## LECTURE 5: NUMPY AND MATPLOTLIB

Introduction to Scientific Python, CME 193
Feb. 6, 2014
Download exercises from:
web.stanford.edu/~ermartin/Teaching/CME193-Winter15
Eileen Martin

Some slides are from Sven Schmit's Fall '14 slides

## Overview

- Numpy: basic objects, methods, functions
- Numpy: linear algebra
- Numpy: random
- Matplotlib: 2D plots
- Matplotlib: 3D plots
- Scipy vs Numpy
- Discuss assignment 4


## Numpy

- Fundamental package for working with N-dimensional array objects (vector, matrix, tensor, ...)
- corn has version 1.9.1, documentation:


## http://docs.scipy.org/doc/numpy/index.html

- Numpy arrays are a fundamental data type for some other packages to use
- Numpy has many specialized modules and functions:

| numpy.linalg (Linear algebra) | numpy.random (Random sampling) |
| :--- | :--- |
| numpy.fft (Discrete Fourier transform) | sorting/searching/counting |
| math functions | numpy.testing (unit test support) |

## Declaring a Numpy array

Each Numpy array has some attributes:
shape (a tuple of the size in each dimension), dtype (data type of entries), size (total \# of entries), ndim (\# of dimensions), T (transpose)
Use these attributes to insert print statements into declaration.py to figure out each object's type, dimensions and entry data type:

```
import numpy as np
x0 = np.array([True,True,False])
x1 = np.array([2,1,4], np.int32)
x2 = np.array([[2,0,4],[3,2,7]])
x3 = np.empty([3,2])
x4 = np.empty_like(x2)
x5 = np.zeros(4, np.complex64)
x6 = np.arange(1,9,2.0)
x7 = np.diag([1, 2, 4])
x8 = np.linspace(0,np.pi,10)
```


## What can you do?

- Add two arrays
- Add all entries in one array
- Multiply two arrays (1D, 2D)
- Take the exponential of each element in an array
- Multiply an array by a scalar
- Get the minimum element of an array
- Print a few elements of an array
- Print a single column or row of an array
- Multiply two arrays via matrix multiplication

Solutions will be posted on website after class

## Array broadcasting:

Automatically make copies of arrays to fill in length 1 dimensions
\(\left.\begin{array}{|l|l|l|l|l|l|l|l|l|}\hline 0 \& 0 \& 0 <br>

10 \& 10 \& 10\end{array} \quad 0 $$
\begin{array}{ll}0 & 1\end{array}
$$\right) \quad\)| 0 | 1 | 2 |
| :--- | :--- | :--- |
| 10 | 11 | 12 |

| 0 | 0 | 0 |
| :--- | :--- | :--- |
| 10 | 10 | 10 |$\quad$| 0 | 1 | 2 |
| :--- | :--- | :--- |
| 0 | 1 | 2 |


| 0 | 1 | 2 |
| :--- | :--- | :--- |
| 10 | 11 | 12 |



## Iterating over an array

- Iteration over all elements of array:
for element in A.flat
- Iteration over multidimensional arrays is done on slices in the first dimension:

$$
\text { for row in } A
$$

- Alternatively, could access entries through indices:
for i in range(A.shape[0]):
for $j$ in range(A.shape[1]):


## Reshaping an array

- Use reshape to modify the dimensions of an array while leaving the total number of elements the same

$$
\begin{gathered}
\text { A }=\text { np.arange }(8) \\
\text { A.reshape }(2,4) \\
\text { \# gives }[[0,1,2,3],[4,5,6,7]]
\end{gathered}
$$

- Use resize to remove elements or append 0's in place (size can change under some circumstances*)
A.resize(2,3)
- Use resize to return a copy with removed elements or repeated copies

$$
b=r e s i z e(a,(2,4))
$$

## Overview

- Numpy: basic objects, methods, functions
- Numpy: linear algebra
- Numpy: random
- Matplotlib: 2D plots
- Matplotlib: 3D plots
- Scipy vs Numpy
- Discuss assignment 4


## Numpy: Linear Algebra

- The numpy.linalg module has many matrix/vector manipulation algorithms
(a subset of these is in the table)

| name | explanation |
| :--- | :--- |
| dot(a,b) | dot product of two arrays |
| kron(a,b) | Kronecker product |
| linalg.norm( $x$ ) | matrix or vector norm |
| linalg.cond(x) | condition number |
| linalg.solve(A,b) | solve linear system $A x=b$ |
| linalg.inv(A) | inverse of A |
| linalg.pinv(A) | pseudo-inverse of A |
| linalg.eig(A) | eigenvalues/vectors of <br> square A |
| linalg.eigvals(A) | eigenvalues of general A |
| trace(A) | trace (diagonal sum) |
| linalg.svd(A) | singular value <br> decomposition |

http://docs.scipy.org/doc/numpy/reference/routines.linalg.html

## Linear algebra exercise: least squares

- In leastSquares.py, you are given a bunch of noisy data points and you want to fit them with a line:

$$
a x_{i}+b=y_{i}
$$

- This can be written in matrix format

$$
\left(\begin{array}{cc}
x_{0} & 1 \\
x_{1} & 1 \\
\cdots & 1 \\
x_{n-1} & 1
\end{array}\right)\binom{a}{b}=\left(\begin{array}{c}
y_{0} \\
y_{1} \\
\cdots \\
y_{n-1}
\end{array}\right)
$$

Solve for $(a, b)$ so that

$$
\min _{a, b}\left\|X\binom{a}{b}-y\right\|_{2}^{2}
$$



- Hint: Try using linalg.solve( $(X, y)$, linalg.pinv $(X)$, or linalg.Istsq( $X, y$ )
http://docs.scipy.org/doc/numpy/reference/routines.linalg.html


## Overview

- Numpy: basic objects, methods, functions
- Numpy: linear algebra
- Numpy: random
- Matplotlib: 2D plots
- Matplotlib: 3D plots
- Scipy vs Numpy
- Discuss assignment 4


## Numpy: Random

- In the linear regression exercise, those 'measurements' were actually generated by numpy.random
$x=$ np.random.randn(50) \# draw 50 numbers from the standard normal dist.
$y=3.5^{*} x+2+n p . r a n d o m \cdot r a n d n(50)^{*} 0.3$ \# apply a linear transform and add noise
- If you run this, you'll get different numbers each time, so you might want to use np. random. seed (some0bject) to reproduce a random experiment


## Numpy: Random

- The numpy.random module has many distributions you can draw from (a very small subset of these is in the table)

| name | explanation |
| :--- | :--- |
| rand(n0,n1, ..) | ndarray of random values from uniform [0,1] |
| randn(n0,n1,...) | random standard normal |
| randint(lo, [hi, size]) | random integers [lo, hi) |
| shuffle(seq) | shuffle sequence randomly |
| choice(seq,[size,replace,p]) | sample $k$ items from a 1D array with or <br> without replacement |
| chisquare(df,[size]) | sample from Chi-squared distribution with df <br> degrees of freedom |
| exponential([scale,size]) | sample from exponential distribution |

http://docs.scipy.org/doc/numpy/reference/routines.random.html

## Overview

- Numpy: basic objects, methods, functions
- Numpy: linear algebra
- Numpy: random
- Matplotlib: 2D plots
- Matplotlib: 3D plots
- Scipy vs Numpy
- Discuss assignment 4


## Matplotlib: 2D plots

- Matplotlib is the 2D Python plotting library
- We'll mostly use matplotlib. pyplot
- There are tons of options, so consult the documentation:
http://matplotlib.org/users/beginner.html
- matplotlib.pyplot can do many types of visualizations including:
- Histograms, bar charts (using hist)
- Error bars on plots, box plots (using boxplot, errorbar)
- Scatterplots (using scatter)
- Line plots (using plot)
- Contour maps (using contour or tricontour)
- Images (matrix to image) (using imshow)
- Stream plots which show derivatives at many locations (streamplot)
- Pie charts, polar charts (using pie, polar)


## Matplotlib: First example

- Run the code in sin.py
- How do we show two curves on the same plot?

```
import numpy as np
import matplotlib.pyplot as plt
# array of evenly spaces points from 0 to pi
x = np.linspace(0,np.pi,100)
# calculate the sine of each of those points
y = np.sin(x)
# create a plot of the sine curve
plt.plot(x,y)
# actually show that plot
plt.show()
```

More examples: http://matplotlib.org/gallery.html
Documentation: http://matplotlib.org/api/pyplot_api.html

## Back to the linear regression example

- Uncomment lines 28-32 and run the code to produce a scatter plot
- At the end of the code create a plot that overlays the scatter plot with a line plot showing your fit: $a x+b=y$
- As an extra challenge, try to color the markers of the data points to reflect their distance from the line


## Overview

- Numpy: basic objects, methods, functions
- Numpy: linear algebra
- Numpy: random
- Matplotlib: 2D plots
- Matplotlib: 3D plots
- Scipy vs Numpy
- Discuss assignment 4


## Matplotlib: 3D plots

- To do 3D plotting, we'll use mpl_toolkits.mplot3d Axes3D class
- Documentation:
http://matplotlib.org/mpl toolkits/mplot3d/tutorial.htm|\#mplot3d-tutorial
- Can do:
- Line plots (use plot)
- Scatter plots (use scatter)
- Wireframe plots (use plot_wireframe)
- Surface plots (use plot_surface)
- Contours (use contour)
- Bar charts (use bar)


## 3D Plots: First example

- Run the code in sin3D.py

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
# arrays of evenly spaces points from 0 to pi
x = np.linspace(0,np.pi,40)
y = np.linspace(0,np.pi*2,80)
x,y = np.meshgrid(x,y)
# calculate the product of sines for each point
z = np.sin(x)*np.sin(y)
# create a plot of the sine product
ax = plt.subplot(111, projection=`3d`)
ax.plot_surface(x,y,z)
# actually show that plot
plt.show()
```

More examples: http://matplotlib.org/gallery.html
Documentation: http://matplotlib.org/mpl_toolkits/mplot3d/

## Overview

- Numpy: basic objects, methods, functions
- Numpy: linear algebra
- Numpy: random
- Matplotlib: 2D plots
- Matplotlib: 3D plots
- Scipy vs Numpy
- Discuss assignment 4


## Scipy vs. Numpy

- Scipy is a library that can work with Numpy arrays, but can achieve better performance and has some more specialized libraries
- linear algebra (scipy.linalg uses BLAS/LAPACK)
- statistics (scipy.stats has hypothesis tests, correlation analysis)
- optimization (scipy.optimize has multiple solvers, gradient checks, simulated annealing)
- sparse matrices (scipy.sparse supports sparse linear algebra, graph analysis, multiple sparse matrix formats)
- signal processing (scipy.signal has convolutions, wavelets, splines, filters)


## Overview

- Numpy: basic objects, methods, functions
- Numpy: linear algebra
- Numpy: random
- Matplotlib: 2D plots
- Matplotlib: 3D plots
- Scipy vs Numpy
- Discuss assignment 4


## Assignment 4 discussion

- Your questions on assignment 4?
- Tips for assignment 5:
- Online documentation is your friend. Don't hesitate to use it!
- Stuck? test smaller, simpler statements in interactive mode
- Build test cases to verify correctness of your code (not every unit test has to fit into the unittest module framework
- Talk to each other. Use the CourseWork Forums.
- Come to office hrs. Mon. 9:30-10:30, Wed. 3:15-4:15

