

## CHAPTER 14: BOND PRICES AND YIELDS

1. a. Effective annual rate for 3-month T-bill:

$$\left(\frac{100,000}{97,645}\right)^4 - 1 = 1.02412^4 - 1 = 0.100 = 10.0\%$$

- b. Effective annual interest rate for coupon bond paying 5% semiannually:

$$(1.05)^2 - 1 = 0.1025 \text{ or } 10.25\%$$

Therefore the coupon bond has the higher effective annual interest rate.

2. The effective annual yield on the semiannual coupon bonds is 8.16%. If the annual coupon bonds are to sell at par they must offer the same yield, which requires an annual coupon rate of 8.16%.
3. The bond callable at 105 should sell at a lower price because the call provision is more valuable to the firm. Therefore, its yield to maturity should be higher.
4. The bond price will be lower. As time passes, the bond price, which is now above par value, will approach par.

5. Yield to maturity: Using a financial calculator, enter the following:

$$n = 3; PV = -953.10; FV = 1000; PMT = 80; \text{COMP } i$$

This results in: YTM = 9.88%

Realized compound yield: First, find the future value (FV) of reinvested coupons and principal:

$$FV = (\$80 \times 1.10 \times 1.12) + (\$80 \times 1.12) + \$1,080 = \$1,268.16$$

Then find the rate ( $y_{\text{realized}}$ ) that makes the FV of the purchase price equal to \$1,268.16:

$$\$953.10 \times (1 + y_{\text{realized}})^3 = \$1,268.16 \Rightarrow y_{\text{realized}} = 9.99\% \text{ or approximately } 10\%$$

6. a. A sinking fund provision requires the early redemption of a bond issue. The provision may be for a specific number of bonds or a percentage of the bond issue over a specified time period. The sinking fund can retire all or a portion of an issue over the life of the issue.
- b. (i) Compared to a bond without a sinking fund, the sinking fund reduces the average life of the overall issue because some of the bonds are retired prior to the stated maturity.
- (ii) The company will make the same total principal payments over the life of the issue, although the timing of these payments will be affected. The total interest payments associated with the issue will be reduced given the early redemption of principal.
- c. From the investor's point of view, the key reason for demanding a sinking fund is to reduce credit risk. Default risk is reduced by the orderly retirement of the issue.
7. a. (i) Current yield = Coupon/Price =  $\$70/\$960 = 0.0729 = 7.29\%$
- (ii) YTM = 3.993% semiannually or 7.986% annual bond equivalent yield. On a financial calculator, enter:  $n = 10$ ;  $PV = -960$ ;  $FV = 1000$ ;  $PMT = 35$ . Compute the interest rate.
- (iii) Realized compound yield is 4.166% (semiannually), or 8.332% annual bond equivalent yield. To obtain this value, first find the future value (FV) of reinvested coupons and principal. There will be six payments of \$35 each, reinvested semiannually at 3% per period. On a financial calculator, enter:  $PV = 0$ ;  $PMT = 35$ ;  $n = 6$ ;  $i = 3\%$ . Compute:  $FV = 226.39$
- Three years from now, the bond will be selling at the par value of \$1,000 because the yield to maturity is forecast to equal the coupon rate. Therefore, total proceeds in three years will be:  $\$226.39 + \$1,000 = \$1,226.39$
- Then find the rate ( $y_{\text{realized}}$ ) that makes the FV of the purchase price equal to \$1,226.39:
- $$\$960 \times (1 + y_{\text{realized}})^6 = \$1,226.39 \Rightarrow y_{\text{realized}} = 4.166\% \text{ (semiannual)}$$
- b. Shortcomings of each measure:
- (i) Current yield does not account for capital gains or losses on bonds bought at prices other than par value. It also does not account for reinvestment income on coupon payments.
- (ii) Yield to maturity assumes the bond is held until maturity and that all coupon income can be reinvested at a rate equal to the yield to maturity.
- (iii) Realized compound yield is affected by the forecast of reinvestment rates, holding period, and yield of the bond at the end of the investor's holding period.

	<u>Zero coupon</u>	<u>8% coupon</u>	<u>10% coupon</u>
8. a. Current prices	\$463.19	\$1,000.00	\$1,134.20
b. Price 1 year from now	\$500.25	\$1,000.00	\$1,124.94
Price increase	\$ 37.06	\$ 0.00	– \$ 9.26
Coupon income	\$ 0.00	\$ 80.00	\$100.00
Pre-tax income	\$ 37.06	\$ 80.00	\$ 90.74
Pre-tax rate of return	8.00%	8.00%	8.00%
Taxes*	\$ 11.12	\$ 24.00	\$ 28.15
After-tax income	\$ 25.94	\$ 56.00	\$ 62.59
After-tax rate of return	5.60%	5.60%	5.52%
c. Price 1 year from now	\$543.93	\$1,065.15	\$1,195.46
Price increase	\$ 80.74	\$ 65.15	\$ 61.26
Coupon income	\$ 0.00	\$ 80.00	\$100.00
Pre-tax income	\$ 80.74	\$145.15	\$161.26
Pre-tax rate of return	17.43%	14.52%	14.22%
Taxes**	\$ 19.86	\$ 37.03	\$ 42.25
After-tax income	\$ 60.88	\$108.12	\$119.01
After-tax rate of return	13.14%	10.81%	10.49%

\* In computing taxes, we assume that the 10% coupon bond was issued at par and that the decrease in price when the bond is sold at year end is treated as a capital loss and therefore is not treated as an offset to ordinary income.

\*\* In computing taxes for the zero coupon bond, \$37.06 is taxed as ordinary income (see part (b)) and the remainder of the price increase is taxed as a capital gain.

9. a. On a financial calculator, enter the following:

$$n = 40; FV = 1000; PV = -950; PMT = 40$$

You will find that the yield to maturity on a semi-annual basis is 4.26%. This implies a bond equivalent yield to maturity equal to:  $4.26\% \times 2 = 8.52\%$

$$\text{Effective annual yield to maturity} = (1.0426)^2 - 1 = 0.0870 = 8.70\%$$

- b. Since the bond is selling at par, the yield to maturity on a semi-annual basis is the same as the semi-annual coupon rate, i.e., 4%. The bond equivalent yield to maturity is 8%.

$$\text{Effective annual yield to maturity} = (1.04)^2 - 1 = 0.0816 = 8.16\%$$

- c. Keeping other inputs unchanged but setting  $PV = -1050$ , we find a bond equivalent yield to maturity of 7.52%, or 3.76% on a semi-annual basis.

$$\text{Effective annual yield to maturity} = (1.0376)^2 - 1 = 0.0766 = 7.66\%$$

10. Since the bond payments are now made annually instead of semi-annually, the bond equivalent yield to maturity is the same as the effective annual yield to maturity. Using a financial calculator, enter:  $n = 20$ ;  $FV = 1000$ ;  $PV = -\text{price}$ ,  $PMT = 80$ . The resulting yields for the three bonds are:

Bond Price	Bond equivalent yield = Effective annual yield
\$950	8.53%
\$1,000	8.00%
\$1,050	7.51%

The yields computed in this case are lower than the yields calculated with semi-annual payments. All else equal, bonds with annual payments are less attractive to investors because more time elapses before payments are received. If the bond price is the same with annual payments, then the bond's yield to maturity is lower.

11.

Price	Maturity (years)	Bond equivalent YTM
\$400.00	20.00	4.688%
\$500.00	20.00	3.526%
\$500.00	10.00	7.177%
\$385.54	10.00	10.000%
\$463.19	10.00	8.000%
\$400.00	11.91	8.000%

12. a. The bond pays \$50 every 6 months. The current price is:

$$[\$50 \times \text{Annuity factor (4\%, 6)}] + [\$1,000 \times \text{PV factor (4\%, 6)}] = \$1,052.42$$

Assuming the market interest rate remains 4% per half year, price six months from now is:

$$[\$50 \times \text{Annuity factor (4\%, 5)}] + [\$1,000 \times \text{PV factor (4\%, 5)}] = \$1,044.52$$

- b. Rate of return =  $\frac{\$50 + (\$1,044.52 - \$1,052.42)}{\$1,052.42} = \frac{\$50 - \$7.90}{\$1,052.42}$   
 $= 0.04 = 4.0\%$  per six months

13. a. Initial price  $P_0 = \$705.46$  [ $n = 20$ ;  $PMT = 50$ ;  $FV = 1000$ ;  $i = 8$ ]  
 Next year's price  $P_1 = \$793.29$  [ $n = 19$ ;  $PMT = 50$ ;  $FV = 1000$ ;  $i = 7$ ]  

$$\text{HPR} = \frac{\$50 + (\$793.29 - \$705.46)}{\$705.46} = 0.1954 = 19.54\%$$

- b. Using OID tax rules, the cost basis and imputed interest under the constant yield method are obtained by discounting bond payments at the *original* 8% yield, and simply reducing maturity by one year at a time:

Constant yield prices (compare these to actual prices to compute capital gains):

$$P_0 = \$705.46$$

$$P_1 = \$711.89 \Rightarrow \text{implicit interest over first year} = \$6.43$$

$$P_2 = \$718.84 \Rightarrow \text{implicit interest over second year} = \$6.95$$

Tax on explicit interest plus implicit interest in first year =

$$0.40 \times (\$50 + \$6.43) = \$22.57$$

Capital gain in first year = Actual price at 7% YTM – constant yield price =

$$\$793.29 - \$711.89 = \$81.40$$

Tax on capital gain =  $0.30 \times \$81.40 = \$24.42$

Total taxes =  $\$22.57 + \$24.42 = \$46.99$

c. After tax HPR =  $\frac{\$50 + (\$793.29 - \$705.46) - \$46.99}{\$705.46} = 0.1288 = 12.88\%$

- d. Value of bond after two years =  $\$798.82$  [using  $n = 18$ ;  $i = 7\%$ ]

Reinvested income from the two coupon interest payments =

$$\$50 \times 1.03 + \$50 = \$101.50$$

Total funds after two years =  $\$798.82 + \$101.50 = \$900.32$

Therefore, the investment of  $\$705.46$  grows to  $\$900.32$  in two years:

$$\$705.46 (1 + r)^2 = \$900.32 \Rightarrow r = 0.1297 = 12.97\%$$

- e. Coupon interest received in first year:  $\$50.00$   
 Less: tax on coupon interest @ 40%:  $- 20.00$   
 Less: tax on imputed interest ( $0.40 \times \$6.43$ ):  $- 2.57$   
 Net cash flow in first year:  $\$27.43$

The year-1 cash flow can be invested at an after-tax rate of:

$$3\% \times (1 - 0.40) = 1.8\%$$

By year 2, this investment will grow to:  $\$27.43 \times 1.018 = \$27.92$

In two years, sell the bond for:	\$798.82	[n = 18; i = 7%]
Less: tax on <i>imputed</i> interest in second year:	- 2.78	[0.40 × \$6.95]
Add: after-tax coupon interest received in second year:	+ 30.00	[\$50 × (1 - 0.40)]
Less: Capital gains tax on (sales price - constant yield value):	- 23.99	[0.30 × (798.82 - 718.84)]
Add: CF from first year's coupon (reinvested):	<u>+ 27.92</u>	[from above]
Total	\$829.97	

$$\$705.46 (1 + r)^2 = \$829.97 \Rightarrow r = 0.0847 = 8.47\%$$

14. The *reported* bond price is: 100  $\frac{2}{32}$  percent of par = \$1,000.625  
However, 15 days have passed since the last semiannual coupon was paid, so:  
accrued interest =  $\$35 \times (15/182) = \$2.885$   
The invoice price is the reported price plus accrued interest: \$1,003.51
15. If the yield to maturity is greater than the current yield, then the bond offers the prospect of price appreciation as it approaches its maturity date. Therefore, the bond must be selling below par value.
16. The coupon rate is less than 9%. If coupon divided by price equals 9%, and price is less than par, then price divided by par is less than 9%.

17.

Time	Inflation in year just ended	Par value	Coupon payment	Principal repayment
0		\$1,000.00		
1	2%	\$1,020.00	\$40.80	\$ 0.00
2	3%	\$1,050.60	\$42.02	\$ 0.00
3	1%	\$1,061.11	\$42.44	\$1,061.11

The *nominal* rate of return and *real* rate of return on the bond in each year are computed as follows:

$$\text{Nominal rate of return} = \frac{\text{interest} + \text{price appreciation}}{\text{initial price}}$$

$$\text{Real rate of return} = \frac{1 + \text{nominal return}}{1 + \text{inflation}} - 1$$

	<u>Second year</u>	<u>Third year</u>
Nominal return	$\frac{\$42.02 + \$30.60}{\$1,020} = 0.071196$	$\frac{\$42.44 + \$10.51}{\$1,050.60} = 0.050400$
Real return	$\frac{1.071196}{1.03} - 1 = 0.040 = 4.0\%$	$\frac{1.050400}{1.01} - 1 = 0.040 = 4.0\%$

The real rate of return in each year is precisely the 4% real yield on the bond.

18. The price schedule is as follows:

Year	Remaining Maturity (T)	Constant yield value $\$1,000/(1.08)^T$	Imputed interest (Increase in constant yield value)
0 (now)	20 years	\$214.55	
1	19	\$231.71	\$17.16
2	18	\$250.25	\$18.54
19	1	\$925.93	
20	0	\$1,000.00	\$74.07

19. The bond is issued at a price of \$800. Therefore, its yield to maturity is: 6.8245%. Therefore, using the constant yield method, we find that the price in one year (when maturity falls to 9 years) will be (at an unchanged yield) \$814.60, representing an increase of \$14.60. Total taxable income is: \$40.00 + \$14.60 = \$54.60

20. a. The bond sells for \$1,124.72 based on the 3.5% yield to *maturity*.  
[n = 60; i = 3.5; FV = 1000; PMT = 40]  
Therefore, yield to *call* is 3.368% semiannually, 6.736% semi-annually.  
[n = 10 semiannual periods; PV = -1124.72; FV = 1100; PMT = 40]
- b. If the call price were \$1,050, we would set FV = 1,050 and redo part (a) to find that yield to call is 2.976% semiannually, 5.952% annually. With a lower call price, the yield to call is lower.
- c. Yield to call is 3.031% semiannually, 6.602% annually.  
[n = 4; PV = -1124.72; FV = 1100; PMT = 40]

21. The stated yield to maturity, based on promised payments, equals 16.075%.  
 [n = 10; PV = -900; FV = 1000; PMT = 140]  
 Based on *expected* coupon payments of \$70 annually, the expected yield to maturity is 8.526%.
22. The bond is selling at par value. Its yield to maturity equals the coupon rate, 10%. If the first-year coupon is reinvested at an interest rate of r percent, then total proceeds at the end of the second year will be:  $[\$100 \times (1 + r)] + \$1,100$   
 Therefore, realized compound yield to maturity is a function of r, as shown in the following table:

r	Total proceeds	Realized YTM = $\sqrt{\text{Proceeds}/1000} - 1$
8%	\$1,208	$\sqrt{1208/1000} - 1 = 0.0991 = 9.91\%$
10%	\$1,210	$\sqrt{1210/1000} - 1 = 0.1000 = 10.00\%$
12%	\$1,212	$\sqrt{1212/1000} - 1 = 0.1009 = 10.09\%$

23. Zero coupon bonds provide no coupons to be reinvested. Therefore, the investor's proceeds from the bond are independent of the rate at which coupons could be reinvested (if they were paid). There is no reinvestment rate uncertainty with zeros.
24. April 15 is midway through the semiannual coupon period. Therefore, the invoice price will be higher than the stated ask price by an amount equal to one-half of the semiannual coupon. The ask price is 101.125 percent of par, so the invoice price is:  
 $\$1,011.25 + (\frac{1}{2} \times \$50) = \$1,036.25$
25. Factors that might make the ABC debt more attractive to investors, therefore justifying a lower coupon rate and yield to maturity, are:
- The ABC debt is a larger issue and therefore may sell with greater liquidity.
  - An option to extend the term from 10 years to 20 years is favorable if interest rates ten years from now are lower than today's interest rates. In contrast, if interest rates increase, the investor can present the bond for payment and reinvest the money for a higher return.
  - In the event of trouble, the ABC debt is a more senior claim. It has more underlying security in the form of a first claim against real property.
  - The call feature on the XYZ bonds makes the ABC bonds relatively more attractive since ABC bonds cannot be called from the investor.
  - The XYZ bond has a sinking fund requiring XYZ to retire part of the issue each year. Since most sinking funds give the firm the option to retire this amount at the lower of par or market value, the sinking fund can be detrimental for bondholders.



26. a. The floating rate note pays a coupon that adjusts to market levels. Therefore, it will not experience dramatic price changes as market yields fluctuate. The fixed rate note will therefore have a greater price range.
- b. Floating rate notes may not sell at par for any of several reasons:  
 The yield spread between one-year Treasury bills and other money market instruments of comparable maturity could be wider (or narrower) than when the bond was issued.  
 The credit standing of the firm may have eroded (or improved) relative to Treasury securities, which have no credit risk. Therefore, the 2% premium would become insufficient to sustain the issue at par.  
 The coupon increases are implemented with a lag, i.e., once every year. During a period of changing interest rates, even this brief lag will be reflected in the price of the security.
- c. The risk of call is low. Because the bond will almost surely not sell for much above par value (given its adjustable coupon rate), it is unlikely that the bond will ever be called.
- d. The fixed-rate note currently sells at only 88% of the call price, so that yield to maturity is greater than the coupon rate. Call risk is currently low, since yields would need to fall substantially for the firm to use its option to call the bond.
- e. The 9% coupon notes currently have a remaining maturity of fifteen years and sell at a yield to maturity of 9.9%. This is the coupon rate that would be needed for a newly-issued fifteen-year maturity bond to sell at par.
- f. Because the floating rate note pays a *variable stream* of interest payments to maturity, the effective maturity for comparative purposes with other debt securities is closer to the next coupon reset date than the final maturity date. Therefore, yield-to-maturity is an indeterminable calculation for a floating rate note, with “yield-to-recoupon date” a more meaningful measure of return.
27. a. The maturity of each bond is ten years, and we assume that coupons are paid semiannually. Since both bonds are selling at par value, the current yield for each bond is equal to its coupon rate.  
 If the yield declines by 1% to 5% (2.5% semiannual yield), the Sentinal bond will increase in value to \$107.79 [ $n=20$ ;  $i = 2.5\%$ ;  $FV = 100$ ;  $PMT = 3$ ].  
 The price of the Colina bond will increase, but only to the call price of 102. The present value of *scheduled* payments is greater than 102, but the call price puts a ceiling on the actual bond price.

- b. If rates are expected to fall, the Sentinal bond is more attractive: since it is not subject to call, its potential capital gains are greater.  
If rates are expected to rise, Colina is a relatively better investment. Its higher coupon (which presumably is compensation to investors for the call feature of the bond) will provide a higher rate of return than the Sentinal bond.
- c. An increase in the volatility of rates will increase the value of the firm's option to call back the Colina bond. If rates go down, the firm can call the bond, which puts a cap on possible capital gains. So, greater volatility makes the option to call back the bond more valuable to the issuer. This makes the bond less attractive to the investor.
28. a. The yield to maturity on the par bond equals its coupon rate, 8.75%. All else equal, the 4% coupon bond would be more attractive because its coupon rate is far below current market yields, and its price is far below the call price. Therefore, if yields fall, capital gains on the bond will not be limited by the call price. In contrast, the 8¾% coupon bond can increase in value to at most \$1,050, offering a maximum possible gain of only 0.5%. The disadvantage of the 8¾% coupon bond, in terms of vulnerability to being called, shows up in its higher *promised* yield to maturity.
- b. If an investor expects yields to fall substantially, the 4% bond offers a greater expected return.
- c. Implicit call protection is offered in the sense that any likely fall in yields would not be nearly enough to make the firm consider calling the bond. In this sense, the call feature is almost irrelevant.
29. Market conversion value = value if converted into stock =  $20.83 \times \$28 = \$583.24$   
Conversion premium = Bond price – market conversion value  
=  $\$775.00 - \$583.24 = \$191.76$
30. a. The call feature requires the firm to offer a higher coupon (or higher promised yield to maturity) on the bond in order to compensate the investor for the firm's option to call back the bond at a specified price if interest rate falls sufficiently. Investors are willing to grant this valuable option to the issuer, but only for a price that reflects the possibility that the bond will be called. That price is the higher promised yield at which they are willing to buy the bond.

- b. The call feature reduces the expected life of the bond. If interest rates fall substantially so that the likelihood of a call increases, investors will treat the bond as if it will "mature" and be paid off at the call date, not at the stated maturity date. On the other hand if rates rise, the bond must be paid off at the maturity date, not later. This asymmetry means that the expected life of the bond is less than the stated maturity.
- c. The advantage of a callable bond is the higher coupon (and higher promised yield to maturity) when the bond is issued. If the bond is never called, then an investor earns a higher realized compound yield on a callable bond issued at par than a non-callable bond issued at par on the same date. The disadvantage of the callable bond is the risk of call. If rates fall and the bond is called, then the investor receives the call price and then has to reinvest the proceeds at interest rates that are lower than the yield to maturity at which the bond originally was issued. In this event, the firm's savings in interest payments is the investor's loss.

31. a. (iii)

- b. (iii) The yield to maturity on the callable bond must compensate the investor for the risk of call.

Choice (i) is wrong because, although the owner of a callable bond receives a premium plus the principal in the event of a call, the interest rate at which he can reinvest will be low. The low interest rate that makes it profitable for the issuer to call the bond also makes it a bad deal for the bond's holder.

Choice (ii) is wrong because a bond is more apt to be called when interest rates are low. Only if rates are low will there be an interest saving for the issuer.

c. (iii)

d. (ii)

e. (iii)