Integrated Mathematics I

Standard 1

Algebra and Functions

CORE STANDARD

Relations and Functions

Determine whether a relation is a function or not a function. Identify the domain and range of a given relation. Translate among tables, graphs, words and equations.

[Standard Indicators: IM1.1.1, IM1.1.2, IM1.1.3]

CORE STANDARD

Linear Equations and Inequalities

Graphing and Writing Linear Equations

Graph linear functions and determine their slopes and *x*- and *y*-intercepts from their graphs and equations. Write a linear function in slope-intercept form. Determine the equation of a line given sufficient information.

[Standard Indicators: IM1.1.4, IM1.1.5, IM1.1.6]

Pairs of Linear Equations in Two Variables

Solve pairs of linear equations in two variables by graphing, substitution or elimination. Solve problems that can be modeled using pairs of linear equations in two variables.

[Standard Indicators: IM1.1.9, IM1.1.11]

Pairs of Linear Inequalities in Two Variables

Graph the solution for pairs of linear inequalities in two variables.

[Standard Indicator: IM1.1.10]

CORE STANDARD

Rational Exponents

Understand and use the laws of exponents for variables with exponents. Multiply, divide and find powers of variables with exponents.

[Standard Indicator: IM1.1.12]

CORE STANDARD

Quadratic Equations and Functions

Solve quadratic equations by graphing, factoring and using the quadratic formula. Graph quadratic functions and understand the relationship between its zeros and the *x*-intercepts of its graph. Solve problems that can be modeled using quadratic equations.

[Standard Indicators: IM1.1.12, IM1.1.13, IM1.1.14, IM1.1.15]

IM1.1.1 Determine whether a relation represented by a table, graph, words or equation is a function or not a function. Translate among tables, graphs, words and equations.

Example: For a square of side x, the area y is given by $y = x^2$. Is y a function of x? Is x a function of y? Answer the same questions for $y = x^2$ if you are told that this holds for negative as well as positive values of x.

IM1.1.2 Identify the domain and range of relations represented by tables, graphs, words and equations.

Example: What is the largest domain for x when $y = x^2$? What is the range of y in this case?

IM1.1.3 Translate among various representations of linear functions like tables, graphs, words and equations.

Example: Use a spreadsheet to create a table of values for the function $y = -\frac{1}{2}x + 5$. Graph the function.

IM1.1.4 Graph linear equations and show they have constant rates of change.

Example: Kathy borrowed $80 from her mother and plans to pay her mother $10 per week until the debt is paid. The equation for the amount of money Kathy owes her mother is $y = 80 - 10x$, where x is the number of weeks after the loan. Graph the equation. What does the slope of the graph represent?

IM1.1.5 Determine the slope, x-intercept and y-intercept of a line given its graph, its equation or two points on the line. Then determine the equation of a line given sufficient information.

Example: Find the slope and y-intercept of the line $4x + 6y = 12$.

IM1.1.6 Write, interpret and translate among equivalent forms of equations for linear functions (i.e., slope-intercept, point-slope and standard). Recognize that equivalent forms reveal more or less information about a given situation.

Example: Write the equation of the line $4x + 6y = 12$ in slope-intercept form. What is the slope of this line? Explain your answer.

IM1.1.7 Solve problems that can be modeled using linear equations and inequalities, interpret the solutions and determine whether the solutions are reasonable.

Example: As your family is traveling along an interstate, you note the distance traveled every five minutes. The distance is approximately the same. You graph the distance traveled as a function of time and assume that what was found for five-minute intervals holds for all time intervals up to two hours. Draw a linear graph representing this trip. Predict the time of a journey of 50 miles. What does the slope of the graph represent?

IM1.1.8 Graph a linear inequality in two variables.

Example: Draw the graph of the inequality $6x + 8y \geq 24$ on a coordinate plane.

IM1.1.9 Understand the relationship between a solution of a pair of linear equations in two variables and the graphs of the corresponding lines. Solve pairs of linear equations in two variables by graphing, substitution or elimination.

Example: Solve the system of equations: $2y + x = 10$ and $x = y + 3$. Graph the two lines and label the point of intersection.

IM1.1.10 Graph with and without technology the solution set for a pair of linear inequalities in two variables. Use the graph to find the solution set.

Example: Graph the inequalities $y \leq 4$ and $x + y \leq 5$. Shade the region where both inequalities are true.

IM1.1.11 Solve problems that can be modeled using pairs of linear equations containing two variables, interpret the solutions and determine whether the solutions are reasonable.

Example: The income a company makes from a certain product can be represented by the equation $y = 10.5x$, and the expenses for that product can be represented by the equation $y = 5.25x + 10{,}500$, where x is the amount of the product sold and y is the number of dollars. How many units of the product must be sold for the company to reach the break-even point?

IM1.1.12 Graph quadratic functions.

Example: Draw the graph of $y = x^2 – 3x + 2$. Using a graphing calculator or a spreadsheet to generate a data set, display the graph to check your work.

IM1.1.13 Solve quadratic equations in the real number system with real number solutions by factoring, by completing the square and by using the quadratic formula.

Example: Solve the equation $x^2 – x + 2 = 0$ in three ways: by factoring the polynomial, by using the quadratic formula and by completing the square. Derive the general quadratic formula by applying the method of completing the square to $ax^2 + bx + c = 0$.

IM1.1.14 Solve problems that can be modeled using quadratic equations, interpret the solutions and determine whether the solutions are reasonable.

Example: A ball falls so that its distance above the ground can be modeled by the equation $s = 100 – 16t^2$, where s is the distance above the ground in feet and t is the time in seconds. According to this model, at what time does the ball hit the ground?

IM1.1.15 Analyze and describe the relationships among the solutions of a quadratic equation, the zeros of a quadratic function, the x-intercepts of a graph of a quadratic function and the factors of a quadratic expression.

Example: A graphing calculator can be used to solve $3x^2 – 5x – 1 = 0$ to the nearest tenth. Justify using the x-intercepts of $y = 3x^2 – 5x – 1$ as the solutions of $3x^2 – 5x – 1 = 0$.

IM1.1.16 Sketch and interpret linear and non-linear graphs representing given situations and identify independent and dependent variables.

Example: The height (h) above water of a diver t seconds after she steps off a 100-foot-high platform is given by the formula $h = 100 – 16t$. Graph the function.

IM1.1.17 Add, subtract, multiply, divide, reduce and evaluate rational expressions with polynomial denominators. Simplify rational expressions with linear and quadratic denominators, including denominators with negative exponents.

Example: Simplify $\frac{x^2-4}{x^5} \div \frac{x^3-8}{x^8}$.

IM1.1.18 Solve equations involving rational expressions.

Example: Solve $\frac{x+5}{4} = \frac{3x+5}{7}$ and $\frac{8}{x} + \frac{28}{x^2-4} = \frac{7}{x-2}$.

IM1.1.19 Simplify radical expressions involving square roots.

Example: Assuming that x and y represent non-negative real numbers, simplify $\sqrt{18}xy^2$.

IM1.1.20 Solve equations that contain radical expressions on only one side of the equation and identify extraneous roots when they occur.

Example: Solve the equation $\sqrt{x+6} = x$.

Standard 2

Geometry and Measurement

CORE STANDARD

Coordinate Geometry

Find slopes, lengths and midpoints of line segments using coordinate geometry.

[Standard Indicators: IM1.2.1]

CORE STANDARD

Polygons

Find the sum of the measures of the interior and exterior angles of convex polygons. Deduce formulas relating lengths and sides, perimeters, and areas of regular polygons. Understand how limiting cases of such formulas leads to expressions for the circumference and the area of a circle.

[Standard Indicators: IM1.2.2, IM1.2.6]

CORE STANDARD

Congruence and Similarity

Solve problems involving congruent and similar polygons and solids.

[Standard Indicators: IM1.2.4]

CORE STANDARD

Transformations

Predict and describe the results of translations, reflections and rotations. Describe a motion or series of motions that will show that two shapes are congruent.

[Standard Indicator: IM1.2.5]

IM1.2.1 Find the length of line segments in one- or two-dimensional coordinate systems, the slopes of line segments in two-dimensional coordinate systems, and the point that is a given fractional distance from one end of the segment to another.

Example: Find the length of the line segment joining $A(3, 8)$ and $B(9, 0)$. Find the midpoint of this segment and the point that is one-third of the way from A to B.

IM1.2.2 Justifying the method used, find and use the sum of the measures of interior and exterior angles of convex polygons.

Example: Calculate the measure of one interior angle of a regular octagon. Explain your method.

IM1.2.3 Identify types of symmetry (i.e., line, point, rotational, self-congruences) of polygons.

Example: *ABCD* is a rhombus. Identify and describe its reflection and rotation symmetry.

IM1.2.4 Solve problems involving congruent and similar polygons.

Example: In the figures below, Figure A ≅ Figure B. Justify this statement: $\angle x \cong \angle y$.

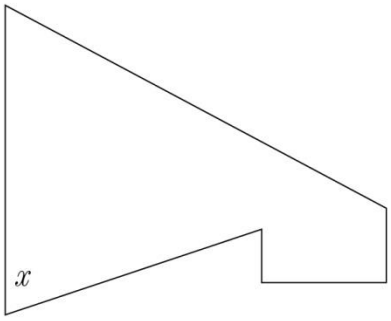

Figure A

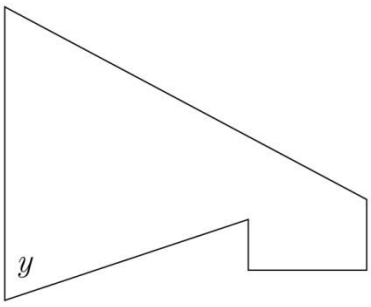

Figure B

IM1.2.5 Predict and describe the results of translations, reflections and rotations on polygons. Describe a motion or series of motions that will show that two shapes are congruent.

Example: Use trigonometric functions to find the perimeter and the area of a regular 12-gon that has been inscribed in a circle with radius r.

IM1.2.6 Deduce formulas relating lengths and sides, perimeters, and areas of regular polygons. Understand how limiting cases of such formulas lead to expressions for the circumference and area of a circle.

Example: Use the formula for the perimeter of a square to write a formula for the area of a square in terms of its perimeter.

IM1.2.7 Recognize and use coordinate geometry to verify properties of polygons such as regularity, congruence and similarity.

Example: Is the polygon formed by connecting the points (2, 1), (6, 2), (5, 6) and (1, 5) a square?

IM1.2.8 Understand the differences among supporting evidence, counterexamples and actual proofs.

Example: Draw and label a figure for the conjecture "If an angle bisector of a triangle is also an altitude, then the triangle is isosceles." Support your conjecture with supporting evidence. Then write a simple proof for your conjecture.

IM1.2.9 Develop simple geometric proofs (i.e., direct proofs, indirect proofs, proofs by contradiction and proofs involving coordinate geometry) using two-column, paragraphs and flow charts formats. Provide reasons for each statement in the proofs.

Example: In ΔLMN, $LM = LN$. Prove that $\angle LMN \cong \angle LNM$.

Standard 3
Data Analysis and Probability

IM1.3.1 Organize and display data using appropriate methods to detect patterns and departures from patterns. Summarize the data using measures of center (i.e., mean, median) and spread (i.e., range, percentiles, variance, standard deviation). Compare data sets using graphs and summary statistics.

Example: Design and conduct a survey about the number of electronic games owned by girls and boys in your school. Organize and display the results of your survey in an appropriate graph. Describe the technique you used to get a random sample. Find the mean, median and mode of your survey data. Which of these gives a useful summary of the data?

IM1.3.2 Distinguish between random and non-random sampling methods, identify possible sources of bias in sampling, describe how such bias can be controlled and reduced, evaluate the characteristics of a good survey and well-designed experiment, design simple experiments or investigations to collect data to answer questions of interest and make inferences from sample results.

Example: To determine what type of videos its customers liked, Drake Video surveyed every tenth person to walk in the store. Describe the sampling method used by Drake Video. Is it an unbiased sampling? Explain your answer.

IM1.3.3 Evaluate reports based on data published in the media by considering the source of the data, the design of the study, the way the data are analyzed and displayed, and whether the report confuses correlation with causation.

Example: Find an example of a graph in a newspaper or magazine that could be considered misleading. Explain why the graph could be misleading.

IM1.3.4 Use the relative frequency of a specified outcome of an event to estimate the probability of the outcome and apply the law of large numbers in simple examples.

Example: Use technology to simulate throwing two dice 500 times. Use the results to estimate the probability of rolling a 7 and then use the diagram of the sample space to find the theoretical probability.

Standard 4
Discrete Mathematics

IM1.4.1 Analyze and apply algorithms for searching (sequential, binary), for sorting (bubble sort, quick sort, bin sort) and for solving optimization problems.

Example: Use bubble sort to put 13, 14, 12, 11, 15 in increasing order.

IM1.4.2 Analyze and interpret relationships defined iteratively and recursively.

Example: Use the recursive definition of the Fibonacci numbers to find the fifth term.

IM1.4.3 Define arithmetic and geometric sequences recursively.

Example: There are 2,500 fish in a pond. Each year the population decreases by 25 percent, but 1,000 fish are added to the pond at the end of each year. Find the population in five years. Also, find the long-term population.

IM1.4.4 Determine the number of ways events can occur using permutations, combinations and the Fundamental Counting Principle.

Example: You are getting dressed one morning when you realize that you have far too many choices. You have six shirts to choose from, four pairs of jeans and three pairs of shoes. Ignoring color coordination, construct a tree diagram or other pictorial representation to show how many different outfits you could assemble.

IM1.4.5 Determine whether two propositions are logically equivalent.

Example: Show that "If today is Sunday, then we have school tomorrow," and "It is not Sunday or we have school tomorrow," are logically equivalent.

IM1.4.6 Distinguish between inductive and deductive reasoning. Identify inductive reasoning as central to the scientific method and deductive reasoning as characteristic of mathematics.

Example: What type of reasoning are you using when you look for a pattern?

PROCESS STANDARDS

Indiana's Academic Standards for Mathematics describe the key content of each grade level and course, and students must develop conceptual understanding of this content. The American Diploma Project noted that, "beyond acquiring procedural mathematical skills with their clear methods and boundaries, students need to master the more subjective skills of reading, interpreting, representing and 'mathematicizing' a problem" (p. 55).

The National Council of Teachers of Mathematics has described five Process Standards

that “highlight ways of acquiring and using content knowledge” (p. 29). The following Process Standards must be addressed throughout the learning and teaching of Indiana’s Academic Standards for Mathematics in all grade levels in mathematics.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate mathematical thinking through communication.
- Communicate mathematical thinking coherently and clearly to peers, teachers and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record and communicate mathematical ideas.
- Select, apply and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social and mathematical phenomena.

In addition, estimation, mental computation and technology are areas that need to be addressed at all grade levels in mathematics.

Estimation and Mental Computation

- Know and apply appropriate methods for estimating the results of computations.
- Use estimation to decide whether answers are reasonable.
- Decide when estimation is an appropriate strategy for solving a problem.
- Determine appropriate accuracy and precision of measurement in problem situations.
- Use properties of numbers and operations to perform mental computation.

- Recognize when the numbers involved in a computation allow for a mental computation strategy.

Technology

- Technology should be used as a tool in mathematics education to support and extend the mathematics curriculum.
- Technology can contribute to concept development, simulation, representation, communication and problem solving.
- The challenge is to ensure that technology supports, but is not a substitute for, the development of skills with basic operations, quantitative reasoning, and problem-solving skills.
    - Graphing calculators should be used to enhance middle school and high school students' understanding and skills.
    - The focus must be on learning mathematics and using technology as a tool rather than as an end unto itself.

References

American Diploma Project (2004). *Ready or not: Creating a high school diploma that counts.* Washington, DC: Achieve, Inc.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston VA: author.