Lecture Notes: Public Economics

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Contents

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Part I

Public Goods

0.1 Introduction

0.1.1 What are Public Goods?

A good is called a pure public good if "each individual's consumption of such a good leads to no subtraction from any other individual's consumption" (Samuelson 1954, p387) This is commonly referred to as *non-rivalry* in use. There are two important areas of economics in which public goods play an important role. The first is the case of spending on things such as national defense: the cost of providing a missile defense system is independent of the number of people inhabiting the protected area, and it is impossible to defend some but not all the inhabitants. This is the prototypical example used to motivate the role of government in providing such public goods.

Historically, this was the set of problems that motivated the interest in public goods. While interesting, more recently a second class of problems in which public goods play an important role is family economics. A married couple jointly consumes many goods and shares the cost of those goods. How a couple decides on the amount of time spent on child rearing and how much each contributes to that amount is central to understanding child development, labor force participation, the degree of matching assortivity and probabilities of divorce. Chapters 2 and 3 in Browning, Chiappori and Weiss (2014) are a good reference to this area.

We will primarily consider the simplest case with a single private good and a single public good.

0.2 The Model

- n consumers, indexed by $i = 1, ..., n$
- x_i : agent i's consumption of private good and denote $\mathbf{x} = (x_1, ..., x_n)$ as the vector of private consumption
- G: the (common) consumption of public good
- Agent i's preference described by the utility function

$$u_i (x_i, G)$$

which is differentiable and increasing in both arguments, quasi-concave and satisfies Inada Condition

- w_i : agent i's endowment of private good and
$$W = \sum_{i=1}^{n} w_i$$
is the total endowment of private good; and public good endowment is taken to be zero

- Public good may be produced from the private good according to a production function $f : R_+ \to R_+$ where $f' > 0$ and $f'' < 0$. That is, if z is the total units of private goods that are used as inputs to produce the public good, the level of public good produced will be
$$G = f(z).$$

0.3 Optimal Provision of Pure Public Good

We first ask the normative question of what is the optimal level of pure public good. We assume that the government of a fully controlled economy chooses the level of G, and the allocation of private goods $\mathbf{x} = (x_1, ..., x_n)$ to agents according to the Pareto criterion.

Definition 1 *An allocation* $(\mathbf{x}, G) \in R_+^{n+1}$ *is feasible if there exists some* $z \geq 0$ *s.t.*

- $\sum_{i=1}^{n} x_i + z \leq W$;
- $G \leq f(z)$.

Alternatively we could define that an allocation $(\mathbf{x}, G) \in R_+^{n+1}$ is feasible if
$$\sum_{i=1}^{n} x_i + f^{-1}(G) \leq W.$$

Definition 2 *A feasible allocation* $(\mathbf{x}, G)$ *is Pareto optimal if there exists no other feasible allocation* $(\mathbf{x}', G')$ *s.t.*
$$u_i\left(x_i', G'\right) \geq u_i(x_i, G)\, \forall i = 1, ..., n$$
and for some $i \in \{1, ..., n\}$,
$$u_i\left(x_i', G'\right) > u_i(x_i, G).$$

That is, a feasible allocation $(\mathbf{x}, G)$ is Pareto optimal if there is no way of making an agent strictly better off without making someone else worse off.

Now we can characterize the set of Pareto optimal allocations. It is the solution to the following problem:

$$\begin{aligned}
&\max_{\{x,G,z\}} u_1 (x_1, G) \\
\text{s.t. } & u_i (x_i, G) - \underline{u}_i \geq 0 \text{ for } i = 2, 3, ..., n, \qquad (\text{multiplier } \gamma_i) \\
& W - \sum_{i=1}^{n} x_i - z \geq 0 \qquad (\text{multiplier } \lambda) \\
& f(z) - G \geq 0 \qquad (\text{multiplier } \mu) \\
& G \geq 0, z \geq 0 \text{ and } x_i \geq 0 \text{ for all } i = 1, ..., n
\end{aligned}$$

where $\underline{u}_i$ are treated as parameters of the problem. Inada conditions on the utility function implies that the non-negativity constraints can be ignored. The necessary and sufficient (sufficiency due to quasi-concavity assumption on u and f) Kuhn-Tucker conditions are:

$$\begin{aligned}
(x_i :) \qquad & \gamma_i \frac{\partial u_i (x_i, G)}{\partial x_i} - \lambda = 0 \qquad (1)\\
(G :) \qquad & \sum_{i=1}^{n} \gamma_i \frac{\partial u_i (x_i, G)}{\partial G} - \mu = 0 \\
(z :) \qquad & -\lambda + \mu f' (z) = 0
\end{aligned}$$

where we have set $\gamma_1 = 1$ by convention.

From the fist n equalities, we obtain

$$\gamma_i = \frac{\lambda}{\partial u_i (x_i, G) / \partial x_i}.$$

From the last equality, we obtain

$$\mu = \frac{\lambda}{f' (z)}$$

Plugging these $n + 1$ equalities into the the middle condition regarding G, we get

$$\sum_{i=1}^{n} \frac{\partial u_i (x_i, G) / \partial G}{\partial u_i (x_i, G) / \partial x_i} = \frac{1}{f' (z)}. \qquad (2)$$

This condition is referred to as the Samuelson condition, the Lindahl-Samuelson condition, or sometimes even the Bowen-Lindahl-Samuelson Condition and is probably familiar to anyone who have taken an intermediate course in public economics.

Interpretations: The left hand of equation (2) is the sum of the marginal rates of substitutions of the n agents. To see this, note that from agent i's indifference curve, the term

$$\frac{\partial u_i\left(x_i, G\right)/\partial G}{\partial u_i\left(x_i, G\right)/\partial x_i}$$

denotes the quantity of private good agent i is willing to give up for a small unit increase in the level of the public good. The right hand of equation (2) is the amount of private good required to produce an additional unit of public good (also known as the marginal rate of transformation). Hence the Samuelson condition says the following: *Any optimal allocation is such that the sum of the quantity of private goods consumers would be willing to give up for an additional unit of public good must equal to the quantity of private good that is actually required to produce the additional unit of public good.*

If there are more than one private goods, say k private goods; and the public good is produced according to

$$f\left(z_1, ..., z_k\right),$$

then the corresponding Samuelson condition for the optimal level of public goods is given by

$$\sum_{i=1}^{n} \frac{\partial u_i\left(x_{ij}, G\right)/\partial G}{\partial u_i\left(x_{ij}, G\right)/\partial x_{ij}} = \frac{1}{\partial f\left(z_1, ..., z_k\right)/\partial z_j} \text{ for all } j = 1, ..., k.$$

A diagrammatic illustration of the Samuelson condition for the case where there are two individuals and two goods is given in Figure 1. In Figure 1, the upper part shows the indifference curves for citizen I and the production constraint AB. Suppose that we fix citizen I on the indifference curve $\underline{u}_I$, then the possibilities for citizen II are shown in the lower part of Figure 1 by CD (which is the difference between AB and $\underline{u}_I$). Clearly Pareto efficiency requires the marginal rate of substitution of the second individual be equal to the slope of the curve CD (i.e. at point E). But this is just the difference between the marginal rate of transformation (the slope of the production possibilities schedule) and the marginal rate of substitution of the first individual (the slope of his indifference curve). Thus we have

$$MRS^{II} = MRT - MRS^{I}.$$

Implementation of the optimal allocation: If the government is able to levy lump sum taxes both to finance the expenditure and to redistribute income, then it is clear that the optimal allocation above can be achieved. If lump sum taxes are not feasible, the government

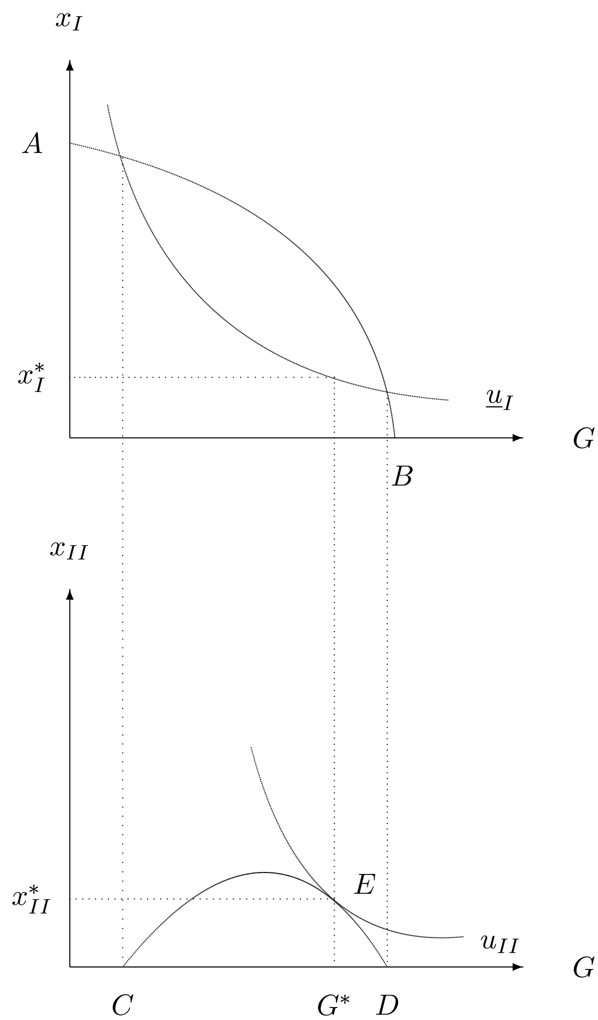


Figure 1: Optimal Provision of Public Goods - The Two Person Example

needs to use distortionary. taxes, for example, tax on labor income, to finance the public goods.

0.4 Can the Optimal Allocation be Decentralized?

Can the optimal allocations characterized by the Samuelson condition be decentralized? Imagine that competitive markets exist for both the private and the public goods. Let the private good be the numeraire.

- Let p denote the price of the public good (in terms of the private good);
- Let g_i denote the quantity of public good purchased by agent i;
- Without loss of generality, we assume that there is a single price-taking profit maximizing firm that operates on the market.

We will make the following (somewhat sloppy) assumption: we assume that all agents are price-takers (i.e. their choice does not affect the price level), but they do take into account that their purchase can affect the aggregate level of public goods.

Given the public good purchases by other agents $\tilde{\mathbf{g}}_{-i} = (g_1, ..., g_{i-1}, g_{i+1}, ..., g_n)$, agent i's best response to $\tilde{\mathbf{g}}_{-i}$ given a price p is defined as

$$\begin{aligned} \beta_i(\tilde{\mathbf{g}}_{-i}, p) &= \arg\max_{\{g_i\}} u_i\left(w_i - pg_i, g_i + \sum_{j \neq i} g_j\right) \\ \text{s.t. } g_i &\geq 0 \\ w_i - pg_i &\geq 0 \end{aligned}$$

Assuming that u_i is strictly quasi-concave, there is a unique solution to the maximization problem for the agent given $\tilde{\mathbf{g}}_{-i}$ and p which is fully characterized by

$$\begin{aligned} -\frac{\partial u_i}{\partial x_i}p + \frac{\partial u_i}{\partial G} + \lambda - \mu p &= 0 \\ \lambda g_i &= 0 \\ \mu(w_i - pg_i) &= 0 \end{aligned}$$

Since u_i satisfies Inada condition, $\mu = 0$. Hence we have

$$p \geq \frac{\partial u_i / \partial G}{\partial u_i / \partial x_i}.$$

The profit maximizing supplier of the public good solves, for a given price p, the following problem

$$\max_{z\geq 0} pf(z) - z$$

which yields the condition that

$$p = \frac{1}{f'(z)}.$$

Definition 3 *A competitive equilibrium consists of* $p^*, G^* = (g_1^*, ..., g_n^*)$ *such that*

1. *For each* i, *given* p^* *and* $\tilde{\mathbf{g}}_{-i}^* = \left(g_1^*, ..., g_{i-1}^*, g_{i+1}^*, ..., g_n^*\right)$,

$$g_i^* \in \beta_i\left(\tilde{\mathbf{g}}_{-i}^*, p^*\right)$$

2. *The firm optimizes, i.e.*

$$p^* = \frac{1}{f'\left(f^{-1}\left(\sum_{i=1}^n g_i^*\right)\right)}.$$

Because of the Inada condition on u_i regarding G, we must have that for some $j \in \{1, ..., n\}$,

$$p = \frac{\partial u_i/\partial G}{\partial u_i/\partial x_i}.$$

Together with the firm's optimization condition, we obtain that for some j,

$$\frac{1}{f'(z)} = \frac{\partial u_j/\partial G}{\partial u_j/\partial x_j}.$$

That is, in the competitive equilibrium, it must be the case that

$$\sum_{i=1}^n \frac{\partial u_i/\partial G}{\partial u_i/\partial x_i} > \frac{1}{f'(z)}.$$

Hence there is under-provision of the public good relative to the level prescribed by the Samuelson condition. The intuition is the following: each agent when deciding how much public good to purchase, does not consider the benefit to other agents of the output he purchased. This is true for each agent and consequently as a group the agents purchase less than the amount desirable for Pareto optimality.

Example 4 *Suppose* $u_i(x_i, g) = \gamma \ln g + \ln x_i$, *and* $w_i = W/n$, *and* $f(z) = z$. *Find the egalitarian Pareto optimal allocation; and the competitive equilibrium allocation. It can be shown that the egalitarian Pareto optimal allocation is given by*

$$\hat{G} = \frac{\gamma W}{1+\gamma}, \hat{x}_i = \frac{W}{n(1+\gamma)}, i = 1, ..., n$$

and the competitive equilibrium allocation is

$$G^* = \frac{\gamma W}{n+\gamma}, x_i^* = \frac{W}{n+\gamma}, i = 1, ..., n.$$

It is clear that as n gets larger, the under-provision of the public good gets more severe.

0.5 Lindahl Equilibria

While the competitive equilibrium with a fixed price of the public good will yield an inefficient allocation, there is a much studied "market institution" that in principle would achieve efficiency. The idea is to think of the amount purchased by each agent as a distinct commodity and have each agent to face a *personalized price* p_i and to have these price chosen in a way such that all agents agree on the level of the public good. Let $s_i \in [0,1]$ be agent i's share of the firm's profit with $\sum_{i=1}^n s_i = 1$

Definition 5 *A Lindahl equilibrium is a vector $\mathbf{p}^* = (p_1^*, ..., p_n^*)$ and an allocation $(x_1^*, ..., x_n^*, G^*)$ such that*

- *The firm maximizes profits, that is,*

$$G^* = \arg\max_{G \geq 0} \left(\sum_i p_i^* \right) G - f^{-1}(G)$$

- *Each consumer maximizes utility, that is,*

$$\begin{aligned} & (x_i^*, G^*) = \arg\max_{x_i, G} u_i(x_i, G) \\ s.t. \qquad & w_i + s_i \left(\sum_i p_i^* G^* - f^{-1}(G^*) \right) - x_i - p_i^* G \geq 0 \end{aligned}$$

- *Market clears, i.e.*

$$\sum_{i=1}^n x_i^* + f^{-1}(G^*) \leq \sum_{i=1}^n w_i.$$

The Lindahl equilibrium is a competitive equilibrium in a fictitious economy where the space of goods has been expanded to $(n+1)$ goods, the private goods and n personalized public goods, that is, the public goods of agent 1 through agent n. These n goods are produced "jointly", so that we must find a vector of prices for which all agents demand equal quantities of the public good. We now show that a Lindahl equilibrium is indeed Pareto optimal. To see this, note that the first order condition for the firm's profit maximization gives

$$\sum_{i=1}^{n} p_i^* = \frac{1}{f'\left(f^{-1}\left(G^*\right)\right)}$$

and the first order condition for individual i's utility maximization is

$$\frac{\partial u_i\left(x_i^*, G^*\right)}{\partial x_i} p_i^* = \frac{\partial u_i\left(x_i^*, G^*\right)}{\partial G} \text{ for all } i = 1, ..., n.$$

Hence

$$\sum_{i=1}^{n} \frac{\partial u_i\left(x_i^*, G^*\right)/\partial G}{\partial u_i\left(x_i^*, G^*\right)/\partial x_i} = \frac{1}{f'\left(f^{-1}\left(G^*\right)\right)}$$

which satisfies the by-now familiar Samuelson condition. Furthermore, all agents' budget sets must hold with equality, which means that market clears with equality. Hence a Lindahl equilibrium is efficient.

Example 6 *Find the Lindahl equilibrium of the economy described by Example 4. Suppose that agent i's personalized price for the public good is p_i. It is easy to solve for i's demand for the public good will be given by*

$$g_i\left(p_i\right) = \frac{1}{p_i} \frac{\gamma W}{\left(\gamma + 1\right) n}.$$

Since the demand of public good must be equal for all the agents in a Lindahl equilibrium it must be the case that in a Lindahl equilibrium, $p_i^* = p_j^*$ for all $i, j \in \{1, ..., n\}$. The firm's profit maximization requires that

$$\sum_{i=1}^{n} p_i^* = 1$$

Hence $p_i^* = 1/n$ for all i. Plugging this individualized price, we obtain that

$$g_i\left(p_i^*\right) = \frac{\gamma W}{\gamma + 1} \textit{ for all } i$$

which is the public good level in the egalitarian Pareto-optimal allocation.

The efficiency of Lindahl equilibrium allocation can be established using an argument which is more or less a copy of the textbook proof of the first welfare theorem.

Proposition 7 *Any Lindahl equilibrium is Pareto optimal.*

Proof. Let $(x_1^*, ..., x_n^*, G^*)$ be a Lindahl equilibrium allocation with corresponding prices $(p_1^*, ..., p_n^*)$ and suppose that there exists a feasible allocation $(x_1', ..., x_n', G')$ that Pareto dominates $(x_1^*, ..., x_n^*, G^*)$. That is,

$$\begin{aligned} u_i\left(x_i', G'\right) &\geq u_i\left(x_i^*, G^*\right) \forall i \\ u_i\left(x_i', G'\right) &> u_i\left(x_i^*, G^*\right) \text{ for at least one agent } i \end{aligned}$$

By revealed preference, this implies that (assuming local non-satiation) that

$$\begin{aligned} x_i' + p_i^* G' &\geq x_i^* + p_i^* G^* \ \forall i \\ x_i' + p_i^* G' &> x_i^* + p_i^* G^* \text{ for at least one agent } i \end{aligned}$$

Hence

$$\begin{aligned} \sum_i x_i' + G' \sum_i p_i^* &> \sum_i x_i^* + G^* \sum_i p_i^* = \sum_i w_i + \sum_i s_i \left[\left(\sum_i p_i^*\right) G^* - f^{-1}\left(G^*\right)\right] \\ &= \sum_i w_i + \left(\sum_i p_i^*\right) G^* - f^{-1}\left(G^*\right) \end{aligned}$$

where the first equality follows since each agent has to fulfill her budget constraint with equality in order to maximize utility.

Moreover, from the first profit maximization, we have

$$\left(\sum_i p_i^*\right) G^* - f^{-1}\left(G^*\right) \geq \left(\sum_i p_i^*\right) G' - f^{-1}\left(G'\right)$$

Combining the above two inequalities, we obtain

$$\sum_i x_i' + f^{-1}\left(G'\right) > \sum_i w_i$$

which contradicts the feasibility of the allocation $(x_1', ..., x_n', G')$. ■

0.5.1 Is Lindahl Equilibrium a Reasonable Market Mechanism?

The Lindahl equilibrium is more a normative prescription for the allocation of public goods than a positive description of the market mechanism. The reason is simple: by the definition of the personalized price in the Lindahl equilibrium, an agent will quickly lean that he should *not* behave competitively (an assumption which has always been justified by the existence of a large number of market participants). He will have incentive to misreport her desire for the public good. Contrary to the case of private goods, where the incentive to reveal false demand functions decreases with the number of agents, an increase in the number of agents in the case of public good only aggregates the problem. We demonstrate this problem by the following example.

Example 8 *Consider n agents with utility function $u_i(x_i, G) = \ln x_i + \alpha_i \ln G$. We suppose that each agent has an endowment of the private good $w_i = 1$ and no public good. Suppose that the technology is linear, i.e. $f(z) = z$ for all $z \geq 0$. Facing a personalized price p_i, it is clear that agent i will demand public good*

$$p_i g_i(p_i) = \frac{\alpha_i}{1+\alpha_i}$$

Since in a Lindahl equilibrium

$$g_i(p_i) = G \, \forall i$$

we have

$$G \sum p_i = \sum_i \frac{\alpha_i}{1+\alpha_i}$$

For the firm's profit maximization problem to have a solution, it must be that

$$\sum p_i = 1$$

hence

$$G = \sum_i \frac{\alpha_i}{1+\alpha_i}$$

and

$$p_i = \frac{\alpha_i / (1+\alpha_i)}{\sum_j \alpha_j / (1+\alpha_j)}$$

Agent i's consumption of private good is

$$x_i = \frac{1}{1+\alpha_i}$$

Suppose that $n = 3$, and $\alpha_i = 1$ for $i = 1, 2, 3$. The Lindahl equilibrium is then

$$p_i^* = \frac{1}{3}, x_i^* = \frac{1}{2}, G^* = \frac{3}{2}$$

so the equilibrium utility level for agent i is

$$\ln x_i^* + \alpha_i \ln G^* = \ln\left(\frac{1}{2}\right) + \ln\left(\frac{3}{2}\right)$$

Now make the following thought experiment: suppose Mr. 2 and 3 report truthfully that their types are $\alpha_i = 1$, but that Mr. 1 lies and claim that $\alpha_1 = 0$. If the planner computes the Lindahl price believing all the agents, the corresponding Lindahl prices and allocations will be

$$p_1 = 0, p_2 = p_3 = \frac{1}{2}, x_1 = 1, x_2 = x_3 = \frac{1}{2}, G = 1.$$

Mr. 1's utility would then be

$$2 \ln (1) = 0.$$

It is easy to see that

$$\ln (1) > \frac{1}{2} \ln \left(\frac{1}{2}\right) + \frac{1}{2} \ln \left(\frac{3}{2}\right)$$

since logarithm is strictly concave. Hence truth telling is not an equilibrium of this game.

While the above example is special, the logic is quite general. If agents have to report preferences (or wealth) they will take into consideration that under-reporting means a lower personalized price, and will generally benefit by misreporting their preferences.

The example may suggest that problems arise in markets for public goods that do not arise in the case of private goods. This is misleading however. One can construct private goods examples analogous to the example. In particular, consider the game in which agents announce preferences, and for each vector of announced preferences, the outcome is the Walrasian equilibrium outcome. (Think of preferences for which there is a unique Walrasian equilibrium for now.) It is generally the case that it is NOT an equilibrium for agents to announce preferences truthfully. In fact, the case is even more serious. Consider all possible mechanisms in which agents announce their preferences, and for each vector of preferences, the outcome is an allocation that is Pareto efficient and individually rational for the economy in which agents' preferences are those announced. For every such mechanism, there will be a pure exchange economy in which at least one agent can benefit by misreporting his preferences if all other agents report truthfully. The result is due to Hurwicz (1972). I will give a simple proof of the result with preferences that are not strictly convex or differentiable. Example 1 in Jackson (A Crash Course in Implementation) gives a slightly more complicated proof with "nicer" preferences.

Example:

There are two agents. Both agents have parallel indifference curves as shown in the figure below. The endowments are $w_1 = (1, 0)$ and $w_2 = (0, 1)$. For these preferences, the Pareto efficient set is the diagonal of the Edgeworth box, and the line segment AB is the subset of the Pareto rational allocations that is individually rational. If there is a mechanism that always selects allocations that are Pareto efficient and individually rational, the outcome must be in the set AB for this economy. Suppose the allocation is C. If agent 1 were to announce preferences with parallel straight line indifference curves steeper than his true indifference curves, the set of Pareto efficient and individually rational allocations for this economy would be in the set DB. The mechanism must choose an allocation in this set if it always selects an allocation that is Pareto efficient and individually rational for the announced preferences. But *all* allocations in DB are preferred by agent 1 to C, hence agent 1 can benefit in this economy by misreporting his preferences.

This argument would fail if the outcome in the initial economy was B, since in that case agent 1 is getting the best point consistent with the outcome being individually rational for agent 2. But it is easy to verify that in this case agent 2 can misreport his preferences and guarantee an outcome preferable to B. Thus, one or both of the agents can beneficially misreport preferences in this economy.

Exercise 9 *Prove that the analog in the public good case of the result that there was no mechanism that selected Pareto efficient and individually rational outcomes for the announced preferences for which truthful announcement was always a Nash equilibrium.*

The results for both the public good and private good case show that there is no hope in either case of finding mechanisms for which truthful announcement of preferences will always be an equilibrium. You know from 701 that in this complete information framework, asking that truthful announcement always be an equilibrium is equivalent to asking for a dominant strategy mechanism. This is the case since if my announcing my true preferences is a best response to all announcements other agents might make, it is a best response to all possible strategies, hence, dominant strategy. I note that this is not the case with asymmetric information since there are some strategies available to agents that are inconsistent with truthful announcement, hence truthful announcement can be a best response for me to all vectors of strategies for other agents that are consistent with truthful announcement, but not be a best response to some nontruthful announcement strategies.

To sum up, it is fruitless to look for dominant strategy mechanisms that give "nice" outcomes (Pareto efficient and individually rational) for either public goods or private goods economies. One possibility is to lower one's sights and ask for a weaker solution concept, say Nash equilibrium. From the discussion above, it is clear that we gain nothing by weakening the solution concept to Nash equilibrium unless we depart from revelation mechanisms.

0.5.2 Nash Implementation

The question is whether we can find a game that has the property that for any public good economy, the Lindahl equilibrium allocation is a Nash equilibrium outcome of the game. The answer is positive, but unsatisfactory.

Example 10 *Consider two person public good economies with the linear technology $f(z) = z$ for all $z \geq 0$ and initial endowments w_1,w_2 Each agent i announces a feasible allocation:*

$$M_i = \{(x_1, x_2, G)\,|x_1 + x_2 + G = w_1 + w_2\}.$$

The outcome function is

$$\begin{aligned} h(m_1, m_2) &= m = m_1 = m_2 \quad \textit{if } m_1 = m_2 \\ \textit{and} \quad h(m_1, m_2) &= (w_1, w_2) \quad \textit{otherwise} \end{aligned}$$

It is easy to see that a Lindahl equilibrium outcome is a Nash equilibrium outcome of the game. If both agents announce the Lindahl equilibrium outcome, that will be the outcome for the announcements. If either unilaterally deviates, the outcome is the initial endowment, (w_1, w_2). This cannot be better for any agent since the Lindahl equilibrium is Pareto efficient.

As mentioned, this is not a very satisfactory game however. While it is true that the Lindahl equilibrium is a Nash equilibrium outcome, so is every other individually rational allocation: if both agents announce such an allocation, no unilateral deviation can benefit an agent. The game is not very interesting since these allocations can be very inefficient.

It is clear from this example that we would like not only for the Lindahl equilibrium to be a Nash equilibrium, but also that there are no other Nash equilibrium outcomes (or at least no inefficient Nash equilibrium outcomes). This leads us to *strong implementation* or *exact implementation* of a social choice function $f : \mathcal{E} \rightarrow \mathcal{A}$, where $\mathcal{E}$ is the domain of the social choice function and $\mathcal{A}$ is the set of possible outcomes. We say that a game *strongly implements* the social choice function f if for every $e \in \mathcal{E}$, $f(e) =$ {Nash equilibrium outcomes given e}.

Monotonicity

I argued above that we would like strong implementation. A necessary condition for strong implementation in complete information is *monotonicity*. Consider first the case that the set of outcomes is finite.

Definition 11 *A social choice correspondence F is a mapping that associates with every profile of preferences a subset of outcomes:*

$$F : \mathcal{P} \to 2^A$$

F is a social choice function if it is single-valued.

Definition 12 *A social choice correspondence F is monotonic if for any $R \in \mathcal{P}$, $\bar{R} \in \mathcal{P}$ and $a \in F(R)$ such that $a \notin F(\bar{R})$, there exists i and b such that aR_ib and $b\bar{P}_ia$.*

Roughly, a social choice correspondence will be monotonic if the following holds. Suppose $a \in F(R)$. Now look at a profile $\bar{R}$ derived from R with the property that for all i the set of outcomes preferred to a in $\bar{R}_i$ is a subset of the preferred set in R_i. Then it must be the case that $a \in F(\bar{R})$.

Theorem 13 *(Maskin) If a social choice correspondence F is Nash implementable then F is monotonic.*

The proof can be found in Jackson and is straightforward, essentially a manipulation of the definition of Nash equilibria.

Monotonicity is a quite strong property, yet many quite plausible social choice correspondences fail to satisfy the property. Thus many interesting social choice correspondences *cannot* be Nash implemented.

Example 14 *(Example 3 in Jackson (Implementation))*

R_1	R_2	R_3	$\bar{R}_3$
a	c	b	c
b	a	c	b
c	b	a	a

Let $R = (R_1, R_2, R_3)$ and consider the social choice correspondence $F(R) = \{a, b, c\}$ and

$F(R_{-3}, \bar{R}_3) = \{c\}$. F is not monotonic ($a \in (F(R) \Longrightarrow a \in F(R_{-3}, \bar{R}_3)$). But F is the Borda rule, plurality rule and most other common voting rules.

Fact: If a social choice function F is monotonic and the domain includes all strict preferences and there are at least three outcomes in the range, there is a dictator, i.e., an agent who always gets his first choice.

Definition 15 *A social choice rule satisfies no veto power if whenever i,R,a are such that aR_ib $\forall j \neq i$ and all $b \in A$, then $a \in F(R)$.*

Theorem 16 *(Maskin) If $n \geq 3$, if a social choice correspondence F satisfies monotonicity and no veto power, then it is Nash implementable.*

One can strongly implement interesting social choice rules in both public goods and private goods problems. The following exercise gives an example of a mechanism does this for public goods problems. Hurwicz, Maskin, Postlewaite (1995) provides an extensive analysis of strong implementation in public and private good economies with and without production.

Exercise 17 *(Hurwicz, Maskin, Postlewaite) Suppose there are 3 agents, $i = 1, 2, 3$. Agent i's utility function is given by $u_i(x_i, G)$ where x_i is the private good and G is the public good. Agent i's endowment of private good is w_i. Suppose that all three agents know all preference parameters, wealth etc. of the other agents, but the planner is uninformed. Suppose the production function for the public good is $f(z) = z$. The planner designs the following mechanism:*

- *Each agent i sends a message from the space*

  $$M_i = \left\{ \left(p_1^i, p_2^i, p_3^i, G^i, n^i\right) \mid \sum_j p_j^i = 1, n^i \in N \right\}$$

  where N denotes the set of natural numbers. [That is, the planner asks each agent to report a price vector, a level of public good and a natural number]

- *The planner will choose the following allocation as a function of the message profile*

$\left(m^1, m^2, m^3\right)$:

$$g\left(m^1, m^2, m^3\right) = \begin{cases} \left\langle \left(w_i - \tilde{p}_i \tilde{G}\right), \tilde{G} = \frac{1}{3}\sum_i G^i \right\rangle_{i=1}^{3} & \textit{if } \exists \tilde{p} = (\tilde{p}_1, \tilde{p}_2, \tilde{p}_3) \textit{ s.t. } p_j^i = \tilde{p}_j \\ & \textit{for at least two agents } i \\ \sum_i w_i \textit{ is divided equally} & \\ \textit{among players who} & \textit{otherwise.} \\ \textit{announced the highest } n^i, & \end{cases}$$

Show that:

1. *Any Lindahl equilibrium allocation is a Nash equilibrium in the game induced by the mechanism* $\langle g, M \rangle$ *constructed above;*

2. *Any interior Nash equilibrium allocation is a Lindahl equilibrium allocation;*

3. *Show by example that the qualifier "interior" is needed above.*

While the example shows that one can strongly implement the Lindahl correspondence (subject to the caveat in part 2), it does not seem particularly attractive. That is, one doesn't have the feeling that if one were to put subjects into the laboratory to play the game, that they would likely arrive at a Nash equilibrium. The most serious problem would seem to be the discontinuity: if agents don't *precisely* agree, the outcome is far away from the Lindahl outcome. One can often find mechanisms that are continuous, but slightly more complicated. (See Postlewaite and Wettstein (1989) for an example in the private goods case.) However, even the continuous mechanisms are not very compelling.

0.6 Positive Models of Private Provision of Public Goods

So far we have discussed that public goods will in general be under-supplied by voluntary contributions. Still, voluntary contributions of public goods constitutes a large fraction of available resources in the economy (approximately 2% of GDP are private donations to charity). Hence it seems to be of importance to have a reasonable positive theory of private provision of public goods from which one can derive policy implications. We may, for instance, be interested in how private provisions change due to changes in the income distribution and how private donations are affected by public provisions.

0.6.1 A Static Model of Private Contributions

This section follows the seminal paper by Bergstrom, Blume and Varian (henceforth BBV, 1986). From previous analysis, we saw that assuming a concave production function $G = f(z)$ does not add any new insights - so we will here assume linear production technology; and as long as we assume that firms are competitive, decentralizing production does not add any further distortion - so we will assume that private good can be turned into a public good by any agent.

Notation

- A set of consumers $N = \{1, ..., n\}$
- w_i : i's (exogenous) wealth
- x_i : i's consumption of private goods
- g_i : i's contribution toward the public good. For ease of notation, write

$$\begin{aligned} G &= \sum_{i=1}^{n} g_i \\ G_{-i} &= \sum_{j \neq i} g_j \end{aligned}$$

  [Note before, we had $G = f\left(\sum_{i=1}^{n} g_i\right)$ where f is the production function for the public good.]

- Consumer i has utility function $u_i(x_i, G)$, increasing in both arguments.
- Timing of the voluntary contribution game: each agent simultaneously chooses $g_i \in [0, w_i]$.

Nash Equilibrium

A *Nash equilibrium* of the voluntary contribution game is a vector of contributions $(g_1^*, ..., g_n^*)$ such that, for all $i \in N$,

$$g_i^* \in \arg \max_{g_i \in [0, w_i]} u_i\left(w_i - g_i, g_i + G_{-i}^*\right)$$

where

$$G^*_{-i} = \sum_{j \neq i} g^*_j.$$

In the above definition of Nash equilibrium we have used the fact that the budget constraint must hold with equality since the utility function is increasing in both arguments.

We can have an alternative expression of agent i's problem as

$$\begin{aligned} & \max_{x_i, G} u_i (x_i, G) \\ \text{s.t.} \quad & x_i + G = w_i + G^*_{-i} \\ & G \geq G^*_{-i} \end{aligned} \tag{3}$$

The point of writing the condition for i playing a best response in this more complicated way is that it should be clear that this is just like any ordinary consumer maximization problem where relative prices equal unity and wealth is given by $w_i + G^*_{-i}$.

A Neutrality Theorem

The first result is that, *if all agents contribute*, then small changes in the distribution of income will leave the allocation unchanged. This result was first obtained by Warr (1983) who used an implicit differentiation argument of the first order condition. BBV extended this argument by a non-calculus approach and could then give a more complete description of how equilibria are affected by changes in the wealth distribution.

Proposition 18 (Neutrality Theorem) *Suppose u_i is quasi-concave for all $i \in N$, and let $(g^*_1, ..., g^*_n)$ be the initial equilibrium. Consider a redistribution of income* among contributing agents $C \equiv \{i : g^*_i > 0\}$ *such that no agents loses more income than his original contribution. Let w'_i be i's post-redistribution wealth. Then the post-redistribution NE $\{g'^*_1, ..., g'^*_n\}$ satisfies*

$$g'^*_i - g^*_i = w'_i - w_i$$

Hence,

$$G'^* \equiv \sum_{i=1}^{n} g'^*_i = G^* \equiv \sum_{i=1}^{n} g^*_i.$$

Proof. Suppose that Δw_i is the change in agent i's wealth caused by the redistribution. Suppose that in a post-redistribution equilibrium, every agent other than agent i changes his contribution by the exact amount of his change in wealth. This implies that

$$G'^*_{-i} = G^*_{-i} - \Delta w_i.$$

Agent i's best response is the solution to the following problem

$$\begin{aligned} & \max_{\{x_i, G\}} u_i\left(x_i, G\right) & & (4) \\ \text{s.t.} \quad & x_i + G = \underbrace{\left(w_i + \Delta w_i\right)}_{\text{new wealth}} + \underbrace{\left(G^*_{-i} - \Delta w_i\right)}_{\text{others' total contribution}} = w_i + G^*_{-i} \\ & G \geq G'^*_{-i} - \Delta w_i \end{aligned}$$

Note that the first constraint in the above problem is the same as that in (3). The difference between (4) and (3) lies in the second constraint.

We consider two cases:

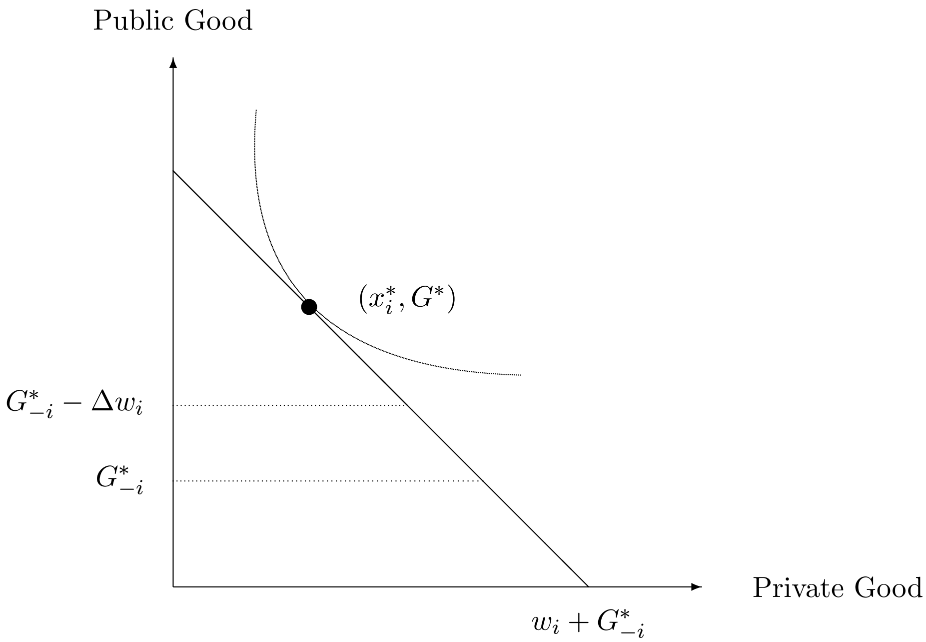
1. Case I: $\Delta w_i < 0$. In this case, the feasible set in (4) is smaller than that in (3). But since by assumption (no consumer loses more income than his original contribution) $g_i^* + \Delta w_i \geq 0$, we know that (x_i^*, G^*) is in the post-redistribution feasible set. By revealed preference, (x_i^*, G^*) is optimal solution to the problem (4);

2. Case II: $\Delta w_i > 0$. In this case, the feasible set in (4) is larger than that in (3). Suppose to the contrary that there is a choice (x_i', G') such that $u_i\left(x_i', G'\right) > u_i\left(x_i^*, G^*\right)$. Then it follows from quasi-concavity that

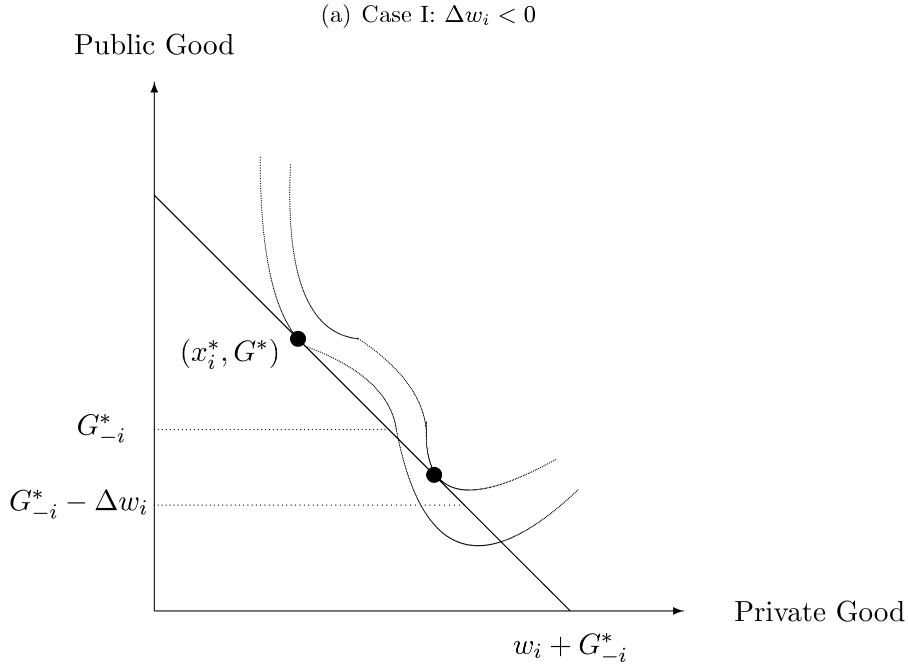
$$u_i\left(\lambda x_i' + (1-\lambda)\, x_i^*, \lambda G' + (1-\lambda)\, G^*\right) > u_i\left(x_i^*, G^*\right)$$

for any $\lambda \in (0, 1)$. Since $i \in C$ by assumption, $x_i^* < w_i$, so for λ small enough the convex combination is feasible before the wealth redistribution. It follows then G^* could not be optimal initially, a contradiction. ■

To summarize, this neutrality theorem says that in the post-redistribution equilibrium each consumer has precisely the same consumption of the private good and the public good as he had before. The optimal responses of the consumers to the wealth transfer have completely offset the effects of the redistribution.



(a) Case I: $\Delta w_i < 0$



(b) Case II: $\Delta w_i > 0$: Convexity Rules this Out

Figure 2: Graph in the Proof of the Neutrality Theorem

A General Characterization of the Set of Nash Equilibria

Consider the problem

$$\begin{aligned} &\max u_i(x_i, G) \\ \text{s.t.} \quad & x_i + G = W \end{aligned}$$

This is a standard consumer optimization problem. Assuming that u_i is strictly quasi-concave, we have a unique solution $f_i(W)$, which in consumer theory language is the demand for the public good G. We make the following assumption:

Assumption: $f_i(\cdot)$ is single-valued, differentiable and satisfy

$$f_i'(W) \in (0,1).$$

This assumption requires that both the private and the public goods are normal goods.

The problem that determines i's best response is of course the problem (3). It is clear that the best response function from the problem (3) is (taking i's contribution to the public good, g_i, as the strategic variable), is

$$\beta_i(G_{-i}) = \max\{f_i(w_i + G_{-i}) - G_{-i}, 0\}. \tag{5}$$

Proposition 19 (Existence of Equilibrium) *A Nash Equilibrium exists.*

Proof. Let $\mathcal{W} = \{z \in R^n : z_i \in [0, w_i]\ \forall i\}$. This is a compact and convex set. Note that

$$(\beta_1(G_{-1}), ..., \beta_n(G_{0n}))$$

is a continuous function from $\mathcal{W}$ to itself. By Brouwer's fixed point theorem, there must exists a fixed point, which is the Nash equilibrium of the voluntary contribution game. ■

Now we investigate a little more of the equilibrium of the voluntary contribution game. Consider an equilibrium $(g_1^*, ..., g_n^*)$. As before, define the set of positive contributors as

$$C^* = \{i \in N : g_i^* > 0\}.$$

The first useful fact simply follows from the best response (5):

FACT 1:

$$\begin{aligned} G^* &= \sum_i g_i^* = f_i\left(w_i + G_{-i}^*\right)\ \forall i \in C^* \qquad (6) \\ G^* &\geq f_j\left(w_j + G_{-j}^*\right) = f_j(w_j + G^*)\ \forall j \notin C^*. \end{aligned}$$

The important implication of FACT 1 is that once we know the level of the public goods G^*, then the set of positive contributors are unique. To see this, note that for a given G^*, we can ask whether $G^* \geq f_j (w_j + G^*)$ holds for all $j \in N$. If it holds, then $j \notin C^*$; otherwise, $j \in C^*$. There is no ambiguity at all. Also once we know G^*, we know g_i^* for every i. To see this, note that

$$g_i^* = w_i + G^* - f_i^{-1} (G^*) \text{ for } i \in C^*$$

and zero otherwise.

A more subtle result is the following:

FACT 2: There exists a real valued function $F (G^*, C^*)$, differentiable and increasing in G^* such that in a Nash equilibrium,

$$F (G^*, C^*) = \sum_{i \in C^*} w_i.$$

Proof. Since by assumption $f_i' \in (0, 1)$, it has a strictly increasing inverse ϕ_i. Moreover, $\phi_i' > 1$. Applying ϕ_i on both sizes of the equality in (6), we obtain

$$\forall i \in C^*, \phi_i (G^*) = w_i + G_{-i}^* = w_i + G^* - g_i^*$$

Summing over all $i \in C^*$, we have

$$\sum_{i \in C^*} \phi_i (G^*) = \sum_{i \in C^*} w_i + (|C^*| - 1) G^*$$

where $|C^*|$ is the cardinality of the set C^*. Now we can define

$$F (G^*, C^*) = \sum_{i \in C^*} \phi_i (G^*) - (|C^*| - 1) G^*.$$

Note that $F (\cdot, C^*)$ is monotonically increasing in G^* since

$$\sum_{i \in C^*} \phi_i' (G^*) > |C^*| . \blacksquare$$

The monotonicity of $F (\cdot, C^*)$ immediately implies that for a fixed set of contributors, there is a unique solution G^* to the equation $F (G^*, C^*) = \sum_{i \in C^*} w_i$.

Combining FACTS 1 and 2, we have the following important conclusion: characterizing the Nash equilibrium allocation is equivalent to characterizing C^* or G^*.

Proposition 20 *[Uniqueness of Equilibrium] Under the assumption that* $f_i' \in (0, 1)$, *there is a unique Nash equilibrium given any distribution of wealth.*

Proof. Suppose that there are two Nash equilibria, with public goods quantities G and G', and corresponding positive contribution sets C and C' respectively. Without loss of generality, assume $G' \leq G$.

Since G' is a Nash equilibrium, by FACT 1, we have

$$G' \geq f_i\left(w_i + G'_{-i}\right) \; \forall i \in N$$

Hence $\phi_i\left(G'\right) \geq w_i + G'_{-i} \; \forall i \in N$ and in particular for all $i \in C$. Hence

$$\begin{aligned} \sum_{i \in C} w_i &\leq \sum_{i \in C} \phi_i\left(G'\right) - \sum_{i \in C} \underbrace{\left(G' - g'_i\right)}_{G'_{-i}} \\ &= \sum_{i \in C} \phi_i\left(G'\right) - |C|\, G' + \sum_{i \in C} g'_i \end{aligned}$$

Since $\sum_{i \in C} g'_i \leq \sum_{i \in C'} g'_i = G'$ by the definition of C and C', we have

$$\sum_{i \in C} w_i \leq \sum_{i \in C} \phi_i\left(G'\right) - |C|\, G' + G' = F\left(G', C\right)$$

Since $F(G, C) = \sum_{i \in C} w_i$, we have

$$F(G, C) \leq F\left(G', C\right).$$

Since $F(\cdot, C)$ is monotonic, we have $G \leq G'$. Hence $G = G'$. But then FACT 1 implies that $C = C'$ and $g_i = g'_i$. ■

Proposition 20 suggests the following algorithm to calculate the Nash equilibrium for any finite set N and wealth distribution:

- Choose an arbitrary subset $C \subseteq N$;
- Use $F(G, C) = \sum_{i \in C} w_i$ to calculate G [a unique G exists since F is monotonic in G];
- If $G \geq f_j(w_j + G)$ for all $j \notin C$ is satisfied, DONE, WE HAVE FOUND THE UNIQUE EQUILIBRIUM; otherwise, continue with a different set C.

0.6.2 Comparative Statics Regarding Changes in the Wealth Distribution

How does the level of the equilibrium public goods provision responds to the change in the wealth distribution? This comparative statics can lead to testable implications. The following proposition turns out to be crucial:

Proposition 21 *Let $\{g_i\}$ and $\{g_i'\}$, $i = 1, ..., n$ be Nash equilibria given the wealth distributions $\{w_i\}$ and $\{w_i'\}$ respectively. Let C and C' be the corresponding sets of contributing agents. Then*

$$F\left(G', C\right) - F\left(G, C\right) \geq \sum_{i \in C} \left(w_i' - w_i\right).$$

Proof. From FACT 1, we have

$$G' \geq f_i\left(w_i' + G_{-i}'\right) \ \forall i \in C$$

Hence

$$w_i' + G_{-i}' \leq \phi_i\left(G'\right) \ \forall i \in C$$

Sum over $i \in C$,

$$\begin{aligned}\sum_{i \in C} w_i' &\leq \sum_{i \in C} \phi_i\left(G'\right) - |C|\, G' + \sum_{i \in C} g_i' \\ &\leq \sum_{i \in C} \phi_i\left(G'\right) - |C|\, G' + G' = F\left(G', C\right)\end{aligned}$$

Since $\sum_{i \in C} w_i = F\left(G, C\right)$, we have the desired conclusion. ■

Exercise 22 *Prove the following implications of the above proposition:*

1. *Any change in the wealth distribution that leaves unchanged the aggregate wealth of current contributors will either increase or leave unchanged the equilibrium supply of the public good;*

2. *Any change in the wealth distribution that increases the aggregate wealth of current contributors will necessarily increase the equilibrium supply of the public good;*

3. *If a redistribution of wealth among current contributors increases the equilibrium supply of the public good, then the set of contributing agents after the redistribution must be a proper subset of the original set of contributors;*

4. *Any simple transfer of wealth from another agent to a currently contributing agent will either increase or leave constant the equilibrium supply of the public good.*

Proof of claim 3: From FACT 1, we know that for all $j \notin C$, $G - f_j\left(w_j + G\right) \geq 0$. Since $f_j' \in (0, 1)$, we know that if $G' > G$, we have $G' - f_j\left(w_j + G'\right) \geq 0$ for all $j \notin C$. Hence $C' \subseteq C$. But if $C' = C$, then if we consider a redistribution from $\{w_i'\}$ to $\{w_i\}$, we will have, by claim 1, $G \geq G'$. A contradiction.

0.6.3 Multiple Public Goods and Spheres of Influence

Many qualitative features of voluntary provision equilibria carry over to the case in which there are multiple private goods and multiple public goods. There is one interesting aspect of the multiple public good case that doesn't arise in the single private good case: which public goods do different agents contribute to? Browning, Chiappori and Lechene (*Economic Journal 2010)* analyze that question for a married couple with multiple public goods. They show that in general there is at most one public good to which both contribute. In other words, for most public goods that are provided by the couple, all the contribution is by one person while the other contributes nothing. This arrangement is termed "spheres of influence" by which they mean that some public goods such as child care are provided solely by one person while other public goods such as household repair are done entirely by the other. We can illustrate the intuition with a simple example with two people, one private good and two public goods.

Example: There are two agents, $i \in \{1, 2\}$, with utility functions $u_i(x_i, g^1, g^2)$ where x_i is the amount of private good and g^j is the combined contribution to public good j. Both agents have income 1. Agent i's problem is

$$\begin{aligned} & \max_{g_i^1, g_i^2} u_i(1 - g_i^1 - g_i^2, g_1^1 + g_2^1, g_2^1 + g_2^2) \\ = \; & \max_{g_i^1, g_i^2} u_i(1 - g_i^1 - g_i^2, G^1, G^2) \end{aligned}$$

where g_i^j is agent i's contribution to public good j. If the equilibrium is interior, both agents are contributing to both public goods and agent i's first order conditions are

$$\begin{aligned} \frac{\partial u_i}{g_1^1} &= -\frac{\partial u_i}{x_1} + \frac{\partial u_i}{G^1} \\ \frac{\partial u_i}{g_1^2} &= -\frac{\partial u_i}{x_1} + \frac{\partial u_i}{G^2} \end{aligned}$$

and hence

$$\frac{\frac{\partial u_i}{G^1}}{\frac{\partial u_i}{G^2}} = 1$$

But generically $\frac{\partial u_1}{G^1} / \frac{\partial u_2}{G^2} \neq \frac{\partial u_2}{G^1} / \frac{\partial u_2}{G^2}$, hence in general, the solution cannot be interior, that is one or the other agent (or both) must not be contributing to one of the public goods.

Note that if there are many public goods, this argument shows that there can be at most one public good to which both contribute.

This is of interest not only for the application of public goods in couples but for general voluntary contribution problems such as charitable contributions. The argument shows that (generically) no two people can both contribute to any pair of charities. Since we should expect that there are many pairs of charities to which many people contribute, the general model voluntary contribution seems not applicable. There is a literature on "warm glow" that assumes agents get utility not only (or possible not even) from the activity of the charity they donate to, but also from the *act* of donating. (See Andreoni, *Economic Journal* 1990 for a seminal paper on the topic.)

0.7 Does Public Provision Crowd Out Private Provision?

It seems intuitively rather obvious that it should be possible to translate the above results regarding income redistribution to results about what happens when the government provides the good publicly, financed by taxes on the citizens. Let

- g_0 be the public provision of the good;
- t_i be the (lump sum) tax on citizen i.

We require budget balance so that $g_0 = \sum_{i=1}^{n} t_i$. From the single consumer's point of view, it is immaterial whether the good is provided by the government or other consumers, so g_0 affects best responses just like any other voluntary contribution. Also, the lump sum tax t_i is just a reduction in the endowment for i. These two facts suggests that taxing everyone less than they contribute and publicly providing the good would not change anything.

Proposition 23 *Let $(g_1^*, ..., g_n^*)$ be the unique equilibrium levels of contribution in the model with no public intervention. Consider a policy $(g_0, t_1, ..., t_n)$ with $\sum_{i=1}^{n} t_i = g_0$. Then*

1. *If $t_i \leq g_i^*$ for all $i \in N$, then there is a unique equilibrium in the model with policy, given by $(g_1', ..., g_n')$ such that $g_i' = g_i^* - t_i \forall i \in N$;*

2. *If $t_j > 0$ for some $j \notin C^*$, then although private contribution may decrease, the equilibrium total supply of the public good must increase;*

3. *If for some* $i \in C^*, t_i > g_i^*$, *then the equilibrium total supply of the public good must increase.*

SKETCH OF PROOF. As before, we can show that under policy $(g_0, t_1, ..., t_n)$, the equilibrium (G, C) must satisfy

$$F(G, C) = \sum_{i \in C} w_i - \sum_{i \in C} t_i + g_0$$

where

$$F(G, C) = \sum_{i \in C} \phi_i(G) - (|C| - 1) G.$$

Claim 1: If $\sum_{i \in C^*} t_i = g_0$, then by revealed preference argument as in the proof of Proposition 18 shows that $G = G^*$. For claim 2, if $g_0 - \sum_{i \in C^*} t_i > 0$, then $F(G, C^*) - F(G^*, C^*) \geq g_0 - \sum_{i \in C^*} t_i > 0$. Hence $G > G^*$. For claim 3: think as follows, first taxing the agent by the exact amount of his contribution, and then taxing him for the extra amount. In the first step, he will reduce his contribution to zero, and leave the total public good unchanged by assertion 1; the second step increases the public good supply by assertion 2.

0.7.1 Guide to the Recent Literature

Bernheim (AER, 1986)

Bernheim (1986) extends the neutrality results concerning the voluntary private funding of public goods. The strong and unbelievable neutrality results lead him to question the assumptions in this literature: first, individuals care about the magnitude of their contributions only insofar as these contributions affect the aggregate level of expenditures; second, chains of operative voluntary transfers and contributions link all individuals.

NOTATION:

- Individuals $i = 1, ..., n$;
- Each individual chooses labor supply l_i; and private good consumption x_i;
- There are two public goods (G, H);
- Individual i's utility function is given by

$$u_i(l_i, x_i, G, H).$$

- Assume that the units of all goods are chosen so that all prices are unity.

TIMING OF THE MODEL:

- Stage 1: Government picks a policy

$$P = (y, \tau, \gamma_0, \eta_0)$$

  where $y \equiv (y_1, ..., y_n)$ is the non-labor income that the government transfers to the agents; $\tau \equiv (\tau_1, ..., \tau_n)$ are a vector of labor income taxes, we allow each τ_i to depend on the vector of labor supply by all the agents $l = (l_1, ..., l_n)$; γ_0, which is also a function of l, is the government's contribution to the first public good; and η_0 is the government's contribution to the second public good. The government's budget constraint is

$$\eta_0 (l) + \gamma_0 (l) = \left(Y - \sum_{i=1}^{n} y_i \right) + \sum_{i=1}^{n} \tau_i (l)$$

  where Y is the government's non-tax resources.

- Stage 2: Each consumer, knowing the government's policy P, simultaneously chooses labor supply; and the government collects its revenue according to τ;

- Stage 3: The government chooses its funding of the public goods as prescribed by its policy P following which the consumers simultaneously decide how much to contribute to the public good and how much to consume as private good.

NASH EQUILIBRIUM:

Assume that all the consumers contribute to the first public good (G) and none to the second public good (H). Assume that non-negativity constraint on the public good provision is non-binding. Given a policy P^*, consumer i's strategy consists of choosing a labor supply level l_i in stage 2 and a function γ_i which prescribes a level of contribution to the first public good for every potential labor supply vector l. Given P^*, $\{l_i^*, \gamma_i^*\}_{i=1}^{n}$ is a Nash equilibrium if

- for all l, $\gamma_i^* (l)$ solves

$$\max_{g_i} u_i \left(l_i, y_i^* + l_i - \tau_i^* (l) - g_i, \sum_{j \neq i} \gamma_j^* (l) + g_i, \eta_0^* (l) \right)$$

  where $j \neq i$ is understood to include $j = 0$ ($j = 0$ stands for the government).

- For each i, l_i^* solves

$$\max_{l_i} u_i \left(l_i, y_i^* + l_i - \tau_i^* \left(l_i, l_{-i}^* \right) - \gamma_i^* \left(l_i, l_{-i}^* \right), \sum_{j=0}^{n} \gamma_j^* \left(l_i, l_{-i}^* \right), \eta_0^* \left(l_i, l_{-i}^* \right) \right).$$

MAIN RESULT:

Proposition 24 *Consider any two policies,* $P = (y, \tau, \gamma_0, \eta_0)$ *and* $P' = (y', \tau', \gamma_0', \eta_0')$. *Suppose that* $\eta_0 = \eta_0'$. *Then any final allocation sustained as an equilibrium under* P *can also be sustained as an equilibrium under* P'.

The proof of this result is left as an exercise. The intuition is simple: if all but one individual acts to offset the policy change for each labor supply profile, then the opportunity set of the remaining individual is unchanged: it is therefore optimal for him as well to neutralize any effects. The assumption that the non-negative private contribution to the public good G is non-binding is clearly important.

Villanacci and Zenginobuz (*JMathEcon*, 2005, JPubEconTheory, 2007)

In "Existence and regularity of equilibria in a general equilibrium model with private provision of a public good" the authors prove existence of equilibria in a general equilibrium model with multiple private goods and a public good. In "On the neutrality of redistribution in a general equilibrium model with public goods" the authors consider a model with multiple private goods, unlike the more commonly treated case with single private good. They show that because of the relative price effects that can arise with redistribution of initial endowments, redistribution may have a significant effect on the provision of public goods.

Andreoni (JPE, 1998)

In the previous studies on the private provision of public goods, there is no role for fund-raising or seed money, because of the crowding out. But in reality, fund raising and seed money, either from a government grant, or from a group of "leadership givers", can generate additional gift. For more empirical motivations, see the paper. Andreoni (1998)'s paper provides a theoretical basis for fund-raisers and seeds to charity.

THE MODEL:

- n individuals, $i = 1, ..., n$
- x_i : $i's$ consumption of private good
- g_i : $i's$ contribution to the public good;
- w_i : $i's$ wealth
- $u_i(x_i, G)$: $i's$ utility function
- [**major departure**] The public good production function is

$$G = \begin{cases} \sum_{i=1}^{n} g_i \text{ if } \sum_{i=1}^{n} g_i \geq \bar{G} \\ 0 \text{ otherwise} \end{cases}$$

[Clearly if $\bar{G} = 0$, then we are back to BBV model]

Because of the minimum threshold $\bar{G}$, it is possible that the simultaneous contribution game has a unique equilibrium at zero contribution. If some leaders are chosen to make binding pledges of contributions, it is possible that the zero contribution equilibrium is eliminated and the public good is completed. This idea is related to the dynamic contribution to the public good that we will discuss below.

Morgan (REStud 2000)

Lotteries are a popular and widespread decentralized mechanism for financing public goods. How do lotteries compare with standard voluntary contributions mechanisms? Morgan (2000)'s paper shows that, relative to the standard voluntary contribution mechanism, lotteries increase the provision of the public good, and are welfare improving. What is the intuition? The reason for the under-provision of public good in voluntary contribution models is that the positive externality of public good is not properly internalized. When purchasing lotteries, however, agents do not properly internalize a negative externality because an agent's lottery purchase not only increases his chance of winning, but also decrease others' chance of winning. This negative externality leads to over-purchase of lotteries, and acts as a compensating externality to ameliorate the free-rider problem in the provision of the public good.

THE MODEL:

- $N = \{1, ..., n\}$: set of individuals (consumers, bettors);
- w_i : individual i's wealth
- utility function is quasi-linear

$$u_i = x_i + h_i(G)$$

where x_i is the (numeraire.) private good, and G is the public good. $h_i' > 0, h_i'' < 0$ for all i.

- The production function for the public good is given by

$$G = f(z) = z.$$

FIRST BEST:

From the Samuelson condition (specialized to the quasi-linear utility function and linear production function), the first best level of public good is determined by

$$\sum_{i=1}^{n} h_i'(G^*) = 1$$

if the public good is desirable, i.e. $G^* > 0$; and 0 if $\sum_{i=1}^{n} h_i'(0) < 1$. It should be clear that voluntary contribution will lead to an under-provision of the public good relative to the first best due to free riding problem.

FIXED-PRIZE RAFFLES

Now suppose that the government or charity chooses a fixed prize R. Each individual chooses a wager $y_i \in [0, w_i]$. Given the wagers of the other contestants, y_{-i}, individual i will win the prize R with probability

$$\pi(y_i, y_{-i}) = \frac{y_i}{\sum_{j=1}^{n} y_j}$$

The government or the charity, will then provide the public good with the total wagers over the prize outlay R. That is,

$$G = \sum_{i=1}^{n} y_i - R.$$

If funds are insufficient to cover the cost of the prize, the raffle will be called off and the wagers will be returned to the contestants. [Also needs a δ amount of financing by the charity to rule out zero wager equilibrium.]

Given the bets $(y_1, ..., y_{i-1}, y_{i+1}, ..., y_n)$, individual i under a fixed prize raffle solves

$$\max_{y_i \in [0,w_i]} w_i - y_i + \frac{y_i}{y_i + \sum_{j \neq i} y_j} R + h_i \left(y_i + \sum_{j \neq i} y_j - R \right).$$

One can show that

Proposition 25 *The fixed prize raffle provides more of the public good than the voluntary contributions.*

PARI-MUTUEL RAFFLES:

Alternatively, the government or the charity can designate a percentage of the total wagers to be placed in a prize pool. This is called pari-mutuel raffles: some percentage, p, of the "handle" (the total bets) is rebated in the form of prizes; and the remainder $1-p$ of the handle is used to fund a public good.

Given the bets $(y_1, ..., y_{i-1}, y_{i+1}, ..., y_n)$, individual i under a pari-mutuel prize raffle solves

$$\begin{aligned}
& \max_{y_i \in [0,w_i]} w_i - y_i + \frac{y_i}{y_i + \sum_{j \neq i} y_j} p \left[y_i + \sum_{j \neq i} y_j \right] + h_i \left((1-p) \left(y_i + \sum_{j \neq i} y_j \right) \right) \\
= & \max_{y_i \in [0,w_i]} w_i - y_i + p \cdot y_i + h_i \left((1-p) \left(y_i + \sum_{j \neq i} y_j \right) \right) \\
= & \max_{y_i \in [0,w_i]} w_i - (1-p) y_i + h_i \left((1-p) \left(y_i + \sum_{j \neq i} y_j \right) \right)
\end{aligned}$$

The FOC for i's problem is

$$\begin{aligned}
(1-p) &= (1-p) \, h_i^{'} \left((1-p) \left(y_i + \sum_{j \neq i} y_j \right) \right) \\
\textit{or} \quad h_i^{'}(G) &= 1.
\end{aligned}$$

Proposition 26 *The equilibrium public goods provision in a pari-mutuel raffle is exactly the same as that obtained through voluntary contribution.*

What is the intuition? While in a fixed prize raffle, individuals do not internalize the negative externality effects of reducing the winning probability of others, here in a pari-mutuel raffle, there is also a positive externality that is internalized: the bets also increases the prize pool available to all other bettors! In the quasi-linear utility function case, these two externalities cancel out exactly.

0.8 Dynamic Voluntary Provision of Public Goods

Prior to the literature on private provision of public goods, Thomas Schelling had the following insightful comment on this issue:

> "If each party agrees to send a million dollars to the red cross on condition that the other does, each may be tempted to cheat if the other contributes first, and each one's anticipation of the other cheating will inhibit agreement. But if the contributions are divided into consecutive small contributions, each can try the other's good faith at a small price. Furthermore, since each can keep the other on short tether to finish, no one ever need risk more than a small contribution at a time."

The idea that dividing the contributions into small sums may help was surprisingly not analyzed until the late 80s.

0.8.1 Admati and Perry (ReStud, 1991)

THE CONTRIBUTION GAME

- Two identical players, 1 and 2;
- The value of a completed project is V to both players;
- The total cost of the completing the project is K. [Note that the project is either completed or not completed];
- Players take turns in making contributions, starting with player 1 in period 1. The project is completed as soon as the total contributions made by both players reach the total cost K.

PRELIMINARIES

- Let c_i^t be the amount of player i's contribution at period t;

- A history at time τ is a sequence of contributions made by agents prior to time τ : $\{c_1^t, c_2^t\}_{t=1}^{\tau-1}$. If it is not i's turn to move in period t, then $c_i^t = 0$;

- A *strategy* for player i specifies the size of his contribution for each history after which it is i's turn to move;

- Players are impatient and discount rates are both $\delta \in (0,1)$. Let T be the first time at which the total contributions reach K, where $T = \infty$ if it is not completed. An outcome of the game is
$$\left\{T, \{c_1^t\}_{t=1}^T, \{c_2^t\}_{t=1}^T\right\}.$$
Player i's payoff from such an outcome is
$$U_i\left(T, \{c_1^t\}_{t=1}^T, \{c_2^t\}_{t=1}^T\right) = \delta^{T-1}V - \sum_{t=1}^{T} \delta^{t-1} W\left(c_i^t\right)$$
where $W(\cdot)$ is the function which measures the disutility from contributing, $W' > 0, W'' > 0$ and $W(0) = 0$.

The main result of the paper utilizes the following variables:

- After a history $\left(\{c_1^t\}_{t=1}^\tau \{c_2^t\}_{t=1}^\tau\right)$, denote X as the total amount of contribution still required for the completion of the project, i.e.,
$$X\left(\{c_1^t\}_{t=1}^\tau \{c_2^t\}_{t=1}^\tau\right) = K - \sum_{t=1}^{\tau} c_1^t - \sum_{t=1}^{\tau} c_2^t.$$

Now we recursively define the amount of contribution that would be made in the equilibrium path if the project is completed in equilibrium, denoted by $\{R_q\}_{q=1}^\infty$.

STEP 1: Define R_1 as the maximum amount that a player is willing to contribute now if by doing so he completes the project ; while if he contributes zero then the project will be completed next period by the other player. That is, R_1 solves
$$V - W(R_1) = \delta V.$$
It is clear that if in a subgame $X < R_1$, then it is a dominant strategy for the player whose turn it is to complete the project. For any $X \in [0, R_1]$, let
$$\begin{aligned} U_a^*(X) &= V - W(X) \\ U_b^*(X) &= V. \end{aligned}$$

$U_a^*(X)$ and $U_b^*(X)$ are respectively the payoffs of the first and second player to move if the remaining size of the project is X and the first player completes the project in his turn.

STEP 2: Let R_2 be the contribution level that makes a player indifferent between (i) contributing R_2 right now under the assumption that the project will be completed in the next period by the other player; and (ii). contributing zero right now and completing the project in two periods by contributing R_1 then (with the rest contributed by the other player in the next move). That is, R_2 solves

$$\delta V - W(R_2) = \delta^2 V - \delta^2 W(R_1),$$

which can be rewritten as

$$\delta U_b^*(S_1) - W(R_2) = \delta^2 U_a^*(S_1)$$

where $S_n = \sum_{q=1}^{n} R_q$ [hence $S_1 = R_1$]. It is clear that if in a subgame $R_1 < X < R_1 + R_2$, it is an iteratively dominant strategy for the player with the current move to contribute $X - R_1$ assuming that the other player will follow his dominant strategy to complete the project (with contribution R_1). For $X \in [R_1, R_1 + R_2]$, or equivalently, for $X \in [S_1, S_2]$, define

$$\begin{aligned} U_a^*(X) &= \delta U_b^*(S_1) - W(X - R_1) \\ U_b^*(X) &= \delta U_a^*(S_1) \end{aligned}$$

$U_a^*(X)$ is the current mover's payoff if he contributes just enough, $X - R_1$, so that after his move the required contribution is R_1 and the second mover completes the project in his turn.

STEP n : Define R_n recursively as the amount that makes player i whose turn it is indifferent between (i) contributing R_n now, and obtaining $U_b^*(S_{n-1})$ in the next period; and (ii) contributing zero now and obtaining $U_a^*(S_{n-1})$ in two periods. That is, R_n solves

$$\delta U_b^*(S_{n-1}) - W(R_n) = \delta^2 U_a^*(S_{n-1}).$$

And for every $X \in [S_{n-1}, S_n]$, define

$$\begin{aligned} U_a^*(X) &= \delta U_b^*(S_{n-1}) - W(X - S_{n-1}) \\ U_b^*(X) &= \delta U_a^*(S_{n-1}). \end{aligned}$$

The essentially unique equilibrium path of the contribution game is described as follows:

Proposition 27 *Let* $S_\infty = \sum_{q=1}^{\infty} R_q$.

1. *Suppose* $S_\infty > K$.

   (a) *If there exists* $N < \infty$ *such that* $S_{N-1} < K < S_N$, *then the unique equilibrium path is: player 1 contributes* $K - S_{N-1}$ *in period 1, and for* $1 < t \leq N$, *the amount contributed in period* t *is* R_{N-t+1}. *Thus the project is completed in* N *rounds.*

   (b) *If there exists* $N < \infty$ *such that* $S_N = K$, *then there are two equilibrium paths. In addition to the path described above, the other equilibrium path is as follows: player 1 contributes zero in period 1, and for* $2 \leq t \leq N$, *the amount contributed in period* t *is* R_{N-t+2}. *Thus the project is completed in* $N + 1$ *periods.*

2. *If* $S_\infty \leq K$, *then the unique equilibrium path is* $c_i^t = 0$ *for all* i *and all* t.

Now we consider a special linear contribution cost function $W(c) = bc$. The equilibrium path described in the above proposition remains an equilibrium path (though uniqueness is lost). For the linear case, we can explicitly solve for S_n (I will let $b = 1$ for the calculations for simplicity):

R_1 must solve

$$\begin{aligned} V - R_1 &= \delta V \\ \text{or} \quad R_1 &= (1-\delta)V \end{aligned}$$

R_2 must solve

$$\begin{aligned} \delta V - R_2 &= \delta^2 V - \delta^2 R_1 \\ &= \delta^2 V - \delta^2((1-\delta)V) \\ &= (\delta^2 - \delta^2 - \delta^3)V \\ \text{or} \quad R_2 &= (\delta - \delta^3)V \\ \text{hence} \quad R_1 + R_2 &= (1 - \delta^3)V \end{aligned}$$

R_3 must solve

$$\begin{aligned}
\delta^2 V - R_3 - \delta^2 R_1 &= \delta^3 V - \delta R_2 \\
&\iff \delta^2 V - R_3 - \delta^2(1-\delta)V = \delta^3 V - \delta^2(\delta - \delta^3)V \\
&\iff (\delta^2 - \delta^2 + \delta^3)V - R_3 = (\delta^3 - \delta^3 + \delta^5)V \\
\textit{or} \quad R_3 &= (\delta^3 - \delta^5)V \\
\textit{hence} \quad R_1 + R_2 + R_3 &= (1 - \delta^5)V
\end{aligned}$$

One can show that the pattern is general:

$$\begin{aligned}
S_n &= V\left(1 - \delta^{2n-1}\right) \ \textit{for } b = 1 \\
\textit{or} \ \ S_n &= \frac{V\left(1 - \delta^{2n-1}\right)}{b} \ \textit{in general}
\end{aligned}$$

hence $S_\infty = V/b$. Hence Proposition 27 implies that the project is completed in that equilibrium if and only if $K < V/b$, which is equivalent to $V > bK$. That is, in a linear case a necessary and sufficient condition for the completion of the project in the above equilibrium is that each player would complete the project immediately if he was the only player. The inefficiency due to delay is stark. However, there *may* be other more efficient equilibria.

More generally, Admati and Perry show that:

Proposition 28 *If $V \leq W'(0)\,K$, then the project is not completed in equilibrium.*

0.8.2 Marx and Matthews (ReStud 2000)

Marx and Matthews (2000) revisit the dynamic voluntary contribution game. They made the following changes to the Admati and Perry (1991) model.

- [One Departure] $N = \{1, ..., n\}$ is the set of agents, $n \geq 2$.
- Time is discrete, periods are indexed by $t = 0, 1, 2, ...$
- [Another Departure] Players can contribute to a public project in each period. Let $z_i(t)$ denote i's contribution in period t.

- Write

$$\begin{aligned} z(t) &= (z_1(t), ..., z_n(t)) \\ Z(t) &= \sum_{j=1}^{n} z_j(t) \\ Z_{-i}(t) &= \sum_{j \neq i} z_j(t) \end{aligned}$$

  Denote i's cumulative contribution up to time t by

$$x_i(t) = \sum_{\tau=1}^{t} z_i(\tau)$$

  and the aggregate cumulative contribution by

$$X(t) = \sum_{j=1}^{n} x_j(t).$$

- [One More Departure] The agents are assumed to receive utility from the public good in each period. If the aggregate cumulative contribution in period t is X and agent i contributes $z_i(t)$, then agent i's period t payoff is $(1-\delta) f(X) - z_i(t)$, where

$$f(X) = \begin{cases} \lambda X \text{ if } X < K \\ V \text{ if } X \geq K. \end{cases}$$

- More generally, given a sequence of contributions $z = \{z(t)\}_{t=0}^{\infty}$, player i's payoff is

$$U_i(z) = \sum_{t=0}^{\infty} \delta^t [(1-\delta) f(X) - z_i(t)].$$

The main results of Marx and Matthews are roughly as follows:

1. Allowing for contributions to be split up in small pieces may alleviate the inefficiencies of static contribution games: equilibria where a project is implemented exists in the dynamic game for some parameterizations where the unique equilibrium in the static game has nobody contributing;

2. There is still an inefficiency due to delay;

3. However, this inefficiency may be rather inconsequential if the time period between contributions are small.

0.9 Provision of Public Goods with Private Information

So far we have assumed perfect information, that is, we assume that the individuals know of others preferences and wealth etc. This is not likely to be the case in reality. When agents' valuations from a public good project are private information, will there exist mechanisms to insure the efficient provision of public good? If not, what is the probability of provision of public goods under the optimal mechanism? The main reference is **Mailath and Postlewaite (ReStud, 1990)**.

0.9.1 An Illustrative Example: two agent case

- 2 individuals, $i = 1, 2$
- v_i : i's valuation for the public good, and assume that $v_i \in \{L, H\}$ and ex ante $v_i = L$ or $v_i = H$ occurs with equal probability; v_1 and v_2 are independent. Let $L = 0$, $H = 10$. v_i is agent i's private information.
- Cost of providing the public good is 8.

Ideally we would like to have a mechanism that has the following three properties:

1. Efficiency: produce the public good if and only if it is efficient to do so. [In our example, to provide the public good unless both agents have valuation L]
2. Individual rationality: participation must be voluntary;
3. Incentive compatibility: each agent has incentive to truthfully report her privately known valuation.
4. Balanced Budget.

We assume for now that the cost is shared if both agents announce H. Does there exist a mechanism with the above three properties? Suppose that agent 2 tells the truth, consider agent 1 with type H. Her payoff matrix if under such a mechanism with the efficiency property is as follows:

	Agent 2's type		
		L	H
Agent 1's announcement	$\tilde{L}$	0	10
	$\tilde{H}$	2	6

The expected payoff for agent 1 when announcing $\tilde{L}$ is $0.5 \times 0 + 0.5 \times 10 = 5$ while announcing $\tilde{H}$ is $0.5 \times 2 + 0.5 \times 6 = 4$. Hence truth telling is not a Bayesian Nash equilibrium.

This simple example illustrates that there may not exist a mechanism that simultaneously satisfies efficiency, individual rationality and incentive compatibility. The intuition is simply the free riding problem: in cases when an individual's announcement will not change the probability of the public good being provided (due to the requirement of efficiency), he will have incentive to report a lower valuation so as to be taxed less. Now it is natural to ask the following question: what efficiency level can we achieve by mechanisms that satisfy IR and IC constraints? In this example, suppose that the public good provision probabilities are given by

$$\rho(L, L) = 0, \rho(H, L) = \rho(L, H) = \rho, \rho(H, H) = 1$$

where $\rho(H, L)$ denotes the probability of the public good being provided when agent 1 announces H and agent 2 announces L. The tax rule remains the same as before. Hence we have the following table:

Announcement Profile	Probability of providing the public good	Taxes if the public good is provided
(L, L)	0	n.a.
(L, H)	ρ	$(0, 8)$
(H, L)	ρ	$(8, 0)$
(H, H)	1	$(4, 4)$.

Suppose that the other agent tells the truth. We first verify the IC constraints for L type:

$$u\left(\tilde{L}|L\right) = 0 > u\left(\tilde{H}|L\right) = \frac{1}{2}(-8 - 4) = -6$$

hence the IC constraints for the low type is satisfied for all p. It is intuitive because if your valuation is 0, why would you lie upward?

Now we verify the IC constraints for H type:

$$\begin{aligned} u\left(\tilde{H}|H\right) &= \frac{1}{2}\left[\rho\left(10-8\right)+\left(10-4\right)\right]=\rho+3 \\ u\left(\tilde{L}|H\right) &= \frac{1}{2}\left(10\rho\right)=5\rho \end{aligned}$$

IC for type H hence requires that

$$u\left(\tilde{H}|H\right)\geq u\left(\tilde{L}|H\right)\Leftrightarrow\rho\leq\frac{3}{4}.$$

Hence IR, IC mechanism will have to sacrifice efficiency.

0.9.2 Impossibility Result for Large Economies (Mailath and Postlewaite, REStud (1990)

It is desirable to have a mechanism that is ex ante efficient, that is, that provides the public good whenever the cost was less than the sum of the benefits to the agents in the economy. When agents' valuations for the public good are privately known, this may be difficult, as we showed in the two agent example above: if agents' cost shares do not depend on their announced values, agents whose valuations exceed their cost shares will have incentive to overstate their valuations; on the other hand, if agents' cost shares *do* depend on their announcements, they may have an incentive to understate their valuations. If we impose an individual rationality constraint that prohibits taxing an agent more than his announced valuation for the public good, we will be in the latter case where agents may have an incentive to understate their valuations. It is intuitive that the understatement problem gets more severe when the number of agents increases. The statement and proof below are for the special case in which we "replicate", that is, look at a sequence of economies that consist of an increasing number of i.i.d. draws from a two-value distribution. Mailath and Postlewaite treat the more general case without this structure.

NOTATION:

- n agents, $i=1...,n$;
- v_i : $i's$ valuation for the public good, $v_i\in\{H,L\}$ which is private information, and ex ante $\Pr\left(v_i=H\right)=p\in\left(0,1\right).$
- Agent i's utility when the public good is provided and he pays tax t is v_i-t;

- The cost of providing the public good in an n person economy is cn [i.e. per capita cost of the public good is constant];
- Assume that $H > c > L$.

DIRECT REVELATION MECHANISM:

In the direct revelation mechanism, the message space for agent i is $M_i = M = \{H, L\}$. The mechanism needs to specify an outcome function (ξ, ρ) where:

- $\xi : M^n \to R_+^n$ is the vector of taxes the agents pay as a function of the announced valuation profile *if* the public good is produced;
- $\rho : M^n \to [0,1]$ is the probability that the public good is produced as a function of the agents' announced valuation profile.

We can further assume (without loss of generality) that $\rho(\cdot)$ depends only on k, the number of agents announcing L, and ignore which specific agents announced L. It can also be shown easily that the tax scheme that provides the greatest incentive to agents with high value H to announce truthfully is

$$\xi_i(v) = \begin{cases} L \text{ if } v_i = L \text{ and } kL + (n-k)H \geq nc \\ \frac{nc-kL}{n-k} \text{ if } v_i = H \text{ and } kL + (n-k)H \geq nc. \end{cases}$$

It is trivial to see that the incentive compatibility constraint for L is satisfied with this tax rule. The incentive constraint for H is

$$\sum_{k=0}^{n-1} p(k)\,\rho(k)\left(H - \frac{nc-kL}{n-k}\right) \geq \sum_{k=0}^{n-1} p(k)\,\rho(k+1)\,(H-L)$$

where $p(\cdot)$ is the probability that there are exactly k agents of type L among the remaining $n-1$ agents [note that the number of type L agents among the $n-1$ remaining agents have a Binomial distribution with parameters $n-1$ and p] [convince yourself that the left (respectively, right) hand side is the agent's expected utility of announcing H (respectively announcing L) assuming all other agents tell truth. This can be re-written as

$$\sum_{k=0}^{n-1} p(k)\,[\rho(k) - \rho(k+1)]\,[H-L] \geq \sum_{k=0}^{n-1} p(k)\,\rho(k)\left(\frac{nc-nL}{n-k}\right).$$

We will show that the left hand side must go to zero as n goes to infinity; and that the probability of provision of the public good in the n person problem must be smaller than the right hand side.

STEP 1: Show that the right hand side goes to zero as n goes to infinity. Because $p(\cdot)$ is the binomial density, it is maximized at an integer $t \in ((n+1)p - 1, (n+1)p]$. For this t, $p(k-1) < p(k)$ for $k < t$ and $p(k-1) > p(k)$ for $k > t$ and $p(t) \geq p(t-1)$.

CLAIM 1: $\sum_{k=0}^{n-1} p(k)[\rho(k) - \rho(k+1)]$ is maximized with $\rho(k) = 1$ for $k \leq t$ and $\rho(k) = 0$ for $k > t$.

Proof. Let $\rho^*(\cdot)$ be the maximizer of the expression. First we show that $\rho^*(t) = 1$ and $\rho^*(t+1) = 0$. Suppose not, let $\rho'(\cdot) = \rho^*(\cdot)$ except let $\rho'(t) = 1$ and $\rho'(t+1) = 0$. Then

$$\begin{aligned} & \sum_{k=0}^{n=1} p(k)\left[\rho'(k) - \rho'(k+1)\right] - \sum_{k=0}^{n=1} p(k)\left[\rho^*(k) - \rho^*(k+1)\right] \\ = \; & [1 - \rho^*(t)][p(t) - p(t-1)] - \rho^*(t+1)[p(t+1) - p(t)] > 0. \end{aligned}$$

A contradiction. Now we can claim that $\rho^*(k) = 1$ for all $k \leq t-1$. Suppose not, then there exist $k \leq t-1$ with $\rho(k) < \rho(k+1)$. Suppose that the smallest such k is k'. Let $\rho'(\cdot) = \rho^*(\cdot)$ except at this k' and let $\rho'(k') = 1$. Then

$$\begin{aligned} & \sum_{k=0}^{n=1} p(k)\left[\rho'(k) - \rho'(k+1)\right] - \sum_{k=0}^{n=1} p(k)\left[\rho^*(k) - \rho^*(k+1)\right] \\ = \; & \left[1 - \rho^*(k')\right]\left[p(k') - p(k'-1)\right] > 0 \end{aligned}$$

a contradiction. Similarly it can be shown that $\rho(k) = 0$ for all $k > t$. ■

The implication of this claim is that

$$\begin{aligned} \sum_{k=0}^{n-1} p(k)[\rho(k) - \rho(k+1)][H - L] &\leq p(t)(H - L) \\ &= (H - L)\frac{(n-1)!}{(n-1-t)!t!}p^t(1-p)^{n-1-t} \end{aligned}$$

Using Sterling's formula which states that

$$\lim_{n\to\infty} \frac{n!}{\sqrt{2\pi} n^{n+1/2} e^{-n}} = 1$$

and using the approximation that $p \approx t/(n-1)$ when n is large, we get

$$\frac{(n-1)!}{(n-1-t)!t!}p^t(1-p)^{n-1-t} \to \frac{1}{\sqrt{2\pi(n-1)p(1-p)}} \to 0 \text{ as } n \to \infty.$$

Hence the left hand side of the incentive constraint goes to zero.

Now we examine the RHS of the incentive constraint:

$$\begin{aligned}\sum_{k=0}^{n-1} p(k)\,\rho(k)\left(\frac{nc-nL}{n-k}\right) &= (c-L)\sum_{k=0}^{n-1} p(k)\,\rho(k)\,\frac{n}{n-k} \\ &\geq (c-L)\sum_{k=0}^{n-1} p(k)\,\rho(k)\,.\end{aligned}$$

Since the LHS goes to zero as n goes to infinity, the right hand side goes to zero as well. Hence

$$\sum_{k=0}^{n-1} p(k)\,\rho(k) \to 0 \text{ as } n \to \infty.$$

But $\sum_{k=0}^{n-1} p(k)\,\rho(k)$ is simply the expected probability of the public good being provided.

This is a situation where there is a qualitative difference between private and public good economies. The theorem above demonstrates that the inefficiency that is due to private information becomes dominant when the number of agents gets large. In the private goods world this is not the case. Gul and Postlewaite ("Asymptotic Efficiency in Large Exchange Economies with Asymmetric Information," *Econometrica* 1992) show that in a replica setting efficient incentive compatible allocations may obtain asymptotically. That paper treats the case of independent values, that is, the case in which agents' types determine their utility functions. For the interdependent value case where agents' utility functions may depend on other agents' private information, McLean and Postlewaite ("Informational Size and Incentive Compatibility," *Econometrica* 2002) provide an analogous result that, roughly, efficient incentive compatible allocations can be obtained asymptotically when the number of agents is sufficiently large.

0.10 Local Public Goods

The under-provision of public goods problem we discussed above is mainly due to the fact that the government can not elicit the citizens' (voters') preferences truthfully. Tiebout (1956) suggests that many public goods are provided by local expenditures, and if there were enough communities, individuals would reveal their true preferences for public goods by the choice of community in which to live (in much the same way as individuals reveal their preferences for private goods by their choices). Where there is a wide range of choices, all those deciding to live in the same community would have essentially the same taste, and there would be no problem of reconciling conflicting preferences. This is an intriguing idea since it suggests that the invisible hand can solve the important problem of under-provision of public goods. Local public goods are public goods that can be enjoyed only by residents in the local community: for example, local public school, beaches, parks, etc. are typical examples of public goods. The crucial idea in Tiebout's hypothesis is residential mobility, which creates competition among communities.

More recently, economists have captured the flavor of local public goods by considering public good provision on networks. A net work can use links between agents to indicate that they are affected by the public good provision of some other agents but not all, and the degree to which they are affected. The two papers by Bramoulle et al. in the bibliography below are nice examples of this work.

0.10.1 A Class of Tiebout Models (Bewley Econometrica 1981)

Bewley (1981) formalizes Tiebout's idea in Arrow-Debreu general equilibrium models:

- Decision Makers in the Model: Consumers, Firms, and Local Governments
- n distinct regions of habitation (communities), $j = 1, ..., n$.
- Each region has a government which provides local public goods, and collects local taxes to pay for them;
- **Crucial Assumption:** There is perfect consumer mobility between regions;
- Consumers are fully informed about prices, taxes, and public goods in each region and choose to live in the region where they can enjoy the highest level of utility;
- Consumers behave competitively, i.e., they do not believe that prices, taxes, or the provision of public goods are influenced by their own choices.
- Local government cannot discriminate among consumers by name or according to taste when levying taxes (otherwise, Lindahl prices may be charged); local government knows about the initial endowments of residents when levying taxes;
- Suppose that there is no spillover effects between regions; the inhabitants of one region are affected by what happens in other regions only through prices and migration;

- **COST OF LOCAL PUBLIC GOODS**: We can think of three possibilities:

    1. The cost is independent of population (called "pure public goods case");
    2. The cost is proportional to the population, hence the per capita cost is constant (called "pure public service case");
    3. The per capita cost of public goods is a U-shaped function of population.

- How does production take place? Two possibilities are considered:

    1. Autarkic Regions Case: all production takes place inside regions and there is no trade between them;
    2. Free Trade Case: production is completely independent of the regional distribution of populations.

- Assume that regions do not directly affect utility or production and there is no land;

- What is the motivation of the government? Two possibilities are considered:

    1. Democratic governments seek to maximize the welfare of their own citizens;
    2. Entrepreneurial governments have objectives which are independent of the welfare of their citizens, may try to repel some inhabitants or attract new ones.

- Decisions: Consumers choose consumption bundles and regions of residence; Firms choose their input-output vectors; Local governments choose the bundle of public goods or services and a tax system: a tax system specifies each inhabitant's tax payment as a function of his initial endowment.

An *allocation* in a Tiebout model specifies the following items:

1. The consumption bundle of each consumer;
2. The input-output vector of each firm;
3. The bundle of public goods provided by each regional government;
4. The region of residence of each consumer.

An allocation is *feasible* if the goods absorbed by consumers and governments may be produced or supplied directly from the consumers' initial endowments. A feasible allocation is *Pareto optimal* if there exists no other feasible allocations which makes every consumer at least as well off and some strictly better off.

A *Tiebout equilibrium* consists of a feasible allocation, a price for each commodity, and a tax system for each region such that the following conditions are satisfied:

1. Each consumer's consumption bundle satisfies his budget constraint given his residence choice and it maximizes the consumer's utility in this budget set;

2. Each consumer chooses the region he prefers (When a consumer compares regions, he assumes that tax systems, public expenditures and prices would not change if he moved);

3. Each firm maximizes profit;

4. Each regional government balances budget (i.e. public goods provided by a regional government must be equal to its tax revenue);

5. Each government's expenditures, tax system and choice of inhabitants are consistent with whatever its objective may be.

0.10.2 Are Tiebout Equilibria Pareto Efficient?

Tiebout equilibria may not be Pareto efficient. Bewley (1981) proceeds by providing a series of examples which highlight the various perspectives which lead to the inefficiency of Tiebout equilibria.

Democratic Government

Example 29 (Democratic Government, Pure Public Goods) *There are two identical consumers and two regions. One public good and one private good which is leisure. Each consumer is endowed with one unit of leisure. The consumer's utility function is $u(l, g) = g$ where g is the level of local public good in the region he inhabits and l is leisure. The production technology for public goods is $g_j = L_j$ where g_j is the public good provided in region j and L_j is the quantity of labor.*

One may wonder, why do consumers behave competitively? If I move from region 1 to 2, shouldn't I expect that the government in region 2 will change its provision of public good. It is true if there are indeed 2 consumers, so we should imagine that there are continuum of consumers with measure two.

One can check that the following is a Tiebout equilibrium: The prices of labor and public good are both 1. The tax system is that a tax of 1 is imposed on any initial endowment. One consumer lives in each region, each consumer sells his leisure to the producer of the public goods and one unit of public good is provided in each region. This Tiebout equilibrium is inefficient since both consumers can be better off if they lived together in one region and two units of public gods are provided in that region and none in the other. The latter is also a Tiebout equilibrium. The punch line: there may be multiple Tiebout equilibria and some Tiebout equilibrium may not be Pareto efficient.

The feature of this example is that the cost of public good provision is independent of the population size and in the inefficient Tiebout equilibrium, the economies of scale is not properly exploited.

The next example illustrates that economies of scale is not necessary for the inefficiency to result:

Example 30 (Democratic Government, Pure Public Service) *Two identical regions, four consumers, and four types of public services, and one private good which is leisure. Each consumer is endowed with one unit of leisure. Regions are labeled by 1 and 2, consumers and public services are labeled by A, B, C, D. Consumer k's utility function is given by*

$$\begin{aligned} u_A(l, g_A, g_B, g_C, g_D) &= 2g_A + g_B \\ u_B(l, g_A, g_B, g_C, g_D) &= g_A + 2g_B \\ u_C(l, g_A, g_B, g_C, g_D) &= 2g_C + g_D \\ u_D(l, g_A, g_B, g_C, g_D) &= g_C + 2g_D \end{aligned}$$

The public service production function in region j is

$$g_{jA} + g_{jB} + g_{jC} + g_{jD} = \frac{L_j}{n_j}$$

where n_j is the number of consumers in region j.

The following situation describes a Tiebout equilibrium: consumer A and C live in region 1 and consumer B and D live in region 2, $(g_{1A}, g_{1B}, g_{1C}, g_{1D}) = (1, 0, 1, 0)$ and

$(g_{2A}, g_{2B}, g_{2C}, g_{2D}) = (0, 1, 0, 1)$. The prices of labor and each type of public service are both 1 in both regions. The tax system in each region is to tax away each consumer's endowment. This is an equilibrium because in this allocation, each consumer gets a utility 2 and if he moves to the other region his utility is reduced to 1.

This Tiebout equilibrium is not Pareto optimal. The following allocation (which is also a Tiebout equilibrium allocation) dominates it: consumers A and B live in region 1, C and D in region 2 and $(g_{1A}, g_{1B}, g_{1C}, g_{1D}) = (1, 1, 0, 0)$ and $(g_{2A}, g_{2B}, g_{2C}, g_{2D}) = (0, 0, 1, 1)$. In this allocation each consumer obtains 3 units of utility.

Entrepreneurial Government

In the above examples local governments do not initiate changes in the supply of public goods when agents move. In Example 29, if one of the regional government proposes a tax system of 1 for every resident and provides 2 units of public goods, such a region will attract all the consumers and the resulting allocation will be efficient; in Example 30, if a local government proposes a tax of one unit for every resident and provides a public service bundle $(g_{1A}, g_{1B}, g_{1C}, g_{1D}) = (1, 1, 0, 0)$, it will attract all the type A and B consumers. Again efficiency will be restored. Is it the case that introducing such entrepreneurial government solves the inefficiency problem? The answer is not necessarily. Here we consider such governments.

One possible motivation for an entrepreneurial government is to maximize the population of the region they control; another possible motivation is to maximize profits: their revenues come from taxes and their expenses are the cost of public goods. The following difficulties must be resolved: First, should we assume that all the regional governments simultaneously choose a public goods/service and tax system mix? Bewley (1981) assumes that each government believes that no other government will change policy when it changes its own. Second, what happens if a change in policy leads to multiple competitive equilibrium in the competitive good market?

Example 31 (Profit-maximizing governments, pure public goods) *Two regions, two consumers, one private good which is leisure, and one public good. Each consumer is endowed with one unit of leisure. Production function for the public good is $g = L$. The utility function of consumer 1 is $u_1(l, g) = g$, while the utility function for consumer 2 is $u_2(l, g) = 3l + g$.*

The following situation is a Tiebout equilibrium of this example. Consumer i lives in region i. In region 1, consumer 1 devotes all his labor to the production of the public good and so consumes no leisure and one unit of the public good; consumer 2 devotes no labor to the production of the public good and consumes one unit of leisure and none of the public good. In each region the price of each good is 1. The tax system in region 1 requires that all consumers pay a tax of 1 and in region 2 no tax is levied. The utility level in this equilibrium of consumer 1 is 1 and that of consumer 2 is 3. Why is this a Tiebout equilibrium? Clearly each consumer optimizes. Suppose that a new regional regime were to established involving both consumers. They would have to pay the same taxes since their initial endowment is the same. Let τ be the tax rate. Then 2τ units of the public good will be provided. To attract consumer 1, it must be the case that $2\tau \geq 1$, hence $\tau \geq 1/2$; to attract consumer 2, it must be the case that $3(1-\tau) + 2\tau \geq 3$, which implies that $\tau \leq 0$. Hence there exists no such τ that can attract both consumers away from the above postulated situation.

This Tiebout equilibrium allocation is not Pareto efficient: it is dominated by the following allocation: consumers live together, consumer 1 pays all his income in taxes and consumer 2 pays none. One unit of public good would be provided, consumer 1 would consume no leisure and consumer 2 would consume one unit of leisure. The utility of consumer 1 in this allocation is 1, as was in the previous equilibrium; but the utility of consumer 2 would be 4, higher than before. The reason that this is not an equilibrium is that we assumed that it is impossible to tax the consumers at different rate.

0.10.3 A Tiebout Model with Efficient Equilibrium

- There are L private goods, and N public goods; The public goods have constant per capita unit provision cost of 1.
- There are J regions;
- There is free trade in both public and private goods, hence the price system is an economy-wide price system. It is given by

$$(p, q) \in \Delta^{L+N-1} = \left\{ p \in R_+^L, q \in R_+^N : \sum_{k=1}^{L} p_k + \sum_{k=1}^{N} q_k = 1 \right\}.$$

- There are T types of consumers: a type t consumer is endowed with $\omega_t \in R_+^L$; type t consumer's utility function is given by $u_t(x, g)$; There are I_t consumers of type t;

- Consumers are indexed by (t, i), where t denotes the type and i distinguishes consumers of the same type. Let $\mathcal{I} = \{(t, i) : t = 1, ..., T, i = 1, ..., I_t\}$.
- A tax system for region j is a function $\tau_j : R^L_+ \to [0, \infty)$, recall that τ_j is a function of the endowment of an individual, that is, if a type t consumer lives in region j, he will be taxed $\tau_j(\omega_t)$;
- The budget set of a consumer of type t if he lives in a region j is

$$B_t(p, \tau_j) = \left\{x \in R^L_+ : p \cdot x \leq p \cdot \omega_t - \tau_j(\omega_t)\right\}.$$

- The production function for the public good is constant returns to scale; hence there are no profits in equilibrium and we do not need to worry about the distribution of profits. The production possibility set is described by Y.

An allocation for the economy is of the form (r, x, g, y) where

- $r : \mathcal{I} \to \{1, ..., J\}$ is a function which assigns consumers to their regions of residence;
- $x = \left(x_{(t,i)}\right)_{(t,i)\in\mathcal{I}}$ is the vector which describes the allocation of private goods, where $x_{(t,i)} \in R^L_+$ for all (t, i);
- $g = (g_1, ..., g_J)$ describes the allocation of public services, where $g_j \in R^N_+$;
- $y \in Y$ is the social production vector.

Given a residence function r, we use n^r_j to denote the number of consumers assigned to region j according to r, i.e.,

$$n^r_j = |(t, i) : r(t, i) = j|.$$

An allocation (r, x, g, y) is feasible if

$$\left(\sum_{(t,i)\in\mathcal{I}} \left(x_{(t,i)} - \omega_t\right), \sum_{j=1}^{J} n^r_j g_j\right) = y.$$

In the above expression, $\left|\sum_{(t,i)\in\mathcal{I}} \left(x_{(t,i)} - \omega_t\right)\right|$ is the excess of the private goods endowments after consumption in the economy; and $\sum_{j=1}^{J} n^r_j g_j$ is the total production cost in the economy.

A Tiebout equilibrium consists of a feasible allocation (r, x, g, y), a price system $(p, q) \in \Delta^{L+N-1}$, and tax system $(\tau_1, ..., \tau_J)$ that satisfies the following conditions:

1. (Profit Maximization): $(p, q) \cdot y = \max \{(p, q) \cdot z : z \in Y\}$;

2. (Utility Maximization and Free Mobility): For all $(t, i) \in \mathcal{I}$,

$$u_t \left(x_{(t,i)}, g_{r(t,i)}\right) = \max \{u_t (x, g_j) : x \in B_t (p, \tau_j), j = 1, ..., J\}$$

   and

$$x_{(t,i)} \in B_t \left(p, \tau_{r(t,i)}\right) \text{ for all } (t, i) \in \mathcal{I}.$$

3. (Perfectly Competitive Governments, or zero profit condition on the government):

$$n_j^r q \cdot g_j = \sum_{\left\{(t,i): r_{(t,i)} = j\right\}} \tau_j (\omega_t).$$

4. (Government Profit Maximization) No local government can propose a tax system τ_j' and a fiscally feasible vector of public services g_j' that makes all of its existing consumers better off than they are under the postulated allocation (r, x, g, y).

We make the following assumptions:

A1. $T \leq J$. That is, there are at least as many regions as types of consumers;

A2. $u_t : R_+^{L+N} \to R$ is continuous, strictly quasi-concave and strictly monotone, for all t.

A3. For all t, $\omega_t \in R_+^L$ and $\omega_t \neq \mathbf{0}$.

A4. $Y \subset R^{L+N}$ and Y is a closed convex cone with apex zero;

A5. $Y \cap R_+^{L+N} = \{\mathbf{0}\}$;

A6. There exists $\bar{y} \in Y$ such that every component of

$$\bar{y} + \left(\sum_{t=1}^{n} \omega_t, 0\right)$$

is positive.

Proposition 32 *Under A1-A6, there exists a Tiebout equilibrium; moreover, every Tiebout equilibrium allocation is Pareto optimal.*

The idea of the proof is to define a corresponding general equilibrium economy with no regions or public services. First, apply a standard existence theorem to show that the new economy has a competitive equilibrium, then show that a Tiebout equilibrium corresponds to this competitive equilibrium.

0.10.4 Empirical Studies Related to Tiebout Hypothesis

There are quite a few empirical papers that attempt to test the implications of Tiebout Hypothesis.

Oates (JPE 1969)

Oates (JPE 1969) used a cross section data of 53 municipalities in New Jersey to estimate the effect of local public finance on property values. The so-called Tiebout hypothesis (which is not clearly spelled out) is as follows: If a community increases its property rate in order to expand its output of public services, then because consumers shop around different communities offering various packages of local public services and selects as a residence the community which offers the tax-expenditure program which best suited to his taste, it is possible that property value needs not decrease, and may well increase, as a result of the increase in the property tax rate. His findings are as follows:

1. Local property values bear a significant negative relationship to the effective tax rate and a significant positive relationship with expenditure per pupil in the public schools;

2. The size of the coefficients suggests that, for an increase in property taxes unaccompanied by an increase in the output of local public services, the bulk of the rise in taxes will be capitalized in the form of reduced property values; on the other hand, if a community increases its tax rates and employs the receipts to improve its school system, the increased benefits from the expenditure side of the budget will roughly offset the depressive effect of the higher tax rates on local property values.

It is a good idea to read this paper and write down your comments and criticisms of this paper. See Epple, Zelenitz and Visscheer (June 1978, JPE) for a systematic study of the testable implications of the Tiebout Hypothesis.

Gramlich and Rubinfeld (JPE 1982)

Gramlich and Rubinfeld (1982) test two implications from a Tiebout model: first, if consumers sort themselves according to their taste of public goods, or public services, then in a Tiebout equilibrium, citizens should have grouped themselves together with others with similar tastes, and hence, the variance of local spending demands within a community should be smaller than the variance throughout the whole state; second, communities should actually respond to the citizens' desired level of public goods (the median voter hypothesis), since otherwise, there is no point for the voters to sort themselves.

Gramlich and Rubinfeld (1982) used a micro survey on demands on public spending from 2001 Michigan households, which also includes information on fiscal, demographic, voting and attitudes etc. To test the first hypothesis, they compare the variance of local spending demands within a community with those throughout the state, after controlling for other factors that may influence spending in all districts. To test the second hypothesis, they compare the desired level of public spending with the actual level of public spending and see if the communities supply the level of public expenditures desired by the median voter in their community. Their results are supportive of Tiebout Hypothesis.

0.11 Public Goods Bibliography

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Part II

Social Arrangements

0.12 A model incorporating social arrangements:

Cole, Mailath, Postlewaite, JPE (1991)

The idea of the paper is that there are things, about which people care, but that are not available in a market. Examples might include college acceptance, grades, and entree into social circles. We would like a model that yields multiple equilibria which could be thought of as associated with different social norms.

0.12.1 Model

- There is a continuum of men indexed by $i \in [0, 1]$ and a continuum of women indexed by $j \in [0, 1]$
- Agents live one period
- In each period, men and women match and decide how much of their endowment to consume. They care about the utility of their male offspring
- All consumption within a matched pair is joint
- Initial endowments:

$$\begin{aligned} \text{Men, period 1} \quad &: \quad k_o : [0, 1] \to R \\ \text{Women, each period} \quad &: \quad e(j) = j \end{aligned}$$

- Technology:

$$c = Ak - k'$$

where c denotes consumption and k the bequest to the son.

- Parents' utility:

$$u(c_0) + \beta(u(c_1) + j$$

where c_0 and c_1 denote respectively the parents' and their son's consumption of the male good and j denotes the endowment of the woman whom their son marries.

Two period example

We will illustrate the model first with a two period example in which the parents have already matched. For this example, we will assume that $k(0) = k$, i.e., the endowment function is constant; all families have equal wealth initially.

Claim 1: No set of couples with positive measure can leave the same bequest.

"Proof" By contradiction. Suppose for some bequest level, there is a positive measure of couples who leave this amount. Some of these couples' sons must be matched with mates who have discretely less endowment than other couples who have left the same bequest. An arbitrarily small increase in the bequest then gives a discrete increase in the child's utility, and hence, in the parents' utility. This is a contradiction.

Claim 2: All first period couples get equal utility, but couples in the second period all have different utility levels.

That all first period couples get equal utility follows immediately from the fact that they all face the same problem. All second period couples must have different utilities since they all receive different bequests.

Note that there are strategic interactions among the first period couples; they face a game, not a decision problem. We will look for a Nash equilibrium. This will essentially be a function $k_1(\cdot) : [0, 1] \to R$ where $k_1(i)$ is the ith families' bequest; it must have the property that $k_1(i)$ is optimal for family i given other families choices, i.e., given $k(\cdot)$. Let $F(k)$ be the CDF for k, i.e., $F(k)$ is the proportion of families with bequest less than or equal to k.

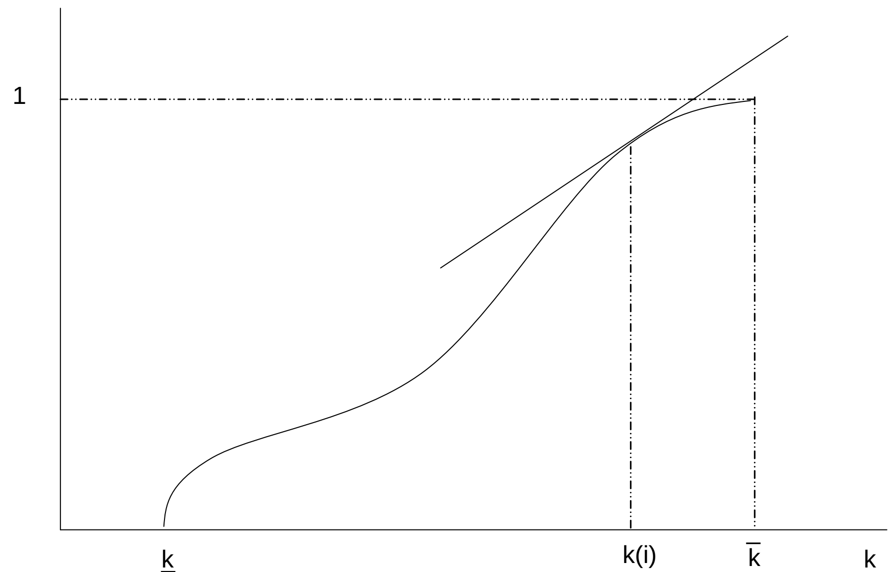
Then for all i

$$k(i) \in \arg\max u(Ak - k(i)) + \beta[u(k(i)) + F(k(i))]$$

since $F(k(i))$ is the rank in the wealth distribution. This implies that $F(k(i))$ is the index of the woman i will match with, and hence, the endowment of his mate. The first order conditions for family i are then (assuming $F(\cdot)$ is differentiable):

$$u'(Ak - k(i)) = A\beta u'(Ak(i)) + \beta F'(k(i))$$

It can be shown that an equilibrium exists for this problem. In addition to the two characteristics mentioned above in claims 1 and 2, we note several things about the equilibrium outcome. First, we see that the first order conditions for a couple's bequest decision when that decision affects the son's match with the first order condition when matching considerations ignored differ only in the additional term $\beta F'(k(i))$ in the former.



Cummulative distribution function of bequests

As can be seen in the figure above, this term is strictly positive and consequently, the marginal utility of parents' consumption is lower when matching is affected by bequests than when it is not. But this implies that consumption is lower in that case, i.e., savings are higher. To summarize, when matching is affected by bequests, people save more.

The second observation is that there will be one couple whose savings are the same whether matching is affected by bequests or not. Some male in the second period must necessarily be matched with the woman whose endowment is zero, namely the son of the couple who saved least in the first period. This couple's bequest must be the same in both cases since if they saved a different amount when matching was affected, they could do strictly better by ignoring matching. Since their son is already matched with the poorest woman, there are no second period consequences of ignoring the matching.

General Model

The two period example above of the general model illustrates how social considerations can affect economic decisions. While this is important in itself, we are also interested in the possibility that there can be a multiplicity of social arrangements with economies that are otherwise identical as a consequence of the different social arrangements. In the present model we can think of different social arrangements meaning different ways that men and women match. We will follow this path by specifying rules, or *social norms*, which prescribe how men and women in a given generation are to match. Our basic idea is to prescribe a matching based on *status.* We assume that men are ranked according to some attribute, which we will interpret as a status indicator. We can think of wealth in the two period example above as the status indicator; in principle, the status indicator could have been education, accent, or inherited social standing. Given a distribution of status across men, we will consider the prescription that the man who is the k^{th} percentile rank in the distribution should match with the woman who is the k^{th} percentile rank according to wealth. Different status indicators will correspond then to different social arrangements.

We want to maintain the conventional economic assumptions, however, that agents optimize. One could, of course, simply specify an arbitrary status indicator for men, and posit that all agents want to follow society's prescribed matching rules. There may well be preferences for following rules, but we won't take this route here for two reasons. First, it's not clear what we will learn by simply assuming that agents care directly about following social rules, and second, there is some advantage in maintaining as much as possible the structure of conventional economic models.

So, we will instead model agents as optimizing with more or less standard preferences. The question is: How can it be optimal for men and women to follow a social norm that prescribes that the wealthiest women match with the highest status men unless status is wealth? If the status indicator is not wealth, then the wealthiest man may well not be the highest status man, and hence, not to match with the wealthiest woman. But in a two period world as in the example above, these two people would clearly not find it in their interests to follow the social norm's prescription.

If we go to an infinite generation model there is the possibility of status rankings of men on the basis other than wealth. While it's true that in any generation the wealthiest men and women will want to match regardless of status, it may be that violating the social norm's prescriptions has consequences for their son, whose utility matters to the parents. We need, of course, an infinite number of generations or the problem unravels from the last period as in a finitely repeated prisoners' dilemma game. The last generation will ignore status and match according to wealth, which eliminates any reason their parents would care about status.

Formal Model

We consider a countably infinite period version of the model described above. In period t, following matching, a couple decides how much to consume. We have $c_t = Ak_t - k_{t+1}, A > 0$. c_t is the consumption of the couple who start with the man's inherited wealth k_t, while k_{t+1} is their bequest. We will assume a particular utility function, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. We will assume for technical reasons that $1 < A\beta < A^{\gamma}$.

Benchmark case: involuntary matching

Let $\{j_t\}$ be a sequence of exogenous matches in the j^{th} family line. The problem this family line faces is then

$$\begin{aligned} & \max_{\{k_t\}} \Sigma_{t=0}^{\infty} \beta^t \{ \frac{1}{1-\gamma} [Ak_t - k_{t+1}]^{1-\gamma} + j_t \\ s.t.\ k_0 &= k(i), Ak_t \geq k_{t+1} \end{aligned}$$

The first order conditions for this problem are

$$(Ak_t - k_{t+1})^{-\gamma} = A\beta[Ak_{t+1} - k_{t+2}]^{-\gamma}$$

Let $\lambda_t \equiv \frac{c_t}{Ak_t}$, the proportion of wealth consumed in period t. Then λ_t satisfies

$$(\lambda_t A k_t)^{-\gamma} = A\beta((1-\lambda_t)\lambda_{t+1} A^2 k_t)^{-\gamma}$$

(Note that $c_{t+1} = \lambda_{t+1} A k_{t+1}$ and $k_{t+1} = A(1-\lambda_t)k_t$). This implies

$$\lambda_t = \lambda^* \equiv 1 - A^{1-\gamma}\beta^{\frac{1}{\gamma}}$$

That is, the family line has at each period a constant marginal propensity to consume. Consequently, the family line's wealth grows at the rate $(1-\lambda^*)A$. Since each family line's wealth grows at this rate, aggregate wealth also grows at this rate.

Social norms

We use the term *status* to refer to an individual's rank in society. Status can, in principle, depend on wealth, education and the status of one's parents among other things. Status will *not* enter directly into individuals' utility functions. Rather, it will serve as a device to coordinate individuals' actions. Hence, individuals will not care directly about their status, but may care about status instrumentally insofar as it might affect the goods and services they or their offspring get through the coordinating role status can play. We will call a *social norm* a specification of matching behavior, savings/bequest behavior, and assignment of status. Individuals may find it in their interest to disregard a social norm's prescriptions. We will refer to a *norm equilibrium* when all individuals find it in their interest to follow the norm's prescriptions when all other agents are doing so. We turn next to the description of two norm equilibria that might exist in the model we are analyzing.

Aristocratic equilibria

An aristocratic ranking is one in which sons inherit their father's rank so long as the son's parents have followed the norm's prescribed behavior, with an exogenously given ranking of men in the first generation. If an couple violates the norm's prescription, either in their matching behavior or in their savings/bequest decision, their son's status will be 0. Given this ranking, the aristocratic social norm prescribes that the k^{th} percentile man in the status distribution is to match with the k^{th} percentile woman, and prescribes that all couples leave bequests as in the benchmark case.

It may or may not be an equilibrium for all family lines to follow this norm's prescriptions. Clearly, when the man who is ranked highest in a given period is poor relative to the richest man in that period, the richest woman has at least a myopic incentive to violate the norm's prescription and match with the rich man. The only thing that may proven is that there is an equilibrium for some given initial data.

Wealth-is-status equilibria

Suppose the distribution of wealth in period t is $k_t : [0,1] \to R_+$. The matching function is then $m_{t+1}(k) = k_{t+1}^{-1}(k)$. The i^{th} family line's problem in period t with wealth k_t is then

$$\max_{k_{t+1}} u(Ak_t - k_{t+1}) + m_t(k_t) + \beta V_{t+1}(k_{t+1}) \equiv V_t(k_t)$$

where $V_t(\cdot)$ and $V_{t+1}(\cdot)$ are the value functions. The first order conditions then imply

$$u'(Ak_t - k_{t+1}) = \beta\{Au'(Ak_{t+1} - k_{t+2}) + m'_{t+1}(k_t)\}$$

Definition: A wealth-is-status equilibrium is a specification of consumption choices for each family for each period such that the choices constitute a Nash equilibrium.

Proposition: Wealth-is-status equilibria exist.

The proof is an adaptation of standard proofs for this sort of game. The only non-standard part of the problem is that for standard fixed-point arguments, you need that the best response functions are upper hemicontinuous as a function of the aggregate of the other agents' strategies. This fails here for aggregate strategies of other agents that have atoms.

Properties of equilibria:

1. For all family lines, $\lambda_t \leq \lambda^*$ with a strict inequality for almost all family lines. That is, almost all family lines save strictly more than they would in the absence of matching concerns for their offspring.

2. All bequest levels are chosen by a zero measure set of agents.

3. If $\gamma < 1$, then $\forall i \in [0,1]$, $\lambda_t(i) \to \lambda^*$ as $t \to \infty$.

4. If $\gamma > 1$ then either $\frac{k_t(i')}{k_t(i)} \to \infty$ or $\lambda_t(i') \to 0$ $\forall i \in [0,1]$ as $t \to \infty$.

5. For family lines close to the bottom of the ranking, λ_t must be close to λ^*. This follows from the fact that these family lines are not getting much from distorting their savings decisions, hence, they will not distort much in equilibrium.

If you are interested in following up on this basic model, the Hopkins and Korienko 2004 *AER* article contains a number of interesting implications of the model.

0.12.2 Related empirical work

Wei and Zhang "The competitive saving motive: Evidence from rising sex ratios in China" JPE 2011

The authors use a model similar to this to investigate why Chinese savings rates are so high. They begin with the observation that China has experienced a rising sex ration imbalance that has resulted in more males than females. They suggest that this leads to increasing competition among males for mates, leading families with sons to increase their savings to make their sons more attractive mates. Using cross-regional and household-level data, they argue that this competition may account for about half of the increase in household savings during 1990-2007.

Du and Wei (2010) "A sexually unbalanced model of current account imbalances" NBER WP 16000

The authors start from the same point as in the previous paper – trying to understand the effect of the sex imbalance on Chinese savings. This paper calibrates an overlapping generation model and finds again that the sex imbalance can have economically significant effects and can account for half the current account imbalances.

Bhaskar, V. (2011) "Sex Selection and Gender Balance" AEJ Micro

Examines equilibrium sex ratios when parents can choose the sex of their child.

0.13 Applications of concern for rank

The wealth is status ranking that we analyzed in the previous section is interesting in itself separate from the role it plays in demonstrating the indeterminateness of economic performance excluding social arrangements. In this section we will illustrate the importance of social arrangements in analyzing economic behavior in two simple models. These applications are taken from Cole, Mailath and Postlewaite (1995).

0.13.1 Effort choice

We consider a simple problem in which there is again a matching concern between men and women. As before, we will assume there are two goods, a male good and a female good. Here, we will have men exogenously endowed while women make a labor-leisure choice decision. Matching between men and women will take place following women's decisions, with matching being based on wealth. That is, the wealthiest women will match with the wealthiest men.

As in the previous model, women's decisions will be distorted by the nonmarket matching considerations: they will choose to work more than they would in the absence of matching concerns. More interesting is that we will be able to point to a difference in individuals' reactions to wage changes when the wage change is general as compared to wage changes that are individual-specific.

Model

Continuum of men indexed by $j \epsilon [0, 1]$

Continuum of women indexed by $i \epsilon [0, 1]$

Two goods, male and female.

Male i endowed exogenously with i units of male good.

Females produce female good with costly effort: $c(e) = e^2$, for effort e.

Output for female i is $a(i) \cdot l(i)$ where $l(i)$ is labor effort and $a(i)$ is a continuous increasing function on $[0, 1]$. You can think of $\alpha(i)$ as the ability or efficiency of female i.

Utility for females: $u(c) + j - \nu(l) \qquad \nu' > 0$, $\nu" > 0$.

Matching is voluntary; $m(c)$associates with each level of female good c, the level of male good in the resulting match. Note that $m(c)$ is just the CDF of the consumption for women.

In Problem Set III you are asked to analyze this model.

0.13.2 Conspicuous consumption

The basic idea is that as in the previous labor supply example, men and women will match and consume jointly the assets the two individuals bring to the match. We will maintain our assumption that men are exogenously endowed with male good, and that the endowments are publicly observable. Unlike that example, however, women's wealth is not directly observable. We are interested in the possibility that women may be able to signal wealth through consumption. Suppose that prior to matching women divide their wealth between consumption and savings. If consumption and saving are correlated and consumption is publicly observable, consumption will signal saving: the higher a woman's consumption, the higher her savings. Since men prefer to match with women with higher savings, women with higher consumption than others will be more desirable mates. Consequently, women will have an incentive to increase their consumption in order to attract more desirable (i.e., wealthier) mates. We interpret this as conspicuous consumption motivated by instrumental concerns.

Model

As in the previous section, there is a male good and a female good. Men are exogenously endowed with male good and women make a labor choice.

l- labor choice

d-amount destroyed

Female's problem:

$$\max_{l,d} u(a(j) \cdot l - d) - \nu(l) + m(d)$$

Note: The difference from effort choice case is that there is no direct benefit from distortion here.

Denote output al by y; the objective function is then

$$u(y - d) - \nu(y/a) + m(d)$$

Signalling equilibrium: A signalling equilibrium is a triple of functions $l : [0,1] \to R_+$, $d : [0,1] \to R_+$, and $m : R_+ \to [0,1]$ satisfying:

1. d and $al - d$ are both strictly increasing functions (i.e. d signals c)
2. $\forall\ j \epsilon\ [0,1] \qquad l(j),\ d(j)\,\mathrm{max}$

$$u\left(a(j)l - d\right) - \nu(l) + m(d)$$

3. $m\left(d(j)\right) = j\ \forall\ j$

Example:

We will consider a special case in which output levels given exogenously. The output of the j^{th} woman is

$$y(j) = e^{\gamma j}, \gamma > 0$$

This eliminates the term $\nu(l)$ from the objective function. The utility for the output good is

$$u(c) = \ln c$$

Female j's problem is then

$$\max_{\hat{d}} \ln\left(y(j) - \hat{d}\right) + m(\hat{d})$$

FOC's

$$\frac{-1}{y(j) - \hat{d}} + m'(\hat{d}) = 0 \text{ (*)}$$

If $d(j)$ is part of equilibrium, $m\left(d(j)\right) = j$, or equivalently, $m(\hat{d}) = d^{-1}(\hat{d})$
$\Longrightarrow$ in equilibrium $m'(\hat{d}) = \left[d'\left(d^{-1}(\hat{d})\right)\right]^{-1}$

Hence (*) can be rewritten

$$y(j) - \hat{d} = d'\left[d^{-1}(\hat{d})\right] = d'(j)$$

Thus we have a differential equation $d'(j) = y(j) - d(j)$; the initial condition is $d(0) = 0$ since the woman at the bottom of the distribution is matched with the lowest man independent of her decision. The solution to the differential equation is

$$d(j) = (1+\gamma)^{-1}\{e^{\gamma j} - e^{-j}\}$$

I leave it to you to verify that this is, in fact, a solution to the differential equation.

Notes:

1.

$$\begin{aligned}\frac{d(j)}{y(j)} &= (1+\gamma)^{-1}\left[\frac{e^{\gamma j} - e^{-j}}{e^{\gamma j}}\right] = \\ & \quad (1+\gamma)^{-1}\left[1 - e^{-j-\gamma j}\right] \\ &= \frac{1}{1+\gamma}\left[1 - \frac{1}{e^{(1+\gamma)j}}\right]\end{aligned}$$

Thus the amount of initial endowment that is destroyed is:

i) increasing in j - i.e., the proportionate distortion is greater for higher output women.

ii) decreasing in γ. This can be interpreted as follows. A small γ means a tight distribution of initial endowments. As γ gets larger, the distribution of initial wealth gets larger and the change in rank that results from a given amount of destruction is reduced. Hence, since the reward to reduction is lowered, the optimal amount destroyed goes down.

0.13.3 Investment: "Investment and concern for relative position," (with H. Cole and G. Mailath) *Review of Economic Design* 6, 241–261 (2001).

Model

Continuum of men indexed by $j \epsilon [0, 1]$

Continuum of women indexed by $i \epsilon [0, 1]$

Three goods, a nonconsumable capital good, a male good and a female good

Male i endowed exogenously with $e(i)$ units of male good, e strictly increasing

Females are endowed with one unit of the capital good, which can be used to produce the female good. There is a common project that has random rate of return a and idiosyncratic projects which have the same random rate of return b_i. All projects are risky and take on values in $[0, 1]$. The returns on all projects are independent and have continuous density functions. Assume that the returns are identical.

Female i decides how to allocate her capital good between project a and project b_i.

The wealth of female agent i is given by $y_i = x_i a + (1 - x_i)b_i$.

Females' preferences are given by $E\{u(\tilde{c}) + \hat{w}(\tilde{e})\}$

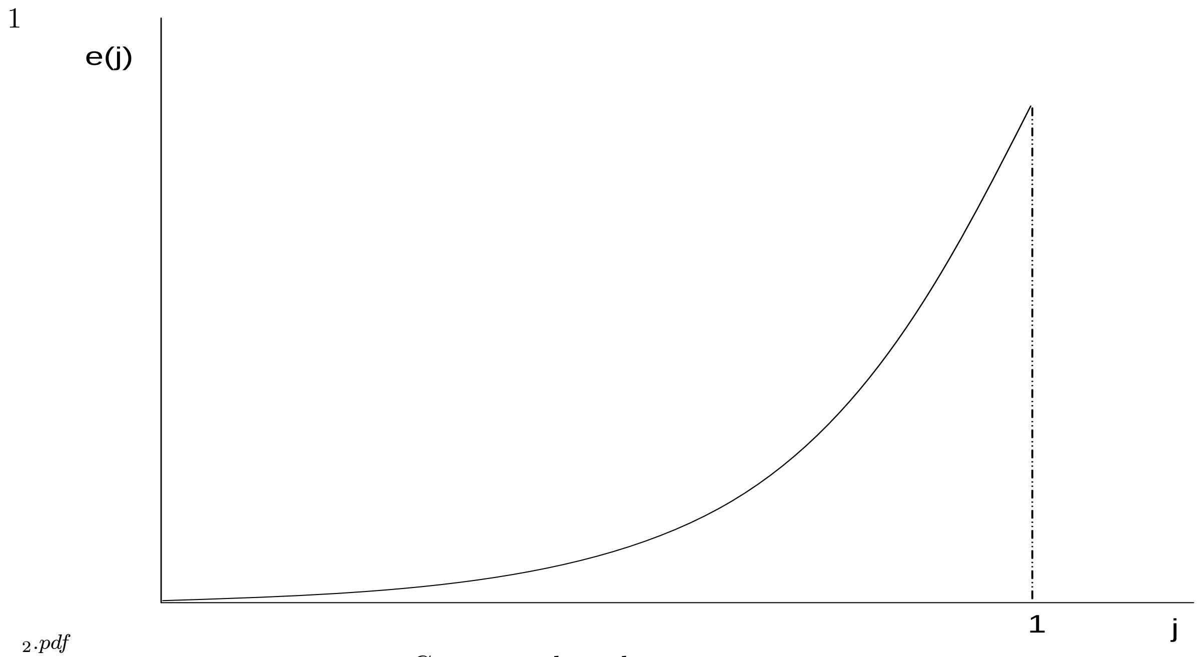
Female preferences over males: $w(j) = \hat{w}(e(j))$

Note that female preferences over males is the composition of female preferences over male good ($\hat{w}$) and the male endowment function (e). Even if females are risk averse in consumption ($\hat{w}'' < 0$) w may be convex if e is sufficiently convex. Male preferences are increasing in both male (predetermined) and female goods. Matching takes place after the realization of the random variables. Since both males and females have strictly increasing preferences, matching will be assortative on wealth.

We consider symmetric choices, i.e., $x_i = x\ \forall i$. Note that unless $x = 1$, no female wealth level is attained by a positive measure of women. A common investment choice x gives a determinate distribution of female wealth, and consequently a determinate distribution of resulting matches and consumption

In the absence of matching concerns, females allocate their endowment equally between the two independent, identically distributed risky projects they face to minimize variance.

Investment



Convex male endowment

If $e(j)$ is sufficiently convex females may choose to invest more in their idiosyncratic projects. Note that females may look as though they are risk-loving if we consider only the monetary rewards even though they are risk averse in consumption. A central point is that the utility function over monetary investments is a reduced-form utility function, that is, it is an equilibrium object that depends on the environment. This suggests that identical people in different environments may look to be differentially risk averse when looking at their investment behavior. It is sometimes said that Europeans are more reluctant to undertake risky ventures than are Americans. It is interesting to look for differences in the consequences of investment outcomes that might explain the difference.

See Nikolai Roussanov, “Diversification and its Discontents: Idiosyncratic and Entrepreneurial Risk in the Quest for Social Status,” (mimeo, March 2008) for a general analysis of a standard economic model in which agents care about their relative standing.

0.14 Social Assets Mailath and Postlewaite IER (2006)

0.14.1 Model:

- Countably infinite number of generations
- Each generation consists of continuua of men and women
- Single nonstorable consumption good
- Individuals have endowments of this good
- In each generation, men and women will match and consume their combined wealth
- Each couple has two offspring
- Common consumption utility function: $U : R \to R$
- Individuals care about their childrens' welfare: the utility to any matched couple is their utility from consumption plus the (discounted at rate β) average utility of their children
- Men and women have high income (H) or low income (L)
- Additionally, there is a nonincome attribute; we denote an agent with the attribute Y and one without N
- The probability that an individual has high income is $1/2 + k$ if he or she has the nonincome attribute and $1/2 - k$ if not; $0 \leq k \leq 1/2$
- The probability that any individual has high income is independent of everything else
- This attribute is heritable: if both parents are Y or both are N, their children will be the same as their parents with probability 1. If only one parent has the attribute, each child will have the attribute with probability $1/2$
- Matching is voluntary, that is, no unmatched pair can increase their utilities by matching (taking into account the consequences to their descendants). We call such a matching *stable.*

Nonproductive attribute ($k = 0$)

- Income is exogenous, either high (H) or low (L), each with probability 1/2
- Each person's income is independent of everything, including parents' incomes
- Matched couple maximizes $U(c)$ plus discounted average utility of their children

How will men and women match?

Definition: We say matching is *assortative on income* (or simply assortative) if all matched couples have the same income.

Assortative matching: Men Women

$$\begin{array}{ccc} H & \longleftrightarrow & H \\ L & \longleftrightarrow & L \end{array}$$

It is clear that people have an incentive to follow the prescribed matching: high income women will want to match with high income men. The value function for an individual who has high income is then

$$V_H^A = U(2H) + \beta[1/2V_H^A + 1/2V_L^A]$$

Similarly, the value function for an individual who has low income is

$$V_L^A = U(2L) + \beta[1/2V_H^A + 1/2V_L^A]$$

We can normalize the utility function U so that $U(2L) = 0$ and $U(2H) = 1$. Denote by u $U(H + L)$. Values of H and L under this matching:

$$\begin{aligned} V_H^A &= \frac{2-\beta}{2(1-\beta)}, \text{ and} \\ V_L^A &= \frac{\beta}{2(1-\beta)} \end{aligned}$$

Both types would be strictly better off if there existed an actuarially fair market to insure offsprings' income.

Mixed matching Definition: We say that a matching is *mixed* if there are matched couples with different incomes.

Consider the following mixed matching:

Men		**Women**
HY	$\longleftrightarrow$	HY
HN	$\longleftrightarrow$	LY
LY	$\longleftrightarrow$	HN
LN	$\longleftrightarrow$	LN

This ranking induces a preference for mates with attribute Y, all other things equal. Will an (H,N) agent prefer matching with an (L,Y) agent to matching with another (H,N) agent?

Incentive constraint for stability of ranking:

$$\begin{aligned} & u + \beta \left[1/4V_{HY}^M + 1/2V_{HN}^M + 1/4V_{LN}^M\right] \\ \geq \quad & 1 + \frac{1}{2}\beta(V_{HN}^M + V_{LN}^M). \end{aligned}$$

Proposition: Mixed matching is stable for the nonproductive attribute case if and only if $u \geq \frac{4-3\beta}{4-2\beta}$.

If the utility function for consumption is sufficiently concave, the proposed matching will be stable. The necessary amount of concavity becomes arbitrarily small as β approaches 1.

Welfare Comparison of Rankings

$$\begin{aligned} V_{HN}^M &= \frac{(1-\frac{1}{2}\beta)u + \frac{\beta}{4}}{1-\beta} \text{ and} \\ V_H^A &= \frac{2-\beta}{2(1-\beta)}. \end{aligned}$$

Hence,

$$\begin{aligned} V_{HN}^M &> V_H^A \iff \\ u &> \frac{4-3\beta}{2(2-\beta)}. \end{aligned}$$

From an ex ante evaluation, mixed attribute is always superior Incentive constraints are interim, hence the welfare superior mixed attribute ranking will not be stable for low levels of risk aversion.

Proposition: Mixed matching is interim welfare superior to assortative matching for the nonproductive attribute case if and only if mixed matching is stable.

Productive Attribute ($k > 0$)

$1/2 + k$ - probability of high income H if attribute is Y

$1/2 - k$ - probability of high income H if attribute is N

Heritability is as before: a child will have the attribute for sure if both parents possessed the attribute, with probability 1/2 if one parent had the attribute. Consider mixed attribute matching: (H, N) matches with (L, Y). Both incentive constraints matter now since the attribute is inherently valuable.

Incentive constraint for (H, N) :

$$u + \beta(\frac{1}{2}V_Y^M + \frac{1}{2}V_N^M) \geq 1 + \beta V_N^M.$$

Incentive constraint for (L, Y) :

$$u + \beta(\frac{1}{2}V_Y^M + \frac{1}{2}V_N^M) \geq \beta V_Y^M.$$

Proposition: Mixed matching is stable for the k-productive attribute case if and only if

$$1 - u \leq \frac{\beta}{2}\left[\frac{1 + 2k}{2 - \beta(1 + 2k)}\right] \leq u.$$

Equilibria that ignore the attribute will not exist if k is large: assortative matching will not be stable if

$$1 - u \leq \frac{2k}{1 - \beta} \leq u.$$

For some values of k, β, and u only mixed matching is stable for the given k, but both mixed matching and assortative on income if $k = 0$. We may have dynamic equilibria in which initially, only mixed matching is stable. However, the asset may become unproductive at some time, and yet the asset remains valuable in equilibrium.

Productive Attributes: Endogenous Choice

Education is the attribute. If both parents are Y, both children will have the attribute with probability $p < 1$; If only one parent is Y, both children will have the attribute with probability $1/2p$. Education can be purchased: if parents pay a cost $c(q)$, both children will have the attribute with probability q. $c'(q) > 0$ and $c''(q) < 0$. This choice is made *after* the realization of whether the child has inherited the attribute. Education is productive at level $k > 0$.

Education valued only for its productive value Consider assortative (on income) matching:

Men		**Women**
HY	$\longleftrightarrow$	HY
HN	$\longleftrightarrow$	HN
LY	$\longleftrightarrow$	LY
LN	$\longleftrightarrow$	LN

If k is not too big and c is not too small, this matching will be stable.

Education additionally valued as a social asset Consider the following mixed matching:

Men		**Women**	
HY	$\longleftrightarrow$	HY	
HN	$\longleftrightarrow$	LY	$(almost)$
LY	$\longleftrightarrow$	HN	
LN	$\longleftrightarrow$	LN	

Proposition: For some k positive but not too large, the steady state proportion of individuals with education is larger under mixed matching than under assortative matching.

There may be higher return to investment in education when there is a social return in addition to the productive return. The higher value of education when there is a social return induces greater investment in education.

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If individuals care about their status, defined as their rank in the distribution of consumption of one "positional" good, then the consumer's problem is strategic as her utility depends on the consumption choices of others. In the symmetric Nash equilibrium, each individual spends an inefficiently high amount on the status good. Using techniques from auction theory, we analyze the effects of exogenous changes in the distribution of income. In a richer society, almost all individuals spend more on conspicuous consumption, and individual utility is lower at each income level. In a more equal society, the poor are worse off.

Hopkins, E. and Tatiana Kornienko, "Inequality and Growth in the Presence of Competition for Status," *Economics Letters* 93 (2006) 291-296. This is a short note that integrates a concern for relative position into a simple endogenous growth model. The authors show that redistribution to reduce inequality may increase inefficient conspicuous consumption.

Hopkins, E. and T. Kornienko (2010), "Which inequality? The inequality of endowments versus the inequality of rewards," *American Economic Journal: Microeconomics.*

Abstract:

We introduce a new distinction between inequality in initial endowments (e.g., ability, inherited wealth) and inequality of what one can obtain as rewards (e.g., prestigious positions, money). We show that, when society allocates resources via tournaments, these two types of inequality have opposing effects on equilibrium behavior and well-being. Greater inequality of rewards hurts most people?both the middle class and the poor?who are forced into greater effort. Conversely, greater inequality of endowments benefits the middle class. Thus, the correctness of our intuitions about the implications of inequalityi s hugelya ffectedb y the typeo f inequalityc onsidered.

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Abstract:

We study the behavioural foundation of interdependent preferences, where the outcomes of others affect the welfare of the decision maker. These preferences are taken as given, not derived from more primitive ones. Our aim is to establish an axiomatic foundation providing the link between observation of choices and a functional representation which is convenient, free of inconsistencies and can provide the basis for measurement. The dependence among preferences may take place in two conceptually different ways, expressing two different views of the nature of interdependent preferences. The first is Festinger's view that the evaluation of peers' outcomes is useful to improve individual choices by learning from the comparison. The second is Veblen's view that interdependent preferences keep track of social status derived from a social value attributed to the goods one consumes. Corresponding to these two different views, we have two different formulations. In the first, the decision maker values his outcomes and those of others on the basis of his own utility. In the second, he ranks outcomes according to a social value function. We give different axiomatic foundations to these two different, but complementary, views of the nature of the interdependence. On the basis of this axiomatic foundation, we build a behavioural theory of comparative statics within subjects and across subjects. We characterize preferences according to the relative importance assigned to gains and losses in social domain, that is pride and envy. This parallels the standard analysis of private gains and losses (as well as that of regret and relief). We give an axiomatic foundation of interpersonal comparison of preferences, ordering individuals according to their sensitivity to social ranking. These characterizations provide the behavioural foundation for applied analysis of market and game equilibria with interdependent preferences.

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Abstract:

The idea that utility or happiness depends on the comparison of one’s own consumption to that of others can be traced back, at the very least, to Veblen (1899). Nevertheless, neoclassical economic theory has typically assumed that an agent’s utility depends solely on the absolute level of consumption. This stance has been harder to maintain in the face of a large body of empirical evidence on an individual’s tendency to evaluate her consumption in comparison with consumption of others, and there is now a growing literature that includes this relative concern, or status, as an additional argument in the utility function. This note considers a preference for status as fundamental. It takes the extreme view that the individual does not care about the consumption of particular goods per se, but only how she ranks in the distribution of consumption. This view makes it possible to straightforwardly reconcile an evolutionary basis for preferences with the obvious fact that absolute consumption levels now vastly exceed any plausible level in hunter-gatherer societies. Despite this apparently fundamentally distinct basis for choice, this paper establishes a startling behavioral equivalence between the relative concern model and an absolute concern model of intertemporal consumption choice. That is, the equilibrium consumption time path followed by individuals with a pure preference for status is the same as that followed by individuals with a logarithm utility function in absolute consumption. This result holds for a large class of preferences for status. There may then be no observable difference at all between a model with a pure concern for status and a conventional model.

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Part III

Public Choice (Silverman)

0.15 Introduction

Modern public economics has recognized that political institutions and motives constrain the implementation of public policies. At the extreme end of the spectrum (something of a caricature of the Buchanan school of thought) are those who essentially view normative theory as useless. From this view, public policies are the outcome of a political process and if someone generates a normative analysis claiming that, say, a tax reform would be beneficial this analysis is both (A) something the policy maker could have figured out himself and (B) not being implemented because of the forces of political equilibrium.

We will discuss the different schools of thought in greater detail later. But while some would argue that only positive analysis makes sense, we note that even economists that see value in normative exercises are using positive models of policy determination more and more.

0.16 Models of Political Competition

0.16.1 The Downsian Model of Political Competition

This simple model of political competition is attributed to Downs (1957), Black (1948), but the basic structure is essentially Hotelling's "location on a beach model". In spite of its simplicity and many unrealistic assumptions it has proved very useful and is still used a lot, in particular in macro-economic models. Assume

- there are n citizens. Set of citizens denoted $N = \{1, ..., n\}$
- a single dimensional policy variable $p \in P = [0, 1]$
- each agent i has single-peaked preferences over p. Denote these by $v_i(p)$
- two political parties, A and B.
- Parties compete in an election for office. Parties care *only about winning the election* and are not motivated by policy at all. Payoffs for party $j \in \{A, B\}$ are

$$u^j = \begin{cases} w^j > 0 & \text{if in office} \\ 0 & \text{if not in office} \end{cases}$$

The timing of the model is as follows:

Stage 1 Parties commit to platforms $(p_A, p_B) \in P \times P$

Stage 2 Voters vote between two parties. Formally a pure voting strategy can be defined as a map $s^i : P \times P \to \{0,1\}$ where $s^i(p_A, p_B) = 0$ means that A gets the vote and $s^i(p_A, p_B) = 1$ means that B gets the vote. A mixed strategy is then a map $\pi^i : P \times P \to \Delta\{0,1\}$ and can thus be represented as a map into $[0,1]$, where we take $\pi^i(p_A, p_B)$ to be the probability that A gets i's vote.

We can now in principle go along and define payoffs as function of *strategies* in order to get the normal form of the game (to appeal to purist game theorists). However, we will take a little bit of a (valid) shortcut and note that in a *subgame* after parties have committed to platforms the payoffs of agent i can be written as

$$u^i(s(p_A, p_B)) = u^i(s_1(p_A, p_B), ..., s_n(p_A, p_B)) = \begin{cases} v_i(p_A) & \text{if } \sum_i s^i(p_A, p_B) \geq \frac{n-1}{2} \\ v_i(p_B) & \sum_i s^i(p_A, p_B) \leq \frac{n-1}{2} \end{cases}$$

we now claim that

Claim *If $v_i(p_A) > v_i(p_B)$ it is a weakly dominant strategy to vote for A and if $v_i(p_A) < v_i(p_B)$ it is a weakly dominant strategy to vote for B.*

This is straightforward. We assume that citizens vote according to their weakly dominant strategy so that A wins if and only if a majority prefer A. Let

$$\begin{aligned} S_A(p_A, p_B) &= \{i \in N | v_i(p_A) > v_i(p_B)\} \\ S_B(p_A, p_B) &= \{i \in N | v_i(p_A) < v_i(p_B)\} \end{aligned} \tag{7}$$

Assuming that a fair coin is tossed in case of an equal number of votes and that citizens vote according to their weakly dominant strategies after any announcement of platforms, then the probability of wining the election for party A is

$$\pi(p_A, p_B) = \begin{cases} 1 & \text{if } |S_A(p_A, p_B)| > |S_B(p_A, p_B)| \\ \frac{1}{2} & \text{if } |S_A(p_A, p_B)| = |S_B(p_A, p_B)| \\ 0 & \text{if } |S_A(p_A, p_B)| < |S_B(p_A, p_B)| \end{cases}$$

Remark 33 *There is a little bit of a subtle issue here. The above derivation is of course OK if there are no indifferences, but if an agent is indifferent between* p_A *and* p_B *any way of casting the vote is consistent with equilibrium. The way to resolve this is by the usual coin flip, but you should note that it may be that* $|S_A(p_A, p_B)| > |S_B(p_A, p_B)|$ *and* A *is losing in this case. Another way of resolving the issue would be to have indifferent voters to abstain, but this means that everyone would abstain if the parties announce the same platform, which means that this is not a very good assumption in this context.*

It is not necessary, but we make the assumption that indifferent agents flip a fair coin and adjust the formula for $\pi(p_A, p_B)$ accordingly when there are ties (as we will see the structure of the model makes sure that we don't really need to write out what the adjusted formula is). Expected payoffs for parties are

$$\pi(p_A, p_B) w_A \text{ for party } A$$
$$(1 - \pi(p_A, p_B)) w_B \text{ for party } B$$

Define

$$p_i^* = \arg\max_{p \in P} v_i(p)$$

for each agent i. We define an equilibrium as an equilibrium of the simultaneous move reduced game where parties chooses platforms and citizens decisions are replaced by the rule to vote for the most preferred platform and flip a coin where indifferent (alternatively, we could just define it as a subgame perfect equilibrium where weakly dominated strategies are eliminated).

Definition 34 *A pair of platforms* (p_A^*, p_B^*) *is an equilibrium if*

$$p_A^* \in \arg\max_{p} \pi(p_A, p_B^*) w_A$$
$$p_B^* \in (1 - \pi(p_A^*, p_B)) w_B$$

Without loss of generality we relabel agents so that $p_1^* \leq p_2^* \leq ... \leq p_n^*$. Assuming n is odd we have:

Proposition 35 *(Median Voter Theorem, Downs (1957), Black (1948)) The unique equilibrium in the Downsian model has* $p_A^* = p_B^* = p_m^*$*, where* $m = \frac{n-1}{2}$ *is the voter who has a most preferred outcome with* $\frac{n-1}{2}$ *voters on each side.*

Proof. ($p_A^* = p_B^* = p_m^*$ is an equilibrium) Suppose $p_B = p_m^*$. If $p_A < p_m^*$ it follows from strict single-peakedness that $v_i(p_A) < v_i(p_B)$ for $i = m, ..., n$. Hence $|S_A(p_A, p_m^*)| < \frac{n-1}{2}$ and $|S_B(p_A, p_m^*)| \geq \frac{n-1}{2}$, so B wins for sure, $\pi(p_A, p_m^*) = 0$. If $p_A > p_m^*$, the same argument applies, so there is no profitable deviation from p_m^* for player A. Since labeling of parties is arbitrary, this proves that (p_m^*, p_m^*) is an equilibrium.

(uniqueness) Suppose without loss of generality that $p_A \neq p_m^*$. Suppose first that A wins with some probability. Then B can't play a best response, since p_m^* wins with probability 1 due to argument above. On the other hand side, suppose A does never win. Then A can't play a best response since if A deviates and play p_m^* the probability of winning is $\frac{1}{2}$ if $p_B = p_m^*$ and 1 otherwise. ■

0.16.2 Criticisms and Weaknesses with the Downsian Model

What has been identified as the main problems are the following:

1. Non-existence of equilibria when single-peakedness fails or if the policy space is not single-dimensional.

2. The fact that the model doesn't work as nicely when there are more than 2 parties.

3. The ad hoc nature of parties: why do they only care about office. In some way, parties are made up of people and people do even in the model care about policies directly.

Example 36 ***Non existence of (pure) equilibria due to multi-dimensional policy space.*** *Let* $N = \{1, 2, 3\}$, $P = \left\{(T_1, T_2, T_3) \in R_+^3 | \sum_i T_i = 1000\right\}$, $u_i(T_1, T_2, T_3) = T_i$. *That is, the policy space are all possible divisions of a 1000 among the three citizens. We claim that for any policy* $T = (T_1, T_2, T_3)$, *there is always a policy* T' *that will defeat* T *for sure, which means that there is no (pure strategy) equilibrium in the Downsian model. The proof is simple. Without loss, assume* $T_1 \geq T_2 \geq T_3$ *and consider* $T' = \left(0, T_2 + \frac{1}{2}T_1, T_3 + \frac{1}{2}T_1\right)$. *Both citizen 2 and 3 are better off under* T', *so it defeats* T. *Since* T *was arbitrary this means that there is no equilibrium in the Downsian model.*

0.16.3 Probabilistic Voting Models (minor footnote)

Mainly to deal with the non-existence issue a class of models called "probabilistic voting models" has been developed and also proved useful in certain applications. The second reason why people have turned to this type of models is that, even when equilibria do exist and given any realistic assumption about voting costs (arbitrarily small, but strictly positive), rational voters will not participate in large elections. We will discuss this second idea, the "paradox of voting" in connection with Ledyard's (1982) model below; but first we'll look briefly at a literature that has, to some extent, given up on the idea of rational voters.

Let $\mu_i(p_A, p_B)$ be the probability that i votes for party A given platforms p_A, p_B. A problem with the Downsian model is that an ϵ change in policy can make this probability switch from 0 to 1, which is what makes profitable deviations so easy to find. The idea is thus to smooth these effects which is done by assuming that

$$\mu_i(p_A, p_B) = G(v_i(p_A) - v_i(p_B))$$

This literature started as an alternative to the rational voter paradigm, so we should not be surprised if we feel that this formulation is a bit ad hoc. However, we might think that parties do not have information about the preferences of every single voter, but rather have some idea about what "types" of voters tend to like different things. If, on top of this voters have direct preferences over parties (ideology?,trust in certain candidates?) one could think of $v_i(p_A) - v_i(p_B)$ as the utility loss (gain) for an agent i, $G(0)$ as the fraction of agents who in case of identical policies like party A and if $v_i(p_A) - v_i(p_B) < 0$ we may then think of $G(v_i(p_A) - v_i(p_B))$ as the fraction of type A supporters who like A enough so that the loss because of the unfortunate choice of policy is smaller than the potential utility loss of voting for the enemy. This is all very loose, but it should be kept in mind that *most authors actually felt that rationalistic models would not work and felt that specifying some "reasonable" voting rules was as good as it could be.* Couglin & Nitzan, who are authorities in the field think of it in a slightly different, more pragmatic, way. They explain $\mu_i(p_A, p_B)$ as a *probabilistic voting estimator* and think of it as some sort of Bayesian estimator of voting behavior as a function of observable characteristics (i.e., this explanation sort of black-boxes individual voting behavior).

These models assume that (there are lots of different variants)

1. G is increasing, $G(v_i(p_A) - v_i(p_B)) \in [0, 1]$ for all $p_A, p_B \in P$

2. $G(0) = \frac{1}{2}$ (symmetry assumption)

The next seemingly ad hoc assumption (which also helps smooth things) is that parties care about the number of votes rather than winning the election. This assumption can be defended on the grounds that it is a good approximation for large elections. The payoffs for the parties are then

$$\begin{aligned} u^A(p_A, p_B) &= \sum_i G(v_i(p_A) - v_i(p_B)) \\ u^B(p_A, p_B) &= n - \sum_i G(v_i(p_A) - v_i(p_B)) \end{aligned}$$

And an equilibrium is defined in the obvious way as a Nash equilibrium in the simultaneous move "platform-choice game" with these payoffs.

We will not show it formally, but given appropriate assumptions on the function G and individual preferences, equilibria can be shown to exist for much more general policy spaces than single-dimensional. However, the assumptions needed are rather restrictive. The problem has to do with the fact that the payoff of one party is negative the others party's payoff so it is hard to get second order conditions to be fulfilled for both A and B.

References: The paper by Couglin & Nitzan (1981) in Journal of Public Economics is instructive because it uses a very typical setup. An easy read is the discussion by Usher (1994). Hinich (1977) shows that for certain large electorates with sufficient noise, the ad hoc specification of payoffs is an innocuous assumption, maximizing the number of votes and the probability of winning is equivalent.

0.16.4 Supermajorities (interesting footnote)

An alternative way to deal with the existence problem and Condorcet cycles is to impose supermajority restrictions. We know that:

- Simple majority rule⇒"voting cycles"
- Unanimity⇒no "cycles"

It seems rather reasonable to ask whether there is some $k-$majority rule with k being in between 50 and 100 percent that rules out voting cycles.

This is exactly what is done in a classic paper by Caplan & Nalebuff (1991). The paper's task is to establish conditions under which the *mean voters* most preferred policy is unbeatable. As in Shepsle (1979), the question asked is *what is needed to support a* *particular policy as an equilibrium given that it is already the status quo.* Hence, Caplan & Nalebuff assume that the mean voter's most preferred point is the status quo and check what supermajority requirement is needed for it to be unbeatable. It should be observed that policies near the status quo will also in general be unbeatable.

Still they get a remarkable result. They found that, under weak assumptions on the distribution of preferences, a 64 percent majority rule will do the job. It is an almost scary coincidence (?) that this is almost exactly 2/3 majority rule, a supermajority requirement that is seen a lot in the real world

0.16.5 Citizen Candidates (important class of models)

An alternative approach to the voting models discussed above is the idea of formalizing an ideal representative democracy. This approach, developed recently by Besley & Coate and Osborne & Slivinski takes as primitives the citizens, the set of available policies and each citizens preferences over the set of policies and assumes that every citizen can run for office and that the holder of office can implement whatever policy he likes. Hence, political outcomes are derived directly from underlying tastes and technology (like in above models), but (unlike above models) no preexisting political actors are assumed. Assume that

- there are n citizens. Set of citizens denoted $N = \{1, ..., n\}$
- One citizen must be selected as policy maker. The set of feasible alteratives when i is in office is denoted by A_i. Let $A = \cup_i A_i$
- If citizen i runs for office she faces a cost $\delta > 0$
- Utility of individual i when policy $x \in A$ is implemented and $j \in N \cup \{0\}$ is in office is $v_i(x, j)$. $j = 0$ refers to the case when nobody is in office.

Timing:

Stage 1 Citizens decide whether to enter as candidates in a costly election

Stage 2 Citizens vote

Stage 3 The winner implements a policy (in her feasible set). The rationale for the citizens to have potentially different sets of feasible policies when in office is to be able to model candidates with different abilities.

0.16.6 Equilibrium in Citizen Candidate Model

The model is a dynamic game with perfect information, so it can be solved by backwards induction.

Stage 3 Suppose i is elected. Then (barring ties) the unique sequentially rational decision for i is to select her most preferred policy. Let

$$x_i^* = \arg\max_{x \in A_i} v_i(x, i)$$

Associated with x_i^* is an implied utility imputation $v_i = (v_{1i}, v_{2i}, .., v_{ni})$ where $v_{ji} = v_j(x_i^*, i)$. If no citizen holds office some default policy x_0 is implemented with associated utility imputation v_0, where $v_{j0} = v_j(x_0, 0)$

Stage 2 After the entry decisions there is an implied set of candidates $C \subset N$. Each citizen can now vote for any one of these (or abstain). Denote i's voting decision by $\alpha_i \in C \cup \{0\}$, where α_i then has the interpretation of being the candidate i votes for (0 means abstention). Let $\alpha = (\alpha_1, ..., \alpha_n)$ and

$$F^i(\alpha) = |j \in N : \alpha_j = i|,$$

that is, the number of votes i receives under voting profile α. Let $W(\alpha, C)$ denote the set of winning candidates when the candidates are C and the voting profile is α,

$$W(C, \alpha) = \left\{ i \in C : F^i(\alpha) \geq F^j(\alpha) \text{ for all } j \in C \right\}$$

Assuming that ties are broken by the toss of a fair coin we have that the probability of winning is

$$P^i(C, \alpha) = \begin{cases} \frac{1}{|W(C,\alpha)|} & \text{if } i \in W(C, \alpha) \\ 0 & \text{otherwise} \end{cases}$$

We'll assume that all agents behave sequentially rationally in the voting stage and eliminate weakly dominated strategies for the usual reasons. We therefore have that equilibria in the subgame starting after a set C of candidates have entered satisfy

$$\alpha_j^*(C) \in \arg\max_{\alpha_j \in C \cup \{0\}} \sum_{i \in C} P^i\left(C, \alpha_{-j}^*, \alpha_j\right) v_{ji}$$

and that there exists no α_j' such that

$$\sum_{i \in C} P^i\left(C, \alpha_{-j}, \alpha_j'\right) v_{ji} \geq \sum_{i \in C} P^i\left(C, \alpha_{-j}, \alpha_j^*(C)\right) v_{ji}$$

for all α_{-j}, with strict inequality for some α_{-j}.

Stage 1 Citizen i's pure action is to select $e_i \in \{0,1\}$ and given action profile $e = (e_1, ..., e_n)$ the set of candidates is $C(e) = (i : e_i = 1)$. Given sequentially rational voting behavior $\alpha^*(\cdot)$ (there may be multiplicity, but as in all equilibrium analysis the agents coordinate on the same continuation play after any particular history), the expected payoff for citizen i can be written

$$U^i(e, \alpha^*(\cdot)) = \sum_{j \in C(e)} P^j(C(e), \alpha^*(C(e)))\, v_{ij} + P^0(C(e))\, v_{i0} - \delta e^i$$

In paper, Besley &Coate uses this framework to study:

1. Efficiency properties of equilibria
2. The question of whether candidates can run just to spoil the chances of winning for some other candidate (like Ross Perot)

Allowing for mixed strategies in the entry stage it follows as a more or less direct application of the Nash existence theorem that an equilibrium exists. In fact, pure strategy equilibria exist quite broadly and unlike the Downsian Model and Ledyard's model, the problem is typically that there are too many of them.

Suppose $\{e^*, \alpha^*(\cdot)\}$ is a subgame perfect equilibrium in pure strategies (we don't include the last stage since this is trivial). Then any candidate who runs must be willing to do so, that is

$$\sum_{j \in C(e)} P^j(C(e^*), \alpha^*(C(e^*)))\, v_{ij} - \delta \geq \sum_{j \in C(e)} P^j(C(e^*) \setminus \{i\}, \alpha^*(C(e^*) \setminus \{i\}))\, v_{ij} + P^0(C(e))\, v_{i0}$$

Also, individuals who stays out must do this for good reason:

$$\sum_{j \in C(e)} P^j(C(e^*), \alpha^*(C(e^*)))\, v_{ij} + P^0(C(e))\, v_{i0} \geq \sum_{j \in C(e)} P^j(C(e^*) \cup \{i\}, \alpha^*(C(e^*) \cup \{i\}))\, v_{ij}$$

Equilibria can then be characterized as *One-Candidate Equilibria, Two Candidate Equilibria* and so on. One-Candidate-Equilibria actually are more interesting than one may think. We can characterize these as follows:

Proposition 37 *An equilibrium where citizen i runs unopposed exists if and only if*

1. $v_{ii} - v_{i0} \geq \delta$

2. *$v_{kk} - v_{ki} \leq \delta$ for any $k \neq i$ such that $v_{kj} > v_{ij}$ for a strict majority (i.e. for any k such that there exists a set $N(k,i) \subset N$ with $|N(k,i)| > |N \backslash N(k,i)|$ such that $j \in N(k) \Rightarrow v_{kj} > v_{ij}$.)*

3. *$\frac{1}{2}(v_{kk} - v_{ki}) \leq \delta$ for any $k \neq i$ such that $v_{kj} > v_{ij}$ for a weak majority (such k can only exist with an even number of players. As above let $N(k,i) = \{j \in N | v_{kj} > v_{ij}\}$. The condition can then be written as $|N(k,i)| = |N \backslash N(k,i)|$, which can obviously only hold if $|N|$ is even.*

Proof. (*Sufficiency).* For any $k \neq i$ let $\alpha^*(\{i,k\})$ be a collection of the n voting rules where

$$\alpha_j^*(\{i,k\}) = \begin{cases} i & \text{if } v_{ij} \geq v_{kj} \\ k & \text{if } v_{ij} < v_{kj} \end{cases}$$

and let $e_i^* = 1$ and $e_j^* = 0$ for all $j \neq i$. For all other candidate sets C we let $\alpha^*(C)$ be an arbitrary voting equilibrium. Now, the voting behavior is clearly sequentially rational. Also, any candidate k that would win given α^* prefers to stay out since $v_{kk} - v_{ki} \leq \delta$ ($\frac{1}{2}(v_{kk} - v_{ki}) \leq \delta$). Finally, any candidate that would want to enter if they could change the outcome will lose under α^* and the first condition ensures that i wants to enter give the continuation strategies.

(*Necessity)* If the first condition fails, citizen i wouldn't want to run against the default option. If the second condition fails for some k, then all voting equilibria with candidate set $\{i,k\}$ is such that k is winning, so k must enter if i is running unopposed (same logic with last condition). ■

For the case where people only care about policy, candidates are equally competent and costs of running are small this result has interesting consequences. A *Condorcet winner* is in general a policy $x \in A$ such that $\left\{i \in N | u^i(x) \geq u^i(x')\right\} \geq \left\{i \in N | u^i(x) < u^i(x')\right\}$ for all $x' \in A$.: a policy that a majority prefers in pairwise majority voting against any other alterative.

Corollary 38 *If $A^i = A$ for all i and $v_i(x,j) = V^j(x)$ for all i,j and $x \in A$. Then*

1. *if δ is sufficiently small and there is an equilibrium where i runs unopposed, x_i^* must be a Condorcet winner relative the set $\{x_1^*, ..., x_n^*\}$*

2. *if x_i^* is a Condorcet winner relative the set $\{x_1^*, ..., x_n^*\}$, then an equilibrium where i runs unopposed exists if δ are sufficiently small.*

To see this one just notes that with the specializations of hypothesizes made above, the conditions approach the conditions for x_i^* to be a Condorcet winner.

Example 39 *(Single-dimensional policy space). Suppose the policy space is* $[0,1]$ *and that each citizen has Euclidean preferences and cares only about policy outcomes,* $v_i(x,j) = v_i(x)$. *Now Euclidean preferences (which are used a lot in political economy/theoretical political science literature) means that each citizen has a distinct ideal point* w_i *and that* $v_i(x) = -|w_i - x|$ *for all* $x \in [0,1]$. *We assume* n *is odd and that the default alternative is, say either* 1 *or* 0. *Now, a Condorcet winner exists, namely the favorite point of the median voter* w_m, *so the corollary above says that if* δ *is sufficiently small, then there exist an equilibrium where the median voter runs unopposed. Thus, the prediction in terms of policy is the same as in the Downsian model. However, under special circumstances, there are other equilibria with more than one candidate, where the policy prediction is different from the Downsian model.*

Besides characterizing equilibria with more than one candidate Besley & Coate studies:

- The possibilities for spoiler candidates: an example is constructed where candidates enter even if their probability of winning is zero. Idea: each spoiler candidate stays in the race just to prevent some other spoiler candidate from winning.

- Efficiency. Quite naturally this model predicts that policies are efficient if all citizens are equally competent. In the case with heterogenous policy making abilities there is a possibility for "political failure", which means that there are *feasible outcomes in the game* that Pareto dominates the equilibrium.

0.17 Comparing Political Institutions

Another research agenda in public choice asks how different political institutions or constitutions influence public decision making, and in particular economic policies. Examples of this sort of research include Persson, Roland, and Tabellini (QJE 1997) and Diermeier and Myerson (AER 1999). An eloquent argument for this type of research is provided in Persson (Econometrica 2002). We'll consider two examples of this sort of research, one theoretical and one empirical

0.17.1 Lizzeri and Persico (AER 2001)

Lizzeri and Persico ask how the rules of elections (winner-take-all, or proportional systems) will affect the provision of public goods when politicians care about the spoils of office. They present a model where candidates face a tradeoff between funneling money to pork-barrel projects, the benefits of which may be targeted to particular voters, and spending on public goods that may benefit all citizens but cannot be targeted. They then ask how alternative political systems affect the provision of public goods and the distribution of resources among a citizenry made up of homogeneous voters.

The Model

There are two candidates, 1 and 2, and a continuum of voters $V = [0,1]$. There are two goods, money and the public good. The public good can only be provided by using all the money in the economy. It is assumed that providing the public good is efficient. Each voter is endowed with one unit of money. The public good yields utility G to each voter. Voters have no taste for either candidate and have linear utility over goods.

Timing: Candidates make binding promises to *each* voter. A candidate can either offer to provide the public good, or he can offer different taxes and transfers to different voters. Voters then vote for the candidate that promises them the highest utility.

Candidate Objectives and Institutional Arrangements: Candidates are motivated to run for office by prospect of spoils of office. In a proportional system the spoils are in proportion to the candidates' shares of the vote. Under this system, candidates therefore seek to maximize their share of the vote. In a winner-take-all system the spoils go to the winner. So if he wins, the candidate get a payoff of one, if he loses his payoff is zero, and if there's a tie his payoff is one half.

Strategies: A candidate's pure strategy specifies whether he offers the public good or pure transfers, and if transfers then the amount for each voters. Formally a strategy is a mapping $\Phi : V \to [-1, \infty)$ such that either (1) $\Phi(v) = G - 1$ for all $v \in V$ or (2) $\int_V \Phi(v)\, dv = 0$ (balanced budget). Note that the utility promised to voter v given the strategy $\Phi(v)$, is $\Phi(v) + 1$. Lizzeri and Persico consider the case where the offer of transfers by candidate i are realizations of the same random variable with CDF $F_i : \Re \to [0,1]$.

With a continuum of voters F_i is the empirical distribution of transfer offers, and $F_i(x)$ is the fraction of voters receiving promises less than x from candidate i. By manipulating F_i, candidate i can target transfers to different parts of the citizenry.

Let $S(F_i, F_h)$ denote the share of the vote of candidate h if he promises to transfer according to F_h and candidate i promises F_i. Thus

$$S(F_i, F_h) = \int_{-1}^{\infty} F_i(x)\, dF_h(x)$$

i.e., the probability that any random voter receives and offer from h that is higher than the offer from i. This is a constant sum game no matter what the assumption on candidate's preferences so, in equilibrium, each candidate must get 50% of the vote in expectation.

Equilibrium

Lizzeri and Persico show that for $G > 2$, the unique equilibrium has both candidates promising to provide the public good, regardless of the assumption on their objectives. Similarly, for $G < 1$, the unique equilibrium outcome has both candidates offering the same expected (zero) transfer to all voters; again this is regardless of the political system. The interesting cases are when $G \in (1, 2)$

Theorem 40 ***(winner-take-all)*** *Suppose $G \in (1,2)$. Under the winner-take-all system the unique equilibrium has both candidates offering the public good with probability $\frac{1}{2}$. When offering money, the transfers satisfy*

$$F^*(x) = \begin{cases} 0 & \text{for } x \leq -1 \\ \frac{1}{2}\left(\frac{x+1}{2-G}\right) & \text{for } x \in [-1, 1-G] \\ \frac{1}{2} & \text{for } x \in [1-G, G-1] \\ \frac{1}{2}\left(1 + \frac{x+1-G}{2-G}\right) & \text{for } x \in [G-1, 1] \\ 1 & \text{for } 1 \leq x \end{cases}$$

Theorem 41 ***(proportional system)*** *Suppose $G \in (1,2)$. Under the proportional system the unique equilibrium has both candidates offering the public good with probability $G - 1$ When offering money, the transfers are made according to the same distribution as above.*

What do these results mean? First, let's look at the distribution of money across voters when transfers are made. As the value of the public good G increases, the distribution becomes more concentrated in the extremes. Second, look at the probability of providing the public good. In the winner-take-all system, the probability is independent of the value of the public good. In the proportional system, the probability is increasing in G. The proportional system provides the good more often iff $G > \frac{3}{2}$.Third, consider efficiency. Lizzeri and Persico point out it is important to distinguish between *ex ante* and *ex post* Pareto efficiency. *Ex ante* ranks allocations taking the view of a voter considering the expected utility of the outcome of the election before receiving any promises. *Ex post* efficiency ranks allocations from the perspective of the voter who has received his consumption.

Proposition 42 *Under both systems the equilibrium outcome is ex post efficient, but for* $G \in (1,2)$ *the outcome is ex ante inefficient. With risk averse or risk neutral voters, when* $G > (<) \frac{3}{2}$ *the proportional (winner take all) system yields higher ex-ante utility for voters.*

0.17.2 Diermeier, Eraslan & Merlo (Econometrica 2003)

This paper is a structural empirical investigation of the effects of institutional features of parliamentary democracies on the formation and dissolution of coalition governments. Parliamentary democracies differ with respect to the rules prescribing how their governments form and terminate. These differences include whether the government actually needs a vote by parliament in order to assume office ("investiture vote"), whether the government must maintain the active support of a majority of parliament ("positive parliamentarism"), whether the tabling of a vote of no-confidence requires an alternative to be prespecified ("constructive vote of no-confidence"), and whether elections must be held at predetermined intervals ("fixed interelection period"). The authors ask how these differences in constitutions affect the duration of government formation, the type and size of government coalitions and the relative durability of governments. Using data from nine West European countries from 1947-1999, Diermeier, Eraslan and Merlo estimate a structural model of bargaining and government coalition formation in parliamentary democracies.

The authors find that the most stable political system (the one with the shortest formation duration and longest government duration) has a positive form of parliamentarism, a constructive vote of no-confidence, a fixed interelection period, but no investiture vote. On the opposite end, the leas stable political system has a positive form of parliamentarism with the investiture vote, but no constructive vote of no-confidence and no fixed inter election period.

Model

- $N = \{1, ..., n\}$ parties

- $\pi \in \Pi$ the parties' relative shares in parliament
- $U_i(x_i, G)$ party i's preferences over the benefits from holding office $x_i \in \Re_+$ and the composition of the government coalition $G \subseteq N$.

$$U_i(x_i, G) = x_i + u_i^G$$

where

$$u_i^G = \begin{cases} \varepsilon_i^G \text{ if } i \in G \\ \eta_i^G \text{ if } i \notin G \end{cases}$$

$\varepsilon_i^G > \eta_i^G$, $\varepsilon_i^G, \eta_i^G \in \Re$. The common discount factor for parties is given by $\beta \in (0,1)$.

Analysis begins just after an election, or the resignation of an incumbent government. The next scheduled election is $\bar{T}$ periods away. The current state of the world is represented by $s \in S$. States of the world are distributed i.i.d. across periods according to the probability distribution function $F_\sigma(\cdot)$.

After the resignation of an incumbent, the head of state chooses a formateur party k to try to form a new government. The probability party i is chosen depends positively on its share of the parliament. The formateur chooses a proto-coalition that bargains over the formation of a new government which, in turn, determines the allocation of cabinet portfolios among the coalition members. The cabinet portfolios generate a surplus in every period the government remains in power.

0.17.3 Literature

Fedderson, Tim, "Rational Choice Theory and the Paradox of Not Voting", *Journal of Economic Perspectives* [0895-3309], 2004 vol:18 iss:1 pg:99 -112.

Part IV

Discrimination and Affirmative Action

0.18 Theoretical Models of Discrimination

0.18.1 Introduction

Wage inequality, and in particular, racial income inequality, is an important question in public economics. Numerous empirical studies have established that when wages are regressed on a bunch of variables that should matter for productivity (schooling, experience, union membership etc.) and dummy variables for race and sex, the coefficient for the dummies usually turn out to be significantly different from zero. While this type of analysis may under- or over- state the extend of racial discrimination, it motivates the theoretical studies on discrimination. [See Cain's (1986) Handbook of Labor Economics survey article for more details of this empirical labor literature, and Section **??** for some problems in interpreting the results in this empirical literature.]

Recently disparate treatment received by different racial and/or gender groups in housing, mortgage lending, retailing, policing, judicial system, and even organ transfers have caused tremendous amount of publicity.

It is essential to understand the extent of and the reasons for disparate treatments to know the effects of policies aimed at helping minorities.

0.19 Taste-Based Discrimination

The modern economics literature on discrimination begins with Becker (1959) and his work is probably still the best known contribution. Becker studied several simple models of discrimination, which shared the feature that the driving force behind discrimination was racist preferences (or racial animus) by some agents in the model. One simple version of the model (due to Arrow 1973) is as follows:

- Price taking firms that produce output from labor input. The production function is

$$y = f(L),$$

  where L is the total labor input;

- Two groups of workers, B and W;

- Each agent supplies labor inelastically at $\bar{L}_B$ and $\bar{L}_W$ respectively for B and W;

- Suppose that the wages for B and W are respectively w_W and w_B;
- Suppose that
- Suppose that workers' labor inputs are perfect substitutes. Then the firms' profit function when is

$$\pi\left(L_B, L_W\right) = f\left(L_B + L_W\right) - w_B L_B - w_W L_W$$

- Suppose that firm owners have identical, but possibly racist preference

$$u\left(\pi, L_B, L_W\right).$$

A utility-maximizing firm owner solves

$$\max_{\{L_B, L_W\}} u\left(\pi\left(L_B, L_W\right), L_B, L_W\right).$$

The first order conditions are

$$\begin{aligned}
\frac{\partial u\left(\pi, L_B, L_W\right)}{\partial \pi}\left[f'\left(L\right) - w_B\right] + \frac{\partial u\left(\pi, L_B, L_W\right)}{\partial L_B} &= 0 \\
\frac{\partial u\left(\pi, L_B, L_W\right)}{\partial \pi}\left[f'\left(L\right) - w_W\right] + \frac{\partial u\left(\pi, L_B, L_W\right)}{\partial L_W} &= 0
\end{aligned}$$

In equilibrium labor supply must equal labor demand, hence

$$w_W - w_B = \frac{1}{\frac{\partial u\left(\pi, \bar{L}_B, \bar{L}_W\right)}{\partial \pi}}\left(\frac{\partial u\left(\pi, \bar{L}_B, \bar{L}_W\right)}{\partial L_W} - \frac{\partial u\left(\pi, \bar{L}_B, \bar{L}_W\right)}{\partial L_B}\right).$$

Hence $w_W - w_B$ whenever

$$\frac{\partial u\left(\pi, \bar{L}_B, \bar{L}_W\right)}{\partial L_W} > \frac{\partial u\left(\pi, \bar{L}_B, \bar{L}_W\right)}{\partial L_B}$$

meaning that whites earn higher wages whenever the employers like whites more than blacks, which is not particularly surprising. However, this does not mean that taste-based discrimination is not important empirically.

0.20 Statistical Discrimination

The literature on statistical discrimination was started by Phelps (1972) and Arrow (1973). There are many variants, but all models have one thing in common: group identity (which the firms do not directly care about) is used as a proxy for productivity (which the firms do care about). The assumption that the firms do not directly care about the workers' racial (or gender) identity is the distinguishing characteristic from the afore-mentioned Becker's model of taste-based discrimination.

0.20.1 Phelps (AER, 1972)

Phelps (1972) is a model of discrimination driven by differences in information technology (to use a recent buzz word), namely, the signal that is used by the firms to infer about a worker's unobserved productivity is less informative for group B members than for group W members.

- Firms are competitive and risk neutral;
- Workers differ both in terms of ability (or productivity) a, which is not observed by the firms. For illustration, assume that in both group B and W, a is distributed according to
$$N\left(\mu_a, \sigma_a^2\right).$$
- Firms, however, observe a noisy signal, θ_i, of worker i's ability, a_i. Specifically,
$$\theta_i = \begin{cases} a_i + \varepsilon_{iB} \text{ if } i \text{ is Black} \\ a_i + \varepsilon_{iW} \text{ if } i \text{ is White.} \end{cases}$$
where $\varepsilon_{iB} \sim N\left(0, \sigma_B^2\right)$, and $\varepsilon_{iW} \sim N\left(0, \sigma_W^2\right)$. Hence θ_i is an unbiased signal of a_i, but the precision of the signal may depend on the group identity;

Since it is assumed that firms are risk neutral and competitive, each worker will be offered a wage which equals his expected productivity (ability) conditional on the signal θ_i. Using the standard results on Bayesian updating (see for example, page 167 of DeGroot 1970), we obtain
$$w\left(\theta_i\right) = \begin{cases} \frac{\sigma_B^2}{\sigma_a^2+\sigma_B^2}\mu_a + \frac{\sigma_a^2}{\sigma_a^2+\sigma_B^2}\theta_i \text{ if } i \text{ is Black} \\ \frac{\sigma_W^2}{\sigma_a^2+\sigma_W^2}\mu_a + \frac{\sigma_a^2}{\sigma_a^2+\sigma_W^2}\theta_i \text{ if } i \text{ is White.} \end{cases}$$
Hence if, $\sigma_B^2 > \sigma_W^2$, (due, possibly, to the fact that most firms are White and they are better in interpreting signals generated by White workers), then we have the following implication: [See Figure 3 for an illustration]

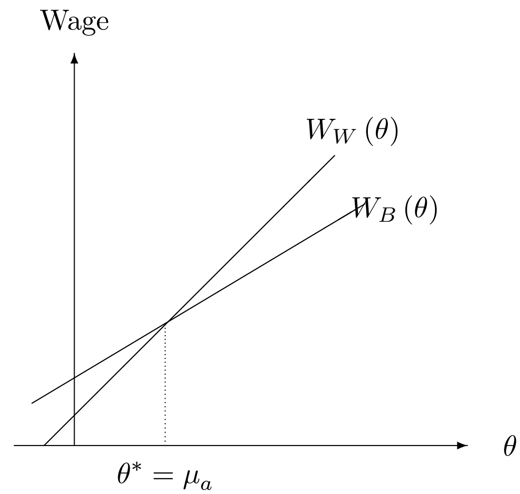

Figure 3: Wage Offers in Phelps Model

- Wages are lower for high scoring blacks than for high scoring whites (signals above the prior mean ability μ_a);
- Wages are higher for low scoring blacks than for low scoring whites (signals below μ_a);
- Average wages are equal for the two groups (unless if there are difference in the distribution of intrinsic ability a across the groups).

We note that white and black workers with the same signals are treated differently depending on group identity, this may, or may not be viewed as discrimination. We note that the fact that the feature that blacks with low score receiving higher wages than whites with the same score is not consistent with most casual notions of racial discrimination. Consequently this paper has had less impact on later work than that of Arrow, discussed in the next section.

0.20.2 Arrow (1973)

Arrow (1973) is the first to lay out the necessary ingredients of a theory of "self-fulfilling prophecy" with endogenous skill acquisition decisions to interpret discriminatory outcomes. He mentioned the following important ingredients of such a statistical discrimination theory: (1) the employers should be able to costlessly observe a worker's race; (2) the employers must incur some cost before he can determine the employee's true productivity (otherwise, there is no need for surrogate information); (3) the employers must have some preconception of the distribution of productivity within each of the two groups of workers. Arrow then proposed the following model:

- Each firm has two kinds of jobs, skilled and unskilled and they are complementary to each other. I think he means that the firm has a production function $f(L_s, L_u)$ where L_s skilled labor and L_u is the unskilled labor, and f is a constant returns to scale production function

- All workers are qualified to perform unskilled jobs; but only a proportion p_w of whites and a proportion of p_b of blacks are skilled. The firm must pay a cost r to find out whether the worker is skilled or not, and the firm knows eventually whether a worker is qualified or not.

- Arrow's model has some problem with the way he perceives the wage determination: his notion of a competitive wage in the skilled job is a contract that pays a worker from group $j = b, w$ a wage $w_j > 0$ if a worker is revealed to be qualified, and 0 otherwise; on the other hand the firm always pays a wage w_u to any worker on the unskilled job.

- Arrow claims that competition among firms will result in a zero profit condition, hence

$$\begin{aligned} r &= p_w \left[f_1 (L_s, L_u) - w_w \right] \\ r &= p_b \left[f_1 (L_s, L_u) - w_b \right]. \end{aligned}$$

  Hence

$$w_w = \frac{p_b}{p_w} w_b + \left(1 - \frac{p_b}{p_w}\right) f_1 (L_s, L_u).$$

  If for some reason $p_b < p_w$, then $w_b < w_w$.

- The wage on the simple task for both groups is

$$w_u = f_2 (L_s, L_u).$$

COMMENTS: Arrow had all the basic ideas correct but his model of wage determination is not consistent: since $w_u > 0$ and any unqualified worker who is hired on the skilled job will eventually get a wage 0, why would any unqualified worker agree to be hired on the skilled job in the first place?

However, neglecting this problem, we have:

- Blacks are paid a lower wage in the skilled task if they are believed to be qualified with a lower probability;

- The explanation of discriminatory behavior is shifted from preferences to beliefs.

Arrow then proceeded to provide an explanation for why p_w and p_b differ in equilibrium even though there are no intrinsic differences between groups. Since Arrow's model is problematic in details we will not go through the details, but he assumes that

- A worker becomes qualified as a result of a costly (unobservable) investment;

- Workers invest in skills if the gains of doing so outweighs the costs. Arrow takes the gains to be $w_j - w_u$ (which is obviously inconsistent with the labor market equilibrium condition). Suppose the distribution of skill investment cost is given by $G(\cdot)$. Then the proportion of skilled workers will be $G(w_j - w_u)$. And equilibrium requires that

$$p_j = G\left(w_j\left(\pi_j\right) - w_u\right).$$

- Arrow then notes that the system can easily have symmetric as well as asymmetric equilibria. The intuition for the asymmetric equilibria is simple: if very few workers invest in a particular group, the firms will rationally perceive this group as unsuitable for the skilled task and equilibrium wages in the skilled task will be low, which will in turn give little incentive for the workers from this group to invest.

Arrow is clearly aware of the logical inconsistency in his model:

> "I believe these results are only the barest fragment of what could be found with better and more detailed systems in which there is an interaction between reality and perception of it. One must consider still more precisely how individual employers acquire knowledge which will modify their initial estimates of distributions as differing between groups and in turn the effects of these perceptions on the market and therefore on any incentives to modify those abilities."

0.20.3 Coate and Loury (AER, 1993)

The main point of Coate and Loury (1993) is to present a theoretical analysis of affirmative actions, mainly to understand the incentive effects of affirmative action policies on agents' incentive to invest in skills. To do that, they first have to present a theoretical model of why market discrimination against the blacks occurred in the first place. They view it as a statistical discrimination. Their model is an improvement upon Arrow, yet still leaves the wage determination exogenous.

MODEL

- There are more than two competitive firms, and a continuum of workers with unit mass;

- The workers belong to one of two identifiable groups, B or W, with $\lambda \in (0,1)$ be the fraction of W in the population;

- [Linear Production Function] There are two tasks, a complex task (task one in the paper) and a simple task (task zero in the paper); the complex task can be successful performed only by qualified workers. The productivities of (or the firms' net return from) qualified and unqualified workers on the two tasks are summarized by the following table:

		Tasks	
		Complex	Simple
Workers	Qualified	$x_q > 0$	0
	Unqualified	$-x_u < 0$	0

- Workers are born unqualified, but they can become qualified through some costly ex ante investment. [The interpretation of investment can be working hard while in school, learning good manners and being disciplined etc.]. The skill investment cost c is heterogenous across workers and is distributed according to CDF $G(\cdot)$ which is assumed to be continuous and differentiable.

- Workers' skill investment decisions are unobservable by the firms. Instead, firms observe a noisy signal θ of the worker's qualification: The signal θ is drawn from $[0,1]$

according to PDF $f_q(\theta)$ if the worker is qualified, and according to $f_u(\theta)$ if he is unqualified. Assume monotone likelihood ratio property on the testing technology:

$$l(\theta) = \frac{f_q(\theta)}{f_u(\theta)}$$

is strictly increasing and continuous [for expositional simplicity only; can be weakened]. This MLRP simply says that a worker with a higher θ is more likely to be qualified than one with a lower θ.

- **[Part of CL Model that is Not Satisfactory]** A worker gets a net benefit ω if he is assigned to the complex task, and 0 if he is assigned to the simple task. [The particular wage values are normalized and hence there is no loss of generality. This exogenous wage assumption prevents them from addressing the issue of how wages will react to affirmative action policies.]

To summarize, in Coate and Loury's model, the two groups are identical in all fundamentals except the group size, which could be equated as well. The only reason $\lambda = 1/2$ is that they later would like to talk about employment quota.

TIMING OF THE GAME:

- STAGE 1: Nature chooses workers' types c;
- STAGE 2: Workers make (unobservable) skill investment decisions;
- STAGE 3: Test results $\theta \in [0, 1]$ observed by firms;
- STAGE 4: Firms decide how to assign the workers to the two tasks.

EQUILIBRIUM

We solve the equilibrium of the model using backward induction.

Consider **STAGE 4.** Suppose that a firm sees a worker with signal θ from a group where a fraction π has invested in skills. The posterior probability that such a worker is qualified is given by [via simple Bayes rule]:

$$p(\theta; \pi) = \frac{\pi f_q(\theta)}{\pi f_q(\theta) + (1 - \pi) f_u(\theta)}.$$

Hence the expected profit for the firm if it assigns such a worker to the complex task is

$$p(\theta; \pi) x_q - [1 - p(\theta; \pi)] x_u$$

while the profit is zero if it assigns the worker to the simple task. Hence the firm will assign this worker to the complex task if and only if

$$\begin{aligned} & p(\theta;\pi)\, x_q - [1 - p(\theta;\pi)]\, x_u \geq 0 \\ \Leftrightarrow \quad & \frac{x_q}{x_u} \geq \frac{1-\pi}{\pi}\frac{f_u(\theta)}{f_q(\theta)}. \end{aligned}$$

Since f_q/f_u is assumed to be monotonically increasing in θ, the above inequality holds if and only if

$$\theta \geq \tilde{\theta}(\pi)$$

where $\tilde{\theta}(\pi)$ is determined as follows:

- If the equation
$$\frac{x_q}{x_u} = \frac{1-\pi}{\pi}\frac{f_u(\theta)}{f_q(\theta)} \tag{8}$$
has a solution [which will be unique due to MLRP], $\tilde{\theta}(\pi)$ is the unique solution;

- If $x_q/x_u > (1-\pi) f_u(\theta)/\pi f_q(\theta)$ for all $\theta \in [0,1]$, then $\tilde{\theta}(\pi) = 0$;

- Otherwise, $\tilde{\theta}(\pi) = 1$.

It is clear that whenever $\tilde{\theta}(\pi)$ is interior, we have

$$\frac{d\tilde{\theta}(\pi)}{d\pi} = -l'\left(\tilde{\theta}(\pi)\right)\frac{x_u}{x_q}\frac{1}{\pi^2} < 0.$$

This is intuitive: the higher the prior probability that a worker is qualified, the more willing firms will be to give the benefit of doubt to the workers.

TO SUMMARIZE: In the task assignment stage, the firm will follow a cutoff rule $\tilde{\theta}(\pi)$: workers with signal θ higher than $\tilde{\theta}$ will be assigned to the complex task and those with signals lower than the cutoff will be assigned to the simple task. Moreover, the cutoff $\tilde{\theta}(\pi)$ is weakly decreasing in π, the fraction of skilled which is weakly decreasing in π, the fraction of skilled workers in that group.

Next we analyze a worker's optimal skill investment decision at STAGE 2, given the firms' sequentially rational behavior in STAGE 4. Suppose that in STAGE 4, the firms follow a cutoff rule at $\tilde{\theta}$. If a worker with cost c decides to invest in skills, his expected payoff will be

$$\left[1 - F_q\left(\tilde{\theta}\right)\right]\omega - c.$$

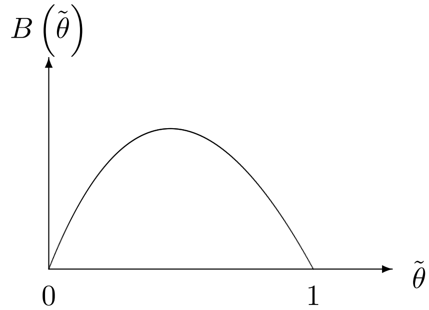

Figure 4: Benefits to Invest in Skills as a Function of the Cutoff $\tilde{\theta}$

If he does not invest in skills, his expected payoff will be

$$\left[1 - F_u\left(\tilde{\theta}\right)\right]\omega.$$

Hence a worker with cost c will invest if and only if

$$c \leq B\left(\tilde{\theta}\right) \equiv \left[F_u\left(\tilde{\theta}\right) - F_q\left(\tilde{\theta}\right)\right]\omega.$$

This implies that the fraction of workers who rationally invest in skills *given a cutoff* $\tilde{\theta}$ is

$$G\left(B\left(\tilde{\theta}\right)\right) = G\left(\left[F_u\left(\tilde{\theta}\right) - F_q\left(\tilde{\theta}\right)\right]\omega\right). \tag{9}$$

A few observations about the benefit function $B(\cdot)$ is useful. Note that

$$B'\left(\tilde{\theta}\right) = \omega\left[f_u\left(\tilde{\theta}\right) - f_q\left(\tilde{\theta}\right)\right]$$

is positive if and only if $l\left(\tilde{\theta}\right) < 1$. Hence it is a single peaked function. Moreover, $B(0) = B(1) = 0$. The function $B(\cdot)$ is depicted in Figure 4.

An *equilibrium* of the game is simply $\left(\tilde{\theta}_j^*, \pi_j^*\right), j = B, W$ such that for each j,

$$\begin{aligned} \tilde{\theta}_j^* &= \tilde{\theta}\left(\pi_j^*\right) \\ \pi_j^* &= G\left(B\left(\tilde{\theta}_j^*\right)\right), \end{aligned}$$

where $\tilde{\theta}(\cdot)$ and $G(B(\cdot))$ are defined by (8) and (9) respectively.

Equivalently, we could redefine the equilibrium of the model as $\pi_j^*, j = B, W$, that satisfy

$$\pi_j^* = G\left(B\left(\tilde{\theta}\left(\pi_j^*\right)\right)\right). \tag{10}$$

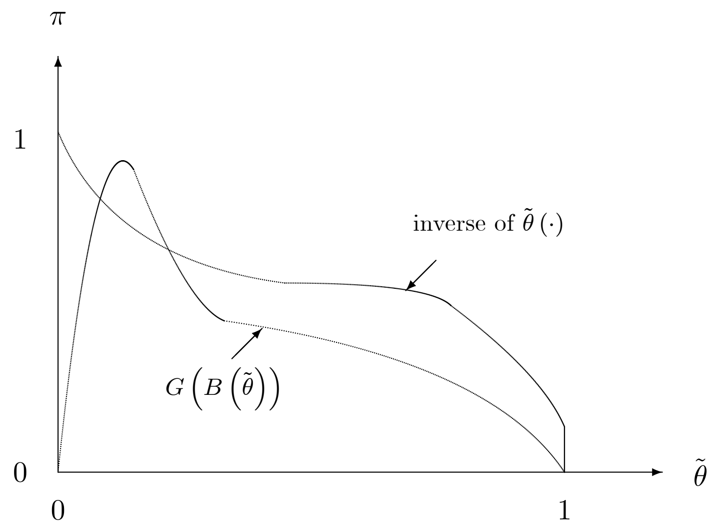

Figure 5: Multiple Equibria

From the above definition of equilibrium, we see that the only way to rationalize discriminatory outcome for the blacks and whites is when the above equation has multiple solutions. In fact, nothing so far guarantees existence of non-trivial equilibria (equilibria where $\pi_j^* \neq 0$). Coate and Loury then proceed to provide (not too special) circumstances under which the model admits discriminatory equilibria, hence it provides an explanation for discrimination with ex ante identical groups. [See Figure 5.]

Proposition 43 *If there exist $\tilde{\theta}$ such that*

$$G\left(B\left(\tilde{\theta}\right)\right) > \frac{f_u\left(\tilde{\theta}\right)/f_q\left(\tilde{\theta}\right)}{x_q/x_u + f_u\left(\tilde{\theta}\right)/f_q\left(\tilde{\theta}\right)},$$

then there exist at least two non-zero solutions to Equation (10).

Thus Coate and Loury demonstrate that statistical discrimination is a logically consistent notion in their model.

- Discrimination in this model can be viewed as a coordination failure: Removing discrimination is achieved if somehow blacks and the firms can all be coordinated on the good equilibrium. There is no conflict of interests among the whites and blacks concerning affirmative actions.

- Equilibria are Pareto ranked: The higher the fraction of investors, the higher are the profits to the firms and the higher are the expected net payoff for the workers.

Exercise 44 *Show that equilibria are Pareto ranked in Coate and Loury Model.*

0.21 Discrimination Due to Inter-Group Interactions

0.21.1 Moro and Norman J. Public Econ (2003)

Moro and Norman's (2001) paper builds upon Coate and Loury. The main changes are:

- They introduce an aggregate production function

  $$y\,(C, S)$$

  where C is the total quantity (measure) of qualified workers who are employed on the complex task, and S is the total quantity of workers on the simple task; Standard neoclassical assumptions are imposed on $y\,(\cdot,\cdot)$: concavity, strictly increasing in both arguments and constant returns to scale.

- Wages are endogenously determined. Specifically, they assume that firms compete for workers by offering wage schedules as a function of θ, the test signal observed.

Note that a linear production function in the form of $y\,(C, S) = C + S$ is special case of the model, we can ask what happens to CL's model if only the wages are endogenized. Indeed with a linear production function, endogenizing wages changes CL's results only slightly: discriminatory equilibrium may still arise as a result of two groups coordinating on different equilibria.

Once $y\,(\cdot,\cdot)$ is not linear, however, new insights emerge. In particular, the two groups now interact with each other through the production function, and the wages offered to blacks will depend not only on the firms' belief about the proportion of skilled workers among blacks, but also on the firms' belief about the proportion of skilled workers among whites. This creates externality between the two groups: in particular, they show that as the proportion of skilled workers in group W increases, the incentive to invest in skills among group B workers decrease. It is this inter-group externality that generates discriminatory outcome as a result of *specialization*: for example, the outcome that whites specializes in the complex task and the blacks in the simple task can be sustained as an equilibrium because of the inter-group externality in the skill investment incentives.

A very nice feature unique to this model is that when affirmative action is imposed, both groups are going to be affected; hence one can naturally rationalize the conflict of interests we see in the debates of affirmative actions. Recall in CL's model, because blacks are treated badly because they are simply in the wrong equilibrium, the whites would not be affected whatsoever if the blacks move to the same equilibrium as the whites. In Moro and Norman, moving to a symmetric equilibrium will lower the whites welfare relative to what they obtains in the asymmetric equilibrium.

0.21.2 Mailath, Samuelson and Shaked (AER, 2000)

Theories of statistical discrimination are based on information friction in the labor market: race-dependent hiring policies are followed because race is used as a proxy for information about the workers' skills. Paradoxically, statistical discrimination models do not yield economic discrimination: all workers are paid their marginal product and given skills, color plays no role in explaining wages. Another feature (with the exception of Moro and Norman 2001 above) is that black and white labor markets are not integrated.

MSS instead propose a model of an integrated labor market and focus on search friction instead of information friction.

THE MODEL

- A continuum of firms and workers. Firms and workers' die with Poisson rate δ and they are replaced by identical new firms and workers [cloning]. The total population of both workers and firms are 1;

- Continuous time, with interest rate r;

- All firms are identical, and workers come with a label, red or green, that has no direct payoff implication. Assume that half of the population has red label;

- Upon entering the market, each worker makes a skill acquisition decision. If one acquires skills, he can enter the skilled sector of the economy. Whether a worker is skilled or not is observed by the firms;

- Workers differ in the opportunity cost of acquiring skills, $c \geq 0$, and in the population c is distributed according to CDF G;

- Each firm can hire at most one worker. If a firm employs a skilled worker, regardless of his color, a surplus flow of x is generated; the flow surplus from hiring an unskilled worker is 0;

- **Search Friction:** Vacant firms (firms without employees) and unemployed workers match through searches. [The firm can choose to search both groups, or only one group, but not the search intensity itself]

  - Suppose that a firm searches for both colors of workers, and suppose that the proportion of the skilled workers in the population is H_W and the unemployment rate of skilled workers is ρ_W, then the process describing meetings between unemployed skilled workers and the searching firm follows a Poisson process with meeting rate $\lambda_F \rho_W H_W$ where the parameter λ_F captures the intensity of firm search.
  - If instead, the firm searches only green workers with intensity λ_F, then the meeting rate between the firm and the green skilled workers is given by $2\lambda_F \rho_G H_G$;
  - Unemployed skilled workers simultaneously search for vacant firms with intensity λ_W and the meetings generated by workers search follow a Poisson process with rate $\lambda_W \rho_F$ where ρ_F is the vacancy rate of the firms.

- When an unemployed worker and a vacant firm match, they bargain over the wage with one of them randomly drawn to propose a take-it-or-leave-it offer.

Besides the symmetric steady state equilibrium, they are interested in asymmetric steady state equilibria. Suppose that firms search only for green workers. Hence skilled green workers will earn higher wages than skilled red workers not only because their match rate will be higher, but also because once matched with a firm, the green worker can demand, or will be offered, a higher wage because his continuation utility is higher. Hence the incentive to invest in skills is higher for the green workers. The question is, will firms find it optimal to only search for green workers? The trade-off is as follows: on the one hand, once a firm is matched with a skilled red worker, the wage a skilled red demands is lower, this will create an incentive to search for red workers; on the other hand, when the red workers' incentive to invest in skills is low, the proportion of skilled red workers will be lower, so the probability of being matched with a red skilled worker if the firm searches also the red workers is low, this creates incentives against searching for red workers. They provide conditions under which asymmetric steady state equilibrium can be sustained.

What are the differences from other papers? In the asymmetric equilibrium of MSS, the skilled red and green workers are equally productive (since their skills are perfectly observable), yet they are offered different wages (which is economic discrimination).

0.21.3 Eeckhout (REStud, 2006)

THE MODEL

- There is a continuum of agents
- Agents will be randomly matched to play the following repeated prisoners dilemma game

	C	D
C	$1, 1$	$-l, 1+g$
D	$1+g, -l$	$0, 0$

$g - l \leq 1$

- Players can separate after any play of the game and rejoin the pool to be rematched
- Consequence: Playing (C, C) always cannot be supported as an equilibrium
- There must be a punishment to deviating in the stage game and rematching
- One possibility: An "Incubation Period" at the start of a match during which (D, D) is played

Example: Suppose there are two groups of equal size. Let $g = .4$ and let the strategies be the standard trigger strategies in the prisoners dilemma game, with one exception: play (D, D) in the first period. It is straightforward that it will be an equilibrium to play (C, C) in every period following the first period of a match if $\delta \geq .4$. Notice, however, that the punishment is typically greater than necessary (that is, the incentive constraint holds with strict inequality if $\delta > .4$. We would have had higher payoff if at the beginning of a match we could randomize between (C, C) and (D, D) in the first period. But this involves correlated strategies, which are not permitted. Nevertheless, Eeckhout shows that there is a mixed strategy (with players mixing independently) that does better than the trigger strategy without mixing described above. There is a problem with independent mixing, however, since this results in (C, D) and (D, C) being played, which is inefficient.

Eeckhout introduces differences between the agents ("colors") that do not affect payoffs. The play described above is "color blind" in that the play in any match does not differ depending on the colors of the agents in a matched pair. Eeckhout shows that there will typically be equilibria with higher efficiency in discriminatory equilibria, that is, equilibria in which agents play differently in a match depending on the color of the other player. Basically, the idea is the following. In a new match, players should play (C, C) in the first period if their colors match and (D, D) if they don't match. Following the first period, they play the cooperative trigger strategies. Notice now that the players' strategies are correlated in the first period via their colors, thus avoiding the "bad" off-diagonal payoffs. Eeckhout shows that typically, there will exist discriminatory equilibria of this type that are more efficient than the best color blind equilibria. For the parameters above, the simple discriminatory equilibrium in which a player plays (C, C) in the first period of a match with a player of the same color and (D, D) otherwise is an equilibrium if $\delta \geq .8$.

Chapter 1

Affirmative Action

1.1 Origins of Affirmative Action

The affirmative action policy developed during the 1960s and 1970s in two phases that embodied conflicting traditions of government regulations:

The first phase, culminating in the Civil Rights Act of 1964 and the Voting Rights Act of 1956, was shaped by the presidency and Congress and emphasized nondiscrimination under a “race-blind Constitution”. The second phase, shaped primarily by federal agencies and courts, witnessed a shift toward minority preferences during the Nixon administration. The development of two new agencies created to enforce the Civil Rights Act, the Equal Employment Opportunity Commission under Title VII and the Office of Federal Contract Compliance under Title VI, demonstrates the tensions between the two regulatory traditions and the evolution of federal policy from non-discrimination to minority preferences under the rubric of affirmative action. The results has strengthened the economic and political base of the civil rights coalition while weakening its moral claims in public opinion.

CIVIL RIGHTS ACT OF 1964: The main intensions of Civil Rights Act of 1964 were “the destruction of legal segregation in the South and a sharp acceleration in the drive for equal rights for women”. Title VII [known as the Fair Employment Commission Title or FEPC title] of the Act would create the Equal Employment Opportunity Commission (EEOC) to police job discrimination in commerce and industry with the intension to destroy the segregated political economy of the South and enforce nondiscrimination throughout the nation. Title VI of the Act [known as the Contract Compliance Title] “prohibit discrimination in programs receiving funds from federal grants, loans or contracts." It clearly bans discrimination: "No person in the United States shall, on the ground of race, color, or national origin, be excluded from participation in, be denied the benefits of, or be subject to discrimination under any program or activity receiving Federal financial assistance." Contract compliance was backed by the authority to cancel the contracts of failed performers and ban the contractors from future contract work. Specter of bureaucrats telling businesses whom to hire under Title VII was raised during the congressional debates prior to the passage of the Civil Rights Act. Majority Leader of the time Hubert Humphrey promised to eat his hat if the civil rights bill ever led to racial preferences. The Civil Rights Act of 1964 was signed by President Lyndon Johnson into law on 2 July.

AFFIRMATIVE ACTION: It turns out that Title VI of the Civil Rights Act of 1964 was the sleeper that leads to the affirmative action. In September 1965, President Johnson issued Executive Order 11246. This order intended to create new enforcement agencies to implement Title VI in the Civil Rights Act, and it repeated nondiscrimination. The Office of Contract Compliance (OFCC) established by the Labor Department to implement Executive Order 11246. It designed a model of contract compliance based on a metropolitan Philadelphia plan, which requires that building contractors submit "pre-award" hiring schedules listing the number of minorities to be hired, with the ultimate goal to make the proportion of blacks in each trade equal to their proportion of metropolitan Philadelphia's workforce (30%). This Philadelphia plan was ruled in November 1968 to violate federal contract law. But in 1971 under the Nixon administration, the Supreme Court affirmed that the minority preferences of the Philadelphia did not violate the Civil Rights Act. The EEOC who is in charge of the implementation of Title VII, followed a similar strategy, it issued guidelines to employers to use statistical proportionality in employee testing. In 1972, Congress extended the EEOC's jurisdiction to state and local governments and education institutions (which were exempt in 1964). Affirmative action is full-blown. Contrary to its original content, Johnson's Executive Order 11246 became known as the beginning of affirmative action.

The original rationale for affirmative action was to right the historical wrong of institutional racism and stressed its temporary nature. In 1978, in Regents of the University of California v. Bakke, Supreme Court Justice Harry Blackmun was apologetic about supporting a government policy of racial exclusion: "I yield to no one in my earnest hope that the time will come when an affirmative action program is unnecessary and is, in truth, only a relic of the past." He expressed the hope that it is a stage of transitional inequality and "within a decade at most, American society must and will reach a stage of maturity where acting along this line is no longer necessary".

1.2 Theoretical Studies of the Effect of Affirmative Action

1.2.1 Coate and Loury's Patronizing Equilibrium

Coate and Loury model affirmative action an employment quota, requiring that the proportion of blacks on the complex task (which pays a higher wage in their model) be equal to the proportion of blacks in the population. As before, suppose λ is the size of white population. Suppose that the proportion of skilled workers are respectively π_B and π_W among blacks and whites. Then facing the employment quota, the firms' task assignment problem becomes

$$\begin{aligned}
\max_{\{\tilde{\theta}_W,\tilde{\theta}_B\}} \quad & \lambda\left\{\pi_W\left[1-F_q\left(\tilde{\theta}_W\right)\right]x_q-(1-\pi_W)\left[1-F_u\left(\tilde{\theta}_W\right)\right]x_u\right\}+ \qquad (1.1)\\
& (1-\lambda)\left\{\pi_B\left[1-F_q\left(\tilde{\theta}_B\right)\right]x_q-(1-\pi_B)\left[1-F_u\left(\tilde{\theta}_B\right)\right]x_u\right\}\\
\text{s.t.} \quad & \overbrace{\pi_W\left[1-F_q\left(\tilde{\theta}_W\right)\right]+(1-\pi_W)\left[1-F_u\left(\tilde{\theta}_W\right)\right]}^{\text{probability a random white assigned to the complex task}}\\
= \quad & \overbrace{\pi_B\left[1-F_q\left(\tilde{\theta}_B\right)\right]+(1-\pi_B)\left[1-F_u\left(\tilde{\theta}_B\right)\right]}^{\text{probability of a random black assigned to the complex task}}
\end{aligned}$$

An equilibrium under affirmative action is a pair of beliefs (π_B^*, π_W^*) and cutoffs $\left(\tilde{\theta}_B^*, \tilde{\theta}_W^*\right)$ such that:

- $\left(\tilde{\theta}_B^*, \tilde{\theta}_W^*\right)$ solves problem (1.1) given (π_B^*, π_W^*);
- $\pi_j^* = G\left(B\left(\tilde{\theta}_j^*\right)\right)$ for $j = B, W$.

CL showed that there are circumstances under which affirmative action removes all discriminatory equilibria. But their conditions are rather difficult to interpret. I think the most important of their analysis is to show that so-called *patronizing equilibrium* may arise as a result of affirmative action. The idea is very simple: to comply with the affirmative action policy (assuming $\pi_B < \pi_W$ is unchanged by the policy for a little while), the standards

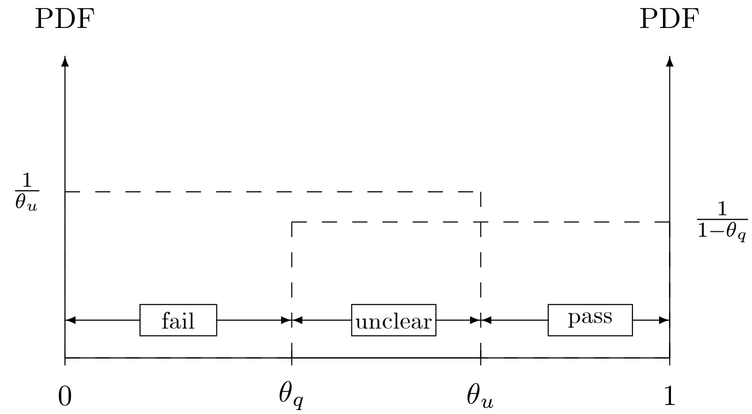

Figure 1.1: Signal Distributions in the Example of Patronizing Equilibrium

for blacks must be lowered and the standards for whites must be raised to comply with the employment quota. Thus, it is now easier for blacks to be assigned to the good job (harder for whites) irrespective of whether a particular worker invested or not. Since the incentives to invest depend on the expected wage difference if one is skilled versus if one is unskilled, whether the above change will increase or decrease blacks' incentive to invest in skills depends on the particularities of the distributions f_q and f_u.

AN EXAMPLE OF PATRONIZING EQUILIBRIUM:

- Suppose that the skill investment cost c is uniform on $[0,1]$;
- The test technology is

$$f_q(\theta) = \begin{cases} \frac{1}{1-\theta_q} \text{ if } \theta \in [\theta_q, 1] \\ 0 \text{ otherwise,} \end{cases}$$

$$f_u(\theta) = \begin{cases} \frac{1}{\theta_u} \text{ if } \theta \in [0, \theta_u] \\ 0 \text{ otherwise,} \end{cases}$$

where $\theta_u > \theta_q$. [See Figure 1.1]

If $\theta > \theta_u$, we say that it is a "pass" score; if $\theta < \theta_q$, we say that it is a "fail" score; otherwise, we say that the score is "unclear".

Discriminatory Equilibrium. We first analyze the equilibrium of this example with no affirmative action. Clearly the firm with assign workers with "pass" score to the complex task and those with "fail" score to the simple task. The decision to make is regarding those workers with "unclear" scores.

- The probability that a qualified worker gets an "unclear" score is

$$p_q = \frac{\theta_u - \theta_q}{1 - \theta_q}$$

and that for an unqualified worker is

$$p_u = \frac{\theta_u - \theta_q}{\theta_u}$$

- Suppose that the prior that a worker is qualified is π. Then the posterior probability that a worker with an unclear score is qualified is

$$\xi = \frac{\pi p_q}{\pi p_q + (1-\pi)\, p_u}.$$

Hence the employer will assign a worker with unclear scores to the complex task if and only if

$$\xi x_q - (1-\xi)\, x_u \geq 0 \Leftrightarrow \pi \geq \hat{\pi} = \frac{p_u/p_q}{x_q/x_u + p_u/p_q}.$$

We will say that a firm follows a *liberal* policy for group i if it assigns all group i workers with unclear test score to the complex task, i.e. if $\tilde{\theta} = \theta_q$; we say that a firm follows a *conservative* policy for group i if it assigns all group i workers with unclear test to the simple task, i.e. if $\tilde{\theta} = \theta_u$.

- When can a liberal policy be an equilibrium? Under a liberal policy, the benefit from skill investment is given by

$$B(\theta_q) = \omega (1 - p_u)$$

because if he is skilled, he will be assigned with probability one to the complex task and if he is unskilled, the probability is p_u. Hence the proportion of skilled workers in response to a liberal policy is

$$\pi_l = B(\theta_q) = \omega (1 - p_u).$$

Similarly, under a conservative policy, the benefit of skill investment is

$$B(\theta_u) = \omega (1 - p_q).$$

Hence the proportion of skilled workers in response to a conservative policy is

$$\pi_c = B(\theta_u) = \omega(1 - p_q).$$

Hence the liberal policy is an equilibrium if $\pi_l \geq \hat{\pi}$ and the conservative policy is an equilibrium if $\pi_c < \hat{\pi}$.

- Hence in the absence of AA, if $\pi_c < \hat{\pi} < \pi_l$, then $(\pi_B, \pi_W) = (\pi_c, \pi_l)$ is an equilibrium outcome. In this equilibrium, firms hold a negative stereotype toward blacks.

Affirmative Action: Suppose that we are in such an equilibrium and affirmative action policy in the form of employment quota is imposed. What happens? Compliance with the AA employment quota requires either more B's or less W's be assigned to the complex task. Given $(\pi_B, \pi_W) = (\pi_c, \pi_l)$, if the firm assigns a failing B to complex task, it loses x_u unit of profits; if the firm assigns an unclear W to the simple task, it loses

$$\underbrace{\frac{\pi_l p_q}{\pi_l p_q + (1-\pi_l)p_u}}_{\equiv \xi_l} x_q - \frac{(1-\pi_l)p_u}{\pi_l p_q + (1-\pi_l)p_u} x_u.$$

If $\lambda[\xi_l x_q - (1-\xi_l)x_u] > (1-\lambda)x_u$, then the firm would rather put failing B's into the complex task than putting unclear W's to the simple task to satisfy the employment quota.

Suppose that the firms still follow the following assignment policies:

- for the whites, the original liberal policy, i.e. assign all pass or unclear W workers to the complex task. Under this policy, we still have $\pi_W = \pi_l$;

- for the blacks the firms follow the following policy: assign all pass or unclear B workers to the complex task, **and** with probability $\alpha(\pi_B)$ assign failing B workers to the complex task, where $\alpha(\pi_B)$ is chosen to satisfy the employment quota requirement:

$$\begin{aligned} \pi_l + (1-\pi_l)p_u &= \pi_B + (1-\pi_B)[p_u + (1-p_u)\alpha(\pi_B)] \\ \Leftrightarrow \quad & \alpha(\pi_B) = \frac{\pi_l - \pi_B}{1-\pi_B}. \end{aligned}$$

We say that the firms are "patronizing" the worker if he assigns a failing worker to the complex task. Hence the firms are patronizing the blacks.

Now consider B's best response to the firms' assignment policy described above. Anticipating being patronized with probability α, the return from investing in skills for a B worker is given by

$$\omega - [p_u + (1 - p_u)\,\alpha] = \omega\,(1-\alpha)\,(1-p_u) = (1-\alpha)\,\pi_l$$

Hence (π_B, π_l) (assuming $\pi_l > 1/2$) can be sustained as an equilibrium under AA if and only if $\pi_B \leq \pi_l$ and π_B satisfies

$$\pi_B = [1 - \alpha\,(\pi_B)]\,\pi_l = \frac{(1-\pi_l)\,\pi_l}{1-\pi_B}.$$

This equation has two solutions: $\pi_B = \pi_l$ or $\pi_B = 1 - \pi_l$. In the first solution, color-blind equilibrium (the employer is liberal toward both groups); in the second solution, the firms continue to see B's as less productive and patronize the B's to fulfil the AA mandates.

1.2.2 Moro and Norman (2001)

Moro and Norman introduced affirmative action (in the form of employment quota as in CL) into their specialization-driven model of statistical discrimination. Because of the inherent interaction between these groups, affirmative action has redistributive consequences (which differs from the effect of affirmative action in CL, in which redistribution between the groups is absent: either blacks are made better off, or the whites are made worse off, but not at the same time.) Because in their model wages are endogenous, they can also study the effect of affirmative action on wages.

In an admittedly styled model, they demonstrate the possibility that affirmative action may increase the inequality between groups. The rough intuition is as follows: the partial equilibrium effect of affirmative action (assuming that affirmative action does not affect the proportion of skilled workers in the population) typically is to reduce the wage in the unskilled job for the discriminated group and increase the wage in the unskilled job for the other group. Changes in investment behavior tend to mitigate the partial equilibrium effects, but nothing guarantees that the responses in terms of changes in human capital investments are large enough to reverse the initial effect.

The message from Moro and Norman is similar to Coate and Loury: we need to worry about possible perverse effects of incentives of affirmative actions, as well as possible unintended consequences to the intended beneficiaries.

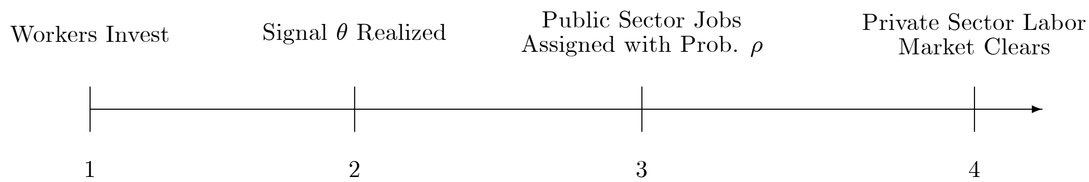

Figure 1.2: Timing of The Events.

1.2.3 Fang and Norman (2001)

We are motivated by the deteriorating (i.e. increasing) Chinese/Malay income inequality in Malaysia despite the fact that wide-ranging preferential policies are granted to the ethic Malays under the New Economic Policy from 1970. Why the preferentially treated Malays did not catch up the Chinese, but instead falls further behind? We make the following assumptions:

- There are two sectors, a private sector and a government sector;
- The private sector is competitive. The technology in the private sector is that a skilled worker can produce β units of output and an unskilled worker can produce 0;
- The public sector offers a fixed wage g to any worker who is hired, but there is rationing of public sector jobs: if applying, the probability of getting hired is $\rho \in [0, 1]$, where ρ is treated as exogenous in our analysis. Workers who apply for but are unsuccessful in obtaining public sector employment can return to and obtain a job in the private sector without waiting.
- A continuum of workers with unit mass in the economy and they have heterogeneous skill investment costs c, distributed in the population according to a continuous cumulative distribution $J(\cdot)$ with support $[\underline{c}, \overline{c}]$.

TIMING:

1. In the first stage, each worker $c \in [\underline{c}, \overline{c}]$ decides whether to invest in the skills. This binary decision is denoted by $s \in \{0, 1\}$ where $s = 0$ stands for no skill investment

and $s = 1$ for skill acquisition. If a worker chooses $s = 1$. Skill acquisitions are *not* perfectly observed by the firms.

2. In the second stage, the worker and the firms observe a noisy signal $\theta \in \{h, l\} \equiv \Theta$ about the worker's skill acquisition decision. We assume:

$$\Pr[\theta = h | s = 1] = \Pr[\theta = l | s = 0] = p > 1/2.$$

3. In the third stage, after observing the noisy signal θ, the worker decides whether to apply for the public sector job. If applying, she is accepted for employment in the public sector with probability ρ.

4. If she did not get employed in the public sector, she will, in the fourth stage, return to the private sector, where firms compete for her services by posting wage offers $w_i : \Theta \to \mathbb{R}_+$. After observing the wage offers, she decides which firm to work for, clearing the private sector labor market.

To make things interesting, we let the wage in the public sector g to be larger than any wage in the private sector, that is, the public sector jobs are good paying jobs. We think this is probably a reason for the government to reserve the public sector jobs to local majorities. We interpret the affirmative action (in the case of Malaysia, government-mandated discrimination) as follows: the government set $\rho = 0$ for the Chinese and $\rho > 0$ for the Malays. We investigate the general equilibrium effect of ρ, a policy imposed on the hiring in the public sector, on the incentives to invest in skills that are useful in the private sector, and in particular, we ask whether it is possible that the Malays may actually be made worse off, on average, by the preferential policy.

We show that giving a group preferential access to high paying public sector jobs dampens the incentives for skill investment valuable in the private sector. If the informational free riding problem in the private labor is sufficiently severe, it is possible that the adverse indirect effect due to the exacerbated informational free riding may dominate the favorable direct effects. The punch line is that affirmative action policy in the form of giving preferential treatments

1.2.4 Fryer (2010)

"Racial Inequality in the 21st Century: The Declining Significance of Discrimination" *Handbook of Labor Economics, 2011.*

1.3 Evaluation of the Effects of Affirmative Action

1.3.1 Donohue and Heckman (1991)

Donohue and Heckman (1991) provide a broad overview of the impacts of Civil Rights policy on the economic status of blacks. There are some important difficulties in assessing the effects of a government policy: first, the most intractable difficulty in assessing the impact of any law is to distinguish the effects of government intervention from those stemming from changes in the underlying attitudes that led to the passage of the legislation in the first place; second, specific to federal intervention such as the Civil Rights Act and affirmative action, the law is applied uniformly in all states, and thus one can not exploit cross-state variations in legislations. [This is in contrast to welfare reform, for example, where there are lots of cross-state variations that can be exploited.]

To partially solve the first difficult, Donohue and Heckman focus on the comparison of the economic progress of the blacks in the South and in the other regions in the U.S. They reason as follows: by noting that black relative improvement was most rapid in the South, they can counter the argument that the laws themselves were mere manifestation of preexisting social changes. Federal activity was imposed on the South and had its greatest apparent effect in the region that resisted it the most.

To partially solve the second difficulty, Donohue and Heckman focus on cross-time variation, in particular, the black economic progress in pre-1964 era and that after 1965.

Their main findings on males are as follows

- There is an upward jump in the time series of black earnings and wages beginning in the mid-1960s;
- The South was the region of the greatest black economic advance in the period 1960-1970, accounting for at least two-thirds of the increase in black economic status over the decade;

- There is evidence of substantial desegregation of firms in the South during 1965-1970 period;
- The black economic progress following the passage of Title VII coincided with a sharp drop in the outflow of blacks from the South and even led to black migration into that region between 1970 and 1980.
- During the seventy-year period from 1920-1990, there are two periods of rapid relative black progress - the period of rebound from the Great Depression brought on by the World War II and the 1965-1975 period;
- Regarding the mechanism underlying the episodic improvement of black economic status in the 1965-1975 period:
  - Black migration contributed little and relative increases in the *quantity* of black education contributed modestly to black progress after 1965;
  - The main cause of the observed black relative economic gains during this period are relative black increases in the returns to education. The reason for the increase in the returns to education could either be relative improvements in schooling quality for the blacks, or changes in the demand for black labor induced by declining racial discrimination, government civil rights policy, or tight labor markets. There is no consensus as to what caused the increase in the blacks' returns to education.

1.3.2 Moro (2001)

Moro (2001) asks the following question: after 1964, the relative economic status of blacks have improved substantially. Was the improvement of the black economic status is a result of the blacks being coordinated on a less discriminatory equilibrium due to the Civil Rights policies?

To this end, Moro estimated a structural model of labor market discrimination for different points in time after 1964 (namely, 1965, 1980, 1995). His structural model is an extension of Moro and Norman's theoretical model allowing for ability heterogeneity. With the estimates of the structural parameters, Moro numerically solves for the other equilibria that can be admitted under these parameter estimates, and ask what equilibrium corresponds to the actual outcome? Is it the least discriminatory, or most discriminatory, or anything in the middle? In particular, has there been change in the severity of discrimination in the selected equilibrium in the three points in time?

He finds that it has always been the equilibrium with the least wage inequality that have been selected whenever estimated parameters are consistent with multiple equilibria. He then concludes that self-fulfilling expectations did not exacerbate wage differentials in the U.S. and that the decline in wage inequalities experienced in the U.S. economy can not be attributed to changes in equilibrium selection, but rather changes in fundamentals of the economy is responsible.

Part V

Welfare Reform and Social Security

Chapter 2

Welfare Reform

2.1 Some Institutional Details

Welfare programs are programs that transfer cash and consumption goods to the poor and destitute. [In medieval Europe the Church took responsibility for the poor.] The milestones in the U.S. welfare programs are:

- In 1935, the federal government established Aid to Families with Dependent Children **(AFDC)** program and Supplemental Security Income (SSI) program under Social Security Act;

- In 1960s under President Johnson "War on Poverty", Medicaid was established to provide medical assistance to poor families, which is now the largest assistance program in dollar terms;

- On August 22, 1996, President Clinton signed the Personal Responsibility and Work Opportunity Reconciliation Act **(PRWORA)**, which changed the name of the assistance program from AFDC to **TANF** (Temporary Assistance for Needy Families). PRWORA and subsequent state legislations changes on welfare constitutes what we meant by welfare reforms.

The major welfare programs include the following:

1. AFDC and TANF: cash program, a *combination of federal and state programs*; states administer AFDC, set the benefit levels and have discretion over rules; federal contribution to the programs varies from 1/2 to 3/4 depending on the per capita income of

the state. There are huge state variation in the level of benefits, for example, Alaska's benefits are seven times higher than Mississippi. From 1970 to 1996, total expenditure increased from $19 billion (in 1996 dollars) to $22 billion, and total number of beneficiaries increased from 7.4 million to 12.6 million [average benefits per family declined almost by half] It is a means-tested program, that is, the benefits are reduced as income rises [this implies that the effective marginal income tax rate could be as high as 100 percent] TANF replaced AFDC from 1997. Two major differences from AFDC: first, matching grants are replaced by block grants, a fixed amount of money; second, TANF focused on moving individuals from welfare to work, the use of TANF funds has to be consistent with federal priorities of strong work requirements, time limits to receiving assistance, a reduction in welfare dependency and the encouragement of two-parent families. [A useful website is the Department of Health and Human Services, Administration for Children and Families at `http://www.acf.dhhs.gov`]

2. EITC [earned income tax credit] supplements the income of low-income families with children by an amount which depends on their income and number of children. Basically if an eligible family's income is below certain level, the government gives it a negative income tax (tax credit) that could be as high as 40%. The rate varies across states and income. After $28,495, one receives no credit for extra dollars earned. [The total expenditure on EITC grew from $1.25 billion in 1975 to $25 billion in 1996]

3. Food Stamps: This is a federal program enacted in 1975 with uniform benefit levels. The benefit level depends on income (adjusted, and housing expenditures are deductible). Food stamp benefits are limited to working-age adults without In 1996, the average monthly benefits were $73.30 per person and $175 per household.

   In 1996, food stamp benefits are limited to working-age adults without children; no more than three months in a thirty-six month period if they have not worked twenty hours a week, completed a job training program or participated in a workfare program.

4. Medicaid:

   - Enacted in 1966, provides medical assistance to the poor; medical care to disabled, and nursing home care to the aged.

- It is a federal-state matching program: The federal portion is about 50-83% (depending on the state's per capita income);
- The states have considerable discretion in determining eligibility and coverage;
- Recently covers about 36 million low-income individuals, including 18 million children;
- Historically, families receiving benefits under AFDC were eligible for medicaid. Hence eligibility for medicaid is based on a threshold test: those with income above the threshold (essentially the cutoff level of AFDC) are not eligible. This creates a welfare lock: because many employers do not provide medical benefits to low income workers, many of those on welfare find themselves in a dilemma: even if they would like to work, they lose eligibility for medicaid benefits if they accept a job. This is particularly important for those with children requiring medical attention.
- A new program, called Children's Health Insurance Program (CHIP) is created to deal with this issue.

An interesting question is why welfare reform commands so much attention recently. There are a couple of factors:

1. It was felt that welfare had created a dependency and there was wide agreement that welfare system has to be reconstructed to help those on welfare get off welfare and be productive in the labor force. Clinton "End welfare as we know it" "A hand up, not a hand out", "making work pay".

2. A misperception: The federal deficit reached record during Bush administration. There was a widespread impression that welfare was largely responsible. [In fact, in 1996 total welfare expenditures account for less than 10% of total federal expenditures, and excluding Medicaid they were only 4%)

3. After the passage of PRWORA, the federal government turned a large surplus, and the welfare rolls declined dramatically. [These two phenomena are correlated, but welfare roll decline did not contribute to the federal surplus at all.]

2.2 Welfare Reform Bill of 1996

Its official name is Personal Responsibility and Work Opportunity Reconciliation Act of 1996. [PRWORA]

Two main changes:

1. It replaced the AFDC system in which the federal government paid a fraction of the state expenditure with a block grants of TANF (Temporary Assistance to Needy Families);

2. It imposed a number of stringent requirements designed to encourage movement from welfare to workfare.

The three most important features of 1996 welfare reform bill are:

- **Block granting:** The federal government gives each state a block grant (which is a fixed amount) in exchange for the state's promise to continue spending on welfare support at least 75% of the amount that they had spent previously on welfare;

- **Time Limits:** Federal rules stipulate that federal TANF funds may not be received by a family which includes an adult who has received sixty months of TANF funds previously;

  - A state may exempt up to 20% of its caseloads from the five year limit based on hardship;
  - States can choose to support all families beyond five years if they choose;
  - Many states adopted a shorter two-year welfare eligibility;
  - The hope is that these time limits would not only push people off the welfare rolls, but also that they would discourage people from joining in the first place.

- **Workfare:** Adults had to engage in some form of work after a maximum of two years of TANF benefits, and to participate, unless the state opted out, in community services after two months (20 hours minimum in 1997-1998, and 30 hours after 2000). This work requirement does not apply to single parents of children under age 6 who can not obtain child care.

2.3 Incentive Effects of the Welfare System (Moffitt 1992)

To be added later.

2.4 Workfare versus Welfare: Besley and Coate (1992)

This is an applied theory paper which addressed the following question: How does work requirements in poverty-alleviation programs provide incentives? Two arguments are provided:

- Screening argument: Workfare will direct poverty-alleviation support toward the truly needy. [For this argument to work, we need to have a model with unobservable types of earning abilities.]

- Deterrent Argument: Work requirement will encourage poverty-reducing investments. [For this argument to work, we need to have a model in which the earnings ability can be changed by incurring some investment.]

THE MODEL

- The government is concerned to ensure that each individual gets a minimum income level z, at minimum fiscal cost;

- n individual. The income generating ability, a is either a_L or a_H with $a_L < a_H$. Thinking of a as individual's wage rates; suppose that in the population a fraction γ of the individuals is of type a_L;

- Worker's utility function over income y and work l is given by

$$y - h(l)$$

  where $h' > 0, h'' > 0$.

- A Poverty-alleviation Program (PAP) is a pair of benefits packages $\{b_i, c_i\}, i = L, H$ where

  - b_i is a cash transfer for individuals with ability i;
  - c_i is the time of the public sector work requirement.

- The public sector work requirement is assumed to be non-productive.

- The government's problem is

$$\begin{aligned} & \min n \left[\gamma b_L + (1-\gamma) b_H\right] \\ \text{s.t.} \quad & y_H \geq z \\ & y_L \geq z, \end{aligned}$$

  where y_i is the income of an individual with type a_i combining the transfer and the income from the labor market.

PRELIMINARY ANALYSIS

- An individual of type a_i chooses which benefit package to claim and the amount of labor to put in the private labor market. Denote $\hat{l}(a_i)$ as the solution to

$$h'\left(\hat{l}(a_i)\right) = a_i,$$

  the amount of labor supplied in the private labor market is

$$l(c; a_i) = \begin{cases} \hat{l}(a_i) - c \text{ if } c \leq \hat{l}(a_i) \\ 0 \text{ otherwise.} \end{cases}$$

  That is, the labor supply in the labor market is independent of b, due to the separable utility function (hence there is no income effect).

- Private sector earning is then

$$y(c, a_i) = \begin{cases} a_i\left[\hat{l}(a_i) - c\right] \text{ if } c \leq \hat{l}(a_i) \\ 0 \text{ otherwise.} \end{cases}$$

- To make the problem interesting, assume that

$$y(0, a_H) > z > y(0, a_L).$$

  That is, without government intervention, only type L group is poor.

- The indirect utility level for type i worker when accepting pair (b, c) is given by

$$v(b, c, a_i) = b + y(c, a_i) - h\left(\hat{l}(a_i)\right).$$

  Note that $v(b, c, a_i)$ is increasing in b, decreasing in c, and increasing in a_i.

BENCHMARK CASE: OBSERVABLE AND EXOGENOUS ABILITY

If ability is unobservable, then the government's problem is

$$\begin{aligned}
&\min_{\{b_L,c_L,b_H,c_H\}\in R_+^4} n\left[\gamma b_L+(1-\gamma)\,b_H\right]\\
\text{s.t.}\quad & v\left(b_H,c_H,a_H\right)\geq v\left(0,0,a_H\right)\\
& v\left(b_L,c_L,a_L\right)\geq v\left(0,0,a_L\right)\\
& b_L+y\left(c_L,a_L\right)\geq z
\end{aligned}$$

Note that we have only the individual rationality constraints, no incentive compatibility constraints are needed because we assume that abilities are observable.

Claim: The optimal solution is given by

$$\begin{aligned}
b_H &= 0, c_H=0\\
c_L &= 0, b_L=z-y\left(0,a_L\right).
\end{aligned}$$

The proof is very simple and omitted. Hence if income generating abilities are observable and exogenous, then the cost minimizing PAP is a welfare program (i.e. imposes no work requirement].

CASE II: UNOBSERVABLE AND EXOGENOUS ABILITIES, UNOBSERVABLE PRIVATE SECTOR EARNINGS

Suppose that the government can not observe the private sector earnings. In this case, the individuals can masquerade and continue to work as much as they like in the private sector. The government's problem is

$$\min n\left[\gamma b_L+(1-\gamma)\,b_H\right]$$

$$\text{s.t.}\quad v\left(b_H,c_H,a_H\right)\geq v\left(b_L,c_L,a_H\right) \tag{2.1}$$

$$v\left(b_L,c_L,a_L\right)\geq v\left(b_H,c_H,a_L\right) \tag{2.2}$$

$$b_L+y\left(c_L,a_L\right)\geq z \tag{2.3}$$

Discussions:

1. If the policy maker can not impose work requirements, i.e. if c_H and c_L are restricted to be zero, then the only IC PAP is $b_L=b_H$ because of the monotonicity of $v\left(\cdot\right)$ in its arguments.

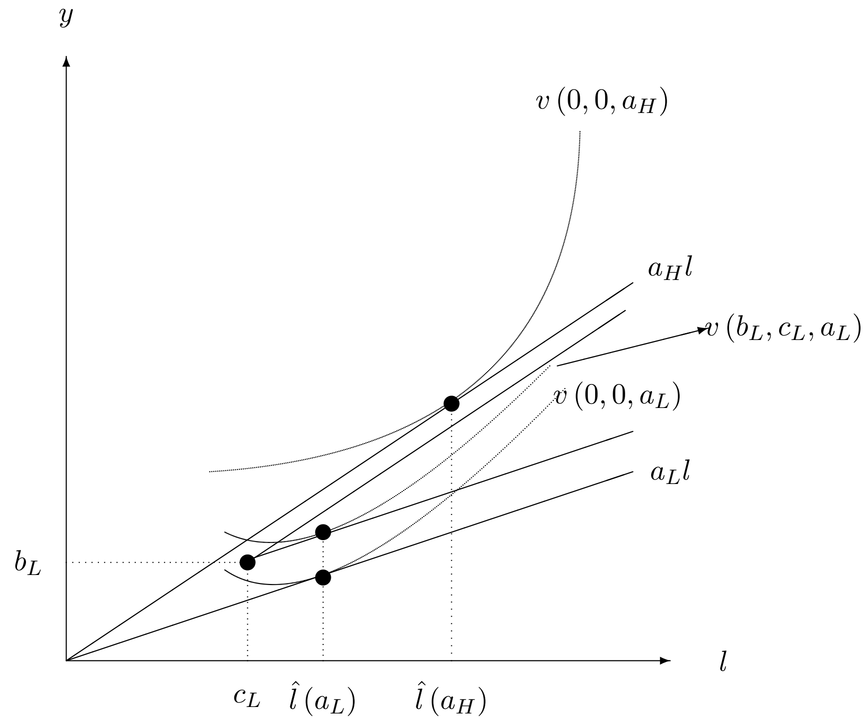

Figure 2.1: Incentive Compatible Benefit Packages

2. The work requirement can be used to screen workers of different types because high ability workers have a higher opportunity cost of satisfying the work requirement. For example, in Figure 2.1, the PAP pairs $\{(b_L, c_L), (0,0)\}$ is an incentive compatible benefits package.

CLAIM: If both ability and private market incomes are unobservable, then one of the following PAP is cost-minimizing:

- **(Welfare)** Impose no work requirement and offer both groups transfers of $z - y(0, a_L)$;
- **(Workfare)** Offer self-categorized a_H individuals no benefits and no work requirement; and offer self-categorized a_L individuals a transfer of $z - y\left(c_L^S, a_L\right)$ in exchange for work requirement of c_L^S, where c_L^S is the unique solution to

$$v(0,0,a_H) = v\left(z - y\left(c_L^S, a_H\right), c_L^S, a_H\right).$$

[See Figure 2 for the illustration]

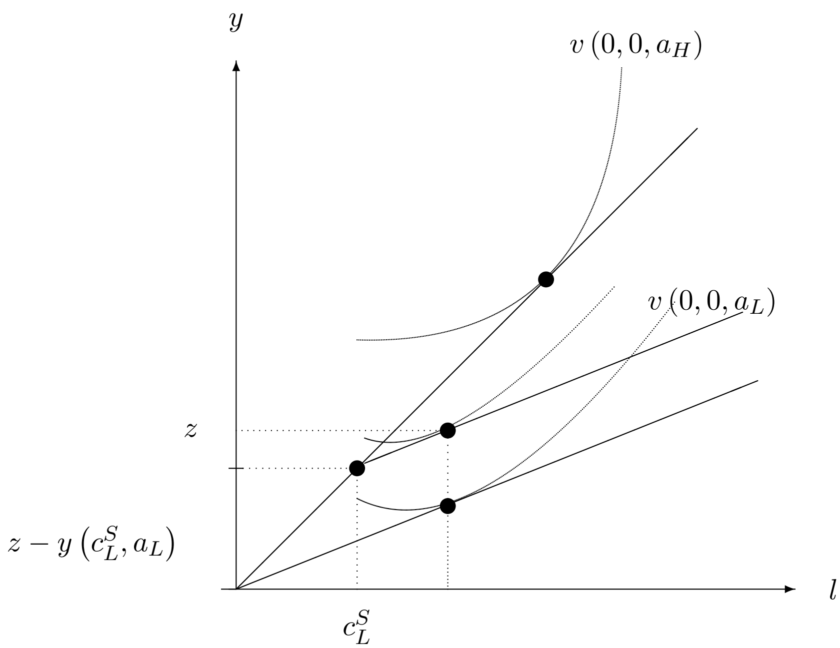

Figure 2.2: Optimal Workfare Packages

- Moreover, the workfare solution is optimal if and only if

$$(1-\gamma)\, a_H > a_{L.}$$

The authors also consider the case where the private sector earnings are observable, and come up with similar characterization of the minimum cost PAP.

THE DETERRENT ARGUMENT: OBSERVABLE AND ENDOGENOUS EARNINGS ABILITIES.

Suppose that individuals can ex ante make efforts to change their earning ability type.

- Assume that the probability an individual is of type a_H is given by $\pi(e)$, where $\pi(\cdot)$ is an increasing and strictly concave function;
- The government first chooses a PAP, then individuals make their effort choices.
- Individuals maximize

$$\pi(e)\, v(b_H, c_H, a_H) + [1-\pi(e)]\, v(b_L, c_L, a_L) - e.$$

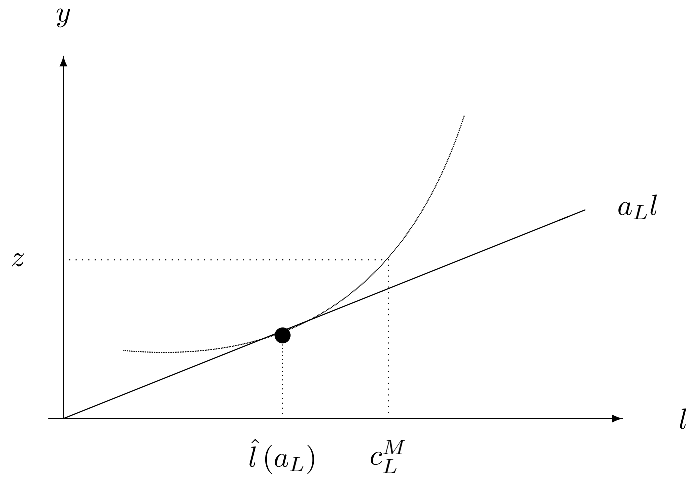

Figure 2.3: Determination of c_L^M

The first order condition is

$$\pi'(e^*)\left[v(b_H, c_H, a_H) - v(b_L, c_L, a_L)\right] = 1$$

Denote $\Delta(b_L, c_L, b_H, c_H) \equiv v(b_H, c_H, a_H) - v(b_L, c_L, a_L)$. We hence have

$$e^* = e^*(\Delta(b_L, c_L, b_H, c_H)).$$

That is, the incentives to exert effort is provided by Δ. If $\Delta = 0$, then $e^* = 0$.

- The expected cost of a PAP is now

$$n\left\{[1 - \pi(e^*(\Delta))] b_L + \pi(e^*(\Delta)) b_H\right\}.$$

- Define the maximal work requirement c_L^M by

$$v\left(z - y\left(c_L^M, a_L\right), c_L^M, a_L\right) = v(0, 0, a_L).$$

That is, c_L^M is the work requirement which, if coupled with a transfer sufficient to get the poor to the poverty line z, would make them just indifferent between status quo and participating in the program. [See Figure 3 for the determination of c_L^M]

CLAIM: Suppose that abilities are observable, but depends partly on effort. Then the cost minimizing PAP either:

1. imposes no work requirement and offers low ability individual a transfer of $z - y(0, a_L)$; or

2. imposes work requirement c_L^M on a_L and offers them a transfer of z.

2.5 A Proposal for Welfare Reform (Keane 1995)

Keane (1995) proposes a novel idea for welfare reform: *All* single mothers who work at least part time receive a $23 per week work subsidy, which is taxed away at a 7% rate as earned income increases. He showed by simulations that such *universal* work subsidy can save costs, make more people work and make women better off.

How can a universal work subsidy save money while also increasing the utility of single mothers? This can be illustrated in Figure 4.

In Figure 4, Line ABC is a typical budget constraint created by the AFDC and food stamp programs. Line EDC is what the budget constraint might look like without any programs (the usual linear budget constraint assumed in the labor supply literature). Thus the distances AE and BD are the benefit amounts at 0 and 20 hours. At 40 hours (at C), benefits go to zero. The introduction of the work subsidy for any single mother who works at least part time in the market shifts the budget constraint to ABB'C'. The distance B'B is the amount of the subsidy for part-time work. Since the subsidy is taxed away with earnings, the subsidy amount for full-time work C'C is smaller than B'B.

As shown in Figure 4, the introduction of the work subsidy will cause women, who were choosing nonwork and full benefits under the original AFDC and Food Stamps program, to shift to part-time work, and move to a higher indifference curve. The work subsidy saves money on these switchers because the combination of subsidy and welfare benefits paid to her if she works part-time, B'D, is smaller than the benefits she was receiving when she did not work, AE.

Of course, the work subsidy is costly because some women who would have worked in the market anyway now receive a subsidy for doing so. This effect causes costs to increase. The net cost implication of such a work subsidy policy depends on the proportion of women who will switch from no-work to part-time work. It turns out that there is a large fraction of single mothers on this margin. So the cost saving effect from the switchers more or less cancels out the cost increasing effect of the workers, will slight cost saving.

It is cost effective to target the subsidy to encourage part-time market work because AFDC and food stamps benefits for a typical single mother drop by roughly two-thirds if she goes from nonwork to part-time work (that is, BD is about 1/3 of AE). Therefore, of the possible savings that accrue to the government from getting welfare recipients to work, most can be achieved by getting them to work just part-time. What is the difference between this work subsidy proposal and the EITC? EITC is more expensive because these tax credit payments are proportional to earned income, which means that they are roughly twice as great for full-time as for part-time work.

2.6 Is Time-limited Welfare Compassionate? (Fang and Silverman 2001)

Proponents of the recent welfare reform often argued for the new eligibility restrictions in the name of compassion for the poor. Prominent law and opinion makers emphasized the value of welfare time limits and other restriction to *recipients*, rather than to taxpayers or to society at large. They, like Dole, claimed that while perhaps "tough," time limits and other welfare eligibility restrictions were truly more compassionate than the previous rules. By encouraging the poor to enter the workforce, the restrictions would not just be good for taxpayers, the reforms would be good for the poor themselves.

The question is: how might time limits on welfare eligibility and work requirements benefit the welfare eligible? It is a fundamental property of standard economic decision problems that adding constraints to an agent's choice set cannot make her better off. Compassionate time limits are therefore precluded by the standard framework. This truism motivates our departure from the standard setup: We allow agents to have *present-biased* preferences, and thus introduce the potential for both problems of self control and utility gains from restricting choice sets.

THE MODEL

- Discrete time, finite horizon model with periods $t = 1, ..., T$.
- In each period, an agent chooses one and only one of the following options:
    - **Welfare.** Welfare benefit is m and it represents cash assistance and the monetary value of in-kind aid such as food stamps, housing subsidies and medical insurance.

The time limit on welfare eligibility is denoted by $L > 0$.

- **Work.** We assume that if an agent works her wage depends only on the cumulative number of periods she has ever worked, denoted by τ. The wage experience profile is denoted by $w(\tau)$.

  ASSUMPTION: There exists $0 < \tau^* < T$ such that $w(\tau) < m$ if $\tau < \tau^*$, and $w(\tau) \geq m$ if $\tau \geq \tau^*$.

- **Home.** The third choice is for an agent to stay home without work or welfare. We assume that by staying home an agent can generate endowment income e, which, without loss of generality, is normalize to zero.

• We adopt a simple, and now familiar formulation of agents' potentially time-inconsistent preferences – (β, δ)-preferences (Phelps and Pollak, 1968, Laibson, 1994):

Definition 45 *(β, δ)-preferences are preferences that can be represented by:*

$$\text{For all } t, U^t(u_t, ..., u_T) \equiv \delta^t u_t + \beta \sum_{s=t+1}^{T} \delta^s u_s, \text{ where } \beta \in (0,1], \delta \in (0,1]. \tag{2.4}$$

We characterize the behavior of agents with time inconsistent agents with and without time limits. Some examples are as follows:

Example 46 ***(Lack of Commitment Outcome)*** *Let $\beta = 1/2$, $\delta = 1$. The wage profile is:*

τ	0	1	2
w	-2	-8	18.5

.

In this example, the agent will be on welfare in all three periods in the absence of time limits. If a time limit of one period is imposed, then the agent will work in all three periods.

Example 47 ***(Now or Never Outcome)*** *Let $\beta = 1/2, \delta = 1$. The wage profile is:*

τ	0	1	2
w	-2	5.6	3

.

In this example, the agent will work from date 1 in the absence of time limits. If a time limit of 1 period is applied, then the agent will be on welfare on period 1, and start working from period 2.