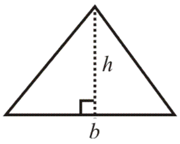
FORM 1 – 3 NOTES

1. SOLID GEOMETRY

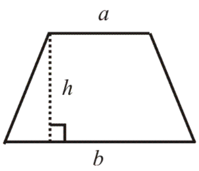
(a) Area and perimeter



Triangle

$A = \frac{1}{2} \times$ base $\times$ height

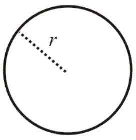
$= \frac{1}{2}\, bh$



Trapezium

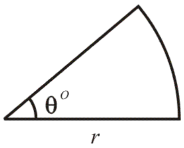
$A = \frac{1}{2}$ (sum of two parallel sides) $\times$ height

$= \frac{1}{2}\,(a + b) \times h$



Circle

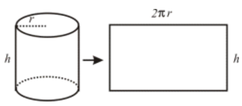
Area $= \pi r^2$
Circumference $= 2\pi r$



Sector

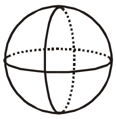
Area of sector $= \frac{\theta}{360} \times \pi r^2$

Length of arc $= \frac{\theta}{360} \times 2\pi r$



Cylinder

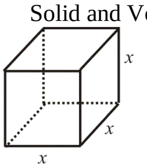
Curve surface area $= 2\pi rh$



Sphere

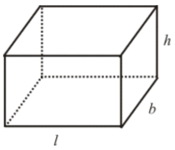
Curve surface area $= 4\pi r^2$

(b) Solid and Volume



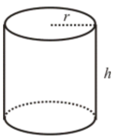
Cube:

V $= x \times x \times x = x^3$



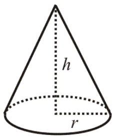
Cuboid:

V $= l \times b \times h$
$= lbh$



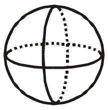
Cylinder

$V = \pi r^2 h$



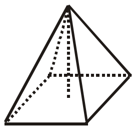
Cone

$V = \frac{1}{3}\pi r^2 h$



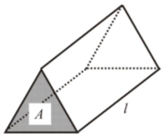
Sphere

$V = \frac{4}{3}\pi r^3$



Pyramid

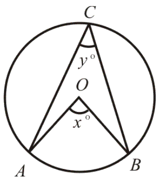
$V = \frac{1}{3} \times$ base area $\times$ height



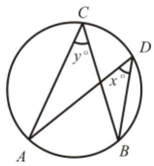
Prism

$V =$ Area of cross section $\times$ length

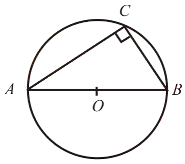
2. CIRCLE THEOREM



Angle at the centre $= 2 \times$ angle at the circumference
$x = 2y$

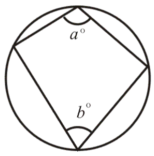


Angles in the same segment are equal
$x = y$



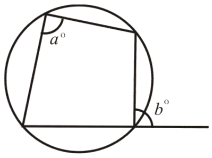
Angle in a semicircle

$\angle ACB = 90^{o}$

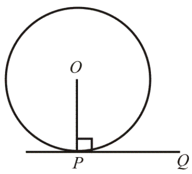


Sum of opposite angles of a cyclic quadrilateral $= 180^{o}$

$a + b = 180^{o}$

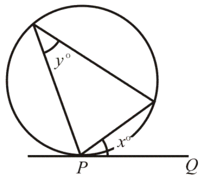


The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.
$b = a$

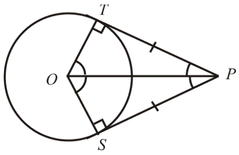


Angle between a tangent and a radius $= 90^{o}$

$\angle OPQ = 90^{o}$



The angle between a tangent and a chord is equal to the angle in the alternate segment.
$x = y$



If PT and PS are tangents to a circle,
$PT = PS$
$\angle TPO = \angle SPO$
$\angle TOP = \angle SOP$

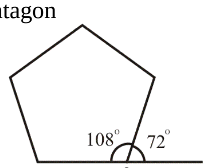
3. **POLYGON**

(a) The sum of the interior angles of a n sided polygon $= (n - 2) \times 180^o$

(b) Sum of exterior angles of a polygon $= 360^o$

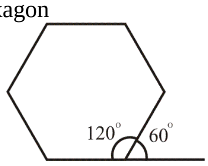
(c) Each exterior angle of a regular n sided polygon $= \dfrac{360^0}{n}$

(d) Regular pentagon



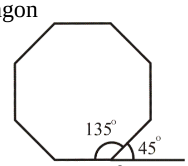
Each exterior angle = 72^o
Each interior angle = 108^o

(e) Regular hexagon



Each exterior angle = 60^o
Each interior angle = 120^o

(f) Regular octagon



Each exterior angle = 45^o
Each interior angle = 135^o

4. **FACTORISATION**

(a) $xy + xz = x(y + z)$

(b) $x^2 - y^2 = (x - y)(x + y)$

(c) $xy + xz + ay + az$
$= x(y + z) + a(y + z)$
$= (y + z)(x + a)$

(d) $x^2 + 4x + 3$
$= (x + 3)(x + 1)$

5. **EXPANSION OF ALGERBRAIC EXPRESSIONS**

(a) $(2x + 1)(x - 3) =$
$2x^2 - 6x + x - 3 = 2x^2 - 5x - 3$

(b) $(x + 3)^2 = x^2 + 2 \times 3 \times x + 3^2$
$= x^2 + 6x + 9$

(c) $(x - y)(x + y) = x^2 + xy - xy - y^2 = x^2 - y^2$

6. **LAW OF INDICES**

(a) $x^m \times x^n = x^{m+n}$

(b) $x^m \div x^n = x^{m-n}$

(c) $(x^m)^n = x^{m \times n}$

(d) $x^{-n} = \dfrac{1}{x^n}$

(e) $x^{\frac{1}{n}} = \sqrt[n]{x}$

(f) $x^{\frac{m}{n}} = (\sqrt[n]{x})^m$

(g) $x^0 = 1$

7. **ALGEBRAIC FRACTION**

Express $\dfrac{1}{2k} - \dfrac{10-k}{6k^2}$ as a fraction in its simplest form.

Solution:

$\dfrac{1}{2k} - \dfrac{10-k}{6k^2} = \dfrac{1 \times 3k - (10-k)}{6k^2}$

$= \dfrac{3k - 10 + k}{6k^2} = \dfrac{4k - 10}{6k^2} = \dfrac{2(k-5)}{6k^2} = \dfrac{k-5}{3k^2}$

8. **LINEAR EQUATION**

Given that $\dfrac{1}{5}(3n + 2) = n - 2$, calculate the value of n.

Solution:

$\dfrac{1}{5}(3n + 2) = n - 2$

$5 \times \dfrac{1}{5}(3n + 2) = 5(n - 2)$

$3n + 2 = 5n - 10$
$2 + 10 = 5n - 3n$ $\quad 2n = 12$ $\quad n = 6$

9. **SIMULTANEOUS LINEAR EQUATIONS**

(a) Substitution Method:
$y = 2x - 5$ --------(1)
$2x + y = 7$ --------(2)
Substitute (1) into (2)
$2x + 2x - 5 = 7$ $\quad 4x = 12$ $\quad x = 3$
Substitute x = 3 into (1), $\quad y = 6 - 5 = 1$

(b) Elimination Method:
Solve:
$3x + 2y = 5$ ----------(1)
$x - 2y = 7$ ----------(2)
(1) + (2), $\quad 4x = 12, \quad x = 3$
Substitute into (1) $\quad 9 + 2y = 5$
$2y = 5 - 9 = -4$

$y = -2$

10. **ALGEBRAIC FORMULAE**

Given that $k - (m + 2) = 3m$, express m in terms of k.

Solution:

$k - (m + 2) = 3m$ $\quad$ $k - m - 2 = 3m$

$k - 2 = 3m + m = 4m$

$m = \dfrac{k-2}{4}$

11. **LINEAR INEQUALITIES**

1. Solve the linear inequality $3x - 2 > 10$.

   Solution:

   $3x - 2 > 10$ $\quad$ $3x > 10 + 2$

   $3x > 12$ $\quad$ $x > 4$

2. List all integer values of x which satisfy the linear inequality $1 \le x + 2 < 4$

   Solution:

   $1 \le x + 2 < 4$

   Subtract 2, $\quad$ $1 - 2 \le x + 2 - 2 < 4 - 2$

   $-1 \le x < 2$

   $\therefore\ x = -1, 0, 1$

3. Solve the simultaneous linear inequalities

   $4p - 3 \le p$ and $p + 2 \ge \dfrac{1}{2}p$

   Solution:

   $4p - 3 \le p$ $\quad$ $4p - p \le 3$ $\quad$ $3p \le 3$

   $p \le 1$

   $p + 2 \ge \dfrac{1}{2}p$ $\quad$ $\times 2,\quad 2p + 4 \ge p$

   $2p - p \ge -4$ $\quad$ $p \ge -4$

   $\therefore$ The solution is $-4 \le p \le 1$.

12. **STATISTICS**

Mean = $\dfrac{\text{sum of data}}{\text{number of data}}$

Mean = $\dfrac{\text{sum of(frequency} \times \text{data)}}{\text{sum of frequency}}$, when the data has frequency.

Mode is the data with the highest frequency

Median is the middle data which is arranged in ascending/descending order.

1. 3, 3, 4, 6, 8

   Mean = $\dfrac{3+3+4+6+8}{5} = 4.8$

   Mode = 3

   Median = 4

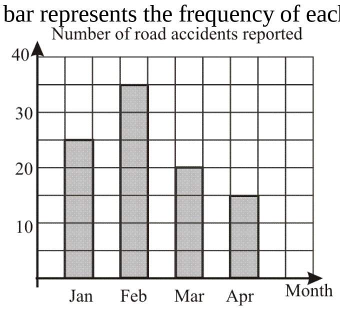
2. 4, 5, 6, 8, 9, 10, there is no middle number, the median is the mean of the two middle numbers.

   Median = $\dfrac{6+8}{2} = 7$

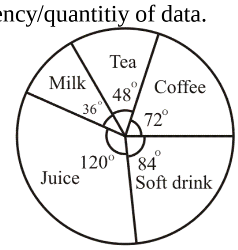
2. A **pictograph** uses symbols to represent a set of data. Each symbol is used to represent certain frequency of the data.

Represents 50 books

3. A **bar chart** uses horizontal or vertical bars to represent a set of data. The length or the height of each bar represents the frequency of each data.



4. A **pie chart** uses the sectors of a circle to represent the frequency/quantitiy of data.



A pie chart showing the favourite drinks of a group of students.

FORM FOUR NOTES

1. **SIGNIFICANT FIGURES AND STANDARD FORM**

<u>Significant Figures</u>

1. Zero in between numbers are significant.
   Example: 3045 (4 significant figures)
2. Zero between whole numbers are not significant figures.
   Example: 4560 (3 significant figures)
3. Zero in front of decimal numbers are not significant.
   Example: 0.00324 (3 significant figures)
4. Zero behind decimal numbers are significant.
   Example: 2.140 (4 significant figures)

<u>Standard Form</u>

Standard form are numbers written in the form $A \times 10^n$, where $1 \le A < 10$ and n are integers.

Example: $340\,000 = 3.4 \times 10^5$

$0.000\,56 = 5.6 \times 10^{-4}$

2. **QUADRATIC EXPRESSION AND QUADRATIC EQUATIONS**

1. Solve quadratic equations by factorization.

   Example: Solve $\dfrac{5k^2 - 8}{3} = 2k$

   $5k^2 - 8 = 6k$ $\quad$ $5k^2 - 6k - 8 = 0$

   $(5k + 4)(k - 2) = 0$

   $k = -\dfrac{4}{5}, 2$

2. Solve qudratic equation by formula:

   Example: Solve $3x^2 - 2x - 2 = 0$

   $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{2 \pm \sqrt{4 - 4(3)(-2)}}{6}$

   $= \dfrac{2 \pm \sqrt{28}}{6}$ $\quad$ $x = 1.215, -0.5486$

3. **SET**

(a) <u>Symbol</u>

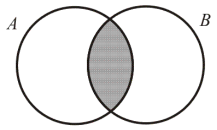
$\cap$ - intersection $\quad$ $\cup$ - union

$\subset$ - subset $\quad$ ξ - universal set

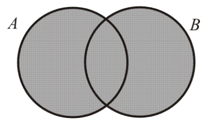
ϕ - empty set $\quad$ $\in$- is a member of

$n(A)$ –number of element in set A.
A' – Complement of set A.

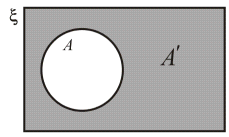
(b) Venn Diagram



$A \cap B$

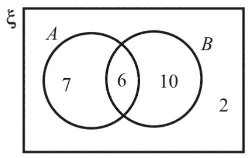


$A \cup B$



A'

Example:



$n(A) = 7 + 6 = 13$
$n(B) = 6 + 10 = 16$
$n(A \cap B) = 6$
$n(A \cup B) = 7 + 6 + 10 = 23$
$n(A \cap B') = 7$
$n(A' \cap B) = 10$
$n(A \cap B)' = 7 + 10 + 2 = 19$
$n(A \cup B)' = 2$

4. **MATHEMATICAL REASONING**

(a) Statement
A mathematical sentence which is either true or false but not both.

(b) Implication
If a, then b
a – antecedent
b – consequent

'p if and only if q' can be written in two implications:
If p, then q
If q, then p

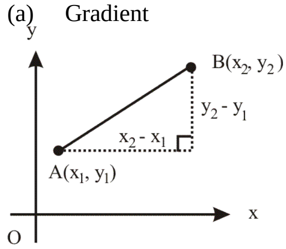
(c) Argument
Three types of argument:
Type I
Premise 1: All A are B
Premise 2 : C is A
Conclusion: C is B

Type II
Premise 1: If A, then B
Premise 2: A is true
Conclusion: B is true.

Type III
Premise 1: If A, then B
Premise 2: Not B is true.
Conclusion: Not A is true.

5. **THE STRAIGHT LINE**

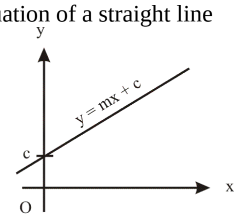
(a) Gradient



Gradient of AB =

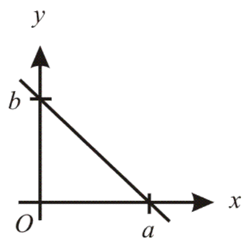
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

(b) Equation of a straight line



Gradient Form:
$y = mx + c$

m = gradient
c = y-intercept



Intercept Form:

$$\frac{x}{a} + \frac{y}{b} = 1$$

a = x–intercept
b = y–intercept

Gradient of straight line $m = -\dfrac{y\text{-intercept}}{x\text{-intercept}}$

$$= -\frac{b}{a}$$

6. **STATISTICS**

(a) Class, Modal Class, Class Interval Size, Midpoint, Cumulative frequency, Ogive
Example :
The table below shows the time taken by 80 students to type a document.

Time (min)	Frequency
10-14	1
15-19	7

20-24	12
25-29	21
30-34	19
35-39	12
40-44	6
45-49	2

For the class 10 – 14 :
Lower limit = 10 min
Upper limit = 14 min

Lower boundary = 9.5 min
Upper boundary = 14.5 min

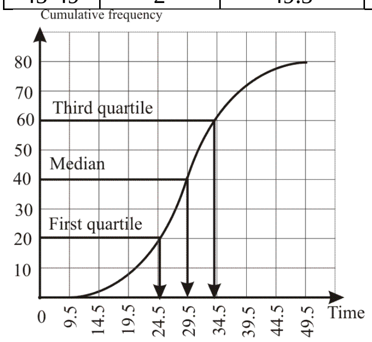
Class interval size = Upper boundary – lower boundary = 14.5 – 9.5 = 5 min

Modal class = 25 – 29 min

Midpoint of modal class = $\frac{25+29}{2}$ = 27

To draw an ogive, a table of upper boundary and cumulative frequency has to be constructed.

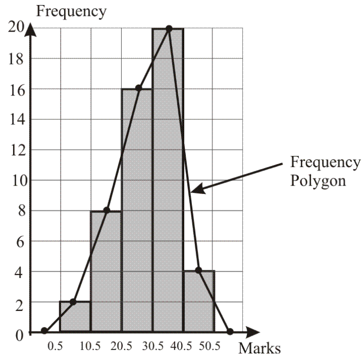
Time (min)	Frequency	Upper boundary	Cumulative frequency
5-9	0	9.5	0
10-14	1	14.5	1
15-19	7	19.5	8
20-24	12	24.5	20
25-29	21	29.5	42
30-34	19	34.5	60
35-39	12	39.5	72
40-44	6	44.5	78
45-49	2	49.5	80



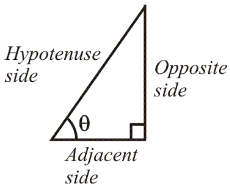
From the ogive :
Median = 29.5 min
First quartile = 24. 5 min
Third quartile = 34 min
Interquartile range = 34 – 24. 5 = 9.5 min.

(b) Histogram, Frequency Polygon
Example:
The table shows the marks obtained by a group of students in a test.

Marks	Frequency
1 – 10	2
11 – 20	8
21 – 30	16
31 – 40	20
41 – 50	4



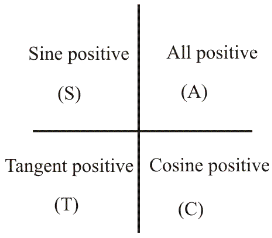
7. TRIGONOMETRY



$\sin\ \theta^o = \frac{\text{Opposite}}{\text{hypotenuse}} = \frac{AB}{AC}$

$\cos\ \theta^o = \frac{adjacent}{hypotenuse} = \frac{BC}{AC}$

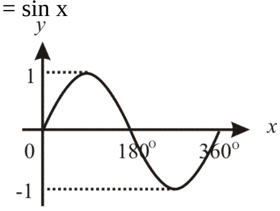
$\tan\ \theta^o = \frac{\text{opposite}}{\text{adjacent}} = \frac{AB}{BC}$



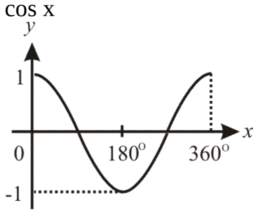
Acronym: "Add Sugar To Coffee"

Trigonometric Graphs

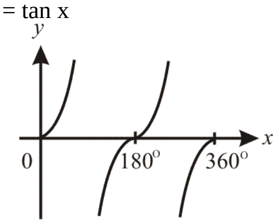
1. y = sin x



2. y = cos x

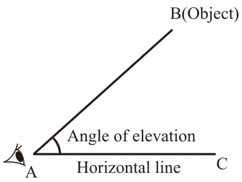


3. y = tan x



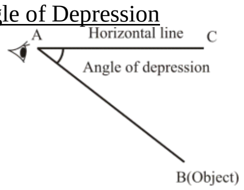
8. ANGLE OF ELEVATION AND DEPRESSION

(a) Angle of Elevation



The angle of elevation is the angle betweeen the horizontal line drawn from the eye of an observer and the line joining the eye of the observer to an object which is higher than the observer.
The angle of elevation of *B* from *A* is $\angle BAC$

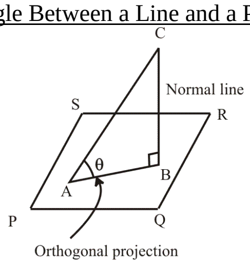
(b) Angle of Depression



The angle of depression is the angle between the horizontal line from the eye of the observer an the line joining the eye of the observer to an object which is lower than the observer.
The angle of depression of *B* from *A* is $\angle BAC$.

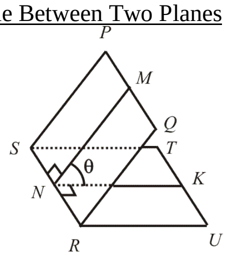
9. **LINES AND PLANES**

(a) Angle Between a Line and a Plane



In the diagram,
(a) *BC* is the normal line to the plane *PQRS*.
(b) *AB* is the orthogonal projection of the line *AC* to the plane *PQRS*.
(c) The angle between the line *AC* and the plane *PQRS* is $\angle BAC$

(b) Angle Between Two Planes



In the diagram,
(a) The plane *PQRS* and the plane *TURS* intersects at the line *RS*.
(b) *MN* and *KN* are any two lines drawn on each plane which are perpendicular to *RS* and intersect at the point *N*.

The angle between the plane *PQRS* and the plane *TURS* is $\angle MNK$.

FORM 5 NOTES

10. **NUMBER BASES**

(a) Convert number in base 10 to a number in base 2, 5 or 8.
Method: Repeated division.
Example:

$$\begin{array}{r|l l}
2 & 34 & \\
2 & 17 & — 0 \\
2 & 8 & — 1 \\
2 & 4 & — 0 \\
2 & 2 & — 0 \\
2 & 1 & — 0 \\
 & 0 & — 1
\end{array} \uparrow$$

$34_{10} = 100010_2$

$$\begin{array}{r|l l}
8 & 34 & \\
8 & 4 & — 2 \\
 & 0 & — 4
\end{array} \uparrow$$

$34_{10} = 42_8$

(b) Convert number in base 2, 5, 8 to number in base 10.
Method: By using place value
Example: (a) $11011_2 =$
$2^4\ 2^3\ 2^2\ 2^1 1$
$1\ 1\ 0\ 1\ 1_2$
$= 2^4 + 2^3 + 2^1 + 1$
$= 27_{10}$
(b) $214_5 =$
$5^2\ 5^1\ 1$
$2\ 1\ 4_5$
$= 2 \times 5^2 + 1 \times 5^1 + 4 \times 1$
$= 59_{10}$

(c) Convert between numbers in base 2, 5 and 8.
Method: Number in base m $\rightarrow$ Number in base 10 $\rightarrow$ Number in base n.
Example: Convert 110011_2 to number in base 5.

$2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 1$
$1\ 1\ 0\ 0\ 1\ 1_2$
$= 2^5 + 2^4 + 2 + 1$
$= 51_{10}$

$$\begin{array}{r|l l}
5 & 51 & \\
5 & 10 & 1 \\
5 & 2 & 0 \\
 & 0 & 2
\end{array} \uparrow$$

Therefore, $110011_2 = 201_5$

(d) Convert number in base two to number in base eight and vice versa.
Using a conversion table

Base 2	Base 8
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

Example :

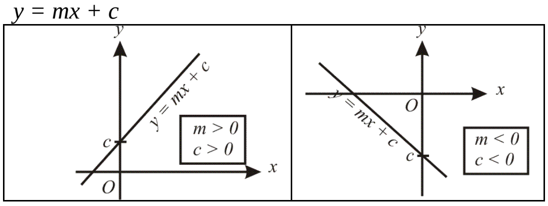
$10\ 011_2 = 23_8$

$45_8 =$ 100 101_2

11. GRAPHS OF FUNCTIONS

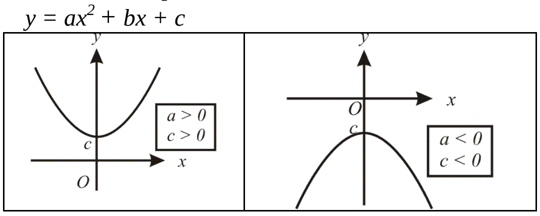
(a) Linear Graph

$y = mx + c$



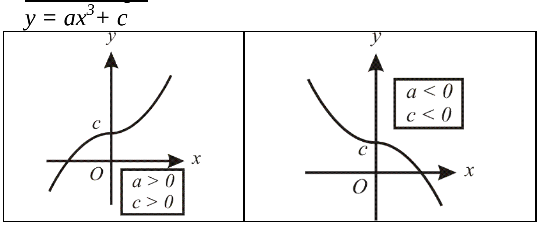
(b) Quadratic Graph

$y = ax^2 + bx + c$



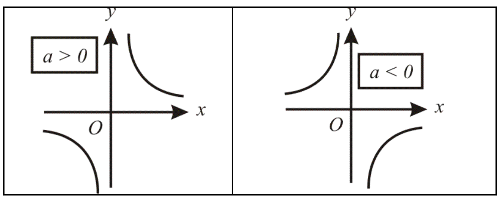
(c) Cubic Graph

$y = ax^3 + c$



(d) Reciprocal Graph

$$y = \frac{a}{x}$$

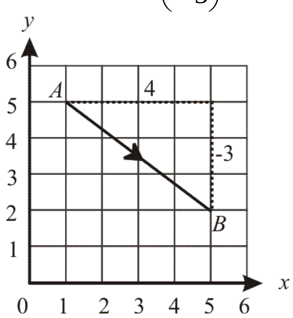


12. TRANSFORMATION

(a) Translastion

Description: Translastion $\begin{pmatrix} h \\ k \end{pmatrix}$

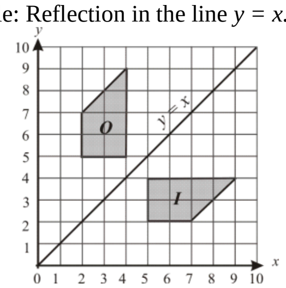
Example : Translastion $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$



(b) Reflection

Description: Reflection in the line ___________

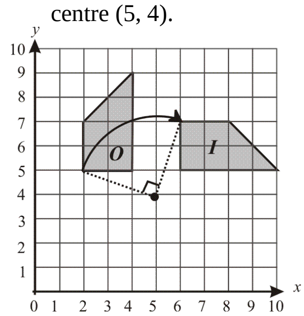
Example: Reflection in the line $y = x$.



(c) Rotation

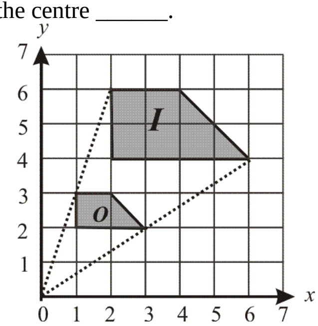
Description: Direction ______rotation of angle______about the centre _______.

Example: A clockwise rotation of 90^{o} about the centre (5, 4).



(d) Enlargement

Description: Enlargement of scale factor ______, with the centre ______.



Example : Enlargement of scale factor 2 with the centre at the origin.

$$\frac{\text{Area of image}}{\text{Area of object}} = k^2$$

k = scale factor

(e) Combined Transformtions

Transformation V followed by transformation W is written as WV.

13. MATRICES

(a) $\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a+c \\ b+d \end{pmatrix}$

(b) $k\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} ka \\ kb \end{pmatrix}$

(c) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}$

(d) If $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then

$M^{-1} = \dfrac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

(e) If $ax + by = h$
$cx + dy = k$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} h \\ k \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \dfrac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\begin{pmatrix} h \\ k \end{pmatrix}$

(f) Matrix $\begin{pmatrix} a & c \\ b & d \end{pmatrix}$ has no inverse if ad – bc = 0

14. VARIATIONS

(a) Direct Variation
If y varies directly as x,
Writtn in mathematical form: $y \propto x$,
Written in equation form: $y = kx$, k is a constant.

(b) Inverse Variation
If y varies inversely as x,

Written in mathematical form: $y \propto \dfrac{1}{x}$

Written in equation form: $y = \dfrac{k}{x}$, k is a constant.

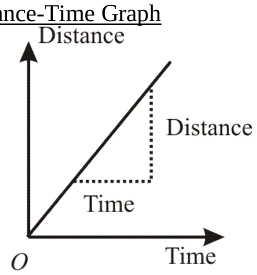
(c) Joint Variation
If y varies directly as x and inversely as z,

Written in mathematical form: $y \propto \dfrac{x}{z}$,

Written in equation form: $y = \dfrac{kx}{z}$, k is a constant.

15. GRADIENT AND AREA UNDER A GRAPH

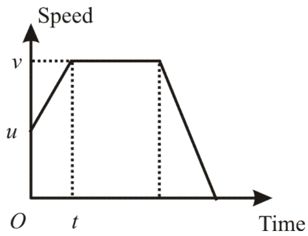
(a) Distance-Time Graph



Gradient = $\dfrac{\text{distance}}{\text{time}}$ = speed

Average speed = $\dfrac{\text{Total distance}}{\text{Total time}}$

(b) Speed-Time Graph



Gradient = Rate of change of speed
$= \dfrac{v-u}{t}$
= acceleration

Distance = Area below speed-time graph

16. PROBABILITY

(a) Definition of Probability
Probability that event A happen,

$P(A) = \dfrac{n(A)}{n(S)}$

S = sample space

(b) Complementary Event
$P(A') = 1 - P(A)$

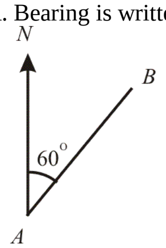
(c) Probability of Combined Events

(i) $P(A \text{ or } B) = P(A \cup B)$

(ii) $P(A \text{ and } B) = P(A \cap B)$

17. BEARING

Bearing
Bearing of point B from A is the angle measured clockwise from the north direction at A to the line joining B to A. Bearing is written in 3 digits.



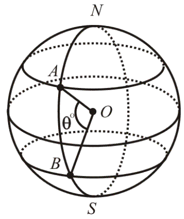
Example : Bearing B from A is 060°

18. THE EARTH AS A SPHERE

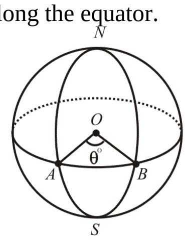
(a) Nautical Miles
1 nautical mile is the length of the arc on a great circle which subtends an angle of 1′ at the centre of the earth.

(b) Distance Between Two Points on a Great Circle.

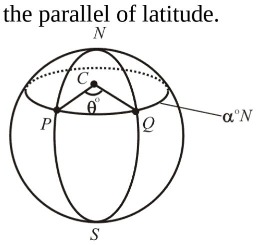
Distance = $\theta \times 60$ nautical miles
θ = angle between the parallels of latitude measured along a meridian of longitude.



θ = angle between the meridians of longitude measured along the equator.



(c) Distance Between Two Points on The Parallel of Latitude.
Distance = $\theta \times 60 \times \cos\ \alpha^{o}$
α = angle of the parallel of latitude.

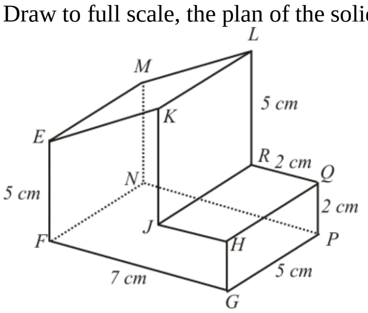


(d) Shortest Distance
The shortest distance between two points on the surface of the earth is the distance between the two points measured along a great circle.

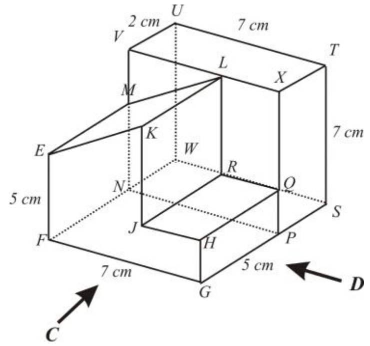
(e) Knot
1 knot = 1 nautical mile per hour.

19. **PLAN AND ELEVATION**

(a) The diagram shows a solid right prism with rectangular base FGPN on a horizontal table. The surface EFGHJK is the uniform cross section. The rectangular surface EKLM is a slanting plane. Rectangle JHQR is a horizontal plane. The edges EF, KJ and HG are vertical.
Draw to full scale, the plan of the solid.

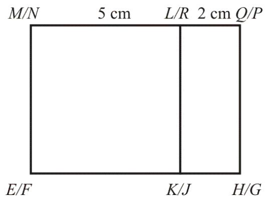


(b) A solid in the form of a cuboid is joined to the solid in (a) at the plane PQRLMN to form a combined solid as shown in the diagram. The square base FGSW is a horizontal plane.
Draw to full scale
(i) the elevation of the combined solid on the vertical plane parallel to FG as viewed from C,
(ii) the elevation of the combined solid on the vertical plane parallel to GPS as viewed from D.



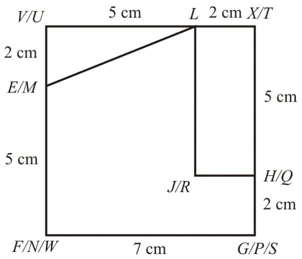
Solution:

(a)



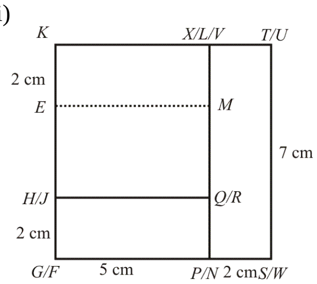
Plan

(b) (i)



C-elevation

(ii)



D-elevation