

Platform or Wholesale? A Strategic Tool for Online Retailers to Benefit from Third-Party Information*

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*We thank the detailed and constructive comments from the review team, which have greatly improved the paper. We also thank participants at International Conference on Information Systems (2012), as well as seminar participants at Tsinghua University, Southern Methodist University, University of Hawaii, and The University of Texas at Dallas for their helpful feedback.

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Abstract

Online retailing is dominated by a channel structure in which a retailer either buys products from competing manufacturers and resells to consumers (wholesale scheme) or lets manufacturers directly sell to consumers on its platform for a commission (platform scheme). Easy access to publicly available third-party information such as product reviews which facilitate consumers' purchase decisions is another distinctive and ubiquitous characteristic of online retailing. We show that retailers can use the upstream pricing scheme, wholesale or platform, as a strategic tool to benefit from third-party information. Information on the quality dimension homogenizes consumers' perceived utility differences between competing products and increases the upstream competition, which benefits the retailer under the wholesale scheme but hurts the retailer under the platform scheme. Information on the fit dimension heterogenizes consumers' estimated fits to the products and softens the upstream competition, which hurts the retailer under the wholesale scheme but benefits the retailer under the platform scheme. Consequently, when the precision of the third-party information is high (low), a retailer can benefit from third-party information by adopting the wholesale (platform) scheme if the quality dimension plays a dominant role and by adopting the platform (wholesale) scheme if the fit dimension plays a dominant role. The results reveal that the quality information and fit information play very different roles in changing the upstream competition, and whether the retailer can benefit from the third-party information depends on its pricing scheme choice, the precision of the third-party information, and the relative importance of quality and fit attributes in consumers' evaluation of products.

Keywords: third-party information; pricing scheme; competition; game theory

1 Introduction

Online retailing has been continuously growing for the past few years. Forrester Research (2012) reported that online retail sales reached \$200 billion in 2011 and accounted for 7 percent of overall retail sales in the U.S. Two distinctive features of online retailing have been well documented (Chen and Xie, 2008; Abhishek et al., 2013). One, in addition to selling products using a wholesale scheme in which the retailer purchases products from manufacturers and then resells to consumers, an online retailer (e.g., Amazon) often lets others sell their products on its platform for a commission fee for each sale which we refer to as platform scheme. Two, regardless of the pricing scheme, online retail platforms and third-party websites (e.g., CNET.com and Consumersearch.com) routinely provide features that enable consumers and experts to post product reviews and read others’ reviews. Such third-party information has been deemed as an influential source for consumer decisions (Deloitte and Touche, 2008; Cone, 2010). In particular, third-party information has become an important information source for consumers to mitigate the uncertainty about the quality of a product and about its fit to their needs (Chen and Xie, 2008).

We use the term third-party information to refer to any publicly available and easily accessible product-related information created by parties other than the sellers—retailers and manufacturers. In the presence of third-party information, an online retailer is forced to rethink not only about how the information affects consumers but also about its impact on upstream firms, especially because a retailer generally cannot control the availability or content of third-party information and its influence on consumers, but a dominant retailer is often able to control its relationships with the upstream manufacturers. Consequently, from a retailer’s perspective, the impact of third-party information on manufacturers and its upstream strategies becomes significant.

One important decision related to a retailer’s upstream relationship with manufacturers is the pricing scheme adopted by the retailer. It appears that the pricing scheme—wholesale or platform—is a key strategic variable for online retailers. For instance, Amazon uses wholesale scheme for only 7% of the more than two million products in the “Electronics” category and the remaining 93% are sold under the platform scheme. On the other hand, Amazon uses wholesale scheme for 64 of the top 100 bestsellers in the electronics category (Jiang et al., 2011). The relative fraction of items sold using these schemes depends on product category too. For instance, while Amazon sells 16.7% of shoes directly, it sells only 3.1% of products in Sports & Outdoors category and 3.2% of products in the Jewelry category. Furthermore, the platform scheme, also referred to as the agency model, has become prevalent in many industries such as the e-book industry (Abhishek et al., 2013; Hagiu and Wright, 2013), and many retailers that operated as traditional reselling intermediaries have adopted platform

scheme in some product categories (e.g., Amazon.com, Alice.com, Jingdong.com).

We show in this paper that a dominant retailer can indeed use the upstream pricing scheme to benefit from the third-party information. Specifically, we show this result by answering the following key questions: How does the third-party information affect competition between upstream sellers in the online retailing industry? How does the pricing scheme affect the impact of the third-party information on the retailer and upstream sellers? How does product category affect this impact?

To address the above questions, we develop a game theoretic model in which a retailer either directly sells two substitutable products produced by different manufacturers or provides a platform for the manufacturers to sell their products. The products differ in both their qualities and the fits to consumers' needs. *Quality* of a product refers to the degree of excellence of the product and consumers agree on the preference order of quality in that they all prefer high quality to low quality. On the contrary, *fit* is consumer-specific—different consumers have different needs, with some consumers perceiving one product more suitable than the other product while others perceiving the opposite way. Each consumer has her own assessment of the quality of each product and its fit to her need. One or more third-parties provide additional information in both the quality and fit dimensions. We distinguish the case in which the quality dimension plays a dominant role in determining consumers' perceived utility differences between the two products, and the case in which the fit dimension plays an important role such that the fit is critical for some consumers. We call the former the *quality-dominates-fit case* and the latter the *fit-dominates-quality case*.

We find the third-party information in the quality dimension and fit dimension has very different effects on the competition between upstream manufacturers. For the quality dimension, we show that the third-party information reduces the heterogeneity between consumers' perceived quality differences which increases the competition between the two manufacturers. We call this reduced heterogeneity resulting from the the third-party information *variance-reducing effect*, which generally hurts manufacturers. This effect is more salient when the information in quality dimension is more precise. In addition, the third-party information shifts the mean perceived quality difference in favor of the product with favorable comments. We call this *mean-shifting effect*, which generally benefits the manufacturer that receives favorable information and the retailer. In contrast, for the fit dimension, we demonstrate that consumers are differentiated further from each other in their perceived fits because of the third-party information. We call this increased heterogeneity resulting from the third-party information *variance-increasing effect*, which softens the competition between the two manufacturers and generally benefits the manufacturers. This effect is more salient when the information in fit dimension is more precise.

Whether the retailer can benefit from the third-party information and how the preci-

sion of this information affects the retailer depends critically on the pricing scheme used by the retailer, in both quality-dominates-fit and fit-dominates-quality cases. In the quality-dominates-fit case, under the platform scheme, on the one hand, the intensified competition resulting from the variance-reducing effect tends to drive down the retail prices as well as manufacturers' total revenue. On the other hand, the mean-shifting effect tends to increase the total revenue generated by the two products. Notice that the retailer takes a fraction of manufacturers' revenues under platform scheme. Whether the retailer benefits from the third-party information depends on the tradeoff between the variance-reducing and mean-shifting effects. If the information regarding the quality of the two products are not very different such that the variance-reducing effect dominates the mean-shifting effect, the retailer's profit decreases as the third-party information becomes more precise. Under the wholesale scheme, in a sharp contrast, the retailer always derives higher profit with more precise third-party information. This is because the variance-reducing effect increases the upstream competition between the two manufacturers, which drives the wholesale prices down and increases the retailer's profit margins. In addition, the strengthened mean-shifting effect with improved precision continues to play a positive role in increasing the total profit from the two products. Consequently, in the quality-dominates-fit case, with the improved precision in third-party information, the retailer's preference may shift from the platform scheme to the wholesale scheme.

In the fit-dominates-quality case, the variance-increasing effect plays a main role in altering the upstream competition, while mean-shifting effect continues to exist if information regarding quality is different for the two products. Again, the pricing scheme plays an important role in how the retailer is affected by third-party information. Under the platform scheme, the reduced competition from the variance-increasing effect tends to increase the a manufacturer's revenue. The precision improvement in the fit dimension strengthens the variance-increasing effect. Contrary to the quality dimension, the improved precision in the fit dimension weakens the mean-shifting effect. Hence, whether the retailer has a higher profit because of the precision improvement depends on the tradeoff between these two effects. Under the wholesale scheme, in contrast, the retailer's profit always decreases as the precision in fit dimension improves, because the more softened upstream competition resulting from the stronger variance-increasing effect tends to drive up the wholesale prices which gives the retailer disadvantage in reselling. The mean-shifting effect also decreases with the better precision in fit dimension. Consequently, in contrast to the quality-dominates-fit case, in the fit-dominates-quality case, the retailer's preference can only shift in favor of platform scheme as the third-party information becomes more precise.

Additionally, we find that consumer surplus is higher under platform scheme than under wholesale scheme in both quality-dominates-fit case and fit-dominates-quality case. While an

improved precision of the third-party information increases the consumer surplus and social welfare when the quality difference between the products suggested by the information is mild in the quality-dominates-fit case, it does not necessarily increase the social welfare or consumer surplus in the fit-dominates-quality case. These findings together suggest that in the quality-dominates-fit case, when the retailer chooses the wholesale scheme over the platform scheme, the retailer benefits from the third-party information at the expense of consumers and the society but an improvement in precision can benefit all parties. On the other hand, in the fit-dominates-quality case, when the retailer chooses the platform scheme over the wholesale scheme, all parties can benefit from the third party information but an improvement in precision may only benefit the retailer.

Stated more generally, our main result is that when the third-party information mitigates consumers' uncertainty about quality and fit of competing products, the retailer's pricing schemes are critical in understanding the effect of third-party information on retailers because it changes the upstream competition. We show that this effect varies depending on whether quality or fit information plays a dominant role. More importantly, the effect of the third-party information on the retailer varies depending on whether the retailer provides a platform or sells directly. Consequently, retailers can use the pricing scheme effectively to their advantage in the presence of third-party information.

The rest of this paper is organized as follows. We review the related literature in the next section. In Section 3, we lay out the model. In Section 4, we derive the results of the effect of the third-party information on the upstream competition, the retailer, as well as consumer surplus and social welfare. In Section 5, we analyze the retailer's preference over the two pricing schemes and how the third-party information might shift the retailer's preference. We also present which pricing scheme is beneficial to consumers and social welfare. In Section 6, we present extensions of our main model and show our main results are generally supported. Section 7 concludes the paper.

2 Literature Review

Our study relates to the work that examines the effect of third-party information which is not controlled by sellers. Several recent studies have analyzed the effect of one of type of third-party information—product reviews—on firms. While some empirical studies find a significant positive association between rating valence and sales (Chevalier and Mayzlin, 2006; Clemons et al., 2006; Duan et al., 2008b), others do not find a relationship between the two (Chen et al., 2004; Liu, 2006; Duan et al., 2008a). Meanwhile, researchers find that the variance of product ratings (Clemons et al., 2006), the volume of ratings (Liu, 2006; Duan et al., 2008a), the reviewer characteristics and product characteristics (Forman et al., 2008;

Zhu and Zhang, 2010; Shen, 2008), and text reviews (Archak et al., 2011) have an impact on sales. These results suggest that sellers may have incentives to manipulate reviews of their products or adjust their marketing-mix strategies directed toward consumers to use reviews to their advantage. Dellarocas (2006) and Mayzlin (2006) analyze sellers’ incentives to manipulate the reviews and show that reviews are informative even under seller manipulation. Recent studies have also started to examine specific aspects of product reviews: the effect of review characteristics and product type on the review helpfulness (Mudambi and Schuff, 2010), the role of product reviews as a new measure of product types (Hong et al., 2012) and as a tool to reduce the uncertainty of product attributes (Hong and Pavlou, 2014), the interaction between promotional marketing and product reviews (Lu et al., 2013), and factors that affect the review posting behavior and review generation (Goes et al., 2014; Dellarocas et al., 2010; Zhu and Zhang, 2010; Rice, 2012; Lee et al., 2014). Different from these studies, we abstract away any specific type or aspect of third-party information and investigate how such information affects online retailers via its impact on upstream players in a channel structure, considering different upstream pricing schemes.

Our study also relates to the existing analytical work that models third-party information which enables consumers to identify products matching their needs (Chen and Xie, 2008) or estimate their true utilities more accurately (Li et al., 2011; Sun, 2012). These studies typically consider the effect of product reviews in a context of sellers directly selling to consumers. For example, Chen and Xie (2008) study how a seller should adjust the amount of information it provides to consumers in response to consumer reviews. They show that the seller can benefit from review supply only when it can ensure sufficiently large number of postings and small size of knowledgeable consumers. We differ from this stream of literature in that we consider a channel structure with a retailer either providing platform for competing manufacturers to sell their products or selling directly to consumers; furthermore, consumers face two dimensional uncertainty about a product—both the product quality and the fit to their needs in our model. Shaffer and Zettelmeyer (2002) analyze the effects of third-party information on the profits of channel members when they bargain over the division of profits. In their model, all consumers have the same product information, additional information has the same qualitative impact (positive or negative) on every consumer, and sellers have perfect knowledge of all product information. In our setting, however, consumers are uncertain about both product quality and the fit to consumers’ needs, and we study how third-party information affects the product competition by changing consumers’ perceived utilities. In particular, we consider that consumers have private estimates of the qualities of products and fits to their needs, and the information such as online product reviews provides public and common additional information about quality and private and idiosyncratic additional information about fit to consumers. More importantly, we focus on the effect of third-party

information on retailers and consumers under wholesale and platform schemes.

Another related stream of research is the recent studies on platform provision in online retailing (Jiang et al., 2011; Abhishek et al., 2013). For instance, Jiang et al. (2011) study a retailer’s choice of products to be sold under platform scheme when product demand is uncertain. Abhishek et al. (2013) study online retailers’ pricing strategies in a context where they compete with a traditional brick-and-mortar channel. They find the online retailers would prefer the platform scheme when online channel cannibalizes the traditional channel whereas they prefer the wholesale scheme when online channel stimulates the demand in traditional channel. Hagiu and Wright (2013) show that intermediary’s platform scheme choice can be affected by the level of marketing effort, and Johnson (2012) suggests that the market outcome under retailers’ different pricing schemes depends on the consumer lock-in effect of competing retailers and the differentiation between them. Foros et al. (2013) show that the intense competition between retailers can maximize the industry profit under the agency model. The focus of these papers is on the selection of pricing mode or a contract term in a channel when retailers compete with other retailers or/and with other channels. However, we focus on the effect of publicly provided third-party information on an online retailer’s pricing scheme preference and show the effect varies depending on the retailer’s pricing scheme choice (wholesale scheme versus platform scheme) and the type of information conveyed by third-party information (in the quality dimension versus in the fit dimension).

3 Model

We consider a retailer R that carries two products, A and B , produced by different manufacturers, and uses one of two pricing schemes—wholesale scheme and platform scheme. Under the wholesale scheme, the manufacturers sell their products to the retailer and the retailer re-sells them to consumers. Under the platform scheme, the manufacturers sell their products directly to consumers on the retailer’s platform, and the retailer charges a commission fee for each sale. Products A and B are imperfect substitutes. We call the manufacturer that produces product A (B) manufacturer A (B). The marginal production cost for each product is assumed to be zero. Each consumer has a unit demand.

Consumer Utility: Each product is characterized by a quality attribute and a fit attribute. The quality attribute represents the vertical dimension in the sense that every consumer prefers high quality to low quality. The fit attribute represents the horizontal dimension in the sense that preferences vary across consumers. The quality of a product determines the maximum value that a consumer derives from the product, which is denoted as x_i , $i \in \{A, B\}$. The products may not have perfect fits to consumers and thus consumers incur misfit costs. As in typical “location” models of product differentiation, the misfit cost is

modeled as the degree of misfit λ , $\lambda \in [0, 1]$, times a unit misfit cost t , and a consumer's degree of misfit to product A is negatively correlated to the misfit to product B . In particular, when the degree of misfit between a consumer and product A is λ , the degree of misfit between the consumer and product B is $(1 - \lambda)$. A consumer's utility from product i , U_i , is the maximum value that the product offers net the misfit cost. A consumer's net utility from a product is the utility net the retail price. Denoting the retail price as p_i , $i \in \{A, B\}$, we can formulate the net utilities derived from products A and B for the consumer with a degree of misfit λ to product A as follows.

$$\begin{cases} V_A = U_A - p_A = x_A - \lambda t - p_A \\ V_B = U_B - p_B = x_B - (1 - \lambda)t - p_B \end{cases}$$

Therefore, the net utility difference between product A and B , $V_A - V_B$, for the consumer with the degree of misfit λ to product A is

$$V_A - V_B = (U_A - U_B) - (p_A - p_B) = (x_A - x_B) + (1 - 2\lambda)t - (p_A - p_B) \quad (1)$$

We also call $(x_A - x_B)$ the quality difference of the two products. Unless otherwise indicated, we call λ the degree of misfit. We assume that the (true) quality difference between the products is zero. A continuum of consumers of measure 1 has different (true) degrees of misfit λ , which satisfies a uniform distribution.

Product Uncertainty and Third-Party Information: Different from the standard vertical differentiation and horizontal differentiation models, consumers are uncertain about both product quality and the misfit, where the third-party information such as online product reviews plays a role as in Chen and Xie (2008). That is, consumers do not know the true quality difference or their true degrees of misfit. In the absence of third-party information, based on the product description and other information sources, each consumer has her own assessment of the quality difference between the two products and of the misfit. We denote a consumer's own assessment of the quality difference as x_C . The third-party information provides public information about the products, and consumers use this information in addition to their own assessments to evaluate the products. We denote as x_R the perceived quality difference in the two products revealed by the third-party information, which is common to all consumers. In the presence of the third-party information, consumers combine their own assessments x_C and the public common assessment x_R to form their judgment of the quality difference between the two products. As shown by Bates and Granger (1969) using the minimum variance estimation, the posterior consumer's expected quality difference conditional

on the two sources of information becomes

$$\gamma x_C + (1 - \gamma)x_R \quad (2)$$

where γ , $\gamma \in (0, 1)$, depends on the relative precision of the two information sources, and the weight on the third-party information, $(1 - \gamma)$, is high when the precision of the third-party information is high.¹ Intuitively, consumers adjust their quality assessments because of the additional information from the third parties, and the extent to which the information affects consumers' assessments depends on the relative precision and confidence between their own assessments and the comments in the third-party information.

Similar to the approach often used in the literature (e.g., Lewis and Sappington, 1994; Ruckes, 2004; Johnson and Myatt, 2006; McCracken, 2011; Petriconi, 2012), the uncertainty in the misfit is modeled as that a consumer observes a signal s , which is equal to the consumer's true degree of misfit with probability β , and with probability $(1 - \beta)$ is uninformative and follows the true distribution; that is, $\Pr(s = y | \lambda = y) = \beta$ and $\Pr(s \neq y | \lambda = y) = 1 - \beta$, where $y \in [0, 1]$. With more accurate information provided by the third-party information, consumers know better about their idiosyncratic fit, which is modeled as a larger β . Based on Bayesian updating, we can derive $\mathbb{E}(\lambda | s = y) = [\beta y + (1 - \beta)/2]$ (see the proof in the appendix).² Substituting $[\gamma x_C + (1 - \gamma)x_R]$ and the expected degree of misfit into Equation (1), the expected utility difference between product A and B for the consumer with perceived quality difference x_C and signal $s = y$ on the degree of misfit is then

$$\mathbb{E}(U_A - U_B | x_C, y) = [\gamma x_C + (1 - \gamma)x_R] + (1 - 2y)\beta t \quad (3)$$

Different consumers perceive different x_C and receive different signals y . We assume that at the aggregate level consumers' perceived quality differences satisfy a uniform distribution over $[-\epsilon, \epsilon]$. The uncertainty model for the signal about misfit implies that the signals satisfy a uniform distribution over $[0, 1]$.³ The retailer and manufacturers do not know an individual consumer's perceived quality difference or signal about the misfit, but know their distributions.

Timing of the Game: The sequence of events under each pricing scheme is as follows. Under the platform scheme, in stage 1 of the game, the retailer announces the commission

¹Bates and Granger (1969) show that $\gamma = \frac{\sigma_R^2}{\sigma_c^2 + \sigma_R^2}$ where σ_R^2 is the variance of x_R and σ_c^2 is the variance of x_C . Denoting the inverse of variance as precision, $(1 - \gamma)$ can be defined as the precision of the third-party quality information as a fraction of the sum of the two precisions.

²Note that β is a proxy for the relative precision of third-party fit information and thus the quality update and fit update expressions are qualitatively equivalent.

³We note that the fit signal follows this uniform distribution only at the aggregate level regardless of the value of β . Conditional on a consumer, the signal distribution varies with β as discussed earlier.

rate k and a fixed fee m . In stage 2, manufacturers set retail prices p_i simultaneously knowing the commission and the fee retailer charges. In stage 3, consumers evaluate the difference of product utility and make their purchase decisions, and the manufacturers pay commission fees to the retailer. Under the wholesale scheme, in stage 1, manufacturers set wholesale prices w_i simultaneously. In stage 2, the retailer sets retail prices p_i . In stage 3, consumers make their purchase decisions by evaluating difference of product utility. Under both pricing scheme, the third-party information is observed by consumers, the manufacturers, and the retailer before they make their decisions.

Finally, we denote the reservation profit of manufacturer i as μ_i . The reservation profit represents the profit a manufacturer can obtain if it sells its product through an alternative channel. We assume that the reservation profits of the two manufacturers are equal when the manufacturers are ex ante symmetric; however, a manufacturer that has a more favorable third-party information about its product will have a higher reservation profit than the one that has a less favorable third-party information. Consumers' own estimates about product quality differences and their misfits are their private information. All other model parameters are common knowledge. All players are risk neutral.

Demand Functions: In the last stage of the game, consumers learn the expected utility differences between two products. Based on Equations (3), we can formulate a consumer's expected net utility difference as

$$\begin{aligned}\mathbb{E}(V_A - V_B|x_C, y) &= \mathbb{E}(U_A - U_B|x_C, y) - (p_A - p_B) \\ &= [\gamma x_C + (1 - \gamma)x_R] + (1 - 2y)\beta t - (p_A - p_B)\end{aligned}\tag{4}$$

in which $\mathbb{E}(U_A - U_B|x_C, y)$ is the consumer's expected utility difference, as defined in Equation (3). Clearly, besides consumers' own assessments, third-party information affects consumers' perceived net utility differences between the two products by changing γ and β . We focus our analysis on the cases in which third-party information plays a mild or moderate role in changing the competition between the two manufacturers such that in equilibrium two manufacturers are comparably competitive. The extreme case in which the additional difference revealed by third-party information is so dramatic such that one manufacturer has a dominant advantage in the market is not considered in this study.

We next distinguish two cases:

- **Quality-dominates-fit case:** in which consumers' own assessment on the quality dimension dominates the fit dimension such that there exist consumers who have the lowest fit with product A but derive higher net utility from it than from product B because their own assessment on the quality dimension is favorable toward product A , and there also exist consumers who have the lowest fit with product B but derive higher

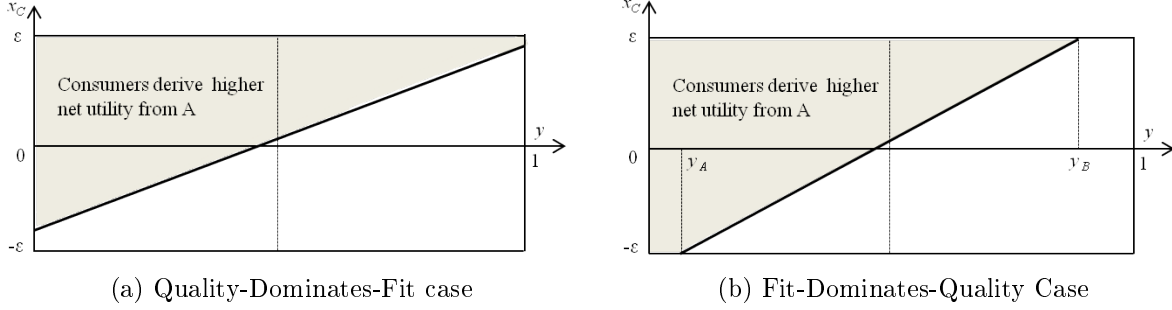


Figure 1: Competitive Cases

net utility from it. Figure 1a illustrates the quality-dominates-fit case.

- Fit-dominates-quality case: in which the fit dimension dominates consumers' own assessment on quality dimension such that consumers who have perfect fit with a product always derive a higher net utility from that product, regardless of their own assessment on the quality dimension. Figure 1b illustrates the fit-dominates-quality case.

The essential features that distinguish the above two cases are similar to those that distinguish *search goods* and *experience goods*—two important concepts of product types discussed widely in the economics literature (e.g., Nelson, 1970; Garvin, 1984; Sutton, 1986). Search goods are likely to come under the quality-dominates-fit case, and experience goods come under the fit-dominates-quality case. In general, products evaluated based on the objective indices such as product performance, reliability, and durability are likely to be quality-dominant (Garvin, 1984). Examples include digital camera, GPS, and hardware. Products evaluated based on subjective consumer-specific indices such as experience attributes, features, and aesthetics are more fit-dominant (Sutton, 1986). Examples include jewelry and video games.

In the quality-dominates-fit case, as illustrated in Figure 1a, for any consumer who receives signal y , $y \in [0, 1]$, her perceived net utility difference would be positive or negative, depending on her perceived quality difference. By Equation (4), if her perceived quality difference is higher than $\frac{1}{\gamma} [(p_A - p_B) - (1 - \gamma)x_R - (1 - 2y)\beta t]$, she derives higher net utility from product A; otherwise, she derives higher net utility from product B. Therefore, we can formulate the base demand for each product as⁴

$$\begin{aligned}\bar{D}_A &= \int_0^1 \int_{\frac{p_A - p_B - (1 - \gamma)x_R - (1 - 2y)\beta t}{\gamma}}^{\epsilon} \frac{1}{2\epsilon} dx dy = \frac{1}{2} - \frac{1}{2\gamma\epsilon} [p_A - p_B - (1 - \gamma)x_R] \\ \bar{D}_B &= \int_0^1 \int_{-\epsilon}^{\frac{p_A - p_B - (1 - \gamma)x_R - (1 - 2y)\beta t}{\gamma}} \frac{1}{2\epsilon} dx dy = \frac{1}{2} + \frac{1}{2\gamma\epsilon} [p_A - p_B - (1 - \gamma)x_R]\end{aligned}\tag{5}$$

⁴The derivation of base demand supposes that the maximum value each product delivers is high enough that consumers derive positive net utility from each product.

where the integral in product i 's base demand measures the consumers who derive higher net utility from product i than from the other product, $i \in \{A, B\}$.

In the fit-dominates-quality case, consumers who perceive a strong fit with product A always derive higher net utility from product A , and consumers who perceive a strong fit with product B always derive higher net utility from product B , regardless of the perceived quality difference. As illustrated in Figure 1b, we can denote the former consumer group as those who receive signal $y \in [0, y_A]$ and the latter as those who receive signal $y \in [y_B, 1]$ along the line, because of the monotonicity between the net utility difference and the fit dimension. The consumers who receive signals between y_A and y_B may derive higher net utility from product A or from product B , depending on their perceived quality differences. The marginal consumer y_A (y_B) is the one who derives the same utility from the two products when perceiving the largest quality difference against product A (B); that is, when $x_C = -\epsilon$ ($x_C = \epsilon$). By Equation (4), we have

$$\begin{aligned} y_A &= \frac{1}{2\beta t} [-\gamma\epsilon + (1 - \gamma)x_R + \beta t - (p_A - p_B)] \\ y_B &= \frac{1}{2\beta t} [\gamma\epsilon + (1 - \gamma)x_R + \beta t - (p_A - p_B)] \end{aligned} \quad (6)$$

We then can formulate the base demand for each product in fit-dominates-quality case as

$$\begin{aligned} \bar{D}_A &= \int_0^{y_A} dy + \int_{y_A}^{y_B} \int_{\frac{p_A - p_B - (1 - \gamma)x_R - (1 - 2y)\beta t}{\gamma}}^{\epsilon} \frac{1}{2\epsilon} dx dy = \frac{1}{2} - \frac{1}{2\beta t} [p_A - p_B - (1 - \gamma)x_R] \\ \bar{D}_B &= \int_{y_A}^{y_B} \int_{-\epsilon}^{\frac{p_A - p_B - (1 - \gamma)x_R - (1 - 2y)\beta t}{\gamma}} \frac{1}{2\epsilon} dx dy + \int_{y_B}^1 dy = \frac{1}{2} + \frac{1}{2\beta t} [p_A - p_B - (1 - \gamma)x_R] \end{aligned} \quad (7)$$

Notice that the above demand functions (5) and (7) are functions of $(p_A - p_B)$. Therefore, if consumers have no outside option, when the retail prices of both products are increased by the same margin, the demand for each product stays the same. We assume that some consumers may find an outside option (e.g., buying from a different retailer) more attractive than buying their preferred product from this focal channel, and that the number of such consumers is increasing in the product prices. This assumption ensures that the retailer or manufacturers suffer a penalty in the form of reduced demand for increasing product prices. We model this outside option using parameter α ($\alpha \in \mathbb{R}^+$) which denotes the marginal decrease in the demand for a product from a marginal increase in its price. Essentially, when the price of a product is increased, the marginal decrease in demand for that product comes from two sources: one, some consumers switch from that product to the other product but still buy from the focal channel (the marginal decrease because of switching between products within the focal retailer is $\frac{1}{2\gamma\epsilon}$ in quality-dominates-fit versus $\frac{1}{2\beta t}$ in fit-dominates-quality), and two, some consumers switch from the focal channel to an outside option (the marginal decrease because of switching to the outside option is α). For example, some consumers who

only consider purchasing a manufacturer's product that they are loyal to, may or may not purchase the product depending on its price. We model this outside option at an aggregate level in order to focus our attention on the effect of third-party information and to ensure tractability, but we can show our results similarly holds by modeling the consumer group explicitly.

From Equations (5) and (7), we notice that each firm's base demands in both cases take the same structure except the coefficients of the terms in the brackets (i.e., $\frac{1}{2\gamma\epsilon}$ in quality-dominates-fit versus $\frac{1}{2\beta t}$ in fit-dominates-quality). As a result, we can uniformly characterize the demand as follows, after incorporating demand loss because of outside option:

$$\begin{aligned} D_A &= \bar{D}_A - \alpha p_A = \left[\frac{1}{2} + \frac{1}{2\tau}(1 - \gamma)x_R \right] - \left(\frac{1}{2\tau} + \alpha \right) p_A + \frac{1}{2\tau} p_B \\ D_B &= \bar{D}_B - \alpha p_B = \left[\frac{1}{2} - \frac{1}{2\tau}(1 - \gamma)x_R \right] - \left(\frac{1}{2\tau} + \alpha \right) p_B + \frac{1}{2\tau} p_A \end{aligned} \quad (8)$$

where $\tau \in \{\gamma\epsilon, \beta t\}$ with $\tau = \gamma\epsilon$ for quality-dominates-fit case and $\tau = \beta t$ for fit-dominates-quality case. This expression evidently demonstrates that the assumptions that we impose on consumers' true and perceived preferences and distribution of consumers' perceived quality difference are equivalent to the assumptions on linear demand functions, which have been commonly used in the literature (e.g., Choi, 1991). Third-party information affects the competition between the two manufacturers via changing the parameters of the above demand functions. Table 1 summarizes the main notations used in the paper.

4 Effect of Third-Party Information

In this section, we first derive the subgame perfect equilibrium using backward induction. We then analyze the effects of third-party information on the competition between the two manufacturers and the retailer under both the platform scheme and the wholesale scheme. Without loss of generality, we consider $x_R \geq 0$; that is, the third-party information favors manufacturer A in the quality dimension.

4.1 Effect of Third-Party Information under Platform Scheme

In stage 2 of the game, the manufacturers maximize their profits by choosing the optimal prices given the commission rate k and the fixed fee m pre-announced by the retailer; that is,

$$\max_{p_i} \pi_i = (1 - k) p_i D_i - m \quad (9)$$

By the first-order conditions, we can derive the manufacturers' optimal retail prices. In stage 1 of the game, the retailer maximizes its profit by choosing the optimal commission rate k

Table 1: Summary of Notations

Notation	Definition and Comments
i	index for products/manufacturers
x_i	true quality of product i
λ	true degree of misfit between a consumer and product A
t	unit misfit cost
U_i	a consumer's utility derived from product i
V_i	a consumer's net utility derived from product i
α	price sensitivity of customers
x_C	a consumer's perceived quality difference between product A and B
ϵ	x_C satisfies a uniform distribution over $[-\epsilon, \epsilon]$
s	misfit signal
y	a consumer's degree of misfit toward product A indicated by misfit signal
x_R	quality difference between product A and B indicated by third-party information
γ	weight assigned to consumers' own assessment of the quality difference
β	probability that the indicated degree of misfit equals a consumer's true degree of misfit
p_i	retail price of product i
w_i	wholesale price of product i
D_i	demand for product i
π_i	manufacturer i 's profit
π_R	retailer's profit
k	commission rate (%) retailer takes per a product sold
m	fixed fee retailer charges to each manufacturer for platform provision
μ_i	manufacturer i 's profit when manufacturer i does not sell through the retailer's platform

and the fixed fee m ; that is,

$$\max_{k,m} \pi_R = k(p_A D_A + p_B D_B) + 2m \quad (10)$$

where $i \in \{A, B\}$ and D_i is specified in Equation (8). Based on the first-order conditions, subject to the condition that the manufacturers receive at least their reservation profits, we can obtain the optimal commission rate k and the fixed fee m . We summarize the equilibrium outcome in the following lemma.

Lemma 1. *Under the platform scheme, the equilibrium retail prices are*

$$p_A = \frac{\tau}{1+4\alpha\tau} + \frac{(1-\gamma)x_R}{3+4\alpha\tau} \quad (11)$$

$$p_B = \frac{\tau}{1+4\alpha\tau} - \frac{(1-\gamma)x_R}{3+4\alpha\tau} \quad (12)$$

and the commission rate and fixed fee are

$$k = 1 - \frac{(1+4\alpha\tau)(3+4\alpha\tau)(\mu_A - \mu_B)}{2(1-\gamma)(1+2\alpha\tau)x_R} \quad (13)$$

$$m = \frac{[\tau(3+4\alpha\tau) - (1-\gamma)(1+4\alpha\tau)x_R]^2 \mu_A - [\tau(3+4\alpha\tau) + (1-\gamma)(1+4\alpha\tau)x_R]^2 \mu_B}{4(1-\gamma)\tau(1+4\alpha\tau)(3+4\alpha\tau)x_R} \quad (14)$$

where $\tau \in \{\beta t, \gamma\epsilon\}$ with $\tau = \gamma\epsilon$ for the quality-dominates-fit case and $\tau = \beta t$ for the fit-dominates-quality case.

Proof. All proofs are in the appendix unless indicated otherwise. \square

4.1.1 Quality-Dominates-Fit Case under Platform Scheme

As explained in Equation (2), consumers combine their own quality assessment information with the third-party information, and the relative weights depend on the relative precision of the two information sources. When third-party information has a higher precision in quality, consumers put less weight (smaller γ) to her own assessment and more weight (higher $(1 - \gamma)$) to the third-party information. Hence, a higher $(1 - \gamma)$ reflects a higher (improved) precision in the third-party information. The following proposition summarizes the effects of precision of third-party information in quality dimension under the platform scheme.

Proposition 1. *Under the platform scheme, in the quality-dominates-fit case, as the third-party information becomes more precise (i.e., $(1 - \gamma)$ increases), product B's retail price decreases (i.e., $\frac{\partial p_B}{\partial(1-\gamma)} < 0$); product A's retail price decreases (i.e., $\frac{\partial p_A}{\partial(1-\gamma)} < 0$) if and only if*

$$x_R < \frac{\epsilon(3+4\alpha\gamma\epsilon)^2}{(3+4\alpha\epsilon)(1+4\alpha\gamma\epsilon)^2} \quad (15)$$

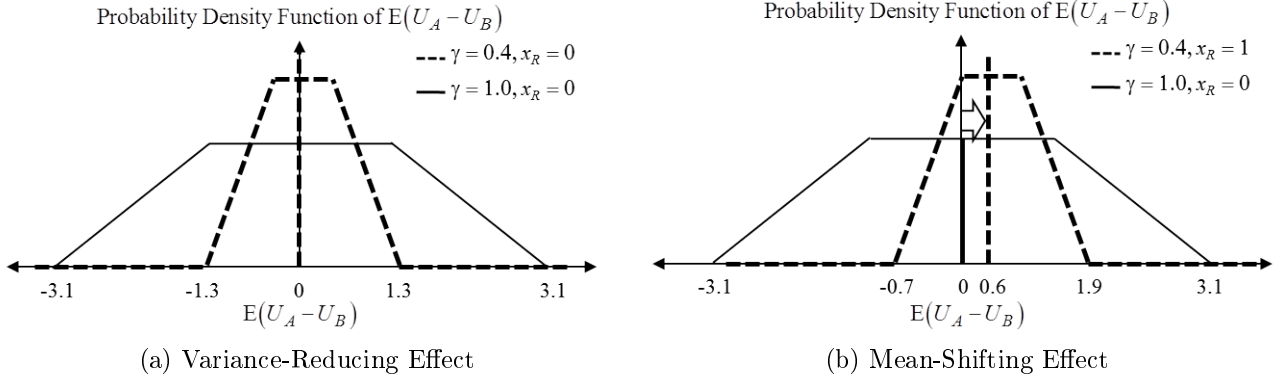


Figure 2: The Effect of Third-Party Information in Quality-Dominates-Fit Case
 $(\epsilon = 3, t = 0.2, \beta_{\gamma=1.0} = 0.5, \text{ and } \beta_{\gamma=0.4} = 0.6)$

The retailer's profit decreases (i.e., $\frac{\partial \pi_R}{\partial (1-\gamma)} < 0$) if and only if

$$x_R < \frac{\gamma \epsilon (3 + 4\alpha \gamma \epsilon) \sqrt{3 + 4\alpha \gamma \epsilon}}{(1 + 4\alpha \gamma \epsilon) \sqrt{(1 - \gamma)(1 + 4\alpha \gamma \epsilon)(3 + \gamma(3 + 4\alpha \epsilon(3 + \gamma(2 + 4\alpha \epsilon))))}} \quad (16)$$

In the symmetric case with $x_R = 0$, the conditions in Inequalities (15) and (16) are apparently satisfied and thus we have the following corollary.

Corollary 1. *Under the platform scheme, in the quality-dominates-fit case, in the presence of the symmetric third-party information in terms of quality (i.e., $x_R = 0$), as the third-party information becomes more precise, the retail prices decrease and retailer's profit decreases.*

The intuition for the symmetric case is as follows. In evaluating the utility difference between the two products, each consumer combines her own assessment with the quality difference assessment revealed by the third-party information. Because the quality difference revealed by the third-party information is public and common to all consumers, the presence of the information reduces the heterogeneity of consumers' perceived quality difference and thus reduces the heterogeneity of their perceived utility differences. Figure 2a illustrates the effect of the third-party information on the perceived utility differences in the symmetric case—when the information about quality difference is more precise, perceived utility differences become more homogenous. We call this the *variance-reducing* effect. The variance-reducing effect becomes stronger as the precision of the third-party information becomes higher (i.e., as $(1 - \gamma)$ increases) because a consumer puts more weight on the third-party information but less weight on her own assessment.

The reduced heterogeneity in consumers' perceived utility differences between the two products makes the two products more substitutable overall and makes consumers more price sensitive to a specific product, and thus it increases the competition between the two manufacturers. The increased substitutability and competition can also be seen from the

demand functions. In this symmetric case, the demand functions in Equation (8) can be rewritten as

$$D_i = \frac{1}{2} - \alpha p_i - \frac{1}{2\gamma\epsilon}(p_i - p_{\bar{i}}) \quad (17)$$

where $\{i, \bar{i}\} = \{A, B\}$. Note that the coefficient of the price difference term (i.e., $\frac{1}{2\gamma\epsilon}$ in this case) measures the substitutability between the two products: the larger the coefficient is, the more substitutable the two products are. The effect of the third-party information on the demand function is that it reduces γ , and thus it increases the substitutability between the two products. The increased competition between the two manufacturers drives their retail prices down as well as the revenues. Notice that the retailer's profit is a fraction of manufacturers' revenues. Therefore, the retailer's profit always decreases as the third-party information provides more accurate information; in other words, the retailer's profit is hurt by the third-party information and the profit decreases as the precision of the third-party information about the product quality difference improves.

In the general case in Proposition 1, the third-party information has asymmetric effect on each manufacturer. The favorable quality information toward product A (i.e., $x_R > 0$) uniformly changes each consumer's perceived quality difference between the two products favorably toward product A . As a result, in the presence of the favorable third-party information toward product A , on average consumers' perceived quality differences and thus their perceived utility differences between the two products are favorable for product A . We call this the *mean-shifting* effect. Figure 2b illustrates such an effect. In the demand functions outlined in Equation (8), the mean shifting is reflected in the shifting from product B 's potential market size to product A 's such that, compared to the symmetric case, manufacturer A 's potential market size increases (from $\frac{1}{2}$ to $[\frac{1}{2} + \frac{1}{2\gamma\epsilon}(1 - \gamma)x_R]$) and manufacturer B 's decreases (from $\frac{1}{2}$ to $[\frac{1}{2} - \frac{1}{2\gamma\epsilon}(1 - \gamma)x_R]$).

Manufacturer B may suffer from the reduced potential market size resulting from unfavorable third-party information, in addition to increased competition resulting from the variance-reducing effect as in the symmetric case. For manufacturer A , the favorable third-party information has a positive effect on its retail price because of the enhanced market potential, whereas the increased competition resulting from the variance-reducing effect has a negative effect. As a result, whether manufacturer A increases its retail price with more precise information depends on the balance between the gain from the mean-shifting effect and the loss from the variance-reducing effect. Inequality (15) pinpoints the condition showing that only if the third-party information is favorable enough, the retail price for product A increases as the third-party information becomes more precise in the quality dimension.

Although it is hurt by the variance-reducing effect, the retailer may benefit from the mean-shifting effect. The mean-shifting effect makes the demands asymmetric in terms of their

potential market sizes, which may increase the total revenue, compared to the symmetric case. For example, shifting the potential demand from product B to product A , per se, allows manufacturer A to charge a higher retail price for product A and receive a higher realized demand, at the cost of a lower retail price with a lower realized demand for product B . Notice the gain from the increased price and increased demand for product A outweighs the loss from the decreased price and decreased demand for product B , because the changes in both the price and demand are more significant for product A than product B due to A 's dominance in the market potential. Therefore, the increase of the degree of the asymmetry in the market potentials can increase the total revenue. When the third-party information is highly favorable to one product, the total revenue could be higher than the total revenue otherwise, despite the variance-reducing effect. With an increased precision, the difference between the impacts of the mean-shifting effect and the variance-reducing effect becomes more salient and the total revenue and thus the retailer's profit can be higher. Inequality (16) essentially shows this condition.

We next use the symmetric case to demonstrate the effect of the third-party information on consumer surplus and social welfare.

Proposition 2. *Under the platform scheme in the quality-dominates-fit case, in the presence of the symmetric third-party information in terms of quality (i.e., $x_R = 0$), as the third-party information becomes more precise, consumer surplus and social welfare increase.*

Consumer surplus depends on three factors: the prices consumers pay, the consumer misfit costs, and the number of consumers who purchase. The increased precision of the third-party information in quality dimension intensifies the competition between manufacturers and this reduces the prices and increases the number of consumers that buy. Furthermore, the improved precision also increases the likelihood of consumers buying their preferred products. Therefore, the precision improvement has a positive impact on all of the above factors, and thus increases the consumer surplus. Social welfare also increases with increased precision because the market size expands and consumer misfit cost decreases.

When x_R is large, however, consumer surplus and social welfare can decrease in the precision because the increase in $(1 - \gamma)$ makes the mean-shifting effect more salient and increases the price of the product with the favorable third-party information. In this case, the aforementioned positive effect by the variance-reducing effect can be offset by the negative effect of the mean-shifting effect.

4.1.2 Fit-Dominates-Quality Case under Platform Scheme

In the fit dimension, the third-party information is modeled as each consumer observes a signal which equals the true degree of misfit with probability β . When the precision of the fit

dimension improves, consumers are more certain about their fits to each product. Therefore, a higher β reflects the improved precision in the fit dimension. The following proposition summarizes the effects of the precision in fit dimension of third-party information under platform scheme.

Proposition 3. *Under the platform scheme, in the fit-dominates-quality case, as the third-party information becomes more precise (i.e., as β increases), product B's retail price increases (i.e., $\frac{\partial p_B}{\partial \beta} > 0$); Product A's retail price increases (i.e., $\frac{\partial p_A}{\partial \beta} > 0$) if and only if*

$$x_R < \frac{(3+4\alpha\beta t)^2}{4\alpha(1-\gamma)(1+4\alpha\beta t)^2} \quad (18)$$

The retailer's profit increases (i.e., $\frac{\partial \pi_R}{\partial \beta} > 0$) if and only if

$$x_R < \frac{\beta t(3+4\alpha\beta t)\sqrt{3+4\alpha\beta t}}{(1-\gamma)(1+4\alpha\beta t)\sqrt{(1+4\alpha\beta t)(3+4\alpha\beta t)(3+4\alpha\beta t)}} \quad (19)$$

In the symmetric case with $x_R = 0$, the condition in Inequality (18) and (19) are satisfied, and we thus have the following corollary.

Corollary 2. *Under the platform scheme in the fit-dominates-quality case, in the presence of the symmetric third-party information in terms of quality (i.e., $x_R = 0$), as the third-party information becomes more precise, the retail prices increase and retailer's profit increases.*

The intuition for the symmetric case is as follows. Different from the quality dimension in which the true quality difference is the same for all consumers and the third-party information adds a common component in evaluating the quality difference across all consumers, in the fit dimension consumers have different true preferences and the third-party information allows them to further calibrate their own fits. With the additional information, consumers become less uncertain about the products' fits to their needs. The reduced uncertainty thus makes consumers more heterogeneous in terms of their perceived fits, which tends to increase the heterogeneity in consumers' perceived utility differences. We call this *variance-increasing* effect. Figure 3a illustrates the effect of the third-party information on the perceived utility differences in this symmetric case. Similar to the quality dimension, this variance-increasing effect becomes stronger as the precision of the third-party information improves (i.e., increase in β).

The increased heterogeneity in consumers' perceived utility differences between the two products makes the two products less substitutable overall and makes consumers less price sensitive to a specific product, and thus it softens the competition between the two manufacturers. The decreased substitutability and competition can also be seen from the demand

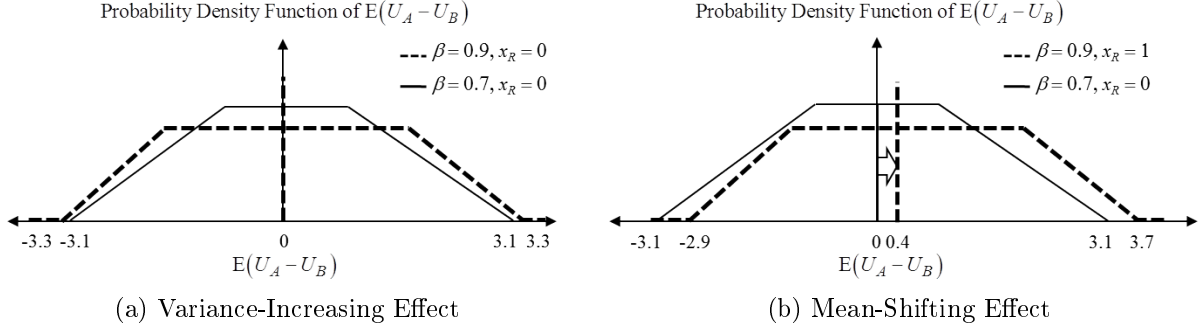


Figure 3: The Effect of Third-Party Information in Fit-Dominates-Quality Case
 $(t = 3, \epsilon = 1, \gamma_{\beta=0.7} = 1.0, \text{ and } \gamma_{\beta=0.9} = 0.6)$

functions. In this symmetric case, the demand functions in Equation (8) can be rewritten as

$$D_i = \frac{1}{2} - \alpha p_i - \frac{1}{2\beta t}(p_i - p_{\bar{i}}) \quad (20)$$

where $\{i, \bar{i}\} = \{A, B\}$. As discussed previously, the larger the coefficient of the price difference term (i.e., $\frac{1}{2\beta t}$ in this case) is, the more substitutable the two products are. The effect of third-party information on the demand function is that it increases β and thus it decreases the substitutability between the two products. The softened competition between the two manufacturers increases their retail prices as well as the revenues. Therefore, the retailer's profit always increases as the third-party information provides more accurate information.

In the general case as prescribed in Proposition 3, the third-party information has asymmetric effect on each manufacturer. As in the quality-dominates-fit case and as illustrated in Figure 3b, the mean-shifting effect continues to exist (i.e., $x_R > 0$). In the demand functions outlined in Equation (8), the mean shifting is reflected in an increase in manufacturer A 's potential market size (from $\frac{1}{2}$ to $[\frac{1}{2} + \frac{1}{2\beta t}(1-r)x_R]$) and a decrease in manufacturer B 's (from $\frac{1}{2}$ to $[\frac{1}{2} - \frac{1}{2\beta t}(1-r)x_R]$). For manufacturer B , the unfavorable third-party information has a negative effect on its retail price because of the reduced appeal in the market, whereas the softened competition resulting from the variance-increasing effect, as in the symmetric case, has a positive effect. Interestingly, the increase in the precision in the fit dimension always increases manufacturer B 's retail price in two ways. First, with the improved precision, the variance-increasing effect becomes more salient, which induces manufacturer B to charge a higher retail price. Second, the improved precision in the fit dimension mitigates the negative effect of the unfavorable third-party information on product B 's market potential (i.e., its market potential $[\frac{1}{2} - \frac{1}{2\beta t}(1-r)x_R]$ increases with the increase of β).

Similarly, the changes in the variance-increasing effect and the mean-shifting effect also influence product A 's retail price and demand: the increased potential market size result-

ing from the favorable information and softened competition resulting from the variance-increasing effect as in the symmetric case. The increased precision in the fit dimension, on the one hand, as that for product B , makes the variance-increasing effect more salient, which tends to drive up product A 's retail price. On the other hand, the improved precision also makes consumers less sensitive to the third-party information for the quality difference and eventually makes the increase of product A 's retail price from the favorable mean-shifting effect less. If the third-party information is strongly favorable, this reduction in the price increase can dominate. As a result, only if the favorable information is mild, manufacturer A increases its retail price from the precision improvement. Inequality (18) illustrates this condition. Therefore, whether the retailer benefits from the improved precision also depends on the magnitude of the mean-shifting effect as presented in Inequality (19).

Proposition 4. *Under the platform scheme in the fit-dominates-quality case, in the presence of the symmetric third-party information in terms of quality (i.e., $x_R = 0$), as the third-party information becomes more precise, consumer surplus decreases if and only if*

$$\gamma^2 < \frac{12\beta^2 t^2 (1+4\alpha\beta t)^2 \left[\frac{2\alpha x}{(1+4\alpha\beta t)^2} + \frac{9+4\alpha\beta t}{8(1+4\alpha\beta t)^3} - \frac{3+8\alpha t(1+\beta+2\alpha\beta^2 t)}{8(1+4\alpha\beta t)^2} \right]}{\epsilon^2 (1+8\alpha\beta t(1+\alpha\beta t))} \quad (21)$$

Social welfare decreases if and only if

$$\gamma^2 < \frac{3\beta^2 t^2 [4\alpha(2x-t(1+\beta+2\alpha\beta^2 t))-1]}{\epsilon^2 (1+8\alpha\beta t(1+\alpha\beta t))} \quad (22)$$

An increase in the precision affects the consumer surplus in the fit-dominates-quality case in the following way. An increase in the precision increases the prices, which has a negative effect on the consumer surplus, decreases the misfit costs, which has a positive effect on the consumer surplus, and reduces the number of consumers that buy, which has a negative effect on consumer surplus. In the fit-dominates-quality case, consumers who receive extreme fit signals which strongly suggest one product over the other buy the product suggested by the signal, regardless of the quality assessment. On the other hand, consumers who receive mild fit signals consider both quality and fit assessment while choosing the product to buy. An improvement in the precision of the fit information will have a larger impact on the consumer surplus of the latter group of consumers than the former. This is because, for consumers in the latter group, a better precision on the fit dimension can offset errors in purchase decisions caused by errors in quality assessment; however, since quality assessment does not play any role in the purchase decisions of the former group, the impact of improvement in fit precision does not have the same magnitude for the former group as the latter group. The size of the first group of consumers—those whose purchase decisions are not affected by quality assessment—increases as γ decreases. Consequently, the reduction in misfit cost

that comes from an improvement in precision is offset by the demand contraction and the price increase when γ is low. A similar reasoning applies for the social welfare also except that in this case only the market size reduction and misfit cost reduction effects are present.

As x_R increases, consumer surplus and social welfare tend to increase because the asymmetry in quality information reduces the price-increasing impact caused by an increase in β . Thus, different from the quality-dominates-fit case, the improved precision in fit dimension tends to increase the social welfare.

4.2 Effect of Third-Party Information under Wholesale Scheme

As in the analysis of the platform scheme, using backward induction, we derive the equilibrium wholesale prices and retail prices under the wholesale scheme. In stage 2 of the game, the retailer maximizes its profit given wholesale prices by choosing the optimal retail price for each product; that is,

$$\max_{p_A, p_B} \pi_R = (p_A - w_A)D_A + (p_B - w_B)D_B \quad (23)$$

By the first-order conditions, we can derive the retailer's optimal prices, which are functions of wholesale prices. In stage 1 of the game, anticipating the retailer's reaction in response to the wholesale prices, the manufacturers maximize their profits by choosing their optimal prices; that is,

$$\max_{w_i} \pi_i = w_i D_i, i \in \{A, B\} \quad (24)$$

Based on the first-order conditions, we can obtain the optimal wholesale price for each manufacturer. We summarize the equilibrium outcome in the following lemma.

Lemma 2. *Under the wholesale scheme, the equilibrium wholesale prices are*

$$w_A = \frac{\tau}{1+4\alpha\tau} + \frac{(1-\gamma)x_R}{3+4\alpha\tau} \quad (25)$$

$$w_B = \frac{\tau}{1+4\alpha\tau} - \frac{(1-\gamma)x_R}{3+4\alpha\tau} \quad (26)$$

and retail prices are

$$p_A = \frac{1+6\alpha\tau}{4\alpha(1+4\alpha\tau)} + \frac{(5+6\alpha\tau)(1-\gamma)x_R}{4(1+\alpha\tau)(3+4\alpha\tau)} \quad (27)$$

$$p_B = \frac{1+6\alpha\tau}{4\alpha(1+4\alpha\tau)} - \frac{(5+6\alpha\tau)(1-\gamma)x_R}{4(1+\alpha\tau)(3+4\alpha\tau)} \quad (28)$$

where $\tau \in \{\beta t, \gamma\epsilon\}$ with $\tau = \gamma\epsilon$ for the quality-dominates-fit case and $\tau = \beta t$ for the fit-dominates-quality case.

4.2.1 Quality-Dominates-Fit Case under Wholesale Scheme

The following proposition summarizes the effects of the precision of third-party information in quality dimension under wholesale scheme.

Proposition 5. *Under the wholesale scheme, in the quality-dominates-fit case, as the third-party information becomes more precise (i.e., as $(1 - \gamma)$ increases)*

(a) *Product B's wholesale price decreases (i.e., $\frac{\partial w_B}{\partial(1-\gamma)} < 0$); Product A's wholesale price decreases (i.e., $\frac{\partial w_A}{\partial(1-\gamma)} < 0$) if and only if*

$$x_R < \frac{\epsilon(3+4\alpha\gamma\epsilon)^2}{(3+4\alpha\epsilon)(1+4\alpha\gamma\epsilon)^2} \quad (29)$$

(b) *Product B's retail price decreases and the retailer's profit increases (i.e., $\frac{\partial p_B}{\partial(1-\gamma)} < 0$ and $\frac{\partial \pi_R}{\partial(1-\gamma)} > 0$); Product A's retail price decreases (i.e., $\frac{\partial p_A}{\partial(1-\gamma)} < 0$) if and only if*

$$x_R < \frac{2\epsilon(1+\alpha\gamma\epsilon)^2(3+4\alpha\gamma\epsilon)^2}{(1+4\alpha\gamma\epsilon)^2(15+\alpha\epsilon(17+2\gamma(18+\alpha\epsilon(20+\gamma(11+12\alpha\epsilon))))))} \quad (30)$$

In the symmetric case with $x_R = 0$, the conditions in Inequalities (29) and (30) are apparently satisfied and thus we have the following corollary.

Corollary 3. *Under the wholesale scheme in the quality-dominates-fit case, in the presence of the symmetric third-party information in terms of quality (i.e., $x_R = 0$), as the third-party information becomes more precise, both the wholesale prices and retail prices decrease and retailer's profit increases.*

The intuition for the effect of the third-party information on upstream competition for the symmetric case is similar to the case under platform scheme. As illustrated in section 4.1.1, the increased upstream competition caused by the *variance-reducing* effect of the third-party information drives their wholesale prices down. As a result, the retailer benefits. With the lower wholesale prices, the retailer can lower its retail prices to increase the demand for each product while increasing its profit margin from each sale at the same time. Therefore, the retailer's profit increases. Because the variance-reducing effect becomes stronger as the precision of the third-party information increases (i.e., as $(1 - \gamma)$ increases), with more precise third-party information, the prices are further reduced and retailer has higher profit from the increased competition.

In the general case (i.e., $x_R > 0$) as prescribed in Proposition 5, the *mean-shifting* effect, as illustrated in section 4.1.1, also plays a role in the equilibrium outcome. For manufacturer B, the unfavorable third-party information, in addition to the increased competition because of the variance-reducing effect has a negative effect. An increase in the precision of the third-party information makes both effects more salient and the manufacturer B charges a

lower wholesale price, which in turn induces the retailer to charge a lower retail price for product B . For manufacturer A , the favorable third-party information has a positive effect on its wholesale price, whereas the increased competition resulting from the variance-reducing effect has a negative effect. Whether the precision improvement increases the wholesale price depends on the magnitude of the two effects. Inequality (29) shows that when the positive third-party information is mild, the variance-reducing effect dominates the mean-shifting effect, and thus the wholesale price for product A is lower with more accurate third-party information. The change in the wholesale price, together with the change in the demand for product A because of the favorable information, changes the retail price of product A in a similar fashion, as characterized by Inequality (30).

The retailer benefits from the third-party information from two sources. First, as in symmetric case, the variance-reducing effect intensifies the upstream competition, which, per se, reduces the wholesale prices and thus increases the retailer's profit. With an increased precision, the variance-reducing effect becomes more salient which increases the retailer's profit further. Second, as illustrated in section 4.1.1, the mean-shifting effect makes the downstream demand asymmetric in terms of their potential market sizes, which engenders more room for the retailer to exploit its market and benefits the retailer. With an increased precision, the market potentials for the two products become even more asymmetric, which further boosts the retailer's profit.

Proposition 6. *Under the wholesale scheme in the quality-dominates-fit case, in the presence of the symmetric third-party information in terms of quality (i.e., $x_R = 0$), as the third-party information becomes more precise, consumer surplus and social welfare increase.*

The impacts of the precision of the third-party information on consumer surplus and social welfare are qualitatively identical under the platform and wholesale schemes. The intuition for the above result is analogous to that discussed following Proposition 2.

4.2.2 Fit-Dominates-Quality Case under Wholesale Scheme

The following proposition summarizes the effects of the precision in fit dimension of third-party information under wholesale scheme.

Proposition 7. *Under the wholesale scheme, in the fit-dominates-quality case, as the third-party information becomes more precise (i.e., β increases):*

(a) *Product B 's wholesale price increases (i.e., $\frac{\partial w_B}{\partial \beta} > 0$); Product A 's wholesale price increases (i.e., $\frac{\partial w_A}{\partial \beta} > 0$) if and only if*

$$x_R < \frac{(3+4\alpha\beta t)^2}{4\alpha(1-\gamma)(1+4\alpha\beta t)^2} \quad (31)$$

(b) *Product B's retail price increases and the retailer's profit decreases (i.e., $\frac{\partial p_B}{\partial \beta} > 0$ and $\frac{\partial \pi_R}{\partial \beta} < 0$); Product A's retail price increases (i.e., $\frac{\partial p_A}{\partial \beta} > 0$) if and only if*

$$x_R < \frac{2(1+\alpha\beta t)^2(3+4\alpha\beta t)^2}{\alpha(1-\gamma)(1+4\alpha\beta t)^2(17+8\alpha\beta t(5+3\alpha\beta t))} \quad (32)$$

In the symmetric case with $x_R = 0$, the conditions in Inequalities (31) and (32) are all satisfied.

Corollary 4. *Under the wholesale scheme in the fit-dominates-quality case, in the presence of the symmetric third-party information in terms of quality (i.e., $x_R = 0$), as the third-party information becomes more precise, both the wholesale prices and retail prices increase and the retailer's profit decreases.*

The intuition of the effect of the third-party information on upstream competition for the symmetric case is similar to the case under the platform scheme. As illustrated in section 4.1.2, with more precise third-party information, the variance-increasing effect becomes stronger and therefore the upstream competition is further relaxed, which leads to higher prices and the lower retailer profit.

In the general case (i.e., $x_R > 0$) as prescribed in Proposition 7, the *mean-shifting* effect, as illustrated in section 4.1.2, also plays a role in the equilibrium outcome. For manufacturer B , the unfavorable information has a negative effect on its wholesale price, whereas the softened competition resulting from the variance-increasing effect has a positive effect. As illustrated in 4.1.2, the improved precision in the fit dimension mitigates the negative effect of the mean-shifting effect, and it also makes the positive effect from the variance-increasing effect becomes more salient (the competition becomes more relaxed). As a result, the precision improvement always allows the manufacturer B to set the higher wholesale price, which in turn induces the retailer to charge a higher retail price for product B . Manufacturer A benefits from the positive effects from the favorable third-party information, as well as from the softened competition because of the variance-increasing effect. With the increased precision in the fit dimension, however, the positive effect of the mean-shifting effect is mitigated. Therefore, whether the product A 's wholesale price increases, which in turn induces the retailer to charge a higher retail price for product A , depends on the magnitude of these two effects. Inequality (31) shows that when the positive third-party information is mild, the variance-increasing effect dominates the mean-shifting effect, and thus the wholesale price for product B is higher with more accurate information. The subsequent change in the retail price of product A is similarly characterized by Inequality (32).

The retailer is affected by the third-party information through both the variance-increasing effect and mean-shifting effect. As third-party information in the fit dimension becomes more

precise, the negative effect of the relaxed upstream competition becomes more salient but the positive effect from the increased asymmetry in market potential becomes less significant. As a result, the retailer has lower profits.

Proposition 8. *Under the wholesale scheme in the fit-dominates-quality case, in the presence of the symmetric quality level from the third-party information (i.e., $x_R = 0$), as the third-party information is more accurate, consumer surplus decreases if and only if*

$$\gamma^2 < \frac{24\beta^2 t(1+4\alpha\beta t)^2 \left[\frac{\alpha t}{(1+4\alpha\beta t)^2} x - \frac{t(4\alpha t(2\beta(\alpha t(\beta(4\alpha\beta t+3)+2)+2)+1)+1)}{8(1+4\alpha\beta t)^3} \right]}{\epsilon^2(1+8\alpha\beta t(1+\alpha\beta t))} \quad (33)$$

Social welfare decreases if and only if

$$\gamma^2 < \frac{24\beta^2 t^2(1+4\alpha\beta t)^2 \left[\frac{\alpha}{(1+4\alpha\beta t)^2} x - \frac{(1+4\alpha t(1+\beta+2\alpha\beta^2 t))}{8(1+4\alpha\beta t)^2} \right]}{\epsilon^2(1+8\alpha\beta t(1+\alpha\beta t))} \quad (34)$$

Again, as in the quality-dominates-fit case, the impacts of the precision of the third-party information on consumer surplus and social welfare are qualitatively identical under the platform and wholesale schemes in the fit-dominates-quality case. The intuition for the above result is analogous to that discussed following Proposition 4.

We summarize the main results of this section in Table 2. The table reveals that the impact of the precision of third-party information depends critically on the pricing scheme as well as whether quality or fit dominates.

Table 2: Summary of Results: Impact of an Improvement in Precision of Third-Party Information when x_R is mild, with an upward arrow indicating an increase and a downward arrow indicating a decrease

	Quality-dominates-Fit case Platform Scheme/Wholesale Scheme	Fit-dominates-Quality case Platform Scheme/Wholesale Scheme
π_R	\downarrow / \uparrow	\uparrow / \downarrow
p_A	\downarrow / \downarrow	\uparrow / \uparrow
p_B	\downarrow / \downarrow	\uparrow / \uparrow
k	\uparrow / na	\downarrow / na
m	\downarrow / na	\uparrow / na
w_A	na / \downarrow	na / \uparrow
w_B	na / \downarrow	na / \uparrow
CS	\uparrow / \uparrow	\downarrow or \uparrow / \downarrow or \uparrow
SW	\uparrow / \uparrow	\downarrow or \uparrow / \downarrow or \uparrow

5 Comparison of Retailer's Profits under Two Pricing Schemes

In this section, we compare the retailer's equilibrium profits under the two different pricing schemes. We show that the third-party information may reverse the retailer's preference over the two schemes in terms of its profit. For ease of exposition, we next use the symmetric case with $x_R = 0$ to illustrate the profit comparison. We use superscript of p and w to indicate the platform scheme and the wholesale scheme, respectively, for the retailer's profit.

Proposition 9. *In the presence of symmetric third-party information in terms of quality, retailer's profit is higher under platform scheme than under wholesale scheme if and only if $\frac{1}{2\tau} < \xi$, where*

$$\xi \triangleq \min \left\{ 3\alpha, \frac{\alpha[3-32\alpha(\mu_A+\mu_B)]}{2\sqrt{[1-10\alpha(\mu_A+\mu_B)]-[1-16\alpha(\mu_A+\mu_B)]}} \right\} \quad (35)$$

and $\tau \in \{\beta t, \gamma \epsilon\}$ with $\tau = \gamma \epsilon$ for the quality-dominates-fit case and $\tau = \beta t$ for the fit-dominates-quality case.⁵

Proposition 9 essentially states that the retailer prefers the platform scheme to wholesale scheme if and only if the upstream competition is mild (a small $\frac{1}{2\tau}$). As explained in Section 3 for Equations (17) and (20), the coefficient of the price difference term in demand functions, $\frac{1}{2\tau}$, measures the substitutability between the two products: the larger the coefficient is, the more substitutable the two products are and more intense the competition between the two manufacturers is. When the upstream competition is low, the retailer prefers the platform scheme to the wholesale scheme, because under the platform scheme manufacturers' profits are high and so is the retailer's, whereas under the wholesale scheme low upstream competition leads to high wholesale prices which leaves the retailer little room to exploit the market. As explained previously, the increase in the upstream competition benefits the retailer under wholesale scheme, but hurts the retailer under the platform scheme. When the upstream competition is high enough, the retailer prefers the wholesale scheme over the platform scheme. The condition $\frac{1}{2\tau} < \xi$ in the proposition essentially requires upstream competition being low enough under which platform scheme is more attractive to the retailer.

Based on the conditions derived in Proposition 9, we can show the retailer's preference over the two pricing schemes may switch from one to the other when the precision of the third-party information improves. In addition, such a switching occurs only in one direction, and the direction is opposite in the quality-dominates-fit case and the fit-dominates-quality case.

⁵We assume that $(\mu_A + \mu_B) < \frac{3}{32\alpha}$. To examine which pricing scheme is attractive to the online retailer, we avoid the trivial case where the competing manufacturers select to not sell through the retailer in either pricing scheme.

Corollary 5. *In the quality-dominates-fit case with the symmetric third-party information, (a) when the precision of the third-party information in quality, $(1 - \gamma)$, increases beyond the point $(1 - \gamma^*)$, where γ^* is the one such that $\frac{1}{2\gamma^*} = \xi$ (ξ defined in Equality (35)), the retailer switches preference from platform scheme to wholesale scheme. (b) The other direction of preference switching cannot occur for the retailer as the precision of the third-party information improves.*

The intuition is as follows. In the quality-dominates-fit case, an increase in the precision of the third-party information increases the upstream competition, which benefits the retailer under the wholesale scheme but hurts the retailer under the platform scheme. When the upstream competition is driven high enough by the increased precision, the retailer's preference switches from the platform scheme to the wholesale scheme, which explains part (a). The preference switching from the wholesale scheme to the platform scheme cannot occur, because, if with the low precision of the third-party information the retailer is better off under the wholesale scheme, then improved precision can only make the retailer even more better off under the wholesale scheme because of the increased upstream competition.

In the fit-dominates-quality case, in contrast, the precision improvement of the third-party information reduces the upstream competition. Analogous to the logic that applies for the quality-dominates-fit case, the retailer's preference over the two pricing schemes may switch from one to the other, but the switching is from the wholesale scheme to the platform scheme, the opposite direction to that in the quality-dominates-fit case. Similarly, the improved precision can switch the preference in one direction only.

Corollary 6. *In the fit-dominates-quality case with the symmetric third-party information, (a) when the precision of the third-party information in fit increases beyond the point β^* , where β^* is the one such that $\frac{1}{2\beta^*} = \xi$ (ξ defined in Equality (35)), the retailer switches preference from wholesale scheme to platform scheme. (b) The other direction of preference switching cannot occur for the retailer as the precisions of the third-party information improves.*

For the general case with asymmetric third-party information (i.e., $x_R > 0$), we can show that the retailer's profit is higher under the wholesale scheme than under the platform scheme if and only if the asymmetry is mild, in addition to the condition on the upstream competition (i.e., $\frac{1}{2\tau} > \xi$). We provide the threshold for x_R for the general case in the proof of Proposition 9 in the Appendix. Figure 4 illustrates that the retailer's preference for one pricing scheme over the other in the general case with asymmetric third-party information is qualitatively similar to that under the symmetric third-party information case discussed in Proposition 9.

We next compare consumer surplus and social welfare under the two pricing schemes.

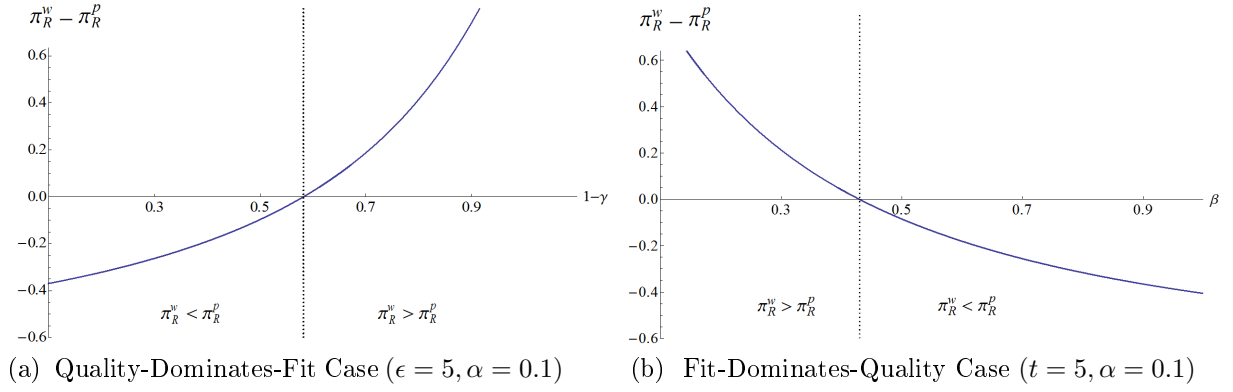


Figure 4: Retailer's Preference of Pricing Scheme($x_R = 0.5$)

Proposition 10. *In both quality-dominates-fit case and fit-dominates-fit case with the symmetric third-party information, consumer surplus and social welfare are higher under platform scheme than under wholesale scheme.*

Consumer surplus is always higher under platform scheme in both quality-dominates-fit case and fit-dominates-quality case because the retail prices are lower under the platform scheme than the wholesale scheme. The reason for the lower prices under the platform scheme than the wholesale scheme is that the wholesale scheme suffers from double marginalization which does not exist in the platform scheme. The lower prices in turn attract more consumers in the platform scheme compared to the wholesale scheme. Similarly the social welfare is higher under the platform scheme than under the wholesale scheme, because more consumers purchase under the platform scheme.

Proposition 10 provides an important insight regarding the recent debate about the agency pricing practice for e-books. At issue is who—authors, publishers or retailer—benefits from agency model. We find that the agency model (i.e., platform scheme) can lead to lower equilibrium prices than the traditional wholesale model. This lower price in the platform scheme also has been found in recent literature (e.g., Abhishek et al., 2013; Tan and Carrillo, 2014), which suggests the positive effect of an agency pricing model on both sellers and consumers. Our results also support the argument that the agency model not only can increase the consumer surplus but might also increase the retailer's profit.

To summarize Sections 4 and 5, the main finding is that online retailers can use the upstream pricing scheme effectively to their advantage in the presence of third-party information. The appropriate pricing scheme depends critically on whether quality or fit plays a dominant role in consumers' assessment of the product, and the precision of the information. Under a high precision, while the retailer will prefer the wholesale scheme if the quality dominates fit, he will prefer the platform scheme if the fit dominates quality. However, consumers and society are always better off under the platform scheme than under the wholesale scheme. A key driver for the above results is the impact of the third-party information on the upstream price competition.

6 Extensions

6.1 Non-Zero Quality Difference

In the baseline model, we assume that the true quality difference between the products is zero and consumers' own assessment of quality difference satisfies a uniform distribution over $[-\epsilon, \epsilon]$. In this section, we extend our model to the case in which the true quality difference is non-zero. Assume that the true quality difference is δ in favor of product A , and that the consumers' assessment of the true quality difference is unbiased at the aggregate level, and satisfies a uniform distribution over $[\delta - \epsilon, \delta + \epsilon]$. All other model parameters remain the same as in our baseline model.

Similar to the demand functions in Equation (5), we can derive the demand functions for the quality-dominates-fit case as:

$$\begin{aligned}\bar{D}_A &= \int_0^1 \int_{\frac{p_A - p_B - (1-\gamma)x_R - (1-2y)\beta t}{\gamma}}^{\delta + \epsilon} \frac{1}{2\epsilon} dx dy = \frac{1}{2} + \frac{\gamma\delta}{2\gamma\epsilon} - \frac{1}{2\gamma\epsilon} [p_A - p_B - (1-\gamma)x_R] \\ \bar{D}_B &= \int_0^1 \int_{\delta - \epsilon}^{\frac{p_A - p_B - (1-\gamma)x_R - (1-2y)\beta t}{\gamma}} \frac{1}{2\epsilon} dx dy = \frac{1}{2} - \frac{\gamma\delta}{2\gamma\epsilon} + \frac{1}{2\gamma\epsilon} [p_A - p_B - (1-\gamma)x_R]\end{aligned}$$

The demand function for the fit-dominates-quality case can be similarly derived as:

$$\begin{aligned}\bar{D}_A &= \int_0^{y_A} dy + \int_{y_A}^{y_B} \int_{\frac{p_A - p_B - (1-\gamma)x_R - (1-2y)\beta t}{\gamma}}^{\delta + \epsilon} \frac{1}{2\epsilon} dx dy = \frac{1}{2} + \frac{\gamma\delta}{2\beta t} - \frac{1}{2\beta t} [p_A - p_B - (1-\gamma)x_R] \\ \bar{D}_B &= \int_{y_A}^{y_B} \int_{\delta - \epsilon}^{\frac{p_A - p_B - (1-\gamma)x_R - (1-2y)\beta t}{\gamma}} \frac{1}{2\epsilon} dx dy + \int_{y_B}^1 dy = \frac{1}{2} - \frac{\gamma\delta}{2\beta t} + \frac{1}{2\beta t} [p_A - p_B - (1-\gamma)x_R]\end{aligned}$$

where the marginal consumer y_A (y_B) is the one who derives the same utility from the two products when perceiving the largest quality difference against product A (B); that is, when $x_C = \delta - \epsilon$ ($x_C = \delta + \epsilon$). In particular, similar to Equation (6), y_A and y_B are derived as:

$$\begin{aligned}y_A &= \frac{1}{2\beta t} [\gamma\delta - \gamma\epsilon + (1-\gamma)x_R + \beta t - (p_A - p_B)] \\ y_B &= \frac{1}{2\beta t} [\gamma\delta + \gamma\epsilon + (1-\gamma)x_R + \beta t - (p_A - p_B)]\end{aligned}$$

Similar to the baseline model, we can uniformly characterize the demand as follows:

$$\begin{aligned}D_A &= \left[\frac{1}{2} + \frac{\gamma\delta + (1-\gamma)x_R}{2\tau} \right] - \left(\frac{1}{2\tau} + \alpha \right) p_A + \frac{1}{2\tau} p_B \\ D_B &= \left[\frac{1}{2} - \frac{\gamma\delta + (1-\gamma)x_R}{2\tau} \right] - \left(\frac{1}{2\tau} + \alpha \right) p_B + \frac{1}{2\tau} p_A\end{aligned}$$

where $\tau \in \{\gamma\epsilon, \beta t\}$ with $\tau = \gamma\epsilon$ for quality-dominates-fit case and $\tau = \beta t$ for fit-dominates-quality case. Based on these demand functions, as in Lemma 1 and Lemma 2, we can similarly derive the equilibrium outcome under the platform and wholesale schemes.

Lemma 3. *Under the platform scheme, the equilibrium retail prices are*

$$p_A = \frac{\tau}{1+4\alpha\tau} + \frac{\gamma\delta}{3+4\alpha\tau} + \frac{(1-\gamma)x_R}{3+4\alpha\tau} \quad (36)$$

$$p_B = \frac{\tau}{1+4\alpha\tau} - \frac{\gamma\delta}{3+4\alpha\tau} - \frac{(1-\gamma)x_R}{3+4\alpha\tau} \quad (37)$$

and the commission rate and fixed fee are

$$k = 1 - \frac{(1+4\alpha\tau)(3+4\alpha\tau)(\mu_A - \mu_B)}{2(1+2\alpha\tau)[\gamma\delta + (1-\gamma)x_R]} \quad (38)$$

$$m = \frac{[\tau(3+4\alpha\tau) - \gamma\delta(1+4\alpha\tau) - (1-\gamma)(1+4\alpha\tau)x_R]^2 \mu_A - [\tau(3+4\alpha\tau) + \gamma\delta(1+4\alpha\tau) + (1-\gamma)(1+4\alpha\tau)x_R]^2 \mu_B}{4\tau(1+4\alpha\tau)(3+4\alpha\tau)[\gamma\delta + (1-\gamma)x_R]} \quad (39)$$

where $\tau \in \{\beta t, \gamma\epsilon\}$ with $\tau = \gamma\epsilon$ for the quality-dominates-fit case and $\tau = \beta t$ for the fit-dominates-quality case.

Lemma 4. *Under the wholesale scheme, the equilibrium wholesale prices are*

$$w_A = \frac{\tau}{1+4\alpha\tau} + \frac{\gamma\delta + (1-\gamma)x_R}{3+4\alpha\tau} \quad (40)$$

$$w_B = \frac{\tau}{1+4\alpha\tau} - \frac{\gamma\delta + (1-\gamma)x_R}{3+4\alpha\tau} \quad (41)$$

and the retail prices are

$$p_A = \frac{(1+6\alpha\tau)}{4\alpha(1+4\alpha\tau)} + \frac{(5+6\alpha\tau)[\gamma\delta + (1-\gamma)x_R]}{4(1+\alpha\tau)(3+4\alpha\tau)} \quad (42)$$

$$p_B = \frac{(1+6\alpha\tau)}{4\alpha(1+4\alpha\tau)} - \frac{(5+6\alpha\tau)[\gamma\delta + (1-\gamma)x_R]}{4(1+\alpha\tau)(3+4\alpha\tau)} \quad (43)$$

where $\tau \in \{\beta t, \gamma\epsilon\}$ with $\tau = \gamma\epsilon$ for quality-dominates-fit case and $\tau = \beta t$ for fit-dominates-quality case.

Notice that when $\delta = 0$, the above two lemmas reduce to the corresponding ones in the baseline case. A comparison of the equilibrium prices in Lemma 3 and Lemma 4 and the corresponding prices in the baseline model reveals that the price for any product under any pricing scheme in the non-zero quality difference model can be obtained from the corresponding price in the baseline model by applying the following simple substitution: replace the variable $(1-\gamma)x_R$ in the baseline model by $(1-\gamma)x_R + \gamma\delta$. In other words, the sellers will price their products based on weighted average of the third-party information and the mean consumer assessment in the non-zero quality difference case, as opposed to only the third-party information in the zero quality difference case. It is easy to see that if we define symmetric third-party information as one that reflects the mean consumer assessment— $x_R = 0$ in the baseline model and $x_R = \delta$ in the non-zero quality difference model—then, all results for the symmetric case are identical regardless of the true quality difference. We leave the detailed

analysis to the Appendix. The analysis shows that all qualitative results and insights of the baseline model carry over to the case when the true quality difference is not zero.

6.2 Different Precisions Among Consumers

In the baseline model, we assume that the precision of consumers' own assessment is same across consumers and consumers incorporate the third-party information in the same fashion (by assigning weight $(1 - \gamma)$ on the third-party information). In this section, we show our results generalize to the case when the consumers have different precisions or assign different weights to the third-party information. For illustration, we consider the case with symmetric third-party information in terms of quality and two groups of consumers with different precisions. We assume the proportions of high-precision and low-precision consumers to be α_H and α_L , respectively, and $a_H + a_L = 1$. The weight assigned by each group for the third-party information is assumed to be $(1 - \gamma_H)$ and $(1 - \gamma_L)$ respectively for high-precision and low-precision consumers in the quality-dominates-fit case (i.e., $(1 - \gamma_H) > (1 - \gamma_L)$) and β_H and β_L in the fit-dominates-quality case (i.e., $\beta_H > \beta_L$), respectively. All other model parameters remain the same as in our baseline model.

Similar to the demand functions in Equation (8), the demand function for the quality-dominates-fit case can be derived when $x_R = 0$ as:

$$\begin{aligned} D_A &= a_H \left[\frac{1}{2} - \frac{p_A - p_B}{2\gamma_H \epsilon} \right] + a_L \left[\frac{1}{2} - \frac{p_A - p_B}{2\gamma_L \epsilon} \right] - \alpha p_A \\ D_B &= a_H \left[\frac{1}{2} + \frac{p_A - p_B}{2\gamma_H \epsilon} \right] + a_L \left[\frac{1}{2} + \frac{p_A - p_B}{2\gamma_L \epsilon} \right] - \alpha p_B \end{aligned}$$

The demand function for the fit-dominates-quality case can be similarly derived when $x_R = 0$ as:

$$\begin{aligned} D_A &= a_H \left[\frac{1}{2} - \frac{p_A - p_B}{2\beta_H t} \right] + a_L \left[\frac{1}{2} - \frac{p_A - p_B}{2\beta_L t} \right] - \alpha p_A \\ D_B &= a_H \left[\frac{1}{2} + \frac{p_A - p_B}{2\beta_H t} \right] + a_L \left[\frac{1}{2} + \frac{p_A - p_B}{2\beta_L t} \right] - \alpha p_B \end{aligned}$$

We can uniformly characterize the demand as follows:

$$\begin{aligned} D_A &= a_H \left[\frac{1}{2} - \frac{p_A - p_B}{2\tau_H} \right] + a_L \left[\frac{1}{2} - \frac{p_A - p_B}{2\tau_L} \right] - \alpha p_A = \frac{1}{2} - \left[\frac{a_H}{2\tau_H} + \frac{a_L}{2\tau_L} \right] (p_A - p_B) - \alpha p_A \\ D_B &= a_H \left[\frac{1}{2} + \frac{p_A - p_B}{2\tau_H} \right] + a_L \left[\frac{1}{2} + \frac{p_A - p_B}{2\tau_L} \right] - \alpha p_B = \frac{1}{2} + \left[\frac{a_H}{2\tau_H} + \frac{a_L}{2\tau_L} \right] (p_A - p_B) - \alpha p_B \end{aligned}$$

in which $\tau_H \in \{\gamma_H \epsilon, \beta_H t\}$ and $\tau_L \in \{\gamma_L \epsilon, \beta_L t\}$ with $\tau_i = \gamma_i \epsilon$ for the quality-dominates-fit case and $\tau_i = \beta_i t$ for the fit-dominates-quality case. We note that the above demand functions are structurally similar to those derived for the baseline model with the difference being the coefficient for $(p_A - p_B)$ —the coefficient of $\frac{1}{2\tau}$ in the baseline model is replaced by the coefficient of $(\frac{a_H}{2\tau_H} + \frac{a_L}{2\tau_L})$ in the model with heterogeneous consumer precisions. We

note that the precision of the third-party information has the same qualitative impact on this coefficient in both heterogeneous consumer precision model and the baseline model. As shown in the Appendix, the qualitative results and insights of the baseline model can be replicated in the model where consumers have different precisions and assign different weights to the third-party information.

7 Conclusion

We examine the effect of third-party information (e.g., online product reviews) in a channel structure with a retailer carrying two substitutable products. The retailer may use wholesale scheme and sell products by itself, or use platform scheme and let manufacturers sell directly to consumers. Consumers face uncertainty in both the product quality and fit to their needs, and third-party information provides additional information that reduces their uncertainty. Consumers agree on the preference order of the attributes in the quality dimension and have idiosyncratic preferences for the same attribute in the fit dimension. We identify the quality-dominates-fit case in which the quality dimension plays a dominant role in determining consumers' perceived utility differences of the competing products and the fit-dominates-quality case in which the fit dimension plays a more important role. We show that retailers can strategically use the upstream pricing scheme to their advantage in the presence of third-party information. The appropriate pricing scheme depends critically on whether quality or fit plays a dominant role in consumers' assessment of the product, and the precision of the information. We identify the impact of the third-party information on the upstream price competition as the primary driving force behind our results.

Our findings have significant implications for online retailers. Online retailers have been deploying a variety of technologies to mitigate consumer uncertainty and match consumers with their preferred products. Online review platforms and recommendation systems are a few examples of these. Our results suggest that the effect of the same third-party information on a retailer can be quite opposite depending on the pricing scheme used. For example, some third-party information may be related to the quality dimension. The retailer should always welcome more precise information of this kind if the retailer uses the wholesale scheme, but might want to avoid such information under platform scheme. The information that provides fit information about products can benefit the retailer more under the platform scheme but may decrease the retailer's profit under the wholesale scheme. The rule of thumb of fostering third-party information is that the intended information to be revealed should be tailored according to the pricing scheme used by a retailer. Under such circumstances where retailer can benefit from this information, retailers should encourage and even induce consumers and/or third parties to generate such information. For instance, retailers can make the

review platform easy to use to facilitate the review generating process, and, in particular, they may provide some review templates to direct users toward generating information about product qualities or fits.

While controlling the third-party information generation and access process, manipulating the third-party information displayed, and adjusting marketing-mix elements including strategic pricing are options that a retailer may have at its disposal to increase benefit from third-party information, these options may also suffer from issues such as a potential decrease in consumers' trust of the retailer. Alternatively, the contract that the retailer enters into with upstream sellers can be a strategic tool that a retailer can use to its advantage. The upstream contract is less likely to suffer from issues related to consumer mistrust. Finally, product category in the sense of whether fit or quality plays a dominant role in consumer decision making significantly affects a retailer's optimal choice. As we noted in the introduction section, anecdotal observations suggest that the relative fraction of products for which Amazon uses the wholesale scheme or platform scheme varies across product categories; however, whether the relative importance of quality and fit varies across these categories and whether this relative importance influences Amazon's choice of pricing scheme are open empirical questions.

Another important implication for online retailers relates to the assessment and measurement of precision of third-party information because the precision affects the choice of the retailer's pricing scheme. One possibility is to collect and use data from consumers about the usefulness of this information. Several online review systems and forums that provide answers to consumers' questions have started to implement a feature that allows consumers to rate the usefulness of reviews and answers. Online retailers can examine this rating data to estimate the precision of third-party information. More importantly, online retailers will find it beneficial to design an appropriate rating form that will enable them to estimate precision more accurately.

Related to the recent debate on the agency model in the e-book industry, we show that consumer surplus and social welfare are higher under the platform scheme than the wholesale scheme. For the product category in which fit dominates quality—e-books is likely to fall under such a category—, as the precision of third-party information increases, the retailer is likely to prefer the platform scheme. Therefore, a win-win outcome that benefits all parties may arise under the platform scheme for such product categories.

This paper suggests a few directions for future research. First, in this paper we consider products can only be sold via one of the two pricing schemes. It will be interesting to see how the results differ if a product is being sold via both the wholesale and platform schemes. Second, our paper provides several testable hypotheses. Rigorous empirical test will complement this work.

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Appendix

A.1 Derivation of Conditional Expectation of Misfit

Proof. Cumulative density function of s , conditional on the consumer's true degree of misfit λ being z , can be formulated as $P(s \leq y | \lambda = z) = (1 - \beta)y + \beta H(y - z)$, where $H(\cdot)$ is the Heaviside step function that evaluates to zero if the argument is negative, and to one otherwise. The corresponding probability density function is $P(s = y | \lambda = z) = (1 - \beta) + \beta \delta(y - z)$, where $\delta(x)$ is the Dirac delta distribution that satisfies

$$\int_{-\infty}^{\infty} \delta(x) dx = 1 \text{ and } \delta(x) = \begin{cases} 0 & \text{for } x \neq 0 \\ \infty & \text{for } x = 0 \end{cases}$$

Using the Bayes' Law,

$$P(\lambda = z | s = y) = \frac{P(s = y | \lambda = z) P(\lambda = z)}{P(s = y)} = (1 - \beta) + \beta \delta(y - z) \quad (44)$$

and the conditional expectation is

$$\mathbb{E}(\lambda | s = y) = \int_0^1 \lambda [(1 - \beta) + \beta \delta(y - \lambda)] d\lambda = \int_0^1 (1 - \beta) \lambda d\lambda + \beta y = \frac{1 - \beta}{2} + \beta y$$

□

A.2 Proof of Lemma 1

Proof. We denote $a_A \equiv \frac{1}{2} + \frac{1}{2\tau}(1 - \gamma)x_R$, $a_B \equiv \frac{1}{2} - \frac{1}{2\tau}(1 - \gamma)x_R$, $b \equiv \frac{1}{2\tau} + \alpha$, and $c \equiv \frac{1}{2\tau}$. The demand functions in Equation (8) then can be rewritten as

$$\begin{aligned} D_A &= a_A - bp_A + cp_B \\ D_B &= a_B - bp_B + cp_A \end{aligned} \quad (45)$$

The retailer's optimization problem in stage 1 is characterized by the first-order conditions as follows:

$$\frac{\partial \pi_A}{\partial k} = p_A(a_A - bp_A + cp_B) + p_B(a_B + cp_A - bp_B) \text{ and } \frac{\partial \pi_R}{\partial m} = 2 \quad (46)$$

The manufacturers' optimization problems in stage 2, given k and m are characterized by the first-order conditions of Equation (9):

$$\begin{aligned} \frac{\partial \pi_A}{\partial p_A} &= (1 - k)(a_A - 2bp_A + cp_B) = 0 \\ \frac{\partial \pi_B}{\partial p_B} &= (1 - k)(a_B - 2bp_B + cp_A) = 0 \end{aligned}$$

from which we can derive the manufacturers' optimal retail prices:

$$\begin{aligned} p_A &= \frac{2a_A b + a_B c}{4b^2 - c^2} \\ p_B &= \frac{2a_B b + a_A c}{4b^2 - c^2} \end{aligned} \quad (47)$$

Substituting the retail prices, we can characterize the retailer's equilibrium profit and the manufacturers' equilibrium profits as:

$$\pi_R = k(p_A D_A + p_B D_B) + 2m = \frac{k[b(a_A^2 + a_B^2)(4b^2 + c^2) + 8a_A a_B b^2 c]}{(4b^2 - c^2)^2} + 2m \quad (48)$$

$$\begin{aligned} \pi_A &= (1 - k)p_A D_A - m = \frac{b(1 - k)(2ba_A + ca_B)^2}{(4b^2 - c^2)^2} - m \\ \pi_B &= (1 - k)p_B D_B - m = \frac{b(1 - k)(ca_A + 2ba_B)^2}{(4b^2 - c^2)^2} - m \end{aligned} \quad (49)$$

Anticipating the manufacturers' participation incentives of selling on the retailer's platform ($\pi_i \geq \mu_i$), the retailer sets the optimal k and m by solving the binding constraints, $\pi_i = \mu_i$, simultaneously:

$$\begin{aligned} k &= 1 - \frac{(4b^2 - c^2)(\mu_A - \mu_B)}{b(a_A^2 - a_B^2)} \\ m &= \frac{\mu_A(ca_A + 2ba_B)^2}{(4b^2 - c^2)(a_A^2 - a_B^2)} - \frac{\mu_B(2ba_A + ca_B)^2}{(4b^2 - c^2)(a_A^2 - a_B^2)} \end{aligned} \quad (50)$$

Substituting the above optimal retail prices, the commission rate, and fixed fee into the retailer's profit:

$$\pi_R = \frac{b(4b^2 + c^2)(a_A^2 + a_B^2) + 8b^2 ca_A a_B}{(4b^2 - c^2)^2} - \mu_A - \mu_B \quad (51)$$

Lemma 1 follows by substituting a_A , a_B , b , and c into the above optimal retail prices, the commission rate, and fixed fee. Similarly, by substituting a_A , a_B , b , and c into Equation (51), the retailer's profit can be derived as

$$\pi_R = \frac{(1 + 2\alpha\tau)}{\tau} \left[\frac{\tau^2}{(1 + 4\alpha\tau)^2} + \frac{(1 - \gamma)^2 x_R^2}{(3 + 4\alpha\tau)^2} \right] - \mu_A - \mu_B \quad (52)$$

□

A.3 Proof of Proposition 1

Proof. We notice

$$\frac{\partial p_B}{\partial(1 - \gamma)} = -\frac{\epsilon}{(1 + 4\alpha\gamma\epsilon)^2} - \frac{(3 + 4\alpha\epsilon)x_R}{(3 + 4\alpha\gamma\epsilon)^2} < 0$$

$$\frac{\partial p_A}{\partial(1 - \gamma)} < 0 \text{ if and only if}$$

$$\frac{\partial p_A}{\partial(1 - \gamma)} = -\frac{\epsilon}{(1 + 4\alpha\gamma\epsilon)^2} + \frac{(3 + 4\alpha\epsilon)x_R}{(3 + 4\alpha\gamma\epsilon)^2} < 0$$

which leads to the condition in Inequality (15).

Substituting $\tau = \gamma\epsilon$ into Equation (52), we have the retailer's profit. $\frac{\partial \pi_R}{\partial (1-\gamma)} < 0$ if and only if

$$\frac{\partial \pi_R}{\partial (1-\gamma)} = -\frac{\epsilon}{(1+4\alpha\gamma\epsilon)^3} + \frac{(1-\gamma)x_R^2[3+\gamma(3+4\alpha\epsilon(3+\gamma(2+4\alpha\epsilon)))]}{\gamma^2\epsilon(3+4\alpha\gamma\epsilon)^3} < 0$$

which leads to the condition in Inequality (16). \square

A.4 Proof of Proposition 2

Proof. Consumer surplus (CS_i) derived from product i can be formulated as:

$$\begin{aligned} CS_A &= \frac{\bar{D}_A - \alpha p_A}{\bar{D}_A} \left(\int_0^1 \int_{\frac{p_A - p_B - (1-\gamma)x_R - (1-2y)\beta t}{\gamma}}^{\epsilon} \frac{[x - (\beta y + \frac{1-\beta}{2})t - p_A]}{2\epsilon} dx dy \right) \\ CS_B &= \frac{\bar{D}_B - \alpha p_B}{\bar{D}_B} \left(\int_0^1 \int_{-\epsilon}^{\frac{p_A - p_B - (1-\gamma)x_R - (1-2y)\beta t}{\gamma}} \frac{[x - (\beta(1-y) + \frac{1-\beta}{2})t - p_A]}{2\epsilon} dx dy \right) \end{aligned} \quad (53)$$

where \bar{D}_A and \bar{D}_B are defined as in Equation (5). The total consumer surplus $CS = CS_A + CS_B$. By substituting the optimal retail prices from Lemma 1, we have

$$CS = \frac{(1+2\alpha\gamma\epsilon)[(1+4\alpha\gamma\epsilon)(3+4\alpha\gamma\epsilon)^2(6\gamma\epsilon x - 3\gamma\epsilon t + \beta^2 t^2) - 6\gamma^2\epsilon^2(3+4\alpha\gamma\epsilon)^2 - 6(1+4\alpha\gamma\epsilon)^2(1-\gamma)^2 x_R^2]}{6\gamma\epsilon(1+4\alpha\gamma\epsilon)^2(3+4\alpha\gamma\epsilon)^2}$$

Similarly, we can derive the social welfare from product i as:

$$\begin{aligned} SW_A &= \frac{\bar{D}_A - \alpha p_A}{\bar{D}_A} \left(\int_0^1 \int_{\frac{p_A - p_B - (1-\gamma)x_R - (1-2y)\beta t}{\gamma}}^{\epsilon} \frac{[x - (\beta y + \frac{1-\beta}{2})t]}{2\epsilon} dx dy \right) \\ SW_B &= \frac{\bar{D}_B - \alpha p_B}{\bar{D}_B} \left(\int_0^1 \int_{-\epsilon}^{\frac{p_A - p_B - (1-\gamma)x_R - (1-2y)\beta t}{\gamma}} \frac{[x - (\beta(1-y) + \frac{1-\beta}{2})t]}{2\epsilon} dx dy \right) \end{aligned} \quad (54)$$

The total social welfare $SW = SW_A + SW_B$. By substituting the optimal retail prices from Lemma 1, we have

$$SW = \frac{[3\gamma\epsilon(2x-t) + t^2\beta^2](1+2\alpha\gamma\epsilon)}{6\gamma\epsilon(1+4\alpha\gamma\epsilon)}$$

When $x_R = 0$, we notice that

$$\begin{aligned} \frac{\partial CS}{\partial (1-\gamma)} &= \frac{6\gamma^2\epsilon^2\alpha(2x-t)(1+4\alpha\gamma\epsilon) + 6\gamma^2\epsilon^2 + t^2\beta^2(1+4\alpha\gamma\epsilon)(1+8\alpha\gamma\epsilon(1+\alpha\gamma\epsilon))}{6\gamma^2\epsilon(1+4\alpha\gamma\epsilon)^3} \\ \frac{\partial SW}{\partial (1-\gamma)} &= \frac{6(2x-t)\alpha\gamma^2\epsilon^2 + t^2\beta^2(1+8\alpha\gamma\epsilon(1+\alpha\gamma\epsilon))}{6\gamma^2\epsilon(1+4\alpha\gamma\epsilon)^2} \end{aligned}$$

We can verify that both are positive using a condition that even the consumer with a signal indicating the largest degree of misfit has incentive to purchase. \square

A.5 Proof of Proposition 3

Proof. We notice

$$\frac{\partial p_B}{\partial \beta} = \frac{t}{(1+4\alpha\beta t)^2} + \frac{4\alpha t(1-\gamma)x_R}{(3+4\alpha\beta t)^2} > 0$$

$\frac{\partial p_A}{\partial \beta} > 0$ if and only if

$$\frac{\partial p_A}{\partial \beta} = \frac{t}{(1+4\alpha\beta t)^2} - \frac{4\alpha t(1-\gamma)x_R}{(3+4\alpha\beta t)^2} > 0$$

which leads to the condition in Inequality (18).

Substituting $\tau = \beta t$ into Equation (52), we have the retailer's profit. $\frac{\partial \pi_R}{\partial \beta} > 0$ if and only if

$$\frac{\partial \pi_R}{\partial \beta} = \frac{t}{(1+4\alpha\beta t)^3} - \frac{(1-\gamma)^2 x_R^2 [3+4\alpha\beta t(3+4\alpha\beta t)]}{\beta^2 t (3+4\alpha\beta t)^3} > 0$$

which leads to the condition in Inequality (19). \square

A.6 Proof of Proposition 4

Proof. Consumer surplus (CS_i) derived from product i can be formulated as:

$$\begin{aligned} CS_A &= \frac{\bar{D}_A - \alpha p_A}{\bar{D}_A} \left(\int_0^{y_A} \left[x - \left(\beta y + \frac{1-\beta}{2} \right) t - p_A \right] dy + \int_{y_A}^{y_B} \int_{-\epsilon}^{\frac{p_A - p_B - (1-\gamma)x_R - (1-2y)\beta t}{\gamma}} \frac{[x - (\beta y + \frac{1-\beta}{2})t - p_A]}{2\epsilon} dx dy \right) \\ CS_B &= \frac{\bar{D}_B - \alpha p_B}{\bar{D}_B} \left(\int_{y_A}^{y_B} \int_{-\epsilon}^{\frac{p_A - p_B - (1-\gamma)x_R - (1-2y)\beta t}{\gamma}} \frac{[x - (\beta(1-y) + \frac{1-\beta}{2})t - p_B]}{2\epsilon} dx dy + \int_{y_B}^1 \left[x - \left(\beta(1-y) + \frac{1-\beta}{2} \right) t - p_B \right] dy \right) \end{aligned} \quad (55)$$

where y_A and y_B are defined as in Equation (6) and \bar{D}_A and \bar{D}_B are defined as in Equation (7). The total consumer surplus $CS = CS_A + CS_B$. By substituting the optimal retail prices from Lemma 1, we have

$$CS = \frac{(1+2\alpha\beta t)[(3+4\alpha\beta t)^2[(1+4\alpha\beta t)(12\beta t x - \gamma^2 \epsilon^2) - 12\alpha(2-\beta)\beta^2 t^3 - 3(3\beta+2)\beta t^2] - 3(1+4\alpha\beta t)^2(5+4\alpha\beta t)(1-\gamma)^2 x_R^2]}{12\beta t(1+4\alpha\beta t)^2(3+4\alpha\beta t)^2}$$

Similarly, we can derive the social welfare (SW_i) derived from product i as:

$$\begin{aligned} SW_A &= \frac{\bar{D}_A - \alpha p_A}{\bar{D}_A} \left(\int_0^{y_A} \left[x - \left(\beta y + \frac{1-\beta}{2} \right) t \right] dy + \int_{y_A}^{y_B} \int_{-\epsilon}^{\frac{p_A - p_B - (1-\gamma)x_R - (1-2y)\beta t}{\gamma}} \frac{[x - (\beta y + \frac{1-\beta}{2})t]}{2\epsilon} dx dy \right) \\ SW_B &= \frac{\bar{D}_B - \alpha p_B}{\bar{D}_B} \left(\int_{y_A}^{y_B} \int_{-\epsilon}^{\frac{p_A - p_B - (1-\gamma)x_R - (1-2y)\beta t}{\gamma}} \frac{[x - (\beta(1-y) + \frac{1-\beta}{2})t]}{2\epsilon} dx dy + \int_{y_B}^1 \left[x - \left(\beta(1-y) + \frac{1-\beta}{2} \right) t \right] dy \right) \end{aligned} \quad (56)$$

The total social welfare $SW = SW_A + SW_B$. by substituting the optimal retail prices from Lemma 1, we have

$$SW = \frac{(1+2\alpha\beta t)[(3+4\alpha\beta t)^2(3\beta t(4x - t(2-\beta)) - \gamma^2 \epsilon^2) - 3(1+4\alpha\beta t)^2(1-\gamma)^2 x_R^2]}{12\beta t(1+4\alpha\beta t)(3+4\alpha\beta t)^2} > 0$$

When $x_R = 0$, $\frac{\partial CS}{\partial \beta} < 0$ if and only if

$$\frac{\partial CS}{\partial \beta} = -\frac{2\alpha t x}{(1+4\alpha\beta t)^2} - \frac{t(9+4\alpha\beta t)}{8(1+4\alpha\beta t)^3} + \frac{t[3+8\alpha t(1+\beta+2\alpha\beta^2 t)]}{8(1+4\alpha\beta t)^2} + \frac{(1+8\alpha\beta t(1+\alpha\beta t))\gamma^2 \epsilon^2}{12\beta^2 t(1+4\alpha\beta t)^2} < 0$$

which leads to the condition in Inequality (21). $\frac{\partial SW}{\partial \beta} < 0$ if and only if

$$\frac{\partial SW}{\partial \beta} = -\frac{2\alpha tx}{(1+4\alpha\beta t)^2} + \frac{t(1+4\alpha t+4\alpha\beta t(1+2\alpha\beta t))}{4(1+4\alpha\beta t)^2} + \frac{(1+8\alpha\beta t(1+\alpha\beta t))\gamma^2\epsilon^2}{12\beta^2 t(1+4\alpha\beta t)^2} < 0$$

which leads to the condition in Inequality (22). □

A.7 Proof of Lemma 2

Proof. We denote $a_A \equiv \frac{1}{2} + \frac{1}{2\tau}(1-\gamma)x_R$, $a_B \equiv \frac{1}{2} - \frac{1}{2\tau}(1-\gamma)x_R$, $b \equiv \frac{1}{2\tau} + \alpha$, and $c \equiv \frac{1}{2\tau}$. The demand functions in Equation (8) then can be rewritten as

$$\begin{aligned} D_A &= a_A - bp_A + cp_B \\ D_B &= a_B - bp_B + cp_A \end{aligned} \tag{57}$$

The retailer's optimization problem in stage 2 is characterized by the first-order conditions of Equation (23):

$$\begin{aligned} \frac{\partial \pi_R}{\partial p_A} &= [a_A - bp_A + cp_B + c(p_B - w_B) - b(p_A - w_A)] = 0 \\ \frac{\partial \pi_R}{\partial p_B} &= [a_B - bp_B + cp_A + c(p_A - w_A) - b(p_B - w_B)] = 0 \end{aligned}$$

from which we can derive the retailer's optimal retail prices as functions of the wholesale prices:

$$\begin{aligned} p_A &= \frac{w_A}{2} + \frac{a_A b + a_B c}{2(b^2 - c^2)} \\ p_B &= \frac{w_B}{2} + \frac{a_B b + a_A c}{2(b^2 - c^2)} \end{aligned} \tag{58}$$

The manufacturers' optimization problems in stage 1 are characterized by the first-order conditions of Equation (24):

$$\begin{aligned} \frac{\partial \pi_A}{\partial w_A} &= \frac{1}{2} (a_A - 2bw_A + cw_B) = 0 \\ \frac{\partial \pi_B}{\partial w_B} &= \frac{1}{2} (a_B - 2bw_B + cw_A) = 0 \end{aligned}$$

from which we can derive the optimal wholesale prices:

$$\begin{aligned} w_A &= \frac{2a_A b + a_B c}{4b^2 - c^2} \\ w_B &= \frac{2a_B b + a_A c}{4b^2 - c^2} \end{aligned} \tag{59}$$

Substituting the above optimal wholesale prices into Equation (58), we derive the optimal retail prices:

$$\begin{aligned} p_A &= \frac{2a_A b + a_B c}{2(4b^2 - c^2)} + \frac{a_A b + a_B c}{2(b^2 - c^2)} \\ p_B &= \frac{2a_B b + a_A c}{2(4b^2 - c^2)} + \frac{a_B b + a_A c}{2(b^2 - c^2)} \end{aligned} \tag{60}$$

With the above equilibrium demands, the optimal wholesale prices in Equation (59), and the optimal retail prices in Equation (60), we have the retailer's equilibrium profit:

$$\pi_R = (p_A - w_A)D_A + (p_B - w_B)D_B = \frac{(a_A - a_B)^2 b^3}{4(b^2 - c^2)(2b + c)^2} + \frac{(2a_A b + a_B c)(2a_B b + a_A c)b^2}{2(b - c)(4b^2 - c^2)^2} \quad (61)$$

Lemma 2 follows by substituting a_A , a_B , b , and c into the above optimal wholesale prices and retail prices. Similarly, by substituting a_A , a_B , b , and c into Equation (61), the retailer's profit can be derived as

$$\pi_R = \left[\frac{(1+2\alpha\tau)^2}{8\alpha(1+4\alpha\tau)^2} + \frac{(1+2\alpha\tau)^2(1-\gamma)^2 x_R^2}{8\tau(1+\alpha\tau)(3+4\alpha\tau)^2} \right] \quad (62)$$

□

A.8 Proof of Proposition 5

Proof. (a) We notice

$$\frac{\partial w_B}{\partial(1-\gamma)} = -\frac{\epsilon}{(1+4\alpha\gamma\epsilon)^2} - \frac{(3+4\alpha\epsilon)x_R}{(3+4\alpha\gamma\epsilon)^2} < 0$$

$\frac{\partial w_A}{\partial(1-\gamma)} < 0$ if and only if

$$\frac{\partial w_A}{\partial(1-\gamma)} = -\frac{\epsilon}{(1+4\alpha\gamma\epsilon)^2} + \frac{(3+4\alpha\epsilon)x_R}{(3+4\alpha\gamma\epsilon)^2} < 0$$

which leads to the condition in Inequality (29).

(b) Substituting $\tau = \gamma\epsilon$ into Equation (62), we have the retailer's profit. $\frac{\partial \pi_R}{\partial(1-\gamma)} > 0$ because

$$\frac{\partial \pi_R}{\partial(1-\gamma)} = \frac{(1+2\alpha\gamma\epsilon)}{8\epsilon} \left(\frac{4\epsilon^2}{(1+4\alpha\gamma\epsilon)^3} + \frac{(1-\gamma)x_R^2[3+\gamma(3+2\alpha\epsilon(6+\gamma(7+2\alpha\epsilon(6+\gamma(3+4\alpha\epsilon)))))]}{\gamma^2(1+\alpha\gamma\epsilon)^2(3+4\alpha\gamma\epsilon)^3} \right) > 0$$

We notice

$$\frac{\partial p_B}{\partial(1-\gamma)} = -\frac{\epsilon}{2(1+4\alpha\gamma\epsilon)^2} - \frac{(15+\alpha\epsilon(17+2\gamma(18+\alpha\epsilon(20+\gamma(11+12\alpha\epsilon))))x_R}{4(1+\alpha\gamma\epsilon)^2(3+4\alpha\gamma\epsilon)^2} < 0$$

$\frac{\partial p_A}{\partial(1-\gamma)} < 0$ if and only if

$$\frac{\partial p_A}{\partial(1-\gamma)} = -\frac{\epsilon}{2(1+4\alpha\gamma\epsilon)^2} + \frac{(15+\alpha\epsilon(17+2\gamma(18+\alpha\epsilon(20+\gamma(11+12\alpha\epsilon))))x_R}{4(1+\alpha\gamma\epsilon)^2(3+4\alpha\gamma\epsilon)^2} < 0$$

which leads to the condition in Inequality (30). □

A.9 Proof of Proposition 6

Proof. As under the platform scheme, we can similarly derive the consumer surplus (CS) and social welfare (SW) by substituting the optimal retail prices in Lemma 2 into Equations

(53) and (54), respectively:

$$CS = \frac{(1+2\alpha\gamma\epsilon)x}{2(1+4\alpha\gamma\epsilon)} - \frac{t}{2} - \frac{(1+2\alpha\gamma\epsilon)(1+6\alpha\gamma\epsilon)}{8\alpha(1+4\alpha\gamma\epsilon)^2} + \frac{t\alpha(1+6\alpha\gamma\epsilon)}{4\alpha(1+4\alpha\gamma\epsilon)} - \frac{(1-\gamma)^2(1+2\alpha\gamma\epsilon)(5+6\alpha\gamma\epsilon)x_R^2}{8\gamma\epsilon(1+\alpha\gamma\epsilon)(3+4\alpha\gamma\epsilon)^2} \\ + \frac{t^2\beta^2}{6\gamma\epsilon} - \frac{t^2\beta^2[2\gamma\epsilon(1+\alpha\gamma\epsilon)^2(3+4\alpha\gamma\epsilon)^2(1+6\alpha\gamma\epsilon) - \alpha(1-\gamma)^2(1+4\alpha\gamma\epsilon)(5+6\alpha\gamma\epsilon)(1+8\alpha\gamma\epsilon(1+\alpha\gamma\epsilon))x_R^2]}{6(1+4\alpha\gamma\epsilon)[4\gamma^2\epsilon^2(1+\alpha\gamma\epsilon)^2(3+4\alpha\gamma\epsilon)^2 - (1-\gamma)^2(1+8\alpha\gamma\epsilon(1+\alpha\gamma\epsilon))^2x_R^2]}$$

$$SW = \frac{(1+2\alpha\gamma\epsilon)}{12\gamma\epsilon} \left[\frac{6\gamma\epsilon x}{1+4\alpha\gamma\epsilon} - \frac{3\gamma\epsilon t}{1+4\alpha\gamma\epsilon} + \frac{2\beta^2 t^2 (1+\alpha\gamma\epsilon)(2\gamma^2\epsilon^2(1+\alpha\gamma\epsilon)(3+4\alpha\gamma\epsilon)^2 - (1-\gamma)^2(1+4\alpha\gamma\epsilon)(1+8\alpha\gamma\epsilon(1+\alpha\gamma\epsilon))x_R^2)}{(1+4\alpha\gamma\epsilon)(4\gamma^2\epsilon^2(1+\alpha\gamma\epsilon)^2(3+4\alpha\gamma\epsilon)^2 - (1-\gamma)^2(1+8\alpha\gamma\epsilon(1+\alpha\gamma\epsilon))^2x_R^2)} \right]$$

When $x_R = 0$, $\frac{\partial CS}{\partial(1-\gamma)} > 0$ because

$$\frac{\partial CS}{\partial(1-\gamma)} = \frac{\alpha\epsilon x}{(1+4\alpha\gamma\epsilon)^2} + \frac{\beta^2 t^2 (1+8\alpha\gamma\epsilon(1+\alpha\gamma\epsilon))}{12\gamma^2\epsilon(1+4\alpha\gamma\epsilon)^2} - \frac{12\alpha\gamma^3\epsilon^3 + t(1+4\alpha\gamma\epsilon)(6\alpha\gamma^2\epsilon^2)}{12\gamma^2\epsilon(1+4\alpha\gamma\epsilon)^3}$$

and we can verify that the above is positive using a condition that even the consumer with a signal indicating the largest degree of misfit has incentive to purchase.

$\frac{\partial SW}{\partial(1-\gamma)} > 0$ because

$$\frac{\partial SW}{\partial(1-\gamma)} = \frac{(2x-t)\alpha\epsilon}{2(1+4\alpha\gamma\epsilon)^2} + \frac{t^2\beta^2(1+8\alpha\gamma\epsilon(1+\alpha\gamma\epsilon))}{12\gamma^2\epsilon(1+4\alpha\gamma\epsilon)^2} > 0$$

□

A.10 Proof of Proposition 7

Proof. (a) We notice

$$\frac{\partial w_B}{\partial\beta} = \frac{t}{(1+4t\alpha\beta)^2} + \frac{4\alpha t(1-\gamma)x_R}{(3+4t\alpha\beta)^2} > 0$$

$\frac{\partial w_A}{\partial\beta} > 0$ if and only if

$$\frac{\partial w_A}{\partial\beta} = \frac{t}{(1+4t\alpha\beta)^2} - \frac{4\alpha t(1-\gamma)x_R}{(3+4t\alpha\beta)^2} > 0$$

which leads to the condition in Inequality (31).

(b) Substituting $\tau = \beta t$ into Equation (62), we have the retailer's profit. $\frac{\partial \pi_R}{\partial\beta} < 0$ because

$$\frac{\partial \pi_R}{\partial\beta} = -\frac{(1+2\alpha\beta t)}{8} \left(\frac{(1-\gamma)^2 x_R^2 [3+4\alpha\beta t(3+2\alpha\beta t)]}{\beta^2 t(1+\alpha\beta t)^2(3+4\alpha\beta t)^3} + \frac{4t}{(1+4\alpha\beta t)^3} \right) < 0$$

We notice

$$\frac{\partial p_B}{\partial\beta} = \frac{t}{2(1+4\alpha\beta t)^2} + \frac{\alpha t(17+8\alpha\beta t(5+3\alpha\beta t))(1-\gamma)x_R}{4(1+\alpha\beta t)^2(3+4\alpha\beta t)^2} > 0$$

$\frac{\partial p_A}{\partial\beta} > 0$ if and only if

$$\frac{\partial p_A}{\partial\beta} = \frac{t}{2(1+4\alpha\beta t)^2} - \frac{\alpha t(17+8\alpha\beta t(5+3\alpha\beta t))(1-\gamma)x_R}{4(1+\alpha\beta t)^2(3+4\alpha\beta t)^2} > 0$$

which leads to the condition in Inequality (32).

□

A.11 Proof of Proposition 8

Proof. As under the platform scheme, we can similarly derive the consumer surplus (CS) and social welfare (SW) by substituting the optimal retail prices in Lemma 2 into Equations (55) and (56), respectively:

$$CS = \frac{(1+2\alpha\beta t)x}{2(1+4\alpha\beta t)} - \frac{t}{2} + \frac{\beta t}{4} - \frac{(1+2\alpha\beta t)(1+6\alpha\beta t)}{8\alpha(1+4\alpha\beta t)^2} + \frac{t(2-\beta)(1+6\alpha\beta t)}{8(1+4\alpha\beta t)} - \frac{(1+2\alpha\beta t)(11+4\alpha\beta t(5+2\alpha\beta t))(1-\gamma)^2 x_R^2}{16\beta t(1+\alpha\beta t)(3+4\alpha\beta t)^2} - \frac{\gamma^2 \epsilon^2}{12\beta t} + \frac{\gamma^2 \epsilon^2 [2\beta t(1+\alpha\beta t)^2(3+4\alpha\beta t)^2(1+6\alpha\beta t) - \alpha(1+4\alpha\beta t)(5+6\alpha\beta t)(1+8\alpha\beta t(1+\alpha\beta t))(1-\gamma)^2 x_R^2]}{12(1+4\alpha\beta t)[4\beta^2 t^2(1+\alpha\beta t)^2(3+4\alpha\beta t)^2 - (1+8\alpha\beta t(1+\alpha\beta t))^2(1-\gamma)^2 x_R^2]}$$

$$SW = \frac{(1+2\alpha\beta t)}{2(1+4\alpha\beta t)} \left[x - \frac{(1+\alpha\beta t)(3(2-\beta)\beta t^2 + \gamma^2 \epsilon^2)(2\beta^2 t^2(1+\alpha\beta t)(3+4\alpha\beta t)^2 - (1+4\alpha\beta t)(1+8\alpha\beta t(1+\alpha\beta t))(1-\gamma)^2 x_R^2)}{6\beta t(4\beta^2 t^2(1+\alpha\beta t)^2(3+4\alpha\beta t)^2 - (1+8\alpha\beta t(1+\alpha\beta t))^2(1-\gamma)^2 x_R^2)} - \frac{(1+8\alpha\beta t(1+\alpha\beta t))(\beta t(3+4\alpha\beta t) + (1+4\alpha\beta t)(1-\gamma)x_R)(4t(1+\alpha\beta t)(3+4\alpha\beta t) + (1+8\alpha\beta t(1+\alpha\beta t))(1-\gamma)x_R)(1-\gamma)x_R}{16\beta t(1+\alpha\beta t)(3+4\alpha\beta t)^2(2\beta t(1+\alpha\beta t)(3+4\alpha\beta t) + (1+8\alpha\beta t(1+\alpha\beta t))(1-\gamma)x_R)} + \frac{(1+8\alpha\beta t(1+\alpha\beta t))(\beta t(3+4\alpha\beta t) - (1+4\alpha\beta t)(1-\gamma)x_R)(4t(1+\alpha\beta t)(3+4\alpha\beta t) - (1+8\alpha\beta t(1+\alpha\beta t))(1-\gamma)x_R)(1-\gamma)x_R}{16\beta t(1+\alpha\beta t)(3+4\alpha\beta t)^2(2\beta t(1+\alpha\beta t)(3+4\alpha\beta t) - (1+8\alpha\beta t(1+\alpha\beta t))(1-\gamma)x_R)} \right]$$

When $x_R = 0$, $\frac{\partial CS}{\partial \beta} < 0$ if and only if

$$\frac{\partial CS}{\partial \beta} = -\frac{\alpha t x}{(1+4\alpha\beta t)^2} + \frac{(1+8\alpha\beta t(1+\alpha\beta t))\gamma^2 \epsilon^2}{24\beta^2 t(1+4\alpha\beta t)^2} + \frac{t(1+4\alpha t(1+2\beta(2+\alpha t(2+\beta(3+4\alpha\beta t))))}{8(1+4\alpha\beta t)^3} < 0$$

which leads to the condition in Inequality (33)

$\frac{\partial SW}{\partial \beta} < 0$ if and only if

$$\frac{\partial SW}{\partial \beta} = -\frac{\alpha t x}{(1+4\alpha\beta t)^2} + \frac{3\beta^2 t^2(1+4\alpha t(1+\beta+2\alpha\beta^2 t)) + (1+8\alpha\beta t(1+\alpha\beta t))\gamma^2 \epsilon^2}{24\beta^2 t(1+4\alpha\beta t)^2} < 0$$

which leads to the condition in Inequality (34)

□

A.12 Proof of Proposition 9

Proof. When $x_R = 0$, the platform scheme generates more profit for the retailer than whole-sale scheme if and only if

$$\pi_R^w - \pi_R^p = \frac{(1+2\alpha\tau)(1-6\alpha\tau) + 8\alpha(1+4\alpha\tau)^2(\mu_A + \mu_B)}{8\alpha(1+4\alpha\tau)^2} < 0 \quad (63)$$

which leads to the condition in Inequality (35).

When $x_R > 0$, we can rewrite Inequality (63) as

$$\pi_R^w - \pi_R^p = \frac{(1-6\alpha\tau)(1+2\alpha\tau)}{8\alpha(1+4\alpha\tau)^2} - \frac{(1-\gamma)^2(7+6\alpha\tau)(1+2\alpha\tau)x_R^2}{8\tau(1+\alpha\tau)(3+4\alpha\tau)^2} + (\mu_A + \mu_B) < 0$$

Therefore, $\pi_R^w < \pi_R^p$ if and only if

$$x_R^2 > \frac{\tau(1+\alpha\tau)(3+4\alpha\tau)^2[(1+2\alpha\tau)(1-6\alpha\tau)+8\alpha(1+4\alpha\tau)^2(\mu_A+\mu_B)]}{\alpha(1+4\alpha\tau)^2(1-\gamma)^2(1+2\alpha\tau)(7+6\alpha\tau)}$$

□

A.13 Proof of Proposition 10

Proof. In the quality-dominates-fit case with $x_R = 0$, $CS^w < CS^p$ because

$$CS^w - CS^p = \frac{(1+2\alpha\gamma\epsilon)[-2\alpha(1+4\alpha\gamma\epsilon)(6\gamma\epsilon x + \beta^2 t^2) - 3\gamma\epsilon + 6\alpha\gamma^2\epsilon^2 + 6\alpha\gamma\epsilon t(1+4\alpha\gamma\epsilon)]}{24\alpha\gamma\epsilon(1+4\alpha\gamma\epsilon)^2}$$

and we can verify that the above is negative using a condition that even the consumer with a signal indicating the largest degree of misfit has incentive to purchase. $SW^w < SW^p$ because

$$SW^w - SW^p = \frac{[-3(2x-t)\gamma\epsilon - \beta^2 t^2](1+2\alpha\gamma\epsilon)}{12\gamma\epsilon(1+4\alpha\gamma\epsilon)} < 0$$

In the fit-dominates-quality case with $x_R = 0$, $CS^w < CS^p$ because

$$CS^w - CS^p = \frac{(1+2\alpha\beta t)[-12\alpha\beta t(1+4\alpha\beta t)x - 3\beta t + 3\beta t(\alpha t(2+(1+4\alpha t(2-\beta))\beta)) + \alpha(1+4\alpha\beta t)\gamma^2\epsilon^2]}{24\alpha\beta t(1+4\alpha\beta t)^2}$$

and we can verify that the above is negative using a condition that even the consumer with a signal indicating the largest degree of misfit has incentive to purchase. $SW^w < SW^p$ because

$$SW^w - SW^p = \frac{(1+2\alpha\beta t)[-12\beta t x + 3\beta t^2(2-\beta) + \gamma^2\epsilon^2]}{24\beta t(1+4\alpha\beta t)} \leq -\frac{(1+2\alpha\beta t)[3\beta t + 9\alpha(3+4\alpha\beta t)\beta^2 t^2 - \alpha(1+4\alpha\beta t)\gamma^2\epsilon^2]}{24\alpha\beta t(1+4\alpha\beta t)^2} < 0$$

where the second inequality is by applying a condition that even the consumer with a signal indicating the largest degree of misfit has incentive to purchase. □