# Math ACT topics (Note: not all topics appear on every test)

## Pre-Algebra (14 questions)

- Order of operations (PEMDAS)
- Place value
- Write generic consecutive integers as x and x+1
- Whole numbers, decimals, fractions, integers
- Odd/even, positive/negative, rational/irrational
- Adding fractions by finding least common denominator
- Multiples, factors, and prime
- Percent, fractions, and decimals
- Ratios and proportions (set up and solve)
- Averages: mean, median, and mode (almost always mean)
- Probability
- Percent
- Fundamental counting principle

## Algebra (19 questions)

- Evaluate algebraic expressions using substitution
- Combining like terms
- Distributing
- Solve equations
- Multiply binomials (FOIL)
- Solve quadratic equations by factoring (usually with special products)
- Solve inequalities, including compound inequalities
- Properties of exponents
- Roots
- Simplify radicals
- Add radicals
- Use variables to write and solve equations (especially total cost = fixed cost + rate*number)
- Write and solve systems of equations (typically using substitution method or setting 2 total cost equations equal to each other and solving)
- Use the quadratic formula (though can often be done by factoring instead)
- Absolute value equations (and occasionally absolute value inequalities)
- Arithmetic and geometric series (must know formulas for terms and sums, but not on most tests)
- Imaginary and complex numbers
- Functions
- Matrices
- Logarithms

## Coordinate Geometry (9 questions)

- number lines, and (x,y) coordinate plane
- Interpret graphs using slope-intercept (y = mx + b)
- Write equations in slope-intercept form starting from: an equation in standard form, a graph, a sentence, a point and a slope, or 2 points
- Find slope from: equation, graph, 2 points, parallel line, perpendicular line
- Distance and midpoint formulas – you must memorize these, and they are on every test at least once.
- Graph inequalities
- Formula for a circle on coordinate plane

## Plane Geometry 14 questions)

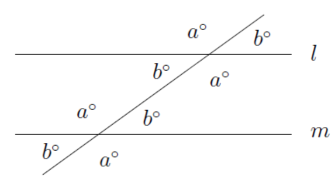
- Angles and lines
- Triangles
- Pythagorean theorem (with special triangles and Pythagorean triples)
- Perimeter and area
- Polygons
- Properties of: Circles, triangles, rectangles, parallelograms, and trapezoids
- Volume and surface area
- Transformations (reflections and rotations)
- Proofs (choose valid reason or conclusion)

## Trigonometry (4 questions)

- Trigonometric functions (especially Soh-Cah-Toa)

## Topics that ARE on EVERY test

- Proportions (set up and solve) – appear multiple times on every test
- % - often appears multiple times
- Average (mean)
- Fundamental counting principle
- Write and solve rate equations (total cost = fixed cost +(rate x number) )
- Solve equation with x on both sides, along with distributing and/or combining like terms
- Rearrange an equation to solve for one variable in terms of other variables
- Multiplication property of exponents, occasionally division or power properties instead
- Multiply binomials with FOIL, or $(2x-3)^2$ type with special patterns
- Solve quadratic equation by factoring (almost always a special product type)
- Solve a square root equation to get 2 answers
- Write and solve system of equations (usually set equal to each other to solve, or use substitution)
- Rearrange formula (usually from standard form) to y = mx + b, then find slope (and/or y-intercept)
- Midpoint formula
- Distance formula
- Triangles total 180°, and straight lines are 180°
- Pythagorean triples
- Special triangles: 30-60-90, and 45-45-90 (isosceles)
- Soh-Cah-Toa - usually multiple times
- Logarithm (though exact question type varies)



Parallel Lines ($l \parallel m$)

**Other tips**

- Draw pictures and label them. Label figures, "a picture is worth a thousand words."
- Read the entire question. Don't give the length of side x if the problem is asking for perimeter.
- I have never seen "cannot be determined from the given information" be the correct answer.
- Use parentheses when substituting.
- Use your calculater wisely. Most questions are answered more quickly and **without** a calculator.
- Figures are not necessarily to scale, but they often are. Use this fact if you are really stuck.
- If solving is too complicated, try plugging in the answers until you find the correct one.
- Plug in convenient numbers for letters (variables) to make a problem more concrete. This strategy is helpful for questions like "how does the volume of a cube change if the side length doubles?"
- Turn words into drawings and/or equations.

# ACT Math Formulas and Review

## Numbers, Sequences, Factors

**Real numbers:** positive and negative integers, rational numbers, and irrational numbers (everything except imaginary and complex numbers)

**Rational numbers:** a number that can be expressed as a fraction or ratio. The numerator and the denominator of the fraction are both integers. When the fraction is divided out, it becomes a terminating or repeating decimal. (The repeating decimal portion may be one number or a billion numbers.)

**Irrational numbers**: cannot be expressed as a fraction. Irrational numbers cannot be represented as terminating or repeating decimals. Irrational numbers are non-terminating, non-repeating decimals. Examples: $\sqrt{2}$ , $\sqrt{3}$ , and $\pi$

**Integer:** a number with no fractional part. Includes positive and negative counting numbers and zero
. . . , -3, -2, -1, 0, 1, 2, 3 . . .
**Whole number:** positive integers and zero

### Imaginary and Complex Numbers

The imaginary number, *i,* equals the square root of negative one.
*A* complex number is of the form a + b*i* When multiplying complex numbers, treat i just like any other variable, except remember to replace powers of i with −1 or 1 as follows:

| | | | |
|---|---|---|---|
| $i^0 = 1$ | $i^1 = i$ | $i^2 = -1$ | $i^3 = -i$ (= −1i) |
| $i^4 = 1$ | $i^5 = i$ | $i^6 = -1$ | $i^7 = -i$ (keeps repeating every 4th exponent) |

**Arithmetic Sequences:** each term is equal to the previous term plus a constant ("common difference") d

- Sometimes you can see the pattern and just follow it; occasionally you will need the formulas.

| | |
|---|---|
| term position<br>$a_n = a_1 + (n-1)d$<br>$n^{th}$ term; first term; common difference | To find the **sum** of a certain number of terms of an arithmetic<br>$S_n = \dfrac{n(a_1 + a_n)}{2}$ |

**Geometric Sequences:** each term is equal to the previous term times r

| | |
|---|---|
| N[th] Term of a Geometric Sequence<br>$a_n = a_1 r^{(n-1)}$<br>$a_1 = a_1 r^0 \quad a_2 = a_1 r^1 \quad a_3 = a_1 r^2, \quad a_n = a_1 r^{n-1}$ | The Sum of a Finite Geometric Sequence<br>$S_n = a_1\left(\dfrac{1-r^n}{1-r}\right)$ $a_1$ = First term of the sequence; $S_n$ = Sum of the first n terms<br>$r \neq 1$ |

**Factors:** the factors of a number divide into that number without a remainder
Example: the factors of 52 are 1, 2, 4, 13, 26, and 52
**Multiples:** the multiples of a number are divisible by that number without a remainder
Example: the positive multiples of 20 are 20, 40, 60, 80, . . .

**Percents:** use the following formula to find part, whole, or percent

$$part = \frac{percent}{100} \times whole$$

## Ratios and proportions

In problems involving proportions, be sure to keep the identical units in the numerators and denominators of the fractions in the proportion. Cross-multiply to solve

**Ex:** A car travels 176 miles on 8 gallons of gas. How far can it go on a tankful of gas if the tank holds 14 gallons?

$\frac{176 \text{ miles}}{8 \text{ gallons}} = \frac{x \text{ miles}}{14 \text{ gallons}}$ Solve:

$$\frac{176}{8} = \frac{x}{14}$$
$$8x = 14 \cdot 176$$
$$8x = 2464$$
$$\frac{8x}{8} = \frac{2464}{8}$$
$$x = 308 \text{ miles}$$

Hence, on 14 gallons, the car n travel a distance of 308 miles.

> ***Math Note:*** *Notice that the numerators of the proportions have the same units, miles, and the denominators have the same units, gallons.*

**Turn % into degrees for circle graph**

- Set up a proportion: $\frac{percent}{100} = \frac{\deg rees}{360}$ , then cross multiply.

**Equations with cross multiplying:** This type of equation appears on about 1/3 of the tests:

"What number added to the numerator and denominator of $\frac{7}{9}$ equals $\frac{1}{2}$?"

Set up as $\frac{7+x}{9+x} = \frac{1}{2}$ , then cross multiply and solve

## Averages, Counting, Statistics, Probability

**average (mean)=** $mean = \frac{sum\ of\ terms}{number\ of\ terms}$

mode = value in the list that appears most often

median = middle value in the list

median of {3, 9, 10, 27, 50} = 10

median of {3, 9, 10, 27} = (9 + 10)/2 = 9.5

> **Typical question:** A student has earned the following scores on four 100-point tests this marking period: 63, 72, 88, and 91. What score must the student earn on the fifth and final 100-point test of the marking period to earn an average test grade of 80 for the five tests?

## Fundamental Counting Principle:

If an event can happen in N ways, and another, independent event can happen in M ways, then both events together can happen in N ×M ways. (Extend this for three or more: $N_1 \times N_2 \times N_3 \ldots$ )

**Ex:** Nancy has 4 pairs of shoes, 5 pairs of pants, and 6 shirts. How many different outfits can she make?

| Shoes | Pants | Shirts |
|---|---|---|
| 4 choices | 5 choices | 6 choices |

To find the number of possible outfits, multiply the number of choices for each item.

4 × 5 × 6 = 120 she can make 120 different outfits.

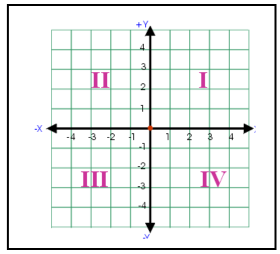
## Probabilty

$$probability = \frac{number\ of\ desired\ outcomes}{number\ of\ total\ possible\ outcomes}$$

**Helpful Hints about Probability**

- The probability of two different events A and B both happening is P(A and B) = P(A) * P(B), as long as the events are independent (the second outcome does not depend on the first outcome)
- If you know the probability of all other events occurring, you can find the probability of the remaining event by adding the known probabilities together and subtracting that sum from 1.

**Quadrants:** Q I x & y both positive
QII x is negative, y is positive
Q III x & y are both negative
QIV x is positive, y is negative




## Powers, exponents, and roots

$x^a \cdot x^b = x^{a+b}$ When multiplying powers, add the exponents

$(x^a)^b = x^{ab}$ power of a power, multiply the exponents

$x^0 = 1$ anything to the zero power equals one

$\frac{x^a}{x^b} = x^{a-b}$ when dividing powers, subtract the exponents

$(xy)^2 = x^2 y^2$ every item in the parentheses is to the power on the parentheses

$\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$

$x^{-b} = \frac{1}{x^b}$

$(-1)^n = +1$, if n is even; $(-1)^n = -1$, if n is odd.
Any number to an even power is positive; a negative to an odd power is negative.

- Radicals with the same radicand (number under the radical symbol) can be combined the same way "like terms" are combined. *Example* $2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$
- When solving equations with exponents, remember to have (or get) the same base.

**Have the perfect squares of numbers from 1 to 13 memorized** since they frequently come up in all types of math problems. The **perfect squares** (in order) are: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169

## Simplify Radicals:

**Example 1** - simplify $\sqrt{8}$

**Step 1)** Find the largest perfect square that is a factor of the radicand

4 is the largest perfect square that is a factor of 8

**Step 2)** Rewrite the radical as a product of the square root of 4 (found in last step) and its matching factor(2)

$\sqrt{8} = \sqrt{4}\sqrt{2}$

**Step 3)** Simplify

$\sqrt{8} = 2\sqrt{2}$

**Example 2** – Simplify $\sqrt{252}$ Using "factor tree" method

| | |
|---|---|
| **Step 1**: Find the prime factorization ("factor tree") of the number inside the radical. | $252 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 7$ |
| **Step 2**: Determine the index of the radical. In this case, the index is two because it is a square root, which means we need two of a kind. | $\sqrt{252} = \sqrt{\underline{2 \cdot 2} \cdot \underline{3 \cdot 3} \cdot 7}$ |
| **Step 3**: Move each group of numbers or variables from inside the radical to outside the radical. In this case, the pair of 2's and 3's moved outside the radical. | $2 \cdot 3\sqrt{7}$ |
| **Step 4**: Simplify the expressions both inside and outside the radical by multiplying. | $6\sqrt{7}$ |

## Equations

- Solve equations with x on both sides, along with distributing and/or combining like terms.
- Solve an equation containing multiple variables in terms of one variable
    - Circle the variable you are trying to isolate, then solve equation by treating remaining variables the same way you would treat a number.
- Write rate equations: total = fixed amoung + (rate x number)

**Equations with infinite or no solutions** (single equations, systems of equations, and inequalities)

- Equations (as well as systems of equations, and inequalities) can possible have no solution, or an infinite number of solutions. When simplifying the equation, the variable(s) disappear from **BOTH** sides.
    - If the remaining statement is false (ex:3 = 7, or 2 > 5), then there are NO solutions (null set)
    - If the statement is true (ex: 6=6, or 9 >4), then there are an infinite number of solutions

## Multiply binomials, Factor, Solve quadratic equations

$(x + a)(x + b) = x^2 + bx + ax + ab = x^2 + (b + a)x + ab$ "FOIL"

$x^2 + (b + a)x + ab = (x + a)(x + b)$ "Factoring"

- Remove GCF (greatest common factor) if possible before factoring
- You can factor a polynomial by thinking about two numbers, a and b, which add to the number in front of the x, and which multiply to give the constant.

## Special Patterns: (Virtually any factoring needed on ACT is one of the special patterns)

$a^2 - b^2 = (a + b)(a - b)$ "Difference Of Squares" [ Remember $x^{16} = (x^8)^2$ ]

$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$ "Perfect Square"

$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$ "Perfect Square"

$(a-b)^3$ may appear, but you will likely be able to find a numerical value for (a-b), and then just cube that value.

## To solve a quadratic equation ($x^2 + bx + c = 0$) by factoring,

- Rearrange to get zero on one side of the equation, then factor
- Set each term (parentheses, and any variable factored out as a GCF) equal to zero, and solve for the variable.
- Solving two linear equations in x and y is geometrically the same as finding where two lines intersect.
- Solve using substitution or elimination (linear combinations). The ACT usually makes substitution the easier choice.
- Two parallel lines will have no solution, and two overlapping lines will have an infinite number of solutions.

## Absolute Value

Recall that both $|9| = 9$ and $|-9| = 9$

You must set up 2 equations to find the value of x that would produce a positive value inside the absolute value sign, and the value of the value of x that would produce a negative value inside the absolute value sign.

$|x + 4| = 9$

$x + 4 = 9$ or $x + 4 = -9$

$x = 5$ or $x = -13$

**solve absolute value inequalities**

- When working with any absolute value inequality, you must create two cases.
    - Case #1: Write the problem without the absolute value sign, and solve the inequality.
    - Case#2: Write the problem without the absolute value sign, reverse the inequality, negate the value NOT inside the absolute value, and solve the inequality.
    - $|x| < a$ becomes the 2 cases: $x < a$ , $x > -a$ connected by either "and' or "or"

- If < or $\leq$, the connecting word is "and".
    - If $a > 0$, then the solutions to $|x| < a$ are ***x < a* and *x >*** - Also written: ***- a < x < a.***
    - If $a < 0$, there is **no solution** to $|x| < a$ . **Think about it:** absolute value is always positive (or zero), so, of course, it cannot be less than a negative number.
- If > or $\geq$, the connecting word is "or".
    - If $a > 0$ (positive), then the solutions to $|x| > a$ are ***x > a* or *x < - a***
    - If $a < 0$, **all real numbers** will satisfy $|x| > a$. **Think about it:** absolute value is always positive (or zero), so, of course, it is greater than any negative number.

## Logarithms

Logarithms are basically the inverse functions of exponentials. The function log x answers the question: b to what power gives x? Here, b is called the logarithmic "base".
So, if $y = \log_b x$, then the logarithm function gives the number y such that $b^y = x$.

**Log Rules:**

$log_b(m \times n) = log_b(m) + log_b(n)$ Multiplication inside the log can be turned into addition outside the log

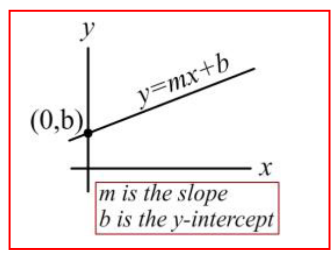
$log_b(^m/_n) = log_b(m) - log_b(n)$ Division inside the log can be turned into subtraction outside the log

$log_b(m^n) = n \cdot log_b(m)$ An exponent on everything inside a log can be moved out front as a multiplier

**Warning:** Just as when you're dealing with exponents, the above rules work *only* if the bases are the same. For instance, the expression "$log_d(m) + log_b(n)$" cannot be simplified, because the bases (the "d" and the "b") are not the same, just as $x^2 \times y^3$ cannot be simplified (because the bases *x* and *y* are not the same).

1. $\log_b 1 = 0$.
2. $\log_b b = 1$.
3. $\log_b b^2 = 2$.
4. $\log_b b^x = x$.
5. $b^{\log_b x} = x$. **The Logarithmic Function can be "undone" by the Exponential Function.**
6. $\log_a b = 1/\log_b a$. (reciprocal property)
7. **Change of base** $\log_b x = \log_a x / \log_a b$.

## Lines (Linear Functions)

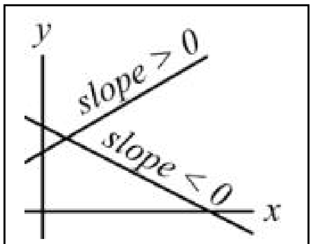



**Steps to graph a linear equation in slope-intercept form:**

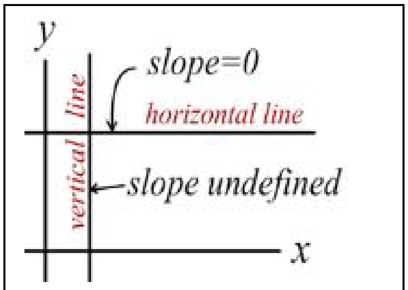
1. Write the equation in slope-intercept form y = mx + b ("solve for y"). Be sure you are **adding b**
   (if you get y = 4x – 3, be sure to change it to **y = 4x + –3**)!!!!!
2. Plot the y-intercept, which is the point (0,b).
3. **Use the slope, m,** to find a second point.
    - Write the slope as a fraction if it is not already a fraction. Ex. change 4 to $\frac{4}{1} = \frac{rise}{run}$
    - **From the y-intercept,** go up the amount of the rise, and over the amount of the run (right for positive, left for negative). This is your second point. Continue this until you run out of room on the graph. Connect the points with a line.

- Slope-intercept form: **y = mx + b** The slope = m, and the y-intercept = b
    - Equations MUST be rearranged into slope-intercept form in order to determine the slope and y-intercept. (This is typically on each ACT multiple times.)
    - **Y-intercept represents value at time = 0**

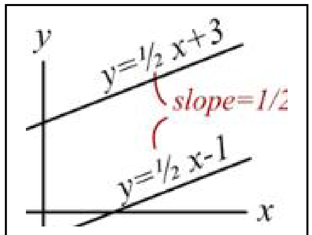
- To find the equation of the line given two points A($x_1$, $y_1$) and B($x_2$, $y_2$), calculate the slope, then use the point-slope form, and finally rearrange into slope-intercept form.
  - **Slope Formula** $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ OR $m = \dfrac{\Delta y}{\Delta x}$ OR $m = \dfrac{change\ in\ y}{change\ in\ x}$ OR $m = \dfrac{rise}{run}$
  - **Point-slope form**: given the slope m and a point (x1, y1) on the line, the equation of the line is **$(y - y_1) = m(x - x_1)$**



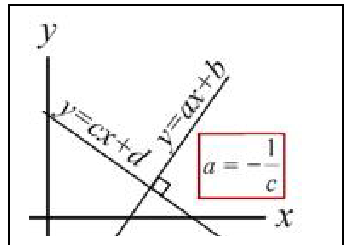

- Positive slope indicates line that rises left to right
- Negative slope indicates line that falls left to right




- Zero slope indicates a horizontal line (equation is y=a constant)
- Undefined slope indicates a vertical line (equation is x = a constant)
  - A line with an undefined slope is NOT a function




- Parallel lines have equal slopes. They will never intersect – there is **no solution** to a system of equations representing parallel lines.




- Perpendicular lines (i.e., those that make a 90° angle where they intersect) have negative reciprocal slopes:
- **$m_1 * m_2 = -1$. Or** $m_2 = \dfrac{-1}{m_1}$

## Shapes of Graphs

- x is to first power (y = mx + b) : linear equation, graph is a straight line
- x is to second power ($y = ax^2 + bx + c$) : quadratic equation, graph is a parabola
  - If a is positive, the graph opens up, if a is negative, the graph opens down
  - C is the y-intercept
- x is in the denominator (y = 1/x) : graph is a curve

### Graphing Inequalities

1. Put into slope-intercept form, and graph the same as if the inequality was an equals sign
   a. If the inequality was a > or < , connect points with a dashed line
   b. If the inequality was a $\geq$ or $\leq$, connect points with a solid line
2. If the inequality was y < mx+b (or y $\leq$ mx+b), shade **under** the line.
   If the inequality was y > mx+b (or y $\geq$ mx+b), shade **above** the line.
3. When graphing compound inequalities:
   a. For "AND," (3x + 7 < y < -2x + 4) the solution is only the region shaded by BOTH inequalities
   b. For "OR," the solution is all of the area shaded

**Solve inequality:** solving an inequality is exactly like solving an equation EXCEPT that you must reverse the inequality sign when multiplying or dividing by a negative.

**Solve compound inequalities**

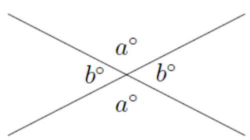
- For compound inequalities with the word "or," just solve each inequality separately, then put the word "or" between the two answers.
- For compound inequalities with the word "and," (or written -3 < 3x – 9 < 12), get the x alone in the middle. Whatever you need to do to the middle section, do the same thing to all three sections.
- Remember to reverse the inequality sign when multiplying or dividing by a negative.

- **Midpoint formula** (just the average of the x-coordinates, and the average of the y-coordinates)

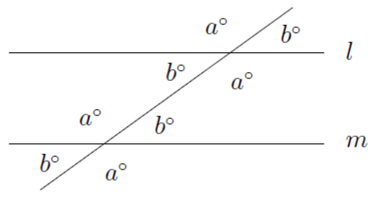
  Mid-point of the segment $\overline{AB}$ : $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

- **Distance formula:** distance from A to B $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

## Angles and Triangles




Intersecting Lines

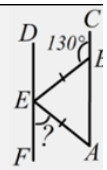



Parallel Lines ($l \parallel m$)

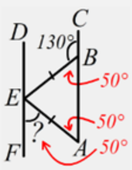
- **Intersecting lines:** opposite angles are equal. Each pair of angles along the same line form a straight angle, so they add to 180°.
  - a + b = 180°
- **Parallel lines:** The angles (a) are equal to each other, and the angles (b) are equal to each other.
  - a + b = 180°
- There are **MULTIPLE** questions on each test using these relationships.

These are two typical questions using the parallel lines diagram

*Ex. Given AC is parallel to DF, AE is congruent to EB, and $\angle CBE$ is 130°, as shown, find $\angle AEF$ .*

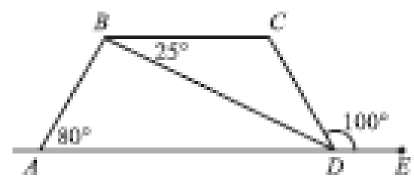



$\angle ABE = 50°$ since it is supplementary to 130°.

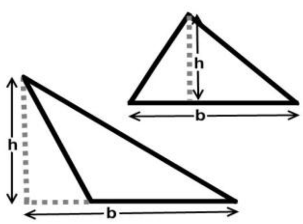



$\Delta ABE$ is isosceles since AE is congruent to BE. $\angle BAE = 50°$ since base angles of isosceles triangle are congruent. AE is a transversal between parallel lines, so alternate interior angles are congruent. Thus $\angle AEF = 50°$

In the figure below, ABCD is a trapezoid. E lies on line AD, and angle measures are as marked. What is the measure of angle CDB?



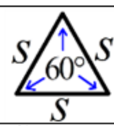
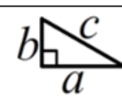
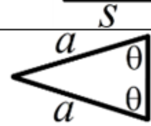
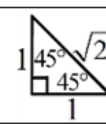
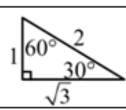
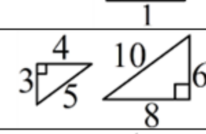
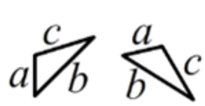
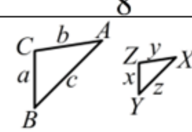

## Triangles



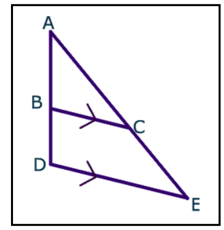

**Area = (1/2) base x height**

**On a coordinate plane**, the base can be found by finding the difference in the x coordinates, and the height by finding the difference in the y coordinates.

Base = $x_2 - x_1$ height = $y_2 - y_1$

| TRIANGLES | |
|---|---|
| For ALL TRIANGLES, the 3 angles add to 180° and Area A=(1/2) *base X height* | |
| **Equilateral Triangle** has 3 identical sides and 3 identical angles of 60° (since 60°+60°+60°=180°).  | **A Right Triangle** has one 90° angle, sides that satisfy $a^2+b^2=c^2$ (Pythag. Thm.), & the other 2 angles are <90°  |
| **Isosceles Triangle** has 2 identical sides & 2 identical angles, as shown.  | **45°–45°–90° Right Triangle** has sides with ratio $1:1:\sqrt{2}$  |
| **30°–60°–90° Right Triangle** has sides with ratio $1:2:\sqrt{3}$ ( hypotenuse = 2 )  | **3 – 4 – 5 Right Triangle** has sides proportional to 3, 4 & 5.  |
| **Congruent Triangles** have identical side lengths and angles. They may be rotated or reflected relative to one another and still be congruent.  | **Similar Triangles** have identical corresponding angles, $\angle A = \angle X$, $\angle B = \angle Y$, $\angle C = \angle Z$ & proportional side lengths, $a/x = b/y = c/z$  |

- The length of any one side of a triangle will always be: less than the sum of the other two sides, and more than the difference of the other two sides.
- "If the measures of two angles of a triangle are unequal, then the longer side is opposite the larger angle, and vice versa."
- An exterior angle of any triangle is equal to the sum of the two remote interior angles (because they form a straight line, 180°, are supplementary, and the angles of a triangle sum to 180°.
- <u>Right Triangle Altitude Theorem</u> - If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and each other.
- If a triangle that contains a **line segment parallel to one side**, this line segment creates a second, smaller, **similar triangle**. In the figure at right, for example, line segment *DE* is parallel to *BC*, and triangle *ABC* is similar to triangle *ADE*.
    - Additionally, this line divides those two sides proportionally. $\frac{AC}{CE} = \frac{AB}{BD}$




## Congruent triangles

- SSS
- SAS
- ASA
- AAS
- Hypotenuse Leg (this is really just a restatement of SAS for right triangles)
- **NOT** ASS (remember: it's a bad word, so don't use it)

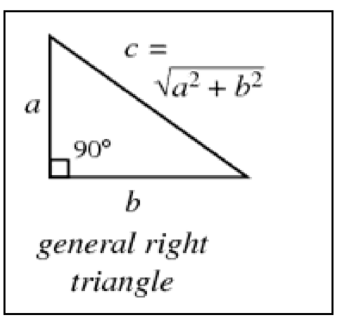
**Types of angles:**

- **Supplementary:** add up to 180° ("straight" line and "supplementary both start with "s")
- **Complementary:** add up to 90° ("corner" and "complementary" both start with "c")
- **Right:** 90° (A right angle is indicated by a little square.)

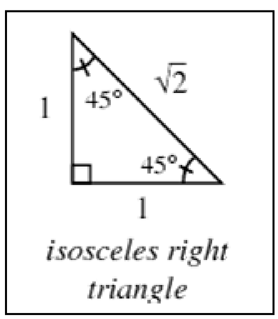
**Types of triangles:**

- **Equilateral:** All three sides are equal, and all three angles are equal (60°).
- **Isosceles:** Two sides are equal, so the angles opposite those two sides are also equal.
- **Similar:** The corresponding angles are equal, and the corresponding sides are in proportion.
- **Congruent:** The corresponding angles, and the corresponding sides are equal.
- **Right:** Has one angle = 90°. The right angle is indicated by a little square.

**Right triangles, and the Pythagorean Theorem**

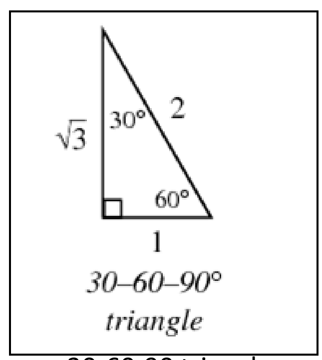



$a^2 + b^2 = c^2$




45-45-90 triangle

(the $\sqrt{2}$ is for the right triangle with 2 angles the same)




30-60-90 triangle

(the $\sqrt{3}$ goes with the 30-60-90)

<u>**Pythagorean triples**</u> are useful for helping you quickly identify right triangles. Some common Pythagorean triples are:

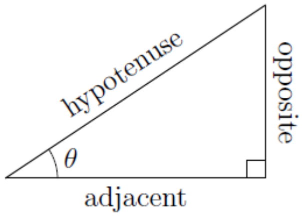
a : b : c **3 : 4 : 5** 5 : 12 : 13 8 : 15 : 17 7 : 24 : 25

**Any multiple of a Pythagorean triple** is also a Pythagorean triple. Therefore, 3 : 4 : 5, then 6 : 8 : 10 and 9 : 12 : 15 are also Pythagorean triples. Multiples of 3 : 4 : 5 are the most commonly used triples

- These are often used when you create right triangles within a problem (ex: a diagonal of a rectangle, or a chord in a circle. See page 13 for an example of this one).

## Trigonometry

Three important functions are defined for angles in a right triangle. Θ, or θ (“theta” is just a generic angle)




$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan\theta = \frac{\text{op}}{\text{adj}}$

$\sin\Theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$ $\csc\Theta = \frac{1}{\sin\Theta}$

$\cos\Theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ $\sec\Theta = \frac{1}{\cos\Theta}$

$\tan\Theta = \frac{\text{Opposite}}{\text{Adjacent}}$ $\cot\Theta = \frac{1}{\tan\Theta}$

Cot is rarely used, I have never seen csc and sec used.

Law of cosines: $b^2 = a^2 + c^2 - 2ac(\cos B)$

This equation is always given whenever it is needed; you just need to be able to use it.

## More Plane Geometry

**QUADRILATERALS**

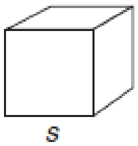
1. *Properties of Parallelograms*
    a. 2 pairs of parallel sides
    b. 2 pairs of opposite sides congruent
    c. 2 pairs of opposite angles congruent
    d. consecutive angles are supplementary
    e. diagonals bisect each other
    f. each diagonal creates 2 congruent triangles
2. *Properties of Rhombi*
    a. All properties of parallelograms
    b. All 4 sides are the same length
    c. Diagonals are perpendicular
    d. Diagonals bisect the angles at each vertex
3. *Properties of Rectangles*
    a. All properties of parallelograms
    b. Contains a right angle (equiangular quadrilateral)
    c. Diagonals are congruent
4. *Properties of Squares*
    a. All properties of rectangles and rhombi
5. *Properties of Trapezoids*
    a. Only one pair of parallel sides
    b. The median of a trapezoid is parallel to both bases and its length is the average of the bases.
6. *Properties of Isosceles Trapezoids*
    a. Non-parallel sides (legs) are congruent
    b. Base angles are congruent
    c. Diagonals are congruent
    d. Opposite angles are supplementary

## Polygons & 3-dimensional shapes

**n-sided Polygon Interior/Exterior Angles:**

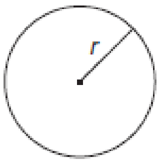
- Sum of int. angles = $180(n - 2)$
- Sum of ext. angles = 360
- Number of diagonals = $n(n-3)/2$

**Cube**



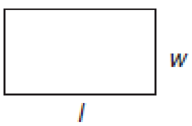

Volume = $s^3$
Surface Area = $6s^2$

**Circle**



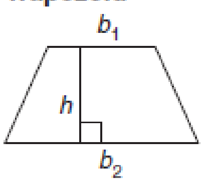

Area = $\pi r^2$
Circumference = $2\pi r$

**Rectangle**



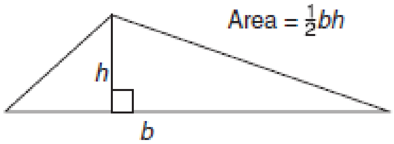

Area = $lw$
Perimeter = $2l + 2w$

**Trapezoid**




Area = $\frac{1}{2}h(b_1 + b_2)$

**Triangle**

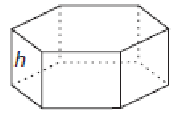



Area = $\frac{1}{2}bh$

**Parallelogram**

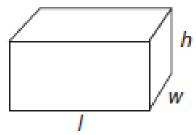

Area = $bh$

**Right Prism**



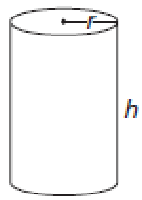

Volume = base area × $h$
Surface Area = base areas + face areas
Lateral Area = sum of face areas

**Rectangular Solid**



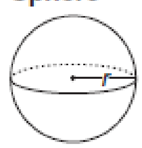

Volume = $lwh$
Surface Area = $2wl + 2lh + 2wh$
Lateral Area = $2(l + w)h$

**Cylinder**



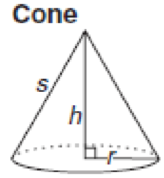

Volume = $\pi r^2 h$
Surface Area = $2\pi r^2 + 2\pi rh$
Lateral Area = $2\pi rh$

**Sphere**




Volume = $\frac{4}{3}\pi r^3$
Surface Area = $4\pi r^2$

**Cone**

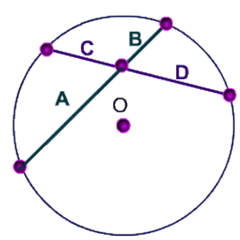



Volume = $\frac{1}{3}\pi r^2 h$
Surface Area = $\pi r^2 + \pi rs$
Lateral Area = $\pi rs$

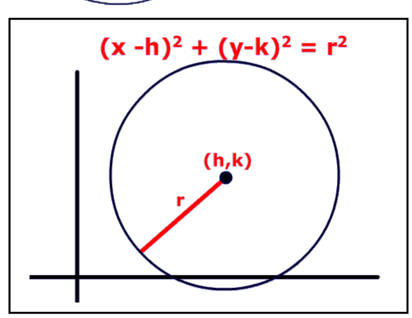
## Circles

- If two chords intersect in a circle, the product of the lengths of the segments of one chord equal the product of the segments of the other.

  **A • B = C • D**

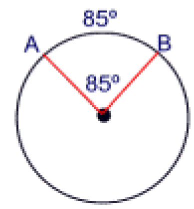



- The standard form equation of a **circle** n the coordinate plane: $\mathbf{(x-h)^2 + (y-k)^2 = r^2}$
  - *h* and *k* are the x and y coordinates of the center of the **circle**
  - $(x-9)^2 + (y-6)^2 = 100$ is a **circle** centered at (9,6) with a radius of 10



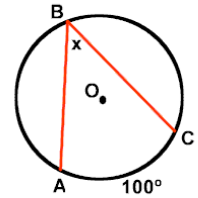
- In a circle, the ***degree measure of an arc*** is equal to the measure of the central angle that intercepts the arc.
- In a circle, the ***length of an arc*** is a portion of the circumference.

$$\text{arc length} = \frac{\text{central angle}}{360^\circ} \bullet \text{circumference}$$



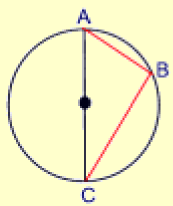

Inscribed Angle = $\frac{1}{2}$ Intercepted Arc

$$m\angle ABC = \frac{1}{2} m\overset{\frown}{AC}$$



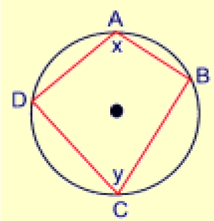

*m<ABC* = 50°

***Special situations involving inscribed angles:***




- **An angle inscribed in a semi-circle is a right angle.**
- **In a circle, inscribed circles that intercept the same arc are congruent.**

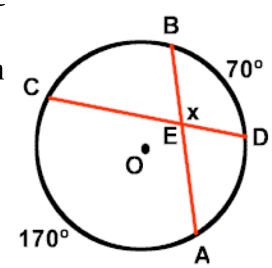
*A quadrilateral inscribed in a circle is called a cyclic quadrilateral.*




- **The opposite angles in a cyclic quadrilateral are supplementary (add up to 180°).**

**Angle Formed by Two Intersecting Chords:**

When two chords intersect "inside" a circle, four angles are formed. At the point of intersection, two sets of vertical angles can be seen in the corners of the X that is formed on the picture. Remember: vertical angles are equal.




Angle Formed Inside by Two Chords = $\frac{1}{2}$ **Sum** of Intercepted Arcs

$$m\angle BED = \frac{1}{2}\left(m\overset{\frown}{AC} + m\overset{\frown}{BD}\right)$$

Once you have found ONE of these angles, you automatically know the sizes of the other three by using your knowledge of vertical angles (being congruent) and adjacent angles forming a straight line (measures adding to 180).

$< BED$ is formed by two intersecting chords.

Its *intercepted arcs* are $\overset{\frown}{BD}$ and $\overset{\frown}{CA}$.

[Note: the intercepted arcs belong to the set of vertical angles.]

$$m\angle BED = \frac{1}{2}(70+170) = \frac{1}{2}(240) = 120^\circ$$

also, *m<CEA* = 120° (vetical angle)

*m<BEC* and *m<DEA* = 60° by straight line.

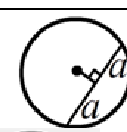
**Angle Formed Outside of a Circle by the Intersection of:**

"Two Tangents," "Two Secants," or "a Tangent and a Secant".

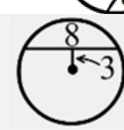
The formulas for all THREE of these situations are the same:

Angle Formed Outside = $\frac{1}{2}$ **Difference** of Intercepted Arcs

The <u>perpendicular bisector of a chord</u> in a circle passes through the center of the circle.




***Ex. A chord in a circle has length 8" and is 3" from the center of the circle. Find the radius of the circle.***




The line segment labeled 3" is perpendicular to the chord, so it bisects it. A right triangle is formed with the hypotenuse equal to the radius. This is a 3-4-5 right triangle, so *r* = 5 in.

# Transformations

## Reflections

- The reflection of the point (***x, y***) across the *x*-axis is the point (***x, -y***).
  $P(x,y) \rightarrow P'(x,-y)$ or $r_{x-axis}(x,y) = (x,-y)$
- The reflection of the point **(*x, y*)** across the *y*-axis is the point **(-*x, y*)**.
  $P(x,y) \rightarrow P'(-x,y)$ or $r_{y-axis}(x,y) = (-x,y)$
- The reflection of the point **(*x, y*)** across the line ***y* = *x*** is the point **(*y, x*)**.
  $P(x,y) \rightarrow P'(y,x)$ or $r_{y=x}(x,y) = (y,x)$
- The reflection of the point **(*x, y*)** across the line ***y* = -*x*** is the point **(-*y*, -*x*).**
  $P(x,y) \rightarrow P'(-y,-x)$ or $r_{y=-x}(x,y) = (-y,-x)$
- The reflection of the point **(*x, y*)** across the origin is the point **(-*x* , -*y*).**
  $P(x, y) \rightarrow P'(-x, -y)$ or $r_{origin}(x, y) = (-x, -y)$

**Rotations**: A positive angle of rotation turns the figure counterclockwise, and a negative angle of rotation turns the figure in a clockwise direction.

| | |
|---|---|
| Rotation of **90°:** | $R_{90^\circ}(x,y) = (-y,x)$ |
| Rotation of **180°:** | $R_{180^\circ}(x,y) = (-x,-y)$ (same as point reflection in origin) |
| Rotation of **270°(or 90°:** | $R_{270^\circ}(x,y) = (y,-x)$ |

**Unit Circle:** Another way to measure angles is radians. $\pi$ Radians = 180° .