Failure, Analysis and Design

Special Cases of Laminates

➢ The symmetry or antisymmetry of a laminate, based on angle, material, and thickness of plies, may zero out some elements of the three stiffness matrices $[A]$, $[B]$, and $[D]$.

➢ These are important to study because they may result in reducing or zeroing out the coupling of forces and bending moments, normal and shear forces, or bending and twisting moments.

➢ This not only simplifies the mechanical analysis, but also gives desired mechanical performance.

Symmetric Laminates

➤ It can be proved that the coupling matrix $[B] = 0$ for symmetric laminates.

➤ Hence the force and moment equations can be decoupled.

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

Symmetric Laminates

➢ If a symmetric laminate is subjected only to forces, it will have zero midplane curvatures.

➢ If it is subjected only to moments it will have zero midplane strains.

➢ Makes analysis much simpler.

➢ Also prevents a laminate from twisting due to thermal loads.

Cross-Ply Laminates

➢ For cross-ply laminates $A_{16} = A_{26} = B_{16} = B_{26} = D_{16} = D_{26} = 0$.

$$
\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\
A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\
0 & 0 & A_{66} & 0 & 0 & B_{66} \\
B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\
B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 \\
0 & 0 & B_{66} & 0 & 0 & D_{66}
\end{bmatrix}
\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}
$$

➢ Hence, there is uncoupling between the normal and shear forces, and also between the bending and twisting moments.

➢ If a cross-ply is also symmetric, then $[B] = 0$ and there will no coupling between the force and moment terms.

Angle-Ply Laminates

➢ If an angle-ply laminate has an even number of plies, then $A_{16} = A_{26} = 0$.

➢ If the number of plies is odd, and it consists of alternating $+\theta$ and $-\theta$ plies, then not only is it symmetric ($[B] = 0$), but also A_{16}, A_{26}, D_{16}, D_{26} → 0 as the number of layers increases for the same laminate thickness.

➢ Similar to symmetric cross-ply laminates, but with higher shear stiffness and shear strength properties.

Antisymmetric Laminates

➤ A laminate is called antisymmetric if the material and thickness of the plies are the same above and below the midplane, but the ply orientations at the same distance above and below the midplane are negative of each other, i.e. +45/60/-60/-45.

$$
\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & B_{26} \\ 0 & 0 & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & 0 \\ B_{16} & B_{26} & B_{66} & 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}
$$

Balanced Laminates

➤ A laminate is balanced when it consists of pairs of layers of the same thickness and material where the angles of the plies are $+\theta$ and $-\theta$.

➤ Thus $A_{16} = A_{26} = 0$.

$$
\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & B_{26} \\ 0 & 0 & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}
$$

➤ If the number of plies in a balanced laminate is odd, it an be made symmetric ($[B] = 0$).

Quasi-Isotropic Laminates

➢ A laminate is called quasi-isotropic if its extensional stiffness matrix [*A*] behaves like that of an isotropic material.

➢ This not only implies $A_{11} = A_{22}$, $A_{16} = A_{26}$, and $A_{66} = (A_{11} - A_{12})/2$, but also that these stiffnesses are independent of the angle of rotation of the laminate.

➢ Called quasi-isotropic and not isotropic because [*B*] and [*D*] may not behave like an isotropic material.

➢ Examples of quasi-isotropic laminates include

$$[0/\pm 60], \ [0/\pm 45/90]_s, \ [0/36/72/-18/-54]$$

Failure Criteria for a Laminate

➢ Laminate failure may not be catastrophic.

➢ It is possible that some layer(s) fail first and the composite continues to take more loads until all the plies fail.

➢ Failed plies may still contribute to the stiffness and strength of the laminate.

Failure Criteria for a Laminate

➤ The degradation of the stiffness and strength properties of each failed lamina depends on the philosophy of the user:

  ➤ When a ply fails, it may have cracks parallel to the fibers. This ply is still capable of taking load parallel to the fibers. The longitudinal modulus and strength remain unchanged, the transverse stiffness and strength as well as the shear strength → 0.

  ➤ When a ply fails, fully discount the ply and replace the ply of near-zero stiffness and strength.

Failure Criteria for a Laminate

- Procedure for finding the successive loads between first ply failure and last ply failure (following the fully discounted method)
  - Use CLT to find the midplane strains and curvatures given the applied load.
  - Find the local stresses and strains in each ply.
  - Apply the failure theories to each ply to determine the strength ratio (SR) for each ply.
  - Multiplying the strength ratio to the applied load gives the load level of the failure of the first ply.
  - Degrade fully the stiffness of damaged ply or plies. Apply the actual load level of the previous failure.
  - Repeat to find the (SR) for the undamaged plies.
    - If the SR>1, multiply to the applied load to obtain the load level of the next ply failure and repeat.
    - If the SR<1, degrade the stiffness and strength properties of all the damaged plies and apply the actual load level of the previous failure.
  - Repeat until all of the plies have failed. The load at which all of the plies have failed is called the last ply failure.

Strength Ratio

➢ In a failure theory, it can be determined whether a lamina has failed if the stress state lies within the failure envelope.

➢ However, it does not give information about how much the load can be increased if the lamina is safe, or how much the load should be decreased if the lamina has failed.

➢ The strength ratio is defined –

$$SR = \frac{\text{Maximum Load Which Can Be Applied}}{\text{Load Applied}}$$

➢ If SR>1, then the lamina is safe and the applied stress can be increased by a factor of SR.

➢ If SR<1, the lamina is unsafe and the applied stress needs to be reduced by a factor of SR.

Example Problem

➢ A [0/90]$_s$ laminate made of glass/epoxy is subjected to an axial load N$_x$ (assume each layer is 0.005" thick.).

   ➢ Determine the first ply failure load (or stress)

Example Problem

➢ 45% fiber volume fraction glass/epoxy composite

➢ $E_1 = 5.60$ Msi, $E_2 = 1.20$ Msi, $G_{12} = 0.60$ Msi

➢ $\nu_{12} = 0.26, \quad \nu_{21} = \nu_{12}\dfrac{E_2}{E_1} = 0.0557$

➢ Compute reduced stiffness terms for each layer,

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}} = 5.682 \text{ Msi}$$

$$Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} = 0.3166 \text{ Msi}$$

$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} = 1.218 \text{ Msi}$$

$$Q_{66} = G_{12} = 0.6 \text{ Msi}$$

Example Problem

➢ Compute transformed reduced stiffness matrices for each layer

$$\left[Q_{xy}\right]_0 = [Q] = \begin{bmatrix} 5.682 & 0.3166 & 0 \\ 0.3166 & 1.218 & 0 \\ 0 & 0 & 0.6 \end{bmatrix}$$

$$\left[Q_{xy}\right]_{90} = \begin{bmatrix} 1.218 & 0.3166 & 0 \\ 0.3166 & 5.682 & 0 \\ 0 & 0 & 0.6 \end{bmatrix}$$

Example Problem

➢ Compute extensional stiffness matrix, [A]

$$[A] = \sum_{k=1}^{4} \left[Q_{xy} \right]_k (z_k - z_{k-1}) = \begin{bmatrix} 6.9(10^4) & 6.332(10^3) & 0 \\ 6.332(10^3) & 6.9(10^4) & 0 \\ 0 & 0 & 1.2(10^4) \end{bmatrix}$$

➢ Compute extensional compliance matrix

   ➢ Laminate is symmetric so [a] = [A]$^{-1}$

$$[a] = \begin{bmatrix} 1.462(10^{-5}) & 1.341(10^{-6}) & 0 \\ 1.341(10^{-6}) & 1.462(10^{-5}) & 0 \\ 0 & 0 & 8.333(10^{-5}) \end{bmatrix}$$

Example Problem

➢ From symmetry, $[B] = 0$, thus midplane strains and curvatures are decoupled.

$$\begin{bmatrix} \varepsilon_x{}^0 \\ \varepsilon_y{}^0 \\ \gamma_{xy}{}^0 \end{bmatrix} = [a] \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} 1.461(10^{-5}) \\ -1.341(10^{-6}) \\ 0 \end{bmatrix} \frac{\text{in.}}{\text{in.}}$$

➢ assuming, $N_x = 1$ lb/in.,

➢ and $Ny = N_{xy} = M_x = M_y = M_{xy} = 0$.

Example Problem

➢ Similarly for the curvatures,

$$\begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = [D]^{-1} \begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix}$$

but $[M] = [0]$

$$\therefore \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \frac{1}{\text{in.}}$$

Example Problem

➢ Compute the global strains at the top surface of the first 0° ply.

$$[\varepsilon] = [\varepsilon_0] + z[\kappa]$$

$$[\varepsilon]_{\text{TOP\_0}} = \begin{bmatrix} 1.462(10^{-5}) \\ -1.341(10^{-6}) \\ 0 \end{bmatrix}$$

➢ Same at the bottom surface

➢ *Same everywhere in the laminate.*

Example Problem

➢ Compute the global stresses at the top surface of the first 0° ply.

$$[\sigma]_k = [Q_{xy}]_k [\varepsilon] = \begin{bmatrix} 82.63 \\ 2.994 \\ 0 \end{bmatrix}$$

➢ Find transformation matrix for each ply angle.

$$[T]_{0°} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad [T]_{90°} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Example Problem

➤ Compute the local stresses at the top surface of the first 0° ply.

$$[\sigma] = \begin{bmatrix} 82.63 \\ 2.994 \\ 0 \end{bmatrix} \text{ psi}$$

➤ Same throughout the first 0° ply *and the other 0 ° ply*.

➤ Similarly for the 90° plies

$$[\sigma] = \begin{bmatrix} -2.994 \\ 17.37 \\ 0 \end{bmatrix} \text{ psi}$$

Example Problem

- Compute strength ratios for all 4 plies
- $SR_0 = 1502$
- $SR_{90} = 259$
- Then N_x @ FPF = 259 psi-in.
- Failure initiates in the 90 plies (*as expected*.)
- Maximum allowable normal stress @ FPF

$$\frac{N_{x\_\text{FPF}}}{h} = 12.94 \text{ ksi}$$

Design Considerations

➢ Design of laminated composites includes selecting a material system or a group of material systems and determining the stacking sequence for the laminate based on applied loads, and constraints on optimizing and constraining factors such as –

  ➢ Cost
  ➢ Mass
  ➢ Stiffness
  ➢ Dimensional Stability

➢ List is similar to that used in designing with monolithic materials; the main issue then comes of understanding the orthotropic nature of composite plies.

Design Considerations

➢ The possibility of different fiber/matrix systems combined with the variables such as V_f dictates the properties of a lamina.

➢ Then laminae can be placed at angles and at particular distances from the midplane of the laminate.

➢ The material systems and the stacking sequence determine the stresses and strains in the laminate.

Design Considerations

➢ Failure may be based on <u>first ply failure</u> or last ply failure.

➢ Laminate selection is a computationally intensive and repetitive task due to the many possibilities of fiber/matrix combinations, material systems, and stacking sequence.

Other Mechanical Design Issues

➢ Hygrothermal Effects

➢ Long-Term Environmental Effects

➢ Interlaminar Stresses

➢ Impact Resistance

➢ Fracture Resistance

➢ Fatigue Resistance

Long-Term Environmental Effects

➤ Corrosive atmospheres and temperatures and humidity variations can lessen the adhesion of the fiber/matrix interface.

➤ Epoxy matrices soften at high temperatures, affecting matrix dominated properties such as transverse and in-plane shear stiffness and strength and flexural strength.

➤ Glass/epoxy composites absorb 1% moisture by weight in just 6 mos. of immersion causing a similar reduction in flexural modulus.

➤ Synthetic fiber composites even worse!

Interlaminar Stresses

➤ Due to the mismatch of angle and elastic moduli between the layers of a multi-axial laminate, interlaminar stresses are developed.

Laminate   Strain Variation Stress Variation
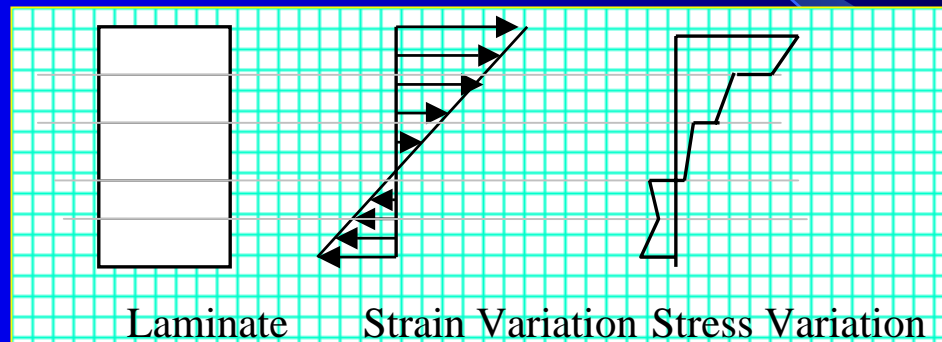
➤ These stresses which are both normal and shear, can be high enough to cause edge delaminations.

➤ Delamination eventually limits usefulness of the composite.

➤ Delamination can be further caused due to non-optimum curing or introduction of foreign bodies in the structure.

Interlaminar Stresses

➢ Some ways to counter the effects
  ➢ Keep the angle/symmetry, and number of plies the same but change the stacking sequence influences the interlaminar stresses.
    ➢ Decrease the the interlaminar shear stresses without increasing the tensile (if any) interlaminar normal stresses.
    ➢ [±30/90]$_s$ ➔ [90/ ±30] produces compressive interlaminar stresses, this stacking sequence is much less prone to delamination.
  ➢ Use toughened resin systems
  ➢ Use interleaved systems where a discrete layer of resin with high toughness and strain to failure is added between the layers.

Impact Resistance

➢ Resistance to impact depends on several factors
  ➢ Material system
  ➢ Interlaminar strengths
  ➢ Stacking sequence
➢ Impact damage reduces strengths of the laminate and also initiates delaminations.
➢ Delamination becomes more problematic since many times visual inspection cannot find delaminations.
➢ Solutions for increasing impact resistance and residual strength include:
  ➢ Toughened epoxies
  ➢ Interleaved laminates

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Fracture Resistance

➢ The mechanics of fracture in composites is very complicated.

  ➢ Cracks can grow in the form of fiber breaks, matrix breaks, fiber/matrix debonding, or debond between layers.

  ➢ No single critical stress intensity factors and/or strain energy release rates to determine the fracture mechanics process.

➢ Fiber breaks may occur because of the brittle nature of fibers (statistics too. . .)

➢ The matrix may then break because of the high strains caused by the fiber breaks.

➢ When a fiber or matrix breaks, the crack behavior is hard to predict

  ➢ It may grow along the interface which blunts the crack and improves fracture resistance

  ➢ It may grow into the next constituent, resulting in uncontrolled failure.

  ➢ Whether a crack grows along the interface or jumps to the adjoining constituent is dependent on the material properties of the fiber, matrix, and the interface, as well as V_f.

Fatigue Resistance

➤ Several factors influence the fatigue properties of a laminate
  - ➤ Fiber and matrix properties
  - ➤ Fiber volume fraction
  - ➤ Stacking sequence
  - ➤ Interfacial bonding

➤ S-N curves are quite different for quasi-isotropic laminates and UD composites.
  - ➤ 90° degree plies develop transverse cracks which influence the elastic moduli and strength of the laminate.
  - ➤ Limited since the 90° degree plies contribute little to the static strength and stiffness, instead stress concentrations caused by these cracks may lead to damage in the 0° degree plies.

Fatigue Resistance

➤ Other Damage Modes include:
  ➤ Fiber and Matrix breaks
  ➤ Interfacial and interlaminar debonding
➤ Laminate stacking sequence influences the onset of edge delaminations
  ➤ $[\pm45/\pm15]_s$ laminate had a higher fatigue life than a $[\pm15/\pm45]_s$ laminate
➤ Loading factors such as tension and/or compression, temperature, moisture, and frequency of loading also determine the fatigue behavior of composites.
  ➤ Carbon/epoxy composites have very poor fatigue resistance when subjected to tension/compression fatigue because of fiber microbuckling.

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Non-Mechanical Issues

➢ Fire Resistance

➢ Smoke Emission

➢ Lightning Strike

➢ Electrical and Thermal Conductivity

➢ Recycling Potential

➢ EMI

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