CLASS NOTES ON CORPORATE FINANCE

by

Yossi Spiegel[*]

Berglas School of Economics, Tel Aviv University

Spring 1999

Copyright © 1999 by Yossi Spiegel

1. The M&M (*AER*, 1958) propositions

Consider a firm that operates for infinitely many periods. For now, we will not worry about what does

the firm do exactly (i.e., which industry it operates in, how does it compete in the market, who manages

the firm, what kind of investment and strategic decisions the managers make, etc.) and instead we will

treat the firm as a black box that somehow generate each year some amount of money. Specifically, we

will assume that each period, the firm generate a random cash flow, $\tilde{X}$, distributed over the interval

$[X_0, X_1]$ according to some distribution function. For now, the exact properties of this distribution

function are not important for us; the only thing we need to know is that the expected cash flow of the

firm in each period is $\hat{X}$. We shall now make some assumptions about the environment in which the firm

operates:

> A1   There are no transactions costs for buying and selling securities and there are no bid-ask
>
>   spreads (i.e., the prices for buying and selling securities are the same).
>
> A2   The capital market is perfectly competitive (firms and investors are all price takers).
>
> A3   There are no bankruptcy costs.
>
> A4   There are no corporate or personal taxes.
>
> A5   All agents (firms and investors) have the same information.

Next, consider two firms, U and L, that are exactly the same (in particularly, they have the same

distribution of cash flows in each period), except for their capital structures. Firm U is all-equity.

Suppose that investors can earn a rate of return ρ on investments which bear the same risk as the risk

associated with the cash flow $\tilde{X}$. Since the capital market is perfectly competitive, and since the entire

cash flow of U accrues to its shareholders, the market value of U, which is the total value of its shares,

is equal to the sum of the discounted value of U's infinite stream of cash flows, with the discount factor being equal to ρ:

$$V_U = \frac{\hat{X}}{1+\rho} + \frac{\hat{X}}{(1+\rho)^2} + \frac{\hat{X}}{(1+\rho)^3} + \frac{\hat{X}}{(1+\rho)^4} + \ldots \tag{1}$$

That is, the market value of U must leave investors indifferent between investing in U's equity and investing in alternative securities that bear exactly the same risk. In the Appendix I show that V_U can be written as follows:

$$V_U = \frac{\hat{X}}{\rho}. \tag{2}$$

Firm L is leveraged. To simplify matters, suppose that L's debt is in the form of consol bonds with face value D, that pay the amount rD in every period, where r is the per-period interest rate on riskless bonds.[1] The assumption that L has consol bonds simplifies the exposition because we do not need to worry about maturity dates and new issues of debt. By Assumption A3, bankruptcy is costless. To simplify matter further, let's assume in addition that the firm's debt is riskless in the sense that $rD \leq X_0$. That is, the firm can meet its debt obligation even in the worst state of nature. As we shall see later on, given that bankruptcy is costless, the results would be just the same even if debt was risky, i.e., even if $rD > X_0$, but then the computations become messier. Given that debt is safe, the market value of L's consol bonds (i.e., the price that investors will be willing to pay for these bonds) must be equal to the present value of all future interest payments, given by

---

[1] Consol bonds (short for consolidated bonds) are bonds that do not have a redemption value and pay an annual interest payments indefinitely (at least in principle). These bonds were first created in England in 1750 by Henry Pelham who was then the British prime minister. Until 1914, consol bonds formed the bulk of the British national debt. Moreover consol bonds enjoyed a reputation for being a safe and highly liquid asset and they formed the largest single security traded on the stock market in England. Today, consol bonds refer to a 2.5% bond issue by the British Government from 1888 (i.e., they pay each year 2.5% on their face value) and they form only a small fraction of the British national debt.

$$B_L = \frac{rD}{1+r} + \frac{rD}{(1+r)^2} + \frac{rD}{(1+r)^3} + \frac{rD}{(1+r)^4} + \dots = \frac{rD}{r} = D. \tag{3}$$

In the Appendix I prove that the infinite sum on the right side of the equation is equal to D. That is, the market and face value of the bonds are the same.[2]

The combined annual payoff to the equityholders of L is $\tilde{X}$-rD. Since rD is deterministic, the equityholders of L get in expectation a cash flow of $\hat{X}$-rD. Using ρ_L to denote the return that investors can get by investing in alternative securities that bear exactly the same risk as the equity of firm L, the market value of L's equity is given as follows:

$$E_L = \frac{\hat{X}-rD}{1+\rho_L} + \frac{\hat{X}-rD}{(1+\rho_L)^2} + \frac{\hat{X}-rD}{(1+\rho_L)^3} + \frac{\hat{X}-rD}{(1+\rho_L)^4} + \dots = \frac{\hat{X}-rD}{\rho_L}. \tag{4}$$

Later in Proposition M&M2 we shall derive the exact relationship between ρ_L and ρ.

The market value of L is the sum of the market value of its equity, E_L, and the market value of its debt, B_L:

$$V_L = E_L + B_L. \tag{5}$$

V_L is the total amount that investors would be willing to pay for the entire equity and debt of firm L. These securities give outsiders the right to receive all of the future cash flows of the firm.

We are now ready to state and prove M&M's first proposition:

M&M Proposition 1 (M&M1): *Given Assumptions A1-A5, the market values of U and L are exactly the same, i.e., $V_U = V_L$.*

---

[2] The assumption that the interest rate on L's bonds is equal to the risk-free interest rate is only meant to simply the exposition and is not essential. For instance, if L pays an annual interest rate of R on its risk free consol bonds instead of r, the market value of its bonds is $B_L = RD/r$, and this expression should be used everywhere instead of D. However, this is exactly as if the firm issued debt with face value RD/r instead of D, so the analysis remains just the same.

Proof: The proof of M&M1 uses a no-arbitrage argument. We show that under Assumptions A1-A5, investors in U can replicate the cash flows of investors in L and vice versa. Thus, the prices of U and L must be same. In what follows I prove the proposition in two parts. In the Appendix, I provide a slightly different proof of the same proposition.

<u>Part 1:</u> Suppose that $V_U > V_L$ and consider an investor who holds a fraction α of firm U. The payoff of this investor each period is $\alpha\tilde{X}$ and the market value of his portfolio is αV_U. Now suppose the investor adopts the following investment strategy:

"Sell your holdings in firm U and buy a fraction α of firm L's equity and debt."

The investor's resulting payoff per period is

$$\alpha(\tilde{X} - rD) + \alpha rD = \alpha \tilde{X}, \tag{6}$$

which is exactly equal to the investor's per-period payoff when he held a fraction α of firm U's equity. The reason for this is that the investor holds equity and debt in equal proportions so the debt payments wash out. What about the investor's wealth? Since there are no transaction costs or bid-ask spreads (Assumption A1), the investor receives αV_U when he sells his holdings in U, and he pays $\alpha(E_L + B_L) = \alpha V_L$ when he buys a fraction α of firm L's equity and debt. Thus, the net change in the investor's wealth is

$$\alpha V_U - \alpha V_L > 0, \tag{7}$$

where the inequality follows because by assumption $V_U > V_L$. Since the investment strategy ensures the investor the same per-period payoff as he had before plus a positive capital gain, the investor should definitely adopt it. But since every investor will do the same and since the capital market is perfectly competitive (Assumption A2), the prices of U's equity will decrease and the prices L's equity and debt will increase until it will no longer be true that $V_U > V_L$.

<u>Part 2:</u> Suppose that $V_U < V_L$ and consider an investor who holds a fraction β of firm L's equity. The

market value of this portfolio is βE_L, and the per-period payoff of the investor is

$$\beta(\tilde{X} - rD). \tag{8}$$

Now suppose the investor readjusts his portfolio according to the following strategy:

"Sell your holdings in L, buy a fraction β of U's equity,

and borrow the amount βD at an annual rate r."

The investor can borrow at an interest rate r since the capital market is competitive and since there are

no bid-ask spreads. If the investor adopts the investment strategy, his per-period payoff becomes

$$\beta\tilde{X} - \beta rD = \beta(\tilde{X} - rD). \tag{9}$$

This payoff is the same as the investor's payoff under his original portfolio. What about the investor's

wealth? Upon selling his holdings in L's equity, the investor receives βE_L. Buying a fraction β of U's

equity costs βV_L, and borrowing money increases the investor's wealth by βD. Thus, the net change in

the investors wealth is

$$\beta E_L - \beta V_U + \beta D = \beta(E_L + D) - \beta V_U = \beta(V_L - V_U) > 0. \tag{10}$$

Since the investment strategy leaves the investor with a net gain he will adopt it. But since every investor

will do the same, the prices of L's securities will decrease and the prices of U's securities will increase

until it will no longer be true that $V_U < V_L$.

From parts 1 and 2 it follows that in equilibrium, $V_U = V_L$. ■

The main idea in M&M1 is that given Assumptions A1-A5, each investor can replicate the cash

flows that he would receive by buying the securities of any firm regardless of how leveraged this firm is.

For instance, an investor can replicate the cash flows that U's equity generate by buying equal fractions

of L's equity and debt. Likewise, an investor can replicate the cash flows that an equityholder of L gets by personally borrowing money (i.e., creating a homemade leverage) and using the money to buy the equity of U. Consequently, the market values of firms with different capital structures should be exactly equal, otherwise there are opportunities to make money (i.e., opportunities for an arbitrage). That is, someone can sell the expensive securities and buy the cheap ones and end up with more wealth than he initially had. Of course, if these arbitrage opportunities are exploited, the prices of the cheap securities will increase and the prices of the expensive ones will fall so that in a competitive equilibrium, the market values of two types of firms will indeed be the same. This result is essentially a particular case of the "law of one price": in competitive equilibrium, identical objects (here securities that yield the same future payoffs) must be traded at exactly the same price.

Another way to interpret M&M1 is that if the capital market is perfectly competitive and there are no taxes and no transaction costs, investors do not need firms to lever or unlever their positions since they can utilize "homemade leverage," by either borrowing money personally to lever up their positions, or lending money personally (by buying the debt of levered firms) and thereby unlever their position. As a result, investors who do not like leverage, will not be willing to pay a premium on the equity of an all-equity firm, and similarly, investors who do like leverage will not be willing to pay a premium on the equity of a leveraged firm. In short, investors will only be interested in the terminal cash flows of firms, not in the way these cash flows are split between equityholders and debtholders. Consequently, firms with equal cash flows should have equal values.

It is important to note that if firms had market power (say due to their size) and were able to borrow at a lower interest rate than individual investors, then investors who want to lever their positions would have been willing to pay a premium on the equity of leveraged firms since these firms allow them to do something they cannot do on their own (at least not for the same price). Likewise, the absence of transaction costs is important since it allows individual investors to replicate the positions that firms take

on their behalf. The importance of the assumption that there are no taxes will become clear later on.

M&M1 shows that the total value of the firm is independent of its capital. But is it also true that the market values of equity and debt (not their combined values) are independent of the firm's capital structure? To answer this question, note first that the market value of riskless consol bonds with face value D is $B_L = D$ (see the Appendix). This value is clearly increasing with D since the more money the firm promises to pay to debtholders, the more money will debtholders be willing to pay for this promise. Now, consider the equity of a firm that issued riskless consol bonds with face value D. Since the total market value of the firm is $V_L = E_L + D$ and since by M&M1, $V_L = V_U = \hat{X}/\rho$, it is clear that the market value of equity is $E_L = \hat{X}/\rho - D$. Thus, E_L is negatively affected by the amount of debt that the firm issued. The reason for this is straightforward: equityholders are residual claimants. Hence, the more money is committed to debtholders, the less money is left for equityholders, so the value of equity is lowered.

Now recall that we used ρ_L to denote the appropriate discount rate for the equity of a leveraged firm. The next proposition explores the relationship between ρ_L and ρ (the latter is the discount rate of the equity of an all-equity firm).

M&M Proposition 2 (M&M2): $\rho_L = \rho + (\rho\text{-}r)D/E_L.$

Proof: Equation (4) implies that:

$$\rho_L = \frac{\hat{X} - rD}{E_L}. \tag{11}$$

But, from M&M1 we know that $V_L = E_L + D = V_U = \hat{X}/\rho$, or $\hat{X} = \rho(E_L + D)$. Substituting in equation (11) and reorganizing terms we obtain:

$$\rho_L = \frac{\rho E_L + \rho D - rD}{E_L} = \rho + (\rho - r)\frac{D}{E_L}. \tag{12}$$

∎

To interpret M&M2, note from equation (11) that ρ_L could be thought of as the expected rate of return (ROR) on the equity of a leveraged firm (i.e., the ratio of expected return to the initial investment). Viewed in this way, M&M2 says that the expected ROR on the equity of a leveraged firm is equal to the expected ROR on the equity of an unlevered firm plus a "financial-risk" premium. This premium is equal to the debt-equity ratio multiplied by the spread between ρ and r. The financial-risk premium shows that for each dollar that investors borrow and invest in the firm's equity, they earn a net return of $(\rho-r)$. Thus their total return when they borrow D dollars is $(\rho-r)D$. Since the cost of this investment is E_L, the ROR on the investment is $(\rho-r)D/E_L$.

Another possible interpretation of ρ_L is that it reflects the firm's "cost of equity." To see why, note that when investors decide how much to pay for the firm's equity, they realize that they can earn ρ_L by investing in alternative securities that bear the same risks as the firm's equity. Hence, they discount the per-period payoffs they get from buying the firm's equity by ρ_L. Thus ρ_L can be viewed as the alternative cost of the firm's equity, i.e., the cost of foregone alternatives to investors. The higher this cost is, the lower is the amount that investors pay for the firm's equity. Therefore ρ_L can also be viewed as the rate of return that the firm is "required to pay" on the firm's equity. Of course, the firm actually pays out its cash flows which are given exogenously but since investors decide how much to pay for these cash flows when they buy the firm's equity, they effectively require the firm to pay them a certain rate of return. Since the market is perfectly competitive, this rate of return is just equal to what investors can get elsewhere when they buy securities with comparable risks.

2. The implications of M&M's propositions

In this section we discuss some of the main implications of M&M1 and M&M2. It should be noted though that the validity of these implications is limited to environments in which the assumptions underlying the M&M propositions hold. In practice of course all the assumptions are violated. Hence

the implications that will be reported in this section should be viewed as benchmark results that hold under very specific assumptions.

2.1 M&M and the weighted average cost of capital

In order to know whether to undertake a project, a firm must take into account the cost of the project and the returns it yields. However since the cost of the project is typically paid before most of the returns are generated, the firm cannot simply sum up the returns and compare them to the cost of the project. For example, a project that costs one dollar and yields a return of one million dollars might sound like a great project if the returns are received shortly after the cost had been paid, but if the return is received, say, two hundred years from now, the project may not sound so attractive anymore.[3] Thus, in order to evaluate projects, the firm needs to figure out what is the alternative cost of the capital invested in the project and take it into account in calculating the net return from the project. In other words, the firm needs to figure out what is the return it can receive on alternative investments that bear the same risks as the project in question and use this return as the discount rate to compute the net present value (NPV) of the project. This rate of return is often referred to as the cost of capital because it measures the opportunity cost of the project borne by investors who can invest their money elsewhere.

In principle each project should have its own cost of capital. In practice however, it is impossible to estimate project-by-project cost of capital, so instead firms estimate a benchmark company or even industry cost of capital, and use these benchmark in evaluating the NPV of projects. Now, if we wish to evaluate projects of a given firm, we would need to know the appropriate discount rate to be used in computing the project's' NPV. One such discount rate that can be used is the Weighted Average Cost of Capital (WACC) defined as:

---

[3] For instance, if investors can invest the one dollar in a security that pays 7.2% a year, then after 200 years their investment will be worth 1,093,800 dollars.

$$WACC = r\frac{D}{V_L} + \rho_L \frac{E_L}{V_L}. \tag{13}$$

The idea is that the firm uses money from a variety of sources and r is the per-period return it pays

debtholders for using their money and ρ_L is how it pays equityholders for using their money.

At first glance it seems that WACC depends on the capital structure of the firm so we would

arrive at different conclusion regarding whether or not to adopt projects based on their capital structures.

However, M&M2 says that the WACC is independent of the firm's capital structure. To see why,

substitute for ρ_L into equation (13). Then,

$$WACC = r\frac{D}{V_L} + \left(\rho + (\rho - r)\frac{D}{E_L}\right)\frac{E_L}{V_L} = \rho\frac{D + E_L}{V_L} = \rho, \tag{14}$$

where the last equality follows from the fact that the value of the firm is the sum of the value of debt and

the value of equity. In other words, in the world of M&M investors should always use ρ as a discount

factor to evaluate the NPV of projects, irrespective of whether they are financed with debt, equity, or both.

This means that the financial side of the firm can be completely separated from its real side.

2.2 M&M and the stock price reaction to recapitalization in an M&M environment

Consider an all-equity firm with N shares outstanding priced at P_0. The market value of this firm is V_U

$= NP_0$. Now suppose that the firm issues debt with market value D and uses the proceeds to repurchase

M outstanding shares for a price P_r per share. In this subsection we ask what is the post-recapitalization

price, P_1?

First, since the transaction takes effect immediately, $P_r = P_1$, otherwise equityholders will either

not tender their shares if $P_r < P_1$, or will tender all their shares if $P_r > P_1$ which in turn drives down P_r.

Now, since the proceeds from the debt issue must cover the cost of the repurchase, it must be that

$$D = MP_r = MP_1. \tag{15}$$

The value of the remaining N-M shares that were not repurchased is

$$E_L = (N - M)P_1. \tag{16}$$

Taking the summation of (13) and (14) we get:

$$E_L + D = (N - M)P_1 + MP_1 = NP_1. \tag{17}$$

Now we also know that by definition, the combined value of equity and debt equals the total value of a

leveraged firm, V_L which in turn, by M&M Proposition 1, equals V_U. Moreover, we also know from

above that $V_U = NP_0$. Taken together this means that

$$E_L + D = V_L = V_U = NP_0. \tag{18}$$

Equation (17) and (18) imply that $P_0 = P_1$. This means that under assumptions A1-A5, a firm cannot boost

its stock prices by repurchasing equity. In other words, the firm cannot make its equityholders better-off

by changing its capital structure. This is to be expected since equityholders can create themselves leverage

by borrowing money personally so they will not pay more for the firm's equity just because the firm did

something that they could have done themselves.

2.3 M&M and the expected EPS response to recapitalization

Now, let us examine the response of expected earnings per share (EPS) to recapitalization. For an all-

equity firm with N shares outstanding,

$$EPS_U = \frac{\hat{X}}{N}. \tag{19}$$

After recapitalization, the expected EPS become

$$EPS_L = \frac{\hat{X} - rD}{N - M}. \tag{20}$$

Since the firm issues debt to finance the M shares it repurchases, $D = MP_1$. Together with the fact that $P_1 = P_0$ (see Section 2.2), it follows that

$$EPS_L = \frac{\hat{X} - rMP_0}{N - M}. \tag{21}$$

Now we also know that

$$NP_0 = V_U = \frac{\hat{X}}{\rho} \Leftrightarrow P_0 = \frac{\hat{X}}{N\rho}. \tag{22}$$

Substituting in (21) yields,

$$EPS_L = \frac{\hat{X} - rM\dfrac{\hat{X}}{N\rho}}{N - M} = \hat{X}\left[\frac{1 - \dfrac{rM}{\rho N}}{N - M}\right]. \tag{23}$$

Substituting from equation (19) for $\hat{X}$, the equation becomes

$$EPS_L = EPS_U N\left[\frac{1 - \dfrac{rM}{\rho N}}{N - M}\right] = EPS_U\left(\frac{\rho N - rM}{\rho N - \rho M}\right) > EPS_U, \tag{24}$$

where the inequality follows because $\rho > r$.

To understand the intuition for (24), note first that in equilibrium, the price of each share must be such that

$$P_0 = \frac{EPS_U}{\rho}; \qquad P_1 = \frac{EPS_L}{\rho_L}, \tag{25}$$

i.e., the price of each share must equal the present value of the future cash flows discounted appropriately. Now, since $P_0 = P_1$, it follows that

$$\frac{EPS_U}{\rho} = \frac{EPS_L}{\rho_L}. \tag{26}$$

Finally, recalling that leverage increases the risk to equityholders, $\rho_L > \rho$ (MM Proposition 2), it must be the case that the expected EPS must increase to ensure that the equation (26) is satisfied. That is, equityholders must be compensated for the added risk that they take. This comes in the form of higher earnings per share.

2.4 M&M and the EPS-price ratios

Practitioners often compare earnings-price ratios across firms in the same industry and then use the following rule: "buy firms with higher EPS-price ratios". The logic given to this rule is that stocks with higher EPS-price ratios yield a higher ROR and hence are better investment (since firms are in the same industry, the risks are presumably the same).

In light of the previous discussion, we can say that this rule is misguided since it ignores the effect of leverage on EPS-price ratios. As we can see from equation (25), $EPS_L/P_1 = \rho_L$. We know from M&M2 that ρ_L increases with D so firms with higher leverage will also have higher EPS-price ratios. Of course, this does not reflect a higher value: it just reflects the need to compensate investors for the added risk associated with higher leverage.

3. The M&M propositions with corporate taxes (*AER 1963)*

In this section we review M&M's second article from 1963 in which they adjusted their original propositions from 1958 by adding corporate taxes into their model. To this end, we will relax Assumption A4 and assume that firms pay corporate taxes on their profits. Let t_C be the corporate tax rate. For the moment we maintain the assumption that individual investors do not pay taxes on their personal income (later, we will relax this assumption too).

In the presence of corporate taxes, the market value of an all-equity firm is given by

$$V_U = \frac{\hat{X}(1 - t_c)}{\rho}. \tag{27}$$

That is, V_U is the present value of the <u>after tax</u> expected cash flow of the firm. In the absence of debt this amount accrues to equityholders.

It turns out that many tax systems (including the 1986 US Tax Code) allow corporations to deduct interest payments from their tax bill as an expense, but do not allow them to deduct dividend payments as an expense, nor do they allow individual investors to deduct personal interest payments from their personal tax bills as an expense. Taken together, these provisions give corporate debt an advantage over equity and over personal debt. In particularly, individual investors can no longer perfectly replicate the cash flows of leveraged firms, since debt is more costly for them given that they cannot deduct interest payments as an expense. Consequently, now individual investors will agree to pay a premium on corporate borrowing because they cannot get these tax savings themselves. The following proposition quantifies this premium.

M&M1 with corporate taxes: *The market value of a leveraged firm equals the market value of an unleveraged firm plus the present value of the tax savings on interest payments, $V_L = V_U + t_C D$.*

Proof: The proof is based again on a no-arbitrage argument. Consider an all-equity firm, U, whose market value is V_U, and a leveraged firm, L, whose market value is $V_L = E_L + D$, and consider the following investment strategies:

Strategy 1:   **"Buy a fraction α of firm U's equity."**

Strategy 2:   **"Buy a fraction α of firm L's equity, E_L, and a fraction $\alpha(1 - t_C)$ of firm L's debt."**

Strategy 1 generates the following per-period payoff:

$$\alpha \tilde{X}(1 - t_C).$$ (28)

Strategy 2 generates the following per-period payoff:

$$\alpha(\tilde{X} - rD)(1 - t_C) + \alpha rD(1 - t_C) = \alpha \tilde{X}(1 - t_C).$$ (29)

The first term on the left side of the equation is the payoff associated with holding the equity of L, while the second term is the payoff associated with holding debt with face value $\alpha(1 - t_C)D$. Since the two investment strategies generate the same per-period payoffs, they must cost the same (otherwise there will be a room for arbitrage).

The cost of the first strategy is αV_U. The cost of the second strategy is $\alpha E_L + \alpha(1 - t_C)D$. Since the two investment strategies must cost the same, it follows that:

$$\alpha V_U = \alpha E_L + \alpha(1 - t_C)D = \alpha(E_L + D) - \alpha t_C D$$

$$\Leftrightarrow$$ (30)

$$V_U = (E_L + D) - t_C D.$$

Substituting from the equation $V_L = E_L + D$, into (30) and rearranging terms, we get

$$V_L = V_U + t_C D.$$ (31)

∎

The intuition for this proposition is as follows: when the firm becomes leveraged, it saves on corporate tax payments, so its value increases by the <u>present value of the tax shield on future interest payments</u>. To see this point, note that the firm's per-period tax payments are $(\tilde{X} - rD)t_C$ rather than $\tilde{X}t_C$ when the firm is all-equity. Hence, leverage leads to a per-period tax saving equal to $t_C rD$. Since these tax savings are constant sums, the appropriate discount rate is r (the risk free interest rate), so their present value is $t_C rD/r = t_C D$. Investors cannot get a similar tax break on homemade leverage since individual interest payments are not tax deductible. Consequently, they are willing to pay a premium for firms who

take leverage.

Now consider the impact of corporate tax on the cost of equity capital.

M&M Proposition 2 with corporate taxes: $\rho_L = \rho + (\rho\text{-}r)D(1\text{-}t_C)/E_L$.

Proof: Recall that ρ_L can be thought of as the expected ROR on a leveraged firm's equity. Since the market value of L's equity is E_L and since the per-period net expected return of U's equity is $(\hat{X}\text{-}rD)(1\text{-} t_C)$, it follows that

$$\rho_L = \frac{(\hat{X} - rD)(1 - t_C)}{E_L}. \tag{32}$$

Similarly, the expected ROR on U's equity is

$$\rho = \frac{\hat{X}(1 - t_C)}{V_U}. \tag{33}$$

Equation (33) implies that $\hat{X} = \rho V_U/(1 - t_C)$. Substituting in (32) yields:

$$\rho_L = \frac{\left(\dfrac{\rho V_U}{1 - t_C} - rD\right)(1 - t_C)}{E_L} = \frac{\rho V_U - rD(1 - t_C)}{E_L}. \tag{34}$$

But, from M&M1 with corporate taxes we know that $V_U = V_L - t_C D$. Using the fact that $V_L = E_L + D$, this equality becomes $V_U = E_L + D(1 - t_C)$. Substituting in (34) yields

$$\rho_L = \frac{\rho(V_L - t_C D) - rD(1 - t_C)}{E_L} = \frac{\rho(E_L + D(1 - t_C)) - rD(1 - t_C)}{E_L}$$

$$= \rho + (\rho - r)\frac{D}{E_L}(1 - t_C). \tag{35}$$

■

To interpret the equityholders' expected ROR, let's rewrite equation (35) as follows:

$$\rho_L = \frac{\rho(E_L - t_C D) + (\rho - r)D + t_C rD}{E_L}.$$ (36)

Now we can decompose the equityholders' expected ROR to three parts:

1. A return of ρ on <u>invested</u> equity funds, $E_L - t_C D$. The amount $t_C D$ is deducted from E_L because it comes from the IRS rather than from equityholders.

2. A financial risk premium, $\rho - r$, on each dollar of borrowed funds D

3. A per-period tax savings of $t_C rD$.

4. The implications of the M&M's propositions in a world with corporate taxes

In this section we discuss some of the main implications of the two propositions we proved above.

4.1 M&M with corporate taxes and the weighted average cost of capital

With corporate taxes, the Weighted Average Cost of Capital (WACC) becomes:

$$WACC = r(1 - t_C)\frac{D}{V} + \rho_L \frac{E_L}{V}.$$ (37)

Substituting for ρ_L into equation (37) yields

$$
\begin{aligned}
WACC &= r(1 - t_C)\frac{D}{V_L} + \left(\rho + (\rho - r)(1 - t_C)\frac{D}{E_L}\right)\frac{E_L}{V_L} = \rho\,\frac{D + E_L - t_C D}{V_L} \\
&= \rho\,\frac{V_L - t_C D}{V_L} = \rho\left(1 - t_C \frac{D}{V_L}\right).
\end{aligned}
$$ (38)

Hence WACC decreases with leverage: a firm with more debt has a lower hurdle for projects and should therefore be willing to adopt more projects than an all-equity firm. Thus, in an M&M world with taxes, one cannot ignore capital structure in project evaluation: investment and financial decisions are linked.

4.2 M&M with corporate taxes and the stock price reaction to recapitalization

Since in the presence of corporate taxation firm value increases with leverage, we can expect stock prices to react positively to increases in leverage. For instance, we can expect firms to be able to boost their stock prices by issuing debt and using the proceeds to repurchase equity. To see why, consider an all-equity firm, U, with N shares outstanding, each of which is priced at P_0. The market value of the firm is $V_U = NP_0$. Now, suppose that the firm issues debt with market value D and uses the proceeds to repurchase M outstanding shares for a price P_r per share.

Since the transaction takes effect immediately, P_r must equal the post-recapitalization price, P_1, otherwise equityholders will either not tender their shares if $P_r < P_1$, or will tender all shares if $P_r > P_1$ thus driving down P_r. Now, since the proceeds from the debt issue must cover the cost of the repurchase, and since the market value of the remaining equity must equal $(N-M)P_1$, the total value of the firm must be:

$$V_L = E_L + D = (N - M)P_1 + MP_1 = NP_1. \tag{39}$$

Now we also from M&M Proposition 1, that $V_L = V_U + t_C D$. Hence,

$$V_L = V_U + t_C D = NP_0 + t_C D. \tag{40}$$

Taken together, (39) and (40) imply that

$$NP_1 = NP_0 + t_C D$$
$$\Leftrightarrow \tag{41}$$
$$P_1 = P_0 + \frac{t_C D}{N}.$$

Equation (41) shows that issuing debt increases the stock prices by $t_C D/N$. The intuition for this result is the following: The entire tax saving, $t_C D$, is added to the value of the <u>old</u> equityholders. Thus, the price per-share increases by this amount divided by N. Since D is used to repurchase M shares at a price P_1, stock price increases by $t_C M P_1/N$, as reflected in the above equation.

4.3 M&M with corporate taxes and the expected EPS response to recapitalization

As we saw in Section 2.3, expected EPS increases with leverage, since equityholders need to receive more earnings to compensate them for the increased risk they bear after the firm increased its leverage. With corporate taxes, expected EPS also increases with leverage, but now this occurs for two reasons: First, just as in the case without taxation, shareholders must get a higher expected payoff per share to compensate them for the financial risk they bear when the firm issues debt. Second, the tax savings on interest payments imply higher net earnings and hence higher EPS ratios.

To see this formally, note that for an all-equity firm with N shares outstanding,

$$EPS_U = \frac{\hat{X}(1-t_C)}{N}. \tag{42}$$

After recapitalization, the expected EPS become

$$EPS_L = \frac{(\hat{X} - r\boldsymbol{D})(1-t_C)}{N-M}. \tag{43}$$

Since the firm issues debt to finance the stock repurchase, $D = MP_1$. Together with the fact that $P_1 = P_0 N/(N - t_C M)$ (see Section 3.2), it follows that

$$EPS_L = \frac{\left[\hat{X} - r\,M\,\boldsymbol{P_0}\left(\dfrac{N}{N-t_C M}\right)\right](1-t_C)}{N-M}. \tag{44}$$

Now we also know that

$$N\boldsymbol{P_0} = V_U = \frac{\hat{X}(1-t_C)}{\rho} \;\Leftrightarrow\; \boldsymbol{P_0} = \frac{\hat{X}(1-t_C)}{N\rho}. \tag{45}$$

Substituting for P_0 in (44) yields,

$$EPS_L = \frac{\left[\hat{X} - rM \dfrac{\hat{X}(1-t_C)}{N\rho}\left(\dfrac{N}{N-t_C M}\right)\right](1-t_C)}{N-M} = \hat{X}(1-t_C)\left[\dfrac{1 - \dfrac{rM(1-t_C)}{\rho(N-t_C M)}}{N-M}\right]. \tag{46}$$

Substituting from equation (42) for $\hat{X}$ and rearranging terms, the equation becomes

$$EPS_L = N\, EPS_U\left[\dfrac{1 - \dfrac{rM(1-t_C)}{\rho(N-t_C M)}}{N-M}\right] = EPS_U\left[\dfrac{\rho N - rM}{\rho(N-M)} + \dfrac{\rho t_C M}{\rho(N-t_C M)}\right]. \tag{47}$$

Note that when $t_C = 0$, equation (47) becomes identical to equation (24), and the expected EPS increases with leverage because shareholders must be compensated for the financial risk they bear. When $t_C > 0$, expected EPS increases with leverage further (the second term inside the square brackets is positive) because the tax shield provided by interest payments imply higher net earnings. The gap between EPS_L and EPS_U increases with the corporate tax rate because then the tax shield becomes larger.

5. A two-period version of M&M with risky debt

One of the less realistic assumptions in M&M is the assumption that debt is always riskless in the sense that the firm can always pay its debtholders in full. In this section, we shall relax this assumption and assume instead that debt is risky: there are states of nature in which the firm's earnings are insufficient to cover its debt payments so debtholders are only partially repaid. We will then examine the implications of this for the capital structure of the firm.

In order to examine the implications of risky debt, we will consider the following two-period version of M&M. The firm is established in period 1 by an entrepreneur who wishes to maximize its value. The firm operates in period 2 and its earnings at the end of that period are represented by a random variable $\tilde{X}$ distributed over the interval $[X_0, X_1]$ according to a cumulative distribution function $F(\tilde{X})$. Let $\hat{X}$ denote the mean earnings of the firm. That is,

$$\hat{X} = \int_{X_0}^{X_1} \tilde{X} \, dF(\tilde{X}), \qquad (48)$$

where $dF(\tilde{X}) \equiv f(\tilde{X})d\tilde{X}$. When the firm is just established, it issues risky debt with face value D. The

assumption that debt is risky means that its face value, D, exceeds X_0 so there are states of nature at which

the firms does not have enough money to repay its debt obligation in full. To simplify matters, let us

assume in addition that $D < X_1$.[4] After the earnings are realized at the end of period 2, the firm is

liquidated and its securityholders are paid according to their respective claims.

Given a debt obligation D, the firm is able to repay its debt in full if and only if $\tilde{X} \geq D$. The firm

then remains solvent: it pays D to debtholders and its equityholders are the residual claimants, receiving

a payment of $\tilde{X} - D$. On the other hand, when $\tilde{X} < D$, the firm declares bankruptcy, and debtholders

become the residual claimants, receiving a payment of $\tilde{X}$. Equityholders are protected by limited liability

so their payoff is 0. In the analysis that follows, we shall maintain assumptions A1-A5 from Section 1

and will add the following assumption:

A6   All investors can diversify risks completely so they behave as if they were risk neutral.

Assumption A6 plays in the current two-period model the same role that the assumption that

individual investors and corporations can borrow and lend at the same rate played in the infinite horizon

version of the M&M model. In particularly, the assumption implies that investors will require a return

r on the firm's securities. It should be pointed out though that Assumption A6 is not essential. We can

assume instead that in period 1 when the firm is just established, all investors have access to a perfect

---

[4] The assumption that $D < X_1$ means that the firm never issues debt to point where it goes bankrupt for sure. Relaxing this assumption complicates the exposition a bit but has no effect on the results. In any event, note that promising debtholders to pay them more than X_1 is not credible since the debtholders realize that at most they are going to get back X_1. Hence, there is no real difference between the promise to pay X_1 and the promise to pay more than X_1.

capital market in which they can trade Arrow-Debreu securities, each of which pays 1 dollar in a particular state of nature and 0 otherwise. Since the market is perfectly competitive, there is a price for each Arrow-Debreu security and agents can either buy these risky securities or instead invest in a risk-free bonds that pay an interest rate r. As we shall see, the entire analysis can be reinterpreted using this alternative assumption so that in fact, there is no need to assume that investors are risk neutral. The main reason for making Assumption A6 is that it seems a bit more natural to talk about investors who can diversify their risks and hence behave as if they risk neutral than talk about "Arrow-Debreu securities" and "state-contingent prices."

Let E(D) and B(D) denote the market values of equity and debt when the firm issues debt with face value D. Since the capital market is perfectly competitive, the market value of the firm's equity must be such that equityholders will be completely indifferent between putting their money in the bank and earning a risk-free interest rate, r, and investing in the firm's equity. In other words, it must be the case that the amount that equityholders would get in period 2 by saving E(D) in a bank, E(D)(1+r), is just equal to what they get in period 2 by investing in the firm's equity. Hence, the market value of equity must be such that

$$E(D) = \frac{1}{1+r} \int_{D}^{X_1} \left[\tilde{X} - D\right] dF(\tilde{X}). \tag{49}$$

The right side of equation (49) represents the discounted value of the firm's expected earnings, net of debt payment over states of nature in which the firm remains solvent. In states of nature between X_0 and D the firm goes bankrupt and since equityholders are protected by limited liability, their payoff is exactly zero.

Similarly, the market value of the debt, B(D), must be such that if investor put B(D) in the bank, the amount that they would get back in period 2, B(D)(1+r), is just equal to what they get in period 2 by investing in the firm's debt. Hence, the market value of debt is given by

$$B(D) = \frac{1}{1+r}\left[\int_0^D \tilde{X}dF(\tilde{X}) + \int_D^{X_1} DdF(\tilde{X})\right]. \tag{50}$$

The first term on the right side of equation (50) represents the discounted value of the expected return to debtholders over states of nature in which the firm goes bankrupt and debtholders become the residual claimants. The second term represent the discounted value of the firm's expected revenues over states of nature in which they are paid in full.

Going back to the Assumption A6, then it is possible to relax this assumption and assume instead that investors might be risk-averse, but they have an access to a capital market with Arrow-Debreu securities. Under this alternative assumption, f(X) can be thought of as the market price of an Arrow-Debreu security that pays 1 dollar when the state of nature is X (i.e., when the cash flow of the firm happens to be equal to X) and 0 otherwise. The equity and debt of the firm are then interpreted as bundles that contain a continuum of Arrow-Debreu securities, each of which is paying money in a particular state of nature. With this interpretation in mind, the integrands in equations (49) and (50) could be viewed as representing the market value of the securities that the firm issue where each state-contingent security is multiplied by its the corresponding market price (since this is a partial equilibrium model, the prices of the Arrow-Debreu securities are given exogenouosly. Equation (49) and (50) then imply that E(D) and B(D) are simply equal to the discounted values of the bundles of state contingent secrurities that the firm issues. The discounting is required to reflect the fact that investors can invest their money in risk-free bonds instead of buying the firm's securities.

When the entrepreneur establishes the firm in period 1 he does not specifically care about the market values of equity and debt. Rather he cares about their combined market values because he sells both of them and put the proceeds from the sale of both equity and debt in his pocket. Combining (49) and (50) reveals that the total value of the firm in period 1 is

$$V(D) = E(D) + B(D) = \frac{1}{1+r} \left[\int_0^D \tilde{X} dF(\tilde{X}) + \int_D^{X_1} \tilde{X} dF(\tilde{X}) \right] = \frac{\hat{X}}{1+r}. \qquad (51)$$

This equation indicates that V(D) is equal to the present value of the firm's earnings. Since V(D) is independent of D, we conclude that M&M proposition 1 result holds in the current two-period model too. In other words, the capital structure of the firm is completely irrelevant. This result is hardly surprising: given our assumptions, corporate debt does not give investors any advantage so they will not agree to pay a premium on the firm's securities once the firm becomes leveraged.

Now suppose that we introduce corporate taxes by replacing Assumptions A4 with the following assumption:

A4   The corporate tax rate is t_C, the amount D is fully tax deductible, and the firm pays taxes only if it remains solvent.

The assumption that D is fully tax deductible simplifies the analysis because it implies that all of D is interest (i.e., D does not include principal payments). This is the equivalent of assuming that the firm issues consol bonds (perpetual debt) in a two-period model. Relaxing this assumption is straightforward, although one has to decide which portion of D represents interest payments and which portion is principal payments. Since D is returned in a lump sum it does not really matter how it is split between interest and principal. However, if interest payments are tax deductible and principal payments are not, then it pays to claim that as large a portion of D represents interest payments to maximize tax savings. The only problem is that the tax authorities might object to this scheme viewing it as a plot to avoid taxes (and of course they will be right). By assuming that all of D is tax deductible, we avoid the need to deal with these issues. The last part of the assumption says the debtholders have an absolute priority over the tax authorities. Given this assumption, the firm goes bankrupt and does not pay taxes if $\tilde{X} < D$, and its remains solvent and pays taxes of $t_C(\tilde{X} - D)$ if $\tilde{X} > D$.

Before considering risky debt, let us consider two benchmarks. First, if the firm is all-equity, then its market value in period 1 is equal to the market value of equity given by

$$V(0) = \frac{1}{1+r} \int_{X_0}^{X_1} \left[\tilde{X} - t_C \tilde{X} \right] dF(\tilde{X}) = \frac{\hat{X}(1-t_C)}{1+r}. \tag{52}$$

That is, the market value of the firm is equal to the present value of its after-tax earnings.

The second benchmark involves riskless debt, i.e., debt whose face value is less than X_0. This debt is non-risky since it can be paid back to debtholders in all states of nature so the firm never goes bankrupt. Since the capital market is perfectly competitive, the market value of equity must adjust to provide equityholders with a return r on their investment, so

$$E(D) = \frac{1}{1+r} \int_{X_0}^{X_1} \left[\tilde{X} - t_C(\tilde{X} - D) - D \right] dF(\tilde{X}) = \frac{(\hat{X} - D)(1-t_C)}{1+r}. \tag{53}$$

Since debt is safe, the market value of the firm's debt is equal to the present value of the face value of debt:

$$B(D) = \frac{\int_{X_0}^{X_1} D\, dF(\tilde{X})}{1+r} = \frac{D}{1+r}. \tag{54}$$

Again, the entrepreneur cares only about the combined value of equity and debt as he sales both types of securities on the capital market. Together, equation (53) and (54) imply that the market value of the firm in period 1 is

$$V(D) = E(D) + B(D) = \frac{\tilde{X}(1-t_C) + t_C D}{1+r} = V(0) + \frac{t_C D}{1+r}. \tag{55}$$

The difference between V(D) and V(0) is the present value of the tax shield. This tax shield is due to the fact that by issuing debt, the firm can deduct debt payments from its tax obligation so less money goes

to the tax authorities. Since the entrepreneur puts in his pocket everything else when he sells the equity

and debt on the capital market, the reduction in the tax payments is a net gain for him. Equation (55) is

the equivalent of M&M Proposition 1 with taxes in our two-period model. The conclusion is as in M&M:

the firm should issue as much safe debt as possible. Here the upper limit on safe debt is X_0, since once

$D > X_0$, debt becomes risky and the analysis has to be modified accordingly. Hence we found that the

optimal debt level, D^*, is <u>at least</u> X_0. We now need to find out whether $D^* > X_0$.

When the firm issues risky debt such that $X_0 < D < X_1$, the market value of its equity in period

1 adjusts again to provide equityholders with a return r on their investment, so

$$E(D) \; = \; \frac{1}{1+r} \int_D^{X_1} \left[\tilde{X} \, - \, t_C(\tilde{X} \, - \, D) \, - \, D \right] dF(\tilde{X}). \qquad (56)$$

The right side of the equation represents the discounted value of the firm's expected earnings, net of

corporate taxes and debt payment over states of nature in which the firm remains solvent. Similarly, the

market value of the firm's debt is such that debtholders earn the risk-free rate on their investment,

$$B(D) \; = \; \frac{1}{1+r} \left[\int_{X_0}^D \tilde{X} dF(\tilde{X}) \; + \; \int_D^{X_1} D dF(\tilde{X}) \right]. \qquad (57)$$

The first term on the right side of this equation represents the discounted value of the expected return to

debtholders over states of nature in which the firm goes bankrupt and debtholders become the residual

claimants. The second term represents the discounted value of the firm's expected revenues over states

of nature in which they are paid in full.

Combining (56) and (57) and using equation (52) reveals that the market value of the firm in

period 1 is

$$V(D) = E(D) + B(D)$$

$$= \frac{1}{1+r} \left[\int_D^{X_1} \left[\tilde{X} - t_C(\tilde{X}-D) - D \right] dF(\tilde{X}) + \int_{X_0}^{D} \tilde{X} dF(\tilde{X}) + \int_D^{X_1} D dF(\tilde{X}) \right]$$

$$= \frac{1}{1+r} \left[\int_{X_0}^{X_1} \tilde{X}(1-t_C) dF(\tilde{X}) + \int_{X_0}^{D} t_C \tilde{X} dF(\tilde{X}) + \int_D^{X_1} t_C D dF(\tilde{X}) \right] \qquad (58)$$

$$= V(0) + \frac{1}{1+r} \left[\int_{X_0}^{D} t_c \tilde{X} dF(\tilde{X}) + t_C D(1-F(D)) \right].$$

The difference between V(D) and V(0), given by the expression inside the square brackets, represents the tax benefits associated with debt payments: the first expression inside the square brackets represents the tax savings that arise because the firm does not need to pay taxes while in bankruptcy, and the second term is the present value of the tax shield that debt payments provide in states of nature in which the firm is solvent. Since the expression in square brackets increases with the firm should issue as much debt as it can. Hence we obtain again the conclusions from M&M proposition 1 in the presence of corporate taxes.

Appendix

The market value of an all-equity firm when the appropriate discount rate is ρ and the mean cash flow of the firm is $\hat{X}$:

The discounted sum of the firm's steam of earnings is:

$$V_U = \frac{\hat{X}}{1+\rho} + \frac{\hat{X}}{(1+\rho)^2} + \frac{\hat{X}}{(1+\rho)^3} + \frac{\hat{X}}{(1+\rho)^4} + \ldots \qquad \text{(A-1)}$$

We can rewrite this sum as follows:

$$V_U = \frac{\hat{X}}{1+\rho} + \frac{1}{1+\rho}\left[\frac{\hat{X}}{1+\rho} + \frac{\hat{X}}{(1+\rho)^2} + \frac{\hat{X}}{(1+\rho)^3} + \ldots\right]. \qquad \text{(A-2)}$$

But since the expression inside the square brackets looks exactly like the right side of equation (A-1), we can write equation (A-2) as follows:

$$V_U = \frac{\hat{X}}{1+\rho} + \frac{V_U}{1+\rho}. \qquad \text{(A-3)}$$

Solving for V_U yields:

$$V_U = \frac{\hat{X}}{\rho}. \qquad \text{(A-4)}$$

∎

The market value riskless consol bonds with face value D when the discount rate is r:

The discounted sum of the infinite steam of debt payments:

$$B_L = \frac{rD}{1+r} + \frac{rD}{(1+r)^2} + \frac{rD}{(1+r)^3} + \frac{rD}{(1+r)^4} + \ldots \qquad \text{(A-5)}$$

We can rewrite this sum as follows:

$$B_L = \frac{rD}{1+r} + \frac{1}{1+r}\left[\frac{rD}{1+r} + \frac{rD}{(1+r)^2} + \frac{rD}{(1+r)^3} + \ldots\right]. \tag{A-6}$$

But since the expression inside the square brackets looks exactly like the right side of equation (A-5), we can write equation (A-6) as follows:

$$B_L = \frac{rD}{1+r} + \frac{B_L}{1+r}. \tag{A-7}$$

Solving for B_L yields:

$$B_L = \frac{rD}{r} = D. \tag{A-8}$$

∎

Alternative proof of M&M Proposition 1:

Consider the following two investment strategies:

Strategy 1:   **"Buy a fraction α of firm U's equity."**

Strategy 2:   **"Buy a fraction α of firm L's equity and debt**."

The cost of strategy 1 is αV_u and the cost of strategy 2 is $\alpha E_L + \alpha B_L = \alpha V_L$. An investor who adopts strategy 1 receives in every period a random payoff of

$$\alpha \tilde{X}. \tag{A-9}$$

If the investor adopts strategy 2, he receives in every period a random payoff of

$$\alpha(\tilde{X} - rD) + \alpha rD = \alpha \tilde{X}. \tag{A-10}$$

Since strategies 1 and 2 yield the same per-period payoffs, they must cost the same, otherwise all investors will prefer the cheap strategy over the expensive one, so the price of the former will increase and the price

of the latter will decrease. Hence, in a competitive equilibrium, it must be that $\alpha V_U = \alpha V_L$, or $V_U = V_L$.

Alternatively, we can consider another pair of strategies.

Strategy 3: **"Buy a fraction β of firm L's equity."**

Strategy 4: **"Buy a fraction β of firm U's equity, and borrow the sum βD at an annual rate r."**

The cost of strategy 3 is βE_L and the cost strategy 4 is $\beta V_U - \beta D$ because the investor spends βV_U on firm U's equity but receives the amount βD when he borrows money. If the investor adopts strategy 3, his per-period payoff is

$$\beta(\tilde{X} - rD). \tag{A-11}$$

If he adopts strategy 4, his per-period payoff is

$$\beta \tilde{X} - \beta rD = \beta(\tilde{X} - rD). \tag{A-12}$$

Again, since the two strategies yield the same per-period payoffs, they must cost the same. Hence,

$$\beta E_L = \beta V_U - \beta D \quad \Leftrightarrow \quad \beta(E_L + D) = \beta V_U. \tag{A-13}$$

Since $E_L + D = V_L$, it follows from the last equation that $V_L = V_U$.

Now, one may wonder about the mechanism that works to equate prices. If securities are just going to be issued, then it is clear that their prices must be equal in a competitive equilibrium. If the securities have been issued in the past and are already owned by investors, then the mechanism is as follows: If $V_U > V_L$, then an investor who holds a fraction α of U's equity, can sell his portfolio for αV_U, and buy a fraction α of firm L's equity and debt, at a cost $\alpha(E_L + B_L) = \alpha V_L < \alpha V_U$. As we saw above, the investor's payoff per period remains the same, so overall his wealth increases by $\alpha(V_U - V_L) > 0$. But, since other investors will do the same, the prices of U's equity will fall while the prices of L's equity and debt will increase until in equilibrium, $V_U = V_L$.

On the other hand, if $V_U < V_L$, then an investor who holds a fraction β of L's equity, can sell his

portfolio for αE_L, and buy a fraction β of U's equity, along with borrowing βD at an annual rate r. As we saw above, the two positions yield the same payoff. The change in the investor's wealth is

$$\beta E_L - \beta V_U + \beta D = \beta(E_L + D) - \beta V_U = \beta(V_L - V_U) > 0. \tag{A-14}$$

Since the investor's wealth increases, he will definitely do the readjustment, so the prices of L's securities will fall and the prices of U's equity will increase until $V_U = V_L$. ∎