REVENUE MANAGEMENT THROUGH DYNAMIC CROSS-SELLING IN CALL CENTERS

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Abstract

This paper models the cross-selling problem of a call center as a dynamic service rate control problem. The question of when and to whom to cross-sell is explored using this model. The analysis shows that under the optimal dynamic policies cross-selling targets may be a function of the operational system state. Structural properties of optimal policies are explored. Sufficient conditions are established for the existence of preferred calls and classes; i.e. calls that will always generate a cross-sell attempt. These provide guidelines in segment formation for marketing managers, and lead to a static heuristic policy. Numerical examples, that are motivated by a real call center, identify call center characteristics that increase the significance of considering dynamic policies rather than static cross-selling rules. The numerical analysis further establishes the value of different types of information, and different types of automation available for cross-selling. Increased staffing for the same call volume is shown to have a positive and increasing return on revenue generation via cross-selling, thus suggesting the need to staff for lower utilization levels in call centers that aim to be revenue generators. Finally, numerical

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examples show that the proposed heuristic leads to near optimal performance both for a loss system and a queueing system.

Keywords: call center; cross-selling; revenue management; customer relationship management; dynamic control; loss system; cross-selling heuristic.

1 Introduction

Many firms in mature industries, like the financial services industry, resort to growth by deepening customer relationships and making them more profitable. A significant part of this profitability comes from revenues generated by the sale of additional products and services to existing customers, in other words through tactics that improve customer life time value. Felvey (1982) states that existing customers are better sales prospects, compared to new customers. Given the growing dislike among consumers for telemarketing, this type of selling is increasingly being performed via cross-selling and up-selling initiatives (Krebsbach, 2002; Walker, 2003). According to Kamakura et al. (1991) cross-selling is emerging as one of the important customer relationship management (CRM) tools used to strengthen relationships. CRM refers to the whole strategy of building relationships and extracting more revenues from existing customers. The global market for CRM systems, service and technology is estimated to be around $ 25 billion (Benjamin, 2001).

Inbound call centers are an important point of contact with the customer, where this type of selling takes place. According to a Tower Group estimate for 2003, in banking, 25 % of transactions are projected to take place in call centers. Given the increasing percentage of these centers that are organized as profit centers, focus is shifting to cross-selling. According to a Wells Fargo executive (The Economist, 2004) 80 % of the bank's growth is coming from selling additional products to existing customers. As the leader in cross-selling, this bank's customers hold an average of a little over four products per household. Given that an average American household has sixteen financial products, the opportunity for cross-selling growth in this industry is apparent.

A major concern for managers is identifying the right person and the right time to attempt a sale. While it is believed that cross-selling ensures that customers acquire multiple products of a firm, improves customer retention (Marple and Zimmerman, 1999) and reduces customer churn (Kamakura et al., 2003), excessive selling can motivate a customer to switch (Kamakura et al., 2003). Database marketing techniques that address this issue are being developed (Paas and Kuijlen, 2001; Kamakura et al. 1991, 2003), and software that helps insurance agents or bankers cross-sell more effectively is becoming more common (Insurance Advocate, 2003; American Banker, 2003) as companies embrace this tactic.

Cross-selling in a call center requires a customer service agent to transform an inbound service call into a sales call. According to an article in the Call Center Magazine (2002), call centers can use integrated predictive analysis and service automation software to make real-time recommendations to banking customers. However, in a review of existing products Chambers (2002) states that real-time automation is relatively immature and many products offer only the option of setting preset business rules that make promotion recommendations based on previously captured and stored data. Common practice is to segment the customer base into groups based on their sales potential, and to target sales to high potential segments. In the absence of real-time automation, the customer service representative will use segment based estimates to determine whether it is appropriate to attempt a cross-sell to a particular call.

Irrespective of the type of automation in place, a cross-sell attempt in a call center implies additional talk time from the agent. Thus cross-selling will influence the load of a call center, as documented in Akşin and Harker (1999). The biggest challenge of a call center manager is to manage the tension between costs and customer service. While for the long-term this corresponds to determining the right number of service representatives to hire, in the shorter term it is resolved through capacity allocation. The primary role of such inbound call centers is service, and demand for service varies during the day. It may be the case that even for calls presenting high revenue potential, cross-selling during peak times is not desirable due to its detrimental effect on capacity and service.

This basic description of cross-selling in a call center identifies a key challenge for managers: When should a cross-selling attempt be made such that revenue generation is maximized while congestion costs are kept as low as possible? Current practice identifies off-peak times during the day for cross-selling. However it is clear that a dynamic policy will utilize valuable capacity more effectively. It is this question of dynamic capacity allocation that motivates the research herein. In a more general setting, Güneş and Akşin (2004) consider this tradeoff between revenue generation and service costs, and analyze the interaction with a market segmentation decision and server incentives. The analysis in that paper does not consider the queue state information in its optimization of the problem. The only other paper that considers a dynamic cross-selling model in call centers is Byers and So (2006). The authors model a call center as a single server Markovian queue, and compare the performance of cross-selling policies that consider queue state information as well as customer profile information. Their analysis extends part of the analysis in Güneş and Akşin (2004) to a dynamic setting. Two recent papers Armony and Gurvich (2006) and Gurvich et al. (2006) explore the cross-selling control problem in conjunction with staffing. We do not explicitly model the staffing problem, however explore the interaction with staffing through a set of numerical examples. Netessine et al. (2004) analyze the dynamic cross-selling problem of an e-commerce retailer, focusing on the packaging of multiple products and their pricing. These aspects of the problem are not considered herein.

We model the cross-selling problem as a dynamic service rate control problem in a multi-server loss system. A customer's revenue potential is modeled as a random variable. For a call center with real-time automation, the realization of this random variable (which is in reality an estimate of the true revenue potential) is observed before a cross-selling decision is made. Otherwise we consider a system where the decision is based on expected revenues. Both of these models are described in the following section. Weber and Stidham (1987) and Stidham and Weber (1989) consider dynamic optimal control of service rates in queueing systems. In a loss system with random rewards the trade-off is between a slow service rate which obtains a high revenue at the expense of, possibly, losing customers who can, potentially, generate more revenue, and a fast rate with low revenue however low probability of losing customers. In this sense, our work relates to dynamic admission control problems, where random rewards have been considered (Ghoneim and Stidham, 1985; Örmeci et al., 2002; Gans and Savin, 2005). All of these papers show the existence of optimal threshold policies. Örmeci et al. (2002) and Gans and Savin (2005) further characterize conditions for the existence of preferred jobs, where preferred jobs are those which are always admitted to the system whenever there is at least one available server. The fact that all calls have to be admitted for service and the decision is to choose a service rate, as opposed to admission control with pre-determined service rates for each class, constitutes the key difference of the model studied herein.

In this paper, preferred calls are defined as calls which always receive a cross-sell attempt, and whenever all calls of a class (segment) are preferred, that class is called as preferred. We derive sufficient conditions for observing preferred calls and segments. For this purpose, we borrow the technique introduced by Örmeci et al. (2002). Our analysis requires a more intricate use of this technique due to two reasons. First, the model considered here involves two types of rewards, random rewards due to cross-selling and a fixed reward for service, so we need to compare not only the cross-sell rewards of market segment(s) but also the rewards of service calls. Second, we consider all policies that can arise under different assumptions about the revenue distributions (a setting where revenues come from one distribution, and another where revenues come from two different distributions). It is shown that unlike the prevailing practice of attempting a cross-sell on all customers in a segment, optimal dynamic policies sometimes dictate that only some customers in a segment, or in some cases even only some customers from each segment receive a cross-sell attempt. Our analysis brings insights on both marketing and operations: The structural results provide guidelines for marketing managers to define segments, such that static segment-based policies might overlap with the optimal ones. To guide the operational policies on cross-selling, we propose an easy-to-implement heuristic based on these structural results.

To assess the value of our results, we develop a set of numerical examples motivated by a real call center. We first explore when dynamic cross-selling is valuable. We then explore parameter settings where state information is preferred over revenue distribution information if only one type of information were available, and settings where real time marketing automation is valuable. A common feature of these settings is found to be the long additional talk times for cross-selling. A numerical analysis of the interaction between cross-selling and staffing in call centers illustrates that providing slack capacity has increasing returns in terms of revenue generation, suggesting that cross-selling call centers should be designed to operate in a lower utilization regime.

The heuristic proposed for cross-selling specifies static rules independent of not only the system state information but also the current arrival rate and the current number of available servers. This feature is especially important in call centers, since both the arrival rate and the number of servers vary throughout the day, and are considered to be random variables. The performance of this heuristic is analyzed numerically both for loss systems and finite capacity queueing systems. It is found to perform uniformly well vis-a-vis the optimal policies in all of the settings considered.

This paper is organized as follows: Section 2 formulates the model with revenue realizations. Section 3 presents sufficient conditions for the existence of preferred classes and preferred calls. In Section 4, we analyze a set of numerical examples, and discuss the implications of our analysis on various aspects of marketing and operations. The paper ends with concluding remarks.

2 A Dynamic Cross-Selling Model

In order to study the dynamic cross-selling problem, we model an inbound call center as a loss system with c identical parallel servers. Call centers have been modeled as loss systems before, particularly to simplify analysis in staffing or routing problems with multi-skill servers (Chevalier and van den Schrieck 2006; Franx et al. 2006). Chevalier and van den Schrieck (2006) numerically illustrate that a loss system captures the basic performance characteristics of corresponding queueing systems quite well. In the cross-selling context, the no-waiting assumption constitutes an acceptable approximation since one cannot sell to a customer who has been waiting for service for a long time. To verify our claim that treating the call center as a loss system does not distort the results, we apply the heuristic that is developed based on the structural properties of optimal policies in the loss systems, to finite capacity queueing systems. The performance of the proposed heuristic in finite-buffer systems is almost the same as that in a loss system, showing that the structures of optimal policies in finite-buffer and loss systems are very similar.

Customers arrive to the system according to a Poisson process. The inbound call center is primarily concerned with service provision, so treats all call requests that are not blocked due to capacity limitations. Each time a call arrives to a system with at least one available server, there will be a decision to attempt a cross-sell or not. If the decision is not to cross-sell, then the call is treated as a service call with a fixed revenue r, which requires an exponentially distributed service time with rate μ. If the decision is to attempt a cross-sell, the call will generate a random revenue

$r + \rho$ and the service time will be distributed exponentially with rate $\mu_1 = \mu - k$, where k is a constant that reflects the impact of the selling activity on the duration of the call. We assume that the random revenue, ρ, which is earned upon a cross-sell attempt, follows a given probability distribution. It is assumed that revenues of successive calls are independent. The objective of the call center is to maximize the total expected discounted revenues over an infinite time horizon and/or maximize long-run average revenue of the center.

2.1 Customer Base and Segmentation

To enable comparison of our optimal cross-selling policies to current practice, we describe next how customers are typically classified by managers for cross-selling purposes. When answering the question of whom to cross-sell, typical approaches do not take the operational capacity tradeoff into account. Instead, marketing managers select cross-selling targets using data on individual customer characteristics. Cross-selling policies are either individual based, where targeting is done at the individual level, or segment-based, where targeting is done at the group level. Segments are formed by aggregating customers in homogeneous groups according to their cross-sell potentials. For example based on demographic, past purchase, and psychographic information a probability of purchase is estimated for each customer. This is then coupled with likely purchase volume and profit margin or revenue information to lead to a customer profitability or revenue potential estimate. In marketing terminology, these estimates, when applied to the entire customer base, constitute a scoring system. In our model, the random revenue potential ρ, which can be a continuous, discrete or mixed random variable, can be thought of as the *score* of each individual customer.

In some marketing settings, the decision of whom to cross-sell is made at the individual level, based on an individual's score ρ. If this is the case, the cross-selling policy that emerges would identify a threshold such that whenever an individual's ρ is above this threshold the decision will be to sell. All individuals scoring above the threshold would be deemed *good-to-sell*. We model this case by assuming that customers' random revenues are coming from one population with a distribution F. We do not have any specific assumptions on the distribution F, however we assume that the random revenues, ρ, have a maximum value of $\bar{\rho} < \infty$ as well as a minimum of $\underline{\rho} \geq 0$. These assumptions are realistic, since rewards gained by selling new products are always finite, and whenever there is a negative reward associated with a possible cross-sell, the server will always choose not to attempt the cross-sell. The cross-selling policies we consider in our control problem will be similarly individual-based, however the threshold will be dynamic since it depends on the operational state of the system.

In other settings, marketing managers will prefer establishing cross-sell decisions for entire groups of customers. Segment-based cross-selling policies will emerge where customers are grouped into *good-to-sell* ($s = G$) and *bad-to-sell* ($s = B$) categories. Whenever a customer is identified as type-G a cross-sell will be attempted. In typical practice, these policies are formed from a value perspective, disregarding the ties to the operational aspect. To establish the link between the optimal dynamic policies and segment-based policies, we next consider how the two-segment setting can be represented in our model. Customers of segment s generate a revenue ρ_s, which follows a probability distribution F_s. We do not have any specific assumptions on the distributions F_s, except for $0 \leq \underline{\rho}_s \leq \rho_s \leq \bar{\rho}_s < \infty$.

We consider two possible scenarios for the segmentation, a discrete and an overlapping one as shown in Figure 1. The segments are characterized in terms of their upper and lower bounds on the ρ scores.

- **Scenario 1:** $\underline{\rho}_B \leq \bar{\rho}_B \leq \underline{\rho}_G \leq \bar{\rho}_G$
- **Scenario 2:** $\underline{\rho}_B \leq \underline{\rho}_G < \bar{\rho}_B \leq \bar{\rho}_G$

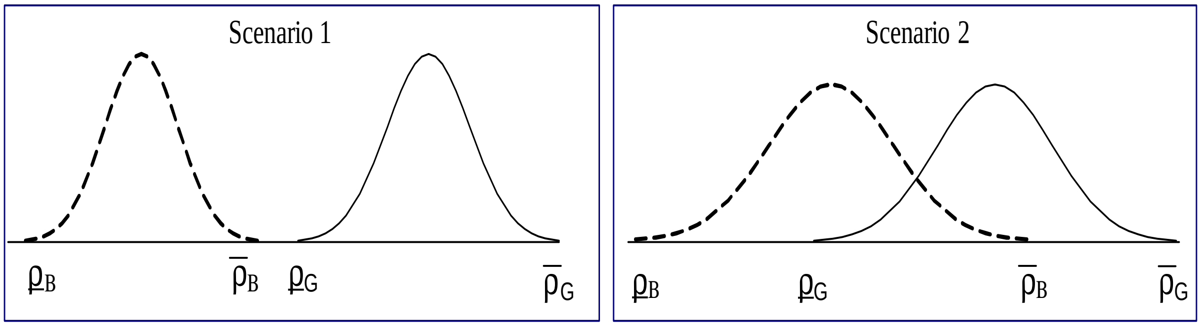

Figure 1: Two possible segmentation scenarios

Segments are formed using statistical procedures that take data on certain customer characteristics as input and group customers based on these. Scenario 1 represents a case where the procedure clearly distinguished between two groups of customers in terms of their cross-selling potential, placing them in non-overlapping segments. Scenario 2 represents the more realistic case. The overlap can be thought of as the type one and type two errors of this statistical procedure. We will show later that optimal dynamic cross-selling policies may or may not be the same as these segment-based policies that aim to cross-sell to all customers of type-G.

2.2 Marketing Automation

As stated in the Introduction, we consider call centers with different estimation capabilities. One where the revenue potential is estimated individually for each customer in real-time and another where only historical averages for the customer base or for a particular segment are estimated. The system we have described so far will be referred to as the *model with revenue realizations* due to the underlying assumption about the possibility to estimate the revenue potential of a customer at the time of the decision: In this case, it is assumed that a server can observe the realization of the random revenue ρ before taking the decision to cross-sell or not. The model with revenue realizations represents the case of a call center where marketing and technology support is such that as soon as a customer call arrives, the system is capable of identifying the customer and displaying its revenue potential. This represents a setting with software that has real-time automation capability as described in Section 1. However, it is also possible that the server takes a decision based on historical segment analysis and then the revenue realization is observed at service completion. As we argue below, this model, which we label as the *model with expected revenues*, is a special case of the model with revenue realizations. In such a setting, managers look at historical data, segment customers and then just use the average for the segment each time a member of this segment is served. How much revenue is eventually realized from a particular customer will be determined at call completion.

When the revenue potentials cannot be estimated for individual customers, all customers of a class will bring the same expected reward. Since the aim of our model is to maximize the total expected reward, the optimal decision will be a result of comparing the expected reward of a class with the additional load that the system will observe due to cross-selling. Then, the model with expected revenues becomes a special case of the model with random revenues: in the segmented case, we set $\underline{\rho}_s = \bar{\rho}_s = \tilde{r}_s = E(\rho_s)$ for $s = G, B$, while we have $\underline{\rho} = \bar{\rho} = \tilde{r} = E(\rho)$ when all customers constitute one population.

2.3 The Discrete Time Model of the System

In this section, we present a discrete time Markov decision process (MDP) for the system with a segmented customer base. The corresponding formulation for a single revenue distribution is straightforward and is omitted. Customers of segment s arrive to the system according to a Poisson process with rate λ_s, and the rewards by successive customers from class s are i.i.d. random variables with probability distribution function F_s.

Our objective is to maximize total expected discounted returns over a finite time horizon with a discount rate β. We define x_1 and x_2 as the number of cross-sell calls and the number of service calls in the system, respectively. The system changes its state at service completions and at arrivals to a system with idle server(s). Because the decision to attempt a cross-sell depends on the customer revenue potential, the state at arrival instants is defined as $(x; s, \rho_s) = (x_1, x_2; s, \rho_s)$. At all other times the state information is described by $x = (x_1, x_2)$.

We interpret discounting as exponential failures with rate β (for the equivalence of discounting and an exponential deadline, see e.g., Walrand, 1988). We use uniformization and normalization to build a discrete time equivalent of the original system by assuming $\lambda_G + \lambda_B + c\mu + \beta = 1$, so that the system will be observed at exponentially distributed intervals with mean 1. There will be an arrival with probability $\lambda_G + \lambda_B$, a real service completion due to a standard service call occurs with probability $x_2\mu$ and due to a cross-sell call with probability $x_1\mu_1$, while a fictitious service completion occurs with probability $c\mu - x_2\mu - x_1\mu_1$.

We next develop the optimality equations for the transformed system over a finite horizon. Let $u_n(x)$ and $v_n(x; s, \rho_s)$ be the maximal expected reward, starting in state x and $(x; s, \rho_s)$, respectively, until n transitions occur. Upon a call arrival, if all servers are busy, then the call has to be rejected. Otherwise, computing $v_n(x; s, \rho_s)$ requires comparison of two actions: cross-selling the incoming class-s call to move to state $x + e_1$ with a reward of $\rho_s + r$, and giving the standard service to enter state $x + e_2$ with a reward of r, where e_j is the two-dimensional unit vector with the j^{th} component equal to 1. The optimality equations of this model are as follows. For $x_1 + x_2 < c$:

$$v_n(x; s, \rho_s) = \max\{\rho_s + r + u_n(x + e_1), r + u_n(x + e_2)\} \tag{1}$$

$$\begin{aligned} u_{n+1}(x) = {} & \lambda_G E[v_n(x; G, \rho_G)] + \lambda_B E[v_n(x; B, \rho_B)] + x_2\mu u_n(x - e_2) + \\ & x_1\mu_1 u_n(x - e_1) + (c\mu - x_2\mu - x_1\mu_1)u_n(x), \end{aligned} \tag{2}$$

where we set $u_n(-1, x_2) = u_n(0, x_2), u_n(x_1, -1) = u_n(x_1, 0)$, and $E[h(x; s, \rho_s)]$ denotes expectation with respect to the probability distribution F_s. We keep the state as $(x; s, \rho_s)$ to prevent any misunderstandings with regard to the expectation $E[v_n(x; s, \rho_s)]$ in equation (2). For $x_1 + x_2 = c$, no calls can be accepted so that $v_n(x; s, \rho_s) = u_n(x)$. We assume that ties in equation (1) are broken by selecting pure service.

We prove certain properties of an optimal policy under the objective of maximizing total expected β-discounted reward for a finite number of transitions, n. The state space of the model is compact, while the action space in each state is finite. Also, we have $E(\rho_s) < \infty$, so that the optimal value functions are bounded. These properties, used with the corresponding results of Hernandez-Lerma and Lasserre (1999), ensure that optimal policies under the expected β-discounted reward over an infinite horizon and/or under the long-run average criterion (when $\beta \to 0$) inherit the structural properties of optimal policies operating with finite n. We refer to the technical report Örmeci and Akşin (2004) for the details.

3 Being Preferred: Definitions, Results, Implications

Preferred calls are those that always generate a cross-sell attempt. A class is called preferred if all calls of that type always generate a cross-sell attempt. In this section, we aim to derive sufficient conditions for having preferred calls and classes. While it is possible to derive several such conditions, we only present one which is both general and provides intuition, thereby forming the basis of our heuristic. A full set including some weaker conditions, as well as all the proofs can be found in the Appendix.

We first specify all possible cross-selling policies in terms of being preferred in the settings with single and two revenue distributions. For a single revenue distribution, the optimal cross-selling policy may be completely dynamic with no preferred calls, or it may select some preferred calls or it may decide to cross-sell to all calls. For the segmented case with two revenue distributions, we have different kinds of policies for Scenarios 1 and 2, which are described below. Given our interest in preferred calls and classes, we will not consider Policy O any further.

Policy O Cross-sell dynamically (no preferred calls)

Policy I Cross-sell attempt to chosen calls of segment G only

Policy II Cross-sell attempt to segment G only

Policy III Cross-sell attempt to segment G and chosen calls of segment B

Policy IV Cross-sell attempt to chosen calls of segment G and B

Policy V Cross-sell attempt to everyone

Our starting point to derive sufficient conditions is equation (1). We observe that it is optimal to cross-sell in a state $(x; s, \rho_s)$ if and only if $u_n(x + e_2) - u_n(x + e_1) < \rho_s$, since we choose "pure service" when both actions are optimal. Now, we let $D_n(21)(x) = u_n(x+e_2) - u_n(x+e_1)$. $D_n(21)(x)$ has a number of interpretations, and will play a key role in our derivation. First of all, $D_n(21)(x)$ is a threshold on the random revenues, so that it is optimal to attempt a cross-sell in state $(x; s, \rho_s)$ if and only if the reward ρ_s exceeds the threshold $D_n(21)(x)$. Moreover, $D_n(21)(x)$ is equal to the relative benefit of starting in state $x + e_2$ versus $x + e_1$, with a horizon of n transitions. Note that the overall speed of service in state $x + e_2$ is higher than that in state $x + e_1$. Hence, we can interpret $D_n(21)(x)$ as the loss in future rewards because of the increased load due to the slow service. Now, we can define being preferred in terms of $D_n(21)(x)$: A call with a reward of ρ is said to be preferred if $D_n(21)(x) < \rho$ for all x. If all calls of class s are preferred, i.e., if $D_n(21)(x) < \rho_s$ for all ρ_s and for all x, then class s is called preferred.

3.1 Structural Results on Preferred Calls and Classes

The main duty of the call center we consider is to answer regular service calls to obtain a fixed reward of r. The call center cannot fulfill this duty if all servers are busy. On the other hand, cross-sell decisions increase the total load of the call center, which, in turn, increases the probability of having all servers busy. Hence, in cross-sell decisions comparison of the fixed reward r with the random reward ρ plays an important role. However, this is not the only aspect. A cross-sell decision for the current call affects the potential cross-sell decision for a future call, which requires a comparison of the random reward offered now with the random reward to be offered in the future. These two comparisons together will determine the sufficient conditions for the existence of preferred calls/classes. We specify these conditions by deriving upper bounds on the relative benefits $D_n(21)(x)$.

The main result of this section identifies an explicit threshold on the cross-sell revenues, which applies to both settings having single revenue and two revenue distributions with the following convention: In the former case, calls arrive according to a Poisson process with rate λ, and revenues have a maximum of $\bar{\rho}$, while in the latter, we set $\lambda = \lambda_G + \lambda_B$ and $\bar{\rho} = \bar{\rho}_G$.

Proposition 1 *All calls bringing at least a reward of ζ are preferred, where*

$$\zeta = \frac{\lambda}{\lambda + \mu_1 + \beta} \frac{\mu - \mu_1}{\mu + \beta} (\bar{\rho} + r).$$

Essentially, Proposition 1 compares the cross-sell revenue of a current call with the potential maximum revenue that can be obtained from a cross-sell call, $\bar{\rho} + r$, by explicitly specifying a threshold on the cross-sell revenues, ζ. For all $\rho > \zeta$, it is always optimal to cross-sell, regardless of the system state. In other words, these calls are preferred. When $\rho < \zeta$, there may be preferred calls with $\rho < \zeta$ but we cannot detect them. For the single distribution case, these observations characterize all possible policies for being preferred. In the two-segment setting, on the other hand, the implication of Proposition 1 is different for each objective function, each scenario and each specific value of ζ. For the model with expected revenues, we have $\bar{\rho}_s = \underline{\rho}_s = \tilde{r}_s$. Then:

(a) If $\zeta < \tilde{r}_G$, class-G is preferred,

(b) If $\zeta < \tilde{r}_B$, both class-G and class-B are preferred.

For the model with random revenue realizations, in Scenario 1:

(a) If $\zeta < \bar{\rho}_G$, there are preferred class-G calls,

(b) If $\zeta < \underline{\rho}_G$, class-$G$ is preferred,

(c) If $\zeta < \bar{\rho}_B$, class-G is preferred, and there are preferred class-B calls,

(d) If $\zeta < \underline{\rho}_B$, both class-$G$ and class-B are preferred.

and in Scenario 2:

(a) If $\zeta < \bar{\rho}_G$, there are preferred class-G calls,

(b) If $\zeta < \bar{\rho}_B$, there are preferred class-G and preferred class-B calls,

(c) If $\zeta < \underline{\rho}_G$, class-$G$ is preferred, and there are preferred class-B calls,

(d) If $\zeta < \underline{\rho}_B$, both class-$G$ and class-B are preferred.

Now, we identify systems with more preferred calls. The threshold decreases when the total arrival rate (λ), the highest possible cross-sell revenue ($\bar{\rho}$), the base service revenue (r) and the service rate difference due to cross-selling ($k = \mu - \mu_1$) decrease, in fact even when only the regular service rate (μ) decreases or only the cross-sell service rate μ_1 increases. Since the arrival rates in call centers are usually high, the first term of the threshold ζ will be very close to 1. Therefore, being preferred mostly depends on the service time and revenue characteristics of the calls. In words, preferred calls exist more easily in environments where service calls and cross-sell calls are relatively similar, the cross-sell revenues are relatively close and the base service revenue r is small. Finally, we note that the threshold is independent of the revenue distribution and only depends on the lower and upper bounds of each segment's revenues.

Identifying preferred calls is useful since these are calls to whom we would attempt to cross-sell irrespective of the system state. Policies like Policy II and V correspond to common practices: sell to everyone in a particular segment or to all customers. Policies I and III where only part of a segment is cross-sold are possible even in the case when discrete segments can be formed. If the initial segmentation is overlapping, then sufficient conditions for a policy of cross-selling to chosen calls from both segments can be stated. This suggests that when the operational aspects of the cross-selling problem are explicitly accounted for, revenue potential based policies used in marketing will not necessarily overlap with the optimal cross-selling policy. Thus Policies I, III and IV suggest that the segment definitions do not correspond to the optimal dynamic policies. If managers want to develop static segment-based cross-selling policies that are also close to the dynamically optimal policies, they can use these structural results to redefine their segments, by shifting the upper and lower bounds of the revenues in the desired direction.

3.2 A Heuristic for Cross-Selling

In this section, we use the expression for the threshold ζ, to construct a heuristic which provides static policies that exploit the structure of the problem. The purpose of introducing this heuristic is to propose an easily implementable control policy, and to improve the information quality provided by the structural results.

Since we would like to develop a heuristic that is independent of arrival rates, we concentrate on the second term of the threshold ζ, which we describe as a function

$$\alpha_1(t) = \frac{\mu - \mu_1}{\mu + \beta}(t + r).$$

Recall that $t = \bar{\rho}$ in ζ, and Proposition 1 compares a current cross-sell revenue with the maximum possible revenue, $\bar{\rho} + r$. Our aim is to replace this benchmark with a more attainable revenue. For this purpose, we define a new quantity $E(R) = E(\rho|\rho > R)$, so that $E(R)$ is the expected reward that we can gain from cross-selling a future customer who offers a revenue of more than R, the current revenue. The current revenue, R, is compared with $\alpha_1(E(R))$, the "expected future gain" when we choose to provide only service to the current customer, and choose to cross-sell a future customer only if s/he offers a revenue of R' with $R' > R$. Then, the heuristic decides to cross-sell the current customer only if $R > \alpha_1(E(R))$. This heuristic can be implemented in settings with more than two customer segments, since the definition of $E(R)$ does not depend on the segments. Moreover, it is in general possible to solve the equation $R = \alpha_1(E(R))$ for R, where we refer to its solution as R^*. Then, the heuristic cross-sells only if the random revenue of an incoming call ρ satisfies $\rho > R^*$.

This heuristic has several attractive features from a managerial point of view. It does not make use of the system state information, and does not depend on the value λ or c takes. The former is an attractive property in settings where the system in place does not enable real time monitoring of the system state. This is not an uncommon feature in many call centers. Furthermore, a cross-selling policy which is static and only depends on call characteristics in terms of revenue and service time is easier to motivate. The independence from call arrival rates and the number of servers is very useful from a practical point of view since both of these quantities exhibit high uncertainty in call center operations, as recently observed by e.g. Avramidis et al. (2004), Brown et al. (2005), Whitt (2006). While the dynamic optimization can be done from a computational feasibility standpoint, its dependence on these variable parameters make it less attractive in practice, motivating the use of a heuristic. Finally, the value of R^* can provide finer information to marketing managers when defining the segments. We discuss the performance of the heuristic in the next section.

4 Numerical Analysis

In this section we have several objectives. First, we would like to understand numerically the difference between the model with expected revenues and the model with revenue realizations. This comparison will provide an assessment of the value of real-time marketing automation. Second, the numerical analysis will illustrate when dynamic cross-selling will be valuable. Subsequently, we will explore the effectiveness of the heuristic. By comparing the heuristic, which makes explicit use of the revenue distribution but not the state of the system, to the model with expected revenues which uses state information but not the revenue distribution, we will analyze under what conditions the different types of information are useful. Using a different set of examples, we will then analyze the interaction between cross-selling and staffing. At the end, the effectiveness of the heuristic will be assessed numerically for finite-buffer systems.

We have developed a set of test problems, all of which are solved to maximize the long-run average revenue, i.e., we set $\beta = 0$. In these problems, we vary base service call lengths (μ), cross-sell durations (k or μ_1), basic service revenues (r), the upper and lower bounds on random revenues ($\underline{\rho}_B, \bar{\rho}_B, \underline{\rho}_G, \bar{\rho}_G$), the size of the call center in terms of number of servers (c), and the load ($\lambda/c\mu$) of the call center. For the numerical analysis, we assume that both G-segment and B-segment revenues have a uniform distribution.

Case 1 (C1) is motivated by a real retail banking call center. Using data estimated for this center, the average length of a service call is taken as 2.7 minutes. Two organizational designs are possible for cross-selling: either the service representative attempts a cross-sell and forwards to a sales department if successful, or the service representative attempts a cross-sell and closes the sale if successful. The latter is expected to take longer in terms of additional talk-time. Motivated by these organizational design options, we form two different talk time scenarios. These are labeled as *attempt-and-forward* (f) and *attempt-and-close* (c) to reflect the original motivation. Increase in talk times will be 27 % for *attempt-and-forward* and 220 % for *attempt-and-close*, again based on estimates from this call center. Average revenue from a call with cross-selling is estimated as 75 units. We take this as the lowest value of the upper limit of G-segment revenues and consider three values $\bar{\rho}_G \in \{75, 125, 175\}$. Revenues from basic service calls are taken as $r = 1$ to reflect the situation that service calls generate very low revenue compared to sales in this call center. For all our examples $\underline{\rho}_B = 0$, $\bar{\rho}_B \in \{0.3\bar{\rho}_G, 0.6\bar{\rho}_G, 0.9\bar{\rho}_G\}$ and $\underline{\rho}_G \in \{0.3\bar{\rho}_G, 0.6\bar{\rho}_G, 0.9\bar{\rho}_G\}$. These values will result in instances with discrete and overlapping segments. We consider call centers with $c \in \{100, 150, 200\}$ servers. Call volumes are obtained to ensure four different loads $(\lambda/c\mu)$, characterizing quality driven (0.75), quality-efficiency driven (0.9), and efficiency driven (1.05, 1.2) centers (for precise definitions of these terms see Gans et al., 2003). The total call volume is split in three different ways, such that $\lambda_G/(\lambda_G + \lambda_B) \in \{0.1, 0.25, 0.4\}$.

Case 2 (C2) is constructed taking C1 as a basis. It is assumed to represent the setting of an insurance call center or an investment bank, where the basic service length is longer. We assume an average service call length of 5.5 minutes, taking the instance of a major insurance call center in the U.S. The percentage increase for *attempt-and-forward* and *attempt-and-close* are taken to be the same as in C1. The only other difference from C1 is the assumption that basic service calls generate more revenue, so that $r = 20$. The two cases result in 3888 problem instances.

The Value of Real-Time Marketing Automation: We compare the performance of the model with expected revenues and the one with revenue realizations. Different assumptions about the marketing automation in place at a call center led to the formulation of these two versions of the model. A comparison of optimal revenues will quantify the value of having real-time automation, as is assumed for the revenue realizations case. In the numerical examples, we set $\tilde{r}_G$ and $\tilde{r}_B$ to the mean revenue of the corresponding revenue distributions. Table 1 displays the averages for each call volume split and overall range of the ratio of the optimal gain from the model with expected revenues to the optimal gain in the model with revenue realization.

		Scenario 1		**Scenario 2**	
		f: 0.1,0.25,0.4	**c:** 0.1,0.25,0.4	**f:** 0.1,0.25,0.4	**c:** 0.1,0.25,0.4
C1	avg. %	99,99,99	85,93,97	99,99,99	87,88,90
	range %	98-100	81-100	99-100	86-90
C2	avg. %	98,98,99	95,98,99	99,99,99	86,90,93
	range %	97-100	81-100	98-100	81-98

Table 1: Ratio of the optimal gain with expected revenues to that with revenue realizations

We observe that the difference in revenues between the two models ranges from less than one percent to nineteen percent. The biggest value from real-time automation is observed in the C1:c and C2:c cases. For these cases, the optimal policy under the model with expected revenues is mostly to sell to everyone, or to good segment calls only whereas the model with random revenues sells more selectively. Similarly, in Scenario 2, $\tilde{r}_G$ and $\tilde{r}_B$ take closer values, thus making it harder for the model with expected revenues to be selective. On the other hand, for the cases with attempt-and-forward type sales organizations, the optimal policy under revenue realizations sells to all G customers and a big proportion of the B customers, thus approaching the selling-to-all policy observed in the expected revenues model. Thus, in these cases the value of revenue realizations is minimal. Finally, we note that the volume split between G-segment and B-segment customers has an impact, as demonstrated by the three averages in each case. The model with expected revenues performs better for higher proportions of G-segment calls. As the two segment's volumes approach each other there is less need for selective selling, which once again helps the model with expected revenues.

The Value of Dynamic Cross-Selling: In order to understand what type of environments lead to more dynamic policies, we compare the difference between the maximum and minimum values

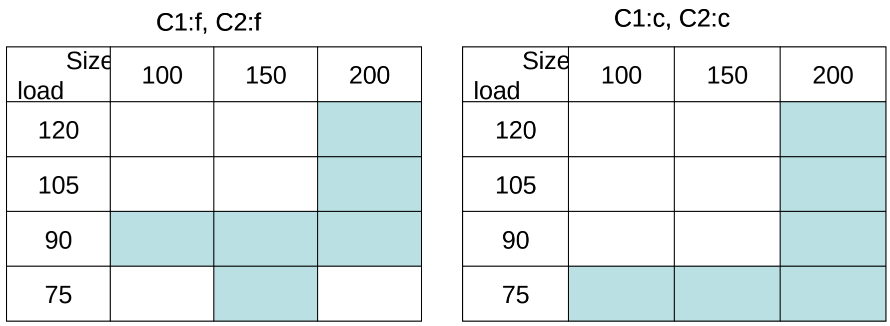

Figure 2: Highest threshold ranges observed in problems with shading

that $D(21)(x)$ takes. Problem instances where this difference, or *threshold range*, is larger, are labeled as more dynamic. The first set of comparisons are made between C1:f, C1:c, and C2:f, C2:c. Comparisons are made such that all other parameters are the same. As expected, all test instances demonstrate that longer talk times and longer cross-selling content lead to more dynamic policies. Ordered from least to most dynamic, one has (C1:f, C1:c, C2:c) and (C1:f, C2:f, C2:c).

We next explore the role that call center load and size play on the dynamic nature of optimal policies. All else being equal, we compare the threshold range for four different loads and three call center sizes. Figure 2 shows the most dynamic instances displayed by call center type. Both for C1:f and C2:f the most dynamic policies are observed for the quality-efficiency regime represented by a load of 90%, and by a large-sized call center represented by 200 servers. In these examples, if the load is set to 75 %, then within the policies with this load those for medium-sized centers with 150 servers are slightly more dynamic than those with 200 servers. It seems the extra slack created by a low load coupled with a large number of servers makes policies less dynamic in this case. For C1:c and C2:c the pattern shifts such that a quality regime represented by a load of 75 % and the largest size with 200 servers lead to the most dynamic policies. The examples demonstrate how the organizational design for cross-selling, and capacity choice impact the dynamic nature of the underlying problem.

The effect of the different cross-selling revenue bounds $(\bar{\rho}_B, \underline{\rho}_G, \bar{\rho}_G)$ on the resulting policies are established via pairwise comparisons of the threshold range within each segmentation scheme (Scenario 1: discrete, Scenario 2: overlapping). The pairwise comparisons are made such that either the G-segment or the B-segment revenue bounds are the same in each pair and the effects of changes in the other segment are explored. The impacts of narrower segment-G revenues, and wider segment-B revenues (more overlapping segment-B revenues in the case of Scenario 2) are analyzed. We observe that for most problem instances, narrower G-segment revenues and wider or more overlapping B-segment revenues lead to more dynamic policies. This effect is less pronounced in C2:c (and to some extent C1:c), which represents call center environments with long base talk times and long cross-sell durations. The volume difference between the two segments also makes an impact: as the G-segment volume proportion decreases from 0.4 to 0.1, the stated properties hold for a higher percentage of instances. In other words, as the G-segment becomes more distinct, both in revenue terms and in volume, observed policies tend to be more dynamic.

We have characterized instances that lead to more dynamic policies. To summarize, these are call centers where talk times are long and the cross-selling portion of calls is significant; capacity is designed such that the number of servers is large and the centers operate in quality or quality-efficiency regimes; and there is a premium good segment.

Performance of the Heuristic: We next implement the heuristic described in Section 3.2. Table 2 reports averages and the overall range of the ratio of the gain obtained with this heuristic to the optimal gain. The heuristic approaches optimal performance on the average consistently across all operating environments considered. Still we need to address its worst case behavior in C1:c under Scenario 1. In this case, all systems with performances less than 70% have a wide range of $(\underline{\rho}_G, \bar{\rho}_G)$ with $\bar{\rho}_B = \underline{\rho}_G$, and $\lambda_G/(\lambda_G + \lambda_B) = 0.1$. We have a total of 36 such incidences out of 216 in C1:c under Scenario 1. When the call volume of the G-segment is low, and the segments are close to each other with a large range, our heuristic becomes very conservative with a high threshold, and so does not perform as well. However, these incidences occur rarely, in 36 out of 3888 instances.

		Scenario 1		**Scenario 2**	
		f: 0.1,0.25,0.4	**c:** 0.1,0.25,0.4	**f:** 0.1,0.25,0.4	**c:** 0.1,0.25,0.4
C1	avg. %	100,100,100	85,95,98	100,100,100	97,98,98
	range %	98-100	55-100	99-100	96-100
C2	avg. %	100,99,99	95,98,98	100,100,100	95,96,97
	range %	96-100	76-100	99-100	75-99

Table 2: Ratio of the gain with the heuristic to the optimal gain

Value of state versus revenue distribution information: The heuristic explicitly uses the probability distributions of the random revenues along with the service rates, while completely ignoring the current state of the system. On the other hand, the model with expected revenues uses the information about the current state of the system, while it has no reference to the random revenue distributions. Hence, comparing their performances may show the value of these two kinds of information. The average performance of the heuristic is almost always superior, where the most significant improvements are observed under Scenario 2 in C1:c and C2:c. In terms of the worst case behavior, the expected revenue model generally performs better, with an exception in C1:c under Scenario 2, and with the most notable case in C1:c under Scenario 1, since 56 out of 216 incidences are below 81% (the worst case of the expected revenue model). As a result, there is no overall winner, and we can conclude that as long as one of the valuable information kinds, the current state of the system or the probability distributions of the revenues, is used effectively, the system will perform very closely to an optimal one. If a single type of information were available, then in our examples, using the state information is more beneficial in the C1:c and C2:c cases of Scenario 1. All other cases are better off relying on the revenue distribution information. From an implementation standpoint, the heuristic has an advantage over the model with expected revenues, by not requiring information on the current arrival rate and the current number of servers.

Interaction with staffing: In our analysis so far, we have taken examples with a given number of servers and specified utilization rates. In order to explore the interaction between cross-selling and staffing, we consider another set of examples where we keep the same utilization rates for given arrival rates and vary the number of servers. We consider the settings where the state information is more valuable, i.e. those with long additional cross-selling times and discrete segments, to be those where staffing would be most valuable. Thus we explore the interaction with staffing for the service rates of C1:c and C2:c, for utilization rates of 75 %, 90 %, 105 %, and 120 %, and for two sets of revenue distributions from before. One where additional revenues can be quite high with $\underline{\rho}_B = 0$, $\bar{\rho}_B = 157.5$, $\underline{\rho}_G = 157.5$, $\bar{\rho}_G = 175$ and another with lower additional revenues with $\underline{\rho}_B = 0$, $\bar{\rho}_B = 22.5$, $\underline{\rho}_G = 22.5$, $\bar{\rho}_G = 75$. Recall that for C1:c the revenue from service calls is 1 whereas this value is 20 for C2:c. We concentrate on the settings where the proportion of good segment calls is 0.1. For the C1:c setting, taking total arrival rate $\lambda = 40$, we compute the number of servers as $144, 120, 102, 90$ for the utilization rates (loads) 75 %, 90 %, 105 %, and 120 % respectively. Similarly for the C2:c setting, taking total arrival rate $\lambda = 21$, we obtain the number of servers as $154, 128, 110, 96$ for the utilization rates 75 %, 90 %, 105 %, and 120 % respectively. We compare total revenues obtained under no cross-selling and optimal cross-selling for all of these cases in Figure 3. We observe that compared to no cross-selling, optimal cross-selling increases revenues for a given staffing level. This benefit is higher for higher cross-sell revenue values, and also for the cases where the service revenue is lower as in C1. As we increase the staffing level (or decrease the utilization rate) the additional revenue benefit obtained from cross-selling vis-a-vis no cross-selling increases. In other words, revenues show an increasing marginal returns feature for percent reduction in utilization. As we go from a utilization rate of 120 % to 75 %, the revenue

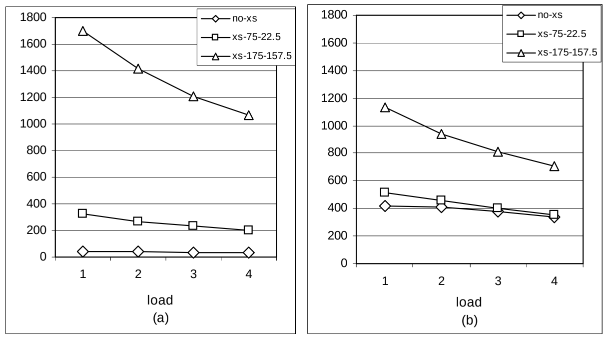

Figure 3: Revenue as a function of the four loads 75 % (1), 90 % (2), 105 % (3), 120 % (4) for C1:c in (a) and C2:c in (b)

difference between the no cross-selling and the optimal cross-selling policies increases. The revenue increase per server for C1:c can be calculated as 2.08 per server and 11.58 per server for the low and high cross-sell revenue settings respectively, while for C2:c these values are 1.32 per server and 5.84 per server respectively.

The result demonstrates that there are operational synergies between service levels and sales effort. If we were to add the customer service related benefits, which we do not capture in our analysis, this effect would be even stronger. Thus in such settings, managers should staff according to a quality regime in order to maximize the benefits obtained from cross-selling.

Heuristic performance under a finite buffer: As a final evaluation, we consider the performance of the heuristic when applied to a queueing system with a finite buffer, as opposed to a loss system. We are mainly concerned with the possible changes in optimal cross-selling policies.

We consider a call center with 50 servers and a buffer capacity of 15. All other parameters are the same as before. We consider the cases with the G-segment volume proportions of 0.1 and 0.4. In order to observe the effect of abandonment, we solve all problems with and without abandonment. In systems with abandonment, we take the abandonment rate as 3μ. Finally, we allow two different cross-selling strategies, one which is not affected by the waiting time of customers, so that it cross-sells to all customers if it is profitable (which we call can-wait-cross-sell), and the other which attempts to cross-sell only to customers who do not wait (which we call no-wait-cross-sell). This gives us a set of 3456 examples. In all the problems, the performance of the heuristic is compared to the optimal policy for the problem with queueing.

Under the can-wait-cross-sell strategy, ignoring the queue has a negligibly small effect on the performance of the heuristic vis-a-vis the optimal gain. For brevity we do not report these results, but note that the averages as well as ranges observed for the different settings being considered are almost identical to those in Table 2, with differences of less than one percent. The overall average of the heuristic to the optimal gain is 97.5 % in systems with no abandonment, while it is 97.3 % in those with abandonment. Under no-wait-cross-sell, the performance of the heuristic improves, yielding an average of 98.8 % in both with and without abandonment. This is due to the limited benefit that can be obtained from optimal cross-selling when the number of customers to be attempted is restricted. The suggested heuristic is very robust since it is performing well with finite buffers in a number of different settings. We can conclude that the structure of an optimal policy in a finite-waiting-room system is similar to that in a loss system, since the heuristic is based on the structural properties of optimal policies in loss systems.

5 Concluding Remarks

This paper formulates and analyzes the first dynamic model of cross-selling in a call center with multiple servers and random revenues. The existence of preferred calls and classes in this type of service rate control problem is demonstrated. The resulting sufficient conditions enable structural comparisons of optimal dynamic policies and prevailing marketing practice. The sufficient conditions are also useful as a building block for a heuristic, which is shown to generate sophisticated static policies that result in near optimal performance both for loss and queueing systems. These static policies are furthermore very easy to understand and implement, making them more valuable in practice. Testing the sensitivity of these results to modeling assumptions like exponential service times is left for future research.

Two versions of the model are analyzed: the model with random realizations and with expected revenues. These represent the two extremes in terms of marketing automation available for cross-selling in call centers, namely real-time automation and historical data analysis. As such, our value comparisons between the optimal dynamic policies under these two versions of the model also characterize the value of real-time automation vis-a-vis historical data analysis. It is shown that in certain operating environments, the difference can be as high as nineteen percent. However, the settings with an attempt-and-forward type sales organization are shown to lead to a small difference, thus underlining the importance of understanding the operating environment of a call center before investing in expensive CRM products. Considering the fact that customer reactions to badly targeted cross-sell attempts are not included in our modeling framework, this difference should be viewed as a lower bound on the real value.

Using the case of a real retail banking call center, an extensive set of numerical examples are developed. These are used to characterize environments where consideration of the dynamic nature of the problem is more important. It is shown that large call centers that operate under a quality-efficiency or quality regime, with call and cross-selling durations that are relatively long, having smaller differences between base service and cross-sell revenues, and where there is a premium good segment of customers who are difficult to segment from the remaining calls, will benefit more from dynamic optimization of their cross-selling policies. Additional analysis that explores the link between cross-selling and staffing demonstrates that there are increasing marginal returns to revenues as a function of utilization. This provides support for a recommendation to improve service concurrently with sales efforts in call centers, if revenue generation is to be maximized. As a caveat, it should be mentioned that the value of dynamic optimization demonstrated through our numerical examples should be viewed as an upper bound on the real value today, since exact estimation of revenue realizations at the beginning of a call represents an idealized view of the current state-of-the-art.

Finally, it is possible to show the optimality of threshold-type policies for this cross-selling problem, when concavity of the value functions u_n in x_1 are assumed. This result can be found in the technical report Örmeci and Akşin (2004). Concavity of value functions even in simpler systems cannot be shown, see Örmeci et al. (2002), although no example of such systems is reported that violated concavity. In the 3888 example test suite that we consider herein, we have similarly not observed a single value function which is not concave.

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