13 + 7 = 120$

120 members

8b. $\sum_{i=1}^{8} f_i = 120$

$\sum_{i=1}^{8} (f_i \cdot X_i) = 2(3) + 8(7) + 15(11) + 6(15) + 38(19) + 31(23) + 13(27) + 7(31)$

$= 2320$

$\overline{X} = \frac{2320}{120}$ or about 19.3

8c.

Visits	Members	Cumulative Members
1–5	2	2
5–9	8	10
9–13	15	25
13–27	6	31
17–21	38	69
21–25	31	100
25–29	13	113
29–33	7	120

Half of the data has been gathered in the 17–21 class. This is the median class.

8d. $69 - 31 = 38$ $\quad 21 - 17 = 4$

$60 - 31 = 29$ $\quad M_d - 17 = x$

$\frac{38}{4} = \frac{29}{x}$

$x \approx 3.052631579$

$M_d - 17 = x$

$M_d - 17 \approx 3.1$

$M_d \approx 20.1$

9a.

stem	leaf
0	6 7 7 7 9
1	3 3 4 4 5 6 7 8 9 9
2	0 0 0 1 1 1 1 1 3 3 8
3	0 1 1 1 2 4 4 6 8
4	1 1 1 2 7

1 | 3 = 13

9b. $\overline{X} = \frac{1}{40}(6 + 3(7) + 9 + 2(13) + 2(14) + 15 + 16 + 17 + 18 + 2(19) + 3(20) + 5(21) + 2(23) + 28 + 30 + 3(31) + 32 + 2(34) + 36 + 38 + 3(41) + 42 + 47)$

$= 23.55$

9c. $M_d = 21$

9d. Mode = 21

9e. Since the mean 23.55, the median 21, and the mode 21 are all representative values, any of them could be used as an average.

Pages 904–907 Exercises

10. $\overline{X} = \frac{1}{4}(140 + 150 + 160 + 170)$ or 155

$M_d = \frac{150 + 160}{2}$ or 155

Mode: none

11. $\overline{X} = \frac{1}{5}(3 + 3 + 3 + 6 + 12)$ or 5.4

$M_d = 3$

Mode = 3

12. $\overline{X} = \frac{1}{4}(17 + 19 + 19 + 21)$ or 19

$M_d = 19$

Mode = 19

13. $\overline{X} = \frac{1}{8}(3 + 5 + 5 + 8 + 14 + 15 + 18 + 18)$

$= 10.75$

$M_d = \frac{8 + 14}{2}$ or 11

Mode = 5 and 18

14. $\overline{X} = \frac{1}{12}(54 + 58 + 62 + 63 + 64 + 70 + 76 + 76 + 87 + 87 + 98)$

$= 73.5$

$M_d = \frac{70 + 76}{2}$ or 73

Mode = 87

15. $\overline{X} = \frac{1}{12}(5 + 6 + 6 + 6 + 7 + 8 + 9 + 10 + 11 + 11 + 11 + 12)$

$= 8.5$

$M_d = \frac{8 + 9}{2}$ or 8.5

Mode = 6 and 11

16a. $\overline{X} = \frac{1}{9}(117 + 124 + 139 + 142 + 145 + 151 + 155 + 160 + 172)$

$= 145$ lb

16b. $M_d = 145$ lb

16c. $\overline{X} = \frac{1}{9}(122 + 129 + 144 + 147 + 150 + 156$
$160 + 165 + 177)$

$= 150$ lb

$M_d = 150$ lb

Each will increase by 5 lb.

17. $\overline{X} = \frac{1}{2}(35 + 2(38) + 39 + 44 + 3(45) + 48 + 2(57) + 59)$

≈ 45.8

$M_d = 45$

Mode = 45

18. $\overline{X} = \frac{1}{14}(5.2 + 5.4 + 5.6 + 6.0 + 6.1 + 6.7 + 6.8$
$+ 6.9 + 7.1 + 7.6 + 8.0 + 8.2 + 8.6 + 9.1)$
$= 6.95$

$M_d = \frac{6.8 + 6.9}{2}$ or 6.85

Mode: none

19. $\overline{X} = \frac{1}{15}(90 + 91 + 97 + 98 + 99 + 105 + 106$
$+ 109 + 113 + 3(118) + 120 + 2(125)) \cdot 10$
$= 1088$

$M_d = 1090$

Mode = 1180

20.

stem	leaf
1	0 5 5 5 5 7 7
2	0 0 0 5 5 5 5 7 8
3	0 0 5 5 5
4	6
5	5

1 | 0 = 10

21a. 135(11) = \$1485; 145(24) = \$3480;
155(30) = \$4650; 165(10) = \$1650;
175(13) = \$2275; 185(8) = \$1480;
195(4) = \$780

21b. 1485 + 3480 + 4650 + 1650 + 2275 + 1480 + 780 = \$15,800

21c. 11 + 24 + 30 + 10 + 13 + 8 + 4 = 100
100 employees

21d. $\overline{X} = \frac{15{,}800}{100}$ or about \$158

21e.

Weekly Wages	Frequency	Cumulative Frequency
\$130–\$140	11	11
\$140–\$150	24	35
\$150–\$160	30	65
\$160–\$170	10	75
\$170–\$180	13	88
\$180–\$190	8	96
\$190–\$200	4	100

Half of the data has been gathered in the \$150–\$160 class. This is the median class.

21f. $65 - 35 = 30$ $\quad 160 - 150 = 10$
$50 - 35 = 15$ $\quad M_d - 150 = x$

$\frac{30}{10} = \frac{15}{x}$

$x = 5$

$M_d - 150 \approx x$

$M_d - 150 \approx 5$

$M_d \approx 155$

21g. Both values represent central values of the data.

22. $7.5 = \frac{1}{5}(2 + 4 + 5 + 8 + x)$

$37.5 = 19 + x$

$18.5 = x$

23. $6 = \frac{1}{4}(x + 2x - 1 + 2x + 3x + 1)$

$24 = 8x$

$3 = x$

24. Order the values from least to greatest. The median lies between the fourth and fifth terms.
2, 3, 3.2, 8, x, 11, 13, 14

$x = 8$, since $M_d = \frac{8 + 8}{2}$ or 8.

25a. $\overline{X} = \frac{1}{179}[9(245) + 14(275) + 23(325) + 30(375)$
$+ 33(425) + 28(475) + 18(525) + 12(575)$
$+ 7(625) + 3(675) + 1(725) + 1(775)]$
≈ 425.6

25b.

Scores	Number of Students	Cumulative Number of Students
200–250	9	9
250–300	14	23
300–350	23	46
350–400	30	76
400–450	33	109
450–500	28	137
500–550	18	155
550–600	12	167
600–650	7	174
650–700	3	177
700–750	1	178
750–800	1	179

$\frac{179}{2} = 89.5$; Half of the data has been gathered in the 400–450 class. This is the median class.

25c. $109 - 76 = 33$ $\quad 450–400 = 50$
$89.5 - 76 = 13.5$ $\quad M_d - 400 = x$

$\frac{33}{50} = \frac{13.5}{x}$

$x = 20.\overline{45}$

$M_d - 400 = x$

$M_d - 400 \approx 20.5$

$M_d \approx 420.5$

26a. $\overline{X} = \frac{1}{5}(3.6 + 3.6 + 3.7 + 3.9 + 4.8)$
$= 3.92$

$M_d = 3.7$

Mode = 3.6

26b. Only the mean would change. It would increase to 4.6.

$\overline{X} = \frac{1}{5}(3.6 + 3.6 + 3.7 + 3.9 + 8.2)$
$= 4.6$

26c. The mean increases slightly; the median increases slightly; the mode stays the same.

$\overline{X} = \frac{1}{4}(3.6 + 3.6 + 3.7 + 3.9)$
$= 3.7$

$M_d = \frac{3.6 + 3.7}{2}$ or 3.65

Mode = 3.6

27a. Sample answer: {1, 2, 2, 2, 3}

27b. Sample answer: {4, 5, 9}

27c. Sample answer: {2, 10, 10, 12}

27d. Sample answer: {3, 4, 5, 6, 9, 9}

28a.

stem	leaf
0	1 1 1 1 1 1 1 2 2 2 2 2 3 3 3
	3 3 4 4 4 5 5 5 5 6 6 7 7 7 8
	8 8 8 9 9 9 9
1	0 1 3 3 3 5 8 9 9
2	5 9
3	2
4	
5	3

5 | 3 = 53

28b. $\overline{X} = \frac{1}{50}[7(1) + 5(2) + 5(3) + 3(4) + 4(5) + 2(6) + 3(7) + 4(8) + 4(9) + 10 + 11 + 3(13) + 15 + 18 + 2(19) + 25 + 29 + 32 + 53]$
$= 8.7$

28c. $M_d = 6$

28d. Mode = 1

28e. The mean 8.7 and the median 6 are representative of the data, but the mode 1 is not representative of the data.

29a. $\overline{X} \approx \frac{1}{27}[1(170) + 6(190) + 10(210) + 6(230) + 3(250) + 1(270)]$
≈ 215.2

29b.

Goals	Number of Teams	Cumulative Number of Teams
160–180	1	1
180–200	6	7
200–220	10	17
220–240	6	23
240–260	3	26
260–280	1	27

$\frac{27}{2} = 13.5$; Half of the data has been gathered in the 200–220 class. This is the median class.

29c. $17 - 7 = 10$ $\quad 220 - 200 = 20$
$13.5 - 7 = 6.5$ $\quad M_d - 200 = x$
$\frac{10}{20} = \frac{6.5}{x}$
$x = 13$
$M_d - 200 = x$
$M_d - 200 = 13$
$M_d \approx 213$

29d. $\overline{X} = \frac{1}{27}(268 + 248 + 245 + 242 + 239 + 239 + 237 + 236 + 231 + 230 + 217 + 215 + 214 + 211 + 210 + 210 + 207 + 205 + 202 + 200 + 196 + 194 + 192 + 190 + 189 + 184 + 179)$
≈ 215.9
$M_d = 211$

29e. The mean calculated using the frequency distribution is very close to the one calculated with the actual data. The median calculated with the actual data is less than the one calculated with the frequency distribution.

30a. Let $\overline{X} = 50$
$\sum_{i=1}^{9}(\overline{X} - f_i) = 0 = (50 - 5) + (50 - 20) + (50 - 37) + (50 - 44) + (50 - 52) + (50 - 68) + (50 - 71) + (50 - 85) + (50 - x)$
$0 = 68 - x$
$x = 68$
The weight should be hung 68 cm from the end.

30b. Let $\overline{X} = 50$
$\sum_{i=1}^{9}(\overline{X} - f_i) = 0 = (50 - 5) + (50 - 20) + (50 - 37) + (50 - 44) + (50 - 52) + (50 - 68) + (50 - 71) + (50 - 85) + (50 - x) + (50 - x)$
$0 = 118 - 2x$
$-118 = -2x$
$59 = x$
The weight should be hung 59 cm from the end.

31a. $\overline{X} = \frac{1}{10}(54 + 55 + 59 + 59 + 61 + 62 + 65 + 75 + 162 + 226) \cdot 1000$
$= \$87{,}800$

31b. $M_d = \frac{61{,}000 + 62{,}000}{2}$ or \$61,500

31c. Mode = \$59,000

31d. The mean, since it is the greatest measure of central tendency.

31e. The mode, since it is the least measure of central tendency.

31f. Median; the mean is affected by the extreme values of \$162,000 and \$226,000, and only two people make less than the mode.

31g. Sample answer: I have been with the company for many years, and I am still making less than the mean salary.

32a. $\overline{X} \approx \frac{1}{100}[12(2.00) + 15(2.50) + 31(3.00) + 37(3.50) + 5(4.00)]$
≈ 30.4

32b.

Grade Point Averages	Frequency	Cumulative Frequency
1.75–2.25	12	12
2.25–2.75	15	27
2.75–3.25	31	58
3.25–3.75	37	95
3.75–4.25	5	100

The median class is 2.75 − 3.25.
$58 - 27 = 31$ $\quad 3.25 - 2.75 = 0.50$
$50 - 27 = 13$ $\quad M_d - 2.75 = x$
$\frac{31}{0.50} = \frac{13}{x}$
$x \approx 0.2096774194$
$M_d - 2.75 = x$
$M_d - 2.75 \approx 0.21$
$M_d \approx 2.96$

33. He is shorter than the mean (5′ 11.6″) and the median (5′ 11.5″).

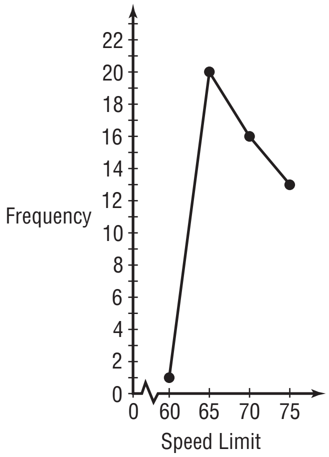
$\overline{X} = \frac{1}{10}(67 + 68 + 69 + 69 + 71 + 72 + 73 + 74 + 75 + 78)$

$= 71.6$ or 5′ 11.6″

$M_d = \frac{71 + 72}{2}$

$= 71.5$ or 5′ 11.5″

34.



35. dependent; $\frac{3}{11} \cdot \frac{2}{10} = \frac{3}{55}$

36. $a_n = \frac{n}{3^n}$; $a_{n+1} = \frac{n+1}{3^{n+1}}$

$$r = \lim_{n\to\infty} \frac{\frac{n+1}{3^{n+1}}}{\frac{n}{3^n}}$$

$$= \lim_{n\to\infty} \frac{3^n(n + 1)}{3^{n+1}(n)}$$

$$= \lim_{n\to\infty} \frac{3^n(n + 1)}{3^n \cdot 3n}$$

$$= \lim_{n\to\infty} \frac{n + 1}{3n}$$

$$= \lim_{n\to\infty} \frac{n}{3n} + \frac{1}{3n}$$

$$= \frac{1}{3} + 0 \text{ or } \frac{1}{3}$$

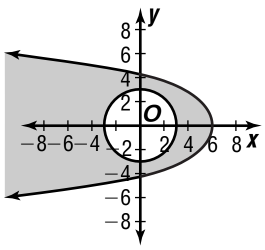
Since $r < 1$, the series is convergent.

37. $F_n = P\left[\frac{(1 + i)^n \cdot 1}{i}\right]$

$= 1500\left[\frac{(1 + 0.03)^{20} - 1}{0.03}\right]$

$= \$40,305.56$

38.



39. To find the area of the triangle use Hero's formula:

Area $= \sqrt{s(s - a)(s - b)(s - c)}$, where $s = \frac{1}{2}(a + b + c)$ and $a = 10$, $b = 7$, and $c = 5$.

So, $s = \frac{1}{2}(10 + 7 + 5) = \frac{1}{2}(22) = 11$.

Area $= \sqrt{11(11 - 10)(11 - 7)(11 - 5)}$

$= \sqrt{11 \cdot 1 \cdot 4 \cdot 6}$ or $\sqrt{264} \approx 16.25$.

The correct choice is A.

14-3 Measures of Variability

Page 914 Check for Understanding

1. The median of the data is 70, Q_1 is 60, and Q_3 is 100. The interquartile range is 40 and the semi-interquartile range is 20. The outliers are 170 and 180. The data in the first two quartiles are close together in range. The last two quartiles are more diverse.

2. square the standard deviation

3. Both the mean deviation and the standard deviation are measures of the average amount by which individual items of data deviate from the mean of all the data. The mean deviation uses the absolute values of the deviations. Standard deviation uses the squares of the deviations.

4. See students' work.

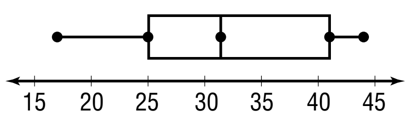
5. interquartile range $= Q_3 - Q_1$

$= 41 - 25$

$= 16$

Semi-interquartile range $= \frac{16}{2}$

$= 8$



6. $\overline{X} = \frac{1}{8}(4.45 + 5.50 + 5.50 + 6.30 + 7.80 + 11.00 + 12.20 + 17.20)$

$= 8.74$

$MD \approx \frac{1}{8}(|-4.29| + |-3.24| + \ldots + |8.46|)$

$\approx \$3.54$

$\sigma \approx \sqrt{\frac{1}{8}(-4.29^2 + (-3.24)^2 + \ldots + 8.46^2}$

$\approx \$4.11$

7. $\overline{X} = \frac{1}{200}[15(5000) + 30(15,000) + 50(25,000) + 60(35,000) + 30(45,000) + 15(55,000)]$

$= 30,250$

$\sigma = \sqrt{\frac{(-25,250)^2 \cdot 15 + (-15,250)^2 \cdot 30 + \ldots + 24,750^2 \cdot 15}{200}}$

$\approx 13,226.39$

8a. $\overline{X} = \frac{1}{12}(65.7 + 65.9 + \ldots + 65.9)$

$= 70.375$

$M_d = \frac{69.0 + 70.3}{2}$ or 69.65

$\sigma = \sqrt{\frac{(-4.875)^2 + (-4.475)^2 + \ldots + 6.225^2}{12}}$

≈ 4.25

8b. $\overline{X} = \frac{1}{12}(57.3 + 63.3 + \ldots + 57.5)$

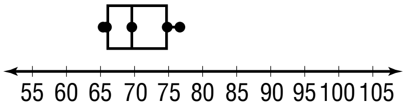
≈ 80.48

$M_d = \frac{77.5 + 82.1}{2}$ or 79.8

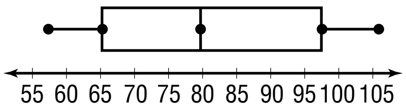
$\sigma \approx \sqrt{\frac{(-23.18)^2 + (-22.98)^2 + \ldots + 25.32^2}{12}}$

≈ 17.06

8c. Los Angeles



Las Vegas



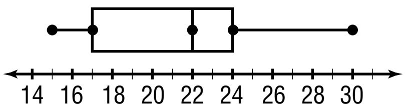
8d. Los Angeles

8e. Los Angeles is near an ocean; Las Vegas is in a desert.

Pages 915–917 Exercises

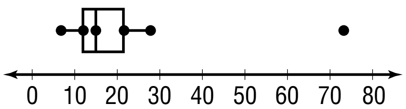
9. interquartile range $= Q_3 - Q_1 = 24 - 17 = 7$

semi-interquartile range $= \frac{7}{2}$ or 3.5



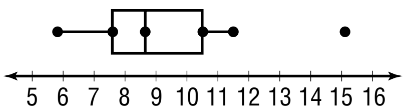
10. interquartile range $= Q_3 - Q_1 = 21.5 - 12 = 9.5$

semi-interquartile range $= \frac{9.5}{2}$ or 4.75

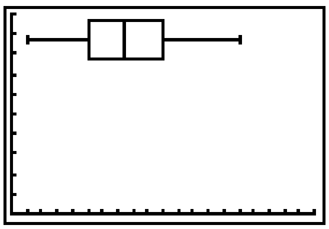


11. interquartile range $= Q_3 - Q_1 = 10.5 - 7.6 = 2.9$

semi-interquartile range $= \frac{2.9}{2}$ or 1.45



12.



13. $\overline{X} = \frac{1}{6}(152 + 158 + \ldots + 721)$

$= 381$

$MD = \frac{1}{6}(|-229| + |-223| + \ldots + |340|)$

$= 211$

$\sigma = \sqrt{\frac{(-229)^2 + (-223)^2 + \ldots + 340^2}{6}}$

≈ 223.14

14. $\overline{X} = \frac{1}{10}(5.7 + 5.7 + \ldots + 3.8)$

$= 4.89$

$MD = \frac{1}{10}(|0.81| + |0.81| + \ldots + |-1.09|)$

$= 0.672$

$\sigma = \sqrt{\frac{0.81^2 + 0.81^2 + \ldots + (-1.09)^2}{10}}$

≈ 0.73

15. $\overline{X} = \frac{1}{12}(369 + 376 + \ldots + 454)$

$= 403.5$

$MD = \frac{1}{12}(|-34.5| + |-27.5| + \ldots + |50.5|)$

$= 20.25$

$\sigma = \sqrt{\frac{(-34.5)^2 + (-27.5)^2 + \ldots + 50.5^2}{12}}$

≈ 25.31

16. $\overline{X} = \frac{1}{5}(13 + 22 + 34 + 55 + 91)$

$= 43$

Variation $= \frac{(-30)^2 + (-21)^2 + (-9)^2 + 12^2 + 48^2}{5}$

$= 774$

17. $\overline{X} = \frac{1}{120}[2(3) + 8(7) + \ldots + 7(31)]$

≈ 19.33

$\sigma \approx \sqrt{\frac{(-16.33)^2 \cdot 2 + (-12.33)^2 \cdot 8 + \ldots + 11.67^2 \cdot 7}{120}}$

≈ 6.48

18. $\overline{X} = \frac{1}{90}[3(57) + 7(65) + \ldots + 12(97)]$

$= 81.8$

$\sigma = \sqrt{\frac{(-24.8)^2 \cdot 3 + (-16.8)^2 \cdot 7 + \ldots + 15.2^2 \cdot 12}{90}}$

≈ 9.69

19. $\overline{X} = \frac{1}{85}[2(80) + 11(100) + \ldots + 7(180)]$

≈ 129.65

$\sigma \approx \sqrt{\frac{(-49.65)^2 \cdot 2 + (-29.65)^2 \cdot 11 + \ldots + 50.35^2 \cdot 7}{85}}$

≈ 23.29

20a. $M_d = 259$ mi

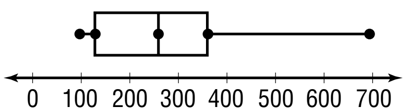
20b. $Q_1 = 129$ mi; $Q_3 = 360$ mi

20c. interquartile range $= Q_3 - Q_1 = 360 - 129 = 231$ mi

20d. semi-interquartile range $= \frac{231}{2} = 115.5$ mi

20e. An outlier would lie 231 + 115.5 or 346.5 mi outside of Q_1 or Q_3. There are no such points.

20f.



20g. The data in the upper quartile is more diverse than the other quartiles.

21. Sample answer: {15, 15, 15, 16, 17, 20, 24, 26, 30, 35, 45}

22a. $M_d = 282$

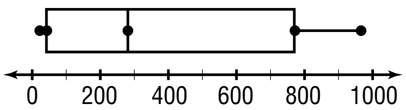
22b. $Q_1 = 42$; $Q_3 = 770$

22c. interquartile range $= Q_3 - Q_1$
$= 770 - 42$
$= 728$

semi-interquartile range $= \frac{728}{2}$
$= 364$

22d. An outlier would lie 728 + 364 or 1092 points outside of Q_1 or Q_3. There are no such points.

22e.



22f. $\overline{X} = \frac{1}{19}(22 + 23 + \ldots + 966)$
≈ 404.42

22g. $MD \approx \frac{1}{19}(|-382.4| + |-381.4| + \ldots + |561.58|)$
≈ 316.97

22h. Variance $\approx \frac{382.4^2 + 381.4^2 + \ldots + 561.58^2}{19}$
$\approx 118{,}712.56$

22i. $\sigma \approx \sqrt{118{,}712.56}$
≈ 344.55

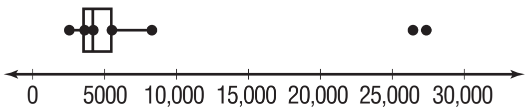
22j. There is a great variability among the number of teams in women's sports.

23a. $Q_1 = \$3616$, $M_d = \$4125$, $Q_3 = \$5664$

23b. interquartile range $= Q_3 - Q_1$
$= 5664 - 3616$
$= 2048$

23c. An outlier would lie 2048 + 1024 or \$3072 outside of Q_1 or Q_3. There are two such values, \$26,954 and \$27,394.

23d.



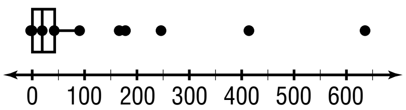
23e. $\overline{X} = \frac{1}{19}(2684 + 2929 + \ldots + 27{,}394)$
≈ 6775.95

$MD \approx \frac{1}{19}(|-4091.95| + |-3846.95| + \ldots + |20{,}618.05|)$
≈ 4463.39

23f. $\sigma \approx \sqrt{\frac{(-4091.95)^2 + (-3846.95)^2 + \ldots + 20{,}618.05^2}{19}}$
≈ 7103.45

23g. The data in the upper quartile is diverse.

24a.



24b. $\overline{X} = \frac{1}{42}(0 + 0 + \ldots + 635)$
≈ 60.40

$MD \approx \frac{1}{42}(|-60.40| + |-60.40| + \ldots + |574.60|)$
≈ 67.87

24c. Variance $\approx \frac{(-60.40)^2 + (-60.40)^2 + \ldots + (514.60)^2}{42}$
$\approx 14{,}065.48$

24d. $\sigma \approx \sqrt{14{,}065.48}$
≈ 118.60

24e. The data in the upper quartile is diverse.

25a. $\overline{X} = \frac{1}{50}[26(9) + 12(11) + \ldots + 2(21)]$
$= 11$

25b. $\sigma = \sqrt{\frac{(9 - 11)^2 \cdot 26 + (11 - 11)^2 \cdot 12 + \ldots + (21 - 11)^2 \cdot 2}{50}}$
≈ 2.94

26. yes; when the standard deviation is less than 1; when both equal 0 or 1

27. See students' work.

28a. $\overline{X} = \frac{1}{35}[2(4.4) + 4.9 + 5.4 + 5.5 + 2(6.2) + 6.4 + 6.5 + 6.9 + 7.1 + 7.4 + 7.5 + 7.6 + 7.7 + 7.8 + 7.9 + 8.0 + 8.2 + 8.4 + 8.5 + 8.6 + 8.7 + 8.8 + 3(8.9) + 9.0 + 9.2 + 2(9.3) + 9.5 + 9.6 + 9.8 + 9.9]$
≈ 7.75

28b. $M_d = 8.0$

28c. Mode = 8.9

29a. range = 68 − 23 or 45

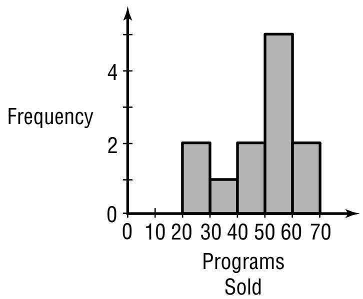
29b. Sample answer: 10

29c. Sample answer: 20, 30, 40, 50, 60, 70

29d. Sample answer:

Programs Sold	Frequency
20–30	2
30–40	1
40–50	2
50–60	5
60–70	2

29e. Sample answer:



30. $\frac{1}{9}(9!) = 40{,}302$ ways

31. $x_1 = 0.5(8) - 1$ or 3
$x_2 = 0.5(3) - 1$ or 0.5
$x_3 = 0.5(0.5) - 1$ or -0.75

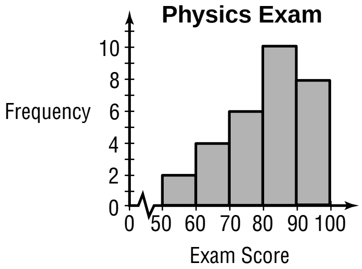
32. 7 ft = 7(12) or 84 in.
84 + 9 = 93 in.
$\frac{93}{3} = 31$ in.
31 in. $= \frac{31}{12}$ or 2 ft 7 in.
The correct choice is C.

Page 917 Mid-Chapter Quiz

1. Sample answer: 10
2. Sample answer:

Exam Scores	Frequency
50–60	2
60–70	4
70–80	6
80–90	10
90–100	8

3. Sample answer:



4.

stem	leaf
5	4 5
6	2 2 4 5
7	1 5 6 7 8 9
8	0 2 4 5 6 7 8 9 9 9
9	0 2 3 3 5 6 8 9

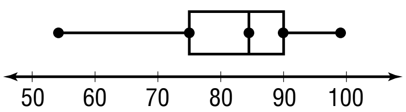
5|4 = 54

5. $\overline{X} = \frac{1}{30}(54 + 55 + \ldots + 99)$
$= 81.1$

6. $M_d = \frac{84 + 85}{2}$ or 84.5

7. Mode = 89

8.



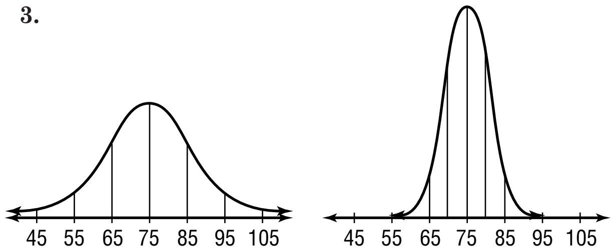
9. $MD = \frac{1}{30}(|-27.1| + |-26.1| + \ldots + |17.9|)$
$= 10.42$

10. Sample answer: The data that are less than the median are more spread out than the data greater than the median.

14-4 The Normal Distribution

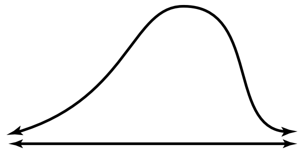
Pages 922–923 Check for Understanding

1. The median, mean, and mode are the same.
2. $\overline{X} \pm 1.5\sigma$
3.



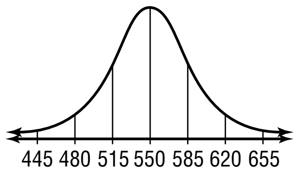
The second curve is less variable.

4. Sample answer:



5. 50th percentile; it contains half of the data.

6a.



6b. Since 515 and 585 are within are standard deviation of the mean, it contains 68.3% of the data.

6c. 99.7% of the data lie within 3 standard deviations of the mean.
$550 \pm 3(35) = 445 - 655$

6d. $550 - 480 = 70$, $620 - 550 = 70$
$t\sigma = 70$
$t(35) = 70$
$t = 2 \rightarrow 95.5\%$
$0.995(200) = 191$ values

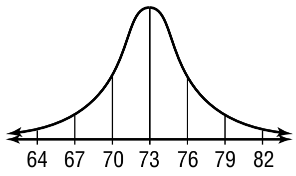
7a. Since 22 and 26 are within one standard deviation of the mean, it contains 68.3% of the data.

7b. $24 - 20.5 = 3.5$, $27.5 - 24 = 3.5$
$t\sigma = 3.5$
$t(2) = 3.5$
$t = 1.75 \rightarrow 92.9\%$

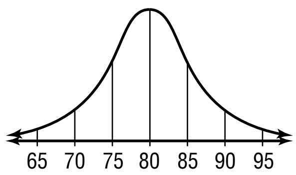
7c. $24 - 0.7(2) = 22.6$ and $24 + 0.7(2) = 25.4$
$22.6 - 25.4$

7d. $24 - 1.96(2) = 20.08$ and $24 + 1.96(2) = 27.92$
$20.08 - 27.92$

8a.



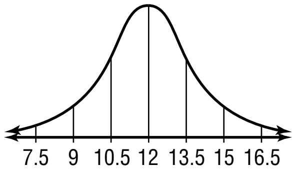
8b.



8c. Chemistry; the chemistry grade is 3 standard deviations above the class mean, while the speech grade is only 2 standard deviations above the class mean.

Pages 923–925 Exercises

9a.



9b. $12 \pm 1(1.5) = 10.5 - 13.5$

9c. $12 - 7.5 = 4.5,\ 16.5 - 12 = 4.5$

$t\sigma = 4.5$

$t(1.5) = 4.5$

$t = 3 \rightarrow 88.7\%$

9d. $12 - 9 = 3,\ 15 - 12 = 3$

$t\sigma = 3$

$t(1.5) = 3$

$t = 2 \rightarrow 95.5\%$

10a. $0.683(200) = 136.6$; about 137

10b. $0.955(200) = 191$

10c. $\frac{0.683}{2}(200) = 68.3$; about 68

11a. 45% corresponds to $t = 0.6$.

$82 \pm 0.6(4) = 79.6 - 84.4$

11b. 80% corresponds to $t = 1.3$

$82 \pm 1.3(4) = 76.8 - 87.2$

11c. $82 - 76 = 6,\ 88 - 82 = 6$

$t\sigma = 6$

$t(4) = 6$

$t = 1.5 \rightarrow 86.6\%$

11d. $82 - 80.5 = 1.5,\ 83.5 - 82 = 1.5$

$t\sigma = 1.5$

$t(4) = 1.5$

$t = 0.375 \rightarrow 31.1\%$

12a. 25% corresponds to $t = 0.3$.

$402 \pm 0.3(36) = 391.2 - 412.8$

12b. $402 - 387 = 15,\ 417 - 402 = 15$

$t\sigma = 15$

$t(36) = 15$

$t = 0.41\overline{6} \rightarrow 31.1\%$

12c. $402 - 362 = 40,\ 442 - 402 = 40$

$t\sigma = 40$

$t(36) = 40$

$t = 1.\overline{1} \rightarrow 72.9\%$

12d. 45% corresponds to $t = 0.6$.

$402 \pm 0.6(36) = 380.4 - 423.6$

13a.

$$\begin{aligned} \overline{X} + t\sigma &= 150 \\ 140 + t(20) &= 150 \\ 20t &= 10 \\ t &= 0.5 \end{aligned} \qquad \begin{aligned} \overline{X} - t\sigma &= 100 \\ 140 - t(20) &= 100 \\ -20t &= -40 \\ t &= 2 \end{aligned}$$

$\frac{38.3\%}{2} = 19.15\%$ $\qquad \frac{95.5\%}{2} = 47.75\%$

$19.15 + 47.75 = 66.9\%$

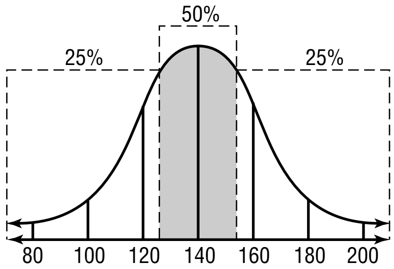
13b.

$$\begin{aligned} \overline{X} + t\sigma &= 180 \\ 140 + t(20) &= 180 \\ 20t &= 40 \\ t &= 2 \end{aligned} \qquad \begin{aligned} \overline{X} + t\sigma &= 150 \\ 140 + t(20) &= 150 \\ 20t &= 10 \\ t &= 0.5 \end{aligned}$$

$\frac{95.5\%}{2} = 47.75\%$ $\qquad \frac{38.3\%}{2} = 19.15\%$

$47.75 - 19.15 = 28.6\%$

13c.



$t = 0.7$ corresponds with 50% of the data centered about the mean.

The upper limit results in 75% of the data.

$140 + 0.7(20) = 154$

14a.

$$\begin{aligned} \overline{X} + t\sigma &= 7 \\ 6 + t(35) &= 7 \\ 3.5t &= 1 \\ t &= 0.29 \end{aligned} \qquad \begin{aligned} \overline{X} + t\sigma &= 6.5 \\ 6 + t(3.5) &= 6.5 \\ 3.5t &= 0.5 \\ t &= 0.14 \end{aligned}$$

$\frac{23.6\%}{2} = 11.8\%$ $\qquad \frac{8\%}{2} = 4\%$

$11.8 - 4 = 7.8\%$

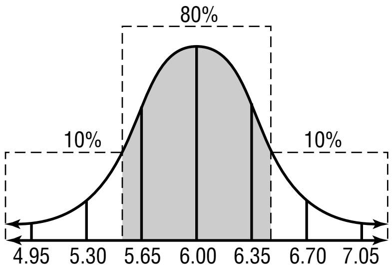
14b.

$$\begin{aligned} \overline{X} + t\sigma &= 6.2 \\ 6 + t(0.35) &= 6.2 \\ 0.35t &= 0.2 \\ t &= 0.57 \end{aligned} \qquad \begin{aligned} \overline{X} - t\sigma &= 5.5 \\ 6 - t(0.35) &= 5.5 \\ -0.35t &= -0.5 \\ t &= 1.43 \end{aligned}$$

$\frac{45.1\%}{2} = 22.55\%$ $\qquad \frac{83.8\%}{2} = 41.9\%$

$22.55 + 41.9 = 64.45\%$

14c.



$t = 1.3$ corresponds with 80% of the data centered about the mean. The lower limit results in the value above which 90% of the data lies.

$6 - 1.3(0.35) = 5.545$

15a. $P(\text{no tails}) = \left(\frac{1}{2}\right)^6$ or $\frac{1}{64}$

$P(\text{one tail}) = 6\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^5$ or $\frac{3}{32}$

$P(\text{two tails}) = 15\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^4$ or $\frac{15}{64}$

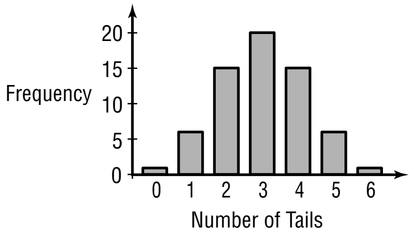
$P(\text{three tails}) = 20\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3$ or $\frac{5}{16}$

$P(\text{four tails}) = 15\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^2$ or $\frac{5}{64}$

$P(\text{five tails}) = 6\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)$ or $\frac{3}{32}$

$P(\text{six tails}) = \left(\frac{1}{2}\right)^6$ or $\frac{1}{64}$

15b.

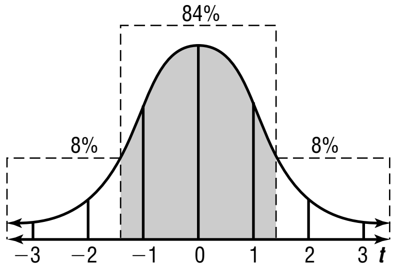


15c. $\overline{X} = \frac{1}{64}[0(1) + 1(6) + 2(15) + 3(20) + 4(15) + 5(6) + 6(1)]$
$= 3$

15d. $\sigma = \sqrt{\frac{(0-3)^2 + (1-3)^2 + \ldots + (6-3)^2}{64}}$
≈ 1.2

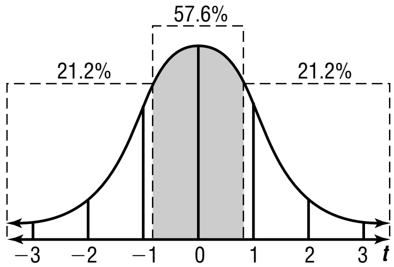
15e. They are similar.

16a.



The 92nd percentile is the upper limit to 84% of the data that is centered about the mean. 84% corresponds to $t = 1.4$. The 92nd percentile is 1.4 standard deviations above the mean.

16b.

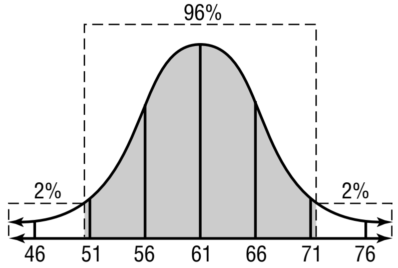


$t = 0.8$ corresponds to 57.6% of the data centered about the mean. $\frac{100 - 57.6}{2} = 21.2$
$21.2 + 57.6 = 78.8$ percentile

17a.
$$\begin{aligned} \overline{X} + t\sigma &= 22.3 \\ 20.4 + t(0.8) &= 22.3 \\ 0.8t &= 1.9 \\ t &= 2.38 \rightarrow 98.4\% \end{aligned}$$
$\frac{100 - 98.4}{2} = 0.8\%$

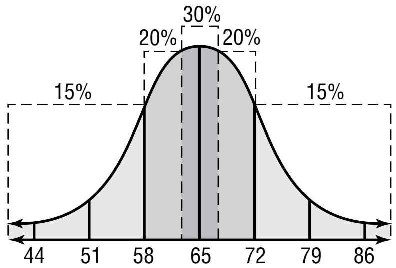
17b. $100 - 0.8 = 99.2\%$

18.



96% corresponds to $t = 2.1$.
$61 - 2.1(5) = 50.5$ months

19a.



70% corresponds to $t = 1.0$.
$65 + 1.0(7) = 72$

19b. $65 - 1.0(7) = 58$

19c. 30% corresponds to $t = 0.4$
$65 + 0.4(7) = 67.8 \approx 68$
The lowest score for an A is 72, so the highest score for a B is 71. The interval for B's is 68−71.

20a. a normal distribution with a small standard deviation

20b. a normal distribution with a large standard deviation

20c. a distribution where values greater than the mean are more spread out than values less than the mean

20d. a distribution where all values occur with the same frequency

21a. 95% corresponds to $t = 1.96$
$$\begin{aligned} \overline{X} + t\sigma &= 260 \\ 255 + 1.96\sigma &= 260 \\ 1.96\sigma &= 5 \\ \sigma &= 2.55 \end{aligned} \qquad \begin{aligned} \overline{X} - t\sigma &= 250 \\ 255 - 1.96\sigma &= 250 \\ -1.96\sigma &= -5 \\ \sigma &\approx 2.55 \end{aligned}$$
about 2.55 mL

21b.
$$\begin{aligned} \overline{X} + t\sigma &= 357 \\ 355 + t(2.55) &= 357 \\ 2.55t &= 2 \\ t &= 0.78 \end{aligned} \qquad \begin{aligned} \overline{X} - t\sigma &= 353 \\ 355 - t(2.55) &= 353 \\ -2.55t &= -2 \\ t &= 0.78 \end{aligned}$$
57.6%

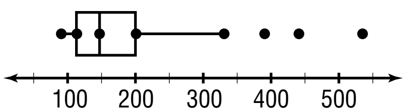
22a. $M_d = \frac{147 + 150}{2}$ or 148.5

22b. $Q_1 = 110$, $Q_3 = 200$

22c. interquartile range $= Q_3 - Q_1$
$= 200 - 110$
$= 90$

22d. semi-interquartile range $= \frac{90}{2}$ or 45

22e.



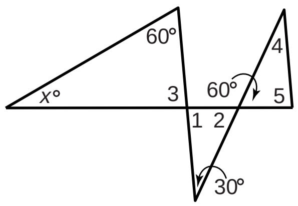
23. $\overline{X} = \frac{1}{9}(19 + 33 + 42 + 42 + 45 + 48 + 55 + 71 + 79)$
≈ 48.2
$M_d = 45$
Mode $= 42$

24. $y = \sec(k\theta + c) + h$
k: $\frac{2\pi}{k} = \frac{\pi}{2}$
$k = 4$
c: $\frac{-c}{k} = -\pi$
$\frac{-c}{4} = -\pi$
$c = 4\pi$
$h = 3$
$y = \sec(4\theta + 4\pi) + 3$

25a. Sample answer: Use a graphing calculator to enter the year data as L1 and the Enrollment data as L2. Then make a scatter plot. The scatter plot indicates that a cubic function would best fit the data. Perform a cubic regression to find the equation
$y = 0.05x^3 - 2.22x^2 + 29.72x + 366.92$.

25b. Sample answer:
$2015 - 1965 = 50$
$f(50) = 0.05 \cdot 50^3 - 2.22 \cdot 50^2 + 29.72 \cdot 50 + 366.92$
$= 2553$ students

26.



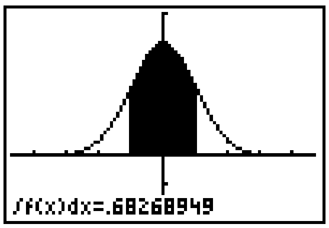
Since vertical angles are equal, $m\angle 2 = 60$.
$m\angle 1 + 60 + 30 = 180$
$m\angle 1 = 90$
Since an exterior angle of a triangle is equal to the measure of the sum of the two remote interior angles, $x + 60 = m\angle 1$. So, $x + 60 = 90$ and $x = 30$. The correct choice is E.

14-4B Graphing Calculator Exploration: The Standard Normal Curve

Page 926

1. The domain of the function is the set of all real numbers. The range is the set of all real numbers y such that $0 < y \leq \frac{1}{\sqrt{2\pi}}$. The graph is symmetric with respect to the y-axis and has the x-axis as a horizontal asymptote. the value of $f(x)$ approaches 0 as x approaches $\pm\infty$.

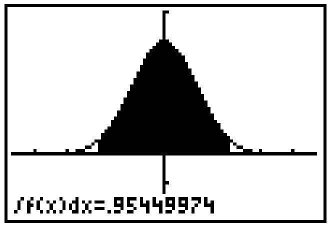
2.



0.68268949

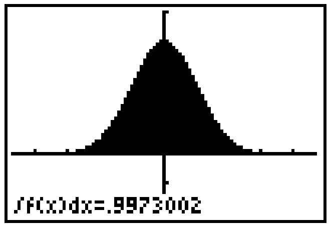
3. About 68.3%; the answer for Exercise 2 can be rounded to 0.683, which equals 68.3%.

4.



0.95449974

5.

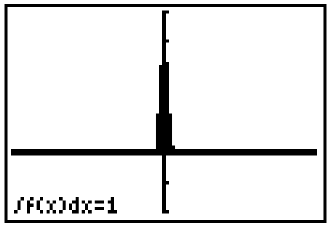


0.9973002

6. The answer for Exercise 3 can be rounded to 0.954, which is about 95.5%. The answer for Exercise 4 can be rounded to 0.997, which equals 99.7%.

7. 0.9999; $t = 4$ corresponds to $P = 0.999$.

8.



1; no; since the curve is approaching the x-axis asymptotically, the area is probably not exactly equal to 1.

14-5 Sample Sets of Data

Page 930 Check for Understanding

1. A sample is a subset of a population. However, a sample must be similar in every way to the population.

2. Divide the standard deviation of the sample by the square root of the number of values in the sample.

3. Use a larger sample.

4. Tyler; every twentieth student to enter the school should produce a representative sample. The senior English class will not represent underclassmen. The track team will not represent students who prefer other types of activities.

5. $\sigma_{\overline{X}} = \frac{73}{\sqrt{100}}$ or 7.3

6. $\sigma_{\overline{X}} = \frac{3.4}{\sqrt{250}}$
≈ 0.22

7. A 1% confidence level is given when $P = 99\%$ and $t = 2.58$.
$\sigma_{\overline{X}} = 5\frac{5}{\sqrt{36}}$ or $0.8\overline{3}$
internal: $\overline{X} \pm t\sigma_{\overline{X}} = 45 \pm 2.58(0.8\overline{3})$
$\approx 42.85 - 47.15$

8. A 5% confidence interval is given when $P = 95\%$ and $t = 1.96$.
$\sigma_{\overline{X}} = \frac{5.6}{\sqrt{300}}$
≈ 0.323316156
internal: $\overline{X} \pm t\sigma_{\overline{X}} = 55 \pm 1.96\sigma_{\overline{X}}$
$\approx 54.37 - 55.63$

9a. $\sigma_{\overline{X}} = \frac{3.5}{\sqrt{150}}$
≈ 0.29

9b. $P = 50\%$ corresponds to $t = 0.7$.
interval: $\overline{X} \pm t\sigma_{\overline{X}} = 27.5 \pm 0.7\sigma_{\overline{X}}$
$\approx 27.30 - 27.70$ min

9c. A 1% confidence level is given when $P = 99\%$ and $t = 2.58$.
interval: $\overline{X} \pm t\sigma_{\overline{X}} = 27.5 \pm 2.58\sigma_{\overline{X}}$
$\approx 27.76 - 28.24$ min

Pages 930–932 Exercises

10. $\sigma_{\overline{X}} = \frac{1.8}{\sqrt{81}}$ or 0.2

11. $\sigma_{\overline{X}} = \frac{5.8}{\sqrt{250}}$
≈ 0.37

12. $\sigma_{\overline{X}} = \frac{7.8}{\sqrt{140}}$
≈ 0.66

13. $\sigma_{\overline{X}} = \frac{14}{\sqrt{700}}$
≈ 0.53

14. $\sigma_{\overline{X}} = \frac{2.7}{\sqrt{130}}$
≈ 0.24

15. $\sigma_{\overline{X}} = \frac{13.5}{\sqrt{375}}$
≈ 0.70

16. $0.056 = \frac{5.6}{\sqrt{N}}$
$\sqrt{N} = 100$
$N = 100{,}000$

17. $\sigma_{\overline{X}} = \frac{5.3}{\sqrt{50}}$
≈ 0.7495331881
interval: $\overline{X} \pm t\sigma_{\overline{X}} = 335 \pm 2.58\sigma_{\overline{X}}$
$\approx 333.07 - 336.93$

18. $\sigma_{\overline{X}} = \frac{40}{\sqrt{64}}$ or 5
interval: $\overline{X} \pm t\sigma_{\overline{X}} = 200 \pm 2.58(5)$
$= 187.1 - 212.9$

19. $\sigma_{\overline{X}} = \frac{12}{\sqrt{200}}$
≈ 0.8485281374
interval: $\overline{X} \pm t\sigma_{\overline{X}} = 80 \pm 2.58\sigma_{\overline{X}}$
$\approx 77.81 - 82.91$

20. $\sigma_{\overline{X}} = \frac{11.12}{\sqrt{1000}}$
≈ 0.3516452758
interval: $\overline{X} \pm t\sigma_{\overline{X}} = 110 \pm 2.58\sigma_{\overline{X}}$
$= 109.09 - 110.91$

21. $P = 90\%$ corresponds to $t = 1.65$.
$\sigma_{\overline{X}} = \frac{4}{\sqrt{100}}$ or 0.4
interval: $\overline{X} \pm t\sigma_{\overline{X}} = 68 \pm 1.65(0.4)$
$= 67.34 - 68.66$ in.

22. $\sigma_{\overline{X}} = \frac{2.4}{\sqrt{100}}$ or 0.24
interval: $\overline{X} \pm t\sigma_{\overline{X}} = 24 \pm 1.96(0.24)$
$\approx 23.53 - 24.47$

23. $\sigma_{\overline{X}} = \frac{17.1}{\sqrt{350}}$
≈ 0.9140334473
interval: $\overline{X} \pm t\sigma_{\overline{X}} = 4526 \pm 1.96\sigma_{\overline{X}}$
$= 4524.21 - 4527.79$

24. $\sigma_{\overline{X}} = \frac{28}{\sqrt{370}}$
≈ 1.455650686
interval: $\overline{X} \pm t\sigma_{\overline{X}} = 678 \pm 1.96\sigma_{\overline{X}}$
$\approx 675.15 - 680.85$

25. $\sigma_{\overline{X}} = \frac{0.67}{\sqrt{80}}$
≈ 0.0749082772
interval: $\overline{X} \pm t\sigma_{\overline{X}} = 5.38 \pm 1.96\sigma_{\overline{X}}$
$\approx 5.23 - 5.53$

26a. $\overline{X} = \frac{1}{64}[1(4) + 3(6) + \ldots + 2(20)]$
$= 12.375$

26b. $\sigma = \sqrt{\frac{(4 - 12.375)^2 + (6 - 12.375)^2 + \ldots + (20 - 12.375)^2}{64}}$
≈ 3.37

26c. $\sigma_{\overline{X}} \approx \frac{3.37}{\sqrt{64}}$
≈ 0.42

26d. $P = 0.95$ corresponds to $t = 1.96$.
interval: $\overline{X} \pm t\sigma_{\overline{X}} = 12.375 \pm 1.96(0.42)$
$\approx 11.55 - 13.20$ min

26e. $t\sigma_{\overline{X}} = 1$
$t(0.42) = 1$
$t \approx 2.38 \rightarrow$ about 98.4%

27. $\sigma_{\overline{X}} = \frac{12}{\sqrt{45}}$
≈ 1.788854382
$t\sigma_{\overline{X}} = 3$
$t = 1.68 \rightarrow 91.1\%$
$100 - 91.1 \approx 8.9\%$

28a. $\sigma_{\overline{X}} = \frac{1.4}{\sqrt{50}}$
≈ 0.1979898987 or about 0.20

28b. A 5% confidence interval is given when $P = 95\%$ and $t = 1.96$.
interval: $\overline{X} \pm t\sigma_{\overline{X}} = 16.2 \pm 1.96\sigma_{\overline{X}}$
$\approx 15.81 - 16.59$ mm

28c. $P = 99\%$ corresponds to $t = 2.58$.
interval: $\overline{X} \pm t\sigma_{\overline{X}} = 16.2 \pm 2.58\sigma_{\overline{X}}$
$\approx 15.69 - 16.71$ mm

28d. $P = 0.80$ corresponds to $t = 1.3$
interval: $\overline{X} \pm t\sigma_{\overline{X}} = 16.2 \pm 1.3\sigma_{\overline{X}}$
$\approx 15.94 - 16.46$ mm

29a. $\sigma_{\overline{X}} = \frac{45}{\sqrt{100}}$ or 4.5

29b. A 1% confidence level is given when $P = 99\%$ and $t = 2.58$.
interval: $\overline{X} \pm t\sigma_{\overline{X}} = 350 \pm 2.58(4.5)$
$= 338.39-361.61$ hours

29c. Sample answer: 338 hours, there is only 0.5% chance the mean is less than this number.

30. $10.2064 - 9.7936 = 0.4128$

$$\frac{0.4128}{2} = 0.2064$$
$$t\sigma_{\overline{X}} = 0.2064$$
$$2.58t\sigma_{\overline{X}} = 0.2064$$
$$\sigma_{\overline{X}} = 0.08$$
$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{N}}$$
$$0.08 = \frac{0.8}{\sqrt{N}}$$
$$\sqrt{N} = 10$$
$$N = 100 \text{ packages}$$

31a. $\sigma_{\overline{X}} = \frac{1.8}{\sqrt{10}}$
≈ 0.57

31b. interval: $\overline{X} \pm t\sigma_{\overline{X}} = 4.1 \pm 1.96(0.57)$
$= 2.98-5.22$ hours
With a 5% level of confidence, the average family in the town will have their televisions on from 2.98 to 5.22 hours.

31c. Sample answer: None; the sample is too small to generalize to the population of the city.

32a. $\sigma_{\overline{X}} = \frac{3.2}{\sqrt{50}}$
≈ 0.45

32b. $P = 50\%$ corresponds to $t = 0.7$.
interval: $\overline{X} \pm t\sigma_{\overline{X}} = 42.7 \pm 0.7\sigma_{\overline{X}}$
$\approx 42.38-43.02$ crackers

32c. Sample answer: No; there is a 50% chance that the true mean is in the interval. However, since 43 is near one end of the interval, they may want to take another sample in the near future.

33a. A 5% confidence interval gives a $P = 95\%$ and $t = 1.96$.
$$\frac{753.136 - 746.864}{} = 3.136$$
$t\sigma_{\overline{X}}$
$$1.960\sigma_{\overline{X}} = 3.136$$
$$\sigma_{\overline{X}} = 1.6$$
$$\overline{X} \pm t\sigma_{\overline{X}} = 753.136$$
$$\overline{X} + 1.96(1.6) = 753.136$$
$$\overline{X} = 750 \text{ h}$$

33b. $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{N}}$
$$1.6 = \frac{\sigma}{\sqrt{1600}}$$
$64 = \sigma$; 64 h

34a. $40{,}000 - 35{,}000 = 5000$, $45{,}000 - 40{,}000 = 5000$
$$t\sigma = 5000$$
$$t(500) = 5000$$
$$t = 1 \rightarrow 68.3\%$$
$0.683(10{,}000) = 6830$ tires

34b.
$$\overline{X} - t\sigma = 30{,}000$$
$$40{,}000 - t(5000) = 30{,}000$$
$$-5000t = -10{,}000$$
$$t = 2 \rightarrow 95.5\%$$
$$\frac{95.5\%}{2} = 47.75\%$$
$0.4775(10{,}000) = 4775$ tires

34c. 50% of $10{,}000 = 0.50(10{,}000)$
$= 5000$ tires

34d.
$$\overline{X} + t\sigma = 50{,}000$$
$$40{,}000 + t(5000) = 50{,}000$$
$$5000t = 10{,}000$$
$$t = 2 \rightarrow 95.5\%$$
$$\frac{100\% - 95.5\%}{2} = 2.25\%$$
$0.0225(10{,}000) = 225$ tires

34e.
$$\overline{X} - t\sigma = 25{,}000$$
$$40{,}000 - t(5000) = 25{,}000$$
$$-5000t = -15{,}000$$
$$t = 3 \rightarrow 99.7\%$$
$$\frac{100\% - 99.7\%}{2} = 0.15\%$$
$0.0015(10{,}000) = 15$ tires

35. $\overline{X} = \frac{1}{8}(44 + 49 + 55 + 58 + 61 + 68 + 71 + 72)$
$= 59.75$
$MD = \frac{1}{8}(|-15.75| + |-10.75| + \ldots + |12.25|)$
$= 8.25$
$$\sigma = \sqrt{\frac{(-15.75)^2 + (-10.75)^2 + \ldots + (12.25)^2}{8}}$$
≈ 9.59

36. $S_n = \frac{a_1 - a_1 r^n}{1 - r}$
$n = 10$, $a_1 = \frac{1}{16}$, $r = 4$
$$S_{10} = \frac{\frac{1}{16} - \frac{1}{16}(4)^{10}}{1 - 4}$$
$= \frac{349{,}525}{16}$ or $21{,}845.3125$

37.
$$x = y$$
$$r\cos\theta = r\sin\theta$$
$$1 = \frac{\sin\theta}{\cos\theta}$$
$$1 = \tan\theta$$
$$\tan^{-1}(1) = \theta$$
$$45° = \theta$$

38.
$$\tan x + \cot x = 2$$
$$\tan x + \frac{1}{\tan x} = 2$$
$$\frac{\tan^2 x + 1}{\tan x} = 2$$
$$\tan^2 x + 1 = 2\tan x$$
$$\tan^2 x - 2\tan x + 1 = 0$$
$$(\tan x - 1)^2 = 0$$
$$\tan x - 1 = 0$$
$$\tan x = 1$$
$$x = 45°$$

39. $*2 = 2^2 - 2(2)$ or 0
$*1 = 1^2 - 2(1)$ or -1
$*2 - *1 = 0 - (-1)$ or 1
The correct choice is C.

Chapter 14 Study Guide and Assessment

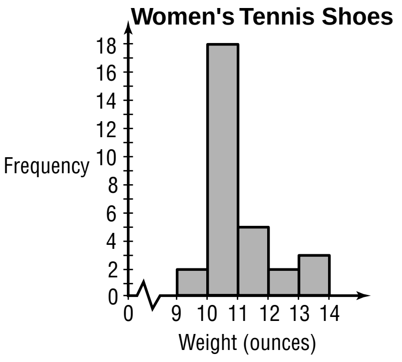
Page 933 Understanding the Vocabulary

1. box-and-whisker plot
2. median
3. standard error of the mean

4. range
5. measure of central tendency
6. population
7. bimodal
8. inferential statistics
9. histogram
10. standard deviation

Pages 934–936 Skills and Concepts

11. range = 14.0 − 9.0 or 5
12. 9.5, 10.5, 11.5, 12.5, 13.5
13.



14. $\overline{X} = \frac{1}{9}(2 + 4 + 4 + 4 + 5 + 5 + 6 + 7 + 8)$
$= 5$
$M_d = 5$
Mode = 4
15. $\overline{X} = \frac{1}{5}(160 + 200 + 200 + 240 + 250)$
$= 210$
$M_d = 200$
Mode = 200
16. $\overline{X} = \frac{1}{5}(11 + 13 + 15 + 16 + 19)$
$= 14.8$
$M_d = 15$
Mode: none
17. $\overline{X} = \frac{1}{8}(5.9 + 6.3 + 6.3 + 6.4 + 6.6 + 6.6 + 6.7 + 6.8)$
$= 6.45$
$M_d = \frac{6.4 + 6.6}{2}$ or 6.5
Mode = 6.3 and 6.6
18. $\overline{X} = \frac{1}{8}(122 + 128 + 130 + 131 + 133 + 135 + 141 + 146)$
$= 133.25$
$M_d = \frac{131 + 133}{2}$ or 132
Mode: none
19. interquartile range $= Q_3 - Q_1$
$= 5 - 2$
$= 3$
20. semi-interquartile range $= \frac{3}{2}$ or 1.5
21. $\overline{X} = \frac{1}{10}(1 + 1 + \ldots + 6)$
$= 3.4$
$MD = \frac{1}{10}(|-2.4| + |-2.4| + \ldots + |2.6|)$
$= 1.6$
22. $\sigma = \sqrt{-2.4)^2 + (-2.4)^2 + \ldots + 2.6^2}$
≈ 1.74
23. $88 - 78 = 10 \quad 98 - 88 = 10$
$t\sigma = 10$
$t(5) = 10$
$t = 2 \rightarrow 95.5\%$
24. $88 - 86 = 2 \quad 90 - 88 = 2$
$t\sigma = 2$
$t(5) = 2$
$t = 0.4 \rightarrow 0.311$
25. 90% corresponds to $t = 1.65$.
interval: $\overline{X} \pm t\sigma = 88 \pm 1.65(5)$
$= 79.75 - 96.25$
26. $0.683(150) = 102.45$
27. $0.955(150) = 143.25$
28. $\frac{0.683}{2}(150) = 51.225$
29. $\sigma_{\overline{X}} = \frac{1.5}{\sqrt{90}}$
≈ 0.16
30. $\sigma_{\overline{X}} = \frac{4.9}{\sqrt{120}}$
≈ 0.45
31. $\sigma_{\overline{X}} = \frac{25}{\sqrt{400}}$ or 1.25
32. $\sigma_{\overline{X}} = \frac{18}{\sqrt{25}}$ or 3.6
33. $\sigma_{\overline{X}} = \frac{15}{\sqrt{50}}$
≈ 2.121320344
interval: $\overline{X} \pm t\sigma_{\overline{X}} = 100 \pm 2.58\sigma_{\overline{X}}$
$\approx 94.53 - 105.47$
34. $\sigma_{\overline{X}} = \frac{30}{\sqrt{15}}$
≈ 7.745966692
interval: $\overline{X} \pm t\sigma_{\overline{X}} = 90 \pm 2.58\sigma_{\overline{X}}$
$\approx 70.02 - 109.98$
35. $\sigma_{\overline{X}} = \frac{24}{\sqrt{200}}$
≈ 1.697056275
interval: $\overline{X} \pm t\sigma_{\overline{X}} = 40 \pm 2.58\sigma_{\overline{X}}$
$\approx 35.62 - 44.38$
36. $\sigma_{\overline{X}} = \frac{0.5}{\sqrt{200}}$
≈ 0.035
37. $P = 0.90$ corresponds to $t = 1.65$.
range: $\overline{X} \pm t\sigma_{\overline{X}} = 1.8 \pm 1.65(0.035)$
$\approx 1.74 - 1.86$ h
38. A 5% confidence level is given when $P = 95\%$ and $t = 1.96$.
range: $\overline{X} \pm t\sigma_{\overline{X}} = 1.8 \pm 1.96(0.035)$
$\approx 1.73 - 1.87$ h
39. A 1% confidence level is given when $P = 99\%$ and $t = 2.58$.
range: $\overline{X} \pm t\sigma_{\overline{X}} = 1.8 \pm 2.58(0.035)$
$\approx 1.71 - 1.89$ h
40. $P = 0.90$ corresponds to $t = 1.65$.
$\sigma_{\overline{X}} = \frac{1.4}{\sqrt{100}}$ or 0.14
range: $\overline{X} \pm t\sigma_{\overline{X}} = 4.6 \pm 1.65(0.14)$
$\approx 4.37 - 4.83$ h

Page 937 Applications and Problem Solving

41a.

stem	leaf
1	0 3 5 6 7 9
2	1 3 4 5
3	9 9

$1|0 = 10$

41b. $\overline{X} = \frac{1}{2}(10 + 13 + 15 + 16 + 17 + 19 + 21 + 23 + 24 + 25 + 39 + 39)$
$= 21.75$

41c. $M_d = \frac{19 + 21}{2}$ or 20

41d. Mode = 39

42. $\overline{X} + t\sigma = 80$
$75 + t(2) = 80$
$2t = 5$
$t = 2.5 \rightarrow 98.8\%$
$\frac{100\% - 98.8\%}{2} = 0.6\%$

Page 937 Open-Ended Assessment

1a. Sample answer: {2, 3, 10, 20, 40}

1b. Sample answer: 15

2. See students' work.

Chapter 14 SAT & ACT Preparation

Page 939 SAT and ACT Practice

1. The percent increase is the ratio of the number increase to the original amount.

Amy $\frac{10}{80} = \frac{1}{8}$

Brad $\frac{30}{70} = \frac{3}{7}$

Cara $\frac{20}{80} = \frac{1}{4}$

Dan $\frac{30}{60} = \frac{1}{2}$

Elsa $\frac{20}{90} = \frac{2}{9}$

The largest fraction is $\frac{1}{2}$. Dan has the greatest percent increase. The correct choice is D.

2. $a = b + bc$

$\frac{b}{a} = \frac{b}{(b + bc)}$

$= \frac{b}{b(1 + c)}$

$= \frac{1}{1 + c}$

The correct choice is B.

3. Method 1:

$0.1\%(m) = 10\%(n)$
$0.001m = 0.1n$
$m = 100n$

$m = x\%(10n)$
$m = \frac{x}{100} \cdot 10n$
$m = 0.1xn$

$100n = 0.1xn$
$100 = x$

Method 2

Let m represent a large number such as 2000.
0.1% of $m = 0.001(2000)$ or 2
10% of $n = 2$, so $0.1n = 2$ or $n = 20$.

$m = x\%(10n)$
$2000 = \frac{x}{100}(10 \cdot 20)$
$1000 = x$

The correct choice is D.

4. The numbers in S are positive numbers that are less than 100 and the square root of each number is an integer. So the set S contains perfect squares between 0 and 100.

Make a list of the numbers, n, in set S. From your list, you can see that the median, or middle value for n, is 25. The correct choice is C.

n	$\sqrt{n}$
1	1
4	2
9	3
16	4
25	5
36	6
49	7
64	8
81	9

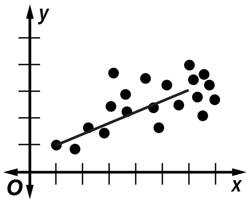
5. $m\angle DBA = 90 - 30 = 60$
$m\angle EBC = 90 - 40 = 50$
$m\angle ABC = 180 - m\angle DBA - m\angle EBC$
$= 180 - (60) - (50)$ or 70

The correct choice is E.

6. $\overline{X} = \frac{1}{6}(10 + 20 + 30 + 35 + 35 + 50)$
$= 30$

There are 3 numbers larger than 30: 35, 35, and 50. The correct choice is D.

7.



The line of best fit has a rise of 2 and a run of 5. So the slope of the line of best fit is $\frac{2}{5}$. The closest answer choice to $\frac{2}{5}$ is $\frac{1}{2}$.

The correct choice is D.

8. Each year the number of employees increases by 300. The last year of data is 2005. The expected employment in 2007, two years later, will be $2 \cdot 300$ more employees than in 2005.
$3100 + 600 = 3700$
The correct choice is D.

9. To find the median of Set A, first rewrite the elements of Set A in order: $-4, -1, 2, 3, 7, 11$. Since the number of elements is even, the median is the average of the middle two elements: 2 and 3. So the median of Set A is 2.5. To find the mean of Set B, add all of the elements together and divide by the number of elements in the set. The sum of the elements is 15, and there are 6 terms. So the mean is $\frac{15}{6}$ or 2.5.

The difference between the median of Set A and the mean of Set B is $2.5 - 2.5$ or 0.

The correct choice is C.

10. $$\begin{aligned}\overline{X} &= \tfrac{1}{10}[-820 + (-65) + (-32) + 0 + 1 + 2 + 3 \\ &\quad + 32 + 64 + 820] \\ &= \tfrac{1}{10}(1 + 2 + 3) \\ &= \tfrac{6}{10}\end{aligned}$$

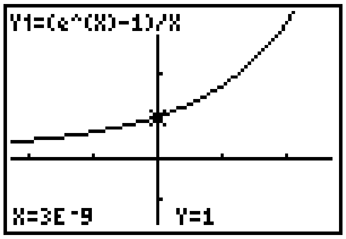
The answer is 0.6, 6/10, or 3/5.

Chapter 15 Introduction to Calculus

15-1 Limits

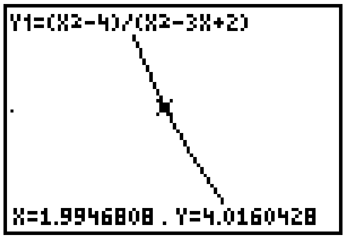
Page 945 Graphing Calculator Exploration

1.



$\lim\limits_{x\to 0} \frac{e^x - 1}{x} = 1$

2.



$\lim\limits_{x\to 2} \frac{x^2 - 4}{x^2 - 3x + 2} = 4$

3. y is undefined when $x = 1$.

4. $$\begin{aligned}\lim_{x\to 2} \frac{x^2-4}{x^2-3x+2} &= \lim_{x\to 2} \frac{(x+2)(x-2)}{(x-2)(x-1)}\\ &= \lim_{x\to 2}\frac{x+2}{x-1}\\ &= \frac{2+2}{2-1} \text{ or } 4\end{aligned}$$

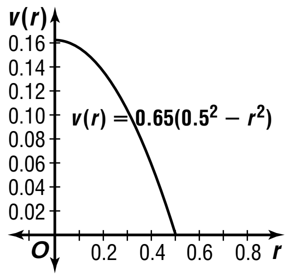
   Yes, the limit is the same.

5. No; if the exact answer is a complicated fraction or an irrational number, you may not be able to tell what it is from the decimals displayed by a calculator.

Page 946 Check for Understanding

1. Sample answer: The limit of $f(x)$ as x approaches a is the number that the values of $f(x)$ get closer to as the values of x gets closer and closer to a.
2. Sample answer: $\lim\limits_{x\to 1} f(x)$ is the number that the values of $f(x)$ approach as x approaches 1. $f(1)$ is the number that you get if you actually plug 1 into the function. They are the same if $f(x)$ is continuous at $x = 1$.
3. Sample answer: If $f(x)$ is continuous at $x = a$ you can plug a into the function. If the function is not continuous, you may be able to simplify it and then plug in a. If neither of these methods work, you can use a calculator. Examples will vary.
4. The closer x is to 0, the closer y is to 3. So, $\lim\limits_{x\to 0} f(x) = 3$. However, there is a point at (0, 1), so $f(0) = 1$.
5. $$\begin{aligned}\lim_{x\to 2}(-4x^2 + 2x - 5) &= -4(2)^2 + 2(2) - 5\\ &= -16 + 4 - 5\\ &= 17\end{aligned}$$
6. $$\begin{aligned}\lim_{x\to 0}(1 + x + 2^x - \cos x) &= 1 + 0 + 2^0 - \cos 0\\ &= 1 + 1 - 1\\ &= 1\end{aligned}$$
7. $$\begin{aligned}\lim_{x\to 2}\frac{x+2}{x^2-4} &= \lim_{x\to 2}\frac{x+2}{(x+2)(x-2)}\\ &= \lim_{x\to 2}\frac{1}{x-2}\\ &= \frac{1}{-2-2} \text{ or } -\frac{1}{4}\end{aligned}$$
8. $$\begin{aligned}\lim_{x\to 0}\frac{x^2-3x}{x^3+4x} &= \lim_{x\to 0}\frac{x(x-3)}{x(x^2+4)}\\ &= \lim_{x\to 0}\frac{x-3}{x^2+4}\\ &= \frac{0-3}{0^2+4} \text{ or } -\frac{3}{4}\end{aligned}$$
9. $$\begin{aligned}\lim_{x\to 3}\frac{x^2+3x-10}{x^2+5x+6} &= \lim_{x\to 3}\frac{(x+5)(x-2)}{(x+3)(x+2)}\\ &= \frac{(3+5)(3-2)}{(3+3)(3+2)}\\ &= \frac{8(1)}{6(5)} \text{ or } \frac{4}{15}\end{aligned}$$
10. $$\begin{aligned}\lim_{x\to -2}\frac{2x^2+5x+2}{x^2+x-2} &= \lim_{x\to -2}\frac{(2x+1)(x+2)}{(x-1)(x+2)}\\ &= \lim_{x\to -2}\frac{2x+1}{x-1}\\ &= \frac{2(-2)+1}{-2-1} \text{ or } 1\end{aligned}$$

11a.



11b. As the molecules get farther from the center and closer to the pipe, r is increasing. As r increases, $v(r)$ gets closer and closer to 0 in./s.

Pages 946–948 Exercises

12. The closer x is to -2, the closer y is to -1. So, $\lim\limits_{x\to -2} f(x) = -1$. Also $f(-2) = -1$.
13. The closer x is to 0, the closer y is to 0. So, $\lim\limits_{x\to 0} f(x) = 0$. However, there is point discontinuity when $x = 0$. So $f(0)$ is undefined.
14. The closer x is to 3, the closer y is to 4. So, $\lim\limits_{x\to 3} f(x) = 4$. However, there is a point at (3, 2). So $f(3) = 2$.
15. $$\begin{aligned}\lim_{x\to 2}(-4x^2 - 3x + 6) &= -4(2)^3 - 3(2) + 6\\ &= -16 - 6 + 6\\ &= -16\end{aligned}$$
16. $$\begin{aligned}\lim_{x\to -1}(-x^3 + 3x^2 - 4) &= -(-1)^3 + 3(-1)^2 - 4\\ &= 1 + 3 - 4\\ &= 0\end{aligned}$$
17. $$\begin{aligned}\lim_{x\to \pi}\frac{\sin x}{x} &= \frac{\sin \pi}{\pi}\\ &= \frac{0}{\pi} \text{ or } 0\end{aligned}$$
18. $$\begin{aligned}\lim_{x\to 0}(x + \cos x) &= 0 + \cos 0\\ &= 0 + 1 \text{ or } 1\end{aligned}$$

19. $\lim\limits_{x\to5}\frac{x^2-25}{x-5}=\lim\limits_{x\to5}\frac{(x+5)(x-5)}{x-5}$
$=\lim\limits_{x\to5}(x+5)$
$=5+5$ or 10

20. $\lim\limits_{x\to0}\frac{2n^2}{n}=\lim\limits_{x\to0}2n$
$=2(0)$ or 0

21. $\lim\limits_{x\to3}\frac{x^2-3x}{x^2+2x-15}=\lim\limits_{x\to3}\frac{x(x-3)}{(x+5)(x-3)}$
$=\lim\limits_{x\to3}\frac{x}{x+5}$
$=\frac{3}{3+5}$ or $\frac{3}{8}$

22. $\lim\limits_{x\to1}\frac{x^3+3x^2-4x+8}{x+6}=\frac{1^3+3(1)^2-4(1)+8}{1+6}$
$=\frac{1+3-4+8}{7}$ or $\frac{8}{7}$

23. $\lim\limits_{h\to-2}\frac{h^2+4h+4}{h+2}=\lim\limits_{h\to-2}\frac{(h+2)(h+2)}{h+2}$
$=\lim\limits_{h\to-2}h+2$
$=-2+2$ or 0

24. $\lim\limits_{x\to3}\frac{2x^2-3x}{x^3-2x^2+x+6}=\frac{2(3)^2-3(3)}{3^3-2(3)^2+3+6}$
$=\frac{18-9}{27-18+9}$ or $\frac{1}{2}$

25. $\lim\limits_{x\to0}\frac{x^3-x^2+2x}{x^3+4x^2-2x}=\lim\limits_{x\to0}\frac{x(x^2-x+2)}{x(x^2+4x-2)}$
$=\lim\limits_{x\to0}\frac{x^2-x+2}{x^2+4x-2}$
$=\frac{0^2-0+2}{0^2+4(0)-2}$ or -1

26. $\lim\limits_{x\to0}\frac{x\cos x}{x^2+x}=\lim\limits_{x\to0}\frac{x\cos x}{x(x+1)}$
$=\lim\limits_{x\to0}\frac{\cos x}{x+1}$
$=\frac{\cos 0}{0+1}$ or 1

27. $\lim\limits_{x\to0}\frac{(x+2)^2-4}{4}=\lim\limits_{x\to0}\frac{x^2+4x-4+4}{x}$
$=\lim\limits_{x\to0}\frac{x(x+4)}{x}$
$=\lim\limits_{x\to0}(x+4)$
$=0+4$ or 4

28. $\lim\limits_{x\to-2}\frac{(x+1)^2-1}{x+2}=\lim\limits_{x\to-2}\frac{x^2+2x+1-1}{x+2}$
$=\lim\limits_{x\to-2}\frac{x(x+2)}{x+2}$
$=\lim\limits_{x\to-2}x$
$=-2$

29. $\lim\limits_{x\to-2}\frac{x^3+8}{x^2-4}=\lim\limits_{x\to-2}\frac{(x+2)(x^2-2x+4)}{(x+2)(x-2)}$
$=\lim\limits_{x\to-2}\frac{x^2-2x+4}{x-2}$
$=\frac{(-2)^2-2(-2)+4}{-2-2}$
$=\frac{4+4+4}{-4}$ or -3

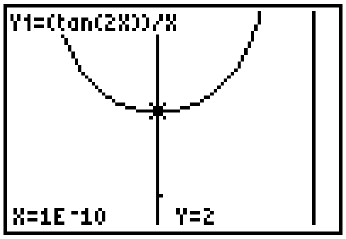
30. $\lim\limits_{x\to4}\frac{2x-8}{x^3-64}=\lim\limits_{x\to4}\frac{2(x-4)}{(x-4)(x^2+4x+16)}$
$=\lim\limits_{x\to4}\frac{2}{x^2+4x+16}$
$=\frac{2}{4^2+4(4)+16}$
$=\frac{2}{48}$ or $\frac{1}{24}$

31. $\lim\limits_{x\to1}\frac{\frac{1}{x}-1}{x-1}=\lim\limits_{x\to1}\frac{\frac{1-x}{x}}{x-1}$
$=\lim\limits_{x\to1}\frac{1-x}{x}\cdot\frac{1}{-(1-x)}$
$=\lim\limits_{x\to1}-\frac{1}{x}$
$=\frac{-1}{1}$ or -1

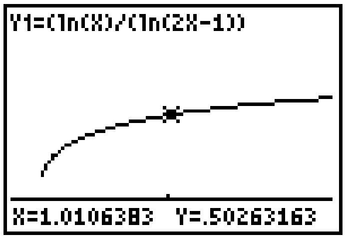
32. $\lim\limits_{x\to4}\frac{x-4}{\sqrt{x}-2}=\lim\limits_{x\to4}\frac{x-4}{\sqrt{x}-2}\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2}$
$=\lim\limits_{x\to4}\frac{(x-4)(\sqrt{x}+2)}{x-4}$
$=\lim\limits_{x\to4}\left(\sqrt{x}+2\right)$
$=\sqrt{4}+2$ or 4

33. $\lim\limits_{h\to0}\frac{2h^3-h^2+5h}{h}=\lim\limits_{h\to0}\frac{h(2h^2-h+5)}{h}$
$=\lim\limits_{h\to0}(2h^2-h+5)$
$=2(0)^2-0+5$ or 5

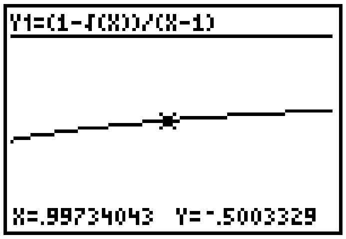
34. $\lim\limits_{x\to0}\frac{x+\pi}{\cos(x+\pi)}=\frac{0+\pi}{\cos(0+\pi)}$
$=\frac{\pi}{\cos(\pi)}$ or $-\pi$

35.


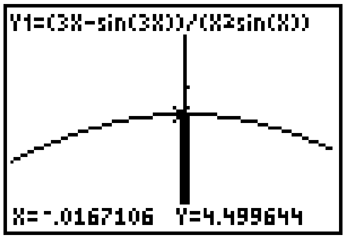
$\lim\limits_{x\to0}\frac{\tan 2x}{x}=2$

36.


$\lim\limits_{x\to1}\frac{\ln x}{\ln(2x-1)}=0.5$

37.


$\lim\limits_{x\to1}\frac{1-\sqrt{8}}{x-1}=-0.5$

38. 

$\lim\limits_{x\to 0} \frac{3x - \sin 3x}{x^2 \sin x} = 4.5$

39. $\lim\limits_{c\to 0} \pi a\sqrt{a^2 - c^2} = \pi a\sqrt{a^2 - 0^2}$
$= \pi a^2$

Letting c approach 0 moves the foci together, so the ellipse becomes a circle. πa^2 is the area of a circle of radius a.

40.

$f(x) = \dfrac{2^{\frac{x}{10}} - 1}{x}$	
x	y
-1	0.06697
-0.1	0.06908
0.1	0.06956
1	0.07177

$\lim\limits_{t\to 0} \frac{2^{\frac{x}{10}} - 1}{t} = 0.07$ or 7%

41. No; the graph of $f(x) = \sin\left(\frac{1}{x}\right)$ oscillates infinitely many times between -1 and 1 as x approaches 0, so the values of the function do not approach a unique number.

42a.

x	1	0.5	0.1	0.01	0.001
$\cos x$	0.540302	0.877583	0.995004	0.999950	1.000000
$1 - \frac{x^2}{2}$	0.5	0.875	0.995	0.99995	1.000000

42b. yes; in the last three columns, all the decimal places of $1 - \frac{x^2}{2}$ agree with those of $\cos x$.

43. $\lim\limits_{t\to 2} \frac{d(t) - d(2)}{t - 2} = \lim\limits_{t\to 2} \frac{16t^2 - 64}{t - 2}$
$= \lim\limits_{t\to 2} \frac{16(t + 2)(t - 2)}{t - 2}$
$= \lim\limits_{t\to 2} 16(t + 2)$
$= 16(2 + 2)$ or 64 ft/s

44a. As x approaches 0, the decimals for the values of $f(x) = (1 + x)^{\frac{1}{x}}$ approach 2.71828 ... , which is the decimal expansion of e.

44b. He ignored the exponent. As x approaches 0 from the positive side, $\frac{1}{x}$ approaches infinity. A number close to 1 raised to a large power need not be close to 1. If x approaches 0 from the negative side, $\frac{1}{x}$ approaches negative infinity. A number close to 1 raised to a large negative power need not be close to 1, either.

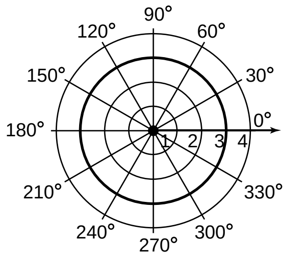
45. When $P = 0.99$, $t = 2.58$.
Find $\sigma_{\overline{X}}$.
$\sigma_{\overline{X}} = \frac{1.4}{\sqrt{50}}$
≈ 0.20
Find the range.
$\overline{X} \pm t\sigma_{\overline{X}} = 16.2 \pm 2.58(0.20)$
$= 15.684 - 16.716$ mm

46. $P(\text{not getting a 7}) = \frac{9}{10}$
$P(\text{never getting a 7 in five spins}) = \left(\frac{9}{10}\right)^5$
$= \frac{59{,}049}{100{,}000}$

47. $(x - 3y)^5 = \sum_{r=0}^{5} \frac{5!}{r!(5 - r)!} (x)^{5-r} (-3y)^r$
To find the third term, evaluate the general term for $r = 2$.
$\frac{5!}{r!(5 - r)!} (x)^{5-r}(-3y)^r = \frac{5!}{2!(5 - 2)!} (x)^{5-2}(-3y)^2$
$= \frac{5!}{2!\,3!} (x^3)(9y^2)$
$= 10(x^3)(9y^2)$
$= 90x^3y^2$

48. $(16y^8)^{\frac{3}{4}} = \left(\sqrt[4]{16}\right)^3 \cdot y^{8\left(\frac{3}{4}\right)}$
$= 2^3 \cdot y^6$
$= 8y^6$

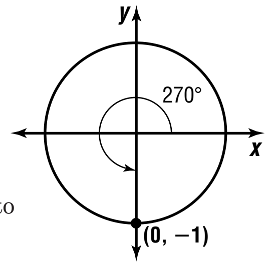
49. center $= \left(\frac{5 + 5}{2}, \frac{-5 + 1}{2}\right)$
$= (5, -2)$
The foci are on the x-axis.
$a = \frac{9 - 1}{2}$ or 4
$b = \frac{1 - (-5)}{2}$ or 3
$\frac{(x - 5)^2}{4^2} + \frac{(y + 2)^2}{3^2} = 1 \rightarrow \frac{(x - 5)^2}{16} + \frac{(y + 2)^2}{9} = 1$

50. 

51. $\overrightarrow{WX} = \langle -3 - 4, -6 - 0\rangle$
$= \langle -7, -6\rangle$
$|\overrightarrow{WX}| = \sqrt{(-3 - 4)^2 + (-6 - 0)^2}$
$= \sqrt{49 + 36}$
$= \sqrt{85}$

52. $C = \frac{25\pi \text{ in.}}{1} \cdot \frac{1 \text{ yd}}{36 \text{ in.}} \cdot \frac{1 \text{ mi}}{1760 \text{ yd}} \approx 0.0012395804$ mi
$\theta = \frac{65}{C} \approx 52437.09741$
$w = \frac{\theta}{t} = \frac{\theta}{3600 \text{ s}}$
≈ 14.6 rps

53. $\csc 270° = \frac{1}{\sin 270°}$
$= \frac{1}{y}$
$= \frac{1}{-1}$ or -1



54. Use a graphing calculator to find the rational roots at $\frac{1}{4}$ and $-\frac{4}{3}$.

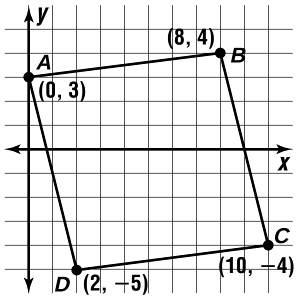
55.

$y = 4x^5 - 2x^2 + 4$	
x	y
-100	-4×10^0
-10	-4×10^5
1	6
10	399804
100	4×10^{10}

$y \to \infty$ as $x \to \infty$, $y \to -\infty$ as $x \to -\infty$

56. $\begin{vmatrix} -1 & -2 \\ 3 & -6 \end{vmatrix} = (-1)(-6) - (-2)(3)$
$= 6 - (-6)$
$= 12$

57. yes; opposite sides have the same slope
$m_{AB} = \frac{1}{8}$
$m_{DC} = \frac{1}{8}$
$m_{BC} = -\frac{1}{4}$
$m_{AD} = -\frac{1}{4}$



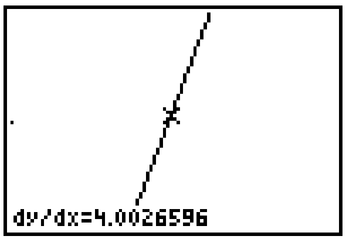
58. If $2^n = 8$, then $n = 3$.
$3^{n+2} = 3^{3+2}$
$= 3^5$ or 243

15-2A Graphing Calculator Exploration: The Slope of a Curve

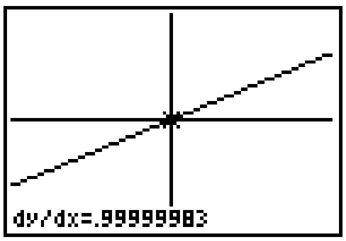
Page 950

1-6. Exact answers are given. Accept all reasonable approximations.

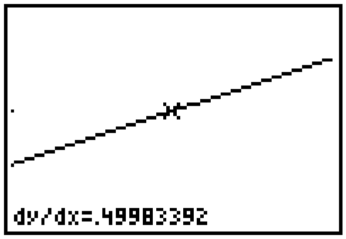
1. 4



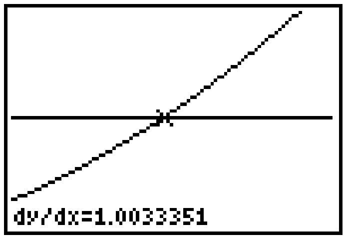
2. 1



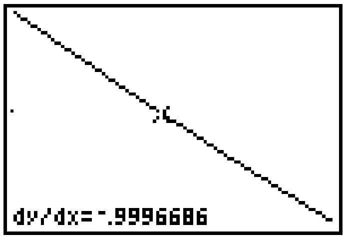
3. 0.5



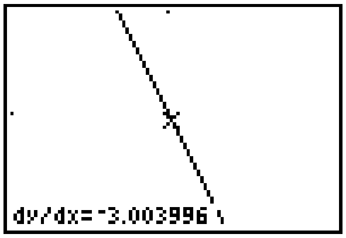
4. 1



5. −1



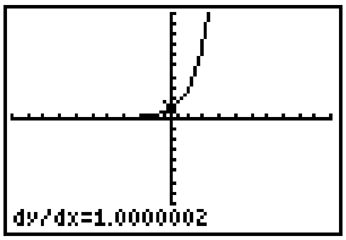
6. −3



7. The graph of a linear function is a line and the methods in this function will result in the calculation of the slope of *that* line.

8. If you zoom in on a maximum or minimum point, the graph appears flat. The slope is 0.

9. At (0, 1), $\frac{dy}{dx} \approx 1$. For other points on the curve, the values for y and $\frac{dy}{dx}$ are approximately the same.



15-2 Derivatives and Antiderivatives

Pages 957–958 Check for Understanding

1. $4x^3$ is the derivative of x^4. x^4 is an antiderivative of $4x^3$.

2. Letting $n = -1$ in the expression $\frac{1}{n+1}x^{n+1}$ results in $\frac{x^0}{0}$, which is undefined.

3. $f(x+h)$ means substitute the quantity $x + h$ into the function. On the other hand, $f(x) + h$ means substitute x into the function, then add h to the result. Using $f(x) + h$ instead of $f(x + h)$ in the definition of the derivative results in:

$$\begin{aligned}\lim_{h\to 0}\frac{f(x)+h-f(x)}{h} &= \lim_{h\to 0}\frac{h}{h}\\ &= \lim_{h\to 0} 1\\ &= 1\end{aligned}$$

You would always get 1.

4. $$\begin{aligned}f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to 0}\frac{3(x+h)+2-(3x+2)}{h}\\ &= \lim_{h\to 0}\frac{3x+3h+2-3x-2}{h}\\ &= \lim_{h\to 0}\frac{3h}{h}\\ &= 3\end{aligned}$$

5. $$\begin{aligned}f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to 0}\frac{(x+h)^2+x+h-(x^2+x)}{h}\\ &= \lim_{h\to 0}\frac{x^2+2xh+h^2+x+h-x^2-x}{h}\\ &= \lim_{h\to 0}\frac{2xh+h^2+h}{h}\\ &= \lim_{h\to 0}\frac{h(2x+h+1)}{h}\\ &= \lim_{h\to 0} 2x+h+1\\ &= 2x+0+1 \text{ or } 2x+1\end{aligned}$$

6. $$\begin{aligned}f(x) &= 2x^2-3x+5\\ f'(x) &= 2\cdot 2x^{2-1}-3\cdot 1x^{1-1}+0\\ &= 4x-3\end{aligned}$$

7. $$\begin{aligned}f(x) &= -x^3-2x^2+3x+6\\ f'(x) &= -1\cdot 3x^{3-1}-2\cdot 2x^{2-1}+3\cdot 1x^{1-1}+0\\ &= -3x^2-4x+3\end{aligned}$$

8. $$\begin{aligned}f(x) &= 3x^4+2x^3-3x-2\\ f'(x) &= 3\cdot 4x^{4-1}+2\cdot 3x^{3-1}-3\cdot 1x^{1-1}-0\\ &= 12x^3+6x^2-3\end{aligned}$$

9. $$\begin{aligned}y &= x^2+2x+3\\ \frac{dy}{dx} &= 2\cdot 1x^{2-1}+2\cdot 1x^{1-1}+0\\ \frac{dy}{dx} &= 2x+2\\ f'(1) &= 2(1)+2\\ &= 4\end{aligned}$$

10. $$\begin{aligned}f(x) &= x^2\\ F(x) &= \frac{1}{2+1}x^{2+1}+C\\ &= \frac{1}{3}x^3+C\end{aligned}$$

11. $$\begin{aligned}f(x) &= x^3+4x^2-x-3\\ F(x) &= \frac{1}{3+1}x^{3+1}+4\left(\frac{1}{2+1}\right)x^{2+1}-\frac{1}{1+1}x^{1+1}\\ &\quad -3x+C\\ &= \frac{1}{4}x^4+\frac{4}{3}x^3-\frac{1}{2}x^2-3x+C\end{aligned}$$

12. $$\begin{aligned}f(x) &= 5x^5+2x^3-x^2+4\\ F(x) &= 5\left(\frac{1}{5+1}\right)x^{5+1}+2\left(\frac{1}{3+1}\right)x^{3+1}-\frac{1}{2\cdot 1}x^{2+1}\\ &\quad +4x+C\\ &= \frac{5}{6}x^6+\frac{1}{2}x^4-\frac{1}{3}x^3+4x+C\end{aligned}$$

13. $$\begin{aligned}C(x) &= 1000+10x-0.001x^2\\ C'(x) &= 0+10\cdot 1x^{1-1}-0.001(2)x^{2-1}\\ C'(x) &= 10-0.002x\\ C'(1000) &= 10-0.002(1000)\\ &= 8\end{aligned}$$

The marginal cost is \$8.

Pages 958–960 Exercises

14. $$\begin{aligned}f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to 0}\frac{2(x+h)-2x}{h}\\ &= \lim_{h\to 0}\frac{2x+2h-3x}{h}\\ &= \lim_{h\to 0}\frac{2h}{h}\\ &= 2\end{aligned}$$

15. $$\begin{aligned}f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to 0}\frac{7(x+h)+4-(7x+4)}{h}\\ &= \lim_{h\to 0}\frac{7x+7h+4-7x-4}{h}\\ &= \lim_{h\to 0}\frac{7h}{h}\\ &= 7\end{aligned}$$

16. $$\begin{aligned}f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to 0}\frac{-3(x+h)-(-3x)}{h}\\ &= \lim_{h\to 0}\frac{-3x-3h+3x}{h}\\ &= \lim_{h\to 0}\frac{-3h}{h}\\ &= -3\end{aligned}$$

17. $$\begin{aligned}f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to 0}\frac{-4(x+h)-9-(-4x-9)}{h}\\ &= \lim_{h\to 0}\frac{-4x-4h-9+4x+9}{h}\\ &= \lim_{h\to 0}\frac{-4h}{h}\\ &= -4\end{aligned}$$

18. $$\begin{aligned}f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to 0}\frac{2(x+h)^2+5(x+h)-(2x^2+5x)}{h}\\ &= \lim_{h\to 0}\frac{2x^2+4xh+2h^2+5x+5h-2x^2-5x}{h}\\ &= \lim_{h\to 0}\frac{4xh+2h^2+5h}{h}\\ &= \lim_{h\to 0}\frac{h(4x+2h+5)}{h}\\ &= 4x+2(0)+5 \text{ or } 4x+5\end{aligned}$$

19. $$\begin{aligned}f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to 0}\frac{(x+h)^3+5(x+h)^2+6-(x^3+5x^2+6)}{h}\\ &= \lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3+5x^2+10xh+5h^2+6-x^3-5x^2-6}{h}\\ &= \lim_{h\to 0}\frac{h(3x^2+3xh+h^2+10x+5h)}{h}\\ &= 3x^2+3x(0)+0^2+10x+5(0)\\ &= 3x^2+10x\end{aligned}$$

20. $f(x) = 8x$
$f'(x) = 8 \cdot 1x^{1-1}$
$= 8$

21. $f(x) = 2x + 6$
$f'(x) = 2 \cdot 1x^{1-1} + 0$
$= 2$

22. $f(x) = \frac{1}{3}x + \frac{4}{5}$
$f'(x) = \frac{1}{3} \cdot 1x^{1-1} + 0$
$= \frac{1}{3}$

23. $f(x) = -3x^2 + 2x + 9$
$f'(x) = -3 \cdot 2x^{2-1} + 2 \cdot 1x^{1-1} + 0$
$= -6x + 2$

24. $f(x) = \frac{1}{2}x^2 - x - 2$
$f'(x) = \frac{1}{2} \cdot 2x^{2-1} - 1 \cdot 1x^{1-1} - 0$
$= x - 1$

25. $f(x) = x^3 - 2x^2 + 5x - 6$
$f'(x) = 3x^{3-1} - 2 \cdot 2x^{2-1} + 5 \cdot 1x^{1-1} - 0$
$= 3x^2 - 4x + 5$

26. $f(x) = 3x^4 + 7x^3 - 2x^2 + 7x - 12$
$f'(x) =$
$3 \cdot 4x^{4-1} + 7 \cdot 3x^{3-1} - 2 \cdot 2x^{2-1} + 7 \cdot 1x^{1-1} - 0$
$= 12x^3 + 21x^2 - 4x + 7$

27. $f(x) = (x^2 + 3)(2x - 7)$
$= 2x^3 - 7x^2 + 6x - 21$
$f'(x) = 2 \cdot 3x^{3-1} - 7 \cdot 2x^{2-1} + 6 \cdot 1x^{1-1} - 0$
$= 6x^2 - 14x + 6$

28. $f(x) = (2x + 4)^2$
$= 4x^2 + 16x + 16$
$f'(x) = 4 \cdot 2x^{2-1} + 16 \cdot 1x^{1-1} + 0$
$= 8x + 16$

29. $f(x) = (3x - 4)^3$
$= 27x^3 - 108x^2 + 144x - 64$
$f'(x) = 27 \cdot 3x^{3-1} - 108 \cdot 2x^{2-1} + 144 \cdot 1x^{1-1} - 0$
$= 81x^2 - 216x + 144$

30. $f(x) = \frac{2}{3}x^3 + \frac{1}{3}x^2 - x - 9$
$f'(x) = \frac{2}{3} \cdot 3x^{3-1} + \frac{1}{3} \cdot 2x^{2-1} - 1 \cdot 1x^{1-1} - 0$
$= 2x^2 + \frac{2}{3}x - 1$

31. $y = x^3$
$\frac{dy}{dx} = 3x^2$
$f'(1) = 3(1)^2$
$= 3$

32. $y = x^3 - 7x^2 + 4x + 9$
$\frac{dy}{dx} = 3x^2 - 14x + 4$
$f'(1) = 3(1)^2 - 14(1) + 4$
$= -7$

33. $y = (x + 1)(x - 2)$
$= x^2 - x - 2$
$\frac{dy}{dx} = 2x - 1$
$f'(1) = 2(1) - 1$
$= 1$

34. $y = (5x^2 + 7)^2$
$= 25x^4 + 70x^2 + 49$
$\frac{dy}{dx} = 100x^3 + 140x$
$f'(1) = 100(1)^3 + 140(1)$
$= 240$

35. $f(x) = x^6$
$F(x) = \frac{1}{6+1}x^{6+1} + C$
$= \frac{1}{7}x^7 + C$

36. $f(x) = 3x + 4$
$F(x) = 3 \cdot \frac{1}{1+1}x^{1+1} + 4x + C$
$= \frac{3}{2}x^2 + 4x + C$

37. $f(x) = 4x^2 - 6x + 7$
$F(x) = 4 \cdot \frac{1}{2+1}x^{2+1} - 6 \cdot \frac{1}{1+1}x^{1+1} + 7x + C$
$= \frac{4}{3}x^3 - 3x^2 + 7x + C$

38. $f(x) = 12x^2 - 6x + 1$
$= 12 \cdot \frac{1}{2+1}x^{2+1} - 6 \cdot \frac{1}{1+1}x^{1+1} + x + C$
$= 4x^3 - 3x^2 + x + C$

39. $f(x) = 8x^3 + 5x^2 - 9x + 3$
$F(x) =$
$8 \cdot \frac{1}{3+1}x^{3+1} + 5 \cdot \frac{1}{2+1}x^{2+1} - 9 \cdot \frac{1}{1+1}x^{1+1} + 3x + C$
$= 2x^4 + \frac{5}{3}x^3 - \frac{9}{2}x^2 + 3x + C$

40. $f(x) = \frac{1}{4}x^4 - \frac{2}{3}x^2 + 4$
$F(x) = \frac{1}{4} \cdot \frac{1}{4+1}x^{4+1} - \frac{2}{3} \cdot \frac{1}{2+1}x^{2+1} + 4x + C$
$= \frac{1}{20}x^5 - \frac{2}{9}x^3 + 4x + C$

41. $f(x) = (2x + 3)(3x - 7)$
$= 6x^2 - 5x - 21$
$F(x) = 6 \cdot \frac{1}{2+1}x^{2+1} - 5 \cdot \frac{1}{1+1}x^{1+1} - 21x + C$
$= 2x^3 - \frac{5}{2}x^2 - 21x + C$

42. $f(x) = x^4(x + 2)^2$
$= x^6 + 4x^5 + 4x^4$
$F(x) = \frac{1}{6+1}x^{6+1} + 4 \cdot \frac{1}{5+1}x^{5+1} + 4 \cdot \frac{1}{4+1}x^{4+1} + C$
$= \frac{1}{7}x^7 + \frac{2}{3}x^6 + \frac{4}{5}x^5 + C$

43. $f(x) = \frac{x^3 + 4x^2 + x}{x}$
$= x^2 + 4x + 1$
$F(x) = \frac{1}{2+1}x^{2+1} + 4 \cdot \frac{1}{1+1}x^{1+1} + x + C$
$= \frac{1}{3}x^3 + 2x^2 + x + C$

44. $f(x) = \frac{2x^2 - 5x - 3}{x - 3}$
$= 2x + 1$
$F(x) = 2 \cdot \frac{1}{1+1}x^{1+1} + x + C$
$= x^2 + x + C$

45. Any function of the form $F(x) = \frac{1}{6}x^6 + \frac{1}{4}x^4 - \frac{1}{3}x^3 - x + C$, where C is a constant.

46a. $v(12) = 15 + 4(12) + \frac{1}{8}(12)^2$
$= 81$ ft/s

46b. $v(t) = 15 + 4t + \frac{1}{8}t^2$

$$v'(t) = 0 + 4 \cdot 1t^{1-1} + \frac{1}{8} \cdot 2t^{2-1}$$
$$= 4 + \frac{1}{4}t$$
$$v'(12) = 4 + \frac{1}{4}(12)$$
$$= 7 \text{ ft/s}^2$$

46c. When $t = 12$ the car's velocity is increasing at a rate of 7 ft/s per second.

46d. $v(t) = 15 + 4t + \frac{1}{8}t^2$

$$s(t) = 15t + 4 \cdot \frac{1}{1+1}t^{1+1} + \frac{1}{8} \cdot \frac{1}{2+1}t^{2+1} + C$$
$$= 15t + 2t^2 + \frac{1}{24}t^3 + C$$

When $t = 0$, $s(t)$ should equal 0, so $C = 0$.

$$s(t) = 15t + 2t^2 + \frac{1}{24}t^3$$

46e. $s(12) = 15(12) + 2(12)^2 + \frac{1}{24}(12)^3$

$$= 540 \text{ ft}$$

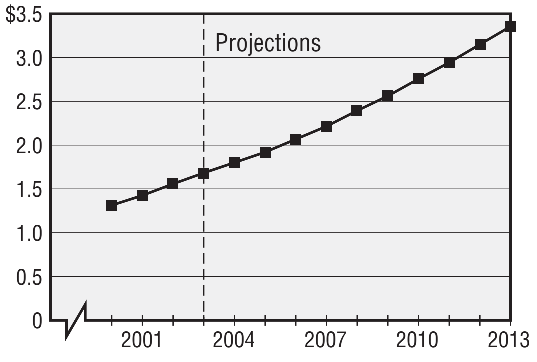
47. $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$
$$= \lim_{h\to 0} \frac{\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}}{h}$$
$$= \lim_{h\to 0} \frac{\frac{-h}{x(x+h)}}{h}$$
$$= \lim_{h\to 0} \frac{-1}{x(x+h)}$$
$$= \frac{-1}{x(x+0)}$$
$$= -\frac{1}{x^2}$$

48a. When $y = 2010$, $I(2010) \approx 2.75$.
The total amount spent in 2010 on health care will be about \$2.75 trillion.

48b. $T(y)$ is approximately linear near (2010, 2.75).

Health Care Spending (Trillions of Dollars)



Source: Centers for Medicare & Medicaid Services

Find the slope of the tangent line at (2010, 2.75).

$$T'(2010) \approx \frac{2.75 - 2.56}{2010 - 2009}$$
$$\approx 0.19$$

In 2010 the amount spent on health care will be increasing at a rate of about \$190 billion per year.

49a. $h(t) = 3 + 80t - 16t^2$

$$h'(t) = v(t) = 0 + 80 \cdot 1t^{1-1} - 16 \cdot 2t^{2-1}$$
$$= 80 - 32t$$

49b. $v(1) = 80 - 32(1)$

$$= 48 \text{ ft/s}$$

49c. At the ball's maximum height, the velocity is 0.

$$0 = 80 - 32t$$
$$-80 = -32t$$
$$2.5 = t;\ 2.5 \text{ s}$$

49d. $h(2.5) = 3 + 80(2.5) - 16(2.5)^2$

$$= 103 \text{ ft}$$

50. $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h\to 0} \frac{e^{x+h} - e^x}{h}$$
$$= \lim_{h\to 0} \frac{e^x \cdot e^h - e^x}{h}$$
$$= \lim_{h\to 0} \frac{e^x \cdot (e^h - 1)}{h}$$
$$= e^x \cdot \lim_{h\to 0} \frac{e^h - 1}{h}$$

A calculator indicates that $\lim_{h\to 0} \frac{e^h - 1}{h} = 1$, so

$f'(x) = e^x \cdot 1 = e^x$. i. e. e^x is its own derivative.

51a. total revenue = cost per cup · number of cups

$$r(p) = p(100 - 2p)$$

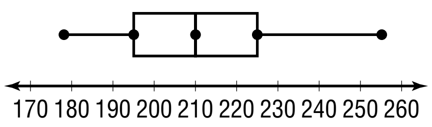
51b. When $r(p)$ is at a maximum, the derivative equals zero.

$$r(p) = p(100 - 2p)$$
$$= 100p - 2p^2$$
$$r'(p) = 100 \cdot 1p^{1-1} - 2 \cdot 2p^{2-1}$$
$$= 100 - 4p$$
$$0 = 100 - 4p$$
$$-100 = -4p$$
$$25 = p;\ 25 \text{ cents}$$

52. $\lim_{x\to 3} \frac{x^2 - 3x - 3}{x - 3} = \lim_{x\to 3} \frac{(x-3)(x+1)}{x-3}$

$$= \lim_{x\to 3} x + 1$$
$$= 3 + 1 \text{ or } 4$$

53a.



53b. See students' work.

54. List all pairs of matching numbers and their sums.
$1 + 1 = 2$; $2 + 2 = 4$; $3 + 3 = 6$; $4 + 4 = 8$;
$5 + 5 = 10$; $6 + 6 = 12$
There are 3 sums out of 6 that are greater than seven.

$$P(\text{sum} > 7 \text{ given that the numbers match}) = \frac{3}{6}$$
$$= \frac{1}{2}$$

55. $a_n = a_1 r^{n-1}$

$$a_6 = 9\left(-\frac{1}{3}\right)^{6-1}$$
$$= -\frac{9}{243} \text{ or } -\frac{1}{27}$$

56. $y = 136e^{-0.06(30)} + 74$

$$\approx 96°$$

57. $x^2 + y^2 + Dx + Ey + F = 0$

$2^2 + (-1)^2 + 2D - E + F = 0$

$\rightarrow 2D - E + F + 5 = 0$

$(-3)^2 + 0^2 - 3D + F = 0 \rightarrow -3D + F + 9 = 0$

$1^2 + 4^2 + D + 4E + F = 0 \rightarrow D + 4E + F + 17 = 0$

$$\begin{array}{rl} 2D - E + F + 5 &= 0 \\ -(-3D \quad + F \quad 9) &= 0 \\ \hline 5D - E \quad - 4 &= 0 \end{array} \qquad \begin{array}{rl} -3D \quad + F + 9 &= 0 \\ -(D + 4E + F + 17) &= 0 \\ \hline -4D - 4E \quad - 8 &= 0 \end{array}$$

$$\begin{array}{rl} 20D - 4E - 16 &= 0 \\ -(-4D - 4E - 8) &= 0 \\ \hline 24D \quad - 8 &= 0 \\ D &= \frac{1}{3} \end{array} \qquad \begin{array}{rl} -4\left(\frac{1}{3}\right) - 4E - 8 &= 0 \\ -4E &= \frac{28}{3} \\ E &= -\frac{7}{3} \\ -3\left(\frac{1}{3}\right) + F + 9 &= 0 \\ F &= -8 \end{array}$$

The solution of the system is $D = \frac{1}{3}$, $E = -\frac{7}{3}$, and $F = -8$

$x^2 + y^2 + \frac{1}{3}x - \frac{7}{3}y - 8 = 0$

$\left(x + \frac{1}{6}\right)^2 + \left(y - \frac{7}{6}\right)^2 = \frac{169}{18}$

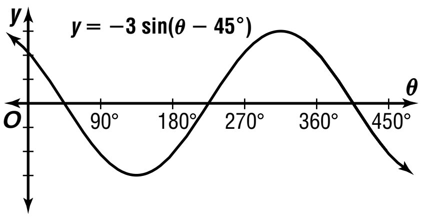
58. $5\left(\cos \frac{5\pi}{6} + \boldsymbol{i} \sin \frac{5\pi}{6}\right) = 5\left(-\frac{\sqrt{3}}{2} + \boldsymbol{i}\left(\frac{1}{2}\right)\right)$

$= -\frac{5\sqrt{3}}{2} + \frac{5}{2}\boldsymbol{i}$

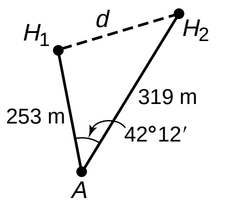
59. $x = x_1 + ta_1 \rightarrow x = 8t - 3$

$y = y_1 + ta_2 \rightarrow y = 3t - 2$

60.



61.



$d^2 = 253^2 + 319^2 - 2(253)(319) \cos 42° \, 12'$

$d = \sqrt{165{,}777 - 161{,}414 \cos 42° \, 12'}$

$d \approx 214.9$ m

62. $\pm\frac{p}{q}$: $\pm\frac{1}{2}$, ± 1, $\pm\frac{3}{2}$, ± 3

$$\begin{array}{r|rrrr} 1 & 2 & 3 & -8 & 3 \\ & & 2 & 5 & -3 \\ \hline -3 & 2 & 5 & -3 & \vert\ 0 \\ & & -6 & 3 & \\ \hline \frac{1}{2} & 2 & -1 & \vert\ 0 & \\ & & 1 & & \\ \hline & 2 & \vert\ 0 & & \end{array}$$

The rational roots are -3, $\frac{1}{2}$, and 1.

63. $90 + x + y + z = 360$

$x + y + z = 270$

The correct choice is D.

Page 960 Mid-Chapter Quiz

1. $\lim\limits_{x \to -3} (2x^2 - 4x + 6) = 2(-3)^2 - 4(-3) + 6$

$= 18 + 12 + 6$

$= 36$

2. $\lim\limits_{x \to 2} \frac{x^2 - 9x + 14}{2x^2 - 7x + 6} = \lim\limits_{x \to 2} \frac{(x - 7)(x - 2)}{(2x - 3)(x - 2)}$

$= \lim\limits_{x \to 2} \frac{x - 7}{2x - 3}$

$= \frac{2 - 7}{2(2) - 3}$ or -5

3.

x	$\frac{\sin x}{x}$
-1	0.9093
-0.1	1.9867
-0.01	1.9999
0.01	1.9999
0.1	1.9867
1	0.9093

$\lim\limits_{x \to 0} \frac{\sin 2x}{x} = 2$

4. $f'(x) = \lim\limits_{h \to 0} \frac{f(x + h) - f(x)}{h}$

$= \lim\limits_{h \to 0} \frac{(x + h)^2 - 3 - (x^2 - 3)}{h}$

$= \lim\limits_{h \to 0} \frac{x^2 + 2xh + h^2 - 3 - x^2 + 3}{h}$

$= \lim\limits_{h \to 0} \frac{2xh + h^2}{h}$

$= \lim\limits_{h \to 0} \frac{h(2x + h)}{h}$

$= 2x + 0$ or $2x$

5. $f(x) = \pi$

$f'(x) = 0$ *π is a constant.*

6. $f(x) = 3x^2 - 5x + 2$

$f'(x) = 3 \cdot 2x^{2-1} - 5 \cdot 1x^{1-1} + 0$

$= 6x - 5$

7. $R(M) = M^2\left(\frac{C}{2} - \frac{M}{3}\right)$

$= \frac{C}{2}M^2 - \frac{1}{3}M^3$

$R'(M) = \frac{C}{2} \cdot 2M^{2-1} - \frac{1}{3} \cdot 3M^{3-2}$

$= CM - M^2$

8. $f(x) = -x^2 + 7x - 6$

$F(x) = -1 \cdot \frac{1}{2 + 1}x^{2+1} + 7 \cdot \frac{1}{1 + 1}x^{1+1} - 6x + C$

$= -\frac{1}{3}x^3 + \frac{7}{2}x^2 - 6x + C$

9. $f(x) = 2x^3 + x^2 + 8$

$F(x) = 2 \cdot \frac{1}{3 + 1}x^{3+1} + \frac{1}{2 + 1}x^{2+1} + 8x + C$

$= \frac{1}{2}x^4 + \frac{1}{3}x^3 + 8x + C$

10. $f(x) = -2x^4 + 6x^3 - 2x - 5$

$F(x) =$

$-2 \cdot \frac{1}{4 + 1}x^{4+1} + 6 \cdot \frac{1}{3 + 1}x^{3+1} - 2 \cdot \frac{1}{1 + 1}x^{1+1} - 5x + C$

$= -\frac{2}{5}x^5 + \frac{3}{2}x^4 - x^2 - 5x + C$

15-3 Area Under a Curve

Page 966 Check for Understanding

1. Sample answer: $y = x^4$

2. Sample answer: Subdivide the interval from a to b into n equal subintervals, draw a rectangle on each subinterval that touches the graph at its upper right corner, add up the areas of the rectangles, and then find the limit of the total area of the rectangles as n approaches infinity.

3. Lorena is correct. If the function is decreasing, then the graph will always be above the tops of the rectangles, so the total area of the rectangles will be less than the area under the graph.

4.
$$\begin{aligned}\int_0^2 x^2dx &= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2i}{n}\right)^2\left(\frac{2}{n}\right)\\ &= \lim_{n\to\infty} \frac{8}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)\\ &= \lim_{n\to\infty} \frac{4}{3}\left(\frac{2n^3+3n^2+n}{n^3}\right)\\ &= \lim_{n\to\infty} \frac{4}{3}\left(2+\frac{3}{n}+\frac{1}{n^2}\right)\\ &= \frac{8}{3} \text{ units}^2\end{aligned}$$

5.
$$\begin{aligned}\int_1^3 x^2dx &= \int_0^3 x^2dx - \int_0^1 x^2dx\\ &= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3i}{n}\right)^2\left(\frac{3}{n}\right) - \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^2\left(\frac{1}{n}\right)\\ &= \lim_{n\to\infty} \frac{27}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)\\ &\quad - \lim_{n\to\infty} \frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)\\ &= \lim_{n\to\infty} \frac{9}{2}\left(\frac{2n^3+3n^2+n}{n^3}\right)\\ &\quad - \lim_{n\to\infty} \frac{1}{6}\left(\frac{2n^3+3n^2+n}{n^3}\right)\\ &= \lim_{n\to\infty} \frac{9}{2}\left(2+\frac{3}{n}+\frac{1}{n^2}\right) - \lim_{n\to\infty} \frac{1}{6}\left(2+\frac{3}{n}+\frac{1}{n^2}\right)\\ &= 9 - \frac{1}{3} \text{ or } \frac{26}{3} \text{ unit}^2\end{aligned}$$

6.
$$\begin{aligned}\int_0^1 x^3dx &= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3\left(\frac{1}{n}\right)\\ &= \lim_{n\to\infty} \frac{1}{n^4}\left(\frac{n^2(n+1)^2}{4}\right)\\ &= \lim_{n\to\infty} \frac{1}{4}\left(\frac{n^2+2n+1}{n^2}\right)\\ &= \lim_{n\to\infty} \frac{1}{4}\left(1+\frac{2}{n}+\frac{1}{n^2}\right)\\ &= \frac{1}{4} \text{ unit}^2\end{aligned}$$

7.
$$\begin{aligned}\int_0^6 x^2dx &= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{6i}{n}\right)^2\left(\frac{6}{n}\right)\\ &= \lim_{n\to\infty} \frac{216}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)\\ &= \lim_{n\to\infty} 36\left(\frac{2n^3+3n^2+n}{n^3}\right)\\ &= \lim_{n\to\infty} 36\left(2+\frac{3}{n}+\frac{1}{n^2}\right)\\ &= 72\end{aligned}$$

8.
$$\begin{aligned}\int_0^3 x^3dx &= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3i}{n}\right)^3\left(\frac{3}{n}\right)\\ &= \lim_{n\to\infty} \frac{81}{n^4}\left(\frac{n^2(n+1)^2}{4}\right)\\ &= \lim_{n\to\infty} \frac{81}{4}\left(\frac{n^2+2n+1}{n^2}\right)\\ &= \lim_{n\to\infty} \frac{81}{4}\left(1+\frac{2}{n}+\frac{1}{n^2}\right)\\ &= \frac{81}{4}\end{aligned}$$

9a.
$$\begin{aligned}\int_0^6 32t\,dt &= \lim_{n\to\infty} \sum_{i=1}^{n} 32\left(\frac{6i}{n}\right)\left(\frac{6}{n}\right)\\ &= \lim_{n\to\infty} \frac{1152}{n^2}\left(\frac{n(n+1)}{2}\right)\\ &= \lim_{n\to\infty} 576\left(\frac{n^2+n}{n^2}\right)\\ &= \lim_{n\to\infty} 576\left(1+\frac{1}{n}\right)\\ &= 576 \text{ ft}\end{aligned}$$

9b.
$$\begin{aligned}\int_0^{10} 32t\,dt &= \lim_{n\to\infty} \sum_{i=1}^{n} 32\left(\frac{10i}{n}\right)\left(\frac{10}{n}\right)\\ &= \lim_{n\to\infty} \frac{3200}{n^2}\left(\frac{n(n+1)}{2}\right)\\ &= \lim_{n\to\infty} 1600\left(\frac{n^2+n}{n^2}\right)\\ &= \lim_{n\to\infty} 1600\left(1+\frac{1}{n}\right)\\ &= 1600 \text{ ft}\end{aligned}$$

Yes; integration shows that the ball would fall 1600 ft in 10 seconds of free-fall. Since this exceeds the height of the building, the ball must hit the ground in less than 10 seconds.

Pages 966-968 Exercises

10.
$$\begin{aligned}\int_0^2 (x+1)dx &= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{2i}{n}\right)+1\right]\left(\frac{2}{n}\right)\\ &= \lim_{n\to\infty} \frac{2}{n}\left[\left(\frac{2(1)}{n}+1\right)+\left(\frac{2(2)}{n}+1\right)+\ldots\right.\\ &\quad \left.+\left(\frac{2\cdot n}{n}+1\right)\right]\\ &= \lim_{n\to\infty} \frac{2}{n}\left[n+\frac{2}{n}(1+2+\ldots+n)\right]\\ &= \lim_{n\to\infty} \frac{2}{n}\left[n+\frac{2}{n}\cdot\frac{n(n+1)}{2}\right]\\ &= \lim_{n\to\infty} \frac{2}{n}(2n+1)\\ &= \lim_{n\to\infty} \frac{4n}{n}+\frac{2}{n}\\ &= 4 \text{ units}^2\end{aligned}$$

11.
$$\begin{aligned}\int_0^3 x^2dx &= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3i}{n}\right)^2\left(\frac{3}{n}\right)\\ &= \lim_{n\to\infty} \frac{27}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)\\ &= \lim_{n\to\infty} \frac{9}{2}\left(\frac{2n^3+3n^2+n}{n^3}\right)\\ &= \lim_{n\to\infty} \frac{9}{2}\left(2+\frac{3}{n}+\frac{1}{n^2}\right)\\ &= 9 \text{ units}^2\end{aligned}$$

12. $\int_{-1}^{2} x^2dx = \int_{-1}^{0} x^2dx + \int_{0}^{2} x^2dx$

$= \int_{0}^{1} x^2dx + \int_{0}^{2} x^2dx$ *By symmetry*

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^2\left(\frac{1}{n}\right) + \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2i}{n}\right)^2\left(\frac{2}{n}\right)$

$= \lim_{n\to\infty} \frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$

$+ \lim_{n\to\infty} \frac{8}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$

$= \lim_{n\to\infty} \frac{1}{6}\left(\frac{2n^3+3n^2+n}{n^3}\right) + \lim_{n\to\infty} \frac{4}{3}\left(\frac{2n^3+3n^2+n}{n^3}\right)$

$= \lim_{n\to\infty} \frac{1}{6}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right) + \lim_{n\to\infty} \frac{4}{3}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right)$

$= \frac{1}{3} + \frac{8}{3}$ or 3 units^2

13. $\int_{1}^{3} xdx = \int_{0}^{3} xdx - \int_{0}^{1} xdx$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3i}{n}\right)\left(\frac{3}{n}\right) - \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)\left(\frac{1}{n}\right)$

$= \lim_{n\to\infty} \frac{9}{n^2}\left(\frac{n(n+1)}{2}\right) - \lim_{n\to\infty} \frac{1}{n^2}\left(\frac{n(n+1)}{2}\right)$

$= \lim_{n\to\infty} \frac{9}{2}\left(\frac{n^2+n}{n^2}\right) - \lim_{n\to\infty} \frac{1}{2}\left(\frac{n^2+n}{n^2}\right)$

$= \lim_{n\to\infty} \frac{9}{2}\left(1 + \frac{1}{n}\right) - \lim_{n\to\infty} \frac{1}{2}\left(1 + \frac{1}{n}\right)$

$= \frac{9}{2} - \frac{1}{2}$ or 4 units^2

14. $\int_{0}^{5} x^2dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{5i}{n}\right)^2\left(\frac{5}{n}\right)$

$= \lim_{n\to\infty} \frac{125}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$

$= \lim_{n\to\infty} \frac{125}{6}\left(\frac{2n^3+3n^2+n}{n^3}\right)$

$= \lim_{n\to\infty} \frac{125}{6}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right)$

$= \frac{125}{3}$ units^2

15. $\int_{1}^{5} 2x^2dx$

$= \int_{0}^{5} 2x^3 - \int_{0}^{1} 2x^3$

$= \lim_{n\to\infty} \sum_{i=1}^{n} 2\left(\frac{5i}{n}\right)^3\left(\frac{5}{n}\right) - \lim_{n\to\infty} \sum_{i=1}^{n} 2\left(\frac{i}{n}\right)^3\left(\frac{1}{n}\right)$

$= \lim_{n\to\infty} \frac{250}{n^4}\left(\frac{n^2(n+1)^2}{4}\right) - \lim_{n\to\infty} \frac{2}{n^4}\left(\frac{n^2(n+1)^2}{4}\right)$

$= \lim_{n\to\infty} \frac{625}{2}\left(\frac{n^2+2n+1}{n^2}\right) - \lim_{n\to\infty} \frac{1}{2}\left(\frac{n^2+2n+1}{n^2}\right)$

$= \lim_{n\to\infty} \frac{625}{2}\left(1 + \frac{2}{n} + \frac{1}{n^2}\right) - \lim_{n\to\infty} \frac{1}{2}\left(1 + \frac{2}{n} + \frac{1}{n^2}\right)$

$= \frac{625}{2} - \frac{1}{2}$ or 312 units^2

16. $\int_{0}^{5} x^4dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{5i}{n}\right)^4\left(\frac{5}{n}\right)$

$= \lim_{n\to\infty} \frac{3125}{n^5}\left(\frac{6n^5+15n^4+10n^3-n}{30}\right)$

$= \lim_{n\to\infty} \frac{625}{6}\left(6 + \frac{15}{n} + \frac{10}{n^2} - \frac{1}{n^4}\right)$

$= 625$ units^2

17. $\int_{0}^{4} (x^2+6x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{4i}{n}\right)^2 + 6\left(\frac{4i}{n}\right)\right]\left(\frac{4}{n}\right)$

$= \lim_{n\to\infty} \frac{4}{n}\left[\left(\left(\frac{4\cdot 1}{n}\right)^2 + 6\left(\frac{4\cdot 1}{n}\right)\right)\right.$

$+ \left(\left(\frac{4\cdot 2}{n}\right)^2 + 6\left(\frac{4\cdot 2}{n}\right)\right) + \ldots$

$\left. + \left(\left(\frac{4\cdot n}{n}\right)^2 + 6\left(\frac{4\cdot n}{n}\right)\right)\right]$

$= \lim_{n\to\infty} \frac{4}{n}\left[\frac{16}{n^2}(1^2 + 2^2 + \ldots + n^2)\right.$

$\left. + \frac{24}{n}(1 + 2 + \ldots + n)\right]$

$= \lim_{n\to\infty} \frac{4}{n}\left[\frac{16}{n^2}\left(\frac{n(n+1)(2n+1)}{6}\right)\right.$

$\left. + \frac{24}{n}\left(\frac{n(n+1)}{2}\right)\right]$

$= \lim_{n\to\infty}\left[\frac{64}{n^3}\left(\frac{2n^3+3n^2+n}{6}\right) + \frac{96}{n^2}\left(\frac{n^2+n}{2}\right)\right]$

$= \lim_{n\to\infty}\left[\frac{32}{3}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right) + 48\left(1 + \frac{1}{n}\right)\right]$

$= \frac{64}{3} + 48$ or $\frac{208}{3}$ units^2

18. $\int_{0}^{3} (x^2 - x + 1)dx$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{3i}{n}\right)^2 - \frac{3i}{n} + 1\right]\left(\frac{3}{n}\right)$

$= \lim_{n\to\infty} \frac{3}{n}\left[\left(\left(\frac{3\cdot 1}{n}\right)^2 - \frac{3\cdot 1}{n} + 1\right)\right.$

$+ \left(\left(\frac{3\cdot 2}{n}\right)^2 - \frac{3\cdot 2}{n} + 1\right) + \ldots$

$\left. + \left(\left(\frac{3\cdot n}{n}\right)^2 - \frac{3\cdot n}{n} + 1\right)\right]$

$= \lim_{n\to\infty} \frac{3}{n}\left[\frac{9}{n^2}(1^2 + 2^2 + \ldots + n^2)\right.$

$\left. - \frac{3}{n}(1 + 2 + \ldots + n) + n\right]$

$= \lim_{n\to\infty} \frac{3}{n}\left[\frac{9}{n^2}\left(\frac{n(n+1)(2n+1)}{6}\right) - \frac{3}{n}\left(\frac{n(n+1)}{2}\right) + n\right]$

$= \lim_{n\to\infty}\left[\frac{27}{n^3}\left(\frac{2n^2+3n^2+n}{6}\right) - \frac{9}{n^2}\left(\frac{n^2+n}{2}\right) + 3\right]$

$= \lim_{n\to\infty}\left[\frac{9}{2}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right) - \frac{9}{2}\left(1 + \frac{1}{n}\right) + 3\right]$

$= 9 - \frac{9}{2} + 3$

$= 12 - \frac{9}{2}$ or $\frac{15}{2}$ units^2

19. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\sin i\frac{\pi}{n}\right) \cdot \frac{\pi}{n}$

20. $\int_{0}^{2} 8xdx = \lim_{n\to\infty} \sum_{i=1}^{n} 8\left(\frac{2i}{n}\right)\left(\frac{2}{n}\right)$

$= \lim_{n\to\infty} \frac{32}{n^2}\left(\frac{n(n+1)}{2}\right)$

$= \lim_{n\to\infty} 16\left(\frac{n^2+n}{n^2}\right)$

$= \lim_{n\to\infty} 16\left(1 + \frac{1}{n}\right)$

$= 16$

21. $\int_1^4 (x+2)dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + \frac{3i}{n} + 2\right)\left(\frac{3}{n}\right)$

$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left(3 + \frac{3i}{n}\right)$

$= \lim_{n\to\infty} \frac{3}{n} \left[\left(3 + \frac{3\cdot 1}{n}\right) + \left(3 + \frac{3\cdot 2}{n}\right) + \ldots + \left(3 + \frac{3\cdot n}{n}\right)\right]$

$= \lim_{n\to\infty} \frac{3}{n} \left[3n + \frac{3}{n}(1 + 2 + \ldots + n)\right]$

$= \lim_{n\to\infty} \frac{3}{n} \left[3n + \frac{3}{n}\left(\frac{n(n+1)}{2}\right)\right]$

$= \lim_{n\to\infty} 9 + \frac{9}{n^2}\left(\frac{n^2+n}{2}\right)$

$= \lim_{n\to\infty} 9 + \frac{9}{2}\left(1 + \frac{1}{n}\right)$

$= 9 + \frac{9}{2}$ or $\frac{27}{2}$

22. $\int_0^4 x^2 dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{4i}{n}\right)^2\left(\frac{4}{n}\right)$

$= \lim_{n\to\infty} \frac{64}{n^3}(1^2 + 2^2 + 3^2 + \ldots + n^2)$

$= \lim_{n\to\infty} \frac{64}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$

$= \lim_{n\to\infty} \frac{32}{3}\left(\frac{2n^3 + 3n^2 + n}{n^3}\right)$

$= \lim_{n\to\infty} \frac{32}{3}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right)$

$= \frac{64}{3}$

23. $\int_3^5 8x^3 dx = \int_0^5 8x^3 dx - \int_0^3 8x^3 dx$

$= \lim_{n\to\infty} \sum_{i=1}^{n} 8\left(\frac{5i}{n}\right)^3\left(\frac{5}{n}\right) - \lim_{n\to\infty} \sum_{i=1}^{n} 8\left(\frac{3i}{n}\right)^3\left(\frac{3}{n}\right)$

$= \lim_{n\to\infty} \frac{5000}{n^4}\left(\frac{n^2(n+1)^2}{4}\right) - \lim_{n\to\infty} \frac{648}{n^4}\left(\frac{n^2(n+1)^2}{4}\right)$

$= \lim_{n\to\infty} 1250\left(\frac{n^2+2n+1}{n^2}\right) - \lim_{n\to\infty} 162\left(\frac{n^2+2n+1}{n^2}\right)$

$= \lim_{n\to\infty} 1250\left(1 + \frac{2}{n} + \frac{1}{n^2}\right) - \lim_{n\to\infty} 162\left(1 + \frac{2}{n} + \frac{1}{n^2}\right)$

$= 1250 - 162$ or 1088

24. $\int_1^4 (x^2 + 4x - 2)dx$

$= \int_0^4 (x^2 + 4x - 2)dx - \int_0^1 (x^2 + 4x - 2)dx$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{4i}{n}\right)^2 + 4\left(\frac{4i}{n}\right) - 2\right]\left(\frac{4}{n}\right) - \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{i}{n}\right)^2 + 4\left(\frac{i}{n}\right) - 2\right]\left(\frac{1}{n}\right)$

$= \lim_{n\to\infty} \frac{4}{n}\left[\left(\left(\frac{4\cdot 1}{n}\right)^2 + 4\left(\frac{4\cdot 1}{n}\right) - 2\right) + \left(\left(\frac{4\cdot 2}{n}\right)^2 + 4\left(\frac{4\cdot 2}{n}\right) - 2\right) + \ldots + \left(\left(\frac{4\cdot n}{n}\right)^2 + 4\left(\frac{4\cdot n}{n}\right) - 2\right)\right] - \lim_{n\to\infty} \frac{1}{n}\left[\left(\left(\frac{1}{n}\right)^2 + 4\left(\frac{1}{n}\right) - 2\right) + \left(\left(\frac{2}{n}\right)^2 + 4\left(\frac{2}{n}\right) - 2\right) + \ldots + \left(\left(\frac{n}{n}\right)^2 + 4\left(\frac{n}{n}\right) - 2\right)\right]$

$= \lim_{n\to\infty} \frac{4}{n}\left[\frac{16}{n^2}(1^2 + 2^2 + \ldots + n^2) + \frac{16}{n}(1 + 2 + \ldots + n) - 2n\right] - \lim_{n\to\infty} \frac{1}{n}\left[\frac{1}{n^2}(1^2 + 2^2 + \ldots + n^2) + \frac{4}{n}(1 + 2 + \ldots + n) - 2n\right]$

$= \lim_{n\to\infty}\left[\frac{64}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) + \frac{64}{n^2}\left(\frac{n(n+1)}{2}\right) - 8\right] - \lim_{n\to\infty}\left[\frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) + \frac{4}{n^2}\left(\frac{n(n+1)}{2}\right) - 2\right]$

$= \lim_{n\to\infty}\left[\frac{32}{3}\left(\frac{2n^3+3n^2+n}{n^3}\right) + 32\left(\frac{n^2+n}{n^2}\right) - 8\right] - \lim_{n\to\infty}\left[\frac{1}{6}\left(\frac{2n^3+3n^2+n}{n^3}\right) + 2\left(\frac{n^2+n}{n^2}\right) - 2\right]$

$= \lim_{n\to\infty}\left[\frac{32}{3}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right) + 32\left(1 + \frac{1}{n}\right) - 8\right] - \lim_{n\to\infty}\left[\frac{1}{6}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right) + 2\left(1 + \frac{1}{n}\right) - 2\right]$

$= \frac{64}{3} + 32 - 8 - \frac{1}{3} - 2 + 2$

$= 24 + \frac{63}{3}$ or 45

25. $\int_0^2 (x^5 + x^2)dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2i}{n}\right)^5\left(\frac{2}{n}\right) + \left(\frac{2i}{n}\right)^2\left(\frac{2}{n}\right)$

$= \lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left(\frac{2i}{n}\right)^5 + \left(\frac{2i}{n}\right)^2$

$= \lim_{n\to\infty} \frac{2}{n}\left[\left(\left(\frac{2\cdot 1}{n}\right)^5 + \left(\frac{2\cdot 1}{n}\right)^2\right) + \left(\left(\frac{2\cdot 2}{n}\right)^5 + \left(\frac{2\cdot 2}{n}\right)^2\right) + \ldots + \left(\left(\frac{2\cdot n}{n}\right)^5 + \left(\frac{2\cdot n}{n}\right)^2\right)\right]$

$= \lim_{n\to\infty} \frac{2}{n}\left[\frac{32}{n^5}(1^5 + 2^5 + \ldots + n^5) + \frac{4}{n^2}(1^2 + 2^2 + \ldots + n^2)\right]$

$= \lim_{n\to\infty}\left[\frac{64}{n^6}\left(\frac{2n^6 + 6n^5 + 5n^4 - n^2}{12}\right) + \frac{8}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)\right]$

$= \lim_{n\to\infty}\left[\frac{16}{3}\left(\frac{2n^6 + 6n^5 + 5n^4 - n^2}{n^6}\right) + \frac{4}{3}\left(\frac{2n^3 + 3n^2 + n}{n^3}\right)\right]$

$= \lim_{n\to\infty}\left[\frac{16}{3}\left(2 + \frac{6}{n} + \frac{5}{n^2} - \frac{1}{n^4}\right) + \frac{4}{3}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right)\right]$

$= \frac{32}{3} + \frac{8}{3}$ or $\frac{40}{3}$

26. $\int_0^5 x^3 dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{5i}{n}\right)^3\left(\frac{5}{n}\right)$

$= \lim_{n\to\infty} \frac{625}{n^4}\left(\frac{n^2(n+1)^2}{4}\right)$

$= \lim_{n\to\infty} \frac{625}{4}\left(\frac{n^2+2n+1}{n^2}\right)$

$= \lim_{n\to\infty} \frac{625}{4}\left(1 + \frac{2}{n} + \frac{1}{n^2}\right)$

$= \frac{625}{4}$

27. $\int_0^3 \frac{1}{2}x^3dx = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{2}\left(\frac{3i}{n}\right)^3\left(\frac{3}{n}\right)$

$= \lim_{n\to\infty} \frac{81}{2n^4}\left(\frac{n^2(n+1)^2}{4}\right)$

$= \lim_{n\to\infty} \frac{81}{8}\left(\frac{n^2+n+1}{n^2}\right)$

$= \lim_{n\to\infty} \frac{81}{8}\left(1+\frac{1}{n}+\frac{1}{n^2}\right)$

$= \frac{81}{8}$ or 10.125 ft^2

28a. $f(20) = 80 - 2(20)$

$= \$40$

28b. $\int_{20}^{40} (80 - 2x)dx$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(80 - 2\left(20 + \frac{20i}{n}\right)\right)\left(\frac{20}{n}\right)$

$= \lim_{n\to\infty} \frac{20}{n} \sum_{i=1}^{n} \left(40 - \frac{40i}{n}\right)$

$= \lim_{n\to\infty} \frac{20}{n}\left[\left(40 - \frac{4\cdot 1}{n}\right) + \left(40 - \frac{4\cdot 2}{n}\right) + \ldots\right.$

$\left. + \left(40 - \frac{4\cdot n}{n}\right)\right]$

$= \lim_{n\to\infty} \frac{20}{n}\left[40n - \frac{40}{n}(1 + 2 + \ldots + n)\right]$

$= \lim_{n\to\infty} \left[800 - \frac{800}{n^2}\left(\frac{n(n+1)}{2}\right)\right]$

$= \lim_{n\to\infty} \left[800 - 400\left(\frac{n^2+n}{n^2}\right)\right]$

$= \lim_{n\to\infty} \left[800 - 400\left(1 + \frac{1}{n}\right)\right]$

$= 800 - 400$ or \$400

29. $\int_0^{10} (6 - 0.06x^2)dx$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(6 - 0.06\left(\frac{10i}{n}\right)^2\right)\left(\frac{10}{n}\right)$

$= \lim_{n\to\infty} \frac{10}{n}\left[\left(6 - 0.06\left(\frac{10\cdot 1}{n}\right)^2\right)\right.$

$+ \left(6 - 0.06\left(\frac{10\cdot 2}{n}\right)^2\right) + \ldots$

$\left. + \left(6 - 0.06\left(\frac{10\cdot n}{n}\right)^2\right)\right]$

$= \lim_{n\to\infty} \frac{10}{n}\left[6n - \frac{6}{n^2}(1^2 + 2^2 + \ldots + n^2)\right]$

$= \lim_{n\to\infty} \left[60 - \frac{60}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)\right]$

$= \lim_{n\to\infty} \left[60 - 10\left(\frac{2n^3+3n^2+n}{n^3}\right)\right]$

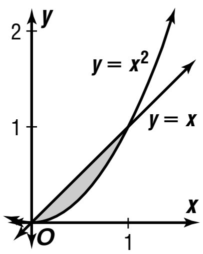
$= \lim_{n\to\infty} \left[60 - 10\left(2 + \frac{3}{n} + \frac{1}{n^2}\right)\right]$

$= 60 - 20$ or 40

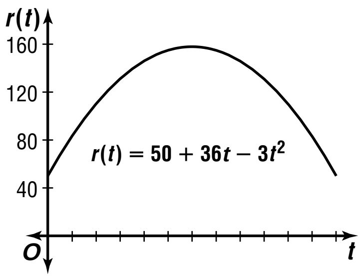
$\int_{-10}^{10} 6 - 0.06x^2dx = 2(40)$ or 80 *By symmetry*

To make a tunnel 100 ft long, multiply 80 by 100.
$80(100) = 8000$ ft^3

30. Setting the two functions equal to each other and solving for x, we find that the curves cross when $x = 0$ and $x = 1$. $x \geq x^2$ for $0 \leq x \leq 1$, so we can find the desired area by subtracting the area between the graph of $y = x^2$ and the x-axis from the area between the graph of $y = x$ and the x-axis. These areas are $\frac{1}{3}$ unit^2 and $\frac{1}{2}$ unit^2, respectively, so the answer is $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ unit^2.



31a.



31b. $\int_0^{12} (50 + 36t - 3t^2)dt$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[50 + 36\left(\frac{12i}{n}\right) - 3\left(\frac{12i}{n}\right)^2\right]\left(\frac{12}{n}\right)$

$= \lim_{n\to\infty} \frac{12}{n}\left[\left(50 + 36\left(\frac{12\cdot 1}{n}\right) - 3\left(\frac{12\cdot 1}{n}\right)^2\right)\right.$

$+ \left(50 + 36\left(\frac{12\cdot 2}{n}\right) - 3\left(\frac{12\cdot 2}{n}\right)^2\right) + \ldots$

$\left. + \left(50 + 36\left(\frac{12\cdot n}{n}\right) - 3\left(\frac{12\cdot n}{n}\right)^2\right)\right]$

$= \lim_{n\to\infty} \frac{12}{n}\left[50n + \frac{432}{n}(1 + 2 + \ldots + n)\right.$

$\left. - \frac{432}{n^2}(1^2 + 2^2 + \ldots + n^2)\right]$

$= \lim_{n\to\infty} \left[600 + \frac{5184}{n^2}\left(\frac{n(n+1)}{2}\right) - \frac{5184}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)\right]$

$= \lim_{n\to\infty} \left[600 + 2592\left(\frac{n^2+n}{n^2}\right) - 864\left(\frac{2n^3+3n^2+n}{n^3}\right)\right]$

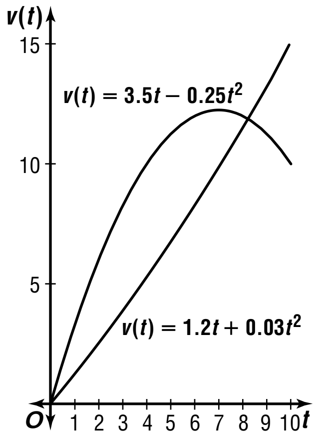
$= \lim_{n\to\infty} \left[600 + 2592\left(1 + \frac{1}{n}\right) - 864\left(2 + \frac{3}{n} + \frac{1}{n^2}\right)\right]$

$= 600 + 2592 - 1728$

$= \$1464$

31c. $\frac{\$1464}{12} = \$122$

32a.



32b. $\int_0^{10} (3.5t - 0.25t^2)dt$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[3.5\left(\frac{10i}{n}\right) - 0.25\left(\frac{10i}{n}\right)^2\right]\left(\frac{10}{n}\right)$

$= \lim_{n\to\infty} \frac{10}{n}\left[\left(3.5\left(\frac{10\cdot 1}{n}\right) - 0.25\left(\frac{10\cdot 1}{n}\right)^2\right)\right.$

$+ \left(3.5\left(\frac{10\cdot 2}{n}\right) - 0.25\left(\frac{10\cdot 2}{n}\right)^2\right) + \ldots$

$\left. + \left(3.5\left(\frac{10\cdot n}{n}\right) - 0.25\left(\frac{10\cdot n}{n}\right)^2\right)\right]$

$= \lim_{n\to\infty} \frac{10}{n}\left[\frac{35}{n}(1 + 2 + \ldots + n) - \frac{25}{n^2}(1^2 + 2^2\right.$

$\left. + \ldots + n^2)\right]$

$= \lim_{n\to\infty} \left[\frac{350}{n^2}\left(\frac{n(n+1)}{2}\right) - \frac{250}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)\right]$

$= \lim_{n\to\infty} \left[175\left(\frac{n^2+n}{n^2}\right) - \frac{125}{3}\left(\frac{2n^3+3n^2+n}{n^3}\right)\right]$

$= \lim_{n\to\infty} \left[175\left(1 + \frac{1}{n}\right) - \frac{125}{3}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right)\right]$

$= 175 - \frac{250}{3}$ or $\frac{275}{3}$ m

$$\int_0^{10} (1.2t + 0.03t^2)dt$$
$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[1.2\left(\frac{10i}{n}\right) + 0.03\left(\frac{10i}{n}\right)^2\right]\left(\frac{10}{n}\right)$$
$$= \lim_{n\to\infty} \left(\frac{10}{n}\right)\left[\left(1.2\left(\frac{10 \cdot 1}{n}\right) + 0.03\left(\frac{10 \cdot 1}{n}\right)^2\right) + \left(1.2\left(\frac{10 \cdot 2}{n}\right) + 0.03\left(\frac{10 \cdot 2}{n}\right)^2\right) + \ldots + \left(1.2\left(\frac{10 \cdot n}{n}\right) + 0.03\left(\frac{10 \cdot n}{n}\right)^2\right)\right]$$
$$= \lim_{n\to\infty} \frac{10}{n}\left[\frac{12}{n}(1 + 2 + \ldots + n) + \frac{3}{n^2}(1^2 + 2^2 + \ldots + n^2)\right]$$
$$= \lim_{n\to\infty} \left[\frac{120}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{30}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)\right]$$
$$= \lim_{n\to\infty} \left[60\left(\frac{n^2+n}{n^2}\right) + 5\left(\frac{2n^3+3n^2+n}{n^3}\right)\right]$$
$$= \lim_{n\to\infty} \left[60\left(1 + \frac{1}{n}\right) + 5\left(2 + \frac{3}{n} + \frac{1}{n^2}\right)\right]$$
$$= 60 + 10 \text{ or } 70 \text{ m}$$

The first one results in a greater distance covered.

33. The equation $y = \sqrt{r^2 - x^2}$ can be rearranged to obtain $x^2 + y^2 = r^2$, which is the circle centered at the origin of radius r. In the equation $y = \sqrt{r^2 - x^2}$, y must be nonnegative, so the graph is only the top half of the circle. Therefore, the value of the integral is $\frac{1}{2}$ the area of a circle of radius r, or $\frac{1}{2}\pi r^2$.

34. $f(x) = -3x^2 + x^2 - 7x$
$$f'(x) = -3 \cdot 3x^{3-1} + 1 \cdot 2x^{2-1} - 7 \cdot 1x^{1-1}$$
$$= -9x^2 + 2x - 7$$

35. $\lim_{x\to 2} \frac{x-2}{x+2} = \frac{2-2}{2+2}$
$$= \frac{0}{4} \text{ or } 0$$

36. $\log_{\frac{1}{3}} x = -3$
$$x = \left(\frac{1}{3}\right)^{-3}$$
$$x = 3^2 \text{ or } 27$$

37. $\vec{\mathbf{u}} = \langle 2, -5, -3\rangle + \langle -3, 4, -7\rangle$
$$= \langle 2 + (-3), -5 + 4, -3 + (-7)\rangle$$
$$= \langle -1, -1, -10\rangle$$
$$= -\vec{\mathbf{i}} - \vec{\mathbf{j}} - 10\vec{\mathbf{k}}$$

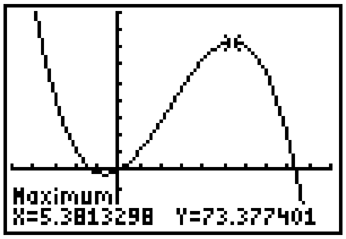
38. $\cos 2\theta = 1 - 2\sin^2\theta$
$$= 1 - 2\left(\frac{3}{5}\right)^2$$
$$= 1 - \frac{18}{25}$$
$$= \frac{7}{25}$$

39. $y = \frac{1}{2}\sin 10\theta$

Amplitude $= \frac{1}{2}$

Period $= \frac{2\pi}{10}$ or $\frac{\pi}{5}$

40. Use a graphing calculator to find the maximum width of $x = 5.4$ cm.



[−5, 10] scl:1 by [−30, 90] scl:1

41. $6^2 + 8^2 \stackrel{?}{=} 10^2$
$$36 + 64 \stackrel{?}{=} 100$$
$$100 = 100$$

$\triangle ABC$ is a right triangle with a base of 6 inches and a height of 8 inches. The area of $\triangle ABC = \frac{1}{2}(6)(8)$ or 24 square inches. If the rectangle has a width of 3 inches, then it has a length of $\frac{24}{3}$ or 8 inches, since $A\ell w$. The perimeter of the rectangle $= 2(3) + 2(8)$ or 22 inches. The correct choice is C.

Page 969 History of Mathematics

1. See students' work. The difference in area should decrease as the number of sides of the polygon increases.
2. The roots of the resulting equation are the zeros of the derivative of the original function.
3. See students' work.

15-4 The Fundamental Theorem of Calculus

Pages 972–973 Check for Understanding

1. $\int f(x)dx$ represents all of the function that have $f(x)$ as their derivative. $\int_a^b f(x)dx$ is a number; it gives the area under the graph of $y = f(x)$ from $x = a$ to $x = b$.
2. Sample answer: Let $a = 0$, $b = 1$, $f(x) = x$, and $g(x) = x$. Then $\int_a^b f(x)g(x)dx = \int_0^1 x^2dx = \frac{1}{3}$, but $\int_a^b f(x)dx \cdot \int_a^b g(x)dx = \int_0^1 xdx \cdot \int_0^1 xdx = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.
3. If the "$+C$" is included in the antiderivative, it will appear as a term in both $F(b)$ and $F(a)$ and will be eliminated when they are subtracted.
4. Rose is correct; the order does matter. Interchanging the order multiplies the result by -1. In symbols, $F(a) - F(b) = -(F(b) - F(a))$. So unless the answer is 0, interchanging the order will give the opposite of the right answer.
5. $\int(2x^2 - 4x + 3)dx = 2 \cdot \frac{1}{3}x^3 - 4 \cdot \frac{1}{2}x^2 + 3x + C$
$$= \frac{2}{3}x^3 - 2x^2 + 3x + C$$
6. $\int(x^3 + 3x + 1)dx = \frac{1}{4}x^4 + 3 \cdot \frac{1}{2}x^2 + x + C$
$$= \frac{1}{4}x^4 + \frac{3}{2}x^2 + x + C$$

7. $\int_{-2}^{0}(4-x^2)dx = 4x - \frac{1}{3}x^3\Big|_{-2}^{0}$
$= \left(4\cdot 0 - \frac{1}{3}\cdot 0^3\right) - \left(4\cdot(-2) - \frac{1}{3}\cdot(-2)^3\right)$
$= 0 - \left(-8 + \frac{8}{3}\right)$
$= \frac{16}{3}$ units^2

8. $\int_{0}^{2} x^4 dx = \frac{1}{5}x^5\Big|_{0}^{2}$
$= \left(\frac{1}{5}\cdot 2^5\right) - \left(\frac{1}{5}\cdot 0^5\right)$
$= \frac{32}{5} - 0$ or $\frac{32}{5}$ units^2

9. $\int_{-1}^{1}(x^2 + 4x + 4)dx$
$= \frac{1}{3}x^3 + 4\cdot\frac{1}{2}x^2 + 4x\Big|_{-1}^{1}$
$= \frac{1}{3}x^3 + 2x^2 + 4x\Big|_{-1}^{1}$
$= \left(\frac{1}{3}\cdot 1^3 + 2\cdot 1^2 + 4\cdot 1\right)$
$- \left(\frac{1}{3}\cdot(-1)^3 + 2\cdot(-1)^2 + 4\cdot(-1)\right)$
$= \frac{19}{3} - \left(-\frac{7}{3}\right)$ or $\frac{26}{3}$ units^2

10. $\int_{1}^{3} 2x^3 dx = 2\cdot\frac{1}{4}x^4\Big|_{1}^{3}$
$= \frac{1}{2}x^4\Big|_{1}^{3}$
$= \left(\frac{1}{2}\cdot 3^4\right) - \left(\frac{1}{2}\cdot 1^4\right)$
$= \frac{81}{2} - \frac{1}{2}$ or 40

11. $\int_{1}^{4}(x^2 - x + 6)dx = \frac{1}{3}x^3 - \frac{1}{2}x^2 + 6x\Big|_{1}^{4}$
$= \left(\frac{1}{3}\cdot 4^3 - \frac{1}{2}\cdot 4^2 + 6\cdot 4\right)$
$- \left(\frac{1}{3}\cdot 1^3 - \frac{1}{2}\cdot 1^2 + 6\cdot 1\right)$
$= \frac{112}{3} - \frac{35}{6}$
$= \frac{224}{6} - \frac{35}{6}$ or $\frac{63}{2}$

12. $\int_{0}^{2}(-2x^2 + 3x + 2)dx = -2\cdot\frac{1}{3}x^3 + 3\cdot\frac{1}{2}x^2 + 2x\Big|_{0}^{2}$
$= -\frac{2}{3}x^3 + \frac{3}{2}x^2 + 2x\Big|_{0}^{2}$
$= \left(-\frac{2}{3}\cdot 2^3 + \frac{3}{2}\cdot 2^2 + 2\cdot 2\right)$
$- \left(-\frac{2}{3}\cdot 0 + \frac{3}{2}\cdot 0 + 2\cdot 0\right)$
$= \frac{14}{3} - 0$ or $\frac{14}{3}$

13. $\int_{2}^{4}(x^3 + x - 6)dx = \frac{1}{4}x^4 + \frac{1}{2}x^2 - 6x\Big|_{2}^{4}$
$= \left(\frac{1}{4}\cdot 4^4 + \frac{1}{2}\cdot 4^2 - 6\cdot 4\right)$
$- \left(\frac{1}{4}\cdot 2^4 + \frac{1}{2}\cdot 2^2 - 6\cdot 2\right)$
$= 48 - (-6)$ or 54

14. $\frac{10\text{ cm}}{1}\cdot\frac{1\text{ m}}{100\text{ cm}} = 0.1$ m
$\int_{0}^{0.1} 500x dx = 500\cdot\frac{1}{2}x^2\Big|_{0}^{0.1}$
$= 250x^2\Big|_{0}^{0.1}$
$= (250\cdot(0.1)^2) - (250\cdot 0^2)$
$= 2.5 - 0$ or 2.5 J

Pages 973–976 Exercises

15. $\int x^5 dx = \frac{1}{6}x^6 + C$

16. $\int 6x^7 dx = 6\cdot\frac{1}{8}x^8 + C$
$= \frac{3}{4}x^8 + C$

17. $\int(x^2 - 2x + 4)dx = \frac{1}{3}x^3 - 2\cdot\frac{1}{2}x^2 + 4x + C$
$= \frac{1}{3}x^3 - x^2 + 4x + C$

18. $\int(-3x^2 - x + 6)dx = -3\cdot\frac{1}{3}x^3 - 1\cdot\frac{1}{2}x^2 + 6x + C$
$= -x^3 - \frac{1}{2}x^2 + 6x + C$

19. $\int(x^4 + 2x^2 - 3)dx = \frac{1}{5}x^5 + 2\cdot\frac{1}{3}x^3 - 3x + C$
$= \frac{1}{5}x^5 + \frac{2}{3}x^3 - 3x + C$

20. $\int(4x^5 - 6x^3 + 7x^2 - 8)dx$
$= 4\cdot\frac{1}{6}x^6 - 6\cdot\frac{1}{4}x^4 + 7\cdot\frac{1}{3}x^3 - 8x + C$
$= \frac{2}{3}x^6 - \frac{3}{2}x^4 + \frac{7}{3}x^3 - 8x + C$

21. $\int(x^2 - 6x + 3)dx = \frac{1}{3}x^3 - 6\cdot\frac{1}{2}x^2 + 3x + C$
$= \frac{1}{3}x^3 - 3x^2 + 3x + C$

22. $\int_{0}^{3} 2x^2 dx = 2\cdot\frac{1}{3}x^3\Big|_{0}^{3}$
$= \frac{2}{3}x^3\Big|_{0}^{3}$
$= \left(\frac{2}{3}\cdot 3^3\right) - \left(\frac{2}{3}\cdot 0^3\right)$
$= 18 - 0$ or 18 units^2

23. $\int_{2}^{3}(x^2 - 2)dx = \frac{1}{3}x^3 - 2x\Big|_{2}^{3}$
$= \left(\frac{1}{3}\cdot 3^3 - 2\cdot 3\right) - \left(\frac{1}{3}\cdot 2^3 - 2\cdot 2\right)$
$= 3 - \left(-\frac{4}{3}\right)$ or $\frac{13}{3}$ units^2

24. $\int_{0}^{2}(4x - x^3)dx = 4\cdot\frac{1}{2}x^2 - 1\cdot\frac{1}{4}x^4\Big|_{0}^{2}$
$= 2x^2 - \frac{1}{4}x^4\Big|_{0}^{2}$
$= \left(2\cdot 2^2 - \frac{1}{4}\cdot 2^4\right) - \left(2\cdot 0^2 - \frac{1}{4}\cdot 0^4\right)$
$= 4 - 0$ or 4 units^2

25. $\int_{0}^{4} x^3 dx = \frac{1}{4}x^4\Big|_{0}^{4}$
$= \left(\frac{1}{4}\cdot 4^4\right) - \left(\frac{1}{4}\cdot 0^4\right)$
$= 64 - 0$, or 64 units^2

26. $\int_{-1}^{1} 3x^6 dx = 3\cdot\frac{1}{7}x^7\Big|_{-1}^{1}$
$= \frac{3}{7}x^7\Big|_{-1}^{1}$
$= \left(\frac{3}{7}\cdot 1^7\right) - \left(\frac{3}{7}\cdot(-1)^7\right)$
$= \frac{3}{7} - \left(-\frac{3}{7}\right)$ or $\frac{6}{7}$ unit^2

27. $\int_{-2}^{0}(x^2 - 2x)dx = \frac{1}{3}x^3 - 2\cdot\frac{1}{2}x^2\Big|_{-2}^{0}$
$= \frac{1}{3}x^3 - x^2\Big|_{-2}^{0}$
$= \left(\frac{1}{3}\cdot 0^3 - 0^2\right) - \left(\frac{1}{3}\cdot(-2)^3 - (-2)^2\right)$
$= 0 - \left(-\frac{20}{3}\right)$ or $\frac{20}{3}$ units^2

28. $\int_1^3 (-x^2 + 2x + 3)dx = -1 \cdot \frac{1}{3}x^3 + 2 \cdot \frac{1}{2}x^2 + 3x \Big|_1^3$

$= -\frac{1}{3}x^3 + x^2 + 3x \Big|_1^3$

$= \left(-\frac{1}{3} \cdot 3^3 + 3^2 + 3 \cdot 3\right) - \left(-\frac{1}{3} \cdot 1^3 + 1^2 + 3 \cdot 1\right)$

$= 9 - \frac{11}{3}$ or $\frac{16}{3}$ units^2

29. $\int_0^1 (x^3 + x)dx = \frac{1}{4}x^4 + \frac{1}{2}x^2 \Big|_0^1$

$= \left(\frac{1}{4} \cdot 1^4 + \frac{1}{2} \cdot 1^2\right) - \left(\frac{1}{4} \cdot 0^4 + \frac{1}{2} \cdot 0^2\right)$

$= \frac{3}{4} - 0$ or $\frac{3}{4}$ unit^2

30. $\int_{-1}^3 (x^3 + 8x + 10)dx$

$= \frac{1}{4}x^4 + 8 \cdot \frac{1}{2}x^2 + 10x \Big|_{-1}^3$

$= \frac{1}{4}x^4 + 4x^2 + 10x \Big|_{-1}^3$

$= \left(\frac{1}{4} \cdot 3^4 + 4 \cdot 3^2 + 10 \cdot 3\right) - \left(\frac{1}{4} \cdot (-1)^4 + 4 \cdot (-1)^2 + 10 \cdot -1\right)$

$= \frac{345}{4} - \left(-\frac{23}{4}\right)$ or 92 units^2

31. $\int_0^7 6x^2dx = 6 \cdot \frac{1}{3}x^3 \Big|_0^7$

$= 2x^3 \Big|_0^7$

$= (2 \cdot 7^3) - (2 \cdot 0^3)$

$= 686 - 0$ or 686

32. $\int_2^4 3x^4dx = 3 \cdot \frac{1}{5}x^5 \Big|_2^4$

$= \frac{3}{5}x^5 \Big|_2^4$

$= \left(\frac{3}{5} \cdot 4^5\right) - \left(\frac{3}{5} \cdot 2^5\right)$

$= \frac{3072}{5} - \frac{96}{5}$ or $\frac{2976}{5}$

33. $\int_{-1}^3 (x + 4)dx = \frac{1}{2}x^2 + 4x \Big|_{-1}^3$

$= \left(\frac{1}{2} \cdot 3^2 + 4 \cdot 3\right) - \left(\frac{1}{2} \cdot (-1)^2 + 4 \cdot -1\right)$

$= \frac{33}{2} - \left(-\frac{7}{2}\right)$ or 20

34. $\int_1^5 (3x^2 - 2x + 1)dx = 3 \cdot \frac{1}{3}x^3 - 2 \cdot \frac{1}{2}x^2 + x \Big|_1^5$

$= x^3 - x^2 + x \Big|_1^5$

$= (5^3 - 5^2 + 5) - (1^3 - 1^2 + 1)$

$= 105 - 1$ or 104

35. $\int_1^3 (x^3 - x^2)dx = \frac{1}{4}x^4 - \frac{1}{3}x^3 \Big|_1^3$

$= \left(\frac{1}{4} \cdot 3^4 - \frac{1}{3} \cdot 3^3\right) - \left(\frac{1}{4} \cdot 1^4 - \frac{1}{3} \cdot 1^3\right)$

$= \frac{45}{4} - \left(-\frac{1}{12}\right)$

$= \frac{135}{12} + \frac{1}{12}$ or $\frac{34}{3}$

36. $\int_0^1 (x^4 + 2x^2 + 1)dx$

$= \frac{1}{5}x^5 + 2 \cdot \frac{1}{3}x^3 + x \Big|_0^1$

$= \frac{1}{5}x^5 + \frac{2}{3}x^3 + x \Big|_0^1$

$= \left(\frac{1}{5} \cdot 1^5 + \frac{2}{3} \cdot 1^3 + 1\right) - \left(\frac{1}{5} \cdot 0^5 + \frac{2}{3} \cdot 0^3 + 0\right)$

$= \frac{28}{15} - 0$ or $\frac{28}{15}$

37. $\int_{-1}^0 (x^4 - x^3)dx$

$= \frac{1}{5}x^5 - \frac{1}{4}x^4 \Big|_{-1}^0$

$= \left(\frac{1}{5} \cdot 0^5 - \frac{1}{4} \cdot 0^4\right) - \left(\frac{1}{5} \cdot (-1)^5 - \frac{1}{4} \cdot (-1)^4\right)$

$= 0 - \left(-\frac{9}{20}\right)$ or $\frac{9}{20}$

38. $\int_0^2 (x^3 + x + 1)dx$

$= \frac{1}{4}x^4 + \frac{1}{2}x^2 + x \Big|_0^2$

$= \left(\frac{1}{4} \cdot 2^4 + \frac{1}{2} \cdot 2^2 + 2\right) - \left(\frac{1}{4} \cdot 0^4 + \frac{1}{2} \cdot 0^2 + 0\right)$

$= 8 - 0$ or 8

39. $\int_{-2}^5 (x^2 - 3x + 8)dx$

$= \frac{1}{3}x^3 - 3 \cdot \frac{1}{2}x^2 + 8x \Big|_{-2}^5$

$= \frac{1}{3}x^3 - \frac{3}{2}x^2 + 8x \Big|_{-2}^5$

$= \left(\frac{1}{3} \cdot 5^3 - \frac{3}{2} \cdot 5^2 + 8 \cdot 5\right) - \left(\frac{1}{3} \cdot (-2)^3 - \frac{3}{2} \cdot (-2)^2 + 8 \cdot -2\right)$

$= \frac{265}{6} - \left(-\frac{74}{3}\right)$

$= \frac{265}{6} + \frac{148}{6}$ or $\frac{413}{6}$

40. $\int_1^3 (x + 3)(x - 1)dx$

$= \int_1^3 (x^2 + 2x - 3)dx$

$= \frac{1}{3}x^3 + 2 \cdot \frac{1}{2}x^2 - 3x \Big|_1^3$

$= \frac{1}{3}x^3 + x^2 - 3x \Big|_1^3$

$= \left(\frac{1}{3} \cdot 3^3 + 3^2 - 3 \cdot 3\right) - \left(\frac{1}{3} \cdot 1^3 + 1^2 - 3 \cdot 1\right)$

$= 9 - \left(-\frac{5}{3}\right)$ or $\frac{32}{3}$

41. $\int_2^3 (x - 1)^3dx = \int_2^3 (x^3 - 3x^2 + 3x - 1)dx$

$= \frac{1}{4}x^4 - 3 \cdot \frac{1}{3}x^3 + 3 \cdot \frac{1}{2}x^2 - x \Big|_2^3$

$= \frac{1}{4}x^4 - x^3 + \frac{3}{2}x^2 - x \Big|_2^3$

$= \left(\frac{1}{4} \cdot 3^4 - 3^3 + \frac{3}{2} \cdot 3^2 - 3\right) - \left(\frac{1}{4} \cdot 2^4 - 2^3 + \frac{3}{2} \cdot 2^2 - 2\right)$

$= \frac{15}{4} - 0$ or $\frac{15}{4}$

42. $\int_0^1 \frac{x^2 - x - 2}{x - 2}dx = \int_0^1 \frac{(x - 2)(x + 1)}{x - 2}dx$

$= \int_0^1 (x + 1)dx$

$= \frac{1}{2}x^2 + x \Big|_0^1$

$= \left(\frac{1}{2} \cdot 1^2 + 1\right) - \left(\frac{1}{2} \cdot 0^2 + 0\right)$

$= \frac{3}{2}$

43. $\int_0^2 x(4x^2 + 1)dx = \int_0^2 (4x^3 + x)dx$

$= 4 \cdot \frac{1}{4}x^4 + \frac{1}{2}x^2 \Big|_0^2$

$= x^4 + \frac{1}{2}x^2 \Big|_0^2$

$= \left(2^4 + \frac{1}{2} \cdot 2^2\right) - \left(0^4 + \frac{1}{2} \cdot 0^2\right)$

$=18 - 0$ or 18

44. $\int_{-1}^{1} (x + 1)(3x + 2)dx$

$= \int_{-1}^{1} (3x^2 + 5x + 2)dx$

$= 3 \cdot \frac{1}{3}x^3 + 5 \cdot \frac{1}{2}x^2 + 2x \Big|_{-1}^{1}$

$= x^3 + \frac{5}{2}x^2 + 2x \Big|_{-1}^{1}$

$= \left(1^3 + \frac{5}{2} \cdot 1^2 + 2 \cdot 1\right)$
$\quad - \left((-1)^3 + \frac{5}{2} \cdot (-1)^2 + 2 \cdot -1\right)$

$= \frac{11}{2} - \left(-\frac{1}{2}\right)$ or 6

45a. $\int_0^{20.5} x^3dx = \frac{1}{4}x^4 \Big|_0^{20.5}$

$= \left(\frac{1}{4} \cdot 20.5^4\right) - \left(\frac{1}{4} \cdot 0^4\right)$

$\approx 44{,}152.52 - 0$ or $44{,}152.52$

$\sum_{i=1}^{20} i^3 = \frac{20^2(20 + 1)^2}{4}$

$= \frac{400(441)}{40}$ or 44,100

45b. $\int_0^{100.5} x^2dx = \frac{1}{3}x^3 \Big|_0^{100.5}$

$= \left(\frac{1}{3} \cdot 100.5^3\right) - \left(\frac{1}{3} \cdot 0^3\right)$

$\approx 338{,}358.38 - 0$ or $338{,}358.38$

$\sum_{i=1}^{100} i^2 = \frac{100(100 + 1)(2 \cdot 100 + 1)}{6}$

$= \frac{100(101)(201)}{6}$ or 338,350

46. $\int_0^2 490{,}000\, xdx = 490{,}000 \cdot \frac{1}{2}x^2 \Big|_0^2$

$= 245{,}000\, x^2 \Big|_0^2$

$= (245{,}000 \cdot 2^2) - (245{,}000 \cdot 0^2)$

$= 980{,}000 - 0$ or $980{,}000$ J

47a. Since the graph is below the x-axis, $f(x)$ is negative. Each $f(x_i)$ is negative and Δx is positive, so each term in the sum $\sum_{i=1}^{n} f(x_i)\Delta x$ is negative. Therefore, the sum is negative. since $\int_a^b f(x)dx$ is a limit of negative sums, it is also negative.

47b. $\int_0^2 (x^2 - 5)dx$

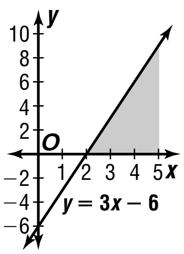
$= \frac{1}{3}x^3 - 5x\Big|_0^2$

$= \left(\frac{1}{3} \cdot 2^3 - 5 \cdot 2\right) - \left(\frac{1}{3} \cdot 0^3 - 5 \cdot 0\right)$

$= -\frac{22}{3}$

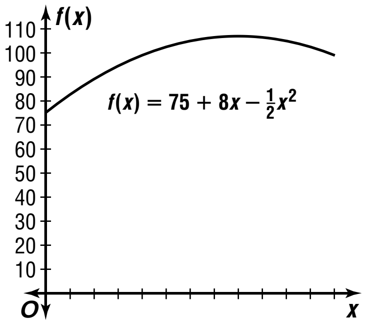
47c. Since the function is negative, the integral in part b gives the opposite of the area of the region. The area is $\frac{22}{3}$.

48.



The integral represents the area of a right triangle. The value is $\frac{1}{2}(3)(9) = \frac{27}{2}$.

49a.



49b. $\frac{1}{6 - 0}\int_0^6 \left(75 + 8x - \frac{1}{2}x^2\right)dx$

$= \frac{1}{6}\left(75x + 8 \cdot \frac{1}{2}x^2 - \frac{1}{2} \cdot \frac{1}{3}x^3\right) \Big|_0^6$

$= \frac{1}{6}\left(75x + 4x^2 - \frac{1}{6}x^3\right) \Big|_0^6$

$= \frac{1}{6}\Big[\left(75 \cdot 6 + 4 \cdot 6^2 - \frac{1}{6} \cdot 6^3\right)$
$\quad - \left(75 \cdot 0 - 4 \cdot 0^2 - \frac{1}{6} \cdot 0^3\right)\Big]$

$= \frac{1}{6}(558 - 0)$

$= \$93$

49c. $\frac{1}{12 - 6}\int_6^{12} \left(75 + 8x - \frac{1}{2}x^2\right)dx$

$= \frac{1}{6}\left(75x + 8 \cdot \frac{1}{2}x^2 - \frac{1}{2} \cdot \frac{1}{3}x^3\right) \Big|_6^{12}$

$= \frac{1}{6}\left(75x + 4x^2 - \frac{1}{6}x^3\right) \Big|_6^{12}$

$= \frac{1}{6}\Big[\left(75 \cdot 12 + 4 \cdot 12^2 - \frac{1}{6} \cdot 12^2\right)$
$\quad - \left(75 \cdot 6 + 4 \cdot 6^2 - \frac{1}{6} \cdot 6^3\right)\Big]$

$= \frac{1}{6}(1188 - 558)$

$= \$105$

50. $\int_{-R}^{R} (\pi R^2 - \pi x^2)dx = \pi R^2x - \pi \cdot \frac{1}{3}x^3 \Big|_{-R}^{R}$

$= \left(\pi R^2 \cdot R - \frac{1}{3}\pi \cdot R^3\right)$
$\quad - \left(\pi R^2 \cdot -R - \frac{1}{3}\pi \cdot (-R)^3\right)$

$= \frac{2}{3}\pi R^3 - \left(-\frac{2}{3}\pi R^3\right)$

$= \frac{4}{3}\pi R^3$

51a. $1000 = k(6.4 \times 10^6)^{-2}$

$1000 = k \cdot \frac{1}{40.96 \times 10^{12}}$

$4.1 \times 10^{16} \approx k$; $4.1 \times 10^{16}\text{Nm}^2$

51b. $\int_{6.4 \times 10^6}^{3.8 \times 10^8} 4.1 \times 10^{16}x^{-2}dx$

$= 4.1 \times 10^{16} \cdot (-1)x^{-1} \Big|_{6.4 \times 10^6}^{3.8 \times 10^8}$

$= -\frac{4.1 \times 10^{16}}{x} \Big|_{6.4 \times 10^6}^{3.8 \times 10^8}$

$= \left(-\frac{4.1 \times 10^{16}}{3.8 \times 10^8}\right) - \left(-\frac{4.1 \times 10^{16}}{6.4 \times 10^6}\right)$

$\approx -1.1 \times 10^8 + 64.1 \times 10^8$

$= 6.3 \times 10^9$ J

52. $\int_0^2 \frac{1}{2}x^2 dx = \lim_{n \to \infty} \sum_{i=1}^{n} \frac{1}{2}\left(\frac{2i}{n}\right)^2\left(\frac{2}{n}\right)$

$= \lim_{n \to \infty} \frac{4 \cdot 2}{2n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$

$= \lim_{n \to \infty} \frac{4}{6}\left(\frac{2n^3 + 3n^2 + n}{n^3}\right)$

$= \lim_{n \to \infty} \frac{2}{3}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right)$

$= \frac{4}{3}$

53. $f(x) = 2x^6 - 3x^2 + 2$

$f'(x) = 2 \cdot 6x^{6-1} - 3 \cdot 2x^{2-1} + 0$

$= 12x^5 - 6x$

54. In a normal distribution, 68.3% of the data lie within 1 standard deviation of the mean. So, 100 − 68.3 or 31.7% of the data lie outside 1 standard deviation of the mean. Thus, 31.7% of the test-takers scored more than 100 points above or below the mean of 500.

55. To begin with 25 out of the 50 numbers are odd. With each consecutive draw, there is 1 fewer odd numbers and 1 fewer tickets.

$P(\text{four odd numbers}) = \frac{25}{50} \cdot \frac{24}{49} \cdot \frac{23}{48} \cdot \frac{22}{47}$

$= \frac{253}{4606}$

56. $A = Pe^{rt}$

$= 600e^{0.06(15)}$

$= \$1475.76$

57. $h = 6$; $k = -1$

$h + p = 3$

$6 + p = 3$ or $p = -3$

$(y - k)^2 = 4p(x - h)$

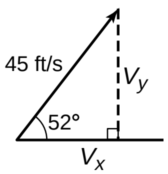
$(y + 1)^2 = -12(x - 6)$

58. $r = \frac{2\sqrt{2}}{\sqrt{2}}$ or 2

$\theta = \frac{2\pi}{3} - \frac{\pi}{3}$ or $\frac{\pi}{3}$

$2\left(\cos\frac{\pi}{3} + \boldsymbol{i}\sin\frac{\pi}{3}\right); 2\left(\frac{1}{2} + \boldsymbol{i}\left(\frac{\sqrt{3}}{2}\right)\right) = 1 + \sqrt{3}\boldsymbol{i}$

59.



$\sin 52° = \frac{v_y}{45}$

$45 \sin 52° = v_y$

$35.46 \approx v_y$; 35.46 ft/s

60. $m\widehat{AE} = m\angle AXE$, so $\angle AXE$ is the smallest angle. If the circle was divided into 5 equal parts, each angle would measure 72°. Since the circle is not divided evenly, the smallest angle AXE is less than 72°. The correct choice is A.

Chapter 15 Study Guide and Assessment

Page 977 Understanding the Vocabulary

1. false; sometimes
2. true
3. false; indefinite
4. true
5. false; secant
6. false; tangent
7. false; derivative
8. true
9. false; rate of change
10. false; derivative

Pages 978–980 Skills and Concepts

11. There is a point at (2, −1) so $f(2) = -1$. However, the closer x is to 2, the closer y is to −3. So, $\lim_{x \to 2} f(x) = -3$.

12. $\lim_{x \to -2} (x^3 - x^2 - 5x + 6) = (-2)^3 - (-2)^2 - 5(-2) + 6$

$= -8 - 4 + 10 + 6$

$= 4$

13. $\lim_{x \to 0} (2x - \cos x) = 2(0) - \cos 0$

$= 0 - 1$

$= -1$

14. $\lim_{x \to 1} \frac{x^2 - 36}{x + 6} = \lim_{x \to 1} \frac{(x + 6)(x - 6)}{x + 1}$

$= \lim_{x \to 1} x - 6$

$= 1 - 6$ or -5

15. $\lim_{x \to 0} \frac{5x^2}{2x} = \lim_{x \to 0} \frac{5}{2}x$

$= \frac{5}{2}(0)$

$= 0$

16. $\lim_{x \to 4} \frac{x^2 + 2x}{x^2 - 3x - 10} = \lim_{x \to 4} 4 \frac{x(x + 2)}{(x - 5)(x + 2)}$

$= \lim_{x \to 4} \frac{x}{x - 5}$

$= \frac{4}{4 - 5}$ or -4

17. $\lim_{x \to 0} (x + \sin x) = 0 + \sin 0$

$= 0$

18. $\lim_{x \to 0} \frac{x^2 + x\cos x}{2x} = \lim_{x \to 0} \frac{x(x + \cos x)}{2x}$

$= \lim_{x \to 0} \frac{x + \cos x}{2}$

$= \frac{0 + \cos 0}{2}$

$= \frac{1}{2}$

19. $\lim_{x \to 2} \frac{x^3 + 2x^2 - 4x - 8}{x^2 - 4} = \lim_{x \to 2} \frac{(x + 2)(x^2 - 4)}{x^2 - 4}$

$= \lim_{x \to 2} x + 2$

$= 2 + 2$ or 4

20. $\lim_{x \to 0} \frac{(x - 3)^2 - 9}{2x} = \lim_{x \to 0} \frac{x^2 - 6x + 9 - 9}{2x}$

$= \lim_{x \to 0} \frac{x(x - 6)}{2x}$

$= \lim_{x \to 0} \frac{x - 6}{2}$

$= \frac{0 - 6}{2}$ or -3

21. $\lim_{x \to 5} \frac{x^2 - 9x + 20}{x^2 - 5x} = \lim_{x \to 5} \frac{(x - 5)(x - 4)}{x(x - 5)}$

$= \lim_{x \to 5} \frac{x - 4}{x}$

$= \frac{5 - 4}{5}$ or $\frac{1}{5}$

22. $f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{2(x + h) + 1 - (2x + 1)}{h}$

$= \lim_{h \to 0} \frac{2x + 2h + 1 - 2x - 1}{h}$

$= \lim_{h \to 0} \frac{2h}{h}$

$= 2$

23. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$
$= \lim_{h\to 0} \frac{4(x+h)^2 + 3(x+h) - 5 - (4x^2+3x-5)}{h}$
$= \lim_{h\to 0} \frac{4x^2 + 8xh + 4h^2 + 3x + 3h - 5 - 4x^2 - 3x + 5}{h}$
$= \lim_{h\to 0} \frac{8xh + 4h^2 + 3h}{h}$
$= \lim_{h\to 0} \frac{h(8x + 4h + 3)}{h}$
$= \lim_{h\to 0} 8x + 4h + 3$
$= 8x + 3$

24. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$
$= \lim_{h\to 0} \frac{(x+h)^3 - 3(x+h) - (x^3 - 3x)}{h}$
$= \lim_{h\to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 3x - 3h - x^3 + 3x}{h}$
$= \lim_{h\to 0} \frac{3x^2h + 3xh^2 + h^3 - 3h}{h}$
$= \lim_{h\to 0} \frac{h(3x^2 + 3xh + h^2 - 3)}{h}$
$= \lim_{h\to 0} 3x^2 + 3xh + h^2 - 3$
$= 3x^2 - 3$

25. $f(x) = 2x^6$
$f'(x) = 2 \cdot 6x^{6-1}$
$= 12x^5$

26. $f(x) = -3x + 7$
$f'(x) = -3 \cdot 1x^{1-1} + 0$
$= -3$

27. $f(x) = 3x^2 - 5x$
$f'(x) = 3 \cdot 2x^{2-1} - 5 \cdot 1x^{1-1}$
$= 6x - 5$

28. $f(x) = \frac{1}{4}x^2 - x + 4$
$f'(x) = \frac{1}{4} \cdot 2x^{2-1} - 1 \cdot 1x^{1-1} + 0$
$= \frac{1}{2}x - 1$

29. $f(x) = \frac{1}{2}x^4 - 2x^3 + \frac{1}{3}x - 4$
$f'(x) = \frac{1}{2} \cdot 4x^{4-1} - 2 \cdot 3x^{3-1} + \frac{1}{3} \cdot 1x^{1-1} - 0$
$= 2x^3 - 6x^2 + \frac{1}{3}$

30. $f(x) = (x+3)(x+4)$
$= x^2 + 7x + 12$
$f'(x) = 2x^{2-1} + 7 \cdot 1x^{1-1} + 0$
$= 2x + 7$

31. $f(x) = 5x^3(x^4 - 3x^2)$
$= 5x^7 - 15x^5$
$f'(x) = 5 \cdot 7x^{7-1} - 15 \cdot 5x^{5-1}$
$= 35x^6 - 75x^4$

32. $f(x) = (x-2)^3$
$= x^3 - 6x^2 + 12x - 8$
$f'(x) = 3x^{3-1} - 6 \cdot 2x^{2-1} + 12 \cdot 1x^{1-1} - 0$
$= 3x^2 - 12x + 12$

33. $f(x) = 8x$
$F(x) = 8 \cdot \frac{1}{1+1}x^{1+1} + C$
$= 4x^2 + C$

34. $f(x) = 3x^2 + 2$
$F(x) = 3 \cdot \frac{1}{2+1}x^{2+1} + 2x + C$
$= x^3 + 2x + C$

35. $f(x) = -\frac{1}{2}x^3 + 2x^2 - 3x - 2$
$F(x) = -\frac{1}{2} \cdot \frac{1}{3+1}x^{3+1} + 2 \cdot \frac{1}{2+1}x^{2+1}$
$- 3 \cdot \frac{1}{1+1}x^{1+1} - 2x + C$
$= -\frac{1}{8}x^4 + \frac{2}{3}x^3 - \frac{3}{2}x^2 - 2x + C$

36. $f(x) = x^4 - 5x^3 + 2x - 6$
$F(x) = \frac{1}{4+1}x^{4+1} - 5 \cdot \frac{1}{3+1}x^{3+1} + 2 \cdot \frac{1}{1+1}x^{1+1}$
$- 6x + C$
$= \frac{1}{5}x^5 - \frac{5}{4}x^4 + x^2 - 6x + C$

37. $f(x) = (x-4)(x+2)$
$= x^2 - 2x - 8$
$F(x) = \frac{1}{2+1}x^{2+1} - 2 \cdot \frac{1}{1+1}x^{1+1} - 8x + C$
$= \frac{1}{3}x^3 - x^2 - 8x + C$

38. $f(x) = \frac{x^2 - x}{x}$
$= x - 1$
$F(x) = \frac{1}{1+1}x^{1+1} - x + C$
$= \frac{1}{2}x^2 - x + C$

39. $\int_0^2 2x\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} 2\left(\frac{2i}{n}\right)\left(\frac{2}{n}\right)$
$= \lim_{n\to\infty} \frac{8}{n^2}\left(\frac{n(n+1)}{2}\right)$
$= \lim_{n\to\infty} 4\left(\frac{n^2+n}{n^2}\right)$
$= \lim_{n\to\infty} 4\left(1 + \frac{1}{n}\right)$
$= 4 \text{ units}^2$

40. $\int_0^1 x^3 dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3\left(\frac{1}{n}\right)$
$= \lim_{n\to\infty} \frac{1}{n^4}\left(\frac{n^2(n+1)^2}{4}\right)$
$= \lim_{n\to\infty} \frac{1}{4}\left(\frac{n^2+2n+1}{n^2}\right)$
$= \lim_{n\to\infty} \frac{1}{4}\left(1 + \frac{2}{n} + \frac{1}{n^2}\right)$
$= \frac{1}{4} \text{ unit}^2$

41. $\int_3^4 x^2 dx = \int_0^4 x^2 dx - \int_0^3 x^2 dx$
$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{4i}{n}\right)^2\left(\frac{4}{n}\right) - \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3i}{n}\right)^2\left(\frac{3}{n}\right)$
$= \lim_{n\to\infty} \frac{64}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$
$- \lim_{n\to\infty} \frac{27}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$
$= \lim_{n\to\infty} \frac{64}{6}\left(\frac{2n^3+3n^2+n}{n^3}\right)$
$- \lim_{n\to\infty} \frac{27}{6}\left(\frac{2n^3+3n^2+n}{n^3}\right)$
$= \lim_{n\to\infty} \frac{64}{6}(2 + \frac{3}{n} + \frac{1}{n^2}) - \lim_{n\to\infty} \frac{27}{6}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right)$
$= \frac{64}{3} - \frac{27}{3}$
$= \frac{37}{3} \text{ units}^2$

42. $\int_1^2 6x^2dx = \int_0^2 6x^2dx - \int_0^1 6.x^2dx$

$= \lim_{n\to\infty}\sum_{i=1}^{n} 6\cdot\left(\frac{2i}{n}\right)^2\left(\frac{2}{n}\right) - \lim_{n\to\infty}\sum_{i=1}^{n} 6\cdot\left(\frac{i}{n}\right)^2\left(\frac{1}{n}\right)$

$= \lim_{n\to\infty}\frac{48}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) - \lim_{n\to\infty}\frac{6}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$

$= \lim_{n\to\infty}\frac{48}{6}\left(\frac{2n^3+3n^2+n}{n^3}\right) - \lim_{n\to\infty}\frac{6}{6}\left(\frac{2n^3+3n^2+n}{n^3}\right)$

$= \lim_{n\to\infty} 8(2+\frac{3}{n}+\frac{1}{n^2}) - \lim_{n\to\infty} 1\left(2+\frac{3}{n}+\frac{1}{n^2}\right)$

$= 16 - 2$ or 14 units^2

43. $\int_2^4 6xdx = 6\cdot\frac{1}{2}x^2\Big|_2^4$

$= 3x^2\Big|_2^4$

$= (3\cdot 4^2) - (3\cdot 2^2)$

$= 48 - 12$ or 36

44. $\int_{-3}^2 3x^2dx = 3\cdot\frac{1}{3}x^3\Big|_{-3}^2$

$= x^3\Big|_{-3}^2$

$= 2^3 - (-3)^2$

$= 8 - (-27)$ or 35

45. $\int_{-2}^2 (3x^2 - x + 3)dx = 3\cdot\frac{1}{3}x^3 - 1\cdot\frac{1}{2}x^2 + 3x\Big|_{-2}^2$

$= x^3 - \frac{1}{2}x^2 + 3x\Big|_{-2}^2$

$= (2^3 - \frac{1}{2}\cdot 2^2 + 3\cdot 2) -$
$\left((-2)^3 - \frac{1}{2}\cdot(-2)^2 + 3\cdot(-2)\right)$

$= 12 - (-16)$ or 28

46. $\int_0^4 (x+2)(2x+3)dx = \int_0^4 (2x^2 + 7x + 6)dx$

$= 2\cdot\frac{1}{3}x^3 + 7\cdot\frac{1}{2}x^2 + 6x\Big|_0^4$

$= \frac{2}{3}x^3 + \frac{7}{2}x^2 + 6x\Big|_0^4$

$= \left(\frac{2}{3}\cdot 4^3 + \frac{7}{2}\cdot 4^2 + 6\cdot 4\right) -$
$\left(\frac{2}{3}\cdot 0^3 + \frac{7}{2}\cdot 0^2 + 6\cdot 0\right)$

$= \frac{368}{3} - 0$ or $\frac{368}{3}$

47. $\int 6x^4dx = 6\cdot\frac{1}{5}x^5 + C$

$= \frac{6}{5}x^5 + C$

48. $\int(-3x^2 + 2x)dx = -3\cdot\frac{1}{3}x^3 + 2\cdot\frac{1}{2}x^2 + C$

$= -x^3 + x^2 + C$

49. $\int(x^2 + 5x - 2)dx = \frac{1}{3}x^3 + 5\cdot\frac{1}{2}x^2 - 2x + C$

$= \frac{1}{3}x^3 + \frac{5}{2}x^2 - 2x + C$

50. $\int(3x^5 + 4x^4 - 7x)dx = 3\cdot\frac{1}{6}x^6 + 4\cdot\frac{1}{5}x^5 - 7\cdot\frac{1}{2}x^2 + C$

$= \frac{1}{2}x^6 + \frac{4}{5}x^5 - \frac{7}{2}x^2 + C$

Page 981 Applications and Problem Solving

51. $\lim_{t\to 100}\left[\frac{1}{2}m\cdot\left(\frac{50}{1+t^2}\right)^2\right] = \frac{1}{2}m\cdot\left(\frac{50}{1+100^2}\right)^2$

≈ 0.0000125 m

52. $c(x) = -9x^5 + 135x^3 + 10{,}000$

$c'(x) = -9\cdot 5x^{5-1} + 135\cdot 3x^{3-1} + 0$

$= -45x^4 + 405x^2$

$c'(2.6) = -45(2.6)^4 + 405(2.6)^2$ or \$681.41

53a. $\frac{60\ \cancel{\text{mi}}}{1\ \cancel{\text{hr}}}\cdot\frac{5280\text{ ft}}{1\ \cancel{\text{mi}}}\cdot\frac{1\ \cancel{\text{hr}}}{3600\text{ s}} = 88$ ft/s

$a = \frac{88\text{ ft/s}}{5\text{ s}} = 17.6$ ft/s^2

53b. $v(t) = \int_0^t 17.6dx$

$= 17.6x\Big|_0^t$

$= 17.6t - 17.6(0)$

$= 17.6t$

53c. $d(t) = \int_0^t 17.6xdx$

$= 17.6\ \cdot\frac{1}{1+1}x^{1+1}\Big|_0^t$

$= 8.8x^2\Big|_0^t$

$= 8.8t^2 - 8.8(0)^2$

$= 8.8t^2$

Page 981 Open-Ended Assessment

1. Sample answer: $f(x) = x^2 + 2x + 2$;

$\lim_{x\to 1}(x^2 + 2x + 2) = 1^2 + 2(1) + 2$

$= 5$

2. Sample answer: $g(x) = 16x^3$;

$\int_0^1 16x^3dx = 4x^4\Big|_0^1$

$= 4\cdot 1^4 - 4\cdot 0^4$

$= 4$

Chapter 15 SAT & ACT Preparation

Page 983 SAT and ACT Practice

1. $\frac{1}{2}\otimes\frac{1}{3} = \frac{1}{\frac{1}{2}-\frac{1}{3}}$

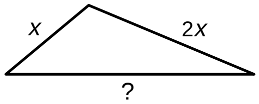
$= \frac{1}{\frac{3}{6}-\frac{2}{6}}$

$= \frac{1}{\frac{1}{6}}$

$= 6$

The correct choice is A.

2. Let x represent the length of the second side of the triangle. Then the first side has length $2x$.

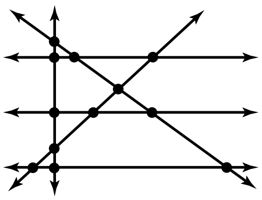


Clearly the perimeter must be greater than $3x$, so eliminate answer choices A and B. Use the Triangle Inequality Theorem.

$x + 2x > ?$	$P < x + 2x + 3x$
$3x > ?$	$P < 6x$
$? + x > 2x$	$P > x + 2x + x$
$? > x$	$P > 4x$

Since the perimeter cannot equal $6x$ or $4x$, eliminate answer choices C and E as well. The only possible answer choice is D.

3. Draw a figure. Begin with the 3 parallel lines. Draw the 3 nonparallel lines in positions that are as general as possible. For example, do not draw perpendicular lines or concurrent lines. Draw the first nonparallel line, and mark the intersections. Then draw the second line, making sure it intersects each of the other lines. Then draw the third line, making sure it intersects each of the others.



There are 12 intersections.
The correct choice is D.

4. The triangles are right triangles. The vertical angles formed by the two triangles each measure 45°, since $180 - 135 = 45$. The hypotenuse of the triangles is the radius of the circle, which is 6, since the diameter is 12. Using the relationships of 45–45–90 right triangles, the length of each height and base must be $\frac{6}{\sqrt{2}}$. The area of one triangle is $\frac{1}{2}\left(\frac{6}{\sqrt{2}}\right)\left(\frac{6}{\sqrt{2}}\right)$ or 9. The area of both triangles is $2 \cdot 9$ or 18. The correct choice is B.

5.
$$x^2 < x + 6$$
$$x^2 - x - 6 < 0$$
$$(x - 3)(x + 2) < 0$$

At $x = 3$ and $x = -2$, the inequality equals 0. Test values that are greater than and less than 3 and -2 to determine for which values the inequality is less than 0.

$x^2 - x - 6 < 0$	
x	y
-3	6
-2	0
0	-6
3	0
4	6

For values of x that are between -2 and 3, the inequality is true.
The correct choice is C.

6. Since 5 is prime, $\boxed{5} = 3(5)$ or 15.

Since 16 is composite, $\boxed{16} = \frac{1}{2}(16)$ or 8.

$\boxed{5} + \boxed{16} = 15 + 8$ or 23.

Now, you need to determine which of the choices is equal to 23. Calculate each one.

$\boxed{21} = 10.5$ $\boxed{23} = 69$ $\boxed{31} = 63$

$\boxed{46} = 23$ $\boxed{69} = 34.5$

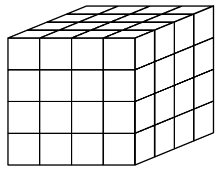
The correct choice is B.

7. The nth term of an arithmetic sequence with first term a_1 and common difference d is given by $a_n = a_1 + (n - 1)d$. $a_1 = 4$, and $d = 3$, so

$$a_{37} = 4 + (37 - 1)(3)$$
$$= 4 + (36)(3) \text{ or } 112$$

The correct choice is C.

8. Draw a figure.



Of the 16 cubes on the front, only the 4 in the center have just one blue face. It is the same on each of the faces. There are 6 faces, so there are $6 \cdot 4$ or 24 cubes with just one blue face. The correct choice is A.

9. To find the value of $\{\{x\}\}$, first find the value of $\{x\}$.
$\{x\} = x^2 - 1$.
So $\{\{x\}\} = \{x^2 - 1\}$.
And $\{x^2 - 1\} = (x^2 - 1)^2 - 1$
$= (x^4 - 2x^2 + 1) - 1$
$= x^4 - 2x^2$
The correct choice is D.

10. There are two distinct prime factors of 20. They are 2 and 5 and their product is 10. There is only one distinct prime factor of 16. It is 2.

$\boxed{20}$

$\boxed{16}$

The answer is 5.

Extra Practice

Lesson 1-1

Page A26

1. $D = \{-2, 1, 2\}$; $R = \{-4, 2, 4\}$; no

2. $D = \{-3, -0.5, 0.5, 3\}$; $R = \{0.5, 3\}$; yes

3. $D = \{-1, 0, 2, 5, 7\}$; $R = \{1, 2, 3, 5, 7\}$; yes

4. $D = \{2, 2.3, 3.2\}$; $R = \{-4, -1, 3, 4\}$; no

5. $f(4) = 4(4) - 2$
$= 16 - 2$ or 14

6. $g(-3) = 2(-3)^2 - (-3) + 5$
$= 2(9) + 3 + 5$
$= 18 + 3 + 5$ or 26

7. $h(1.5) = \frac{3}{2(1.5)}$
$= \frac{3}{3}$ or 1

8. $k(5m) = |3(5m)^2 - 3|$
$= |3(25m^2) - 3|$
$= |75m^2 - 3|$

Lesson 1-2

Page A26

1. $f(x) + g(x) = 2x - 1 + x^2 + 3x - 1$
$= x^2 + 5x - 2$
$f(x) - g(x) = 2x - 1 - x^2 - 3x + 1$
$= -x^2 - x$
$f(x) \cdot g(x) = (2x - 1)(x^2 + 3x - 1)$
$= 2x^3 + 5x^2 - 5x + 1$
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$
$= \frac{2x - 1}{x^2 + 3x - 1}$

2. $[f \circ g](x) = f(g(x))$
$= f(4x^2)$
$= 3 - 4x^2$
$[g \circ f](x) = f(f(x))$
$= g(3 - x)$
$= 4(3 - x)^2$
$= 36 - 24x + 4x^2$

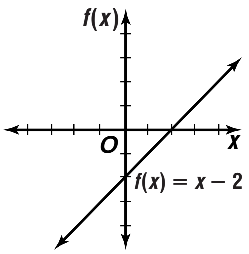
3. $[f \circ g](x) = f(g(x))$
$= f(x + 9)$
$= \frac{1}{3}(x + 9) - 1$
$= \frac{1}{3}x + 2$
$[g \circ f](x) = g(f(x))$
$= g\left(\frac{1}{3}x - 1\right)$
$= \frac{1}{3}x - 1 + 9$
$= \frac{1}{3}x + 8$

4. $[f \circ g](x) = f(g(x))$
$= f(2x^3 - x^2 + x - 1)$
$= -2(2x^3 - x^2 + x - 1)$
$= -4x^3 + 2x^2 - 2x + 2$
$[g \circ f](x) = g(f(x))$
$= g(-2x)$
$= 2(-2x)^3 - (-2x)^2 + (-2x) - 1$
$= -16x^3 - 4x^2 - 2x - 1$

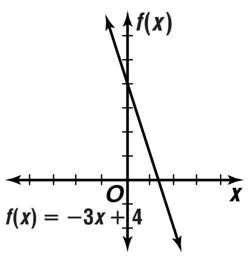
Lesson 1-3

Page A26

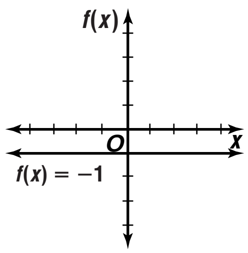
1. $x - 2 = 0$
$x = 2$



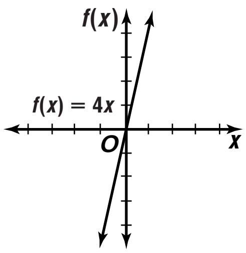
2. $-3x + 4 = 0$
$-3x = -4$
$x = \frac{4}{3}$



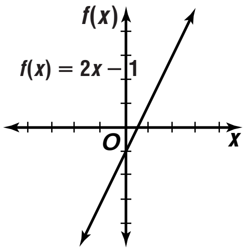
3. $-1 = 0$, false
none



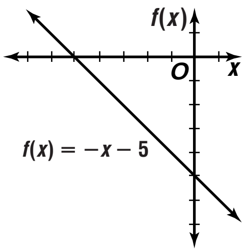
4. $4x = 0$
$x = 0$



5. $2x - 1 = 0$
$2x = 1$
$x = \frac{1}{2}$



6. $x - 5 = 0$
$-x = 5$
$x = -5$



Lesson 1-4

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1. $y = mx + b \rightarrow y = 2x + 1$

2. $y - 2 = -1(x - 1)$
$y - 2 = -x + 1$
$y = -x + 3$

3. $y = mx + b \rightarrow y = -\frac{1}{4}x - 3$

4. $y - (-4) = 0(x - (-2))$
$y + 4 = 0$
$y = -4$

5. $m = \frac{3 - 1}{-2 - 2}$
$= \frac{2}{-4}$ or $-\frac{1}{2}$
$y - 1 = -\frac{1}{2}(x - 2)$
$y - 1 = -\frac{1}{2}x + 1$
$y = -\frac{1}{2}x + 2$

6. $m = \frac{6 - 0}{0 - (-1)}$
$= \frac{6}{1}$ or 6
$y = mx + b \rightarrow y = 6x + 6$

7. $y = 0$

8. $y - 0 = 1.5(x - 10)$
$y = 1.5x - 15$

Lesson 1-5

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1. None of these; the slopes are neither the same nor opposite reciprocals.

2. $y - (-2) = 1(x - 0)$
$y + 2 = x$
$x - y - 2 = 0$

3. $y - 3 = 2(x - 1)$
$y - 3 = 2x - 2$
$2x - y + 1 = 0$

4. $y = -1$ is a vertical line; parallel slope is undefined.
$y = 12$ or $y - 12 = 0$

5. $m = -\left(\frac{-1}{-3}\right)$ or $-\frac{1}{3}$
$y - 6 = -\frac{1}{3}(x - (-2))$
$-3y + 18 = x + 2$
$x + 3y - 16 = 0$

6. $x = 10$ is a horizontal line; perpendicular slope is undefined.
$y = -15$ or $y + 15 = 0$

7. $m = -\left(\frac{-2}{-5}\right)$ or $-\frac{2}{5}$
$y - (-7) = -\frac{2}{5}(x - 3)$
$5y + 35 = -2x + 6$
$2x + 5y + 29 = 0$

Lesson 1-6

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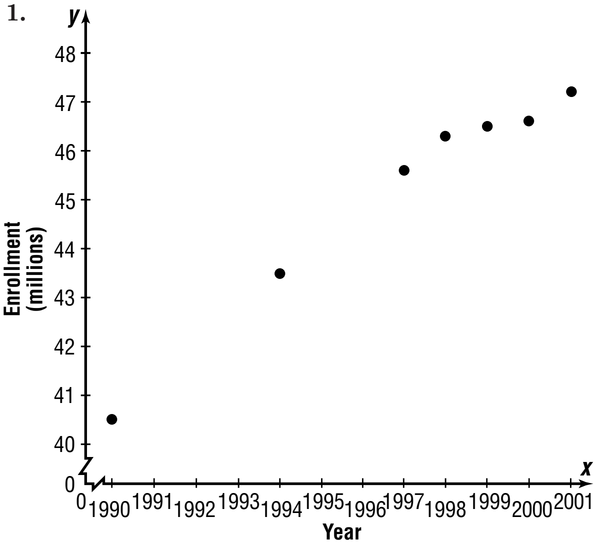
1.



2. Sample answer: $y = 0.6091x - 1171.6$
$m = \frac{47.2 - 40.5}{2001 - 1990}$
≈ 0.6091
$y - 40.5 = 0.6091(x - 1990)$
$y = 0.6091x - 1171.6$

3. Sample answer: $y = 0.6125x - 1178$; $r \approx 0.99$
Enter the School Year data as List 1.
Enter the Enrollment data as List 2.
Perform a linear regression on the graphing calculator.

4. Sample answer: 53.7 million; yes; the correlation coefficient shows a strong correlation.
$f(2011) = 0.6125(2011) - 1178$
≈ 53.7 million

Lesson 1-7

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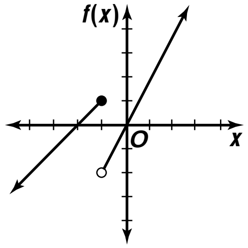
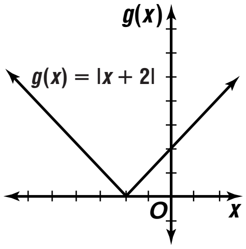
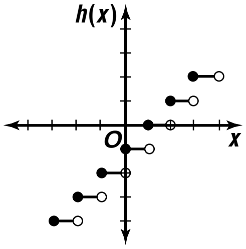
1.



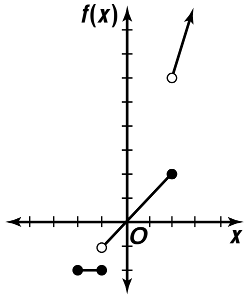
2.



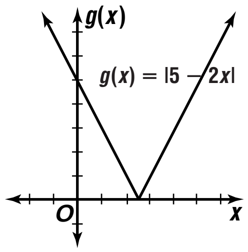
3.



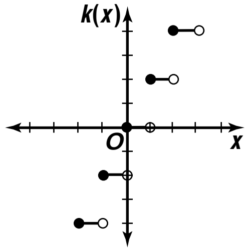
4.



5.



6.



Lesson 1-8

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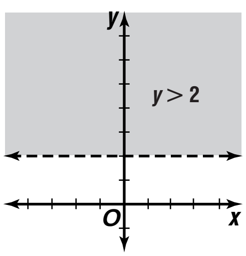
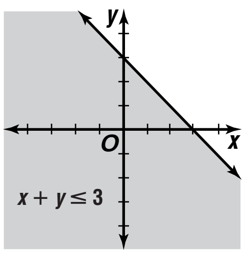
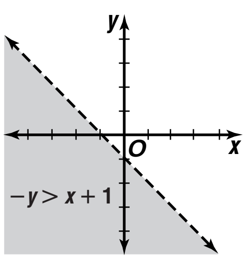
1.



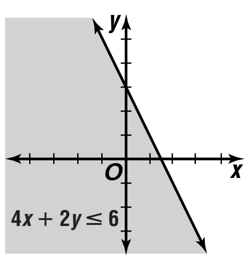
2.



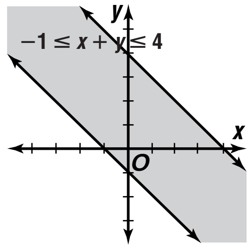
3.



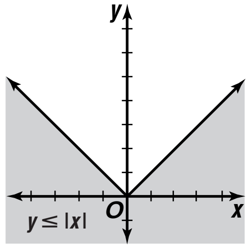
4.



5.



6.



Lesson 2-1

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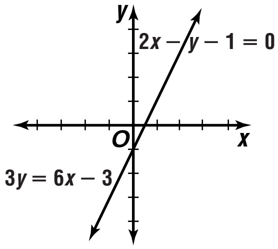
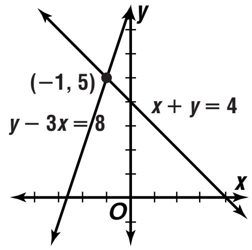
1.



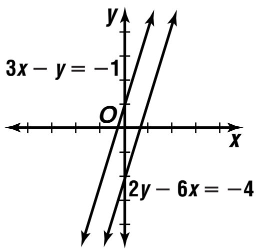
$y = 2x - 1$; consistent and dependent

2.



$(-1, 5)$; consistent and independent

3.



no solution; inconsistent

4.

$$\begin{array}{rl} 5x + 2y &= 1 \\ x + 2y &= 5 \\ \hline 4x &= -4 \\ x &= -1 \end{array} \qquad \begin{array}{rl} x + 2y &= 5 \\ -1 + 2y &= 5 \\ 2y &= 6 \\ y &= 3 \end{array}$$

$(-1, 3)$

5.

$$\begin{array}{rl} 2x + 4y &= 8 \\ 2x + 3y &= 8 \\ \hline y &= 0 \end{array} \qquad \begin{array}{rl} 2x + 3y &= 8 \\ 2x + 3(0) &= 8 \\ 2x &= 8 \\ x &= 4 \end{array}$$

$(4, 0)$

6.
$$\begin{array}{ll} 8x + 2y = 2 & \rightarrow \\ 3x - 4y = -23 & \end{array} \qquad \begin{array}{rcl} 16x + 4y &=& 4 \\ 3x - 4y &=& -23 \\ \hline 19x &=& -19 \\ x &=& -1 \end{array}$$

$$\begin{aligned} 8x + 2y &= 2 \\ 8(-1) + 2y &= 2 \\ 2y &= 10 \\ y &= 5 \end{aligned}$$

$(-1, 5)$

Lesson 2-2

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1.
$$\begin{array}{rcl} x + y &=& 6 \\ x + z &=& -2 \\ \hline y - z &=& 8 \end{array} \qquad \begin{array}{rcl} y - z &=& 8 \\ y + z &=& 2 \\ \hline 2y &=& 10 \\ y &=& 5 \end{array}$$

$$\begin{aligned} x + y &= 6 \\ x + 5 &= 6 \\ x &= 1 \end{aligned} \qquad \begin{aligned} x + z &= -2 \\ 1 + z &= -2 \\ z &= -3 \end{aligned}$$

$(1, 5, -3)$

2.
$$\begin{array}{ll} 2x - 2y - z = 6 & \rightarrow \\ x + y - 2z = -6 & \end{array} \qquad \begin{array}{rcl} 4x - 4y - 2z &=& 12 \\ x + y - 2z &=& -6 \\ \hline 3x - 5y &=& 18 \end{array}$$

$$\begin{array}{rcl} x + 2y - z &=& -7 \\ 2x - 2y - z &=& 6 \\ \hline -x + 4y &=& -13 \end{array}$$

$$\begin{array}{ll} 3x - 5y = 18 & \rightarrow \\ -x + 4y = -13 & \end{array} \qquad \begin{array}{rcl} 3x - 5y &=& 18 \\ -3x + 12y &=& -39 \\ \hline 7y &=& -21 \\ y &=& -3 \end{array}$$

$$\begin{aligned} -x + 4y &= -13 \\ -x + 4(-3) &= -13 \\ x &= 1 \end{aligned} \qquad \begin{aligned} x + y - 2z &= -6 \\ 1 + (-3) - 2z &= -6 \\ -2z &= -4 \\ z &= 2 \end{aligned}$$

$(1, -3, 2)$

3.
$$\begin{array}{rcl} 2x - 3y + z &=& 1 \\ x + y - z &=& -4 \\ \hline 3x - 2y &=& 3 \end{array}$$

$$\begin{array}{ll} x + y - z = -4 & \rightarrow \\ 3x - 2y + 2z = 3 & \end{array} \qquad \begin{array}{rcl} 2x + 2y - 2z &=& -8 \\ 3x - 2y + 2z &=& 3 \\ \hline 5x &=& -5 \\ x &=& 1 \end{array}$$

$$\begin{aligned} 3x - 2y &= -3 \\ 3(-1) - 2y &= -3 \\ -2y &= 0 \\ y &= 0 \end{aligned} \qquad \begin{aligned} x + y - z &= -4 \\ -1 + 0 - z &= -4 \\ -z &= -3 \\ z &= 3 \end{aligned}$$

$(-1, 0, 3)$

Lesson 2-3

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1.
$$\begin{aligned} 2x + y &= -1 \\ y &= -1 - 2x \end{aligned} \qquad \begin{aligned} x + 2y &= 1 \\ x + 2(-1 - 2x) &= 1 \\ -3x - 2 &= 1 \\ -3x &= 3 \\ x &= -1 \end{aligned}$$

$$\begin{aligned} y &= -1 - 2(-1) \\ &= 1 \end{aligned}$$

$(-1, 1)$

2.
$$\begin{array}{rcl} x + 2y &=& 5 \\ 2x - 2y &=& -2 \\ \hline 3x &=& 3 \\ x &=& 1 \end{array} \qquad \begin{aligned} x + 2y &= 5 \\ 1 + 2y &= 5 \\ 2y &= 4 \\ y &= 2 \end{aligned}$$

$(1, 2)$

3.
$$\begin{aligned} 3x &= y - 7 \\ 3x + 7 &= y \end{aligned} \qquad \begin{aligned} 4y &= 5x \\ 4(3x + 7) &= 5x \\ 12x + 28 &= 5x \\ 7x &= -28 \\ x &= -4 \end{aligned}$$

$$\begin{aligned} y &= 3(-4) + 7 \\ &= -5 \end{aligned}$$

$(-4, -5)$

4.
$$A + C = \begin{bmatrix} 4 + 7 & -1 + (-5) \\ 1 + 0 & 5 + 1 \\ 2 + 8 & 6 + 4 \end{bmatrix} = \begin{bmatrix} 11 & -6 \\ 1 & 6 \\ 10 & 10 \end{bmatrix}$$

5.
$$D - E = \begin{bmatrix} -4 - 1 & 1 - (-5) \\ 2 - (-3) & 3 - 2 \end{bmatrix} = \begin{bmatrix} -5 & 6 \\ 5 & 1 \end{bmatrix}$$

6.
$$4B = \begin{bmatrix} 4(2) & 4(0) & 4(-3) \\ 4(4) & 4(-3) & 4(2) \end{bmatrix} = \begin{bmatrix} 8 & 0 & -12 \\ 16 & -12 & 8 \end{bmatrix}$$

7. impossible

8.
$$2C + 3A = \begin{bmatrix} 2(7) & 2(-5) \\ 2(0) & 2(1) \\ 2(8) & 2(4) \end{bmatrix} + \begin{bmatrix} 3(4) & 3(-1) \\ 3(1) & 3(5) \\ 3(2) & 3(6) \end{bmatrix} = \begin{bmatrix} 14 + 12 & -10 + (-3) \\ 0 + 3 & 2 + 15 \\ 16 + 6 & 8 + 18 \end{bmatrix} = \begin{bmatrix} 26 & -13 \\ 3 & 17 \\ 22 & 26 \end{bmatrix}$$

9. impossible

10.
$$ED = \begin{bmatrix} 1 & -5 \\ -3 & 2 \end{bmatrix} \cdot \begin{bmatrix} -4 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1(-4) + (-5)(2) & 1(1) + (-5)(3) \\ -3(-4) + 2(2) & -3(1) + 2(3) \end{bmatrix} = \begin{bmatrix} -14 & -14 \\ 16 & 3 \end{bmatrix}$$

11. $BC = \begin{bmatrix} 2 & 0 & -3 \\ 4 & -3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 7 & -5 \\ 0 & 1 \\ 8 & 4 \end{bmatrix}$

$= \begin{bmatrix} 2(7) + 0(0) + (-3)(8) & 2(-5) + 0(1) + (-3)(4) \\ 4(7) + (-3)(0) + 2(8) & 4(-5) + (-3)(1) + 2(4) \end{bmatrix}$

$= \begin{bmatrix} -10 & -22 \\ 44 & -15 \end{bmatrix}$

$BC - D = \begin{bmatrix} -10 - (-4) & -22 - 1 \\ 44 - 2 & -15 - 3 \end{bmatrix}$

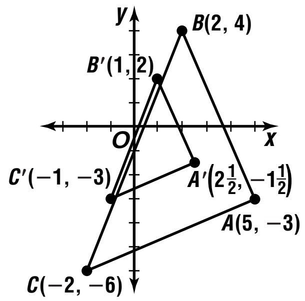
$= \begin{bmatrix} -6 & -23 \\ 42 & -18 \end{bmatrix}$

Lesson 2-4

Page A28

1. $0.5\begin{bmatrix} 5 & 2 & -2 \\ -3 & 4 & -6 \end{bmatrix} = \begin{bmatrix} 2\frac{1}{2} & 1 & -1 \\ -1\frac{1}{2} & 2 & 3 \end{bmatrix}$

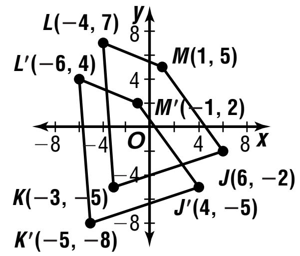
$A'\left(2\frac{1}{2}, -1\frac{1}{2}\right)$, $B'(1, 2)$, $C'(-1, 3)$



2. $\begin{bmatrix} 6 & -3 & -4 & 1 \\ -2 & -5 & 7 & 5 \end{bmatrix} + \begin{bmatrix} -2 & -2 & -2 & -2 \\ -3 & -3 & -3 & -3 \end{bmatrix}$

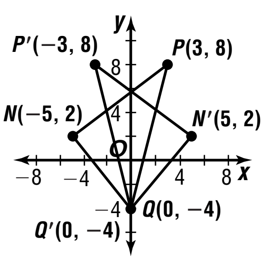
$= \begin{bmatrix} 4 & -5 & -6 & -1 \\ -5 & -8 & 4 & 2 \end{bmatrix}$

$J'(4, -5)$, $K'(-5, -8)$, $L'(-6, 4)$, $M'(-1, 2)$



3. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -5 & 3 & 0 \\ 2 & 8 & -4 \end{bmatrix} = \begin{bmatrix} 5 & -3 & 0 \\ 2 & 8 & -4 \end{bmatrix}$

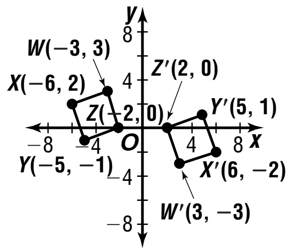
$N'(5, 2)$, $P'(-3, 8)$, $Q'(0, -4)$



4. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -3 & -6 & -5 & -2 \\ 3 & 2 & -1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 3 & 6 & 5 & 2 \\ -3 & -2 & 1 & 0 \end{bmatrix}$

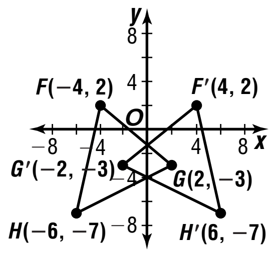
$W'(3, -3)$, $X'(6, -2)$, $Y'(5, 1)$, $Z'(2, 0)$



5. $Rot_{90} \cdot R_{y=x} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -4 & 2 & -6 \\ 2 & -3 & -7 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 6 \\ 2 & -3 & -7 \end{bmatrix}$

$F'(4, 2)$, $G'(-2, -3)$, $H'(6, -7)$



Lesson 2-5

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1. $\begin{vmatrix} 3 & 7 \\ -11 & 2 \end{vmatrix} = 3(2) - (-11)(7)$

$= 83$

2. $\begin{vmatrix} -3 & -5 \\ -7 & -2 \end{vmatrix} = -3(-2) - (-7)(-5)$

$= -29$

3. $\begin{vmatrix} -5 & 0 \\ -\frac{1}{2} & -6 \end{vmatrix} = -5(-6) - \left(-\frac{1}{2}\right)(0)$

$= 30$

4. $\begin{vmatrix} -1 & 0 & 2 \\ -3 & 1 & -2 \\ 5 & -1 & -3 \end{vmatrix}$

$= -1\begin{vmatrix} 1 & -2 \\ -1 & -3 \end{vmatrix} - 0\begin{vmatrix} -3 & -2 \\ 5 & -3 \end{vmatrix} + 2\begin{vmatrix} -3 & 1 \\ 5 & -1 \end{vmatrix}$

$= -1(-5)$
$- 0(19) + 2(-2)$
$= 1$

5. $\begin{vmatrix} -1 & 3 & 2 \\ 4 & -2 & 1 \\ 3 & -3 & -4 \end{vmatrix}$

$= -1\begin{vmatrix} -2 & 1 \\ -3 & -4 \end{vmatrix} - 3\begin{vmatrix} 4 & 1 \\ 3 & -4 \end{vmatrix} + 2\begin{vmatrix} 4 & -2 \\ 3 & -3 \end{vmatrix}$

$= -1(11) - 3(-19) + 2(-6)$
$= 34$

6. $\begin{vmatrix} 4 & 0 & -1 \\ 5 & 3 & 6 \\ -2 & -5 & 2 \end{vmatrix}$

$= 4\begin{vmatrix} 3 & 6 \\ -5 & 2 \end{vmatrix} - 0\begin{vmatrix} 5 & 6 \\ -2 & 2 \end{vmatrix} + (-1)\begin{vmatrix} 5 & 3 \\ -2 & -5 \end{vmatrix}$

$= 4(36) - 0(22) - 1(-19)$

$= 163$

7. $\frac{1}{\begin{vmatrix} 2 & 1 \\ 1 & 5 \end{vmatrix}} \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} \frac{3}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix}$

8. $\frac{1}{\begin{vmatrix} 10 & 0 \\ 5 & 4 \end{vmatrix}} \begin{bmatrix} 4 & 0 \\ -5 & 10 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 4 & 0 \\ -5 & 10 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{10} & 0 \\ -\frac{1}{8} & \frac{1}{4} \end{bmatrix}$

9. $\frac{1}{\begin{vmatrix} 5 & -6 \\ -3 & 4 \end{vmatrix}} \begin{bmatrix} 4 & 6 \\ 3 & 5 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 4 & 6 \\ 3 & 5 \end{bmatrix}$

$= \begin{bmatrix} 2 & 3 \\ \frac{3}{2} & \frac{5}{2} \end{bmatrix}$

10. $\frac{1}{\begin{vmatrix} 3 & -5 \\ 6 & 1 \end{vmatrix}} \begin{bmatrix} 1 & 5 \\ -6 & 3 \end{bmatrix} = \frac{1}{33}\begin{bmatrix} 1 & 5 \\ -6 & 3 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{33} & \frac{5}{33} \\ -\frac{2}{11} & \frac{1}{11} \end{bmatrix}$

11. $\begin{bmatrix} 3 & 2 \\ 1 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 22 \\ -6 \end{bmatrix}$

$\frac{1}{\begin{vmatrix} 3 & 2 \\ 1 & -2 \end{vmatrix}} \begin{bmatrix} -2 & -2 \\ -1 & 3 \end{bmatrix} = -\frac{1}{8}\begin{bmatrix} -2 & -2 \\ -1 & 3 \end{bmatrix}$

$-\frac{1}{8}\begin{bmatrix} -2 & -2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{8}\begin{bmatrix} -2 & -2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 22 \\ -6 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

(4, 5)

12. $\begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -7 \end{bmatrix}$

$\frac{1}{\begin{vmatrix} 4 & -2 \\ 3 & 1 \end{vmatrix}} \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$

$\frac{1}{10}\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} -6 \\ -7 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$

(−2, −1)

13. $\begin{bmatrix} 2 & 1 \\ 4 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 13 \end{bmatrix}$

$\frac{1}{\begin{vmatrix} 2 & 1 \\ 4 & -3 \end{vmatrix}} \begin{bmatrix} -3 & -1 \\ -4 & 2 \end{bmatrix} = -\frac{1}{10}\begin{bmatrix} -3 & -1 \\ -4 & 2 \end{bmatrix}$

$-\frac{1}{10}\begin{bmatrix} -3 & -1 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{10}\begin{bmatrix} -3 & -1 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 13 \end{bmatrix}$

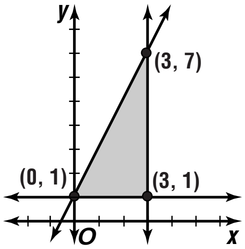
$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{5}{2} \\ -1 \end{bmatrix}$

$\left(\frac{5}{2}, -1\right)$

Lesson 2-6

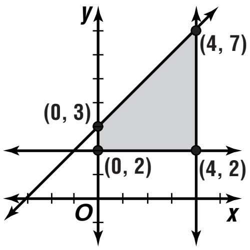
Page A29

1.



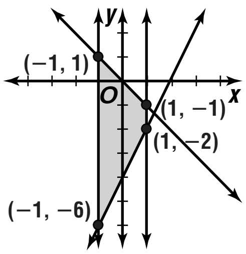
vertices: (3, 1), (0, 1), (3, 7)
$f(x, y) = 4x + 3y$
$f(3, 1) = 4(3) + 3(1)$ or 15
$f(0, 1) = 4(0) + 3(1)$ or $3 \rightarrow$ minimum
$f(3, 7) = 4(3) + 3(7)$ or $33 \rightarrow$ maximum

2.



vertices: (0, 3), (4, 7), (4, 2), (0, 2)
$f(x, y) = 2x - y$
$f(0, 3) = 2(0) - 3$ or $-3 \rightarrow$ minimum
$f(4, 7) = 2(4) - 7$ or 1
$f(4, 2) = 2(4) - 2$ or $6 \rightarrow$ maximum
$f(0, 2) = 2(0) - 2$ or -2

3.

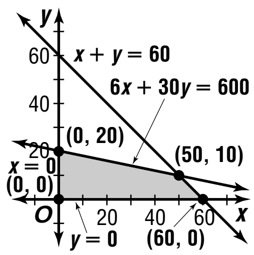


vertices: (−1, 1), (−1, −6), (1, −2), (1, −1)
$f(x, y) = -x - y$
$f(-1, 1) = -(-1) - 1$ or $0 \rightarrow$ minimum
$f(-1, 6) = -(-1) - (-6)$ or $7 \rightarrow$ maximum
$f(1, -2) = -1 - (-2)$ or 1
$f(1, -1) = -1 - (-1)$ or $0 \rightarrow$ minimum

Lesson 2-7

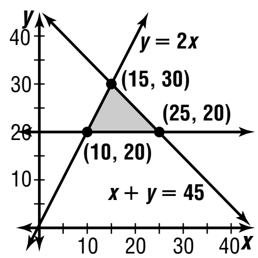
Page A29

1. Let x = the number of cars.
Let y = the number of buses.
$6x + 30y \leq 600$
$x + y \leq 60$
$x \geq 0$
$y \geq 0$



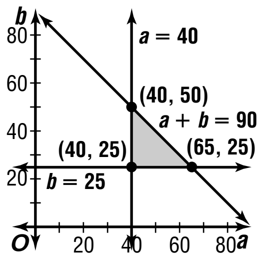
$C(x, y) = 3x + 8y$
$C(0, 0) = 3(0) + 8(0)$ or 0
$C(0, 20) = 3(0) + 8(20)$ or 160
$C(50, 10) = 3(50) + 8(10)$ or 230
$C(60, 0) = 3(60) + 8(0)$ or 180
The maximum income is with 50 cars and 5 buses.

2. Let x = number of gallons of black walnut.
Let y = number of gallons of chocolate mint.
$y \leq 2x$
$y \geq 20$ thousand
$x + y \leq 45$ thousand



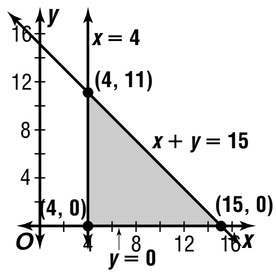
$C(x, y) = 2.95x + 2.95y$
$C(10, 20) = 2.95(10) + 2.95(20)$ or 88.50
$C(15, 30) = 2.95(15) + 2.95(30)$ or 132.75
$C(25, 20) = 2.95(25) + 2.95(20)$ or 132.75
alternate optimal solutions

3. Let a = the number of company A's cards.
Let b = the number of company B's cards.
$a + b \leq 90$
$a \geq 40$
$b \geq 25$



$C(a, b) = 0.30a + 0.32b$
$C(40, 25) = 0.30(40) + 0.32(25)$ or 20
$C(40, 50) = 0.30(40) + 0.32(50)$ or 28
$C(65, 25) = 0.30(65) + 0.32(25)$ or 27.5
The maximum profit is with 40 cards from company A and 50 cards from company B.

4. Let x = the number of video store hours.
Let y = the number of landscaping company hours.
$y \geq 0$
$x \geq 4$
$x + y \leq 15$



$C(x, y) = 5x + 7y$
$C(4, 0) = 5(4) + 7(0)$ or 20
$C(4, 11) = 5(4) + 7(11)$ or 97
$C(15, 0) = 5(15) + 7(0)$ or 75
The maximum earnings is \$97.

Lesson 3-1

Page A30

1. $f(x) = -4x$
$f(-x) = -4(-x)$ $\qquad -f(x) = -(-4x)$
$f(-x) = 4x$ $\qquad -f(x) = 4x$
yes

2. $f(x) = x^2 + 3$
$f(-x) = (-x)^2 + 3$ $\qquad -f(x) = -(x^2 + 3)$
$f(-x) = x^2 + 3$ $\qquad -f(x) = -x^2 - 3$
no

3. $f(x) = \frac{1}{3x^3}$
$f(-x) = \frac{1}{3(-x)^3}$ $\qquad -f(x) = -\left(\frac{1}{3x^3}\right)$
$f(-x) = -\frac{1}{3x^3}$ $\qquad -f(x) = -\frac{1}{3x^3}$
yes

4. $xy = 2 \quad \rightarrow \quad ab = 2$
x-axis $\qquad a(-b) = 2$
$-ab = 2$
$ab = -2$; no
y-axis $\qquad (-a)b = 2$
$-ab = 2$
$ab = -2$; no
$y = x$ $\qquad (b)(a) = 2$
$ab = 2$; yes
$y = -x$ $\qquad (-b)(-a) = 2$
$ab = 2$; yes
$y = x, y = -x$

5. $y + x^2 = 3 \quad \rightarrow \quad b + a^2 = 3$
x-axis $\qquad (-b) + a^2 = 3$
$-b + a^2 = 3$; no
y-axis $\qquad b + (-a)^2 = 3$
$b + a^2 = 3$; yes
$y = x$ $\qquad (a) + (b)^2 = 3$
$a + b^2 = 3$; no
$y = -x$ $\qquad (-a) + (-b)^2 = 3$
$-a + b^2 = 3$; no
y-axis

6. $y^2 = \frac{2x^2}{7} + 1 \quad \rightarrow \quad b^2 = \frac{2a^2}{7} + 1$

x-axis $\quad (-b)^2 = \frac{2a^2}{7} + 1$

$b^2 = \frac{2a^2}{7} + 1$; yes

y-axis $\quad b^2 = \frac{2(-a)^2}{7} + 1$

$b^2 = \frac{2a^2}{7} + 1$; yes

$y = x \quad (a)^2 = \frac{2(b)^2}{7} + 1$

$a^2 = \frac{2b^2}{7} + 1$; no

$y = -x \quad (-a)^2 = \frac{2(-b)^2}{7} + 1$

$a^2 = \frac{2b^2}{7} + 1$; no

x-axis, y-axis

7. $|x| = 4y \quad \rightarrow \quad |a| = 4b$

x-axis $\quad |a| = 4(-b)$

$|a| = -4b$; no

y-axis $\quad |(-a)| = 4b$

$|a| = 4b$; yes

$y = x \quad |(b)| = 4(a)$

$|b| = 4a$; no

$y = -x \quad |(-b)| = 4(-a)$

$|b| = -4a$; no

y-axis

8. $y = 3x \quad \rightarrow \quad b = 3a$

x-axis $\quad (-b) = 3a$

$-b = 3a$; no

y-axis $\quad b = 3(-a)$

$b = -3a$; no

$y = x \quad (a) = 3(b)$

$a = 3b$; no

$y = -x \quad (-a) = 3(-b)$

$-a = -3b$

$a = 3b$; no

none of these

9. $y = \pm\sqrt{x^2 - 1} \quad \rightarrow \quad b = \pm\sqrt{a^2 - 1}$

x-axis $\quad (-b) = \pm\sqrt{a^2 - 1}$

$-b = \pm\sqrt{a^2 - 1}$

$b = \mp\sqrt{a^2 - 1}$; yes

y-axis $\quad b = \pm\sqrt{(-a)^2 - 1}$

$b = \pm\sqrt{a^2 - 1}$; yes

$y = x \quad (a) = \pm\sqrt{(b)^2 - 1}$

$a = \pm\sqrt{(b)^2 - 1}$; no

$y = -x \quad (-a) = \pm\sqrt{(-b)^2 - 1}$

$-a = \pm\sqrt{b^2 - 1}$

$a = \mp\sqrt{b^2 - 1}$; no

x-axis, y-axis

Lesson 3-2

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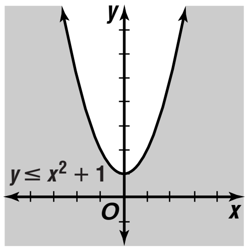
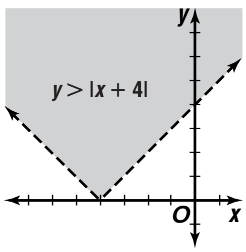
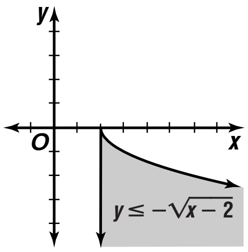
1. $g(x)$ is a translation of $f(x)$ up 2 units.
2. $g(x)$ is the graph of $f(x)$ expanded vertically by a factor of 3.
3. $g(x)$ is a translation of $f(x)$ left 4 units and down 3 units.
4. $g(x)$ is a translation of $f(x)$ right 1 unit and compressed vertically by a factor of 2.
5. $y = -\left[\left[\frac{1}{2}x\right]\right] - 5$

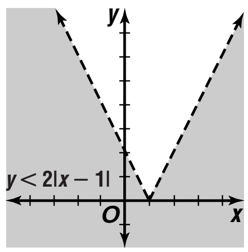
Lesson 3-3

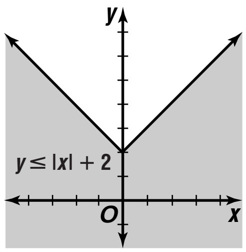
Page A30

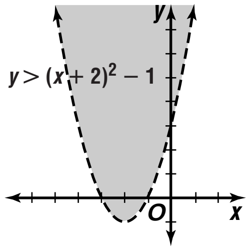
1.


2.


3.


4.


5.


6.


7. Case 1

$|x - 2| \leq 3$

$-(x - 2) \leq 3$

$x - 2 \geq -3$

$x \geq -1$

$\{x \mid -1 \leq x \leq 5\}$

Case 2

$|x - 2| \leq 3$

$x - 2 \leq 3$

$x \leq 5$

8. Case 1

$|4x - 2| \geq 18$

$-(4x - 2) \geq 18$

$4x - 2 \leq -18$

$4x \leq -16$

$x \leq -4$

$\{x \mid x \leq -4 \text{ or } x \geq 5\}$

Case 2

$|4x - 2| \geq 18$

$4x - 2 \geq 18$

$4x \geq 20$

$x \geq 5$

9. Case 1

$|5 - 2x| < 9$

$-(5 - 2x) < 9$

$5 - 2x > -9$

$-2x > -14$

$x < 7$

$\{x \mid -2 < x < 7\}$

Case 2

$|5 - 2x| < 9$

$5 - 2x < 9$

$-2x < 4$

$x > -2$

10. Case 1

$$\begin{aligned} |x + 1| - 3 &> 1 \\ -(x + 1) - 3 &> 1 \\ -(x + 1) &> 4 \\ x + 1 &< -4 \\ x &< -5 \end{aligned}$$

Case 2

$$\begin{aligned} |x + 1| - 3 &> 1 \\ (x + 1) - 3 &> 1 \\ x &> 3 \end{aligned}$$

$\{x \mid x > -5 \text{ or } x > 3\}$

11. Case 1

$$\begin{aligned} |2x + 3| &< 27 \\ -(2x + 3) &< 27 \\ 2x + 3 &> -27 \\ 2x &> -30 \\ x &> -15 \end{aligned}$$

Case 2

$$\begin{aligned} |2x + 3| &< 27 \\ 2x + 3 &< 27 \\ 2x &< 24 \\ x &< 12 \end{aligned}$$

$\{x \mid -15 < x < 12\}$

12. Case 1

$$\begin{aligned} |3x + 4| - 3x &\geq 0 \\ -(3x - 4) - 3x &\geq 0 \\ -6x + 4 &\geq 0 \\ -6x &\geq -4 \\ x &\leq \tfrac{2}{3} \end{aligned}$$

Case 2

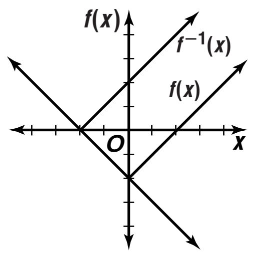
$$\begin{aligned} |3x + 4| - 3x &\geq 0 \\ (3x + 4) - 3x &\geq 0 \\ 4 &\geq 0; \text{ true} \end{aligned}$$

all real numbers

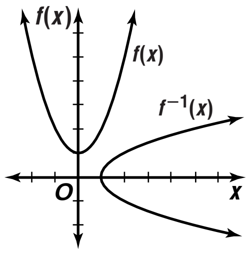
Lesson 3-4

Page A30

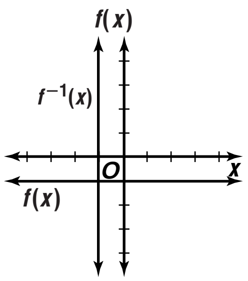
1.



2.



3.



4.

$$\begin{aligned} f(x) &= 4x - 5 \\ y &= 4x - 5 \\ x &= 4y - 5 \\ x + 5 &= 4y \\ y &= \tfrac{x + 5}{4} \\ f^{-1}(x) &= \tfrac{x + 5}{4}; \text{ Yes, it is a function.} \end{aligned}$$

5.

$$\begin{aligned} f(x) &= -2x + 2 \\ y &= -2x + 2 \\ x &= -2y + 2 \\ x - 2 &= -2y \\ y &= \tfrac{x - 2}{-2} \\ f^{-1}(x) &= \tfrac{x - 2}{-2}; \text{ Yes, it is a function.} \end{aligned}$$

6.

$$\begin{aligned} f(x) &= x^2 + 6 \\ y &= x^2 + 6 \\ x &= y^2 + 6 \\ x - 6 &= y^2 \\ y &= \pm\sqrt{x - 6} \\ f^{-1}(x) &= \pm\sqrt{x} - 6; \text{ No, it is not a function.} \end{aligned}$$

7.

$$\begin{aligned} f(x) &= (x - 2)^2 \\ y &= (x - 2)^2 \\ x &= (y - 2)^2 \\ \pm\sqrt{x} &= y - 2 \\ y &= 2 \pm \sqrt{x} \\ f^{-1}(x) &= 2 \pm \sqrt{x}; \text{ No, it is not a function.} \end{aligned}$$

8.

$$\begin{aligned} f(x) &= -\tfrac{x}{2} \\ y &= -\tfrac{x}{2} \\ x &= -\tfrac{y}{2} \\ 2x &= -y \\ y &= -2x \\ f^{-1}(x) &= -2x; \text{ Yes, it is a function.} \end{aligned}$$

9.

$$\begin{aligned} f(x) &= \tfrac{1}{x - 4} \\ y &= \tfrac{1}{x - 4} \\ x &= \tfrac{1}{y - 4} \\ y - 4 &= \tfrac{1}{x} \\ y &= \tfrac{1}{x} + 4 \\ f^{-1}(x) &= \tfrac{1}{x} + 4; \text{ Yes, it is a function.} \end{aligned}$$

10.

$$\begin{aligned} f(x) &= x^2 + 8x - 2 \\ y &= x^2 + 8x - 2 \\ x &= y^2 + 8y - 2 \\ x + 2 &= y^2 + 8y \\ x + 2 + 16 &= (y + 4)^2 \\ \pm\sqrt{x + 18} &= y + 4 \\ -4 \pm \sqrt{x + 18} &= y \\ f^{-1}(x) &= -4 \pm \sqrt{x + 18}; \text{ No, it is not a function.} \end{aligned}$$

11.

$$\begin{aligned} f(x) &= x^3 + 4 \\ y &= x^3 + 4 \\ x &= y^3 + 4 \\ x - 4 &= y^3 \\ \sqrt[3]{x - 4} &= y \\ f^{-1}(x) &= \sqrt[3]{x - 4}; \text{ Yes, it is a function.} \end{aligned}$$

12.

$$\begin{aligned} f(x) &= -\tfrac{3}{(x + 1)^2} \\ y &= -\tfrac{3}{(x + 1)^2} \\ x &= -\tfrac{3}{(y + 1)^2} \\ (y + 1)^2 &= -\tfrac{3}{x} \\ y + 1 &= \pm\sqrt{-\tfrac{3}{x}} \\ y &= -1 \pm \sqrt{-\tfrac{3}{x}} \\ f^{-1}(x) &= -1 \pm \sqrt{-\tfrac{3}{x}}; \text{ No, it is not a function.} \end{aligned}$$

Lesson 3-5

Page A31

1. Yes; the function approaches 1 as x approaches 2 from both sides.
2. No; the function is undefined when $x = -3$.
3. No; the function is undefined when $x = 1$.
4. Yes; the function approaches 1 as x approaches 3 from both sides.
5. jump discontinuity
6. $y \to -\infty$ as $x \to -\infty$, $y \to \infty$ as $x \to \infty$
7. $y \to -\infty$ as $x \to -\infty$, $y \to -\infty$ as $x \to \infty$
8. $y \to 0$ as $x \to -\infty$, $y \to 0$ as $x \to \infty$

Lesson 3-6

Page A31

1. abs. max.: $(-1, 2)$
2. rel. min.: $(-3, 0)$, rel max.: $(-1, 3)$, abs. min.: $(2, -1)$

Lesson 3-7

Page A31

1. $x = 2$

$$y = \frac{3x}{x-2}$$
$$y = \frac{\frac{3x}{x}}{\frac{x}{x} - \frac{2}{x}}$$
$$y = \frac{3}{1 - \frac{2}{x}}$$

as $x \to \infty$, $y \to 3$; $y = 3$

2. $x = -3$

$$y = \frac{2x^2}{x+3}$$
$$y = \frac{\frac{2x^2}{x^2}}{\frac{x}{x^2} + \frac{3}{x^2}}$$
$$y = \frac{2}{\frac{1}{x} + \frac{3}{x^2}}$$

no horizontal asymptotes since as $x \to \infty$, y is undefined

3. $h(x) = \frac{x-5}{x^2+6x+5}$

$= \frac{x-5}{(x+5)(x+1)}$

$x = -5$, $x = -1$

$$y = \frac{x-5}{x^2+6x+5}$$
$$y = \frac{\frac{x}{x^2} - \frac{5}{x^2}}{\frac{x^2}{x^2} + \frac{6x}{x^2} + \frac{5}{x^2}}$$
$$y = \frac{\frac{1}{x} - \frac{5}{x^2}}{1 + \frac{6}{x} + \frac{5}{x^2}}$$

as $x \to \infty$, $y \to 0$; $y = 0$

4. yes $y = x + 5$

$$\begin{array}{r} x + 5 \\ x - 3\overline{)3x^2 + 2x + 1} \\ \underline{x^2 - 3x} \\ 5x + 1 \\ \underline{5x - 15} \\ 16 \end{array}$$

$\to y = x + 5 + \frac{16}{x-3}$

As $x \to \infty$, $\frac{16}{x-3} \to 0$.
So, the graph of $f(x)$ will approach that of $y = x + 5$.

Lesson 3-8

Page A31

1. $y = kx$
$8 = k(2)$
$4 = k$

$y = 4x$
$y = 4(9)$
$y = 36$

2. $g = kw$
$10 = k(-3)$
$-\frac{10}{3} = k$

$g = -\frac{10}{3}w$
$4 = -\frac{10}{3}w$
$-\frac{6}{5} = w$

3. $t = \frac{k}{r}$
$-6 = \frac{k}{14}$
$-84 = k$

$t = \frac{-84}{r}$
$rt = -84$
$r(-7) = -84$
$r = 12$

4. $y = kxz$
$60 = k(5)(4)$
$3 = k$

$y = 3xz$
$y = 3(5)(10)$
$y = 150$

5. $y = \frac{k}{x^2}$
$27 = \frac{k}{(3)^2}$
$243 = k$

$y = \frac{243}{x^2}$
$yx^2 = 243$
$y(5)^2 = 243$
$y = 9.72$

6. $a = kbc^3$
$-36 = k(3)(2)^3$
$-1.5 = k$

$a = -1.5bc^3$
$a = -1.5(5)(3)^3$
$a = -202.5$

Lesson 4-1

Page A32

1. yes; $f(x) = x^3 - 7x^2 + 2x + 40$
$f(-2) = (-2)^3 - 7(-2)^2 + 2(-2) + 40$
$= -8 - 28 - 4 + 40$
$= 0$
2. no; $f(x) = x^3 - 7x^2 + 2x + 40$
$f(1) = (1)^3 - 7(1)^2 + 2(1) + 40$
$= 1 - 7 + 2 + 40$
$= 36$
3. no; $f(x) = x^3 - 7x^2 + 2x + 40$
$f(2) = (2)^3 - 7(2)^2 + 2(2) + 40$
$= 8 - 28 + 4 + 40$
$= 24$
4. yes; $f(x) = x^3 - 7x^2 + 2x + 40$
$f(5) = (5)^3 - 7(5)^2 + 2(5) + 40$
$= 125 - 175 + 10 + 40$
$= 0$

5. $(x-3)(x-4)=0$
$x^2-7x+12=0$; even; 2

6. $(x-(-2))(x-(-1))(x-2)=0$
$(x+2)(x+1)(x-2)=0$
$(x+2)(x^2-x-2)=0$
$x^3+x^2-4x-4=0$; odd; 3

7. $(x-(-1.5))(x-(-1))(x-1)=0$
$(x+1.5)(x+1)(x-1)=0$
$(x+1.5)(x^2-1)=0$
$x^3+1.5x^2-x-1.5=0$; odd; 3

8. $(x-(-2))(x-(-\boldsymbol{i}))(x-\boldsymbol{i})=0$
$(x+2)(x+\boldsymbol{i})(x-\boldsymbol{i})=0$
$(x+2)(x^2+1)=0$
$x^3+2x^2+x+2=0$; odd; 1

9. $(x-(-3\boldsymbol{i}))(x-3\boldsymbol{i})(x-(-\boldsymbol{i}))(x-\boldsymbol{i})=0$
$(x+3\boldsymbol{i})(x-3\boldsymbol{i})(x+\boldsymbol{i})(x-\boldsymbol{i})=0$
$(x^2+9)(x^2+1)=0$
$x^4+10x^2+9=0$; even; 0

10. $(x-(-1))(x-1)(x-2)(x-3)=0$
$(x+1)(x-1)(x-2)(x-3)=0$
$(x^2-1)(x^2-5x+6)=0$
$x^4-5x^3+5x^2+5x-6=0$; even; 4

Lesson 4-2

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1. $x^2-4x-5=0$
$x^2-4x=5$
$x^2-4x+4=5+4$
$(x-2)^2=9$
$x-2=\pm 3$
$x-2=3 \qquad x-2=-3$
$x=5 \qquad x=-1$

2. $x^2+6x+8=0$
$x^2+6x=-8$
$x^2+6x+9=-8+9$
$(x+3)^2=1$
$x+3=\pm 1$
$x+3=1 \qquad x+3=-1$
$x=-2 \qquad x=-4$

3. $m^2+3m-2=0$
$m^2+3m=2$
$m^2+3m+\frac{9}{4}=2+\frac{9}{4}$
$\left(m+\frac{3}{2}\right)^2=\frac{17}{4}$
$m+\frac{3}{2}=\pm\frac{\sqrt{17}}{2}$
$m=-\frac{3}{2}\pm\frac{\sqrt{17}}{2}$

4. $2a^2-8a-6=0$
$a^2-4a-3=0$
$a^2-4a=3$
$a^2-4a+4=3+4$
$(a-2)^2=7$
$a-2=\pm\sqrt{7}$
$a=2\pm\sqrt{7}$

5. $h^2-12h=4$
$h^2-12h+36=4+36$
$(h-6)^2=40$
$h-6=\pm 2\sqrt{10}$
$h=6\pm 2\sqrt{10}$

6. $x^2-9x+1=0$
$x^2-9x=-1$
$x^2-9x+\frac{81}{4}=-1+\frac{81}{4}$
$\left(x-\frac{9}{2}\right)^2=\frac{77}{4}$
$x-\frac{9}{2}=\pm\frac{\sqrt{77}}{2}$
$x=\frac{9}{2}\pm\frac{\sqrt{77}}{2}$

7. $b^2-4ac=(-3)^2-4(4)(-7)$ or 121; 2 real
$x=-\frac{(-3)\pm\sqrt{121}}{2(4)}$
$x=\frac{3\pm 11}{8}$
$x=\frac{7}{4}$ or $x=-1$

8. $b^2-4ac=(2)^2-4(1)(-10)$ or 44; 2 real
$w=\frac{-2\pm\sqrt{44}}{2(1)}$
$w=-1\pm\sqrt{11}$

9. $b^2-4ac=(-5)^2-4(12)(6)$ or -263; 2 imaginary
$t=\frac{-(-5)\pm\sqrt{-263}}{2(12)}$
$t=\frac{5\pm \boldsymbol{i}\sqrt{263}}{24}$

10. $b^2-4ac=(6)^2-4(1)(-13)$ or 88; 2 real
$x=\frac{-6\pm\sqrt{88}}{2(1)}$
$x=-3\pm\sqrt{22}$

11. $b^2-4ac=(-4)^2-4(4)(1)$ or 0; 1 real
$n=\frac{-(-4)\pm\sqrt{0}}{2(4)}$
$n=\frac{1}{2}$

12. $b^2-4ac=(6)^2-4(4)(-15)$ or 276; 2 real
$x=\frac{-6\pm\sqrt{276}}{2(4)}$
$x=\frac{-6\pm 2\sqrt{69}}{8}$
$x=\frac{-3\pm\sqrt{69}}{4}$

Lesson 4-3

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1.
$$\begin{array}{r|rrr} \underline{-2} & 1 & 10 & 8 \\ & & -2 & -16 \\ \hline & 1 & 8 & -8 \end{array}$$
$x+8$, R-8

2.
$$\begin{array}{r|rrrr} \underline{1} & 1 & -3 & 4 & -1 \\ & & 1 & -2 & 2 \\ \hline & 1 & -2 & 2 & 1 \end{array}$$
x^2-2x+2, R1

3.
$$\begin{array}{r|rrrr} \underline{-1} & 1 & 0 & -3 & -5 \\ & & -1 & 1 & 2 \\ \hline & 1 & -1 & -2 & -3 \end{array}$$
x^2-x-2, R-3

4. $\begin{array}{r|rrrrr} 4 & 1 & -2 & -7 & -3 & -4 \\ & & 4 & 8 & 4 & 4 \\ \hline & 1 & 2 & 1 & 1 & 0 \end{array}$

$x^3 + 2x^2 + x + 1$

5. $f(x) = x^2 + 2x - 8$
$f(-4) = (-4)^2 + 2(-4) - 8$
$= 16 - 8 - 8$
$= 0$; yes

6. $f(x) = x^3 + 12$
$f(1) = (1)^3 + 12$
$= 1 + 12$ or 13; no

7. $f(x) = 4x^3 + 2x^2 + 6x + 1$
$f(-1) = 4(-1)^3 + 2(-1)^2 + 6(-1) + 1$
$= -4 + 2 - 6 + 1$
$= -7$; no

8. $f(x) = x^4 - 4x^2 + 16$
$f(4) = (4)^4 - 4(4)^2 + 16$
$= 256 - 64 + 16$
$=208$; no

Lesson 4-4

Page A32

1. p: $\pm1, \pm2, \pm3, \pm6$
q: ±1
$\frac{p}{q}$: $\pm1, \pm2, \pm3, \pm6$

r	1	2	-5	-6
1	1	3	-2	-8
-1	1	1	-6	0
2	1	4	3	0
-2	1	0	-5	-16
3	1	5	10	24
-3	1	-1	-2	0

rational roots: $-3, -1, 2$

2. p: ±1
q: $\pm1, \pm2$
$\frac{p}{q}$: $\pm1, \pm\frac{1}{2}$

r	2	-1	2	-3	1
1	2	1	3	0	1
-1	2	-3	5	-8	9
$\frac{1}{2}$	2	0	2	-2	0
$-\frac{1}{2}$	2	-2	3	$-\frac{9}{2}$	$\frac{13}{4}$

rational root: $\frac{1}{2}$

3. p: $\pm1, \pm2$
q: ±1
$\frac{p}{q}$: $\pm1, \pm2$

r	1	1	0	-2
1	1	2	2	0
-1	1	0	0	-2
2	1	3	6	10
-2	1	-1	2	-6

rational root: 1

4. p: $\pm1, \pm2, \pm4, \pm8$
q: $\pm1, \pm2, \pm3, \pm6$
$\frac{p}{q}$: $\pm1, \pm2, \pm4, \pm8, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}, \pm\frac{1}{6}$

r	6	1	22	4	-8
1	6	7	29	33	25
-1	6	-5	27	-23	15
⋮	⋮	⋮	⋮	⋮	⋮
$-\frac{2}{3}$	6	-3	24	-12	0
$\frac{1}{2}$	6	4	24	16	0

rational roots: $-\frac{2}{3}, \frac{1}{2}$

5. 2 or 0 positive
$f(-x) = -x^3 - 4x^2 + x + 4$
1 negative

r	1	-4	-1	4
-1	1	-5	4	0

$x^2 - 5x + 4 = 0$
$(x - 4)(x - 1) = 0$
$x = 4, x = 1$
rational zeros: $-1, 1, 4$

6. 2 or 0 positive
$f(-x) = x^4 - x^3 + 3x^2 + 5x + 10$
2 or 0 negative

r	1	1	3	-5	10
1	1	2	5	0	10
⋮	⋮	⋮	⋮	⋮	⋮
-10	1	-9	93	-935	9360

rational zeros: none

7. 2 or 0 positive
$f(-x) = -4x^3 + 7x + 3$
1 negative

r	4	0	-7	3
$-\frac{3}{2}$	4	-6	2	0

$4x^2 - 6x + 2 = 0$
$(4x - 2)(x - 1) = 0$
$x = \frac{1}{2}, x = 1$
rational zeros: $-\frac{3}{2}, \frac{1}{2}, 1$

8. 3 or 1 positive
$f(-x) = x^4 + x^3 - 4x - 4$
1 negative

r	1	-1	0	-4	-4
1	1	0	0	4	0
⋮	⋮	⋮	⋮	⋮	⋮
-4	1	-5	20	-76	300

rational zero: 1

Lesson 4-5

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1.

r	2	-4	-5
-2	2	-8	11
-1	2	-6	1
0	2	-4	-5
1	2	-2	-7
2	2	0	-5
3	2	2	1

-1 and 0, 2 and 3

2.

r	1	0	0	-5
-2	1	-2	4	-13
-1	1	-1	1	-6
0	1	0	0	-5
1	1	1	1	-4
2	1	2	4	3

1 and 2

3.

r	1	0	1	4	2
-2	1	-2	3	-2	6
-1	1	-1	0	4	-2
0	1	0	-1	4	2
1	1	1	0	4	6
2	1	2	3	10	22

-2 and -1, -1 and 0

4-6. Use the TABLE feature of a graphing calculator.

4. 0.3, 1.3

5. -2.2, 0.3, 1.2

6. -1.3, 1.3

Lesson 4-6

Page A33

1.
$$\frac{6}{x} + x = 5$$
$$6 + x^2 = 5x$$
$$x^2 - 5x + 6 = 0$$
$$(x - 2)(x - 3) = 0$$
$$x - 2 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = 2 \qquad\qquad x = 3$$

2.
$$\frac{7}{y-1} - \frac{4}{y} = \frac{y}{y-1}$$
$$7y - 4(y - 1) = y^2$$
$$3y + 4 = y^2$$
$$y^2 - 3y - 4 = 0$$
$$(y - 4)(y + 1) = 0$$
$$y - 4 = 0 \quad \text{or} \quad y + 1 = 0$$
$$y = 4 \qquad\qquad y = -1$$

3.
$$\frac{5}{r+1} - \frac{4}{r-1} = \frac{1}{r^2-1}$$
$$5(r - 1) - 4(r + 1) = 1$$
$$5r - 5 - 4r - 4 = 1$$
$$r = 10$$

4.
$$2 = \frac{1}{2-t} + \frac{4}{t-2}$$
$$2 = \frac{4}{t-2} - \frac{1}{t-2}$$
$$2(t - 2) = 4 - 1$$
$$2t - 4 = 3$$
$$2t = 7$$
$$t = 3.5$$

5.
$$\frac{1}{3w} + \frac{4}{5w} = \frac{1}{15}$$
$$5 + 12 = w$$
$$17 = w;\ w \neq 0$$

Test $w = -1$: $\frac{1}{3(-1)} + \frac{4}{5(-1)} \overset{?}{\leq} \frac{1}{15}$
$-\frac{1}{3} - \frac{4}{5} \overset{?}{\leq} \frac{1}{15}$
$-\frac{17}{15} \overset{?}{\leq} \frac{1}{15}$; true

Test $w = 1$: $\frac{1}{3(1)} + \frac{1}{5(1)} \overset{?}{\leq} \frac{1}{15}$
$\frac{1}{3} + \frac{1}{5} \overset{?}{\leq} \frac{1}{15}$
$\frac{8}{15} \overset{?}{\leq} \frac{1}{15}$; false

Test $w = 18$: $\frac{1}{3(18)} + \frac{1}{5(18)} \overset{?}{\leq} \frac{1}{15}$
$\frac{1}{54} + \frac{1}{90} \overset{?}{\leq} \frac{1}{15}$
$\frac{4}{135} \overset{?}{\leq} \frac{1}{15}$; true

$w < 0$ or $w \geq 17$

6. $\frac{x-2}{x} = \frac{x-4}{x-6}$

$(x-6)(x-2) = x(x-4)$

$x^2 - 8x + 12 = x^2 - 4x$

$12 = 4x$

$3 = x;\ x \neq 0$ or 6

Test $x = -1$: $\frac{(-1)-2}{-1} \overset{?}{<} \frac{(-1)-4}{(-1)-6}$

$3 \overset{?}{<} \frac{5}{7}$; false

Test $x = 1$: $\frac{1-2}{1} \overset{?}{<} \frac{1-4}{1-6}$

$-1 \overset{?}{<} -\frac{3}{5}$; true

Test $x = 4$: $\frac{4-2}{4} \overset{?}{<} \frac{4-4}{4-6}$

$\frac{1}{2} \overset{?}{<} 0$; false

Test $x = 7$: $\frac{7-2}{7} \overset{?}{<} \frac{7-4}{7-6}$

$\frac{5}{7} \overset{?}{<} 3$; true

$0 < x < 3$ or $x > 6$

Lesson 4-7

Page A33

1. $\sqrt{2+3t} = 4$

$2 + 3t = 16$

$3t = 14$

$t = \frac{14}{3}$

Check: $\sqrt{2+3\left(\frac{14}{3}\right)} \overset{?}{=} 4$

$\sqrt{16} \overset{?}{=} 4$

$4 = 4$ ✓

2. $4 - \sqrt{x-2} = 1$

$-\sqrt{x-2} = -3$

$x - 2 = 9$

$x = 11$

Check: $4 - \sqrt{11-2} \overset{?}{=} 1$

$4 - 3 \overset{?}{=} 1$

$1 = 1$ ✓

3. $\sqrt[3]{y-7} + 10 = 2$

$\sqrt[3]{y-7} = -8$

$y - 7 = -512$

$y = -505$

Check: $\sqrt[3]{-505-7} + 10 \overset{?}{=} 2$

$-8 + 10 \overset{?}{=} 2$

$2 = 2$ ✓

4. $\sqrt{a-1} - 5 = \sqrt{a-6}$

$a - 1 - 10\sqrt{a-1} + 25 = a - 6$

$-10\sqrt{a-1} = -30$

$\sqrt{a-1} = 3$

$a - 1 = 9$

$a = 10$

Check: $\sqrt{10-1} - 5 \overset{?}{=} \sqrt{10-6}$

$3 - 5 \overset{?}{=} 2$

$-2 \neq 2$

no real solution

5. $\sqrt{2x+3} \leq 2$

$2x + 3 \leq 4$

$2x \leq 1$

$x \leq \frac{1}{2}$

$2x + 3 \geq 0$

$2x \geq -3$

$x \geq -\frac{3}{2}$

Test $x = -2$: $\sqrt{2(-2)+3} \overset{?}{\leq} 2$

$\sqrt{-1} \overset{?}{\leq} 2$; meaningless

Test $x = 0$: $\sqrt{2(0)+3} \overset{?}{\leq} 2$

$\sqrt{3} \overset{?}{\leq} 2$; true

Test $x = 1$: $\sqrt{2(1)+3} \overset{?}{\leq} 2$

$\sqrt{5} \overset{?}{\leq} 2$; false

Solution: $-\frac{3}{2} \leq x \leq \frac{1}{2}$

6. $\sqrt[4]{6a-2} > 4$

$6a - 2 > 256$

$6a > 258$

$a > 43$

$6a - 2 \geq 0$

$6a \geq 2$

$a \geq \frac{1}{3}$

Test $a = 0$: $\sqrt[4]{6(0)-2} \overset{?}{>} 4$

$\sqrt[4]{-2} \overset{?}{>} 4$; meaningless

Test $a = 1$: $\sqrt[4]{6(1)-2} \overset{?}{>} 4$

$\sqrt[4]{4} \overset{?}{>} 4$; false

Test $a = 44$: $\sqrt[4]{6(44)-2} \overset{?}{>} 4$

$\sqrt[4]{262} \overset{?}{>} 4$; true

Solution: $a > 43$

Lesson 4-8

Page A33

1. $f(x) = 0.75x - 2$
2. $f(x) = x^3 + x^2 - x + 2$
3. Sample answer: $f(x) = 0.51x^2 + 0.02x - 0.79$

4a. Sample answer: $y = 1.632x + 99.275$

4b. Sample answer: about 122.123 thousand

$f(x) = 1.632x + 99.275$

$f(15) = 1.632(14) + 99.275 = 122.123$

Lesson 5-1

Page A34

1. $13.75° = 13° + (0.75 \cdot 60)'$

$= 13° + 45'$

$= 13°\ 45'$

2. $75.72° = 75° + (0.72 \cdot 60)'$
$= 75° + 43.2'$
$= 75° + 43' + (0.2 \cdot 60)''$
$= 75° + 43' + 12''$
$= 75° 43' 12''$

3. $-29.44° = -29° + (0.44 \cdot 60)'$
$= -29° + 26.4'$
$= -29° + 26' + (0.4 \cdot 60)''$
$= -29° + 26' + 24''$
$= -29° 26' 24''$

4. $87.81° = 87° + (0.81 \cdot 60)'$
$= 87° + 48.6'$
$= 87° + 48' + (0.6 \cdot 60)''$
$= 87° + 48' + 36''$
$= 87° 48' 36''$

5. $144° 12' 30'' = 144° + 12'\left(\frac{1°}{60'}\right) + 30''\left(\frac{1°}{3600''}\right)$
$= 144.208°$

6. $-38° 15' 10'' = -\left(38° + 15'\left(\frac{1°}{60'}\right) + 10''\left(\frac{1°}{3600''}\right)\right)$
$= -38.253°$

7. $-107° 12' 45'' = -\left(107° + 12'\left(\frac{1°}{60'}\right) + 45\left(\frac{1°}{3600''}\right)\right)$
$= -107.213°$

8. $51° 14' 32'' = 51° + 14'\left(\frac{1°}{60'}\right) + 32\left(\frac{1°}{3600''}\right)$
$= 51.242°$

9. $850° - 360° = 490°$
$490° - 360° = 130°$
130°; II

10. $-65° + 360° = 295°$
295°; IV

11. $1012° - 360° = 652°$
$652° - 360° = 292°$
292°; IV

12. $578° - 360° = 218°$
218°; III

13. $180° - 126° = 54°$

14. $-480° + 360° = -120°$
$-120° + 360° = 240°$
$240° - 180° = 60°$
60°

15. $642° - 360° = 282°$
$360° - 282° = 78°$
78°

16. $1154° - 360° = 794°$
$794° - 360° = 434°$
$434° - 360° = 74°$
74°

Lesson 5-2

Page A34

1. $\cot \theta = \frac{1}{\tan \theta}$
$\cot \theta = \frac{1}{\frac{5}{6}}$ or $\frac{6}{5}$

2. $\sin \theta = \frac{1}{\csc \theta}$
$\sin \theta = \frac{1}{2.5}$ or 0.4

3. $(AC)^2 + (BC)^2 = (AB)^2$
$14^2 + 16^2 = (AB)^2$
$452 = (AB)^2$
$\sqrt{452} = AB$; $\sqrt{452}$ or $2\sqrt{113}$
$\sin A = \frac{\text{side opposite}}{\text{hypotenuse}}$ $\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}}$
$\sin A = \frac{16}{2\sqrt{113}}$ or $\frac{8\sqrt{113}}{113}$ $\cos A = \frac{14}{2\sqrt{113}}$ or $\frac{7\sqrt{113}}{113}$
$\tan A = \frac{\text{side opposite}}{\text{side adjacent}}$ $\csc A = \frac{\text{hypotenuse}}{\text{side opposite}}$
$\tan A = \frac{16}{14}$ or $\frac{8}{7}$ $\csc A = \frac{2\sqrt{113}}{16}$ or $\frac{\sqrt{113}}{8}$
$\sec A = \frac{\text{hypotenuse}}{\text{side adjacent}}$ $\cot A = \frac{\text{side adjacent}}{\text{side opposite}}$
$\sec A = \frac{2\sqrt{113}}{14}$ or $\frac{\sqrt{113}}{7}$ $\cot A = \frac{14}{16}$ or $\frac{7}{8}$

4. $(AC)^2 + (BC)^2 = (AB)^2$
$25^2 + (BC)^2 = 28^2$
$BC^2 = 159$
$BC = \sqrt{159}$
$\sin A = \frac{\text{side opposite}}{\text{hypotenuse}}$ $\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}}$
$\sin A = \frac{\sqrt{159}}{28}$ $\cos A = \frac{25}{28}$
$\tan A = \frac{\text{side opposite}}{\text{side adjacent}}$ $\csc A = \frac{\text{hypotenuse}}{\text{side opposite}}$
$\tan A = \frac{\sqrt{159}}{25}$ $\csc A = \frac{28}{\sqrt{159}}$ or $\frac{28\sqrt{159}}{159}$
$\sec A = \frac{\text{hypotenuse}}{\text{side adjacent}}$ $\cot A = \frac{\text{side adjacent}}{\text{side opposite}}$
$\sec A = \frac{28}{25}$ $\cot A = \frac{25}{\sqrt{159}}$ or $\frac{25\sqrt{159}}{159}$

5. $(AC)^2 + (BC)^2 = (AB)^2$
$9^2 + 6^2 = (AB)^2$
$117 = (AB)^2$
$\sqrt{117} = AB$; $\sqrt{117}$ or $3\sqrt{13}$
$\sin A = \frac{\text{side opposite}}{\text{hypotenuse}}$ $\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}}$
$\sin A = \frac{6}{3\sqrt{13}}$ or $\frac{2\sqrt{13}}{13}$ $\cos A = \frac{9}{3\sqrt{13}}$ or $\frac{3\sqrt{13}}{13}$
$\tan A = \frac{\text{side opposite}}{\text{side adjacent}}$ $\csc A = \frac{\text{hypotenuse}}{\text{side opposite}}$
$\tan A = \frac{6}{9}$ or $\frac{2}{3}$ $\csc A = \frac{3\sqrt{13}}{6}$ or $\frac{\sqrt{13}}{2}$
$\sec A = \frac{\text{hypotenuse}}{\text{side adjacent}}$ $\cot A = \frac{\text{side adjacent}}{\text{side opposite}}$
$\sec A = \frac{3\sqrt{13}}{9}$ or $\frac{\sqrt{13}}{3}$ $\cot A = \frac{9}{6}$ or $\frac{3}{2}$

Lesson 5-3

Page A34

1. $\tan \theta = \frac{y}{x}$
Since $\tan \theta = 0$, $y = 0$.
$\cot \theta = \frac{x}{y} = \frac{x}{0}$
$\cot \theta$ is undefined.

2. Sample answers: 90°, 270°
$\cos \theta = \frac{x}{r}$
Since $\cos \theta = 0$, $x = 0$.
On the unit circle, $x = 0$ when $\theta = 90°$ or $\theta = 270°$.

3. $r = \sqrt{x^2 + y^2}$
$= \sqrt{(-1)^2 + (-2)^2}$
$= \sqrt{5}$

$\sin\theta = \frac{y}{r}$ | $\cos\theta = \frac{x}{r}$

$\sin\theta = \frac{-2}{\sqrt{5}}$ or $-\frac{2\sqrt{5}}{5}$ | $\cos\theta = \frac{-1}{\sqrt{5}}$ or $-\frac{\sqrt{5}}{5}$

$\tan\theta = \frac{y}{x}$ | $\csc\theta = \frac{r}{y}$

$\tan\theta = \frac{-2}{-1}$ or 2 | $\csc\theta = -\frac{\sqrt{5}}{2}$

$\sec\theta = \frac{r}{x}$ | $\cot\theta = \frac{x}{y}$

$\sec\theta = \frac{\sqrt{5}}{-1}$ or $-\sqrt{5}$ | $\cot\theta = \frac{-1}{-2}$ or $\frac{1}{2}$

4. $r = \sqrt{x^2 + y^2}$
$= \sqrt{(-2)^2 + (2)^2}$
$= 2\sqrt{2}$

$\sin\theta = \frac{y}{r}$ | $\cos\theta = \frac{x}{r}$

$\sin\theta = \frac{2}{2\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ | $\cos\theta = \frac{-2}{2\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$

$\tan\theta = \frac{y}{x}$ | $\csc\theta = \frac{r}{y}$

$\tan\theta = \frac{2}{-2}$ or -1 | $\csc\theta = \frac{2\sqrt{2}}{2}$ or $\sqrt{2}$

$\sec\theta = \frac{r}{x}$ | $\cot\theta = \frac{x}{y}$

$\sec\theta = \frac{2\sqrt{2}}{-2}$ or $-\sqrt{2}$ | $\cot\theta = \frac{-2}{2}$ or -1

5. $r = \sqrt{x^2 + y^2}$
$= \sqrt{(5)^2 + (2)^2}$
$= \sqrt{29}$

$\sin\theta = \frac{y}{r}$ | $\cos\theta = \frac{x}{r}$

$\sin\theta = \frac{2}{\sqrt{29}}$ or $\frac{2\sqrt{29}}{29}$ | $\cos\theta = \frac{5}{\sqrt{29}}$ or $\frac{5\sqrt{29}}{29}$

$\tan\theta = \frac{y}{x}$ | $\csc\theta = \frac{r}{y}$

$\tan\theta = \frac{2}{5}$ | $\csc\theta = \frac{\sqrt{29}}{2}$

$\sec\theta = \frac{r}{x}$ | $\cot\theta = \frac{x}{y}$

$\sec\theta = \frac{\sqrt{29}}{5}$ | $\cot\theta = \frac{5}{2}$

6. $r = \sqrt{x^2 + y^2}$
$= \sqrt{(-4)^2 + (3)^2}$
$= 5$

$\sin\theta = \frac{y}{r}$ | $\cos\theta = \frac{x}{r}$

$\sin\theta = \frac{3}{5}$ | $\cos\theta = -\frac{4}{5}$

$\tan\theta = \frac{y}{x}$ | $\csc\theta = \frac{r}{y}$

$\tan\theta = -\frac{3}{4}$ | $\csc\theta = \frac{5}{3}$

$\sec\theta = \frac{r}{x}$ | $\cot\theta = \frac{x}{y}$

$\sec\theta = -\frac{5}{4}$ | $\cot\theta = -\frac{4}{3}$

Lesson 5-4

Page A34

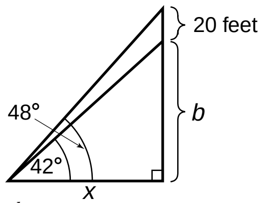
1. $\tan A = \frac{a}{b}$
$\tan 38° = \frac{a}{15}$
$15 \tan 38° = a$
$11.7 \approx a$

2. $\cos B = \frac{a}{c}$
$\cos 87° = \frac{a}{19}$
$19 \cos 87° = a$
$1.0 \approx a$

3. $\cos B = \frac{a}{c}$
$\cos 65.4° = \frac{16.5}{c}$
$c = \frac{16.5}{\cos 65.4°}$
$c \approx 39.6$

4. $\tan B = \frac{b}{a}$
$\tan 42.5° = \frac{12}{a}$
$a = \frac{12}{\tan 42.5°}$
$a \approx 13.1$

5. $\sin B = \frac{b}{c}$
$\sin 75° = \frac{b}{5.8}$
$5.8 \sin 75° = b$
$5.6 \approx b$



6. $\tan 48° = \frac{20 + b}{x}$
$x = \frac{20 + b}{\tan 48°}$
$\tan 42° = \frac{b}{x}$
$\tan 42° = \frac{b}{\frac{20 + b}{\tan 48°}}$
$20 \tan 42° + b \tan 42° = b \tan 48°$
$20 \tan 42° = b(\tan 48° - \tan 42°)$
$\frac{20 \tan 42°}{(\tan 48° - \tan 42°)} = b$
$85.7 \approx b$; 85.7 ft

Lesson 5-5

Page A35

1. Let $A = \arcsin \frac{3}{4}$. Then $\sin A = \frac{3}{4}$.
$\sin\left(\arcsin \frac{3}{4}\right) = \frac{3}{4}$

2. Let $A = \cos^{-1} \frac{1}{2}$. Then $\cos A = \frac{1}{2}$.
$\sec A = \frac{1}{\cos A}$
$\sec A = \frac{1}{\frac{1}{2}}$ or 2

$\sec\Big(\cos$

3. Let $A = \tan^{-1} 1$. Then $\tan A = 1$.
$\tan(\tan^{-1} 1) = 1$

4. $\tan A = \frac{a}{b}$
$\tan A = \frac{38}{25}$
$A = \tan^{-1} \frac{38}{25}$
$A \approx 56.7°$

5. $\sin B = \frac{b}{c}$
$\sin B = \frac{17}{19}$
$B = \sin^{-1} \frac{17}{19}$
$B \approx 63.5°$

6. $\cos B = \frac{a}{c}$

$\cos B = \frac{24}{30}$

$B = \cos^{-1} \frac{24}{30}$

$B \approx 36.9°$

7. $\cos B = \frac{a}{c}$

$\cos B = \frac{9.2}{12.6}$

$B = \cos^{-1} \frac{9.2}{12.6}$

$B \approx 43.1°$

8. $\tan A = \frac{a}{b}$

$\tan A = \frac{28.4}{36.5}$

$A = \tan^{-1} \frac{28.4}{36.5}$

$A \approx 37.9°$

Lesson 5-6

Page A35

1. $C = 180° - (75° + 50°)$ or $55°$

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{7}{\sin 75°} = b \sin 50°$

$b = \frac{7 \sin 50°}{\sin 75°}$

$b \approx 5.551472956$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{7}{\sin 75°} = \frac{c}{\sin 55°}$

$c = \frac{7 \sin 55°}{\sin 75°}$

$c \approx 5.936340197$

$C = 55°$, $b = 5.6$, $c = 5.9$

2. $B = 180° - (97° + 42°)$ or $41°$

$\frac{c}{\sin C} = \frac{b}{\sin B}$

$\frac{12}{\sin 42°} = \frac{b}{\sin 41°}$

$b = \frac{12 \sin 41°}{\sin 42°}$

$b \approx 11.76557801$

$\frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{12}{\sin 42°} = \frac{a}{\sin 97°}$

$a = \frac{12 \sin 97°}{\sin 42°}$

$a \approx 17.80004338$

$B = 41°$, $a = 17.8$, $b = 11.8$

3. $A = 180° - (49° + 32°)$ or $99°$

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{10}{\sin 99°} = \frac{b}{\sin 49°}$

$b = \frac{10 \sin 49°}{\sin 99°}$

$b \approx 7.641171301$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{10}{\sin 99°} = \frac{c}{\sin 32°}$

$c = \frac{10 \sin 32°}{\sin 99°}$

$c \approx 5.365247745$

$A = 99°$, $b = 7.6$, $c = 5.4$

4. $B = 180° - (22° + 41°)$ or $117°$

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{25}{\sin 117°} = \frac{a}{\sin 22°}$

$a = \frac{25 \sin 22°}{\sin 117°}$

$a \approx 10.51077021$

$\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{25}{\sin 117°} = \frac{c}{\sin 41°}$

$c = \frac{25 \sin 41°}{\sin 117°}$

$c \approx 18.40780654$

$B = 117°$, $a = 10.5$, $c = 18.4$

5. $K = \frac{1}{2}bc \sin A$

$K = \frac{1}{2}(12)(6) \sin 34°$

$K \approx 20.1 \text{ units}^2$

6. $C = 180° - (87° + 56.8°)$ or $36.2°$

$K = \frac{1}{2}c^2 \frac{\sin A \sin B}{\sin C}$

$K = \frac{1}{2}(6.8)^2 \frac{\sin 87° \sin 56.8°}{\sin 36.2°}$

$K \approx 32.7 \text{ units}^2$

7. $A = 180° - (60° + 75°)$ or $45°$

$K = \frac{1}{2}a^2 \frac{\sin B \sin C}{\sin A}$

$K = \frac{1}{2}(8)^2 \frac{\sin 60° \sin 75°}{\sin 45°}$

$K \approx 37.9 \text{ units}^2$

8. $K = \frac{1}{2}bc \sin A$

$K = \frac{1}{2}(16)(12) \sin 43°$

$K \approx 65.5 \text{ units}^2$

Lesson 5-7

Page A35

1. Since $145° > 90°$, consider Case II.
$5 \leq 10$; no solution

2. Since $25° < 90°$, consider Case I.

$b \sin A = 10 \sin 25°$

≈ 4.226182617

$9 > 4.226182617$; 2 solutions

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{6}{\sin 25°} = \frac{10}{\sin B}$

$6 \sin B = 10 \sin 25°$

$B = \sin^{-1}\left(\frac{10 \sin 25°}{6}\right)$

$B \approx 44.77816685$

$B \approx 180° - 44.8°$ or $135.2°$

Solution 1

$C \approx 180° - (25° + 44.8°)$ or $110.2°$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{6}{\sin 25°} \approx \frac{c}{\sin 110.2°}$

$c \approx \frac{6 \sin 110.2°}{\sin 25°}$

$c \approx 13.32398206$

$B = 44.8°$, $C = 110.2°$, $c = 13.3$

Solution 2

$C \approx 180° - (25° + 135.2°)$ or $19.8°$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{6}{\sin 25°} \approx \frac{c}{\sin 19.8°}$

$c \approx \frac{6 \sin 19.8°}{\sin 25°}$

$c \approx 4.809133219$

$B = 135.2°$, $C = 19.8°$, $c = 4.8$

3. Since $56° < 90°$, consider Case I.

$C \sin B = 50 \sin 56°$

≈ 41.45187863

$34 < 41.5$; no solution

4. $C = 180° - (45° + 85°)$ or $50°$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{a}{\sin 45°} = \frac{15}{\sin 50°}$

$a = \frac{15 \sin 45°}{\sin 50°}$

$a \approx 13.8459352$

$\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{b}{\sin 85°} = \frac{15}{\sin 50°}$

$b = \frac{15 \sin 85°}{\sin 50°}$

$b \approx 19.50659731$

$C = 50°, a = 13.8, b = 19.5$

Lesson 5-8

Page A35

1. $a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 6^2 + 8^2 - 2(6)(8) \cos 62°$

$a^2 \approx 54.93072997$

$a \approx 7.411526831$

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{a}{\sin 62°} = \frac{6}{\sin B}$

$\sin B = \frac{6 \sin 62°}{a}$

$B = \sin^{-1}\left(\frac{6 \sin 62°}{a}\right)$

$B \approx 45.62599479$

$C \approx 180° - (62° + 45.6°)$ or $72.4°$

$a = 7.4, B = 45.6°, C = 72.4°$

2. $a^2 = b^2 + c^2 - 2bc \cos A$

$9^2 = 7^2 + 12^2 - 2(7)(12) \cos A$

$-112 = -168 \cos A$

$A = \cos^{-1}\left(\frac{-112}{-168}\right)$

$A \approx 48.1896851$

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{9}{\sin A} = \frac{7}{\sin B}$

$\sin B = \frac{7 \sin A}{9}$

$B = \sin^{-1}\left(\frac{7 \sin A}{9}\right)$

$B \approx 35.43094469$

$C \approx 180° - (48.2° + 35.4°)$ or $96.4°$

$A = 48.2°, B = 35.4°, C = 96.4°$

3. $b^2 = a^2 + c^2 - 2ac \cos B$

$b^2 = 14^2 + 18^2 - 2(14)(18) \cos 48°$

$b^2 \approx 182.7581744$

$b \approx 13.51880817$

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{14}{\sin A} = \frac{b}{\sin 48°}$

$\sin A = \frac{14 \sin 48°}{b}$

$A = \sin^{-1}\left(\frac{14 \sin 48°}{b}\right)$

$A \approx 50.31729382$

$C \approx 180° - (50.3° + 48°)$ or $81.7°$

$b = 13.5, A = 50.3°, C = 81.7°$

4. $c^2 = a^2 + b^2 - 2ab \cos C$

$c^2 = (14.2)^2 + (24.5)^2 - 2(14.2)(24.5) \cos 85.3°$

$c^2 \approx 744.8771857$

$c \approx 27.29243825$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{14.2}{\sin A} = \frac{c}{\sin 85.3°}$

$\sin A = \frac{14.2 \sin 85.3°}{c}$

$A = \sin^{-1}\left(\frac{14.2 \sin 85.3°}{c}\right)$

$A \approx 31.23444201$

$B \approx 180° - (31.2° + 85.3°)$ or $63.5°$

$c = 27.3, A = 31.2°, B = 63.5°$

5. $s = \frac{1}{2}(a + b + c)$

$s = \frac{1}{2}(4 + 7 + 10)$

$s = 10.5$

$K = \sqrt{s(s - a)(s - b)(s - c)}$

$K = \sqrt{10.5(10.5 - 4)(10.5 - 7)(10.5 - 10)}$

$K = \sqrt{119.4375}$

$K \approx 10.9$ units^2

6. $s = \frac{1}{2}(a + b + c)$

$s = \frac{1}{2}(4 + 6 + 5)$

$s = 7.5$

$K = \sqrt{s(s - a)(s - b)(s - c)}$

$K = \sqrt{7.5(7.5 - 4)(7.5 - 6)(7.5 - 5)}$

$K = \sqrt{98.4375}$

$K \approx 9.9$ unit^2

7. $s = \frac{1}{2}(a + b + c)$

$s = \frac{1}{2}(12.4 + 8.6 + 14.2)$

$s = 17.6$

$K = \sqrt{s(s - a)(s - b)(s - c)}$

$K = \sqrt{17.6(17.6 - 12.4)(17.6 - 8.6)(17.6 - 14.2)}$

$K = \sqrt{2800.512}$

$K \approx 52.9$ units^2

8. $s = \frac{1}{2}(a + b + c)$

$s = \frac{1}{2}(150 + 124 + 190)$

$s = 232$

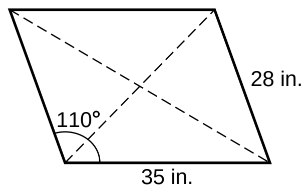
$K = \sqrt{s(s - a)(s - b)(s - c)}$

$K = \sqrt{232(232 - 150)(232 - 124)(232 - 190)}$

$K = \sqrt{86{,}292{,}864}$

$K \approx 9289.4$ units^2

9a.



$d^2 = 28^2 + 35^2 - 2(28)(35)\cos 110°$
$d^2 \approx 2679.359481$
$d \approx 51.8$ in.

9b. Area $= 2\left[\frac{1}{2}(28)(35)\sin 110°\right]$
≈ 920.9 in^2

Lesson 6-1

Page A36

1. $120° = 120° \cdot \frac{\pi}{180°} = \frac{2\pi}{3}$

2. $280° = 280° \cdot \frac{\pi}{180°} = \frac{14\pi}{9}$

3. $-440° = -440° \cdot \frac{\pi}{180°} = -\frac{22\pi}{9}$

4. $-150° = -150° \cdot \frac{\pi}{180°} = -\frac{5\pi}{6}$

5. $\frac{8\pi}{3} = \frac{8\pi}{3} \cdot \frac{180°}{\pi} = 480°$

6. $\frac{5\pi}{12} = \frac{5\pi}{12} \cdot \frac{180°}{\pi} = 75°$

7. $-2 = -2 \cdot \frac{180°}{\pi} = -114.6°$

8. $10.5 = 10.5 \cdot \frac{180°}{\pi} = 601.6°$

9. reference angle: $\pi - \frac{5\pi}{6} = \frac{\pi}{6}$; Quadrant II
$\sin \frac{5\pi}{6} = \frac{1}{2}$

10. reference angle: $\frac{4\pi}{3} - \pi = \frac{\pi}{3}$; Quadrant III
$\sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$

11. $\frac{9\pi}{4}$ is coterminal with $\frac{\pi}{4}$; Quadrant I
$\cos \frac{9\pi}{4} = \frac{\sqrt{2}}{2}$

12. $\frac{-3\pi}{2}$ is coterminal with $\frac{\pi}{2}$
$\cos\left(-\frac{3\pi}{2}\right) = 0$

13. If the diameter = 10 in., the radius = 5 in.
$80° = 80° \cdot \frac{\pi}{180} = \frac{4\pi}{9}$
$s = r\theta$
$s = 5\left(\frac{4\pi}{9}\right)$
$s \approx 7.0$ in.

Lesson 6-2

Page A36

1. $5 \cdot 2\pi = 10\pi$ or about 31.4 radians

2. $3.8 \cdot 2\pi = 7.6\pi$ or about 23.9 radians

3. $14.2 \cdot 2\pi = 28.4\pi$ or about 89.2 radians

4. $2.1 \cdot 2\pi = 4.2\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{4.2\pi}{5}$
$\omega \approx 2.6$ radians/s

5. $1.5 \cdot 2\pi = 3\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{3\pi}{2}$
$\omega \approx 4.7$ radians/min

6. $15.8 \cdot 2\pi = 31.6\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{31.6\pi}{18}$
$\omega \approx 5.5$ radians/s

7. $140 \cdot 2\pi = 280\pi$
$\omega = \frac{\theta}{t}$
$\omega = \frac{280\pi}{20}$
$\omega \approx 44.0$ radians/min

8. $\omega = \frac{\theta}{t}$
$\omega = \frac{2\pi}{30}$
$\omega \approx$ about 0.2 radian/s

Lesson 6-3

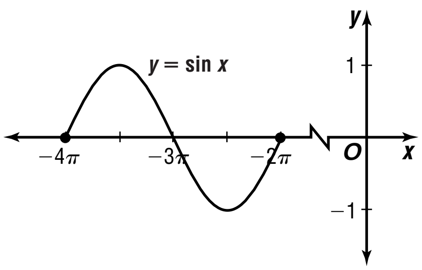
Page A36

1. 1

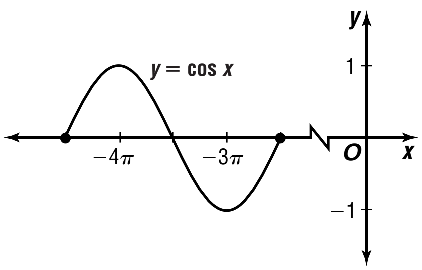
2. 0

3. -1

4.



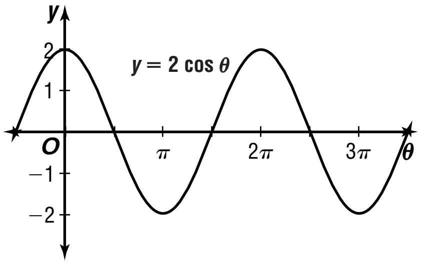
5.



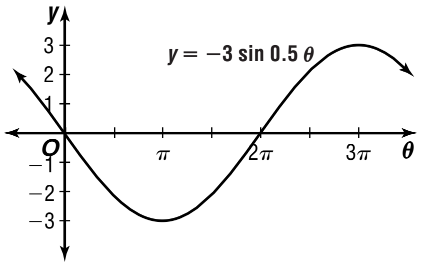
Lesson 6-4

Page A36

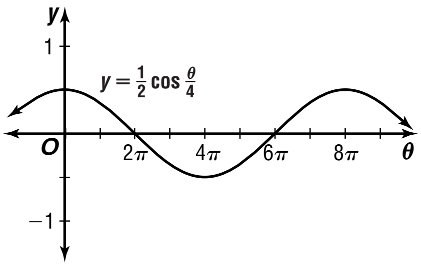
1. $|2| = 2; \frac{2\pi}{1} = 2\pi$



2. $|-3| = 3; \frac{2\pi}{0.5} = 4\pi$



3. $\left|\frac{1}{2}\right| = \frac{1}{2}; \dfrac{2\pi}{\frac{1}{4}} = 8\pi$



4. $A = |0.5|$ $\qquad \frac{2\pi}{k} = 6\pi$

$A = \pm 0.5$ $\qquad k = \frac{2\pi}{6\pi}$ or $\frac{1}{3}$

$y = \pm 0.5 \sin \frac{\theta}{3}$

5. $A = |2|$ $\qquad \frac{2\pi}{k} = \frac{\pi}{3}$

$A = \pm 2$ $\qquad k = \dfrac{2\pi}{\frac{\pi}{3}}$ or 6

$y = \pm 2 \sin 6\theta$

6. $A = \left|\frac{3}{5}\right|$ $\qquad \frac{2\pi}{k} = 4\pi$

$A = \pm\frac{3}{5}$ $\qquad k = \frac{2\pi}{4\pi}$ or $\frac{1}{2}$

$y = \pm\frac{3}{5}\cos\frac{\theta}{2}$

7. $A = |0.25|$ $\qquad \frac{2\pi}{k} = 8$

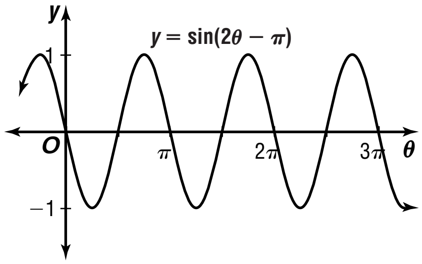
$A = \pm 0.25$ $\qquad k = \frac{2\pi}{8}$ or $\frac{\pi}{4}$

$y = \pm 0.25 \cos \frac{\pi}{4}\theta$

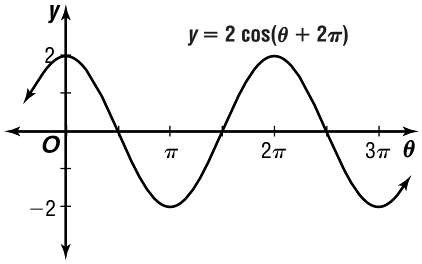
Lesson 6-5

Page A37

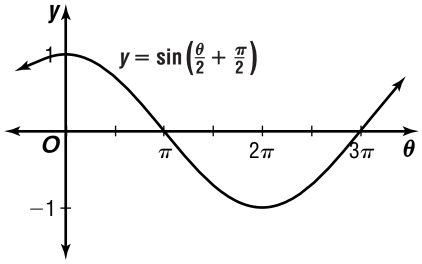
1. $-\frac{c}{k} = -\left(-\frac{\pi}{2}\right)$ or $\frac{\pi}{2}$



2. $-\frac{c}{k} = -\frac{2\pi}{1}$ or -2π



3. $-\frac{c}{k} = -\left(\dfrac{\frac{\pi}{2}}{\frac{1}{2}}\right)$ or $-\pi$



4. $A = |2|$ $\qquad \frac{2\pi}{k} = \frac{\pi}{2}$ $\qquad -\frac{c}{1} = \pi$ $\qquad h = -1$

$A = \pm 2$ $\qquad k = \dfrac{2\pi}{\frac{\pi}{2}}$ or 1 $\qquad c = -\pi$

$y = \pm 2 \sin(\theta - \pi) - 1$

5. $A = |0.5|$ $\qquad \frac{2\pi}{k} = \frac{\pi}{4}$ $\qquad -\frac{c}{8} = 0$ $\qquad h = 3$

$A = \pm 0.5$ $\qquad k = \frac{2\pi}{\pi 4}$ or 8 $\qquad c = 0$

$y = \pm 0.5 \sin 8\theta + 3$

6. $A = |20|$ $\qquad \frac{2\pi}{k} = \frac{\pi}{2}$ $\qquad -\frac{c}{4} = 2\pi$ $\qquad h = 4$

$A = \pm 20$ $\qquad k = \dfrac{2\pi}{\frac{\pi}{2}}$ or 4 $\qquad c = -8\pi$

$y = \pm 20 \cos(4\theta - 8\pi) + 4$

7. $A = \left|\frac{3}{4}\right|$ $\qquad \frac{2\pi}{k} = 10$ $\qquad -\dfrac{c}{\frac{\pi}{5}} = 0$ $\qquad h = \frac{1}{2}$

$A = \pm\frac{3}{4}$ $\qquad k = \frac{2\pi}{10}$ or $\frac{\pi}{5}$ $\qquad c = 0$

$y = \pm\frac{3}{4}\cos\frac{\pi}{5}\theta + \frac{1}{2}$

Lesson 6-6

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1a. $12.1 - 2.7 = 9.4$ h

1b. $12.1 + 2.7 = 14.8$ h

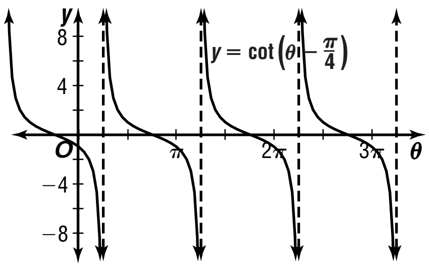
1c. $m = 10$ represents the middle of October.
$d = 2.7 \sin(0.5(10) - 1.4) + 12.1$
$d \approx 10.9$ h

2. $A = \frac{6}{2}$ or 3; $\frac{2\pi}{14} = \frac{\pi}{7}$
$y = 3\cos\left(\frac{\pi}{7}t\right)$

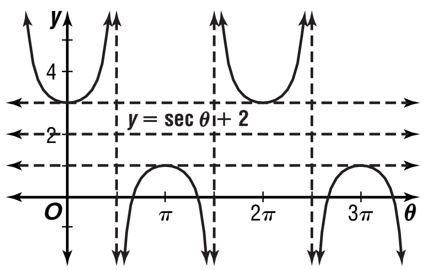
Lesson 6-7

Page A37

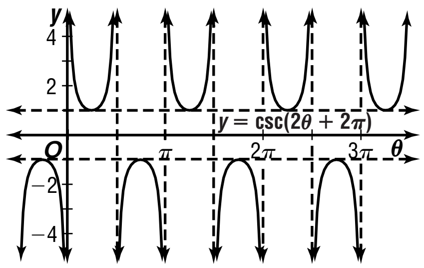
1.



2.



3.



Lesson 6-8

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1. Let $\theta = \text{Cos}^{-1} 0$.
$\text{Cos}\,\theta = 0$
$\theta = \frac{\pi}{2}$

2. Let $\theta = \text{Arcsin } 0$.
$\text{Sin}\,\theta = 0$
$\theta = 0$

3. Let $\theta = \text{Tan}^{-1} 1$.
$\text{Tan}\,\theta = 1$
$\theta = \frac{\pi}{4}$

$\cos(\text{Tan}^{-1} 1) = \cos\frac{\pi}{4}$
$= \sqrt{\frac{2}{2}}$

4. If $y = \tan\frac{3\pi}{4}$, then $y = -1$.

$$\begin{aligned}\text{Cos}^{-1}\left(\tan\frac{3\pi}{4}\right) &= \text{Cos}^{-1} y\\ &= \text{Cos}^{-1}(-1)\\ &= \pi\end{aligned}$$

5. Let $\alpha = \text{Cos}^{-1}\frac{1}{2}$ and $\beta = \text{Sin}^{-1} 0$.

$\text{Cos}\,\alpha = \frac{1}{2}$ $\quad$ $\text{Sin}\,\beta = 0$

$\alpha = \frac{\pi}{3}$ $\quad$ $\beta = 0$

$$\begin{aligned}\sin\left(\text{Cos}^{-1}\frac{1}{2} + \text{Sin}^{-1} 0\right) &= \sin(\alpha + \beta)\\ &= \sin\left(\frac{\pi}{3} + 0\right)\\ &= \sin\frac{\pi}{3}\\ &= \frac{\sqrt{3}}{2}\end{aligned}$$

6. Let $\theta = \sin^{-1}\frac{\sqrt{3}}{2}$.
$\sin\theta = \frac{\sqrt{3}}{2}$
$\theta = \frac{\pi}{3}$

$$\begin{aligned}\cos\left(2\,\text{Sin}^{-1}\frac{\sqrt{3}}{2}\right) &= \cos(2\theta)\\ &= \cos\left(2\cdot\frac{\pi}{3}\right)\\ &= \cos\frac{2\pi}{3}\\ &= -\frac{1}{2}\end{aligned}$$

Lesson 7-1

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1.
$$\begin{aligned}\csc\theta &= \frac{1}{\sin\theta}\\ &= \frac{1}{\sqrt{1-\cos^2\theta}}\\ &= \frac{1}{\sqrt{1-\left(\frac{1}{4}\right)^2}}\\ &= \frac{1}{\sqrt{\frac{15}{16}}}\\ &= \frac{1}{\frac{\sqrt{15}}{4}}\\ &= \frac{4}{\sqrt{15}}\cdot\frac{\sqrt{15}}{\sqrt{15}}\\ &= \frac{4\sqrt{15}}{15}\end{aligned}$$

2.
$$\begin{aligned}\tan\theta &= \frac{1}{\cot\theta}\\ &= \frac{1}{-\frac{\sqrt{6}}{3}}\\ &= -\frac{3}{\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}}\\ &= -\frac{\sqrt{6}}{2}\end{aligned}$$

3.
$$\begin{aligned}\cos\frac{13\pi}{6} &= \cos\left(\frac{13\pi}{6} - 2\pi\right)\\ &= \cos\frac{\pi}{6}\end{aligned}$$

4.
$$\begin{aligned}\tan(-315°) &= \frac{\sin(-315°)}{\cos(-315°)}\\ &= \frac{\sin(45° + (-360°))}{\cos(45° + (-360°))}\\ &= \frac{\sin 45°}{\cos 45°}\\ &= \tan 45°\end{aligned}$$

5. $\csc(-930°) = \frac{1}{\sin(-930°)}$

$= \frac{1}{\sin(-360°(2) - 210°)}$

$= \frac{1}{-\sin(210°)}$

$= \frac{1}{-\sin(-30°)}$

$= \frac{1}{\sin 30°}$

$= \csc 30°$

6. $\frac{\tan\theta}{\sin\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{\sin\theta}$

$= \frac{1}{\cos\theta}$

$= \sec\theta$

7. $\cot\theta\tan\theta + \sin\theta\sec\theta = \frac{\cos\theta}{\sin\theta}\cdot\frac{\sin\theta}{\cos\theta} + \sin\theta\cdot\frac{1}{\cos\theta}$

$= 1 + \frac{\sin\theta}{\cos\theta}$

$= 1 + \tan\theta$

8. $(1 + \sin x)(1 - \sin x) = 1 - \sin^2 x$

$= \cos^2 x$

9. $\frac{\cot x \sin x}{\csc x \cos x} = \frac{\frac{\cos x}{\sin x}\cdot\sin x}{\frac{1}{\sin x}\cdot\cos x}$

$= \frac{\cos x}{\frac{\cos x}{\sin x}}$

$= \sin x$

Lesson 7-2

Page A38

1. $\csc^2\theta \stackrel{?}{=} \cot^2\theta + \sin\theta\csc\theta$

$\csc^2\theta \stackrel{?}{=} \cot^2\theta + \sin\theta\cdot\frac{1}{\sin\theta}$

$\csc^2\theta \stackrel{?}{=} \cot^2\theta + 1$

$\csc^2\theta = \csc^2\theta$

2. $\frac{\sec\theta - \csc\theta}{\csc\theta\sec\theta} \stackrel{?}{=} \sin\theta - \cos\theta$

$\frac{\sec\theta}{\csc\theta\sec\theta} - \frac{\csc\theta}{\csc\theta\sec\theta} \stackrel{?}{=} \sin\theta - \cos\theta$

$\frac{1}{\csc\theta} - \frac{1}{\sec\theta} \stackrel{?}{=} \sin\theta - \cos\theta$

$\sin\theta - \cos\theta = \sin\theta - \cos\theta$

3. $\sin^2 x + \cos^2 x \stackrel{?}{=} \sec^2 x - \tan^2 x$

$1 \stackrel{?}{=} \tan^2 x + 1 - \tan^2 x$

$1 = 1$

4. $\sec A - \cos A \stackrel{?}{=} \tan A \sin A$

$\frac{1}{\cos A} - \cos A \stackrel{?}{=} \tan A \sin A$

$\frac{1}{\cos A} - \frac{\cos^2 A}{\cos A} \stackrel{?}{=} \tan A \sin A$

$\frac{1 - \cos^2 A}{\cos A} \stackrel{?}{=} \tan A \sin A$

$\frac{\sin^2 A}{\cos A} \stackrel{?}{=} \tan A \sin A$

$\frac{\sin A}{\cos A}\sin A \stackrel{?}{=} \tan A \sin A$

$\tan A \sin A = \tan A \sin A$

5. Sample answer: $\cos x = 1$

$\frac{\cot x}{\csc x} = 1$

$\frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}} = 1$

$\cos x = 1$

6. Sample answer: $\cot x = 2$

$2\tan x \sin x + 2\cos x = \csc x$

$2\left(\frac{\sin x}{\cos x}\cdot\sin x + \cos x\right) = \frac{1}{\sin x}$

$2\left(\frac{\sin^2 x + \cos^2 x}{\cos x}\right) = \frac{1}{\sin x}$

$2\left(\frac{1}{\cos x}\right) = \frac{1}{\sin x}$

$2 = \frac{\cos x}{\sin x}$

$2 = \cot x$

Lesson 7-3

Page A38

1. $\cos 75° = \cos(45° + 30°)$

$= \cos 45°\cos 30° - \sin 45°\sin 30°$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

$= \frac{6 - \sqrt{2}}{4}$

2. $\sin 105° = \sin(60° + 45°)$

$= \sin 60°\sin 45° + \cos 60°\cos 45°$

$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{6 + \sqrt{2}}{4}$

3. $\tan\frac{\pi}{12} = \tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= \frac{\tan\frac{\pi}{4} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{4}\tan\frac{\pi}{6}}$

$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + 1\cdot\frac{\sqrt{3}}{3}}$

$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}\cdot\frac{1 - \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3}}$

$= 2 - \sqrt{3}$

4. $\tan\frac{7\pi}{12} = \tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

$= \frac{\tan\frac{\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{3}\tan\frac{\pi}{4}}$

$= \frac{\frac{\sqrt{3}}{1} + 1}{1 - \frac{\sqrt{3}}{1}\cdot 1}$

$= \frac{\sqrt{3} + 1}{} \cdot \frac{1 + \sqrt{3}}{}$

$= -2 - \sqrt{3}$

5. $\sec \frac{29\pi}{12} = \sec \frac{5\pi}{12}$
$$= \frac{1}{\cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right)}$$
$$= \frac{1}{\cos \frac{\pi}{4} \cos \frac{\pi}{6} - \sin \frac{\pi}{4} \sin \frac{\pi}{6}}$$
$$= \frac{1}{\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}}$$
$$= \frac{1}{\frac{\sqrt{6} - \sqrt{2}}{4}}$$
$$= \frac{4}{\sqrt{6} - \sqrt{2}} \cdot \frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}}$$
$$= \sqrt{6} + \sqrt{2}$$

6. $\cot 375° = \cot 15°$
$$= \frac{1}{\tan(45° - 30°)}$$
$$= \frac{1}{\frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}}$$
$$= \frac{1}{\frac{1 - \frac{\sqrt{3}}{3}}{1 + 1\frac{\sqrt{3}}{3}}}$$
$$= \frac{1 + \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3}} \cdot \frac{1 + \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$$
$$= 2 + \sqrt{3}$$

7. $\sin x = \sqrt{1 - \left(\frac{2}{5}\right)^2}$ $\qquad$ $\cos y = \sqrt{1 - \left(\frac{3}{4}\right)^2}$
$$= \sqrt{\frac{21}{25}} \qquad = \sqrt{\frac{7}{16}}$$
$$= \sqrt{\frac{21}{5}} \qquad = \sqrt{\frac{7}{4}}$$
$\sin(x + y) = \sin x \cos y + \cos x \sin y$
$$= \frac{\sqrt{21}}{5} \cdot \frac{\sqrt{7}}{4} + \frac{2}{5} \cdot \frac{3}{4}$$
$$= \frac{6 + 7\sqrt{3}}{20}$$

8. $\sqrt{12^2 - 5^2} = \sqrt{119}$ $\qquad$ $\sqrt{12^2 - 11^2} = \sqrt{23}$
$\sin x = \frac{\sqrt{119}}{12}$ $\qquad$ $\sin y = \frac{\sqrt{23}}{12}$
$\cos(x - y) = \cos x \cos y + \sin x \sin y$
$$= \frac{5}{12} \cdot \frac{11}{12} + \frac{\sqrt{119}}{12} \cdot \frac{\sqrt{23}}{12}$$
$$= \frac{55 + \sqrt{2737}}{144}$$

9. If $\cot x = \frac{4}{3}$, then $\tan x = \frac{3}{4}$.
$\sec^2 y = 1 + \tan^2 y$
$\left(\frac{5}{4}\right)^2 = 1 + \tan^2 y$
$\frac{9}{16} = \tan^2 y$
$\frac{3}{4} = \tan y$
$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$
$$= \frac{\frac{3}{4} + \frac{3}{4}}{1 - \frac{3}{4} \cdot \frac{3}{4}}$$
$$= \frac{\frac{6}{4}}{\frac{7}{16}}$$
$$= \frac{24}{7}$$

10. $\sqrt{7^2 + 6^2} = \sqrt{85}$; $\cos x = \frac{6\sqrt{85}}{85}$; $\sin x = \frac{7\sqrt{85}}{85}$
$\csc y = \frac{8}{5}$ $\qquad$ $\sqrt{8^2 - 5^2} = \sqrt{39}$
$\frac{1}{\sin y} = \frac{8}{5}$ $\qquad$ $\cos y = \frac{\sqrt{39}}{8}$
$\sin y = \frac{5}{8}$
$$\sec(x - y) = \frac{1}{\cos(x - y)}$$
$$= \frac{1}{\cos x \cos y + \sin x \sin y}$$
$$= \frac{1}{\frac{6\sqrt{85}}{85} \cdot \frac{\sqrt{39}}{8} + \frac{7\sqrt{85}}{85} \cdot \frac{5}{8}}$$
$$= \frac{680}{6\sqrt{3315} + 35\sqrt{85}} \cdot \frac{6\sqrt{3315} - 35\sqrt{85}}{6\sqrt{3315} - 35\sqrt{85}}$$
$$= \frac{48\sqrt{3315} - 280\sqrt{85}}{179}$$

Lesson 7-4

Page A39

1. $\sin 15° = \sin \frac{30°}{2}$
$$= \pm\sqrt{\frac{1 - \cos 30°}{2}}$$
$$= \pm\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}}$$
$$= \pm\frac{\sqrt{2 - \sqrt{3}}}{2}$$
Since 15° is in Quadrant I, $\sin 15° = \frac{\sqrt{2 - \sqrt{3}}}{2}$.

2. $\cos 75° = \cos \frac{150°}{2}$
$$= \pm\sqrt{\frac{1 + \cos 150°}{2}}$$
$$= \pm\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}}$$
$$= \pm\frac{\sqrt{2 - \sqrt{3}}}{2}$$
Since 75° is in Quadrant I, $\cos 75° = \frac{\sqrt{2 - \sqrt{3}}}{2}$.

3. $\tan\frac{\pi}{12} = \tan\frac{\frac{\pi}{6}}{2}$

$$= \pm\sqrt{\frac{1-\cos\frac{\pi}{6}}{1+\cos\frac{\pi}{6}}}$$

$$= \pm\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}}$$

$$= \pm\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}\cdot\frac{2-\sqrt{3}}{2-\sqrt{3}}}$$

$$= \pm\sqrt{\frac{(2-\sqrt{3})^2}{1}} \text{ or } \pm\left(2-\sqrt{3}\right)$$

Since $\frac{\pi}{12}$ is in Quadrant I, $\tan\frac{\pi}{12} = 2-\sqrt{3}$.

4. $\cos 22.5° = \cos\frac{45°}{2}$

$$= \pm\sqrt{\frac{1+\cos 45°}{2}}$$

$$= \pm\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}$$

$$= \pm\frac{\sqrt{2+\sqrt{2}}}{2}$$

Since 22.5° is in Quadrant I, $\cos 22.5° = \frac{\sqrt{2+\sqrt{2}}}{2}$.

5. $\sin\frac{5\pi}{12} = \sin\frac{\frac{5\pi}{6}}{2}$

$$= \pm\sqrt{\frac{1-\cos\frac{5\pi}{6}}{2}}$$

$$= \pm\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$$

$$= \pm\frac{\sqrt{2+\sqrt{3}}}{2}$$

Since $\frac{5\pi}{12}$ is in Quadrant I, $\sin\frac{5\pi}{12} = \frac{\sqrt{2+\sqrt{3}}}{2}$.

6. $\tan 112.5° = \tan 225°$

$$= \pm\sqrt{\frac{1-\cos 225°}{1+\cos 225°}}$$

$$= \pm\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}}}$$

$$= \pm\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}\cdot\frac{2+\sqrt{2}}{2+\sqrt{2}}}$$

$$= \pm\sqrt{\frac{(2+\sqrt{2})^2}{2}} \text{ or } \pm\left(1+\sqrt{2}\right)$$

Since 112.5° is in Quadrant II,
$\tan 112.5° = -1-\sqrt{2}$.

7. $\sqrt{7^2-2^2} = 3\sqrt{5}$; $\sin\theta = \frac{3\sqrt{5}}{7}$; $\tan\theta = \frac{3\sqrt{5}}{2}$

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2\left(\frac{3\sqrt{5}}{7}\right)\left(\frac{2}{7}\right)$$
$$= \frac{12\sqrt{5}}{49}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= \left(\frac{2}{7}\right)^2 - \left(\frac{3\sqrt{5}}{7}\right)^2$$
$$= -\frac{41}{49}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$
$$= \frac{2\left(\frac{3\sqrt{5}}{2}\right)}{1-\left(\frac{3\sqrt{5}}{2}\right)^2}$$
$$= -\frac{12\sqrt{5}}{41}$$

8. $\sqrt{3^2-2^2} = \sqrt{5}$; $\cos\theta = \frac{\sqrt{5}}{3}$, $\tan\theta = \frac{2\sqrt{5}}{5}$

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2\left(\frac{2}{3}\right)\left(\frac{\sqrt{5}}{3}\right)$$
$$= \frac{4\sqrt{5}}{9}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= \left(\frac{\sqrt{5}}{3}\right)^2 - \left(\frac{2}{3}\right)^2$$
$$= \frac{1}{9}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$
$$= \frac{2\left(\frac{2\sqrt{5}}{5}\right)}{1-\left(\frac{2\sqrt{5}}{5}\right)^2}$$
$$= 4\sqrt{5}$$

9. $\sqrt{(3)^2+(-1)^2} = \sqrt{10}$; $\sin\theta = \frac{3\sqrt{10}}{10}$; $\cos\theta = -\frac{\sqrt{10}}{10}$

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2\left(\frac{3\sqrt{10}}{10}\right)\left(-\frac{\sqrt{10}}{10}\right)$$
$$= -\frac{3}{5}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= \left(-\frac{\sqrt{10}}{10}\right)^2 - \left(\frac{3\sqrt{10}}{10}\right)^2$$
$$= -\frac{4}{5}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$
$$= \frac{2(-3)}{1-(-3)^2}$$
$$= \frac{3}{4}$$

10. $\csc \theta = -\frac{3}{2}$

$\frac{1}{\sin \theta} = -\frac{3}{2}$

$\sin \theta = -\frac{2}{3}$

$\sqrt{3^2 - (-2)^2} = \sqrt{5}$

$\cos \theta = \frac{\sqrt{5}}{3}$

$\tan \theta = -\frac{2\sqrt{5}}{5}$

$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta \\ &= 2\left(-\frac{2}{3}\right)\left(\frac{\sqrt{5}}{3}\right) \\ &= -\frac{4\sqrt{5}}{9}\end{aligned}$$

$$\begin{aligned}\cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= \left(\frac{\sqrt{5}}{3}\right)^2 - \left(-\frac{2}{3}\right)^2 \\ &= \frac{1}{9}\end{aligned}$$

$$\begin{aligned}\tan 2\theta &= \frac{2 \tan \theta}{1 - \tan^2 \theta} \\ &= \frac{2\left(-\frac{2\sqrt{5}}{5}\right)}{1 - \left(-\frac{2\sqrt{5}}{5}\right)^2} \\ &= -4\sqrt{5}\end{aligned}$$

Lesson 7-5

Page A39

1. $4 \cos^2 x - 2 = 0$

$$\cos^2 x = \frac{1}{2}$$

$$\cos x = \pm \frac{\sqrt{2}}{2}$$

$x = 45°, 135°, 225°, 315°$

2. $\sin^2 x \csc x - 1 = 0$

$$\sin^2 x \cdot \frac{1}{\sin} x - 1 = 0$$

$$\sin x = 1$$

$x = 90°$

3.

$$\sqrt{3} \cos x = 2 \cos x$$

$$\sqrt{3} \cdot \frac{\cos x}{\sin x} = 2 \cos x$$

$$\sqrt{3} \cdot \frac{\cos x}{\sin x} - 2 \cos x = 0$$

$$\cos x\left(\frac{\sqrt{3}}{\sin x} - 2\right) = 0$$

$\cos x = 0$ or $\frac{\sqrt{3}}{\sin x} - 2 = 0$

$x = 90°, 270°$

$$\frac{\sqrt{3}}{\sin x} = 2$$

$$\frac{\sqrt{3}}{2} = \sin x$$

$x = 60°, 120°$

4.

$$3 \cos^2 x = 6 \cos x - 3$$

$$3 \cos^2 x - 6 \cos x + 3 = 0$$

$$3(\cos^2 x - 2 \cos x + 1) = 0$$

$$3(\cos x - 1)(\cos x - 1) = 0$$

$$\cos x - 1 = 0$$

$$\cos x = 1$$

$x = 0°$

Lesson 7-6

Page A39

1. $x \cos 30° + y \sin 30° - 12 = 0$

$$\frac{\sqrt{3}}{2x} + \frac{1}{2} y - 12 = 0$$

$$\sqrt{3}x + y - 24 = 0$$

2. $x \cos \frac{\pi}{3} + y \sin \frac{\pi}{3} - 2 = 0$

$$\frac{1}{2} x + \frac{\sqrt{3}}{2} y - 2 = 0$$

$$x + \sqrt{3}y - 4 = 0$$

3. $x \cos 150° + y \sin 150° - \frac{1}{2} = 0$

$$-\frac{\sqrt{3}}{2} x + \frac{1}{2} y - \frac{1}{2} = 0$$

$$-\sqrt{3}x + y - 1 = 0$$

4. $-\sqrt{A^2 + B^2} = -\sqrt{4^2 + 10^2} = -2\sqrt{29}$

$$\frac{4x}{-2\sqrt{29}} + \frac{10y}{-2\sqrt{29}} + \frac{10}{-2\sqrt{29}} = 0$$

$$-\frac{2\sqrt{29}}{29}x - \frac{5\sqrt{29}}{29}y - \frac{5\sqrt{29}}{29} = 0$$

$p = \frac{5\sqrt{29}}{29}$

$\sin \phi = -\frac{5\sqrt{29}}{29}$, $\cos \phi = -\frac{2\sqrt{29}}{29}$

$\tan \phi = \frac{5}{2}$

$$\phi = \text{Arctan}\left(\frac{5}{2}\right) + 180°$$

$$\phi \approx 248°$$

5. $\sqrt{A^2 + B^2} = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$$\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} - \frac{2}{\sqrt{2}} = 0$$

$$\frac{\sqrt{2}}{2} x - \frac{\sqrt{2}}{2} y - \sqrt{2} = 0$$

$p = \sqrt{2}$

$\sin \phi = -\frac{\sqrt{2}}{2}$, $\cos \phi = \frac{\sqrt{2}}{2}$

$\tan \phi = -1$

$$\phi = \text{Arctan}(-1) + 360°$$

$$\phi = 315°$$

6. $\sqrt{A^2 + B^2} = \sqrt{2^2 + 3^2} = \sqrt{13}$

$$\frac{2}{\sqrt{13}} x + \frac{3}{\sqrt{13}} y - \frac{12}{\sqrt{13}} = 0$$

$$\frac{2\sqrt{13}}{13} x + \frac{3\sqrt{13}}{13} y - \frac{12\sqrt{13}}{13} = 0$$

$p = \frac{12\sqrt{13}}{13}$

$\sin \phi = \frac{3\sqrt{13}}{13}$, $\cos \phi = \frac{2\sqrt{13}}{13}$

$\tan \phi = \frac{3}{2}$

$$\phi = \text{Arctan}\,\frac{3}{2}$$

$$\phi \approx 56°$$

Lesson 7-7

Page A39

1. $\frac{3(2) - 2(1) - 2}{\sqrt{3^2 + (-2)^2}} = \frac{2}{\sqrt{13}}$
$= \frac{2\sqrt{13}}{13}$

2. $\frac{2(3) + 4(0) - 2}{\sqrt{2^2 + 4^2}} = \frac{4}{\sqrt{20}}$
$= \frac{4}{2\sqrt{5}}$
$= \frac{2\sqrt{5}}{5}$

3. $\frac{-3(-1) + (-4) - 1}{\sqrt{(-3)^2 + 1^2}} = \frac{-2}{\sqrt{10}}$
$= -\frac{\sqrt{10}}{5}$; $|d| = \frac{\sqrt{10}}{5}$

4. $y = \frac{2}{3}x - 2 \rightarrow 2x - 3y - 6 = 0$
$\frac{2(4) - 3(-2) - 6}{\sqrt{2^2 + (-3)^2}} = \frac{8}{\sqrt{13}}$
$= \frac{8\sqrt{13}}{13}$

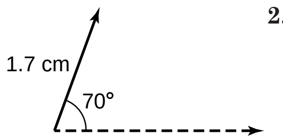
5. $(0, -1)$
$d = \frac{0 + 2(-1) - 4}{\sqrt{1^2 + 2^2}}$
$= \frac{-6}{\sqrt{5}}$
$= -\frac{6\sqrt{5}}{5}$; $|d| = \frac{6\sqrt{5}}{5}$

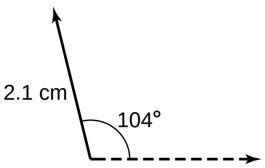
6. $(0, 3)$
$d = \frac{3(3) - 2(0) - 7}{\sqrt{3^2 + (-2)^2}}$
$= \frac{2}{\sqrt{13}}$
$= \frac{2\sqrt{13}}{13}$

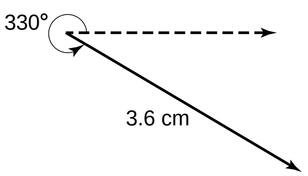
7. $(2, -1)$
$d = \frac{2(2) + 5(-1) - 4}{\sqrt{2^2 + 5^2}}$
$= \frac{-5}{\sqrt{29}}$
≈ -0.9; $|d| \approx 0.9$ unit

Lesson 8-1

Page A40

1. 

2. 

3. 

4. 3.6 cm; 89°
5. 2.6 cm; 23°
6. 3.7 cm; 357°
7. 1.2 cm; 342°
8. 7.2 cm; 330°
9. 8.8 cm; 340°

10. $\cos 70° = \frac{h}{1.7}$ $\quad$ $\sin 70° = \frac{x}{1.7}$
$1.7 \cos 70° = h$ $\quad$ $1.7 \sin 70° = v$
$0.58 \approx h$ $\quad$ $1.60 \approx v$

11. $\cos 76° = \frac{h}{2.1}$ $\quad$ $\sin 76° = \frac{v}{2.1}$
$2.1 \cos 76° = h$ $\quad$ $2.1 \sin 76° = v$
$0.51 \approx h$ $\quad$ $2.04 \approx v$

12. $\cos 30° = \frac{h}{3.6}$ $\quad$ $\sin 30° = \frac{v}{3.6}$
$3.6 \cos 30° = h$ $\quad$ $3.6 \sin 30° = v$
$3.12 \approx h$ $\quad$ $1.8 = v$

Lesson 8-2

Page A40

1. $\overrightarrow{AB} = \langle 4 - 3, 1 - 6\rangle$
$= \langle 1, -5\rangle$
$|\overrightarrow{AB}| = \sqrt{1^2 + (-5)^2}$
$= \sqrt{26}$

2. $\overrightarrow{AB} = \langle -2 - (-1), 2 - 3\rangle$
$= \langle -1, -1\rangle$
$|\overrightarrow{AB}| = \sqrt{(-1)^2 + (-1)^2}$
$= \sqrt{2}$

3. $\overrightarrow{AB} = \langle -1 - 0, -8 - (-4)\rangle$
$= \langle -1, -4\rangle$
$|\overrightarrow{AB}| = \sqrt{(-1)^2 + (-4)^2}$
$= \sqrt{17}$

4. $\overrightarrow{AB} = \langle 3 - 1, -9 - 10\rangle$
$= \langle 2, -19\rangle$
$|\overrightarrow{AB}| = \sqrt{2^2 + (-19)^2}$
$= \sqrt{365}$

5. $\overrightarrow{AB} = \langle -3 - (-6), -6 - 0\rangle$
$= \langle 3, -6\rangle$
$|\overrightarrow{AB}| = \sqrt{3^2 + (-6)^2}$
$= \sqrt{45}$
$= 3\sqrt{5}$

6. $\overrightarrow{AB} = \langle 0 - 4, 7 - (-5)\rangle$
$= \langle -4, 12\rangle$
$|\overrightarrow{AB}| = \sqrt{(-4)^2 + 12^2}$
$= \sqrt{160}$
$= 4\sqrt{10}$

7. $|\langle 5, 6\rangle| = \sqrt{5^2 + 6^2}$
$= \sqrt{61}$
$5\vec{i} + 6\vec{j}$

8. $|\langle -2, 4\rangle| = \sqrt{(-2)^2 + 4^2}$
$= \sqrt{20}$
$= 2\sqrt{5}$
$-2\vec{i} + 4\vec{j}$

9. $|\langle -10, -5\rangle| = \sqrt{(-10)^2 + (-5)^2}$
$= \sqrt{125}$
$= 5\sqrt{5}$
$-10\vec{i} - 5\vec{j}$

10. $|\langle 2.5, 6\rangle| = \sqrt{(2.5)^2 + 6^2}$
$= \sqrt{42.25}$
$= 6.5$
$2.5\vec{i} + 6\vec{j}$

11. $|\langle 2, -6\rangle| = \sqrt{2^2 + (-6)^2}$
$= \sqrt{40}$
$= 2\sqrt{10}$

$2\vec{i} - 6\vec{j}$

12. $|\langle -15, -12\rangle| = \sqrt{(-15)^2 + (-12)^2}$
$= \sqrt{369}$
$= 3\sqrt{41}$

$-15\vec{i} - 12\vec{j}$

Lesson 8-3

Page A40

1. $\vec{p} = 2\langle 1, 2, -1\rangle + 3\langle -4, -3, 0\rangle$
$= \langle 2, 4, -2\rangle + \langle -12, -9, 0\rangle$
$= \langle -10, -5, -2\rangle$

2. $\vec{p} = \langle 1, 2, -1\rangle - \frac{1}{2}\langle -2, 2, 4\rangle + \langle -4, -3, 0\rangle$
$= \langle 1, 2, -1\rangle + \langle 1, -1, -2\rangle + \langle -4, -3, 0\rangle$
$= \langle -2, -2, -3\rangle$

3. $\vec{p} = -2\langle -2, 2, 4\rangle + \langle -4, -3, 0\rangle$
$= \langle 4, -4, -8\rangle + \langle -4, -3, 0\rangle$
$= \langle 0, -7, -8\rangle$

4. $\vec{p} = \frac{3}{4}\langle -4, -3, 0\rangle + 2\langle 1, 2, -1\rangle$
$= \left\langle -3, -\frac{9}{4}, 0\right\rangle + \langle 2, 4, -2\rangle$
$= \left\langle -1, 1\frac{3}{4}, -2\right\rangle$

5. $\langle 2, -4, 1\rangle + \langle 5, 4, 3\rangle + \vec{v} = \langle 0, 0, 0\rangle$
$\langle 7, 0, 4\rangle + \vec{v} = \langle 0, 0, 0\rangle$
$\vec{v} = \langle 0 - 7, 0 - 0, 0 - 4\rangle$
$\vec{v} = \langle -7, 0, -4\rangle$

Lesson 8-4

Page A41

1. $\langle 3, 4\rangle \cdot \langle 2, 5\rangle = 3 \cdot 2 + 4 \cdot 5$
$= 26$; no

2. $\langle -3, 2\rangle \cdot \langle 4, 6\rangle = -3 \cdot 4 + 2 \cdot 6$
$= 0$; yes

3. $\langle -5, 3\rangle \cdot \langle 2, -3\rangle = -5 \cdot 2 + 3 \cdot (-3)$
$= -19$; no

4. $\langle 8, 6\rangle \cdot \langle -2, -3\rangle = 8 \cdot (-2) + 6 \cdot (-3)$
$= -34$; no

5. $\langle 3, 4, 0\rangle \cdot \langle 4, -3, 6\rangle = 3 \cdot 4 + 4 \cdot (-3) + 0 \cdot 6$
$= 0$; yes

6. $\langle 4, 5, 1\rangle \cdot \langle -1, -2, 3\rangle = 4 \cdot (-1) + 5 \cdot (-2) + 1 \cdot 3$
$= -11$; no

7. $\langle 1, 0, 3\rangle \times \langle 1, 1, 2\rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 3 \\ 1 & 1 & 2 \end{vmatrix}$
$= -3\vec{i} + \vec{j} + \vec{k}$
$= \langle -3, 1, 1\rangle$
$\langle -3, 1, 1\rangle \cdot \langle 1, 0, 3\rangle = -3 \cdot 1 + 1 \cdot 0 + 1 \cdot 3$
$= 0$; yes
$\langle -3, 1, 1\rangle \cdot \langle 1, 1, 2\rangle = -3 \cdot 1 + 1 \cdot 1 + 1 \cdot 2$
$= 0$; yes

8. $\langle 3, 0, 4\rangle \times \langle -1, 5, 2\rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 0 & 4 \\ -1 & 5 & 2 \end{vmatrix}$
$= -20\vec{i} - 10\vec{j} + 15\vec{k}$
$= \langle -20, -10, 15\rangle$
$\langle -20, -10, 15\rangle \cdot \langle 3, 0, 4\rangle = -20 \cdot 3 + (-10) \cdot 0 + 15 \cdot 4$
$= 0$; yes
$\langle -20, -10, 15\rangle \cdot \langle -1, 5, 2\rangle = -20 \cdot (-1) + (-10) \cdot 5 + 15 \cdot 2$
$= 0$; yes

9. $\langle -1, 1, 0\rangle \times \langle 2, 1, 3\rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 0 \\ 2 & 1 & 3 \end{vmatrix}$
$= 3\vec{i} + 3\vec{j} - 3\vec{k}$
$= \langle 3, 3, -3\rangle$
$\langle 3, 3, -3\rangle \cdot \langle -1, 1, 0\rangle = 3 \cdot (-1) + 3 \cdot 1 + 1\,(-3) \cdot 0$
$= 0$; yes
$\langle 3, 3, -3\rangle \cdot \langle 2, 1, 3\rangle = 3 \cdot 2 + 3 \cdot 1 + (-3) \cdot 3$
$= 0$; yes

10. $\langle -1, -3, 2\rangle \times \langle 6, -1, -2\rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -3 & 2 \\ 6 & -1 & -2 \end{vmatrix}$
$= 8\vec{i} + 10\vec{j} + 19\vec{k}$
$= \langle 8, 10, 19\rangle$
$\langle 8, 10, 19\rangle \cdot \langle -1, -3, 2\rangle = 8 \cdot (-1) + 10 \cdot (-3) + 19 \cdot 2$
$= 0$; yes
$\langle 8, 10, 19\rangle \cdot \langle 6, -1, -2\rangle = 8 \cdot 6 + 10 \cdot (-1) + 19 \cdot (-2)$
$= 0$; yes

Lesson 8-5

Page A41

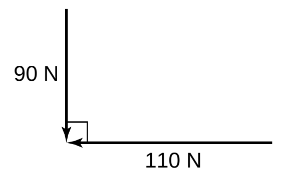
1. magnitude $= \sqrt{200^2 + 220^2}$
≈ 297.32 N

direction: $\tan\theta = \frac{200}{220}$
$\theta \approx 42.3°$

2. $|\vec{r}|^2 = 50^2 + 50^2 - 2(50)(50)\cos 120°$
$|\vec{r}|^2 = 7500$
$|\vec{r}| \approx 86.60$ mph; 30°

3. $|\vec{r}|^2 = 350^2 + 280^2 - 2(350)(280)\cos 135°$
$|\vec{r}| \approx 339{,}492.9291$
$|\vec{r}| \approx 582.66$ N
$\frac{582.66}{\sin 135°} \approx \frac{280}{\sin\theta}$
$\sin\theta \approx \frac{280 \sin 135°}{582.66}$
$\sin\theta \approx 0.34$
$\theta \approx 19.9°$

4. $|\vec{r}| = \sqrt{90^2 + 110^2}$
$|\vec{r}| \approx 142.13$ N



Lesson 8-6

Page A41

1. $\langle x - 2, y - 3\rangle = t\langle 1, 0\rangle$

$x - 2 = t \qquad\qquad y - 3 = 0$

$x = 2 + t \qquad\qquad y = 3$

2. $\langle x - (-1), y - (-4)\rangle = t\langle 5, 2\rangle$

$\langle x + 1, y + 4\rangle = t\langle 5, 2\rangle$

$x + 1 = 5t \qquad\qquad y + 4 = 2t$

$x = -1 + 5t \qquad\qquad y = -4 + 2t$

3. $\langle x - (-3), y - 6\rangle = t\langle -2, 4\rangle$

$\langle x + 3, y - 6\rangle = t\langle -2, 4\rangle$

$x + 3 = -2t \qquad\qquad y - 6 = 4t$

$x = -3 - 2t \qquad\qquad y = 6 + 4t$

4. $\langle x - 3, y - 0\rangle = t\langle 0, -1\rangle$

$x - 3 = 0 \qquad\qquad y - 0 = -t$

$x = 3 \qquad\qquad y = -t$

5. $x = 3t \quad \rightarrow \quad t = \frac{x}{3}$

$y = 2 + t \quad \rightarrow \quad t = y - 2$

$y - 2 = \frac{x}{3}$

$y = \frac{1}{3}x + 2$

6. $x = -1 + 2t \quad \rightarrow \quad t = \frac{x + 1}{2}$

$y = 4t \quad \rightarrow \quad t = \frac{y}{4}$

$\frac{y}{4} = \frac{x}{2} + \frac{1}{2}$

$y = 2x + 2$

7. $x = 3t - 10 \quad \rightarrow \quad t = \frac{x + 10}{3}$

$y = t - 1 \quad \rightarrow \quad t = y + 1$

$y + 1 = \frac{1}{3}x + \frac{10}{3}$

$y = \frac{1}{3}x + \frac{7}{3}$

Lesson 8-7

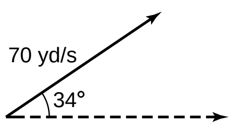
Page A41

1. $v_y = 70 \sin 34°$

≈ 39.14 yd/s

$v_x = 70 \cos 34°$

≈ 58.03 yd/s



2a. $x = t\,|\vec{v}|\cos\theta$

$= 75t \cos 25°$

$y = t\,|\vec{v}|\sin\theta + \frac{1}{2}gt^2 + h$

$= 75t \sin 25° + \frac{1}{2}(-32)t^2 + 5$

$= 5 + 75t \sin 25° - 16t^2$

2b. $y = 0$ when $t = ?$

$t = \frac{-75 \sin 25° \pm \sqrt{(75 \sin 25°)^2 - 4(16)(5)}}{2(-16)}$

$t \approx$

-0.1468595989 or $t \approx 2.127882701$

$t = 2.13$ s

$x = 75t \cos 25°$

$= 75(2.13) \cos 25°$

≈ 145 ft

2c. $y = 5 + 75t \sin 25° - 16t^2$

$= 5 + 75\left(\frac{2.13}{2}\right) \sin 25° - 16\left(\frac{2.13}{2}\right)^2$

≈ 20.6 ft

Lesson 8-8

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1. The figure is 4 times the original size and reflected over the *xy*-plane.

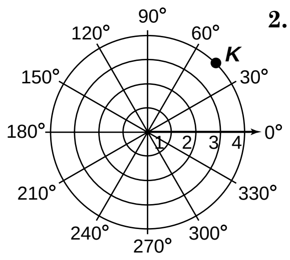
2. The figure is half the original size.

3. The figure is 1.5 times the original size and reflected over the *yz*-plane.

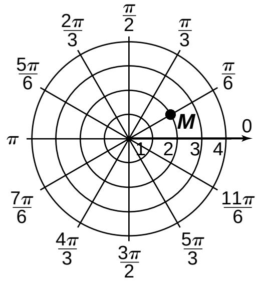
Lesson 9-1

Page A42

1.



2.



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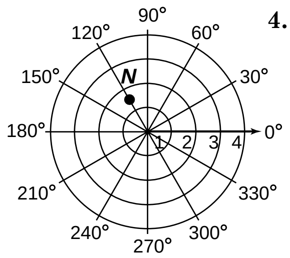

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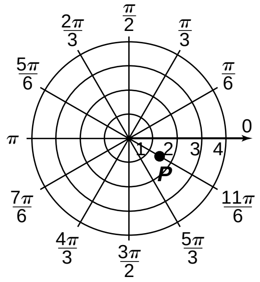

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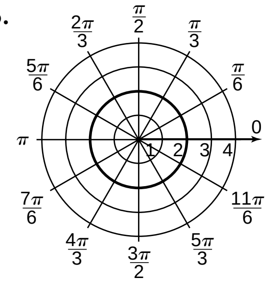
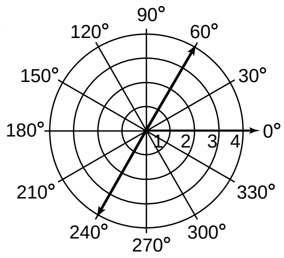


6.



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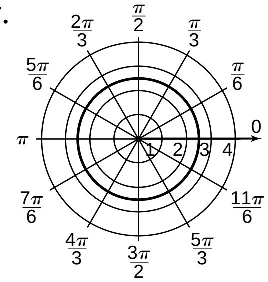
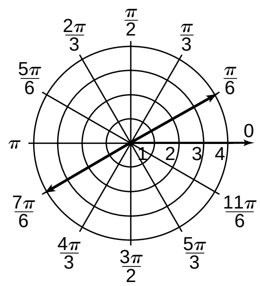


8.

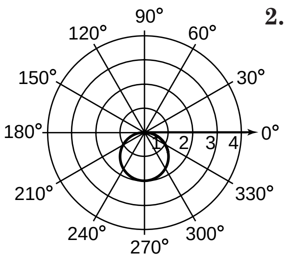


9. $r = \sqrt{5}$ or $r = -\sqrt{5}$

Lesson 9-2

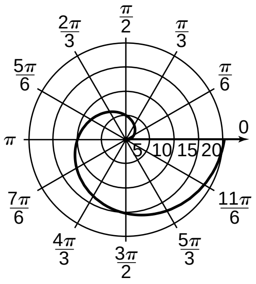
Page A42

1.



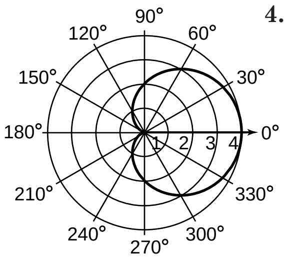
circle

2.



spiral of Archimedes

3.



cardioid

4. Sample answer: $r = \sin 5\theta$

Lesson 9-3

Page A42

1. $r = \sqrt{1^2 + (-1)^2}$ $= \sqrt{2}$ $\left(\sqrt{2}, \frac{7\pi}{4}\right)$ $\theta = \text{Arctan}\left(\frac{-1}{1}\right) + 2\pi$ $= \frac{7\pi}{4}$

2. $r = \sqrt{3^2 + 0^2}$ $= \sqrt{9}$ or 3 (3, 0) $\theta = \text{Arctan}\left(\frac{0}{3}\right)$ $= 0$

3. $r = \sqrt{2^2 + (\sqrt{2})^2}$ $= \sqrt{6}$ or about 2.45 (2.45, 0.62) $\theta = \text{Arctan}\left(\frac{\sqrt{2}}{2}\right)$ ≈ 0.62

4. $x = 2 \cos \frac{\pi}{4}$ $= 2\left(\frac{\sqrt{2}}{2}\right)$ $= \sqrt{2}$ $\left(\sqrt{2}, \sqrt{2}\right)$ $y = 2 \sin \frac{\pi}{4}$ $= 2\left(\frac{\sqrt{2}}{2}\right)$ $= \sqrt{2}$

5. $x = \frac{1}{4} \cos \frac{\pi}{2}$ $= \frac{1}{4}(0)$ $= 0$ $\left(0, \frac{1}{4}\right)$ $y = \frac{1}{4} \sin \frac{\pi}{2}$ $= \frac{1}{4}(1)$ $= \frac{1}{4}$

6. $x = 5 \cos 240°$ $= 5\left(-\frac{1}{2}\right)$ $= -\frac{5}{2}$ or -2.5 $y = 5 \sin 240°$ $= 5\left(-\frac{\sqrt{3}}{2}\right)$ $= -\frac{5\sqrt{3}}{2}$ or about -4.33 $(-2.5, -4.33)$

7. $x = -2$ $r \cos \theta = -2$ $r = -\frac{2}{\cos \theta}$ $r = -2 \sec \theta$

8. $y = 6$ $r \sin \theta = 6$ $r = \frac{6}{\sin \theta}$ $r = 6 \csc \theta$

9. $x^2 + y^2 = 36$ $(r \cos \theta)^2 + (r \sin \theta)^2 = 36$ $r^2 (\cos^2 \theta + \sin^2 \theta) = 36$ $r^2 = 36$ $r = 6$ or $r = -6$

10. $x^2 + y^2 = 3y$ $(r \cos \theta)^2 + (r \sin \theta)^2 = 3r \sin \theta$ $r^2 (\cos^2 \theta + \sin^2 \theta) = 3r \sin \theta$ $r^2 = 3r \sin \theta$ $r = 3 \sin \theta$

11. $r = 4$ $r^2 = 16$ $x^2 + y^2 = 16$

12. $r = 4 \cos \theta$ $r^2 = 4r \cos \theta$ $x^2 + y^2 = 4x$

Lesson 9-4

Page A42

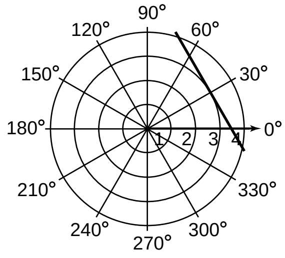
1. $-\sqrt{A^2 + B^2} = -\sqrt{6^2 + (-5)^2}$ $= -\sqrt{61}$ $-\frac{6}{\sqrt{61}}x + \frac{5}{\sqrt{61}}y - \frac{6}{\sqrt{61}} = 0$ $\cos \phi = -\frac{6}{\sqrt{61}}, \sin \phi = \frac{5}{\sqrt{61}}, p = \frac{6\sqrt{61}}{61}$ $\phi = \text{Arctan}\left(-\frac{5}{6}\right) + 180°$ $\approx 140°$ $\frac{6\sqrt{61}}{61} = r \cos (\theta - 140°)$

2. $\sqrt{A^2 + B^2} = \sqrt{3^2 + 9^2}$ $= 3\sqrt{10}$ $\frac{3}{3\sqrt{10}}x + \frac{9}{3\sqrt{10}}y - \frac{90}{3\sqrt{10}} = 0$ $\cos \phi = \frac{\sqrt{10}}{10}, \sin \phi = \frac{3\sqrt{10}}{10}, p = 3\sqrt{10}$ $\phi = \text{Arctan } 3$ $\approx 72°$ $3\sqrt{10} = r \cos (\theta - 72°)$

3. $8 = r \cos (\theta - 30°)$ $0 = r(\cos \theta \cos 30° + \sin \theta \sin 30°) - 8$ $0 = \frac{\sqrt{3}}{2} r \cos \theta + \frac{1}{2} r \sin \theta - 8$ $0 = \frac{\sqrt{3}}{2} x + \frac{1}{2}y - 8$ $0 = \sqrt{3}x + 2y - 16$

4. $1 = r \cos (\theta + \pi)$ $0 = r (\cos \theta \cos \pi - \sin \theta \sin \pi) - 1$ $0 = -r \cos \theta - 0 - 1$ $0 = -x - 1$ $x = 1$

5.



Lesson 9-5

Page A43

1. $i^{-10} = (i^4)^{-3} \cdot i^2$
$= 1^{-3} \cdot (-1)$
$= -1$

2. $i^{17} = (i^4)^4 \cdot i$
$= 1^4 \cdot i$
$= i$

3. $i^{1000} = (i^4)^{250}$
$= 1^{250}$
$= 1$

4. $i^{12} + i^4 = (i^4)^3 + (i^4)^{-1}$
$= 1^3 + 1^{-1}$
$= 2$

5. $(4 - i) + (-3 + 5i) = (4 + (-3)) + (-i + 5i)$
$= 1 + 4i$

6. $(6 + 6i) - (2 + 4i) = \big(6 + (-2)\big) + (6i + (-4i))$
$= 4 + 2i$

7. $(3 + i)(5 - 3i) = 15 - 4i - 3i^2$
$= 18 - 4i$

8. $(2 + 5i)^2 = (2 + 5i)(2 + 5i)$
$= 4 + 20i + 25i^2$
$= -21 + 20i$

9. $(1 - \sqrt{2}i)(-3 - \sqrt{8}i) = -3 - \sqrt{8}i + 3\sqrt{2}i + \sqrt{16}i^2$
$= -3 - 2\sqrt{2}i + 3\sqrt{2}i + 4i^2$
$= -7 + \sqrt{2}i$

10. $\frac{4 + i}{1 - i} = \frac{4 + i}{1 - i} \cdot \frac{1 + i}{1 + i}$
$= \frac{4 + 5i + i^2}{1 - i^2}$
$= \frac{3 + 5i}{2}$
$= \frac{3}{2} + \frac{5}{2}i$

11. $\frac{6 + 2i}{-2 + i} = \frac{6 + 2i}{-2 + i} \cdot \frac{-2 - i}{-2 - i}$
$= \frac{-12 - 10i - 2i^2}{4 - i^2}$
$= \frac{-10 - 10i}{5}$
$= -2 - 2i$

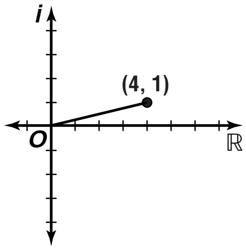
12. $\frac{(i - 2)^2}{4 + 2i} = \frac{i^2 - 4i + 4}{4 + 2i}$
$= \frac{3 - 4i}{4 + 2i} \cdot \frac{4 - 2i}{4 - 2i}$
$= \frac{12 - 22i + 8i^2}{16 - 4i^2}$
$= \frac{4 - 22i}{20}$
$= \frac{1}{5} - \frac{11}{10i}$

Lesson 9-6

Page A43

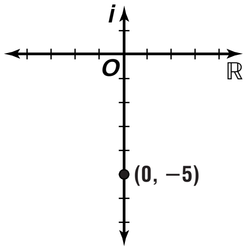
1. $4x - 6yi = 14 + 12i$
$4x = 14$ $\qquad$ $-6y = 12$
$x = \frac{14}{4}$ $\qquad$ $y = -\frac{12}{6}$
$x = 3.5$ $\qquad$ $y = -2$

2.



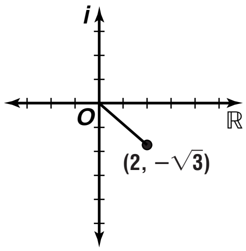
$|z| = \sqrt{4^2 + 1^2}$
$= \sqrt{17}$

3.



$|z| = \sqrt{0^2 + (-5)^2}$
$= \sqrt{25}$ or 5

4.



$|z| = \sqrt{2^2 + (-\sqrt{3})^2}$
$= \sqrt{7}$

5. $r = \sqrt{4^2 + 4^2}$ $\qquad$ $\theta = \text{Arctan}\left(\frac{4}{4}\right)$
$= \sqrt{32}$ or $4\sqrt{2}$ $\qquad$ $= \frac{\pi}{4}$
$4 + 4i = 4\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

6. $r = \sqrt{(-2)^2 + 1^2}$ $\qquad$ $\theta = \text{Arctan}\left(\frac{1}{-2}\right) + \pi$
$= \sqrt{5}$ $\qquad$ ≈ 2.68
$-2 + i = \sqrt{5}(\cos 2.68 + i \sin 2.68)$

7. $r = \sqrt{4^2 + (-\sqrt{2})^2}$ $\qquad$ $\theta = \text{Arctan}\left(-\frac{\sqrt{2}}{4}\right) + 2\pi$
$= \sqrt{18}$ or $3\sqrt{2}$ $\qquad$ ≈ 5.94
$4 - \sqrt{2}i = 3\sqrt{2}\,(\cos 5.94 + i \sin 5.94)$

8a. $5(\cos 0.9 + i \sin 0.9) \approx 3.11 + 3.92j$
$8(\cos 0.4 + j \sin 0.4) \approx 7.37 + 3.12j$

8b. $(3.11 + 3.92j) + (7.37 + 3.12j)$
$= (3.11 + 7.37) + (3.92j + 3.12j)$
$= 10.48 + 7.04j$ ohms

8c. $r = \sqrt{(10.48)^2 + (7.04)^2}$ $\qquad$ $\theta = \text{Arctan}\left(\frac{7.04}{10.48}\right)$
≈ 12.63 $\qquad$ ≈ 0.59
$10.48 + 7.04j = 12.63\,(\cos 0.59 + j \sin 0.59)$ ohms

Lesson 9-7

Page A43

1. $r = 6 \cdot 4$ or 24 $\qquad$ $\theta = \frac{\pi}{2} + \frac{\pi}{4}$
$= \frac{3\pi}{4}$
$24\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) = 24\left(-\frac{\sqrt{2}}{2} + i\left(\frac{\sqrt{2}}{2}\right)\right)$
$= -12\sqrt{2} + 12\sqrt{2}i$

2. $r = \dfrac{3}{\frac{1}{2}}$ or 6 $\qquad \theta = \frac{3\pi}{4} + \pi$

$= \frac{7\pi}{4}$

$6\left(\cos \frac{7\pi}{4} + \boldsymbol{i} \sin \frac{7\pi}{4}\right) = 6\left(\frac{\sqrt{2}}{2} + \boldsymbol{i}\left(-\frac{\sqrt{2}}{2}\right)\right)$

$= 3\sqrt{2} - 3\sqrt{2}\boldsymbol{i}$

3. $r = 5 \cdot 2$ or 10 $\qquad \theta = 135° + 45°$

$= 180°$

$10(\cos 180° + \boldsymbol{i} \sin 180°) = 10(-1 + \boldsymbol{i}(0))$

$= -10$

Lesson 9-8

Page A43

1. $4^4\left(\cos (4)\left(\frac{\pi}{2}\right) + \boldsymbol{i} \sin (4)\left(\frac{\pi}{2}\right)\right)$

$= 256(\cos 2\pi + \boldsymbol{i} \sin 2\pi)$

$= 256(1 + \boldsymbol{i}(0))$

$= 256$

2. $r = \sqrt{12^2 + (-5)^2} \qquad \theta = \text{Arctan}\left(\frac{12}{-5}\right) + \pi$

$= 13 \qquad \approx 1.965587446$

$13^3(\cos (3)(\theta) + \boldsymbol{i} \sin (3)(\theta)) = 2035 - 828\boldsymbol{i}$

3. $r = \sqrt{1^2 + 1^2} \qquad \theta = \text{Arctan}\left(\frac{1}{1}\right)$

$= \sqrt{2} \qquad = \frac{\pi}{4}$

$\sqrt{2}^{\frac{1}{3}}\left(\cos \left(\frac{1}{3}\right)\left(\frac{\pi}{4}\right) + \boldsymbol{i} \sin \left(\frac{1}{3}\right)\left(\frac{\pi}{12}\right)\right)$

$= \sqrt{2}^{\frac{1}{3}}\left(\cos \frac{\pi}{12} + \boldsymbol{i} \sin \frac{\pi}{12}\right)$

$\approx 1.08 + 0.29\boldsymbol{i}$

4. $r = \sqrt{(-1)^2 + 0^2} \qquad \theta = \pi$

$= 1$

$1^{\frac{1}{5}}\left(\cos \left(\frac{1}{5}\right)(\pi) + \boldsymbol{i} \sin \left(\frac{1}{5}\right)(\pi)\right)$

$= 1\left(\cos \frac{\pi}{5} + \boldsymbol{i} \sin \frac{\pi}{5}\right)$

$\approx 0.81 + 0.59\boldsymbol{i}$

Lesson 10-1

Page A44

1. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(4 - (-2))^2 + (5 - 2)^2}$

$d = \sqrt{6^2 + 3^2}$

$d = \sqrt{45}$ or $3\sqrt{5}$

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 4}{2}, \frac{2 + 5}{2}\right)$

$= (1, 3.5)$

2. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(8 - (-3))^2 + (-1 - 6)^2}$

$d = \sqrt{11^2 + (-7)^2}$

$d = \sqrt{170}$

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-3 + 8}{2}, \frac{6 + (-1)}{2}\right)$

$= (2.5, 2.5)$

3. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(r - r)^2 + (-2 - 6)^2}$

$d = \sqrt{0^2 + (-8)^2}$

$d = \sqrt{64}$ or 8

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{r + r}{2}, \frac{6 + (-2)}{2}\right)$

$= (r, 2)$

4. $\left(\frac{6 + x_2}{2}, \frac{2 + y_2}{2}\right) = (-5, 8)$

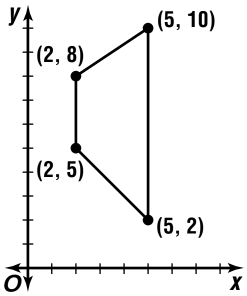
$\frac{6 + x_2}{2} = -5 \qquad \frac{2 + y_2}{2} = 8$

$6 + x_2 = -10 \qquad 2 + y_2 = 16$

$x_2 = -16 \qquad y_2 = 14$

Then A has coordinates $(-16, 14)$.

5. A quadrilateral is a parallelogram if both pairs of opposite sides are parallel. Since only one pair of opposite sides are parallel, the quadrilateral is not a parallelogram.



6a. $\left(\frac{0 + 50}{2}, \frac{0 + 40}{2}\right) = (25, 20)$

6b. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(25 - 0)^2 + (20 - 0)^2}$

$d = \sqrt{25^2 + 20^2}$

$d = \sqrt{1025}$

$d = 5\sqrt{41}$ or about 32 ft

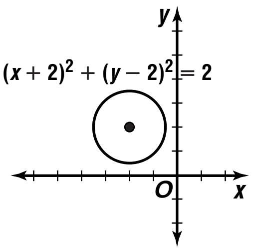
Lesson 10-2

Page A44

1. $(x - h)^2 + (y - k)^2 = r^2$

$[x - (-2)]^2 + (y - 2)^2 = \left(\sqrt{2}\right)^2$

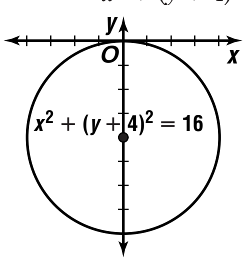
$(x + 2)^2 + (y - 2)^2 = 2$



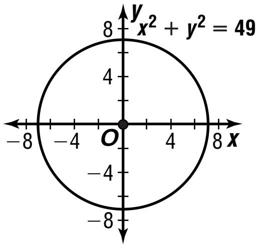
2. $(x - h)^2 + (y - k)^2 = r^2$

$(x - 0)^2 + (y - (-4))^2 = 4^2$

$x^2 + (y + 4)^2 = 16$



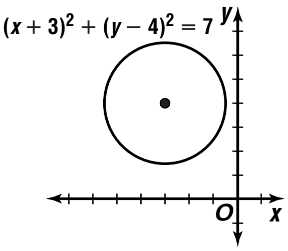
3. $x^2 = 49 - y^2 \to x^2 + y^2 = 49$



4. $x^2 + y^2 + 6x - 8y + 18 = 0$

$(x^2 + 6x + 9) + (y^2 - 8y + 16) = -18 + 9 + 16$

$(x + 3)^2 + (y - 4)^2 = 7$



5. $x^2 + y^2 + Dx + Ey + F = 0$

$2^2 + (-2)^2 + 2D - 2E + F = 0$

$\to 2D - 2E + F = -8$

$0^2 + (-4)^2 + 0(D) - 4E + F = 0$

$\to -4E + F = -16$

$(-2)^2 + (-2)^2 - 2D - 2E + F = 0$

$\to -2D - 2E + F = -8$

$$\begin{array}{rl} 2D - 2E + F &= -8 \\ -2D - 2E + F &= -8 \\ \hline 4D &= 0 \\ D &= 0 \end{array} \qquad \begin{array}{rl} 2(2 \cdot 0 - 2E + F) &= 2(-8) \\ -4E + F &= -16 \\ \hline F &= 0 \end{array}$$

$-4E + (0) = -16$

$-4E = -16$

$E = 4$

$x^2 + y^2 + 4y = 0$ center: $(0, -2)$

$x^2 + (y^2 + 4y + 4) = 0 + 4$ radius: 2

$x^2 + (y + 2)^2 = 4$

6. $x^2 + y^2 + Dx + Ey + F = 0$

$(-1)^2 + 3^2 - D + 3E + F = 0$

$\to -D + 3E + F = -10$

$(-4)^2 + 6^2 - 4D + 6E + F = 0$

$\to -4D + 6E + F = -52$

$(-7)^2 + 3^2 - 7D + 3E + F = 0$

$\to -7D + 3E + F = -58$

$$\begin{array}{rl} 2(-D + 3E + F) &= 2(-10) \\ -4D + 6E + F &= -52 \\ \hline 2D + F &= 32 \end{array} \qquad \begin{array}{rl} -4D + 6E + F &= -52 \\ 2(-7D + 3E + F) &= 2(-58) \\ \hline 10D - F &= 64 \end{array}$$

$$\begin{array}{rl} 2D + F &= 32 \\ 10D - F &= 64 \\ \hline 12D &= 96 \\ D &= 8 \end{array} \qquad \begin{array}{rl} -(8) + 3E + F &= -10 \\ -4(8) + 6E + F &= -52 \\ \hline 24 - 3E &= 42 \\ -3E &= 18 \\ E &= -6 \end{array}$$

$-(8) + 3(-6) + F = -10$

$-26 + F = -10$

$F = 16$

$x^2 + y^2 + 8x - 6y + 16 = 0$

$(x^2 + 8x + 16) + (y^2 - 6y + 9) = -16 + 16 + 9$

$(x + 4)^2 + (y - 3)^2 = 9$

center: $(-4, 3)$

radius: 3

7. $r = \sqrt{(4 - 2)^2 + (0 - (-5))^2}$

$r = \sqrt{2^2 + 5^2}$

$r = \sqrt{29}$; $r^2 = 29$

$(x - 4)^2 + (y - 0)^2 = 29$

$(x - 4)^2 + y^2 = 29$

Lesson 10-3

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1. center: $(h, k) = (0, 0)$

$a = \frac{10}{2}$ or 5

$b = \frac{6}{2}$ or 3

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$\frac{(x-0)^2}{5^2} + \frac{(y-0)^2}{3^2} = 1$

$\frac{x^2}{25} + \frac{y^2}{9} = 1$

$c^2 = a^2 - b^2$

$c^2 = 25 - 9$

$c^2 = 16$

$c = 4$

foci: $(\pm 4, 0)$

2. center: $(h, k) = (-2, 1)$

$a = \frac{8}{2}$ or 4

$b = \frac{4}{2}$ or 2

$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$

$\frac{(y-1)^2}{4^2} + \frac{[x-(-2)]^2}{2^2} = 1$

$\frac{(y-1)^2}{16} + \frac{(x+2)^2}{4} = 1$

$c^2 = a^2 - b^2$

$c^2 = 16 - 4$

$c^2 = 12$

$c = \sqrt{12}$ or $2\sqrt{3}$

foci: $(-2, 1 \pm 2\sqrt{3})$

3. The major axis contains the foci and it is located on the x-axis.

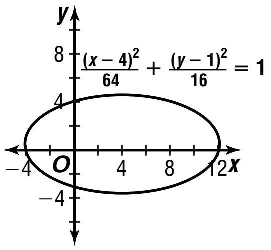
center: $(h, k) = (4, 1)$

$c^2 = a^2 + b^2$

$c^2 = 64 + 16$

$c^2 = 80$

$c = \sqrt{80}$ or $4\sqrt{5}$



foci: $(h \pm c, k) = (4 \pm 4\sqrt{5}, 1)$

major axis vertices: $(h + a, k) = (4 \pm 8, 1)$
$= (12, 1)$ and $(-4, 1)$

minor axis vertices: $(h, k \pm b) = (4, 1 \pm 4)$
$= (4, 5)$ and $(4, -3)$

Lesson 10-4

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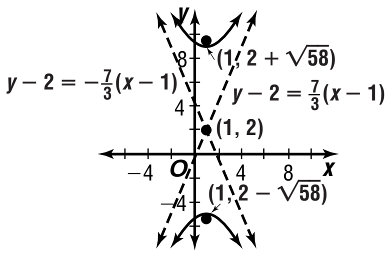
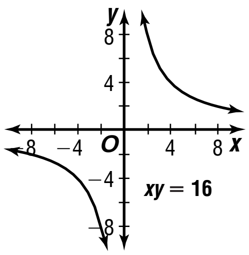
1.



2.



3. center: $(h, k) = (-4, 3)$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{[x-(-4)]^2}{3^2} - \frac{(y-3)^2}{2^2} = 1$$

$$\frac{(x+4)^2}{9} - \frac{(y-3)^2}{4} = 1$$

4. transverse axis: $x = h = 2$

foci: $(h, k + c) = (2, 7)$ $\quad k + c = 7$

$(h, k - c) = (2, -3)$ $\quad k - c = -3$

$$\begin{array}{rl} k + c = & 7 \\ \underline{k - c =} & \underline{-3} \\ 2k \quad = & 4 \end{array}$$

$k = 2; c = 5$

vertices: $(h, k + a) = (2, 5)$ $\quad 2 + a = 5$

$(h, k - a) = (2, -1)$ $\quad a = 3$

$a^2 + b^2 = c^2$

$3^2 + b^2 = 5^2$

$b^2 = 16$

$b = 4$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\frac{(y-2)^2}{3^2} - \frac{(x-2)^2}{4^2} = 1$$

$$\frac{(y-2)^2}{9} - \frac{(x-2)^2}{16} = 1$$

Lesson 10-5

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1. vertex $= (h, k) = (0, 0)$

$4p = 4$

$p = 1$

focus: $(h + p, k) = (0 + 1, 0)$ or $(1, 0)$

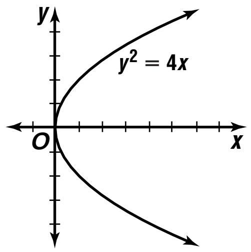
directrix: $x = h - p$

$x = 0 - 1$

$x = -1$

axis of symmetry: $y = k$

$y = 0$



2. $x^2 - 4x + 4 = 12y - 12$

$(x - 2)^2 = 12(y - 1)$

vertex $= (h, k) = (2, 1)$

$4p = 12$

$p = 3$

focus: $(h, k + p) = (2, 1 + 3)$ or $(2, 4)$

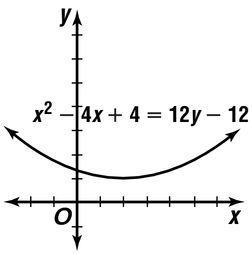
directrix: $y = k - p$

$y = 1 - 3$

$y = -2$

axis of symmetry: $x = h$

$x = 2$



3. vertex: $(h, k) = (-2, 3)$

focus: $(h + p, k) = (0, 3)$

$h + p = 0, k = 3$

$-2 + p = 0$

$p = 2$

$(y - k)^2 = 4p(x - h)$

$(y - 3)^2 = 4(2)[x - (-2)]$

$(y - 3)^2 = 8(x + 2)$

4. directrix: $y = k - p = -3$

focus: $(h, k + p) = (0, -2)$

$$\begin{array}{rl} k - p = & -3 \\ \underline{k + p =} & \underline{-2} \\ 2k \quad = & -5 \end{array}$$

$k = -2.5$

$-2.5 + p = -2$

$p = 0.5$

$(x - h)^2 = 4p(y - k)$

$(x - 0)^2 = 4(0.5)[y - (-2.5)]$

$x^2 = 2(y + 2.5)$

Lesson 10-6

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1. $A = 1$, $C = 1$; since $A = C$, the conic is a circle.
$$x^2 + y^2 - 8x + 2y + 13 = 0$$
$$(x^2 - 8x + 16) + (y^2 + 2y + 1) = -13 + 16 + 1$$
$$(x - 4)^2 + (y + 1)^2 = 4$$

2. $A = 1$, $C = -4$; since A and C have different signs, the conic is a hyperbola.
$$x^2 - 4y^2 + 10x - 16y = -5$$
$$(x^2 + 10x + 25) - 4(y^2 + 4y + 4) = -5 + 25 - 16$$
$$(x + 5)^2 - 4(y + 2)^2 = 4$$
$$\frac{(x + 5)^2}{4} - \frac{(y + 2)^2}{1} = 1$$

3. $A = 0$, $C = 1$; since $A = 0$, the conic is a parabola.
$$y^2 - 5x - 6y + 9 = 0$$
$$(y^2 - 6y + 9) = 5x - 9 + 9$$
$$(y - 3)^2 = 5x$$

4. $A = 1$, $C = 2$; since A and C have the same sign
$$x^2 + 2y^2 + 2x + 8y = 15$$
$$(x^2 + 2x + 1) + 2(y^2 + 4y + 4) = 15 + 1 + 8$$
$$(x + 1)^2 + 2(y + 2)^2 = 24$$
$$\frac{(x + 1)^2}{24} + \frac{(y + 2)^2}{12} = 1$$

Lesson 10-7

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1. $B^2 - 4AC = 0^2 - 4(1)(1)$
$= 0 - 4$ or -4

Since $-4 < 0$ and $A = C$, the graph is a circle.
$$x^2 + y^2 = 9$$
$$(x - 1)^2 + [y - (-1)]^2 = 9$$
$$(x - 1)^2 + (y + 1)^2 = 9$$
$$x^2 - 2x + 1 + y^2 + 2y + 1 = 9$$
$$x^2 + y^2 - 2x + 2y - 7 = 0$$

2. $B^2 - 4AC = 0^2 - 4(4)(1)$
$= 0 - 16$ or -16

Since $-16 < 0$ and $A \neq C$, the graph is an ellipse.
$$4x^2 + y^2 = 16$$
$$4[x - (-3)]^2 + [y - (-2)]^2 = 16$$
$$4(x + 3)^2 + (y + 2)^2 = 16$$
$$4(x^2 + 6x + 9) + (y^2 + 4y + 4) = 16$$
$$4x^2 + y^2 + 24x + 4y + 24 = 0$$

3. $B^2 - 4AC = 0^2 - 4(49)(-16)$
$= 0 + 3136$ or 3136

Since $3136 > 0$, the graph is a hyperbola.
$$49x^2 - 16y^2 = 784$$
$$49\left(x' \cos \tfrac{\pi}{4} + y' \sin \tfrac{\pi}{4}\right)^2 - 16\left(-x' \sin \tfrac{\pi}{4} + y' \cos \tfrac{\pi}{4}\right)^2 = 784$$
$$49\left(\tfrac{\sqrt{2}}{2}x' + \tfrac{\sqrt{2}}{2}y'\right)^2 - 16\left(-\tfrac{\sqrt{2}}{2}x' + \tfrac{\sqrt{2}}{2}y'\right)^2 = 784$$
$$49\left[\tfrac{1}{2}(x')^2 + x'y' + \tfrac{1}{2}(y')^2\right] - 16\left[\tfrac{1}{2}(x')^2 - x'y' + \tfrac{1}{2}(y')^2\right] = 784$$
$$\tfrac{49}{2}(x')^2 + 49x'y' + \tfrac{49}{2}(y')^2 - 8(x')^2 + 16x'y' - 8(y')^2 = 784$$
$$49(x')^2 + 98x'y' + 49(y')^2 - 16(x')^2 + 32x'y' - 16(y')^2 = 1568$$
$$33(x')^2 + 130x'y' + 33(y')^2 - 1568 = 0$$

4. $B^2 - 4AC = 0^2 - 4(4)(-25)$
$= 0 + 400$ or 400

Since $400 > 0$, the graph is a hyperbola.
$$4x^2 - 25y^2 = 64$$
$$4(x' \cos 90° + y' \sin 90°)^2 - 25(-x' \sin 90° + y' \cos 90°)^2 = 64$$
$$4(0 + y')^2 - 25(-x' + 0)^2 = 64$$
$$4(y')^2 - 25(x')^2 - 64 = 0$$

5. $B^2 - 4AC = \left(-2\sqrt{2}\right)^2 - 4(1)(2)$
$= 8 - 8$ or 0

parabola

$\tan 2\theta = \frac{B}{A - C}$

$\tan 2\theta = \frac{-2\sqrt{2}}{1 - 2}$

$\tan 2\theta = 2\sqrt{2}$

$2\theta \approx 70.52877937°$

$\theta \approx 35, -35°$

6. $B^2 - 4AC = 5^2 - 4(15)(5)$
$= 25 - 300$ or -275

Since $-275 < 0$ and $A \neq C$, the graph is an ellipse.

$\tan 2\theta = \frac{B}{A - C}$

$\tan 2\theta = \frac{5}{15 - 5}$

$\tan 2\theta = \frac{1}{2}$

$2\theta \approx 26.56505118°$

$\theta \approx 13°$

Lesson 10-8

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1. $xy = 3$

$y = \frac{3}{x}$
$$x^2 - y^2 = 8$$
$$x^2 - \left(\frac{3}{x}\right)^2 = 8$$
$$x^2 - \frac{9}{x^2} = 8$$
$$x^4 - 9 = 8x^2$$
$$x^4 - 8x^2 - 9 = 0$$
$$(x^2 - 9)(x^2 + 1) = 0$$

$x^2 - 9 = 0$ or $x^2 + 1 = 0$

$x^2 = 9$ $\quad$ $x^2 = -1$

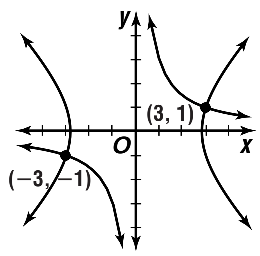
$x = \pm 3$ $\quad$ $x = \sqrt{-1}$ or $\boldsymbol{i}$

If $x = 3$, then $y = \frac{3}{(3)}$ or 1.

If $x = -3$, then $y = \frac{3}{(-3)}$ or -1.

Since $x = \sqrt{-1}$ is an imaginary number, disregard this solution.

$(3, 1), (-3, -1)$



2. $x - y = 4$

$x = y + 4$

$$x^2 = 10y^2 + 10$$
$$(y + 4)^2 = 10y^2 + 10$$
$$y^2 + 8y + 16 = 10y^2 + 10$$
$$9y^2 - 8y - 6 = 0$$

$y = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(9)(-6)}}{}$

$y = \frac{8 \pm 2\sqrt{70}}{18}$

$y = \frac{4 \pm \sqrt{70}}{9}$

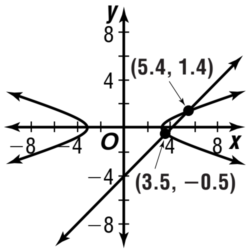
$y = \frac{4 \pm \sqrt{70}}{9}$ or $y = \frac{4 - \sqrt{70}}{9}$

$y \approx 1.4$ $\quad$ $y \approx -0.5$

If $y = 1.4$, then $x = (1.4) + 4$ or 5.4.

If $y = -0.5$, then $x = (-0.5) + 4$ or 3.5.

$(5.4, 1.4), (3.5, -0.5)$



Lesson 11-1

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1. $(-12)^{-2} = \frac{1}{(-12)^2} = \frac{1}{144}$

2. $-12^{-2} = -\frac{1}{12^2} = -\frac{1}{144}$

3. $(4 \cdot 6)^3 = 4^3 \cdot 6^3 = 64 \cdot 216 = 13{,}824$

4. $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}$

5. $\dfrac{16}{16^{\frac{1}{2}}} = \dfrac{16}{(4^2)^{\frac{1}{2}}} = \frac{16}{4}$ or 4

6. $27^{\frac{1}{2}} \cdot 20^{\frac{1}{2}} = (3^2 \cdot 3)^{\frac{1}{2}} \cdot (2^2 \cdot 5)^{\frac{1}{2}}$
$= (3^2)^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} \cdot (2^2)^{\frac{1}{2}} \cdot 5^{\frac{1}{2}}$
$= 3 \cdot 2 \cdot 15^{\frac{1}{2}}$
$= 6\sqrt{15}$

7. $\left(\sqrt[4]{625}\right)^2 = 625^{\frac{2}{4}} = 625^{\frac{1}{2}} = \sqrt{625}$ or 25

8. $\dfrac{1}{\sqrt[3]{(15)^6}} = \dfrac{1}{15^{\frac{6}{3}}} = \frac{1}{15^2} = \frac{1}{225}$

9. $(2a^4)^2 = 2^2 \cdot (a^4)^2 = 4a^8$

10. $(x^4)^3 \cdot x^5 = x^{12} \cdot x^5 = x^{17}$

11. $\left((3f)^{-2}\right)^3 = (3f)^{-6} = \frac{1}{(3f)^6} = \frac{1}{3^6 \cdot f^6} = \frac{1}{729f^6}$

12. $\left(\frac{c^{-3a}}{c^{4a}}\right)^2 = \frac{c^{-6a}}{c^{8a}} = c^{-14a} = \frac{1}{c^{14a}}$

13. $(2n^{\frac{1}{3}} \cdot 3n^{\frac{1}{2}})^6 = 2^6 n^{\frac{6}{3}} \cdot 3^6 n^{\frac{6}{2}}$
$= 64n^2 \cdot 799n^3$
$= 46{,}656n^5$

14. $\left(\frac{h^6}{216h^{-3}}\right)^{-\frac{1}{3}} = \left(\frac{h^9}{216}\right)^{-\frac{1}{3}}$
$= \dfrac{h^{-\frac{9}{3}}}{216^{-\frac{1}{3}}}$
$= \dfrac{216^{\frac{1}{3}}}{h^3}$
$= \frac{\sqrt[3]{216}}{h^3}$
$= \frac{6}{h^3}$

15. $\sqrt[3]{z^4(z^4)^{\frac{1}{2}}} = \sqrt[3]{z^4 \cdot z^2} = \sqrt[3]{z^6} = z^{\frac{6}{3}} = z^2$

16. $(4r^2t^5)(16r^4t^8)^{\frac{1}{4}} = \left(4r^2t^5\right)(16^{\frac{1}{4}}r^{\frac{4}{4}}t^{\frac{8}{4}})$
$= (4r^2t^5)(2rt^2)$
$= 8r^3t^7$

17. $\sqrt{a^3b^5} = a^{\frac{3}{2}}b^{\frac{5}{2}}$

18. $\sqrt[3]{64m^9n^6} = 64^{\frac{1}{3}}m^{\frac{9}{3}}n^{\frac{6}{3}}$
$= (4^3)^{\frac{1}{3}}m^3n^2$
$= 4m^3n^2$

19. $15\sqrt[3]{r^{12}t^2} = 15r^{\frac{12}{3}}t^{\frac{2}{3}} = 15r^4t^{\frac{2}{3}}$

20. $\sqrt[8]{256x^2y^{16}} = 256^{\frac{1}{8}}x^{\frac{2}{3}}y^{\frac{16}{8}}$
$= (2^8)^{\frac{1}{8}}x^{\frac{1}{4}}y^2$
$= 2x^{\frac{1}{4}}y^2$

Lesson 11-2

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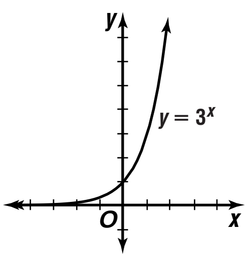
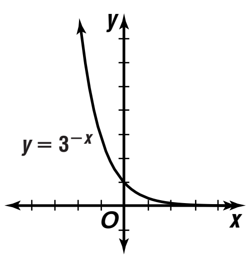
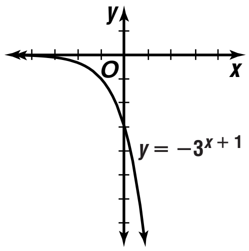
1.



2.



3.



Lesson 11-3

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1. $p = (100 - a)e^{-bt} + a$
$p = (100 - 18)e^{-0.6(2)} + 18$
$\approx 42.7\%$

2. $y = ae^{-kt} + c$
$y = 140e^{-0.01(10)} + 70$
$\approx 197°\text{ F}$

3a. $y = 6.7e^{\frac{-48.1}{t}}$
$y = 6.7e^{\frac{-48.1}{15}}$
$y \approx 0.271292$ millions of cubic feet
$y \approx 271{,}292\text{ ft}^3$

3b. $y = 6.7e^{\frac{-48.1}{t}}$
$y = 6.7e^{\frac{-48.1}{50}}$
$y \approx 2.560257$ millions of cubic feet
$y \approx 2{,}256{,}275\text{ ft}^3$

4.

Continuously	Semiannually
$A = Pe^{rt}$	$A = \text{P}\left(1 + \frac{r}{n}\right)^{nt}$
$A = 5000e^{0.058(20)}$	$A = 5000\left(1 + \frac{0.058}{2}\right)^{2(20)}$
$A = \$15{,}949.67$	$A = \$15{,}688.63$

The account that compounds continuously would earn \$261.04 more than the account compounded semiannually.

Lesson 11-4

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1. $16^{\frac{1}{4}} = 2$

2. $\left(\frac{1}{2}\right)^{-3} = 8$

3. $4^{-1} = \frac{1}{4}$

4. $\log_8 x = -2$

5. $\log_x 32 = 5$

6. $\log_{\frac{1}{4}} 16 = -2$

7. $\log_5 \frac{1}{5} = x$
$5^x = \frac{1}{5}$
$5^x = 5^{-1}$
$x = -1$

8. $\log_3 27 = x$
$3^x = 27$
$3^x = 3^3$
$x = 3$

9. $\log_{36} 6 = x$
$36^x = 6$
$(6^2)^x = 6$
$6^{2x} = 6^1$
$2x = 1$
$x = \frac{1}{2}$

10. $\log_3 y = 4$
$3^4 = y$
$81 = y$

11. $\log_5 r = \log_5 8$
$r = 8$

12. $\log_5 35 - \log_5 d = \log_5 5$
$\log_5 \frac{35}{d} = \log_5 5$
$\frac{35}{d} = 5$
$7 = d$

13. $\log_4 \sqrt{4} = x$
$\log_4 4^{\frac{1}{2}} = x$
$\frac{1}{2}\log_4 4 = x$
$\frac{1}{2} = x$

14. $\log_4 (2x + 3) = \log_4 15$
$2x + 3 = 15$
$2x = 12$
$x = 6$

15. $4 \log_8 2 + \frac{1}{3} \log_8 27 = \log_8 a$
$\log_8 2^4 + \log_8 27^{\frac{1}{3}} = \log_8 a$
$\log_8 16 + \log_8 3 = \log_8 a$
$\log_8 48 = \log_8 a$
$48 = a$

Lesson 11-5

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1. $\log 5000 = \log (5 \cdot 1000)$
$= \log 5 + \log 10^3$
$= \log 5 + 3 \log 10$
$= 0.6990 + 3$
$= 3.6990$

2. $\log 0.0008 = \log (0.0001 \cdot 8)$
$= \log 10^{-4} + \log 8$
$= -4 \log 10 + \log 8$
$= -4 + 0.9031$
$= -\ 3.0969$

3. $\log 0.14 = \log (0.01 \cdot 14)$
$= \log 10^{-2} + \log 14$
$= -2 \log 10 + \log 14$
$= -2 + 1.1461$
$= -0.8539$

4. $\log_3 81 = \frac{\log 81}{\log 3}$
$= 4$

5. $\log_6 12 = \frac{\log 12}{\log 6}$
≈ 1.3869

6. $\log_5 29 = \frac{\log 29}{\log 5}$
≈ 2.0922

7. $3^x = 45$
$x \log 3 = \log 45$
$x = \frac{\log 45}{\log 3}$
$x \approx 3.4650$

8. $6^x = 2^{x-1}$
$x \log 6 = (x - 1) \log 2$
$x \log 6 = x \log 2 - \log 2$
$x \log 6 - x \log 2 = -\log 2$
$x (\log 6 - \log 2) = -\log 2$
$x = \frac{-\log 2}{\log 6 - \log 2}$
$x \approx -0.6309$

9. $5 \log y = \log 32$
$\log y^5 = \log 32$
$y^5 = 32$
$y^5 = 2^5$
$y = 2$

Lesson 11-6

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1. 3.5553
2. -0.5763
3. 3.4398
4. $\log_{15} 10 = \frac{\ln 10}{\ln 15}$
≈ 0.8503
5. $\log_3 14 = \frac{\ln 14}{\ln 3}$
≈ 2.4022
6. $\log_8 350 = \frac{\ln 350}{\ln 8}$
≈ 2.8171
7. $5^x = 90$
$x \ln 5 = \ln 90$
$x = \frac{\ln 90}{\ln 5}$
$x \approx 2.7959$
8. $7^{x+2} = 5.25$
$(x + 2) \ln 7 = \ln 5.25$
$x \ln 7 + 2 \ln 7 = \ln 5.25$
$x \ln 7 = \ln 5.25 - 2 \ln 7$
$x = \frac{\ln 5.25 - 2 \ln 7}{\ln 7}$
$x \approx -1.1478$
9. $4^x = 4\sqrt{3}$
$x \ln 4 = \ln 4\sqrt{3}$
$x = \frac{\ln 4\sqrt{3}}{\ln 4}$
$x \approx 1.3962$
10. $6e^x = 48$
$e^x = 8$
$x \ln e = \ln 8$
$x \approx 2.0794$
11. $50.2 < e^{0.2x}$
$\ln 50.2 < 0.2x \ln e$
$\frac{\ln 50.2}{0.2} < x$
$x > 19.5801$
12. $16 = 10(1 + e^x)$
$1.6 = 1 + e^x$
$0.6 = e^x$
$\ln 0.6 = x \ln e$
$-0.5108 \approx x$

Lesson 11-7

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1. $t = \frac{\ln 2}{0.045}$
≈ 15.40 yr
2. $t = \frac{\ln 2}{0.06}$
≈ 11.55 yr
3. $t = \frac{\ln 2}{0.08125}$
≈ 8.53 yr

4a. $y = 5.2449(1.5524)^x$

4b. $y = 5.2449(e^{\ln 1.5524})^x$
$y \approx 5.2449e^{0.4398x}$

4c. Use $t = \frac{\ln 2}{k}$; $k = 0.4398$
$t = \frac{\ln 2}{0.4398}$
≈ 1.58 hr

Lesson 12-1

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1. $d = 3 - 7$ or -4
$-1 + (-4) = -5, -5 + (-4) = -9,$
$-9 + (-4) = -13, -13 + (-4) = -17$
$-5, -9, -13, -17$
2. $d = -1 - 0.5$ or -1.5
$-2.5 + (-1.5) = -4, -4 + (-1.5) = -5.5,$
$-5.5 + (-1.5) = -7, -7 + (-1.5) = -8.5$
$-4, -5.5, -7, -8.5$
3. $d = -8 - (-14)$ or 6
$-2 + 6 = 4, 4 + 6 = 10,$
$10 + 6 = 16, 16 + 6 = 22$
4, 10, 16, 22
4. $d = 2.8 - 3$ or -0.2
$2.6 + (-0.2) = 2.4, 2.4 + (-0.2) = 2.2,$
$2.2 + (-0.2) = 2, 2 + (-0.2) = 1.8$
2.4, 2.2, 2, 1.8
5. $d = -x - 4x$ or $-5x$
$-6x + (-5x) = -11x, -11x + (-5x) = -16x,$
$-16x + (-5x) = -21x, -21x + (-5x) = -26x$
$-11x, -16x, -21x, -26x$
6. $d = (2y - 2) - (2y - 4)$
$= 2y - 2y + (-2) - (-4)$
$= 2$
$2y + 2, 2y + 2 + 2 = 2y + 4,$
$2y + 4 + 2 = 2y + 6, 2y + 6 + 2 = 2y + 8$
$2y + 2, 2y + 4, 2y + 6, 2y + 8$

7. $a_n = a_1 + (n-1)d$
$a_{16} = 2 + (16-1)5$
$= 77$

8. $-20 = 6 + (n-1)(-2)$
$-26 = -2n + 2$
$-28 = -2n$
$14 = n$

9. $42 = a_1 + (12-1)4$
$42 = a_1 + 44$
$-2 = a_1$

10. $30 = 7 + (13-1)d$
$23 = 12d$
$1\frac{11}{12} = d$

11. $d = 10 - 10.5$ or -0.5
$a_{24} = 10.5 + (24-1)(-0.5)$
$a_{24} = -1$

12. $S_n = \frac{n}{2}[2a_1 + (n-1)d]$; $d = 2.8 - 2$ or 0.8
$S_{12} = \frac{12}{2}[2 \cdot 2 + (12-1)(0.8)]$
$= 6(4 + 8.8)$
$= 76.8$

13. $S^n = \frac{n}{2}[2a_1 + (n-1)d]$
$80 = \frac{n}{2}[2 \cdot (-4) + (n-1)4]$
$160 = n(-8 + 4n - 4)$
$160 = -12n + 4n^2$
$0 = 4(n^2 - 3n - 40)$
$0 = 4(n-8)(n+5)$
$n = 8$ or $n = -5$
Since n cannot be negative, $n = 8$.

Lesson 12-2

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1. $r = \frac{7}{14}$ or 0.5
$3.5(0.5) = 1.75$, $1.75(0.5) = 0.875$,
$0.875(0.5) = 0.4375$
$1.75, 0.875, 0.4375$

2. $r = \frac{4}{-2}$ or -2
$-8(-2) = 16$, $16(-2) = -32$, $-32(-2) = 64$
$16, -32, 64$

3. $r = \frac{\frac{3}{8}}{\frac{1}{2}}$ or $\frac{3}{4}$
$\frac{3}{8}\left(\frac{3}{4}\right) = \frac{9}{32}$, $\frac{9}{32}\left(\frac{3}{4}\right) = \frac{27}{128}$, $\frac{27}{128}\left(\frac{3}{4}\right) = \frac{81}{512}$
$\frac{9}{32}, \frac{27}{128}, \frac{81}{512}$

4. $r = \frac{-5}{10}$ or -0.5
$2.5(-0.5) = -1.25$, $-1.25(-0.5) = 0.625$,
$0.625(-0.5) = -0.3125$
$-1.25, 0.625, -0.3125$

5. $r = \frac{8\sqrt{2}}{8}$ or $\sqrt{2}$
$16\left(\sqrt{2}\right) = 16\sqrt{2}$, $16\sqrt{2}\left(\sqrt{2}\right) = 32$, $32\left(\sqrt{2}\right) = 32\sqrt{2}$
$16\sqrt{2}, 32, 32\sqrt{2}$

6. $r = \frac{a^8}{a^{10}}$ or $\frac{1}{a^2}$
$a^6\left(\frac{1}{a^2}\right) = a^4$, $a^4\left(\frac{1}{a^2}\right) = a^2$, $a^2\left(\frac{1}{a^2}\right) = 1$
$a^4, a^2, 1$

7. $a_n = a_1 r^{n-1}$
$a_6 = 9(2)^{6-1}$
$= 288$

8. $100 = a_1(4)^{8-1}$
$100 = 16{,}384\, a_1$
$\frac{25}{4096} = a_1$

9. $10 = a_1\left(-\frac{1}{2}\right)^{5-1}$
$10 = \frac{1}{16}a_1$
$160 = a_1$
$a_2 = 160\left(-\frac{1}{2}\right)$ or -80
$a_3 = -80\left(-\frac{1}{2}\right)$ or 40
$160, -80, 40$

10. $256 = 4r^{4-1}$
$64 = r^3$
$4 = r$
$4(4) = 16$, $16(4) = 64$
$4, 16, 64, 256$

11. $S_n = \frac{a_1 - a_1 r^n}{1 - r}$; $r = \frac{9}{3}$ or 3
$S_6 = \frac{3 - 3(3)^6}{1 - 3}$
$= \frac{3 - 2187}{-2}$
$= 1092$

12a. There are four 15-minute periods in an hour, so $r = 2^4$ or 16.
$b_t = b_0 \cdot 16^t$

12b. $b_4 = 12 \cdot 16^4$
$= 786{,}432$

Lesson 12-3

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1. $\lim\limits_{n\to\infty} \frac{4 + 2n}{3n} = \lim\limits_{n\to\infty} \frac{4}{3n} + \lim\limits_{n\to\infty} \frac{2n}{3n}$
$= 0 + \frac{2}{3}$
$= \frac{2}{3}$

2. Limit does not exist. $\lim\limits_{n\to\infty} \frac{n^4 - 3n}{n^3} = \lim\limits_{n\to\infty}\left(n - \frac{3}{n^2}\right)$
$= \lim\limits_{n\to\infty} n$. But as n approaches infinity, n becomes increasingly large, so there is no limit.

3. $\lim\limits_{n\to\infty} \frac{8n^2 + 6n - 2}{4n^2} = \lim\limits_{n\to\infty} \frac{8n^3}{4n^2} + \lim\limits_{n\to\infty} \frac{6n}{4n^2} + \lim\limits_{n\to\infty} \frac{-2}{4n^2}$
$= 2 + 0 + 0$
$= 2$

4. $\lim\limits_{n\to\infty} \frac{4n^2 - 2n + 1}{n^2 + 2} = \lim\limits_{n\to\infty} \frac{\frac{4n^2}{n^2} - \frac{2n}{n^2} + \frac{1}{n^2}}{\frac{n^2}{n^2} + \frac{2}{n^2}}$
$= \frac{4 - 0 + 0}{1 + 0}$
$= 4$

5. $\lim_{n\to\infty} \frac{n^3 - n^2 + 4}{5 + 2n^3} = \lim_{n\to\infty} \frac{\frac{n^3}{n^3} - \frac{n^2}{n^3} + \frac{4}{n^3}}{\frac{5}{n^3} + \frac{2n^3}{n^3}}$
$= \frac{1 - 0 + 0}{0 + 2}$
$= \frac{1}{2}$

6. Limit does not exist.
$\lim_{n\to\infty} \frac{2^n n}{2 + n} = \lim_{n\to\infty} \left(2^n \frac{n}{2 + n}\right) = \lim_{n\to\infty} \left(2^n \frac{1}{\frac{2}{n} + 1}\right) =$
$\lim_{n\to\infty} (2^n \cdot 1)$ or $\lim_{n\to\infty} 2^n$ As n approaches infinity, 2^n becomes increasingly large, so there is no limit.

7. $0.\overline{09} = \frac{9}{100} + \frac{9}{10{,}000} + \ldots$
$a_1 = \frac{9}{100}, r = \frac{1}{100}$
$S_n = \frac{\frac{9}{100}}{1 - \frac{1}{100}}$
$= \frac{1}{11}$

8. $0.1\overline{3} = \frac{1}{10} + \frac{3}{100} + \frac{3}{1000} + \ldots$
$a_1 = \frac{3}{100}, r = \frac{1}{10}$
$S_n = -\frac{1}{10} + \frac{\frac{3}{100}}{1 - \frac{1}{10}}$
$= \frac{2}{15}$

9. $7.\overline{407} = 7 + \frac{407}{1000} + \frac{407}{1{,}000{,}000} + \ldots$
$a_1 = \frac{407}{1000}, r = \frac{1}{1000}$
$S_n = 7 + \frac{\frac{407}{1000}}{1 - \frac{1}{1000}}$
$= 7\frac{11}{27}$

10. $a_1 = \frac{1}{20}, r = \frac{\frac{1}{40}}{\frac{1}{20}}$ or $\frac{1}{2}$
$S_n = \frac{\frac{1}{20}}{1 - \frac{1}{2}}$
$= \frac{2}{20}$ or $\frac{1}{10}$

11. S_n does not exist. This series is geometric with a common ratio of 2. Since this ratio is greater than 1, the sum of the series does not exist.

Lesson 12-4

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1. $a_n = (2^n)^2, a_{n+1} = (2^{n+1})^2$
$= 2^{2n} \qquad = 2^{2n+2}$
$r = \lim_{n\to\infty} \frac{2^{2n+2}}{2^{2n}}$
$= \lim_{n\to\infty} \frac{2^{2n} \cdot 2^2}{2^{2n}}$
$= 4$
divergent

2. $a_n = \frac{n}{3}, a_{n+1} = \frac{n+1}{3}$
$r = \lim_{n\to\infty} \frac{\frac{n+1}{3}}{\frac{n}{3}}$
$= \lim_{n\to\infty} \frac{n+1}{n}$
$= \lim_{n\to\infty} \frac{n}{n} + \lim_{n\to\infty} \frac{1}{n}$
$= 1 + 0$
$= 1$
test provides no information

3. $a_n = \frac{1}{3^n(n+1)^3}, a_{n+1} = \frac{1}{3^{n+1}(n+1+1)^3}$
$r = \lim_{n\to\infty} \frac{\frac{1}{3^{n+1}(n+2)^3}}{\frac{1}{3^n(n+1)^3}}$
$= \lim_{n\to\infty} \frac{3^n (n^3 + 3n^2 + 3n + 1)}{3^n \cdot 3(n^3 + 6n^2 + 12n + 8)}$
$= \lim_{n\to\infty} \frac{\frac{n^3}{n^3} + \frac{3n^2}{n^3} + \frac{3n}{n^3} + \frac{1}{n^3}}{\frac{3n^3}{n^3} + \frac{6n^2}{n^3} + \frac{12n}{n^3} + \frac{8}{n^3}}$
$= \frac{1 + 0 + 0 + 0}{3 + 0 + 0 + 0}$
$= \frac{1}{3}$
convergent

4. $a_n = \frac{4}{2^n}, a_{n+1} = \frac{4}{2^{n+1}}$
$r = \lim_{n\to\infty} \frac{\frac{4}{2^{n+1}}}{\frac{4}{2^n}}$
$= \lim_{n\to\infty} \frac{2^n}{2^n \cdot 2}$
$= \frac{1}{2}$
convergent

5. The general term is $\frac{7}{6n+1}$.
$\frac{7}{6n+1} > \frac{1}{n}$ for all n, so divergent

6. The general term is $\frac{1}{(2n)^2}$ or $\frac{1}{4n^2}$.
$\frac{1}{4n^2} \leq \frac{1}{n}$ for all n, so convergent.

7. $a_n = \frac{1}{2^n + 1}, a_{n+1} = \frac{1}{2^{n+1} + 1}$
$r = \lim_{n\to\infty} \frac{\frac{1}{2^{n+1} + 1}}{\frac{1}{2^n + 1}}$
$= \lim_{n\to\infty} \frac{2^n + 1}{2^n \cdot 2 + 1}$
$= \lim_{n\to\infty} \frac{\frac{2^n}{2^n} + \frac{1}{2^n}}{\frac{2^n \cdot 2}{2^n} + \frac{1}{2^n}}$
$= \frac{1 + 0}{2 + 0}$
$= \frac{1}{2}$
convergent

8. $a_n = \frac{2^n}{n+2}$, $a_{n+1} = \frac{2^{n+1}}{(n+1)+2}$

$$r = \lim_{n\to\infty} \frac{\frac{2^{n+1}}{n+3}}{\frac{2^n}{n+2}}$$
$$= \lim_{n\to\infty} \frac{2^n \cdot 2(n+2)}{2^n\,(n+3)}$$
$$= \lim_{n\to\infty} \frac{2n+2}{n+3}$$
$$= \lim_{n\to\infty} \frac{\frac{2n}{n}+\frac{2}{n}}{\frac{n}{n}+\frac{3}{n}}$$
$$= \frac{2+0}{1+0}$$
$$= 2$$

divergent

Lesson 12-5

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1. $\sum_{n=1}^{5} (3n-1) = (3\cdot 1-1) + (3\cdot 2-1) + (3\cdot 3-1) + (3\cdot 4-1) + (3\cdot 5-1)$
$= 2 + 5 + 8 + 11 + 14$
$= 40$

2. $\sum_{k=3}^{6} 4a = 4\cdot 3 + 4\cdot 4 + 4\cdot 5 + 4\cdot 6$
$= 12 + 16 + 20 + 24$
$= 72$

3. $\sum_{k=3}^{7} (k^2-2) = (3^2-2) + (4^2-2) + (5^2-2) + (6^2-2) + (7^2-2)$
$= 7 + 14 + 23 + 34 + 47$
$= 125$

4. $\sum_{j=4}^{8} \frac{j}{j+3} = \frac{4}{4+3} + \frac{5}{5+3} + \frac{6}{6+3} + \frac{7}{7+3} + \frac{8}{8+3}$
$= \frac{4}{7} + \frac{5}{8} + \frac{6}{9} + \frac{7}{10} + \frac{8}{11}$
$= 3\frac{2683}{9240}$

5. $\sum_{p=0}^{4} 3^p = 3^0 + 3^1 + 3^2 + 3^3 + 3^4$
$= 1 + 3 + 9 + 27 + 81$
$= 121$

6. $\sum_{n=1}^{\infty} 2\cdot\left(\frac{3}{4}\right)^n = 2\cdot\left(\frac{3}{4}\right)^1 + 2\cdot\left(\frac{3}{4}\right)^2 + 2\cdot\left(\frac{3}{4}\right)^3 + \ldots + 2\cdot\left(\frac{3}{4}\right)^\infty$
$= 1\frac{1}{2} + 1\frac{1}{8} + 2\frac{7}{32} + \ldots + 2\cdot\left(\frac{3}{4}\right)^\infty$

$$S = \frac{1\frac{1}{2}}{1-\frac{3}{4}}$$
$$= 6$$

7. $\sum_{n=1}^{4} (3n+2)$

8. $\sum_{r=2}^{10} -4r$

9. $\sum_{m=0}^{9} 4^{m-1}$

10. $\sum_{w=1}^{\infty} w!$

Lesson 12-6

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1. $(2+x)^4 = 16 + 32x + 24x^2 + 8x^3 + x^4$
2. $(n+m)^5 = n^5 + 5n^4m + 10n^3m^2 + 10n^2m^4 + 5nm^4 + m^5$
3. $(4a-b)^3 = 64a^3 - 48a^2b + 12ab^2 - b^3$
4. $(m-3)^6 = m^6(-3)^0 + 6\cdot m^5(-3)^1 + \frac{6\cdot 5\cdot m^4(-3)^2}{2\cdot 1} + \frac{6\cdot 5\cdot 4\cdot m^3(-3)^3}{3\cdot 2\cdot 1} + \frac{6\cdot 5\cdot 4\cdot 3\cdot m^2(-3)^4}{4\cdot 3\cdot 2\cdot 1} + \frac{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot m^1(-3)^5}{5\cdot 4\cdot 3\cdot 2\cdot 1} + \frac{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1\cdot m^0(-3)^6}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}$
$= m^6 - 18m^5 + 135m^4 - 540m^3 + 1215m^2 - 1458m + 729$
5. $(2r+s)^4 = (2r)^4(s)^0 + 4(2r)^3(s)^1 + \frac{4\cdot 3(2r)^2(s)^2}{2\cdot 1} + \frac{4\cdot 3\cdot 2\,(2r)^1(s)^3}{3\cdot 2\cdot 1} + \frac{4\cdot 3\cdot 2\cdot 1(2r)^0(s)^4}{4\cdot 3\cdot 2\cdot 1}$
$= 16r^4 + 32r^3s + 24r^2s^2 + 8rs^3 + s^4$
6. $(5x-4y)^3 = (5x)^3(-4y)^0 + 3(5x)^2(-4y)^1 + \frac{3\cdot 2(5x)^1(-4y)^2}{2\cdot 1} + \frac{3\cdot 2\cdot 1(5x)^0(-4y)^3}{3\cdot 2\cdot 1}$
$= 125x^3 - 300x^2y + 240xy^2 - 64y^3$
7. $\frac{8!}{5!(8-5)!}x^{8-5}y^5 = \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 3\cdot 2\cdot 1}x^3y^5$
$= 56x^3y^5$
8. $\frac{7!}{4!(7-4)!}b^{7-4}\cdot\sqrt{3^4} = \frac{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{4\cdot 3\cdot 2\cdot 1\cdot 3\cdot 2\cdot 1}b^3\cdot 9$
$= 315b^3$
9. $\frac{10!}{2!(10-2)!}(4z)^{10-2}(-w)^2$
$= \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{2\cdot 1\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}(4z)^8(-w)^2$
$= 45\cdot 65{,}536z^8\cdot w^2$
$= 2{,}949{,}120z^8\cdot w^2$
10. $\frac{12!}{7!(12-7)!}(2h)^{12-7}(-k)^7$
$= \frac{12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{7\cdot 6\cdot 5\cdot 3\cdot 2\cdot 1\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}(2h)^5(-k)^7$
$= 792\cdot 32h^5\cdot(-k)^7$
$= -25{,}344h^5k^7$

Lesson 12-7

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1. $\ln(-3) = \ln(-1) + \ln(3)$
$= \boldsymbol{i}\pi + 1.0986$
2. $\ln(-4.6) = \ln(-1) + \ln(4.6)$
$= \boldsymbol{i}\pi + 1.5261$
3. $\ln(-0{,}75) = \ln(-1) + \ln(0.75)$
$= \boldsymbol{i}\pi - 0.2877$
4. $e^{1.2} = 1 + 1.2 + \frac{(1.2)^2}{2!} + \frac{(1.2)^3}{3!} + \frac{(1.2)^4}{4!}$
$= 1 + 1.2 + 0.72 + 0.288 + 0.0864$
≈ 3.29
5. $e^{-0.7} = 1 + (-0.7) + \frac{(-0.7)^2}{2!} + \frac{(-0.7)^3}{3!} + \frac{(-0.7)^4}{4!}$
$\approx 1 - 0.7 + 0.245 - 0.057 + 0.010$
≈ 0.50

6. $e^{3.65} = 1 + 3.65 + \frac{(3.65)^2}{2!} + \frac{(3.65)^3}{3!} + \frac{(3.65)^4}{4!}$
$\approx 1 + 3.65 + 6.661 + 8.105 + 7.395$
≈ 26.81

7. $\cos x \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!}$
$\cos \frac{\pi}{4} \approx \cos 0.7854$
$\approx 1 - \frac{(0.7854)^2}{2!} + \frac{(0.7854)^4}{4!} - \frac{(0.7854)^6}{6!} + \frac{(0.7854)^8}{8!}$
$\approx 1 - \frac{0.6169}{2} + \frac{0.3805}{24} - \frac{0.2347}{720} + \frac{0.1448}{40{,}320}$
≈ 0.7071
actual value: $\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} \approx 0.7071$

8. $\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$
$\sin \frac{\pi}{6} \approx \sin 0.5236$
$\approx 0.5236 - \frac{(0.5236)^3}{3!} + \frac{(0.5236)^5}{5!} - \frac{(0.5236)^7}{7!}$
$+ \frac{(0.5236)^9}{9!}$
$\approx 0.5236 - \frac{0.1435}{6} + \frac{0.0394}{120} - \frac{0.0108}{5040} + \frac{0.0030}{362{,}880}$
≈ 0.5000
actual value: $\sin \frac{\pi}{6} = 0.5$

9. $\cos \frac{\pi}{3} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!}$
$\cos \frac{\pi}{3} \approx \cos 1.0472$
$\approx 1 - \frac{(1.0472)^2}{2!} + \frac{(1.0472)^4}{4!} - \frac{(1.0472)^6}{6!} + \frac{(1.0472)^8}{8!}$
$\approx 1 - \frac{1.0966}{2} + \frac{1.2026}{24} - \frac{1.3188}{720} + \frac{1.4462}{40{,}320}$
≈ 0.5000
actual value: $\cos \frac{\pi}{3} = 0.5$

Lesson 12-8

Page A50

1. $f(-2) = 2 \cdot (-2)$ or -4
$f(-4) = 2 \cdot (-4)$ or -8
$f(-8) = 2 \cdot (-8)$ or -16
$f(-16) = 2 \cdot (-16)$ or -32
$-4, -8, -16, -32$

2. $f(4) = 4^2$ or 16
$f(16) = 16^2$ or 256
$f(256) = 256^2$ or 65,536
$f(65{,}536) = 65{,}536^2$ or 4,294,967,296
16, 256, 65,536, 4,294,967,296

3. $z_0 = 2\boldsymbol{i}$
$z_1 = 0.5(2\boldsymbol{i}) + \boldsymbol{i}$
$= 2\boldsymbol{i}$
$z_2 = 0.5(2\boldsymbol{i}) + \boldsymbol{i}$
$= 2\boldsymbol{i}$
$z_3 = 0.5(2\boldsymbol{i}) + \boldsymbol{i}$
$= 2\boldsymbol{i}$
$2\boldsymbol{i}, 2\boldsymbol{i}, 2\boldsymbol{i}$

4. $z_0 = 4 + 4\boldsymbol{i}$
$z_1 = 0.5(4 + 4\boldsymbol{i}) + \boldsymbol{i}$
$= 2 + 2\boldsymbol{i} + \boldsymbol{i}$
$= 2 + 3\boldsymbol{i}$
$z_2 = 0.5(2 + 3\boldsymbol{i}) + \boldsymbol{i}$
$= 1 + 1.5\boldsymbol{i} + \boldsymbol{i}$
$= 1 + 2.5\boldsymbol{i}$
$z_3 = 0.5(1 + 2.5\boldsymbol{i}) + \boldsymbol{i}$
$= 0.5 + 1.25\boldsymbol{i} + \boldsymbol{i}$
$= 0.5 + 2.25\boldsymbol{i}$
$2 + 3\boldsymbol{i}, 1 + 2.5\boldsymbol{i}, 0.5 + 2.25\boldsymbol{i}$

5. $p_1 = p_0 + rp_0$
$p_1 = 4000 + (0.054)4000$
$= 4216$
$p_2 = 4216 + (0.054)4216$
$= 4443.66$
$p_3 = 4443.66 + (0.054)4443.66$
$= 4683.62$
$p_4 = 4683.62 + (0.054)4683.62$
$= 4936.54$
$p_5 = 4936.54 + (0.054)4936.54$
$= 5203.11$
\$4216, \$4443.66, \$4683.62, \$4936.54, \$5203.11

Lesson 12-9

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1. Step 1: Verify that the formula is valid for $n = 1$.
Since $S_1 = 2$ and $1(1 + 1) = 2$, the formula is valid for $n = 1$.
Step 2: Assume the formula is valid for $n = k$ and show that it is also valid for $n = k + 1$.
$2 + 4 + 6 + \ldots + 2k = k(k - 1)$
Derive the formula for $n = k + 1$ by adding $2(k + 1)$ to each side.
$2 + 4 + 6 + \ldots + 2k + 2(k + 1)$
$= k(k + 1) + 2(k + 1)$
$2 + 4 + 6 + \ldots + 2k + 2(k + 1)$
$= (k + 1)(k + 2)$
Apply the original formula for $n = k + 1$.
$S_{k+1} = (k + 1)((k + 1) + 1)$ or $(k + 1)(k + 2)$
The formula gives the same result as adding the $(k + 1)$ term directly. Thus, if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since S_n is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Hence the formula is valid for all positive integral values of n.

2. Step 1: Verify that the formula is valid for $n = 1$.
Since $S_1 = 1$ and $\frac{1(1+1)(1+2)}{6} = 1$, the formula is valid for $n = 1$.
Step 2: Assume the formula is valid for $n = k$ and show that it is also valid for $n = k + 1$.
$1 + 3 + 6 + \ldots + \frac{k(k+1)}{2} = \frac{k(k+1)(k+2)}{6}$
Derive the formula for $n = k + 1$ by adding $\frac{(k+1)(k+2)}{2}$ to each side.
$1 + 3 + 6 + \ldots + \frac{k(k+1)}{2} + \frac{(k+1)(k+2)}{2}$
$= \frac{k(k+1)(k+2)}{6} + \frac{(k+1)(k+2)}{2}$
$1 + 3 + 6 + \ldots + \frac{k(k+1)}{2} + \frac{(k+1)(k+2)}{2}$
$= \frac{k(k+1)(k+2) + 3(k+1)(k+2)}{6}$
$1 + 3 + 6 + \ldots + \frac{k(k+1)}{2} + \frac{(k+1)(k+2)}{2}$
$= \frac{(k+1)(k+2)(k+3)}{6}$
Apply the original formula for $n = k + 1$.
$S_{k+1} = \frac{(k+1)((k+1)+1)((k+1)+2)}{6}$
or $\frac{(k+1)(k+2)(k+3)}{6}$
The formula gives the same result as adding the $k + 1$ term directly. Thus, if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Hence, the formula is valid for all positive integral values of n.

3. S_n: $5^n - 1 = 2r$ for some integer r.
Step 1: Verify that S_n is valid for $n = 1$.
$S_1 = 5^1 - 1$ or 4. Since $4 = 2 \cdot 2$, S_n is valid for $n = 1$.
Step 2: Assume that S_n is valid for $n = k$ and show that it is also valid for $n = k + 1$.
$S_k \to 5^k - 1 = 2r$ for some integer t
$S_{k+1} \to 5^{k+1} - 1 = 2t$ for some integer t
$5^k - 1 = 2r$
$5(5^k - 1) = 5(2r)$
$5^{k+1} - 5 = 10r$
$5^{k+1} - 1 = 10r + 4$
$5^{k+1} - 1 = 2(5r + 2)$
Thus, $5^{k+1} - 1 = 2t$, where $t = (5r + 2)$ is an integer. Thus if S_k is valid, then S_{k+1} is also valid. Since S_n is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on indefinitely. Hence, $5^n - 1$ is even for all positive integral values of n.

Lesson 13-1

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1. Using the Basic Counting Principle, $6 \cdot 6 \cdot 6 = 216$.
2. $P(8, 8) = \frac{8!}{(8-8)!}$
$= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1}$
$= 40{,}320$
3. independent
4. dependent
5. dependent
6. $P(5, 5) = \frac{5!}{(5-5)!}$
$= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1}$
$= 120$
7. $P(8, 3) = \frac{8!}{(8-3)!}$
$= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 336$
8. $P(4, 1) = \frac{4!}{(4-1)!}$
$= \frac{4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1}$
$= 4$
9. $P(10, 9) = \frac{10!}{(10-9)!}$
$= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1}$
$= 3{,}628{,}800$
10. $P(9, 6) = \frac{9!}{(9-6)!}$
$= \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1}$
$= 60{,}480$
11. $P(7, 3) = \frac{7!}{(7-3)!}$
$= \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1}$
$= 210$
12. $\frac{P(5, 2)}{P(2, 1)} = \frac{\frac{5!}{(5-2)!}}{\frac{2!}{(2-1)!}}$
$= \frac{5!\,1!}{3!\,2!}$
$= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}$
$= 10$
13. $\frac{P(8, 6)}{P(7, 4)} = \frac{\frac{8!}{(8-6)!}}{\frac{7!}{(7-4)!}}$
$= \frac{8!\,3!}{7!\,2!}$
$= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}$
$= 24$
14. $\frac{P(5, 2) \cdot P(8, 4)}{P(10, 1)}$
$= \frac{\frac{5!}{(5-2)!} \cdot \frac{8!}{(8-4)!}}{\frac{10!}{(10-1)!}} = \frac{9!\,8!\,5!}{10!\,4!\,3!}$
$= \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}$
$= 3360$
15. $C(4, 2) = \frac{4!}{(4-2)!\,2!}$
$= \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1}$
$= 6$
16. $C(10, 7) = \frac{10!}{(10-7)!\,7!}$
$= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 120$

17. $C(6, 5) = \frac{6!}{(6-5)!\,5!}$
$= \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 6$

18. $C(4, 3) \cdot C(7, 3) = \frac{4!}{(4-3)!\,3!} \cdot \frac{7!}{(7-3)!\,3!}$
$= \frac{4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 3 \cdot 2 \cdot 1} \cdot \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}$
$= 140$

19. $C(3, 1) \cdot C(8, 7) = \frac{3!}{(3-1)!\,1!} \cdot \frac{8!}{(8-7)!\,7!}$
$= \frac{3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 1} \cdot \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 24$

20. $C(9, 5) \cdot C(4, 3) = \frac{9!}{(9-5)!\,5!} \cdot \frac{4!}{(4-3)!\,3!}$
$= \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \cdot \frac{4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 2 \cdot 1}$
$= 504$

Lesson 13-2

Page A51

1. $\frac{7!}{0!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1}$
$= 5040$
2. $\frac{8!}{2!\,2!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1}$
$= 10{,}080$
3. $\frac{7!}{3!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1}$
$= 840$
4. $\frac{10!}{2!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1}$
$= 1{,}814{,}400$
5. $\frac{4!}{2!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1}$
$= 12$
6. circular; $(4 - 1)!$ or 6
7. circular; since the bracelet can be turned over there are $\frac{(9-1)!}{2}$ or 20,160 permutations
8. linear; 5! or 120

Lesson 13-3

Page A51

1. $P(\text{ace}) = \frac{4}{52}$ or $\frac{1}{13}$
2. $P(\text{a card of 5 or less}) = \frac{4+4+4+4+4}{52}$
$= \frac{20}{52}$ or $\frac{5}{13}$
3. $P(\text{a red face card}) = \frac{6}{52}$ or $\frac{3}{26}$
4. $P(\text{not a queen}) = 1 - P(\text{a queen})$
$= 1 - \frac{4}{52}$
$= \frac{48}{52}$ or $\frac{12}{13}$
5. $P(\text{blue}) = \frac{2}{8+4+2} = \frac{2}{14}$ or $\frac{1}{7}$
6. $P(\text{green}) = \frac{4}{8+4+2} = \frac{4}{14}$ or $\frac{2}{7}$
7. $P(\text{not red}) = \frac{4+2}{8+4+2} = \frac{6}{14}$ or $\frac{3}{7}$
8. $P(\text{red or green}) = \frac{8+4}{8+4 \cdot 2} = \frac{12}{14}$ or $\frac{6}{7}$
9. $P(s) = \frac{4}{16} \cdot \frac{3}{15}$
$= \frac{1}{20}$

$P(f) = 1 - P(s)$
$= 1 - \frac{1}{20}$ or $\frac{19}{20}$

odds $= \frac{\frac{1}{20}}{\frac{19}{20}}$ or $\frac{1}{19}$

10. $P(s) = \frac{6}{16} \cdot \frac{5}{15}$
$= \frac{1}{8}$

$P(f) = 1 - P(s)$
$= 1 - \frac{1}{8}$ or $\frac{7}{8}$

odds $= \frac{\frac{1}{8}}{\frac{7}{8}}$ or $\frac{1}{7}$

11. $P(s) = \frac{C(2, 1) \cdot C(4, 1)}{C(16, 2)}$
$= \frac{2 \cdot 4}{120}$
$= \frac{1}{15}$

$P(f) = 1 - P(s)$
$= 1 - \frac{1}{15}$
$= \frac{14}{15}$

odds $= \frac{\frac{1}{15}}{\frac{14}{15}}$ or $\frac{1}{14}$

12. $P(s) = P(\text{1 white, 1 yellow}) + P(\text{1 yellow, 1 red}) + P(\text{1 white, 1 red})$
$= \frac{C(2, 1) \cdot C(4, 1)}{C(16, 2)} + \frac{C(4, 1) \cdot C(10, 1)}{C(16, 2)} + \frac{C(2, 1) \cdot C(10, 1)}{C(16, 2)}$
$= \frac{2 \cdot 4}{120} + \frac{4 \cdot 10}{120} + \frac{2 \cdot 10}{120}$
$= \frac{1}{15} + \frac{1}{3} + \frac{1}{6}$
$= \frac{17}{30}$

$P(f) = 1 - P(s)$
$= 1 - \frac{17}{30}$ or $\frac{13}{30}$

odds $= \frac{\frac{17}{30}}{\frac{13}{30}}$ or $\frac{17}{13}$

Lesson 13-4

Page A52

1. dependent; $\frac{6}{8} \cdot \frac{2}{7} = \frac{3}{14}$
2. independent; $\frac{10}{18} \cdot \frac{10}{18} = \frac{25}{81}$
3. independent; $\frac{1}{2} \cdot \frac{6}{10} = \frac{3}{10}$
4. inclusive; $\frac{1}{6} + \frac{1}{6} - \frac{1}{36} = \frac{11}{36}$
5. exclusive; $\frac{4}{52} + \frac{4}{52} = \frac{8}{52}$ or $\frac{2}{13}$

Lesson 13-5

Page A52

1. $P(\text{second clip is blue} \mid \text{first clip was red})$

$$= \frac{\frac{8}{12} \cdot \frac{4}{11}}{\frac{8}{12}}$$

$$= \frac{4}{11}$$

2. $P(\text{second clip is blue} \mid \text{first clip was blue})$

$$= \frac{\frac{4}{12} \cdot \frac{3}{11}}{\frac{4}{12}}$$

$$= \frac{3}{11}$$

3. $P(\text{numbers on dice match} \mid \text{sum is greater than 7})$

$$= \frac{\frac{3}{36}}{\frac{15}{36}}$$

$$= \frac{3}{15} \text{ or } \frac{1}{5}$$

4. $P(\text{sum is greater than 7} \mid \text{numbers match})$

$$= \frac{\frac{3}{36}}{\frac{6}{36}}$$

$$= \frac{3}{6} \text{ or } \frac{1}{2}$$

5. $P(\text{ball is from second box} \mid \text{ball is white})$

$$= \frac{P(\text{2nd box and white})}{P(\text{white})}$$

$$= \frac{\frac{1}{2} \cdot \frac{3}{8}}{\frac{1}{2} \cdot \frac{4}{7} + \frac{1}{2} \cdot \frac{3}{8}}$$

$$= \frac{21}{53}$$

Lesson 13-6

Page A52

1. $P(\text{all heads}) = C(3, 3)\left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^0$

$$= 1 \cdot \frac{1}{8} \cdot 1$$

$$= \frac{1}{8}$$

2. $P(\text{exactly 2 tails}) = C(3, 2)\left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1$

$$= 3 \cdot \frac{1}{4} \cdot \frac{1}{2}$$

$$= \frac{3}{8}$$

3. $P(\text{at least 2 heads})$

$$= P(\text{2 heads}) + P(\text{3 heads})$$

$$= C(3, 2)\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1 + C(3, 3)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0$$

$$= 3 \cdot \frac{1}{4} \cdot \frac{1}{2} + 1 \cdot \frac{1}{8} \cdot 1$$

$$= \frac{3}{8} + \frac{1}{8}$$

$$= \frac{4}{8} \text{ or } \frac{1}{2}$$

4. $P(\text{exactly 1 hit}) = C(5, 1)\left(\frac{200}{1000}\right)^1\left(\frac{800}{1000}\right)^4$

$$= 5 \cdot \frac{1}{5} \cdot \frac{256}{625}$$

$$= \frac{256}{625} \text{ or } 0.4096$$

5. $P(\text{exactly 3 hits}) = C(5, 3)\left(\frac{200}{1000}\right)^3\left(\frac{800}{1000}\right)^2$

$$= 10 \cdot \frac{1}{125} \cdot \frac{16}{25}$$

$$= \frac{32}{625} \text{ or } 0.0512$$

6. $P(\text{at least 4 hits})$

$$= P(\text{4 hits}) + P(\text{5 hits})$$

$$= C(5, 4)\left(\frac{200}{1000}\right)^4\left(\frac{800}{1000}\right)^1 + C(5, 5)\left(\frac{200}{1000}\right)^5\left(\frac{800}{1000}\right)^0$$

$$= 5 \cdot \frac{1}{625} \cdot \frac{4}{5} + 1 \cdot \frac{1}{3125} \cdot 1$$

$$= \frac{21}{3125} \text{ or } 0.00672$$

Lesson 14-1

Page A53

1. range = 70 − 22 or 48

2. Sample answer: 10

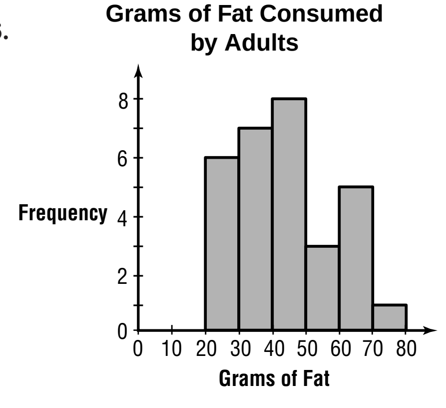
3. Sample answer: 20, 30, 40, 50, 60, 70, 80

4. Sample answer: 25, 35, 45, 55, 65, 75

5.

Grams of Fat	Frequency
20-30	6
30-40	7
40-50	8
50-60	3
60-70	5
70-80	1

6.



7. Sample answer: 40–50

Lesson 14-2

Page A53

1. $\overline{X} = \frac{1}{4}(130 + 150 + 180 + 190)$

$= 162.5$

$M_d = \frac{150 + 180}{2}$ or 165

Mode: none

2. $\overline{X} = \frac{1}{6}(15 + 16 + 17 + 18 + 18 + 19)$

≈ 17.2

$M_d = \frac{17 + 18}{2}$ or 17.5

Mode = 18

3. $\overline{X} = \frac{1}{6}(25 + 28 + 30 + 36 + 38 + 42)$

≈ 33.2

$M_d = \frac{30 + 36}{2}$ or 33

Mode: none

4. $\overline{X} = \frac{1}{10}(1 + 2 + 3 + 4 + 5 + 5 + 6 + 9 + 9 + 10)$

$= 5.4$

$M_d = 5$

Mode = 5 and 9

5. $\overline{X} = \frac{1}{12}(2.3 + 2.5 + 4 + 2(5.6) + 6 + 6.4 + 6.5 + 2(7) + 8 + 10)$

≈ 5.9

$M_d = \frac{6 + 6.4}{2}$ or 6.2

Mode = 5.6 and 7

6. $\overline{X} = \frac{1}{12}(14 + 2(15) + 16 + 20 + 21 + 24 + 27 + 28 + 36 + 2(39))$

$= 24.5$

$M_d = \frac{21 + 24}{2}$ or 22.5

Mode = 15 and 39

7. $\overline{X} = \frac{1}{18}(3.0 + 3.4 + 3.6 + 5.2 + 2(5.4) + 2(5.6) + 5.7 + 6.2 + 6.3 + 6.8 + 7.0 + 7.1 + 7.6 + 7.7 + 8.2)$

≈ 5.9

$M_d = 5.7$

Mode = 5.4 and 5.6

8. $\overline{X} = \frac{1}{14}(800 + 820 + 830 + 890 + 960 + 970 + 980 + 1040 + 2(1050) + 1080 + 1110 + 1170 + 1180)$

$= 995$

$M_d = \frac{980 + 1040}{2}$ or 1010

Mode = 1050

9.

stem	leaf
1	2 7 9
2	3 4 4 6 8 9 9
3	4 4 5 6 7 9
4	0 2 4
5	5

1 | 2 = 12

10. $8 = \frac{1}{6}(4 + 5 + 6 + 9 + 10 + x)$

$48 = 34 + x$

$14 = x$

11. Order the values from least to greatest. The median lies between the fourth and fifth terms.

3, 4, 4, 7, x, 12, 16, 19

$7.5 = \frac{7 + x}{2}$

$15 = 7 + x$

$8 = x$

Lesson 14-3

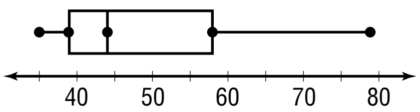
Page A54

1. interquartile range $= Q_3 - Q_1$

$= 58 - 39$

$= 19$

semi-interquartile range $= \frac{19}{2}$ or 9.5

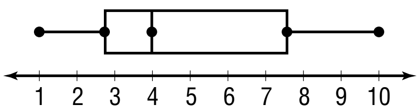


2. interquartile range $= Q_3 - Q_1$

$= 7.65 - 2.75$

$= 4.9$

semi-interquartile range $= \frac{4.9}{2}$ or 2.45



3. $\overline{X} = \frac{1}{9}(150 + 175 + 180 + 180 + 195 + 200 + 212 + 220 + 250)$

$= 195.\overline{7}$

$MD = \frac{1}{9}(|-45.8| + |-20.8| + |-15.8| + \ldots + |54.2|)$

≈ 21.98

$\sigma = \sqrt{\frac{(-45.8)^2 + (-20.8)^2 + \ldots + 54.2^2}{9}}$

≈ 27.56

4. $\overline{X} = \frac{1}{11}(1.4 + 2 + 2.4 + 2.9 + 3 + 3.5 + 3.7 + 4.2 + 4.6 + 5.3 + 5.5)$

$= 3.5$

$\text{MD} = \frac{1}{11}(|-2.1| + |-1.5| + |-1.1| + \ldots + |2|)$

≈ 1.05

$\sigma = \sqrt{\frac{(-2.1)^2 + (-1.5)^2 + (-1.1)^2 + \ldots + 2^2}{11}}$

≈ 1.26

5a. $M_d = 18$

5b. $Q_1 = \frac{16 + 18}{2}$ or 17

$Q_3 = 20$

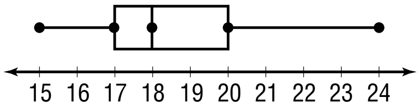
5c. interquartile range $= Q_3 - Q_1$

$= 20 - 17$

$= 3$

5d. semi-interquartile range $= \frac{3}{2}$ or 1.5

5e. Any points less than $17 - 1.5(3)$ or 12.5 and any points greater than $20 + 1.5(3)$ or 24.5 are considered outliers. There are no such points.

5f. 

Lesson 14-4

Page A54

1a. 25% corresponds to $t = 0.3$.
$10 \pm 0.3(2) = 9.4 - 10.6$

1b. $10 - 8 = 2,\ 14 - 10 = 4$

$$t\sigma = 2 \qquad t\sigma = 4$$
$$t(2) = 2 \qquad t(2) = 4$$
$$t = 1 \qquad t = 2$$
$$\frac{68.3\%}{2} = 34.15\% \qquad \frac{95.5\%}{2} = 47.75\%$$

$34.15\% + 47.75\% = 81.9\%$

1c. $10 - 7 = 3,\ 10 - 10 = 0$

$$t\sigma = 3 \qquad t\sigma = 0$$
$$t(2) = 3 \qquad t(2) = 0$$
$$t = 1.5 \qquad t = 0$$
$$\frac{86.6\%}{2} = 43.3\%$$

1d. 80% corresponds to $t = 1.3$.
$10 \pm 1.3(2) = 7.4 - 12.6$

2a. $0.683(400) = 273.2$

2b. $0.955(400) = 382$

2c. $\frac{0.683}{2}(400) = 136.6$

Lesson 14-5

Page A54

1. $\sigma_{\overline{X}} = \frac{1.2}{\sqrt{90}}$ or about 0.13

2. $\sigma_{\overline{X}} = \frac{3.4}{\sqrt{100}}$ or 0.34

3. $\sigma_{\overline{X}} = \frac{12.4}{\sqrt{240}}$ or about 0.80

4. A 1% confidence level is given when $P = 99\%$ and $t = 2.58$.

$$\sigma_{\overline{X}} = \frac{4.2}{\sqrt{40}} \approx 0.6640783086$$

interval: $\overline{X} \pm t\sigma_{\overline{X}} = 150 \pm 2.58\sigma_{\overline{X}} \approx 148.29 - 151.71$

5. A 1% confidence level is given when $P = 99\%$ and $t = 2.58$.

$$\sigma_{\overline{X}} = \frac{10}{\sqrt{78}} \approx 1.132277034$$

interval: $\overline{X} \pm t\sigma_{\overline{X}} = 320 \pm 2.58\sigma_{\overline{X}} \approx 317.08 - 322.92$

Lesson 15-1

Page A55

1. $\lim\limits_{x \to 4} (x^2 + 2x - 2) = 4^2 + 2(4) - 2 = 22$

2. $\lim\limits_{x \to -1} (-x^4 + x^3 - 2x + 1) = -(-1)^4 + (-1)^3 - 2(-1) + 1 = 1$

3. $\lim\limits_{x \to 0} (x + \sin x) = 0 + \sin 0 = 0$

4.
$$\lim_{x \to -4} \frac{x^2 - 16}{x + 4} = \lim_{x \to -4} \frac{(x + 4)(x - 4)}{x + 4} = \lim_{x \to -4} (x - 4) = -4 - 4 = -8$$

5.
$$\lim_{x \to -2} \frac{x^2 + 5x + 6}{x^2 + x - 2} = \lim_{x \to -2} \frac{(x + 3)(x + 2)}{(x - 1)(x + 2)} = \lim_{x \to -2} \frac{x + 3}{x - 1} = \frac{-2 + 3}{-2 - 1} = -\frac{1}{3}$$

6.
$$\lim_{x \to 2} \frac{3x + 9}{x^2 - 5x - 24} = \lim_{x \to 2} \frac{3(x + 3)}{(x - 8)(x + 3)} = \lim_{x \to 2} \frac{3}{x - 8} = \frac{3}{2 - 8} = -\frac{3}{6} \text{ or } -\frac{1}{2}$$

Lesson 15-2

Page A55

1.
$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{5(x + h) - 5x}{h} = \lim_{h \to 0} \frac{5x + 5h - 5x}{h} = \lim_{h \to 0} \frac{5h}{h} = 5$$

2.
$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{9(x + h) - 2 - (9x - 2)}{h} = \lim_{h \to 0} \frac{9x + 9h - 2 - 9x + 2}{h} = \lim_{h \to 0} \frac{9h}{h} = 9$$

3.
$$f(x) = \frac{1}{2}x + \frac{2}{3}$$
$$f'(x) = \frac{1}{2} \cdot 1x^{1-1} + 0 = \frac{1}{2}$$

4.
$$f(x) = x^2 + 4x + 8$$
$$f'(x) = 2x^{2-1} + 4 \cdot 1x^{1-1} + 0 = 2x + 4$$

5. $f(x) = x^5$

$F(x) = \frac{1}{5+1}x^{5+1} + C$

$= \frac{1}{6}x^6 + C$

6. $f(x) = 2x^2 - 8x + 2$

$F(x) = 2 \cdot \frac{1}{2+1}x^{2+1} - 8 \cdot \frac{1}{1+1}x^{1+1} + 2x + C$

$= \frac{2}{3}x^3 - 4x^2 + 2x + C$

7. $f(x) = \frac{1}{5}x^3 - \frac{3}{4}x - 1$

$F(x) = \frac{1}{5} \cdot \frac{1}{3+1}x^{3+1} - \frac{3}{4} \cdot \frac{1}{1+1}x^{1+1} - x + C$

$= \frac{1}{20}x^4 - \frac{3}{8}x^2 - x + C$

8. $f(x) = \frac{x^3 - 2x^2 + x}{x}$

$= x^2 - 2x + 1$

$F(x) = \frac{1}{2+1}x^{2+1} - 2 \cdot \frac{1}{1+1}x^{1+1} + x + C$

$= \frac{1}{3}x^3 - x^2 + x + C$

Lesson 15-3

Page A55

1. $\int_0^3 5x\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} 5\left(\frac{3i}{n}\right)\left(\frac{3}{n}\right)$

$= \lim_{n\to\infty} \frac{45}{n^2}\left(\frac{n(n+1)}{2}\right)$

$= \lim_{n\to\infty} \frac{45}{2n^2}(n^2 + n)$

$= \lim_{n\to\infty} \frac{45n^2}{2n^2} + \lim_{n\to\infty} \frac{45n}{2n^2}$

$= \frac{45}{2} + 0$

$= \frac{45}{2}$

2. $\int_1^5 (x+1)dx = \int_0^5 (x+1)dx - \int_0^1 (x+1)dx$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{5i}{n}\right) + 1\right]\left(\frac{5}{n}\right) - \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{i}{n}\right) + 1\right]\left(\frac{1}{n}\right)$

$= \lim_{n\to\infty} \frac{5}{n}\left[\left(\frac{5\cdot 1}{n} + 1\right) + \left(\frac{5\cdot 2}{n} + 1\right) + \ldots + \left(\frac{5\cdot n}{n} + 1\right)\right]$

$-\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n} + 1\right) + \left(\frac{2}{n} + 1\right) + \ldots + \left(\frac{n}{n} + 1\right)\right]$

$= \lim_{n\to\infty} \frac{5}{n}\left[n + \frac{5}{n}(1 + 2 + \ldots + n)\right]$

$- \lim_{n\to\infty} \frac{1}{n}\left[n + \frac{1}{n}(1 + 2 + \ldots + n)\right]$

$= \lim_{n\to\infty} \frac{5}{n}\left[n + \frac{5}{n} \cdot \frac{n(n+1)}{2}\right] - \lim_{n\to\infty} \frac{1}{n}\left[n + \frac{1}{n} \cdot \frac{n(n+1)}{2}\right]$

$= \lim_{n\to\infty} \frac{5}{n}\left(\frac{7}{2}n + \frac{5}{2}\right) - \lim_{n\to\infty} \frac{1}{n}\left(\frac{3}{2}n + \frac{1}{2}\right)$

$= \lim_{n\to\infty} \left(\frac{35}{2} + \frac{25}{2n}\right) - \lim_{n\to\infty} \left(\frac{3}{2} + \frac{1}{2n}\right)$

$= \frac{35}{2} + 0 - \frac{3}{2} - 0$

$= \frac{32}{2}$ or 16

3. $\int_0^2 (x^2 + 4x + 4)dx$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{2i}{n}\right)^2 + 4 \cdot \left(\frac{2i}{n}\right) + 4\right]\left(\frac{2}{n}\right)$

$= \lim_{n\to\infty} \frac{2}{n}\left[\left(\left(\frac{2\cdot 1}{n}\right)^2 + 4\left(\frac{2\cdot 1}{n}\right) + 4\right) + \left(\left(\frac{2\cdot 2}{n}\right)^2 + \right.\right.$

$\left.\left. 4\left(\frac{2\cdot 2}{n}\right) + 4\right) + \ldots + \left(\left(\frac{2\cdot 3}{n}\right)^2 + 4\left(\frac{2\cdot 3}{n}\right) + 4\right)\right]$

$= \lim_{n\to\infty} \frac{2}{n}\left[\frac{4}{n^2}(1^2 + 2^2 + \ldots + n^2)\right.$

$\left. + \frac{8}{n}(1 + 2 + \ldots + n) + 4n\right]$

$= \lim_{n\to\infty} \frac{2}{n}\left[\frac{4}{n^2}\left(\frac{n(n+1)(2n+1)}{6}\right) + \frac{8}{n}\left(\frac{n(n+1)}{2}\right) + 4n\right]$

$= \lim_{n\to\infty} \left[\frac{8}{n^3}\left(\frac{2n^3 + 3n^2 + n}{6}\right) + \frac{16}{n^2}\left(\frac{n^2 + n}{2}\right) + 8\right]$

$= \lim_{n\to\infty} \left[\frac{4}{3}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right) + 8\left(1 + \frac{1}{n}\right) + 8\right]$

$= \frac{8}{3} + 8 + 8$

$= \frac{56}{3}$

4. $\int_{1500}^{2000} (6 - 0.002x)dx$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[6 - 0.002\left(1500 + \frac{500i}{n}\right)\right]\left(\frac{500}{n}\right)$

$= \lim_{n\to\infty} \frac{500}{n} \sum_{i=1}^{n} \left(3 - \frac{i}{n}\right)$

$= \lim_{n\to\infty} \frac{500}{n}\left[\left(3 - \frac{1}{n}\right) + \left(3 - \frac{2}{n}\right) + \ldots + \left(3 - \frac{n}{n}\right)\right]$

$= \lim_{n\to\infty} \frac{500}{n}\left[3n - \frac{1}{n}(1 + 2 + \ldots + n)\right]$

$= \lim_{n\to\infty} \left[1500 - \frac{500}{n^2}\left(\frac{n(n+1)}{2}\right)\right]$

$= \lim_{n\to\infty} \left[1500 - 250\left(1 + \frac{1}{n}\right)\right]$

$= 1500 - 250$ or \$1250

Lesson 15-4

Page A55

1. $\int x^6 dx = \frac{1}{7}x^7 + C$

2. $\int 5x^4 dx = 5 \cdot \frac{1}{5}x^5 + C$

$= x^5 + C$

3. $\int (x^2 - x + 5)dx = \frac{1}{3}x^3 - \frac{1}{2}x^2 + 5x + C$

4. $\int (-4x^4 + x^2 - 6)dx = -4 \cdot \frac{1}{5}x^5 + \frac{1}{3}x^3 - 6x + C$

$= -\frac{4}{5}x^5 + \frac{1}{3}x^3 - 6x + C$

5. $\int_{-2}^{2} 14x^6 dx = 14 \cdot \frac{1}{7}x^7 \Big|_{-2}^{2}$

$= 2x^7 \Big|_{-2}^{2}$

$= (2 \cdot 2^7) - (2 \cdot (-2)^7)$

$= 256 - (-256)$

$= 512$

6. $\int_0^6 (x + 2)dx = \frac{1}{2}x^2 + 2x \Big|_0^6$

$= \left(\frac{1}{2} \cdot 6^2 + 2 \cdot 6\right) - \left(\frac{1}{2} \cdot 0^2 + 2 \cdot 0\right)$

$= 30 - 0$

$= 30$

7. $\int_2^4 (x^2 - 4)dx = \frac{1}{3}x^3 - 4x \Big|_2^4$

$= \left(\frac{1}{3} \cdot 4^3 - 4 \cdot 4\right) - \left(\frac{1}{3} \cdot 2^3 - 4 \cdot 2\right)$

$= \frac{16}{3} - \left(-\frac{16}{3}\right)$

$= \frac{32}{3}$

8. $\int_4^5 (x - 4)(x + 2)dx$

$= \int_4^5 (x^2 - 2x - 8)dx$

$= \frac{1}{3}x^3 - 2 \cdot \frac{1}{2}x^2 - 8x \Big|_4^5$

$= \frac{1}{3}x^3 - x^2 - 8x \Big|_4^5$

$= \left(\frac{1}{3} \cdot 5^3 - 5^2 - 8 \cdot 5\right) - \left(\frac{1}{3} \cdot 4^3 - 4^2 - 8 \cdot 4\right)$

$= -\frac{70}{3} - \left(-\frac{80}{3}\right)$

$= \frac{10}{3}$

Chapter 1 Test

Page A56

1. $D = \{-1, 0, 2, 3\}$; $R = \{2, 4, 5\}$; yes

2. $D = \{-5, 4, 6, 7\}$; $R = \{-3, -2, 0, 2, 7\}$; no

3. $f(4) = 4 - 3 \cdot 4^2$
$= 4 - 48$ or -44

4. $f(-7) = -7 - 3(-7)^2$
$= -7 - 147$ or -154

5. $f(a + 2) = a + 2 - 3(a + 2)^2$
$= a + 2 - 3(a^2 + 4a + 4)$
$= a + 2 - 3a^2 - 12a - 12$
$= -3a^2 - 11a - 10$

6a. $E = \frac{P}{4\pi d^2}$
$= \frac{1200}{4\pi(0.9)^2}$
≈ 117.89 Im/m^2

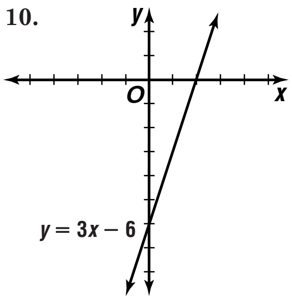
6b. d cannot be negative or zero.

7. $f(x) + g(x) = x^2 - 7 + x + 3$
$= x^2 + x - 4$
$f(x) - g(x) = x^2 - 7 - (x + 3)$
$= x^2 - x - 10$
$f(x) \cdot g(x) = (x^2 - 7)(x + 3)$
$= x^3 + 3x^2 - 7x - 21$
$\frac{f(x)}{g(x)} = \frac{x^2 - 7}{x + 3}$

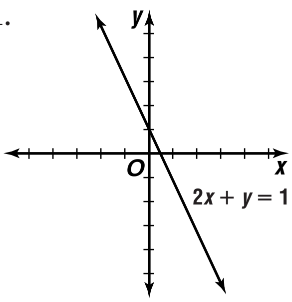
8. $[f \circ g](x) = f(g(x))$
$= f(4x - 5)$
$= 4x - 5 + 1$
$= 4x - 4$
$[g \circ f](x) = g(f(x))$
$= g(x + 1)$
$= 4(x + 1) - 5$
$= 4x + 4 - 5$
$= 4x - 1$

9. $[f \circ g](x) = f(g(x))$
$= f(2x^2 + 6)$
$= 5(2x^2 + 6)$
$= 10x^2 + 30$
$[g \circ f](x) = g(f(x))$
$= g(5x)$
$= 2(5x)^2 + 6$
$= 50x^2 + 6$

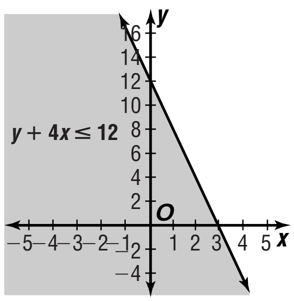
10.



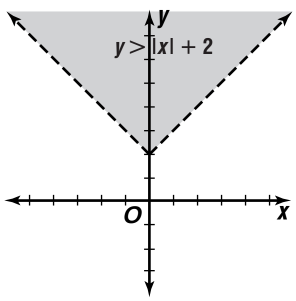
11.



12.



13.



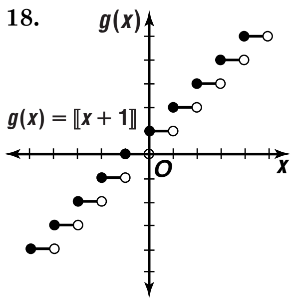
14. $y - 3 = \frac{5}{3}(x - (-1))$
$y - 3 = \frac{5}{3}(x + 1)$
$y - 3 = \frac{5}{3}x + \frac{5}{3}$
$y = \frac{5}{3}x + \frac{14}{3}$

15. $m = \frac{-2 - 4}{8 - 0}$
$= \frac{-6}{8}$ or $-\frac{3}{4}$
$y - 4 = -\frac{3}{4}(x - 0)$
$y - 4 = -\frac{3}{4}x$
$y = -\frac{3}{4}x + 4$

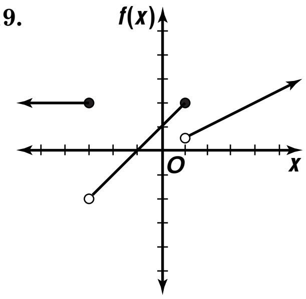
16. parallel: $y - 2 = 4(x - 0)$
$y - 2 = 4x$
$4x - y + 2 = 0$
perpendicular: $y - 2 = -\frac{1}{4}(x - 0)$
$y - 2 = -\frac{1}{4}x$
$4y - 8 = -x$
$x + 4y - 8 = 0$

17. $x - 5y = 3 \Rightarrow m = \frac{1}{5}$
parallel: $y - 2 = \frac{1}{5}(x - (-1))$
$y - 2 = \frac{1}{5}(x + 1)$
$5y - 10 = x + 1$
$x - 5y + 11 = 0$
perpendicular: $y - 2 = -\frac{5}{1}(x - (-1))$
$y - 2 = -5(x + 1)$
$y - 2 = -5x - 5$
$5x + y + 3 = 0$

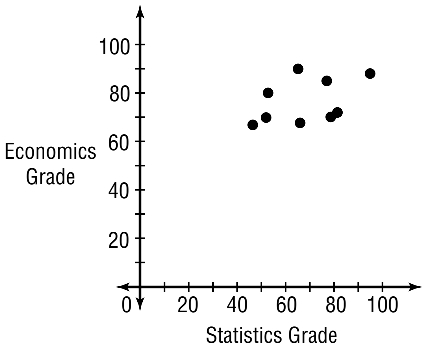
18.



19.



20a.



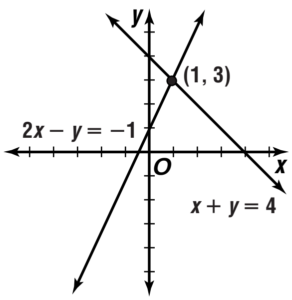
20b. Sample answer: Using (47, 67) and (95, 88), $y = 0.44x + 46.44$

$$m = \frac{88 - 67}{95 - 47}$$
$$= 0.4375$$
$$y - 67 = 0.4375(x - 47)$$
$$y - 67 = 0.4375x - 20.5625$$
$$y = 0.4375x + 46.4375$$
$$y \approx 0.44x + 46.44$$

Chapter 2 Test

Page A57

1.



2. $3x + y = 7 \rightarrow 6x + 2y = 14$
$5x + 2y = 12 \qquad 5x + 2y = 12$
$x = 2$

$3x + y = 7$
$3(2) + y = 7$
$y = 1$
$(2, 1)$

3. $4x - 5y = -2 \rightarrow 12x - 15y = -6$
$3x + 2y = -13 \qquad 12x + 8y = -52$
$-23y = 46$
$y = -2$

$4x - 5y = -2$
$4x - 5(-2) = -2$
$4x = -12$
$x = -3$
$(-3, -2)$

4. $4x + 6y - 3z = 20 \rightarrow 4x + 6y - 3z = 20$
$x - 5y + z = -15 \qquad 3x - 15y + 3z = -45$
$7x - 9y = -25$

$x - 5y + z = -15 \rightarrow 2x - 10y + 2z = -30$
$-7x + y + 2z = 1 \qquad -7x + y + 2z = 1$
$9x - 11y = -31$

$7x - 9y = -25 \rightarrow 63x - 81y = -225$
$9x - 11y = -31 \qquad 63x - 77y = -217$
$-4y = -8$
$y = 2$

$7x - 9y = -25$
$7x - 9(2) = -25$
$7x = -7$
$x = -1$

$x - 5y + z = -15$
$-1 - 5(2) + z = -15$
$-11 + z = -15$
$z = -4$

$(-1, 2, -4)$

5. $y = 12 - 2x$

$3x - 4 = 2y$
$3x - 4 = 2(12 - 2x)$
$3x - 4 = 24 - 4x$
$7x = 28$
$x = 4$

$y = 12 - 2(4)$
$= 4$
$(4, 4)$

6. $$A + B = \begin{bmatrix} -5 + (-2) & 1 + 3 \\ 4 + (-3) & 2 + 0 \end{bmatrix} = \begin{bmatrix} -7 & 4 \\ 1 & 2 \end{bmatrix}$$

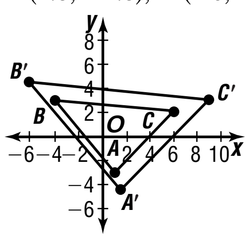
7. $$-3C = \begin{bmatrix} -3(-1) & -3(0) & -3\ (3) \\ -3\ (3) & -3(2) & -3(-4) \end{bmatrix} = \begin{bmatrix} 3 & 0 & -9 \\ -9 & -6 & 12 \end{bmatrix}$$

8. $$2B = \begin{bmatrix} 2(-2) & 2(3) \\ 2(-3) & 2(0) \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -6 & 0 \end{bmatrix}$$

$$2B - A = \begin{bmatrix} -4 - (-5) & 6 - 1 \\ -6 - \ 4 & 0 - 2 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ -10 & -2 \end{bmatrix}$$

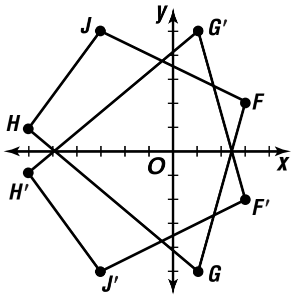
9. $$BC = \begin{bmatrix} -2 & 3 \\ -3 & 0 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 & 3 \\ 3 & 2 & -4 \end{bmatrix}$$
$$= \begin{bmatrix} -2(-1) + 3(3) & -2(0) + 3(2) & -2(3) + 3(-4) \\ -3(-1) + 0(3) & -3(0) + 0(2) & -3(3) + 0(-4) \end{bmatrix}$$
$$= \begin{bmatrix} 11 & 6 & -18 \\ 3 & 0 & -9 \end{bmatrix}$$

10. $$1.5\begin{bmatrix} 1 & -4 & 6 \\ -3 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 1.5 & -6 & 9 \\ -4.5 & 4.5 & 3 \end{bmatrix}$$
$A'(1.5, -4.5)$, $B'(-6, 4.5)$, and $C'(9, 3)$



11. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 & -6 & -3 \\ 2 & -5 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 1 & -6 & -3 \\ -2 & 5 & -1 & -5 \end{bmatrix}$

$F'(3, -2)$, $G'(1, 5)$, $H'(-6, -1)$, $J'(-3, -5)$



12. $\begin{vmatrix} 5 & 7 \\ 3 & 6 \end{vmatrix} = 5(6) - 3(7)$
$= 9$

13. $\begin{vmatrix} 2 & 4 \\ -10 & 9 \end{vmatrix} = 2(9) - (-10)(4)$
$= 58$

14. $\begin{vmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 4 & 1 & -2 \end{vmatrix} = 1\begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix} - 2\begin{vmatrix} 3 & 1 \\ 4 & -2 \end{vmatrix} + (-1)\begin{vmatrix} 3 & 0 \\ 4 & 1 \end{vmatrix}$
$= 1(-1) - 2(-10) - 1(3)$
$= 16$

15. $\dfrac{1}{\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix}} \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$
$= \begin{bmatrix} \frac{3}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix}$

16. $\dfrac{1}{\begin{vmatrix} 3 & -4 \\ 4 & -2 \end{vmatrix}} \begin{bmatrix} -2 & 4 \\ -4 & 3 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} -2 & 4 \\ -4 & 3 \end{bmatrix}$
$= \begin{bmatrix} -\frac{1}{5} & \frac{2}{5} \\ -\frac{2}{5} & \frac{3}{10} \end{bmatrix}$

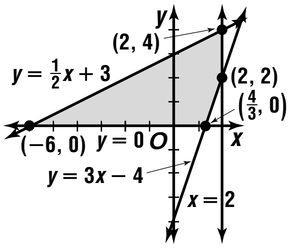
17. The inverse does not exist since $\begin{vmatrix} -5 & 4 \\ -15 & 12 \end{vmatrix} = 0$.

18. $\begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

$\dfrac{1}{\begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix}} \begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}$

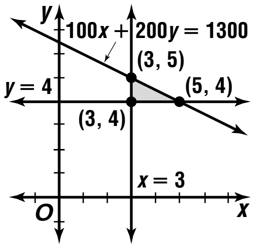
$\frac{1}{5}\begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 7 \\ 3 \end{bmatrix}$
$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

19.



$f(x, y) = 2x - y$
$f(-6, 0) = 2(-6) - 0$ or $-12 \Rightarrow$ minimum
$f(2, 4) = 2(2) - 4$ or 0
$f(2, 2) = 2(2) - 2$ or 2
$f\left(\frac{4}{3}, 0\right) = 2\left(\frac{4}{3}\right) - 0$ or $\frac{8}{3} \Rightarrow$ maximum

20. Let x = number of ads.
Let y = number of commercial minutes.
$x \geq 3$
$y \geq 4$
$100x + 200y \leq 1300$



$f(x, y) = 12{,}000x + 16{,}000y$
$f(3, 5) = 12{,}000(3) + 16{,}000(5)$
$= 116{,}000$
$f(3, 4) = 12{,}000(3) + 16{,}000(4)$
$= 100{,}000$
$f(5, 4) = 12{,}000(5) + 16{,}000(4)$
$= 124{,}000$

The company reaches the most people with 5 ads and 4 commercial minutes.

Chapter 3 Test

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1. $y = 2x + 1 \quad \rightarrow \quad b = 2a + 1$

x-axis: $-b = 2a + 1$
$b = -2a - 1$ no

y-axis: $b = 2(-a) + 1$
$b = -2a + 1$ no

$y = x$: $a = 2b + 1$
$b = \frac{a-1}{2}$ no

$y = -x$: $-a = 2(-b) + 1$
$a = 2b - 1$
$b = \frac{a+1}{2}$ no

none of these

2. $y = -\frac{2}{x^2}$ $\rightarrow$ $b = -\frac{2}{a^2}$

x-axis $\quad -b = -\frac{2}{a^2}$

$b = \frac{2}{a^2}$ no

y-axis $\quad b = -\frac{2}{(a)^2}$

$b = -\frac{2}{a^2}$ yes

$y = x$ $\quad a = -\frac{2}{b^2}$

$b = \sqrt{-\frac{2}{a}}$ no

$y = -x$ $\quad -a = -\frac{2}{(-b)^2}$

$-a = -\frac{2}{b^2}$

$b = \sqrt{\frac{2}{a}}$ no; y-axis

3. $x = y^2 + 3$ $\rightarrow$ $a = b^2 + 3$

x-axis $\quad a = (-b)^2 + 3$

$a = b^2 + 3$ yes

y-axis $\quad -a = b^2 + 3$

$a = -b^2 - 3$ no

$y = x$ $\quad b = a^2 + 3$

$a = \sqrt{b - 3}$ no

$y = -x$ $\quad -b = (-a)^2 + 3$

$-b = a^2 + 3$

$a = \sqrt{-b - 3}$

no; x-axis

4. $xy = 5$ $\rightarrow$ $ab = 5$

x-axis $\quad a(-b) = 5$

$-ab = 5$ no

y-axis $\quad (-a)b = 5$

$-ab = 5$ no

$y = x$ $\quad ba = 5$

$ab = 5$ yes

$y = -x$ $\quad (-b)(-a) = 5$

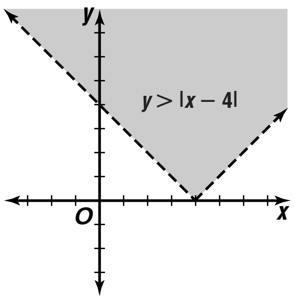
$ab = 5$ yes

$y = x, y = -x$

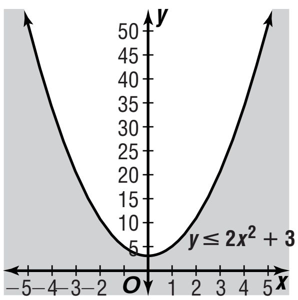
5. The graph of $g(x)$ is the graph of $f(x)$ translated left 3 units and reflected over the x-axis.

6. The graph of $g(x)$ is the graph of $f(x)$ stretched vertically by a factor of 4 and then translated down 2 units.

7.



8.



9. $f(x) = 5x - 4$

$y = 5x - 4$

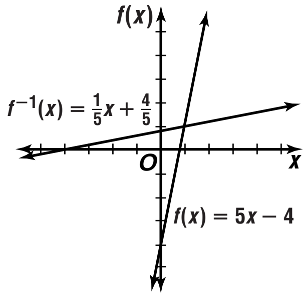
$x = 5y - 4$

$x + 4 = 5y$

$y = \frac{x + 4}{5}$

$f^{-1}(x) = \frac{1}{5}x + \frac{4}{5}$

Yes, it is a function.



10. $f(x) = \frac{3}{x - 2}$

$y = \frac{3}{x - 2}$

$x = \frac{3}{y - 2}$

$y - 2 = \frac{3}{x}$

$y = \frac{3}{x} + 2$

$f^{-1}(x) = \frac{3}{x} + 2$; Yes, it is a function.

11. $f(2) = \frac{2^2}{2 - 2} = \frac{4}{0}$

No; the function is undefined when $x = 2$.

12. Yes; the function is defined when $x = 0$, the function approaches 1 as x approaches 0 from both sides, and $f(0) = 1$.

13. $y \rightarrow \infty$ as $x \rightarrow \infty$, $y \rightarrow \infty$ as $x \rightarrow -\infty$

14.

x	y
3	−11
4	−12
5	−11

At $x = 4$, y is at a minimum value, since $f(4) < f(3)$ and $f(4) < f(5)$.

15.

x	y
0	−1
1	−2
2	−3

At $x = 1$, y is at a point of inflection, since $f(0) > f(1) > f(2)$.

16. $x = 1$

$$y = \frac{4x}{x - 1}$$

$$y = \frac{\frac{4x}{x}}{\frac{x}{x} - \frac{1}{x}}$$

$$y = \frac{4}{1 - \frac{1}{x}}$$

as $x \rightarrow \infty$, $y \rightarrow 4$; $y = 4$

17. $x^2 - 4 = 0$

$(x + 2)(x - 2) = 0$

$x = \pm 2$

$$y = \frac{x}{x^2 - 4}$$

$$y = \frac{\frac{x}{x^2}}{\frac{x^2}{x^2} - \frac{4}{x^2}}$$

$$y = \frac{\frac{1}{x}}{1 - \frac{4}{x^2}}$$

as $x \rightarrow \infty$, $y \rightarrow 0$; $y = 0$

18a. $y = \dfrac{0.25 + 0.004x}{0.25 + 0.001x}$

$y = \dfrac{\frac{0.25}{x} + \frac{0.004x}{x}}{\frac{0.25}{x} + \frac{0.001x}{x}}$

$y = \dfrac{\frac{0.25}{x} + 0.004}{\frac{0.25}{x} + 0.001}$

as $x \to \infty$, $y \to \frac{0.004}{0.001}$ or 4; $y = 4$

18b. As the amount of 4 molar solution added increases, the molarity of the mixture approaches 4.

19. $y = kx$ | $y = 0.25x$

$0.5 = k(2)$ | $y = 0.25(10)$

$0.25 = k$ | $y = 2.5$

20. $y = \frac{k}{x^2}$ | $y = \frac{72}{x^2}$

$8 = \frac{k}{(3)^2}$ | $18 = \frac{72}{x^2}$

$72 = k$ | $x^2 = 4$

$x = \pm 2$

Chapter 4 Test

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1. $(x - 4)(x - \boldsymbol{i})(x - (-\boldsymbol{i})) = (x - 4)(x - \boldsymbol{i})(x + \boldsymbol{i})$
$= (x - 4)(x^2 - \boldsymbol{i}^2)$
$= (x - 4)(x^2 + 1)$
$= x^3 - 4x^2 + x - 4$

2. $b^2 - 4ac = (-5)^2 - 4(1)(4)$
$= 9$

Since $b^2 - 4ac > 0$, there are 2 real roots.

$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$n = \frac{-(-5) \pm \sqrt{9}}{2(1)}$

$n = \frac{5 \pm 3}{2}$

$n = \frac{5 + 3}{2}$ or $n = \frac{5 - 3}{2}$

$n = 4$ $\quad n = 1$

3. $b^2 - 4ac = (-7)^2 - 4(1)(-3)$
$= 61$

Since $b^2 - 4ac > 0$, there are 2 real roots.

$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$z = \frac{-(-7) \pm \sqrt{61}}{2(1)}$

$z = \frac{7 \pm \sqrt{61}}{2}$

4. $b^2 - 4ac = (-5)^2 - 4(2)(4)$
$= -7$

Since $b^2 - 4ac < 0$, there are 2 imaginary roots.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-5) \pm \sqrt{-7}}{2(2)}$

$x = 5 \pm \frac{\sqrt{7}\boldsymbol{i}}{4}$

5.
$$\begin{array}{r|rrrr} 2 & 2 & -3 & 3 & -4 \\ & & 4 & 2 & 10 \\ \hline & 2 & 1 & 5 & 6 \end{array}$$

$2x^2 + x + 5$, R6

6.
$$\begin{array}{r|rrrrr} -1 & 1 & -5 & -13 & 53 & 60 \\ & & -1 & 6 & 7 & -60 \\ \hline & 1 & -6 & -7 & 60 & 0 \end{array}$$

$x^3 - 6x^2 - 7x + 60$

7. $f(x) = x^3 + 8x^2 + 2x - 11$
$f(-2) = (-2)^3 + 8(-2)^2 + 2(-2) - 11$
$= -8 + 32 - 4 - 11$
$= 9$; no

8. $f(x) = 4x^4 - 2x^2 + x - 3$
$f(1) = 4(1)^4 - 2(1)^2 + 1 - 3$
$= 4 - 2 - 2$
$= 0$; yes

9. 1 positive
$f(-x) = -6x^3 + 11x^2 + 3x - 2$
2 or 0 negative

r	6	11	−3	−2
$\frac{1}{2}$	6	14	4	0
$-\frac{1}{3}$	6	9	−6	0
−2	6	−1	−1	0

rational zeros: $-2, -\frac{1}{3}, \frac{1}{2}$

10. 1 positive
$f(-x) = x^4 - x^3 - 9x^2 + 17x - 8$
3 or 1 negative

r	1	1	−9	−17	−8
8	1	9	63	487	3888
−8	1	−7	47	−393	3136
⋮	⋮	⋮	⋮	⋮	⋮
1	1	2	−7	−24	−32
−1	1	0	−9	−8	0

rational zero: −1

11. Use the TABLE function of a graphing calculator; −0.8, 3.8.

12. Use the TABLE function of a graphing calculator: −1.3.

13. Sample answer: 2; −5

r	1	3	−5	−9
1	1	4	−1	−10
2	1	5	5	1

upper bound: 2
$f(-x) = -x^3 + 3x^2 + 5x - 9$

r	−1	3	5	−9
1	−1	2	7	−2
2	−1	1	7	5
3	−1	0	5	6
4	−1	−1	1	−5
5	−1	−2	−5	−34

lower bound: −5

14. Sample answer: 1; -2

r	2	3	-1	1	1
1	2	5	4	5	6

upper bound: 1

$f(-x) = 2x^4 - 3x^3 - x^2 - x + 1$

r	2	-3	-1	-1	1
1	2	-1	-2	-3	-2
2	2	1	1	1	3

lower bound: -2

15. $\frac{1}{80} + \frac{1}{a} = \frac{1}{10}$

$\frac{1}{a} = \frac{7}{80}$

$a = \frac{80}{7}$

16. $\frac{4}{x-2} = \frac{3}{x^2-4} - \frac{1}{4}$

$\frac{4}{x-2} = \frac{3}{(x+2)(x-2)} - \frac{1}{4}$

$4(x+2) = 3 - \frac{(x+2)(x-2)}{4}$

$16(x+2) = 12 - (x+2)(x-2)$

$16x + 32 = 12 - x^2 + 4$

$x^2 + 16x + 16 = 0$

$x = \frac{-16 \pm \sqrt{16^2 - 4(1)(16)}}{2(1)}$

$x = \frac{-16 \pm 8\sqrt{3}}{2}$

$x = -8 \pm 4\sqrt{3}$

17. $\frac{5}{x+2} = \frac{5}{x} + \frac{2}{3x}$

$15x = 15(x+2) + 2(x+2)$

$15x = 15x + 30 + 2x + 4$

$-2x = 34$

$x = -17;\ x \neq -2 \text{ or } 0$

Test $x = -18$: $\frac{5}{-18+2} \overset{?}{>} \frac{5}{-18} + \frac{2}{3(-18)}$

$-\frac{5}{16} \overset{?}{>} -\frac{5}{18} - \frac{1}{27}$

$-0.3125 \overset{?}{>} -0.3148$ true

Test $x = -3$: $\frac{5}{-3+2} \overset{?}{>} \frac{5}{-3} + \frac{2}{3(-3)}$

$-5 \overset{?}{>} -\frac{5}{3} - \frac{2}{9}$

$-5 \overset{?}{>} -1\frac{8}{9}$ false

Test $x = -1$: $\frac{5}{-1+2} \overset{?}{>} \frac{5}{-1} + \frac{2}{3(-1)}$

$5 \overset{?}{>} -5 - \frac{2}{3}$

$5 \overset{?}{>} -5\frac{2}{3}$ true

Test $x = 1$: $\frac{5}{1+2} \overset{?}{>} \frac{5}{1} + \frac{2}{3(1)}$

$\frac{5}{3} \overset{?}{>} 5 + \frac{2}{3}$

$1\frac{2}{3} \overset{?}{>} 5\frac{2}{3}$ false

$x < -17,\ -2 < x < 0$

18. $\sqrt{y-2} - 3 = 0$

$\sqrt{y-2} = 3$

$y - 2 = 9$

$y = 11$

Check: $\sqrt{11-2} - 3 \overset{?}{=} 0$

$\sqrt{9} - 3 \overset{?}{=} 0$

$3 - 3 \overset{?}{=} 0$

$0 = 0$ ✓

19. $\sqrt{2x+2} = \sqrt{3x-5}$

$2x + 2 = 3x - 5$

$7 = x$

Check: $\sqrt{2(7)+2} \overset{?}{=} \sqrt{3(7)-5}$

$\sqrt{16} = \sqrt{16}$ ✓

20. $\sqrt{11 - 10m} > 9$

$11 - 10m > 81$

$-10m > 70$

$m < -7$

$11 - 10m \geq 0$

$-10m \geq -11$

$m \leq \frac{11}{10}$

Test $m = -8$: $\sqrt{11 - 10(-8)} \overset{?}{>} 9$

$\sqrt{91} \overset{?}{>} 9$

$9.54 \overset{?}{>} 9$ true

Test $m = 0$: $\sqrt{11 - 10(0)} \overset{?}{>} 9$

$\sqrt{11} \overset{?}{>} 9$

$3.32 \overset{?}{>} 9$ false

Solution: $m < -7$

21. $\frac{5z-11}{2z^2+z-6} = \frac{5z-11}{(z+2)(2z-3)}$

$\frac{5z-11}{2z^2+z-6} = \frac{A}{z+2} + \frac{B}{2z-3}$

$5z - 11 = A(2z-3) + B(z+2)$

Let $z = \frac{3}{2}$.

$5\left(\frac{3}{2}\right) - 11 = A\left(2 \cdot \frac{3}{2} - 3\right) + B\left(\frac{3}{2} + 2\right)$

$-\frac{7}{2} = \frac{7}{2}B$

$-1 = B$

Let $z = -2$.

$5(-2) - 11 = A(2 \cdot (-2) - 3) + B(-2 + 2)$

$-21 = -7A$

$3 = A$

$\frac{5z-11}{2z^2+z-6} = \frac{3}{z+2} - \frac{1}{2z-3}$

22. $\frac{7x^2+18x-1}{(x^2-1)(x+2)} = \frac{7x^2+18x-1}{(x+1)(x-1)(x+2)}$

$\frac{7x^2+18x-1}{(x^2-1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x-1}$

$7x^2 + 18x - 1 = A(x-1)(x+2) + B(x+1)(x-1) + C(x+1)(x+2)$

Let $x = 1$.

$7(1)^2 + 18(1) - 1 = A(1-1)(1+2) + B(1+1)(1-1) + C(1+1)(1+2)$

$24 = 6C$

$4 = C$

Let $x = -2$.

$7(-2)^2 + 18(-2) - 1 = A(-2-1)(-2+2) + B(-2+1)(-2-1) + C(-2+1)(-2+2)$

$-9 = 3B$

$-3 = B$

Let $x = -1$.

$7(-1)^2 + 18(-1) - 1 = A(-1-1)(-1+2) + B(-1+1)(-1-1) + C(-1+1)(-1+2)$

$-12 = -2A$

$6 = A$

$\frac{7x^2+18x-1}{(x^2-1)(x+2)} = \frac{6}{x+1} - \frac{3}{x+2} + \frac{4}{x-1}$

23. quadratic

24. Let x = the height.
Then $6x$ = the length and $6x - 7$ = the width.
$V = x(6x)(6x - 7)$
$120 = 36x^3 - 42x^2$
$0 = 6x^3 - 7x^2 - 20$
Use a graphing calculator to graph the related function.
$V(x) = 6x^3 - 7x^2 - 20$
It appears as if the zero is at $x = 2$.
Use the Factor Theorem to check
$V(2) = 6(2)^3 - 7(2)^2 - 20 = 0$.
The height is 2 cm, the length is 6(2) or 12 cm and the width is 12 − 7 or 5 cm.
12 cm by 5 cm by 2 cm

25. Let s = the speed of the freight train.
Then $30 + s$ = the speed of the car.
Car: $500 = (30 + s)t \Rightarrow t = \frac{500}{30 + s}$
Train: $350 = st \Rightarrow t = \frac{350}{s}$
$\frac{500}{30 + s} = \frac{350}{s}$
$500\,s = 10{,}500 + 350s$
$150s = 10{,}500$
$s = 70$ km/h

Chapter 5 Test

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1. $\frac{995°}{360°} \approx 2.76$
$\alpha + 360°(2) = 995°$
$\alpha + 720° = 995°$
$x = 275°$; IV

2. $\frac{-234°}{360°} = -0.65$
$\alpha + 360°(-1) = -234°$
$\alpha - 360° = -234°$
$\alpha = 126°$; II

3. $\frac{410°}{360°} \approx 1.14$
$\alpha + 360°(1) = 410°$
$\alpha + 360° = 410°$
$\alpha = 50°$; I

4. $\frac{-1245°}{360°} \approx -3.46$
$\alpha + 360°(-4) = -1245°$
$\alpha - 1440° = -1245°$
$\alpha = 195°$; III

5. $(RS)^2 + (ST)^2 = (RT)^2$
$(RS)^2 + 12^2 = 15^2$
$RS^2 = 225 - 144$
$RS = \sqrt{81}$ or 9 in.

$\sin R = \frac{\text{side opposite}}{\text{hypotenuse}}$	$\cos R = \frac{\text{side adjacent}}{\text{hypotenuse}}$
$\sin R = \frac{12}{15}$ or $\frac{4}{5}$	$\cos R = \frac{9}{15}$ or $\frac{3}{5}$
$\tan R = \frac{\text{side opposite}}{\text{side adjacent}}$	$\csc R = \frac{\text{hypotenuse}}{\text{side opposite}}$
$\tan R = \frac{12}{9}$ or $\frac{4}{3}$	$\csc R = \frac{15}{12}$ or $\frac{5}{4}$
$\sec R = \frac{\text{hypotenuse}}{\text{side adjacent}}$	$\cot R = \frac{\text{side adjacent}}{\text{side opposite}}$
$\sec R = \frac{15}{9}$ or $\frac{5}{3}$	$\cos R = \frac{9}{12}$ or $\frac{3}{4}$

6. $\tan 60° = \frac{y}{x}$
$\tan 60° = \frac{\sqrt{3}}{1}$ or $\sqrt{3}$

7. $\sec 270° = \frac{r}{x}$
$\sec 270° = \frac{1}{0}$; undefined

8. $\sin(-405°) = \sin(-45°)$
$\sin(-45°) = -y$
$\sin(-45°) = -\frac{\sqrt{2}}{2}$

9. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{3^2 + 5^2}$
$r = \sqrt{34}$

$\sin\theta = \frac{y}{r}$	$\cos\theta = \frac{x}{r}$	$\tan\theta = \frac{y}{x}$
$\sin\theta = \frac{5}{\sqrt{34}}$	$\cos\theta = \frac{3}{\sqrt{34}}$	$\tan\theta = \frac{5}{3}$
$\sin\theta = \frac{5\sqrt{34}}{34}$	$\cos\theta = \frac{3\sqrt{34}}{34}$	
$\csc\theta = \frac{r}{y}$	$\sec\theta = \frac{r}{x}$	$\cot\theta = \frac{x}{y}$
$\csc\theta = \frac{\sqrt{34}}{5}$	$\sec\theta = \frac{\sqrt{34}}{3}$	$\cot\theta = \frac{3}{5}$

10. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{(-4)^2 + 2^2}$
$r = \sqrt{20}$ or $2\sqrt{5}$

$\sin\theta = \frac{y}{r}$	$\cos\theta = \frac{x}{r}$	$\tan\theta = \frac{y}{x}$
$\sin\theta = \frac{2}{2\sqrt{5}}$	$\cos\theta = \frac{-4}{2\sqrt{5}}$	$\tan\theta = \frac{2}{-4}$
$\sin\theta = \frac{\sqrt{5}}{5}$	$\cos\theta = -\frac{2\sqrt{5}}{5}$	$\tan\theta = -\frac{1}{2}$
$\csc\theta = \frac{r}{y}$	$\sec\theta = \frac{r}{x}$	$\cot\theta = \frac{x}{y}$
$\csc\theta = \frac{2\sqrt{5}}{2}$	$\sec\theta = \frac{2\sqrt{5}}{-4}$	$\cot\theta = \frac{-4}{2}$
$\csc\theta = \sqrt{5}$	$\sec\theta = -\frac{\sqrt{5}}{2}$	$\cot\theta = -2$

11. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{0^2 + (-3)^2}$
$r = \sqrt{9}$ or 3

$\sin\theta = \frac{y}{r}$	$\cos\theta = \frac{x}{r}$	$\tan\theta = \frac{y}{x}$
$\sin\theta = \frac{-3}{3}$	$\cos\theta = \frac{0}{3}$	$\tan\theta = \frac{-3}{0}$
$\sin\theta = -1$	$\cos\theta = 0$	undefined
$\csc\theta = \frac{r}{y}$	$\sec\theta = \frac{r}{x}$	$\cot\theta = \frac{x}{y}$
$\csc\theta = \frac{3}{-3}$	$\sec\theta = \frac{3}{0}$	$\cot\theta = \frac{0}{-3}$
$\csc\theta\ -1$	undefined	$\cot\theta = 0$

12. $\cos A = \frac{b}{c}$

$\cos \pi° = \frac{42}{c}$

$c = \frac{42}{\cos 77°}$

$c \approx 186.7$

13. $\tan B = \frac{b}{a}$

$\tan 27° = \frac{b}{13}$

$b = 13 \tan 27°$

$b \approx 6.6$

14. $\sin A = \frac{a}{c}$

$\sin 32° \, 17' = \frac{a}{14}$

$a = 14 \sin 32° \, 17°$

$a \approx 7.5$

15. $\cos B = \frac{a}{c}$

$\cos B = \frac{23}{37}$

$B = \cos^{-1} \frac{23}{37}$

$B \approx 51.6°$

16. $\tan A = \frac{a}{b}$

$\tan A = \frac{3}{11}$

$A = \tan^{-1} \frac{3}{11}$

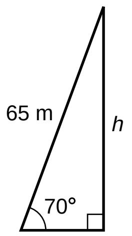
$A \approx 15.3°$

17. Let h = the height.

$\sin 70° = \frac{h}{65}$

$h = 65 \sin 70°$

$h \approx 61.1$ m



18. $B = 180° - 36° - 87°$ or $57°$

$K = \frac{1}{2}a^2 \frac{\sin B \sin C}{\sin A}$

$K = \frac{1}{2}(24)^2\left(\frac{\sin 57° \sin 87°}{\sin 36°}\right)$

$K \approx 410.4$ units^2

19. $K = \frac{1}{2}bc \sin A$

$K = \frac{1}{2}(56.4)(92.5) \sin 58.4°$

$K \approx 2221.7$ units^2

20. $\frac{180° - 36°}{2} = 72°$

$\frac{\sin 36°}{22} = \frac{\sin 72°}{x}$

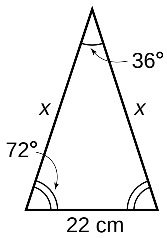
$x \sin 36° = 22 \sin 72°$

$s = \frac{22 \sin 72°}{\sin 36°}$

$x \approx 35.6$ cm

Perimeter $\approx 22 + 35.6 + 35.6$

≈ 93.2 cm



21. Since $98° \geq 90°$, consider Case II.

$c > a$; one solution

$\frac{\sin 98°}{90} = \frac{\sin A}{64}$

$64 \sin 98° = 90 \sin A$

$A = \sin^{-1} \frac{64 \sin 98°}{90}$

$A \approx 44.8°$

$B \approx 180° - 98° - 44.8°$ or $37.2°$

$\frac{\sin 37.2}{b} \approx \frac{\sin 98°}{90}$

$90 \sin 37.2° \approx b \sin 98°$

$b \approx \frac{90 \sin 37.2°}{\sin 98°}$

$b \approx 54.9$

$A = 44.8°$, $B = 37.2°$, $b = 54.9$

22. Since $31° \leq 90°$, consider Case I.

$a \stackrel{?}{\geq} b \sin A$

$9 \stackrel{?}{\geq} 20 \sin 31°$

$9 < 10.3$; none

23. $a^2 = b^2 + c^2 - 2bc \cos A$

$13^2 = 7^2 + 15^2 - 2(7)(15) \cos A$

$169 = 274 - 210 \cos A$

$-105 = -210 \cos A$

$A = \cos^{-1}\left(\frac{105}{210}\right)$

$A = 60°$

$\frac{\sin 60°}{13} = \frac{\sin B}{7}$

$7 \sin 60° = 13 \sin B$

$B = \sin^{-1} \frac{7 \sin 60°}{13}$

$B \approx 27.8°$

$C \approx 180° - 60° - 27.8°$ or $92.2°$

$A = 60°$, $B = 27.8°$, $C = 92.2°$

24. $b^2 = a^2 + c^2 - 2ac \cos B$

$b = \sqrt{20^2 + 24^2 - 2(20)(24) \cos 47°}$

$b \approx 17.92432912$

$\frac{\sin 47°}{b} = \frac{\sin C}{24}$

$24 \sin 47° = b \sin C$

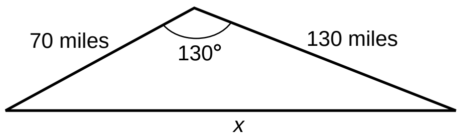
$C = \sin^{-1} \frac{24 \sin 47°}{b}$

$C \approx 78.3°$

$A \approx 180° - 47° - 78.3°$ or $54.7°$

$b = 17.9$, $C = 78.3°$, $A = 54.7°$

25. Let x = the distance between the transmitters.



$x^2 = 70° + 130^2 - 2(70)(130) \cos 130°$

$x = \sqrt{21{,}800 - 18{,}200 \cos 130°}$

$x \approx 183.0$ miles

Chapter 6 Test

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1. $225° = 225° \cdot \frac{\pi}{180°}$

$= \frac{5\pi}{4}$

2. $480° = 480° \cdot \frac{\pi}{180°}$

$= \frac{8\pi}{3}$

3. reference angle: $\pi - \frac{5\pi}{6} = \frac{\pi}{6}$; Quadrant 2

$\sin \frac{5\pi}{6} = \frac{1}{2}$

4. reference angle: $\frac{5\pi}{4} - \pi = \frac{\pi}{4}$; Quadrant 3

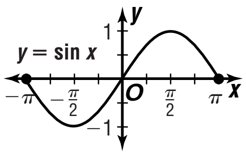
$\tan \frac{5\pi}{4} = 1$

5. $v = r\,\frac{\theta}{t}$

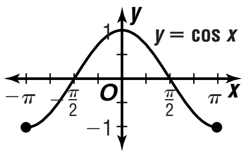
$v = 12 \cdot \left(\frac{7.1}{1}\right)$

$v = 85.2$ cm/s

6.



7.



8. $|3| = 3;\ \frac{2\pi}{4}$ or $\frac{\pi}{2}$

9. $|-2| = 2;\ \frac{2\pi}{\frac{3\pi}{2}}$ or $\frac{4}{3}$

10. $|A| = 4$ $\qquad \frac{2\pi}{k} = 4\pi$

$A = \pm 4$ $\qquad k = \frac{2\pi}{4\pi}$ or 0.5

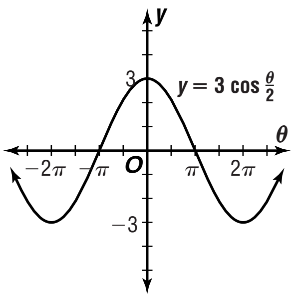
$y = \pm 4 \sin 0.5\theta - 2$

11. $|A| = 0.5$ $\qquad \frac{2\pi}{k} = \frac{\pi}{2}$ $\qquad -\frac{c}{4} = \frac{\pi}{4}$

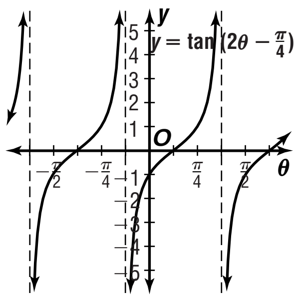
$A = \pm 0.5$ $\qquad k = \frac{2\pi}{\frac{\pi}{2}}$ or 4 $\qquad c = -\pi$

$y = \pm 0.5 \cos(4\theta - \pi) + 1$

12.



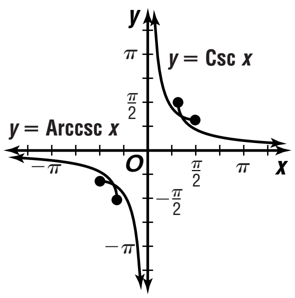
13.



14. $y = \text{Arccsc } x$

$x = \text{Arccsc } y$

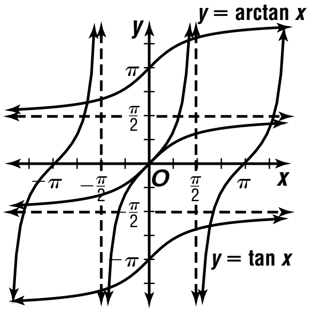
$\text{Csc } x = y$ or $y = \text{Csc } x$



15. $y = \tan x$

$x = \tan y$

$\arctan x = y$ or $y = \arctan x$



16. $\sin\left(\text{Arccos } \frac{1}{2}\right) = \sin 60°$

$= \frac{\sqrt{3}}{2}$

17. $\tan\left(\pi + \sin^{-1}\frac{1}{2}\right) = \tan\left(\pi + \frac{\pi}{6}\right)$

$= \tan\left(\frac{7\pi}{6}\right)$

$= \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$

18. $v = r\,\frac{\theta}{t}$

$2300 = 240{,}000\left(\frac{2\pi}{t}\right)$

$t = \frac{240{,}000(2\pi)}{2300}$

$t \approx 656$ hours or 27.3 days

19. $|A| = \frac{73 - 21}{2}$ $\qquad h = \frac{73 + 21}{2}$ $\qquad \frac{2\pi}{k} = 12$

$|A| = 26$ $\qquad h = 47$ $\qquad k = \frac{2\pi}{12}$

$A = \pm 26$ $\qquad k = \frac{\pi}{6}$

$y = 26 \sin\left(\frac{\pi}{6}t + c\right) + 47$

$21 = 26 \sin\left(\frac{\pi}{6} \cdot 1 + c\right) + 47$

$-26 = 26 \sin\left(\frac{\pi}{6} + c\right)$

$-1 = \sin\left(\frac{\pi}{6} + c\right)$

$\sin^{-1}(-1) = \frac{\pi}{6} + c$

$-\frac{\pi}{2} = \frac{\pi}{6} + c$

$-\frac{4\pi}{6} = c$

$-\frac{2\pi}{3} = c$

Sample answer: $y = 26 \sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right) + 47$

20. $\frac{65 \text{ miles}}{1 \text{ hour}} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}} \cdot \frac{1 \text{ hour}}{3600 \text{ seconds}} = 95.\overline{3} \text{ ft/s}$

$\tan \theta = \frac{v^2}{rg}$

$\theta = \tan^{-1} \frac{(95.\overline{3})^2}{1200(32)}$

$\theta \approx 0.23 \text{ radians}$

Chapter 7 Test

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1. $\sin^2 \theta + \cos^2 \theta = 1$

$\left(\frac{1}{3}\right)^2 + \cos^2 \theta = 1$

$\cos^2 \theta = \frac{8}{9}$

$\cos \theta = \frac{2\sqrt{2}}{3}$

2. $\tan^2 \theta + 1 = \sec^2 \theta$

$\tan^2 \theta + 1 = (-2)^2$

$\tan^2 \theta = 3$

$\tan \theta = -\sqrt{3}$

3. $\sin^2 \theta + \cos^2 \theta = 1$

$\left(-\frac{4}{5}\right)^2 + \cos^2 \theta = 1$

$\cos^2 \theta = \frac{9}{25}$

$\cos \theta = -\frac{3}{5}$

$\sec \theta = \frac{1}{\cos \theta}$

$\sec \theta = \frac{1}{-\frac{3}{5}}$ or $-\frac{5}{3}$

4. $\sin \theta = \frac{1}{\csc \theta}$

$\sin \theta = \frac{1}{-\frac{5}{3}}$ or $-\frac{3}{5}$

$\sin^2 \theta + \cos^2 \theta = 1$

$\left(-\frac{3}{5}\right)^2 + \cos^2 \theta = 1$

$\cos^2 \theta = \frac{16}{25}$

$\cos \theta = \frac{4}{5}$

5. $\tan(-420°) = \frac{\sin(-420°)}{\cos(-420°)}$

$= \frac{\sin(-360° - 60°)}{\cos(-360° - 60°)}$

$= \frac{-\sin 60°}{\cos 60°}$

$= -\tan 60°$

6. $\tan \theta(\cot \theta + \tan \theta) \stackrel{?}{=} \sec^2 \theta$

$\tan \theta \cot \theta + \tan^2 \theta \stackrel{?}{=} \sec^2 \theta$

$1 + \tan^2 \theta \stackrel{?}{=} \sec^2 \theta$

$\sec^2 \theta = \sec^2 \theta$

7. $\sin^2 A \cos^2 A = (1 - \sin A)(1 + \sin A)$

$\sin^2 A \cdot \frac{\cos^2 A}{\sin^2 A} \stackrel{?}{=} (1 - \sin A)(1 + \sin A)$

$\cos^2 A \stackrel{?}{=} (1 - \sin A)(1 + \sin A)$

$1 - \sin^2 A = (1 - \sin A)(1 + \sin A)$

$(1 - \sin A)(1 + \sin A) = (1 - \sin A)(1 + \sin A)$

8. $\frac{\sec x}{\sin x} - \frac{\sin x}{\cos x} \stackrel{?}{=} \cot x$

$\frac{\sec x \cos x - \sin^2 x}{\sin x \cos x} \stackrel{?}{=} \cot x$

$\frac{1 - \sin^2 x}{\sin x \cos x} \stackrel{?}{=} \cot x$

$\frac{\cos x}{\sin x} \stackrel{?}{=} \cot x$

$\cot x = \cot x$

9. $\frac{\cos x}{1 + \sin x} + \frac{\cos x}{1 - \sin x} \stackrel{?}{=} 2 \sec x$

$\frac{\cos x - \cos x \sin x + \cos x + \sin x \cos x}{1 - \sin^2 x} \stackrel{?}{=} 2 \sec x$

$\frac{2 \cos x}{1 - \sin^2 x} \stackrel{?}{=} 2 \sec x$

$\frac{2 \cos x}{\cos^2 x} \stackrel{?}{=} 2 \sec x$

$\frac{2}{\cos x} \stackrel{?}{=} 2 \sec x$

$2 \sec x = 2 \sec x$

10. $\csc(A - B) \stackrel{?}{=} \frac{\sec B}{\sin A - \cos A \tan B}$

$\csc(A - B) \stackrel{?}{=} \frac{\frac{1}{\cos B}}{\sin A - \cos A \frac{\sin B}{\cos B}}$

$\csc(A - B) \stackrel{?}{=} \frac{1}{\sin A \cos B - \cos A \sin B}$

$\csc(A - B) \stackrel{?}{=} \frac{1}{\sin(A - B)}$

$\csc(A - B) = \csc(A - B)$

11. $\cot 2\theta \stackrel{?}{=} \frac{1}{2} \cot \theta - \frac{1}{2} \tan \theta$

$\cos 2\theta \stackrel{?}{=} \frac{\cos \theta}{2 \sin \theta} - \frac{\sin \theta}{2 \cos \theta}$

$\cot 2\theta \stackrel{?}{=} \frac{\cos^2 \theta - \sin^2 \theta}{2 \sin \theta \cos \theta}$

$\cot 2\theta \stackrel{?}{=} \frac{\cos 2\theta}{\sin 2\theta}$

$\cot 2\theta = \cot 2\theta$

12. $\sin 255° = \sin(225° + 30°)$

$= \sin 225° \cos 30° + \cos 225° \sin 30°$

$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \left(-\frac{\sqrt{2}}{2}\right) \cdot \frac{1}{2}$

$= \frac{-\sqrt{6} - \sqrt{2}}{4}$

$= -\frac{\sqrt{2} + \sqrt{6}}{4}$

13. $\tan \frac{5\pi}{12} = \tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$

$= \frac{\tan \frac{\pi}{6} + \tan \frac{\pi}{4}}{1 - \tan \frac{\pi}{6} \tan \frac{\pi}{4}}$

$= \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}} \cdot 1}$

$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} \cdot \frac{1 + \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$

$= \frac{1 + \frac{2}{\sqrt{3}} + \frac{1}{3}}{1 - \frac{1}{3}}$

$= \frac{3}{2}\left(\frac{4}{3} + \frac{2\sqrt{3}}{3}\right)$

$= 2 + \sqrt{3}$

14. $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(\frac{3}{4}\right)^2 = 1$

$\sin^2\theta = \frac{7}{16}$

$\sin\theta = -\frac{\sqrt{7}}{4}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{-\frac{\sqrt{7}}{4}}{\frac{3}{4}}$

$= -\frac{\sqrt{7}}{3}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(-\frac{\sqrt{7}}{4}\right)\left(\frac{3}{4}\right)$

$= -\frac{3\sqrt{7}}{8}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{3}{4}\right)^2 - \left(-\frac{\sqrt{7}}{4}\right)^2$

$= \frac{9}{16} - \frac{7}{16}$

$= \frac{2}{16}$ or $\frac{1}{8}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\left(-\frac{\sqrt{7}}{3}\right)}{1-\left(-\frac{\sqrt{7}}{3}\right)^2}$

$= \frac{-\frac{2\sqrt{7}}{3}}{\frac{2}{9}}$

$= -3\sqrt{7}$

15. $\cos(22.5°) = \cos\left(\frac{45°}{2}\right)$

$= +\sqrt{\frac{1+\cos 45°}{2}}$

$= \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}$

$= \sqrt{\frac{2+\sqrt{2}}{4}}$

$= \frac{\sqrt{2+\sqrt{2}}}{2}$

16. $\tan^2 x = \sqrt{3}\tan x$

$\tan^2 x - \sqrt{3}\tan x = 0$

$\tan x(\tan x - \sqrt{3}) = 0$

$\tan x = 0$ or $\tan x - \sqrt{3} = 0$

$x = 0°$ $\quad \tan x = \sqrt{3}$

$x = 60°$

17. $\cos 2x - \cos x = 0$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$2\cos x + 1 = 0$ or $\cos x - 1 = 0$

$\cos x = -\frac{1}{2}$ $\quad \cos x = 1$

$x = 120°$ $\quad x = 0°$

18. $\sin x - \cos x = 0$

$\sin x = \cos x$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1$

$x = 45°$ or $x = 225°$

19. $2\cos^2 x + 3\sin x = 3$

$2(1-\sin^2 x) + 3\sin x - 3 = 0$

$2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$2\sin x - 1 = 0$ or $\sin x - 1 = 0$

$\sin x = \frac{1}{2}$ $\quad \sin x = 1$

$x = 30°, 150°$ $\quad x = 90°$

20. $y = x + 3$

$-x + y - 3 = 0$

$\sqrt{A^2+B^2} = \sqrt{(-1)^2+1^2}$ or $\sqrt{2}$

$-\frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{3}{\sqrt{2}} = 0$

$-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y - \frac{3\sqrt{2}}{2} = 0$

$\sin\phi = \frac{\sqrt{2}}{2}$, $\cos\phi = -\frac{\sqrt{2}}{2}$, $p = \frac{3\sqrt{2}}{2}$; Quadrant II

$\tan\phi = \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}$ or -1

$\phi = \tan^{-1}(-1) + \pi$

$= 135°$

21. $5 + 5y = 10x$

$0 = 10x - 5y - 5$

$\sqrt{A^2+B^2} = \sqrt{10^2+(-5)^2}$ or $5\sqrt{5}$

$\frac{10}{5\sqrt{5}}x - \frac{5}{5\sqrt{5}}y - \frac{5}{5\sqrt{5}} = 0$

$\frac{2\sqrt{5}}{5}x - \frac{\sqrt{5}}{5}y - \frac{\sqrt{5}}{5} = 0$

$\sin\phi = -\frac{\sqrt{5}}{5}$, $\cos\phi = \frac{2\sqrt{5}}{5}$, $p = -\frac{\sqrt{5}}{5}$; Quadrant IV

$\tan\phi = \frac{-\frac{\sqrt{5}}{5}}{\frac{2\sqrt{5}}{5}}$ or $-\frac{1}{2}$

$\phi = \tan^{-1}\left(-\frac{1}{2}\right) + 2\pi$

$\approx 333°$

22. $2x + y = 6 \rightarrow 2x + y - 6 = 0$

$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}}$

$d = \frac{2(-5) + 1(8) - 6}{\sqrt{2^2+1^2}}$

$d = -\frac{8}{\sqrt{5}}$ or $-\frac{8\sqrt{5}}{5}$

$\frac{8\sqrt{5}}{5}$

23. $d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2+B^2}}$

$d = \frac{3(-6) + 4(8) + 2}{-\sqrt{3^2+4^2}}$

$d = -\frac{16}{5}$

$\frac{16}{5}$

24. $5x + 2y = 7 \rightarrow 5x + 2y - 7 = 0$

$y = -\frac{3}{4}x + 1 \rightarrow -3x - 4y + 4 = 0$

$d_1 = \frac{5x_1 + 2y_1 - 7}{\sqrt{5^2+2^2}}$ $\quad d_2 = \frac{-3x_1 - 4y_1 + 4}{\sqrt{3^2+4^2}}$

$\frac{5x_1 + 2y_1 - 7}{\sqrt{29}} = \frac{-3x_1 - 4y_1 + 4}{5}$

$25x_1 + 10y_1 - 35 = -3\sqrt{29}x_1 - 4\sqrt{29}y_1 + 4\sqrt{29}$

$(25 + 3\sqrt{29})x + (10 + 4\sqrt{29})y$

$- 35 - 4\sqrt{29} = 0$

25. $R = \frac{v_0^2}{g}\sin 2\theta$

$R = \frac{v_0^2}{g}\cdot 2\sin\theta\cos\theta$

$R = \frac{88^2}{32}\cdot 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$R = 232.32$ ft

Chapter 8 Test

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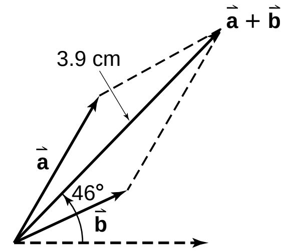
1. 2.5 cm, 60°

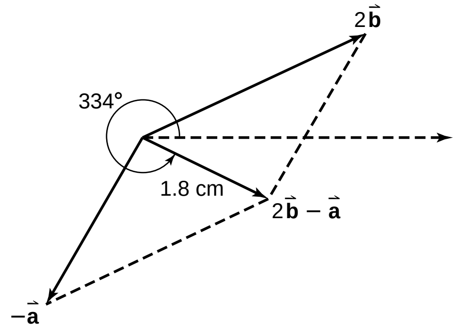
2. 1.6 cm, 25°

3.



3.9 cm, 46°

4.



1.8 cm, 334°

5. $\overrightarrow{AB} = \langle -1 - 3, 9 - 6\rangle$
$= \langle -4, 3\rangle$
$|\overrightarrow{AB}| = \sqrt{(-4)^2 + (3)^2}$
$= \sqrt{25}$ or 5

6. $\overrightarrow{AB} = \langle 3 - (-2), 10 - 7\rangle$
$= \langle 5, 3\rangle$
$|\overrightarrow{AB}| = \sqrt{5^2 + 3^2}$
$= \sqrt{34}$

7. $\overrightarrow{AB} = \langle 9 - 2, -3 - (-4), 7 - 5\rangle$
$= \langle 7, 1, 2\rangle$
$|\overrightarrow{AB}| = \sqrt{7^2 + 1^2 + 2^2}$
$= \sqrt{54}$ or $3\sqrt{6}$

8. $\overrightarrow{AB} = \langle -8 - (-4), -10 - (-8), 2 - (-2)\rangle$
$= \langle -4, -2, 4\rangle$
$|\overrightarrow{AB}| = \sqrt{(-4)^2 + (-2)^2 + 4^2}$
$= \sqrt{36}$ or 6

9. $\vec{\mathbf{r}} - \vec{\mathbf{s}} = \langle -1 - 4, 3 - 3, 4 - (-6)\rangle$
$= \langle -5, 0, 10\rangle$

10. $3\vec{\mathbf{s}} = \langle 3 \cdot 4, 3 \cdot 3, 3 \cdot -6\rangle$
$= \langle 12, 9, -18\rangle$
$2\vec{\mathbf{r}} = \langle 2 \cdot (-1), 2 \cdot 3, 2 \cdot 4\rangle$
$= \langle -2, 6, 8\rangle$
$3\vec{\mathbf{s}} - 2\vec{\mathbf{r}} = \langle 12 - (-2), 9 - 6, -18 - 8\rangle$
$= \langle 14, 3, -26\rangle$

11. $\vec{\mathbf{r}} + 3\vec{\mathbf{s}} = \langle -1 + 12, 3 + 9, 4 + (-18)\rangle$
$= \langle 11, 12, -14\rangle$

12. $|\vec{\mathbf{r}}| = \sqrt{(-1)^2 + 3^2 + 4^2}$
$= \sqrt{26}$

13. $|\vec{\mathbf{s}}| = \sqrt{4^2 + 3^2 + (-6)^2}$
$= \sqrt{61}$

14. $\vec{\mathbf{r}} = -\vec{\mathbf{i}} + 3\vec{\mathbf{j}} + 4\vec{\mathbf{k}}$

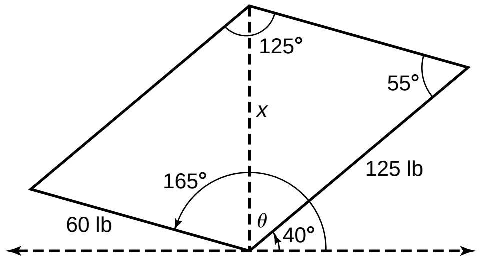
15. $\vec{\mathbf{s}} = 4\vec{\mathbf{i}} + 3\vec{\mathbf{j}} - 6\vec{\mathbf{k}}$

16. $\vec{\mathbf{r}} \cdot \vec{\mathbf{s}} = -1(4) + 3(3) + 4(-6)$
$= -19$

17. $\vec{\mathbf{r}} \times \vec{\mathbf{s}} = \begin{vmatrix} 3 & 4 \\ 3 & -6 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} -1 & 4 \\ 4 & -6 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} -1 & 3 \\ 4 & 3 \end{vmatrix}\vec{\mathbf{k}}$
$= -30\vec{\mathbf{i}} + 10\vec{\mathbf{j}} - 15\vec{\mathbf{k}}$
$= \langle -30, 10, -15\rangle$

18. No; since $\vec{\mathbf{r}} \cdot \vec{\mathbf{s}} = -19$ and not 0.

19.



$x^2 = 125^2 + 60^2 - 2(125)(60)\cos 55°$
$x = \sqrt{19{,}225 - 15{,}000\cos 55°}$
$x \approx 103.06$ lb
$\frac{\sin 55°}{103.06} = \frac{\sin\theta}{60}$
$103.06\sin\theta = 60\sin 55°$
$\theta = \sin^{-1}\frac{60\sin 55°}{103.06}$
$\theta \approx 28.48°$
$\theta + 40° = 28.48° + 40°$ or $68.48°$

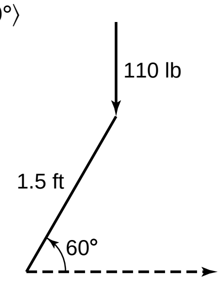
20. $x = x_1 + ta_1$ $\qquad$ $y = y_1 + ta_2$
$x = 3 + t(2)$ $\qquad$ $y = 11 + t(-5)$
$x = 2t + 3$ $\qquad$ $y = -5t + 11$

21. $x = 2t + 3$ $\qquad$ $y = t + 1$
$\frac{x - 3}{2} = t$ $\qquad$ $y - 1 = t$
$y - 1 = \frac{x - 3}{2}$
$y = \frac{1}{2}x - \frac{1}{2}$

22. $v_x = 100\cos 2°$ $\qquad$ $v_y = 100\sin 2°$
$v_x \approx 99.94$ mph $\qquad$ $v_y \approx 3.49$ mph

23. The figure is four times the original size and reflected over the yz-plane.

24. $\overrightarrow{AB} = \langle 1.5\cos 60°, 0, 1.5\sin 60°\rangle$
$= \langle 0.75, 0, 0.75\sqrt{3}\rangle$
$\vec{\mathbf{F}} = \langle 0, 0, -110\rangle$



$\vec{\mathbf{T}} = \overrightarrow{AB} \times \vec{\mathbf{F}}$
$= \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 0.75 & 0 & 0.75\sqrt{3} \\ 0 & 0 & -110 \end{vmatrix}$
$= \begin{vmatrix} 0 & 0.75\sqrt{3} \\ 0 & -110 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 0.75 & 0.75\sqrt{3} \\ 0 & -110 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 0.75 & 0 \\ 0 & 0 \end{vmatrix}\vec{\mathbf{k}}$
$= 0\vec{\mathbf{i}} + 82.5\vec{\mathbf{j}} + 0\vec{\mathbf{k}}$
$|\vec{T}| = \sqrt{0^2 + 82.5^2 + 0^2}$
$= 82.5$ lb-ft

25. $y = t|\vec{\mathbf{v}}| \sin\theta - \frac{1}{2}gt^2$ $\qquad x = t|\vec{\mathbf{v}}| \cos\theta$

$y = 28t \sin 35° - \frac{1}{2}(32)t^2$ $\qquad x = 28t \cos 35°$

$0 = 4t(7 \sin 35° - 4t)$ $\qquad x \approx 23.02$ feet

$4t = 0$ or $7 \sin 35° - 4t = 0$

$t = 0$ $\qquad t = \frac{-7 \sin 35°}{-4}$

$t \approx 1.003758764$

Chapter 9 Test

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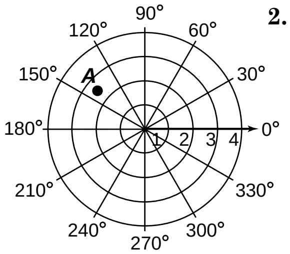
1.



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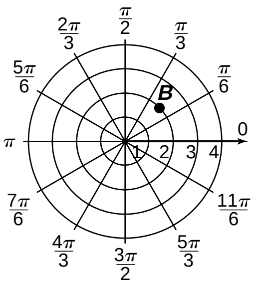

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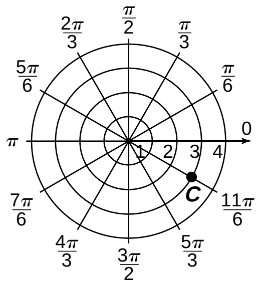

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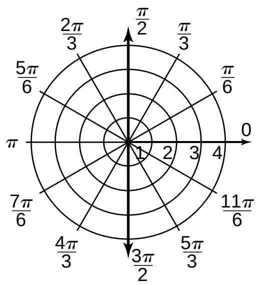

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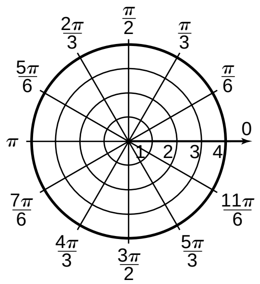

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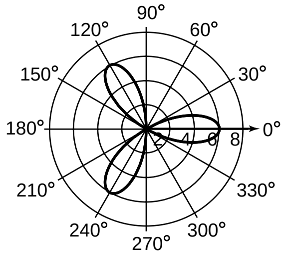

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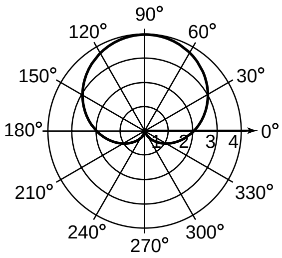


8. $r = \sqrt{2^2 + 2^2}$ $\qquad \theta = \text{Arctan}\,\frac{2}{2}$

$= \sqrt{8}$ or $2\sqrt{2}$ $\qquad = \frac{\pi}{4}$

$\left(2\sqrt{2}, \frac{\pi}{4}\right)$

9. $r = \sqrt{(-6)^2 + 0^2}$

$= \sqrt{36}$ or 6

Since $x < 0$ and $y = 0$, $\theta = \pi$.

$(6, \pi)$

10. $r = \sqrt{(-2)^2 + (-3)^2}$ $\qquad \theta = \text{Arctan}\left(\frac{-3}{-2}\right) + \pi$

$= \sqrt{13}$ $\qquad \approx 4.12$

≈ 3.61

$(3.61, 4.12)$

11. $x = r\cos\theta$ $\qquad y = r\sin\theta$

$= 3\cos\left(-\frac{5\pi}{4}\right)$ $\qquad = 3\sin\left(-\frac{5\pi}{4}\right)$

$= 3\left(-\frac{\sqrt{2}}{2}\right)$ $\qquad = 3\left(\frac{\sqrt{2}}{2}\right)$

$= -\frac{3\sqrt{2}}{2}$ $\qquad = \frac{3\sqrt{2}}{2}$

$\left(-\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$

12. $x = r\cos\theta$ $\qquad y = r\sin\theta$

$= 2\cos\frac{7\pi}{6}$ $\qquad = 2\sin\frac{7\pi}{6}$

$= 2\left(-\frac{\sqrt{3}}{2}\right)$ $\qquad = 2\left(-\frac{1}{2}\right)$

$= -\sqrt{3}$ $\qquad = -1$

$(-\sqrt{3}, -1)$

13. $x = r\cos\theta$ $\qquad y = r\sin\theta$

$= -4\cos 1.4$ $\qquad = -4\sin 1.4$

≈ -0.68 $\qquad \approx -3.94$

$(-0.68, -3.94)$

14. $y = -3$

$r\sin\theta = -3$

$r = \frac{-3}{\sin\theta}$

$r = -3\csc\theta$

15. $x^2 + y^2 = 3x$

$r^2 = 3r\cos\theta$

$r = 3\cos\theta$

16. $r = 7$

$r^2 = 49$

$x^2 + y^2 = 49$

17. $5 = r\cos(\theta - 45°)$

$0 = r\cos(\theta - 45°) - 5$

$0 = r(\cos\theta\cos 45° + \sin\theta\sin 45°) - 5$

$0 = \frac{\sqrt{2}}{2}r\cos\theta + \frac{\sqrt{2}}{2}r\sin\theta - 5$

$0 = \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y - 5$

$0 = \sqrt{2}x + \sqrt{2}y - 10$

18. $-\sqrt{A^2 + B^2} = -\sqrt{5^2 + 3^2}$

$= -\sqrt{34}$

$-\frac{5}{\sqrt{34}}x - \frac{3}{\sqrt{34}}y - \frac{3}{\sqrt{34}} = 0$

$\cos\phi = -\frac{5\sqrt{34}}{34}$, $\sin\phi = -\frac{3\sqrt{34}}{34}$, $p = \frac{3\sqrt{34}}{34}$

$\phi = \text{Arctan}\left(\frac{-3}{-5}\right) + 180°$

$\approx 211°$

$p = r\cos(\theta - \phi)$

$\frac{3\sqrt{34}}{34} = r\cos(\theta - 211°)$

19. $+\sqrt{A^2 + B^2} = \sqrt{2^2 + (-4)^2}$

$= \sqrt{20}$ or $2\sqrt{5}$

$\frac{2}{2\sqrt{5}}x - \frac{4}{2\sqrt{5}}y - \frac{1}{2\sqrt{5}} = 0$

$\cos\phi = \frac{\sqrt{5}}{5}$, $\sin\phi = -\frac{2\sqrt{5}}{5}$, $p = \frac{\sqrt{5}}{10}$

$\phi = \text{Arctan}\left(\frac{-2}{1}\right) + 360°$

$\approx 297°$

$p = r\cos(\theta - \phi)$

$\frac{\sqrt{5}}{10} = r\cos(\theta - 297°)$

20. $\boldsymbol{i}^{93} = (\boldsymbol{i}^4)^{23} \cdot \boldsymbol{i}$

$= 1^{23} \cdot \boldsymbol{i}$

$= \boldsymbol{i}$

21. $(2 - 5\boldsymbol{i}) + (-2 + 4\boldsymbol{i}) = (2 + (-2)) + (-5\boldsymbol{i} + 4\boldsymbol{i})$
$= 0 + (-\boldsymbol{i})$
$= -\boldsymbol{i}$

22. $-6\boldsymbol{i} - (-3 + 2\boldsymbol{i}) = -6\boldsymbol{i} + 3 - 2\boldsymbol{i}$
$= 8\boldsymbol{i} + 3$

23. $(3 + 5\boldsymbol{i})(3 - 2\boldsymbol{i}) = 9 + 9\boldsymbol{i} - 10\boldsymbol{i}^2$
$= 19 + 9\boldsymbol{i}$

24. $(1 - 3\boldsymbol{i})(2 - \boldsymbol{i})(1 + 2\boldsymbol{i}) = (2 - 7\boldsymbol{i} + 3\boldsymbol{i}^2)(1 + 2\boldsymbol{i})$
$= (-1 - 7\boldsymbol{i})(1 + 2\boldsymbol{i})$
$= -1 - 9\boldsymbol{i} - 14\boldsymbol{i}^2$
$= 13 - 9\boldsymbol{i}$

25. $\frac{6 - 2\boldsymbol{i}}{2 + \boldsymbol{i}} = \frac{6 - 2\boldsymbol{i}}{2 + \boldsymbol{i}} \cdot \frac{2 - \boldsymbol{i}}{2 - \boldsymbol{i}}$
$= \frac{12 - 10\boldsymbol{i} + 2\boldsymbol{i}^2}{4 - \boldsymbol{i}^2}$
$= \frac{10 - 10\boldsymbol{i}}{5}$
$= 2 - 2\boldsymbol{i}$

26. $r = \sqrt{(-4)^2 + 4^2}$ $\qquad \theta = \text{Arctan}\left(\frac{4}{-4}\right) + \pi$
$= \sqrt{32}$ or $4\sqrt{2}$ $\qquad = \frac{3\pi}{4}$
$4\sqrt{2}\left(\cos \frac{3\pi}{4} + \boldsymbol{i} \sin \frac{3\pi}{4}\right)$

27. $r = \sqrt{(-5)^2 + 0^2}$ $\qquad \theta = \text{Arctan}\left(\frac{0}{-5}\right) + \pi$
$= \sqrt{25}$ or 5 $\qquad = \pi$
$5(\cos \pi + \boldsymbol{i} \sin \pi)$

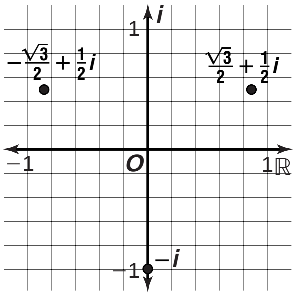
28. $r = 4 \cdot 3$ $\qquad \theta = \frac{3\pi}{2} + \frac{\pi}{4}$
$= 12$ $\qquad = \frac{6\pi + \pi}{4}$ or $\frac{7\pi}{4}$
$12\left(\cos \frac{7\pi}{4} + \boldsymbol{i} \sin \frac{7\pi}{4}\right) = 12\left(\frac{\sqrt{2}}{2} + \boldsymbol{i}\left(-\frac{\sqrt{2}}{2}\right)\right)$
$= 6\sqrt{2} - 6\sqrt{2}\boldsymbol{i}$

29. $r = \frac{2\sqrt{3}}{\sqrt{3}}$ $\qquad \theta = \frac{2\pi}{3} + \frac{\pi}{6}$
$= 2$ $\qquad = \frac{3\pi}{6}$ or $\frac{\pi}{2}$
$2\left(\cos \frac{\pi}{2} + \boldsymbol{i} \sin \frac{\pi}{2}\right) = 2(0 + \boldsymbol{i})$
$= 2\boldsymbol{i}$

30. $r = \sqrt{1^2 + (-1)^2}$ $\qquad \theta = \text{Arctan}\left(\frac{-1}{1}\right) + 2\pi$
$= \sqrt{2}$ $\qquad = \frac{7\pi}{4}$
$(1 - \boldsymbol{i})^8 = (\sqrt{2})^8\left[\cos (8)\left(\frac{7\pi}{4}\right) + \boldsymbol{i} \sin (8)\left(\frac{7\pi}{4}\right)\right]$
$= 16 (\cos 14\pi + \boldsymbol{i} \sin 14\pi)$
$= 16 (1 + 0)$
$= 16$

31. $r = \sqrt{0^2 + (-27)^2}$ $\qquad \theta = -\frac{\pi}{2}$
$= \sqrt{729}$ or 27
$\sqrt[3]{-27\boldsymbol{i}} = (0 - 27\boldsymbol{i})^{\frac{1}{3}}$
$= 27^{\frac{1}{3}}\left[\cos \left(\frac{1}{3}\right)\left(-\frac{\pi}{2}\right) + \boldsymbol{i} \sin \left(\frac{1}{3}\right)\left(-\frac{\pi}{2}\right)\right]$
$= 3\left[\cos \left(-\frac{\pi}{6}\right) + \boldsymbol{i} \sin \left(-\frac{\pi}{6}\right)\right]$
$= 3\left[\frac{\sqrt{3}}{2} + \boldsymbol{i}\left(-\frac{1}{2}\right)\right]$
$= \frac{3\sqrt{3}}{2} - \frac{3}{2}\boldsymbol{i}$

32. $x^3 - \boldsymbol{i} = 0 \rightarrow x^3 = \boldsymbol{i}$
Find the cube roots of $\boldsymbol{i}$.
$r = \sqrt{0^2 + 1^2}$ $\qquad \theta = \frac{\pi}{2}$
$= \sqrt{1}$ or 1
$(0 + \boldsymbol{i})^{\frac{1}{3}} = \left[1\left(\cos\left(\frac{\pi}{2} + 2n\pi\right) + \boldsymbol{i} \sin\left(\frac{\pi}{2} + 2n\pi\right)\right)\right]^{\frac{1}{3}}$
$= 1^{\frac{1}{5}}\left[\cos\left(\frac{\pi}{6} + \frac{2n\pi}{3}\right) + \boldsymbol{i} \sin\left(\frac{\pi}{6} + \frac{2n\pi}{3}\right)\right]$
$= \cos \frac{\pi + 4n\pi}{6} + \boldsymbol{i} \sin \frac{\pi + 4n\pi}{6}$
$x_1 = \cos \frac{\pi}{6} + \boldsymbol{i} \sin \frac{\pi}{6} = \frac{\sqrt{3}}{2} + \frac{1}{2}\boldsymbol{i}$
$x_2 = \cos \frac{5\pi}{6} + \boldsymbol{i} \sin \frac{5\pi}{6} = -\frac{\sqrt{3}}{2} + \frac{1}{2}\boldsymbol{i}$
$x_3 = \cos \frac{9\pi}{6} + \boldsymbol{i} \sin \frac{9\pi}{6} = -\boldsymbol{i}$



33. $E = I \cdot Z$
$= 8 (\cos 307° + \boldsymbol{j} \sin 307°)$
$\cdot\, 20 (\cos 115° + \boldsymbol{j} \sin 115°)$
$= 8 \cdot 20 [\cos (307° + 115°) + \boldsymbol{j} \sin (307° + 115°)]$
$= 160 (\cos 422° + \boldsymbol{j} \sin 422°)$
$= 160 (\cos 62° + \boldsymbol{j} \sin 62°)$

Chapter 10 Test

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1. $d = \sqrt{(x_2 - y_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{3 - (-1))^2 + (1 - 2)^2}$
$d = \sqrt{4^2 + (-1)^2}$
$d = \sqrt{17}$
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-1 + 3}{2}, \frac{2 + 1}{2}\right)$
$= \left(1, \frac{3}{2}\right)$

2. $d = \sqrt{(x_2 - y_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(2k - 3k)^2 + (k - 1 - (k + 1))^2}$
$d = \sqrt{(-k)^2 + (-2)^2}$
$d = \sqrt{k^2 + 4}$
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3k + 2k}{2}, \frac{k + 1 + k - 1}{2}\right)$
$= \left(\frac{5}{2}k, k\right)$

3. $r = \sqrt{(-8 - (-6)^2 + (3 - (-4))^2}$
$r = \sqrt{(-2)^2 + (7)^2}$
$r = \sqrt{53}$
$(x - h)^2 + (y - k)^2 = r^2$
$(x - (-8)^2 + (y - 3)^2 = \left(\sqrt{53}\right)^2$
$(x + 8)^2 + (y - 3)^2 = 53$

4. center: $(h, k) = (0, 0)$

$a^2 = 10 \qquad b^2 = 6 \qquad c = \sqrt{a^2 - b^2}$

$a = \sqrt{10} \qquad b = \sqrt{6} \qquad c = \sqrt{10 - 6}$ or 2

foci: $(h, k \pm c) = (0, \pm 2)$

major axis vertices: $(h, k \pm a) = \left(0, \pm\sqrt{10}\right)$

minor axis vertices: $(h \pm b, k) = (\pm\sqrt{6}, 0)$

5. $e = \frac{c}{a} = \frac{1}{2}$

If $c = \frac{1}{2}$, then $a = 1$.

$c = \sqrt{a^2 - b^2}$

$\frac{1}{2} = \sqrt{1^2 - b^2}$

$\frac{1}{4} = 1 - b^2$

$b^2 = \frac{3}{4}$

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$\frac{(x-0)^2}{1} + \frac{(y-0)^2}{\frac{3}{4}} = 1$

$x^2 + \frac{4y^2}{3} = 1$

6. center: $(h, k) = (-4, 2)$

$a^2 = 16 \qquad b^2 = 7 \qquad b^2 = c^2 - a^2$

$a = 4 \qquad b = \sqrt{7} \qquad 7 = c^2 - 16 \rightarrow c = \sqrt{23}$

foci: $(h, k \pm c) = \left(-4, 2 \pm \sqrt{23}\right)$

vertices: $(h, k \pm a) = (-4, 2 \pm 4)$

$= (-4, 6)$ and $(-4, -2)$

asymptotes: $y - k = \pm\frac{a}{b}(x - h)$

$y - 2 = \pm\frac{4}{\sqrt{7}}(x - (-4))$

$y - 2 = \pm\frac{4\sqrt{7}}{7}(x + 4)$

7. foci: $(h, k \pm c) \Rightarrow h = -5$

$$\begin{array}{l} k + c = 4 \\ \underline{k - c = -2} \\ 2k = 2 \end{array}$$

$k = 1; c = 3$

$e = \frac{3}{2} = \frac{c}{a}$

$b^2 = c^2 - a^2$

$b^2 = 3^2 - 2^2$

$b^2 = 5$

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

$\frac{(y-1)^2}{2^2} - \frac{(x-(-5))^2}{5} = 1$

$\frac{(y-1)^2}{4} - \frac{(x+5)^2}{5} = 1$

8. vertex: $(h, k) = (0, -3)$

$4p = 8 \Rightarrow p = 2$

focus: $(h + p, k) = (0 + 2, -3) = (2, -3)$

directrix: $x = h - p$

$x = 0 - 2$

$x = -2$

axis of symmetry: $y = k$

$y = -3$

9.
$$\begin{array}{l} k + p = -5 \\ \underline{k - p = -2} \\ 2k = -7 \end{array}$$

$k = -\frac{7}{2}, p = -\frac{3}{2}$

foci: $(h, k + p) = (3, -5)$

directrix: $y = k - p = -2$

$(x - h)^2 = 4p(y - k)$

$(x - 3)^2 = 4 \cdot \left(-\frac{3}{2}\right)\left(y - \left(-\frac{7}{2}\right)\right)$

$(x - 3)^2 = -6\left(y + \frac{7}{2}\right)$

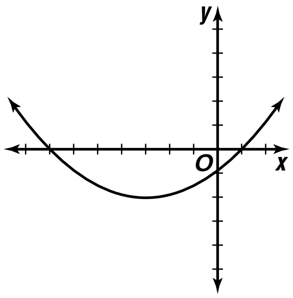
10. $A = 1, C = 0$; since $C = 0$, the conic is a parabola.

$x^2 + 6x - 8y - 7 = 0$

$x^2 + 6x + 9 = 8y + 7 + 9$

$(x + 3)^2 = 8y + 16$

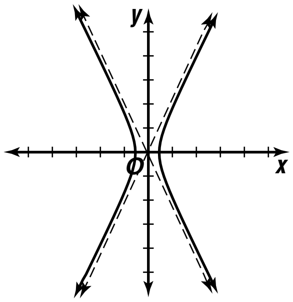
$(x + 3)^2 = 8(y + 2)$



11. $A = 4, C = -1$; since A and C have opposite signs, the conic is a hyperbola.

$4x^2 - y^2 = 1$

$\frac{x^2}{\frac{1}{4}} - \frac{y^2}{1} = 1$

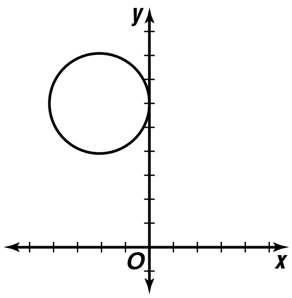


12. $A = 1, C = 1$; since $A = C$, the conic is a circle.

$x^2 + y^2 + 4x - 12y + 36 = 0$

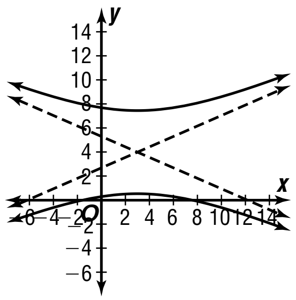
$x^2 + 4x + 4 + y^2 - 12y + 36 = 4$

$(x + 2)^2 + (y - 6)^2 = 4$



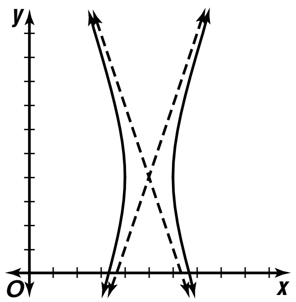
13. $A = 3$, $C = -16$; since A and C have opposite signs, the conic is a hyperbola.

$$\begin{aligned} 3x^2 - 16y^2 - 18x + 128y - 37 &= 0 \\ 3(x^2 - 6x + 9) - 16(y^2 - 8y + 16) &= 37 + 27 - 256 \\ 3(x - 3)^2 - 16(y - 4)^2 &= -192 \\ \frac{(y - 4)^2}{12} - \frac{(x - 3)^2}{64} &= 1 \end{aligned}$$



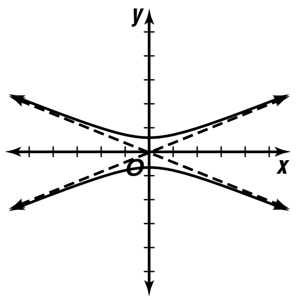
14. $A = 9$, $C = -1$; since A and C have opposite signs, the conic is a hyperbola.

$$\begin{aligned} 9x^2 - y^2 - 90x + 8y + 200 &= 0 \\ 9(x^2 - 10x + 25) - (y^2 - 8y + 16) &= -200 + 225 - 16 \\ 9(x - 5)^2 - (y - 4)^2 &= 9 \\ \frac{(x - 5)^2}{1} - \frac{(y - 4)^2}{9} &= 1 \end{aligned}$$



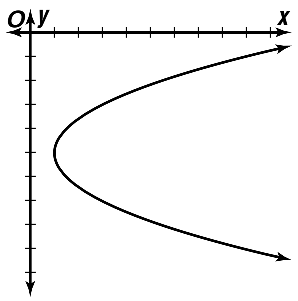
15. $A = 2$, $C = -13$; since A and C have opposite signs, the conic is a hyperbola.

$$\begin{aligned} 2x^2 - 13y^2 + 5 &= 0 \\ 2x^2 - 13y^2 &= -5 \\ \frac{13y^2}{5} - \frac{2x^2}{5} &= 1 \\ \frac{y^2}{\frac{5}{13}} - \frac{x^2}{\frac{5}{2}} &= 1 \end{aligned}$$



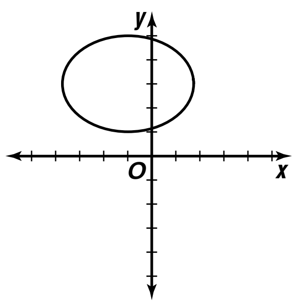
16. $A = 0$, $C = 1$; since $A = 0$, the conic is a parabola.

$$\begin{aligned} y^2 - 2x + 10y + 27 &= 0 \\ y^2 + 10y + 25 &= 2x - 27 + 25 \\ (y + 5)^2 &= 2x - 2 \\ (y + 5)^2 &= 2(x - 1) \end{aligned}$$



17. $A = 1$, $C = 2$; since A and C have the same sign and $A \neq C$, the conic is an ellipse.

$$\begin{aligned} x^2 + 2y^2 + 2x - 12y + 11 &= 0 \\ (x^2 + 2x + 1) + 2(y^2 - 6y + 9) &= -11 + 1 + 18 \\ (x + 1)^2 + 2(y - 3)^2 &= 8 \\ \frac{(x + 1)^2}{8} + \frac{(y - 3)^2}{4} &= 1 \end{aligned}$$



18. $y = 2x^2 + x$

19. $x = 2 \cos t$ $\qquad$ $y = 2 \sin t$

$\frac{x}{2} = \cos t$ $\qquad$ $\frac{y}{2} = \sin t$

$$\begin{aligned} \cos^2 t + \sin^2 t &= 1 \\ \left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2 &= 1 \\ \frac{x^2}{4} + \frac{y^2}{4} &= 1 \\ x^2 + y^2 &= 4 \end{aligned}$$

20. $B^2 - 4AC = 0^2 - 4(4)(1)$

$= -16$

$A \neq C$; ellipse

$$\begin{aligned} 4(x + 1)^2 + (y - 3)^2 &= 36 \\ 4(x + 1 - 3)^2 + (y - 3 - (-5))^2 &= 36 \\ 4(x - 2)^2 + (y + 2)^2 &= 36 \\ 4x^2 - 16x + 16 + y^2 + 4y + 4 &= 36 \\ 4x^2 + y^2 - 16x + 4y - 16 &= 0 \end{aligned}$$

21. $B^2 - 4AC = 0^2 - 4(2)(-1)$
$= 8$

hyperbola

Replace x with $x' \cos 60° + y' \sin 60°$ or $\frac{1}{2}x' + \frac{\sqrt{3}}{2}y'$.

Replace y with $-x' \sin 60° + y' \cos 60°$ or $-\frac{\sqrt{3}}{2}x' + \frac{1}{2}y'$.

$$2\left(\frac{1}{2}x' + \frac{\sqrt{3}}{2}y'\right)^2 - \left(-\frac{\sqrt{3}}{2}x' + \frac{1}{2}y'\right)^2 = 8$$

$$2\left[\frac{1}{4}(x')^2 + \frac{\sqrt{3}}{2}x'y' + \frac{3}{4}(y')^2\right] - \left[\frac{3}{4}(x')^2 - \frac{\sqrt{3}}{2}x'y' + \frac{1}{4}(y')^2\right] = 8$$

$$\frac{1}{2}(x')^2 + \sqrt{3}x'y' + \frac{3}{2}(y')^2 - \frac{3}{4}(x')^2 + \frac{\sqrt{3}}{2}x'y' - \frac{1}{4}(y')^2 = 8$$

$$-(x')^2 + 6\sqrt{3}x'y' + 5(y')^2 - 32 = 0$$

$$(x')^2 - 6\sqrt{3}x'y' - 5(y')^2 + 32 = 0$$

22.

$$\begin{aligned} (x-1)^2 + y^2 &= 1 \\ x^2 - 2x + 1 + y^2 &= 1 \\ x^2 + y^2 - 2x &= 0 \\ y^2 &= -x^2 + 2x \end{aligned}$$

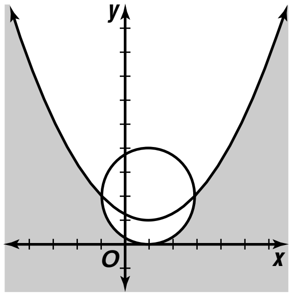
$$\begin{aligned} x^2 + 4y^2 &= 4 \\ x^2 + 4(-x^2 + 2x) &= 4 \\ x^2 - 4x^2 + 8x &= 4 \\ 3x^2 - 8x + 4 &= 0 \\ (3x-2)(x-2) &= 0 \\ x = \frac{2}{3} \text{ or } x &= 2 \end{aligned}$$

If $x = \frac{2}{3}$, $y = \pm\sqrt{-\left(\frac{2}{3}\right)^2 + 2\left(\frac{2}{3}\right)} \approx \pm\, 0.9$.

If $x = 2$, $y = \pm\sqrt{-(2)^2 + 2(2)} = 0$.

(2, 0), (0.7, ±0.9)

23.



24. $x^2 + y^2 + Dx + Ey + F = 0$

$0^2 + 1^2 + D(0) + E(1) + F = 0$
$\Rightarrow E + F = -1$

$(-2)^2 + 3^2 + D(-2) + E(3) + F = 0$
$\Rightarrow -2D + 3E + F = -13$

$4^2 + 5^2 + D(4) + E(5) + F = 0$
$\Rightarrow 4D + 5E + F = -41$

$$\begin{aligned} -4D + 6E + 2E &= -26 \\ 4D + 5E + F &= -41 \\ \hline 11E + 3F &= -67 \end{aligned} \qquad \begin{aligned} 3E + 3F &= -3 \\ 11E + 3F &= -67 \\ \hline -8E &= 64 \\ E &= -8 \end{aligned}$$

$$\begin{aligned} E + F &= -1 \\ -8 + F &= -1 \\ F &= 7 \end{aligned} \qquad \begin{aligned} -2D + 3E + F &= -13 \\ -2D + 3(-8) + 7 &= 13 \\ -2D &= 4 \\ D &= -2 \end{aligned}$$

$$\begin{aligned} x^2 + y^2 - 2x - 8y + 7 &= 0 \\ x^2 - 2x + 1 + y^2 - 8y + 16 &= -7 + 1 + 16 \\ (x-1)^2 + (y-4)^2 &= 10 \end{aligned}$$

center: $(h, k) = (1, 4)$

radius $= \sqrt{10}$

25.

$$\begin{aligned} x^2 + y^2 &= 90 \\ x^2 + (2x-3)^2 &= 9 \\ x^2 + 4x^2 - 12x + 9 &= 90 \\ 5x^2 - 12x - 81 &= 0 \\ (5x-27)(x+3) &= 0 \\ x = \frac{27}{5} \text{ or } x &= -3 \end{aligned}$$

$$\begin{aligned} y &= 2x - 3 \\ y &= 2\left(\frac{27}{5}\right) - 3 \text{ or } \frac{39}{5} \\ y &= 2(-3) - 3 \text{ or } -9 \end{aligned}$$

If a person is walking northeast, the person will first hit the motion detector at $(-3, -9)$.

Chapter 11 Test

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1. $343^{\frac{2}{3}} = \left(\sqrt[3]{343}\right)^2$
$= 7^2$
$= 49$

2. $64^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{64}}$
$= \frac{1}{4}$

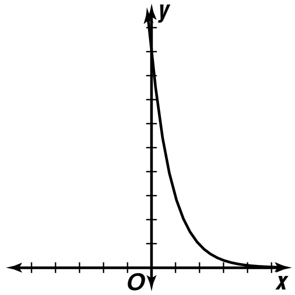
3. $((2a)^3)^{-2} = (2a)^{-6}$
$= \frac{1}{(2a)^6}$
$= \frac{1}{2^6a^6}$
$= \frac{1}{64a^6}$

4. $\left(x^{\frac{3}{2}}y^2z^{\frac{5}{4}}\right)^4 = x^{\frac{12}{2}}y^8z^{\frac{20}{4}}$
$= x^6y^8z^5$

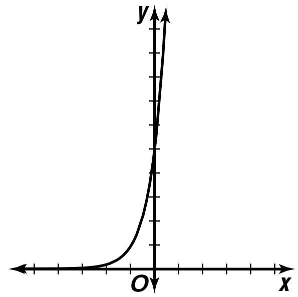
5. $\sqrt[3]{27a^6b^{12}} = (3^3)^{\frac{1}{3}}a^{\frac{6}{3}}b^{\frac{12}{3}}$
$= 3a^2b^4$

6. $m^{\frac{1}{2}}n^{\frac{2}{3}} = m^{\frac{3}{6}}n^{\frac{4}{6}}$
$= \sqrt[6]{m^3n^4}$

7.



8.



9. $4^{\frac{1}{2}} = 2$

10. $\left(\frac{1}{6}\right)^{-3} = 216$

11. $\log_5 625 = 4$

12. $\log_8 m = 5$

13. $\log_x 32 = -5$
$x^{-5} = 32$
$\frac{1}{x^5} = 32$
$\frac{1}{32} = x^5$
$\left(\frac{1}{2}\right)^5 = x^5$
$\frac{1}{2} = x$

14. $\log_5 (2x) = \log_5 (3x - 4)$
$2x = 3x - 4$
$4 = x$

15. $3.6^x = 72.4$
$x \log 3.6 = \log 72.4$
$x = \frac{\log 72.4}{\log 3.6}$
$x \approx 3.3430$

16. $6^{x-1} = 8^{2-x}$
$(x - 1) \log 6 = (2 - x) \log 8$
$x \log 6 - \log 6 = 2 \log 8 - x \log 8$
$x \log 6 + x \log 8 = 2 \log 8 + \log 6$
$x(\log 6 + \log 8) = 2 \log 8 + \log 6$
$x = \frac{2 \log 8 + \log 6}{\log 6 + \log 8}$

17. $\log_4 15 = \frac{\log 15}{\log 4}$
≈ 1.9534

18. $\log_3 0.9375 = \frac{\log 0.9375}{\log 3}$
≈ -0.0587

19. $\log_{81} 3 = \frac{\log 3}{\log 81}$
$= \frac{1}{4}$

20. $\log 542 \approx 2.7340$ **21.** $\ln 0.248 \approx -1.3943$

22. antiln $(-1.9101) \approx 0.1481$

23. $t = \frac{\ln 2}{k}$
$t = \frac{\ln 2}{0.054}$
$t \approx 12.84$ yr

24. Let x = the original number of bacteria.
$3x = xe^{k(6)}$
$3 = e^{6k}$
$\ln 3 = 6k$
$k = \frac{\ln 3}{6}$
$k \approx 0.1831020481$
$8x = xe^{0.1831t}$
$8 = e^{0.1831t}$
$\ln 8 = 0.1831t$
$t = \frac{\ln 8}{0.1831}$
$t \approx 11.3568626$ hours
$0.3568626(60) \approx 21$ minutes
11 hours, 21 minutes

25. $Q(t) = Qe^{-\frac{t}{BC}}$
$1 \times 10^{-6} = (5 \times 10^{-6})e^{-\frac{t}{3(2 \times 10^{-6})}}$
$\frac{1}{5} = e^{-\frac{t}{6 \times 10^{-6}}}$
$\ln 0.2 = -\frac{t}{6 \times 10^{-6}}$
$t = \ln 0.2(-6 \times 10^{-6})$
$t \approx 9.66 \times 10^{-6}$ s

Chapter 12 Test

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1. $d = 4.5 - 2$ or 2.5
$7 + 2.5 = 9.5,\ 9.5 + 2.5 = 12,\ 12 + 2.5 = 14.5,$
$14.5 + 2.5 = 17$
$9.5, 12, 14.5, 17$

2. $d = -1 - (-6)$ or 5
$a_{24} = -6 + (24 - 1)5$
$a_{24} = 109$

3. $8 = -4 + (5 - 1)d$
$12 = 4d$
$3 = d$
$-4 + 3 = -1,\ -1 + 3 = 2,\ 2 + 3 = 5$
$-4, -1, 2, 5, 8$

4. $345 = \frac{n}{2}(2(12) + (n - 1)5)$
$690 = 24n + 5n^2 - 5n$
$0 = 5n^2 + 19n - 690$
$0 = (5n + 69)(n - 10)$
$n = -\frac{69}{5}$ or $n = 10$
Since there cannot be a fractional number of terms, $n = 10$.

5. $r = \frac{\frac{1}{10}}{\frac{1}{4}}$ or $\frac{2}{5}$
$\frac{1}{25}\left(\frac{2}{5}\right) = \frac{2}{125},\ \frac{2}{125}\left(\frac{2}{5}\right) = \frac{4}{625},\ \frac{4}{625}\left(\frac{2}{5}\right) = \frac{8}{3125}$
$\frac{2}{125}, \frac{4}{625}, \frac{8}{3125}$

6. $1 = 16r^{5-1}$
$\frac{1}{16} = r^4$
$\pm\frac{1}{2} = r$
$16\left(\pm\frac{1}{2}\right) = \pm 8,\ \pm 8\left(\pm\frac{1}{2}\right) = 4,\ 4\left(\pm\frac{1}{2}\right) = \pm 2$
$16, \pm 8, 4, \pm 2, 1$

7. $r = \frac{5}{\frac{5}{2}}$ or 2
$S_{10} = \frac{\frac{5}{2} - \frac{5}{2}(2)^{10}}{1 - 2}$
$= \frac{\frac{5}{2} - \frac{5120}{2}}{-1}$
$= \frac{5115}{2}$

8. does not exist;
$\lim_{n\to\infty} \frac{n^3 + 3}{3n^2 + 1} = \lim_{n\to\infty} \frac{n + \frac{3}{n^2}}{3 + \frac{1}{n^2}}$, $\lim_{n\to\infty} 3 = 3$, and $\lim_{n\to\infty} \frac{1}{n^2} = 0$,
so the denominator approaches 3. As n approaches infinity, the n term in the numerator makes the whole numerator approach infinity, so the entire fraction has no limit.

9. $\lim_{n\to\infty} \frac{n^3 + 4}{2n^3 + 3n} = \lim_{n\to\infty} \frac{\frac{n^3}{n^3} + \frac{4}{n^3}}{\frac{2n^3}{n^3} + \frac{3n}{n^3}}$
$= \frac{1 + 0}{2 + 0}$
$= \frac{1}{2}$

10. The general term is $\frac{1}{3n^2}$.
$\frac{1}{3n^2} \le \frac{1}{n}$ for all n, so convergent

11. The series is arithmetic, so it is divergent.

12. Sample answer: $\sum_{k=1}^{19} 5k$

13. Sample answer: $\sum_{k=1}^{\infty} 6\left(\frac{3}{2}\right)^{k-1}$

14. $(2a - 3b)^5$
$= (2a)^5(-3b)^0 + 5(2a)^4(-3b)^1 + \frac{5 \cdot 4(2a)^3(-3h)^2}{2 \cdot 1} + \frac{5 \cdot 4 \cdot 3(2a)^2(-3b)^3}{3 \cdot 2 \cdot 1} + \frac{5 \cdot 4 \cdot 3 \cdot 2\,(2a)^1(-3b)^4}{4 \cdot 3 \cdot 2 \cdot 1} + \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1(2a)^0(-3b)^5}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 32a^5 - 240a^4b + 720a^3b^2 - 1080a^2b^3 + 810ab^4 - 243b^5$

15. $\frac{10!}{5!(10-5)!}a^{10-5} \cdot 2^5 = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} a^5 \cdot 32$
$= 8064a^5$

16. $\frac{8!}{4!(8-4)!}(3x)^{8-4}(-y)^4$
$= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1}(3x)^4(-y)^4$
$= 70(81x^4)(y^4)$
$= 5670x^4y^4$

17. $r = \sqrt{(-2)^2 + 2^2}$ or $2\sqrt{2}$
$\theta = \text{Arctan}\left(\frac{2}{-2}\right) + \pi$ or $\frac{3\pi}{4}$
$-2 + 2\boldsymbol{i} = 2\sqrt{2}\left(\cos\frac{3\pi}{4} + \boldsymbol{i}\sin\frac{3\pi}{4}\right) = 2\sqrt{2}\,e^{\boldsymbol{i}\frac{3\pi}{4}}$

18. $z_0 = 2\boldsymbol{i}$
$z_1 = 3(2\boldsymbol{i}) + (2 - \boldsymbol{i})$
$= 6\boldsymbol{i} + 2 - \boldsymbol{i}$
$= 2 + 5\boldsymbol{i}$
$z_2 = 3(2 + 5\boldsymbol{i}) + (2 - \boldsymbol{i})$
$= 6 + 15\boldsymbol{i} + 2 - \boldsymbol{i}$
$= 8 + 14\boldsymbol{i}$
$z_3 = 3(8 + 14\boldsymbol{i}) + (2 - \boldsymbol{i})$
$= 24 + 42\boldsymbol{i} + 2 - \boldsymbol{i}$
$= 26 + 41\boldsymbol{i}$

19. Step 1: Verify that the formula is valid for $n = 1$.
Since $2 \cdot 1(2 \cdot 1 + 1) = 6$ and $\frac{1(1+1)(4 \cdot 1 + 5)}{3} = 6$, the formula is valid for $n = 1$.
Step 2: Assume that the formula is valid for $n = k$ and prove that it is valid for $n = k + 1$.
$2 \cdot 3 + 4 \cdot 5 + 6 \cdot 7 + \ldots + 2k(2k+1)$
$= \frac{k(k+1)(4k+5)}{3}$
$2 \cdot 3 + 4 \cdot 5 + 6 \cdot 7 + \ldots + 2k(2k+1) + 2(k+1)(2k+3)$
$= \frac{k(k=1)(4k+5)}{3} + 2(k+1)(2k+3)$
$= \frac{k(k+1)(4k+5)}{3} + \frac{6(k+1)(2k+3)}{3}$
$= \frac{k(k+1)(4k+5) + 6(k+1)(2k+3)}{3}$
$= \frac{(k+1)(k(4k+5) + 6(2k+3)}{3}$
$= \frac{(k+1)(4k^2 + 17k + 18)}{3}$
$= \frac{(k+1)(k+2)(4k+9)}{3}$

Evaluate the original formula for $n = k + 1$.
$\frac{(k+1)[(k+1)+1][4(k+1)+5]}{3} = \frac{(k+1)(k+2)(4k+9)}{3}$
The formula gives the same result as adding the $(k + 1)$ term directly. Thus, if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 1$, it is also valid for $n = 2$. Since it is valid for $n = 2$, it is also valid for $n = 3$, and so on, indefinitely. Thus, the formula is valid for all positive integral values of n.

20. There are 4 groups of three months in a year. There are $4 \cdot 10$ or 40 groups of three months in ten years. So, $n = 40$. Since the interest is compounded quarterly, the rate per period is $\frac{0.08}{4}$ or 0.02. The common ratio r is 1.02.
$a_1 = 200(1.02) = 204$
$S_n = \frac{a_1 - a_1r^n}{1 - r}$
$S_{40} = \frac{204 - 204(1.02)^{40}}{1 - 1.02}$
$S_{40} = \$12{,}322.00$

Chapter 13 Test

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1. $P(6, 2) = \frac{6!}{(6-2)!}$
$= \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1}$
$= 30$

2. $P(7, 5) = \frac{7!}{(7-5)!}$
$= \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1}$
$= 2520$

3. $C(8, 3) = \frac{8!}{(8-3)!\,3!}$
$= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}$
$= 56$

4. $C(5, 4) = \frac{5!}{(5-4)!\,4!}$
$= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 5$

5. Using the Basic Counting Principle, $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$.

6. $P(5, 3) = \frac{5!}{(5-3)!}$
$= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1}$
$= 60$

7. $(7 - 1)! = 6! = 720$

8. $C(3, 1) \cdot C(12, 8)$
$= \frac{3!}{(3-1)!\,1!} \cdot \frac{12!}{(12-8)!\,8!}$
$= \frac{3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 1} \cdot \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 1485$

9. $C(4, 2) \cdot C(6, 3) = \frac{4!}{(4-2)!\,2!} \cdot \frac{6!}{(6-3)!\,3!}$
$= \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1} \cdot \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}$
$= 120$

10. $P(2 \text{ white}) = \frac{6}{10} \cdot \frac{5}{9}$
$= \frac{1}{3}$

11. $P(s) = \frac{1}{4}$ $\quad P(f) = 1 - P(s)$
$= 1 - \frac{1}{4}$ or $\frac{3}{4}$

$\text{odds} = \dfrac{\frac{1}{4}}{\frac{3}{4}}$ or $\frac{1}{3}$

12. $P(5 \text{ clubs or } 5 \text{ hearts or } 5 \text{ spades or } 5 \text{ diamonds})$
$= P(5 \text{ clubs}) + P(5 \text{ hearts}) + P(5 \text{ spades}) + P(5 \text{ diamonds})$
$= 4\left(\frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} \cdot \frac{9}{48}\right)$
$= \frac{33}{16{,}660}$

13. $P(\text{sum of } 8) \cdot P(\text{sum of } 4) = \frac{5}{36} \cdot \frac{3}{36}$
$= \frac{15}{1296}$ or $\frac{5}{432}$

14. $P(3 \text{ odd digits}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$
$= \frac{1}{8}$

15. $P(\text{all red or all blue}) = P(\text{all red}) + P(\text{all blue})$
$= \frac{3}{12} \cdot \frac{2}{11} \cdot \frac{1}{10} + \frac{5}{12} \cdot \frac{4}{11} \cdot \frac{3}{10}$
$= \frac{1}{220} + \frac{1}{22}$
$= \frac{1}{20}$

16. $P(\text{ace or black card})$
$= P(\text{ace}) + P(\text{black card}) - P(\text{black ace})$
$= \frac{4}{52} + \frac{26}{52} - \frac{2}{52}$
$= \frac{28}{52}$ or $\frac{7}{13}$

17. $P(\text{3's together} \mid \text{odd number})$
$= \dfrac{P(\text{3's together and odd number})}{P(\text{odd number})}$
$= \frac{5}{18}$

18. $P(\text{both even} \mid \text{even product})$
$= \dfrac{P(\text{both even and even product})}{P(\text{even product})}$
$= \frac{6}{26}$ or $\frac{3}{13}$

19. $P(\text{no more than two heads})$
$= P(0 \text{ heads}) + P(1 \text{ head}) + P(2 \text{ heads})$
$= C(5, 0)\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^5 + C(5, 1)\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^4$
$+ C(5, 2)\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^3$
$= \frac{1}{243} + \frac{10}{243} + \frac{40}{243}$
$= \frac{51}{243}$ or $\frac{17}{81}$

20. $C(7, 4)\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)^3 = \frac{1792}{15{,}625}$ or 0.114688

Chapter 14 Test

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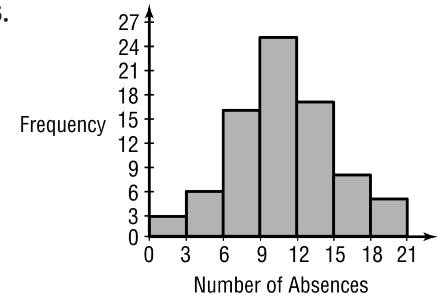
1. range = 20 − 1 or 19
2. Sample answer: 3
3. Sample answer: 0, 3, 6, 9, 12, 15, 18, 21
4. Sample answer: 1.5, 4.5, 7.5, 10.5, 13.5, 16.5, 19.5
5. Sample answer:

Number of Absences	Tallies	Frequency
0–3	|||	3
3–6	卌		6
6–9	卌 卌 卌		16
9–12	卌 卌 卌 卌 卌	25
12–15	卌 卌 卌 ||	17
15–18	卌 |||	8
18–21	卌	5

6. 

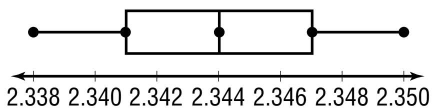
7. $\overline{X} = \frac{1}{80}(6 + 16 + 12 + 7 + \ldots + 8 + 9 + 7 + 9)$
≈ 10.44

8. Order the data from least to greatest. Since there are 80 terms, the median is the average of the 40th term and the 41st term.
$M_d = \frac{10 + 10}{2}$ or 10

9. Order the values from least to greatest. Since there are 15 terms, the median is the $\frac{15 + 1}{2}$ or 8th term.
$M_d = 2.344$

10. $Q_1 = 2.341$; $Q_3 = 2.347$

11. interquartile range $= Q_3 - Q_1$
$= 2.347 - 2.341$
$= 0.006$
semi-interquartile range $= \frac{0.006}{2}$
$= 0.003$

12. 

13. $\overline{X} = \frac{1}{15}(2.334 + 2.338 + \ldots + 2.350)$
≈ 2.34393333
$MD = \frac{1}{15}(|2.34393333 - 2.334| + \ldots + |2.34393333 - 2.350|)$
≈ 0.0025

14. $\sigma = \sqrt{\dfrac{(2.34393333 - 2.334)^2 + \ldots + (2.34393333 - 2.350)^2}{15}}$
≈ 0.0031

15. 68.3% of the data lie within 1 standard deviation of the mean.
$24 \pm 2.8 = 21.2 - 26.8$

16. 90% corresponds to $t = 1.65$.
$24 \pm 1.65(2.8) = 19.38 - 28.62$

17. $24 - 18.4 = 5.6$
$t\sigma = 5.6$
$t(2.8) = 5.6$
$t = 2$
$\frac{95.5\%}{2} = 47.75\%$

$32.4 - 24 = 8.4$
$t\sigma = 8.4$
$t(2.8) = 8.4$
$t = 3$
$\frac{99.7\%}{2} = 49.85\%$

$47.75\% + 49.85\% = 97.6\%$

18. $29.6 - 24 = 5.6$
$t\sigma = 5.6$
$t(2.8) = 5.6$
$t = 2$
$\frac{95.5\%}{2} = 47.75\%$

19. $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{N}}$
$\sigma_{\overline{X}} = \frac{3.6}{\sqrt{400}}$
$\sigma_{\overline{X}} = 0.18$

20. A 5% level of confidence is given when $P = 95\%$.
95% corresponds to $t = 1.96$.
$\overline{X} \pm t\sigma_{\overline{X}} = 57 \pm 1.96(0.18)$
$= 56.65–57.35$

Chapter 15 Test

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1. The closer x is to -1, the closer y is to 1.
So, $\lim_{x\to -1} f(x) = 1$. Also, $f(-1) = 1$.

2. The closer x is to 2, the closer y is to 4.
So, $\lim_{x\to 2} f(x) = 4$. However, there is a point at $(2, -1)$.
So $f(2) = -1$.

3. $$\lim_{x\to 0} \frac{x^2 + 3x}{x} = \lim_{x\to 0} (x + 3) = 3$$

4. $$\lim_{x\to 1} \frac{x^2 - 3x + 2}{x - 1} = \lim_{x\to 1} \frac{(x - 2)(x - 1)}{x - 1} = \lim_{x\to 1} (x - 2) = -1$$

5. $$\lim_{x\to 3} \frac{x^2 - 9}{x^3 - 27} = \lim_{x\to 3} \frac{(x + 3)(x - 3)}{(x - 3)(x^2 + 3x + 9)} = \lim_{x\to 3} \frac{x + 3}{x^2 + 3x + 9} = \frac{3 + 3}{3^2 + 3(3) + 9} = \frac{6}{27} \text{ or } \frac{2}{9}$$

6. $$\lim_{x\to 1} \frac{x^2 - 2x + 3}{3x^2 - 5} = \frac{1^2 - 2(1) + 3}{3(1)^2 - 5} = \frac{2}{-2} \text{ or } -1$$

7. $$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x + h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{(x + h)^2 - 2(x + h) - (x^2 - 2x)}{h} \\ &= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 2x - 2h - x^2 + 2x}{h} \\ &= \lim_{h\to 0} \frac{2xh + h^2 - 2h}{h} \\ &= \lim_{h\to 0} (2x + h - 2) \\ &= 2x - 2 \end{aligned}$$

8. $$\begin{aligned} f(x) &= \tfrac{1}{2}x^2 - 7x + 1 \\ f'(x) &= \tfrac{1}{2} \cdot 2x^{2-1} - 7 \cdot 1x^{1-1} + 0 \\ &= x - 7 \end{aligned}$$

9. $$\begin{aligned} f(x) &= 4x^3 - 4 \\ f'(x) &= 4 \cdot 3x^{3-1} - 0 \\ &= 12x^2 \end{aligned}$$

10. $$\begin{aligned} f(x) &= 6x^4 - 2x^2 - 30 \\ f'(x) &= 6 \cdot 4x^{4-1} - 2 \cdot 2x^{2-1} - 0 \\ &= 24x^3 - 4x \end{aligned}$$

11. $$\begin{aligned} f(x) &= 2x^5 - 4x^3 + \tfrac{2}{5}x^2 - 6 \\ f'(x) &= 2 \cdot 5x^{5-1} - 4 \cdot 3x^{3-1} + \tfrac{2}{5} \cdot 2x^{2-1} - 0 \\ &= 10x^4 - 12x^2 + \tfrac{4}{5}x \end{aligned}$$

12. $$\begin{aligned} f(x) &= 2x^4(x^3 + 3x^2) \\ &= 2x^7 + 6x^6 \\ f'(x) &= 2 \cdot 7x^{7-1} + 6 \cdot 6x^{6-1} \\ &= 14x^6 + 36x^5 \end{aligned}$$

13. $$\begin{aligned} f(x) &= (x + 3)^2 \\ &= x^2 + 6x + 9 \\ f'(x) &= 2x^{2-1} + 6 \cdot 1x^{1-1} + 0 \\ &= 2x + 6 \end{aligned}$$

14. $$\begin{aligned} f(x) &= -2x + 6 \\ F(x) &= \tfrac{-2}{1 + 1}x^{1+1} + 6x + C \\ &= -x^2 + 6x + C \end{aligned}$$

15. $$\begin{aligned} f(x) &= -x^3 + 4x^2 - x + 4 \\ F(x) &= \tfrac{-1}{3 + 1}x^{3+1} + \tfrac{4}{2 + 1}x^{2+1} - \tfrac{1}{1 + 1}x^{1+1} + 4x + C \\ &= -\tfrac{1}{4}x^4 + \tfrac{4}{3}x^3 - \tfrac{1}{2}x^2 + 4x + C \end{aligned}$$

16. $$\begin{aligned} f(x) &= \tfrac{1}{2}x^3 - \tfrac{2}{7}x + 5 \\ F(x) &= \tfrac{1}{2} \cdot \tfrac{1}{3 + 1}x^{3+1} - \tfrac{2}{7} \cdot \tfrac{1}{1 + 1}x^{1+1} + 5x + C \\ &= \tfrac{1}{8}x^4 - \tfrac{2}{14}x^2 + 5x + C \\ &= \tfrac{1}{8}x^4 - \tfrac{1}{7}x^2 + 5x + C \end{aligned}$$

17. $$\begin{aligned} f(x) &= \frac{x^3 - 4x^2 + x}{x} \\ &= x^2 - 4x + 1 \\ F(x) &= \tfrac{1}{2 + 1}x^{2+1} - \tfrac{4}{1 + 1}x^{1+1} + x + C \\ &= \tfrac{1}{3}x^3 - 2x^2 + x + C \end{aligned}$$

18. $$\begin{aligned} \int_0^2 x^3 dx &= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2i}{n}\right)^3 \left(\frac{2}{n}\right) \\ &= \lim_{n\to\infty} \frac{16}{n^4}\left(\frac{n^2(n + 1)^2}{4}\right) \\ &= \lim_{n\to\infty} 4\left(\frac{n^2 + 2n + 1}{n^2}\right) \\ &= \lim_{n\to\infty} 4\left(1 + \frac{2}{n} + \frac{1}{n^2}\right) \\ &= 4 \text{ units}^2 \end{aligned}$$

19. $$\begin{aligned} \int_1^3 3x^2 dx &= 3 \cdot \tfrac{1}{3}x^3 \Big|_1^3 \\ &= x^3 \Big|_1^3 \\ &= 3^3 - 1^3 \\ &= 26 \text{ units}^2 \end{aligned}$$

20. $$\begin{aligned} \int_0^1 (2x + 3)dx &= 2 \cdot \tfrac{1}{2}x^2 + 3x \Big|_0^1 \\ &= x^2 + 3x \Big|_0^1 \\ &= [1^2 + 3(1)] - [0^2 + 3(0)] \\ &= 4 \end{aligned}$$

21. $\int_1^3 (-x^2 + x + 6)\, dx = -\frac{1}{3}x^3 + \frac{1}{2}x^2 + 6x \Big|_1^3$

$= \left[-\frac{1}{3}(3)^3 + \frac{1}{2}(3)^2 + 6(3)\right] - \left[-\frac{1}{3}(1)^3 + \frac{1}{2}(1)^2 + 6(1)\right]$

$= \frac{27}{2} - \frac{37}{6}$

$= \frac{22}{3}$

22. $\int (1 - 2x)dx = x - \frac{2}{2}x^2 + C$

$= x - x^2 + C$

23. $\int (3x^2 + 4x + 7) = \frac{3}{3}x^3 + \frac{4}{2}x^2 + 7x + C$

$= x^3 + 2x^2 + 7x + C$

24. $h(t) = 3 + 95t - 16t^2$

$h'(t) = v(t) = 0 + 95 \cdot 1t^{1-1} - 16 \cdot 2t^{2-1}$

$= 95 - 32t$

$h'(2) = v(2) = 95 - 32(2)$

$= 31$ ft/s

25. $V = 2\pi \int_0^r \left(-\frac{b}{r}x^2 + hx\right)dx$

$= 2\pi \left[-\frac{h}{r} \cdot \frac{1}{3}x^3 + h \cdot \frac{1}{2}x^2 \Big|_0^r\right]$

$= 2\pi \left[\left(-\frac{h}{3r} \cdot r^3 + \frac{h}{2} \cdot r^2\right) - \left(-\frac{h}{3r} \cdot 0^3 + \frac{h}{2} \cdot 0^2\right)\right]$

$= 2\pi \left(-\frac{hr^2}{3} + \frac{hr^2}{2}\right)$

$V(h, r) = V(2, 3) = 2\pi \left(-\frac{2 \cdot 3^2}{3} + \frac{2 \cdot 3^2}{2}\right)$

$= 6\pi$ units^3