

Statistics 1 – Normal Distribution Exam Questions

- 3 The weights of bags of red gravel may be modelled by a normal distribution with mean 25.8 kg and standard deviation 0.5 kg.

- (a) Determine the probability that a randomly selected bag of red gravel will weigh:
- (i) less than 25 kg; *(3 marks)*
 - (ii) between 25.5 kg and 26.5 kg. *(4 marks)*
- (b) Determine, to two decimal places, the weight exceeded by 75% of bags. *(4 marks)*

- 6 (a) The time, X minutes, taken by Fred Fast to install a satellite dish may be assumed to be a normal random variable with mean 134 and standard deviation 16.

- (i) Determine $P(X < 150)$. *(3 marks)*
- (ii) Determine, to one decimal place, the time exceeded by 10 per cent of installations. *(4 marks)*

- (b) The time, Y minutes, taken by Sid Slow to install a satellite dish may also be assumed to be a normal random variable, but with

$$P(Y < 170) = 0.14 \quad \text{and} \quad P(Y > 200) = 0.03.$$

Determine, to the nearest minute, values for the mean and standard deviation of Y . *(6 marks)*

- 1 The volume, L litres, of emulsion paint in a plastic tub may be assumed to be normally distributed with mean 10.25 and variance σ^2 .

- (a) Assuming that $\sigma^2 = 0.04$, determine $P(L < 10)$. *(4 marks)*
- (b) Find the value of σ so that 98% of tubs contain more than 10 litres of emulsion paint. *(4 marks)*

- 1 The length, X centimetres, of eels in a river may be assumed to be normally distributed with mean 48 and standard deviation 8.

An angler catches an eel from the river. Determine the probability that the length of the eel is:

- (a) exactly 60 cm; *(1 mark)*
- (b) less than 60 cm; *(2 marks)*
- (c) within 5% of the mean length. *(5 marks)*

1 The content, in milligrams, of vitamin C in a litre carton of cranberry juice can be modelled by a normal distribution with a mean of 32 and a standard deviation of 2.

(a) Determine the probability that, for a carton chosen at random, the vitamin C content is less than 30 mg. *(3 marks)*

(b) Find, to the nearest milligram, the value of the mean required to ensure that the **percentage** of cartons with a vitamin C content of less than 30 mg is 2.5. *(5 marks)*

1 Garden canes have lengths that are normally distributed with mean 208.5 cm and standard deviation 2.5 cm.

(a) Show that the probability of the length of a randomly selected cane being between 205 cm and 210 cm is 0.645, correct to three decimal places. *(4 marks)*

(b) Ten canes are selected at random.

Calculate the probability that exactly 6 of these canes have lengths between 205 cm and 210 cm. *(3 marks)*

8 A gas supplier maintains a team of engineers who are available to deal with leaks reported by customers. Most reported leaks can be dealt with quickly but some require a long time. The time (excluding travelling time) taken to deal with reported leaks is found to have a mean of 65 minutes and a standard deviation of 60 minutes.

(a) Assuming that the times may be modelled by a normal distribution, estimate the probability that:

(i) it will take more than 185 minutes to deal with a reported leak; *(3 marks)*

(ii) it will take between 50 minutes and 125 minutes to deal with a reported leak; *(4 marks)*

(iii) the mean time to deal with a random sample of 90 reported leaks is less than 70 minutes. *(4 marks)*

(b) A statistician, consulted by the gas supplier, stated that, as the times had a mean of 65 minutes and a standard deviation of 60 minutes, the normal distribution would not provide an adequate model.

(i) Explain the reason for the statistician's statement. *(2 marks)*

(ii) Give a reason why, despite the statistician's statement, your answer to part (a)(iii) is still valid. *(2 marks)*

- 7 Kevin uses his mobile phone for X minutes each day. X is a random variable which may be modelled by a normal distribution with mean 28 minutes and standard deviation 8 minutes.
- (a) Find the probability that on a particular day Kevin uses his mobile phone for:
- (i) less than 30 minutes; *(4 marks)*
- (ii) between 10 and 20 minutes. *(4 marks)*
- (b) Calculate an interval, symmetrical about 28 minutes, within which X will lie on 80% of days. *(4 marks)*
- (c) Find the probability that on 7 randomly selected days the mean time Kevin spends on his mobile phone is at least 30 minutes. *(4 marks)*
- 5 The heights of female students attending a sixth form college have a mean of 168.0 cm and a standard deviation of 4.5 cm. The heights can be modelled by a normal distribution.
- (a) Find the probability that the height of a randomly selected female student attending this college is:
- (i) less than 172.5 cm;
- (ii) between 159 cm and 163.5 cm. *(7 marks)*
- (b) Find the probability that the mean height of a random sample of 11 female students from this college exceeds 172 cm. *(4 marks)*
- (c) The college was represented by 11 players in a ladies football match. Comment on the fact that the mean height of these players was 173.0 cm. *(2 marks)*
- 6 The distance, in kilometres, travelled to work by the employees of a city council may be modelled by a normal distribution with mean 7.5 and standard deviation 2.5.
- (a) Find the probability that the distance travelled to work by a randomly selected employee of the city council is:
- (i) less than 11.0 km; *(3 marks)*
- (ii) between 5.5 km and 10.5 km. *(4 marks)*
- (b) Find d such that 10% of the council's employees travel less than d kilometres to work. *(4 marks)*
- (c) Find the probability that the mean distance travelled to work by a random sample of 6 of the council's employees is less than 5.0 km. *(4 marks)*
- (d) **Without further calculation**, comment on the fact that the mean distance travelled to work by the 6 people employed to clean the council's offices is 4.4 km. *(2 marks)*

- 4 The number of miles that Anita's motorbike will travel on one gallon of petrol may be modelled by a normal distribution with mean 135 and standard deviation 12.
- (a) Given that Anita starts a journey with one gallon of petrol in her motorbike's tank, find the probability that, without refuelling, she can travel:
- (i) more than 111 miles; *(3 marks)*
- (ii) between 141 and 150 miles. *(4 marks)*
- (b) Find the longest journey that Anita can undertake, if she is to have a probability of at least 0.9 of completing it on one gallon of petrol. *(4 marks)*
- 8 The weights, in grams, of the contents of tins of mackerel fillets are normally distributed with mean μ and standard deviation 2.5. The value of μ may be adjusted as required.
- (a) Find the proportion of tins with contents weighing between 125.0 grams and 130.0 grams when $\mu = 129.0$. *(5 marks)*
- (b) (i) State, without proof, the value of μ which would maximise the proportion of tins with contents weighing between 125.0 grams and 130.0 grams. *(1 mark)*
- (ii) Find the proportion of tins with contents weighing between 125.0 grams and 130.0 grams when μ is equal to the value you have specified in part (b)(i). *(3 marks)*
- (c) Find, to one decimal place, the value of μ such that 99% of the tins have contents weighing more than 125.0 grams. *(4 marks)*
- (d) The normal distribution provides a good model for many continuous distributions which arise in production processes or in nature. Explain why the Central Limit Theorem provides another reason for the importance of the normal distribution. *(2 marks)*
- 4 The White Hot Peppers is a traditional jazz band. The length, in minutes, of each piece of music played by the band may be modelled by a normal distribution with mean 5 and standard deviation 1.5, and may be assumed to be independent of the lengths of all other pieces.
- (a) Find the probability that a particular piece will last between 3.5 minutes and 7.25 minutes. *(5 marks)*
- (b) Find the probability that the mean length of the next six pieces to be played by the band will be less than 4 minutes. *(4 marks)*
- (c) There is an interval of exactly one minute between the band finishing one piece and starting the next. The band starts to play its last six pieces at 10.31 pm. Using your answer to part (b), state whether you think it likely that the band will still be playing after 11.00 pm. Justify your answer. *(3 marks)*
- (d) Give a reason why the assumption that the lengths of all pieces are independent may be unrealistic. *(1 mark)*

Statistics 1 – Normal Distribution Exam Questions Mark Scheme

Q	Solution	Marks	Total	Comments
3	Weights $W \sim N(25.8, 0.5^2)$			
(a)(i)	$P(W < 25) = P\left(Z < \frac{25.0 - 25.8}{0.5}\right)$ $= P(Z < -1.6) = P(Z > 1.6) = 1 - \Phi(1.6)$ $1 - 0.94520 = 0.05480 \Rightarrow 0.054 \text{ to } 0.055$	M1 m1 A1	3	<p style="text-align: right;">27.1 26.6 26.1</p> Standardising 24.5, 25.0, or 25.5 with $\sqrt{0.5}$, 0.5, or 0.5^2 and/or $(25.8 - x)$ Area change AFWW
(ii)	$P(25.5 < W < 26.5)$ $P\left(\frac{25.5 - 25.8}{0.5} < Z < \frac{26.5 - 25.8}{0.5}\right) =$ $P(-0.6 < Z < 1.4) =$ $\Phi(1.4) - (1 - \Phi(0.6)) =$ $0.91924 - 1 + 0.72575 = 0.64499 \Rightarrow$ $0.644 \text{ to } 0.646$	M1 A1 m1 A1	4	Standardising 26.1, 25.5 or 26.5 with $\sqrt{0.5}$, 0.5 or 0.5^2 and/or $(25.8 - x)$ CAO both; ignore signs Area difference AFWW
(b)	$P(W > w) = 75\%$ $z_{0.75} = -0.6745$ <p style="text-align: center;">and $z = \frac{w - 25.8}{0.5}$</p> $\therefore \frac{w - 25.8}{0.5} = -0.6745$ $\therefore w = 25.46 \text{ to } 25.47$	B1 M1 m1 A1	4	AFWW ± 0.674 to ± 0.675 not 0.67 or 0.68 Standardising w with $\sqrt{0.5}$, 0.5, or 0.5^2 and/or $(25.8 - w)$ Equating z -value & z -term; not using 0.75, 0.25 or $1 - z$ AFWW; can be gained from using -0.67 or -0.68
Total			11	

Q	Solution	Marks	Total	Comments
6 (a)	Time $X \sim N(134, 16^2)$			
(i)	$P(X < 150) = P\left(Z < \frac{150 - 134}{16}\right)$	M1		standardising (149.5, 150 or 150.5 with $\sqrt{16}$, 16 or 16^2 and/or $(134 - x)$)
	$= P(Z < 1)$	A1		CAO; ignore sign
	$= 0.841(34)$	A1	3	AWRT
(ii)	$P(X > x) = 10\%$			
	$z_{0.10} = 1.2816$	B1		AWFW 1.28 to 1.29; ignore sign
	and $z = \frac{x - 134}{16}$	M1		standardising x with $\sqrt{16}$, 16 or 16^2 and/or $(134 - x)$
	$\therefore \frac{x - 134}{16} = 1.2816$	m1		equating z -value to z -term not using 0.10, 0.90 or $ 1 - z $
	$\therefore x = 154.5$	A1	4	AWRT
(b)	Time $Y \sim N(\mu, \sigma^2)$			
	$z_{0.14} = -1.0803$	B1		AWRT -1.08; ignore sign; not -1.085
	$z_{0.03} = +1.8808$	B1		AWRT +1.88; ignore sign; not +1.885
	$P(Y < 170) = P\left(Z < \frac{170 - \mu}{\sigma}\right)$			
	and	M1		use of either; not with ± 0.5
	$P(Y > 200) = P\left(Z > \frac{200 - \mu}{\sigma}\right)$			
	$\therefore 170 - \mu = -1.0803\sigma$			
	and $200 - \mu = +1.8808\sigma$	m1		simultaneous equations involving μ , σ and z -values; not probabilities etc
	$\therefore 30 = 2.9611\sigma$			
	$\therefore \sigma = 9.9$ to 10.2	A1		AWFW
	$\therefore \mu = 180.8$ to 181.2	A1	6	AWFW
	Total		13	

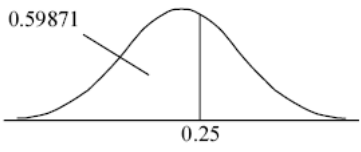
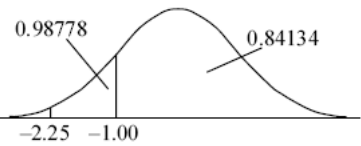
Q	Solution	Marks	Total	Comments
1(a)	$L \sim N(10.25, \sigma^2)$			
	$P(L < 10) = P\left(Z < \frac{10-10.25}{\sqrt{0.04}}\right) =$	M1		standardising (9.5, 10 or 10.5) with ($\sqrt{0.04}$, 0.04 or 0.04^2) and/or $(10.25 - 10)$
	$P(Z < -1.25) =$	A1		cao; ignore sign
	$1 - \Phi(1.25) =$	m1		area change
	$1 - 0.89435 =$			
	0.105 to 0.106	A1	4	awfw
(b)	$P(L > 10) = 0.98$			
	$z_{0.98} = -2.0537$	B1		awfw 2.05 to 2.06; ignore sign
	Also $z = \frac{10-10.25}{\sigma}$	M1		standardising (10 or 10.5) with 10.25 and σ ; allow $(10.25 - 10)$
	Thus $\frac{10-10.25}{\sigma} = -2.0537$	m1		equating z-term to z-value; not using 0.98, 0.02 or $ 1-z $
	Thus $\sigma = 0.121$ to 0.122	A1	4	awfw; do not ignore sign (A0 if negative sign dropped)
Total			8	

Q	Solution	Marks	Total	Comments
1(a)	Length $X \sim N(48, 8^2)$			
	$P(X = 60) = 0$	B1	1	CAO
(b)	$P(X < 60) = P\left(Z < \frac{60-48}{8}\right)$	M1		standardising (59.5, 60 or 60.5) with 48 and ($\sqrt{8}$, 8 or 8^2) and/or $(48 - x)$
	$= P(Z < 1.5) = 0.933$	A1	2	AWRT (0.93319)
(c)	5% of 48 = 2.4	B1		CAO; OE
	$P(48 - x < X < 48 + x)$	M1		attempt at the probability of an interval, symmetric about 48
	$= P(-0.3 < Z < 0.3)$	A1		CAO 0.3
	$= \Phi(0.3) - (1 - \Phi(0.3))$	m1		area change
	$= 2 \times 0.61791 - 1$			
	$= 0.235$ to 0.236	A1	5	AWFW (0.23582)
Total			8	

Q	Solution	Marks	Total	Comments	
1	(a) Vitamin C $X \sim N(32, 2^2)$ $P(X < 30) = P\left(Z < \frac{30-32}{2}\right) =$ $P(Z < -1) = 1 - P(Z < 1) =$ $1 - 0.84134 = 0.15866 = 0.159$	M1	3	Standardising 29.5, 30 or 30.5 with 2, 2 ² or $\sqrt{2}$ and/or (32-x) Area change AWRT	
		m1			
		A1			
	(b) $P(X < 30) = P\left(Z < \frac{30-\mu}{2}\right)$ $z_{0.025} = -1.96$ Thus $\frac{30-\mu}{2} = -1.96$ Thus $\mu = 34$ mg	M1	5	Standardising 29.5, 30 or 30.5 with 2, 2 ² or $\sqrt{2}$ and/or ($\mu-x$) CAO ± 1.96 Equating two z-values (m0 for using $1 - z$ -value) fully correct equality AWRT 34.0; ignore units dependent upon previous A1	
		B1			
		m1			
		A1			
		Total		8	

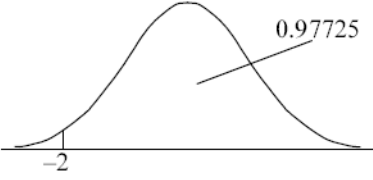
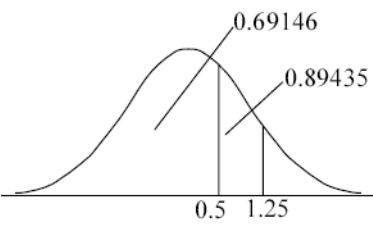
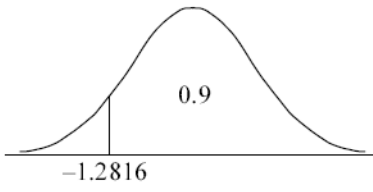
Q	Solution	Marks	Total	Comments
1	(a) $L \sim N(208.5, 2.5^2)$ $P(205 < L < 210)$ $= P\left(\frac{205-208.5}{2.5} < Z < \frac{210-208.5}{2.5}\right)$ $= P(-1.4 < Z < 0.6)$ $= \Phi(0.6) - (1 - \Phi(1.4))$ $= 0.72575 - (1 - 0.91924)$ $= 0.72575 - 0.08076 = 0.64499$ or 0.6448 to 0.6452	M1	4	Standardising (204.5, 205, 205.5 or 209.5, 210, 210.5) with $\sqrt{2.5}$, 2.5 or 2.5 ² ; allow (208.5 - l) CAO; either ignoring sign Area difference AWFW; AG of 0.645
		A1		
		m1		
		A1		
		A1		
		A1		
	(b) Binomial $P(R = 6)$ $= \binom{10}{6} (0.645)^6 ((1-0.645)=0.355)^4$ $= 210 \times 0.07200 \times 0.01588$ = 0.239 to 0.241	M1	3	Use of Correct expression; may be implied AWFW
		A1		
		A1		
		A1		
	Total		7	

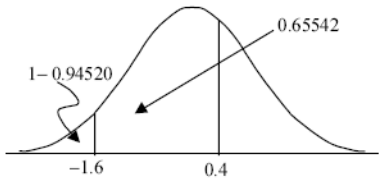
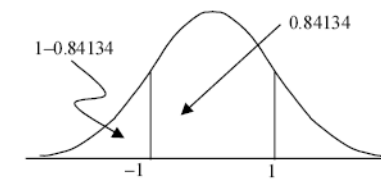
8(a)(i)	$z = (185 - 65)/60 = 2.0$ probability $< 185 = 1 - 0.97725 = 0.02275$	M1 M1 A1	3	method for z completely correct method 0.0227 – 0.023
(ii)	$z_1 = (50 - 65)/60 = -0.25$ $z_2 = (125 - 65)/60 = 1.0$ Probability between 50 and 125 minutes $= 0.84134 - (1 - 0.59871)$ $= 0.440$	M1 M1 M1 A1	4	method for both z's including correct sign any correct use of normal tables completely correct method 0.439 – 0.441
(iii)	$z = (70 - 65)/(60/\sqrt{90}) = 0.791$ Probability mean of 90 < 60 $= 0.786$	M1 M1 M1 A1	4	attempted use of $60/\sqrt{90}$ correct method for z completely correct method 0.78 – 0.79
(b)(i)	Mean only just over one standard deviation above zero. Normal distribution would give a substantial chance of negative time which is impossible.	B2	2	allow both marks if point clearly made
(ii)	Large sample \rightarrow mean normally distributed by central limit theorem.	B1 B1	2	
Total			15	

Q	Solution	Marks	Total	Comments
7(a)				
(i)	$z = \frac{(30-28)}{8} = 0.25$ 	M1		Method for z- ignore sign
		M1 m1		Any correct use of normal tables Completely correct method
		A1	4	0.599 (0.598 - 0.6)
	P(<30 mins) = 0.599	M1		Method for z's - ignore signs if consistent
(ii)	$z = \frac{10-28}{8} = \frac{20-28}{8} = -2.25$ $z = \frac{10-28}{8} = \frac{20-28}{8} = -1.00$ 	m1		Correct signs for z or comment or diagram
	Prob between 10 and 20 0.98778 - 0.84134 = 0.146	M1 A1	4	Correct method - requires first M mark only 0.146 (0.146 - 0.147)
(b)	80% within $28 \pm 1.2816 \times 8$	B1 M1		1.2816 or 1.282 or 1.28 Their $z \times \sigma$
	17.7 to 38.3 mins	M1 A1	4	Method using their z 28 ± 10.3 (10.2 - 10.3) or 17.7 (17.7 - 17.8) and 38.3 (38.2 - 38.3)
(c)	$z = \frac{(30-28)}{\frac{8}{\sqrt{7}}} = 0.661$	B1 M1 m1		$\frac{8}{\sqrt{7}}$ Method for z Completely correct method
	Prob mean of 7 $\geq 30 = 1 - 0.746$ = 0.254	A1	4	0.254 (0.251 - 0.258)
		Total	16	

5(a)(i)	$z = \frac{172.5 - 168}{4.5} = 1.0$ $P(<172.5) = 0.841$	M1 M1 A1	3	method of standardising – ignore sign A correct use of normal tables 0.841 (0.841 , 0.842)
(ii)	$z_1 = \frac{159 - 168}{4.5} = -2.0$ $z_2 = \frac{163.5 - 168}{4.5} = -1.0$ Probability between 159 and 163.5 is $0.97725 - 0.84134 = 0.136$	M1 m1 M1 A1	4	method of standardising – consistent signs Signs of z clearly correct Correct method – depends on M1 only (0.1355 , 0.1365)
(b)	$z = \frac{172 - 168}{\frac{4.5}{\sqrt{11}}}$ $= 2.948$ $P(\text{mean} > 172) = 1 - 0.9984 = 0.0016$	B1 M1 m1 A1	4	Use of $\frac{4.5}{\sqrt{11}}$ method for z Completely correct method 0.0016 (0.0015 , 0.0017)
(c)	Very unlikely 11 randomly selected female students would have a mean height as great as 172cm	E2,1	2	Clear explanation scores 2 marks
Total			13	

Question Number and Part	Solution	Marks	Total	Comments
6(a)(i)	$z = \frac{11 - 7.5}{2.5} = 1.4$ $P(<11) = 0.919$	M1 M1 A1	3	method for z - ignore sign a correct use of normal tables 0.919 (0.919 ~ 0.92)
(ii)	$z_1 = \frac{5.5 - 7.5}{2.5} = -0.8$ $z_2 = \frac{10.5 - 7.5}{2.5} = 1.2$ Probability between 5.5 and 10.5 is $0.88493 - (1 - 0.78814) = 0.673$	M1 M1 m1 A1	4	method for z 's - both signs correct correct methods, their z 's completely correct method 0.673 (0.6725 ~ 0.6735) implies full marks
(b)	$7.5 - 1.2816 \times 2.5 = 4.30$	B1 M1 m1 A1	4	(1.28 ~ 1.29) (their z) $\times 2.5$ completely correct method 4.30 (4.29 ~ 4.3)
(c)	$z = \frac{5.0 - 7.5}{\frac{2.5}{\sqrt{6}}} = -2.449$ probability mean less than 5.0 $= 1 - 0.9928 = 0.0072$	M1 m1 m1 A1	4	use of $\frac{2.5}{\sqrt{6}}$ correct method for z completely correct method 0.0072 (0.007 ~ 0.0073)
(d)	Very unlikely for a random sample of employees. Suggests that cleaners live nearer their place of work, on average, than council employees as a whole.	E1✓ E1	2	Unlikely reason/conclusion
Total			17	

Question Number and part	Solution	Marks	Total marks	Comments
4(a)(i)	$z = \frac{111 - 135}{12} = -2$  $P(>111) = 0.977$	M1 M1 A1	3	Attempt at z, ignore sign, disallow variance Any correct use of normal tables 0.977 – 0.978
(ii)	$z_1 = \frac{141 - 135}{12} = 0.5$ $z_2 = \frac{150 - 135}{12} = 1.25$  Probability between 141 and 150 $0.89435 - 0.69146 = 0.203$	M1 m1 m1 A1	4	Correct method, both zs, ignore signs Correct use of normal tables Completely correct method 0.202 – 0.204
(b)	 Longest journey $135 - 1.2816 \times 12$ $= 120$ miles	B1 M1 m1 A1	4	1.28 – 1.29 ignore sign Correct method – allow incorrect sign Completely correct method 119 – 120
Total			11	

Question Number and part	Solution	Marks	Total	Comments
8(a)	$z_1 = \frac{125 - 129}{2.5} = -1.6$ $z_2 = \frac{130 - 129}{2.5} = 0.4$  <p>Probability between 125 and 130 $= 0.65542 - (1 - 0.94520) = 0.601$</p>	M1 m1 M1	5	Method for z ignore sign; ignore attempt at continuity correction both z's correct sign A correct use of normal distribution – generous. Allow method marks for ‘correct’ continuity correction (ie not 124.5)
(b)(i)	127.5	B1	1	127.5 or 127 or 128 only
(ii)	$z_1 = \frac{130 - 127.5}{2.5} = 1$ $z_2 = \frac{125 - 127.5}{2.5} = -1$  <p>Probability between 125 and 130 $= 0.84134 - (1 - 0.84134) = 0.683$</p>	M1	3	Their mean Completely correct method (dependent on M1 and 127.5) 0.683 (0.68 – 0.685)
(c)	$125 + 2.3263 \times 2.5 = 130.8$	B1 M1 m1 A1	4	2.3263 (2.32 – 2.33) their $z \times 2.5$ m1 correct method 130.8 (130.8 – 130.82) or 131
(d)	Mean of large samples is approximately normally distributed for any parent distribution.	E1 E1	2	Mean normal Large sample
	Total		15	

Question Number and Part	Solution	Marks	Total	Comments
4(a)	$z_1 = \frac{3.5 - 5.0}{1.5} = -1$	M1	5	Method for z – ignore sign
	$z_2 = \frac{7.25 - 5.0}{1.5} = 1.5$	m1		Both signs correct or correct diagram
	$P(3.50 < X < 7.25)$	M1		Any correct use of normal tables - generous
	$= 0.93319 - (1 - 0.84134) = 0.775$	m1		Completely correct method – not dependent on previous m1
		A1		0.775 (0.774 to 0.775)
(b)	$z = \frac{4 - 5}{\frac{1.5}{\sqrt{6}}} = -1.633$	M1	4	Use of $\frac{1.5}{\sqrt{6}}$
		m1		Correct method for z ignore sign
	$P(\text{mean} < 4) = 1 - 0.9488 = 0.0512$	m1 A1		Completely correct method 0.0512 (0.051 to 0.052)
(c)	To finish before 11.00pm band will need to play 6 pieces in $29 - 5 = 24$ minutes i.e. mean 4 minutes. Low probability as shown in (b).	E1 E1 E1✓	3	Attempt to find necessary mean, or verify Correct mean found Correct conclusion, generous
(d)	After playing a long piece the band may choose to play a short piece.	E1	1	Reason
Total			13	