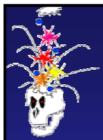

Z-Scores

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Z-Scores

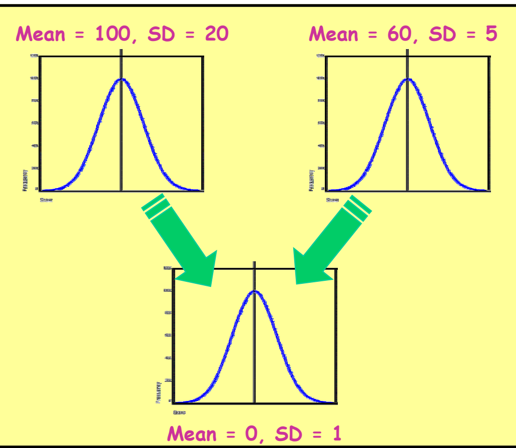
- Z-scores are a way of standardising a score with respect to the other scores in the group.
- This is done by taking account of the mean and SD of the group.
- A Z-score expresses a particular score in terms of how many Standard Deviations it is away from the mean.

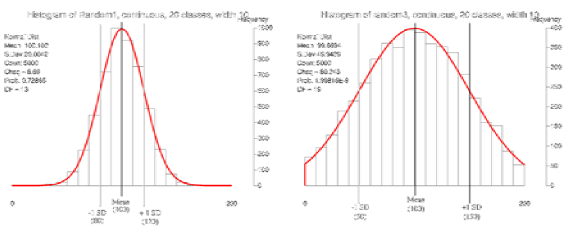
- By converting a raw score to a z-score, we are expressing that score on a z-score scale, which always has a mean of 0 and a standard deviation of 1.
- In short, we are re-defining each raw score in terms of how far away it is from the group mean.

Calculating a Z-score

- First, we find the difference between the raw score and the mean score (this tells us how far away the raw score is from the average score)
- Second, we divide by the standard deviation (this tells us how many standard deviations the raw score is away from the average score)

$$z = \frac{X - \bar{X}}{s}$$





Advantages of Using Z-scores

❶ **Clarity**: The relationship between a raw score and the distribution of scores is much clearer. It is possible to get an idea of how good or bad a score is relative to the entire group.

❷ **Comparison**: You can compare scores measured on different scales.

❸ ***Area Under The Curve***: We know various properties of the normal distribution.

- ☺ By converting to a normal distribution of z-scores, we can see how many scores should fall between certain limits.
- ☺ We can, therefore, calculate the probability of a given score occurring.

Area Under the Normal Curve

- In most Statistics text books you can find a table of numbers labelled *area under the normal curve.*
- This table allows us to discover things about any set of scores provided that we first convert them to z-scores.

- ***Area between the mean and z***: This part of the table tells us the proportion of scores that lie between the mean and a given z-score (this proportion is the area under the curve between those points).
- ***Area beyond z***: This part of the table tells us the proportion of scores that were greater than a given z-score

These areas can be used to find out:

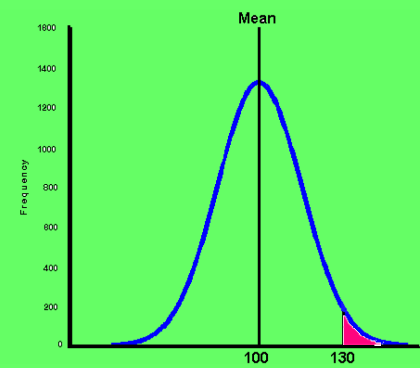
❶ The proportion of Scores that were greater than a particular score on a test.

❷ What proportion of scores lie between the mean and a given test score.

❸ What proportion lie between two scores

Example 1

- A social-skills scale had a mean of 100 and a standard deviation of 15.
- 263 people at RH were tested.
- A psychology Student scores 130.
- What proportion of people got a higher score than this? How many people is this?



Convert the Raw Score to a Z-score

$$z = \frac{X - \bar{X}}{s}$$

$$z = \frac{130 - 100}{15}$$

$$z = \frac{30}{15}$$

$$z = 2$$

Look up the proportion in the z-score table

- The diagram shows that we are interested in the area above 130 (shaded).
- Look in column labelled *area above z.*
- when z = 2, area beyond = **0.0228**.

$$Percentage = 100 \times 0.0228 = 2.28\%$$

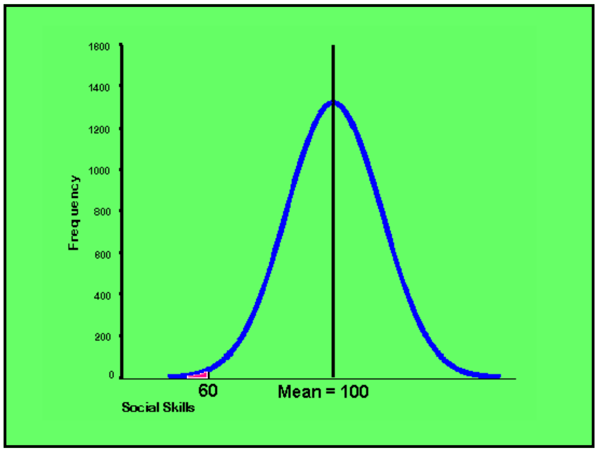
Conclusion

- 2.28% of people had better social skills than our psychology student.
- We can work out how many people this was by multiplying the proportion by the number of scores collected:

$$263 \times 0.0228 = 6\,\text{people}$$

Example 2

- A social-skills scale had a mean of 100 and a standard deviation of 15.
- 263 people at RH were tested.
- A statistic lecturer scores of 60.
- What proportion of people got a lower score than this? How many people is this?



Convert the Raw Score to a Z-score

$$z = \frac{X - \bar{X}}{s}$$

$$z = \frac{60 - 100}{15}$$

$$z = \frac{-40}{15}$$

$$z = -2.67$$

Look up the proportion in the z-score table

- The diagram shows that we are interested in the area below 40 (shaded).
- Look in column labelled *area above z*.
- when z = 2.67, area beyond = **0.0038**.

$$Percentage = 100 \times 0.0038 = 0.38\%$$

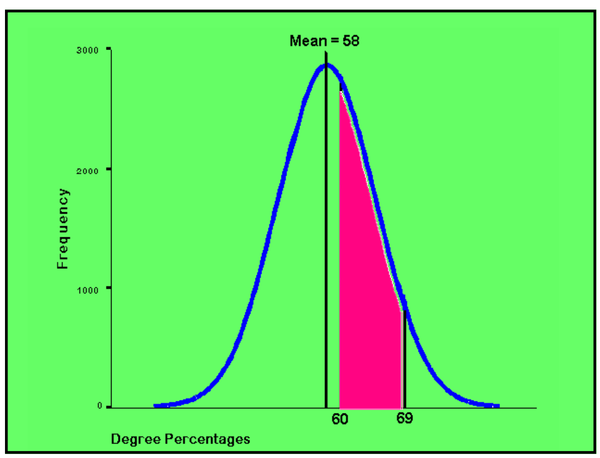
Conclusion

- 0.38% of people had worse social skills than our statistic lecturer.
- We can work out how many people this is by multiplying the proportion by the number of scores collected:

$$263 \times 0.0038 = 1 \textbf{ person}$$

Example 3

- 130 Students' degree percentages were recorded.
- The mean percentage was 58% with a standard deviation of 7.
- What proportion of students received a 2:1? How many people is this?
- Hint 2:1 = between 60% and 69%



Convert the Raw Scores to Z-scores

$$z_{60} = \frac{60-58}{7}$$

$$z_{60} = \frac{2}{7}$$

$$z_{60} = 0.29$$

$$z_{69} = \frac{69-58}{7}$$

$$z_{69} = \frac{11}{7}$$

$$z_{69} = 1.57$$

Look up the proportions in the z-score table

- The diagram shows that we are interested in the area above both scores.
- Look in column labelled *area above z*.
- when z = 0.29, area beyond = **0.386**.
- when z = 1.57, area beyond = **0.058**.

Calculate Shaded Area

$$\textbf{Shaded Area} = \textbf{Area Beyond } Z_{60} - \textbf{Area Beyond } Z_{69}$$

$$\textbf{Shaded Area} = 0.386 - 0.058$$

$$\textbf{Shaded Area} = 0.328$$

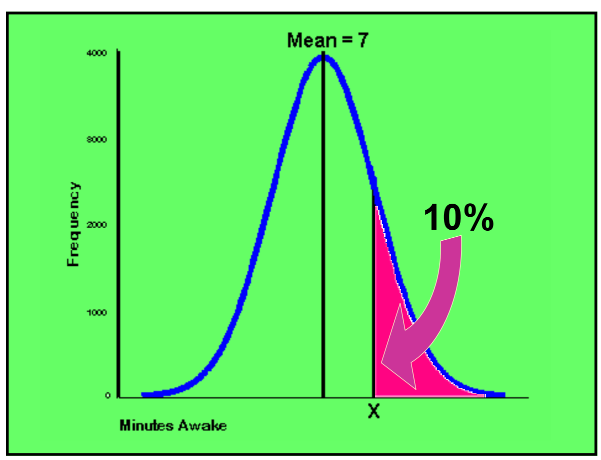
Conclusion

- 32.8% of students received a 2:1.
- We can work out how many people this was by multiplying the proportion by the number of scores collected:

$$130 \times 0.328 = 43 \textbf{ people}$$

Example 4

- The time taken for a lecturer to bore their audience to sleep was measured.
- The average time was 7 minutes, with a standard deviation of 2.
- What is the minimum time that the audience stayed awake for the most interesting 10% of lecturers?



Convert the Proportion to a Z-score

- 10% as a proportion is 0.10.
- Look in column labelled *area beyond z*.
- Find the value 0.10 in that column.
- Read off the corresponding z-score:

$$z = 1.28$$

$$Z = \frac{X - \bar{X}}{s}$$

$$1.28 = \frac{X - 7}{2}$$

$$1.28 \times 2 = X - 7$$

$$X = 2.56 + 7$$

$$X = 9.56$$

Conclusion

- The time that cuts off the most entertaining 10% of lecturers is 9.5 minutes.