



Appendix 1 to Chapter 9

Duration Gap Analysis

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Duration Gap Analysis

Examines the sensitivity of the market value of the financial institution's net worth to changes in interest rates

$$\% \Delta P \approx -DUR \times \frac{\Delta i}{1+i}$$

$$\% \Delta P = \frac{(P_{t+1} - P_t)}{P_t} = \text{percent change in market value of the security}$$

DUR = duration (calculated for each asset and liability)

i = interest rate

After calculating the duration for each asset and liability, the weighted duration is determined by multiplying the duration times the amount of the asset divided by total assets

Adding all the weighted duration figures up yields the average duration of either the assets or the liabilities



TABLE 1 **Duration of the First National Bank's Assets and Liabilities**

	Amount (\$ millions)	Duration (years)	Weighted Duration (years)
Assets			
Reserves and cash items	5	0.0	0.00
Securities			
Less than 1 year	5	0.4	0.02
1 to 2 years	5	1.6	0.08
Greater than 2 years	10	7.0	0.70
Residential mortgages			
Variable-rate	10	0.5	0.05
Fixed-rate (30-year)	10	6.0	0.60
Commercial loans			
Less than 1 year	15	0.7	0.11
1 to 2 years	10	1.4	0.14
Greater than 2 years	25	4.0	1.00
Physical capital	5	0.0	<u>0.00</u>
<i>Average duration</i>			2.70
Liabilities			
Checkable deposits	15	2.0	0.32
Money market deposit accounts	5	0.1	0.01
Savings deposits	15	1.0	0.16
CDs			
Variable-rate	10	0.5	0.05
Less than 1 year	15	0.2	0.03
1 to 2 years	5	1.2	0.06
Greater than 2 years	5	2.7	0.14
Fed funds	5	0.0	0.00
Borrowings			
Less than 1 year	10	0.3	0.03
1 to 2 years	5	1.3	0.07
Greater than 2 years	5	3.1	<u>0.16</u>
<i>Average duration</i>			1.03



Example 1: Duration Gap Analysis

What happens when interest rates rise from 10% to 11%?

Total asset value = \$100 M and total liabilities = \$95 M

For assets

$$DUR = 2.70$$

$$\Delta i = 0.01$$

$$i = 0.10$$

$$\% \Delta P \approx -2.70 \times \frac{0.01}{1 + 0.10} = -0.025 = -2.5\%$$

For liabilities

$$DUR = 1.03$$

$$\Delta i = 0.01$$

$$i = 0.10$$

$$\% \Delta P \approx -1.03 \times \frac{0.01}{1 + 0.10} = -0.009 = -0.9\%$$

The net worth of the bank would decline by \$1.6 M



Example 2: Duration Gap Analysis

A quicker way to arrive at the answer can be done by calculating the

Duration Gap

$$DUR_{gap} = DUR_a - \left(\frac{L}{A} \times DUR_l\right)$$

DUR_a = average duration of assets

DUR_l = average duration of liabilities

L = market value of liabilities

A = market value of assets

For the previous example

$$DUR_{gap} = 2.70 - \left(\frac{95}{100} \times 1.03\right) = 1.72 \text{ years}$$



Example 3: Duration Gap Analysis

Using the DUR_{gap} calculation to obtain the change in the market value of net worth as a percentage of total assets

$$\frac{\Delta NW}{A} \approx -DUR_{gap} \times \frac{\Delta i}{1+i}$$

Using the previous figures for an interest rate rise from 10% to 11%

$$\frac{\Delta NW}{A} \approx -1.72 \times \frac{0.01}{1+0.10} = -0.016 = -1.6\%$$

With assets of \$100M this is a fall in the market value of \$1.6M which is the same amount we found with the first example



TABLE 2 Duration of the Friendly Finance Company's Assets and Liabilities

	Amount (\$ millions)	Duration (years)	Weighted Duration (years)
Assets			
Cash and deposits	3	0.0	0.00
Securities			
Less than 1 year	5	0.5	0.05
1 to 2 years	1	1.7	0.02
Greater than 2 years	1	9.0	0.09
Consumer loans			
Less than 1 year	50	0.5	0.25
1 to 2 years	20	1.5	0.30
Greater than 2 years	15	3.0	0.45
Physical capital	5	0.0	<u>0.00</u>
<i>Average duration</i>			1.16
Liabilities			
Commercial paper	40	0.2	0.09
Bank loans			
Less than 1 year	3	0.3	0.01
1 to 2 years	2	1.6	0.04
Greater than 2 years	5	3.5	0.19
Long-term bonds and other long-term debt	40	5.5	<u>2.44</u>
<i>Average duration</i>			2.77



Example of a Nonbanking Financial Institution

Rate-sensitive assets equal \$5M of securities with maturities less than one year plus \$50M of consumer loans with maturities of less than one year

Rate-sensitive liabilities equal \$40M of commercial paper plus \$3M of bank loans both of which have maturities of less than one year

$$GAP = RSA - RSL = \$55M - \$43M = \$12M$$

The effect on income if interest rates rise by 1% is

$$\Delta I = GAP \times \Delta i = \$12M \times 1\% = \$120,000$$

Income will rise instead of fall as with the bank because there are more rate-sensitive assets than rate-sensitive liabilities



Nonbanking Financial Institution Duration Gap Analysis

Total assets = \$100M; Duration for assets = 1.16 years

Total liabilities = \$90M; Duration for liabilities = 2.77 years

$$DUR_{gap} = 1.16 - \left(\frac{90}{100} \times 2.77\right) = -1.33 \text{ years}$$

$$\frac{\Delta NW}{A} = -(-1.33) \times \frac{0.01}{1 + 0.10} = 0.012 = 1.2\%$$

With assets of \$100M, net worth will rise in market value by \$1.2M



Problems with Income and Duration Gap Analysis

- We assume that when the level of interest rates changes, interest rates on all maturities change by exactly the same amount \Rightarrow the slope of the yield curve remains unchanged
- For the duration gap, if the interest rates for all maturities are the same \Rightarrow the yield curve is flat
- Duration gap analysis only works well for small interest rate changes
- Must estimate the proportion that may be rate-sensitive
- Cash payments are uncertain
- Good first assessment



Strategies for Managing Interest-Rate Risk—1st National Bank

- Eliminate income gap
 - ◆ Increase rate-sensitive assets to \$49.5M
 - ◆ Reduce rate-sensitive liabilities to \$32M
- Immunize the market value of the bank's net worth by adjusting assets and liabilities so the duration gap is zero
 - ◆ Reduce average duration of asset to 0.98 year
 - ◆ Increasing average duration of liabilities to 2.84 years
- These strategies can all be very costly in the short run