Simple Interest, Compound Interest, and Effective Yield

Simple Interest

The formula that gives the amount of simple interest (also known as **add-on** interest) owed on a Principal P (also known as **present value**), with annual interest rate r, over time (in years) t is

$$I = Prt$$

In calculating with interest formulas, be sure to change the percent rate r to a decimal number. Be sure the time t is in years.

Example

Find the simple interest paid to borrow \$4800 for 6 months at 7%.

Solution

$I = Prt = \$4800(.07)(6/12) = \$168$

Remember that 6 months is 6/12 of a year.

The total amount to be repaid is \$4800 + \$168 = \$4968. This total amount is called the **maturity value** or **future value** or **future amount** of the loan.

Future Value for Simple Interest

If a principal P is borrowed at simple interest for t years at an annual interest rate of r, then the **future value** of the loan, denoted **A**, is given by

$$A = P(1 + rt)$$

Example

Find the future value of \$460 in 8 months if the annual interest rate is 12% (great rate!).

Solution

$A = P(1 + rt) = \$460\left(1 + .12\left(\frac{8}{12}\right)\right) = \$496.80$

Example

If you can earn 6% interest, what lump sum must be deposited now so that its value will be \$3500 after 9 months?

Solution

$A = P(1 + rt)$

$3500 = P\left(1 + .06\left(\frac{9}{12}\right)\right)$

$P = \frac{\$3500}{1.045} \cong \$3349.28$

Compound Interest

Interest paid on principal plus interest is called compound interest. After a certain period, the interest earned so far is credited (added) to the account, and the sum (principal plus interest) then earns interest during the next period.

Compounding Period

Interest can be credited to an account at time intervals other than 1 year. For example, it can be done semiannually, quarterly, monthly, or daily. This time interval is called the **compounding period** (or the **period**).

Future Value for Compound Interest

If P dollars are deposited at an annual interest rate of r, compounded m times per year, and the money is left on deposit for a total of n periods, then the **future value**, **A** (the final amount on deposit), is given by

$$A = P\left(1 + \frac{r}{m}\right)^{n}$$

where $n = mt$.

Example: **Future Value for Compound Interest**

Find the future value of \$8560 at 4% compounded quarterly for 8 years.

Solution

$P = \$8560,\ r = 4\% = .04,\ m = 4.$

Over 8 years $n = 8m = 8(4) = 32.$

$A = P\left(1 + \frac{r}{m}\right)^{n} = \$8560\left(1 + \frac{.04}{4}\right)^{32} \approx \$11769.49.$

Example: **Present Value for Compound Interest**

What amount must be deposited today, at 5% compounded monthly, so that it will be \$18,000 in 20 years?

Solution

$\$18000 = P\left(1 + \frac{.05}{12}\right)^{240}$

$P = \dfrac{\$18000}{\left(1 + \frac{.05}{12}\right)^{240}} \approx \$6635.60$

Do these calculations in ONE calculator step and round off.to the nearest cent. Here's a screenshot from a TI-84 (83 is similar).

```
8560(1+.04/4)^32
       11769.49221
18000/(1+.05/12)
^240
       6635.601519
```

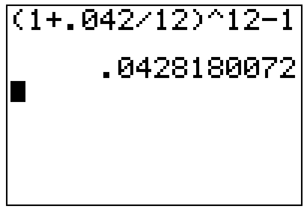
Effective Annual Yield

Savings institutions often give two quantities when advertising the rates. The first, the actual annualized interest rate, is the **nominal rate** (the "stated" rate). The second quantity is the equivalent rate that would produce the same final amount, or future value, at the end of 1 year if the interest being paid were simple rather than compound. This is called the "effective rate," or the **effective annual yield**.

A nominal interest rate of r, compounded m times per year, is equivalent to an **effective annual yield of**

$$Y = \left(1 + \frac{r}{m}\right)^{m} - 1$$

Example: **Effective Annual Yield**
What is the effective annual yield of an account paying a nominal rate of 4.2%, compounded monthly?
Solution
$Y = \left(1 + \frac{.042}{12}\right)^{12} - 1 \approx .0428 = 4.28\%$
Again, do this calculation in ONE calculator step and round off.to the nearest cent. Here's a screenshot from a TI-84 (83 is similar).



Inflation
In terms of the equivalent number of goods or services that a given amount of money will buy, it is normally more today than it will be later. In this sense, the money *loses value over time*. This periodic increase in the cost of living is called **inflation**.

Unlike account values under interest compounding, which make sudden jumps at certain points, price levels tend to fluctuate gradually over time. It is appropriate, for inflationary estimates, to use a formula for continuous compounding.

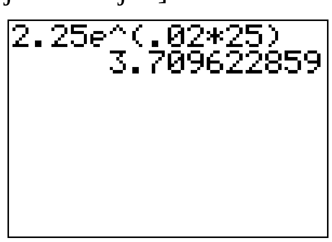
Inflation in an economy usually is expressed as a monthly or annual rate of price increases, estimated by government agencies in a systematic way. In the United States, the Bureau of Labor Statistics publishes consumer price index (CPI) figures, which reflect the prices of certain items purchased by large numbers of people (see table on page 862 of the text).

Future Value for Continuous Compounding (Inflation)
If an initial deposit of P dollars earns continuously compounded interest at an annual rate r for a period of t years, then the **future value**, **A** is given by

$$A = Pe^{rt}$$

where $e \cong 2.71828\ldots$ This is a special constant and is built into the calculator. The base-*e* exponential function is accessed by typing **2nd / LN**.

Example
Suppose that a cup of your favorite coffee is $2.25. If the inflation rate persists at 2% over time, find the approximate cost of the coffee in 25 years.
Solution
$A = Pe^{rt} = \$2.25e^{(.02)(25)} \approx \$3.71$
The coffee will cost about $3.71.
["ONE calculator step, Vasily, one calculator step only."
http://www.youtube.com/watch?v=jr0JaXfKj68]



Inflation Proportion

For a consumer product or service subject to average inflation,

$$\frac{\text{Price in year A}}{\text{Price in year B}} = \frac{\text{CPI in year A}}{\text{CPI in year B}}$$

Example: **Inflation**

If your mother paid $3,000 in tuition in 1980 at the same college that you will be attending and paying $9,300 in 2005, compare the school's tuition increase to inflation over that same period of time.

Solution

Let *x* represent what we expect the tuition to be in 2005 if it had increased at the average rate since 1980.

$$\frac{\text{Price in year 2005}}{\text{Price in year 1980}} = \frac{\text{CPI in year 2005}}{\text{CPI in year 1980}}$$

$$\frac{x}{\$3000} = \frac{195.3}{82.4}$$

$$x = \frac{(195.3)(\$3000)}{82.4}$$

$$x \approx \$7110.44.$$

Now compare with the actual 2005 tuition.

$$\frac{\$9300}{\$7128.64} \approx 1.30$$

Tuition at the school increased approximately 30% more than the average CPI-U rate.

Doubling Time for Inflation

An estimation of the **years to double**, which is the number of years it takes for the general level of prices to double for a given annual rate of inflation, is given by

$$\text{years to double} \approx \frac{70}{\text{annual inflation rate}}$$

Use the **percent number** here, not the decimal number. This is called the "Rule of 70." You may see this in some textbooks as the "Rule of 72." (The number 72 is easier to work with in division, and you get approximately the same result.)

Example

Estimate the number of years to double for an annual inflation rate of 2.1%.

Solution

Years to double $\approx \frac{70}{2.1} = 33.33$

Rounding up to the nearest integer, we see that prices would double in about 34 years.

Special Note

The Rule of 70 also works for compound interest future value. The amount of time to double the present value in compound interest is

$$\text{years to double} \approx \frac{70}{\text{interest rate}}$$

where you use the **percent number**.

Example

How long does it take to double the present value P if $r = 4\%$? Here we get only an approximate number and the number of compounding period per year does not matter.

years to double $\approx \frac{70}{4} = 17.5$ (LMNC)

Now let's check this result using the information in the previous example on Future Value of Compound Interest. In this we had $P = \$8560$, $r = 4\% = .04$, $m = 4$, and in this case $n = mt = (4)(17.5) = 70$. Then

$$A = \$8560\left(1 + \frac{.04}{4}\right)^{70}$$

$$A \approx \$17177.89$$

(You should check this with your calculator.) This result is reasonably close to $2 \cdot \$8560 = \$17120.00$.