

Grade 8 Math

Unit 5 – Percents, Ratios and Rates Study Guide

Percent means out of 100.

Ex. 37% of our students love apples means 37 out of every 100 students love apples.

A percent can also be represented as a fraction or a decimal or a ratio.

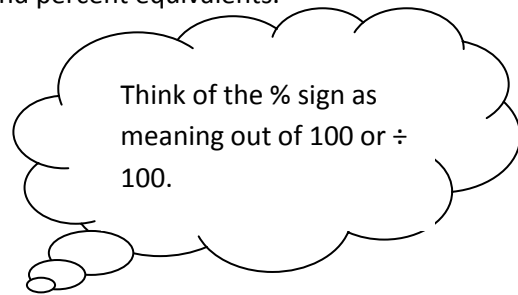
Ex.

percent	fraction	Decimal	Ratio
37%	$\frac{37}{100}$	0.37 (37 hundredths)	37:100

Number sense with decimals and percents:

The following pattern might help us recognize decimal and percent equivalents.

0.0001 = 0.01%	0.0025 = 0.25%
0.001 = 0.1%	0.025 = 2.5%
0.01 = 1%	0.25 = 25%
0.1 = 10%	2.5 = 250%



To convert a percent to a decimal - percent \div 100 = decimal

To convert decimal to percent - decimal \times 100 = percent

Number sense with fractions and percents

When a fraction has a denominator 100, the numerator is the percent.

Ex. $\frac{45}{100} = 45\%$ and $\frac{350}{100} = 350\%$

Often it is easiest to write your fraction as a decimal and then as a percent.

Ex. $\frac{3}{8} = 3 \div 8 = 0.375 = 37.5\%$

Note: any fraction can be converted to a decimal using division

Decimal = numerator \div denominator

A ratio is another way to express the part out of 100. The ratio expresses numerator : denominator

Practice A – complete the following table

Fraction	Decimal	Percent	Ratio
	0.007		
$\frac{12}{25}$			
		375%	
			3:5
		8%	
	2.5		
$\frac{1}{250}$			
			5:3

Percent Problems

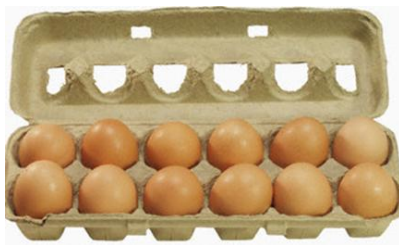
Consider the following

A full carton of eggs has 12 eggs.

So, 100% of 1 carton has 12 eggs.

Then 50% of 1 carton has 6 eggs because $50\% \times 12 \text{ eggs} = 0.5 \times 12 = 6 \text{ eggs}$

And so 150% of 1 carton is 18 eggs because $150\% \times 12 = 1.5 \times 12 = 18 \text{ eggs}$



Note: percent x whole amount = part

Practice B

Calculate the following. When necessary, round your answer to the nearest tenth.

- a. 28% of 40
- b. 3% of 20
- c. 234% of 8
- d. $3\frac{1}{2}\%$ of 50
- e. 0.2% of 15 000
- f. 8.25% of 62

Other percent problems

Sometimes we might be given a part and asked to find the whole amount.

Ex 1. 22% of what number is 40? Ans: $40 \div 22\% = 40 \div 0.22 =$

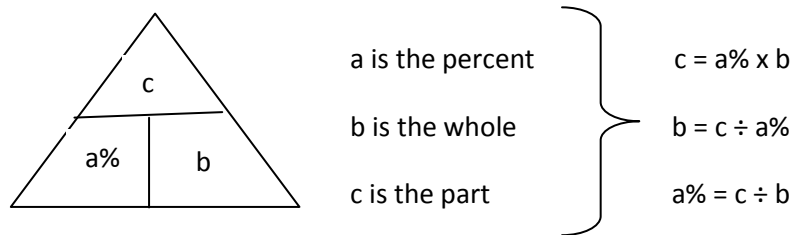
Ex 2. 20% of all students walk to school. In my class, 4 students walk. How many students are in my class? Ans: $4 \div 20\% = 4 \div 0.20 = 20$

Or we may be given the part and whole and asked for the percent.

Ex 3. 25 out of 30 is what percent? Ans: $25 \div 30 = 0.83333... = 83.3\%$

Ex 4. 45 is what percent of 15? Ans: $45 \div 15 = 3 = 300\%$

The triangle will help you to determine if you need to multiply or divide to solve the problem.



Practice C – Mixed problems – be sure to show your work!

1. What is 45% of 16?
2. 24 is what percent of 10?
3. 78% of what number is 22?
4. A salesman gets paid 35% commissions. How much commission does he make on sales of \$700?
5. Mary has borrowed 48 books from the library. This is 22% of all of the books in the library. How many books are in the library?
6. Harry bought a shirt that was on sale for \$29 after a 33% discount. What was the original price?
7. A cell phone case which regularly sells for \$39 is on sale for 40% off. How much would you pay for this case, including NL sales tax?
- *8. A box of cereal used to contain 450g. The new package has 400g? What is the percent decrease in the size of the package?

* **Percent increase/decrease** can be found using the rule

$$\text{Percent increase/decrease} = \frac{\text{change}}{\text{original}} \times 100$$

A ratio is a comparison of two or more quantities.

Ex. A hockey score of 3 to 2

- Ratios are usually written using a colon to separate the terms.

Ex. 3:2

- A ratio can have more than two terms .

Ex. A box contains 3 red, 4 green and 5 blue shirts.

The ratio of red to green shirts is 3:4

The ratio of green to red to blue shirts is 4:3:5

- There are many equivalent forms of any ratio, just as there are many equivalent forms of a fraction. Equivalent ratios are found by multiplying or dividing each term in a ratio by the same number.

Ex. $2:3 = 4:6 = 6:9 = \dots$

A statement of equal ratios is called a **proportion**.

Ex. $45:100 = 9:20$

- The lowest term ratios, highlighted above, are often the most useful.
- Note that when the second term of the ratio is 100, we have our percent.

Practice D

- Complete the equivalent ratios.
a. $3:8 = 15: \underline{\hspace{1cm}}$ b. $9:18 = \underline{\hspace{1cm}}:2$ c. $1:2:3 = 6: \underline{\hspace{1cm}}: \underline{\hspace{1cm}}$
- Write each ratio in lowest terms.
a. 4:22 b. 160:24 c. 25:2500 d. 148:42:100
- A sock drawer has 5 black socks, 12 white socks and 1 pink sock. Write the lowest term ratio of
a. Black to white socks c. pink to black socks
b. pink to all socks d. All socks to white socks
- In # 3 above, what percent of socks are white?
- Use proportions to solve the following problems.
A. The ratio of ducks to ducklings at Bowring park is 7:4. If there are 350 ducks, how many ducklings are there?
B. The ratio of blue to red pens in my desk is 4:1. If there are 25 pens all together, how many are red and how many are blue?
C. The scale on a map is 1:50 000. If the distance on a map is 12cm, what is the actual distance, in km?
D. The scale for a drawing is 10:1. If the actual length of a leaf is 3.4 cm, how long is its drawing?

A **rate** is used when you are comparing two quantities with different units.

Ex. 50 km in 2 hours.

A **unit rate** is when the second term of a rate is 1.

Ex. 25 km in 1 hour. Usually we say “25 km per hour” and we write 25km/hr

Ex. A 15 kg bag of potatoes costs \$40. What is the unit cost?

$$\text{Ans: } \frac{\$40}{15kg} = \frac{\$40 \div 15}{15 kg \div 15} = \$2.67/kg$$

Ex. One athlete can ski 35km in 2hr. Another can ski 45 km in 2.5 hr. Which athlete is faster?

$$\text{Ans: First athlete unit rate} = \frac{35km}{2 hr} = \frac{35 km \div 2}{2 hr \div 2} = 17.5 km/hr$$

$$\text{Second athlete unit rate} = \frac{45 km}{2.5 hr} = \frac{45 km \div 2.5}{2.5 hr \div 2.5} = 18 km/hr$$

The second athlete is faster!

Practice E

- Tell if the following are examples of ratios or rates.
 - Basketball score 45 to 50
 - Scale on a map 1:10000
 - Scale on a map 1cm:3km
 - Apples \$2.99/lb
- Write each as a unit rate.
 - 40 bars in 5 boxes
 - 48 slices in 4 pizzas
 - \$8 for 5 kg
- Which is the better buy?
 - 3.5L juice for \$7.49 or B. 1.5 L juice for \$2.99
- A diver descends 30m in 8 min. How long will it take her to descend 100m?

Answers to Practice Questions

Practice A

Fraction	Decimal	Percent	Ratio
$\frac{7}{1000}$	0.007	0.7%	7:1000
$\frac{12}{25}$	0.48	48%	12:25
$\frac{375}{100} = \frac{15}{4} = 3\frac{3}{4}$	3.75	375%	15:4
$\frac{3}{5}$	0.6	60%	3:5
$\frac{8}{100} = \frac{2}{25}$	0.08	8%	2:25
$\frac{250}{100} = \frac{5}{2} = 2\frac{1}{2}$	2.5	250%	5:2
$\frac{1}{250}$	0.004	0.4 %	1:250
$\frac{5}{3}$	1.66666...	166.7%	5:3

Practice B

Calculate the following. When necessary, round your answer to the nearest tenth.

- a. 28% of 40 = $0.28 \times 40 = 11.2$ b. 3% of 20 = $0.03 \times 20 = 0.6$
c. 234% of 8 = $2.34 \times 8 = 18.72$ d. $3\frac{1}{2}\%$ of 50 = $0.035 \times 50 = 1.75$
e. 0.2% of 15 000 = $0.002 \times 15\ 000 = 30$ f. 8.25% of 62 = $0.0825 \times 62 = 5.115$

Practice C

- $0.45 \times 16 = 7.2$
- $24 \div 10 = 2.4 = 240\%$
- $22 \div 78\% = 22 \div 0.78 = 28.2$
- $35\% \times 700 = 0.35 \times 700 = 245$
- $48 \div 22\% = 48 \div 0.22 = 218.2$
- 100% - 33% = 67% of the price was left after the sale.
 $29 \div 67\% = 29 \div 0.67 = 43.3$

7. Sale Price L 40% off means $100\% - 40\% = 60\%$ left. 60% of 39 = $0.60 \times 39 = 23.40$

Price with tax is $100\% + 13\% = 113\%$ of sale price. $113\% \times 23.40 = 1.13 \times 23.40 = \26.44

*8. $450 - 400 = 50$ g change. $50 \div 450 = 0.111111 = 11.1\%$

Practice D

1. a. $3 : 8 = 15 : \underline{40}$ b. $9 : 18 = \underline{1} : 2$ c. $1 : 2 : 3 = 6 : \underline{12} : \underline{18}$

2. a. 2:11 b. 20:3 c. 1:100 d. 74:21:50

3. a. 5:12 b. 1:18 c. 1 : 5 d. 18:12 = 3:2

4. $12 \div 18 = 0.66666 = 66.7\%$

5. A. $7 : 4 = 350 : x$, $7 \times 50 = 350$ so $x = 4 \times 50 = 200$ ducklings

B. 4 blue + 1 red = 5 pens all together. 4 blue:5 pens = b:25 pens, $5 \times 5 = 25$ so $b = 4 \times 5 = 20$ blue pens. 25 pens – 20 blue = 5 red pens

C. $1 : 50\,000 = 12 \text{ cm} : d$, $50\,000 \times 12 \text{ cm} = 600\,000 \text{ cm} = 60 \text{ km}$

D. $10 : 1 = d : 3.4 \text{ cm}$, $10 \times 3.4 = 34 \text{ cm}$ long drawing

Practice E

1. a. Ratio b. Ratio c. Rate d. Rate

2. a. $\frac{40 \text{ bars} \div 5}{5 \text{ boxes} \div 5} = 8 \text{ bars/box}$ b. $\frac{48 \text{ slices} \div 4}{4 \text{ pizzas} \div 4} = 12 \text{ slices/pizza}$ c. $\frac{\$8 \div 5}{5 \text{ kg} \div 5} = \$1.60/\text{kg}$

3. A. $\$7.49 \div 3.5 \text{ L} = \$2.14/\text{L}$ B. $\$2.99 \div 1.5 \text{ L} = \$1.99/\text{L}$. B is cheaper and the better buy

4. $\frac{30 \text{ m} \div 30}{8 \text{ min} \div 30} = \frac{1 \text{ m}}{0.27 \text{ min}}$ $\frac{1 \text{ m} \times 100}{0.27 \text{ min} \times 100} = \frac{100 \text{ m}}{27 \text{ min}}$