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ROLES AND POSITIONS: A CRITIQUE AND EXTENSION OF THE BLOCKMODELING APPROACH

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The concept of role is fundamental to both sociological theory and empirical analysis.[1] While most sociologists agree on its impor-

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[1] This chapter brings together ideas and work that have been presented elsewhere in unpublished form. The shortcomings of the notion of structural

tance, this has not prevented them from using it in a great variety of different ways (Gross, Mason, and McEachern, 1957), including some that are excessively uncritical and abstract (Jackson, 1972; White, 1970).

Recent work in network analysis has attempted to address these problems by providing a firmer understanding of roles. In particular, White, Boorman, and Breiger (1976) and others (Burt, 1976) have suggested an operational definition of role based on identifying actors who share the same structural position within a network of social relations. Their research—drawing on Nadel's *The Theory of Social Structure* (1957) and on formal models of kinship (for example, White, 1963)—focused on the relational rather than the normative aspects of roles.

By emphasizing the importance of concrete and observable relations, this approach can yield a theoretically coherent definition of *position.*[2] The core concept is that two individuals occupy the same position in a social structure if and only if they are related to the same individuals in the same way—that is, if they are *structurally equivalent.* With appropriate modifications, the notion of structural equivalence leads to the empirically applicable procedures of blockmodel analysis (White, Boorman, and Breiger, 1976).

However, having a definition of *position* is not the same as having a definition of *role.* Positions can be thought of as specific locations in a particular social structure; roles, in contrast, should provide a way of

equivalence and the motivation for the approach discussed here can be found in Bernard (1971). The basic analytic scheme in this chapter and its relationship to the mathematical concept of an automorphism is described in Winship (1974). The full development and operationalization of that scheme is found in Mandel (1978). The approach carried out in this chapter has been termed elsewhere the Winship–Pattison approach (Mandel, 1978). In a companion paper, (Mandel, 1983), an alternative formulation based on individual-level "algebras" is described. This approach has been labeled in other work the Winship–Mandel approach (Mandel, 1978). An earlier description of the work reported here is found in Mandel and Winship (1979). Parallel and independent work has been carried out recently by Douglas White (White, 1982; White and Reitz, 1983).

[2] We are not suggesting that structural equivalence is the only possible definition of position. For example, the density and span of a person's network describes something about the person's place in the social structure not necessarily captured by structural equivalence. However, such measures are in a much different spirit than the conceptualization presented in this chapter.

classifying positions across any number of distinct social networks, or within different parts of the same network. Common everyday role labels like foreman, leader, or mother are not used to identify a single position in a particular organization or family (as is the term *president of the United States*). Instead, they are applied to many different positions.[3]

Approaches based on structural equivalence have generally defined positions much more satisfactorily than roles. This is because structural equivalence, as will be shown, is inherently population-specific. Consequently, blockmodel analysis and related methods have no way of comparing the roles of actors who are not actually occupying the same position.[4]

Some of the previous literature implicitly recognizes this problem. White, Boorman, and Breiger's (1976) analysis of the Bank Wiring Room data (Homans, 1950; Roethlisberger and Dickson, 1939) partitions the population into six blocks (positions)—two cliques of three blocks each, with each clique including a *core*, a *hangers-on*, and a *marginal* block. The terminology implies that the men in the two core positions, for example, have something in common—but structural equivalence and related concepts cannot bring this out. (Also see Burt's (1976) typology of positions.)

In this chapter, our intention is to incorporate the basic blockmodel approach into a framework that deals with the problems just outlined. The fundamental premise is that a position can be character-

[3] The importance of roles as mechanisms by which people identify similar positions across different situations has received little attention within sociology. This idea is implicit in Simmel's classic work, "How Is Society Possible?" (Levine, 1971). There, Simmel talks about the importance of human types as general categories in a way that parallels our discussion of roles. Simmel argues that human types are necessary in order for individuals to perceive similarity in different situations. Only by understanding individuals in terms of general categories does society become possible. The importance of Simmel's work to a study of roles is discussed at length by Popitz (1972).

[4] Previous comparative work has focused on the algebras for whole populations (Boorman and White, 1976; Breiger and Pattison, 1978; Bonacich, 1980; Bonacich and McConaghy, 1979). This approach complements the one developed here by providing for the comparison of whole networks rather than positions within those networks. In this chapter we have little to say about this approach and its relationship to models discussed here. A companion paper by one of the authors (Mandel, 1983) treats the comparison in depth.

ized by its associated pattern of relationships, rather than by the identities of the specific actors involved in those relationships (as in the structural equivalence approach). Different positions, either from the same population or from separate ones, can be associated with the same pattern of relationships. Thus, it is natural to identify roles with particular patterns of relationship.

The key to such a definition is, of course, how the relational patterns of a position are to be described. The method used here is based on Merton's (1959) concepts of role relation and role set. Other sociologists have employed these ideas in network analysis (White, Boorman, and Breiger, 1976; Breiger and Pattison, 1978; Burt, 1977a, b), but the framework developed here uses these concepts in a way that makes possible comparisons of roles across populations.

The importance of our method is twofold. As previously argued, roles provide the means of identifying individuals who are in similar positions but in different populations. One goal of this chapter is to show how this identification can be achieved, and in so doing, to provide a formal definition of role that is distinct from the blockmodeling definition of position. The second goal of our work is to develop a set of methodological tools that can be used for the comparison of positions in different populations or for the comparison of positions in a single population. The formal model we develop does allow the researcher to do this, and illustrative analyses are carried out.

The following section examines the problems with the blockmodel approach in more detail. Following that, our definition of role is developed, along with the needed theoretical concepts. In particular, we show how the notions of role set and role relation can be used to describe positions in social networks. We then carry out a number of illustrative empirical analyses, based on distance measures that indicates how similar the roles of different actors are. Finally, directions for future work are outlined.

STRUCTURAL EQUIVALENCE AND THE LIMITATIONS OF BLOCKMODEL ANALYSIS

Structural equivalence (Lorrain and White, 1971) lies at the heart of both blockmodel analysis and the positional analysis of Burt (1976). It is defined as follows:

DEFINITION: *Two individuals or actors are* structurally equivalent *if and only if both have the same relations with each individual or actor (in the population).*

Let a social network be represented by a family of binary matrices (each matrix corresonds to a different social relationship, and a 1 denotes the presence of a relationship of that type between the appropriate individuals). Then the structural equivalence of two individuals means that in each matrix, the row of the first individual is identical to the row of the second, and their two columns are identical as well.

Structural equivalence has proved to be too restrictive for most purposes. Most data sets do not contain any actors who are structurally equivalent in the strict sense of the definition. It is, however, common to find individuals or actors who are nearly structurally equivalent. Therefore, the basic objective of blockmodel analysis is to divide the population into clusters, each of which contains actors that are nearly structurally equivalent.

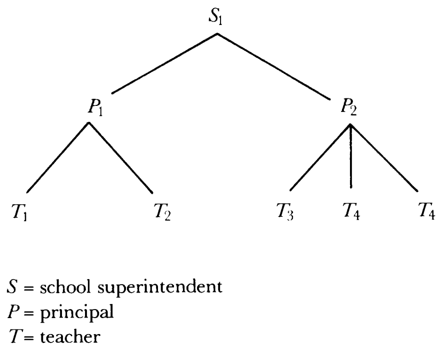
However, whether used in the strict sense or in the weakened blockmodel sense, the concept of structural equivalence does not allow individuals from two separate populations to be compared. There is

FIGURE 1. Relation matrix: school system example, supervision relation.

	S_1	P_1	P_2	T_1	T_2	T_3	T_4	T_5
S_1	0	1	1	0	0	0	0	0
P_1	0	0	0	1	1	0	0	0
P_2	0	0	0	0	0	1	1	1
T_1	0	0	0	0	0	0	0	0
T_2	0	0	0	0	0	0	0	0
T_3	0	0	0	0	0	0	0	0
T_4	0	0	0	0	0	0	0	0
T_5	0	0	0	0	0	0	0	0

S = school superintendent
P = principal
T = teacher

FIGURE 2. Supervision relationship for School System 1.



no possibility that they can have the same relationships with the same actors; hence, there is no way they can be structurally equivalent.

Consider, for instance, a simple example in which the roles and positions are obvious: two distinct school systems with the same formal supervisory structure. Even if both school systems have a single superintendent supervising several principals, who in turn supervise several teachers, the corresponding positions in the two school systems are not structurally equivalent.

The same problem also arises in many cases where the positions being compared are in the same population. Figure 1 gives the relation matrix for a single school system. In this example, all the teachers under a single principal are structurally equivalent (for example, T_1 and T_2). However, the two principals are not structurally equivalent, even though it is clear that they are both playing the same role (see Figure 2).[5]

In effect, structural equivalence places an extreme emphasis on the relational aspects of roles, eliminating considerations of content not

[5] The blockmodel approach and its associated algorithims, CONCOR and BLOCKER, impose a global partition on the population and thus define positions in terms of the entire social structure. The result is that individuals playing the same role but in different positions cannot be put into the same block without collapsing the entire social structure (and losing large amounts of information).

only from the definition of position but from the definition of relation as well. To specify the structural equivalence class that an actor belongs to, one must specify not only what relations the individual has but also with whom the individual has them. Structural equivalence demands that relations be classified by both type and recipient. As such, cross-population analysis is not possible using the principle of structural equivalence.

ROLE RELATIONS AND ROLE SETS

Role Relations. Structural equivalence was formulated in such a restrictive manner because it was intended to provide an operational definition of position.[6] To keep this desirable property while moving beyond some of the limitations of structural equivalence, it is necessary to specify some alternative way of describing the position that an actor occupies in a social structure—ideally with the description not being bound to a particular population.[7]

Consider two individuals in the same population. Enumerate first the direct ties between them: friendship, admiration, business partners, and so on.[8] Now consider the indirect or compound relationships between them—whether they share friends in common, or whether one admires the business partner of the other. The existence

[6] In the text we have attempted to minimize the amount of formal mathematics. A more formal treatment of the ideas presented can be found in Winship (1974) and Mandel (1978). In the footnotes we briefly develop the mathematical theory of automorphisms as it applies to the analysis of roles. This theory provides the basic conceptual insights for the model of roles developed here. More detailed discussions of automorphisms can be found in Winship (1974), Mandel (1978), and Pattison (1980).

[7] The approach to roles developed in this section generalizes the notion of an automorphism. In general, an automorphism is a functional mapping of a structure onto itself that preserves the relationships between elements in the structure. In the network context, the automorphism that is of interest is defined as follows:

> DEFINITION: *An automorphism,* f, *is a one-to-one onto mapping of a population onto itself such that for any pair of individuals* i, j *and all relations* A, iAj *if and only if* f(i)Af(j).

[8] In practice, the direct relations are simply the ones that the data collector recorded.

or nonexistence of these compound relations contains information about the social network in which the two individuals are enmeshed.

Formally, the total set of all relationships between two individuals i and j can be represented by a binary vector—to be denoted $\mathbf{R}_{ij}$—where a 1 signifies the presence of a particular type of relation.[9] The vector will include all direct relations, plus all compound relations up to some specified length.[10]

This binary vector $\mathbf{R}_{ij}$—to be called a *role relation*—plays a key part in the framework being developed here. It is closely analogous to the idea of role relation as used by Merton (1959). That is, the vector characterizes the relationships (both direct and indirect) that exist between a pair of actors as the result of the roles that they occupy with respect to each other.[11]

As an example, consider once more the school system of Figure 1. The superintendent has eight role relations: one with himself or herself, one with each of the two principals, and one with each of the five teachers (see Figure 3). Note that the role relations from the superintendent to each principal are identical, as are the role relations from the superintendent to each teacher. In effect, the role relations indicate which individuals are equivalent from the perspective of the

[9] We use conventional matrix notation. The prime notation is used to represent the converse of a relation or equivalently the transpose of matrix. Thus, for symmetric relations, $Sij = S'ji$ for all i and j. The product notion, AB, as in most network applications, represents the existence of a compound relation. That is, $ABij = 1$ if and only if for i and j there exists some k such that $Aik = 1$ and $Bkj = 1$. As shown in Lorrain and White (1971), we can derive compound relations by taking the binary product of the two matrices that represent the relations A and B.

[10] Potentially, the number of compound relations is infinite. Substantively, though, long chains of relationships are seldom salient (Hammer, 1980; Friedkin, 1983). We have not gone beyond compound relations of length 4 in our work because going from length 3 to length 4 seems in general to add little new information. Often, a relation box consisting only of direct and length 2 relations is sufficient.

[11] The notion of role relation presented here is asymmetric in nature in that the role relation that i has with j will usually differ from the role relation that j has with i. If we only consider symmetric relationships, the two role relations will necessarily be identical. If we consider sets of relations that include the complement of each relation, then knowing what i's role relation is with j tells us what j's role relation is with i. This follows from the fact that if $Sij = 1$, then $S'ji = 1$, and similarly with $Sij = 0$.

FIGURE 3. Role relations of school superintendent in Figure 1.

S	0	1	1	0	0	0	0	0
S'	0	0	0	0	0	0	0	0
S^2	0	0	0	1	1	1	1	1
S'^2	0	0	0	0	0	0	0	0
SS'	1	0	0	0	0	0	0	0
$S'S$	0	0	0	0	0	0	0	0
	S_1	P_1	P_2	T_1	T_2	T_3	T_4	T_5

S = supervisory relation
S' = supervised relation (transpose of S)
Each column is a role relation.

superintendent's role, yielding three classes of actors; the superintendent, the principals, and the teachers.[12]

Principal P_1, on the other hand, has five distinct role relations, corresponding to the classes of individuals that are distinct from his or her perspective; himself or herself, the teachers under principal P_1, the superintendent, the other principal, and the teachers under the other principal (see Figure 4).[13] In this sense, the social structure "looks" different from the superintendent's perspective as opposed to the principal's perspective. The superintendent "sees" three equivalence classes of individuals, whereas the principal "sees" five. Additionally, the two individuals have similar relations with different individuals. For instance, the superintendent is in the same role relation with the

[12] The example that we present here is particularly simple in that we have only considered one direct relationship and its complement (supervises and is supervised by) and the network presented is very sparse. In general, we would expect the role relations of an individual to be considerably more complicated than that presented here. The reader is invited to construct more sophisticated examples and to test the ideas presented here. We have not done so in order to avoid complicating the exposition.

[13] For this example, adding higher-order compound relationships would not change anything important. Both the superintendent and the principal would still have the same number of distinct relationships. However, the numerical results of the distance measure would be affected slightly.

FIGURE 4. Role relations of Principal P_1.

S	0	0	0	1	1	0	0	0
S'	1	0	0	0	0	0	0	0
S^2	0	0	0	0	0	0	0	0
S'^2	0	0	0	0	0	0	0	0
SS'	0	1	0	0	0	0	0	0
$S'S$	0	1	1	0	0	0	0	0
	S_1	P_1	P_2	T_1	T_2	T_3	T_4	T_5

principal that the principal is with each of his or her teachers. In this sense, role relations provide a way of characterizing differences and similarities in ego-centered networks.

In comparison to the approach described here, blockmodel analysis enforces a global partition of the population into blocks. However, it is important to point out the link between structural equivalence and role relations. In this example, T_1 and T_2 have the same role relations with S_1. They also have the same role relations with P_1. This is not a coincidence. In general, if individuals j and k are structurally equivalent, then for all individuals i in the population, $\mathbf{R}_{ij}$ will be identical to $\mathbf{R}_{ik}$. Thus, the definition of role being developed here will be consistent with the structural equivalence definition of position. Structural equivalence will imply role equivalence, though the reverse will not be true.

The importance of role relations lies in the fact that they allow for comparisons across populations. If we have two role relations from either the same or different populations and they are defined over the same relations, then we can compare them to see how close they are to being identical. As shown in the next section, this provides a basis for the comparison of role sets.

Role Sets. Merton describes role relations as the component parts that, when taken together, constitute roles. A role is characterized by its role set: "the complement of role relations in which persons are involved by virtue of occupying a particular status" (Merton, 1959,

p. 110).[14] The role set will be used to construct a definition of role that does not have the flaws of structural equivalence.

Role relations were defined as binary vectors. Viewed as such, role relations are not inherently linked to any particular population, even if they summarize an aspect of the overall social network. Moreover, it is not important for determining the role of the superintendent, for example, that we know which of the two identical role relations is going to which principal — so that the identities of the recipients of the role relations do not matter.

This observation leads to the key assumption of this approach, which is that the set of role relations that an individual has — his or her role set — can adequately characterize the individual's role. More formally:

> DEFINITION: *Two individuals are* role-equivalent *if their role sets contain the same role relations. That is, for every role relation associated with each individual, there is at least one (and perhaps more) identical role relation associated with the other individual.*[15]

This definition of role has several interesting features. First, since it is formulated in terms of role relations — treated as binary vectors and not particular recipients of ties — the definition is not limited to a single population. It can be used to compare the roles of actors from different populations or from different sections of the same population. Second, note that the types of role relations associated

[14] Our usage of the term *role* in this chapter is similar to the traditional usage of the word *status*. Over the last couple of decades, the term *status* has come to strictly denote a position within a hierarchical structure, which is why we have not used the term in this chapter.

[15] Role equivalence is a generalization of automorphic equivalence (Winship, 1974). Two individuals i and j are automorphically equivalent if there is an automorphism f such that $f(i) = j$. Note that automorphic equivalence is much stronger than role equivalence, since a necessary condition for two people to be automorphically equivalent is that their role sets must contain not only the same types of role relations but also the same number of each.

An idea related to the idea of a role relation is automorphic equivalence with respect to an individual (Winship, 1974; Mandel, 1978). Two individuals i and j are automorphically equivalent with respect to k if there exists an automorphism f such that $f(k) = k$ and $f(i) = j$. Individuals who are automorphically equivalent with respect to every other individual in a network are structurally equivalent (Winship, 1974).

with a position are important, but not the number of role relations of any particular type (as long as there is at least one). Intuitively, this makes sense. A principal is still a principal, whether he or she supervises one teacher or many.

Consider the school system in Figure 1 again. Now consider another school system in which there are similar relations except that each principal supervises four teachers. The role relations for this second superintendent will be identical to those of the first except that there will be eight teacher role relations rather than five. In every other way, the two superintendents will have exactly the same role sets. In terms of the preceding definition, the two superintendents are role-equivalent, since their role sets contain the same relations.

As this simple example shows, we now have a principle for comparing positions across populations. As such, we now have provided a formal definition of role that differs from the blockmodel definition of position.[16] As shown in the following sections, the ideas of role relation and role set also provide the basis for an empirical analysis of roles as opposed to positions in social networks.

The Relation Box. The construction of role sets can also be approached from a slightly different direction. Recall that each relationship—both direct and compound—on a social network can be represented by an $N \times N$ binary matrix (where N is the size of the population). Denote each of these relation matrices by $\mathbf{R}_{..k}$. Now line up these relation matrices, one behind the other, like so many record albums in a box. This three-dimensional binary array is called a *relation box.*[17] Figure 5 illustrates the relation box for the school system example. Table 1 provides a glossary of terms.

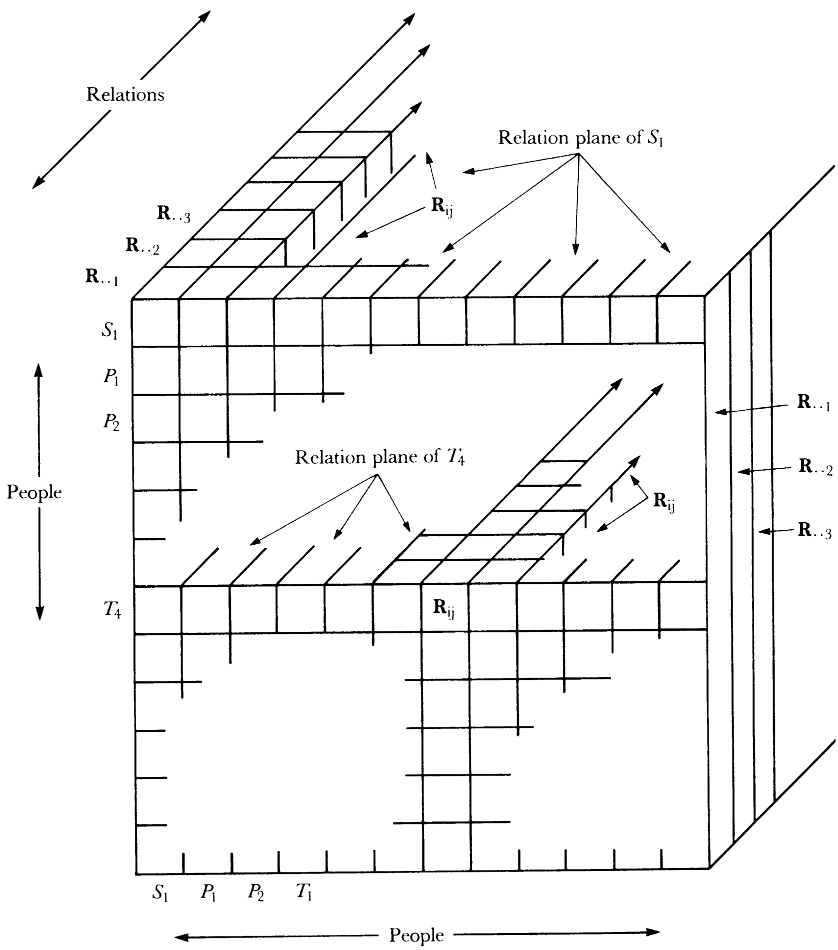
The dimensions of the relation box are $N \times N \times T$ (where T is the number of different types of relations).[18] Which relation matrices does the relation box contain? In general, the first few stacked relation matrices are the direct relations and their converses. For the school

[16] This also demonstrates how individuals in society might identify positions in different populations as having the same role. We, however, have not investigated the plausibility of our model as a social psychological process.

[17] In earlier work, *relation box* has been termed the Winship box (Mandel, 1978; Pattison, 1980).

[18] The same relation box can contain more than one distinct population by the simple expedient of treating them as if they were in the same network, but without any ties between the two populations. In this case, $N = N_1 + N_2$, where N_1 and N_2 are the number of individuals in the two populations.

FIGURE 5. Relation box.



system example, these are the relations of supervising and being supervised, S and S'. These are followed by the compound relations of length 2—S^2, SS', $S'S$, S'^2—and so on, for as long as desired.

A role relation, then, is a vector $\mathbf{R}_{ij}$ that runs through the relation matrices. For example, the vector $\mathbf{R}_{12}$ in the upper left-hand

TABLE 1
Key to Figure 5 and Glossary of Terms

Relation box: (**R**)	The three-dimensional matrix of zeros and 1's representing the direct and indirect relationships between individuals.
Relation matrix: ($\mathbf{R}_{..r}$)	The $n \times n$ matrix of zeros and 1's representing which individuals are in relation r.
	Examples: Figure 1, which indicates the pattern of supervisory relations in the school system example. $\mathbf{R}_{..1}$, $\mathbf{R}_{..2}$, $\mathbf{R}_{..3}$, and so on, in Figure 5.
Role relation: ($\mathbf{R}_{ij}$)	The vector of zeros and 1's representing the direct and indirect relations that exist between i and j.
	Examples: The columns of Figures 3 and 4, which are the role relations of the superintendent and the principal. The two vectors labeled $\mathbf{R}_{ij}$ in Figure 5.
Relation plane: ($\overline{R}_i$)	The matrix of role relations an individual has with other individuals.
	Examples: S_1's relation plane is shown in Figure 3. P_1's relation plane is shown in Figure 4. The top of the box in Figure 5 is also S_1's relation plane.
Role set: ($\mathbf{R}_1$)	An individual's set of role relations, with duplicates removed.

corner of Figure 5 is the role relation from S_1 to P_1.[19] The *relation plane* $\overline{R}_1$ is the set of all role relations of an actor, including duplicate role relations. The role set of an actor, $\mathbf{R}_i$, is derived from his or her relation plane by identifying role relations that are the same. Thus, in Figure 5, the top of the box is the relation plane of S_1.

It is easy to see the difference between structural equivalence and role equivalence in terms of the relation box. Two individuals i and j are structurally equivalent only if the role relations they have with each individual in the population are identical ($\mathbf{R}_{ip} = \mathbf{R}_{jp}$ for all p). Thus, when we look down the relation box from above, the two relation planes look identical. Role equivalence is a much weaker concept, since the two planes need not be identical. Instead, for every role relation in one individual's plane, there must be at least one other

[19] Note that as long as the relations generating the relation box either are symmetric or include the transpose of every nonsymmetric relation, the role relation $\mathbf{R}_{ij}$ actually contains the same information as $\mathbf{R}_{ji}$ (though rearranged).

identical role relation somewhere in the other individual's plane; that is, for every p, there must exist p' and p'' such that $\mathbf{R}_{ip} = \mathbf{R}_{jp'}$ and $\mathbf{R}_{jp} = \mathbf{R}''_{ip}$. Thus, structural equivalence implies role equivalence.

The relation box also provides a framework for comparing role sets across populations. Consider two social networks on distinct populations. Each network will generate a relation box.[20] For role relations from individuals in the two populations to be comparable, the ordering and type of the relation matrices in the two boxes must match. A role relation from the school system relation box, for example, could only be compared with role relations from a relation box generated by a supervising relation and converse of a supervising relation. In some cases, it may be necessary to drop some relations from one data set (or otherwise modify the given data) to make the two relation boxes comparable.

EMPIRICAL ANALYSIS

A Distance Measure. Just as most social networks have few actors who are strictly structurally equivalent, so are there few strictly role-equivalent actors in most empirical data. It is therefore necessary to have a distance measure that indicates how dissimilar the role sets of two individuals are. This section describes the distance measure used in the remainder of the chapter; it is simple and straightforward and yields reasonable results, but alternative measures are of course possible.[21]

The first stage in constructing our distance measure is to provide a way of calculating the distance between two role relations (assumed to

[20] It is possible to put the two populations into a single relation box, as in footnote 18, but it only makes sense under the same conditions needed for comparing two relation boxes. It is merely a matter of expository convenience whether the two populations are placed in the same or in different relation boxes.

[21] Different methods are not explored in this chapter. The purpose in this section is to provide illustrative analyses, not to map out the definitive approach to role analysis. In the actual analysis of a particular data set, the analyst would be wise to try out a number of different methods. As with blockmodels, different methods may well give different results. Only considerable experience will suggest what metrics and algorithms give the best results or are the most robust. For an initial attempt to carry out such a comparative analysis in blockmodeling framework, see Breiger, Arabie, and Boorman, 1975.

be of the same length and comparable, in the sense just described). The "city-block" metric seems the simplest one. In this case, the distance between role relations $\mathbf{R}_{ij}$ and $\mathbf{R}_{kl}$ is given by

$$d(\mathbf{R}_{ij}, \mathbf{R}_{kl}) = \sum_{r=1}^{T} |\mathbf{R}_{ijr} - \mathbf{R}_{klr}| \qquad (1)$$

where T is the number of relation matrices in the relation box (T is the length of the role relation). Note that the summation in the equation is across relations, not individuals. In effect, this metric counts the number of places in which the two role relations differ.[22]

Given two relation planes $\overline{R}_i$ and $\overline{R}_j$, our next step in constructing the distance measure is to specify how the role relations in one relation plane will be compared with the role relations in the other. One procedure is, for each role relation, to identify the role relation in the other relation plane that is the most similar. The sum of these minimum distances over all the role relations in both relation planes is the distance between the two actors' roles.

Formally, the distance measure D is defined as

$$D(\overline{R}_i, \overline{R}_k) = \sum_{j=1}^{N_1} \min_{l} [d(\mathbf{R}_{ij}, \mathbf{R}_{kl})] + \sum_{l=1}^{N_2} \min_{j} [d(\mathbf{R}_{kl}, \mathbf{R}_{ij})] \qquad (2)$$

where N_1 is the number of role relations in $\overline{R}_i$ (size of population 1), N_2 is the number of role relations in $\overline{R}_k$ (size of population 2), and where if $\overline{R}_i$ and $\overline{R}_k$ are in the same population, then $N_1 = N_2$.[23] This distance measure tells us how closely we can match the role relations in the two role planes.

To see how this measure works, consider again the school system in Figure 1. Figure 6 shows the distances between different actors. In the Appendix, we illustrate how these distances are calculated for one pair of individuals. As can be seen from the figure, individuals who have the same role labels (superintendent, principal, teacher) all are of

[22] One alternative at this point, which we have not explored, would be to cluster role relations into specific types and then use these types as the basis for determining how close individuals are to being role-equivalent. The approach discussed in the chapter is more direct.

[23] Note that the distance measure is defined on relation planes and not role sets. When two individuals are role-equivalent, this difference does not matter (since their distance is zero in any case). By a slight misuse of language, we will sometimes speak of the distance between the roles of two individuals.

FIGURE 6. Distance matrix for school system example.

	S_1	P_1	P_2	T_1	T_2	T_3	T_4	T_5
S_1	0	14	13	20	20	21	21	21
P_1	14	0	0	4	4	4	4	4
P_2	13	0	0	4	4	4	4	4
T_1	20	4	4	0	0	0	0	0
T_2	20	4	4	0	0	0	0	0
T_3	21	4	4	0	0	0	0	0
T_4	21	4	4	0	0	0	0	0
T_5	21	4	4	0	0	0	0	0

distance zero from their counterparts, thus indicating that they are role-equivalent. This results from the fact that for each individual, it is possible to perfectly match each role relation with at least one of the role relations in his or her counterpart's relation plane and that, vice versa, there is a perfect match for each of the counterpart's role relations in the individual's relation plane. If we had two school systems with similar structures, a similar set of matches would occur both within each school system and across the two school systems.

The distances in Figure 6 between individuals who are not in the same role are also informative.[24] The greatest distance is between the superintendents and teachers. This is logical, since their roles are the most dissimilar. Superintendents (in this example) are not supervised by anyone, and teachers (again, in this example) do not supervise anyone. Also note that the role of teacher is closer to that of principal than the role of principal is to that of superintendent. This can be understood by considering a variant of the distance measure given.

Note that Equation (2) is symmetric in i and j. The first term

[24] Note that the distances between similar "role pairs" are not identical in Figure 6. For instance, the superintendent is a distance 20 from teacher 1, but is a distance 21 from teacher 3. This is because our distance measure is defined on relation planes rather than role sets. In the present example, it would probably make more sense to use a distance measure defined on role sets. In actual empirical analysis, however, we have found that defining distances on the relation planes works better.

measures how closely the role relations of i can be matched to those of j, while the other term measures how closely the role relations of j can be matched to those of i. If the first term but not the second term is zero, then the role set of j includes the role set of i and not the reverse. In some sense, the role of i is simpler than that of j, and the role of i will be said to be *nested* in the role of j. The nested distance from i to j is

$$\overline{D}(\overline{R}_i, \overline{R}_k) = \sum_{j}^{N_1} \min_{1} [d(\mathbf{R}_{ij}, \mathbf{R}_{k1})] \tag{3}$$

In Figure 1, the nested distance from teacher to principal is 1, indicating that the role of teacher is almost perfectly nested within that of principal. This is because for each role relation in the teacher's role set, there is an equivalent role relation in the principal's role set with one exception. The superintendent is the teacher's supervisor's supervisor; but there is no comparable individual in the role set of the principal.

Applications. The purpose of this section is to illustrate how the role set definition of role, together with an associated distance measure, can be used to analyze empirical data. Several different types of applications will be presented: comparison of the roles of positions, both within and across populations; comparison of the roles of individuals; and comparison of the roles of positions with the roles of individuals (via the nesting concept).

First, however, it is necessary to comment on the difference between using positions as the units of analysis and using individuals. If positions were really determined by strict structural equivalence, then it would not matter which one was chosen. All individuals in the same position would be role-equivalent as well as structurally equivalent and would therefore have the same role set.

In practice, however, positions will be identified by weaker variants of structural equivalence, such as blockmodeling. The role set associated with such a position is determined by the blockmodel images on the reduced network. Individuals may be assigned to the same position who are not actually structurally equivalent and who may not be role-equivalent to either the other individuals in the same position or to the block (treated as an actor in the reduced network). Therefore, analysis on the position level and on the individual level can yield different results. Each level has its place, though, so this section will present examples of both.

FIGURE 7. Blockmodel images for the Bank Wiring group data.

Like

	A	B	C	D	E	F
A	1	1	0	0	1	0
B	1	0	0	0	0	0
C	0	0	0	0	0	0
D	0	0	0	1	1	0
E	1	0	0	1	0	0
F	0	0	0	0	0	0

Antagonism

	A	B	C	D	E	F
A	0	0	1	0	0	0
B	0	0	1	0	0	0
C	1	1	1	1	1	1
D	0	0	1	0	0	0
E	0	0	1	0	0	1
F	0	0	1	0	1	0

SOURCE: Adapted from White, Boorman, and Breiger (1976).

The sample data for the first set of examples will be drawn from the Bank Wiring Room study, collected by Roethlisberger and Dickson (1939) and further analyzed by Homans (1950). They recorded information on several different types of relations among the 14 workers in a factory assembly area. These men were labeled $W_1 - W_9$ (wiremen); S_1, S_2, S_4 (soldermen); and I_1, I_3 (inspectors).

Blockmodel analysis, following Homans, identified six positions in the population, labeled by $A-F$ (White, Boorman, and Breiger, 1976):

$$\underset{A}{(W_3\ S_1\ W_4)}\underset{B}{(W_1\ I_1)}\underset{C}{(W_2\ W_5\ I_3)}\underset{D}{(W_8\ W_9)}\underset{E}{(W_7\ S_4)}\underset{F}{(W_6\ S_2)}$$

The positions reflect the existence of two cliques in the population. The men in blocks *A-B-C* are in one clique, and the men in blocks *D-E-F* are in the other. Within the cliques, blocks *A* and *D* include the core members, blocks *B* and *E* the hangers-on, and blocks *C* and *F* the marginal members.

Each of these pairs of blocks—*A-D*, *B-E*, *C-F*—are cases where two distinct positions actually have the same roles. For instance, men in *A* and in *D* are playing the role "leader of a clique." Yet because they have ties to different individuals—each only likes men in his own clique—they are not in the same position in the population.

However, in the reduced network, blocks *A* and *D* do have very similar role sets, so our methods identify them as having similar roles. The same is true for the pairs *B-E* and *C-F*. These results were obtained by taking the blockmodel images for like and antagonism (as suggested by White, Boorman, and Breiger, 1976), and using them as the relations on the reduced network—the blockmodel images. The two images are shown in Figure 7.

A relation box with compounds up to length 4 was constructed using these images as generators.[25] Since both relations are symmetric, transposes were not included. As a result, the relation box includes 30 relation matrices: the 2 like and antagonism images, L and A; the 4 matrices corresponding to compound relations of length 2 (L^2, LA, AL, A^2); the 8 matrices of length 3; and the 16 matrices of length 4. Each position has six role relations in its relation plane, and each role relation is a binary vector with 30 1's or zeros.

The distance measure we have described can be used to calculate a matrix distance among the six positions. A clustering was then produced using Johnson's maximum-link method (Johnson, 1967):

$$(A\ D)\ (B\ E)\ (C\ F)$$

Blocks A and D were almost strictly role-equivalent, since $D(A, D) = 1$. Similarly, blocks B and E also have almost identical role sets, since $D(B, E) = 3$. This is exactly what we hoped to find, as previously explained. Blocks C and F are clustered together, but they have roles that are comparatively far apart: $D(C, F) = 11$. In fact, they are further apart than the (maximum) diameter of the A-D-B-F cluster, which supports Homans's (1950) suggestion that there was a much bigger difference between the two marginal positions than between the core and hangers-on positions. Moreover, the distance between the marginal positions and the others was on the order of 140—so that the marginal positions differ from the core and hangers-on positions much more than they differ from each other.

On the individual level, the role set approach produces similar results.[26] A relation box with compounds up to length 2 was con-

[25] The same results were obtained from a relation box of length 3, with only 14 relation matrices. An important question at this point is whether two relations should be both included in the relation box if they have identical matrices. In general, the answer to this question is yes. Although this equivalence may occur in this particular population, it may not be true of others. Although two populations may differ in this respect, it may still be the case that they have individuals who are in similar roles. To examine this, it is necessary that relations have not been collapsed within population.

[26] Clustering the relation box generated by like and antagonism on the individual level also produced similar results. Conversely, analyzing the relation box generated by the images of like, help, and the transpose of help on the position level yielded similar results as well, but it was harder to distinguish the core positions from the hangers-on positions.

structed using as generators the relations like, help, and transpose of help.[27] This relation box includes 12 relation matrices.

The distance measure was used to calculate how close the role sets of the 14 men were, and then the Johnson maximum-link method was used to get a clustering.[28] Choosing a distance cutoff of $D = 48$ yields the following clusterings:[29]

$$\begin{array}{l}(W_4\ W_8)(W_3\ W_9\ W_7)(W_1\ S_4\ S_1)(W_5\ S_2\ I_3\ I_1)(W_2\ W_6)\\ A\ \ D\ \ A\ \ D\ \ E\ \ B\ \ E\ \ A\ \ C\ \ F\ \ C\ \ B\ \ C\ \ F\end{array}$$

The letter below each individual identifies his block (as assigned in White, Boorman, and Breiger, 1976). The first two clusters consist predominantly of core members, the third cluster is mostly hangers-on, and the fourth and fifth clusters are marginal members. Note that every cluster contains members from both cliques. Moreover, since the previous analysis shows that the core and hangers-on positions are closer to each other than to the marginal positions, there is only one major difference between the position clustering and the individual clustering. That difference is the inclusion of I_1 with the marginal members. This anomaly will be discussed further in the next section.

Cross-Population Analysis. The point of the previous section is that there may be more than one position with the same role in a population. Moreover, our procedures can, for the well-studied data set given, pick out these role-equivalent positions in an intuitively reasonable way.

A potentially more interesting use of the role equivalence concept is in comparing the roles of actors from different populations. The example to be discussed here uses the Bank Wiring Room data and the network data collected by Kapferer (1972) on an African clothing factory. We describe this latter data set briefly.

Kapferer's clothing factory contained approximately 50 workers performing several different types of jobs: supervisors, line 1 tailors, line 2 tailors, line 3 tailors, and unskilled workers (in roughly

[27] "Help" means that one worker helps another with work tasks.

[28] Other clustering methods, like the Johnson minimum-link procedure, yielded different groupings.

[29] The distance cutoff was chosen to produce reasonable-looking clusters. The two closest roles in the population were those of $W5$ and $S2$ ($D = 7$). If the distance cutoff had been increased, the third and fourth groups would have been clustered together next.

FIGURE 8. Reduced network for Kapferer data, time 1. (Positions are, in order, supervisors, line 1 tailors, line 2 tailors, and all others.)

	A′	B′	C′	D′
A′	1	1	0	1
B′	1	1	0	0
C′	0	0	0	0
D′	0	0	0	0

Instrumental image

	A′	B′	C′	D′
A′	1	1	1	1
B′	1	1	0	0
C′	1	0	0	0
D′	1	0	0	1

Sociational image

decreasing order of skills). Kapferer recorded in binary matrix form two types of interactions over a period of months: nonsymmetric instrumental ties (that is, lending money or helping at work) and symmetric sociational ties (that is, conversation and friendship).

Kapferer suggested that the population could be divided into three clusters (positions): supervisors, line 1 tailors, and all others. Mandel (1977) refined this partition by distinguishing among four positions: supervisors, line 1 tailors, line 2 tailors, and all others. This partition did not produce a clean blockmodel image when imposed on the original network, but combining the images with Kapferer's analysis did yield a reduced network for the four-position population (A', B', C', D') (see Figure 8).[30]

The simplest way of comparing positions from two different populations is to treat them as if they were part of the same population and then proceed as in the previous section. However, it is essential to make sure that relations are comparable. The relations on the Kapferer data set are sociational, instrumental, and transpose of instrumental. Clearly, sociational ties are comparable to like ties on the Bank Wiring Room network. It is not obvious, though, whether instrumental ties should correspond to help ties or to the transpose of help ties. In the Bank Wiring Room, individuals in leadership positions (that is, W_3) receive help ties. On the other hand, leaders in the Kapferer population, like the supervisors, give out instrumental ties.

In the following analysis, therefore, the instrumental relation will be matched up with the transpose of help because they seem to have

[30] Using CONCOR on this population never yielded a satisfactory partition.

the same consequences for roles. There is as yet no formal procedure for determining when relations from two different data sets are comparable. It depends on the judgment of the analyst.

With this correspondence, two relation boxes with compounds up to length 2 were generated. One was on the six-block Bank Wiring Room population, using the reduced networks on like, transpose of help, and help as generators. The other was on the four-block Kapferer population, using the reduced network on sociational, instrumental, and transpose of instrumental (from Figure 8). Calculating the distance matrix for the 10 members of the combined population and using Johnson's maximum-link clustering method yield

$$(A\ B\ D\ E)\ (A'\ B'\ D')\ (C\ F)\ (C')$$

This clustering was generated by a distance cutoff of 18 (meaning that all clusters have a diameter of 18 or less, as measured by the maximum of the distances between all members). As the cutoff is increased up to 34, the first two groups are put together. The third cluster is about equidistant from the first two, at a distance of approximately 50. The fourth cluster—consisting only of C'—has a quite different role from the others and only gets merged with the other groups when the distance cutoff goes up to 70.

How can these results be interpreted? They show that the roles of the nonmarginal positions in the Bank Wiring Room are relatively closer to the roles of the non-line 2 positions (A', B', D') in the Kapferer factory—closer, at least, than to the roles of the marginal positions in their own population. Leaders have more in common with other leaders, even from different groups, than they do with the bottom members of their own group.

Equally important, the clustering shows that the line 2 tailors—position C'—have a much different role from the other positions in the Kapferer population. This is consistent with Kapferer's observations, since the line 2 tailors' place in the factory's social structure differed from that of other men. In a formal institutional sense, they had high prestige, since they had almost as much skill in tailoring as the line 1 tailors in a situation where tailoring ability was a major measure of a man's worth. However, unlike the other positions, their place in the informal social network did not match their level of formal prestige because the line 1 tailors refused to recognize them as their peers. Thus, as the men in the factory population struggled to increase their influence within the group, the line 2 tailors were being deprived of the benefits of their formal prestige. This will be further examined later in this section.

What about on the individual level? Theoretically, the 50 members of the Kapferer population could be put together with the 14 members of the Bank Wiring Room and then the clustering done as above. However, the relation planes from the Kapferer population will have 50 role relations in them, while the relation planes from the Bank Wiring Room will have only 14. As a result, one term in Equation (2) will invariably drown out the other.[31]

One approach to correcting this is to use the concept of nesting, as described earlier. The basic assumption underlying nesting is that if all the role relations in one role set are also found in another role set, but not the reverse, then the first role is a simplification of the second. The first role is nested in the second.

The natural application for the nesting concept is in comparing the role sets of positions with the role sets of individuals. In general, the role set of a position should provide a good model for the role sets of the men in that position. Moreover, in general, the relation planes of positions contain far fewer role relations than those of individuals. It is reasonable, then, that the role set of a position should be nested in—and thus be a simplification of—the role sets of the men in that position.

With this assumption, the nesting distance measure (Equation 3) can be used to assign individuals to positions. Given a network of ties among a set of positions, an individual can be matched with that position whose role provides the best model for the role of the individual—the role that is closest to being nested in the individual's role. The network of positions can be derived empirically from that popula-

[31] An alternative measure that would solve this problem would be to divide each term in Equation (2) by the number of summations, giving

$$D = \frac{1}{N_1} \sum_{j=1}^{N_1} \min_{l} \, [d(\mathbf{R}_{ij}, \mathbf{R}_{kl})] + \frac{1}{N_2} \sum_{l=1}^{N_2} \min_{j} \, [d(\mathbf{R}_{ij}, \mathbf{R}_{kl})]$$

For comparing two individuals from the same population, this change does not alter anything. However, it makes a big difference when the two individuals being compared come from different-sized populations. It is our position, however, that it is not in general a good idea to make ad hoc changes in an algorithm. The nesting principle provides a much more theoretically interesting way of dealing with the imbalance.

tion, can be derived from a network of another population, or can even be proposed for purely theoretical reasons. In each of these cases, the procedure sorts individuals into positions.

Alternately, the nesting principle can be used to choose among different blockmodels (reduced networks on positions). Posit, as above, that if a particular partition of a population accurately reflects its social structure, then the role set of each position (obtained by imposing the partition on the data and generating a blockmodel image) should be nested in the role sets of each of its members. Blockmodels can then be ranked by how well they fit this criterion. In fact, this was the procedure used to show that the four-position blockmodel of the Kapferer data set was much better than the three-position blockmodel (Mandel, 1978).

There is also a third way of using the nesting principle, and that is the one that will be demonstrated here. It is actually somewhat better suited for comparing individuals' roles with positions from a different population because it does not assume that the roles of all the men will have counterparts in the other social structure. In fact, the object of this procedure is to determine which individuals do in fact have roles that can be matched to positions from the second data set.

The "target" blockmodel, in the example to be presented here, will be the four-position reduced network on the Kapferer data set. The individuals will be men from the Bank Wiring Room, with like, help, and transpose of help generating a relation box of length 2. For each position-man pair, a nesting distance was calculated using Equation (3), and the resulting distance matrix examined.

The position-individual pair that came closest to being nested was C'-I_1, with only three errors ($\overline{D} = 3$). For almost every other man, the closest fitting position was 10 to 15 errors away from being nested. Recall from earlier analysis in this section that C'—the line 2 tailors—had a much different role from the other positions in the Kapferer population. Similarly, when the members of the Bank Wiring Room were clustered using their role sets, I_1 was the sole anomaly.

The close match in roles between C' and I_1 sheds some light on these earlier results. As described earlier, the line 2 tailors combined formal prestige with a lack of informal influence. Man I_1 was an inspector in the Bank Wiring Room. He therefore had a job where he had supervisory power, but as an emissary of management he probably exerted little influence in the informal social structure (at least in this situation).

This apparent parallel in the roles of C' and I_1 shows up in the relational data when the role sets are compared, even though the rest of the social structure they are each embedded in differs. Moreover, I_1 was hard to place in a cluster with other Bank Wiring Room individuals because his role was unique in the population. He did have analogs in the Kapferer factory, though.

What about the rest of the men in the Bank Wiring Room? For the marginal members of the population, the block C' also came closest to providing a model for their roles, but with a $\bar{D}$ of around 10. The block C' was thus not very close to being nested in their roles, but it was closer than the other blocks (with a $\bar{D}$ of between 25 and 30). By comparison, the core members and hangers-on did not fit very closely at all into the Kapferer social structure—no position stood out as providing a better model than any other.

Conversely, the supervisor position A' was not closely nested in the role of any member of the Bank Wiring Room. The man whose role it modeled best was W_8 (a core member), with $\bar{D} = 13$. This lack of "supervisors" in the Bank Wiring Room is reasonable, since no man in that population was really in the same position of power as the Kapferer supervisors were, combining as they did both formal prestige and informal influence. In fact, the position B', line 1 tailors, consistently provided a slightly better model for the roles of the core and hangers-on than did the block A'.

The conclusion is that while I_1 and perhaps the marginal members have roles with counterparts in the Kapferer factory social structure, the roles of the other members of the Bank Wiring Room do not clearly fit into it. This result reflects a difference in the overall social structures of the two populations large enough to overpower whatever similarities might exist between the roles of the Bank Wiring Room and the roles of the Kapferer factory.

CONCLUSION

Defining roles by means of role sets and role relations has two major advantages. The advantage on the theoretical level is that a clear distinction can be made between positions and roles. Positions are specific locations within particular social structures, containing individuals who are structurally equivalent or nearly so. Roles, in contrast, consist of individuals who have similar patterns of relations, although they may occupy different positions.

Empirically, the role set approach does not require knowing anything about the roles before comparing the roles of individuals either within or across populations. Thus, it is possible to ask whether different formal positions in a data set have the same role. Conversely, it is also possible to discover when positions that are in different populations but identified by the same label actually have different roles. The second advantage of the role set approach is that it rests solely on relational considerations.[32]

Both the theoretical and the empirical advantages of the role set approach derive from its willingness to give an explicit characterization of roles. In this chapter, role sets were built up from role relations and then compared with each other. The next step is to study *which* role relations two role sets share, rather than just looking at a numerical measure of dissimilarity. This will give substantive content to the role sets and role relations themselves.

Along the same lines, the role set approach suggests the notion of a local social structure. That is, if a person has identical role relations with some set of individuals, then those individuals are in some sense equivalent with respect to that person—whether or not they are equivalent with respect to any other person in the population. This idea, as well as the idea that ego-centered networks can be compared by looking at the similarities and differences in individuals' role sets, is an interesting one that needs to be developed further.

Finally, more empirical work needs to be done to test the power and robustness of the methods we have proposed. Tests need to be developed and carried out of the goodness of fit of results. The sensitivity of our approach to the metric and algorithm used should also be examined. Additionally, the potential effects of measurement error deserve scrutiny. Finally, we need to test whether the definition of role equivalence specified in this chapter works across other data sets before we can be convinced of its usefulness. This chapter has outlined a basic

[32] Although our approach to roles has been relational as opposed to normative, the ideas presented here are useful for normative questions as well. A critical question for a normative theory is how people know how to apply norms across different positions. We would argue that roles, as defined in this chapter, are capable of providing the needed identification. To quote Popitz (1972, p. 22), "We can only act in conformity with our roles if we can give ourselves and those concerned at the moment the correct social classification, if we can recognize therefore the particular social positions (roles) to which they are bound."

approach to a relational analysis of roles. It is for future research to judge its utility.

APPENDIX: ILLUSTRATIVE CALCULATION OF DISTANCES BETWEEN TWO RELATION PLANES

Figure 3 shows the relation plane of the school superintendent, S_1, in the school system example (Figure 1), and Figure 4 shows the relation plane of the principal, P_1, for the same example. Figure 9 indicates the distances between each of these two individuals' role relations (the columns in Figures 3 and 4) as defined by Equation (1) in the text. This distance is simply the number of entries by which two columns differ. For example, if we take the superintendent's role relation with teacher T_1 and compare it with principal P_1's role relation

FIGURE 9. Distance between superintendent's and Principal P_1's role relations in school system example.

		Principal P_1								
		S_1	P_1	P_2	T_1	T_2	T_3	T_4	T_5	Minimum of Each Row
School superintendent S_1	S_1	2	1	2	2	2	1	1	1	1
	P_1	2	3	2	0	0	1	1	1	0
	P_2	2	3	2	0	0	1	1	1	0
	T_1	2	3	2	2	2	1	1	1	1
	T_2	2	3	2	2	2	1	1	1	1
	T_3	2	3	2	2	2	1	1	1	1
	T_4	2	3	2	2	2	1	1	1	1
	T_5	2	2	2	2	2	1	1	1	1
Minimum of each column		2	1	2	0	0	1	1	1	

Sum of row minimums = nested distance from S_1 to P_1	6
Sum of column minimums = nested distance from P_1 to S_1	8
Sum of two nested distances = distance between S_1 and P_1	14

with P_2, we note that they differ for two different relations, S^2 and $S'S$. Thus, the distance between the two role relations is 2.

Equation (2) in the text defines the distance between two relation planes. The two minimum functions in the equation indicate how good the best match is between each role relation in one person's relation plane to a role relation in the other person's plane. In Figure 9, the column labeled "Minimum of Each Row" indicates how closely each role relation in the superintendent's relation plane can be matched to a role relation in the principal's relation plane. Similarly, the row labeled "Minimum of Each Column" indicates how closely each role relation in the principal's relation plane can be matched to a role relation in the superintendent's relation plane.

The sum of these minimum distances indicates how well each individual's relation plane is "nested" in the other individual's relation plane. These sums are equal to the nested distance from one individual to another as defined by Equation (3) in the text.

In the example illustrated in Figure 9, the nested distance from S_1 to P_1 is 6 and the nested distance from P_1 to S_1 is 8. The distance between S_1 and P_1 as defined by Equation (2) is then the sum of these two nested distances, 14.

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