

STAT 250
Dr. Kari Lock Morgan

Hypothesis Testing: Hypotheses

SECTION 4.1

- Hypothesis test
- Null and alternative hypotheses
- Statistical significance

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Tea and the Immune System



- L-theanine is an amino acid found in tea
 - Black tea: about 20mg per cup
 - Green tea (standard): varies, as low as 5mg per cup
 - Green tea (shade grown): varies, up to 46mg per cup (Shade grown green tea examples: Gyokuro, Matcha)
- Gamma delta T cells are important for helping the immune system fend off infection
- It is thought that L-theanine primes T cells, activating them to a state of readiness and making them better able to respond to future antigens.
- Does drinking tea actually boost your immunity?

Antigens in tea-Beverage Prime Human V α 2V β 2 T Cells in vitro and in vivo for Memory and Non-memory Antibacterial Cytokine Responses, Kamath et.al., Proceedings of the National Academy of Sciences, May 13, 2003.

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Tea and the Immune System



- Participants were randomized to drink five or six cups of either tea (black) or coffee every day for two weeks (both drinks have caffeine but only tea has L-theanine)
- After two weeks, blood samples were exposed to an antigen, and production of interferon gamma (immune system response) was measured
- Explanatory variable: tea or coffee
- Response variable: measure of interferon gamma

Antigens in tea-Beverage Prime Human V α 2V β 2 T Cells in vitro and in vivo for Memory and Non-memory Antibacterial Cytokine Responses, Kamath et.al., Proceedings of the National Academy of Sciences, May 13, 2003.

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Tea and the Immune System



If the tea drinkers have significantly higher levels of interferon gamma, can we conclude that drinking tea rather than coffee *caused* an increase in this aspect of the immune response?

- Yes
- No

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Tea and Immune System



The explanatory variable is tea or coffee, and the response variable is immune system response measured in amount of interferon gamma produced. How could we visualize this data?

- Bar chart
- Histogram
- Side-by-side boxplots
- Scatterplot

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Tea and Immune System



The explanatory variable is tea or coffee, and the response variable is immune system response measured in amount of interferon gamma produced. How might we summarize this data?

- Mean
- Proportion
- Difference in means
- Difference in proportions
- Correlation

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Hypothesis Test

- One mean is higher than the other *in the sample*
- Is this difference large enough to conclude the difference is real, and holds for the true population parameters?

A ***hypothesis test*** uses data from a sample to assess a claim about a population

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Hypotheses

- Hypothesis tests are framed formally in terms of two competing hypotheses:

Null Hypothesis (H_0): Claim that there is no effect or difference.

Alternative Hypothesis (H_a): Claim for which we seek evidence.

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Tea and Immune Respose

- Null Hypothesis (H_0): No difference between drinking tea and coffee regarding interferon gamma
- Alternative Hypothesis (H_a): Drinking tea increases interferon gamma production more than drinking coffee

No "effect" or no "difference"

Claim we seek "evidence" for

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Hypotheses: parameters

- More formal hypotheses:
- μ_T = true mean interferon gamma response after drinking tea
- μ_C = true mean interferon gamma response after drinking coffee

$$\begin{aligned} H_0: \mu_T &= \mu_C \\ H_a: \mu_T &> \mu_C \end{aligned}$$

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Difference in Hypotheses

- Note: the following two sets of hypotheses are equivalent, and can be used interchangeably:

$$\begin{aligned} H_0: \mu_1 &= \mu_2 & H_0: \mu_1 - \mu_2 &= 0 \\ H_a: \mu_1 &\neq \mu_2 & H_a: \mu_1 - \mu_2 &\neq 0 \end{aligned}$$

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Hypothesis Helpful Hints

- Hypotheses are always about population parameters, not sample statistics
- The null hypothesis always contains an equality
- The alternative hypothesis always contains an inequality ($<$, $>$, \neq)
- The type of inequality in the alternative comes from the wording of the question of interest

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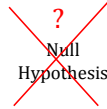
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Statistical Hypotheses



ALL POSSIBILITIES

Usually the null is a very specific statement

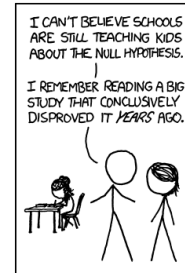


Can we reject the null hypothesis?

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Null Hypothesis



<http://xkcd.com/892/>

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Sleep versus Caffeine



- Students were given words to memorize, then randomly assigned to take either a 90 min nap, or a caffeine pill. 2 ½ hours later, they were tested on their recall ability.
- Explanatory variable: sleep or caffeine
- Response variable: number of words recalled
- Is sleep or caffeine better for memory?

Mednick, Cai, Kanady, and Drummond (2008). "Comparing the benefits of caffeine, naps and placebo on verbal, motor and perceptual memory," *Behavioral Brain Research*, 193, 79-86.

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Sleep versus Caffeine

What is the parameter of interest in the sleep versus caffeine experiment?

- Proportion
- Difference in proportions
- Mean
- Difference in means
- Correlation

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Sleep versus Caffeine

- Let μ_s and μ_c be the true mean number of words recalled after sleeping and after caffeine.
- Is there a difference in average word recall between sleep and caffeine?
- What are the null and alternative hypotheses?
 - $H_0: \mu_s \neq \mu_c, H_a: \mu_s = \mu_c$
 - $H_0: \mu_s = \mu_c, H_a: \mu_s \neq \mu_c$
 - $H_0: \mu_s \neq \mu_c, H_a: \mu_s > \mu_c$
 - $H_0: \mu_s = \mu_c, H_a: \mu_s > \mu_c$
 - $H_0: \mu_s = \mu_c, H_a: \mu_s < \mu_c$

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Hypotheses

- Define the parameter(s) and state the hypotheses.
 - Does the proportion of people who buy organic food when possible differ between males and females?
 - Is the average hours of sleep per night for college students less than 7?
 - Is amount of time spent studying positively associated with numeric grade in STAT 250?

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Your Own Hypotheses

- Come up with a situation where you want to establish a claim based on data
- What parameter(s) are you interested in?
- What would the null and alternative hypotheses be?
- What type of data would lead you to believe the null hypothesis is probably not true?

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Two Plausible Explanations

- If the sample data support the alternative, there are two plausible explanations:
 1. The alternative hypothesis (H_a) is true
 2. The null hypothesis (H_0) is true, and the sample results were just due to random chance
- Key question: Do the data provide enough evidence to rule out #2?

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Two Plausible Explanations

- Why might the tea drinkers have higher levels of interferon gamma?
- Two plausible explanations:
 - **Alternative true**: Tea causes increase in interferon gamma production
 - **Null true, random chance**: the people who got randomly assigned to the tea group have better immune systems than those who got randomly assigned to the coffee group

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Hypothesis Testing

- In **hypothesis testing**, the goal is determine whether random chance can be ruled out as a plausible explanation.
- **Key idea: How unlikely would it be to see a difference in means this large, just by random chance?**

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Statistical Significance

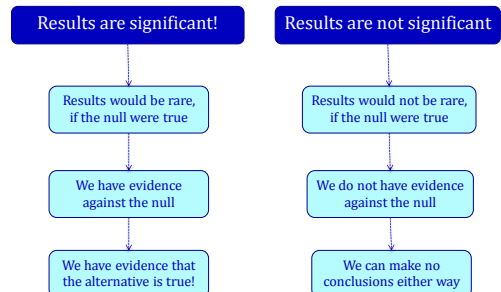
When results as extreme as the observed sample statistic are *unlikely* to occur by random chance alone (assuming the null hypothesis is true), we say the sample results are **statistically significant**

- If our sample is statistically significant, we have convincing evidence against H_0 , in favor of H_a
- If our sample is not statistically significant, our test is inconclusive

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Statistical Significance



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Note on Statistical Significance

- Statistical significance is a difficult concept, but also one of the most fundamental concepts of the course
- We return to this concept almost every class for the rest of the semester, so
 - it will get easier!
 - it's worth thinking deeply about!

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Sleep versus Caffeine

μ_s and μ_c : mean number of words recalled after sleeping and after caffeine

$$H_0: \mu_s = \mu_c, H_a: \mu_s \neq \mu_c$$

If the difference is statistically significant...

- a) we have evidence against the null hypothesis, in favor of the alternative
- b) we do not have evidence against the null hypothesis

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μ_s and μ_c : mean number of words recalled after sleeping and after caffeine

$$H_0: \mu_s = \mu_c, H_a: \mu_s \neq \mu_c$$

If the difference is statistically significant...

- a) we have evidence that there is a difference between sleep and caffeine for memory
- b) we do not have evidence that there is a difference between sleep and caffeine for memory

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Sleep versus Caffeine

μ_s and μ_c : mean number of words recalled after sleeping and after caffeine

$$H_0: \mu_s = \mu_c, H_a: \mu_s \neq \mu_c$$

If the difference is not statistically significant...

- a) we have evidence that there is a difference between sleep and caffeine for memory
- b) we do not have evidence that there is a difference between sleep and caffeine for memory

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Sleep versus Caffeine

μ_s and μ_c : mean number of words recalled after sleeping and after caffeine

$$H_0: \mu_s = \mu_c, H_a: \mu_s \neq \mu_c$$

If the difference is not statistically significant, we could conclude...

- a) there is a difference between sleep and caffeine for memory (and data show sleep is better)
- b) there is not a difference between sleep and caffeine for memory
- c) nothing

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Hours of Sleep per Night

In testing whether the mean number of hours of sleep per night, μ , for college students is less than 7, we have

$$H_0: \mu = 7 \text{ vs } H_a: \mu < 7$$

If the results of the test are statistically significant, we can conclude...

- a) There is evidence that the mean is equal to 7.
- b) There is evidence that the mean is less than 7.
- c) There is evidence that the mean is greater than 7.
- d) There is no evidence of anything.
- e) College students get lots of sleep.

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Hours of Sleep per Night

In testing whether the mean number of hours of sleep per night, μ , for college students is less than 7, we have

$$H_0: \mu = 7 \text{ vs } H_a: \mu < 7$$

If the results of the test are not statistically significant, we can conclude...

- a) There is evidence that the mean is equal to 7.
- b) There is evidence that the mean is less than 7.
- c) There is evidence that the mean is greater than 7.
- d) There is no evidence of anything.
- e) College students get lots of sleep.

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Summary

- Statistical tests use data from a sample to assess a claim about a population
- Statistical tests are usually formalized with competing hypotheses:
 - Null hypothesis (H_0): *no effect or no difference*
 - Alternative hypothesis (H_a): *what we seek evidence for*
- If it would be unusual to get results as extreme as that observed, just by random chance, if the null were true, then the data is statistically significant
- If data are statistically significant, we have convincing evidence against the null hypothesis, and in favor of the alternative

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To Do

- Read Section 4.1
- HW 4.1 due Friday, 3/6

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