Extension

Distance and Midpoint Formula in the Complex Plane

The modulus of the complex number $a + bi$ is $|a + bi| = \sqrt{a^2 + b^2}$. This is the distance between the origin $(0, 0)$ and the point (a, b) in the complex plane. For two points in the complex plane, the distance between the points is the modulus of the difference of the two complex numbers.

Let (a, b) and (s, t) be points in the complex plane. The difference of the complex numbers is $(s + ti) - (a + bi) = (s - a) + (t - b)i$. The modulus of the difference is

$$|(s - a) + (t - b)i| = \sqrt{(s - a)^2 + (t - b)^2}.$$

So, $d = \sqrt{(s - a)^2 + (t - b)^2}$ is the difference between the two points in the complex plane.

Distance Formula in the Complex Plane

The difference between the points (a, b) and (s, t) in the complex plane is $d = \sqrt{(s - a)^2 + (t - b)^2}$.

Figure 1.1 shows the points represented as vectors. The magnitude of the vector $\mathbf{u} - \mathbf{v}$ is the distance between (a, b) and (s, t).

$$\mathbf{u} - \mathbf{v} = \langle s - a, t - b \rangle$$
$$\|\mathbf{u} - \mathbf{v}\| = \sqrt{(s - a)^2 + (t - b)^2}$$

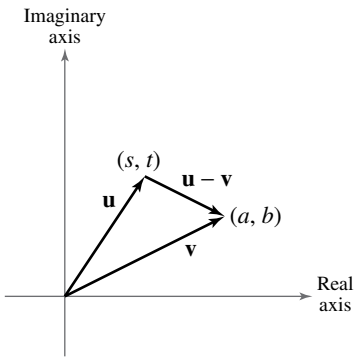


Figure 1.1

EXAMPLE 1 Finding the Distance Between Points in the Complex Plane

Find the distance between the points $2 + 3i$ and $5 - 2i$ in the complex plane.

Solution

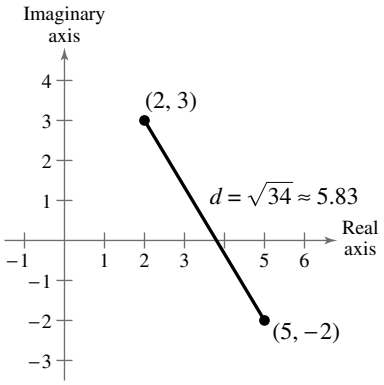
Let $a + bi = 2 + 3i$ and $s + ti = 5 - 2i$. The difference between the complex numbers is

$$(5 - 2i) - (2 + 3i) = (5 - 2) + (-2 - 3)i = 3 - 5i.$$

The distance is

$$d = \sqrt{3^2 + (-5)^2} = \sqrt{34} \approx 5.83 \text{ units}$$

as shown in the figure below.



The midpoint of the line segment joining two complex numbers $a + bi$ and $s + ti$ is the average of the numbers at the endpoints.

Midpoint Formula in the Complex Plane

The Midpoint Formula is

$$\text{Midpoint} = \frac{a + s}{2} + \left(\frac{b + t}{2}\right)i.$$

EXAMPLE 2 Finding the Midpoint of a Line Segment in the Complex Plane

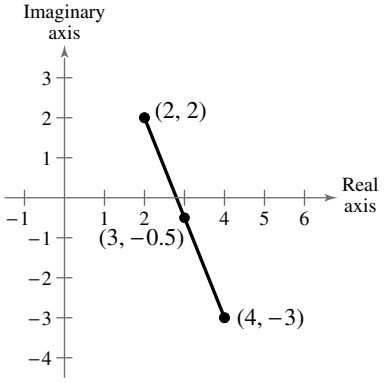
Find the midpoint of the line segment joining the points $4 - 3i$ and $2 + 2i$.

Solution

Let $a + bi = 4 - 3i$ and $s + ti = 2 + 2i$. Apply the Midpoint Formula.

$$\text{Midpoint} = \frac{a + s}{2} + \left(\frac{b + t}{2}\right)i = \frac{4 + 2}{2} + \left(\frac{(-3) + 2}{2}\right)i = 3 - 0.5i$$

The midpoint of the line segment joining the points $4 - 3i$ and $2 + 2i$ is $3 - 0.5i$, as shown in the figure below.



Exercises

Finding the Distance Between Points in the Complex Plane **In Exercises 1–4, find the distance between the points in the complex plane.**

1. $1 + 2i, -1 + 4i$

2. $-5 + i, -2 + 5i$

3. $6i, 3 - 4i$

4. $-7 - 3i, 3 + 5i$

Finding the Midpoint of a Line Segment in the Complex Plane **In Exercises 5–8, find the midpoint of the line segment joining the points.**

5. $2 + i, 6 + 5i$

6. $-3 + 4i, 1 - 2i$

7. $7i, 9 - 10i$

8. $-1 - \frac{3}{4}i, \frac{1}{2} + \frac{1}{4}i$