Math 1313 Section 1.5

Linear Cost, Revenue and Profit Functions:

If x is the number of units of a product manufactured or sold at a firm then,

The **cost function**, C(x), is the total cost of manufacturing x units of the product.

Fixed costs are the costs that remain regardless of the company's activity.

Examples: building fees (rent or mortgage), executive salaries

Variable costs are costs that vary with the production or sales.

Examples; wages of production staff, raw materials

The **revenue function**, R(x), is the total revenue realized from the sale of x units of the product.

The **profit function**, P(x), is the total profit realized from the manufacturing and sale of the x units of product.

Formulas: Suppose a firm has fixed cost of F dollars, production cost of c dollars per unit and selling price of s dollars per unit then

$C(x) = cx + F$

$R(x) = sx$

$P(x) = R(x) - C(x) = (s-c)x - F$

Where x is the number of units of the commodity produced and sold.

Example 3: A manufacturer has a monthly fixed cost of \$150,000 and a production cost of \$18 for each unit produced. The product sells for \$24 per unit.

a. What is the cost function? $C(x) = 18x + 150{,}000$

b. What is the revenue function? $R(x) = 24x$

c. What is the profit function?

$P(x) = R - C = (24-18)x - 150{,}000 = 6x - 150000$

d. Compute the profit (loss) corresponding to production levels of 22,000 and 28,000.

$P(22000) = 6(22000) - 150{,}000$
$= 132{,}000 - 150{,}000$
$= -18{,}000$ (Loss)

$P(28000) = 6(28000) - 150{,}000$
$= 168{,}000 - 150{,}000$
$= 18{,}000$ (Profit)/(Gain)

e. How many units must the company produce and sell if they wish to make a profit of \$40,000?

$P(x) = 40{,}000$

$6x - 150{,}000 = 40000$

$6x = 190{,}000$

$x = \dfrac{190{,}000}{6} = \boxed{\dfrac{95000}{3}}$

CASA Answer

Math 1313 Section 1.5

Popper 2
Question 1
An office building worth \$1 million when completed in 2000 is being depreciated linearly over 50 years. (Assume scrap value is \$100,000) What is the linear depreciation?

a. \$20000
b. \$18000
c. \$2000
d. None of the above

Example 4: Auto Time, a manufacturer of 24-hour variable timers, has a fixed monthly cost of \$56000 and a production cost of \$10 per unit manufactured. The timers sell for \$17 each.

a. What is the cost function?

$C(x) = 10x + 56000$

b. What is the revenue function?

$R(x) = 17x$

c. What is the profit function?

$P(x) = 7x - 56000$

d. Compute the profit (loss) corresponding to the production and sale of 4,000, 8,000 and 10,000 timers.

$P(4000) = 7(4000) - 56000 = -28000$ Loss

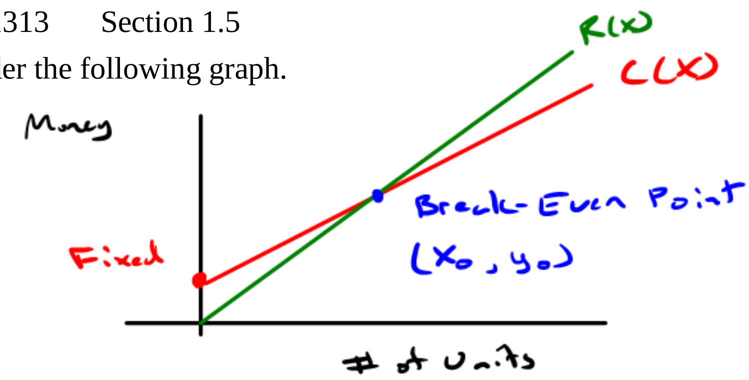
$P(8000) = 7(8000) - 56000 = 0$ Break-Even

$P(10,000) = 7(10000) - 56000 = 14000$ Profit

Break-Even Point

The **break-even level of operation-** is when the company neither makes a profit nor sustains a loss.
***Note*:** The break-even level of operation is represented by the point of intersection of two lines.
The break-even level of production means the profit is zero.

Consider the following graph.



The point (x_o , y_o) is referred to as the break-even point.

x_o = break even quantity

y_o = break even revenue

If $x < x_o$ then R(x) < C(x), therefore P(x) =R(x) –C(x) < 0 so you will have a loss.

If $x > x_o$ then R(x) > C(x), therefore P(x) = R(x) – C(x) > 0 so you will have a profit.

Example 5: Find the break-even quantity and break-even revenue if C(x) = 110x + 40,000 and R(x) = 150x.

$R(x) = C(x)$

$150x = 110x + 40{,}000$

$40x = 40000$

$x = 1000$

Break Even Quantity

$R(x) = 150x$

$R(1000) = 150(1000)$

$= 150{,}000$

Break-Even Revenue

Example 6: The XYZ Company has a fixed cost of 20,000, a production cost of $12 for each unit produced and a selling price of $20 for each unit produced.

$C(x) = 12x + 20{,}000$ $R(x) = 20x$

a. Find the break-even point for the firm.

$R(x) = C(x)$

$20x = 12x + 20000$

$8x = 20000$

$x = 2500$

B.E.Q.

$R(2500) = 20(2500)$

$= 50000$ BER

$\boxed{(2500 , 50000)}$

b. If the company produces and sells 2000 units, would they obtain a profit or loss?

$2000 <$ BEQ (2500)

$\boxed{\text{LOSS}}$

c. If the company produces and sells 3000 units, would they obtain a profit or loss?

3000 > BEQ (2500)

$\boxed{\text{Profit}}$

Popper 2
Question 3
A division of Carter Enterprises produces "Personal Income Tax" diaries. Each diary sells for \$18. The monthly fixed costs incurred by the division are \$35,000, and the variable cost of producing each diary is \$4. What is the profit function?

a. $P(x) = 22x + 3500$
b. $P(x) = 14x + 35000$
c. $P(x) = 14x - 35000$
d. None of the above

Popper 2
Question 4
Find the break-even point for the previous problem.

$R(x) = C(x)$

a. $(8333.33, 25000)$
b. $(3125, 25000)$
c. $(2500, 45000)$
d. None of the above

Example 7: Given the following profit function P(x) = 6x -12,000.

a. How many units should be produced in order to realize a profit of $9,000?

$P(x) = 9000$

$6x - 12000 = 9000$

$6x = 21{,}000$

$\boxed{x = 3500}$

b. What is the profit or loss if 1,000 units are produced?

$P(1000) = 6(1000) - 12000$

$= -6000$

Loss of $6000

Example 8: A bicycle manufacturer experiences fixed monthly costs of $124,992 and variable costs of $52 per standard model bicycle produced. The bicycles sell for $100 each. How many bicycles must he produce and sell each month to break even? What is his total revenue at the point where he breaks even?

$C(x) = 52x + 124{,}992$

$R(x) = 100x$

$R(x) = C(x)$

$100x = 52x + 124{,}992$

$48x = 124{,}992$

$x = 2{,}604$

Break Even Quantity

$R(2604) = 100(2604)$

$= 260{,}400$

Break Even Revenue

Math 1313 Section 2.1

Section 2.1: Solving Linear Programming Problems

Definitions:

An **objective function** is subject to a system of constraints to be optimized (maximized or minimized)

Constraints are a system of equalities or inequalities to which an objective function is subject to.

A **linear programming problem** consists of an objective function subject to a system of constraints

Example of what they look like:

An **objective function** is max P(x,y) = 3x + 2y or min C(x,y) = 4x + 8y

Constraints are: $5x + 3y \le 120$, $2x + 6y \le 60$

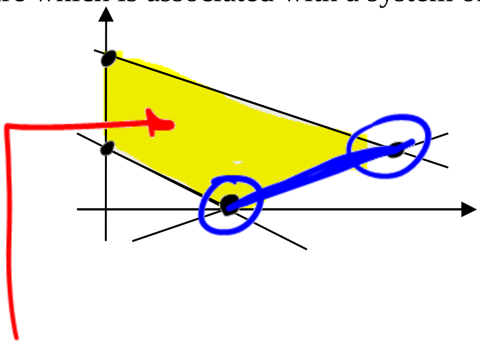
A **linear programming problem** consists of a both the objective function subject to restraints.

$Max\ \ P(x,y) = 3x + 2y$

$$\text{St:}\quad \begin{aligned} x + y &\le 4 \\ 2x + 5y &\le 80 \\ x, y &\ge 0 \end{aligned}$$

Just positive values

Consider the following figure which is associated with a system of linear inequalities:



Definitions:

The region is called a **feasible set**. Each point in the region is a candidate for the solution of the problem and is called a feasible **solution**.

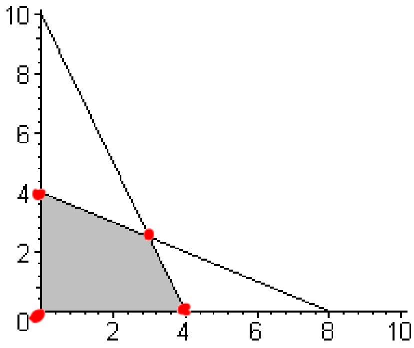
The point(s) in region that optimizes (maximizes or minimizes) the objective function is called the **optimal solution.**

Fundamental Theorem of Linear Programming

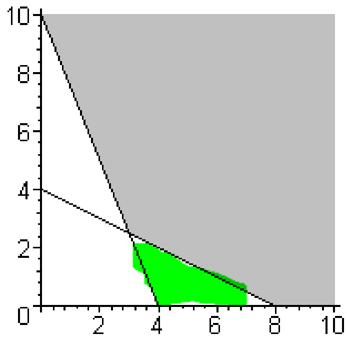
- Given that an optimal solution to a linear programming problem exists, it must occur at a vertex of the feasible set.
- If the optimal solution occurs at two adjacent vertices of the feasible set, then the linear programming problem has infinitely many solutions. Any point on the line segment joining the two vertices is also a solution.

Math 1313 Section 2.1

This theorem is referring to a solution set like the one that follows:



Maximum



Minimum

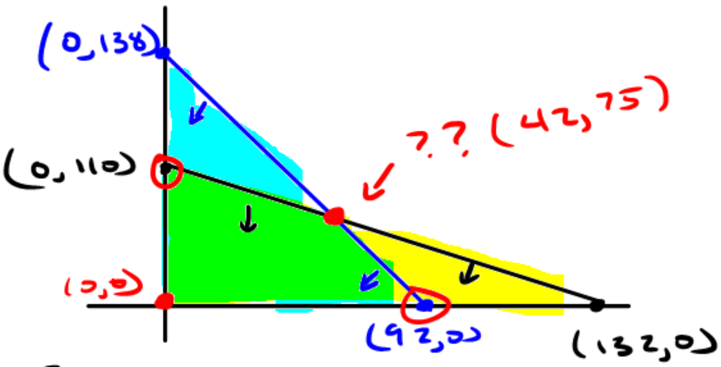
The Method of Corners

1. Graph the feasible set.
2. Find the coordinates of all corner points (vertices) of the feasible set.
3. Evaluate the objective function at each corner points.
4. Find the vertex that renders the objective function a maximum (minimum). If there is only one such vertex, then this vertex constitutes a unique solution to the problem. If the objective function is maximized (minimized) at two adjacent corner points of S, there are infinitely many optimal solutions given by the points on the line segment determined by these two vertices.

Example 1: Given the following Linear Program, Determine the vertices of the feasible set.

Max profit $P(x,y)=12x+10y$

Subject to:
$15x+10y\le 1380$ ①
$10x+12y\le 1320$ ②
$x,y\ge 0$



Line 1

x-int
$y=0$
$15x=1380$
$x=92$
$(92,0)$

y-int
$x=0$
$10y=1380$
$y=138$
$(0,138)$

$15x+10y\le 1380$
$10y\le -15x+1380$
$y\le -\frac{3}{2}x+138$

Line 2

x-int
$y=0$
$10x=1320$
$x=132$
$(132,0)$

y-int
$x=0$
$12y=1320$
$y=110$
$(0,110)$

$10x+12y\le 1320$
$12y\le -10x+1320$
$y\le -\frac{5}{6}x+110$

$-\frac{3}{2}x+138=-\frac{5}{6}x+110$
$-\frac{4}{6}x=-28$
$x=42$

$y=75$

$$\boxed{\begin{array}{c}(0,110)\\(42,75)\\(92,0)\\(0,0)\end{array}}$$

Math 1313 Section 2.1

Example 2: Given the following Linear Program, Determine the vertices of the feasible set

$$\text{Min} \quad D_3 = 3x + y$$

Subject to:
$$\begin{aligned} 10x + 2y &\geq 84 \\ 8x + 4y &\geq 120 \\ x, y &\geq 0 \end{aligned}$$